DETERMINATION OF DISCHARGE AND HEAD LOSS USING A FLOW-MEASURING APPARATUS

1. **INTRODUCTION**

Through use of the Flow-Measuring Apparatus, this experiment is designed to accustom students to typical methods of measuring the discharge of an essentially incompressible fluid, whilst at the same time giving applications of the Steady-Flow Energy Equation (Bernoulli's Equation). The discharge is determined using a Venturi Meter, an Orifice Plate Meter and a Rotameter.

Also, head losses associated with each meter are determined and compared as well as those arising in two fittings (a rapid enlargement and a 90-degree elbow). The loss coefficients associated with these fittings can be determined.

The unit is designed for use with TecQuipment's Hydraulic Bench H1, which provides the necessary liquid service and gravimetric evaluation of flow rate. See Appendix B for a description of the Hydraulic Bench's operating procedures.

1.1 Description of Apparatus

The Flow-Measuring Apparatus is shown in Fig 1. Water from the Hydraulic Bench enters the equipment through a perspex Venturi Meter, which consists of a gradually-converging section, followed by a throat, and a long gradually-diverging section. After a change in cross-section through a rapidly diverging section, the flow continues along a settling length and through an Orifice Plate Meter. The Orifice Plate Meter is manufactured in accordance with B.S. 1042, from a plate with a hole of reduced diameter through which the fluid flows.

Following a further settling length and a right-angled bend, the flow enters the Rotameter. This consists of a transparent tube in which a float takes up an equilibrium position. The position of this float is a measure of the flow rate.

After the Rotameter, the water returns via a control valve to the Hydraulic Bench and the weightank. The equipment has nine pressure tappings as detailed in Fig 2, each of which is connected to its own manometer for immediate read out.

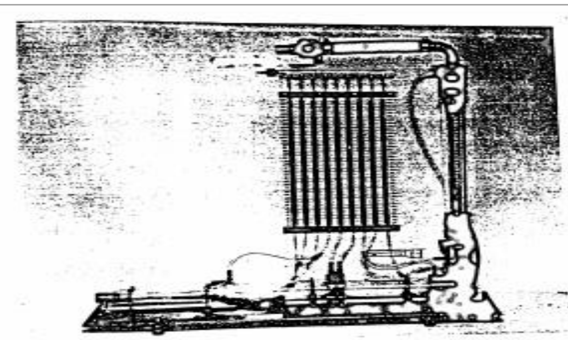


Fig 1 Flow Measuring Apparatus

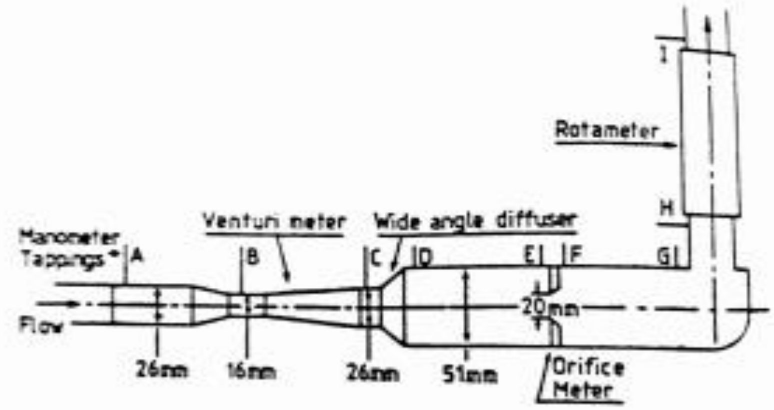


Fig 2 Explanatory Diagram of Flow Measuring Apparatus

1.2 Installation and Preparation

a) Remove the transit wire from the glass rotameter by first removing the control valve, elbow assembly from the top of the Rotameter. Replace the valve, elbow assembly securely.

b) Connect the supply hose from the Hydraulic Bench to inlet of the Venturi Meter and secure with a hose clip. Connect a hose to the control valve outlet and direct its free end into the central hole in the Bench. Before continuing, refer to the Hydraulics Bench Manual to find method of flow evaluation by weighing.

c) With the air purge-valve closed, close the apparatus valve then open it by about 1/3. Switch on the Bench and slowly open its valve until water starts to flow. Allow the apparatus to fill with water. Continue to open the bench valve until it is fully open. Close the apparatus valve fully. Couple the bicycle pump to the purge salve and pump down until all the manometer read approximately 280 mm. Dislodge entrained air from the manometers by gentle tapping with the fingers. Check that the water levels are constant. A steady rise in Levels will be seen if the purge valve is leaking.

d) Check that the tube ferrules and the top manifold are free from water blockage, (this will suppress the manometer level). Ferrules blockage can be cleared by a sharp burst of pressure from the bicycle pump.

1.3 **Routine Care and Maintenance**

a) When not in use water should not be allowed to stand in the apparatus for long periods. After use, fully drain the apparatus and dry externally with a lint-free cloth.

b) If the control valve shows signs of leaking, the procedure for checking and inspecting is as detailed in the H1 Hydraulic Bench technical manual.

c) If plastic manometer tubes become discolored, a stain and Deposit Remover is available

2. THEORY

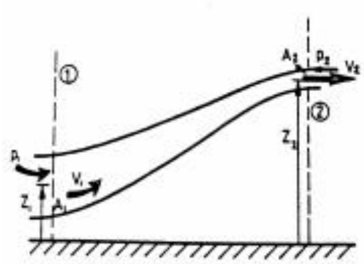


Fig 3 The steady-flow Energy Equation

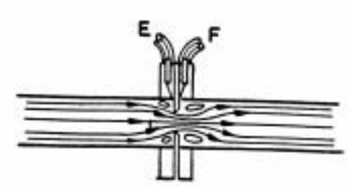


Fig 4 Construction of the Orifice Meter

2.1 Bernoulli's Equation

For steady, adiabatic flow of an incompressible fluid along a stream tube (see fig 3). Bernoulli's equation can be written in the form;

$$\frac{P_1}{\rho g}+\frac{{V_1}^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{{V_2}^2}{2g}+Z_2+\Delta H_{12} \qquad \text{(Eq. 4-1)}$$

Where $\frac{P}{\rho g}$ is termed the hydrostatic head or pressure head.

$\frac{V^2}{2g}$ is termed the kinetic head or velocity head (V is the mean velocity i.e. the ratio of volumetric discharge to cross-sectional area of tube).

Z is termed potential head or elevation head

$\frac{P}{\rho g}+\frac{V^2}{2g}+Z$ represents the total head.

The head loss ΔH_{12}may be assumed to arise as a consequence of vorticity in the stream. Because the flow is viscous, a wall shear stress exists and a pressure force must be applied to overcome it. The consequent increase in flow work appears as increased internal energy. Also, because the flow is viscous, the velocity profile at any section is non-uniform. The kinetic energy per unit mass at any section is then greater than $v^2/2g$ and Bernoulli's equation incorrectly assesses this term. The fluid mechanics entailed in all but the very simplest internal flow problems is too complex to permit the head loss ΔH to be obtained by other than experimental means. Since a contraction of stream boundaries can be shown (with incompressible fluids) to increase flow uniformity and a divergence correspondingly decreases it, ΔH is typically negligibly small between the ends of a contracting duct but is normally significant when the duct walls diverge.

2.2. **Fitting Head Loss Coefficients**

2.2.1. Expansion

Friction head losses through an expansion (wide angled diffuser) may be estimated using an equation of the form:

$$h_L=K_E\frac{{v_1}^2}{2g} \qquad \text{(Eq 4 - 2)}$$

Where $\frac{{v_1}^2}{2g}$ is the approach velocity head and K_E is a constant head loss coefficient associated with the ratio of the diameters and the angle of the expansion.

2.2.2. **Elbow**

Friction head losses through an elbow (90° bend) may be estimated using an equation of the form

$$h_L = K_b \frac{v^2}{2g} \qquad \text{(Eq. 4-3)}$$

where v^2/2g is the velocity head of the flow and K_b is a constant head loss coefficient associated with the sharpness of the bend.

3. **EXPERIMENTAL PROCEDURE**

Step 1: With the equipment set as in Section 1.2, measurements can be taken in the following manner. Open the apparatus valve until the rotameter shows a reading of about 10 mm. When a steady flow is maintained measure the flow with the Hydraulic Bench as outlined in its manual (Appendix B). During this period, record the readings of the manometers in a table of the form of Fig 8. Repeat this procedure for a number of equidistant values of rotameter readings up to a maximum of approximately 220 mm. At least 6 sets of data should be taken. Finally, record a set of readings with no flow.

Step 2: For each set of data, compute the mass flow rates for the venturi, orifice, rotameter and weigh tank.

Step 3: For each set of data, compute the head losses associated with each meter and the fittings. Discuss findings. Plot head losses vs flow rate for each meter. Also, plot head loss per kinetic head vs kinetic head (see Fig 9) for each meter.

Step 4: Compute the friction head losses associated with the two fittings. a. For each set of data, determine the velocity head and the head loss (ΔH_{CD}) across the expansion Plot h_L, versus the velocity head. Determine the slope (best fit) of the line through these points. Determine K_E from your results and compare with published values.

b. For each set of data, determine the velocity head and the head loss (ΔH_{GH}) across the elbow Plot h_L, versus the velocity head. Determine the slope (best fit) of the line through these points. Determine K_b from your results and compare with published values.

Step 5: Analysis of findings

a. Based on your findings, discuss the accuracy and limitations of each method of flow measurement

b. Review head loss findings

c. Review head loss coefficient findings for fittings.

4. **RESULTS AND CALCULATIONS**

4.1 **Calculations of Discharge**

The Venturi meter, the orifice plate Meter and the Rotameter are all dependent upon Bernoulli's equation for their principle of operation. The following have been prepared for a typical set of results to show in form of calculations.

4.1.1 Venturi Meter

Since ΔH_{12} is negligibly small between the ends of a contracting duct it, along with the Z terms, can be omitted from equation 1 between stations A and B.

From continuity ($\rho V_A A_A = \rho V_B A_B$)

The discharge, $Q = A_B V_B$

$$= A_B \frac{2g}{1-(\frac{A_B}{A_A})^2} \frac{P_A}{\rho g} - \frac{P_B}{\rho g} \quad ^{1/2}$$

With the apparatus provided, the bores of the meter at A and B are 26 mm and 16 mm respectively. Thus:- A_B/A_A = 0.38 and $A_B =$ 2.01 x 10^{-4} m^2, since g=9.81m/s^2 and P_A/ρ_g , P_B/ρ_g are respectively heights of the manometric tubes A and B meters, we have from equation 2:

$$Q = .962 \times 10^{-4}(h_A - h_B)^{1/2} \ m^3/s$$

Taking the density of water as 1000 kg/m^3, the mass flow will be

$$m = 0.962(h_A - h_B)^{1/2} \ kg/s$$

e.g. if h_A = 372mm, h_B = 116mm then $[(h_A - h_B)/1000^{1/2}$ = 0.51

and m = 0.962 x 0.51 = 0.49kg/s

(The corresponding weightank assessment was 0.47kg/s)

4.1.2 Orifice Meter

Between tappings (E) and (F), ΔH_{12} in equation 1 is by no means negligible. Re-writing the equation with the appropriate symbols,

$$\frac{V_F{}^2}{2g} - \frac{V_F{}^2}{2g} = \frac{P_F}{\rho g} - \frac{P_F}{\rho g} - \Delta H_{12}$$

i.e. the effect of the head loss is to make the difference in manometric height (h_E - h_F) less than it would otherwise be.

An alternative expression is $\frac{V_F{}^2}{2g} - \frac{V_F{}^2}{2g} = K^2 \frac{P_F}{\rho g} - \frac{P_F}{pg}$

Where the coefficient of discharge K is given by previous experience in B.S.1042(1943)* for the particular geometry of the Orifice Meter. For the apparatus provided K is given as 0.601

Reducing the expression in exactly the same way as for venturi meter,

$$Q = A_F V_F$$

$$= KA_F[\frac{2g}{1-\left(\frac{A_F}{A_E}\right)^2}\left(\frac{P_E}{\rho g}-\frac{P_F}{\rho g}\right)]^{\frac{1}{2}}$$

Since the apparatus provided, the bore at E is 51 mm and at F is 20 mm.

$$Q = 1.89\ x\ 10^{-4} x \frac{(2\ x\ 9.81)^{\frac{1}{2}}}{1-0.155}(h_E - h_F)^{1/2}$$

$Q = 9.10\ x\ 10^{-4}(h_E - h_F)^{1/2}\ \ m^3/s$

Thus m=0.910$(h_E - h_F)^{\frac{1}{2}}\ \ kg/s$

e.g. If $h_E = 354\ mm,\ h_F = 44\ mm\ then\ [\frac{h_E - h_F}{1000}]^{\frac{1}{2}} = 0.55$

and m = 0.910 x 0.55 =0.50 kg/s

(The corresponding weightank assessment was 0.47 kg/s)

*N.B. It is found that the value of C given in the 1943 BS1042 publication gives better results over the velocity range of the apparatus than the figures given in later edictions and has thus been retained for use in this manual.

4.1.3 Rotameter

Observation of recordings for the pressure drop across the Rotameter, (H) - (I), shows that this difference is large and virtually independent of discharge. Though there is a term which arises because of wall shear stresses and which is therefore velocity dependent, since the Rotameter is of large bore this term is small. Most of the observed pressure difference is required to maintain the float in equilibrium and as the float is of constant weight, this pressure difference is independent of discharge.

The cause of this pressure difference is the head loss associated with the high velocity of water around the float periphery. Since this head loss is constant then the peripheral velocity is constant. To maintain a constant velocity with a varying discharge rate, the cross sectional area will arise as the float moves up and down the tapered Rotameter tube.

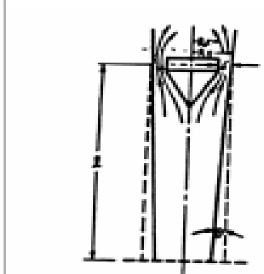


Fig 5 Principle of the Rotameter

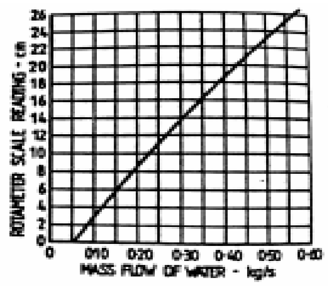


Fig 6 Typical Rotameter Calibration Curve

From Fig 5, if the float radius is R_f and the local bore of the Rotameter tube is $2R_\tau$ then,

$\pi\left({R_\tau}^2 - {R_f}^2\right) = 2\pi R_f \delta$ = Cross sectional area

= Discharge/Constant peripheral velocity

Now $\delta = 10$, where 1 is the distance from datum to the cross section at which the local bore is R_τ and 0 is the semi-angle of tube taper. Hence 1 is proportional to discharge. An approximately linear calibration characteristic would be anticipated for the Rotameter.

4.2 <u>Calculations of Head Loss-Meters</u>

<u>B</u>y reference to equation 1 the head loss associated with each meter can be evaluates.

4.2.1 Venturi Meter

Applying the equation between pressure tappings (A) and (C)

$$\frac{P_A}{\rho g} - \frac{P_c}{\rho g} = \Delta H_{AC}$$

$$\text{i.e. } h_A - h_c = \Delta H_{AC}$$

This can be made dimensionless by dividing it by the inlet kinetic head ${V_A}^2/2g$

Now ${V_B}^2 = \frac{2g}{1-(A_B/A_A)^2}\left[\frac{P_A}{\rho g} - \frac{P_B}{\rho g}\right]$

And ${V_A}^2 = {V_B}^2(A_B/A_A)^2$

Thus $\frac{{V_A}^2}{2g} = \frac{{A_B}^2}{A_A}\left[\frac{1}{1-(A_B/A_A)^2}\left(\frac{P_A}{\rho g} - \frac{P_B}{\rho g}\right)\right]$

With the apparatus provided, A_B/A_A=0.38 and thus the inlet kinetic head

$$\frac{V_A{}^2}{2g} = 0.144\ x\ 1.16\ \left(\frac{p_A}{\rho g} - \frac{P_B}{\rho g}\right) = 0.167(h_A - h_B)$$

e.g. If h_A=372 mm, h_B=116 mm and h_c= 332 mm then:

$$\Delta H_{AC} = h_A - h_c = 40\ mm$$

$$\frac{V_A{}^2}{2g} = 0.167(h_A - h_B) = 0.167\ X\ 256 = 42.75\ mm$$

Head Loss = $\frac{\Delta H_{AC}}{V_A{}^2/2g} = \frac{40\ mm}{42.75\ mm} = 0.936\ inlet\ kinetic\ heads$

4.2.2 Orifice Meter

Applying equation 1 between E and F by substituting kinetic and hydrostatic heads would give an elevated value to the head loss for the meter. This is because, at an obstruction such as an orifice plate, there is a small increase in pressure on the pipe wall due to part of the impact pressure on the plate being conveyed to the pipe wall. BS 1042 (Section 1.1 1981) gives an approximate expression for finding the head loss and generally this can be taken as 0.83 times the measured head difference.

Therefore: $\Delta H_{EF} = 0.83(h_E - h_F)mm$

= 0.83(354-44) mm

= 257 mm

The orifice plate diameter is approximately twice the venture inlet diameter, therefore the orifice inlet kinetic head is approximately 1/16 that of venturi = 1/16 X 42.75 mm = 2.67 mm

Therefore $\Delta H_{EF} = \frac{257}{2.67} = 96$ inlet kinetic heads

4.2.3 Rotameter

For this meter, application of equation 1 gives

$$\frac{P_H}{\rho g} + Z_H - \frac{P_1}{\rho g} + Z_1 = \Delta H_{HI}$$

Then as shown in fig 7: $h_H - h_I = \Delta H_{HI}$

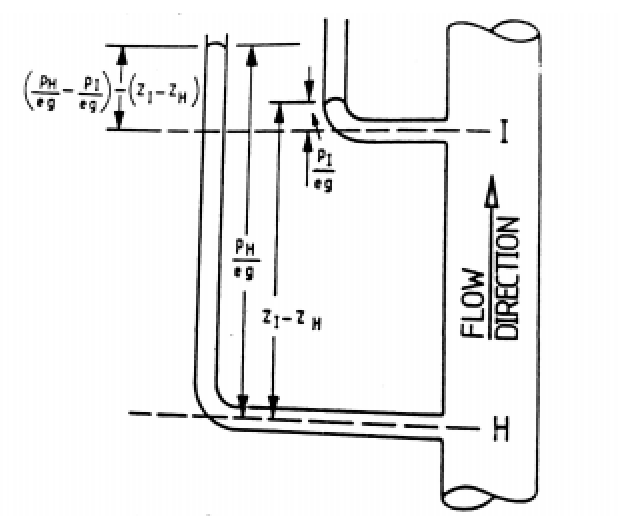


Fig 7 Rotameter Head Loss

	Manometric levels(mm)													M(kg/s)				Δ Inlet kinetic head				
Test no	A	B	C	D	E	F	G	H	I	Rotameter (c	Hydr Bench	Waterw (kg	T (seconds)	Venturi (8)	Orifice (11)	Rotameter Fig	Weigh tank	Venturi	Orifice (14)	Rotamet	Diffuser (1	Elbow (1

Inspection of the table of experimental results show that this head loss is virtually independent or discharge and has a constant value of about 100 mm of water. As has already been shown, this is a characteristic property of the Rotameter. For comparative purposes it could be expressed in ter : of the inlet kinetic head. For example, since the connecting tube has a 26 min bore, with the test results under consideration the inlet kinetic head is 42.75 mm of water as it is with the Venturi Meter Hence the Rotameter head loss is then 2.3 inlet kinetic heads. However when the velocity is very low the head loss remains the same and thus becomes many, many times the inlet kinetic head.

It is instructive to compare the head losses associated with the three meters with those associated with the rapidly diverging section, or wide-angled diffuser, and with the right-angled bend or elbow. The same procedure is adopted to evaluate these losses.

4.**2.4 Wide angled diffuser**

The inlet to the diffuser may be considered to be at (c) and the outlet at (D). Applying equation 1,

$$\frac{P_c}{\rho g} + \frac{{v_c}^2}{2g} = \frac{P_D}{\rho g} + \frac{{V_D}^2}{2g} + \Delta H_{CD}$$

Since the area ratio, inlet to outlet, of the diffuser is 1:4, the outlet kinetic head is one-sixteenth of the inlet kinetic head.

e.g. if h_A= 372 mm, h_B= 116 mm, h_c=332 mm and h_D= 337 mm
the inlet kinetic head = 42.75 mm, (see Venturi meter head loss calculations).
The corresponding outlet kinetic head = 1/16 x 42.75 = 2.67 mm
And thus ΔH_{CD}= (332-337) + (42.75 – 2.67) = -5 + 40.08 = 35.08 mm of water.
Head loss = 35.08/42.75 = 0.821 inlet kinetic heads

4.2.5 **Right angled bend**

The inlet to the bend is at (G) where the pipe bore is 51 mm and outlet is at (H) where the bore is 26 mm.

Applying equation 1: $\frac{P_G}{\rho g} + \frac{{V_G}^2}{2g} = \frac{P_H}{\rho g} + \frac{{V_H}^2}{2g} + \Delta H_{GH}$

The outlet kinetic head is now approximately sixteen times the inlet kinetic head.
e.g. if h_A=372 mm, h_B=116 mm, h_G= 94 mm and h_H= 33 mm
The inlet kinetic head = 2.67 mm and the outlet kinetic head =42.75 mm
And thus ΔH_{GH}= (94-33)+(2.67-42.75) = 61 – 40.08 = 20.92 mm of water.
Head loss = 20.92/2.67 = 7.8 inlet kinetic heads.

4.3 Calculations of Head Loss coefficients

4.3.1 Expansion

Measured values - H_c= 277 mm

H_D = 285 mm

Weightank mass flow rate = .40 Kg/sec

Q=m/p =$\frac{.40\ kg/sec}{.998\ kg/m^3}$ = .00040 m^3/sec

A = π(.026)^2/4 = .000531m^2

v_1=$\frac{Q}{A}$= $\frac{.00040}{.000531}$ = .753 m/sec

$\frac{{v_1}^2}{2g} = \frac{(.753)^2}{2\ x\ 9.81}$ = .029 m

ΔH_{CD}= 12 mm = .012 m

Experimental $K_E = \Delta H_{CD}$/velocity head = $\frac{.012}{.029}$ = .41

For D_1/D_2 = $\frac{26\ mm}{51\ mm}$ = .50, and 0 =80°

Theoretical K_E -0.35

5. DISCUSSIONS

5.1. Discussion of the Meter Characteristics

There is little to choose in accuracy of discharge measurement between the Venturi Meter, the Orifice Meter and the Rotameter. All are dependent upon the same principle. Discharge coefficients and the Rotameter calibration are largely dependent on the way the stream forms a `vena contracta' or actual throat of smaller cross-sectional area than that of the constraining tube. This effect is negligibly small where a controlled contraction takes place in a Venturi Meter but is significant in the Orifice Meter. The Orifice meter discharge coefficient is also dependent on the precise location of the pressure tappings (E) and (F). Such data is given in B.S. 1042 which also emphasizes the dependence of the meter's behavior on the uniformity of the flow upstream and downstream of the meter. In order to keep the apparatus as compact as possible, the dimensions ache equipment in the neighborhood of the Orifice Meter have been reduced to their limit. Consequently, some inaccuracy in the assumed value of its discharge coefficient may be anticipated.

The considerable difference in head loss between the Orifice Meter and the Venturi Meter should be noted. The Orifice Meter is much simpler to make and use, for it is comparatively easy to manufacture a suitable orifice plate and insert it between two existing pipe flanges which have been appropriately pressure-tapped for the purpose. In contrast, the Venturi Meter is large, comparatively difficult to manufacture and complicated to fit into an existing flow system. But the low head loss associated with I the controlled expansion occurring in the Venturi Meter gives it an obvious superiority in applications where power to overcome flow losses may be limiting.

Rotameters and other flow-measuring instruments which depend on the displacement of floats in tapered tubes may be selected from a very wide range of specifications. They are unlikely to be comparable with the Venturi Meter from the standpoint of head loss but, provided the discharge range is not extreme, the ease of reading the instrument may well compensate for the somewhat higher head loss associated with it.

The head losses associated with the wide-angled diffuser and the right-angled bend are not untypical. Both could be reduced Wit were desirable to do so. The diffuser head loss would be minimized if the total expansion angle of about 50 degrees were reduced to about 10 degrees. The right-angled bend loss would be substantially reduced if' the channel, through which the water flows, was shaped in the arc of a circle having a large radius compared with the bore of the tube containing the fluid.

Large losses in internal flow systems are associated with uncontrolled expansions of the stream. Attention should always be paid to increases in cross-sectional area and changes of direction of the stream as these parts of the system are most responsive, in terms of associated head loss, to small improvements in design.

5.2 Discussion of Results

If mass flow results are plotted against mass flow rates from the weighing tank method, the accuracy of the various methods can be compared Since all are derived from Equation 1, similar results would be expected from the three methods. The differential mass flow measurement ($m_{meter} - m_{weightank}$)) could be plotted against the weighing tank mass flow results for a better appraisal of accuracy.

Some overestimation in the Venturi Meter determination can he anticipated because its vena contracta has been assumed to be negligibly small. Similarly, the Rotameter determination may well be sensitive to the proximity of the elbow and the associated inlet velocity distribution. The Orifice Meter is likely to be sensitive to the inlet flow which is associated with the separation induced in the wide-angle diffuser upstream of it Thus both the Rotameter and the Orifice Meter calibrations would be likely to change if a longer length of straight pipe were introduced upstream of them.

In the calculations the head losses associated with the various meters and flow components have been made dimensionless by dividing by the appropriate inlet kinetic heads. The advantage of the Venturi Meter over the Orifice Meter and Rotameter is evident, though over a considerable range of inlet kinetic head, the loss associated with the Rotameter is sufficiently small to consider that it would be more than compensated by the relative ease in evaluation of mass flow from this instrument.

It should also be noted from Fig 9 that the dimensionless head losses of the Venturi Meter and the Orifice Meter are Reynolds Number dependent. This effect is also noticeable with the dimensionless head loss of the elbow.

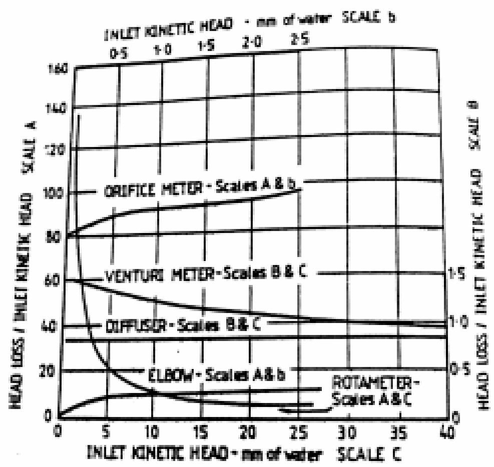


Fig 9 Typical Head Loss Graphs

Fig 9 Typical Head Loss Graphs

6. CONCLUSIONS

The most direct measurement of fluid discharge is by the weightank principle. In installations where this is impracticable (e.g. on account of size of installation or gaseous fluid flow), one of the three discharge meters described may be used instead.

The Venturi Meter offers the best control to the fluid. Its discharge coefficient is little different from unity and the head loss it offers is minimal. But it is relatively expensive to manufacture and could be difficult to install in existing pipework.

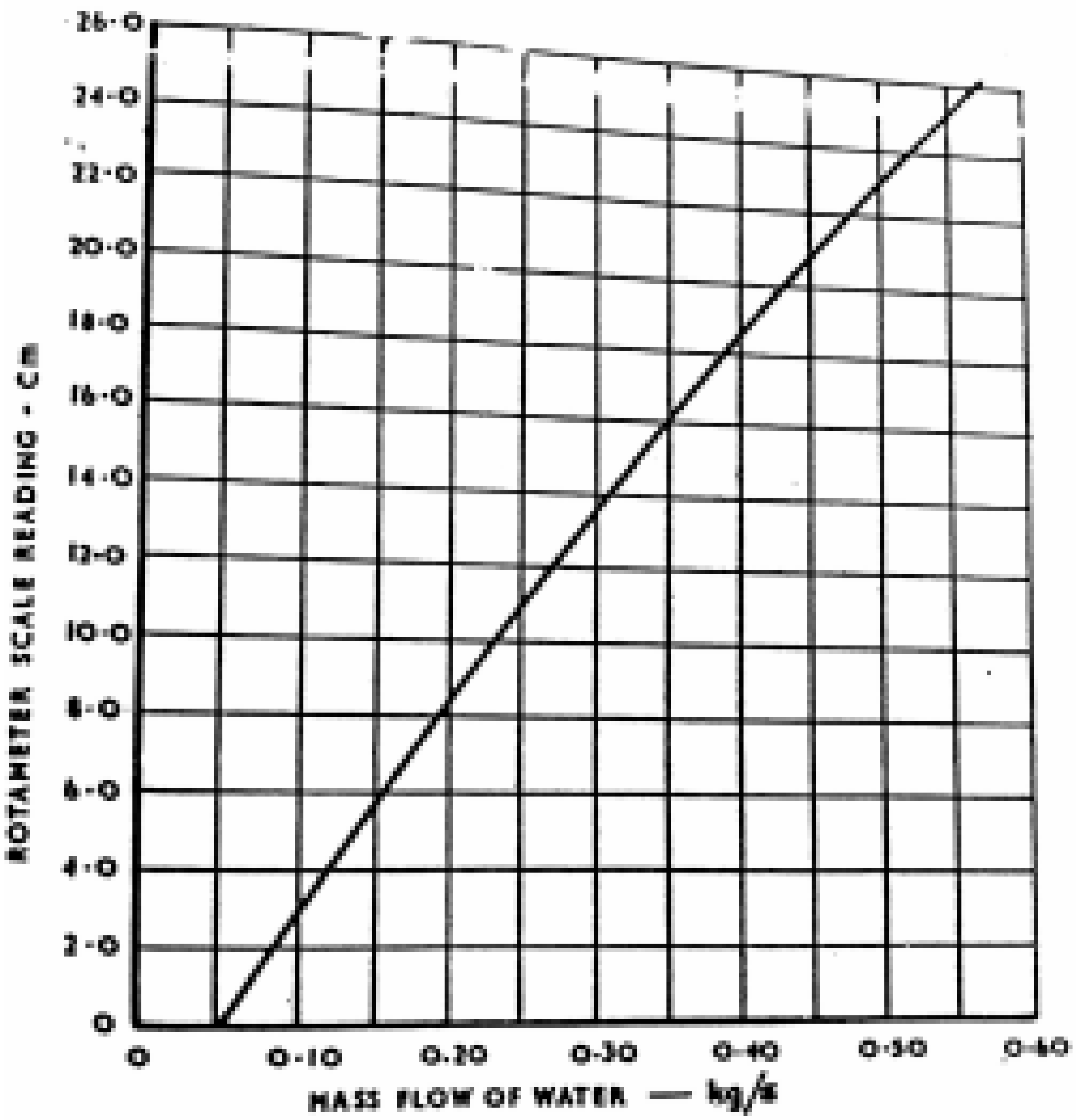
The Orifice Meter is easiest to install between pipe flanges and, provided it is manufactured and erected in accordance with B.S. 1041, will give accurate measurements. But the head loss associated with it is very large compared with that of the Venture Meter.

The Rotameter given the easiest derivation of discharge, dependent only on sighting the float and reading a calibration curve. It needs to be chosen judiciously, however, so that the associated head loss is not excessive.

7. REFERENCES

Flow Measurement B.S. 1042 British Standards Institution 1943.

Flow Measurement B.S. 1042 British Standards Institution Section 1.1 1981.



Rotameter calibration curve