

EXPERIMENT 2

Hooke's Law and Comparing Measurements with Uncertainty

EXPERIMENTAL GOAL

Determine if achieving high-precision results with this equipment will allow you to detect a measurable deviation from Hooke's Law for small (Δx_{small}) and large (Δx_{large}) compressions.

INTRODUCTION

Hooke's Law states that the deformation (elongation or compression) of a spring is proportional to the force applied ($|F| = k|x|$). There are real-world limitations to this model, one of which is that the spring constant, k , is no longer constant when the spring experiences large deformations. In order to explore the real-world limitations of this model you will attempt to distinguish between two values of k , determined from two different stretches of the spring.

LEARNING GOALS

You will:

- Become familiar with a framework for how physics experiments work;
- Use quantitative approaches to compare between experimental results and evaluate the quality of experimental results;
- Use an iterative process to minimize the overall uncertainty in the final result of an experiment;
- Develop your lab notebook skills related to reflecting on your numerical results and comparisons.
- Develop your lab notebook skills related to planning for future/improved measurements.
- Use Hooke's law to determine the spring constant of your spring.

PART 1 – YOUR FIRST HOOKE’S LAW MEASUREMENT

Hooke’s law tells us that the change in force (ΔF) needed to extend or compress a spring by some distance (Δx) is proportional to that distance,

$$k = |\Delta F|/|\Delta x|,$$

where the proportionality constant k is referred to as the spring constant and is a characteristic of that particular spring.

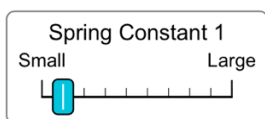
Your 40-minute task is to determine k for your spring (in units of N/m, with measurement uncertainty $u[k]$) using only the following simulations:

https://phet.colorado.edu/sims/html/masses-and-springs/latest/masses-and-springs_en.html

<https://www.phas.ubc.ca/~james/shmscale.html>

Requirements:

- Use the Lab setting in the simulation
- Set the spring constant in the simulation to the second smallest.



- Choose a mass that has the **spring stretch no more than 30 cm** in the simulation.
- Focus on figuring out $u[k]$ (the measurement uncertainty in your spring constant) for your first trial before trying to take multiple trials.
- Record in your lab notebook justifications for your choices for your measurement methods and the uncertainties for each measurement.

Some things that you might find useful:

1. The mass (m) displayed on the scale can be related to force (F) by $F = mg$, where g is the acceleration due to gravity at our altitude in Vancouver, $g = 9.809$ m/s.
2. There are numerous approaches that you can take with your measurements. Below is a list of uncertainty propagation equations which should cover most approaches. Please ask your instructor/TAs for any others that you might need.

$k = \frac{ \Delta F }{ \Delta x }$	$u[k] = k \sqrt{\left(\frac{u[\Delta F]}{\Delta F}\right)^2 + \left(\frac{u[\Delta x]}{\Delta x}\right)^2}$
$ \Delta x = x_2 - x_1 $	$u[\Delta x] = \sqrt{(u[x_1])^2 + (u[x_2])^2}$
$ \Delta F = F_2 - F_1 $	$u[\Delta F] = \sqrt{(u[F_1])^2 + (u[F_2])^2}$
$ \Delta F = g \Delta m $	$u[\Delta F] = g \cdot u[\Delta m]$
$ F = g \cdot m$	$u[F] = g \cdot u[m]$
$ \Delta m = m_2 - m_1 $	$u[\Delta m] = \sqrt{(u[m_1])^2 + (u[m_2])^2}$

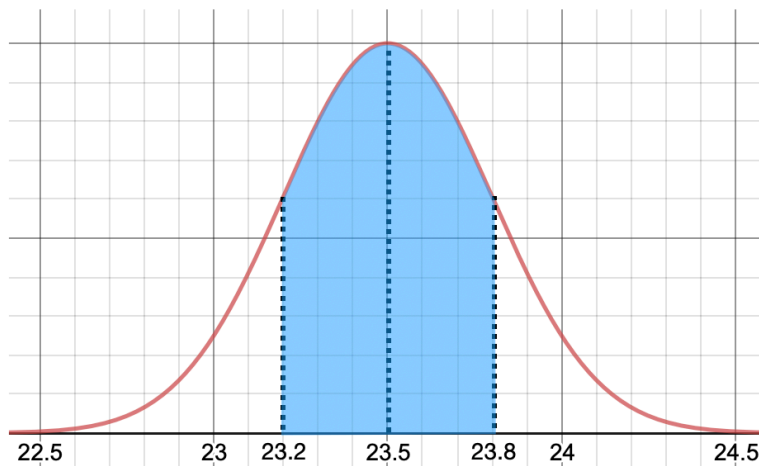
THEORY INTERLUDE – PROBABILITY DENSITY FUNCTIONS

The point of this lab is comparisons, so we need to make a second measurement. But before we do that, let's think about what the measurement we just did means. Imagine this was our measurement for the spring constant

$$k = 23.5 \pm 0.3 \text{ N/m.}$$

But this isn't the real value of k. The real value of k is something like 23.45768.... N/m. What our measurement actually describes how likely it is for the real value of the spring constant to be in a certain range of numbers.

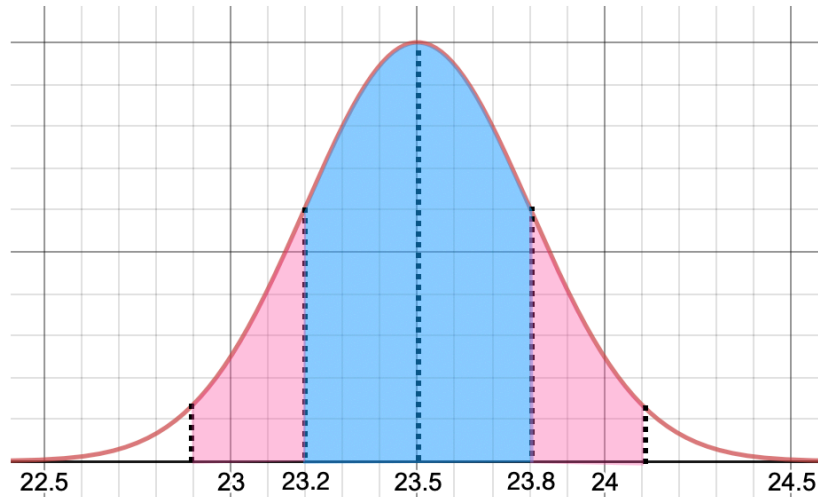
It turns out that uncertainty in science is modelled well using a Gaussian distribution. The value we measured 23.3 is called the mean of this distribution and 0.3 is called the standard deviation. Plotted, it looks like this:



The blue region, which is one standard deviation of either side of the mean, is 68% of the total area under the red curve. What our measurement with uncertainty is telling us is that there is a

68% chance that the real value of k is between 23.2 N/m and 23.8 N/m. However, there is a 32% chance that it could be outside that region.

A very common term used in science, particularly the Life Sciences, is the 95% confidence interval. That comes from this distribution as well. Let's draw the region in which we go another 0.3 N/m (2 standard deviations) out from either side of our mean.



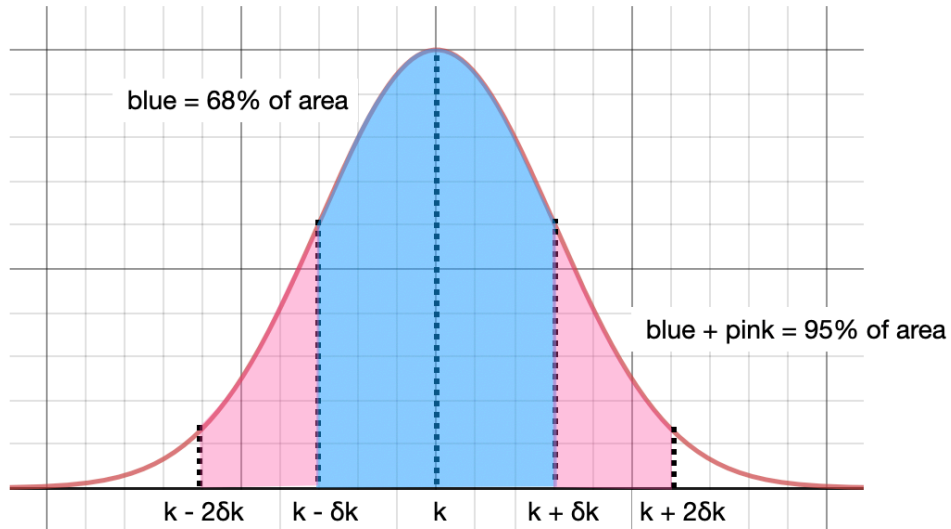
The pink region shows 2 standard deviations away from the mean. The area in the pink and the blue region now accounts for 95% of the total area under the red line.

What does this mean in terms of the real value of the spring constant? It means that there is a 68% chance that the real value is between 23.2 and 23.8 N/m. However, it also means that there is a 2.5% chance that the actual value is greater than 24.1 N/m. It's unlikely, but it could happen.

Let's generalize this a little with a measurement

$$k \pm \delta k$$

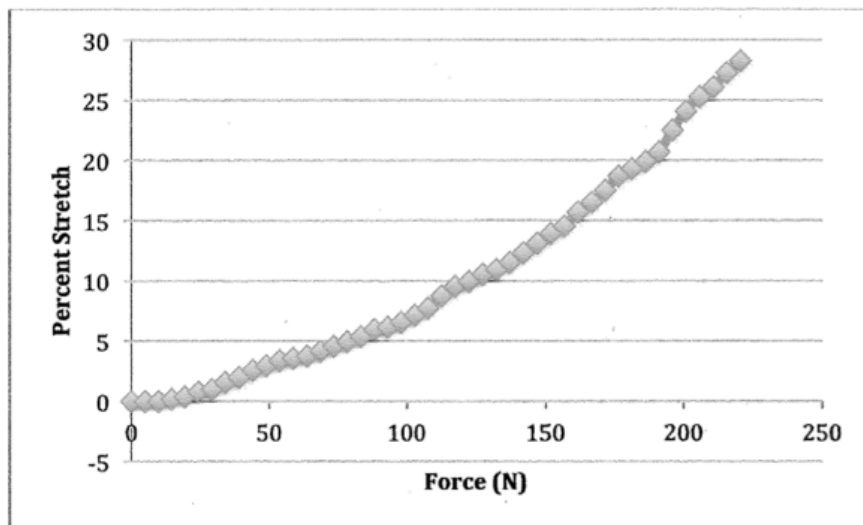
The distribution of this looks like the following:



PART 2 – YOUR SECOND HOOKE’S LAW MEASUREMENT

So, know that we have an idea of what the measurement and uncertainty are modelling, let’s take your second measurement.

As mentioned in the introduction, Hooke’s Law has limitations in the real world. For instance, look at this kelp stretching data from a lab at the Bamfield Marine Science Centre.



The fact that the slope changes as more force is applied tells us that the spring constant

$$k = |\Delta F|/|\Delta x| = dF/dx,$$

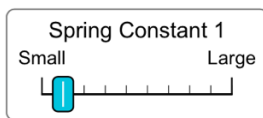
must be changing as the kelp is stretched. Let's see if this is true for our simulation. Does a spring stretched to greater than 60cm have a similar k to one stretched to less than 30cm?

Your 20-minute task is to determine k again for your spring (in units of N/m, with measurement uncertainty $u[k]$) using only the following simulations:

https://phet.colorado.edu/sims/html/masses-and-springs/latest/masses-and-springs_en.html
<https://www.phas.ubc.ca/~james/shmscale.html>

Requirements:

- Use the Lab setting in the simulation
- Set the spring constant in the simulation to the second smallest.



- Choose a mass that has the **spring stretch more than 60 cm** in the simulation

THEORY INTERLUDE – COMPARING YOUR MEASUREMENTS

At this point you should have two measurements for your spring constant. We are going to use the t-test to compare them. The t-test was originally developed by Guinness Brewery in the late 1800s to ensure that the quality of their materials were consistent. At the time, consistency of product was by far the most important element in selling beer. If you ordered a Guinness, you better get a Guinness.

You also might have heard of the p-value or hypothesis testing. We won't use p-values in this course, but understating how the t-test works will give you a lot of insight into how hypothesis testing works.

The t-test we will be using has the mathematical form

$$t = \frac{|A - B|}{\sqrt{(\delta A)^2 + (\delta B)^2}}$$

where A and B are two independently measured values and δA and δB are their associated uncertainties. Let's break it down:

1. The numerator $A - B$ is simply the difference between the two values.
2. The denominator is the propagated uncertainty in that difference. Remember, when we subtract two values, we add their uncertainty in quadrature.

3. The t score, which is the ratio of these two things, is unitless! It is then interpreted as how many widths of uncertainty (or standard deviations) are these two values separated by.

Let's illustrate with an actual example. Let's compare the two values

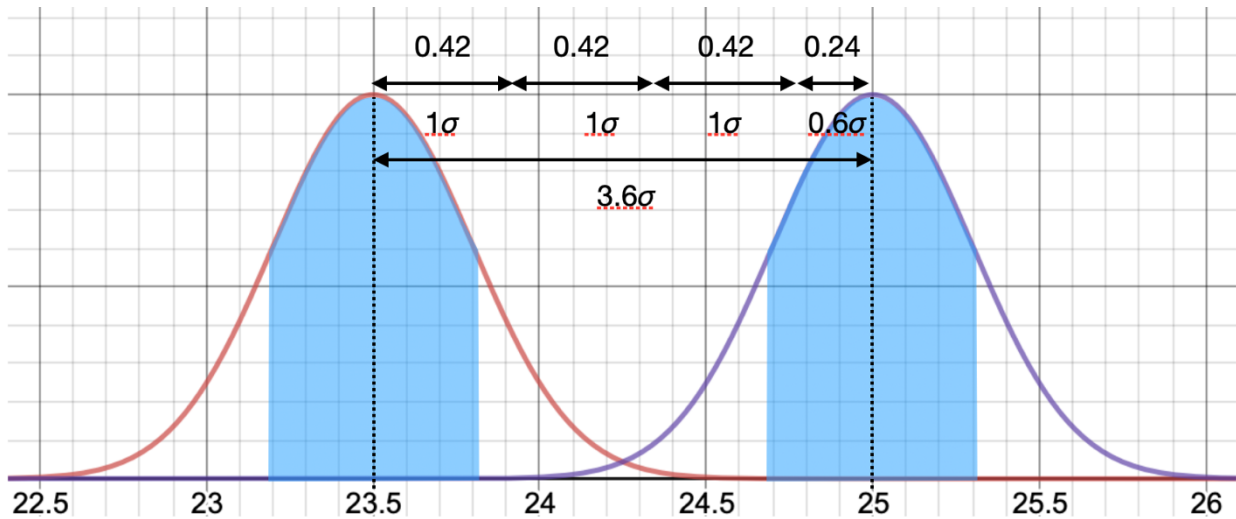
$$k_1 = 23.5 \pm 0.3 \text{ N/m},$$

$$k_2 = 25.0 \pm 0.3 \text{ N/m}.$$

The t-test says that

$$t = \frac{25.0 - 23.5}{\sqrt{(0.3)^2 + (0.3)^2}} = \frac{1.5}{0.42} = 3.6$$

Let's look at what this means in terms of our Gaussian model of uncertainty. You can use DESMOS (<https://www.desmos.com/calculator/ve4v2f3f9z>) to make Gaussian distributions like this.



The graph above shows the two measurements with their uncertainty shown by the shaded region. The units in the graph are suppressed for clarity. The difference between these values is shown in units of combined uncertainty of the two values $\sigma = \delta(k_1 - k_2) = 0.41 \text{ N/m}$. We see that the values are 3.6σ apart, which is our t-score. The t-score given a measure of how far apart two values are in units of the uncertainty of that difference.

A t-score of > 3 like this shows that these are unlikely to be the same value. The distributions very little overlap, but they still overlap. There is a small chance that these are both measuring the same spring constant between 24.2 N/m and 24.3 because their distributions both are non-zero at this point. However, this is unlikely.

What does it look like when the measurements are closer? Imagine we have the two values

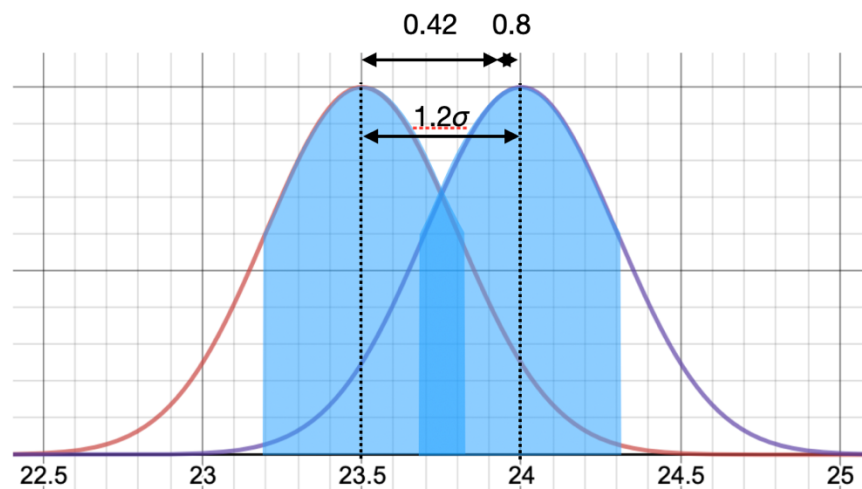
$$k_1 = 23.5 \pm 0.3 \text{ N/m,}$$

$$k_2 = 24.0 \pm 0.3 \text{ N/m.}$$

The t-test says that

$$t = \frac{24.0 - 23.5}{\sqrt{(0.3)^2 + (0.3)^2}} = \frac{0.5}{0.42} = 1.2$$

There is not much separation between these two values. If we look at the graph below we see there is a lot of overlap in their distributions.



Sure, they are separated a bit, but it seems quite likely that both of these measurements could be measuring the same spring constant that has a value around 23.25 N/m.

PART 3 – COMPARE YOUR NUMBERS

Use the t-test and the reasoning above to compare your two measurements for your spring constant.

Requirements for report:

- Calculate your t-score for your two measurements.
- Justify why your values are the same or different.
- Explain what this means about our spring simulation.

Some things that you might find useful:

1. You can make graphs like the ones I've made in DESMOS:

<https://www.desmos.com/calculator/ve4v2f3f9z>

I added the lines and colouring in Keynote.

