Summary lecture IX

- **The electron-light Hamilton operator** reads in second quantization

$$H_{\text{e-l}}^{\boldsymbol{p}\cdot\boldsymbol{A}} = \frac{i\hbar e_0}{m_e}\sum_{\boldsymbol{l},\boldsymbol{l}'} \boldsymbol{M}_{\boldsymbol{l},\boldsymbol{l}'}\cdot\boldsymbol{A}(\boldsymbol{r}_0,t)\, a_{\boldsymbol{l}}^+ a_{\boldsymbol{l}'} \qquad H_{\text{e-l}}^{\boldsymbol{r}\cdot\boldsymbol{E}} = \sum_{\boldsymbol{l},\boldsymbol{l}'} d_{\boldsymbol{l},\boldsymbol{l}'}\cdot\boldsymbol{E}(\boldsymbol{r}_0,t)\, a_{\boldsymbol{l}}^+ a_{\boldsymbol{l}'}$$

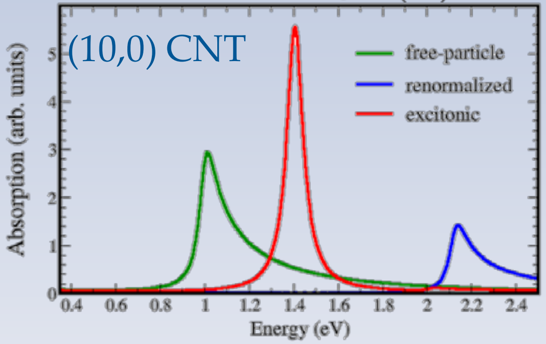
- **Absorption coefficient** $\alpha(\omega)$ is given by the **optical susceptibility** $\chi(\omega)$

$$\alpha(\omega) = \frac{\omega}{n(\omega)\, c_0}\text{Im}\chi(\omega)$$

that is determined by microscopic polarization $\chi(\omega) = \dfrac{P(\omega)}{\varepsilon_0 E(\omega)}$

- Bloch equation in the limit of linear optics

$$\dot{p}_{\mathbf{k}}(t) = \frac{i}{\hbar}\Delta\tilde{\varepsilon}_{\boldsymbol{k}}^{vc}\, p_{\boldsymbol{k}}(t) + i\tilde{\Omega}_{\boldsymbol{k}}(t) - \gamma\, p_{\boldsymbol{k}}(t)$$



Chapter VI

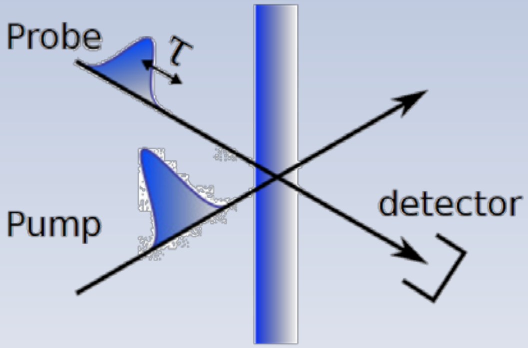
Learning outcomes lecture X

- Recognize the importance of **pump-probe experiments** for revealing carrier dynamics
- Explain the **photon statistics** for different light sources

3. Differential transmission spectra

- Bloch equations provide **microscopic access to carrier dynamics**, however it is **difficult to directly measure** the carrier occupation
- In **pump-probe** experiments**, differential transmission spectra (DTS)** are measured, where a **pump pulse** creates a **non-equilibrium distribution** and a weaker **probe pulse** measures the **dynamics** of excited carriers

$$\frac{T - T_0}{T_0}(\tau, \omega) \propto \left(\alpha^{(t)}(\omega) - \alpha^{(p+t)}(\tau, \omega)\right)$$



- Exploit the relation $T = I/I_0$ with the intensity $I = I_0 e^{-\alpha z}$ and assume that the absorption coefficient α is relatively small

Differential transmission spectra

3. Differential transmission spectra

- Assuming a **delta-shaped probe pulse** and exploiting the Bloch equations, we obtain for the differential transmission

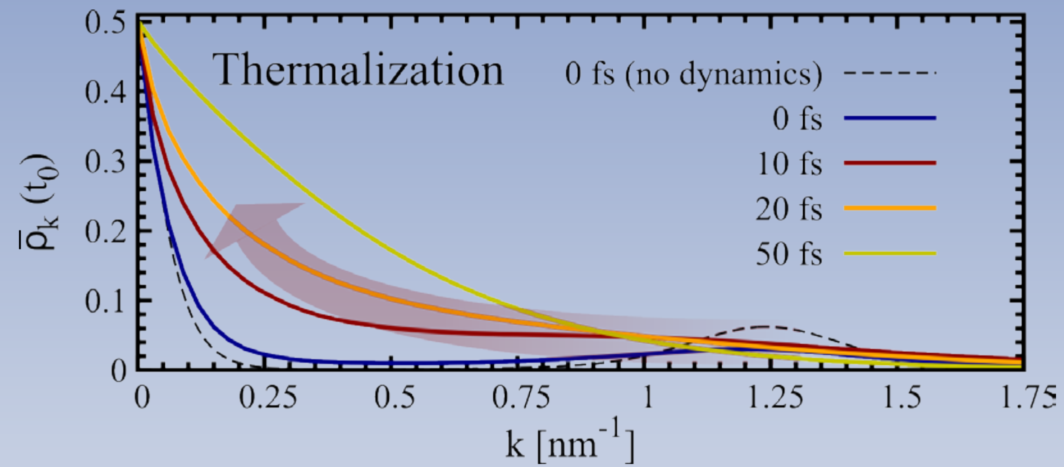
$$\Delta T/T_0(\tau,\omega) \propto \int d\phi |\boldsymbol{M}_{k_0,\phi}|^2 \Delta\rho_{k_0,\phi}(\tau) \approx \left(\rho^{(p)}_{k_0}(\tau) - \rho_{k_0}(-\infty)\right)$$

  with the optical matrix element $\boldsymbol{M}_{k_t,\phi}$ in polar coordinates and with $\Delta\rho_{k_t,\phi}(\tau) = \rho^{(p)}_{k_0,\phi}(\tau) - \rho_{k_0,\phi}(-\infty)$ where $\rho_{k_0,\phi}(-\infty)$ is the **carrier occupation before the pump pulse** and $\rho^{(p)}_{k_0,\phi}(\tau)$ the **pump-induced carrier occupation** (momentum k_0 corresponds to the pumped state)

- The crucial quantity is the carrier occupation $\rho_{\boldsymbol{k}}(t)$

Carrier thermalization

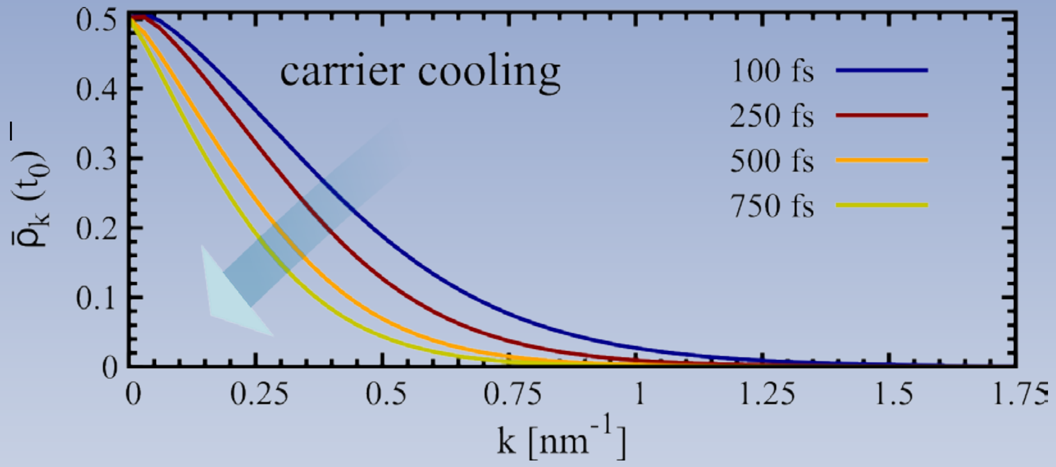
3. Differential transmission spectra



- Significant **relaxation** takes place already **during** the excitation **pulse**
- Coulomb-induced carrier-carrier scattering is the dominant channel
- **Thermalized Fermi distribution** reached within the first **50 fs**

Carrier cooling

3. Differential transmission spectra



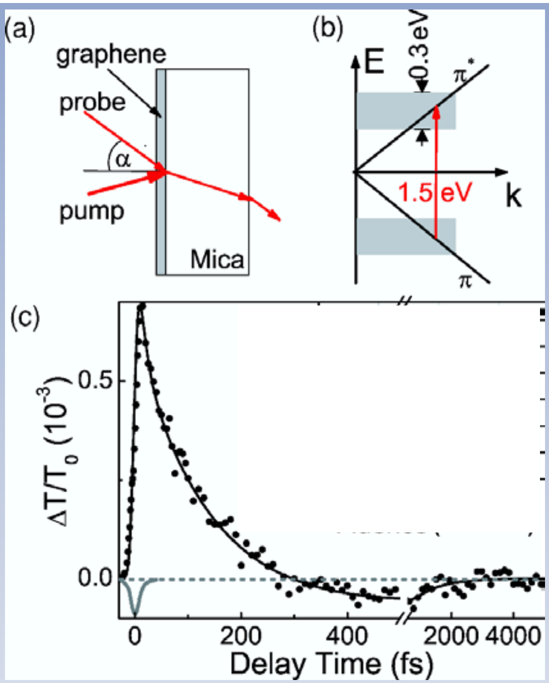
- **Carrier cooling** takes place on a **picosecond time scale**
- **Optical phonons** (in particular ΓLO, ΓTO and *K* phonons) are more **efficient** than acoustic phonons

Pump-probe experiment in the infrared

3. Differential transmission spectra

- Pump-probe-experiment measuring **differential transmission** in graphene
- **Excitation energy** is **1.5 eV,** temporal **resolution** is **10 fs**
- Initial increase of transmission is due to the **absorption bleaching**
- Following **decay** is characterized by **two time constants**:

τ_1 = 140 fs; τ_2 = 0.8 ps



Experiments performed by **Thomas Elsaesser** (Max-Born Institut, Berlin)

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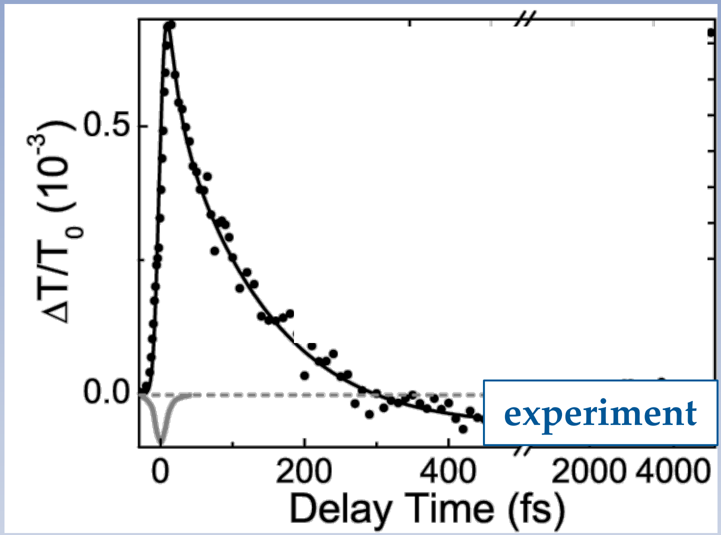
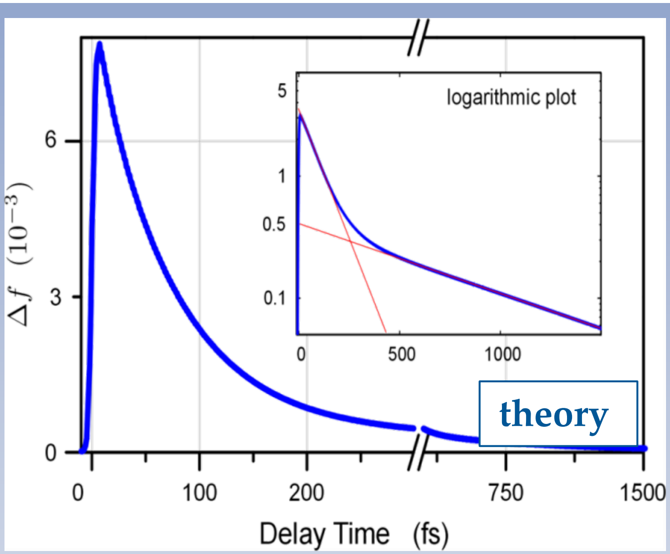
Experiment-theory comparison

3. Differential transmission spectra





τ_1 = 140 fs, τ_2 = 0.8 ps **two decay times** τ_1 = 104 fs, τ_2 = 0.7 ps

- Theory is in **good agreement** with experiment: τ_1 corresponds to carrier **thermalization**, τ_2 describes **carrier cooling**

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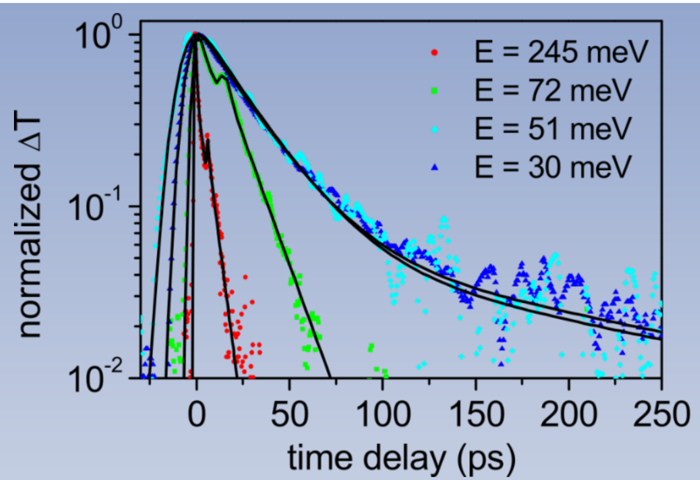
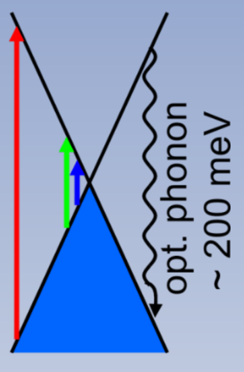
Pump-probe experiment close to the Dirac point

3. Differential transmission spectra



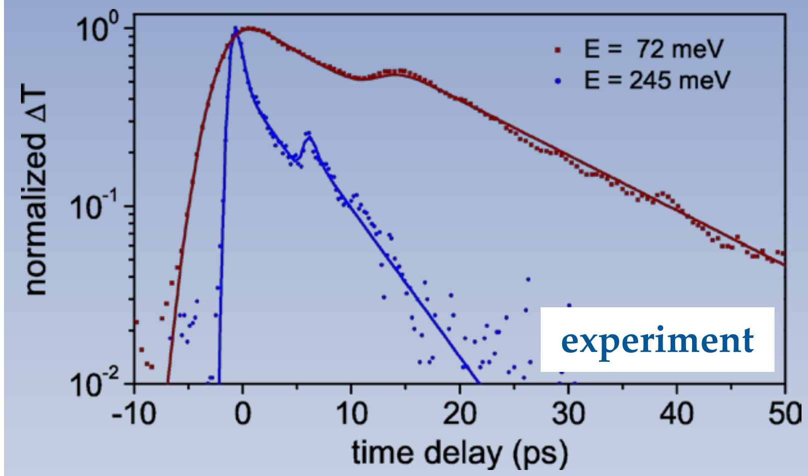


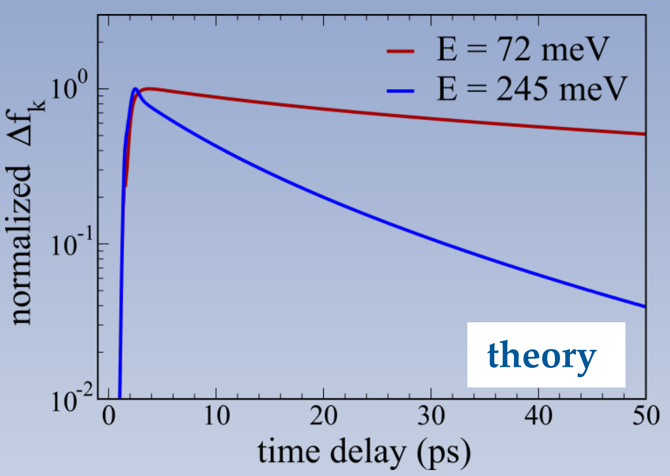
experiments performed by Manfred Helm (Helmholtz-Zentrum Dresden)

- Transmission in the vicinity of Dirac point and **below** the **energy** of **optical phonons** (~ 200 meV) ⟶ acoustic phonons dominant?
- Relaxation **dynamics** is **slowed down** (5 ps at 245 meV, **25 ps** at 30 meV)

Experiment-theory comparison

3. Differential transmission spectra





- Theory in **good agreement** with experiment (slowed-down dynamics):

  → **Optical phonons** remain the **dominant** relaxation channel, since carrier-carrier scattering leads to a **spectrally broad distribution**

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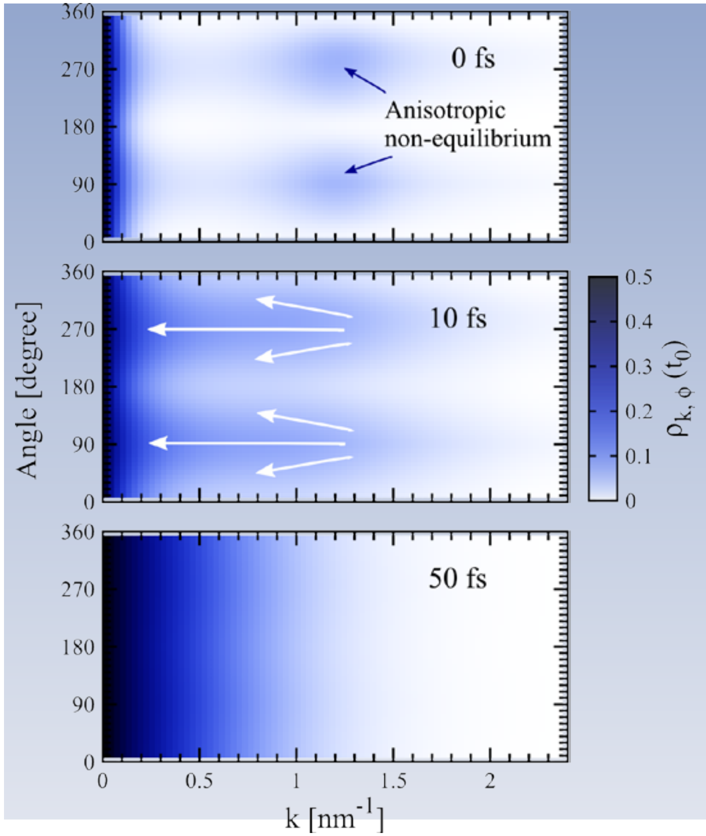
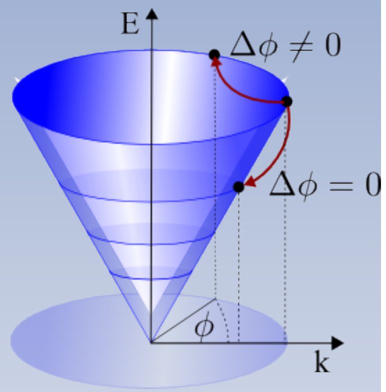
Anisotropic carrier dynamics

3. Differential transmission spectra

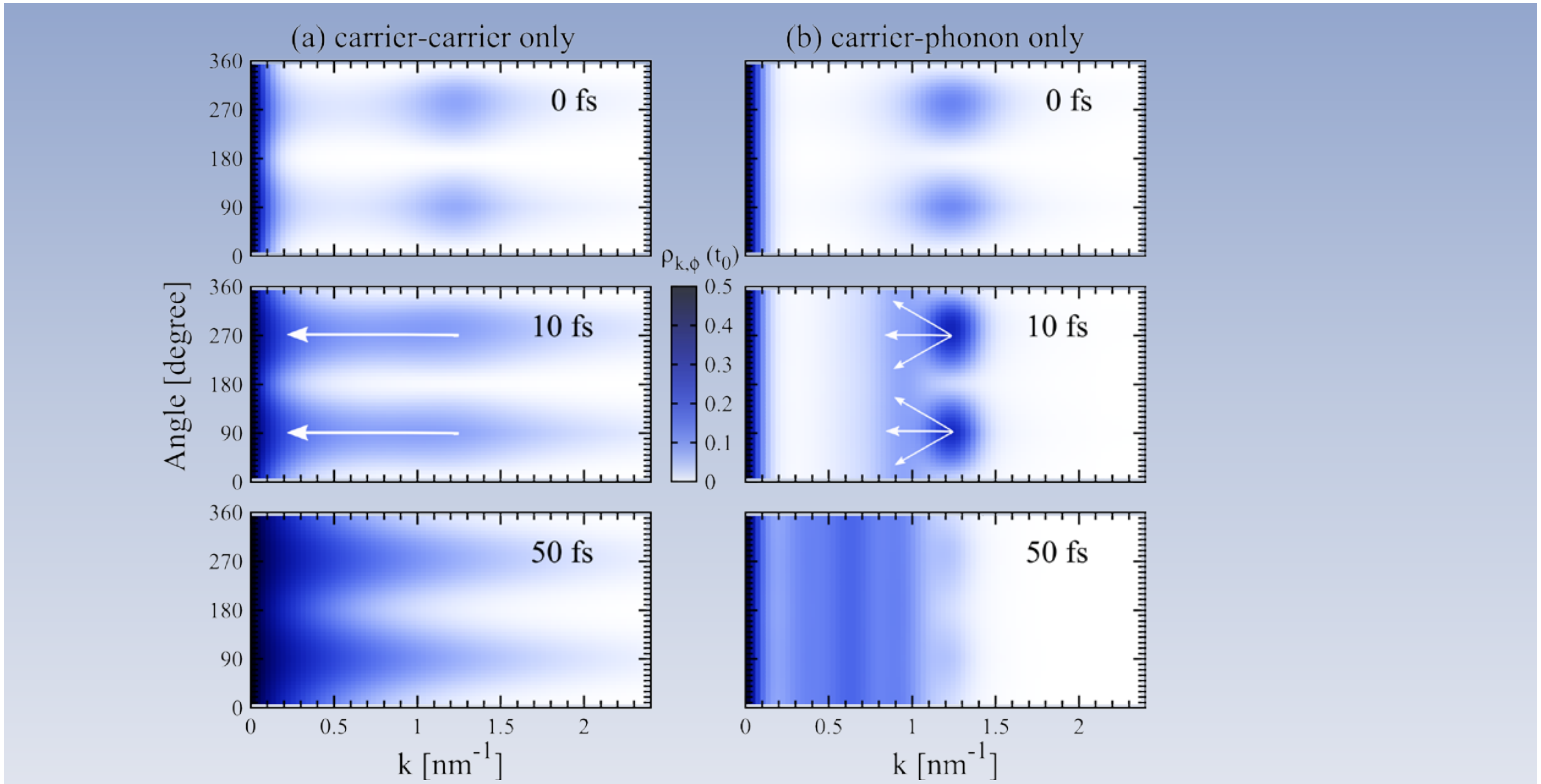


- **Anisotropy** of the **carrier-light coupling** element
- Scatering **across** the Dirac **cone reduces anisotropy**
- Carrier distribution becomes entirely **isotropic** within the first **50 fs**



Microscopic mechanism

3. Differential transmission spectra



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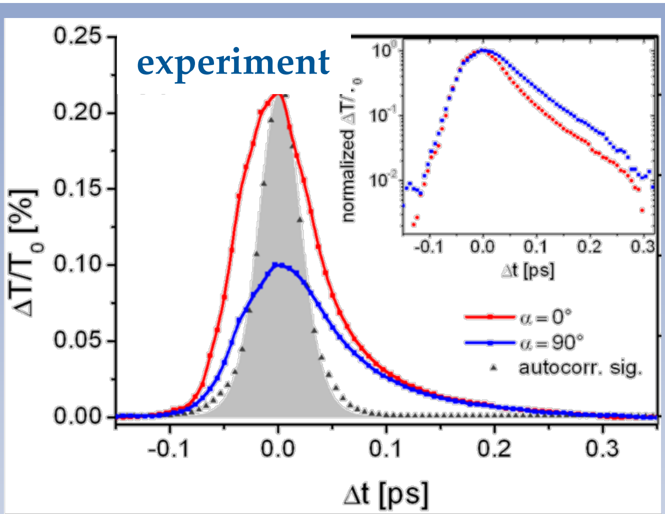
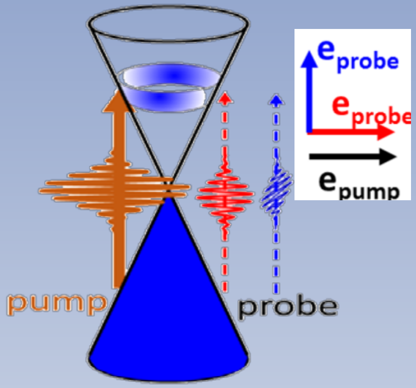
Polarization-dependent pump-probe experiment

3. Differential transmission spectra

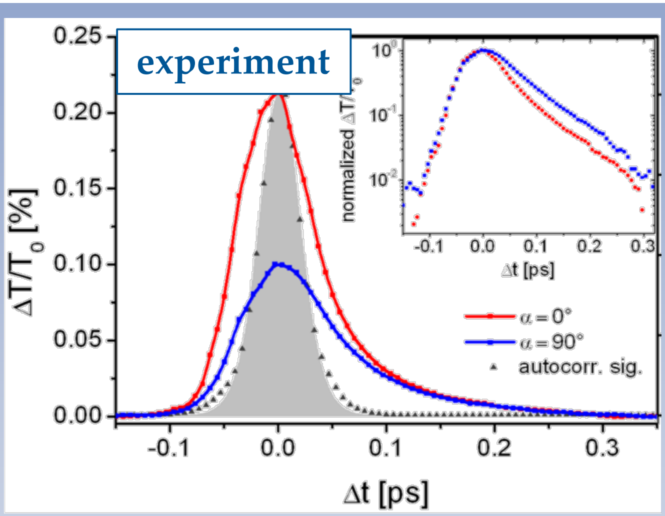


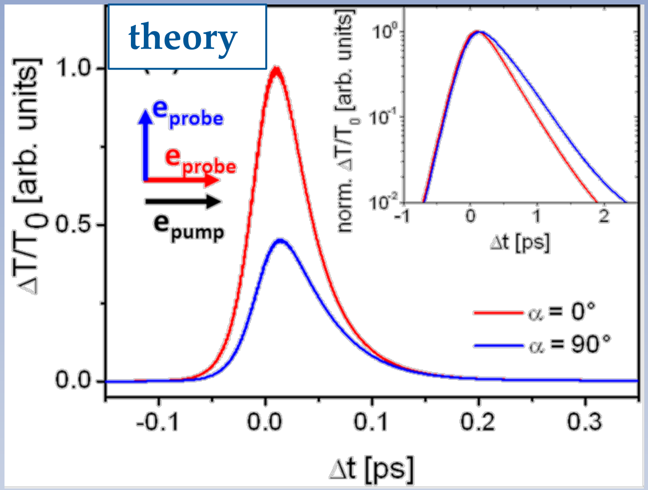


- **Polarization-dependent** high-resolution **pump-probe** experiments performed by Manfred Helm (Helmholtz-Zentrum Dresden)
- Variation of the **relative polarization** of the **pump** and the **probe** pulse

Experiment-theory comparison

3. Differential transmission spectra





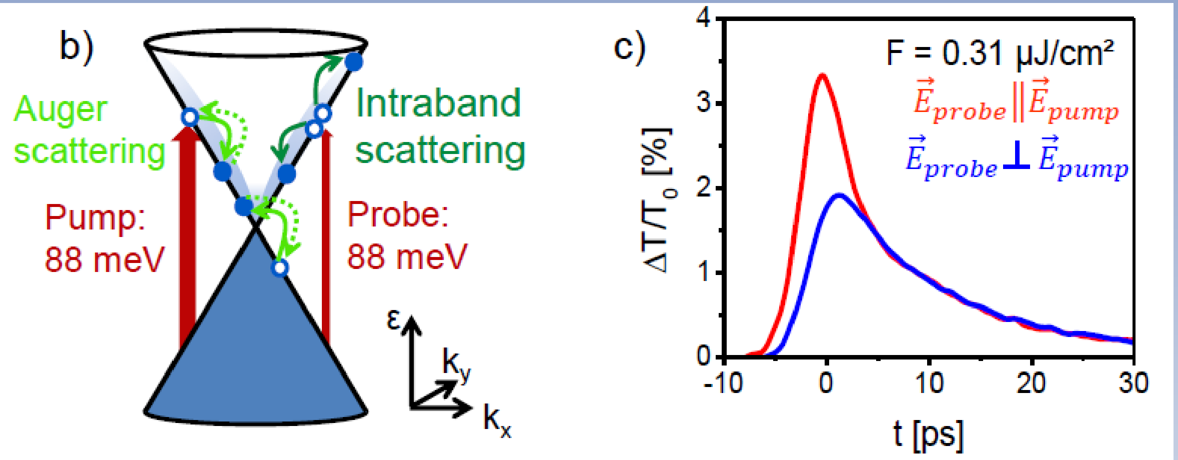
- Theoretical prediction is in **excellent agreement** with experiment:

  ➞ **Anisotropic differential transmission** can be observed within the **first 100 fs**

Nano Lett. 14, 1504 (2014)

Anisotropy close to the Dirac point

3. Differential transmission spectra

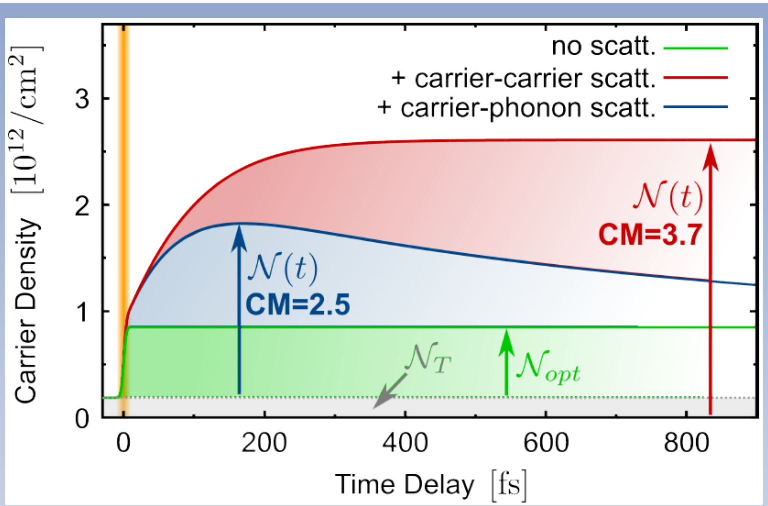


- **Optical excitation below** the optical phonon energy of **200 meV** strongly **suppress carrier-phonon scattering**

  → **Isotropic carrier** distribution is reached via **carrier-carrier scattering** on a much smaller **picosecond time scale**

Carrier multiplication

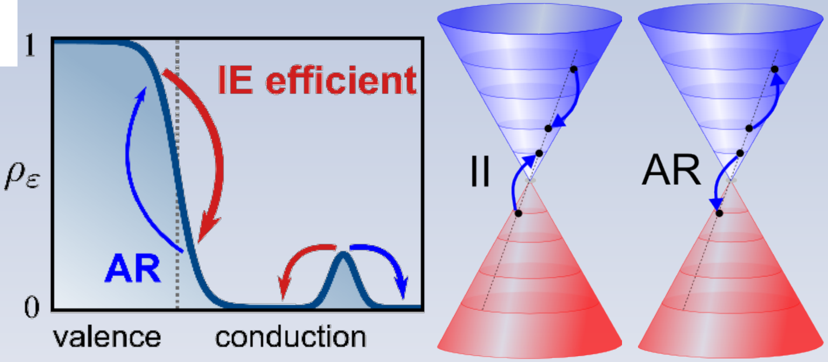
3. Differential transmission spectra



- **Carrier density increases** during the excitation pulse
- Auger scattering leads to **carrier multiplication (CM)**

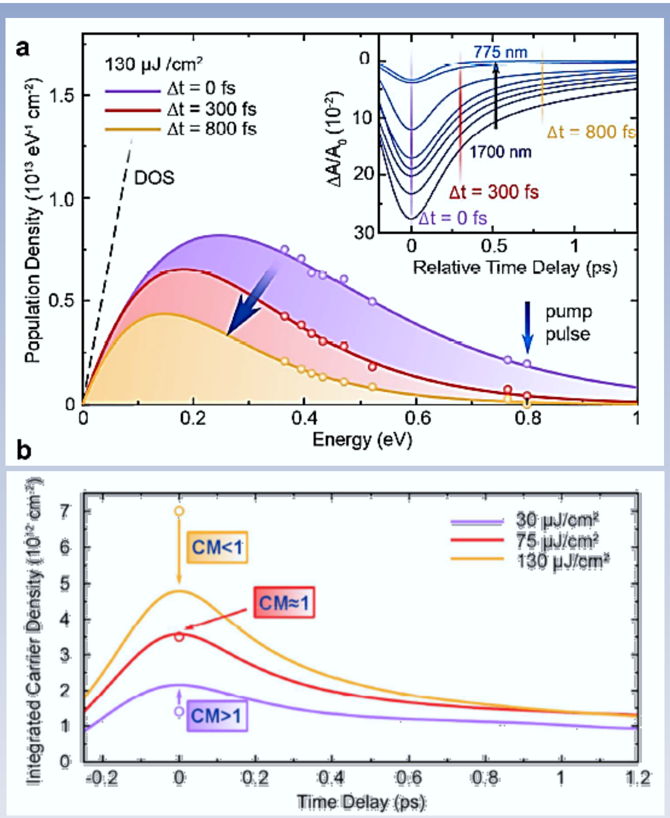
$$CM = \frac{\mathcal{N}(t) - \mathcal{N}_T}{\mathcal{N}_{opt}}$$

- **Carrier-phonon** scattering **reduces CM** on a ps time scale



High-resolution multi-color pump-probe experiment

3. Differential transmission spectra

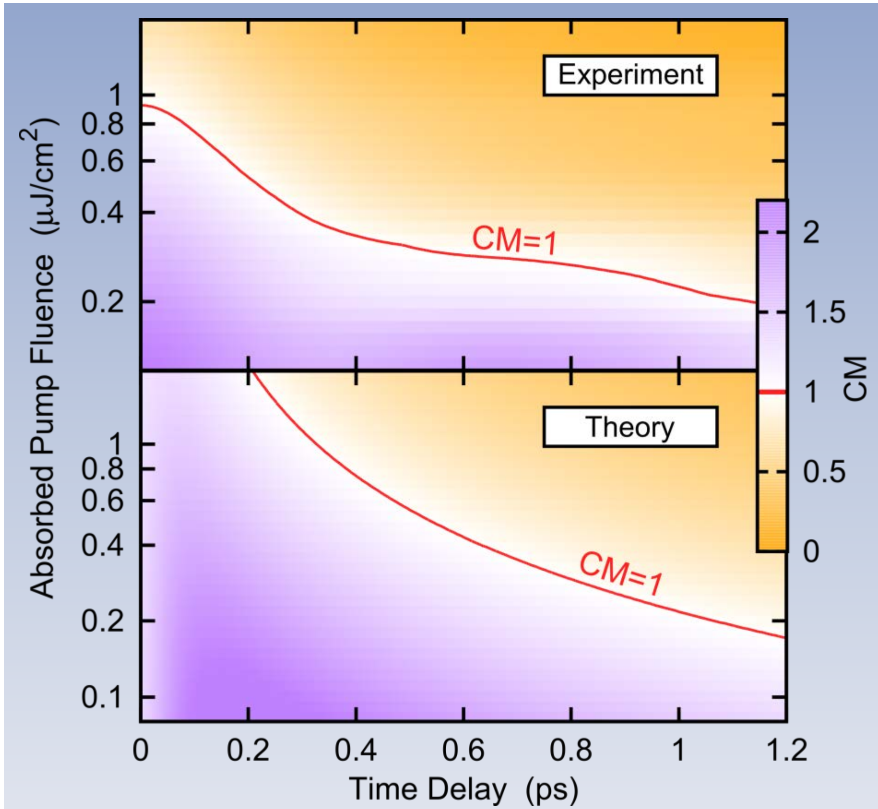


- Experiment performed by Daniel Neumaier andHeinrich Kurz, AMO Aachen
- Extract temporal evolution of the carrier density from **multi-color pump-probe** measurements and assume a **quick thermalization**
- Estimate the optically injected carrier density

→ **Carrier multiplication** in dependence of pump fluence

Experiment-theory comparison

3. Differential transmission spectra



- Theoretical prediction is in **excellent agreement** with experiment:

Appearance of long-lived **CM** in the **weak excitation regime**

Distinct fluence dependence found both in theory and and experiment

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4. Statistics of light

- Light consists of **electromagnetic waves** with the energy

$$H = \frac{1}{2}\int \left(\varepsilon_0 E(r)^2 + \frac{1}{\mu_0} B(r)^2 \right) dr = \sum_j \left(\frac{1}{2} m_j \omega_j^2 q_j^2 + \frac{p_j^2}{2m_j} \right)$$

→ corresponds to the energy of **uncoupled harmonic oscillators**

- **Quantization of light** through annihilation and creation operators

$$c_j = \frac{1}{\sqrt{2m_j \hbar \omega_j}} \left(m_j \omega_j \hat{q}_j + i\hat{p}_j \right), \quad c_j^+ = \frac{1}{\sqrt{2m_j \hbar \omega_j}} \left(m_j \omega_j \hat{q}_j - i\hat{p}_j \right)$$

- They fulfil the fundamental **commutation relations for bosons**

$$[\hat{q}_j, \hat{p}_{j'}] = i\hbar \delta_{j,j'} \qquad [\hat{q}_j, \hat{q}_{j'}] = [\hat{p}_j, \hat{p}_{j'}] = 0$$

$$\Rightarrow$$

$$[c_j, c_{j'}^+] = \delta_{j,j'} \qquad [c_j, c_{j'}] = [c_j^+, c_{j'}^+] = 0$$

Photons

4. Statistics of light

- **Hamilton operator** for photons reads

$$H = \sum_j \hbar\omega_j \left(c_j^+ c_j + \frac{1}{2} \right)$$

- **Eigen states** are the **Fock states**

$$|n\rangle = \frac{1}{\sqrt{n!}} (c^+)^n |0\rangle$$

- **Eigen energies** are **quantized**

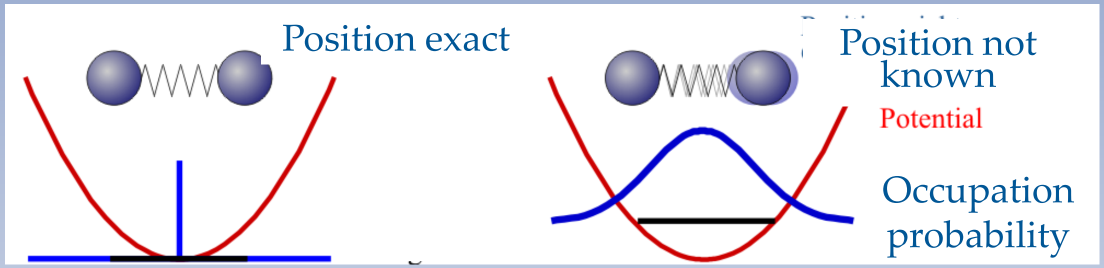
$$E_n = \sum_j \hbar\omega_j \left(n + \frac{1}{2} \right)$$

with the **photon number n** and the **zero-point energy** $\frac{1}{2}\hbar\omega_j$

Vacuum fluctuations

4. Statistics of light

Classical oscillator **quantum mechanical oscillator**



- Zero-point energy corresponds $\frac{1}{2}\hbar\omega_j$ to vacuum fluctuations and results from the Heisenberg uncertainty principle $\Delta q \Delta p \geq \frac{\hbar}{2}$

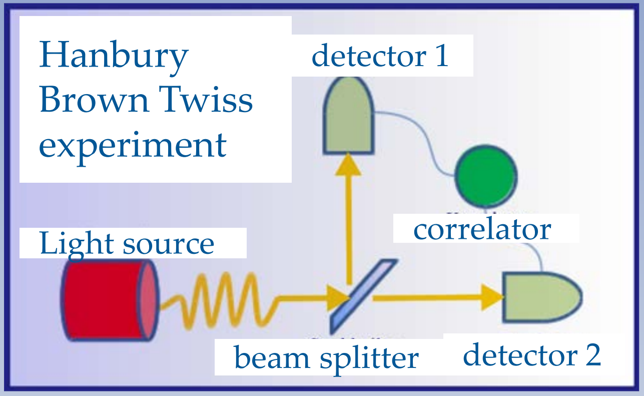
- Vacuum fluctuations give rise to non-classical phenomena

  → **spontaneous emission (light emission without excitation)**

  → **Lamb-Shift** (Energy shift in hydrogen atom)

Hanbury Brown – Twiss Experiment

4. Statistics of light



R. Hanbury Brown and R. Twiss, Nature 178, 1046 (1956)

- **1956**: **Hanbury Brown** und **Twiss** measure correlations between starlight intentisities with two separate detectors (star radius determination)

- The signal is proportional to $\langle I_1(t) I_2(t) \rangle$ where $I_1(t)$ is the intensity measured at detector 1

  → **characteristic photon statistics** for different light sources

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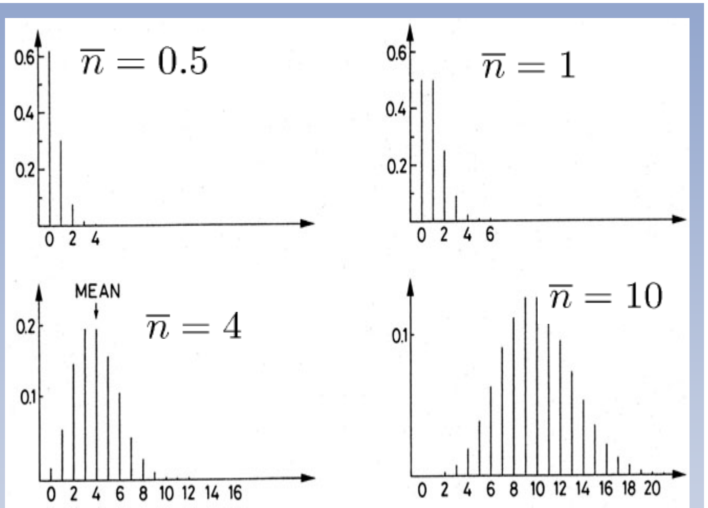
Poisson distribution

4. Statistics of light

- For **coherent light** with constant frequency, phase, and amplitude, the **signal** at both detectors is **uncorrelated**
- Photon statistics corresponds to the **Poisson distribution**

$$\mathcal{P}(n) = \frac{\overline{n}^n}{n!} e^{-\overline{n}} \quad (n = 0, 1, 2, 3, \ldots)$$

probability to measure n photons at an averaged photon number $\overline{n}$



M. Fox, Quantum Optics, Oxford University Press (2006)

- For large $\overline{n}$ goes into a **Gaussian distribution**
- **Standard deviation** is $\Delta n = \sqrt{\overline{n}}$

Super-Poisson distribution

4. Statistics of light

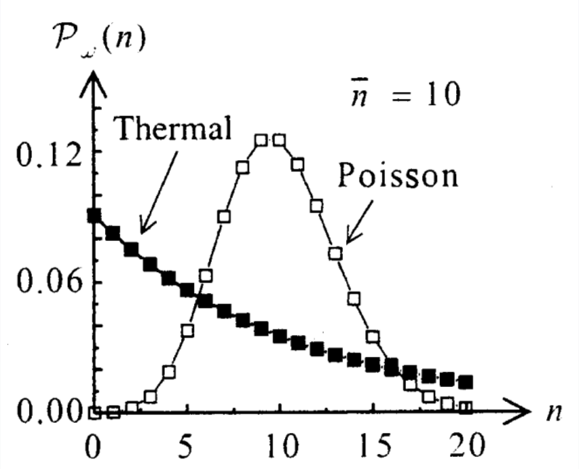
- **Thermal light** shows **intensity fluctuations**

  → **Super-Poisson** distribution

  $$\Delta n > \sqrt{\overline{n}}$$

- Thermal light corresponds to **Bose-Einstein** distribution

$$\mathcal{P}(n) = \frac{\overline{n}^n}{(\overline{n}+1)^{n+1}} \quad \text{with } \overline{n} = \left(e^{\hbar\omega/k_B T} - 1\right)^{-1}$$



M. Fox, Quantum Optics

- **Thermal light** is characterized by a **broader distribution** compared to coherent light (Poisson distribution)

- Bose-Einstein distribution has a **standard deviation** of $\Delta n = \sqrt{\overline{n}^2 + \overline{n}}$

Sub-Poisson distribution

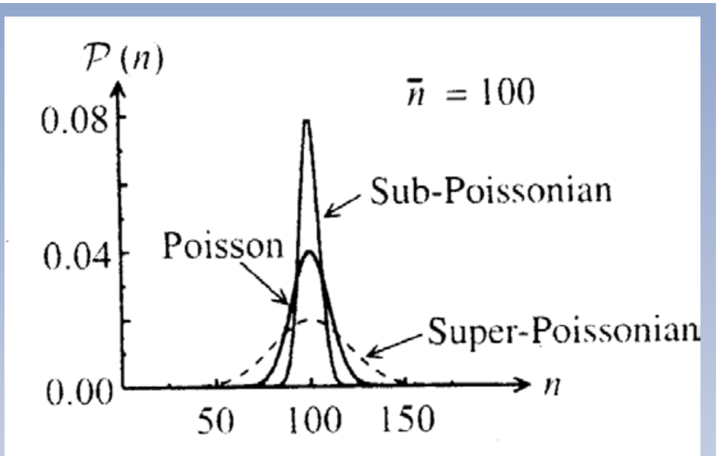
4. Statistics of light

- **Sub-Poisson distribution** with

$$\Delta n < \sqrt{\bar{n}}$$

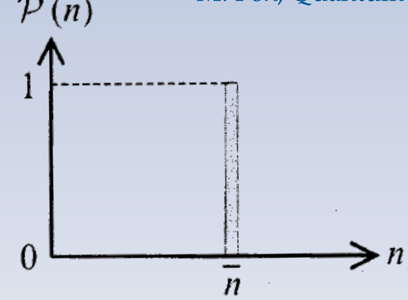
is **narrower** compared to Poisson (**less fluctuations** thancoherent light)

→ **non-classical light**



M. Fox, Quantum Optics

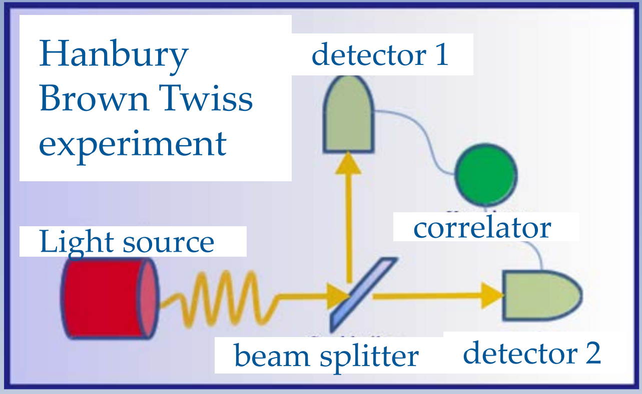
- **Fock states** with $\Delta n = 0$ correspond to purest non-classical light



- Detection of such light is a proof of **quantum nature of light,** but difficult to measure, since all losses destroy the Poisson statistics

Second-order correlation function $g^{(2)}$

4. Statistics of light



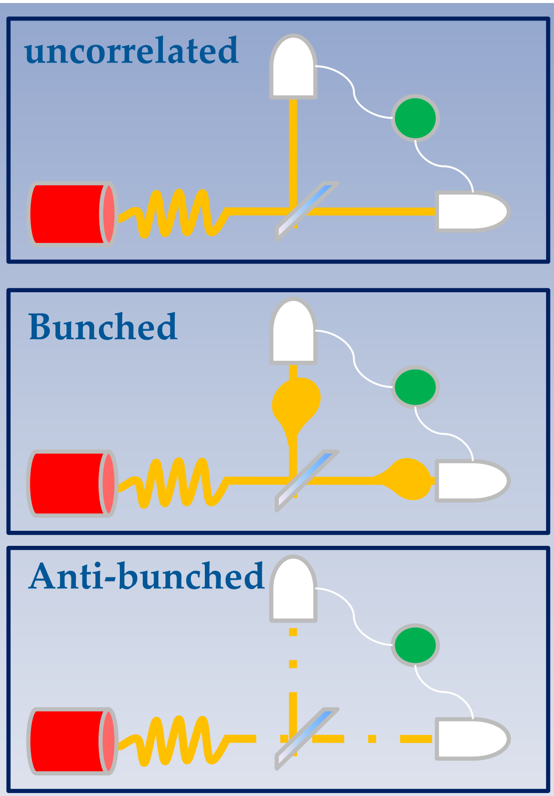
$$g^{(2)}(\tau) = \frac{\langle I_1(t) I_2(t+\tau)\rangle}{\langle I_1(t)\rangle \langle I_2(t+\tau)\rangle}$$

with intensities $I_1(t)$ and $I_2(t+\tau)$ measured at detectors 1 and 2 with a time delay τ

- $g^{(2)}$ is a measure for the probability to detect a photon at the detector 2, after a photon has already been measured at the detector 1

- The **correlation vanishes for $\tau \gg \tau_c$** with the **coherence time τ_c**, since on this time scale the intensity fluctuations become small

4. Statistics of light



- Measurements at detectors are uncorrelated

  ⇒ **coherent light**

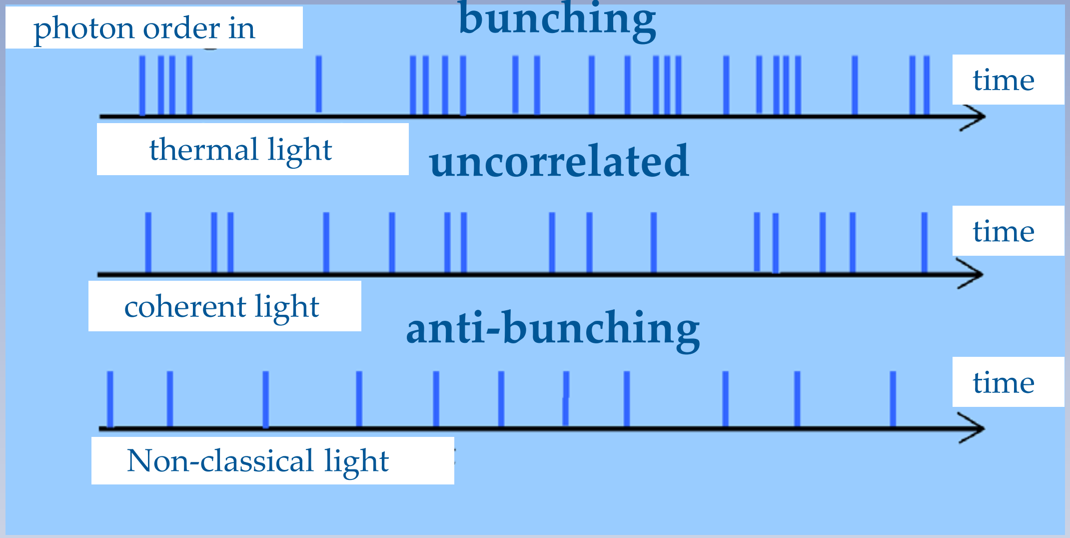
- **Positive correlation** between the measurements **(bunching)**

  ⇒ **thermal light**

- **Negative correlation** between the measurements (**anti-bunching**)

  ⇒ **non-classical light**

4. Statistics of light



forphys.de

- Classification of light via **$g^{(2)}$ – function** (quantitative determination of photon correlation in different light sources)
- Evaluaation at **τ = 0** sufficient, since here the **correlation the strongest**

Photon statistics via $g^{(2)}$

4. Statistics of light

$$g^{(2)}(0) = \frac{\langle c^+ c^+ c\ c\rangle}{\langle c^+ c\rangle\ \langle c^+ c\rangle} = \frac{\langle (\Delta n)^2\rangle - \bar{n}}{\bar{n}^2} + 1$$

$$\bar{n} = \langle c^+ c\rangle$$

$$\langle (\Delta n)^2\rangle = \langle c^+ c c^+ c\rangle - \langle c^+ c\rangle^2$$

- **Coherent light**
  Poisson distribution $\Delta n = \sqrt{\bar{n}}$
  Example: **laser**

  $g^{(2)} = 1$

  uncorrelated

- **Thermal light**
  Super-Poisson distribution $\Delta n > \sqrt{\bar{n}}$
  Exampel: **sun light** $\Delta n = \sqrt{\bar{n}^2 + \bar{n}}$ → $g^{(2)} = 2$

  $g^{(2)} > 1$

  bunching

- **Non-classical light**
  Sub-Poisson distribution $\Delta n < \sqrt{\bar{n}}$
  Example: **Fock state** $\Delta n = 0$ → $g^{(2)} = 1 - \frac{1}{\bar{n}}$

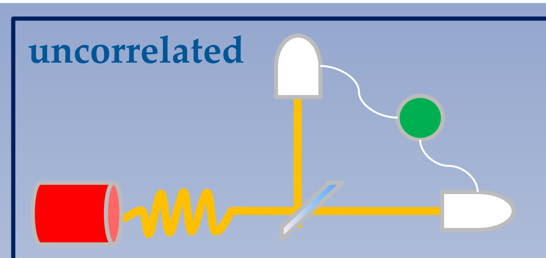
  $g^{(2)} < 1$

  anti-bunching

Photon statistics at one sight

4. Statistics of light

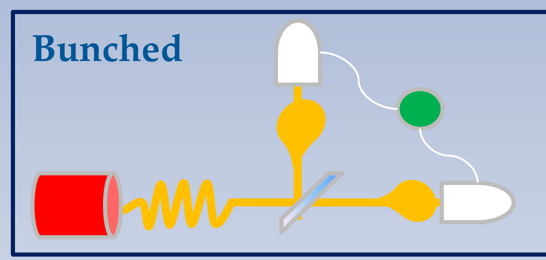
$g^{(2)} = 1$



Poisson

Coherent light
(e.g. laser light)

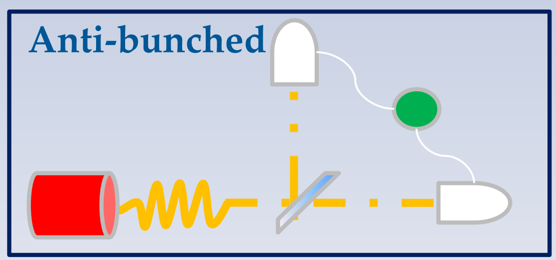
$g^{(2)} > 1$



Super-Poisson

thermal light
(e.g. sun light)

$g^{(2)} < 1$



Sub-Poisson

Non-classical light
(e.g. Fock states)

Learning outcomes lecture X

- Recognize the importance of **pump-probe experiments** for revealing carrier dynamics
- Explain the **photon statistics** for different light sources

Contents: FKA091 Part I

Contents: FKA091 Part I

- IV. Density matrix theory
  1. Statistic operator
  2. Bloch equations
  3. Boltzmann equation

- V. Density functional theory (guest lecture by Paul Erhart)

- VI. Optical properties of solids
  1. Electron-light interaction
  2. Absorption spectra
  3. Differential transmission spectra
  4. Statistics of light

Learning Outcomes

- Recognize the **main concepts** of condensed matter physics including introduction of **quasi-particles** (such as excitons, plasmons)
- Realize the importance of **Born-Oppenheimer, Hartree-Fock**, and **Markov approximations**
- Explain the **Bloch theorem** and calculate the **electronic band structure**
- Define **Hamilton operator** in the formalism of **second quantization**
- Realize the potential of **density matrix** and **density functional theory**
- Explain the semiconductor **Bloch** and **Boltzmann equations**
- Recognize the **optical finger print** of nanomaterials

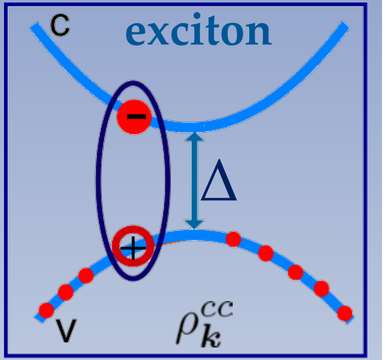
Recap chapter I

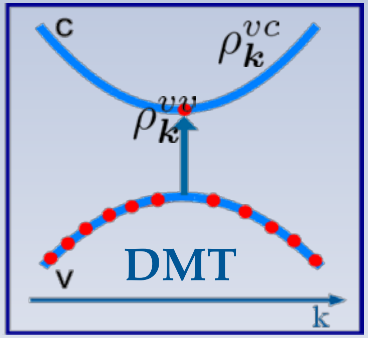
Born-Oppenheimer approximation

- Introduction of **non-interacting quasi-particles** (eg excitons) is an important concept of condensed matter physics to deal with many-particle systems

- Main theoretical approaches include **density matrix theory** (DMT) with **semiconductor Bloch** and **Boltzmann scattering equations** and **density functional theory** (DFT) with the central Hohenberg-Kohn theorem

- In **Born-Oppenheimer approximation, electron** and **ion dynamics** is **separated** based on the much larger mass and slower motion of ions



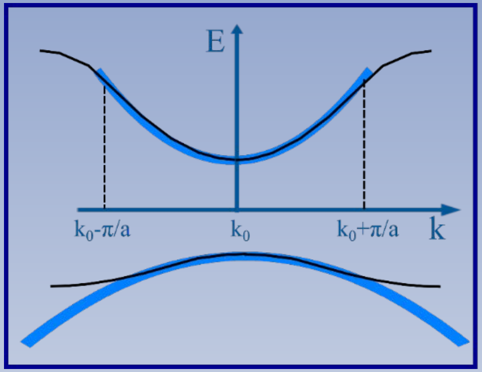


Recap chapter II

Bloch theorem

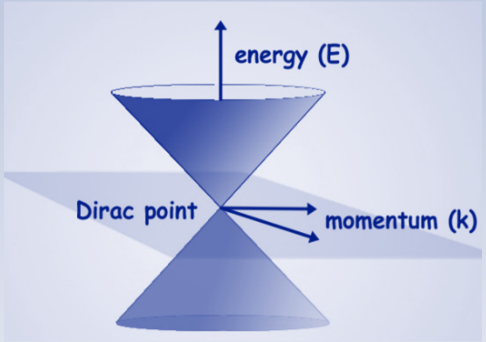
- **Bloch theorem: eigen functions** of an electron in a periodic potential have the shape of **plane waves modulated with a periodic Bloch factor**

$$\Psi_{\boldsymbol{k}}(\boldsymbol{r}) = \frac{1}{\sqrt{V}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} u_{\boldsymbol{k}}(\boldsymbol{r})$$



- **Eigen energies** are expressed in material-specific **electronic band structure that** can be calculated in **effective mass** or **tight-binding approximation**

- **Graphene** exhibits a remarkable **linear and gapless** electronic band structure $E_\lambda(\boldsymbol{k}) = \sigma_\lambda \hbar \nu_F |\boldsymbol{k}|$



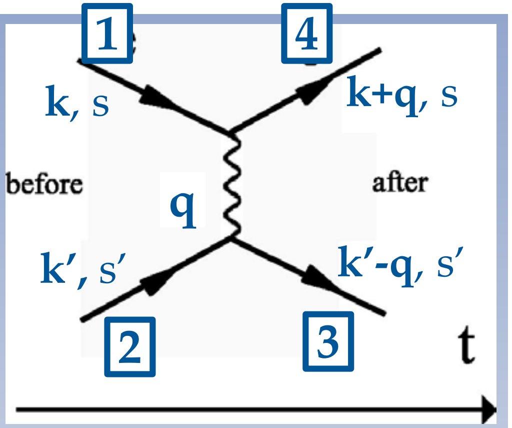
Second quantization

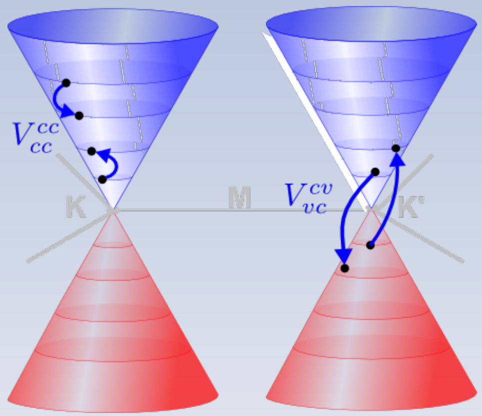
- **Electron-electron interaction** is driven by the **repulsive Coulomb potential**

$$V_q^{3D} = \frac{e_0^2}{\varepsilon_0 L^3}\frac{1}{q^2}\delta_{\mathbf{k}_1+\mathbf{k}_2,\mathbf{k}_3+\mathbf{k}_4}\delta_{s_1,s_4}\delta_{s_2,s_3}$$

- Second quantization **avoids (anti-)symmetrisation** of many-particle states and mirrors the physics in **fundamental commutation relations**

$$[a_\alpha, a_\beta^+]_\pm = a_\alpha a_\beta^+ \pm a_\beta^+ a_\alpha = \delta_{\alpha\beta}$$
$$[a_\alpha^+, a_\beta^+]_\pm = [a_\alpha, a_\beta]_\pm = 0$$





Recap chapter III

Jellium/Hubbard model

- **Jellium** model assumes **interacting electrons** in a **smeared potential** of ions (no lattice structure considered)

$$H_J = \sum_{\boldsymbol{k}s} \varepsilon_{\boldsymbol{k}s} a^+_{\boldsymbol{k}s} a_{\boldsymbol{k}s} + \frac{1}{2} \sum_{\boldsymbol{k}\boldsymbol{k'}\boldsymbol{q}}^{q\neq 0} \sum_{ss'} V_q \, a^+_{\boldsymbol{k}+\boldsymbol{q}s} a^+_{\boldsymbol{k'}-\boldsymbol{q}s'} a_{\boldsymbol{k'}s'} a_{\boldsymbol{k}s}$$

- **Hubbard** model assumes **strong lattice potential** and **tightly bound electrons**

$$H_H = \sum_{ijs} T_{ijs} a^+_{is} a_{js} + \frac{1}{2} U \sum_i \hat{n}_i \left(\hat{n}_i - 1 \right)$$

Recap chapter III

Hartree-Fock approximation

- **Hartree-Fock approximation** introduces an **effective one-particle problem** by considering a single particle in a **mean-field potential** generated by all other particles

$$H_{\text{eff}} = \sum_{l_1} \left[\varepsilon_{l_1} + \sum_{l_2} \left(V^{l_1 l_2}_{l_2 l_1} - V^{l_1 l_2}_{l_1 l_2} \right) \langle a^+_{l_2} a_{l_2} \rangle_{\text{eff}} \right] a^+_{l} a_{l}$$

Hartree term (direct term) **Fock term** (exchange term)

- Most important approximation is the **factorization of 2-particle into single-particle** expectation values

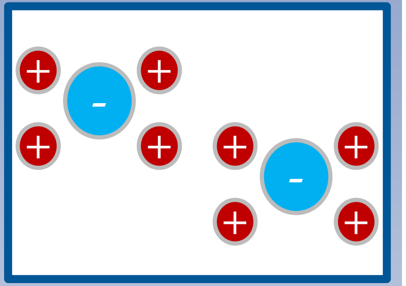
$$\langle a^+_A a^+_B a_C a_D \rangle = \langle a^+_A a_D \rangle \langle a^+_B a_C \rangle \delta_{A,D} \delta_{B,C} - \langle a^+_A a_C \rangle \langle a^+_B a_D \rangle \delta_{A,C} \delta_{B,D}$$

Recap chapter III

Screening

- **Lindhard equation** describes the screening of the Coulomb interaction due to the presence of many particles

$$W(\boldsymbol{q}) = \frac{V(\boldsymbol{q})}{\varepsilon(\boldsymbol{q})}, \quad \varepsilon(\boldsymbol{q}) = 1 - V(\boldsymbol{q}) \sum_{\boldsymbol{k}} \frac{\rho_{\boldsymbol{k}+\boldsymbol{q}} - \rho_{\boldsymbol{k}}}{\varepsilon_{\boldsymbol{k}+\boldsymbol{q}} - \varepsilon_{\boldsymbol{k}} - \hbar\omega}$$



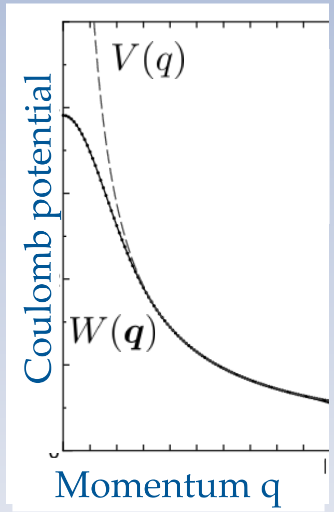
- In the **static and long-wavelength limit** we find

$$W(\boldsymbol{q}) = \frac{e_0^2}{\varepsilon_0 V} \frac{1}{q^2 + \kappa^2}$$

with the screening length

$$\kappa = \sqrt{\frac{e_0^2}{\varepsilon_0} \partial_\mu n}$$

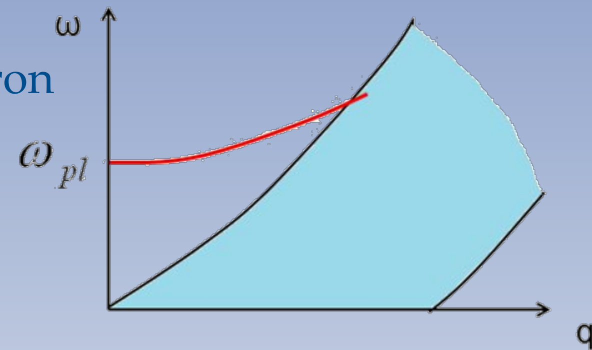
- Screening **removes** the **divergence** of Coulomb potential for $q \to 0$



Plasmons and excitons

- Besides a continuum of electron-hole excitations, there is a **collective oscillation** of the entire electron plasma with the characteristic **plasma frequency**

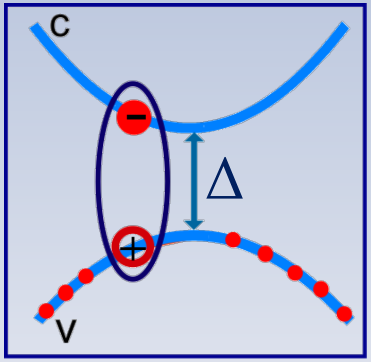
$$\omega_{pl} = \sqrt{\frac{ne_0^2}{\varepsilon_0 m_e}}$$



- **Excitonic binding energy** reads

$$E_b = \frac{\mu e_0^4}{2\hbar^2 \varepsilon_{bg}^2}$$

with the reduced mass $\mu = \frac{m_c m_v}{M}$
and the dielectric background constant



Recap chapter IV

Statistical operator (density matrix)

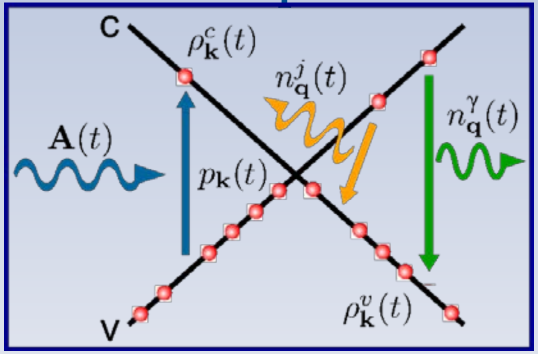
- **Statistical operator** (density matrix) characterizes quantum systems in a **mixed state**

$$\rho = \sum_n p_n |\Psi_n\rangle\langle\Psi_n|$$

and builds the **expectation value of observables**

$$\langle O \rangle = tr\,(\rho\, O)$$

- In the limiting case of one-particle systems, the **diagonal (non-diagonal) elements** of the statistical operator correspond to the **carrier occupation probability (microscopic polarization)**



- **Carrier occupation** $\rho^\lambda_{\boldsymbol{k}}(t) = \langle a^+_{\lambda\boldsymbol{k}} a_{\lambda\boldsymbol{k}}\rangle(t)$

- **Microscopic polarization** $p_{\boldsymbol{k}}(t) = \langle a^+_{c\boldsymbol{k}} a_{v\boldsymbol{k}}\rangle(t)$

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Semiconductor Bloch equations

- To tackle the many-particle-induced hierarchy problem, we perform the correlation expansion followed by a **systematic truncation** resulting in **semiconductor Bloch equations** on Hartree-Fock level

$$\dot{\rho}^{\lambda}_{\mathbf{k}}(t) = -2\mathrm{Im}\big(\Omega^*_{\mathrm{k}}(\mathrm{t})\,\mathrm{p}_{\mathrm{k}}(\mathrm{t})\big) + \frac{2}{\hbar}\sum_{\boldsymbol{k}'} V^{\boldsymbol{k}\; v,\boldsymbol{k}'c}_{\boldsymbol{k}'v,\boldsymbol{k}\; c}\, p_{\boldsymbol{k}'}(t)\, p^*_{\boldsymbol{k}}(t)$$

$$\dot{p}_{\mathbf{k}}(t) = \frac{i}{\hbar}\left(\varepsilon^v_{\boldsymbol{k}} - \varepsilon^c_{\boldsymbol{k}}\right) p_{\boldsymbol{k}}(t) - i\Omega_{\boldsymbol{k}}(t)\big(\rho^c_{\boldsymbol{k}}(t) - \rho^v_{\boldsymbol{k}}(t)\big) - \gamma_{\boldsymbol{k}}(t)\, p_{\boldsymbol{k}}(t)$$

$$+\frac{i}{\hbar}\sum_{\boldsymbol{k}'}\left[V^{\boldsymbol{k}\boldsymbol{k}'}_{\mathrm{ren}}\big(\rho^v_{\boldsymbol{k}'}(t) - \rho^c_{\boldsymbol{k}'}(t)\big)p_{\boldsymbol{k}}(t) - V^{\boldsymbol{k}\boldsymbol{k}'}_{\mathrm{exc}}\big(\rho^c_{\boldsymbol{k}}(t) - \rho^v_{\boldsymbol{k}}(t)\big)p_{\boldsymbol{k}'}(t)\right]$$

$$H = H_0 + H_{e-l} + H_{e-e}$$

Recap chapter IV

Boltzmann scattering equation

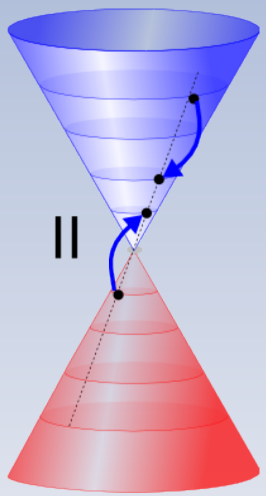
- **Boltzmann scattering equation** reads in **second-order Born-Markov approximation**

$$\dot{\rho}^{\lambda}_{\mathbf{k}}(t) = \Gamma^{in}_{\boldsymbol{k},\lambda}(t)\left(1-\rho^{\lambda}_{\boldsymbol{k}}(t)\right) - \Gamma^{out}_{\boldsymbol{k},\lambda}(t)\rho^{\lambda}_{\boldsymbol{k}}(t)$$

  and describes **time- and momentum-resolved electron scattering** dynamics in non-equilibrium

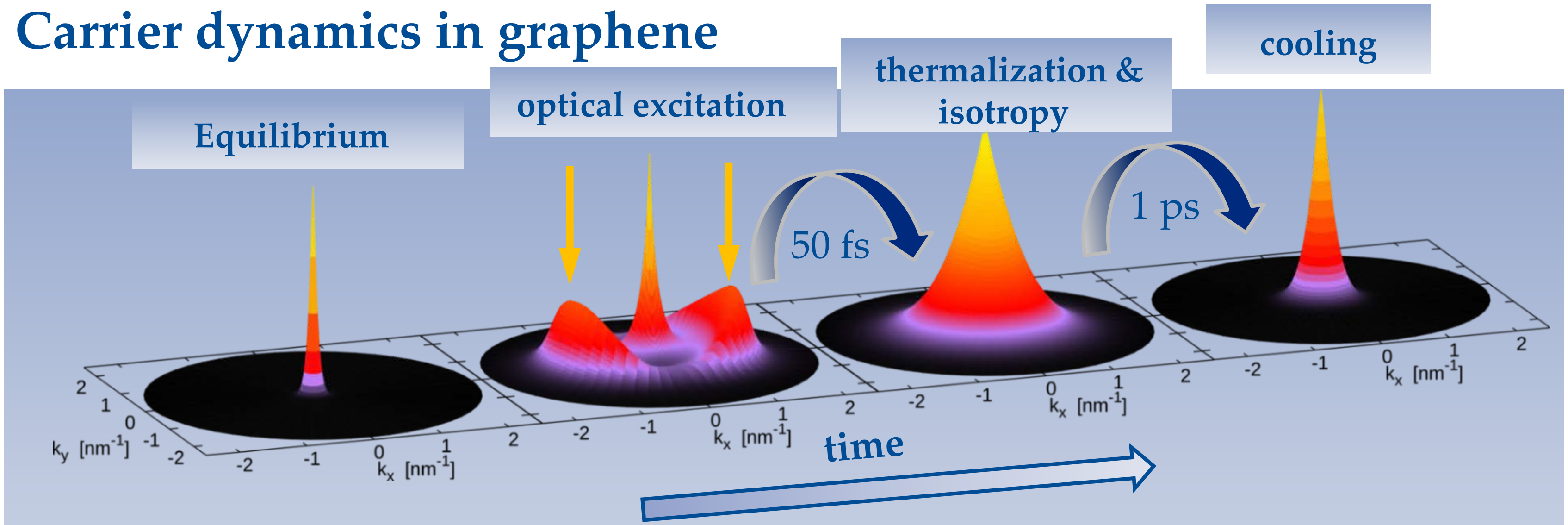
- **Markov approximation** neglects **quantum-mechanical memory effects** stemming from energy-time uncertainty

- In graphene, **carrier multiplication** can take place due to **efficient Auger scattering** channels



Recap chapter IV

Carrier dynamics in graphene



- Optically generated **anisotropic non-equilibrium** carrier distribution
- **Carrier-phonon** scattering accounts for **isotropy,** while **carrier-carrier** scattering leads to a spectrally broad **thermalized** distribution in the first **50 fs**
- **Carrier-phonon** scattering gives rise to carrier **cooling** on **ps time scale**

Recap chapter V

Density functional theory

- **Density functional theory** aims at the calculation of the quantum mechanic **ground state** $\Psi_0(\boldsymbol{x_1}, \boldsymbol{x_2}, ..., \boldsymbol{x_N})$ of a many-particle system

- Ground state can be unambiguously determined from the **electron density** $n_0(\boldsymbol{r}) = \sum_i |\Psi_i(\boldsymbol{r})|^2$ (**Hohenberg-Kohn Theorem**)

  → full Schrödinger equation with N^3 degrees of freedom does not need to be solved

- Electron density is solved through **Kohn-Sham equations** assuming an effective one-particle Hamilton operator

Recap chapter VI

Electron-light interaction

- **The electron-light Hamilton operator** reads in second quantization

$$H^{\boldsymbol{p}\cdot\boldsymbol{A}}_{\text{e-l}} = \frac{i\hbar e_0}{m_e}\sum_{\boldsymbol{l},\boldsymbol{l}'} \boldsymbol{M}_{\boldsymbol{l},\boldsymbol{l}'} \cdot \boldsymbol{A}(\boldsymbol{r}_0,t)\, a^+_{\boldsymbol{l}} a_{\boldsymbol{l}'} \qquad H^{\boldsymbol{r}\cdot\boldsymbol{E}}_{\text{e-l}} = \sum_{\boldsymbol{l},\boldsymbol{l}'} d_{\boldsymbol{l},\boldsymbol{l}'} \cdot \boldsymbol{E}(\boldsymbol{r}_0,t)\, a^+_{\boldsymbol{l}} a_{\boldsymbol{l}'}$$

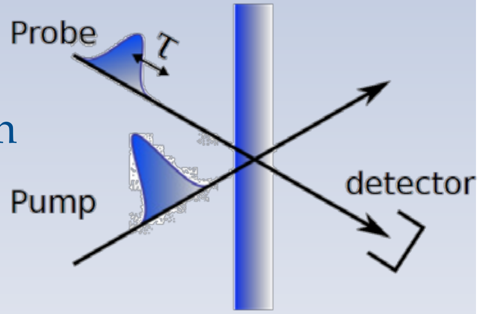
- **Absorption coefficient** $\alpha(\omega)$ is given by the **optical susceptibility** $\chi(\omega)$

$$\alpha(\omega) = \frac{\omega}{n(\omega)\, c_0}\text{Im}\chi(\omega)$$

that is determined by microscopic polarization

- In **pump-probe experiments, differential transmission** is measured, where pump pulse creates non-equilibrium and weaker probe pulse measures the carrier dynamics

$$\Delta T/T_0(\tau,\omega) \propto \left(\rho^{(p)}_{k_0}(\tau) - \rho_{k_0}(-\infty)\right)$$



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Recap chapter VI

Semiconductor Bloch equation (linear optics)

- **Graphene Bloch equation in the limit of linear optics**

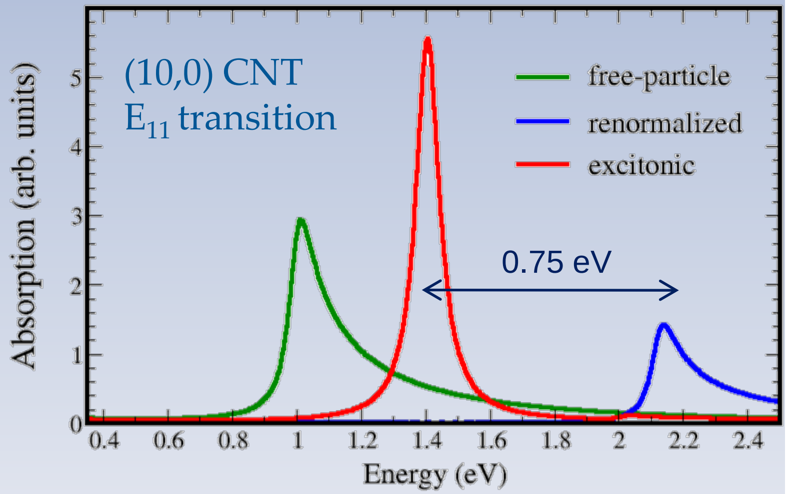
$$\dot{p}_{\mathbf{k}}(t) = \frac{i}{\hbar}\Delta\tilde{\varepsilon}^{vc}_{\boldsymbol{k}}\, p_{\boldsymbol{k}}(t) + i\tilde{\Omega}_{\boldsymbol{k}}(t) - \gamma\, p_{\boldsymbol{k}}(t)$$

- Electron-electron interaction leads to a **renormalization** of the **energy**

$$\Delta\tilde{\varepsilon}^{vc}_{\boldsymbol{k}} = (\varepsilon^{v}_{\boldsymbol{k}} - \varepsilon^{c}_{\boldsymbol{k}}) + \frac{i}{\hbar}\sum_{\boldsymbol{k}'} V^{\boldsymbol{kk}'}_{\text{ren}}$$

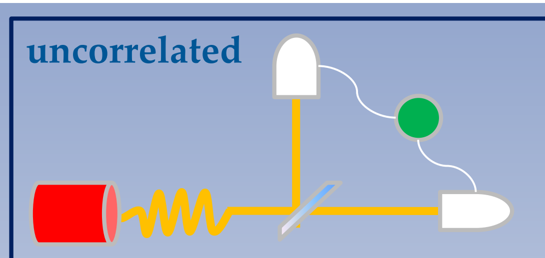
and to a **renormalization** of the **Rabi frequency (excitons)**

$$\tilde{\Omega}_{\boldsymbol{k}}(t) = \Omega_{\boldsymbol{k}}(t) + \frac{i}{\hbar}\sum_{\boldsymbol{k}'} V^{\boldsymbol{kk}'}_{\text{exc}}\, p_{\boldsymbol{k}'}(t)$$



Photon statistics

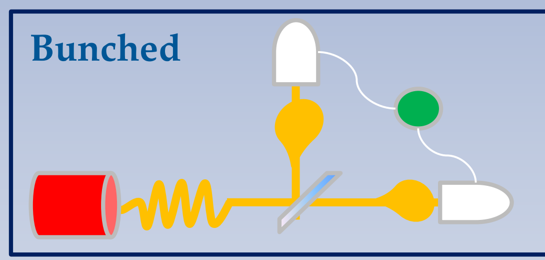
$g^{(2)} = 1$



Poisson

Coherent light
(e.g. laser light)

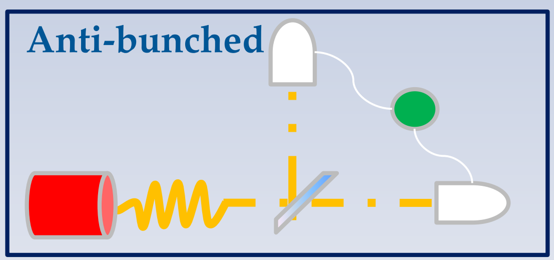
$g^{(2)} > 1$



Super-Poisson

thermal light
(e.g. sun light)

$g^{(2)} < 1$



Sub-Poisson

non-classical light
(e.g. Fock states)

Learning Outcomes

- Recognize the **main concepts** of condensed matter physics including introduction of **quasi-particles** (such as excitons, plasmons)
- Realize the importance of **Born-Oppenheimer, Hartree-Fock**, and **Markov approximations**
- Explain the **Bloch theorem** and calculate the **electronic band structure**
- Define **Hamilton operator** in the formalism of **second quantization**
- Realize the potential of **density matrix** and **density functional theory**
- Explain the semiconductor **Bloch** and **Boltzmann equations**
- Recognize the **optical finger print** of nanomaterials