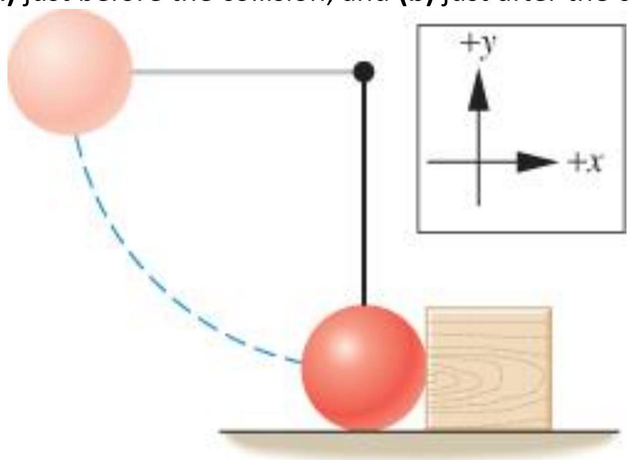
21. A ball is attached to one end of a wire, the other end being fastened to the ceiling. The wire is held horizontal, and the ball is released from rest (see the drawing). It swings downward and strikes a block initially at rest on a horizontal frictionless surface. Air resistance is negligible, and the collision is elastic. The masses of the ball and block are, respectively, 1.60 kg and 2.40 kg, and the length of the wire is 1.20 m. Find the velocity (magnitude and direction) of the ball **(a)** just before the collision, and **(b)** just after the collision.



To find velocity of the ball just before the collision is a conservation of energy problem

$$E_{before} = mgh = E_{after} = \frac{1}{2}mv_0^2$$

From diagram we can see h = L

$$\frac{1}{2}mv_0^2 = mgL$$

$$v_0^2 = 2gL$$

$$v_0 = \sqrt{2gL} = \sqrt{2\left(9.80\ {}^{m}/_{s^2}\right)(1.20\ m)} = 4.85\ {}^{m}/_{s}$$

From diagram the velocity is in the $\widehat{+x}$ direction

Now after the collision we find the velocity by conserving both momentum and kinetic energy

$$p_{before} = m_{ball}v_0 = p_{after} = m_{ball}v_{f\ ball} + m_{block}v_{f\ block}$$

$$m_{ball}v_{f\ ball} + m_{block}v_{f\ block} = m_{ball}v_0$$

$$m_{block}v_{f\ block} = m_{ball}v_0 - m_{ball}v_{f\ ball}$$

$$v_{f\ block} = \left(\frac{m_{ball}}{m_{block}}\right)(v_0 - v_{f\ ball})$$

Now Conserve kinetic energy

$$K_{before} = \frac{1}{2} m_{ball} v_0^2 = K_{after} = \frac{1}{2} m_{ball} v_{f\ ball}^2 + \frac{1}{2} m_{block} v_{f\ block}^2$$

$$\frac{1}{2} m_{block} v_{f\ block}^2 = \frac{1}{2} m_{ball} v_0^2 - \frac{1}{2} m_{ball} v_{f\ ball}^2 = \frac{1}{2} m_{ball} \left(v_0^2 - v_{f\ ball}^2\right)$$

$$\frac{1}{2} m_{block} v_{f\ block}^2 = \frac{1}{2} m_{ball} \left(v_0^2 - v_{f\ ball}^2\right)$$

$$v_{f\ block}^2 = \frac{m_{ball}}{m_{block}} \left(v_0^2 - v_{f\ ball}^2\right)$$

Now use the equation from conservation of momentum

$$v_{f\ block} = \left(\frac{m_{ball}}{m_{block}}\right) \left(v_0 - v_{f\ ball}\right)$$

$$\left(v_{f\ block}\right)^2 = \left(\frac{m_{ball}}{m_{block}}\right)^2 \left(v_0 - v_{f\ ball}\right)^2$$

Now equate the two relationships

$$\left(v_{f\ block}\right)^2 = \left(\frac{m_{ball}}{m_{block}}\right)^2 \left(v_0 - v_{f\ ball}\right)^2 = \frac{m_{ball}}{m_{block}} \left(v_0^2 - v_{f\ ball}^2\right)$$

$$\frac{m_{ball}}{m_{block}} \left(v_0 - v_{f\ ball}\right)^2 = \left(v_0^2 - v_{f\ ball}^2\right) = \left(v_0 + v_{f\ ball}\right)\left(v_0 - v_{f\ ball}\right)$$

$$\frac{m_{ball}}{m_{block}} \left(v_0 - v_{f\ ball}\right) = \left(v_0 + v_{f\ ball}\right)$$

$$\frac{m_{ball}}{m_{block}} v_0 - \frac{m_{ball}}{m_{block}} v_{f\ ball} = v_0 + v_{f\ ball}$$

$$\frac{m_{ball}}{m_{block}} v_{f\ ball} + v_{f\ ball} = \frac{m_{ball}}{m_{block}} v_0 - v_0$$

$$v_{f\ ball} \left(\frac{m_{ball}}{m_{block}} + 1\right) = v_0 \left(\frac{m_{ball}}{m_{block}} - 1\right)$$

$$v_{f\ ball} \left(\frac{m_{ball} + m_{block}}{m_{block}}\right) = v_0 \left(\frac{m_{ball} - m_{block}}{m_{block}}\right)$$

$$v_{f\,ball} = v_0\left(\frac{m_{ball} - m_{block}}{m_{ball} + m_{block}}\right)$$

$$v_{f\,ball} = (4.85\ {}^{m}/_{s})\left(\frac{1.60\ kg - 2.40\ kg}{1.60\ kg + 2.40\ kg}\right) = (4.85\ {}^{m}/_{s})(-0.200) = -0.970\ {}^{m}/_{s}$$

Before collision with block	$v_{0\,ball} = 4.85\ {}^{m}/_{s}\ \widehat{+x}$
After collision with block	$v_{f\,ball} = -0.970\ {}^{m}/_{s}\ \widehat{+x}$

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