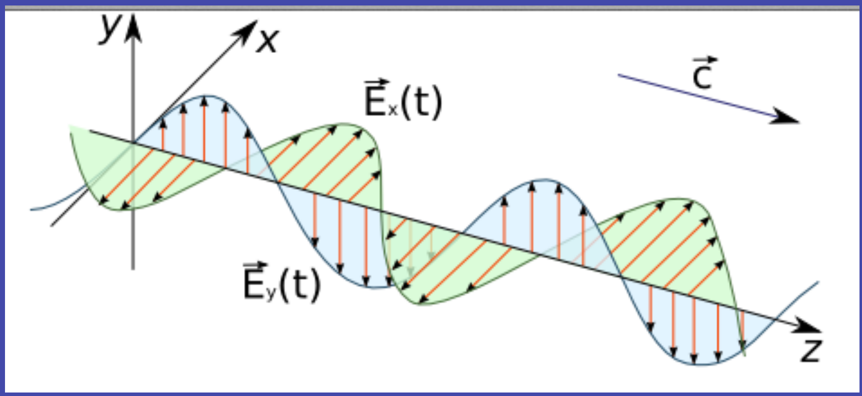
Circularly and elliptically polarized waves

- When there is a *phase difference* between orthogonal polarization vectors in a light wave their vector sum **E-field** will be ellipically or circularly polarized
  - The orthogonal vectors must be 90° out of phase and have equal amplitudes for the vector sum to be *circularly* polarized!
  - Other phase differences or relative amplitudes produce *ellliptic* polarization, of which *linear* polarization is a limiting case



E is vector sum of $\mathbf{E_x}$ and $\mathbf{E_y}$. Tip of **E**-vector will trace out helix in z at given time or circle in x-y plane at a given z

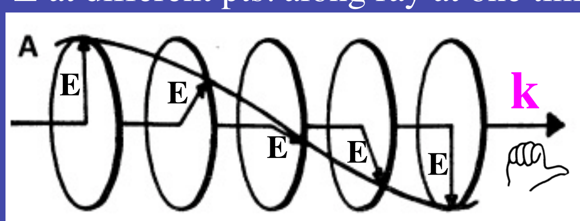
More explanation of circular polarization

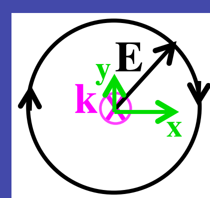
k parallel to thumb

Left/right handedness determined by **pointing left or right thumb in wave propagation direction**, and matching curl of one's fingers to direction of rotation of field at one z. Convention of non-optics physics community (Plasma physics, high energy, etc)

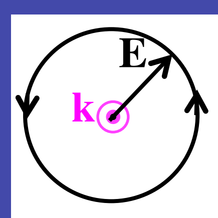
For k into paper, clockwise E(t) at one z **= RT** CIRC Polarized

E at different pts. along ray at one time





Looking from other side, k points out of paper, thumb still ‖ to k, but now counter-clockwise
STILL RT CIRC Polarized.

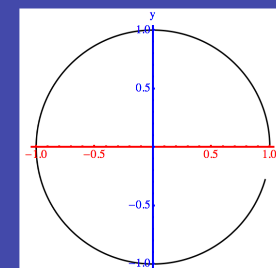


Eqns. describing t-dependence for **thumb ‖ to k**

$$E_x(z,t) = A\cos(kz - \omega t + \varphi),$$

$$E_y(z,t) = -A\sin(kz - \omega t + \varphi),$$

$E_x^2 + E_y^2 = A^2$ for all z, t.



At z such that $kz + \varphi = 0$,

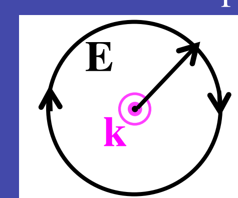
$$\boxed{E_x(\omega t) = A\cos\omega t, \quad E_y(\omega t) = A\sin\omega t}$$

k antiparallel to thumb

Alternative convention, — left/right handednes determined by pointing left/right thumb **toward −k**, and matching curl of fingers to rotation of **E**
Many optics textbooks use this second convention

E at *different times* at one pt

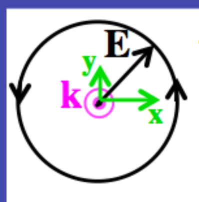
For k out of paper, clockwise = RT CIRC Polarized



Conventions for circularly polarized light

Two different conventions

- Summary of conventions



thumb ∥k	RightCP	$E_x(z,t)=A\cos(kz-\omega t),\ E_y(z,t)=-A\sin(kz-\omega t)$
thumb ∥k	LeftCP	$E_x(z,t)=A\cos(kz-\omega t),\ E_y(z,t)=+A\sin(kz-\omega t)$
thumb anti-∥k	RightCP	$E_x(z,t)=A\cos(kz-\omega t),\ E_y(z,t)=+A\sin(kz-\omega t)$
thumb anti-∥k	LeftCP	$E_x(z,t)=A\cos(kz-\omega t),\ E_y(z,t)=-A\sin(kz-\omega t)$

$$\begin{aligned}\mathbf{E}(z,t) &= \mathrm{Re}\left[E_x e^{i(kz-\omega t)}\right]\hat{\mathbf{x}}+\mathrm{Re}\left[e^{i\pi/2}E_x e^{i(kz-\omega t)}\right]\hat{\mathbf{y}}\\ &= E_x\cos(kz-\omega t)\hat{\mathbf{x}}+E_x\cos(kz-\omega t+\pi/2)\hat{\mathbf{y}}\\ &= E_x\left[\cos(kz-\omega t)\hat{\mathbf{x}}-\sin(kz-\omega t)\hat{\mathbf{y}}\right]\end{aligned} \quad \text{(left circular)} \quad (6.3)$$

- Both conventions in physics (Jackson, *EM Theory*) ⇒
  - **Parallel** convention is consistent with positive helicity

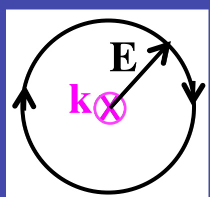
$$\left.\begin{aligned}E_x(\mathbf{x},t) &= E_0\cos(kz-\omega t)\\ E_y(\mathbf{x},t) &= \mp E_0\sin(kz-\omega t)\end{aligned}\right\} \quad (7.21)$$

At a *fixed point in space*, the fields (7.21) are such that the electric vector is constant in magnitude, but sweeps around in a circle at a frequency ω, as shown in Fig. 7.3. For the upper sign $(\boldsymbol{\epsilon}_1+i\boldsymbol{\epsilon}_2)$, the rotation is counterclockwise when the observer is facing into the oncoming wave. This wave is called *left circularly polarized* in optics. In the terminology of modern physics, however, such a wave is said to have *positive helicity*. The latter description seems more appropriate because such a wave has a positive projection of angular momentum on the z axis (see Problem 6.12). For the lower sign $(\boldsymbol{\epsilon}_1-i\boldsymbol{\epsilon}_2)$, the rotation of $\mathbb{E}$ is clockwise when looking into

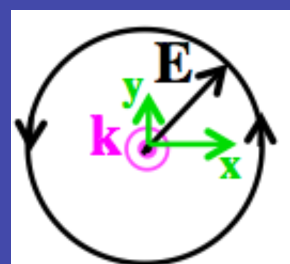
Screw sense of spatial helix to parallel **k** when use ***antiparallel*** convention for finding rotation in time

k parallel to thumb ⇒ t-dependence

E at *different times* at one pt



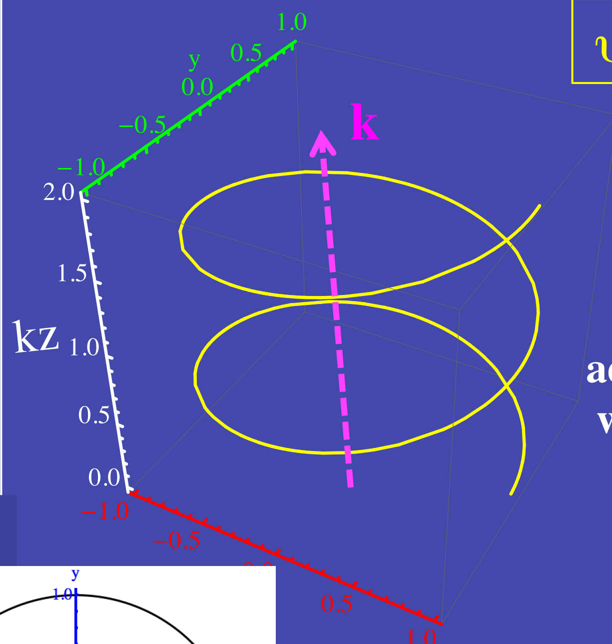
For **k into paper,**
clockwise = RIGHT
CIRC Polarized



k antiparallel to thumb ⇒ t-dependence

ParametricPlot3D[{Cos[ψ], Sin[ψ], ψ/2π}, {θ,0,4π}]

$\psi = kz - \omega t$



k is in direction of helix advancement with correct RH rule

Thumb anti-parallel to k:

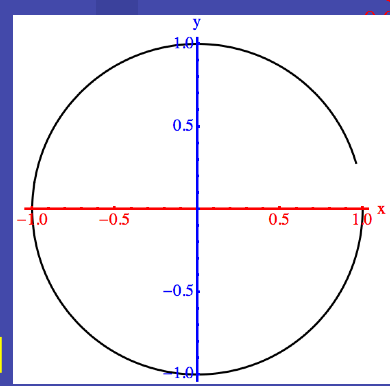
Eqns. for t-dependence

$E_x(z,t) = A\cos(kz - \omega t + \varphi),$

$E_y(z,t) = A\sin(kz - \omega t + \varphi), \quad \text{At, } kz + \varphi = 0,$

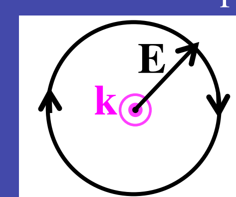
$$\boxed{E_x(\omega t) = A\cos\omega t,\ \ E_y(\omega t) = -A\sin\omega t}$$

ParametricPlot[{Cos[θ], -Sin[θ]]}, {θ, 0, 6}]



E at *different times* at one pt

For **k out of paper,**
clockwise = RIGHT
CIRC Polarized



Elliptically polarized light

- Two ways to make **elliptically** polarized light
  - Let E_x and E_y have different amplitudes

thumb ‖ k	**RightEP**	$E_x(z,t) = A_x \cos(kz - \omega t),\ E_y(z,t) = -A_y \sin(kz - \omega t)$
thumb ‖ k	**LeftEP**	$E_x(z,t) = A_x \cos(kz - \omega t),\ E_y(z,t) = +A_y \sin(kz - \omega t)$
thumb anti-‖ k	**RightEP**	$E_x(z,t) = A_x \cos(kz - \omega t),\ E_y(z,t) = +A_y \sin(kz - \omega t)$
thumb anti-‖ k	**LeftEP**	$E_x(z,t) = A_x \cos(kz - \omega t),\ E_y(z,t) = -A_y \sin(kz - \omega t)$

$$\left(\frac{E_x}{A_x}\right)^2 + \left(\frac{E_y}{A_y}\right)^2 = 1 \quad \text{Eqn of ellipse}$$

  - Let phase difference between equal-amplitude E_x and E_y be different from 90° (next slide)

Phase **difference** between two orthogonal equal-amplitude vector components of **E**-field determine kind of polarization

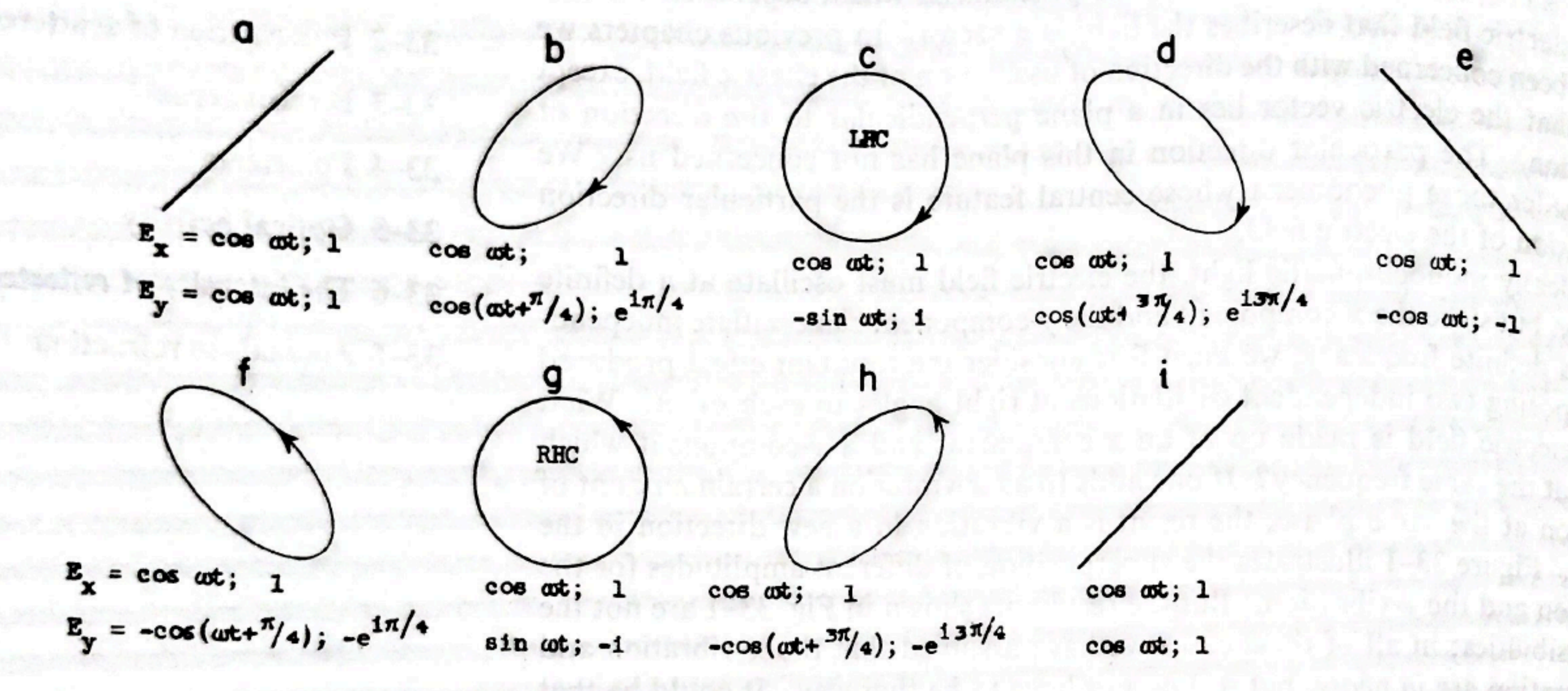
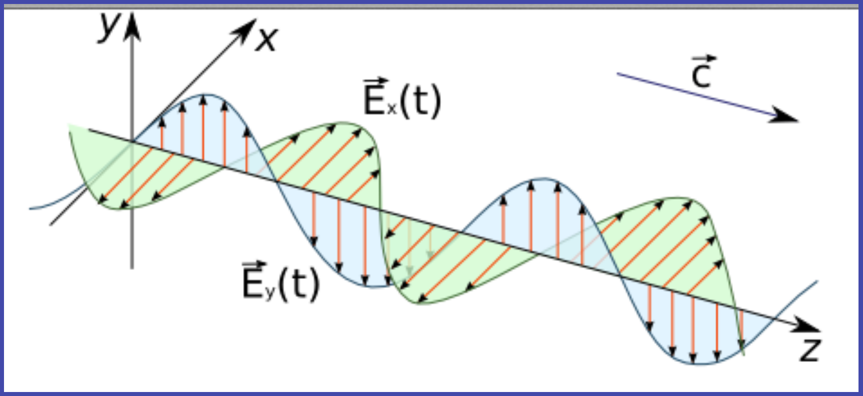
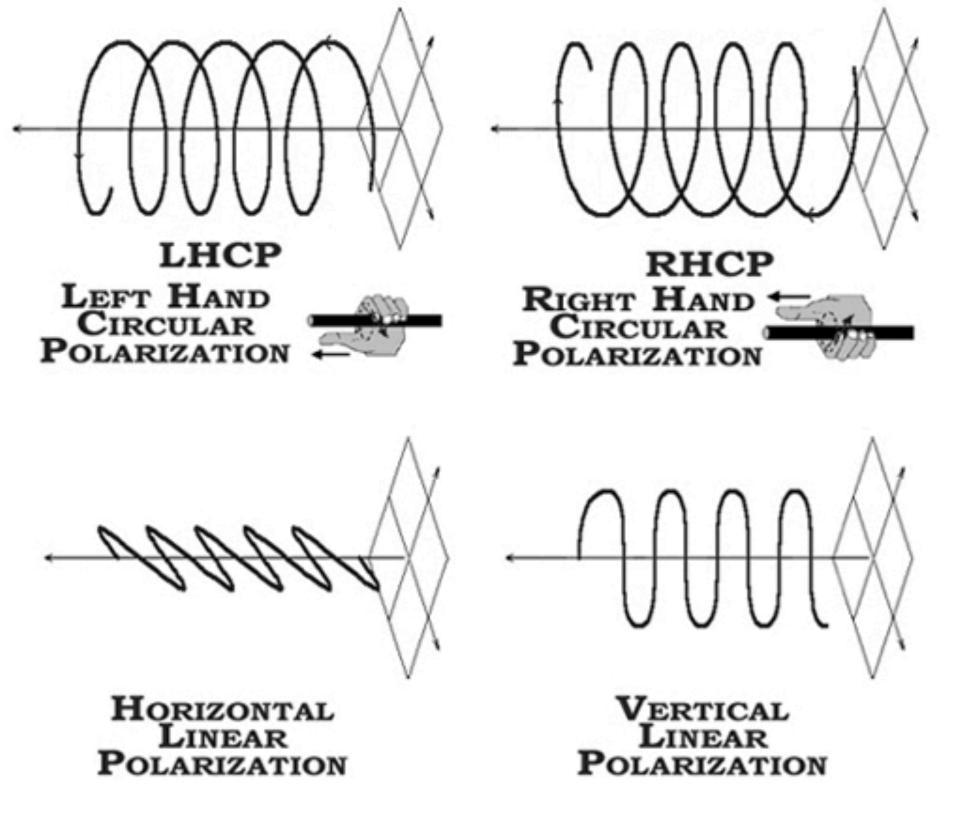


Fig. 33–2. Superposition of *x*-vibrations and *y*-vibrations with equal amplitudes but various relative phases. The components E_x and E_y are expressed in both real and complex notations.

Circularly polarized light

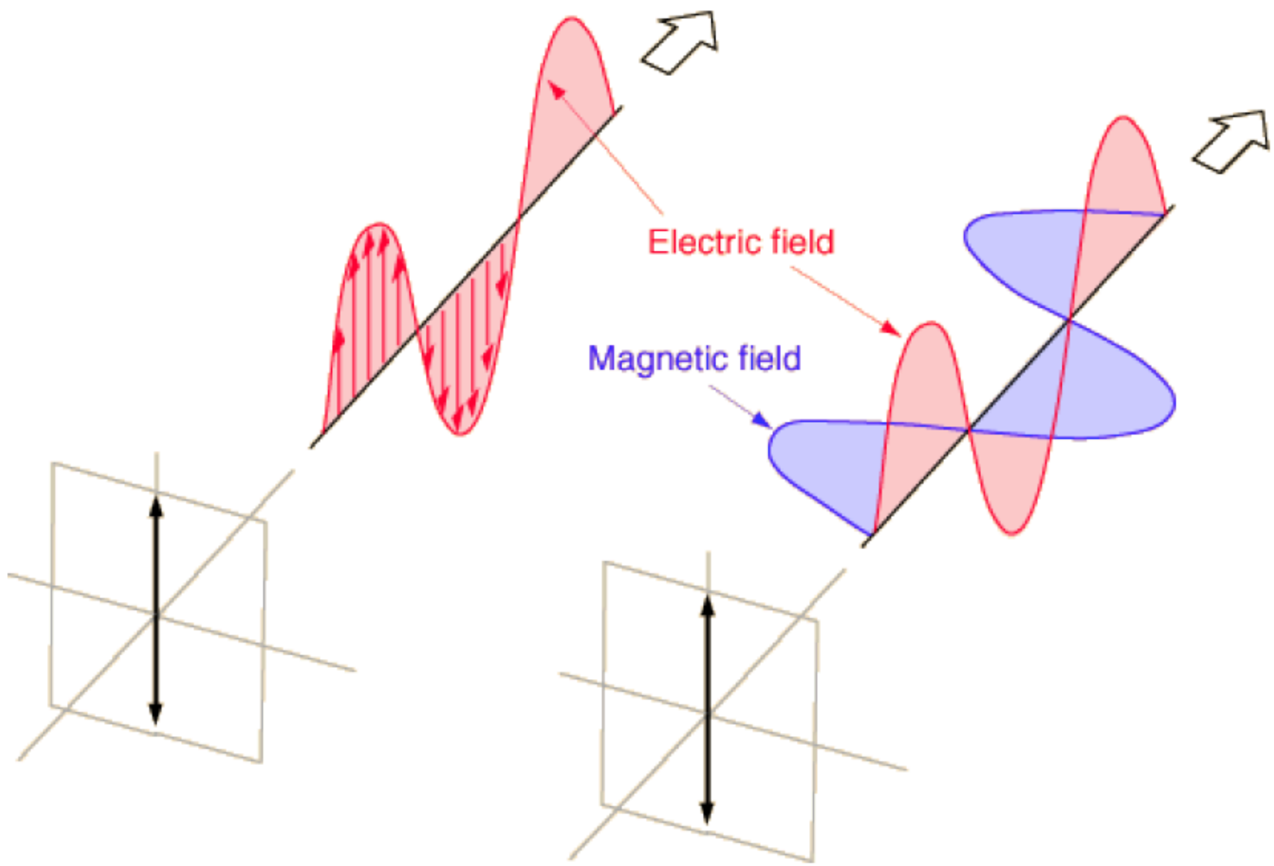


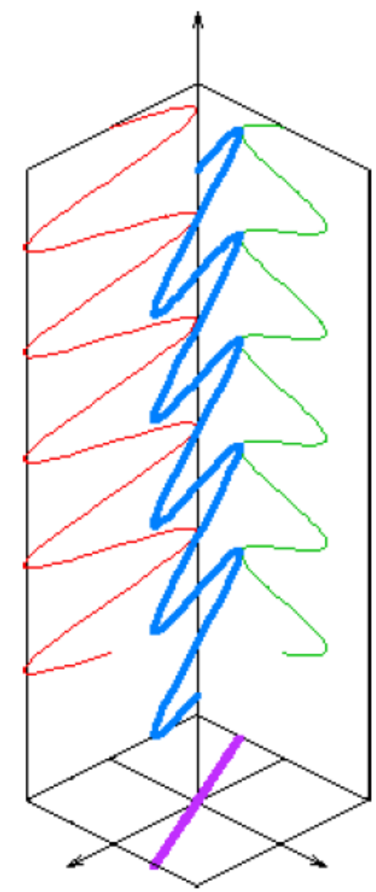


Summary of linear, circular and elliptic polarization

- Can understand all of these polarizations as well as unpolarized light by thinking of the electric field vector as the *sum of two orthogonal component vectors*.
  - Type of polarization depends on relative amplitude and phase of components

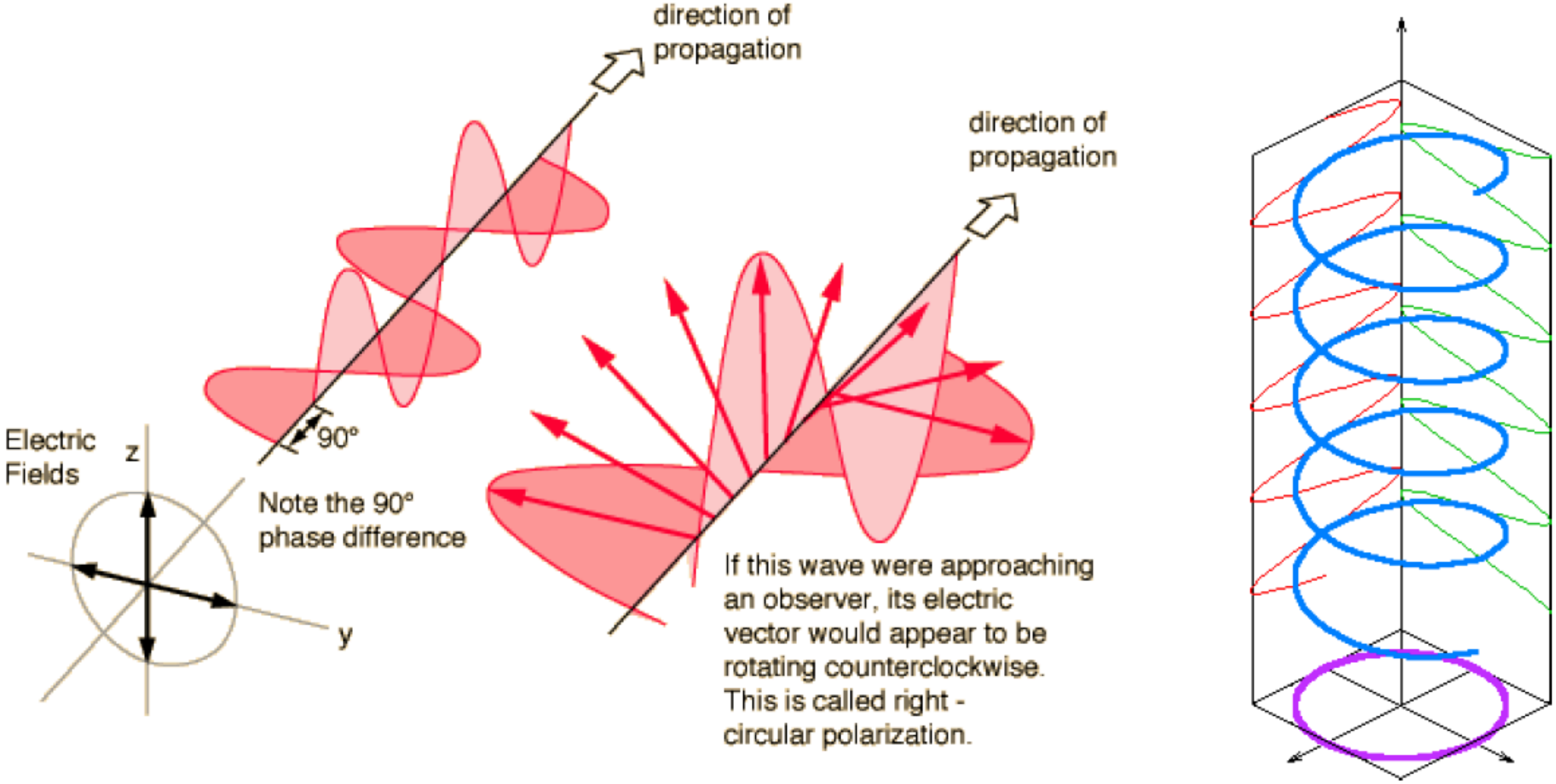
Linear Polarization





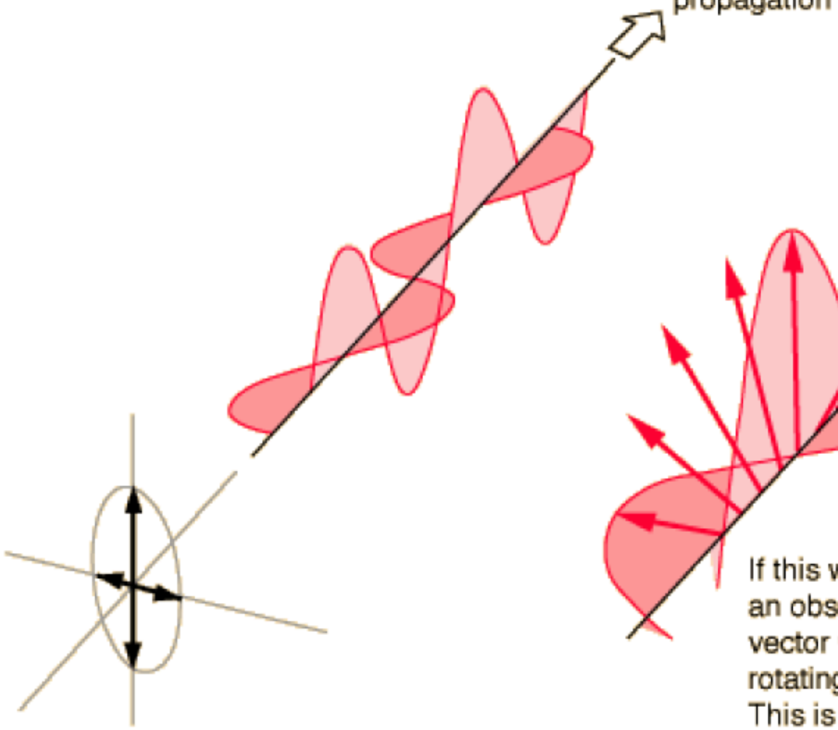
- Plane EM wave – linearly polarized
- Trace of electric field vector is linear
- Also called plane-polarized light
- Convention is to refer to the electric field vector

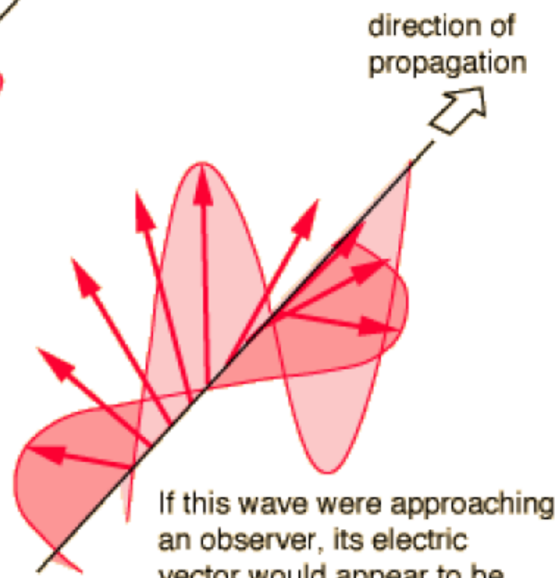
Circular Polarization



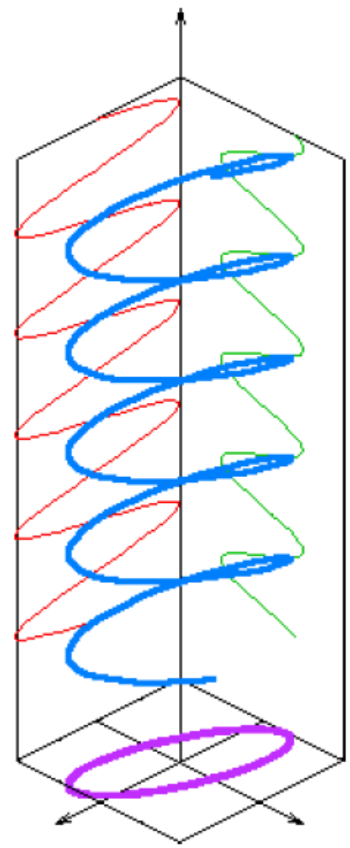
- Two perpendicular electric field components of equal amplitude with 90° difference in phase
- Electric vector rotates counterclockwise ➔ right-hand circular polarization
- Electric vector rotates clockwise ➔ left-hand circular polarization

Elliptical Polarization





If this wave were approaching an observer, its electric vector would appear to be rotating counterclockwise. This is called right - elliptic polarization.



- Two perpendicular eletric field components not in phase, either with different amplitudes and/or not 90º out of phase
- Electric vector rotates counterclockwise ➔ right-hand elliptical polarization
- Electric vector rotates clockwise ➔ left-hand elliptical polarization
- The most general state of complete polarization is elliptical

Birefringence

Polarization in birefringent crystals
(half and quarter wave plates)

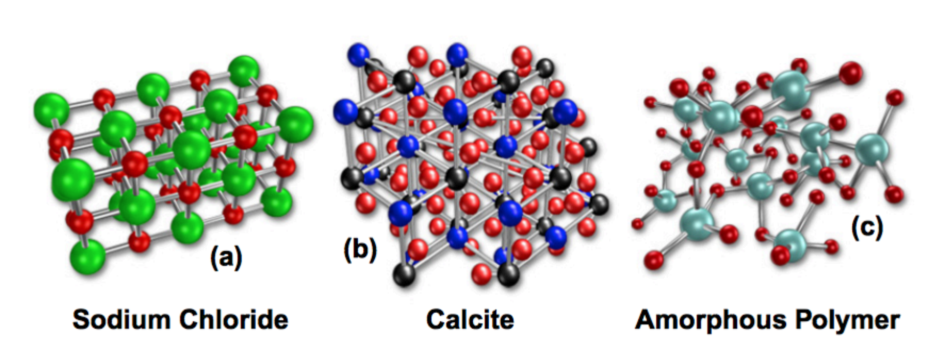
- Different indices of refraction depending on direction of wave electric field vector with respect to crystal axis

- Birefringent materials
  - Anisotropic (e.g., stressed)
  - Can produce two shifted images
  - Can create circular or elliptic polarization
  - Can rotate plane of polarization of light wave
  - Can display different colors

Anisotropic media have different properties in different direction

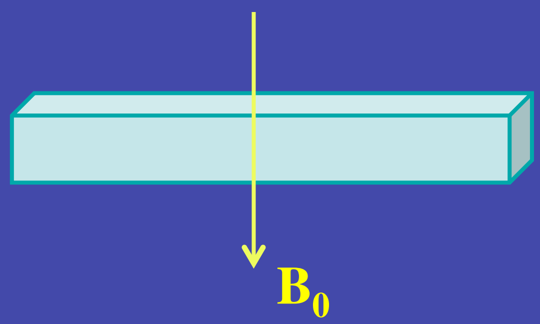
- Calcite

Figure 1 - Crystalline Structure of Isotropic and Anisotropic Materials



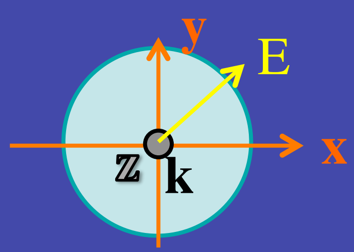
Structured but isotropic | **Anisotropic** | **Unstructured and isotropic**

- Free electrons in a DC magnetic field, B_0
  - Plasma in B-field
  - Thin metal film in B-field

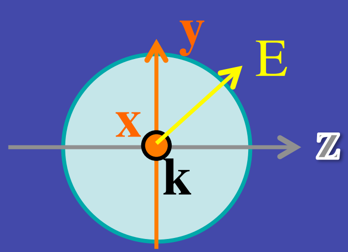


Birefringent anisotropic media have a special direction called the optic axis

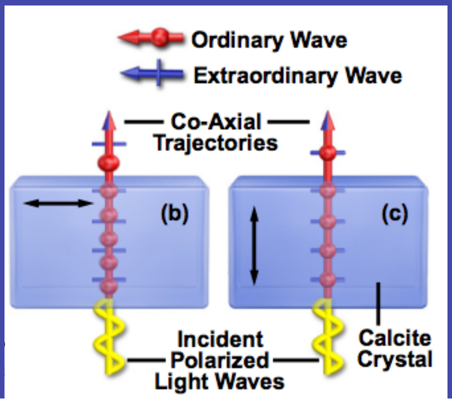
- *Normally incident* monochromatic polarized light with **k** || optic axis has same index of refraction, n, for both components of **E**
- *Normally incident* light with **k** ⊥ optic axis has different n for wave with component of **E** || optic axis ($E_{||} = E_{extraordinary}$) and wave with component ⊥ optic axis ($E_{\perp} = E_{ordinary}$)
  - $n_{ext} = ck_{ext}/\omega \neq n_{ord} = ck_{ord}/\omega$
  - Different wavenumbers, $k_{ext} \neq k_{ord}$
  - Different wavelengths, $\lambda_{ext} \neq \lambda_{ord}$
  - Different phase velocities, $\omega/k_{ext} \neq \omega/k_{ord}$
  - Slow versus fast wave

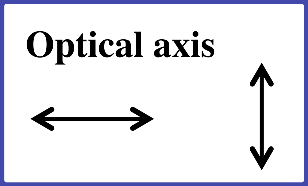


Optical axis along z
Propagation along z
E_x and E_y waves see **same** index of refraction



Optical axis along z
Propagation along x
E_z (= $E_{||}$) waves and E_y (= $E_{\perp}$) waves see **different** indices of refraction

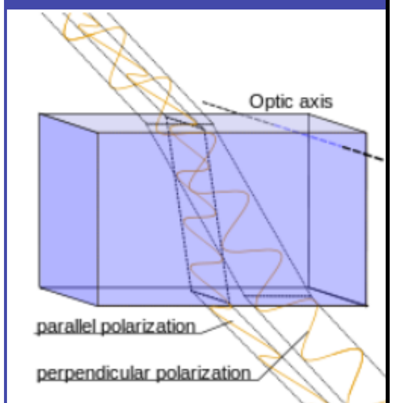




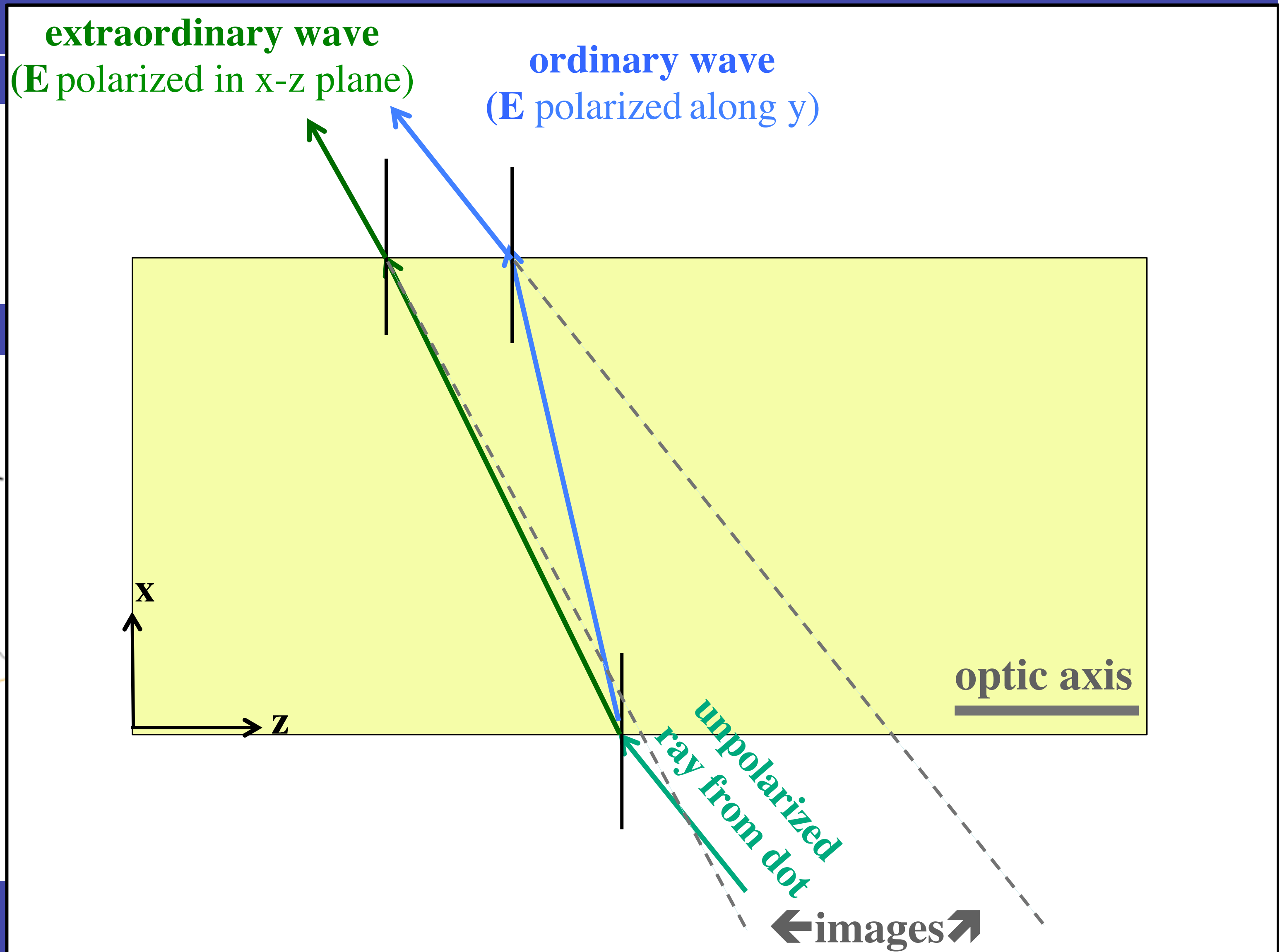
Double image of object viewed through birefringent crystal

- Two images
- Wolfram Demo

Wikipedia Theory



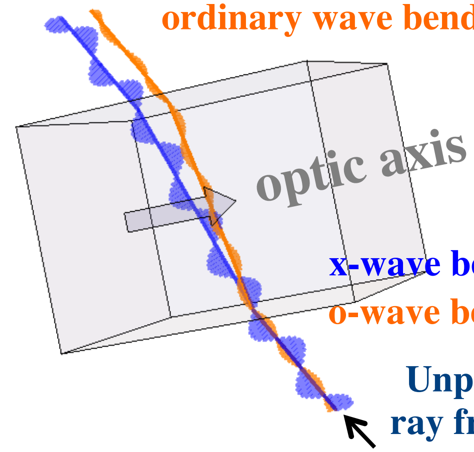
Incoming light in the parallel (*s*) polarization sees a different effective index of refraction than light in the perpendicular (*p*) polarization, and is thus refracted at a different angle.



Double image of object viewed through birefringent crystal

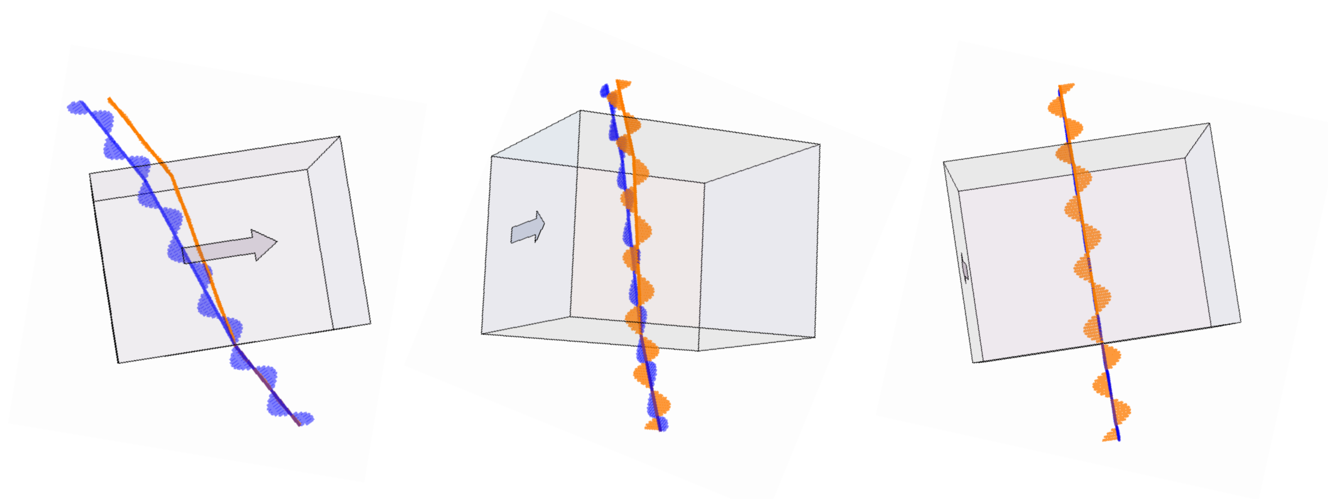
- Two images
- Wolfram Demo

extraordinary-wave bends away from normal
ordinary wave bends more away from normal



x-wave bends towards normal
o-wave bends more towards normal

Unpolarized light ray from dot enters crystal



Theory of EM waves in an anisotropic birefractive media

Wave eqn: $\left(k^2 - \frac{\omega^2}{c^2}\right)\mathbf{E} - \mathbf{k}\mathbf{k}\cdot\mathbf{E} = 4\pi i\omega\mathbf{J}$. $\mathbf{J}$ = oscillating dipole current in medium

Isotropic medium, $\mathbf{J} = \sigma\mathbf{E}$, $\varepsilon = 1 + \frac{4\pi i\sigma}{\omega}$, $\boxed{k^2\mathbf{E} - \mathbf{k}\mathbf{k}\cdot\mathbf{E} = \frac{\omega^2\varepsilon}{c^2}\mathbf{E} = \frac{\omega^2 n^2}{c^2}\mathbf{E}}$

Anisotropic medium, $\mathbf{J} = \overleftrightarrow{\sigma}\cdot\mathbf{E}$, $\overleftrightarrow{\varepsilon} = 1 + \frac{4\pi i\overleftrightarrow{\sigma}}{\omega}$, $\boxed{k^2\mathbf{E} - \mathbf{k}\mathbf{k}\cdot\mathbf{E} = \frac{\omega^2}{c^2}\overleftrightarrow{\varepsilon}\cdot\mathbf{E}}$

Take **optic axis** in z-direction, $\overleftrightarrow{\varepsilon} = \begin{bmatrix} n_x^2 & 0 & 0 \\ 0 & n_y^2 & 0 \\ 0 & 0 & n_z^2 \end{bmatrix}$, where, $n_x^2 = n_y^2 = n_o^2$, $n_z^2 = n_e^2$

Assume $k_y = 0$: Propagation ($\mathbf{k}$) in x-z plane. Wave eqn becomes:

$$\begin{pmatrix} k_z^2 - \omega^2 n_o^2/c^2 & 0 & -k_x k_z \\ 0 & k^2 - \omega^2 n_o^2/c^2 & 0 \\ -k_x k_z & 0 & k_x^2 - \omega^2 n_e^2/c^2 \end{pmatrix}\cdot\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0$$

Ordinary wave: $\boxed{\mathbf{E} = E_y \neq 0, \quad \frac{k^2}{n_o^2} = \frac{\omega^2}{c^2}}$ Index refraction = n_o

Extraordinary wave: $E_y = 0$, $-\left(\omega^2/c^2\right)\left(k_x^2 n_o^2 + k_z^2 n_e^2 - \omega^2 n_e^2 n_o^2/c^2\right) = 0$, or,

$\boxed{\mathbf{E} \text{ in x-z plane}, \quad \frac{k_x^2}{n_e^2} + \frac{k_z^2}{n_o^2} = \frac{\omega^2}{c^2}}$ Effective index of refraction, n_{eff} between n_e and n_0

depending on angle, θ, between $\mathbf{k}$ and z-axis: $\boxed{\frac{1}{n_{eff}^2} = \frac{\omega^2}{k^2c^2} = \frac{\sin^2\theta}{n_e^2} + \frac{\cos^2\theta}{n_o^2}}$