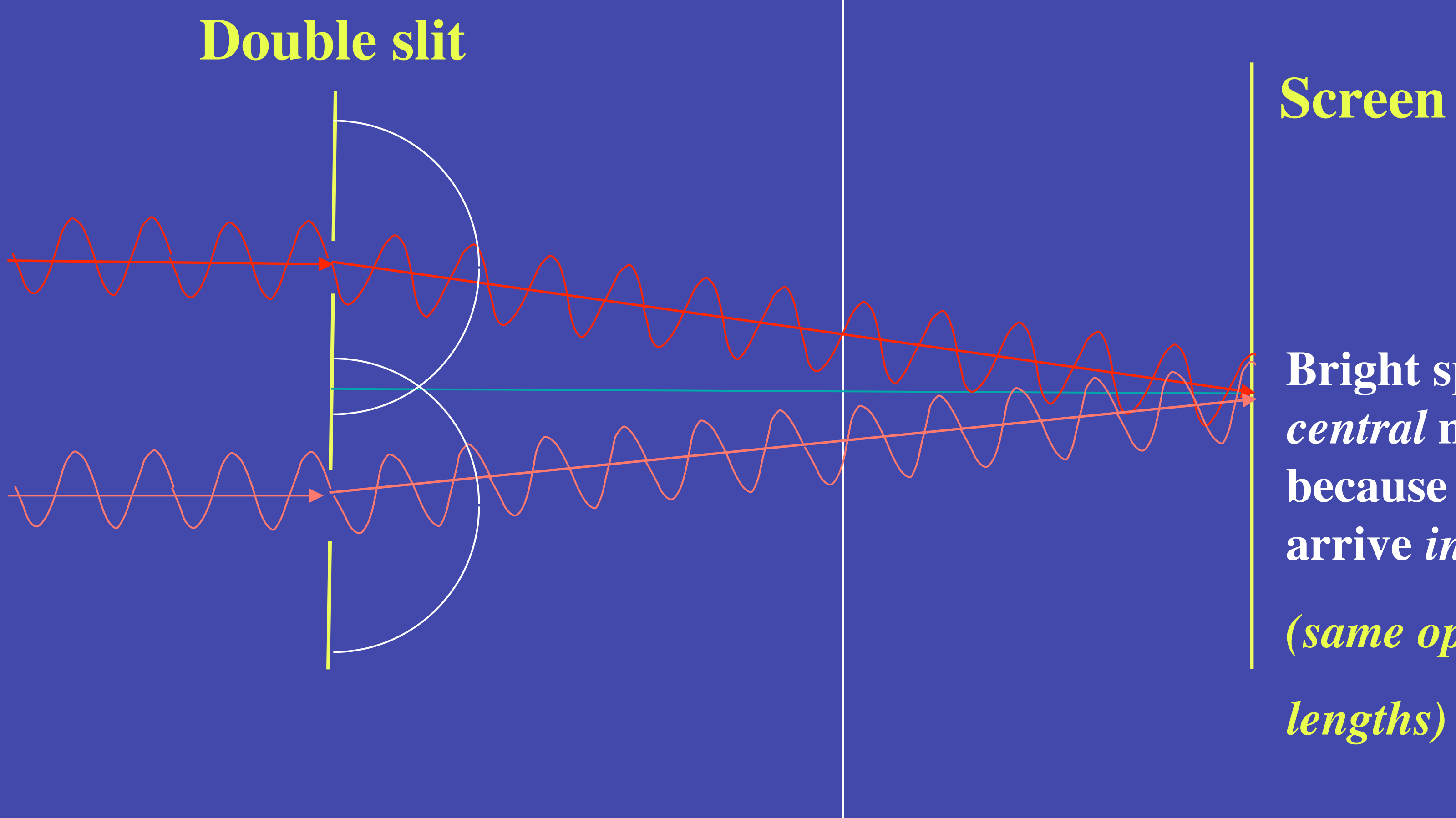
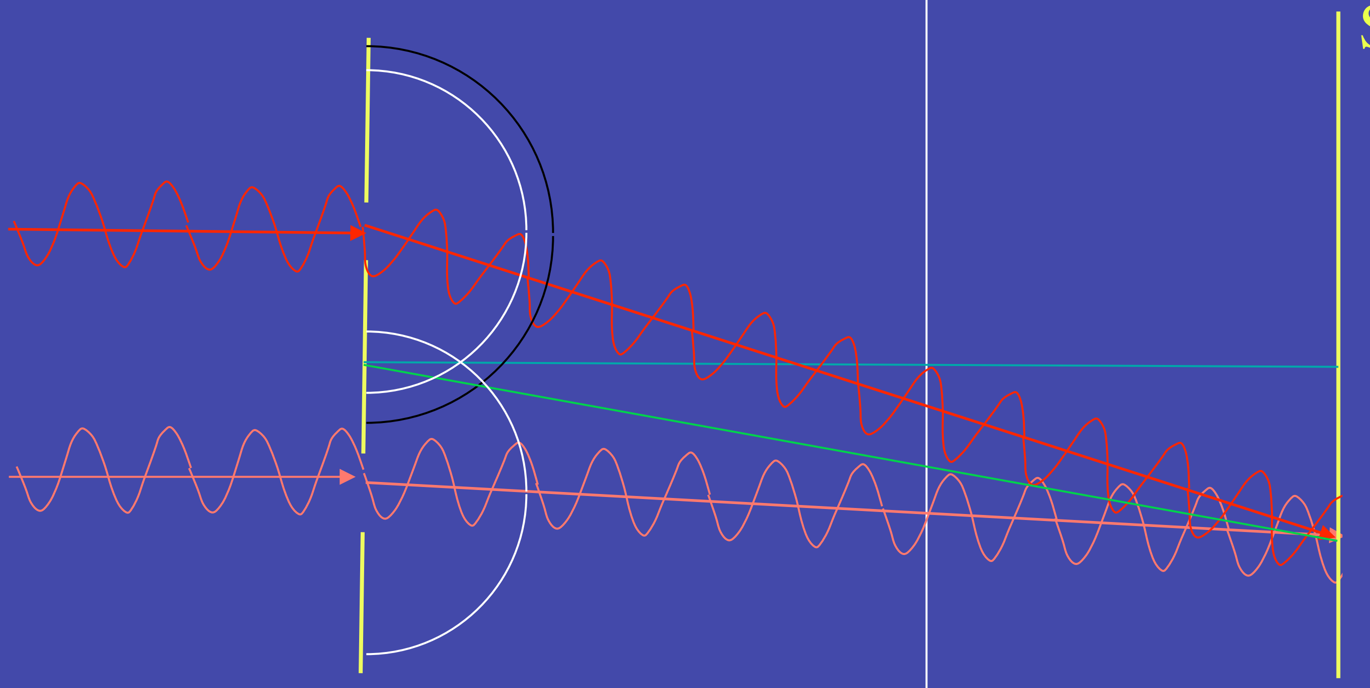
Constructive interference of monochromatic light from two thin slits



Bright spot at *central* maximum because waves arrive *in phase*

(same optical path lengths)

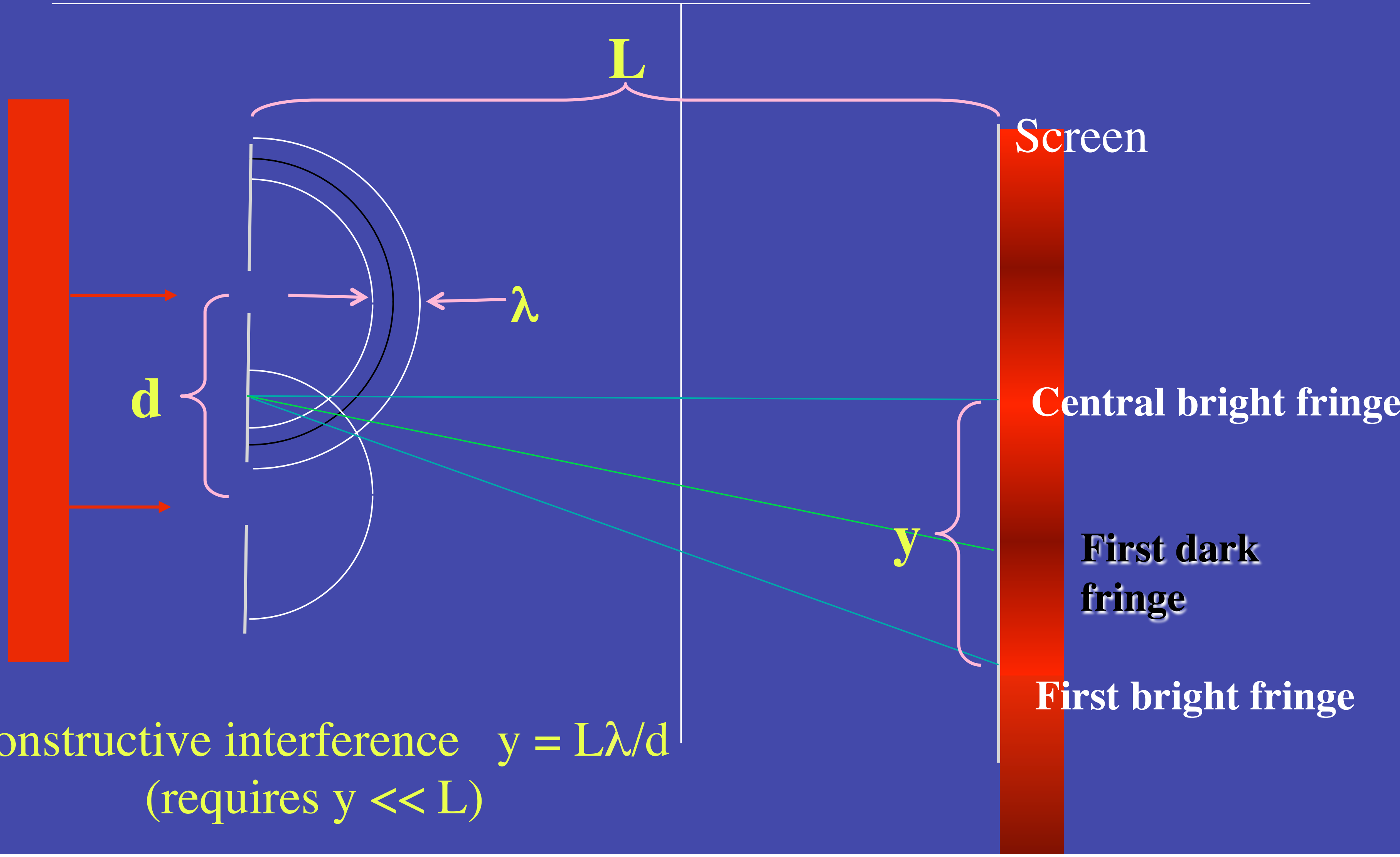
Destructive interference at a different location on the screen from the same two thin slits



Dark spot (fringe) at point of destructive interference because waves are *out of phase*

$$\sin\theta_i \approx \frac{y_{idark}}{L} = \frac{\lambda}{d}\left[j + \frac{1}{2}\right]$$

Distance, y, between *bright* fringes on screen a distance L from 2 thin slits separated by d << L is $y = \lambda L/d$, where λ is ***light wavelength***

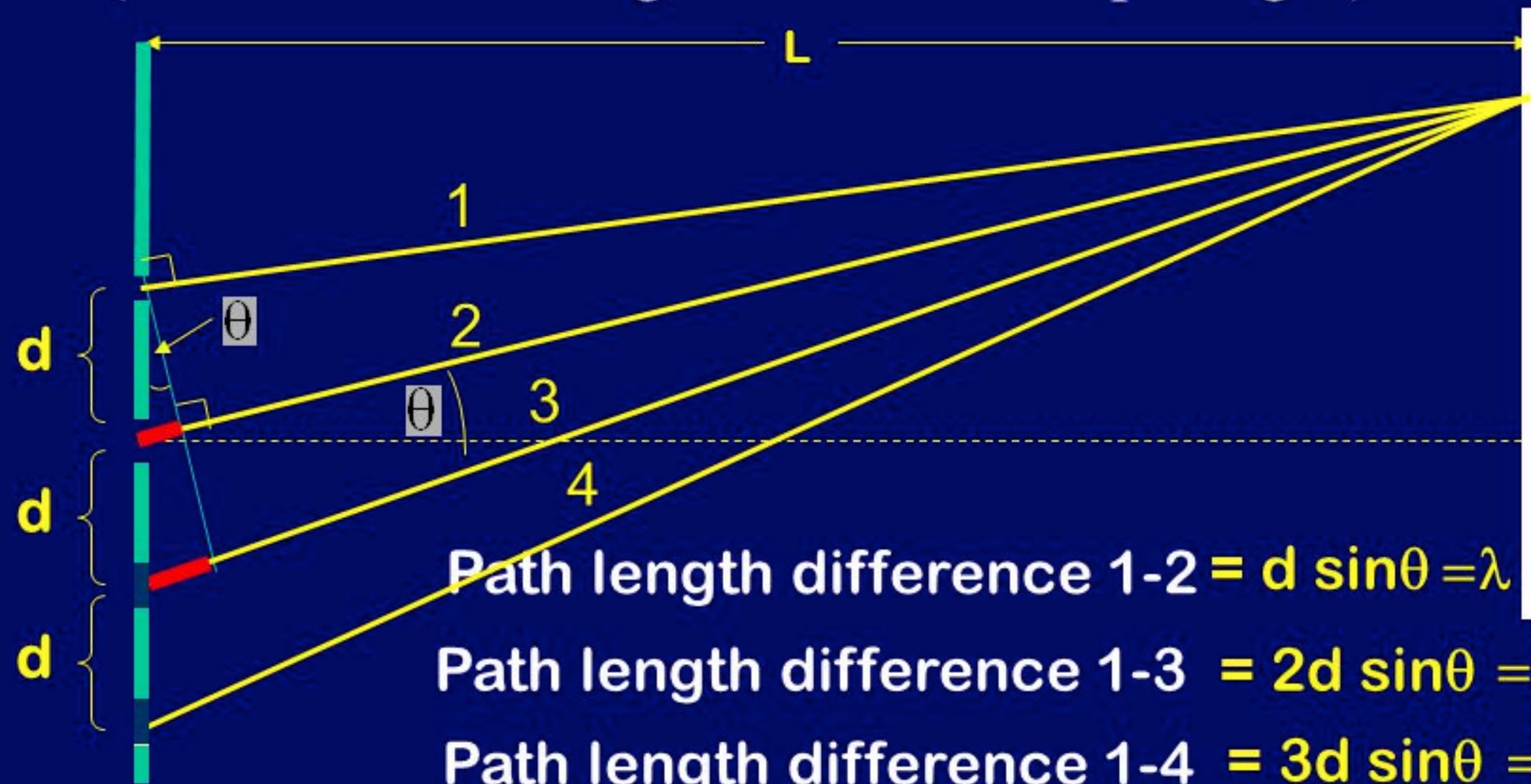


Constructive interference $y = L\lambda/d$

(requires y << L)

Multiple Slits

(Diffraction Grating – N slits with spacing d)



Path length difference 1-2 $= d \sin\theta = \lambda$

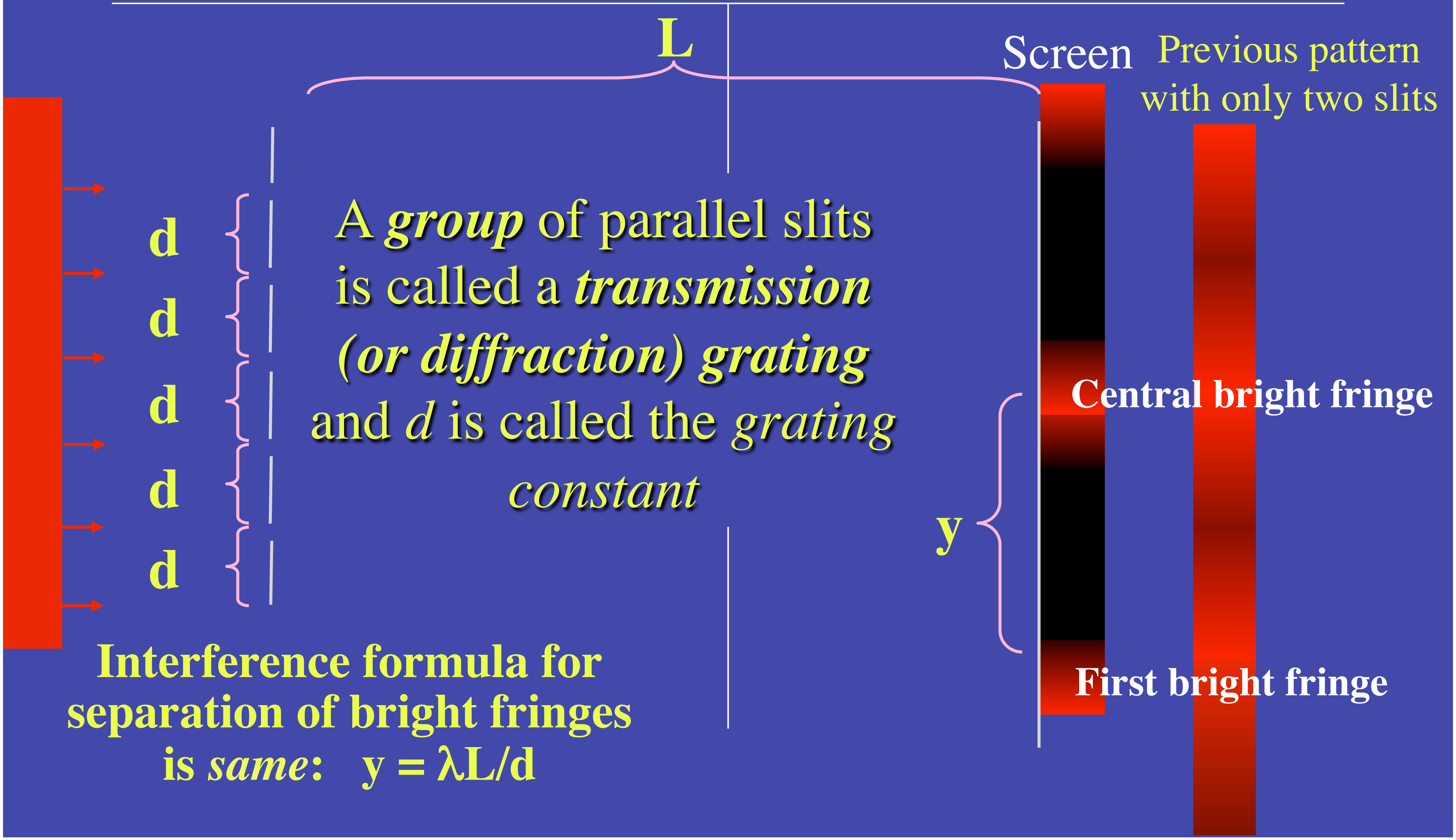
Path length difference 1-3 $= 2d \sin\theta = 2\lambda$

Path length difference 1-4 $= 3d \sin\theta = 3\lambda$

Constructive interference for all paths when

$$d \sin\theta = m\lambda$$

Effect of *many* slits separated by d instead of just two is simply that the bright fringes are *narrower*, but in *same place for same d, L and λ*



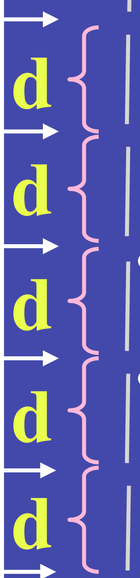
A ***group*** of parallel slits is called a ***transmission (or diffraction) grating*** and *d* is called the *grating constant*

Interference formula for separation of bright fringes is *same*: $y = \lambda L/d$

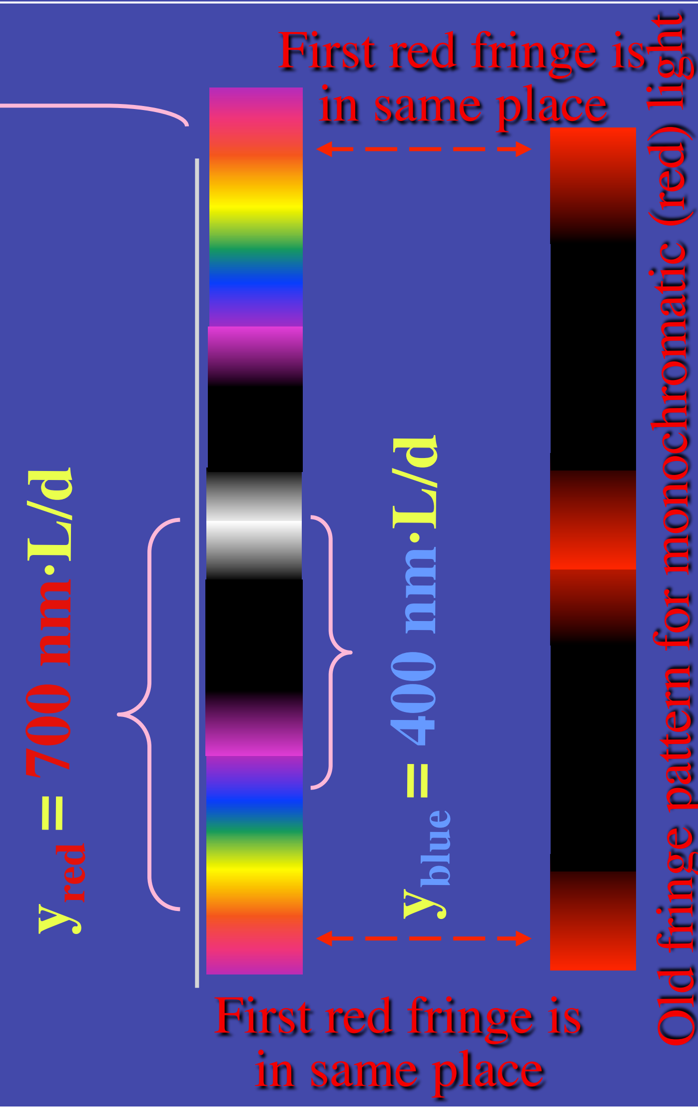
What do you see if ***white*** light or mixed colored lights goes through a *transmission grating*?

$y/L = \lambda/d$

L



- Fringe separation formula applies to each wavelength in white light!
  - *All* wavelengths contribute to *central* (white) fringe since all wavelengths are in phase at center
- 1[st] bright red fringe (long wavelength) is in same place
- 1st bright fringes of *shorter* wavelengths *closer* to central bright fringe because **y** is proportional to wavelength, **λ**.
- Transmission grating white light or color mixture into spatially spread colors like a prism!
  - However physics is based on *interference* not *dispersion*



How to calculate intensity pattern from n-slit interference

Sum of fields on screen from n_{slits} intersecting rays

(See notes on Brightness and Phase) Assume rays close, almost parallel, same polarization

$$\mathbf{E}_{tot-n_{slits}}(t) = \sum_{j=1}^{n} \mathbf{E}_j ,$$

$$\mathbf{E}_j = \mathbf{A}\cos\left[\left(\mathbf{k}\cdot\mathbf{r} - \omega t\right) + \varphi_j\right],\ \text{A is real}$$

Let r = 0 at point on screen:

$$\mathbf{E}_{tot-n_{slits}}(t) = \mathbf{A}\sum_{j=1}^{n_{slits}} \cos\left[-\omega t + \varphi_j\right] = \mathbf{A}\,\mathrm{Re}\left\{e^{-i\omega t}\sum_{j=1}^{n_{slits}} e^{i\varphi_j}\right\}$$

But $\varphi_j = j\varphi$, $\boxed{\varphi = k\delta = kd\sin\theta \approx kd\frac{y}{L},\ \text{for}\ \ \theta << 1}$

$$\boxed{\mathbf{E}_{tot-n_{slits}}(t) = \mathbf{A}\,\mathrm{Re}\left\{e^{-i\omega t}\sum_{j=1}^{n_{slits}}\left(e^{i\varphi}\right)^j\right\}}$$

$$S_n = x + x^2 + \ldots x^n$$

$$xS_n = \quad x^2 + \ldots . x^n + x^{n+1}$$

$$xS_n - S_n = x^{n+1} - x$$

$\boxed{S_n = x\frac{x^n - 1}{x - 1}}$ Now let $x = e^{i\varphi}$:

$$S_n = e^{i\varphi}\frac{e^{in\varphi} - 1}{e^{i\varphi} - 1} = e^{i\varphi}\frac{e^{in\varphi/2}}{e^{i\varphi/2}}\frac{e^{in\varphi/2} - e^{-in\varphi/2}}{e^{i\varphi/2} - e^{-i\varphi/2}}$$

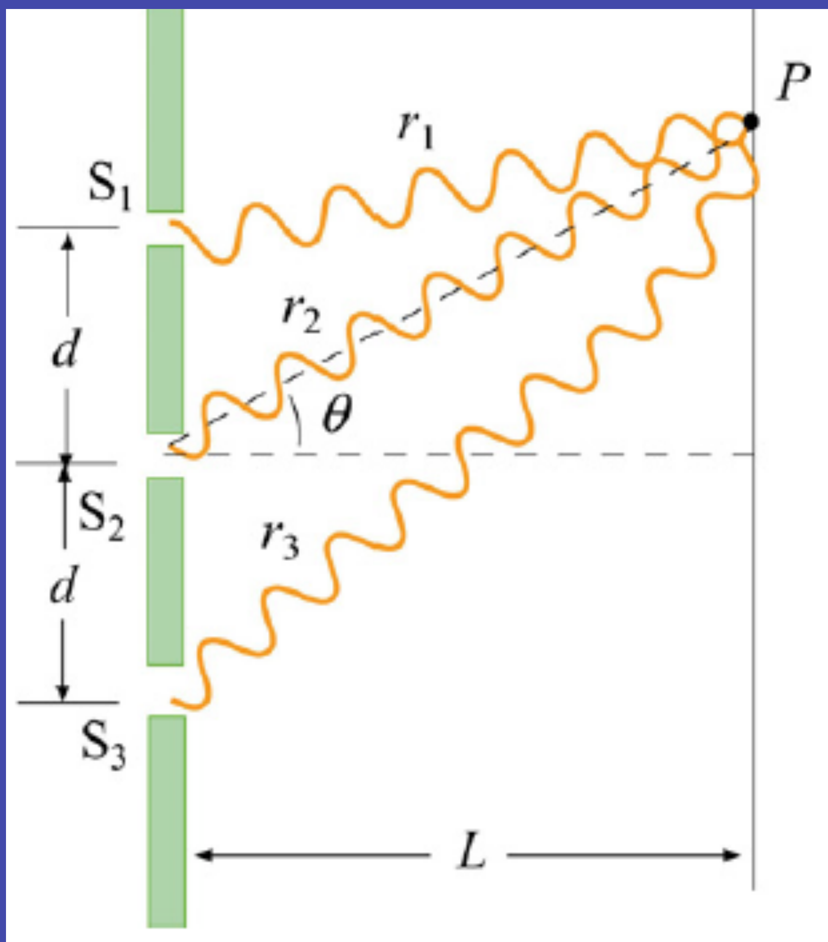
$$S_n = e^{i(n+1)\varphi/2}\frac{\sin(n\varphi/2)}{\sin(\varphi/2)}$$

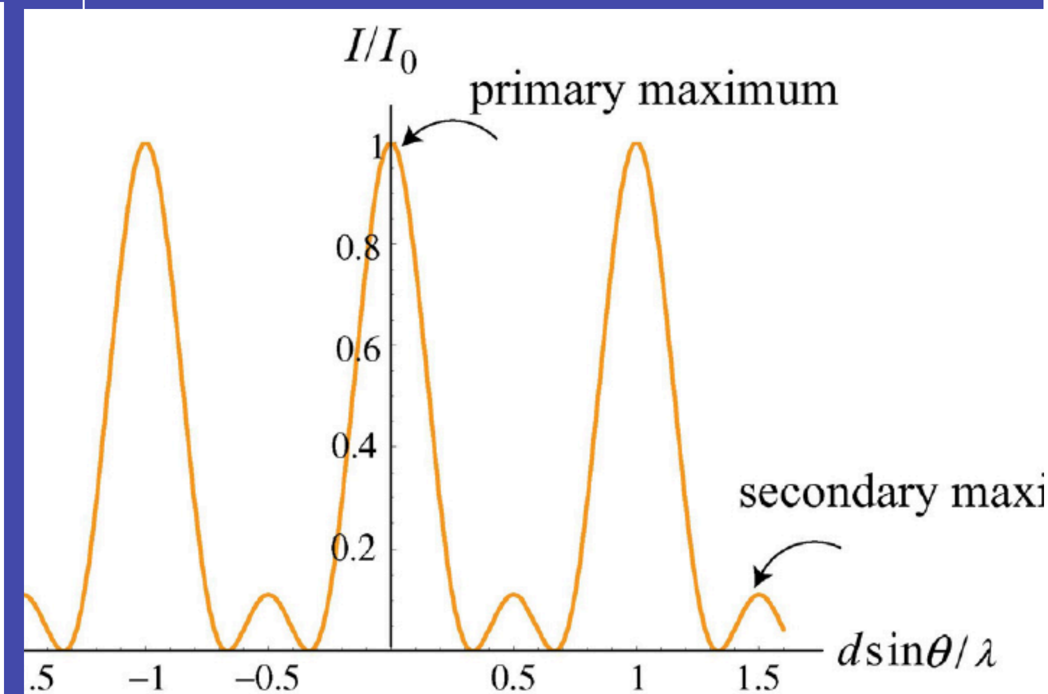
$\mathbf{E}_{tot-n}(t) = \mathbf{A}\,\mathrm{Re}\left\{e^{-i\omega t}\sum_{j=1}^{n}\left(e^{i\varphi}\right)^j\right\}$ gives

$$\boxed{\mathbf{E}_{tot-n}(t) = \mathbf{A}\cos\left(-\omega t + [n+1]\varphi/2\right)\frac{\sin(n_{slits}\varphi/2)}{\sin(\varphi/2)}}$$

Intensity $\boxed{\mathrm{I} \propto \left\langle \mathbf{E}_{tot-n}(t)^2 \right\rangle = \frac{A^2}{2}\frac{\sin^2(n_{slits}\varphi/2)}{\sin^2(\varphi/2)}}$

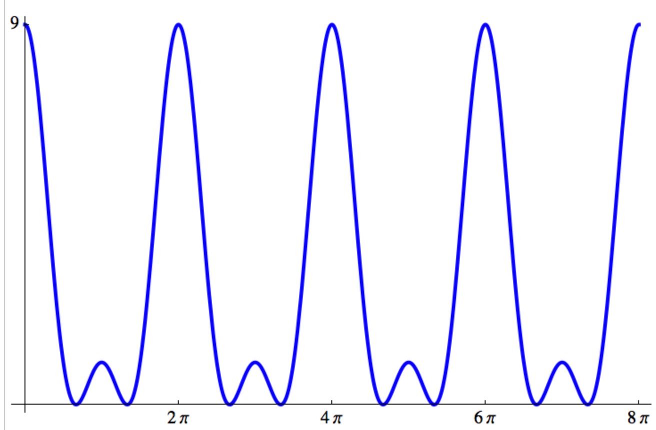
Three slit interference





Interference intensity patterns for $n_{slits} = 3, 4, 7$

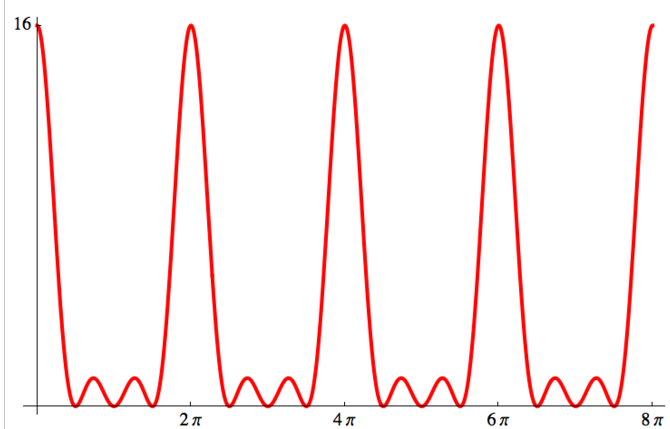
```
Plot[(Sin[3 ϕ / 2] / Sin[ϕ / 2])^2, {ϕ, 0, 8 π}, PlotStyle → {Thick, Blue},
 Ticks → {{0, 2 π, 4 π, 6 π, 8 π}, {0, 9}}]
```



$n_{slits} = 3$

ϕ➔

```
Plot[(Sin[4 ϕ / 2] / Sin[ϕ / 2])^2, {ϕ, 0, 8 π}, PlotStyle → {Thick, Red},
 Ticks → {{0, 2 π, 4 π, 6 π, 8 π}, {0, 16}}]
```



$n_{slits} = 4$

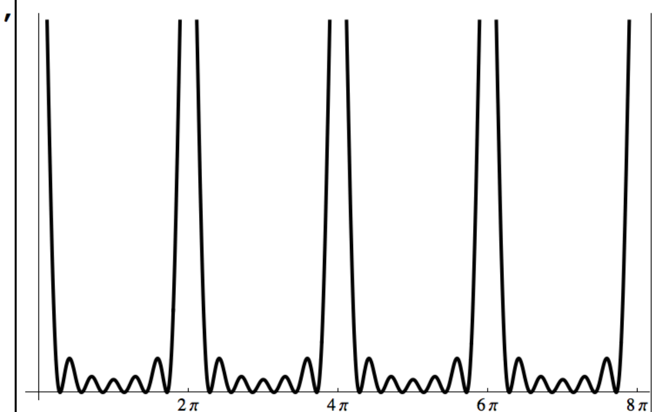
ϕ➔

Intensity $I \propto \left\langle \mathbf{E}_{tot-n_{slits}}(t)^2 \right\rangle = \frac{A^2}{2} \frac{\sin^2(n_{slits}\varphi/2)}{\sin^2(\varphi/2)}$

Peaks at $\varphi = 0$ mod 2π. Value at peak $= \frac{A^2 n_{slits}^2}{2}$

$\varphi = k\delta = kd\sin\theta \approx kd\frac{y}{L}$, for $\theta << 1$

```
Plot[(Sin[7 ϕ / 2] / Sin[ϕ / 2])^2, {ϕ, 0, 8 π}, PlotStyle → {Thick, Black},
 Ticks → {{0, 2 π, 4 π, 6 π, 8 π}, {0, 49}}]
```



$n_{slits} = 7$

ϕ➔

Reflection "grating"

- Interference from a *reflection grating* works the same way as interference from a *transmission grating*
  - Monochromatic light incident on a reflection grating reflects a pattern of bright fringes whose separation, y, is larger when the grating spacing, d, is smaller.
  - White light incident on a reflection grating reflects a spectrum of the colors in the white light with white in the center and short wavelengths closer to the center
- We see this effect in the colors we observe when we look at the surface of a CD in bright white light
  - Examples of toy gratings, real one and CD surface
- The shifting colors we see from peacocks and butterflies are also due to organic naturally occurring reflection gratings on the surfaces of animals and insects
  - This is called *iridescence*.