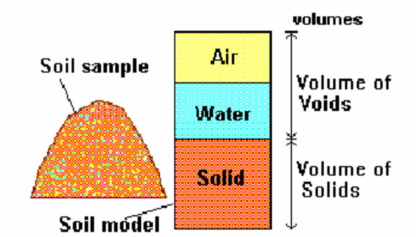
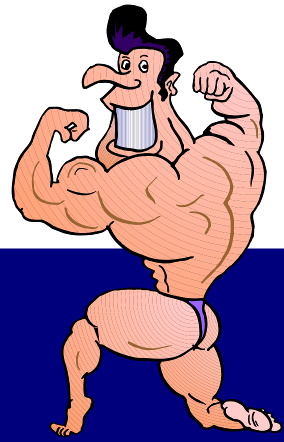
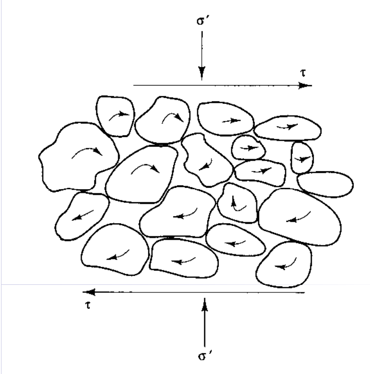




Soil Mechanics –

Shear Strength of Soils



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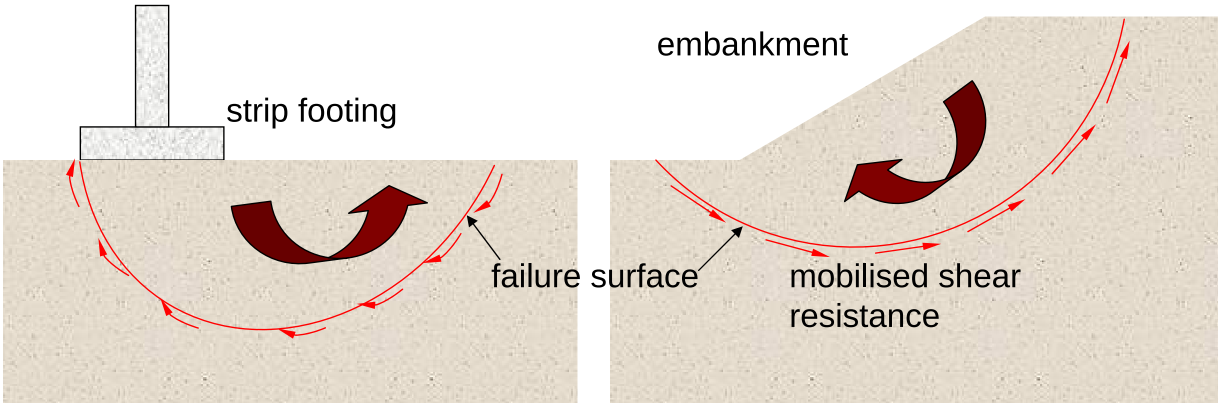
Outline

- Shear Failure
- Soil Strength
  - Mohr-Coulomb Failure Criterion
- Laboratory Shear Strength Test
  - Direct shear
  - Triaxial
- Stress Path
- Pore Pressure Parameters

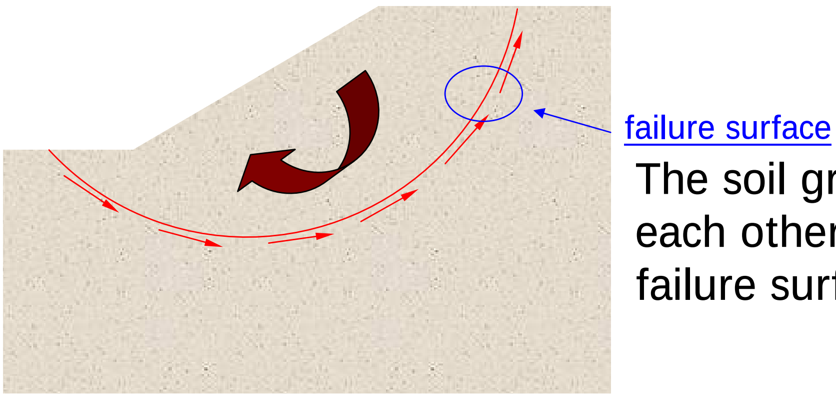
Shear failure

Soils generally fail in shear



At failure, shear stress along the failure surface reaches the shear strength.

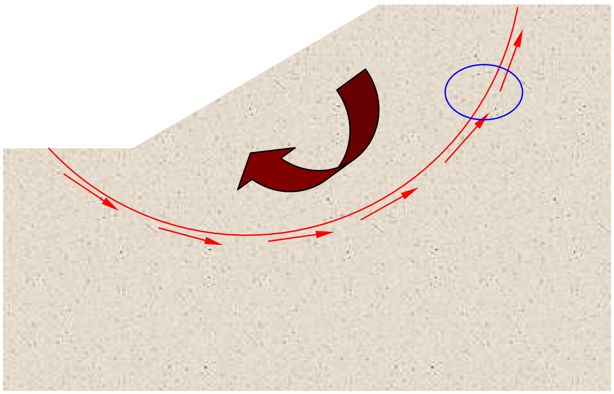
Shear failure

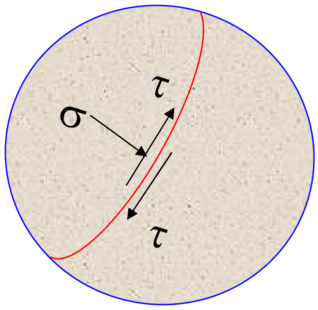


The soil grains slide over each other along the failure surface.

No crushing of individual grains.

Shear failure



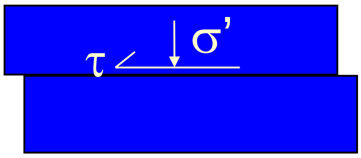


At failure, shear stress along the failure surface (τ) reaches the shear strength (τ_f).

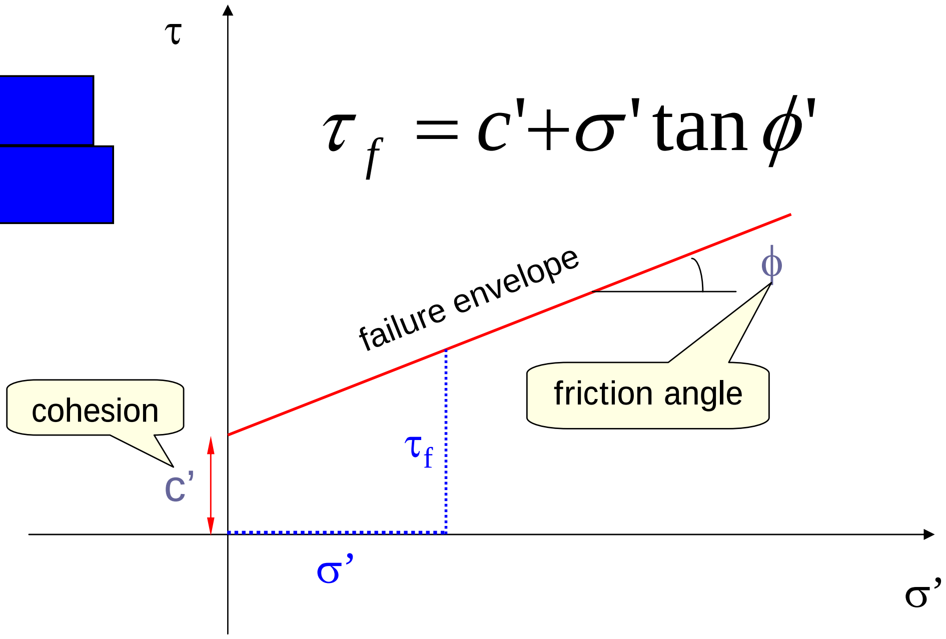
Soil (Shear) Strength

- Soils are essentially frictional materials
  - the strength depends on the applied stress
- Strength is controlled by effective stresses
  - water pressures are required
- Soil strength depends on drainage
  - different strengths will be measured for a given soil
    a) deforms at constant volume (undrained) and
    b) deforms without developing excess pore pressures (drained)

Mohr-Coulomb Failure Criterion



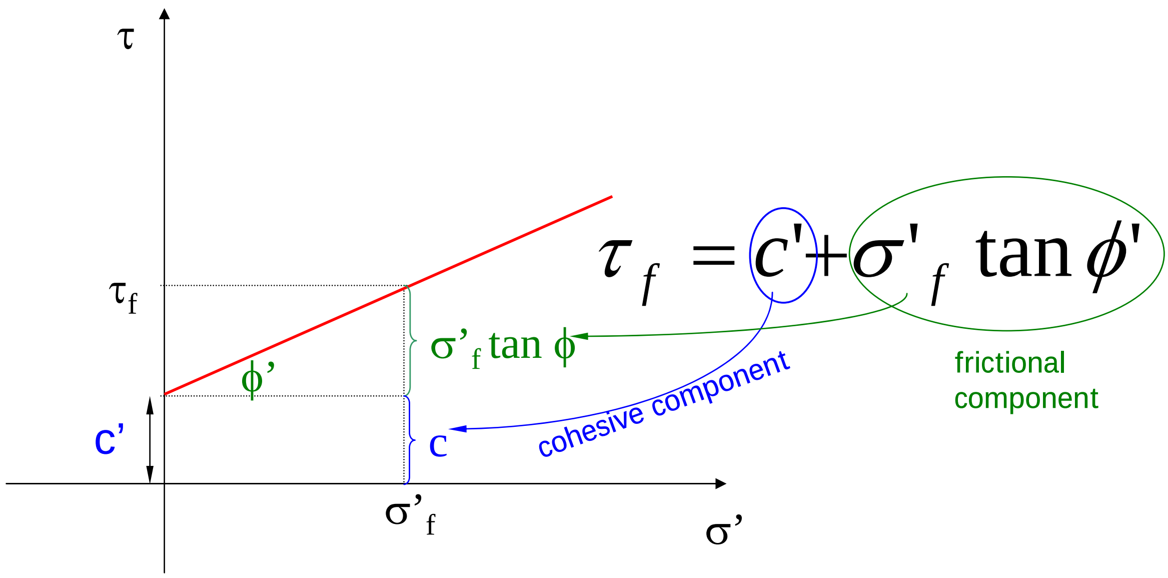
$$\tau_f = c' + \sigma' \tan \phi'$$



τ_f is the maximum shear stress the soil can take without failure, under normal stress of σ'.

Mohr-Coulomb Failure Criterion

Shear strength consists of two components: cohesive and frictional.



c' and ϕ' are measures of shear strength.

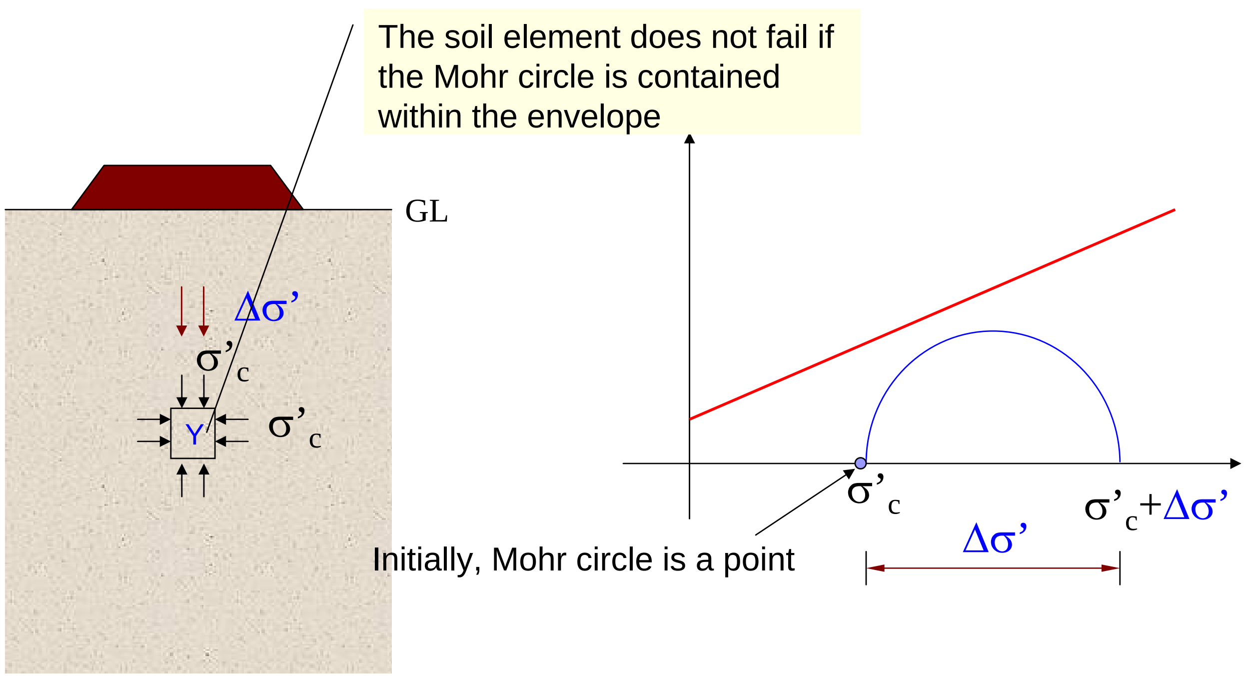
Higher the values, higher the shear strength.

The parameters c', ϕ' depend on

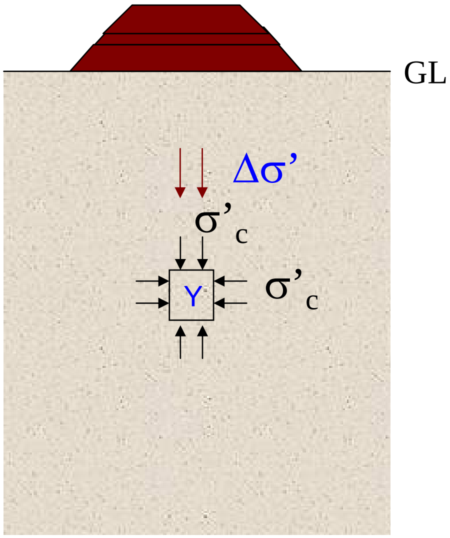
- Soil composition
- Stress state of the soil (OCR)

The Mohr-Coulomb criterion is an empirical criterion, and the failure locus is only locally linear. Extrapolation outside the range of normal stresses for which it has been determined is likely to be unreliable.

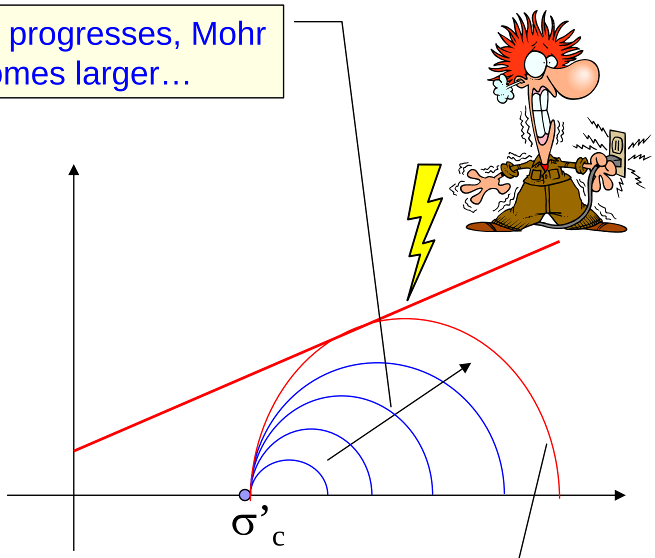
Mohr Circles & Failure Envelope



Mohr Circles & Failure Envelope

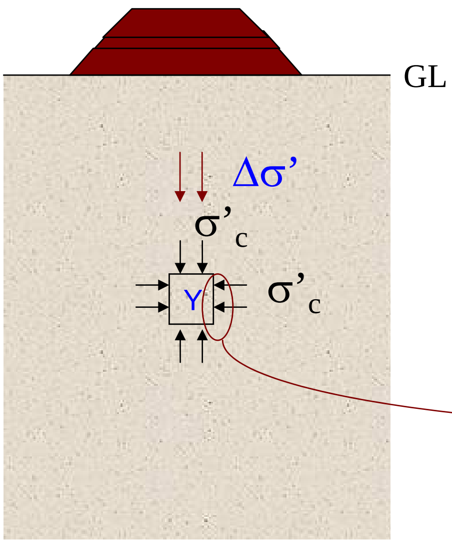


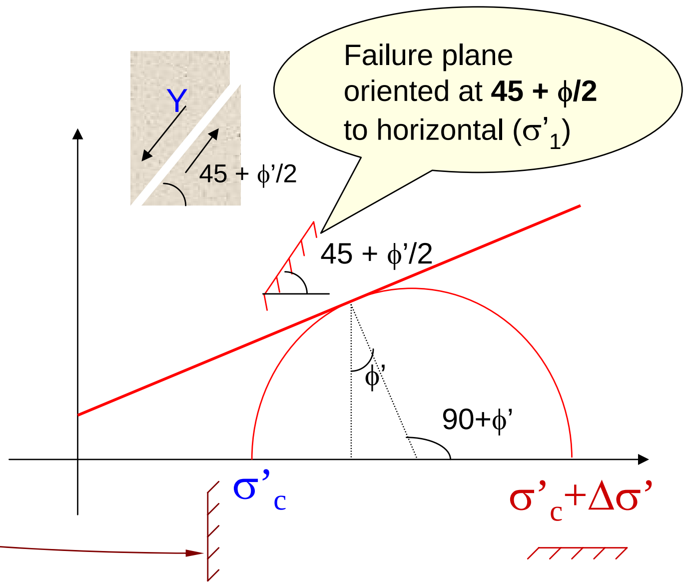
As loading progresses, Mohr circle becomes larger…



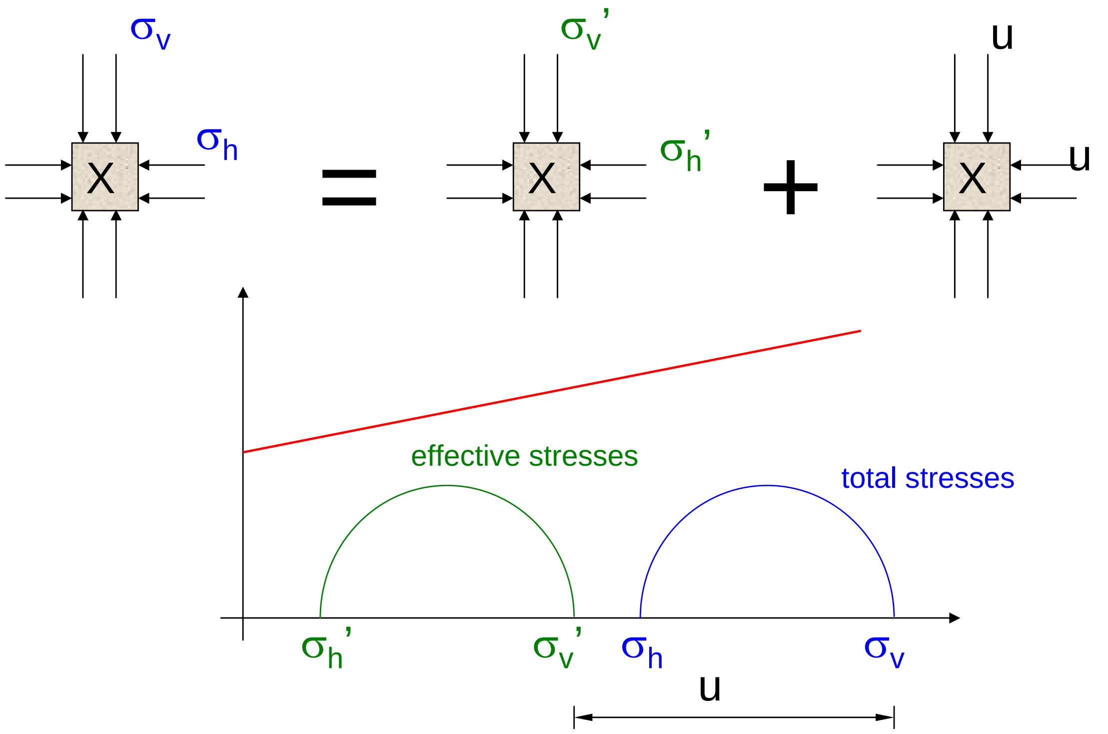
.. and finally failure occurs when Mohr circle touches the envelope

Orientation of Failure Plane



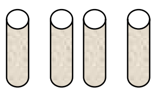


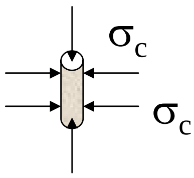
Mohr circles in terms of σ & σ'



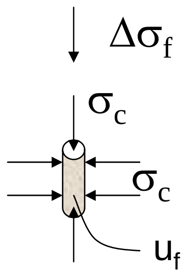
Envelopes in terms of σ & σ'

Identical specimens initially subjected to different isotropic stresses (σ_c) and then loaded axially to failure





Initially…

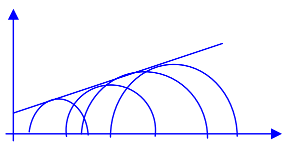


Failure

At failure,

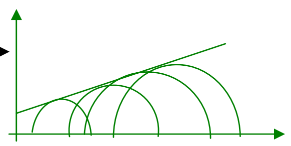
$\sigma_3 = \sigma_c$; $\sigma_1 = \sigma_c + \Delta\sigma_f$

$\sigma_3' = \sigma_3 - u_f$; $\sigma_1' = \sigma_1 - u_f$



c, ϕ

in terms of σ



c', ϕ'

in terms of σ'

Effective stress failure criterion

If the soil is at failure the effective stress failure criterion will always be satisfied.

$$\tau = c' + \sigma' \tan\phi'$$

c' and ϕ' are known as the effective (or drained) strength parameters.

Soil behaviour is controlled by effective stresses, and the effective strength parameters are the fundamental strength parameters. But they are not necessarily soil constants.

Total stress failure criterion

If the soil is taken to failure at constant volume (undrained) then the failure criterion can be written in terms of total stress as

$$\tau = c_u + \sigma \tan\phi_u$$

c_u and ϕ_u are known as the undrained strength parameters

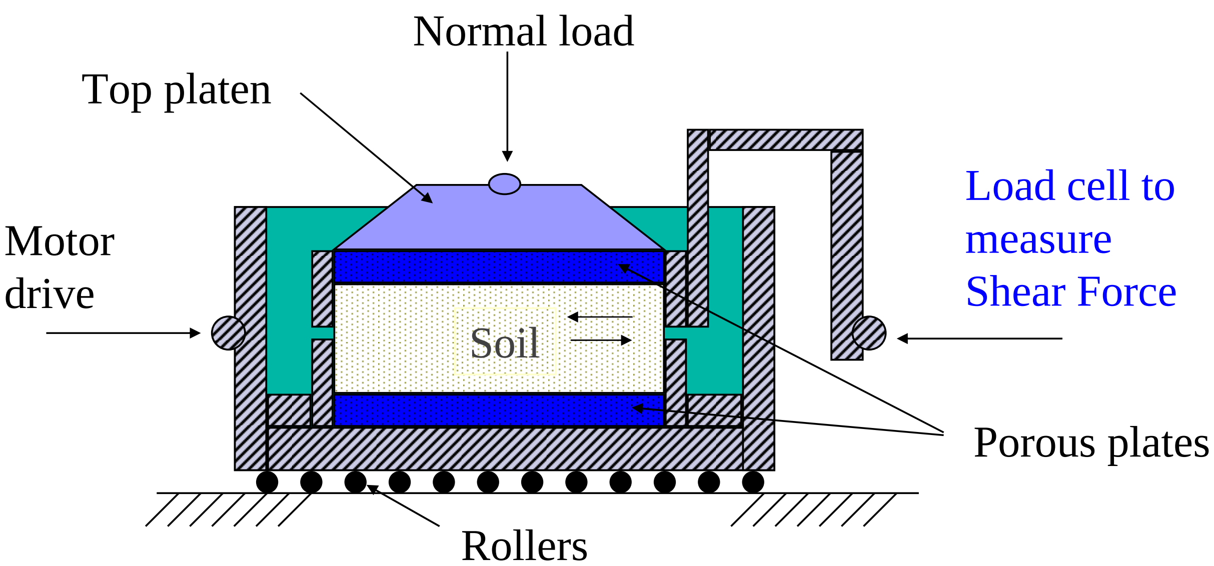
These parameters are not soil constants, they depend strongly on the moisture content of the soil.

The undrained strength is only relevant in practice to clayey soils that in the short term remain undrained. Note that as the pore pressures are unknown for undrained loading the effective stress failure criterion cannot be used.

Laboratory Tests for Shear Strength Parameters

- Direct shear test
- Triaxial test
- Direct simple shear test
- Plane strain triaxial test
- Torsional ring shear test

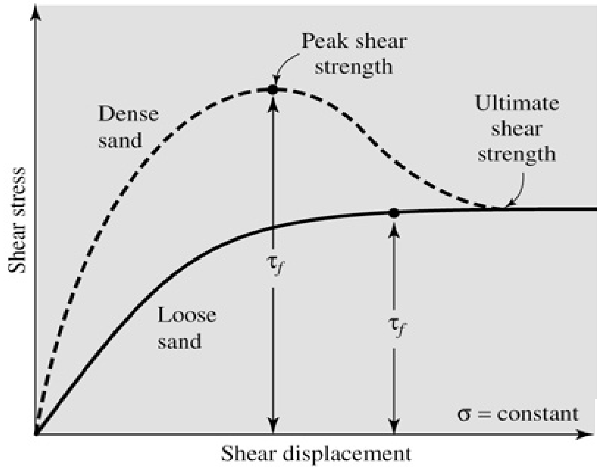
Direct Shear Test

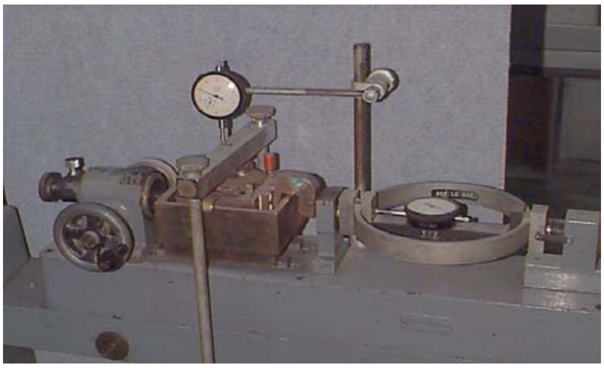


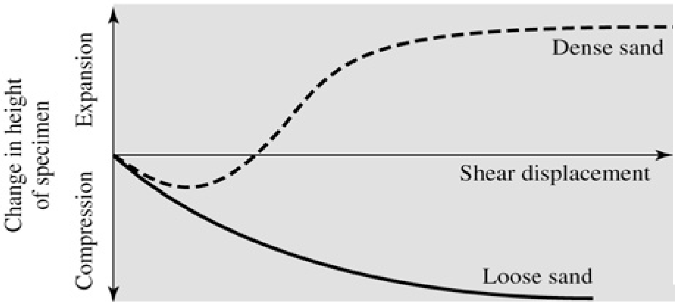
Measure relative horizontal displacement, dx

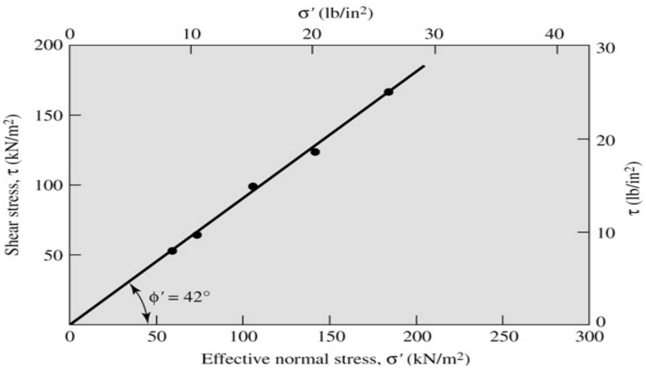
vertical displacement of top platen, dy

Sand

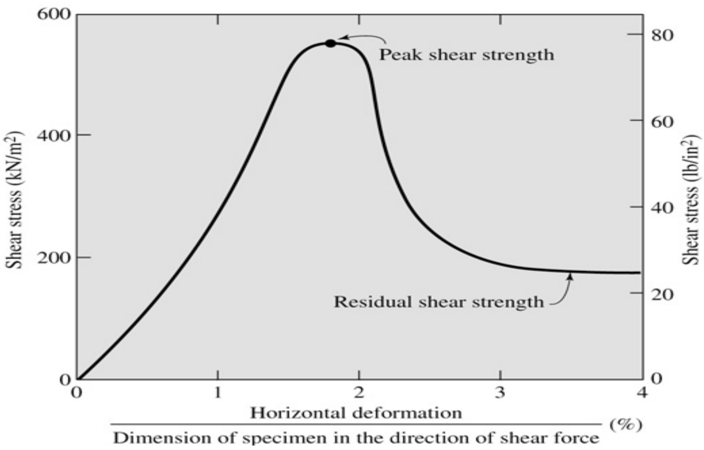

Peak shear
strength
Dense
sand
Ultimate
shear
strength
Shear stress
τ_f
Loose
sand
τ_f
σ = constant
Shear displacement

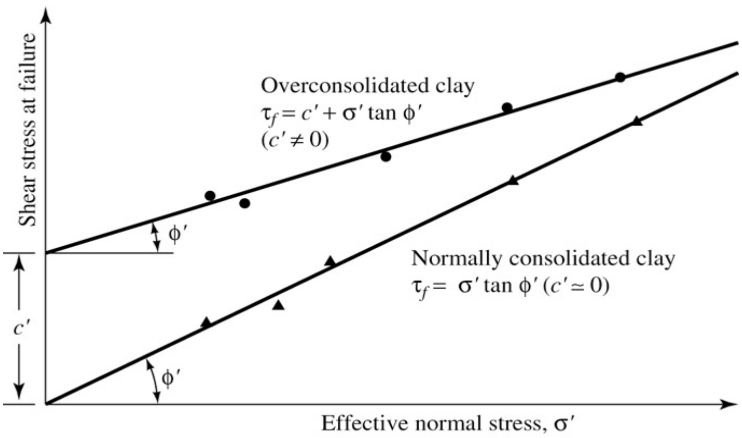


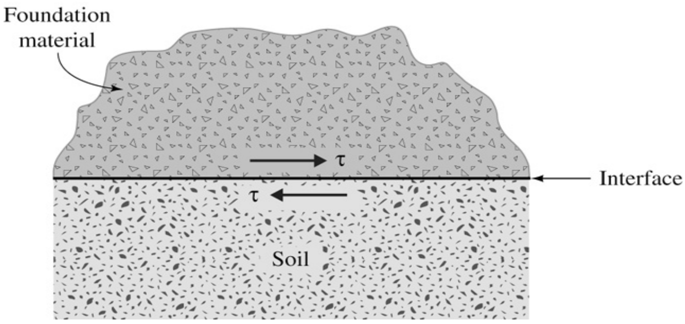

Change in height
of specimen
Expansion
Compression
Dense sand
Shear displacement
Loose sand


σ′ (lb/in²)
0
10
20
30
40
200
150
100
50
30
20
10
0
Shear stress, τ (kN/m²)
τ (lb/in²)
φ′ = 42°
0
50
100
150
200
250
300
Effective normal stress, σ′ (kN/m²)

Clay


600
400
200
0
Shear stress (kN/m²)
80
60
40
20
0
Shear stress (lb/in²)
Peak shear strength
Residual shear strength
0
1
2
3
4
Horizontal deformation
Dimension of specimen in the direction of shear force
(%)


Shear stress at failure
Overconsolidated clay
$\tau_f = c' + \sigma' \tan \phi'$
$(c' \neq 0)$
φ′
c′
Normally consolidated clay
$\tau_f = \sigma' \tan \phi' \; (c' \simeq 0)$
φ′
Effective normal stress, σ′



Pros:

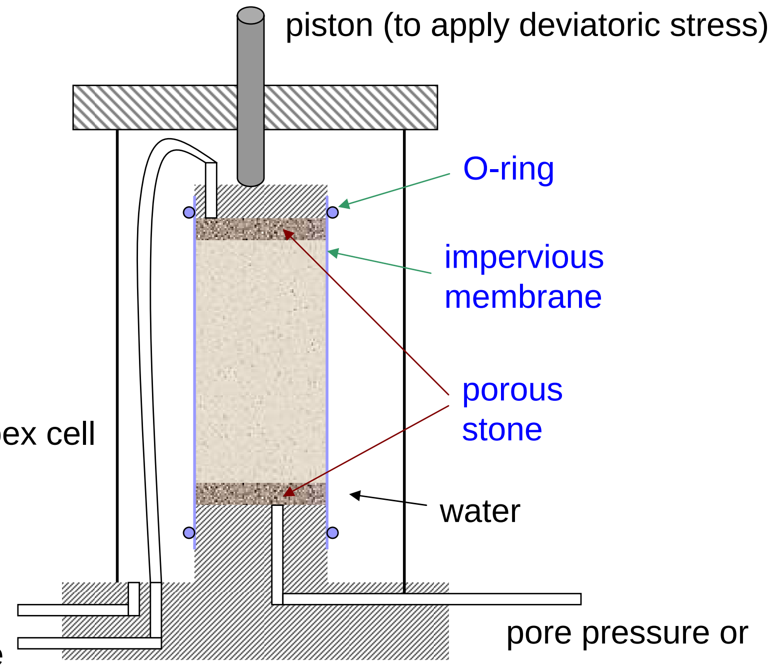
- Simplest and most economical for sandy soil
- Applicable for soil/structure interface

Cons:

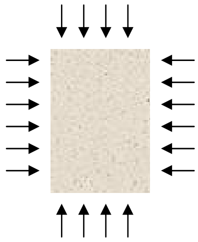
- Soil not allowed to fail along the weakest plane.
- Shear stress distribution is not uniform.

Triaxial Test Apparatus

soil sample at failure



Types of Triaxial Tests

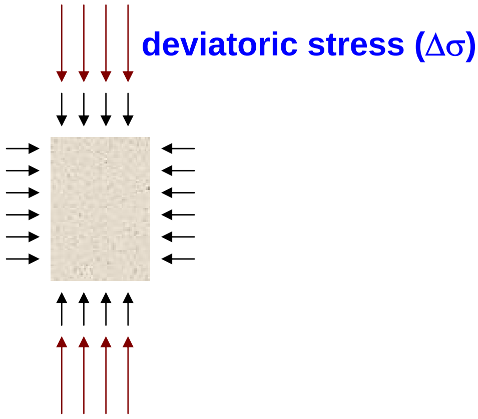


Under all-around cell pressure σ_c

Is the drainage valve open?

yes → **Consolidated** sample

no → **Unconsolidated** sample



Shearing (loading)

Is the drainage valve open?

yes → **Drained** loading

no → **Undrained** loading

Types of Triaxial Tests

Depending on whether drainage is allowed or not during

- initial isotropic cell pressure application, and
- shearing,

there are three special types of triaxial tests that have practical significances. They are:

Consolidated Drained (CD) test
Consolidated Undrained (CU) test
Unconsolidated Undrained (UU) test

For unconsolidated undrained test, in terms of total stresses, $\phi_u = 0$

Granular soils have no cohesion.
c = 0 & c'= 0

For normally consolidated clays, c' = 0 & c = 0.

CD, CU and UU Triaxial Tests

Consolidated Drained (CD) Test

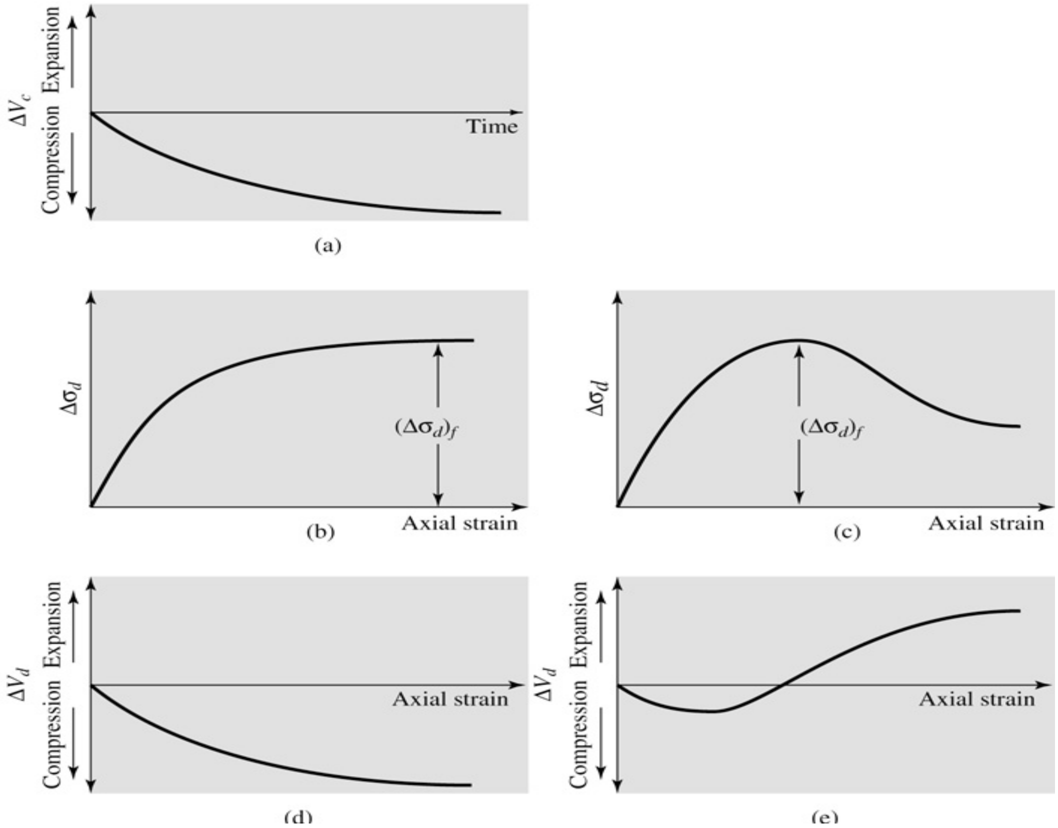
- no excess pore pressure throughout the test
- very slow shearing to avoid build-up of pore pressure

Can be days!
∴ not desirable

- gives c' and ϕ'

Use c' and ϕ' for analysing fully drained situations (e.g., long term stability, very slow loading)

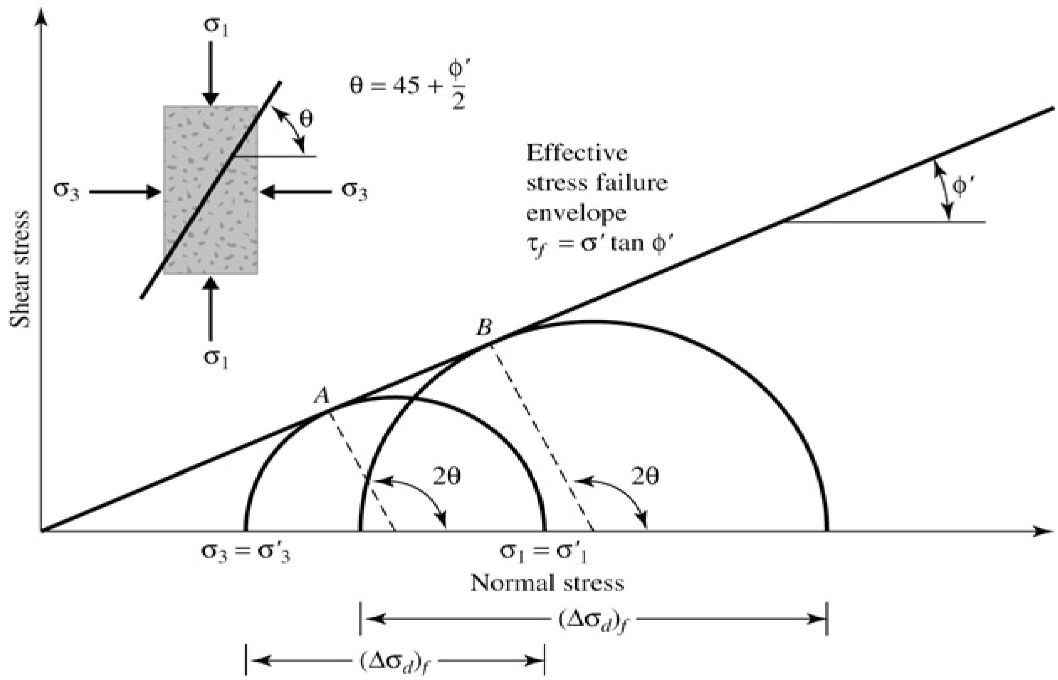
Triaxial

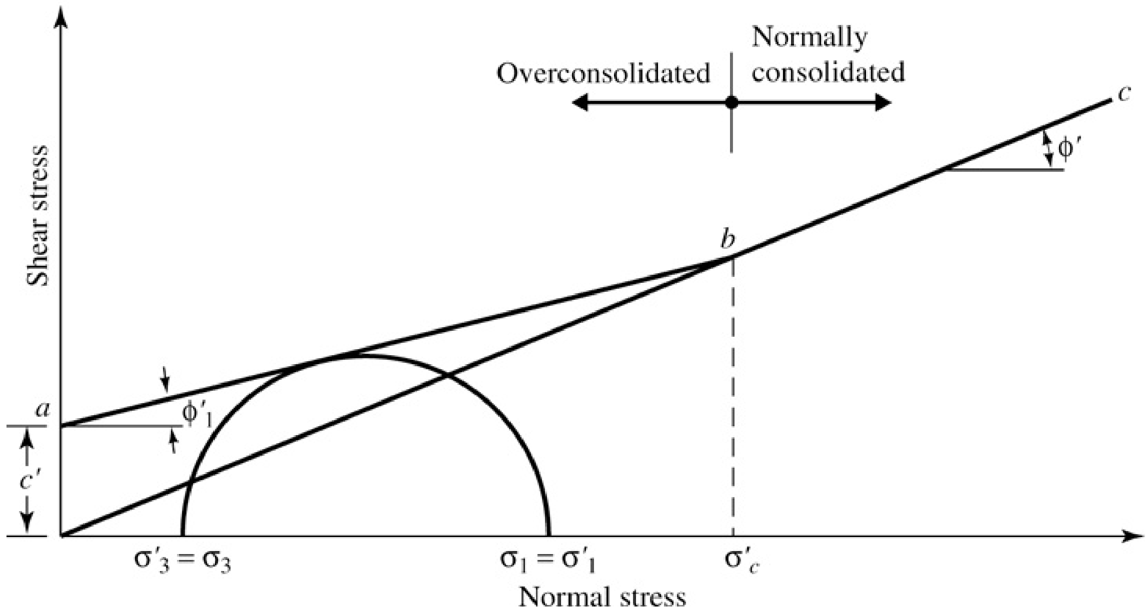


Loose sand / NC clay

Dense sand / OC Clay

Triaxial





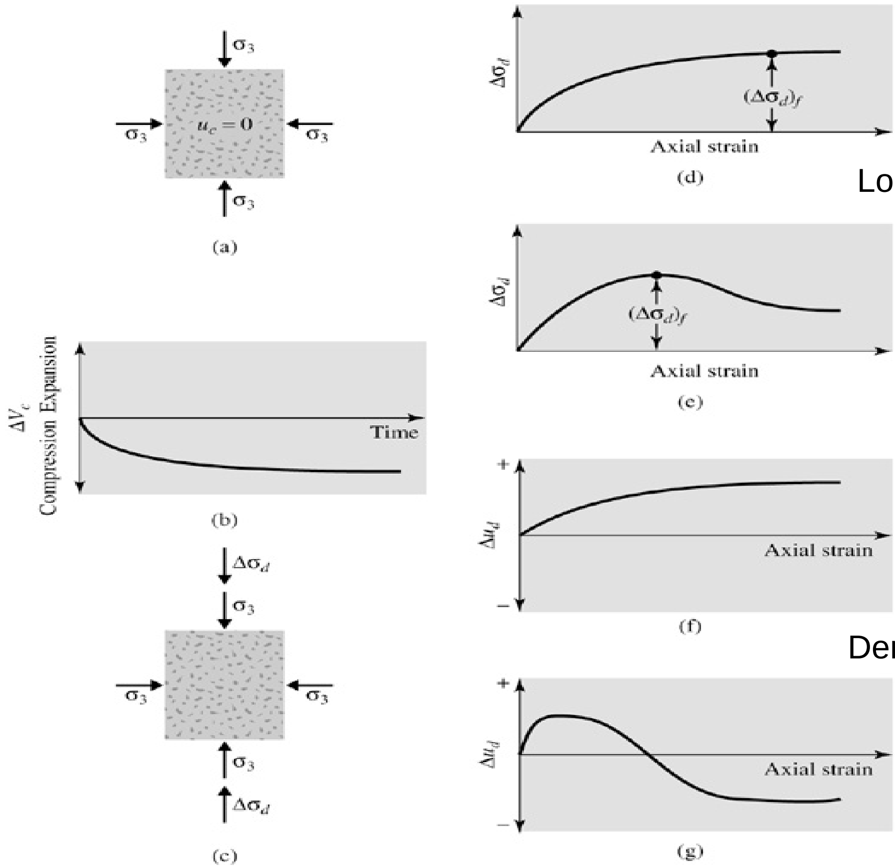
CD, CU and UU Triaxial Tests

Consolidated Undrained (CU) Test

- pore pressure develops during shear

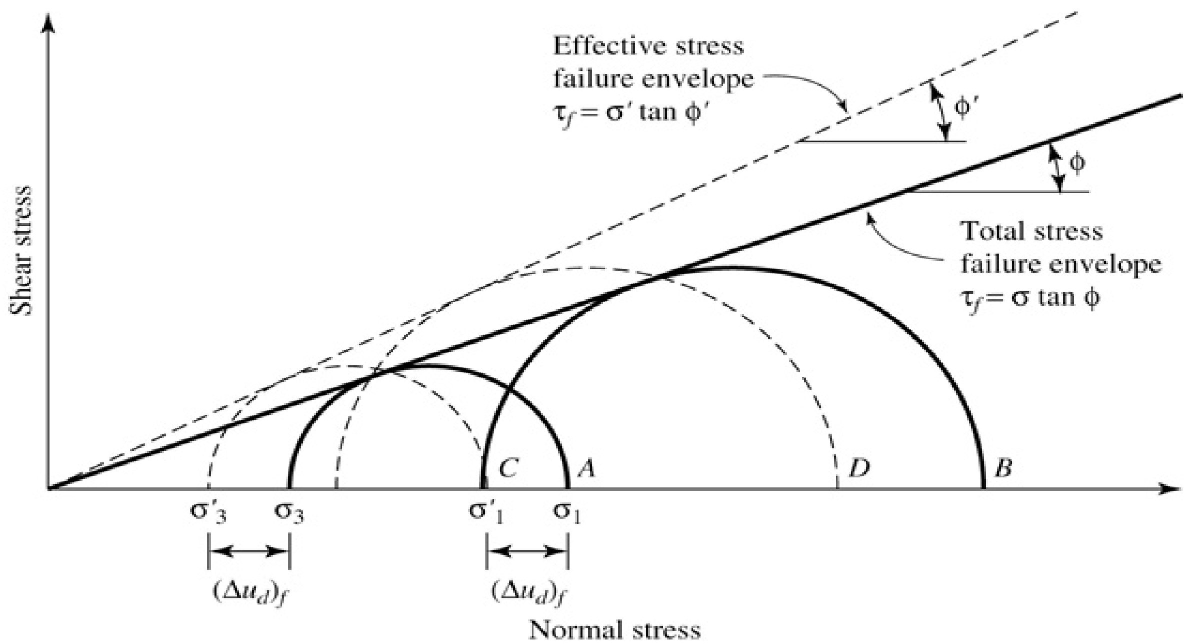
  Measure ➔ σ'

- gives c' and ϕ'
- faster than CD (∴preferred way to find c' and ϕ')



Loose sand / NC clay

Dense sand / OC Clay



CD, CU and UU Triaxial Tests

Unconsolidated Undrained (UU) Test

- pore pressure develops during shear

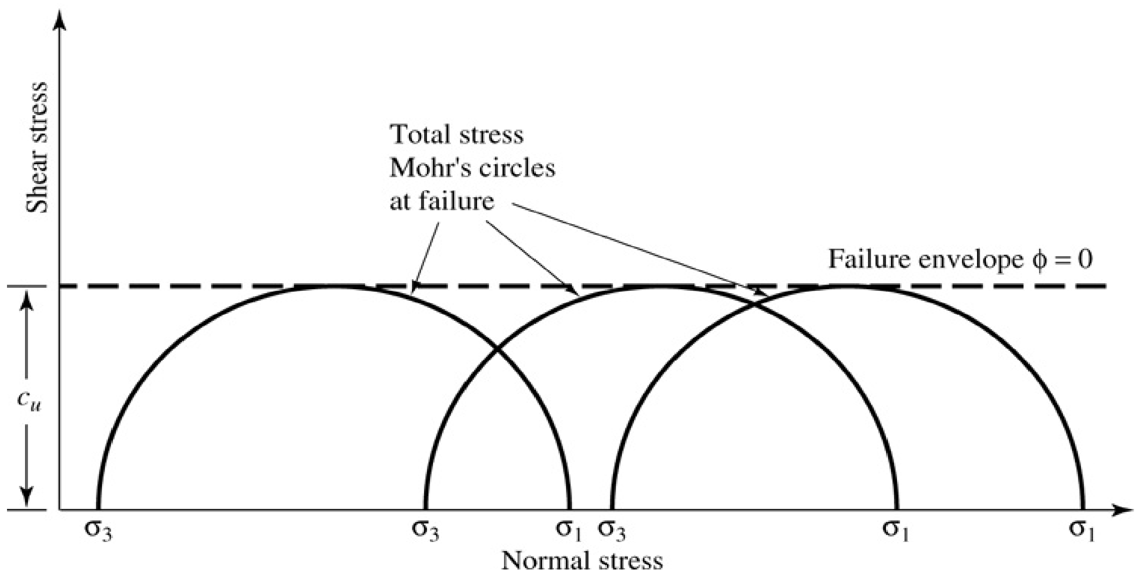
Not measured
∴σ' unknown

= **0**; i.e., failure envelope is horizontal

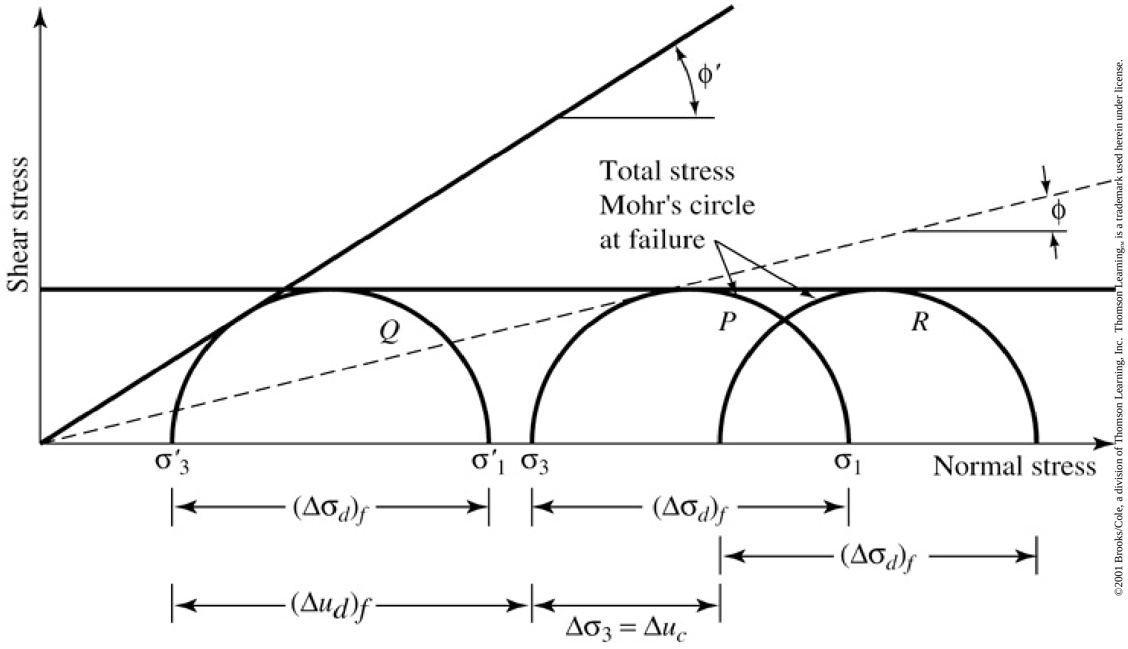
- analyze in terms of σ ➔ gives c_u and ϕ_u
- very quick test

Use c_u and ϕ_u for analysing undrained situations (e.g., short term stability, quick loading)

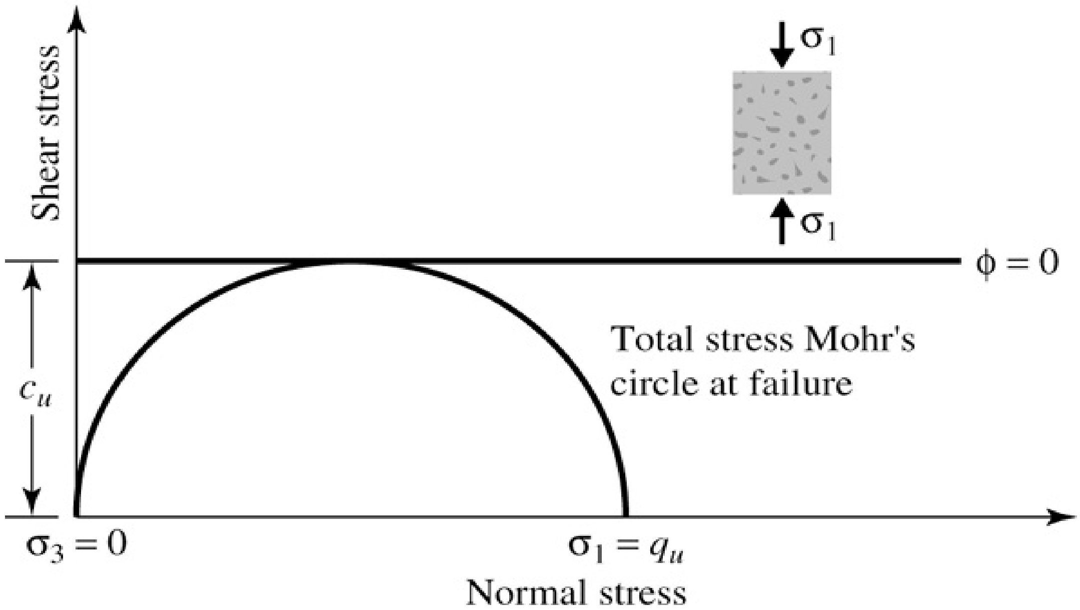
UU test on saturated clay



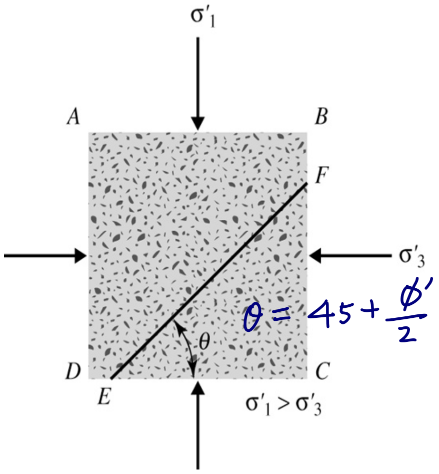
The ϕ=0 concept

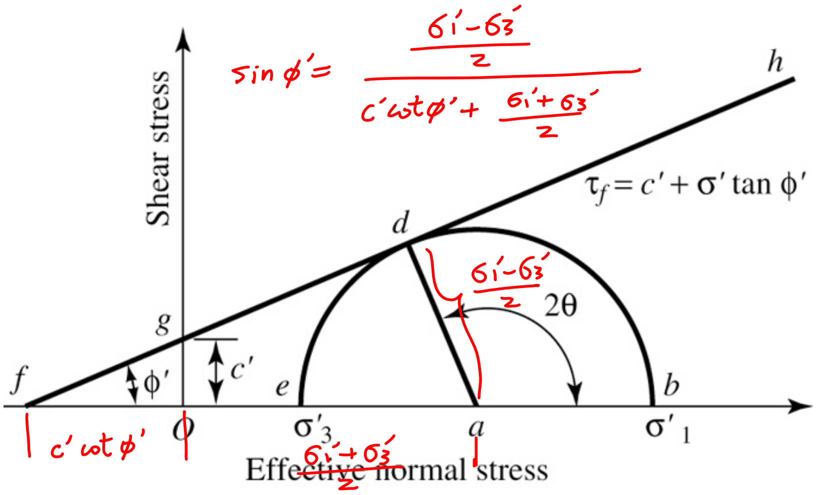


Unconfined compression test on saturated clay



σ'_1 and σ'_3 at Failure





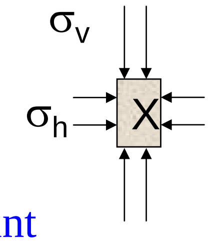
$$\sigma'_1 = \sigma'_3 \tan^2(45+\phi'/2) + 2c' \tan(45+\phi'/2)$$

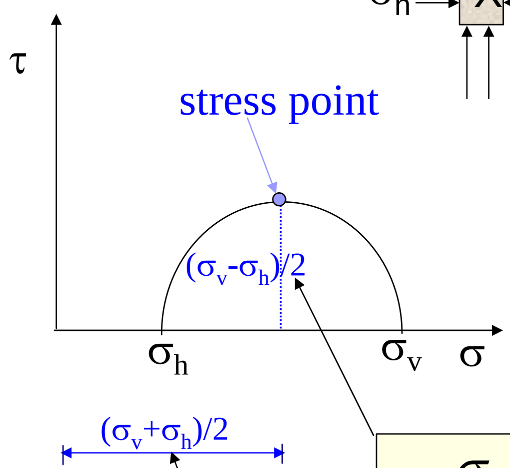
$$\sigma'_3 = \sigma'_1 \tan^2(45-\phi'/2) - 2c' \tan(45-\phi'/2)$$

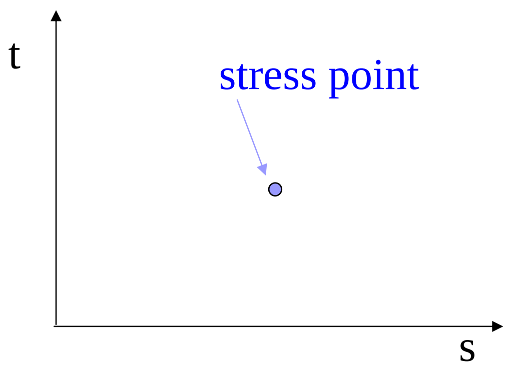
$$\sigma'_1 = \sigma'_3 \left(\frac{1+\sin\phi'}{1-\sin\phi'}\right) + 2c'\left(\frac{\cos\phi'}{1-\sin\phi'}\right)$$

$$\sigma'_3 = \sigma'_1 \left(\frac{1-\sin\phi'}{1+\sin\phi'}\right) - 2c'\left(\frac{\cos\phi'}{1+\sin\phi'}\right)$$

Stress Point





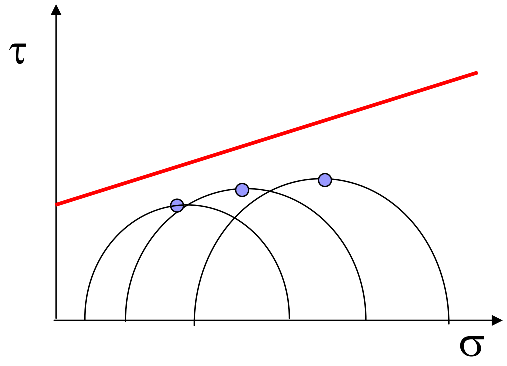


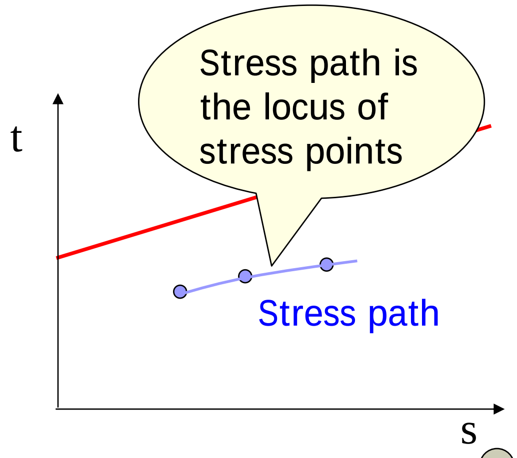
$$t = \frac{\sigma_v - \sigma_h}{2}$$

$$s = \frac{\sigma_v + \sigma_h}{2}$$

Stress Path

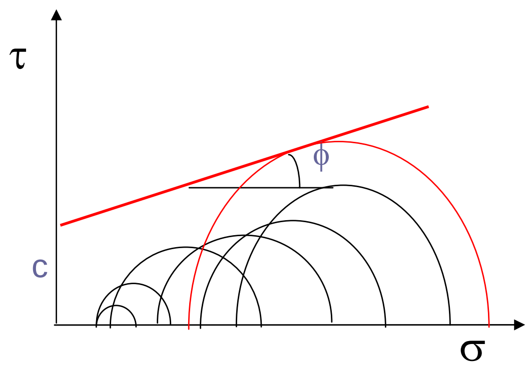
During loading…

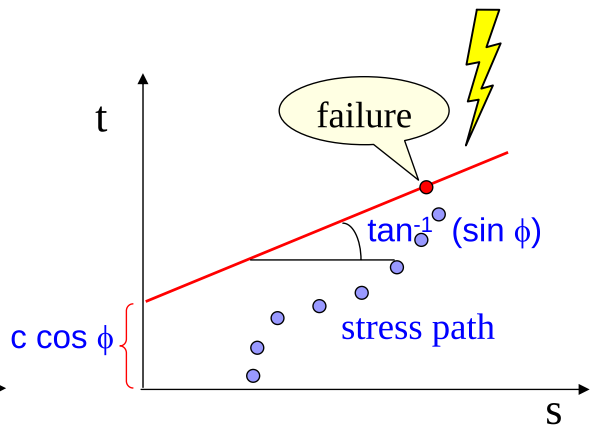




Stress path is a convenient way to keep track of the progress in loading with respect to failure envelope.

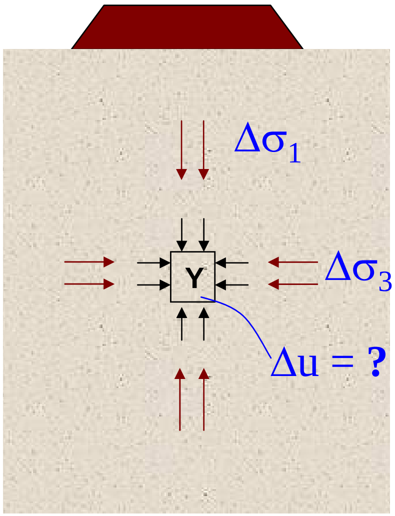
Failure Envelopes





During loading (shearing)….

Pore Pressure Parameters



A simple way to estimate the pore pressure change in undrained loading, in terms of total stress changes ~ after Skempton (1954)

$$\Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$$

Skempton's pore pressure parameters A and B

Pore Pressure Parameters

B-parameter

B = f (saturation,..)

For saturated soils, B ≈ 1.

A-parameter at failure (A_f)

$A_f = f(OCR)$

For normally consolidated clays $A_f \approx 1$.

For heavily overconsolidated clays A_f is negative.