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CHARACTERS OF INDUCED REPRESENTATIONS AND WEIGHTED ORBITAL INTEGRALS

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The main result of this paper is a formula relating characters of principal series representations of a reductive Lie group to weighted orbital integrals of wave packets.

1. Introduction. Let G be a reductive Lie group satisfying Harish-Chandra's general assumptions [**2**]. Let $P = MAN$ be the Langlands decomposition of a cuspidal parabolic subgroup of G. Denote by $\varepsilon_2(M)$ the set of equivalence classes of irreducible unitary square integrable representations of M. For $\omega \in \varepsilon_2(M)$ and $\nu \in \mathscr{F} = \mathfrak{a}^*$, the real dual of the Lie algebra of A, let $\pi_{\omega,\nu}$ be the corresponding unitary representation of G induced from P. Let f be a wave packet corresponding to ω. Then the integral of f over any regular (semisimple) orbit of G which can be represented by an element of $L = MA$ has been evaluated by Harish-Chandra in terms of the character $\Theta_{\omega,\nu}$ of $\pi_{\omega,\nu}$ [**4**].

Let γ be a regular element of G contained in a Cartan subgroup H of L. Write $H = H_K H_p$ where H_K is compact, H_p is split, and $A \subseteq H_p$. Then for suitable normalizations of the G-invariant measure $d\dot{x}$ on $H_p \setminus G$ and Haar measure $d\nu$ on $\mathscr{F}$.

$$(1.1) \quad \int_{H_p \setminus G} f(x^{-1}\gamma x)\, d\dot{x} = \varepsilon(A, H)[W(\omega)]^{-1} \int_{\mathscr{F}} \langle \Theta_{\omega,\nu}, f \rangle \Theta_{\omega,\nu}(\gamma)\, d\nu$$

where $W(\omega) = \{s \in N_G(A)/L | s\omega = \omega\}$ and $\varepsilon(A, H)$ is 1 if $H_p = A$ and is 0 otherwise. This formula can be interpreted as giving the value of $\Theta_{\omega,\nu}$ on regular elements γ of a fundamental Cartan subgroup of L in terms of the integral of a wave packet for ω over the orbit of γ. It also gives the Fourier inversion formula for the tempered invariant distribution

$$f \to \langle \Lambda(\gamma), f \rangle = \int_{H_p \setminus G} f(x^{-1}\gamma x)\, d\dot{x}$$

restricted to the subspace of $\mathscr{C}(G)$, the Schwartz space of G, spanned by wave packets corresponding to representations induced from cuspidal parabolic subgroups $P = MAN$ with $A \subseteq H_p$. The complete Fourier inversion formula for $\Lambda(\gamma)$ is much more complicated. (See [**5**].)

In the case that $P = G$ is cuspidal and $\omega \in \varepsilon_2(G)$, then Θ_ω is a discrete series character of G, and f is a matrix coefficient corresponding to ω. Formula (1.1) becomes

$$(1.2) \qquad \int_{H_p\backslash G} f(x^{-1}\gamma x)\, d\dot{x} = \varepsilon(1, H)\langle \Theta_\omega, f\rangle \Theta_\omega(\gamma).$$

Arthur has obtained the following generalization of (1.2) [**1**]. Let A be the split component of a parabolic subgroup of G. Let L be the centralizer in G of A. Corresponding to A, Arthur defines a function v_A on G which is left L-invariant. Let γ be a regular element of G contained in a Cartan subgroup $H = H_K H_p$ of L. Let $\omega \in \varepsilon_2(G)$, and let f be a matrix coefficient for ω. Then Arthur's formula is

$$(1.3) \qquad \int_{H_p\backslash G} f(x^{-1}\gamma x) v_A(x)\, d\dot{x} = (-1)^p \varepsilon(A, H)\langle \Theta_\omega, f\rangle \Theta_\omega(\gamma)$$

where p is the dimension of A. This formula gives the value of the character Θ_ω on the nonelliptic element γ in terms of a weighted orbital integral of a matrix coefficient of ω. It also gives the Fourier inversion formula for the tempered distribution

$$f \to \langle r_A(\gamma), f\rangle = \int_{H_p\backslash G} f(x^{-1}\gamma x) v_A(x)\, d\dot{x}$$

restricted to the space ${}^0\mathscr{C}(G)$ of cusp forms on G. The distributions $r_A(\gamma)$ occur in the Selberg trace formula for $\Gamma \backslash G$, Γ a discrete subgroup of G for which $\Gamma \backslash G$ has finite volume but is not compact. As formula (1.3) shows, $r_A(\gamma)$ is invariant on ${}^0\mathscr{C}(G)$. However, $r_A(\gamma)$ is not an invariant distribution on $\mathscr{C}(G)$, and the full Fourier inversion formula for $r_A(\gamma)$ is not known.

Authur's formula can be generalized to the setting of induced representations and wave packets. Let $P = MAN$ be a cuspidal parabolic subgroup of G, and let A_1 be the split component of a parabolic subgroup of $L = MA$, L_1 its centralizer in G. Let γ be a regular element of G contained in a Cartan subgroup $H = H_K H_p$ of L_1. We will define a left L_1-invariant function $v_{A_1}^P$ on G with the following properties.

If f' is a wave packet coming from a cuspidal parabolic subgroup $P' = M'A'N'$ of G with $\dim A' \leq \dim A$ and A not conjugate to A', then

$$(1.4) \qquad \int_{H_p\backslash G} f'(x^{-1}\gamma x) v_{A_1}^P(x)\, d\dot{x} = 0.$$

Now let f be a wave packet corresponding to $\omega \in \varepsilon_2(M)$. Then

$$(1.5) \qquad \int_{H_p\backslash G} f(x^{-1}\gamma x) v_{A_1}^P(x)\, d\dot{x} = 0 \quad \text{if } H_p \neq A_1.$$

If $H_p = A_1$, let $\gamma = \gamma_1, \gamma_2, \ldots, \gamma_k$ be a complete set of elements of L for which $\gamma_i = x_i \gamma x_i^{-1}$ for some $x_i \in G$, but γ_i and γ_j are not conjugate in L for $1 \leq i \neq j \leq k$. Let $A_i = x_i A_1 x_i^{-1}$. Then

$$(1.6) \qquad \begin{aligned} &\int_{H_p\backslash G} f(x^{-1}\gamma x) \sum_{i=1}^{k} v_{A_i}^P(x_i x)\, d\dot{x} \\ &\qquad = (-1)^{p_1}[W(\omega)]^{-1} \int_{\mathscr{F}} \langle \Theta_{\omega,\nu}, f\rangle \Theta_{\omega,\nu}(\gamma)\, d\nu \end{aligned}$$

where p_1 is the dimension of $A_1 \cap M$.

Formulas (1.4)–(1.6) are proved by using Arthur's formula and results of Harish-Chandra relating characters and orbital integrals on G to those on M and L. Any unexplained notation follows that of Harish-Chandra [**2**, **3**, **4**].

2. Background material. Let G be a real reductive Lie group, $\mathfrak{g}$ the Lie algebra of G. Let K be a maximal compact subgroup of G, θ the Cartan involution of G corresponding to K, and B a real symmetric bilinear form on $\mathfrak{g}$. Assume that (G, K, θ, B) satisfy the general assumptions of Harish-Chandra in [**2**] and that Haar measures are normalized as in [**2**]. Given a θ-stable Cartan subgroup H of G, we will write $H = H_K H_p$ where $H_K = H \cap K$ and H_p is a vector subgroup with Lie algebra $\mathfrak{h}_p$ contained in the -1 eigenspace for θ. Let G' be the set of regular semisimple elements of G, $H' = H \cap G'$. If J is any subgroup of G, we will write $N_G(J)$ and $C_G(J)$ for the normalizer and centralizer of J in G, respectively, and $W(G, J) = N_G(J)/C_G(J)$.

We will first review some definitions and formulas of Harish-Chandra from [**2**, **3**, **4**]. Fix a double unitary representation τ of K on a finite-dimensional Hilbert space V. Let $\mathscr{C}(G, \tau)$ and ${}^0\mathscr{C}(G, \tau)$ denote the τ-spherical functions in the spaces of V-valued Schwartz functions $\mathscr{C}(G, V)$ and V-valued cusp forms ${}^0\mathscr{C}(G, V)$ respectively. Let F_0 be the operator on V given by

$$F_0 v = \int_K \tau(k^{-1}) v \tau(k)\, dk, \qquad v \in V.$$

For $f \in \mathscr{C}(G, V)$ and $x \in G$, define $\bar{f}(x) = \int_K f(k^{-1}xk)\, dk$. Then if $f \in \mathscr{C}(G, \tau)$, $\bar{f}(x) = F_0 f(x)$, $x \in G$.

Fix a cuspidal parabolic subgroup $P = MAN$, that is, a parabolic subgroup of G with $\varepsilon_2(M) \neq \varnothing$. Let τ_M be the restriction of τ to $K_M = K \cap M$. For any $f \in \mathscr{C}(G, V)$, $m \in M$, and $a \in A$, let

$$(2.1) \qquad f^{(P)}(ma) = f_a^{(P)}(m) = \delta_P^{1/2}(a) \int_N f(man)\, dn$$

where δ_P is the module of P. Then $f_a^{(P)} \in \mathscr{C}(M, V)$, $f^{(P)} \in \mathscr{C}(MA, V)$, and the following relationships between f and $f^{(P)}$ can be found in or easily derived from results in [**2**, **3**, **4**].

Let H be a θ-stable Cartan subgroup of L. for $f \in \mathscr{C}(G, V)$ and $h \in H'$,

$$(2.2) \qquad \int_N f(n^{-1}hn)\, dn = \Delta_+^G(h)^{-1} \Delta_+^L(h) f^{(P)}(h)$$

where Δ_+^L and Δ_+^G are the functions Δ_+ on H, considered as a Cartan subgroup of L and G respectively, defined by Harish-Chandra in [**2**].

For $\nu \in \mathscr{F} = \mathfrak{a}^*$ and $m \in M$, define

$$(2.3) \qquad f_\nu^{(P)}(m) = \int_A f^{(P)}(ma) e^{-i\nu(\log a)}\, da.$$

Then because $d\nu$ is the dual measure to da on A and $f^{(P)}$ is rapidly decreasing in the A variable,

$$(2.4) \qquad f^{(P)}(ma) = \int_{\mathscr{F}} f_\nu^{(P)}(m) e^{i\nu(\log a)}\, d\nu.$$

For $\omega \in \varepsilon_2(M)$ and $\nu \in \mathscr{F}$, let $\pi_{\omega,\nu}$ be the tempered unitary representation of G induced from $\omega \otimes e^{i\nu} \otimes 1$ on MAN. Let $\Theta_{\omega,\nu}$ and Θ_ω denote the characters of $\pi_{\omega,\nu}$ and ω considered as functions on G' and M' respectively. For $f \in \mathscr{C}(G, \tau)$, $g \in \mathscr{C}(M, \tau_M)$, define

$$\langle \Theta_{\omega,\nu}, f \rangle = \int_G f(x) \overline{\Theta_{\omega,\nu}(x)} dx \quad \text{and} \quad \langle \Theta_\omega, g \rangle = \int_M g(m) \overline{\Theta_\omega(m)} dm.$$

Then, for $f \in \mathscr{C}(G, \tau)$, $\nu \in \mathscr{F}$, $f_\nu^{(P)} \in \mathscr{C}(M, \tau_M)$ and

$$(2.5) \qquad \langle \Theta_{\omega,\nu}, f \rangle = F_0 \langle \Theta_\omega, f_\nu^{(P)} \rangle.$$

For $\omega \in \varepsilon_2(M)$, let $L(\omega) = {}^0\mathscr{C}(M, \tau_M) \cap \mathfrak{h}_\omega \otimes V$ where $\mathfrak{h}_\omega$ is the closed subspace of $L^2(M)$ spanned by matrix coefficients for ω. For $\psi \in L(\omega)$, $\alpha \in C_c^\infty(\mathscr{F})$, and $x \in G$, define

$$(2.6) \qquad \varphi_\alpha(x) = \int_{\mathscr{F}} \alpha(\nu) E(P : \psi : \nu : x) \mu(\omega : \nu)\, d\nu$$

where $E(P:\psi:\nu)$ is the Eisenstein integral defined in [2], and $\mu(\omega:\nu)$ is the Plancherel factor corresponding to $\pi_{\omega,\nu}$. Then $\varphi_\alpha \in \mathscr{C}(G,\tau)$ is called a wave packet for $\omega \in \varepsilon_2(M)$, and for $\nu \in \mathscr{F}$, $(\varphi_\alpha)_\nu^{(P)}$ belongs to $\sum_{s\in W(G,A)} L(s\omega)$ and is supported on a compact subset of $\mathscr{F}$.

We now turn to Arthur's results. Let A be a special vector subgroup of G, that is, the split component of a parabolic subgroup of G. Write $\mathscr{P}(A)$ for the (finite) set of all parabolic subgroups of G having A as split component. For $P \in \mathscr{P}(A)$ let Φ_P denote the set of simple roots of (P, A). We identify $\mathfrak{a}$, the Lie algebra of A, and its dual via the bilinear form B. A set $\mathscr{Y} = \{Y_P | P \in \mathscr{P}(A)\}$ of points in $\mathfrak{a}$ is called A-orthogonal if for any pair of adjacent parabolic subgroups $P, P' \in \mathscr{P}(A)$, $Y_P - Y_{P'} = r\alpha$, $r \in \mathbf{R}$, where α is the unique element of Φ_P with $-\alpha \in \Phi_{P'}$. Let

$$\mathfrak{a}^0 = \{H \in \mathfrak{a} | \langle \alpha, H\rangle = 0 \text{ for every root } \alpha \text{ of } (\mathfrak{g}, \mathfrak{a})\},$$

$\mathfrak{a}^1$ its orthogonal complement in A. Let p be the dimension of $\mathfrak{a}^1$, and let $c_A = |\det C|^{1/2}$ where C is the Cartan matrix for the roots of $(\mathfrak{g}, \mathfrak{a})$. For any $P = MAN \in \mathscr{P}(A)$ and $x \in G$, write

$$x = m(x)\exp(H_P(x))n(x)k(x)$$

where $m(x) \in M$, $n(x) \in N$, $k(x) \in K$, and $H_P(x) \in \mathfrak{a}$. For any A-orthogonal set $\mathscr{Y}$ and $\lambda \in \mathfrak{a}^1_{\mathbf{C}}$, define

$$v(x:\mathscr{Y}) = c_A(p!)^{-1} \sum_{P\in\mathscr{P}(A)} \frac{\langle \lambda, Y_P - H_P(x)\rangle^p}{\prod_{\alpha\in\Phi_P}\langle\lambda,\alpha\rangle}. \tag{2.7}$$

Then $v(x:\mathscr{Y})$ is independent of λ and is left-invariant under $L = C_G(A)$. It is also clearly right K-invariant. If $v_A(x) = v(x:\mathscr{Y})$ for any A-orthogonal set $\mathscr{Y}$, then (1.3) is valid.

3. The distributions. Fix a cuspidal parabolic subgroup $P = MAN$ of G. Let A_1^M be a special vector subgroup of M, $A_1 = A_1^M A$. Let $\mathscr{Y}_1$ be an A_1^M-orthogonal set, and let $v_1^M(m) = v(m:\mathscr{Y}_1)$, $m \in M$, be the function on M defined as in (2.7) with respect to A_1^M and $\mathscr{Y}_1$. Extend v_1^M to a function v_1 on G by setting

$$v_1(mank) = [W(G,A)]^{-1} v_1^M(m), \qquad m \in M, a \in A, n \in N, k \in K. \tag{3.1}$$

This extension is well defined since v_1^M is right K_M-invariant. Since v_1^M if left-invariant under $L_1^M = C_M(A_1^M)$, v_1 is left-invariant under $L_1 = L_1^M A = C_G(A_1)$.

Let H be a θ-stable Cartan subgroup of G with $A_1 \subseteq H_p$. Write $J = H \cap M$. Let $h \in H'$. For $f \in C_c^\infty(G, V)$, define

$$\langle r_1(h), f\rangle = \int_{H_p\backslash G} f(x^{-1}hx)v_1(x)\, d\dot{x}. \tag{3.2}$$

LEMMA 3.3. *For any $h \in H'$ the distribution $r_1(h)$ is tempered. For any $f \in \mathscr{C}(G, V)$, $\int_{H_p\backslash G} f(x^{-1}hx)v_1(x)\, d\dot{x}$ is absolutely convergent and*

$$\begin{aligned}\langle r_1(h), f\rangle &= \int_{H_p\backslash G} f(x^{-1}hx)v_1(x)\, d\dot{x}\\ &= [W(G, A)]^{-1}\Delta_+^G(h)^{-1}\Delta_+^L(h)\int_{J_p\backslash M} \bar{f}^{(P)}(m^{-1}hm)v_1^M(m)\, d\dot{m}.\end{aligned}$$

Proof. Let $f \in \mathscr{C}(G, V)$. Write $h = ja$ where $j \in J'$, $a \in A$. Then using (2.2) and (3.1),

$$\begin{aligned}&\int_{H_p\backslash G} |f(x^{-1}hx)v_1(x)|\,d\dot{x}\\ &\quad= [W(G, A)]^{-1}\int_{J_p\backslash M} |v_1^M(m)|\int_{NK} |f(k^{-1}n^{-1}m^{-1}hmnk)|\,dn\, dk\, d\dot{m}\\ &\quad= [W(G, A)]^{-1}\Delta_+^G(h)^{-1}\Delta_+^L(h)\int_{J_p\backslash M} |v_1^M(m)\bar{f}_a^{(P)}(m^{-1}jm)|\,d\dot{m}\end{aligned}$$

since Δ_+^G and Δ_+^L are invariant under conjugation by M. The lemma now follows since for any $a \in A$, $f \to \bar{f}_a^{(P)}$ is a continuous map from $\mathscr{C}(G, V)$ to $\mathscr{C}(M, V)$ [**2**]. Further, for $g \in \mathscr{C}(M, V)$, $j \in J'$,

$$\int_{J_p\backslash M} g(m^{-1}jm)v_1^M(m)\, d\dot{m}$$

is absolutely convergent and defines a tempered distribution [**1**]. □

COROLLARY 3.4. *Let A' be a special vector subgroup of G with* dim A' $\leq$ dim A. *Let $P' = M'A'N' \in \mathscr{P}(A')$, $\omega' \in \varepsilon_2(M')$. Let f be a wave packet defined as in* (2.6) *with respect to ω' and P'. Then $\langle r_1(h), f\rangle = 0$ unless A' is conjugate to A under K.*

Proof. In this case $\bar{f}^{(P)} = 0$ [**4**]. Thus the result follows from (3.3). □

LEMMA 3.5 *Suppose that* $f = \varphi_\alpha$ *is a wave packet associated to* $\omega \in \varepsilon_2(M)$. *Let* $h \in H'$. *Then*

$$\langle r_1(h), f\rangle = [W(G, A)]^{-1}(-1)^{p_1}\varepsilon(A_1, H)[W(\omega)]^{-1}\Delta_+^G(h)^{-1}\Delta_+^L(h) \cdot \int_{\mathscr{F}} \langle \Theta_{\omega,\nu}, f\rangle \sum_{s\in W(G,A)} (\Theta_{s\omega} \otimes e^{is\nu})(h)\, d\nu$$

where $p_1 = \dim A_1^M$.

Proof. Using (3.3) and (2.4),

$$\langle r_1(h), f\rangle = [W(G,A)]^{-1}\Delta_+^G(h)^{-1}\Delta_+^L(h) F_0 \cdot \int_{J_p\backslash M} v_1^M(m) \int_{\mathscr{F}} e^{i\nu(\log a)} f_\nu^{(P)}(m^{-1}jm)\, d\nu\, d\dot{m}.$$

Since $f_\nu^{(P)} \in \mathscr{C}(M, V)$ and is supported on a compact subset of $\mathscr{F}$, we can interchange the order of integration. Let $W = W(G, A)$, and write $f_\nu^{(P)} = \sum_{s\in W/W(\omega)} g_s$ where $g_s \in L(s\omega)$. Then, using (1.3),

$$\int_{J_p\backslash M} g_s(m^{-1}jm) v_1^M(m)\, d\dot{m} = (-1)^{p_1}\varepsilon(A_1^M, J)\langle \Theta_{s\omega}, g_s\rangle \Theta_{s\omega}(j).$$

But $\varepsilon(A_1^M, J) = \varepsilon(A_1, H)$, and $\langle \Theta_{s\omega}, g_{s'}\rangle = 0$ if $s\omega \neq s'\omega$. Thus using (2.5),

$$\begin{aligned} F_0 \int_{J_p\backslash M} & f_\nu^{(P)}(m^{-1}jm) v_1^M(m)\, d\dot{m} \\ &= (-1)^{p_1}\varepsilon(A_1, H) \sum_{s\in W/W(\omega)} F_0\langle \Theta_{s\omega}, f_\nu^{(P)}\rangle \Theta_{s\omega}(j) \\ &= (-1)^{p_1}\varepsilon(A_1, H)[W(\omega)]^{-1} \sum_{s\in W} \langle \Theta_{s\omega,\nu}, f\rangle \Theta_{s\omega}(j). \end{aligned}$$

Now for each $s \in W$,

$$\int_{\mathscr{F}} e^{i\nu(\log a)}\Theta_{s\omega}(j)\langle \Theta_{s\omega,\nu}, f\rangle d\nu = \int_{\mathscr{F}} e^{is\nu(\log a)}\Theta_{s\omega}(j)\langle \Theta_{\omega,\nu}, f\rangle d\nu$$

since $\Theta_{s\omega,s\nu} = \Theta_{\omega,\nu}$. □

Now suppose that H is a Cartan subgroup of G with $H_p = A_1$, and fix $h \in H'$. Let $h_i = x_i h x_i^{-1}$, $1 \leq i \leq k$, be defined as in (1.6). Then using results from **[6, 7]**, for $\omega \in \varepsilon_2(M)$ and $\nu \in \mathscr{F}$,

$$(3.6)\qquad \Theta_{\omega,\nu}(h) = \sum_{i=1}^{k} \Delta_+^G(h_i)^{-1}\Delta_+^L(h_i)(\Theta_\omega \otimes e^{i\nu})(h_i).$$

Fix $1 \le i \le k$. Let $A_i = x_i A_1 x_i^{-1}$, $A_i^M = A_i \cap M$. Let $L_i = C_G(A_i)$. Then A_i^M is a special vector subgroup of M. Let $\mathscr{Y}_i$ be any A_i^M-orthogonal set, and define v_i on G as in (3.1) starting from v_i^M. Then v_i is left L_i-invariant so that $x \to v_i(x_i x)$ is left L_1-invariant. For $x \in G$ define

$$(3.7)\qquad \langle r(h), f\rangle = \int_{H_p\backslash G} f(x^{-1}hx) \sum_{i=1}^{k} v_i(x_i x)\, d\dot{x}.$$

THEOREM 3.8. *Let H be a Cartan subgroup of G with $H_p = A_1$, $h \in H'$. Then $r(h)$ is a tempered distribution, and for f a wave packet corresponding to $\omega \in \varepsilon_2(M)$,*

$$\langle r(h), f\rangle = (-1)^{p_1}[W(\omega)]^{-1}\int_{\mathscr{F}} \langle \Theta_{\omega,\nu}, f\rangle \Theta_{\omega,\nu}(h)\, d\nu.$$

Proof. Define $x_1,\ldots,x_k$ and $h_1,\ldots,h_k$ as in (3.6). Then for $1 \le i \le k$, $H_i = x_i H x_i^{-1}$ is a Cartan subgroup of G with $A_i = (H_i)_p$ so that using (3.3),

$$\begin{aligned}\langle r(h), f\rangle &= \sum_{i=1}^{k} \int_{H_p\backslash G} f(x^{-1}hx) v_i(x_i x)\, d\dot{x}\\ &= \sum_{i=1}^{k} \int_{(H_i)_p\backslash G} f(x^{-1}h_i x) v_i(x)\, d\dot{x}\\ &= (-1)^{p_1}\varepsilon(A_1, H)[W(\omega)]^{-1}\int_{\mathscr{F}} \langle \Theta_{\omega,\nu}, f\rangle \varphi(\omega,\nu,h)\, d\nu\end{aligned}$$

where

$$\begin{aligned}\varphi(\omega,\nu,h) &= [W]^{-1}\sum_{s\in W}\sum_{i=1}^{k} \Delta_+^G(h_i)^{-1}\Delta_+^L(h_i)\Theta_{s\omega} \otimes e^{is\nu}(h_i)\\ &= [W]^{-1}\sum_{s\in W} \Theta_{s\omega,s\nu}(h) = \Theta_{\omega,\nu}(h).\end{aligned}$$

□

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