Surveying

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Achievement:

- **Selected Scientist, NEERI-CSIR, Govt. of India.**
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UNIT-VI

1) Tacheometer

2) Tacheometric surveying

3) Classification

4) Principle of stadia method

5) Distance calculation by stadia method

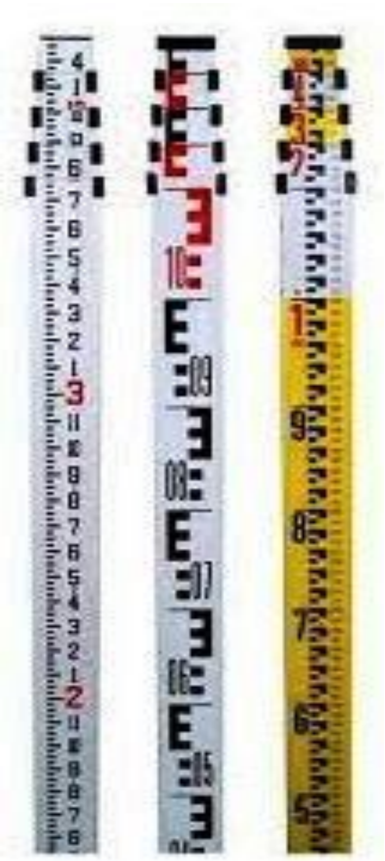
6) Elevation calculation by stadia method

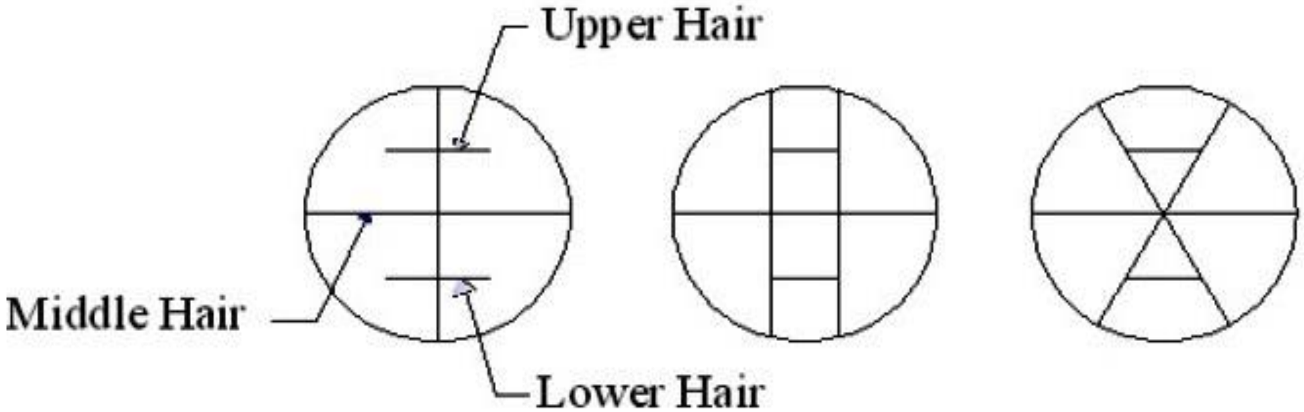
References:-

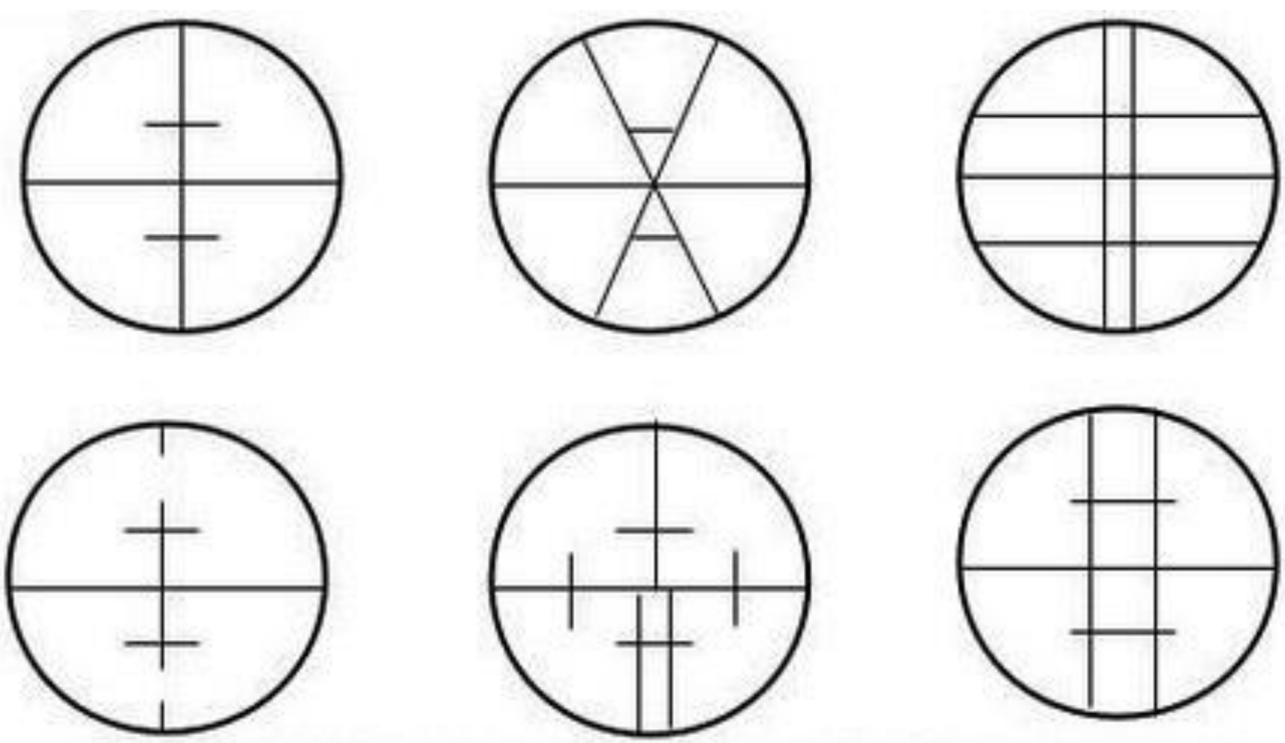
SN	Title	Authors	Publisher
1	Surveying and Leveling (Vol-I & II)	Kanitkar T. P., Kulkarni S. V.	Pune Vidyarthi Griha Prakashan, Pune.
2	Surveying and Leveling (Vol–I & II)	Punmia B. C., Jain A. K., Jain A. K.	Laxmi Publication(P) Ltd. New Delhi.
3	Surveying & Leveling	Basak N. N.	Tata McGraw Hill Pub. Co. Ltd. New Delhi

Tacheometry Surveying:

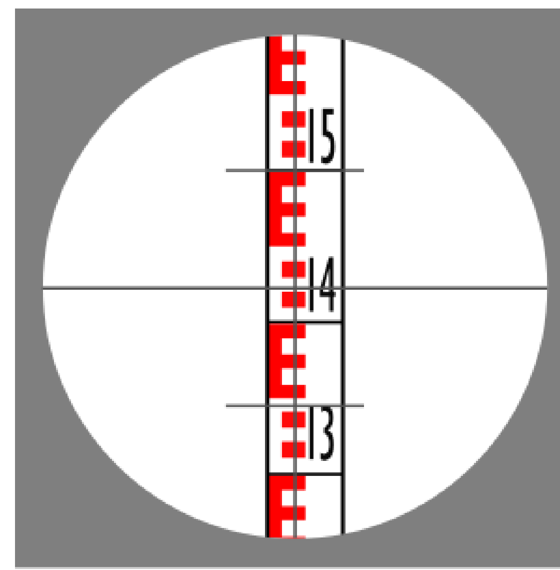
- It is branched of surveying in which horizontal and vertical distance are determined by taking **angular observations** with an instrument known as a tacheometer.
- The **chaining** operation is completely **eliminated.**
- It is adopted in **rough and difficult terrain** where direct leveling and chaining are either not possible or very tedious.
- It is also in location survey for railways, roads, reservoir, etc.







Different forms of stadia diaphragm commonly used



Tacheometer and Its Characteristics:

1) It is a **Transit Theodolite** fitted with **stadia diaphragm**.

2) Telescope should be fitted with an **anallactic lens**.

3) **Magnification power** of the eyepiece is kept high.

4) Value of multiplying constant, k=100.

5) Value of the additive constant, C=0

Stadia Rod:

1) It is also called as **vertical stave.**

2) It is a **5-15m** long rod for distance more than **100m.**

3) It can be held **vertical or normal** to the **line of sight.**

Tacheometry:

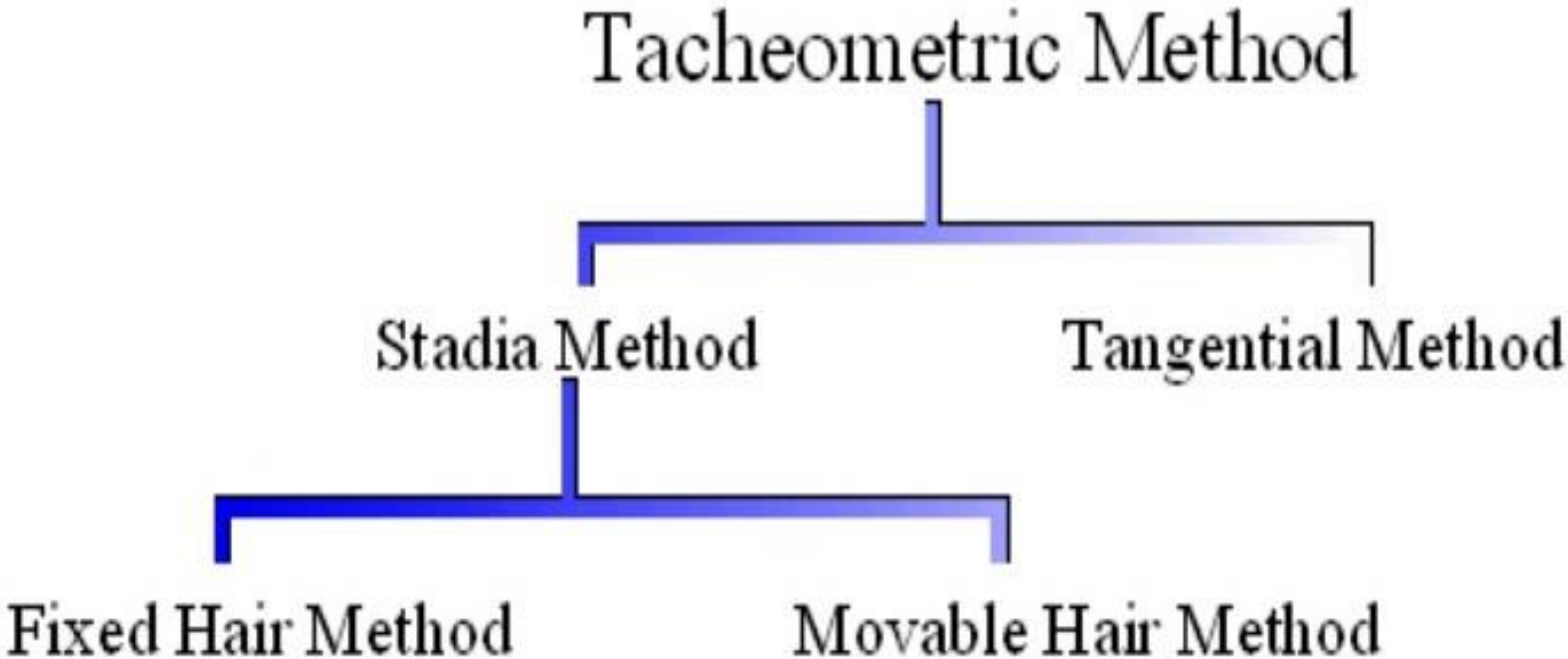
1) It is defined as an **optical distance measurement** method.
2) Tachymetry or Telemetry
3) As compared to chaining on flat grounds, the accuracy of tacheometric distance is low but on **rough and steep grounds the accuracy is more**.

Tacheometry Surveying: Why we do?

1) To measure **horizontal distance** which can't be measured by tape or chain as in hilly area.
2) To measure **vertical distance** by angular observations.
3) In **difficult areas** where leveling and other types of surveys are tedious to conduct
4) Suitable for **overcoming obstacles** such as rough & broken ground deep rivers ponds etc.
5) For preparing **contour plans**.
6) For conducting hydrographic and land survey.
7) For traversing in **difficult terrain**.

Methods of Tacheometry:

Tacheometry are of **two types** mainly: **Stadia & Tangential Method**



11.5 METHODS OF TACHEOMETRY

Tacheometry involves mainly two methods:

1. The stadia method
2. The tangential method.

1. The stadia method In this method the diaphragm of the tacheometer is provided with two stadia hairs (upper and lower). Looking through the telescope the stadia hair readings are taken. The difference in these readings gives the staff intercept. To determine the distance between the station and the staff, the staff intercept is multiplied by the stadia constant (i.e. multiplying constant, 100). The stadia method may, in turn, be of two kinds.

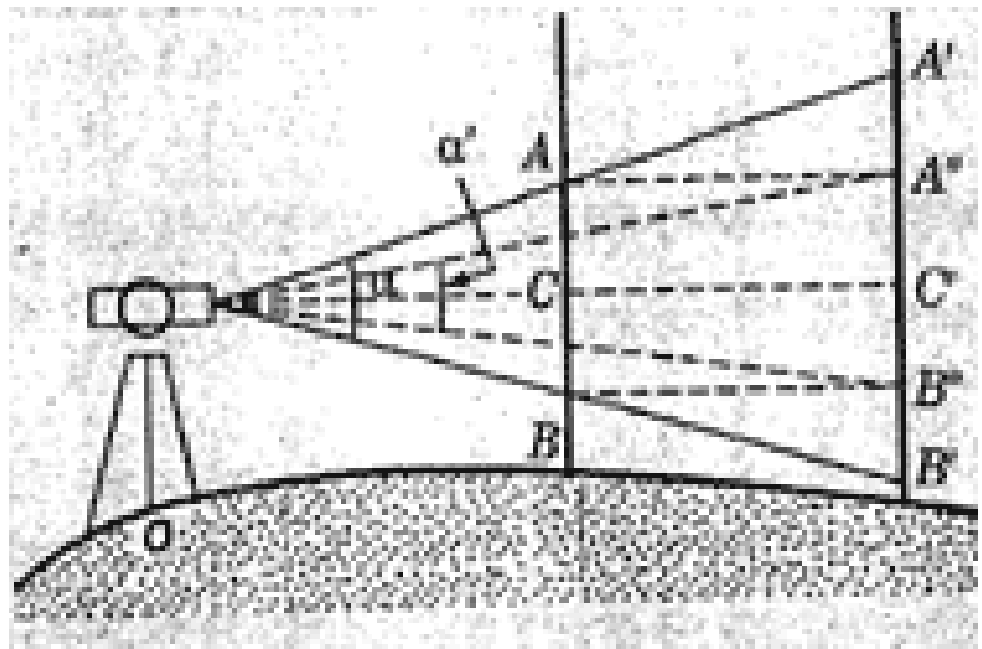
(a) The Fixed Hair Method The distance between the stadia hairs is fixed in this method, which is the one commonly used. When the staff is sighted through the telescope, a certain portion of the staff is intercepted by the upper and lower stadia. The value of the staff intercept varies with the distance. However, the distance between the station and the staff can be obtained by multiplying the staff intercept by the stadia constant.

2. The tangential method In this method, the diaphragm of the tacheometer is not provided with stadia hair. The readings are taken by the single horizontal hair.

Stadia Method of Tacheometry: (**Fixed Hair** & Moveable Hair Method)

1) Various wires, in addition to the cross wires on the diaphragm are known as stadia wires and the vertical distance between these stadia wires is termed as stadia interval.

2) When the **parallactic angle α ,** defined with the help of stadia wires, is kept **fixed** and the **staff intercept** is **varied,** ex. AB and A'B', the method is known as fixed hair method.

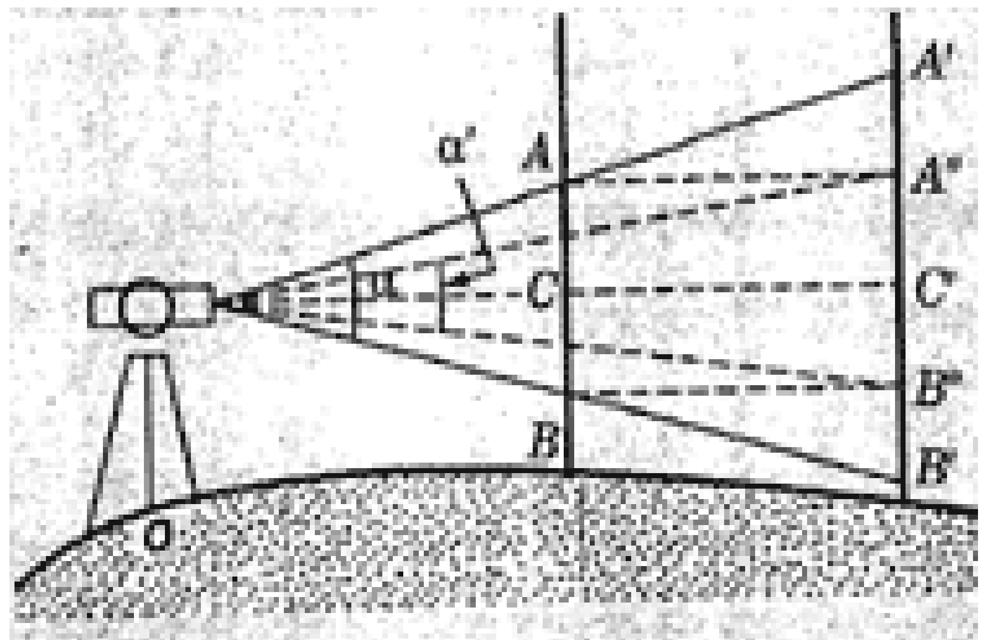


Stadia method

Stadia Method of Tacheometry: (Fixed Hair **& Moveable Hair Method)**

1) Another way used to make observation is to keep the **staff intercept fixed** ex. AB and A”B” and **vary the parallactic angle**, ex. α & α’

2) In this method, **stadia wires** will have to be **moved** and is accordingly called as **movable hair** or subtense method.

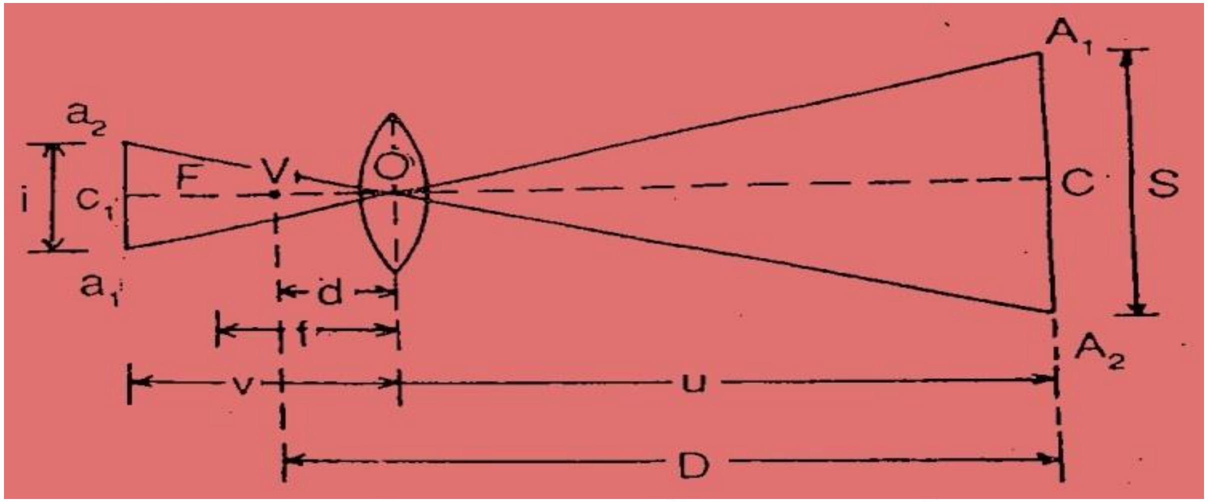


Stadia method

Tangential Method of Tacheometry:

1) Observations are made for vertical angles and staff intercepts are obtained with the cross wires only.
2) This method of tacheometry is quite **similar** to the method of **trignometrical levelling.**

Theory of stadia Tacheometery



A_1, A_2, C = readings on staff cut by three hairs

a_1, a_2, C = bottom, top and central hairs of diaphragm

$a_1a_2 = i$ = length of image

$A_1A_2 = S$ = staff intercept

F = focus

V = vertical axis of instrument

f = focal length of object glass

d = distance between optical centre and vertical axis of instrument

u = distance between optical centre and staff

v = distance between optical centre and image

3. *Principle of tacheometry* The principle of tacheometry is based on the property of isosceles triangles, where the ratio of the distance of the base from the apex and the length of the base is always constant.

In Fig. 11.2, $qo_1a_1a_2$, $qo_1b_1b_2$, and $qo_1c_1c_2$ are all isosceles triangles where D_1, D_2 and D_3 are the distances of the bases from the apices, and S_1, S_2 and S_3 are the lengths of the bases (staff intercepts).

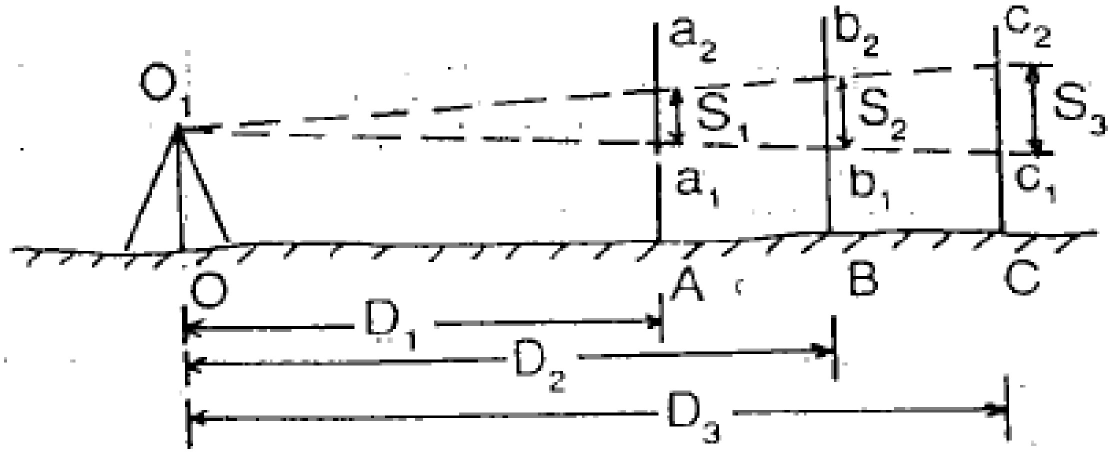


Fig. 11.2

So, according to the stated principle,

$$\frac{D_1}{S_1} = \frac{D_2}{S_2} = \frac{D_3}{S_3} = \frac{f}{i} \qquad \text{(constant)}$$

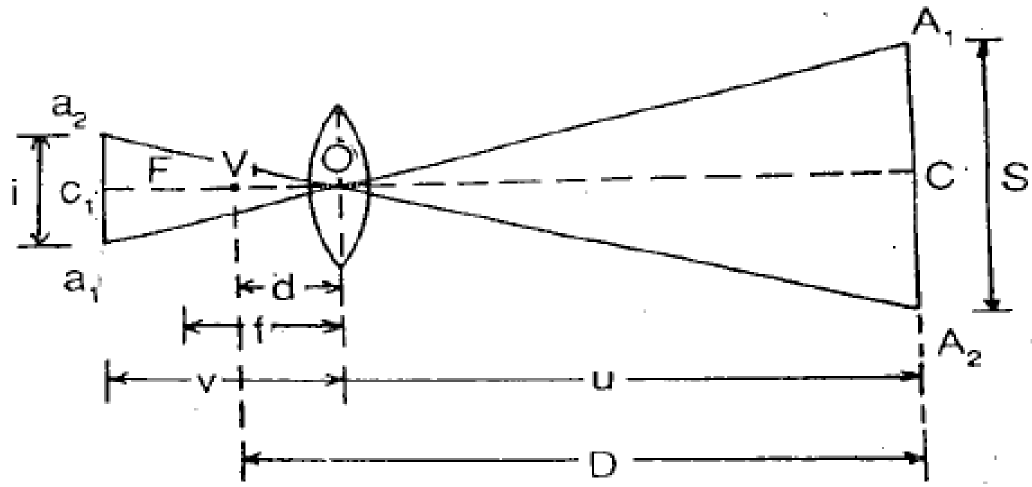
The constant f/i is known as the multiplying constant,

where, f = focal length of objective and i = stadia intercept

11.2 THEORY OF STADIA TACHEOMETRY

The following is the notation used in stadia tacheometry (Fig. 11.3):

O = optical centre of object glass



A_1, A_2, C = readings on staff cut by three hairs
a_1, a_2, C = bottom, top and central hairs of diaphragm
$a_1a_2 = i$ = length of image
$A_1A_2 = S$ = staff intercept
F = focus
V = vertical axis of instrument
f = focal length of object glass
d = distance between optical centre and vertical axis of instrument
u = distance between optical centre and staff
v = distance between optical centre and image

From similar triangles a_1oa_2 and A_1OA_2, $\frac{i}{s} = \frac{v}{u}$

or $$v = \frac{iu}{s} \tag{1}$$

From the properties of lenses,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \tag{2}$$

Putting the value of v in Eq. (2),

Putting the value of v in Eq. (2),

$$\frac{1}{iu/s} + \frac{1}{u} = \frac{1}{f}$$

or

$$\frac{s}{iu} + \frac{1}{u} = \frac{1}{f}$$

or

$$\frac{1}{u}\left(\frac{s}{i} + 1\right) = \frac{1}{f}$$

or

$$u = \left(\frac{s}{i} + 1\right)f \tag{3}$$

But

$$D = u + d$$

so,

$$D = \left(\frac{s}{i} + 1\right) f + d$$

$$= \frac{s}{i} \times f + f + d = \left(\frac{f}{i}\right) \times s + (f + d)$$

The quantities (f/i) and $(f + d)$ are known as tacheometric constants. (f/i) is called the multiplying constant, as already stated, and $(f + d)$ the additive constant.

By adopting an anallatic lens in the telescope of a tacheometer, the multiplying constant is made 100, and the additive constant zero.

2. Field measurement

(a) A fairly level ground is selected. The tacheometer is set up at O and pegs are fixed at A_1, A_2 and A_3 known distances apart (see Fig. 11.4).

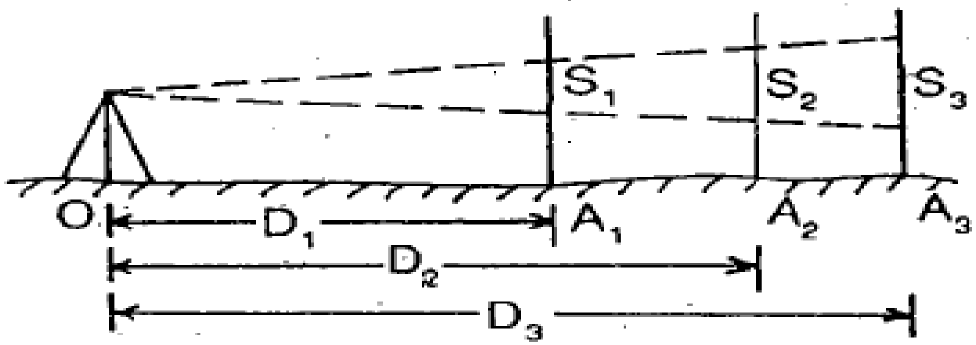


Fig. 11.4

(b) The staff intercepts (stadia hair readings) are noted at each of the pegs. Let these intercepts be S_1, S_2 and S_3.

(c) The horizontal distances of the pegs from O are accurately measured. Let these distances be D_1, D_2 and D_3.

(d) By substituting the values of D_1, D_2, ... and S_1, S_2, ... in the general equation

$$D = \left(\frac{f}{i}\right) S + (f + d)$$

we get a number of equations, as follows:

$$D_1 = \left(\frac{f}{i}\right) S_1 + (f + d)$$

$$D_2 = \left(\frac{f}{i}\right) S_2 + (f + d) \quad \text{and so on.}$$

(e) By solving the equations in pairs, several values of (f/i) and $(f + d)$ are obtained. The mean of these values gives the required constant.

$$D = \left(\frac{f}{i}\right)s + (f + d)$$

$$\boxed{D = Ks + C}$$

- Above equation is *Tacheometric Distance Equation*, K and C are the multiplying and additive constants respectively.
- The multiplying constant $K = \left(\frac{f}{i}\right)$ is also called as *stadia interval factor.*
- Its value depends upon the stadia interval and the principal focal length of the objective.

- For ease of calculation of distances, the stadia wires are spaced such that the multiplying constant $K = 100$
- The additive constant $C = (f + d)$. Practically speaking, it is a constant value for a tacheometer, because the value of d varies by only a small and negligible amount when focussing the telescope on different objects.
- Value of C ranges from 0.25 to 0.35 m
- If an anallactic lens is used into the telescope, it is so placed that all the observations are reduced to the centre of the instrument and the constant C becomes zero and the Tacheometric distance equation is simplified as $D = Ks$.
- Hence, if the staff intercept is known, the horizontal distance can be readily obtained by multiplying it by 100.

The staff consists of two vanes or targets a known distance apart. To measure the staff intercept, two pointings are required. The angles of elevation or depression are measured and their tangents are used for finding the horizontal distances and elevations.

This method is also not generally used. The stadia method requires only one observation, but the tangential method requires two pointings of the telescope.

11.6 FIXED HAIR METHOD

Case 1 *When line of sight is horizontal and staff is held vertically.*

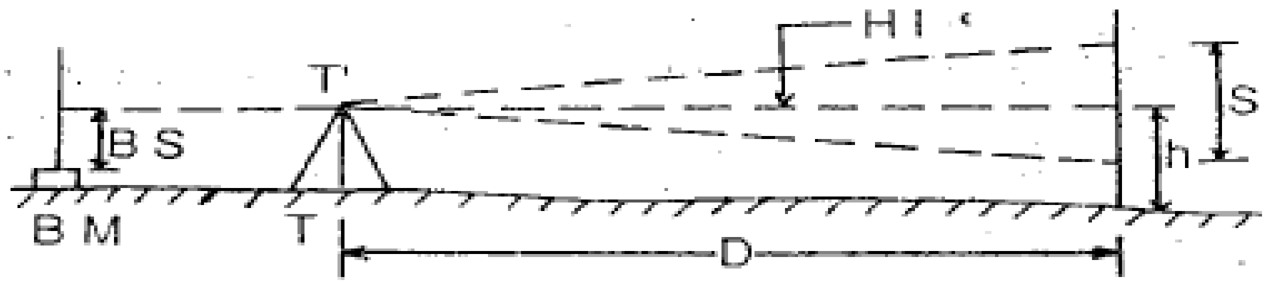


Fig. 11.6

When the line of sight is horizontal, the general tacheometric equation for distance is given by

$$D = \left(\frac{f}{i}\right) S + (f + d)$$

The multiplying constant (f/i) is 100, and additive constant ($f + d$) is generally zero.

RL of staff station P = HI − h

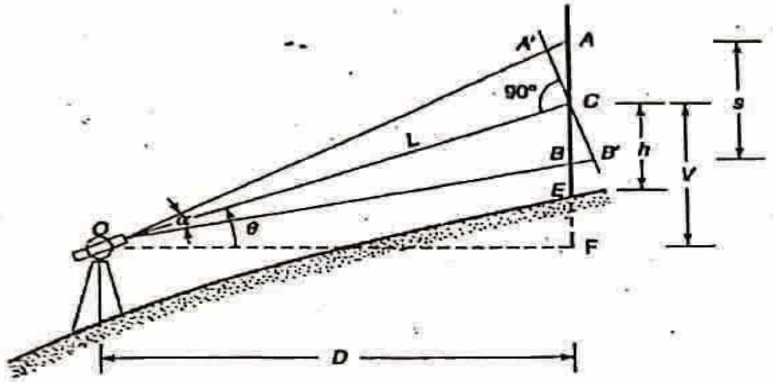
where HI = RL of BM + BS (HI = height of instrument

h = central hair reading BS = backsight)

Distance and Elevation Formular for Inclined Sights

1. Staff Vertical

- As the staff is held vertical, the staff intercept AB is not normal to the line of sight OC. Draw a line A'B' passing through C and perpendicular to OC, cutting OA at A' and OB at B'.



Staff vertical (angle of elevation)

In right angle triangle OFC

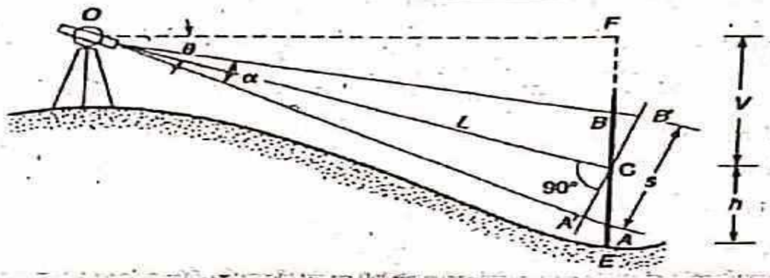
$$\angle OCF = 90° - \theta$$
$$\angle BCB' = \theta \text{ (as } CB' \text{ is perpendicular to } OC)$$
$$\angle A'CA = \angle B'CB = \theta$$

Let the stadia-hairs subtend an angle α,

$$\angle COA' = \alpha/2$$
$$\angle CAO = 90° - (\alpha/2)$$
$$\angle CA'A = 180° - (90° - \alpha/2)$$
$$= 90° + \alpha/2$$

We know that the value of $\alpha/2$ (its value being 17′11″ for K = 100) is very small. Hence, th triangles AA′C and BB′C may be assumed to be right angled triangles.

$\therefore$
$$A'B' = A'C + B'C$$
$$= AC \cos\theta + BC \cos\theta$$
$$= (AC + BC) \cos\theta$$
$$= s \cos\theta$$



Staff vertical (angle of depression).

Inclined distance OC,

$$L = K \cdot A'B' + C$$

$$L = Ks \cos\theta + C$$

$$\because \quad D = L \cos\theta$$

$$= (Ks \cos\theta + C)\cos\theta$$

$$\boxed{D = Ks \cos^2\theta + C\cos\theta}$$

$$V = FC = L \sin\theta$$

$$= (Ks \cos\theta + C)\sin\theta$$

$$= Ks \cos\theta \sin\theta + C \sin\theta$$

$$\boxed{V = \frac{1}{2} Ks \sin 2\theta + C \sin\theta}$$

$\therefore$ Elevation of staff station for angle of elevation θ

$= H.I. + V - h$

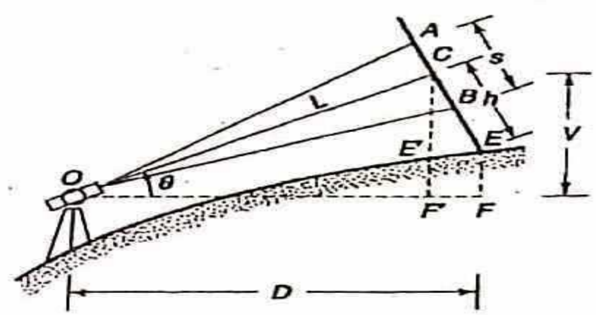
$\therefore$ Elevation of staff station for angle of depression θ

$= H.I. - V - h$

2. Staff Normal

- As the staff at E is held normal to the line of sight AC therefore the staff intercept AB is normal to the line of sight OC.

Case I: Line of sight at an angle of elevation.



Staff normal (angle of elevation)

Where,

$AB = s$ = staff intercept,

$CE = h$ = central hair reading,

θ = angle of elevation, and

$OC = L$ = inclined distance.

Draw a perpendicular CF' to OF.

$$L = Ks + C$$

Elevation

$$OF' = (Ks + C)\cos\theta$$

$$\because \quad D = OF' + F'F$$

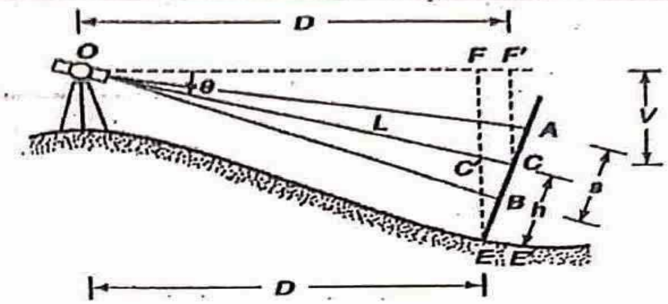
$$\boxed{D = (Ks + C)\cos\theta + h\sin\theta}$$

Elevation of the staff station, $V = OC\sin\theta$

$$= L\sin\theta$$

$$\boxed{V = (Ks + C)\sin\theta}$$

Elevation of staff station = H.I. + V − h cos θ



Staff normal (angle of depression)

Case II: Line of sight at an angle of depression

$$L = Ks + C$$

$$OF' = L\cos\theta = (Ks + C)\cos\theta$$

$$\therefore \quad D = OF' - FF' = OF' - EE'$$

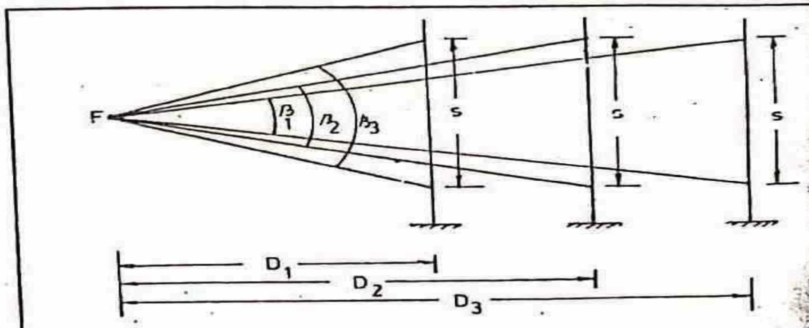
$$\boxed{D = (Ks + C)\cos\theta - h\sin\theta}$$

Elevation of staff station,

$$V = OC\sin\theta = L\sin\theta = \boxed{(Ks + C)\sin\theta = V}$$

Elevation of staff station = H.I. − V − h cos θ.

Movable-Hair Method



- As discussed if the stadia hairs are fixed, the angle β between the ray along the upper stadia and that along the lower stadia is also fixed.
- The staff intercept varies with the distance of the staff from the instrument if the stadia hairs are movable, and the staff whereas intercept 's' is kept fixed, the tacheometer angle β changes with the staff position.
- The diaphragm has an arrangement for the measurement of the stadia interval i accurately. Each hair of the stadia diaphragm can be moved independently by a separate sliding frame.

Example 1.

The following readings were taken with a tachometer with the line of sight horizontal on a staff held vertical.

0.950, 1.285, 1.620 m

Determine the horizontal distance from the instrument station to the staff station if k = 100 and C = 0.15 m.

Also determine the R.L. of the staff station if the R.L. of the instrument station is 101.580 m and the height of the trunnion axis is 1.460 m.

Sol.

Horizontal distance $D = ks + C$

$k = 100$, $C = 0.15m$

Hence, $D = 100(1.620 - 0.950) + 0.15 = 67.15$ m

R.L. of instrument station = 101.58 m

Height of trunnion axis = 1.460 m

Hence, R.L. of line of collimation = 101.58 + 1.46 = 103.04 m

Now, R.L. of staff station = 103.04 − 1.285 = 101.755 m

Example 2.

Determine the distance between the instrument station P and the staff station Q from the following data:

Height of instrument = 1.490 m

*Vertical angle = +4°30′

Staff reading (staff vertical) = 0.645; 0.998; 1.351 m

Also determine the R.L. of Q if that of P is 200.410 m.

Take k = 100, and C = 0.0.

Sol.

$D = K \cdot S \cos^2\theta + C\cos\theta$

$$\text{Horizontal distance } D = ks\cos^2\theta + 0.0$$

$$= 100(1.351 - 0.645)\cos^2(4°30') = 70.165 \text{ m}$$

$$\text{Vertical distance } V = \frac{1}{2}ks\sin 2\theta + 0.0$$

$V = \frac{1}{2}K \cdot S \cdot \sin 2\theta + C\sin\theta$

$$= \frac{1}{2} \times 100 \times (1.351 - 0.645)\sin 9° = 5.522 \text{ m}$$

R.L. of Q = R.L. of P + H.I. + V – Staff reading

= (200.410 + 1.490) + 5.522 – 0.998

= **206.424 m**

Example 3.

Determine the distance between the instrument station P and the staff station Q from the following data:

R.L. of the line of colimation = 200.150 m

* Vertical angle = $-3°45'$

Staff readings = 1.450; 0.900; 0.350 m

Also determine the R.L. of Q

Take k = 100 and C = 0.0

Sol.

$D = K \cdot S \cdot \cos^2\theta + C \cdot \cos\theta$

Horizontal distance $D = ks\cos^2\theta$

$$= 100 \times (1.45 - 0.35) \times \cos^2(3°45') = 109.529 \text{ m}$$

Veritcal distance $V = \frac{1}{2}ks\sin(2\theta)$

$V = \frac{1}{2}k \cdot S \cdot \sin 2\theta + C\sin\theta$

$$= \frac{1}{2} \times 100 \times (1.45 - 0.35)\sin(7°30') = 7.179 \text{ m}$$

R.L. of Q = R.L. of line of colimation – V – staff reading

= 200.150 – 7.179 – 0.9 = 192.071 m

Example 4.

The following observations were taken with a tacheometer at the station P to a staff at Q held normal to the line of sight. If the staff readings are 1.71, 2.64 and 3.57 m, and the angle of inclination is 29°30′, determine the horizontal distance between P and Q.

Also determine the elevation of Q if the line of colimation is at R.L. of 200.00 m. Take k = 100 and C = 0.50.

Sol.

$D = (k \cdot s + c) \cos\theta + h \sin\theta$

Here, $r = 2.64$ m

Horizontal distance $D = (ks + C)\cos\theta + r\sin\theta$

$= [100\,(3.57-1.71) + 0.5)\cos(29°30') + 2.64\sin(29°30')$

$= 163.621$ m

$V = (k \cdot s + c) \sin\theta$

Vertical distance $V = (ks + C)\ \sin\theta$

$= (100 \times 1.86 + 0.5)\sin(29°30') = 91.3837$ m

R.L. of Q = R.L. of P + H.I. + V – Staff reading × cosθ

= R.L. of Line collimation + V – Staff reading × cosθ

$= 200.00 + 91.837 - 2.298 = 289.539$ m

Example 5.

The following observations were taken with a tacheometer at the station P to a staff at Q held normal to the line of sight.

Staff readings = 1.450; 1.915; 2.380 m

Angle of depression = 15°30′

R.L. of P = 201.45 m

Height of trunnion axis above the peg at P = 1.315 m

Determine the horizontal distance between P and Q, and the R.L. of Q. Take k = 100 and C = 0.0

Sol.

$r = 1.915$ m

Horizontal distance $D = (ks + C)\cos\theta - r\sin\theta$

$= [100 \times (2.38-1.45) + 0) \cos(15°30') - 1.915 \sin(15°30')$

$= 89.106$ m

Vertical distance $V = (ks + C)\sin\theta$

$= [100 \times (2.38 - 1.45)] \sin(15°30') = 24.853$ m

R.L. of Q = R.L. of P + H.I. $- V - r\cos\theta$

$= 201.45 + 1.315 - 24.853 - 1.915 \cos(15° 30')$

$= 176.067$ m

Example 8.

To determine the gradient between two points A and B, a tacheometer was set up at another station C and the following observatios were taken, keeping the staff vertical.

Staff at	Vertical angle	Stadia readings
A	+4°20′0″	1.300, 1.610, 1.920
B	+0°10′40″	1.100, 1.410, 1.720

If the horizontal angle ACB is 35°20′, determine the average gradient between A and B.

k = 100, C = 0.0

Sol. Horizontal distance

$$H_{CA} = ks\cos^2\theta$$

$$\Rightarrow \quad H_{CA} = 100 \times 0.620 \times \cos^2(4°20'0'') = 61.646 \text{ m}$$

Vertical distacne

$$V_{CA} = \frac{1}{2} ks \sin(2\theta)$$

$$\Rightarrow \quad V_{CA} = \frac{1}{2} \times 100 \times 0.620 \sin 8^\circ 40' = 4.671 \text{ m}$$

R.L. of A = R.L. of line of colimination + $V_{CA} - r_1$

Let R.L. of line of colimination is 100.00 m

R.L. of A = 100 + 4.671 − 1.610 = 103.061 m

Staff at B

Horizontal distance

$$H_{CB} = 100 \times 0.62 \cos^2(0^\circ 10' 40'')$$

$$\Rightarrow \quad H_{CB} = 62.0(1 - \sin^2 10' 40'') = 61.999 \text{ m}$$

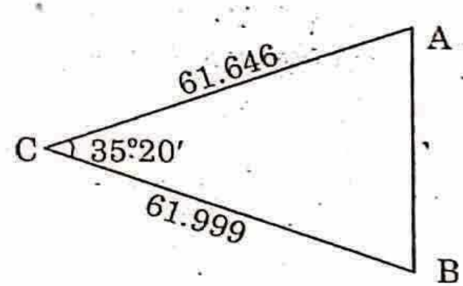
Vertical distance

$$V_{CB} = \frac{1}{2} \times 100 \times 0.62 \sin 21' 20'' = 0.192$$

R.L. of B = 100 + 0.192 − 1.410 = 98.782 m

Difference of R.L. of A and B = 103.061 − 98.782 = 4.279 m

The distance AB can be calculated from cosine law,



$$\therefore \quad \text{Cos } 35^\circ 20' = \frac{(61.3646)^2 + (61.999)^2 - (AB)^2}{2 \times 61.646 \times 61.999}$$

$$0.8158(7643.98) = 3800.23 + 3843.876 - AB^2$$

$$AB = 37.52 \text{ m}$$

$$\text{Gradient of AB} = \frac{4.279}{37.52} = 0.114 = 1 \text{ in } 8.77$$

Example 9.

Following observations were taken for determining the R.L. of station A.

Instrument station	Height of instrument	Staff station	Vertical angle	Staff readings	Remark
Q	1.600 m	B.M.	+6°12′	0.945, 1.675, 2.405	R.L. of B.M. = 421.625
Q	1.600 m	P	–4°12′	1.450, 2.380, 3.310	
A	1.650 m	P	+7°0′	'X', 0.655, 1.255	Reading 'X' could not be observed

The instrument was fitted with an anallactic lens, and the value of constant was 100.00.

Calculate the R.L. of station A.

Sol.

Horizontal Distance between B.M. and Q

$$D_1 = 100(2.405 - 0.945)\cos^2(6°12')$$

$$D_1 = 144.297 \text{ m}$$

Vertical distance between BM & Q

$$V_1 = \frac{1}{2}ks\sin 2\theta$$

$$= \frac{1}{2}\times 100\times(2.405-0.945)\sin(12°24')$$

$$V_1 = 15.676 \text{ m}$$

$$\text{R.L. of B.M.} = \text{R.L. of Q} + 1.600 + 15.676 - 1.675$$

$$421.625 = \text{R.L. of Q} + 15.601$$

$$\text{R.L. of Q} = 406.024 \text{ m}$$

Horizontal Distance D_2 between Q and P

$$D_2 = 100(3.310 - 1.450)\cos^2(4°12') = 185.002 \text{ m}$$

Vertical distance between P and Q

$$V_2 = \frac{100}{2}(3.310 - 1.450)\sin(8°24')$$

$$V_2 = 13.586 \text{ m}$$

$$\text{R.L. of P} = \text{R.L. of Q} + 1.600 - 13.586 - 2.380$$

$$= 406.024 + 1.600 - 13.586 - 2.380 = 391.658 \text{ m}$$

Instrument at A

$$\text{Intercept, } s = (1.255 - 0.655)\times 2 = 1.200$$

Vertical distance between A & P $= \dfrac{100}{2}\times 1.2\sin(14°) = 14.515$ m

$$\text{R.L. of P} = (\text{R.L. of A} + 1.650) + 14.515 - 0.655$$

$$\Rightarrow \quad 391.658 = \text{R.L. of A} + 15.510$$

$$\Rightarrow \quad \text{R.L. of A} = 376.148 \text{ m}$$

Problem 1 A tacheometer was set up at a station C and the following readings were obtained on a staff vertically held.

Inst. station	Staff station	Vertical angle	Hair readings (m)	Remark
C	BM	$-5°20'$	1.50, 1.800, 2.450	RL of BM = 750.50 m
C	D	$+8°12'$	0.750, 1.500, 2.250	

Calculate the horizontal distance CD and RL of D, when the constants of instrument are 100 and 0.15.

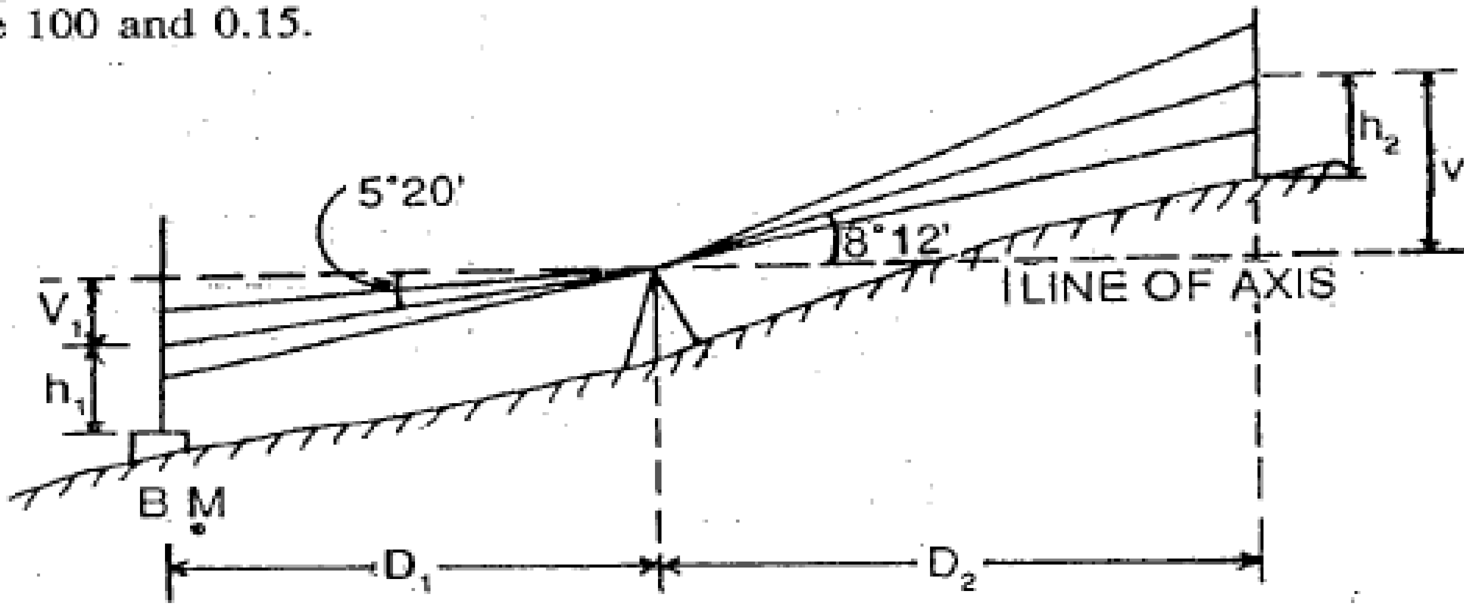


Fig. P.11.1

When the staff is held vertically, the horizontal and vertical distances are given by the relations

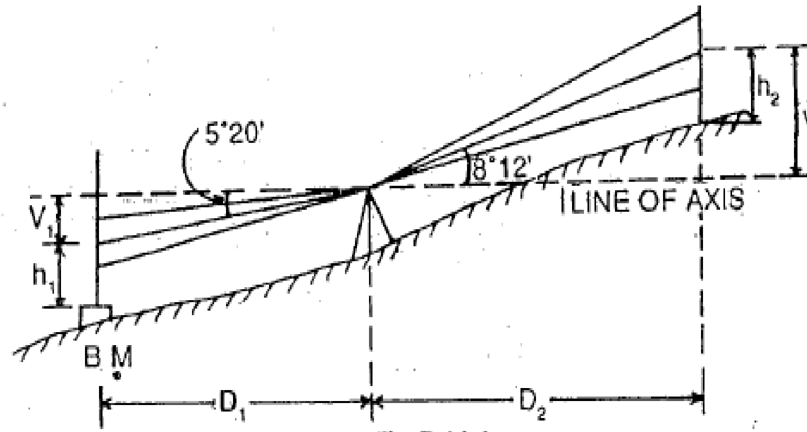


Fig. P.11.1

$$D = \frac{f}{i} \times S\cos^2\theta + (f+d)\cos\theta$$

$$V = \frac{f}{i} \times S \times \frac{\sin 2\theta}{2} + (f+d)\sin\theta$$

Here $\quad \frac{f}{i} = 100$ and $(f+d) = 0.15$

In the first observation, $\quad S_1 = 2.450 - 1.150 = 1.300$ m

$\theta_1 = 5°20'$ (depression)

$$V_1 = 100 \times 1.300 \times \frac{\sin 10°40'}{2} + 0.15 \times \sin 5°20' = 12.045 \text{ m}$$

In the second observation, $\quad S_2 = 2.250 - 0.750 = 1.500$

$\theta_2 = 8.12'$ (elevation)

$$V_2 = 100 \times 1.500 \times \frac{\sin 16°24'}{2} + 0.15 \times \sin 8°12' = 21.197 \text{ m}$$

$$D_2 = 100 \times 1.500 \times \cos^2 8°12' + 0.15 \times \cos 8°12' = 147.097 \text{ m}$$

RL of instrument axis = RL of BM + h_1 + V_1

$= 750.500 + 1.800 + 12.045 = 764.345$ m

$$\text{RL of } D = \text{RL of inst. axis} + V_2 - h_2$$
$$= 764.345 + 21.197 - 1.500 = 784.042 \text{ m}$$

So, the distance CD = 147.097 m and RL of D = 784.042 m.

Problem 2 The following observations were taken with a tacheometer fitted with an anallatic lens, the staff being held vertically. The constant of the tacheometer is 100.

Int. station	Height of instrument	Staff station	Vertical angle	Staff readings (m)	Remark
P	1.255	BM	– 4°20′	1.325, 1.825, 2.325	RL of BM = 255.750 m
P	1.255	A	+ 6°30′	0.850, 1.600, 2.350	
B	1.450	A	– 7°24′	1.715, 2.315, 2.915	

Calculate the RL of B and the distance between A and B.

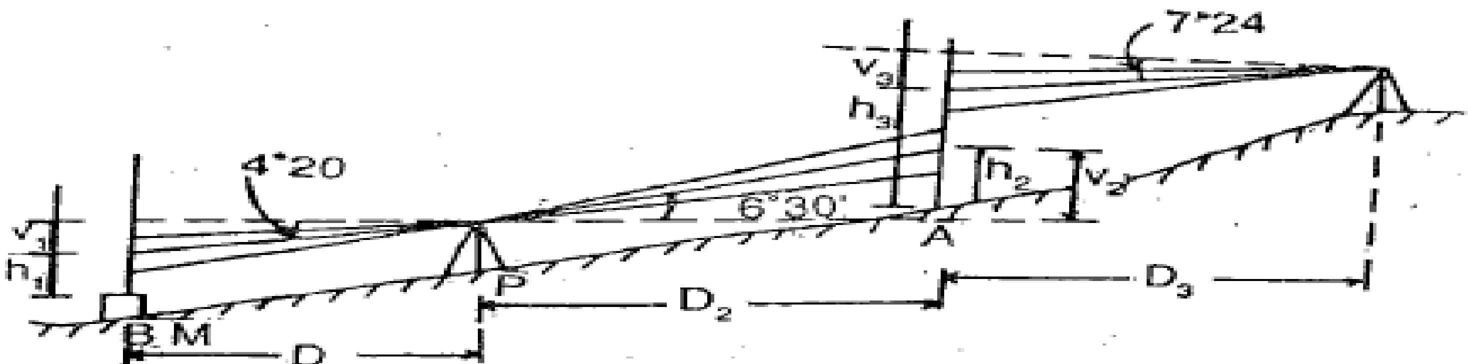


Fig. P.11.2

Solution

Here, Multiplying constant, $f/i = 100$ and Additive constant, $f + d = 0$

Since, the staff is held vertically, the vertical distance is given by

$$V = \frac{f}{i} \times S \times \frac{\sin 2\theta}{2}$$

In the first observation,

$$V_1 = 100\,(2.325 - 1.325) \times \frac{\sin 8°40'}{2} = 7.534 \text{ m}$$

In the second observation,

$$V_2 = 100\,(2.350 - 0.850) \times \frac{\sin 13°0'}{2} = 16.871 \text{ m}$$

In the third observation,

$$V_3 = 100\,(2.915 - 1.715) \times \frac{\sin 14°48'}{2} = 15.326 \text{ m}$$

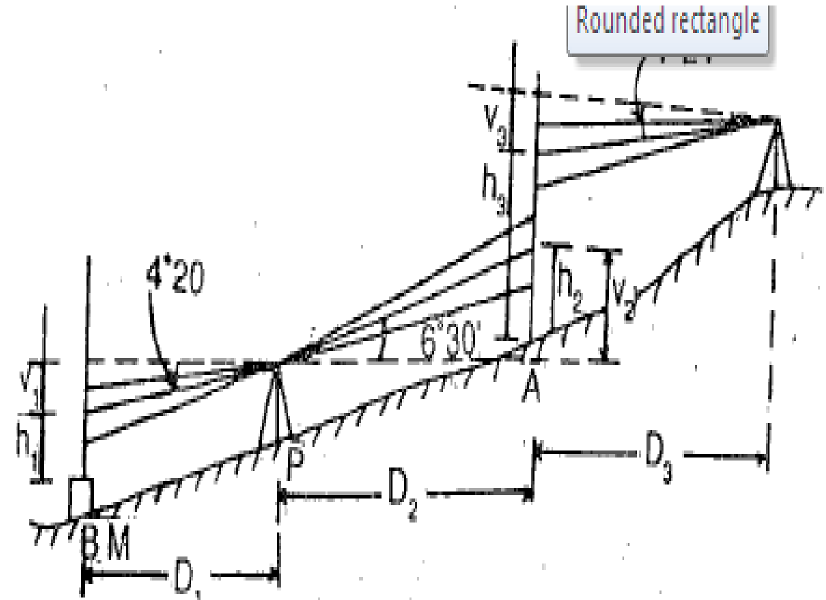


Fig. P.11.2

$$\begin{aligned}
\text{RL of axis when Inst. at P} &= \text{RL of BM} + h_1 + V_1 \\
&= 255.750 + 1.825 + 7.534 = 265.109 \text{ m} \\
\text{RL of A} &= 265.109 + V_2 - h_2 \\
&= 265.109 + 16.871 - 1.600 = 280.380 \text{ m} \\
\text{RL of axis when inst. at B} &= 280.380 + h_3 + V_3 \\
&= 280.380 + 2.315 + 15.326 = 298.021 \text{ m} \\
\text{RL of B} &= 298.021 - \text{HI} \\
&= 298.021 - 1.450 = 296.571 \text{ m} \\
\text{Distance between A and B, } D_3 &= 100\,(2.915 - 1.715) \times \cos^2 7°24' \\
&= 118.009 \text{ m}
\end{aligned}$$

Problem 3 The following observations were made in a tacheometric survey.

Inst. station	Height of axis	Staff station	Vertical angle	Hair readings (m)	Remark
A	1.345	BM	− 5°30′	0.905, 1.455, 2.005	RL of RM = 450.500 m
A	1.345	B	+ 8°0′	0.755, 1.655, 2.555	
B	1.550	C	+ 10°0′	1.500, 2.250, 3.000	

Calculate the RLs of A, B and C, and the horizontal distances AB and BC. The tacheometer is fitted with an anallatic lens and the multiplying constant is 100.

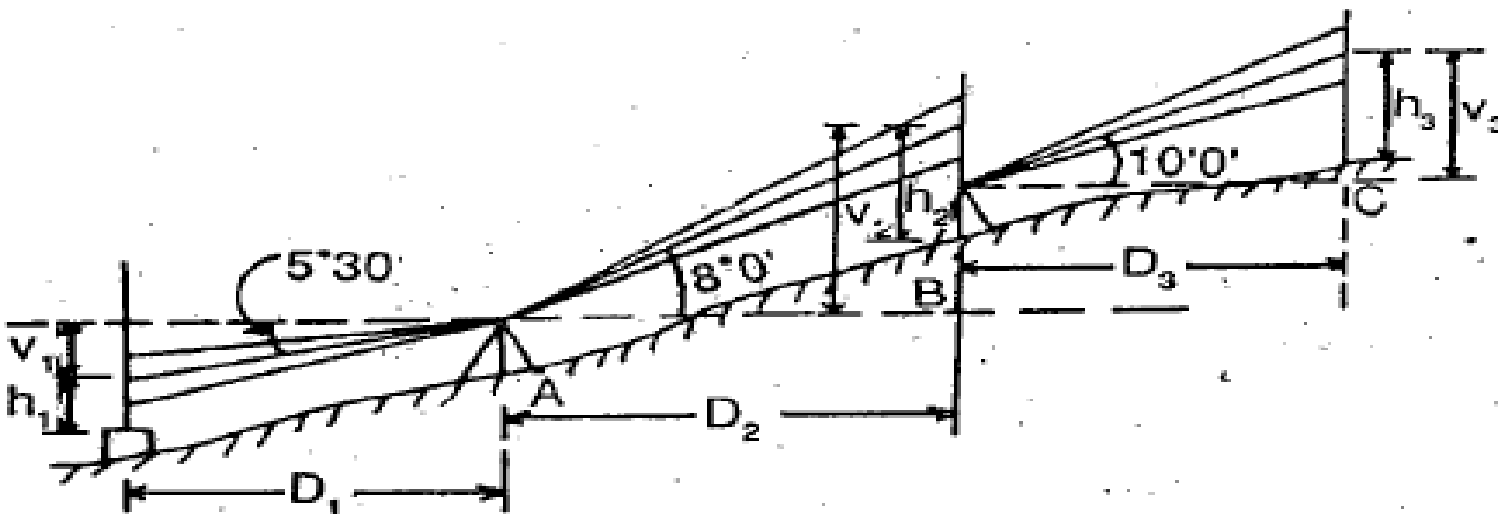


Fig. P. 11.3

Solution

Here $\frac{f}{i} = 100$ and $(f + d) = 0$

Since the staff is held vertically,

$$\text{Horizontal distance } D = \left(\frac{f}{i}\right) \times S \cos^2 \theta$$

$$\text{Vertical distance } V = \left(\frac{f}{i}\right) \times S \times \frac{\sin^2 \theta}{2}$$

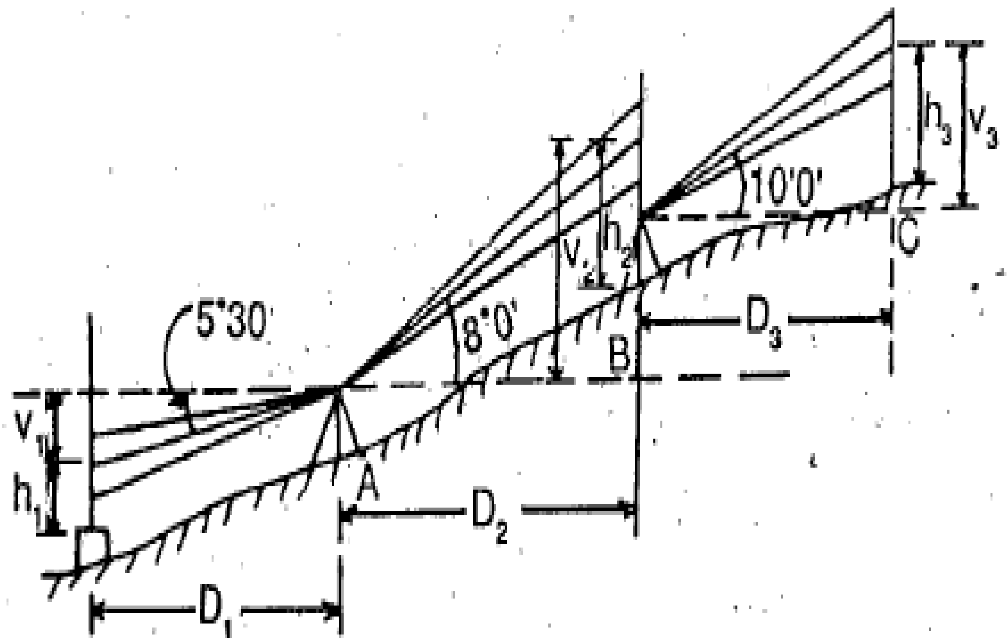


Fig. P. 11.3

In the first observation,

$$V_1 = 100 \times (2.005 - 0.905) \times \frac{\sin 11^\circ}{2} = 10.494 \text{ m}$$

$$D_1 = 100 \times (2.005 - 0.905) \times \cos^2 5^\circ 30' = 108.989 \text{ m}$$

In the second observation,

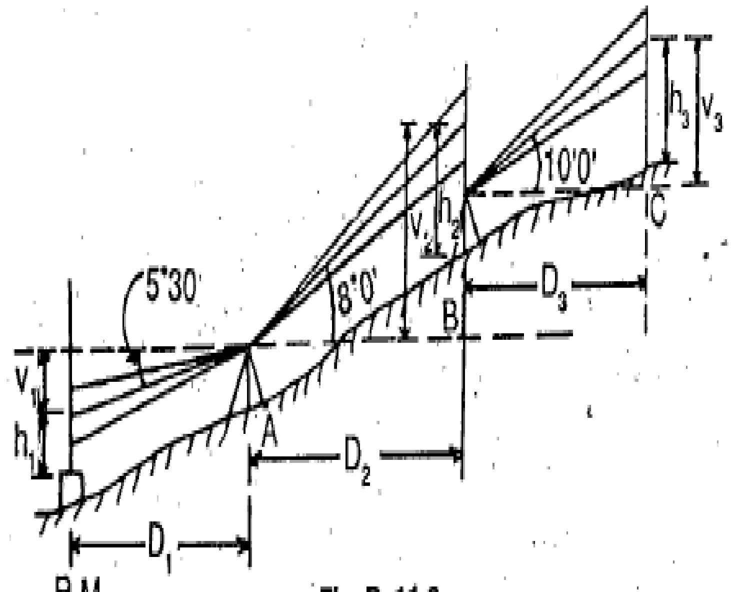


Fig. P. 11.3

$$V_2 = 100 (2.555 - 0.755) \times \frac{\sin 16^\circ}{2} = 24.807 \text{ m}$$

$$D_2 = 100 (2.555 - 0.755) \times \cos^2 8^\circ = 176.514 \text{ m}$$

In the third observation,

$$V_3 = 100 \times (3.000 - 1.500) \times \frac{\sin 20^\circ}{2} = 25.652 \text{ m}$$

$$D_3 = 100 \times (3.000 - 1.500) \times \cos^2 10^\circ = 145.477 \text{ m}$$

Distance AB = D_2 = 176.514 m

Distance BC = D_3 = 145.477 m

RL of axis when inst. at A = RL of BM + V_1 + h_1

$$= 450.500 + 10.494 + 1.455 = 462.449 \text{ m}$$

RL of A = 462.449 height of axis

$$= 462.449 - 1.345 = 461.104 \text{ m}$$

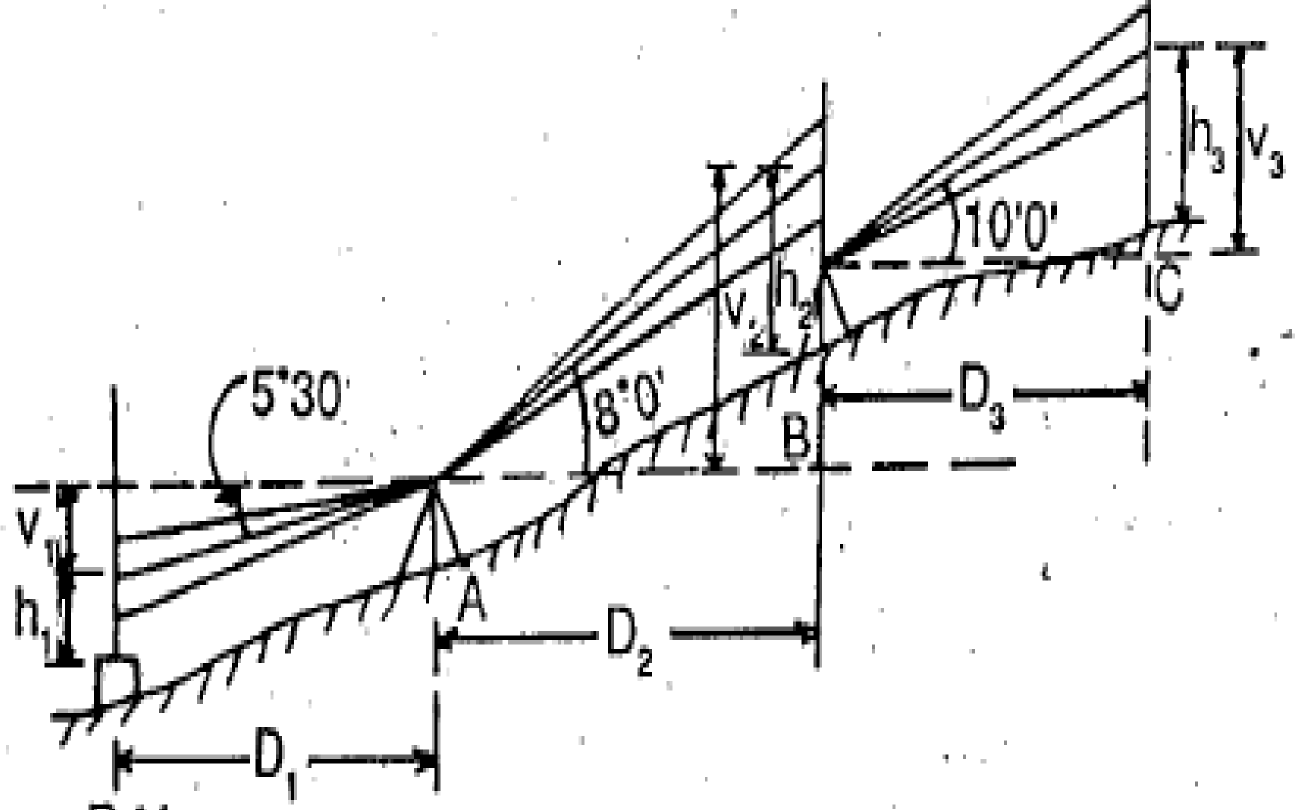


Fig. P. 11.3

$$\text{RL of B} = 462.449 + V_2 - h_2$$
$$= 462.449 + 24.807 - 1.655 = 485.601 \text{ m}$$

$$\text{RL of axis when inst. at B} = 485.601 + 1.550 = 487.151 \text{ m}$$

$$\text{RL of C} = 487.151 + v_3 - h_3$$
$$= 487.151 + 25.652 - 2.250 = 510.553 \text{ m}$$

Problem 4 The following observation were made using a tacheometer fitted with an anallatic lens, the multiplying constant being 100.

Inst. station	Height of inst.	Staff station	WCB	Vertical angle	Hair readings	Remarks
0	1.550	A	30°30′	4°30′	1.155, 1.755 2.355	RL of O = 150.000
		B	75°30′	10°15′	1.250, 2.000, 2.750	

Calculate the distance AB, and the RLs of A and B. Find also the gradient of the line AB.

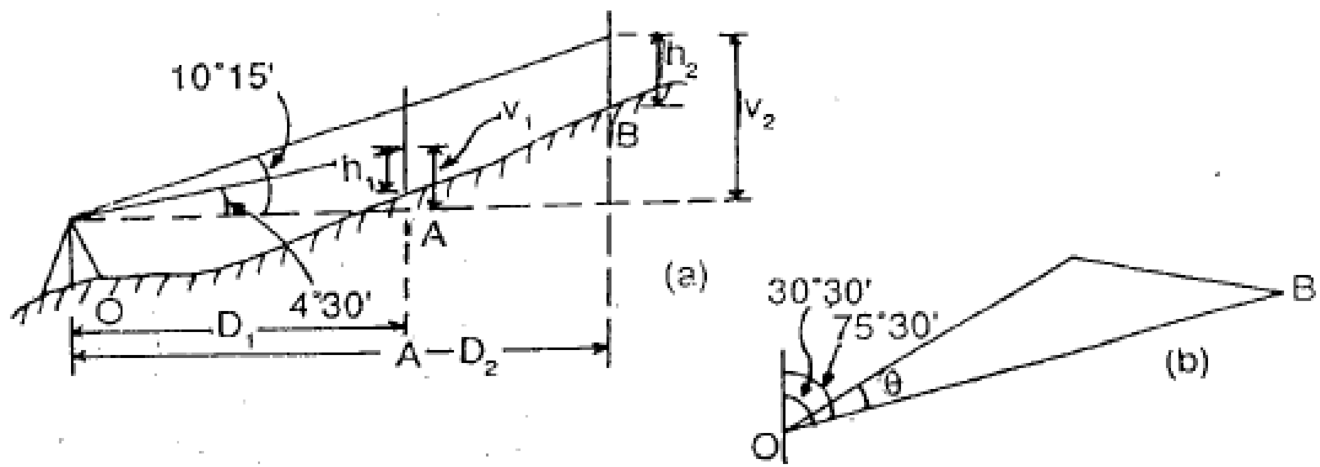


Fig. P.11.4

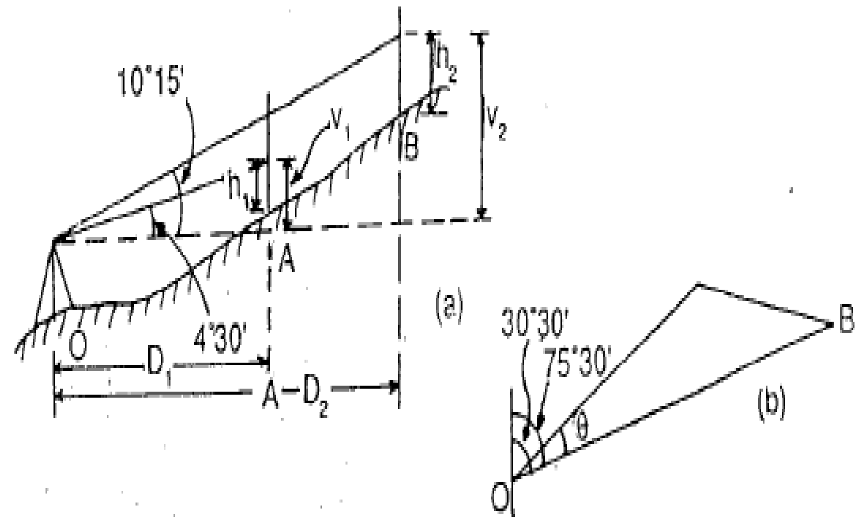


Fig. P.11.4

In the first observation:

$$V_1 = 100 \times (2.355 - 1.155) \times \frac{\sin 9^\circ}{2} = 9.386 \text{ m}$$

$$D_1 = 100 \times (2.355 - 1.155) \times \cos^2 4^\circ 30' = 119.261 \text{ m}$$

In the second observation:

$$V_2 = 100\ (2.750 - 1.250) \times \frac{\sin 20^\circ 30'}{2} = 26.265 \text{ m}$$

$$D_2 = 100\ (2.750 - 1.250) \times \cos^2 10^\circ 15' = 145.250 \text{ m}$$

$$\begin{aligned}
\text{RL of axis} &= \text{RL of O} + \text{height of inst.} \\
&= 150.000 + 1.550 = 151.550 \text{ m} \\
\text{RL of A} &= 151.550 + V_1 - h_1 \\
&= 151.550 + 9.386 - 1.755 = 159.181 \text{ m} \\
\text{RL of B} &= 151.550 + V_2 - h_2 \\
&= 151.550 + 26.265 - 2.000 = 175.815 \text{ m}
\end{aligned}$$

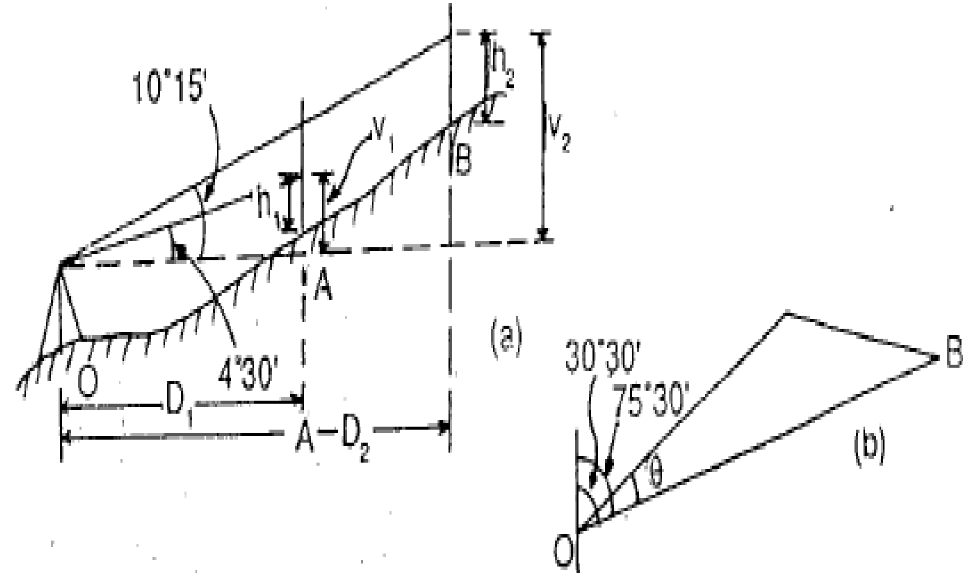


Fig. P.11.4

$$OA = D_1 = 119.261 \text{ m}$$

$$OB = D_2 = 145.250 \text{ m}$$

$$\theta = 75°30' - 30°30' = 45°0'$$

$$AB = \sqrt{OA^2 + OB^2 - 2 \times OA \times OB \times \cos 45°}$$

$$= \sqrt{(119.261)^2 + (145.250)^2 - 2 \times 119.261 \times 145.250 \times 0.707}$$

$$= 104.05 \text{ m}$$

Difference of level between

A and B = 175.815 − 159.181 = 16.634 m (rise from A to B)

$$\text{Gradient of AB (rising)} = \frac{16.634}{104.05} = \frac{1}{6.25} \quad \text{i.e. 1 in 6.25.}$$

Problem 5 Two points A and B are on opposite sides of a summit. The tacheometer was set up at P on top of the summit, and the following readings were taken.

Inst. station	Height of inst.	Staff station	Vertical angle	Hair readings	Remark
P	1.500	A	– 10°0′	1.150, 2.050, 2.950	RL of P = 450.500 m
P	1.500	B	– 12°0′	0.855, 1.605, 2.355	

The tacheometer is fitted with an anallatic lens, the multiplying constant being 100. The staff was held normal to the line of sight.

Find: (a) The distance between A and B, and

(b) The gradients of lines PA and PB.

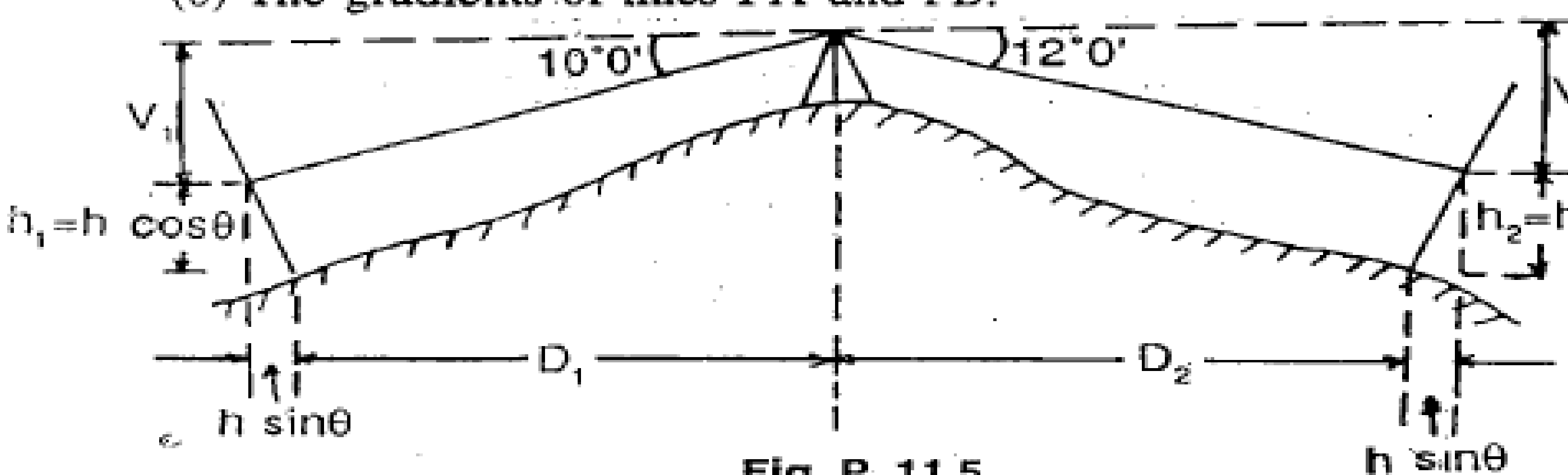


Fig. P. 11.5

Solution

We know that when the staff is held normal to the line of sight, the vertical distance is given by

$$V = \frac{f}{i} \times S \sin\theta + (f + d)\sin\theta$$

Here $\frac{f}{i} = 100$ and $(f + d) = 0$

$\theta_1 = 10°$ and $\theta_2 = 12°$

From Eq. (11)

$$V_1 = \frac{f}{i} \times S \sin\theta_1 = 100 \times (2.950 - 1.150) \times \sin 10° = 31.256 \text{ m}$$

Similarly, $V_2 = 100\,(2.355 - 0.855) \sin 12° = 31.186$ m

$h_1 = 2.050 \times \cos 10° = 2.018$ m

$h_2 = 1.605 \times \cos 12° = 1.569$ m

RL of A inst. axis = 450.500 + 1.500 = 452.000 m

RL of A = RL of inst. axis − V_1 − h_1

= 452.000 − 31.256 − 2.018 = 418.726 m

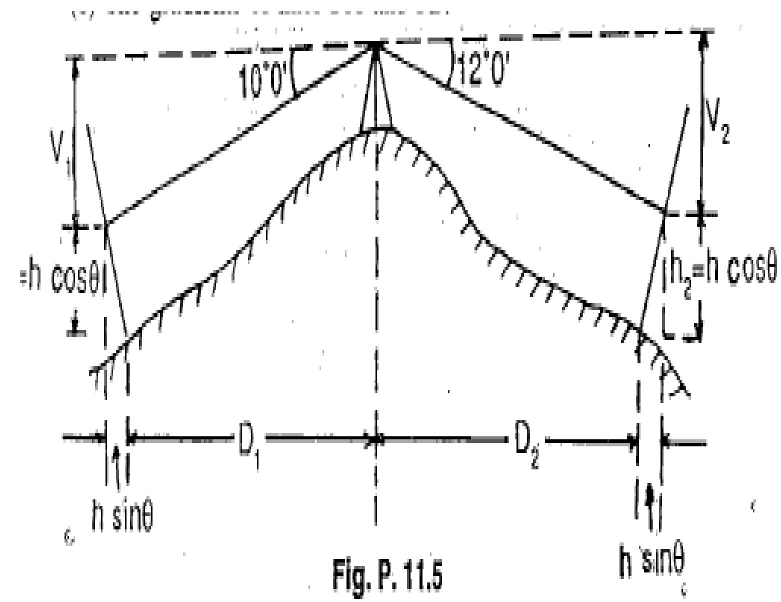


Fig. P. 11.5

$$\begin{aligned}\text{RL of B} &= 452.000 - V_2 - h_2\\ &= 452.000 - 31.186 - 1.569 = 419.245 \text{ m}\end{aligned}$$

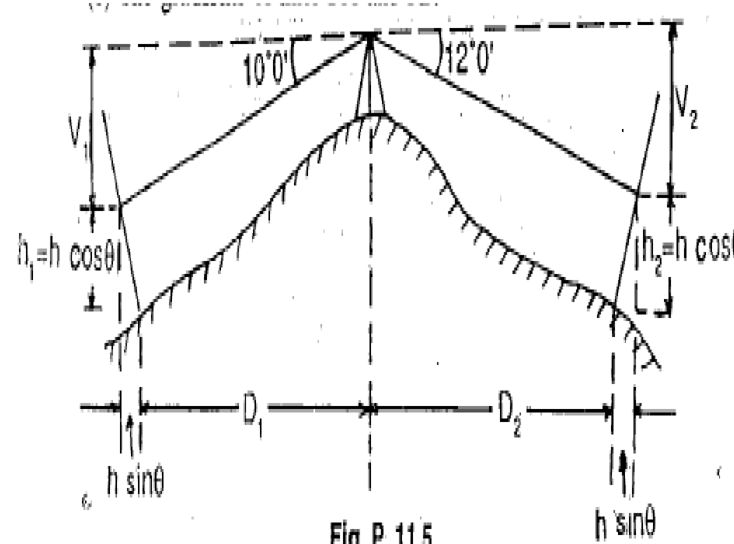


Fig. P. 11.5

The horizontal distances are given by equation,

$$D = \frac{f}{i} \times S\cos\theta + (f+d)\cos\theta - h\sin\theta$$

Hence,
$$\begin{aligned}D_1 &= 100 \times (2.950 - 1.150)\cos 10° - 2.050 \sin 10°\\ &= 177.265 - 0.355 = 176.91 \text{ m}\\ D_2 &= 100\,(2.355 - 0.855)\cos 12° - 1.605 \text{ in } 12°\\ &= 146.722 - 0.333 = 146.389 \text{ m}\end{aligned}$$

$$\begin{aligned}\text{Distance between A and B} &= D_1 + D_2\\ &= 176.910 + 146.389 = 323.299 \text{ m}\end{aligned}$$

$$\text{Gradient of PA (falling)} = \frac{450.500 - 418.726}{176.910} = \frac{1}{5.56} \qquad (1 \text{ in } 5.56)$$

$$\text{Gradient of PB (falling)} = \frac{450.500 - 419.245}{146.389} = \frac{1}{4.68} \qquad (1 \text{ in } 4.68)$$

11.8 THE MOVEABLE HAIR METHOD

In this method the staff intercept is kept constant, but the distance between the stadia wires is variable. The staff is provided with two targets a known distance apart and a third target in the middle. The theodolite is provided with a special type of diaphragm shown in Fig. 11.11. This type of theodolite is known as subtense theodolite. The diaphragm consists of a central wire fixed with the axis of the telescope. The upper and lower stadia wires can be moved by micrometer screws in a vertical plane. The distance by which the stadia wires are moved is measured according to the number of turns of the micrometer screws. The number of complete turns is read on the scale, and the fractional parts are read on the drum of the micrometer screws provided one above and one below the eye-piece. The sum of the micrometer readings is taken in order to obtain the total distance moved by the stadia wires.

For taking the observation, the middle target is first bisected by the central wire. Then the micrometer screws are simultaneously turned to move the stadia wires until the upper and lower targets are bisected.

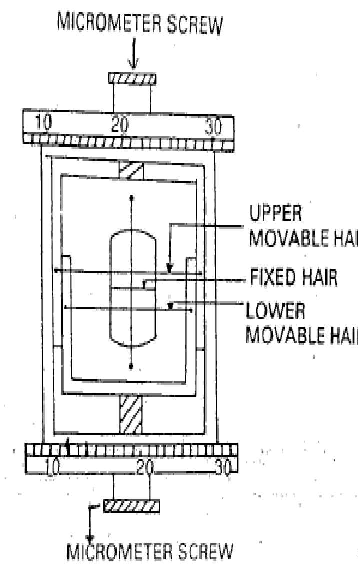


Fig. 11.11

When the line of sight is horizontal, the distance is given by

$$D = \frac{C \times S}{n} + (f + d)$$

where C = constant varying from 600 to 1,000

n = sum of the readings in the micrometer

S = staff intercept (distance between targets)

When the line of sight is inclined, the distance is calculated by a formula similar to the one used for the fixed hair method, namely

$$D = \frac{C \times S \times \cos^2 \theta}{n} + (f + d) \cos \theta$$

Example 1 The micrometer readings of a subtense theodolite are 3.455 and 3.405. The distance between the targets is 3 m. The constants of the instrument are 600 and 0.5 m. Calculate the distance between the instrument and the staff.

Solution

Here

$$C = 600 \qquad (f + d) = 0.5$$
$$n = 3.455 + 3.405 = 6.860$$
$$S = 3 \text{ m}$$

$$\text{Distance} = \frac{C \times S}{n} + (f + d)$$

$$= \frac{600 \times 3}{6.860} + 0.5 = 262.89 \text{ m}$$

Example 2 The same distance was measured by a tacheometer and a subtense theodolite.

Records of the tacheometer reading are as follows:

Staff intercept = 1.255 m, Angle of elevation = 5°0′

$$\frac{f}{i} = 100 \qquad (f + d) = 0.2$$

Subtense theodolite readings:

Staff intercept = 2 m Angle of elevation = 5°30′

Constants = 1000 and 0.3

Find the total number of turns in micrometer.

Solution

In the first case

$$D = \frac{f}{i} \times S \cos^2 \theta + (f + d) \cos \theta$$

$$= 100 \times 1.255 \times \cos^2 5° + 0.2 \cos 5° = 124.5 + 0.20$$

$$= 124.75 \text{ m}$$

In the second case $n = ?$, $C = 1{,}000$, $S = 2$ m, $(f + d) = 0.3$

$\theta = 5°30'$

$$D = \frac{C \times S \times \cos^2 5°30'}{n} + (f + d) \cos 5°30'$$

$$= \frac{1{,}000 \times 2 \times 0.99}{n} + 0.3 \times 0.9953$$

$$= \frac{1{,}980}{n} + 0.2985$$

From Eqs (1) and (2), $\dfrac{1{,}980}{n} + 0.2985 = 124.75$

or $\dfrac{1{,}980}{n} = 124.45 \qquad n = \dfrac{1{,}980}{124.45} = 15.91$

Que. The following observations were taken with a tacheometer at the station P to a staff at Q held normal to the line of sight. If the staff readings are 1.71, 2.64, and 3.57m, and the angle of inclination is $29^0 30'$, determine the horizontal distance between P and Q. Also determine the elevation of Q if the line of collimation is at RL of 200m. Take k=100 and C=0.5.

Que. The To determine the gradient between two points A and B, a tacheometer was set up at another station C and the following observation were taken, keeping the staff vertical. If the horizontal angle ACB is $35^0 20'$, determine the average gradient between A and B Take k=100 and C=0.0

Staff at	Vertical angle	Stadia readings
A	$+4^0 20' 0''$	1.300, 1.610, 1.920
B	$+0^0 10' 40''$	1.100, 1.410, 1.720

Assignment-VI:-

1) Derive the distance and elevation formula for inclined sight when staff is normal?10-Marks

2) Derive the distance and elevation formula for inclined sight when staff is vertical?10-Marks

3) Explain moveable hair method? 10-Marks

4) What are the advantages and disadvantage of movable hair method? 05-Marks

5) Explain tangential method of tacheometry? 10-Marks

6) Explain stadia method of tacheometry? 10-Marks

7) Determine the distance between the instrument station P and the staff station Q from the following data: Height of instrument= 1.490m, vertical angle= $+4^{0}30'$, Staff reading (staff vertical) = 0.645, 0.998 & 1.351m. Also determine the RL of Q if that of P is 200.410m. 10-Marks

8) Determine the distance between the instrument station P and the staff station Q from following data: RL of line of collimation = 200.150m, Vertical angle = $-3^{0}45'$, Staff readings= 1.450, 0.900 & 0.350m. Also determine the RL of Q. 10-Marks