## Kinematic (stick or skeleton) Diagrams

A striped-down (simplified) drawing showing the essentials needed for kinematics analysis. All links are numbered while the joints are lettered.


Mechanism to open and close a window


Kinematic diagram

## Kinematic (stick or skeleton) Diagrams



## Kinematic (stick or skeleton) Diagrams



## Type of Joints - Kinematic Pairs

Lower Pairs - motion is transmitted through an area contact, pin and slider joints.

Higher Pairs - motion is transmitted through a line or a point contact; gears, rollers, and spherical joints.

## Joints

## The Revolute joint (pin or hinge joint) - one degree of freedom

It allows pure rotation between the two links that it connects (R joints)


## Joints

## The Sliding joint (prism or piston joint) - one degree of freedom

It allows linear sliding between the two links that it connects (P joint)


## Joints

## The Helical joint (helix or screw joint) - one degree of freedom

The sliding and rotational motions are related by the helix angle of the thread (H joint)


## Joints

The Cylindrical (cylindric) joint - two degrees of freedom
It permits both angular rotation and an independent sliding motion (C joint)


## Joints

## The Spherical (spheric) - Three degree of freedom

It permits rotational motion about all three axes, a ball-and-socket joint (S joint)



Spherical (S) joint-3 DOF

## Joints

## The Planar (flat) - Three degree of freedom

It permits rotational motion about the $Z$ axes axis and sliding motion in $x$ and $y$ axes (F joint), used seldom in design


## Joints

A cam joint allows both rotation and sliding between two links.


A gear connection also allows both rotation and sliding as the gear teeth mesh


## Degrees of Freedom (DOF) - Type of Joints, Higher Pairs

Roll-slide contact, 2 DOF


Rolling contact (no sliding), 1 DOF


Gears - sliding and rotation motion between two teeth, 2 DOF


## Degrees of Freedom (DOF) - Type of Joints, Higher Pairs

Belt and pulley (no sliding) or chain and sprocket - 1 DOF


Spring - no effect on mechanism DOF


## Degrees of Freedom (DOF) - Type of Joints, Lower Pairs

| Name of Pair | $\underbrace{\substack{\text { Cemome } \\ \text { form }}}_{\text {ceomeric }}$ | ${ }_{\text {Seperemanatic }}^{\text {Sen }}$ | ¢ Depres |
| :---: | :---: | :---: | :---: |
| 1. Reotule |  | Clloler | 1 |
| 2. Cyinder $\begin{gathered}\text { (c) } \\ \text { der }\end{gathered}$ | OE | $0 \sqrt{5}$ | 2 |
| 3. Prism ${ }_{(P)}$ | R | 0 <br> + | 1 |
| 4. Sophere | 54 |  | 3 |
| 5. ${ }_{\text {chelix }}^{\text {(fi) }}$ |  |  | 1 |
| 6. ${ }_{\text {come }}^{\text {Pape }}$ (p) |  | $\sigma$ | 3 |

Each pin connection removes two degrees of freedom of relative motion between two successive links.

Two degrees of freedom joints are sometimes called a half a joint (Norton).

A slider is constrained against moving in the vertical direction as well as being constrained from rotating in the plane.

A spheric pair is a ball and socket joint, 3 DOF.

The helical pair has the sliding and rotational motion related by the helix angle of the screw.

Planar pair is seldom used

## Degrees of Freedom

An object in space has six degrees of freedom.

- Translation - movement along $\mathrm{X}, \mathrm{Y}$, and Z axis (three degrees of freedom)
- Rotation - rotate about $\mathrm{X}, \mathrm{Y}$, and Z axis (three degrees of freedom)



## Degrees of Freedom (DOF)

Planar (2D) mechanisms

Degrees of Freedom - number of independent coordinates required to completely specify the position of the link

Three independent coordinates needed to specify the location of the link $A B, x_{A}, y_{A}$, and angle $\theta$


An unconstrained link in a plane has three degrees of freedom, a mechanism with $L$ links has $3 L$ degrees of freedom

## Degrees of Freedom (DOF)

Kutzbach's (modified Groubler) equation

$$
\begin{aligned}
\text { DOF } & =3(L-1)-2 J_{1}-J_{2} \\
\text { DOF } & =\text { degree of freedom or mobility } \\
L & =\text { number of links, including ground link } \\
J_{1} & =\text { number of } 1 \text { DOF joints (full joints) } \\
J_{2} & =\text { number of } 2 \text { DOF joints (half joints) }
\end{aligned}
$$

$$
\begin{array}{lll}
\mathrm{DOF} \leq 0 & \longrightarrow & \text { structure } \\
\mathrm{DOF}>0 & \longrightarrow \text { mechanism }
\end{array}
$$

## Degree of Freedom (DOF) - example

## Four Bar mechanism

$L=4, J_{1}=4$ pin connections, $J_{2}=0$
DOF $=3(L-1)-2 J_{1}-J_{2}$
DOF $=3(4-1)-2(4)-(0)=1$

## Slider crank mechanism


$L=4, J_{1}=3$ pin connections +1 slider $=4$
$J_{2}=0$
DOF $=3(4-1)-2(4)-(0)=1$

1 DOF means only one input (power source) is needed to control the mechanism


## Degrees of Freedom (DOF) - trench hoe

Number of links, $L=12$, Number of one DOF joints, $J_{1}=12$ (pins) +3 (slider) $=15$,


$$
\text { DOF }=3(L-1)-2 J_{1}-J_{2}=3(12-1)-2(15)=3
$$

3 hydraulics are used to control the position of the bucket.


## Degree of Freedom (DOF) - example

Number of links, $L=7$,
Number of one DOF joints, $J_{1}=6$ (pins) +1 (slider) $=7$,
Number of two DOF joints, $J_{2}=1$ (fork joint)
DOF $=3(L-1)-2 J_{1}-J_{2}=3(7-1)-2(7)-1=3$

Three input sources are needed to control the mechanism

Fork Joint

1.

## Paradoxes

Two rollers in contact, no slipping

## Two gears in contact



$$
\begin{array}{ll}
L=3, J_{1}=3, J_{2}=0 & L=3, J_{1}=2, J_{2}=1 \\
\text { DOF }=3(3-1)-2(3)=0 & \text { DOF }=3(3-1)-2(2)-1=1
\end{array}
$$

## Redundant support



$$
\begin{aligned}
& L=5, J_{1}=6, J_{2}=0 \\
& \text { DOF }=3(5-1)-2(6)=0
\end{aligned}
$$

Eliminate the redundancy before determining DOF

