MOHR – COULOMB FAILURE THEORY:

Of the many theories of failure that have been proposed the Mohr strength theory and Mohr theory have been well accepted by soil engineers. The following are the essential points of Mohr's strength theory.

i)   Materials fails by shear
ii) The ultimate strength of the material is determined by the stresses in the failure plane.
iii) When the material subjected to three principal stresses ($\sigma 1$, $\sigma 2$, $\sigma 3$),the intermediate principal stresses does not have any influence on the strength of the material.
     The theory was first expressed by coulomb and later generalized by Mohr.

The Mohr theory can be expressed by the equation,

$\tau_f = S = F(\sigma)$

τ_f = Shear Resistance of material

$F(\sigma)$ = Function of normal stress

If the normal and shear corresponding to failure are plotted then a curve is obtained. The plot (or) the curve is called the sheen envelope.

$$\boxed{S \text{ or } \tau = C + \sigma \tan \varphi}$$

Coulomb defined the function $F(\sigma)$ as a **linear function** of σ **and c**.

C- Cohesion ϕ - Angle of internal friction (or) Angle of shearing resistancerespectively.

FAILURE ENVELOPES

COULOMB ENVELOPE
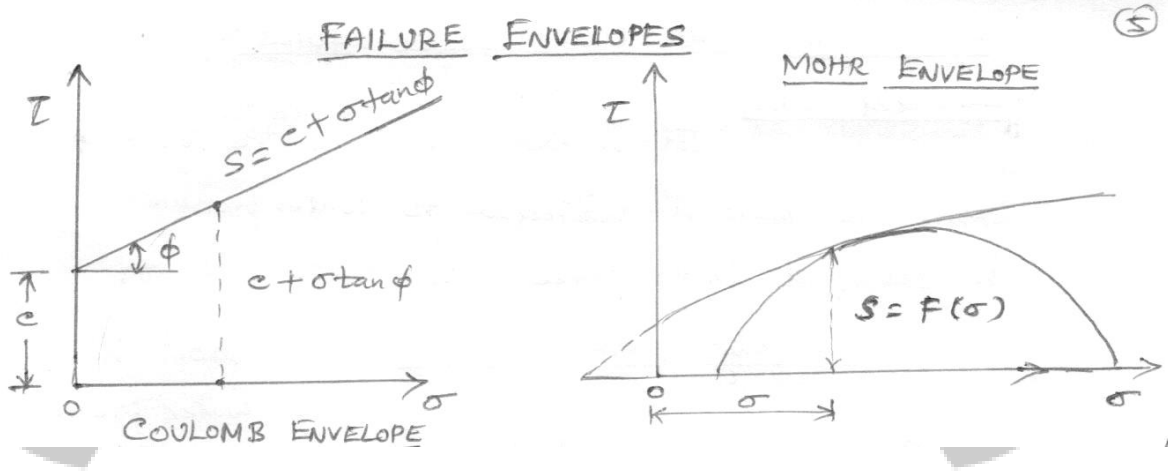
MOHR ENVELOPE

Coulomb considered, that the relationship between shear strength and **normal** stress, is represented by a **straight line**.

In **Mohr** theory, the **shear** strength and normal stress gives **non linear curve**. the curved failure envelope of Mohr is referred to as a straight line for most cases.

For an ideal pure friction material, straight line passes through the origin.

The Mohr envelope can be considered to be straight of the angle of internal friction is assumed to be constant. Depending upon the properties of a material the failure envelope may be straight (or) curved, and it may pass through the origin (or) it may intersect the stress axis.

COHESIONLESS SOIL   COHESIVE SOIL
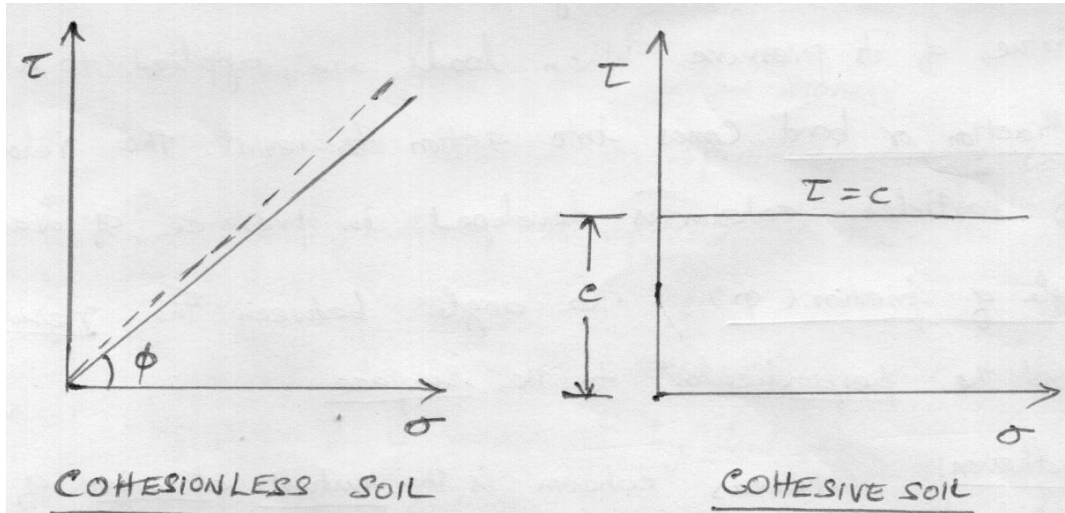
SATURATED AND UNSATURATED SOILS :

Saturated soils :

It is soil, in which the voids are filled by water. Entire soil mass is subjected to the water pressure .Normally this soil is classified into three types
1.Partially saturated soil : The soil is partially subjected to water pressure.
2.Fully saturated soil :   The water is filled in voids. ($s_{r=100\%}$)
3.Submerged soils :   The water is floating over the soil. Soil is in the submerged condition . The permeability property is low.
Unsaturated soils:
The soil contains no water. But all types of soil have some amount of initial moisture content (2% to 5%) The permeability property is high due to dry condition, it absorb more water.

STRENGTH PARAMETER:
The parameters c and ϕ termed as strength parameters
$c = cohesion$

φ $= angle\ of\ internal\ friction\ (or)\ angle\ of\ shearing\ resistance$

Cohesion (c) :

        The shearing strength in which a soil passes by virtue of its pressure When loads are applied to a cohesive soil **attraction or bond** comes in to action to resist. The relative displacement of particles. Cohesion is developed in presence of water.

Angle of friction (ϕ) : The angle between the resultant force and the **perpendicular** to the **surface.**

Adhesion: where as cohesion is the mutual attraction of two different parts of a Clay mass to each other, clay often also exhibit the property of ' adhesion' which is a Propensity to adhere to other materials at a common surface. This has no relation to the normal pressure. This is of particular interest in relation to the supporting capacity of friction piling in clays and to the lateral pressure on retaining walls.
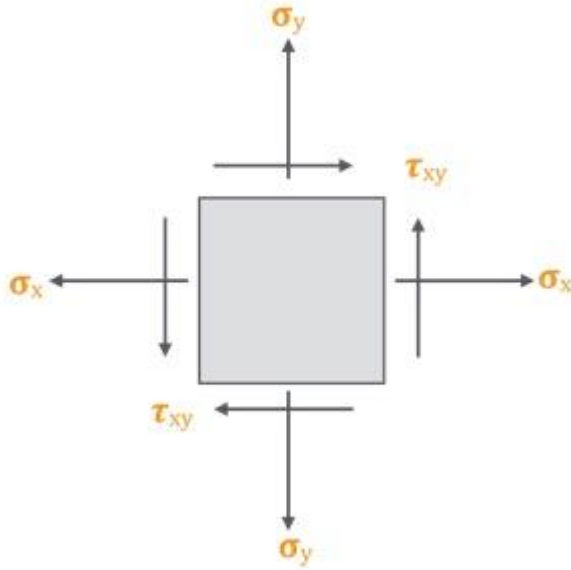
Mohr's Circle :

Mohr's Circle is graphical tool that is commonly used by engineers to graphically analyze the principal and maximum shear stresses on any plane, as well as provide graphical coordinates of these shear stresses.
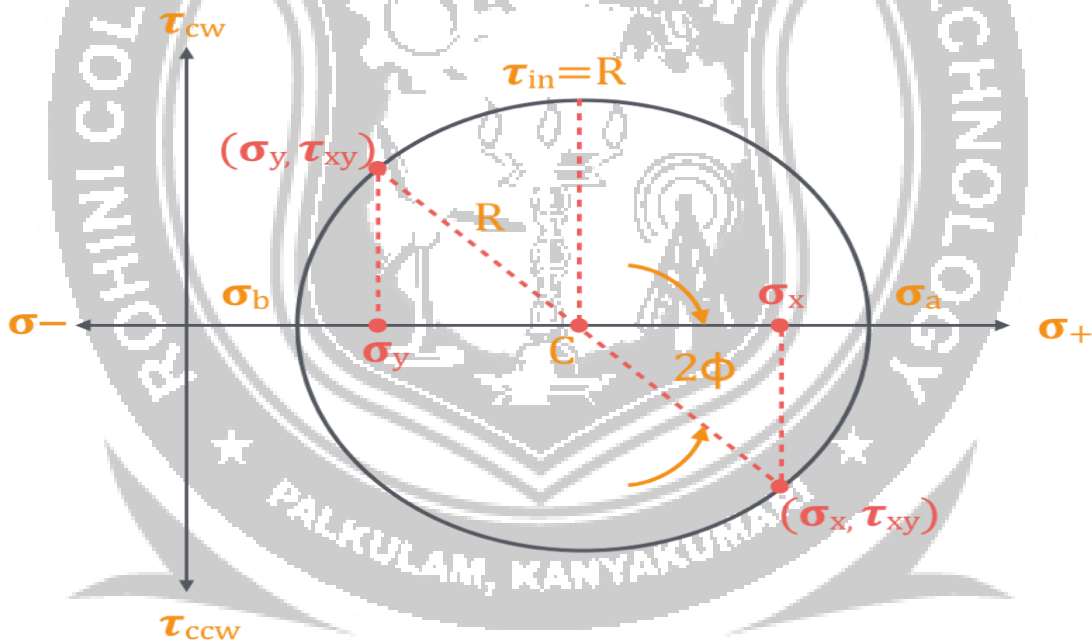
However, there can be infinite number of planes passing through a point, and the normal stress on each plane will vary.

The **PRINCIPAL PLANE** or maximum principal plane is the plane on which the normal stress value is at a MAXIMUM, with this value being referred to as the **MAXIMUM PRINCIPAL STRESS**.

A typical 2D stress element is shown below with all indicated components shown in their positive sense:

Using a graphical approach, we are able to determine the PRINCIPAL STRESSES on each plane using a MOHR'S CIRCLE.

Mohr's circle is a geometric representation of the 2-D transformation of stresses, in which the component stresses and are found as the coordinates of a point whose location depends upon the angle to determine the aspect of the cross section.

Mohr's circle is used to determine the principle angles (orientations) of the principal stresses without have to plug an angle into stress transformation equations

To draw a Mohr's Circle for a typical 2-D element, we can use the following **procedure** to determine the principal stresses.

1.Define The Shear Stress Coordinate System:

Define the coordinate system for the normal and shear axes – Tensile normal stress components are plotted on the horizontal axis and are considered positive. Compressive normal stress components are also plotted on the horizontal axis and are negative.

2.Define The Torsional Coordinate System:

For the construction of a Mohr's circle, shearing stresses are plotted ABOVE the normal stress axis when the pair of shearing stresses, acting on opposite and parallel faces of an element, form a

CLOCKWISE (cw) couple. Shearing stresses are plotted BELOW the normal axis when the shear stresses form a COUNTERCLOCKWISE (ccw) couple.


3.Plot The Shear Stress values:

Plot the shear stress values given in the problem statement, or plot generic points on the for σ_x-axis and σ_y as shown below.


4. Plot the magnitude of the couple:

Plot the magnitude of the couple given in the problem statement with a clockwise (cw) couple being plotted above the σ_x-axis, and a counter clockwise (ccw) couple being plotted below the σ_x-axis.If not values are provided for the moment plot generic points above and below the σ-axis for τ_{xy} as shown below
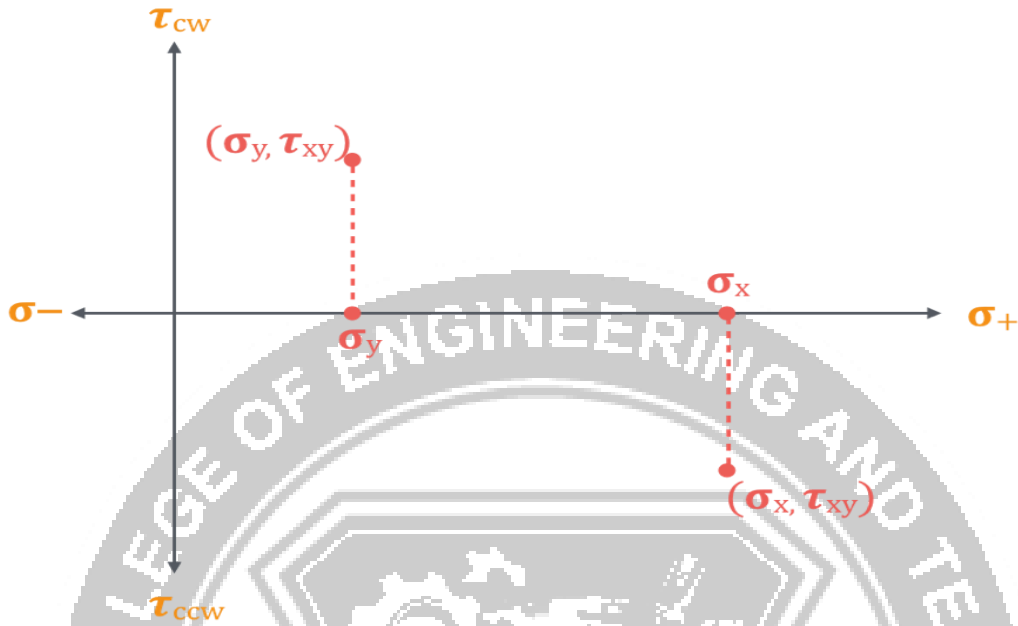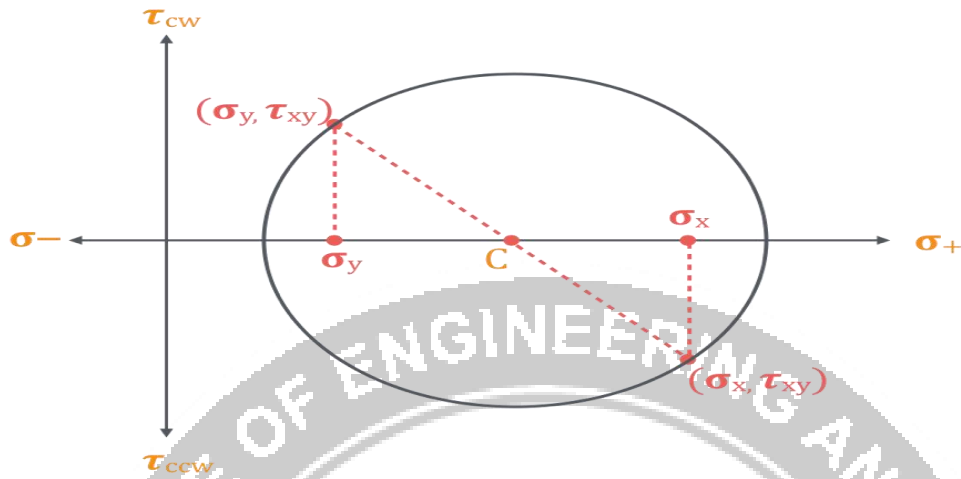
5. Obtain the center of the mohr's circle:

The center of the Mohr's circle is obtained graphically by plotting the two points representing the two known states of stress, and drawing a straight line between the two points. The intersection of this straight line and the -axis is the location of the center of the circle.
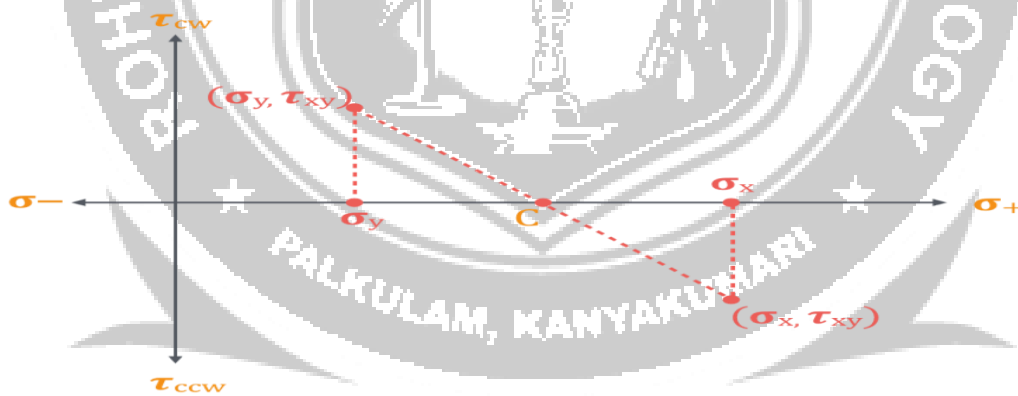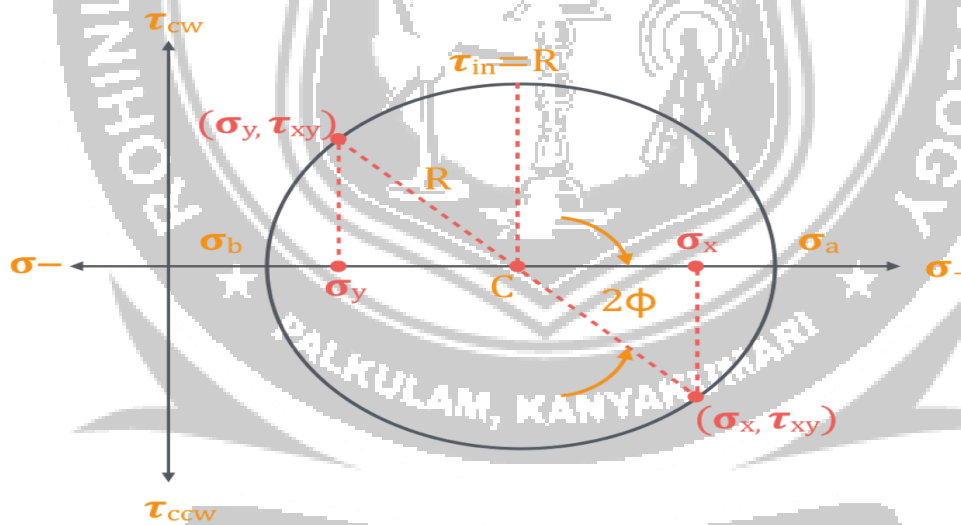

6. Draw The Mohr's circle:

Draw the Mohr's circle assuming the connection line as the diameter of the circle, using the intersection of the diagonal straight line and the σ-axis as the center of the circle

$\boldsymbol{\tau}_{cw}$

$(\boldsymbol{\sigma}_y, \boldsymbol{\tau}_{xy})$

$\boldsymbol{\sigma}_x$

$\sigma-$

$\boldsymbol{\sigma}_y$ C $\sigma+$

$(\boldsymbol{\sigma}_x, \boldsymbol{\tau}_{xy})$

$\boldsymbol{\tau}_{ccw}$

7. Stress Analysis With The Mohr's circle

Stress Analysis on Mohr's circle – To get normal and shear stress values at any plane theta, take angle 2φ in the Mohr's circle starting from diagonal of the circle and locate a peripheral point as as shown. Shear stress value will be on the y-axis and normal stress values will be on the x-axis

$\boldsymbol{\tau}_{cw}$

$\boldsymbol{\tau}_{in} = R$

$(\boldsymbol{\sigma}_y, \boldsymbol{\tau}_{xy})$

R

$\boldsymbol{\sigma}_b$ $\boldsymbol{\sigma}_x$ $\boldsymbol{\sigma}_a$

$\sigma-$

$\boldsymbol{\sigma}_y$ C   2Φ $\sigma+$

$(\boldsymbol{\sigma}_x, \boldsymbol{\tau}_{xy})$

$\boldsymbol{\tau}_{ccw}$