CE 626

Where does groundwater occur?

Aquifer Characteristics and Darcy's Law

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Lecture 2

Definition

Groundwater hydrology may be defined as the science of the occurrence, distribution and movement of water below the surface of the earth.

Todd , 1980

Today we will learn about…

- Characterization of earth materials
- Darcy's experiment
- Aquifers and their characteristics

CHARACTERIZATION OF EARTH MATERIALS

Some basics

Physical quantity	Brief definition	Formula	Dimensions (Mass, Length, Time)	Units
Work	Work is done when a force is applied to a fluid while the fluid is moving	W=FD (force x displacement)	ML^2T^{-2}	N.m or Joule
Weight	Gravitational force exerted on it by the Earth	W=mg	MLT^{-2}	N (same unit as force)
Density	Mass per unit volume	$\rho=m/V$	ML^{-3}	kgm^{-3}
Specific weight	Weight per unit volume	$\gamma=W/V$	$ML^{-2}T^{-2}$	Nm^{-3}
Pressure	Force per unit area (perpendicular to the direction of the force)	P=F/A	$ML^{-1}T^{-2}$	Pa or Nm^{-2}
Dynamic viscosity	Resistance to relative motion	$\mu=\tau/(du/dy)$	??	Poise, Nsm^{-2}
Bulk modulus	ratio of change pressure to relative change in density	$K_c=\rho(dP/d\rho)$	$ML^{-1}T^{-2}$	Nm^{-2}

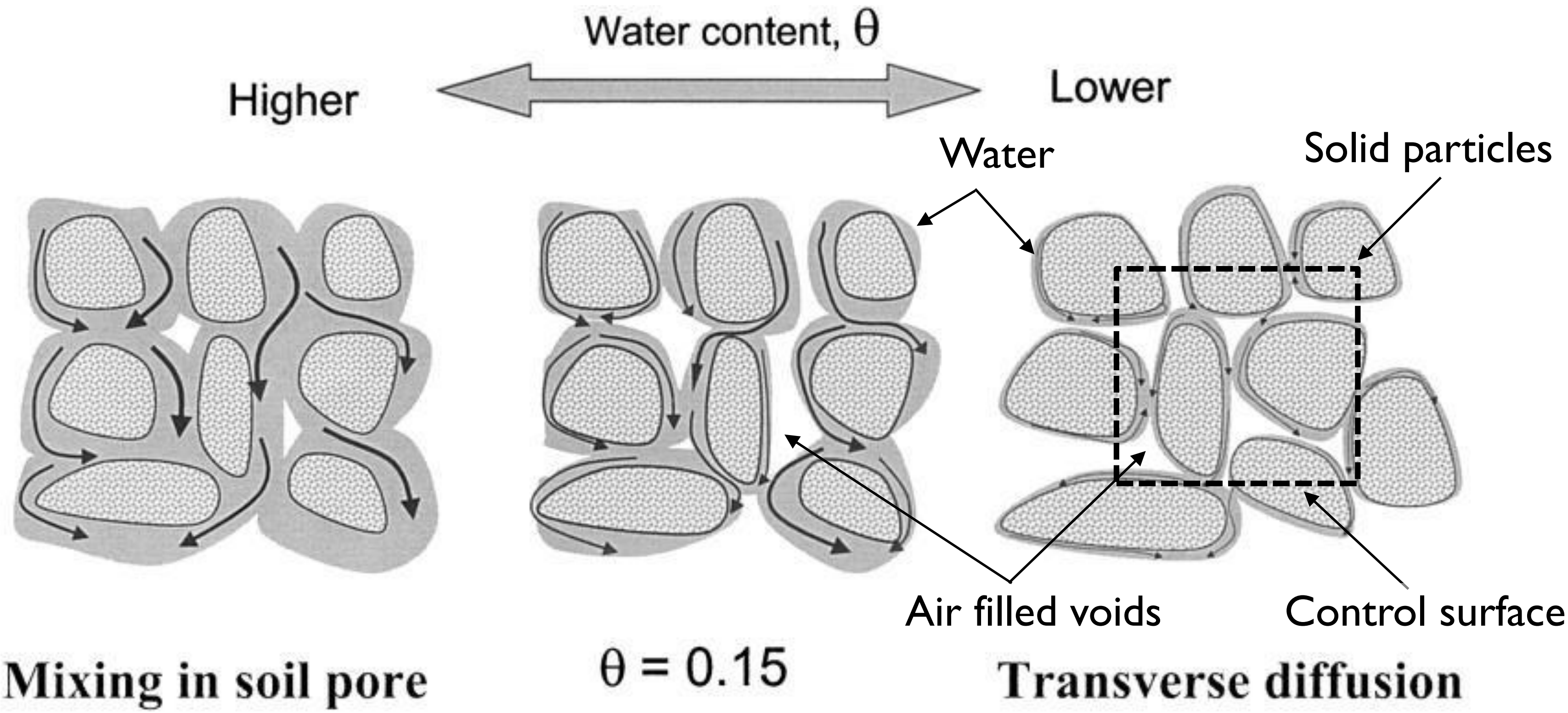
Porosity: pores* in rocks or soil material which can be occupied by water

- Pores are characterized by size, shape, irregularity, and distribution
- *Original* interstices were created during the formation of rock, found in sedimentary and igneous rocks
- *Secondary* interstices develop after the rock is formed (joints, fractures, openings due to chemical or live agents)
- Size classification: capillary (surface tension holds water), super-capillary (larger than capillary), and sub-capillary (water held by adhesive forces)
- Connection based: communicating or isolated

*voids, interstices, pore space

Image: https://upload.wikimedia.org/wikipedia/commons/thumb/d/d6/VessicularBasalt1.JPG/220px-VessicularBasalt1.JPG (top)
http://www.monotaro.sg/g/01010117/(bottom)

Characterizing the porous medium: porosity and soil moisture content



Cross section through an unsaturated porous medium

$$\eta = \frac{volume\ of\ \ voids}{total\ volume}$$

$$\theta = \frac{volume\ of\ \ water}{total\ volume}$$

Based on Figure 4.1.2 in Chow et al. (2010)

Image: https://www.soils.org/images/publications/sssaj/67/3/703f11.jpeg

Porosity: relationships and effective porosity

$$\eta = \frac{volume\ of\ voids}{total\ volume} = \frac{V_i}{V}$$

$$= \frac{\rho_m - \rho_d}{\rho_m} = 1 - \frac{\rho_d}{\rho_m}$$

Where, ρ_m is the density of the mineral particles, grain density
and ρ_d is the bulk density

>Effective porosity is the amount of interconnected pore space available for fluid flow.
For sediments: effective porosity == porosity
> Primary porosity and secondary porosity

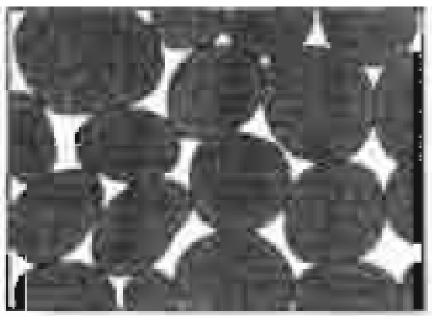
Table: Porosity ranges for rocks and sediments

Sandstone	33%-37%	Well-sorted sand or gravel	25-50%
Limestone	30%	Sand and gravel, mixed	20-35%
Dolomite	26%	Glacial till	10-20%
Peat	92%	Silt	35-50%
Shale	6%	Clay	33-60%

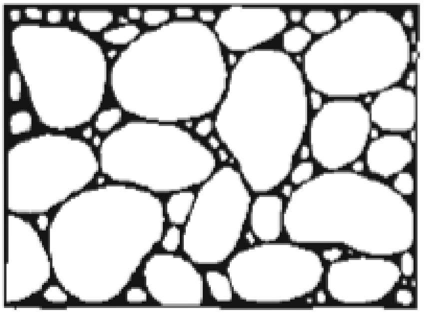
From: Fetter and Todd

Relationship of interstices with porosity

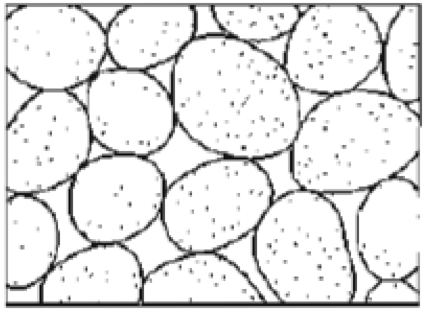
Well-sorted sedimentary deposit with high porosity



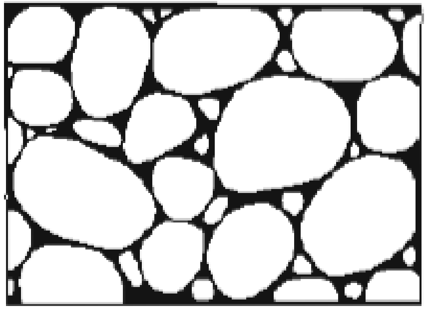
Poorly sorted sedimentary deposit with low porosity



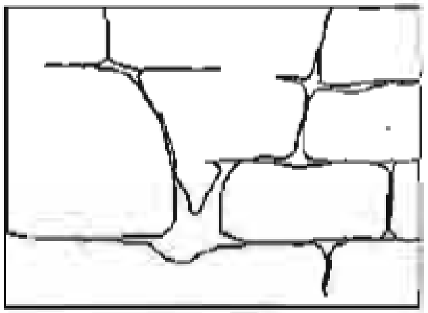
Well-sorted sedimentary deposit with porous pebbles, very high porosity



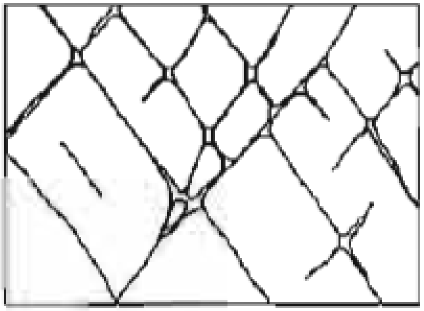
Reduced porosity due to deposition of minerals in interstices



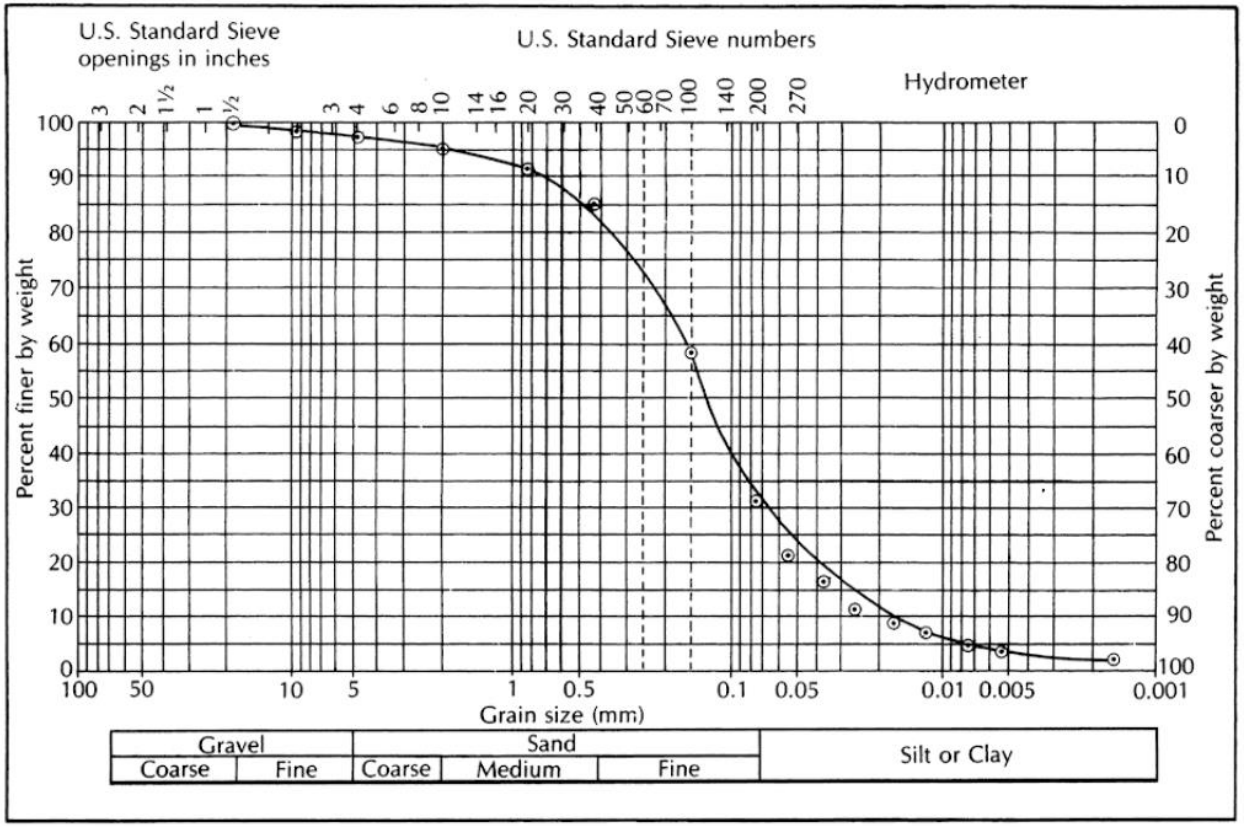
Solid rocks rendered porous by solution



Solid rocks rendered porous by fracturing



Grain size distribution



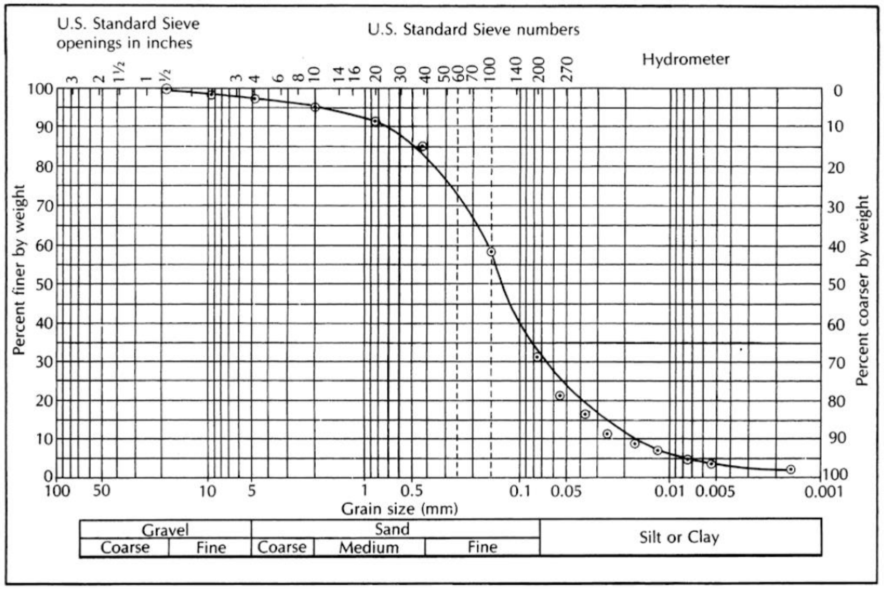
▲ FIGURE 3.4
Grain-size distribution curve of a silty fine to medium sand.

- Particles passing 200-mesh (0.075 mm) screen are called fines
- Sieves for sizes > 0.075 mm
- Hydrometer test for fines (<0.075 mm)
- Uniformity coefficient:

$$C_u = \frac{d_{60}}{d_{10}}$$

$C_u < 4 \rightarrow$ well sorted
$C_u > 6 \rightarrow$ poorly sorted
d_{10} : effective grain size

Poorly sorted vs. well sorted



▲ FIGURE 3.4
Grain-size distribution curve of a silty fine to medium sand.

Poorly sorted

C_u = 8.3

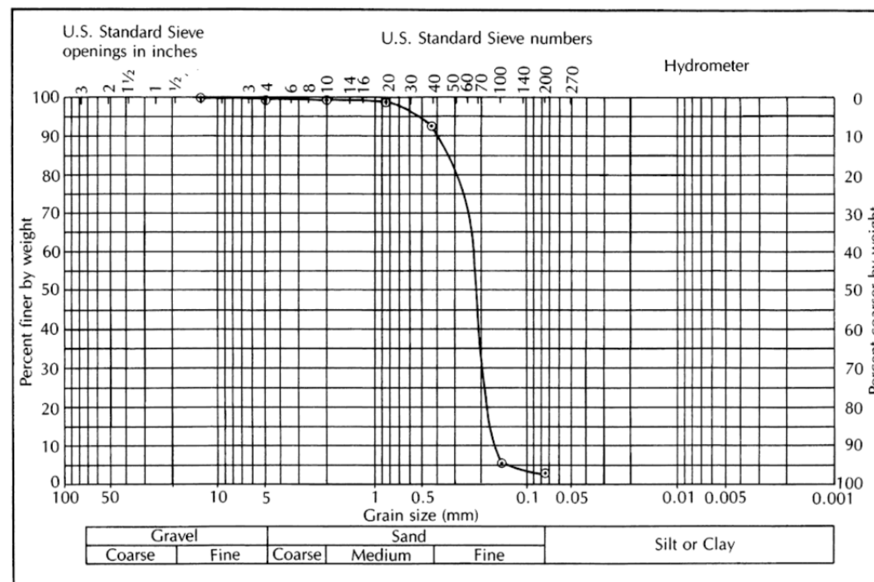
Silty fine to coarse sand

Well sorted

C_u = 1.4

Fine sand

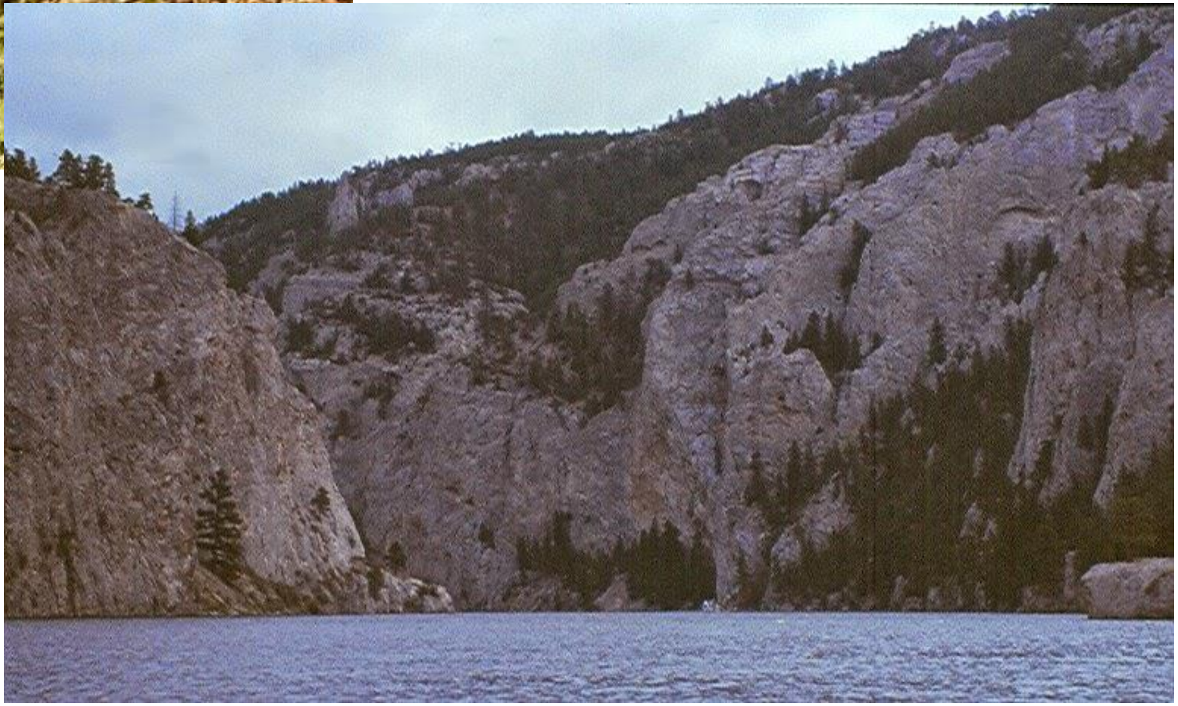
No need to perform hydrometer test as <5% fines



▲ FIGURE 3.5
Grain-size distribution curve of a fine sand.

Outcrop of the Minnelusa Formation at Ranch A near Beulah, Wyoming.

Image: https://www.usgs.gov/media/images/minnelusa-formation

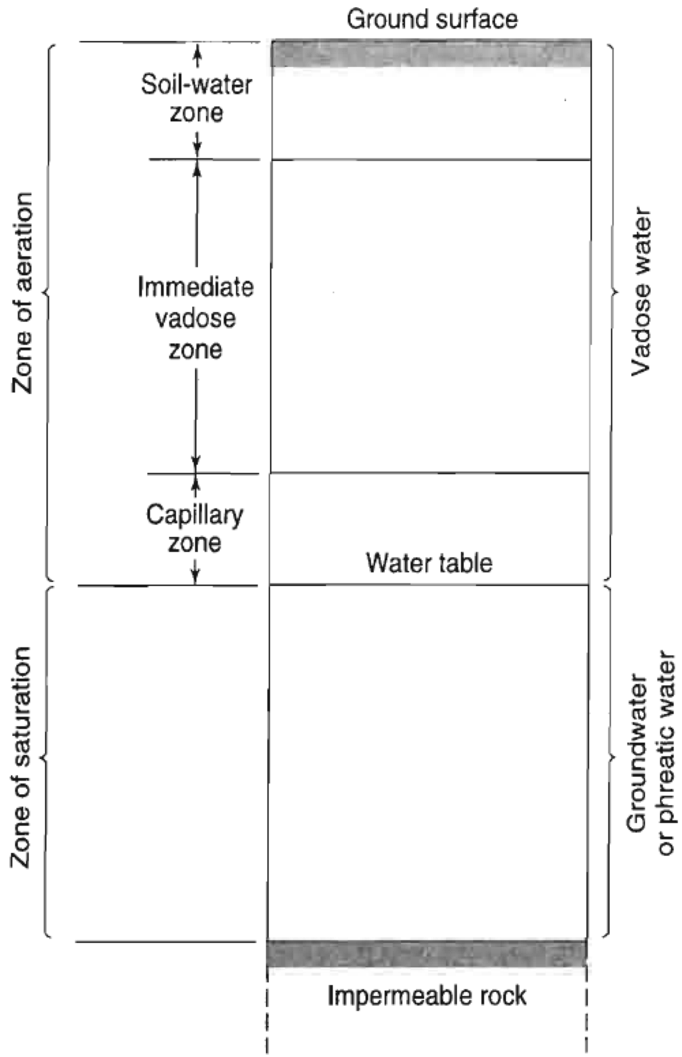


Madison Limestone at Gates of the Mountains

Image: http://1.bp.blogspot.com/-v027d0KK6Io/U5XDAB_CwrI/AAAAAAAACrM/XCJ_BjrX3Tk/s1600/Gatesofmts.jpg

AQUIFERS AND THEIR CHARACTERISTICS

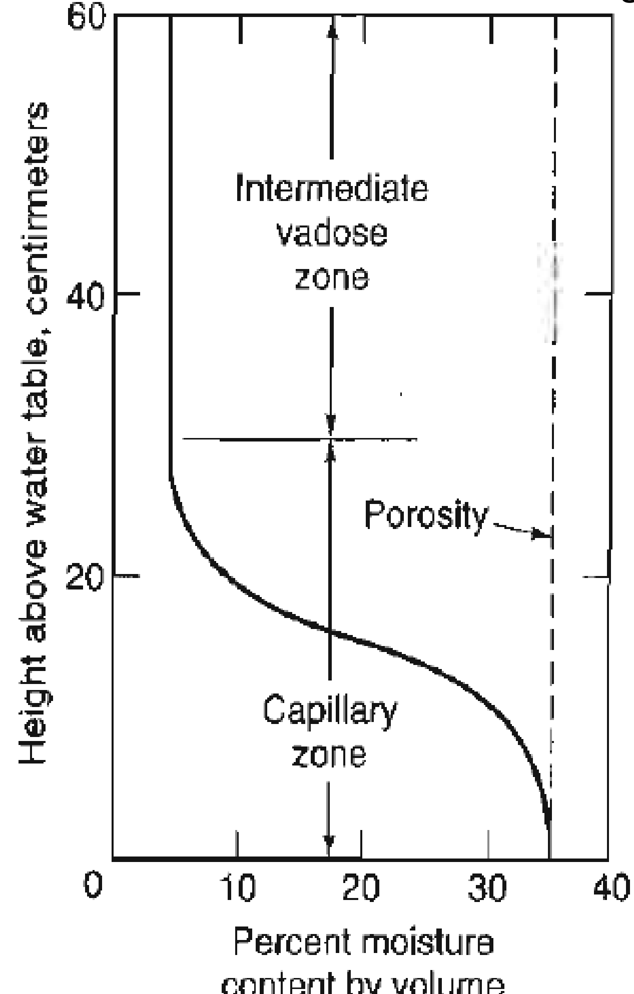
Vertical distribution of groundwater



From: Todd

Figure 2.3.1. Divisions of subsurface water.

Distribution water in a coarse sand above the water table after drainage

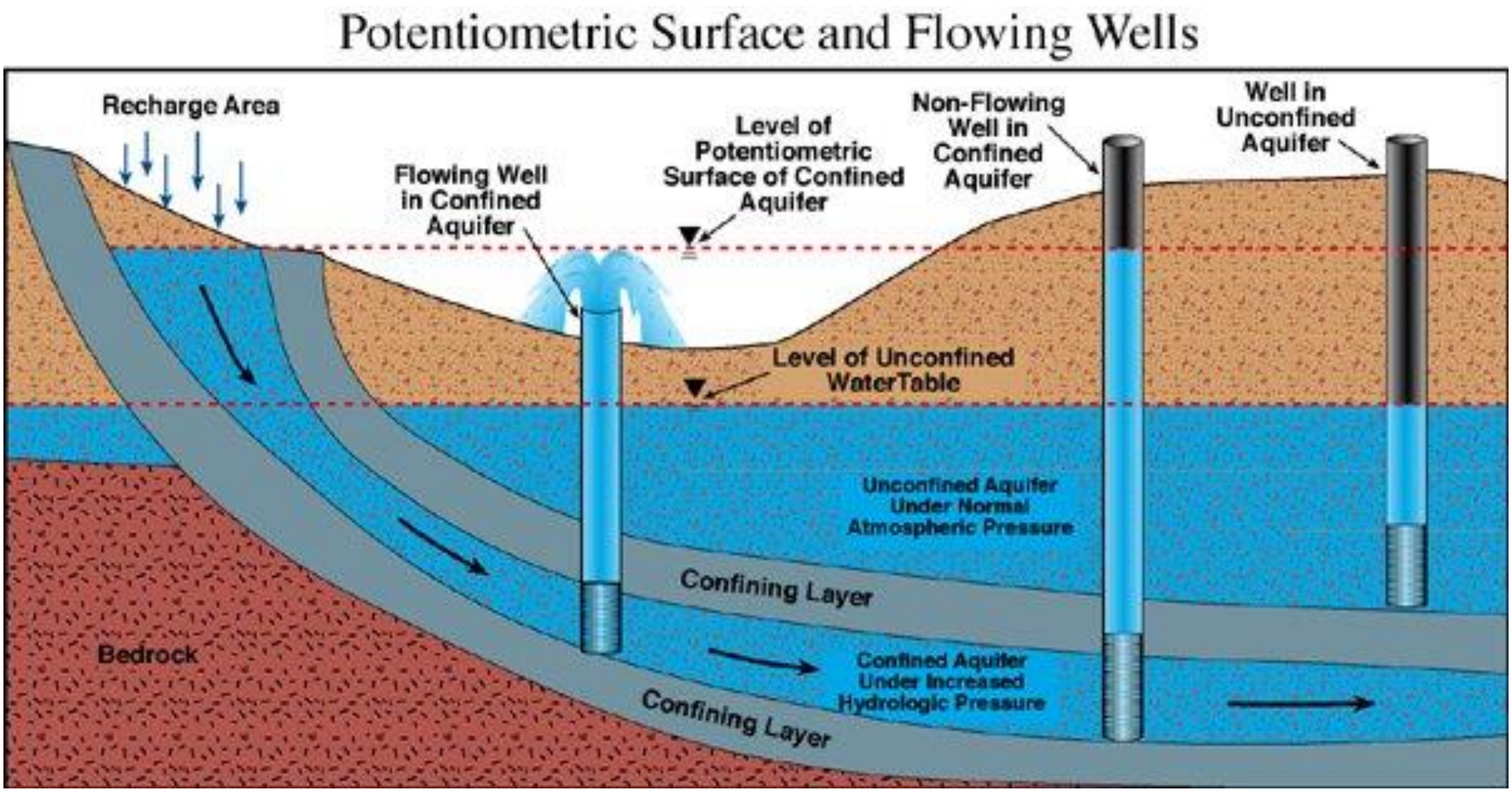


Saturated formations can be classified into four categories based on their porosity (ability to hold water) and permeability (ability to transmit water):

1. Aquifer*: stores and transmits water. Sand.
2. Aquitard: slow transmission with some storage. Sand mixed with clay. *(leaky confining layer)*
3. Aquiclude: water can be stored but not transmitted. Impermeable, closed to water movement. Porosity can be high causing large amounts of water to be stored but not transmitted. Clay. *(confining layer)*
4. Aquifuge: neither stores nor transmits. Material is neither porous nor permeable. Compact rocks without any fractures such as solid granite. *(confining layer)*

*groundwater reservoir or water bearing formation

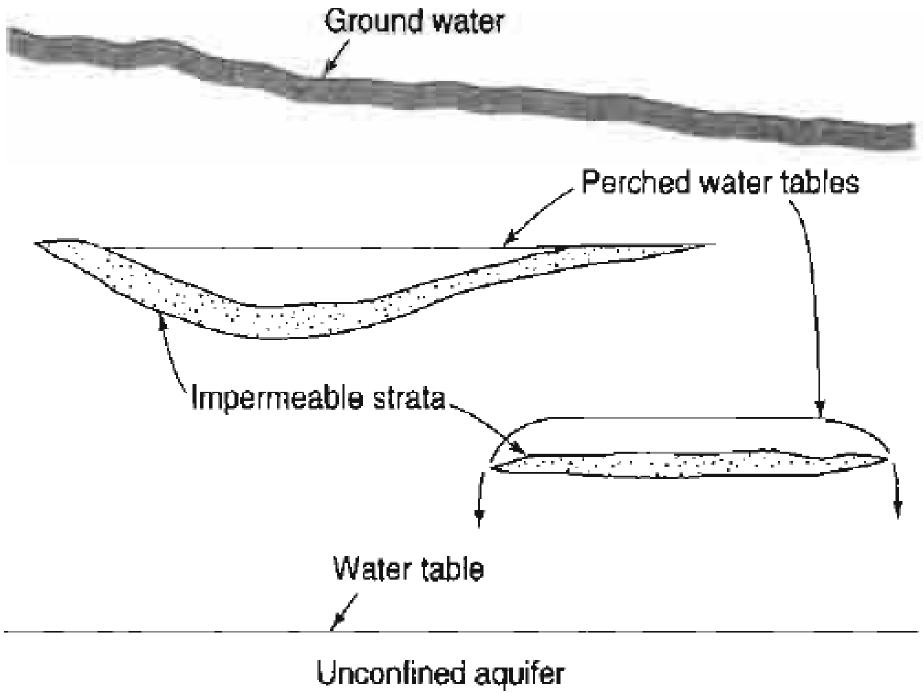
An ***aquifer*** is a geologic unit that can store enough water and transmit it at a rate fast enough to be hydrologically significant.

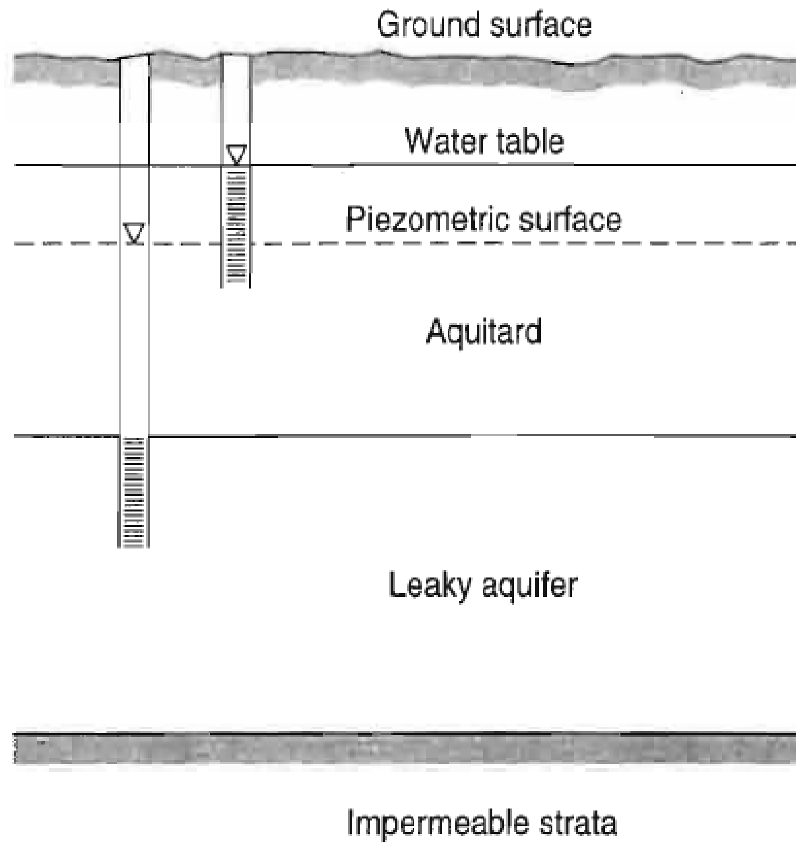


There are two types of aquifers: *unconfined* and *confined (artesian or pressure aquifers)*.

Image: http://www.waterwatchalliance.us/Large%20images/potentiometricsurface_flow_well_sml.jpg

Other types: perched, leaky/ semi-confined aquifers



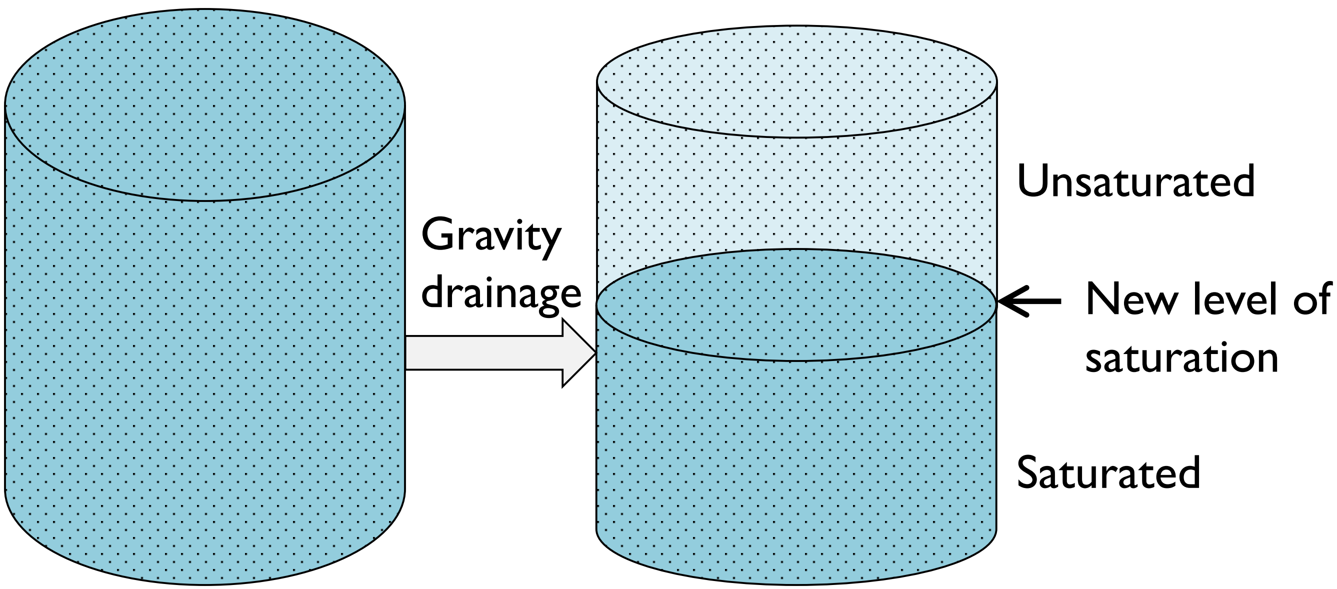


Pumping removes water from both layers (horizontal flow in aquifer, vertical flow through aquitard)

From: Todd

Saturated formations are characterized by specific yield

Ratio of the volume of water that drains from a saturated medium owing to the attraction of gravity to the total volume of the medium



Surface tension vs. gravity forces

Increasing grain size increases surface tension

Volume of rock saturated with water

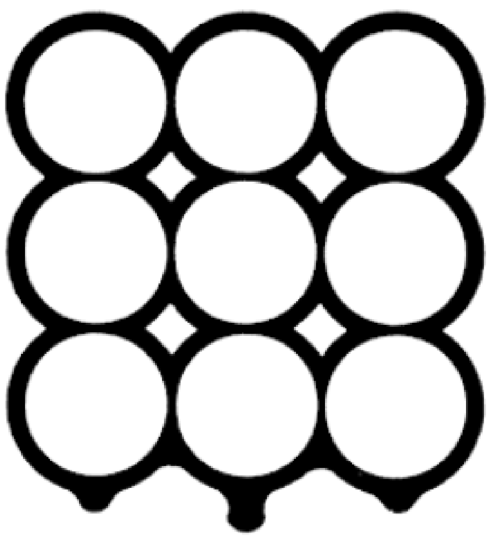
Volume of rock saturated after dewatering

Specific retention:

Ratio of the volume of water that a saturated medium retains against gravity to the total volume of the medium

$$n = S_y + S_r$$

Specific retention increases with decreasing grain size.



◀ FIGURE 3.9
Pendular water clinging to spheres owing to surface tension. Gravity attraction is pulling the water downward.

Table 3.5 Specific Yields in Percent

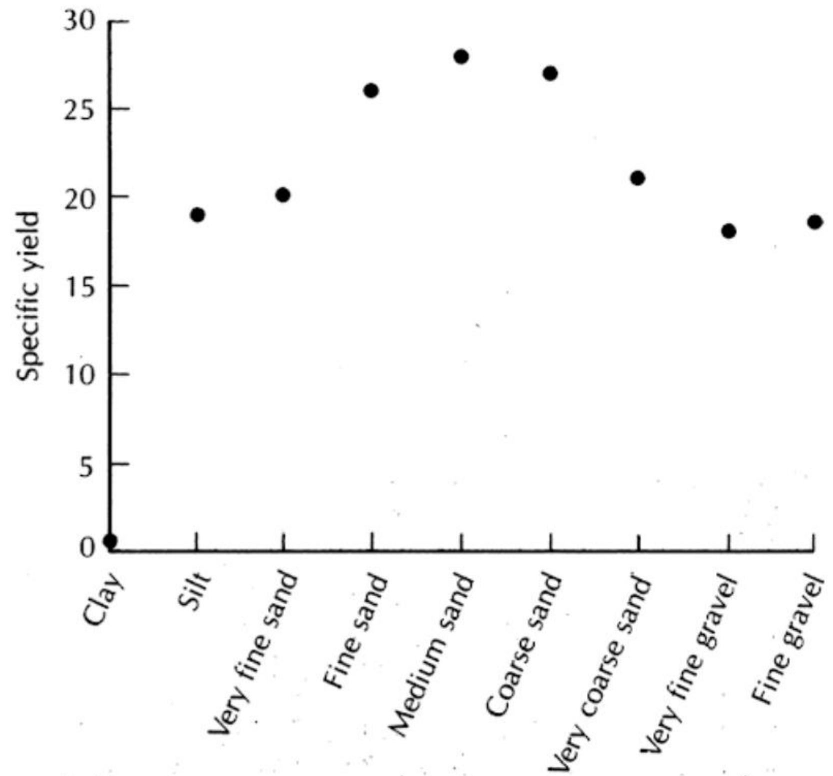
Material	Maximum	Specific Yield Minimum	Average
Clay	5	0	2
Sandy clay	12	3	7
Silt	19	3	18
Fine sand	28	10	21
Medium sand	32	15	26
Coarse sand	35	20	27
Gravelly sand	35	20	25
Fine gravel	35	21	25
Medium gravel	26	13	23
Coarse gravel	26	12	22

Source: Johnson (1967).

From: Fetter

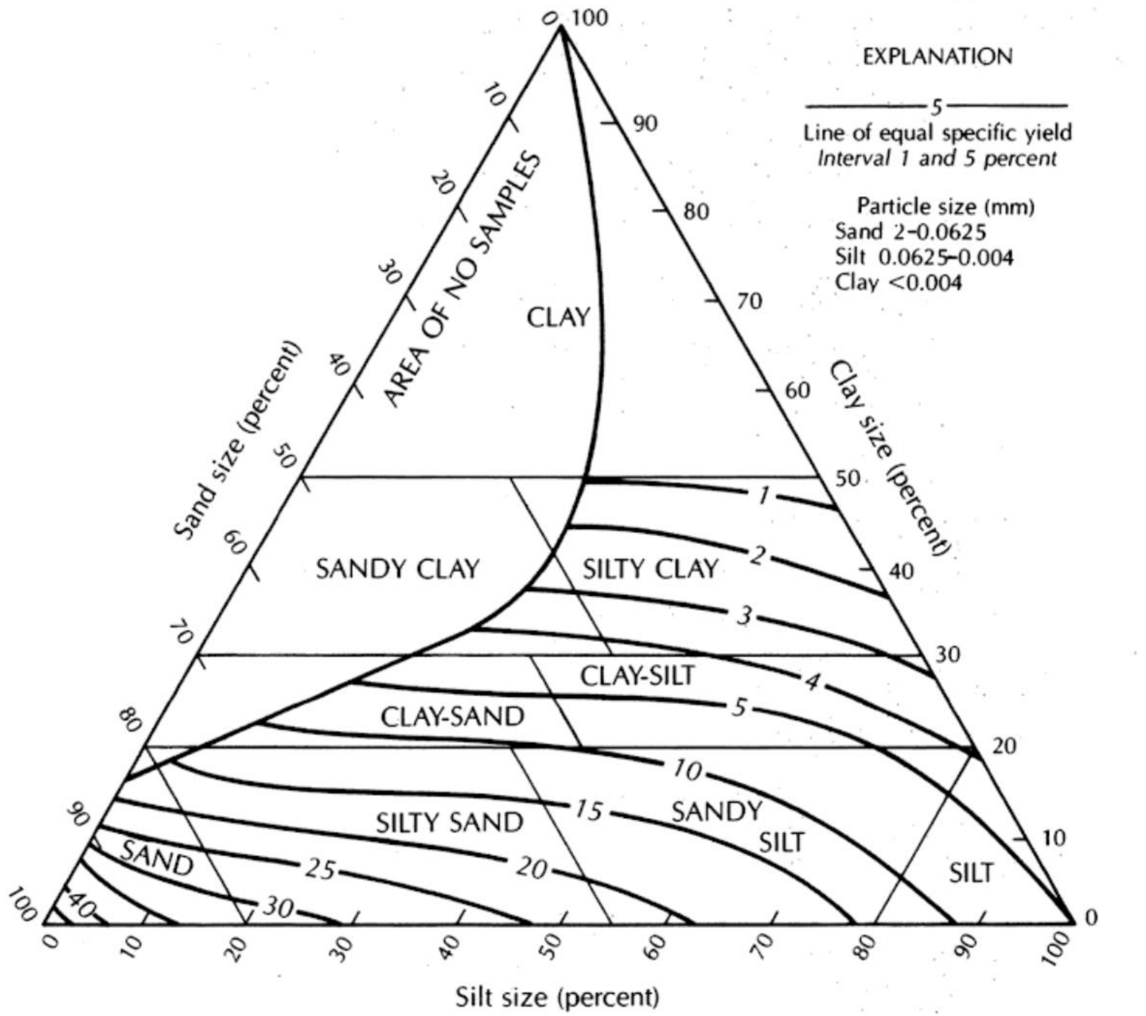
Specific yield varies with particle size

▶ FIGURE 3.10
Specific yield of sediments from the Humboldt River Valley of Nevada as a function of the median grain size.
Source: Data from P. Cohen, U.S. Geological Survey Water-Supply Paper *1975, 1965.*



From: Fetter

Specific yield with particle size



▲ FIGURE 3.11
Textural classification triangle for unconsolidated materials showing the relation between particle size and specific yield. *Source: A. I. Johnson*, U.S. Geological Survey Water-Supply Paper *1662–D, 1967.*

From: Fetter

Aquifers are further characterized on their ability to hold, release and transmit water.

1. *Porosity* ϕ: (characterizes storage) Volume of water stored in a saturated porous medium per unit volume of the medium
2. *Storage coefficient or storativity:* (characterizes release) The volume of water that a unit area of aquifer releases (takes up) in response to a unit decrease (increase) in head

$$S[-] = \frac{Volume\ of\ water \left\{ \frac{taken\ into}{released\ from} \right\} storage[L^3]}{Surface\ area\ of\ aquifer\ x \left\{ \frac{increase}{decrease} \right\} in\ head[L]}$$

3. *Specific storage:* (also characterizes release) The volume of water that a unit volume of aquifer releases (takes up) in response to a unit decrease (increase) in head, while remaining fully saturated

$$S_s[L^{-1}] = \frac{Volume\ of\ water \left\{ \frac{taken\ into}{released\ from} \right\} storage[L^3]}{Volume\ of\ aquifer\ x \left\{ \frac{increase}{decrease} \right\} in\ head[L]}$$

ρ_w: density of water volume (kg/m^3)

g: acceleration due to gravity (m/s^2)

α: compressibility of aquifer skeleton (N/m^2)

η: porosity

β:compressibility of water (N/m^2)

$$S_s = \rho_w g \left(\alpha + \eta \beta \right)$$

Water is stored or released due to the compressibility of the mineral skeleton and the pore water per unit change in head.

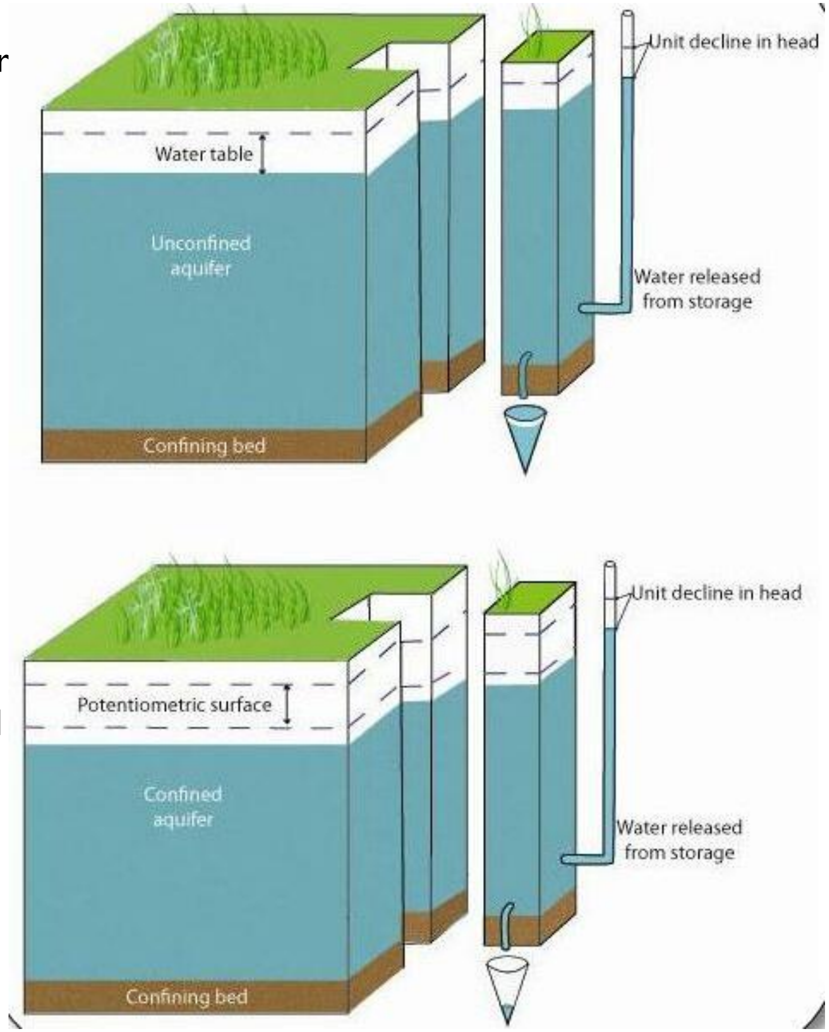
Storage coefficient is an important property of aquifers

For unconfined aquifers, a change in head is reflected in change of water table with a corresponding change in water content (saturated to unsaturated) of the aquifer.

Specific yield: Storage coefficient of unconfined aquifers (0.05 to 0.35).

$$S = S_y + bS_s ; S_y >> bS_s$$

$$\rightarrow S = S_y$$



For confined aquifers, a unit change in head is reflected in a change of piezometric surface but the aquifer remains saturated. Storage coefficient of confined aquifers is termed *storativity* (5×10^{-5} to 5×10^{-3}).

$$S = H.S_s$$

The change in storage is due to:

1. Compression/expansion due to change in porosity
2. Change in pressure causing slight expansion/compression of water

Image: https://www.dwaf.gov.za/Groundwater/Groundwater_Dictionary/storage%20coefficient.jpg

Aquifer characterization: Transmission properties

1. *Hydraulic conductivity, K:* Appears in the Darcy's law. Defined as the rate of flow under a unit hydraulic gradient across a unit area of the aquifer. Varies with both properties of the fluid (dynamic viscosity, density) and medium (grain size, configuration of water transmitting pathways, etc.).
2. *Transmissivity, T:* Rate at which water moves through a unit width of an aquifer under a unit hydraulic gradient. If H is the vertical thickness of the porous media: $T = H.K$

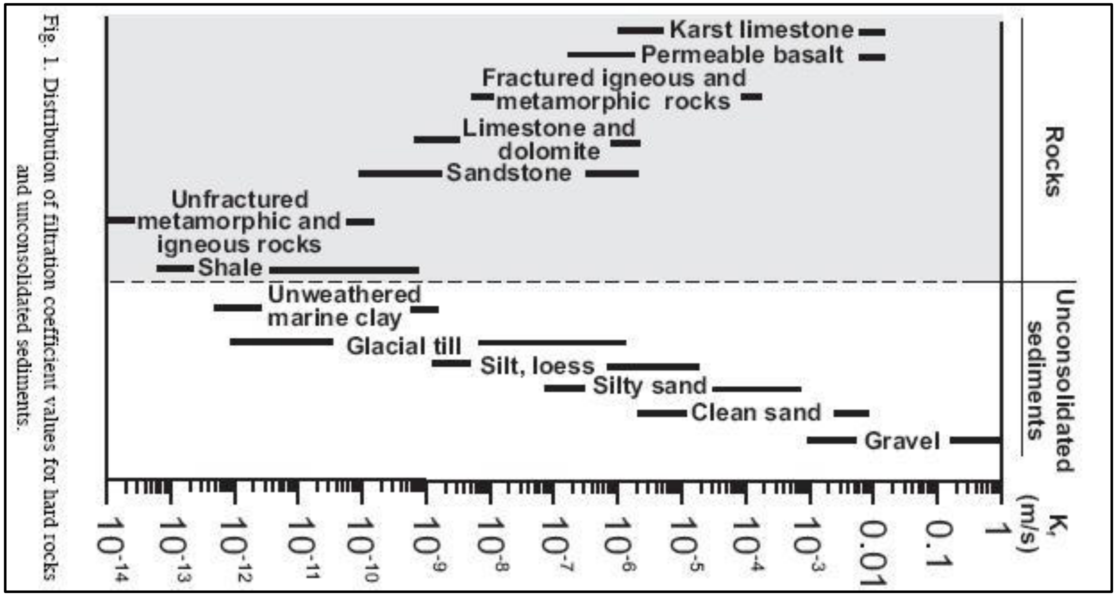
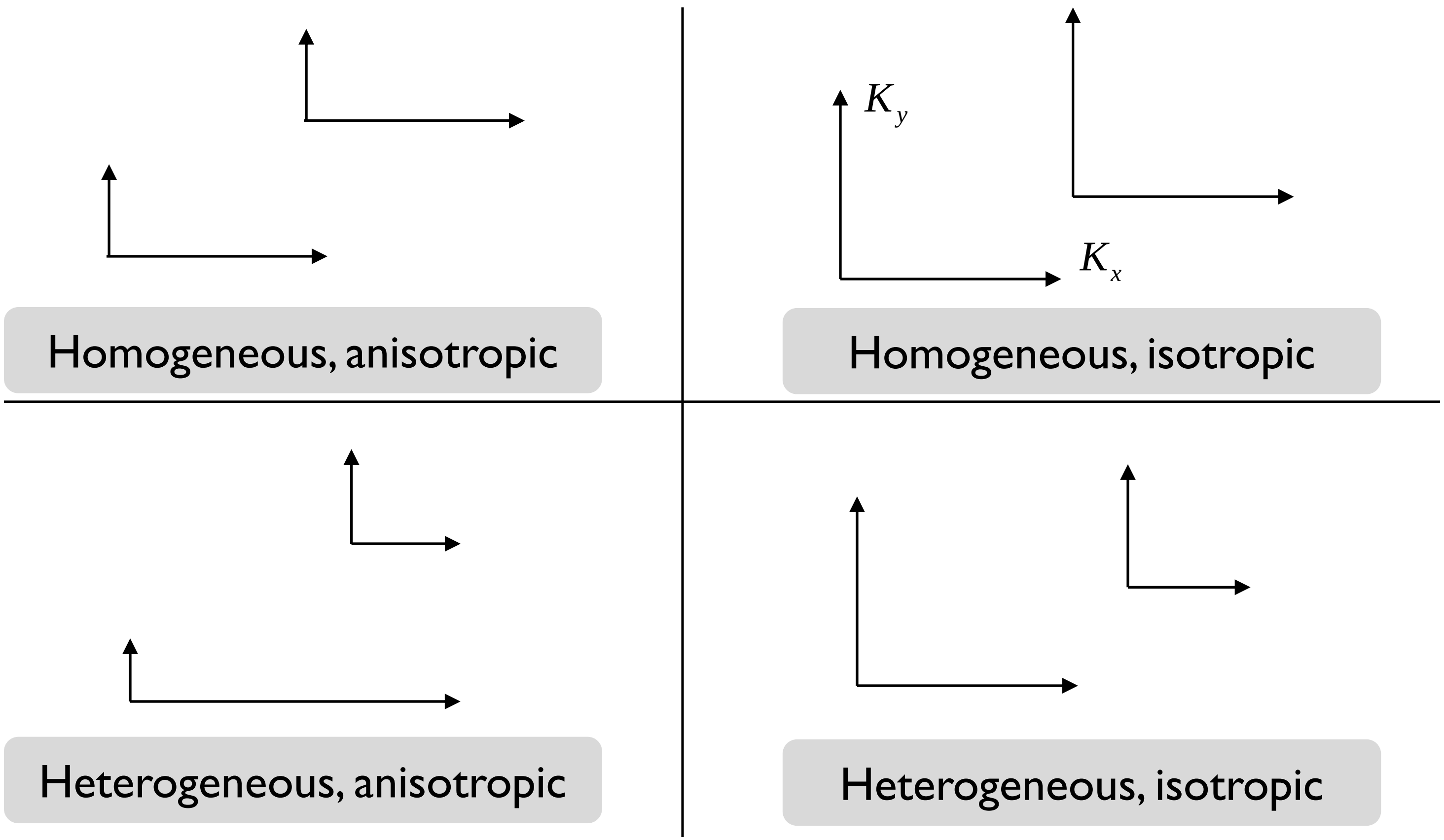


Image: http://www.scielo.org.mx/img/revistas/geoint/v45n3/a4f1.jpg

Characterizing variability in the sub-surface:

1. *Homogeneous/ Heterogeneous:* Property in a given direction is the same at all points, the medium is homogeneous, otherwise, heterogeneous
2. *Isotropic/ Anisotropic:* If the property at a point is the same for all directions, the medium is isotropic; if it differs for different directions, it is anisotropic



Major Indian aquifer systems

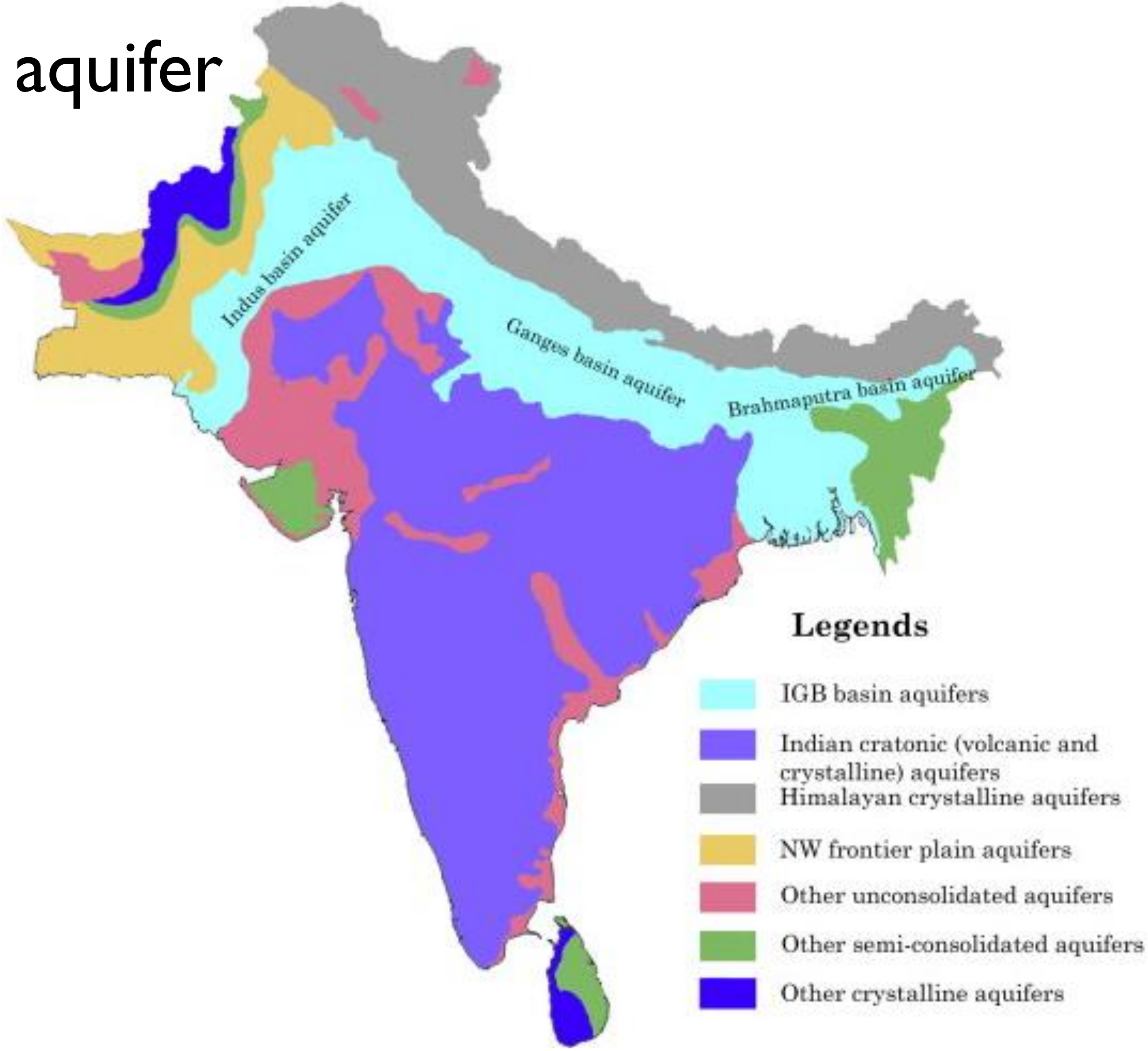
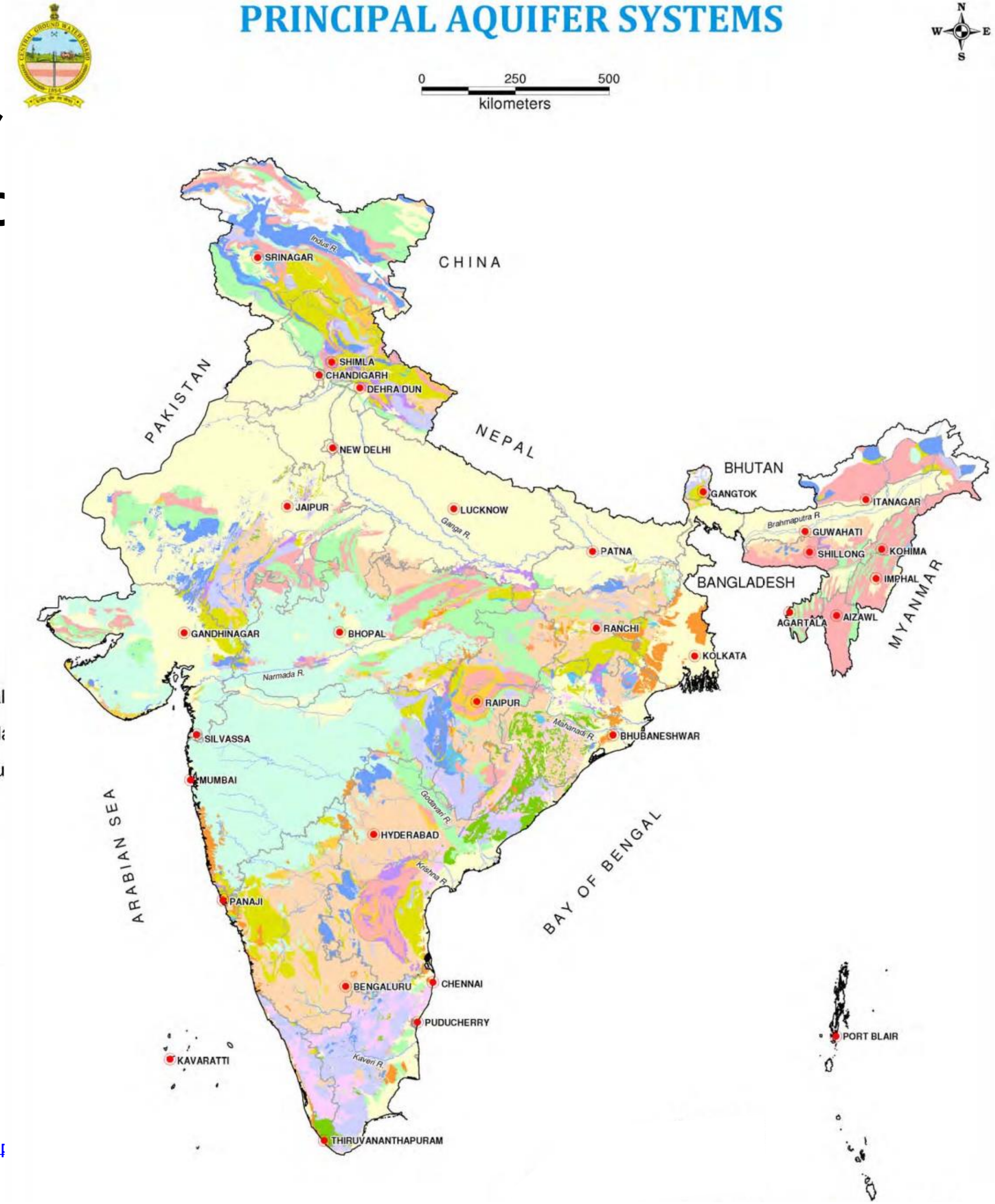


Image: https://www.researchgate.net/profile/Kazi_Matin_Ahmed/publication/275103720/figure/fig1/AS:267548671344704@1440799982393/Map-of-the-major-aquifers-of-the-Indian-Sub-Continent-The-figure-is-not-to-scale-and.png

Major Indian aquifer systems: a closer loc



Aquifers

- Alluvium
- Laterite
- Basalt
- Sandstone
- Shale
- Limestone
- Granite
- Schist
- Quartzite
- Charnockite
- Khondalite
- Gneiss
- Intrusives
- Banded Gneissic Complex
- Unclassified

- State Capital
- State Bound
- National Bou
- River

Image:
http://www.indiawaterportal.org/sites/indiawaterportal.org/files/aquifer_systems.j

Central Ground Water Board, Ministry of Water Resources, Government of India

DARCY'S EXPERIMENT

Henry Darcy carried out experiments to understand the motion of water through bed of sand used for water filtration
He reported (1856):

I have attempted by precise experiments to determine the law of the flow of water through filters…The experiments demonstrate positively that the volume of water which passes through a bed of sand of a given nature is proportional to the pressure and inversely proportional to the thickness of the bed traversed; thus, in calling ***s*** the surface area of a filter, ***k*** a coefficient depending on the nature of the sand, **e** the thickness of the sand bed, $\boldsymbol{P-H_o}$ the pressure below the filtering bed, $\boldsymbol{P+H}$ the atmospheric pressure added to the depth of water on the filter; one has for the flow of this last condition $Q=(ks/e)(H+e+H_o)$, which reduces to $Q=(ks/e)(H+e)$ when $H_o=0$, or when the pressure below the filter is equal to the weight of the atmosphere.

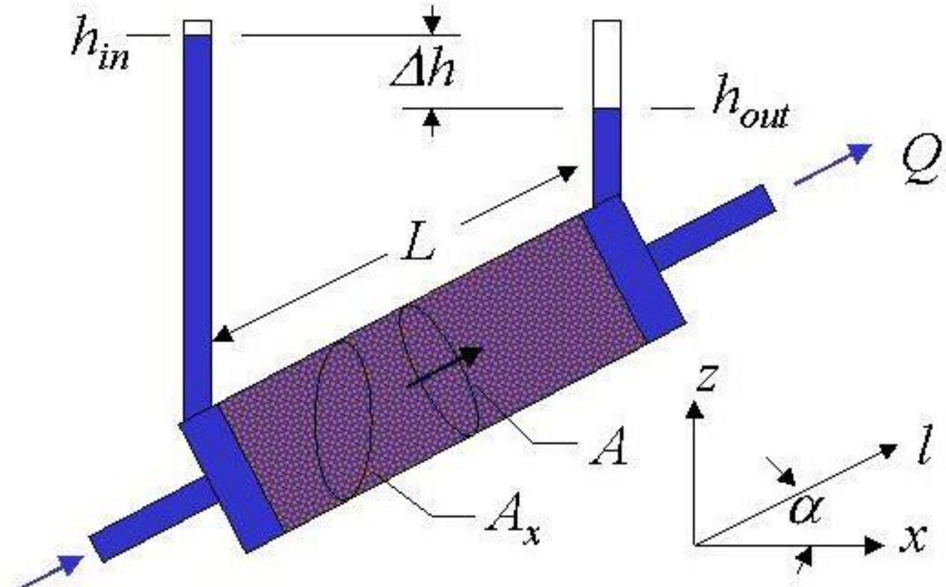
From: Todd

Image: http://en.wikipedia.org/wiki/Henry_Darcy (right)

Henry Darcy carried out experiments to understand the motion of water through bed of sand used for water filtration

Henry Darcy

q is volumetric flow rate per unit cross-sectional area, or, specific discharge [LT^{-1}]
z is the elevation above a datum [L]
p is the water pressure [FL^{-2}]
K is the hydraulic conductivity of the medium [LT^{-1}]
γ_w is the weight density of water [FL^{-3}]



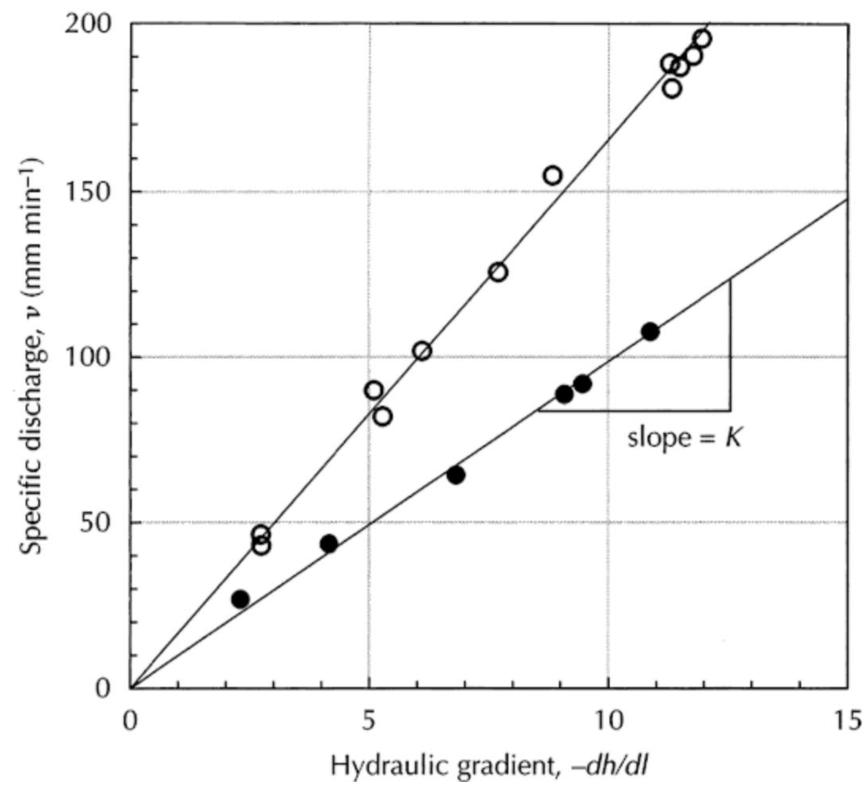


Image: http://en.wikipedia.org/wiki/Henry_Darcy (right)
http://biosystems.okstate.edu/darcy/LaLoi/figure2.jpg (left)

Quantifying flow in saturated porous medium: *Darcy's law* states that flow is proportional to the gradient of the mechanical potential energy (gravitational and pressure)

Henry Darcy

q is volumetric flow rate per unit cross-sectional area, or, specific discharge [LT^{-1}]
z is the elevation above a datum [L]
p is the water pressure [FL^{-2}]
K is the hydraulic conductivity of the medium [LT^{-1}]
γ_w is the weight density of water [FL^{-3}]

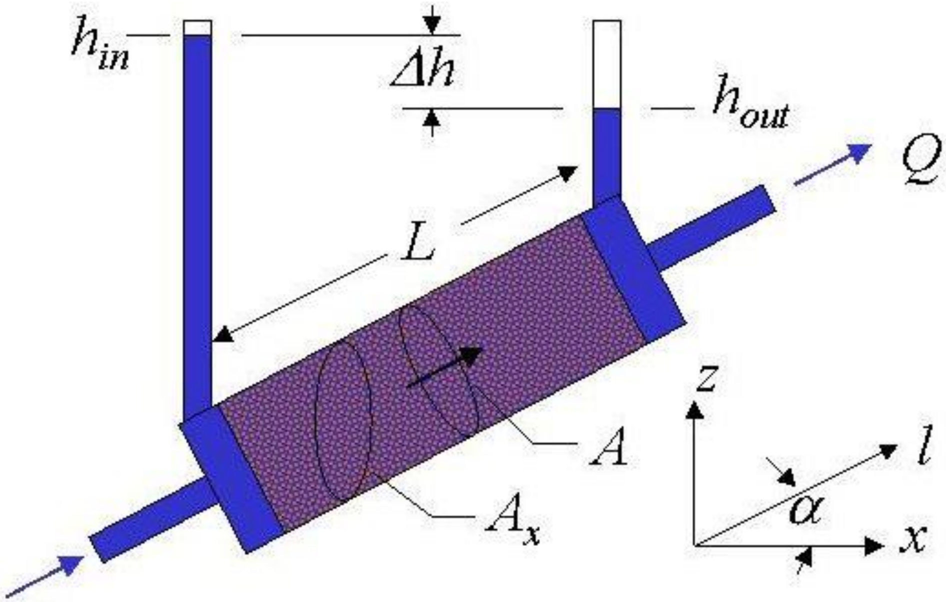
$$Q \propto h_{in} - h_{out}$$

$$Q \propto 1/L$$

$$Q = -KA\left(\frac{h_{in} - h_{out}}{L}\right)$$

$$Q = -KA\frac{dh}{dl}$$

$$Q = -KA\frac{d\left(z + p/\gamma_w\right)}{dl}$$



Darcy's law is only applicable to laminar flows

Image: http://en.wikipedia.org/wiki/Henry_Darcy (right)
http://biosystems.okstate.edu/darcy/LaLoi/figure2.jpg (left)

$$v = \frac{Q}{A} = -K\frac{dh}{dl} = -K\frac{d\left(z + p/\gamma_w\right)}{dl}$$

Specific discharge or Darcy flux; not a true velocity as the cross-sectional area of flow is partially blocked with soil material.

$$v_x = \frac{Q}{\eta_e A} = -\frac{K}{\eta_e}\frac{dh}{dl}$$

Seepage velocity or average linear velocity; η_e is the effective porosity

$$K = \frac{-Q}{A\left(dh/dl\right)}$$

Hydraulic conductivity, or, coefficient of permeability; dimensions: L/T

$$Q \propto \gamma, \ Q \propto \frac{1}{\mu}$$

$$Q \propto d^2$$

K is a function of properties of porous medium and fluid passing through it

>more viscous fluid → greater resistance to fluid flow

>greater specific weight → greater discharge

>greater dynamic viscosity → lower discharge

$$Q = -\frac{Cd^2\gamma A}{\mu}\frac{dh}{dl}$$

C: shape factor, property of porous media

d: particle size, property of porous media

μ, γ: properties of fluid

Intrinsic permeability:

$$Q = -\frac{Cd^2 \gamma A}{\mu}\frac{dh}{dl}$$

C: shape factor, property of porous media

d: particle size, property of porous media

μ, γ: properties of fluid

$$K_i = Cd^2$$

K_i: intrinsic permeability

d: mean pore diameter

C: constant that describes the overall effect of the shape of the pore spaces

$$K = K_i\left(\frac{\gamma}{\mu}\right) = K_i\left(\frac{\rho g}{\mu}\right)$$

$$\gamma, \mu = f(temperature, salinity)$$

Standard value of hydraulic conductivity is defined for pure water at a temperature of 15.6°C.

Permeability of sediments

Table 3.7 Ranges of Intrinsic Permeabilities and Hydraulic Conductivities for Unconsolidated Sediments

Material	Intrinsic Permeability (darcys)	Hydraulic Conductivity (cm/s)
Clay	$10^{-6} - 10^{-3}$	$10^{-9} - 10^{-6}$
Silt, sandy silts, clayey sands, till	$10^{-3} - 10^{-1}$	$10^{-6} - 10^{-4}$
Silty sands, fine sands	$10^{-2} - 1$	$10^{-5} - 10^{-3}$
Well-sorted sands, glacial outwash	$1 - 10^{2}$	$10^{-3} - 10^{-1}$
Well-sorted gravel	$10 - 10^{3}$	$10^{-2} - 1$

>Intrinsic permeability is a function of the size of pore opening

>Smaller size of sediment grains, larger surface area, increasing frictional resistance, reducing intrinsic permeability

>as median grain size increases, permeability also increases

>permeability decreases for a given median diameter as standard deviation of particle size increases

>unimodal samples have a greater permeability than bimodal samples

Hazen method for hydraulic conductivity

$$K = C\left(d_{10}\right)^2$$

Applicable to sands where effective grain size (d_{10}) is between 0.1 and 3.0 mm

K: hydraulic conductivity, cm/s

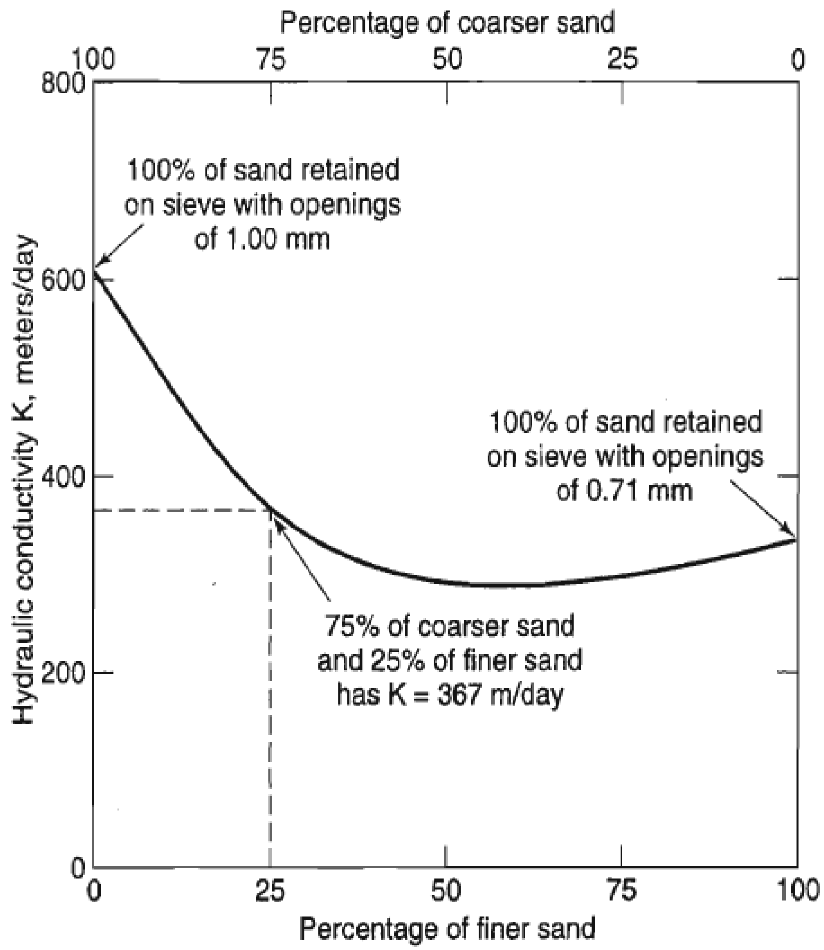
d_{10}: effective grain size, cm

C: coefficient based on the following table:

Very fine sand, poorly sorted	40–80
Fine sand with appreciable fines	40–80
Medium sand, well sorted	80–120
Coarse sand, poorly sorted	80–120
Coarse sand, well sorted, clean	120–150

(Applicable to sediments)

Variation of hydraulic conductivity with particle size and composition



From: Todd

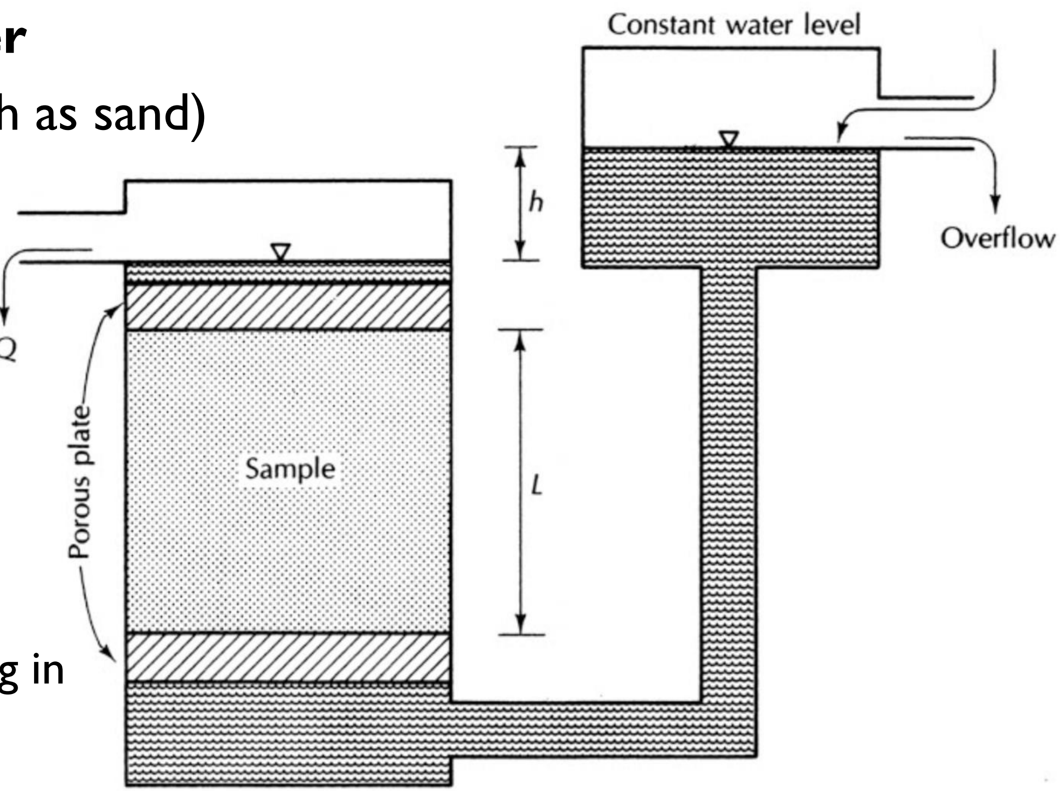
Lab determination of hydraulic conductivity:

Constant head permeameter

(for non cohesive sediments such as sand)

$$V = Qt = -\frac{KA\left(h_A - h_B\right)}{L}$$

$$\rightarrow K = \frac{VL}{Ath}$$



V: volume (cm^3) of water discharging in time t (s)

L: length of the sample (cm)

A: cross sectional area of the sample(cm^2)

h: hydraulic head (cm)

K: hydraulic conductivity (cm/s)

Lab determination of hydraulic conductivity:

Falling head permeameter

(for cohesive sediments)

$$q_{in} = -A_t \frac{dh}{dt}, q_{out} = -\frac{KA_c h}{L}$$

$$\rightarrow K = -\frac{A_t L}{A_c}\frac{dh}{dt}$$

q_{in}/q_{out}: rate at which water enters/ leaves

h_0: initial head in the falling tube (cm)

h: final head in the falling tube at time t (cm)

d_t: diameter of falling head tube

d_c : diameter of the sample

L: length of the sample (cm)

Integrate from t=0 to t: $$K = \frac{d_t^2 L}{d_c^2 t}\ln\frac{h_0}{h}$$