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Vehicle routing with cumulative objectives: A state of the art and analysis

Karina Corona-Gutiérrez [a], Samuel Nucamendi-Guillén [a], Eduardo Lalla-Ruiz [b,*]

[a] Facultad de Ingeniería, Universidad Panamericana Campus Guadalajara, Álvaro del Portillo 49, Zapopan, Jalisco, Mexico
[b] Department of Industrial Engineering and Business Information Systems, University of Twente, the Netherlands

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ABSTRACT

In the past decades, there has been an increasing body of research in vehicle routing problems involving cumulative costs. These problems consider cumulative objective functions such as the sum of arrival times to customers or the cumulative costs based on the vehicles' load throughout the routes. This paper reviews this type of routing problem by means of the cumulative vehicle routing problem (Cum-VRP) and cumulative capacitated vehicle routing problem (CCVRP). In doing so, we present and discuss all related problem variants with regard to the different problem features proposed over time. Moreover, we provide an analysis of related papers concerning solution algorithms and used benchmark instances. Finally, an overview of the trends and promising areas for further research are also provided.

1. Introduction

Cumulative vehicle routing problems are an extension of the classic capacitated vehicle routing problem (VRP, Dantzig & Ramser (1959)) aiming to find a set of delivery routes that optimizes a given objective function considering cost accumulation in the course of the planning realization. The first cumulative VRP, i.e., Cum-VRP, was proposed in 2008 by Kara, Kara, and Yetiş (2008), as a way to incorporate the flow of freights throughout the routes within two applications, namely, energy minimizing and school bus routing. In doing so, the authors quantify the objective function as the product of the vehicle's load and the cost of the arc traversed to reach the node in which the requested demand is delivered/collected. Given that, the objective of the Cum-VRP prefers to traverse the most distant arcs as the vehicle gets lighter. Later, that approach was extended to consider the arrival of customers as the accumulative component in such a way that customers have to wait the least time possible (see Ngueveu, Prins, & Wolfler Calvo (2010)). In this sense, the authors compute the arrival time to the node is as the total distance traveled to reach the incumbent node. That is, the sum of the arcs traversed to reach the incumbent node. That variant is known as the cumulative capacitated vehicle routing problem (CCVRP) and received relevant attention due to several applications on health-care, disaster relief operations, maintenance, and customer-centred logistics operations.

Due to the wide spectrum of possible applications, this type of routing problem has attracted the interest of researchers and practitioners, resulting in several works continuing and extending these two cumulative-oriented routing problems. Recently, Nucamendi-Guillén, Flores-Díaz, Olivares-Benitez, and Mendoza (2020) extended the problem by incorporating priority indexes to cover commercial contexts such as the delivery of perishable goods requiring to differentiate between orders (based on the preference in the attention of the customers). Liu and Jiang (2019) extended the CCVRP proposed in (Ngueveu et al., 2010) to consider customers' time windows, they referred to this problem as Cum-VRPTW[1]. Lalla-Ruiz and Voß (2020) considered the problem with multiple depots to handle logistics contexts involving vehicles departing from different depots. Wang, Choi, Liu, and Yue (2018) studied the problem as an emergency transportation problem in a disaster relief supply chain where fast response and fairness are of major importance. As can be noticed, variants considering practical features already known in the VRP literature are also starting to be considered with cumulative objectives. In that regard, we can observe that, in some cases, the use of acronyms to distinguish the version of the problem is not consistently used due to the close relationship between both objectives. Besides this, the increasing number of vehicle routing works with cumulative objectives lead to the necessity of a literature study that permits positioning and mapping current research on this type of problem while also providing insights concerning successful or efficient solutions approaches developed so far.

This work presents a literature review on cumulative vehicle routing

* Corresponding author.
E-mail addresses: kcorona@up.edu.mx (K. Corona-Gutiérrez), snucamendi@up.edu.mx (S. Nucamendi-Guillén), e.a.lalla@utwente.nl (E. Lalla-Ruiz).

[1] Notice that a more accurate acronym should have been CCVRP-TW instead of Cum-VRPTW as the cumulative VRP variant extended was CCVRP.

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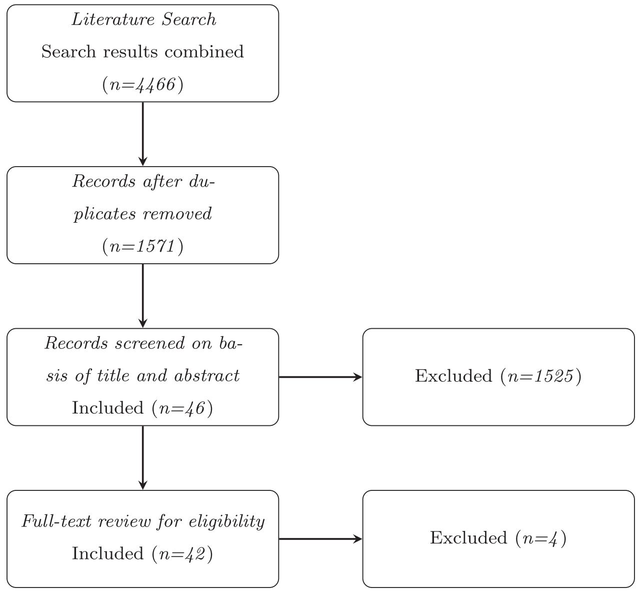


Fig. 1. Search strategy results.

problems by means of the CCVRP and Cum-VRP and provides an analysis of their latest advances and directions with regards to problem variants and quantitative approaches.

To the best of our knowledge, no published work summarizes the state-of-the-art of research in cumulative-oriented vehicle routing problems. Below, we outline the main contributions:

- A comprehensive and extensive review of recent cumulative vehicle routing problems originated from the CCVRP and Cum-VRP that include new features, e.g., time windows, limited duration, multiple depots, among others. From the collected works, a scheme of variants permitting to map and relate current and future works is provided.
- An up-to-date literature analysis, including summarizing tables of all related cumulative routing approaches. In the case of new problem variants, this permits positioning the problem at hand and finding the most suitable method based on the problem features.
- Overview of proposed and used benchmarks in cumulative vehicle routing problems as well as analysis of solution methods' performance on most commonly used instances. This serves as a study of these methods over time while indicating current challenges regarding problem instances.

1.1. Review methodology

For conducting this state-of-the-art review, we conducted a systematic search to identify the related literature to Cum-VRP and CCVRP by employing the SCOPUS and Web of Science (WoS) databases and using the following search terms: *cumulative capacitated vehicle, latency location* [2]*, cumulative routing, cumulative vehicle, cumulative VRP, multi-depot cumulative, cumulative VRP, cumulative-oriented routing, cumulative objective* and *latency time*.

After conducting the search and collecting the works, the results from both databases were compared and duplicates were manually removed. The remaining studies were then screened based on title and abstract to identify and exclude those not meeting the selection criteria. Namely, the excluded records either belong to a different field of study (e.g., medicine, cloud computing), were identified as a non-routing vehicle problem or not related to Cum-VRP nor CCVRP.

Table 1
Cumulative vehicle routing works over time.

Publication type	2008–2012	2013–2016	2017–2021	Total
Journals	3	9	17	**29**
Proceedings	1	4	4	**9**
Book chapter	1	0	3	**4**
Total	**5**	**13**	**24**	**42**

A full-text review was performed on the remaining records to identify the problems' main features and used objective functions. Since we focused our attention on those works studying cumulative vehicle routing problems, we limit the collection of works to those addressing the CCVRP or Cum-VRP. Thus, we discarded two final records related to the Travelling Repairman Problem (TRP), whose features, although associated with the CCVRP, do not fit our inclusion criteria. As a complement, Fig. 1 explains the used search strategy and shows the records obtained at each step of the process.

The results obtained from the applied search methodology can be found in Table 1. As can be seen, despite the fact that this problem type is quite recent, the number of works addressing the CCVRP and Cum-VRP over time follows an increasing trend.

1.2. Organization of this paper

The rest of this paper is organized as follows. Section 2 describes two main cumulative problems (i.e., Cum-VRP and CCVRP) and all those variants originated from them. Afterwards, Section 3 reviews and classifies the solution approaches proposed for these problems. Analysis and discussions are provided in Section 4. Finally, Section 5 draws the main conclusions extracted from this work and provides some lines and challenges for further research.

2. Cumulative vehicle routing problems

As discussed in the introduction, two main routing problems address the VRP involving cumulative objectives, i.e., Cum-VRP and CCVRP. In

[2] *This term was used for obtaining CCVRP related works*

the following, the problem description and mathematical formulation for each of the above-mentioned problems are provided.

2.1. Problem definition and mathematical formulation for the Cum-VRP

The Cum-VRP (Kara et al., 2008) is defined over a graph $G = (V,A)$, where $V = \{0, 1, 2, \dots, n\}$ is the node set, the node 0 corresponds to the depot, and $A = \{(i,j) : i,j \in V, i \neq j\}$ represents the set of arcs. There is a fleet of $|K|$ vehicles (where K represents the set of available vehicles), all with identical capacity that can be used to serve customers, given that each customer $i \in V \setminus \{0\}$ has a demand q_i. For each pair of nodes $(i, j) \in A$, there is a travel time or travel distance (c_{ij}) associated. There is a parameter Q_0 that denotes the initial value of flow from the origin to the first node of the tour (e.g., representing the tare of truck), in addition, parameter M represents the flow capacity of the arcs of the network (maximal value of the flow on any arc of the network, i.e., capacity plus tare of the trucks in the case of carrying goods). The objective of the problem consists of determining a set of tours, each starting at the depot and covering all customers (visiting each node only once and by exactly one vehicle) before ending at the depot while minimizing the total cost. In this problem, the cost is determined as the product of the distance of the selected arc (i, j) and the flow on this arc. This problem can be addressed from two different perspectives: collection and delivery.

The definition of the variables is as follows. Let x_{ij} be a binary variable equal to 1 if the arc (i,j) is on the tour of a vehicle, zero otherwise. Further, let y_{ij} be a variable that denotes the flow arc (i,j) if the vehicle travels from i to j, zero otherwise. The definition of the variables y_{ij} is key for this VRP variant, since they indicate the cumulative load of the vehicle after departing from node i either in the collection or delivery case (for further information, see Kara et al., 2008).

The mathematical formulation for the Cum-VRP (for the collection case) is defined as follows:

$$min \sum_{i=0}^{n} \sum_{j=0}^{n} c_{ij} \cdot y_{ij} \tag{1}$$

subject to :

$$\sum_{i=1}^{n} x_{0i} = |K| \tag{2}$$

$$\sum_{i=1}^{n} x_{i0} = |K| \tag{3}$$

$$\sum_{i=0}^{n} x_{ij} = 1 \quad \forall j = \{0, 1, 2, \dots, n\} \tag{4}$$

$$\sum_{j=0}^{n} x_{ij} = 1 \quad \forall i = \{0, 1, 2, \dots, n\} \tag{5}$$

$$\sum_{\substack{j=0 \\ i \neq j}}^{n} y_{ij} - \sum_{\substack{j=0 \\ i \neq j}}^{n} y_{ji} = q_i \quad \forall i = \{1, 2, \dots, n\} \tag{6}$$

$$y_{0i} = Q_0 \cdot x_{0i} \quad \forall i = \{1, 2, \dots, n\} \tag{7}$$

$$y_{ij} \leqslant (M - q_j) x_{ij} \quad \forall (i,j) \in A \tag{8}$$

$$y_{ij} \geqslant (Q_0 + q_i) x_{ij} \quad \forall (i,j) \in A \tag{9}$$

$$x_{ij} \in \{0, 1\} \quad \forall (i,j) \in A \tag{10}$$

$$y_{ij} \geqslant 0 \quad \forall (i,j) \in A \tag{11}$$

The objective function given in (1) defines the cumulative cost function based on the arc distance and flow. Constraints (2) and (3) ensure that $|K|$ vehicles are used. Notice that "=" can be replaced by "⩽" to model the case where at most $|K|$ vehicles can be used. Constraints (4) and (5) are used to estimate the degree of each node. Together with (2) and (3), they are called assignment constraints of the formulation. Constraints (6) are the classical conservation of flow constraint for balancing inflow and outflow at each node. They guarantee that the flow variables of each tour perform an increasing step function. Since the formulation operates for the collection case, the values of y_{ij} will increase in dependency of the capacity on the next customer j included in the route. These constraints also contribute to avoiding sub-tours. Constraints (7) initialize the flow on the first arc of each route. Constraints (8) model the capacity restrictions and force y_{ij} to zero when the arc (i,j) is not on any route. Constraints (9) produce lower bounds for the flow of any arc. The nature of the decision variables is defined in (10) and (11). It is important to remark that non-negativity constraints for variables y_{ij} become redundant because of lower bounds provided in (9).

The model for the delivery case can be obtained by removing constraints (6)–(9) and replacing them by the following:

$$\sum_{\substack{j=0 \\ i \neq j}}^{n} y_{ji} - \sum_{\substack{j=0 \\ i \neq j}}^{n} y_{ij} = q_i \quad \forall i = \{1, 2, \dots, n\} \tag{12}$$

$$y_{i0} = Q_0 \cdot x_{i0} \quad \forall i = \{1, 2, \dots, n\} \tag{13}$$

$$y_{ij} \leqslant (M - q_i) x_{ij} \quad \forall (i,j) \in A \tag{14}$$

$$y_{ij} \geqslant (Q_0 + q_j) x_{ij} \quad \forall (i,j) \in A \tag{15}$$

where Q_0 indicates the final value of flow from the last node of the tour to the origin (e.g. tare of the truck). In this delivery approach, constraints (12) denote the conservation flow whereas constraints (13) define the final flow value on the last node of the tour. Specifically constraints (12) work in an opposite way to the collection case, decreasing the values of y_{ij} in dependency of the demand delivered on customer i in the route. Finally, constraints (14) and (15) determine the capacity restrictions for the delivery case. Consequently, the formulation for the delivery case is obtained by using constraints (1)–(7) in conjunction with constraints (12)–(15), (10), (11).

2.2. Problem definition and mathematical formulation for the CCVRP

The CCVRP (Ngueveu et al., 2010) is defined on a graph $G = (V, A)$ where $V = \{0, \dots, n, n+1\}$ is the node set (nodes 0 and $n+1$ correspond to the depot and $V' = V \setminus \{0, n+1\}$ to the set of customers), $A = \{(i,j) : i, j \in V, i \neq j\}$ is the arc set. Each arc $(i,j) \in A$ has associated a travel time c_{ij} between nodes i and j. Parameter Q denotes the vehicle capacity, K represents the fleet set, and each customer $i \in V'$ has a demand q_i. The objective consists of identifying a set of routes so that every customer is visited exactly once and the sum of arrival times at customers is minimized.

A route is defined as a cycle, starting and ending at the depot, in which the serviced demand does not exceed the vehicle capacity Q. Let t_i^k be the arrival time of vehicle k at customer i and x_{ij}^k a binary variable equal to 1 if vehicle k traverses edge (i,j) going from i to j.

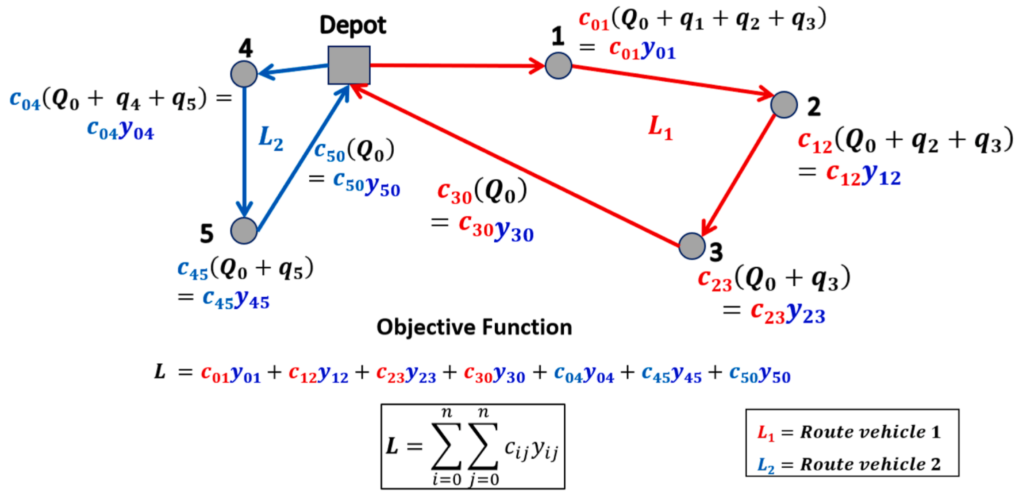
The mathematical formulation for the CCVRP is defined as follows:

$$min \sum_{k=1}^{|K|} \sum_{i=1}^{n} t_i^k \tag{16}$$

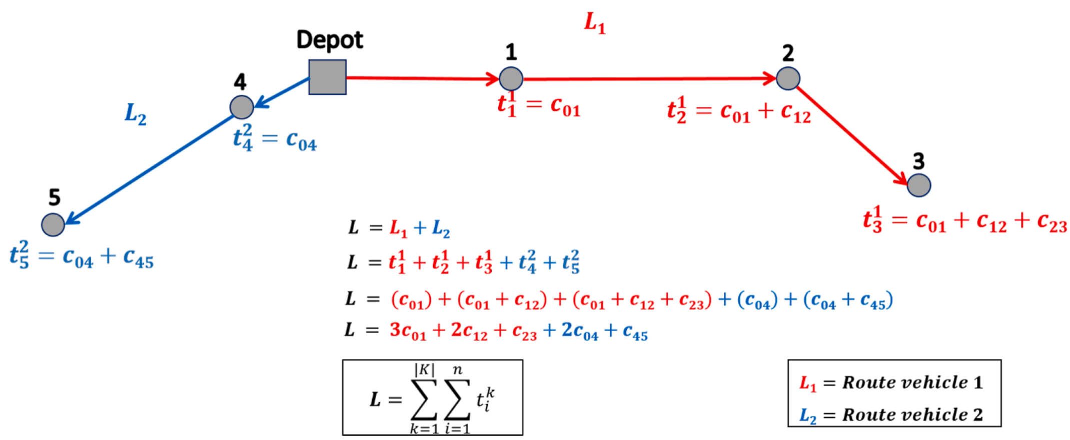
subject to :

$$\sum_{i=0}^{n+1} x_{ij}^k = \sum_{i=0}^{n+1} x_{ji}^k \quad \forall j = \{1, \dots, n\}, \quad \forall k \in \{1, \dots, |K|\} \tag{17}$$

$$\sum_{k=1}^{|K|} \sum_{j=1}^{n} x_{ij}^k = 1 \quad \forall i = \{1, \dots, n\} \tag{18}$$



(a) Cum-VRP Solution



(b) CCVRP Solution

Fig. 2. Comparison of solutions for the CCVRP and Cum-VRP.

$$\sum_{i=1}^{n}\sum_{j=0}^{n+1} x_{ij}^k q_i \leqslant Q \quad \forall k \in \{1,\dots,|K|\} \tag{19}$$

$$\sum_{j=0}^{n+1} x_{0j}^k = 1 \quad \forall k \in \{1,\dots,|K|\} \tag{20}$$

$$\sum_{i=0}^{n+1} x_{i,n+1}^k = 1 \quad \forall k \in \{1,\dots,|K|\} \tag{21}$$

$$t_i^k + w_{ij} - M(1 - x_{ij}^k) \leqslant t_j^k \quad \forall i = \{0,\dots,n\}, \quad \forall j = \{0,\dots,n+1\}, \quad \forall k \in \{1,\dots,|K|\} \tag{22}$$

$$t_i^k \geqslant 0 \forall i = \{0,\dots,n+1\}, \quad \forall k \in \{1,\dots,|K|\} \tag{23}$$

$$x_{ij}^k \in \{0,1\} \forall i,j = \{0,\dots,n+1\}, i \neq j, \quad \forall k \in \{1,\dots,|K|\} \tag{24}$$

In this model, the objective function (16) minimizes the total latency of the customers (i.e., the sum of arrival times at the n customer locations). Constraints (17) and (18) specify that each customer is visited by only one vehicle. Constraints (19) ensure that the capacity of each vehicle is not exceeded. Constraints (20) and (21) force that routes start and end at the depot. Constraints (22) compute the arrival times at each node and prevent sub-tours (using the big-M technique). Finally, constraints (23) and (24) establish the nature of the variables.

Although both models, Cum-VRP and CCVRP, consider cumulative functions, the difference between them relies on what is accumulated. With the aim of illustrating that, Fig. 2 shows a solution for the Cum-VRP and CCVRP for the case of 5 customers. Regarding the Cum-VRP, it considers the relation of the distance (c_{ij}) between each pair of nodes (i, j) and the cumulative load of the vehicle (sum of the demands of remaining customers scheduled in the route, y_{ij}) at the moment of reaching node j ($\sum_{i=0}^{n}\sum_{j=0}^{n} c_{ij} \cdot y_{ij}$), as observed in Fig. (2a). With respect to the CCVRP, the objective function $\sum_{k=1}^{|K|}\sum_{i=1}^{n} t_i^k$ calculates the cumulative arrival time of the vehicle $k \in K$ to the node $i \in V'$, as can be observed in Fig. (2b). Finally, it is relevant to indicate that due to the type of accumulation considered in Cum-VRP, both problems also differ in the route structure, that is, while in Cum-VRP the routes are closed, in the CCVRP they are open.

With respect to the CCVRP objective function, different variants have considered related cumulative functions. For instance, Nucamendi-Guillén et al. (2018) proposed the objective function $\sum\sum c_{ij}u_{ij}$, where u_{ij} denotes the number of remaining nodes in the route after visiting node i. That objective quantifies the sum of the arrival times equivalently as Ngueveu et al. (2010). It is worth noting that, since the sum of the values of t_i^k is minimized, the minimum possible value for the arrival time at node i corresponds to the sum of the traveling arcs of the previously visited nodes (see Fig. 2b). Additionally, in the objective $\sum t_i^k$ presented in Ngueveu et al. (2010), the index k represents the vehicle

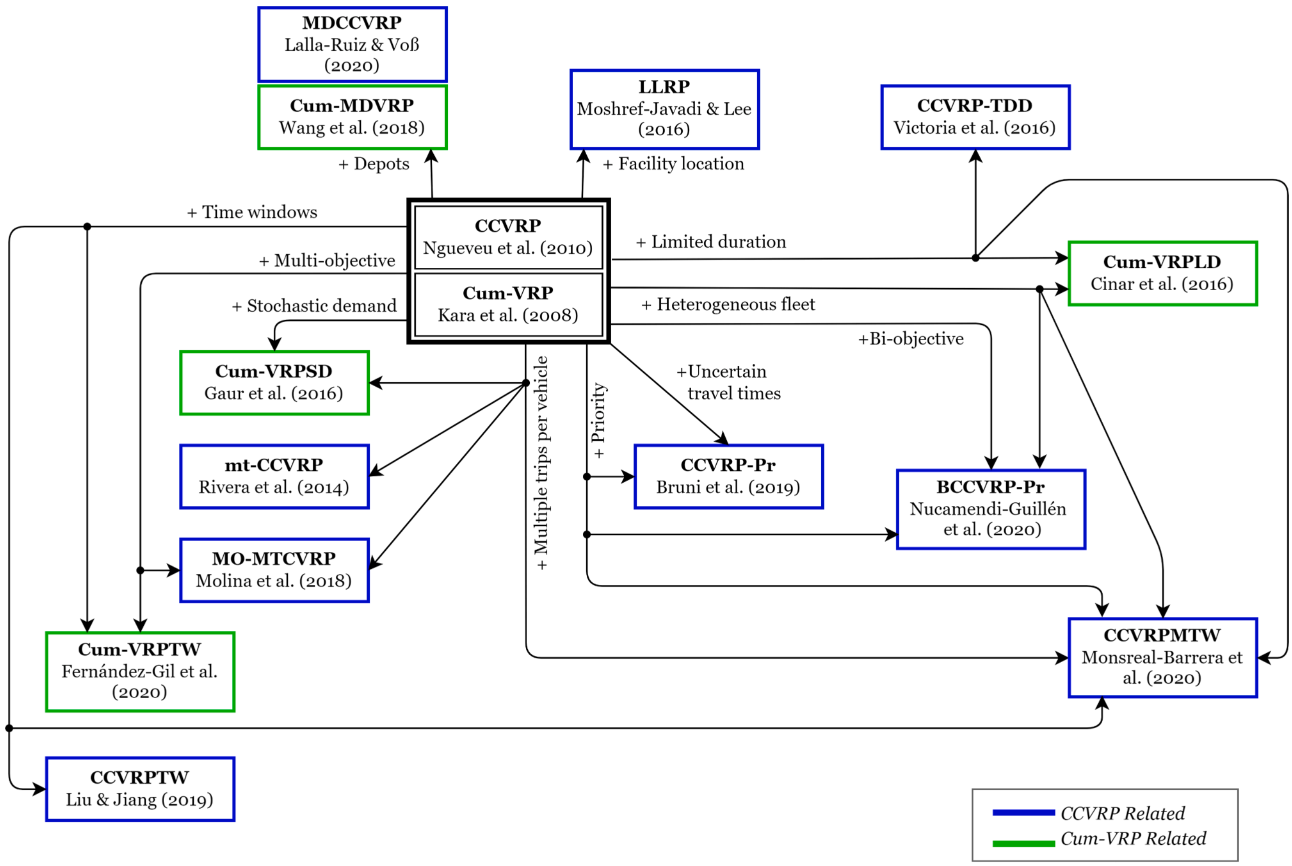


Fig. 3. CCVRP and Cum-VRP variants scheme.

used, and the only difference with the objective function provided in (16) is that the sum of the arrival times is quantified by vehicle. Also, notice that if the objective function (16) is changed by (1), we obtain the model proposed by Kara et al. (2008), since the feasible region denoted by the constraints is the same.

Other variants of the CCVRP concerning objective functions are:

- $\sum\sum(t_i^k q_i)$. This objective function multiplies the arrival times t_i by the demand q_i of each node i. This objective function might be partially similar to $\sum\sum c_{ij}y_{ij}$ since it multiplies the cumulative load y_{ij} by the value of the traversing arc c_{ij}.
- $\sum\sum(c_{ij}q_i)$. This cumulative objective is expressed as the product of the traversing arc between nodes i and j and the demand of node i. Even when this objective function is not directly equivalent to those proposed in (Kara et al., 2008; Ngueveu et al., 2010), it computes weighted arcs, which can be seen, thus, as a weighted objective function.

2.3. Problem variants

Several variants of the CVRP involving cumulative costs (i.e., Cum-VRP and CCVRP) have been studied in the related literature. Fig. 3 shows the evolution of the problem approaches based on the addition of different problem features. The figure shows with green lines all related Cum-VRPs, while CCVRP related ones are delimited with blue ones. The features necessary to add to the originating problems are provided near the arcs. This way Cum-VRP with limited duration leads to the CumVRP-LD. Similarly, if we add a limited duration to the Cum-VRP, then the Cum-VRPLD variant is obtained. From this scheme, it can be observed that Cum-VRP and CCVRP have similar variants as the same feature was added to their corresponding problem, e.g., CCVRP-TW and Cum-VRPTW, MDCCVRP and Cum-MDVRP, etc. Lastly, as all variants consider vehicle's capacity restrictions, that feature is not explicitly shown in the figure.

Table 2 summarizes the different contributions addressed and their defining components (e.g, objective function, additional constraints, fleet type, etc.). In the table, column 1 indicates the reference of the cumulative routing-related work. Columns 2, 3, 4, and 5 display the type of objective function conducted (either for the Cum-VRP or CCVRP). If the study has more than one objective, columns 6 and 7 indicate if the study conducts a bi-objective or multi-objective approach. Columns 8 and 9 are used to denote the type of sources considered, whereas the following two columns indicate the type of fleet addressed. Columns 12 to 16 indicate the type of specific constraints addressed, such as time windows, prioritization, limited duration, or multiple trips. Finally, the last two columns indicate the type of solution approach considered for each problem variant.

From the information shown in Table 2, it can be observed that both main problems (i.e., Cum-VRP and CCVRP) have been continuously studied over the years. Moreover, there is a slight preference of researchers for studying the problem proposed by Ngueveu et al. (2010) since around 71% of the total papers (30 out of 42) directly addressed the CCVRP or proposed a related variant. In the case of the Cum-VRP introduced by Kara et al. (2008), it can be noticed that it was mainly continued by researchers: Cinar, Gakis, and Pardalos (2016), Cinar, Cayir Ervural, Gakis, and Pardalos (2017) and Gaur, Mudgal, and Singh (2013), Gaur, Mudgal, and Singh (2020), Gaur and Singh (2015), Gaur and Singh (2017). It can also be observed from the works that incorporate a MILP technique that 13 out of 15 formulations were developed for the CCVRP.

New variants incorporating time windows (Monsreal-Barrera, Cruz-Mejia, & Marmolejo-Saucedo, 2020) and a sustainability perspective (Fernández Gil, Gómez Sánchez, Lalla-Ruiz, & Castro, 2020) have been recently proposed. Monsreal-Barrera et al. (2020) analyzed the

Table 2
Taxonomic review of the Cum-VRP and CCVRP works.

Reference	Objective functions				Multi-Objective		# Depots		Fleet Type		TW		Additional Constraints			Solution Method	
	$\sum\sum t_i^k$	$\sum\sum c_{ij}y_{ij}$	$\sum maxt_i^k$	$\sum\sum\sum d_{ij}(a_k) + \sum(b_k q_{ijk})$	Bi	Multi	Single	Multi	Ho	Ht	Hard	Soft	Priorities	Route Duration	Multiple Trips	Exact	Approx
Kara et al. (2008)		•					•		•							•	
Ngueveu et al. (2009)	•						•		•								•
Ngueveu et al. (2010)	•						•		•							•	•
Chen et al. (2012)	•						•		•								•
Mattos-Ribeiro and Laporte (2012)	•						•		•								•
Gaur et al. (2013)		•		•			•		•								•
Ke and Feng (2013)	•						•		•								•
Ozsoydan and Sipahioglu (2013)	•						•		•								•
Lysgaard and Wøhlk (2014)	•						•		•							•	
Rivera et al. (2014)	•						•		•						•	•	•
Gaur and Singh (2015)				•			•		•								•
Martínez-Salazar et al. (2015)	•						•		•						•	•	•
Rivera et al. (2015)	•						•		•						•	•	•
Cinar et al. (2016)				•			•			•				•			•
Gaur et al. (2016)				•			•		•						•		•
Moshref-Javadi and Lee (2016)	•							•	•							•	•
Rivera et al. (2016)	•						•		•						•	•	•
Victoria et al. (2016)	•						•		•					•		•	•
Cinar et al. (2017)				•			•			•				•			•
Flores-Garza et al. (2017)	•						•		•					•		•	•
Gaur and Singh (2017)				•			•		•						•		•
Singh and Gaur (2017)				•			•		•								
Sze et al. (2017)	•		•				•		•								•
Ke (2018)	•						•		•								•
Lenis and Rivera (2018)		•					•		•								•
Molina et al. (2018)	•					•	•		•						•		•
Nucamendi-Guillén et al. (2018)	•						•		•							•	•
Wang et al. (2018)		•						•	•								•
Bruni et al. (2019)	•						•		•				•			•	•
Liu and Jiang (2019)	•						•		•		•						•
Ramadhan and Imran (2019)	•						•		•								•
Fernández Gil et al. (2020)		•				•	•		•		•	•				•	•
Gaur et al. (2020)				•			•		•								•
Lalla-Ruiz and Voß (2020)	•							•	•							•	•
Monsreal-Barrera et al. (2020)	•						•			•		•	•	•	•	•	•
Nucamendi-Guillén et al. (2020)	•				•		•			•			•			•	•
Smiti et al. (2020)	•						•		•								•
Wang et al. (2020)	•							•	•								•
Damião et al. (2021)	•						•	•	•							•	
Kyriakakis et al. (2021)	•						•		•								•
Niu et al. (2021)	•							•	•								•
Osorio-Mora et al. (2021)	•							•	•							•	

processes of collecting used non-returnable beverage packaging to improve the recycling of material. A reverse logistics network was determined according to the current situation and packaging reverse flows. The packaging collection was designed using routing algorithms to identify how it can be carried out while determining the involved cost. Afterwards, the recovery center's required processes were analyzed for conditioning the materials before transferring them to the recycler. Osorio-Mora, Soto-Bustos, Gatica, Palominos, and Linfati (2021) considered mandatory visit times and introduced the concept of delayed latency related to the overtime hours in which the patients are visited. The overtime is computed when the arrival time to customers exceeds the maximum desirable quota by which vehicles can visit clients before being penalized. The penalized overtimes becomes the criterion to minimize. Previously, Fernández Gil et al. (2020) investigated the Cum-VRP with hard and soft time windows in order to analyze the trade-off between environmental costs and customer dissatisfaction.

The first variant considered for the CCVRP involved multiple trips (Rivera, Afsar, & Prins, 2014; Rivera, Afsar, & Prins, 2015; Monsreal-Barrera et al., 2020), this extension was proposed in the context of disaster relief where arrival times have a critical impact (Campbell, Vandenbussche, & Hermann, 2008). Similarly, the CCVRP was generalized to multiple depots (Lalla-Ruiz & Voß, 2020; Wang, Choi, Li, & Shao, 2020). In Moshref-Javadi and Lee (2016) the location routing version was introduced. Concerning additional physical constraints, the problem has been addressed for cases where a limited travel duration has to be considered (Victoria, Afsar, & Prins, 2016; Flores-Garza, Salazar-Aguilar, Ngueveu, & Laporte, 2017; Monsreal-Barrera et al., 2020) as well as time windows (Liu & Jiang, 2019; Monsreal-Barrera et al., 2020). Another point to highlight related to the previous point as well as to vehicles' capacity is the consideration of heterogeneous fleet (Monsreal-Barrera et al., 2020; Nucamendi-Guillén et al., 2020). In particular, Monsreal-Barrera et al. (2020) considered two types of vehicles for transporting glass and aluminium containers for recycling.

Concerning the Cum-VRP, Cinar et al. (2016) and Cinar et al. (2017) considered the case of total load transported per vehicle and incorporated coefficients per load and distance in the objective function. The authors proposed a heterogenous variant, however, their computational experiments were conducted using instances that involved identical vehicles (i.e., homogeneous fleet). Lenis and Rivera (2018) considered a Cum-VRP with an objective function that incorporates the vehicle weight times the traveled distance, the cumulative load of the vehicle, and the cost related to demand of the edges being traversed.

As can be observed in Table 2, different related objective functions have been used for the CCVRP and Cum-VRP. They can be classified into four groups:

- $\sum\sum t_i^k$: Quantifies the sum of arrival times to customer locations for all routes.
- $\sum\sum c_{ij}y_{ij}$: Quantifies the sum of the product between the cost and flow on the traversed arcs.
- $\sum max\ t_i^k$: Quantifies the sum of the maximum (i..e, latest) arrival time to each customer in each route.
- $\sum\sum\sum d_{ij}(a_k) + \sum(b_k q_{ijk})$: Quantifies the weighted sum of the distance traveled and the load transported multiplied by weight parameters, i.e., a_k and b_k.

Table 3 shows the distribution in the literature of the aforementioned cumulative objective functions. It can be observed that the objective function related to the CCVRP is the most used one, followed by that same function but involving the incorporation of weights.

Concerning multiple objective functions, three studies developed multi-objective variants, namely, multi-objective multi-trip cumulative capacitated vehicle routing problem (MO-MTCCVRP) (Molina et al., 2018), the bi-objective cumulative capacitated vehicle routing problem including priority indexes (BCCVRP-Pr) (Nucamendi-Guillén et al., 2020), and the cumulative vehicle routing problem with time windows (Cum-VRPTW) (Fernández Gil et al., 2020). In the MO-MTCCVRP, the objectives under consideration are the minimization of used vehicles, total travel cost, and maximum latency. In this situation, the authors considered maximum latency to be more appropriate than classic latency criteria since reducing the last affected waiting time is critical for survival when any disaster strikes. Regarding the bi-objective CCVRP including priority indexes (BCCVRP-Pr), the authors proposed a bi-objective approach to model the case when an importance index (weight) is associated with each customer. This way, the weights were used for denoting the customer importance (i.e., the higher, the more critical). The application of this variant arises in situations such as affected zones after a natural disaster, where the indices can indicate the critical level of the affected zones'. That problem, therefore, aims at serving demand points by following their priorities but procuring, at the same time, to minimize the sum of the arrival times to the customers. Lastly, the Cum-VRPTW incorporates soft and hard time windows into the Cum-VRP and considers objectives related to cumulative costs and

Table 3
Summary of papers that addressed each objective function.

Objective Function	Number of articles	References
$\sum\sum t_i^k$	30	(Ngueveu et al., 2009; Ngueveu et al., 2010; Chen et al., 2012; Mattos-Ribeiro and Laporte, 2012; Ozsoydan & Sipahioglu, 2013; Ke & Feng, 2013; Lysgaard & Wøhlk, 2014; Rivera et al., 2014; Rivera et al., 2015; Martínez-Salazar et al., 2015; Victoria et al., 2016; Moshref-Javadi & Lee, 2016; Rivera et al., 2016; Flores-Garza et al., 2017; Sze et al., 2017; Ke, 2018; Molina et al., 2018; Nucamendi-Guillén et al., 2018; Bruni et al., 2019; Liu & Jiang, 2019; Ramadhan & Imran, 2019; Nucamendi-Guillén et al., 2020; Lalla-Ruiz & Voß, 2020; Smiti et al., 2020; Monsreal-Barrera et al., 2020; Wang et al., 2020; Damião et al., 2021; Kyriakakis et al., 2021; Niu et al., 2021; Osorio-Mora et al., 2021)
$\sum\sum\sum d_{ij}(a_k) + \sum(b_k q_{ijk})$	8	(Gaur et al., 2013; Gaur and Singh, 2015; Cinar et al., 2016; Gaur et al., 2016; Cinar et al., 2017; Gaur and Singh, 2017; Singh and Gaur, 2017; Gaur et al., 2020)
$\sum\sum c_{ij}y_{ij}$	5	(Kara et al., 2008; Gaur et al., 2013; Lenis & Rivera, 2018; Wang et al., 2018; Fernández Gil et al., 2020)
$\sum max t_i^k$	1	(Sze et al., 2017)

Table 4
Most relevant problem variants addressed during the last 5 years.

Variant	Number of articles	References
Multi-Depot	7	(Moshref-Javadi & Lee, 2016; Wang et al., 2018; Lalla-Ruiz & Voß, 2020; Wang et al., 2020; Damião et al., 2021; Niu et al., 2021; Osorio-Mora et al., 2021)
Multiple Trips	5	(Gaur et al., 2016; Rivera et al., 2016; Gaur & Singh, 2017; Molina et al., 2018; Monsreal-Barrera et al., 2020)
Limited Duration	5	(Cinar et al., 2016; Victoria et al., 2016; Cinar et al., 2017; Flores-Garza et al., 2017; Monsreal-Barrera et al., 2020)
Heterogeneous Fleet	4	(Cinar et al., 2016; Cinar et al., 2017; Monsreal-Barrera et al., 2020; Nucamendi-Guillén et al., 2020)
Time Windows	3	(Liu & Jiang, 2019; Fernández Gil et al., 2020; Monsreal-Barrera et al., 2020)
Priorities	3	(Bruni et al., 2019; Monsreal-Barrera et al., 2020; Nucamendi-Guillén et al., 2020)

Table 5
Compilation of solution methods proposed for the Cum-VRP and CCVRP.

Reference	Type	Exact Methods			Approximate Methods																								
					Approximation		Heuristic		Metaheuristic Algorithms																			Matheuristics	
									Population-based					Single-solution based															
		MILP	B&C&P	Solver	CFAA	CG	PVA	NNH	GA	PSO	MA	BSO	ACO	ALNS	VNS	TS	IG	ILS	PLS	GRASP	TPH	MS-ELS	RGA	SA	MS-INS	RRT	SVNS	GRASP-MILP	POPMUSIC
Kara et al. (2008)	Cum-VRP	•		CPLEX 8.1																									
Ngueveu et al. (2010)	CCVRP	•		No results							•																		
Chen et al. (2012)	CCVRP																	•											
Mattos-Ribeiro and Laporte (2012)	CCVRP													•															
Gaur et al. (2013)	Cum-VRP				•																								
Ke and Feng (2013)	CCVRP																				•								
Ozsoydan and Sipahioglu (2013)	CCVRP								•	•						•													
Lysgaard and Wøhlk (2014)	CCVRP		•	CPLEX 12.2																									
Rivera et al. (2014)	CCVRP	•		CPLEX 12.4																		•							
Gaur and Singh (2015)	Cum-VRP					•																							
Martínez-Salazar et al. (2015)	CCVRP	•		CPLEX 12.4																•									
Rivera et al. (2015)	CCVRP	•		CPLEX 12.4														•		•									
Cinar et al. (2016)	Cum-VRP																				•								
Gaur et al. (2016)	Cum-VRP				•																								
Moshref-Javadi and Lee (2016)	CCVRP	•		No results							•												•						
Rivera et al. (2016)	CCVRP	•		CPLEX 12.5																									
Victoria et al. (2016)	CCVRP	•		CPLEX 12.5																	•								
Cinar et al. (2017)	Cum-VRP								•															•					
Flores-Garza et al. (2017)	CCVRP	•		CPLEX 12.6																•									
Gaur and Singh (2017)	Cum-VRP					•																							
Sze et al. (2017)	CCVRP														•														
Ke (2018)	CCVRP											•																	
Lenis and Rivera (2018)	Cum-VRP																			•									
Molina et al. (2018)	CCVRP																								•				
Nucamendi-Guillén et al. (2018)	CCVRP	•		CPLEX 12.4													•												
Wang et al. (2018)	Cum-VRP												•																
Bruni et al. (2019)	CCVRP	•		SCIP 3.2.0													•												
Liu and Jiang (2019)	CCVRP								•					•															
Ramadhan and Imran (2019)	CCVRP																									•			
Fernández Gil et al. (2020)	Cum-VRP	•		CPLEX 12.9																								•	
Gaur et al. (2020)	Cum-VRP				•																								
Lalla-Ruiz and Voß (2020)	CCVRP	•		CPLEX 12.6																									•
Monsreal-Barrera et al. (2020)	CCVRP						•	•																					
Nucamendi-Guillén et al. (2020)	CCVRP	•		Gurobi 9.0							•																		
Smiti et al. (2020)	CCVRP																										•		
Wang et al. (2020)	CCVRP																		•										
Damião et al. (2021)	CCVRP		•	CPLEX 12.9																									
Kyriakakis et al. (2021)	CCVRP												•																
Niu et al. (2021)	CCVRP												•																
Osorio-Mora et al. (2021)	CCVRP	•		CPLEX 12.9; Gurobi 8.1.0																									

the number of used vehicles as well as penalties for the service delay in the case of soft time windows.

With regards to practical cases, the works of Kara et al. (2008), Molina et al. (2018) and Monsreal-Barrera et al. (2020) addressed real-world applications in their studies. Kara et al. (2008) implemented their model focusing on the collection case of the energy minimizing and testing realistic instances by using the data from the Turkish highway map. Molina et al. (2018) analyzed the real case of a flood in Villahermosa (México) in 2007 that affected a large part of its territory (65%). The water reached unexpected levels of more than four meters, challenging authorities by the magnitude of the humanitarian logistics required. The authors developed a model able to define evacuation routes for the victims procuring the minimization of the waiting time for evacuation. The fleet size and capacity of the vehicles was limited, justifying the need for performing multiple trips per vehicle. However, their experimentation was conducted over benchmark instances from the literature and no data about the case study was provided. Recently, Monsreal-Barrera et al. (2020) considered the CCVRP by incorporating multiple trips with time windows (CCVRPMTW) to optimize recycling networks, achieve higher volumes of recycled material, and decrease the total cost of collection. In this variant, a profit was associated with each node in terms of a specified threshold or break-even point.

Table 4 shows the most studied problem features of the CCVRP and Cum-VRP in terms of number of works from the last 5 years (period 2016–2021). Based on this, we can observe that variants incorpating features such as allowing multiple depots, multiple trips per vehicle, establishing a travel limit, considering heterogeneous fleet, or setting time windows raised relevant attention with regards to the number of works. These features are generally used in VRPs, thus their proposition goes in line with its related literature.

3. Solution Methods

This section is devoted to listing, classifying, and discussing all solution methods proposed for cumulative VRPs. Considering the classification of methods provided in Talbi (2009), Table 5 shows the different approaches classified by type, i.e., exact or approximate, and the main base problem addressed (i.e., Cum-VRP and CCVRP). Complementary, Fig. 4 illustrates the distribution of works based on such classification. After the research methods are classified, analysis and discussion for each category (i.e., exact and approximate) are provided in Sections 3.1 and 3.2. This section finalizes providing an overall figure concerning the different approximate approaches per type of problem. The notation used to list the different solutions methods is provided in Appendix A.

As shown in Table 5, 17 out of 42 papers used exact methods (i.e., 15 MILP and 2 B&C&P). In the majority of those papers, it coincides that the MILP model is provided when a new variant of either CCVRP and Cum-VRP is introduced. Among the 37 papers that proposed approximation procedures, 2 heuristic algorithms, 33 metaheuristic methods and 2 matheuristic methods were developed. To better illustrate this, Fig. 4 displays the distribution per type of solution method.

Exact approaches for solving these problems are mainly MILP formulations using CPLEX as the main engine. Only two works used other optimization engines, i.e., Gurobi and SCIP. Another aspect to highlight is that, from these papers, nine of them also proposed approximate procedures due to the limitations of the solvers when addressing medium or large scenarios.

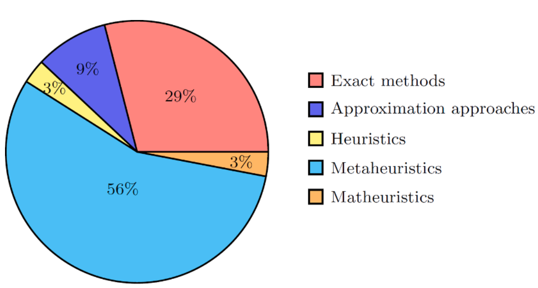


Fig. 4. Summary of the different solution methods proposed in the literature.

3.1. Formulations and exact approaches

The exact methods proposed for the Cum-VRP were those related to the MILP formulations. Kara et al. (2008) proposed the first two formulations, one for dealing with the collection case and the other related to the delivery case. The proposed formulations were tested over realistic instances using data from the Turkish highway map and analyzing the effect of energy consumption compared to the number of vehicles used. Instances involving 24 and 31 cities were tested, varying the number of trucks between 4 and 9. The results indicated that increasing the number of vehicles decreases the total energy used. This goes in line with what it was demonstrated for the CCVRP and MDCCVRP with regards to the number of vehicles. Since the proposal of Cum-VRP, further contributions to this topic have been made in the context of approximation procedures (i.e., analytical methods that provide provable solution quality and provable run-time bounds).

Concerning the CCVRP, the first mathematical formulation for this problem was introduced by Ngueveu, Prins, and Wolfler-Calvo (2009) and further discussed in Ngueveu et al. (2010). Together with the formulation, the authors provided some problem properties such as (i) the traveling repairman problem does not provide a lower bound for the CCVRP, (ii) optimal CCVRP use exactly $min\{|K|,n\}$, and (iii) a route get a different cost when reversed. The authors also provided two lower bounds for the problem, one based on unrestricted vehicle fleet size and the other by approximating the edge costs and customers coefficients. Although those lower bounds were used to evaluate the approximate solutions, there is no numerical experimentation on the models' performance. Later, Rivera et al. (2015) proposed a mixed-integer formulation based on replenishment arcs (Boland, Clarke, & Nemhauser, 2000; Mak & Boland, 2000) to deal with the multi-trip variant, i.e., mt-CCVRP. The replenishment arcs were used to replace the trips of a multi-trip with a single trip (see Rivera et al. (2015) for details). The authors also proposed valid inequalities for the formulation which were shown useful to reduce running time when the model was solved by a commercial solver. The model without valid inequalities solved 6 instances with up to 15 nodes, whereas the one with those inequalities led to 8 instances while reducing computational times. In a follow-up paper, for that same variant, Rivera, Murat Afsar, and Prins (2016) presented two improved formulations, one flow-based and another based on set-partitioning. In addition, the authors developed valid inequalities to enhance the performance of the models and an exact procedure based on a resource-constrained shortest path approach. The resulting problem was solved via an adaptation of the Bellman-Ford algorithm. The results indicated that the formulation solved a minority of instances for 20 locations. In contrast, the improved formulation could tackle instances with up to 40 locations, proving that good initial solutions, dominance rules, and lower bounds enhance the solution procedure.

Two tractable formulations for the CCVRP were proposed by Nucamendi-Guillén et al. (2018). The first formulation is based on the flow-based model for the m-TSP (Gavish and Graves, 1978) and further adapted to the related Multiple minimum latency problem (mMLP) (Angel-Bello, Cardona-Valdés, & Álvarez, 2019). It consists of adding new integer variables y_{ij} to indicate the amount of flow on arc (i,j). The variable y_{ij} is equal to the number of nodes in a path after node i when $x_{ij} = 1$ and it is equal to 0 when $x_{ij} = 0$. In addition, the classical MTZ constraints were incorporated to prevent subtours. Consequently, the objective function was reformulated based on these flow variables (in addition, a similar definition of the variables and constraints that estimate the cumulative load of a vehicle was provided). The main benefit of the proposed formulation is that it avoids the use of the big M method and provides tight linear bounds. The computational experiments showed that the model was able to solve instances of up to 25 nodes. In the same work, the authors proposed a second formulation based on a time-dependent approach for the traveling salesman problem and its relation to the tardiness problem in one-machine scheduling (see Picard & Queyranne, 1978) together with a multilevel network scheme as shown in (Angel-Bello, Alvarez, & García, 2013). In this way, level activation and linking variables were defined and assignment constraints were reformulated to avoid sub-tours. Moreover, another aspect to highlight is that, as in Angel-Bello et al. (2013), only the variables associated with the nodes on each level were preserved as binary whereas the linking variables were relaxed to be continuous (non-negative). These improvements reduced the number of binary variables significantly and allowed their formulation to solve instances up to 44 nodes.

Victoria et al. (2016) developed a mathematical formulation to solve the problem of CCVRP considering time-dependent demands (CCVRP-TDD) in the context of humanitarian logistics. The authors also considered that demand was dynamic and nodes were divided into critical and non-critical nodes. In addition, a demand variation was also considered for the arcs from critical nodes to non-critical nodes. This variation corresponded to the number of people per time unit who flee from a critical city before the arrival of relief of goods. This problem variant aimed to minimize the sum of arrival times at critical nodes. The formulation was able to solve some instances of up to 17 nodes. On the other hand, Flores-Garza et al. (2017) presented a mixed-integer linear formulation for the multi-vehicle cumulative covering tour problem for efficient distribution of humanitarian aid and when damage to the communication infrastructure make some locations unreachable. This problem was treated as CCVRP where not necessarily all locations have to be visited. This way, the problem aims at determining a set of tours such that the sum of arrival times is minimized, where mandatory locations must be included and optional ones can be included if and only if they are required to cover unreachable locations. This way, each unreachable location must be covered by at least one optional one. The model provided feasible solutions for some instances up to 200 nodes but reported large gaps depending on the characteristics of the instance, especially when the number of vehicles increases.

Recently, Lalla-Ruiz and Voß (2020) developed a mixed-integer formulation for the multi-depot CCVRP, i.e., MDCCVRP, and proposed lower bound inequalities to solve instances up to 10 nodes while providing feasible solutions for instances up to 100 customers. Considering the topology of the problem (i.e., multiple depots), the authors proposed a matheuristic decomposition approach that utilizes the existing formulation. Osorio-Mora et al. (2021) proposed a mixed-integer linear programming model for a variant of the MDCCVRP that incorporates mandatory visit times (MDCCVRmvt). The problem aims to minimize the total delayed latency. To evaluate the performance of the proposed formulation, the authors generated 165 test instances of size ranging from 10–50 nodes and from 2 to 4 depots. The model could solve instances with 10 and 20 nodes with relative ease, but in the case of instances over 40 nodes, the solver reached the time limit without finding the optimal solution in most cases.

Considering the CCVRP involving priorities, Bruni et al. (2019) presented a mixed-integer formulation for the CCVRP-Pr that, in the first component of the objective function, maximizes a stochastic revenue, expressed as the sum of the profits collected at visited nodes minus the expected arrival time at those nodes, whereas the second component accounts for the standard deviation of the total arrival time. The model is implemented using the sum-weighted method. The formulation was solved using SCIP 3.2.0 and reported solutions for instances up to 23 nodes. On the other hand, Nucamendi-Guillén et al. (2020) introduced a variant of the CCVRP that considers an importance index (weight), associated with each node. Those weights denote the priority of each customer to be served. However, since the purpose of the CCVRP is to minimize the sum of arrival times to customers, this variant defines a second objective based on a tardiness measure. This measure is computed for each pair of nodes (i,j) as $max\{0, t_j - t_i\}$, where t_i and t_j represent the arrival times to the nodes i and j, being node j the one with the highest weight. For this variant, the authors developed a bi-objective formulation that seeks for the trade-off between the total latency and tardiness of the system. The model is based on the single flow perspective proposed in Nucamendi-Guillén et al. (2018) and incorporates the constraints to compute the tardiness. The formulation was implemented following the AUGMECON-2 method (Mavrotas and Florios, 2013), preserving the latency of the system into the objective function and moving the tardiness objective into the set of the constraints (as ϵ-constraint). The model solved instances up to 15 customers but required computational times of around 4 h.

Apart from MILP related developments discussed above, Lysgaard and Wøhlk (2014) proposed a B&C&P to deal with the CCVRP. The authors also included the analysis to investigate the effect of including an extra route (i.e., increasing vehicles' availability by one unit) over the savings in the total objective. Their method followed the modeling approach of Fukasawa et al. (2006) for the CVRP. However, to assess the cost of traversing an edge (i,j), besides knowing the traveling time (t_{ij}) along with (i,j), it is also needed to know the number of customers remaining to visit. Since this information is unavailable when creating a route from the source vertex, the authors used a backward labeling algorithm to solve the pricing problem. In this way, the customers remaining to be serviced after traversing an edge are the ones that have been visited in the current backward path. Thus, the reduced cost of traversing an edge can easily be computed. The method solved up to 70 customers' instances for the traditional approach in reasonable computational times. In contrast, the solved instances' size increased to up to 100 nodes with the additional vehicle consideration. This empirically shows the importance of the number of vehicles on the solving times as also later reported for the multi-depot version (Lalla-Ruiz and Voß, 2020). More recently, Damião, Silva, and Uchoa (2021) proposed a B&C&P using a VRPSolver package (see Pessoa, Sadykov, Uchoa, & Vanderbeck, 2020) for both the CCVRP and MDCCVRP. The authors compared their results with those from Lysgaard and Wøhlk (2014) showing computational superiority with previous results. Similarly, the authors provided optimal values for several instances proposed in the MDCCVRP benchmark set proposed in Lalla-Ruiz and Voß (2020).

3.2. Approximate and Approximation Methods

As previously reported in Table 2, around 88% of the papers (37 out of 42) included a heuristic or metaheuristic procedure in their study. From those, five papers conducted approximation methods to address the Cum-VRP, while the rest of the works developed approximate methods for the CCVRP (see Table 5). Regarding the approximated procedures' classification, several approaches, such as constant factor approximation procedures, heuristic, metaheuristic algorithms, and matheuristic procedures have been proposed. A detailed description and analysis are provided next.

Fig. 5 shows the distribution of the type of approximate method. As

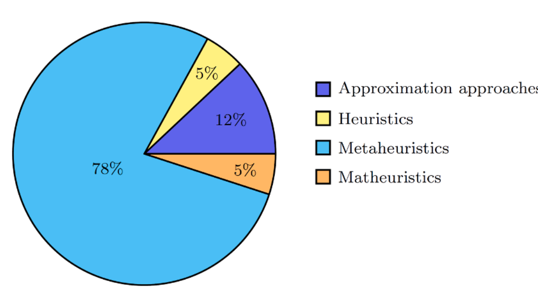
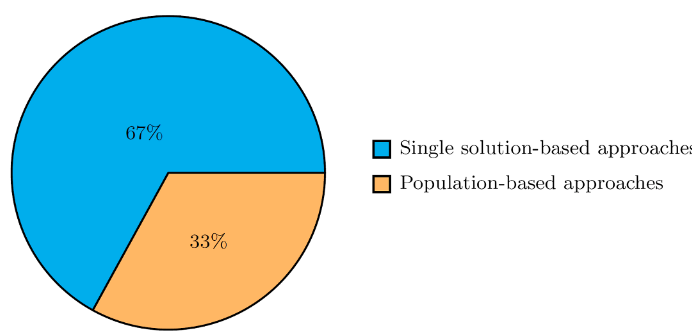


Fig. 5. Distribution of the type of approximate methods (incl. approximation).

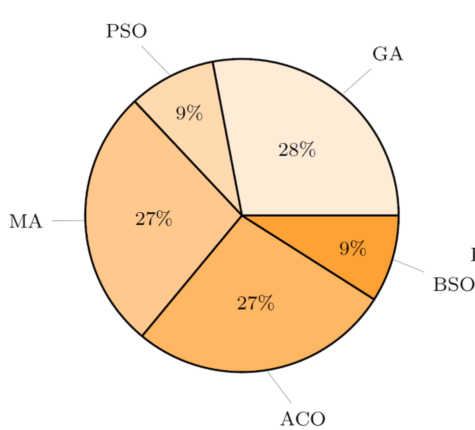
can be observed, there is a relevant percentage of the studies that contribute via metaheuristic algorithms. Concerning the other types of approaches (i.e., constant factor approximations, heuristics and matheuristics) heuristics and matheuristics show a similar percentage of papers, while approximation approaches stand out. A detailed description of each solution method is presented next.

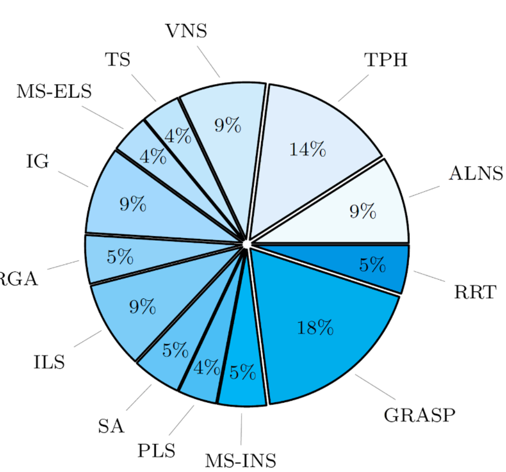
Concerning constant factor approximation algorithms, in Gaur et al. (2013) the authors developed approximation factors based on the iterated tour partitioning technique of Haimovich and Rinnooy Kan (1985) where the travel schedule for the vehicle was computed using a variation of dynamic programming on a traveling salesperson tour (Beasley, 1983, Mole, Johnson, & Wells, 1983). They evaluated four different Cum-VRP cases obtained by varying the vehicles capacity and the distribution of demand: (1) each vehicle has infinite capacity and all customers have equal demands, (2) each vehicle has infinite capacity and the customers have unequal demands, (3) each vehicle has a capacity Q and the customers have equal demands and (4) each vehicle has a capacity Q and the customers have unequal demands. For the above four cases, the authors obtained approximation algorithms with factors 2.5, 2.5, 3.186, and 4, respectively. Gaur, Mudgal, and Singh (2016) studied the Cum-VRP considering stochastic demands (Cum-VRPSD) and split and unsplit deliveries. For these two variants, the authors adapted the algorithm presented in Gupta, Nagarajan, and Ravi (2012) and developed constant factor approximation algorithms for the metric version of the problem (i.e., where the edge matrix respect the triangle inequality). For the Cum-VRPSD without split deliveries, the algorithm obtained a bound of 7 on the approximation ratio, whereas a ratio of 4 was provided for the Cum-VRPSD with split deliveries. Four years later, Gaur et al. (2020) improved their algorithms proposed in Gaur et al. (2016), mainly by incorporating computable a priori tours and revising the methods to estimate lower and upper bounds for the cumulative vehicle routing problem with stochastic demands, considering split and unsplit deliveries. The results of that paper proved that the improved randomized algorithm provides an approximation ratio of $\max\{1+1.5\alpha, 3\}$ for split delivery Cum-VRPSD (where α represents the approximation ratio to the TSP tour obtained by Christofides (1976). For the case of unsplit deliveries, the authors provided a bound of 6 on the approximation ratio. A second approximation approach was proposed by Gaur et al. (2016) and Gaur and Singh (2017) by means of a column-generation-based approximation algorithm. The numerical experiments indicated that their results were better than the worst-case bounds provided by the tour partitioning technique.

The only study that presents heuristic algorithms as the main contribution is the one proposed by Monsreal-Barrera et al. (2020) who developed two algorithms based on the nearest neighborhood strategy: nearest-neighbor heuristic (NNH) and a profitable visit algorithm (PVA). The PVA uses the nearest neighbor algorithm to calculate an initial solution. Thus, a vehicle is assigned to the closest client and, successively, visits the nearest neighbor procuring each time not to exceed the transport capacity of the vehicle nor the route limit (including the time to return to the depot). Then, the initial solution is improved by employing the shift-and-route reduction algorithm. One important remark reported by the authors is that this type of algorithm lacks a sense of direction since the nearest neighbor is chosen exclusively based



(a) Distribution of approximate approaches for the CCVRP and Cum-VRP





(b) Distribution of population-based approaches (c) Distribution of single solution-based approaches

Fig. 6. Distribution of the diverse metaheuristic approaches implemented.

on distances. The effectiveness of the proposed approach was assessed via a case study on beverage package recycling. The results indicated that the PVA outperforms the NNH in objective value.

With respect to metaheuristic algorithms, these were the first type of methods developed to deal with large size instances. Fig. 6 summarizes the results per type of approach. In total, 11 (33%) contributions were presented in the field of population-based algorithms whereas 22 (67%) of the contributions were developed following single-solution based schemes (see Fig. 6a).

As it can be observed in Fig. 6b, within population-based algorithms, 55% of the studies implemented solution approaches based on genetic algorithms. Namely, 28% implemented genetic algorithms (GA) whereas 27% concerned memetic algorithms (MA). The other 45% contributed with particle swarm optimization (PSO), ant colony optimization (ACO) and brain storm optimization (BSO) algorithms. Regarding single solution-based algorithms, we can highlight that many more approaches have been developed for cumulative vehicle routing problems. From there, it can be observed a preference for greedy randomized adaptive search procedure (GRASP) and the two-phase heuristics (TPH) (see Fig. 6c). It is important to remark that, in the case of TPH approaches, several two-phase versions have been developed by implementing different perturbations and local search procedures, but preserving the overall two-phase structure.

Several metaheuristics have been proposed for the Cum-VRP. For instance, Cinar et al. (2016) proposed a modified Clarke & Wright algorithm (mC&W) and also proposed a two-phase heuristic to deal with the cumulative VRP with a limited duration (Cum-VRPLD). In the mC&W algorithm, the savings depend on the distances between vertices and the load of the tours based on the demands of the corresponding assigned vertices. On the other hand, for the two-phase heuristic, the constructive procedure is based on a one-dimensional K-means clustering mechanism, while the mC&W algorithm is used to create routes. The computational experiments performed on data sets from the literature showed that the two-phase heuristic improved the C&W algorithm by reducing the computational time by 89% on average, with only a 0.89% loss in solution quality. One year later, Cinar et al. (2017) incorporated simulated annealing (SA) and a genetic algorithm (GA) into the improvement phase to enhance the performance of the previously developed approaches. The experimental analysis conducted over instances up to 483 nodes showed an improvement when considering previous results. In this sense, the results indicated that the performance of SA was significantly affected by the initial solution generator, whereas the GA was less affected. In general, the SA observed the best performance when the initial solution was generated through the mC&W algorithm. Wang et al. (2018) proposed a hybrid ant colony optimization with multiple rounds (ACOMR) algorithm for the multi-depot Cum-VRP (Cum-MDVRP). In their proposed algorithm, ants were allowed to go in and out of the virtual central depot multiple times so that the path of each could be easily converted to a feasible solution. In addition, the algorithm incorporated a tabu list during the constructive procedure in pursuit of diversifying the search space. The obtained solution was further updated via a two steps *2-opt* mechanism. Computational experiments on standard benchmark instances (up to 360 nodes) revealed that the Cum-MDVRP for emergency transportation was more effective than the standard MDVRP and that ACOMR yielded more stable solutions with regards to their standard deviation when compared to existing heuristics. In addition, two different algorithms from the literature were adapted to solve this problem. Namely, the TPH proposed in (Ke & Feng, 2013) adds the multi-depot restriction and the parallel improved ant colony optimization (PIACO) (Yu, Yang, & Xie, 2011) considering the objective function modified. The experimental results indicated that the ACOMR outperformed the two adapted algorithms by yielding the best solutions for 29 out of 33 instances, whereas from the two adapted methods, the PIACO performed better than TPH since it was already prepared to deal with the multi-depot feature. Lenis and Rivera (2018) proposed a GRASP + VND algorithm with a post-optimization procedure. For the constructive phase of the algorithm, three different procedures were designed (two based on route-first cluster-second and one that operates randomly). Regarding the local search procedure, 4 different local search strategies, i.e., pick-up in last repeated age, reverse loops, edge exchange, and path reconstruction, were implemented under a VND scheme. Finally, a post-optimization procedure based on a set-covering mathematical model was implemented. Since the approach is relatively new, there were no benchmark solutions to compare with.

With respect to the CCVRP, Ozsoydan and Sipahioglu (2013) presented the first two population-based metaheuristics (i.e., GA and PSO) and a third one that considered a tabu search (TS) scheme. The authors conducted a comparative analysis with 39 benchmark instances taken from the literature. Their results indicated that the procedure based on the PSO approach remarkably outperformed those from the GA, whereas the TS was more successful in obtaining the best results. Rivera et al. (2016) developed a hybrid metaheuristic based on a multi-start evolutionary local search (MS-ELS) that incorporated a constructive strategy based on a splitting procedure and used an improvement strategy based on a variable neighborhood approach. The algorithm was capable of obtaining high-quality solutions for instances up to 480 nodes. An improved version of both the formulation and the metaheuristic procedure was presented in Rivera et al. (2015).

Martínez-Salazar, Angel-Bello, and Alvarez (2015) proposed a greedy randomized adaptive search procedure (GRASP) algorithm for the mt-CCVRP. The metaheuristic algorithm provided high-quality solutions in short computational times for instances up to 480 nodes, reporting solutions that improved their implemented model's values up to 37.68%. Victoria et al. (2016) presented a two-phase metaheuristic (TPH) for the CCVRP considering time-dependent demands (CCVRP-TDD). The procedure starts by creating an initial solution considering only the critical nodes. After that, a local search procedure (based on *swap*, *2-opt*, and *3-opt* moves) is applied to improve the initial routes before trying to insert the non-critical nodes, preferably preserving feasibility in terms of capacity and route duration. In case of infeasibility, a repair procedure is executed. On the other hand, Flores-Garza et al. (2017) presented a GRASP for the multi-vehicle cumulative covering tour problem. To evaluate the efficiency of the proposed algorithm, it was assessed over instances up to 150 nodes and reported solutions that improved the best solution reported by their model by at least 0.54% by spending less than 15 min, in comparison with the model that required up to 6 h. Sze, Salhi, and Wassan (2017) developed a two-stage adaptive variable neighborhood search (AVNS) with a large neighborhood search (LNS) as a guided diversification for the CCVRP with min–max objective. The authors also validates the algorithm's efficiency by adapting it to consider the min-sum objective and comparing it with the best-known results so far. The authors adapted the metaheuristic to consider a min–max objective. This adaptation provided the first results on this approach and showed the flexibility and effectiveness of the proposed metaheuristic. It is relevant to highlight that this was the first work that provided new best-known results for instances ranging from 560 to 1200 nodes.

One year later, Nucamendi-Guillén et al. (2018) proposed an iterated greedy approach that incorporated a random variable neighborhood descent (RVND) during the improvement procedure for the classical CCVRP. For the constructive procedure, two different strategies were considered: a parallel route building strategy (Potvin & Rousseau, 1993) and a clustering strategy based mechanism (Mulvey & Beck, 1984). Regarding the improvement phase, the RVND strategy was implemented (Subramanian, Drummond, Bentes, Ochi, & Farias, 2010). According to their results, the algorithm was able to find all the best-known solutions at that time for the small size instances within shorter times. Also, they reported competitive results for large-size instances. Subsequently, Ke (2018) presented a brain storm optimization (BSO) approach to solve the CCVRP. His procedure consists of a constructive mechanism, based on the strategy presented by Mattos-Ribeiro and Laporte (2012) and a single route local search procedure based on *2-opt*, *3-opt*, and *4-opt* operators for the improvement phase. After that, convergent and divergent procedures were applied. During the convergent procedure, the best-so-far solution was perturbed via a partial destruction-reconstruction mechanism to intensify the search. Concerning the divergent procedure, it was based on implementing problem-dependent operators to generate new partial solutions that were further improved separately and then reassembled into a new global solution. As a result, the algorithm was able to report new best solutions for large-size benchmark instances (larger than 199 nodes) in competitive execution times. Bruni et al. (2019) proposed an iterated local search procedure considering profits and stochastic travel times. the metaheuristic approach integrates a constructive procedure based on a regret cost insertion and a local search mechanism considering well-known neighborhood structures to evaluate changes of nodes inside the route and between routes. Finally, a diversification procedure is implemented by decreasing the number of customers to visit in pursuit to minimize the deviation in the arrival time (and therefore, maximizing the global revenue). Furthermore, Ramadhan and Imran (2019) developed a record-to-record travel (RRT) algorithm. Their procedure operates in three main steps: (i) a constructive procedure based on the least cost insertion mechanism, (ii) an up-hill move to perturb the solution in order to expand the search in the solution space. The algorithm allows to temporarily accept poor solutions within a maximum allowed threshold based on the best solution obtained so far, (iii) a down-hill move by using local search, in order to reach the best possible solution. Smiti, Dhiaf, Jarboui, and Hanafi (2020), presented a skewed variable neighborhood search procedure composed of two heuristics. The first consisted of a constructive heuristic to generate an initial solution, whereas the second was the skewed variable neighborhood search (SVNS) heuristic. The SVNS considered three phases to improve the solution of the CCVRP: (i) a perturbation phase with three movement procedures to generate a random neighbor, (ii) a local search phase that proposed three neighborhood structures in descent neighborhood search to generate a local optimum and (iii) an acceptance criteria phase (skewed move) using a distance function to improve the exploration of the solution space.

Recently, Kyriakakis, Marinaki, and Marinakis (2021) proposed two metaheuristic algorithms based on the ACO for the CCVRP. Their algorithms hybridize the well-known ant colony approach with the VND procedure in order to develop swarm intelligence algorithms. The first algorithm was based on an ant colony system, whereas the second one considered the max–min ant system. Both algorithms used memory-based operators to construct the solutions and a common improvement procedure based on a VND strategy that incorporates 7 local search operators (2 intra-route and 5 inter-route operators) based on relocation and swap mechanisms. The experimental results indicate that, in general, the ACO-VND algorithms were able to obtain the best-known solutions for 92 of them, reporting an average deviation of 0.35% from the best solutions obtained so far and a maximum deviation of 0.98%. Additionally, the algorithms outperform 4 of the approaches already presented in the literature (MA, (Ngueveu et al., 2010); ALNS, (Mattos-Ribeiro & Laporte, 2012); TPH, (Ke & Feng, 2013); IG, (Nucamendi, Cardona-Valdes, & Angel-Bello Acosta, 2015) and report two new best solutions for benchmark instances.

Regarding multi-objective variants for the CCVRP, Molina et al. (2018) developed a multi-start algorithm with intelligent neighborhood selection (MS-INS). The algorithm operates in two phases. The first phase generates feasible solutions, whereas the second phase improves them via an intelligent local search scheme, in which the neighborhood to be used is selected according to its performance (according to predefined success indicators). In addition, a selection mechanism was designed to ensure that each neighborhood is used at least once in each local search in such a way the local search process is not finished until all the neighborhood structures have been discarded. The metaheuristic was compared against the well-known NSGA-II procedure and the results indicated that the proposed algorithm performed better regarding multi-objective metrics. Nucamendi-Guillén et al. (2020) proposed two versions of a memetic algorithm with random keys to deal with the problem. In both versions, the construction of the initial population was performed via random key genetic algorithms (RKGA). However, the difference between the two versions lies in the improvement phase. In the first version, all of the chromosomes created by the RKGA are sent to the local search improvement procedure, while the second version applies a selecting mechanism to send only a percentage of them. The computational experiments were performed on instances up to 100 nodes and revealed that the elitist version of the memetic algorithm outperformed the first version in both the quality of the obtained solutions and computational time.

Only three approximate approaches have been proposed for the MDCCVRP. Lalla-Ruiz and Voß (2020) proposed and applied the matheuristc version of POPMUSIC [3] proposed in (Lalla-Ruiz & Voß, 2016) for the MDCCVRP. The proposed solution procedure decomposes the problem into reduced versions of it in order to solve the overall problem. The matheuristic POPMUSIC version solves those reduced problems by means of an exact approach (Lalla-Ruiz & Voß, 2016). Different configurations for building the sub-problems were proposed (i.e., lexicographic and based on distance). This way, once the problem is decomposed into sub-problems, they were solved by means of an exact algorithm, in (Lalla-Ruiz & Voß, 2020) that was performed through the optimization model. Wang et al. (2020) proposed a perturbation local search heuristic, a fast algorithm designed to handle large size problems. The algorithm starts by constructing virtual tours based on a regret insertion cost. Then, the initial tours are subsequently improved by local search operators: reallocation, exchange, *2-opt*, arc-node exchange, *or-opt* move and cross-exchange. To extend the search space, perturbation operators (based on random reallocation) are applied. For comparison, the authors adapted a VNS originally developed for the multi-depot vehicle routing problem with loading cost (Kuo & Wang, 2012) and also used a POPMUSIC approach to compare with. Their results indicated that PLS outperformed the previous solutions presented in (Lalla-Ruiz & Voß, 2020) by providing new best solutions. Finally, Fernández Gil et al. (2020) proposed a matheuristic approach for solving a multi-objective Cum-VRP with soft and hard time windows. Their algorithm proposal was based on combining the MILP model with a GRASP. In their method, a feasible solution was built within the constructive part of the GRASP and optimized by using the MILP model. That is, the optimization model was applied for solving a set of sub-instances defined by each vehicle tour. After the constructive part, a local search method based on hill-climbing with the first improvement strategy was applied.

Based on the information shown in Fig. 7, it can be observed that, concerning the single-solution based algorithms, the most preferred algorithms are those that incorporate constructive and local search improve mechanisms (such as GRASP, ILS or TPH), followed by algorithms that incorporate variable neighborhood searches during the improvement phase (VNS and ALNS). On the other hand, regarding the population-based metaheuristics, the most implemented procedures are those based on evolutionary approaches, such as genetic or memetic algorithms (GA and MA), relegating bio-inspired procedures to the second place.

[3] The acronym stands for Partial OPtimization Metaheuristic Under Specific Intensification Conditions, Taillard (1993)

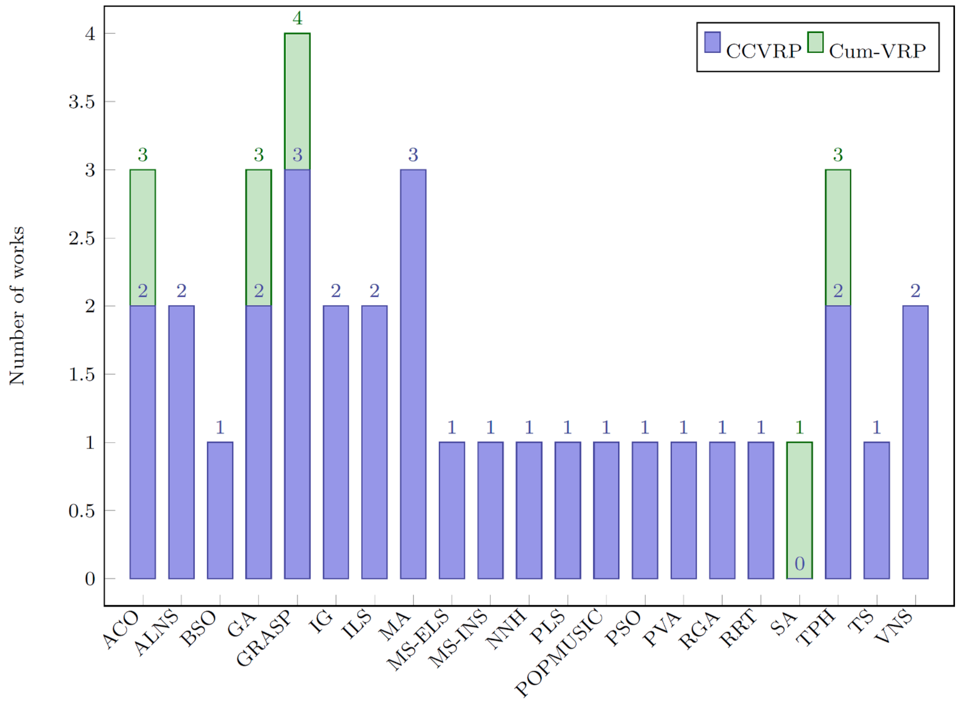


Fig. 7. Distribution of approximate approaches for the CCVRP and Cum-VRP.

4. Instances and best-known results

As part of the analysis of solution methods in the related literature, in the following, we provide a detailed review of all instance sets proposed so far for evaluating the performance of the different approaches developed for solving cumulative routing problems.

Table 6 provides an overview of proposed benchmark suites. The first column of the table corresponds to the work where the instances were proposed and the second column indicates the corresponding CCVRP or Cum-VRP variant in which the instance set has been used. Columns 3 and 4 indicate the number of instances and the range in terms of the number of nodes, respectively. Finally, a cross-reference between the instance sets and the works using them is shown in the rest of the columns, except for the last one that provides the sum of all works using each instance set.

From the information displayed in Table 6, it can be observed that some instances are from classic VRP works and, thus, have been widely used like Christofides et al., (1979), Gillet and Johnson (1976), Taillard (1993), Golden et al., (1998), etc., while others have been used by fewer authors, mainly because some of them were generated for the specific variant, e.g., CCVRP with time windows or mt-CCVRP. In the following, we refer to the sets proposed by Christofides et al. (1979) (i.e., CMT) and Golden et al. (1998) (GWKC) as the *Legacy instances* given that they have been frequently used in cumulative VRPs, especially in the CCVRP. This legacy instance set is used as a reference point to analyze the different solution methods collected in this review.

Focusing the attention on the mentioned *Legacy instances*, they have been mainly used in numerical experiments related to the CCVRP and some of its variants. This can be explained by the fact that those instances were the first to be used by the two first articles addressing the CCVRP. Another aspect to highlight is that the set of instances proposed by Augerat et al. (1995) have been also used by the papers addressing both the CCVRP and the CumVRP.

Table 7 reports the best-known solution (BKS) obtained so far for the legacy instances. Columns 1, 2, and 3 of the table denote the name of the instance, the number of customers, and the number of vehicles involved, respectively. Column 4 reports the best-known solution value for each instance. The rest of the columns display the works that have used the instances and whether they could report the best-known solution.

Regarding the CMT dataset, most of the authors have reported the BKS. From the information shown in Table 7, it can be observed that all the referenced authors at least reported one BKS for the CMT benchmarks. Three works, in particular, Sze et al. (2017), Ke (2018) and Kyriakakis et al. (2021), found all the BKSs for all considered cases. For most of the instances, the BKS has been preserved since 2012 (some of them since 2010), except for the instances CMT11 and CMT5 where the BKS were reported in 2013 and 2018, respectively. For the GKWC instances, only a few works have reported the BKSs. Most of them are distributed between the works of Sze et al. (2017), Ke (2018), and Smiti et al. (2020). With respect to exact methods, only one of the works, i.e., Lysgaard and Wøhlk (2014), used an exact method (branch-and-cut-and-price) to solve the sets. The rest of the authors utilized an approximation approach, where 75% correspond to population-based methods and 25% to single solution-based ones. Regarding the methods that provided all BKSs, Ke (2018) developed a population-based algorithm (i.e., BSO), whereas Smiti et al. (2020) implemented a single-solution metaheuristic (i.e., SVNS).

From the analysis of works using the GWKC instances, 7 out of 9 approaches (77.8%) belong to single-solution based algorithms, i.e., (Mattos-Ribeiro & Laporte, 2012; Ke & Feng, 2013; Rivera et al., 2015; Sze et al., 2017; Nucamendi-Guillén et al., 2018; Ke, 2018; Smiti et al., 2020), and two studies addressed population-based procedures, (Ke, 2018; Kyriakakis et al., 2021). Moreover, 5 out of those 9 works provide the current best-known results (Ke & Feng, 2013; Sze et al., 2017; Ke, 2018; Smiti et al., 2020; Kyriakakis et al., 2021). Three of them correspond to metaheuristic methods using a single-solution based approach

Table 6
Compilation of instances used in the reviewed literature. (See below-mentioned references for further information)

Benchmark set	Problem variant	#	Nodes	Kara et al. (2008)	Ngueveu et al. (2009)	Ngueveu et al. (2010)	Chen et al. (2012)	Mattos Ribeiro & Laporte (2012)	Ozsoydan & Sipahioglu (2013)	Ke & Feng (2013)	Lysgaard & Wøhlk (2014)	Rivera et al. (2014)	Rivera et al. (2015)	Gaur & Singh (2015)	Martínez-Salazar et al. (2015)	Victoria et al. (2016)	Moshref-Javadi & Lee (2016)	Cinar et al. (2016)	Rivera et al. (2016)	Cinar et al. (2017)	Flores-Garza et al. (2017)	Gaur & Singh (2017)	Sze et al. (2017)	Ke (2018)	Lenis & Rivera (2018)	Molina et al. (2018)	Nucamendi-Guillén et al. (2018)	Wang et al. (2018)	Bruni et al. (2019)	Liu & Jiang (2019)	Ramadhan & Imran (2019)	Wang et al. (2020)	Fernández Gil et al. (2020)	Lalla-Ruiz & Voß (2020)	Nucamendi-Guillén et al. (2020)	Smiti et al. (2020)	Damião et al. (2021)	Kyriakakis et al. (2021)	Niu et al. (2021)	Osorio-Mora et al. (2021)	Total
Christofides & Eilon (1969)	CCVRP, Cum-VRP, CCVRP-Pr	13	12-100						•					•								•							•												4
Gillett & Johnson (1976)	BCCVRP-Pr	6	75-100																																•						1
Christofides et al. (1979)	CCVRP, mt-CCVRP, CCVRP-TDD, Cum-VRPLD, MO-MTCVRP	14	50-199		•	•	•	•	•	•	•	•	•		•	•		•	•	•			•	•		•	•			•	•					•	•	•			23
Golden et al. (1983)	CCARP	23	7-27																						•																1
Rinaldi & Yarrow (1985)	Cum-VRP	1	48											•								•																			2
Taillard (1993)	Cum-VRPLD	13	75-385															•		•																					2
Fisher (1994)	CCVRP, MO-MTCVRP	3	44-134						•														•			•															3
Augerat et al. (1995)	CCVRP, Cum-VRP, CCVRP-Pr, BCCVRP-Pr	A:27 B:23 P:24	A:31-79 B:30-77 P:15-100						•					•								•							•						•						5
Cordeau et al. (1997)	Cum-MDVRP, MDC-CVRP	33	48-360																									•				•		•					•		4
Golden et al. (1998)	CCVRP, mt-CCVRP, Cum-VRPLD	20	200-483					•		•		•	•		•			•		•			•	•			•			•						•		•			13
Tuzun & Burke (1999)	LLRP	36	100-200														•																								1
dos Santos Barreto (2004)	LLRP	10	21-134														•																								1
Li et al. (2005)	CCVRP, Cum-VRPLD	12	560-1200															•					•	•																	3
Prins et al. (2006)	LLRP	30	20-200														•																								1
Tan & Kara (2007)	Cum-VRP	1	81	•																																					1
Salehipour et al. (2008)	CCVRP, Cum-VRP	140	10-1000		•	•									•																										3
Chunyu & Xiaobo (2010)	BCCVRP-Pr	1	20																																•						1
Demir et al. (2012)	Cum-VRPTW	180	10-200																														•								1
Rivera et al. (2014)	mt-CCVRP	12	15									•																													1
Hà et al. (2013)	m-CCTP	96	100-200																		•																				1
Angel-Bello et al. (2013b)	CCVRP	150	10-40																								•														1
Lysgaard & Wøhlk (2014)	CCVRP	A:27 B:23 P:24 E:13 M:5	A:31-79 B:30-77 P:15-100 E:12-100 M:100-199								•																•			•							•	•			5
Koulaeian et al. (2015)	BCCVRP-Pr	1	12																																•						1
Rivera et al. (2015)	mt-CCVRP	12	15										•																												1
Martínez-Salazar et al. (2015)	mt-CCVRP	125	10-30												•																										1
Victoria et al. (2016)	CCVRP-TDD	11	10-17													•																									1
Lalla-Ruiz & Voß (2020)	MDCCVRP	18	10-100																													•		•					•		3
Osorio-Mora et al. (2021)	MDCCVRP	165	10-50																																					•	1

(TPH (Ke & Feng, 2013), VNS/ALNS (Sze et al., 2017), and SVNS (Smiti et al., 2020)). On the other hand, the population-based approaches that provided some BKS are the BSO (Ke, 2018) and the ACO (Kyriakakis et al., 2021). In particular, there are four recent algorithms that report recent BKS for instances greater than or equal to 200 nodes (Sze et al., 2017; Ke, 2018; Smiti et al., 2020; Kyriakakis et al., 2021), where two of them belong to the category of single-solution algorithms (Sze et al., 2017; Smiti et al., 2020) and two to the category of population-based ones, i.e., (Ke, 2018) and Kyriakakis et al. (2021). It is important to highlight that, except for the instance GKWC5, where Kyriakakis et al.

Table 7
Best-known values for the legacy instances.

Instance	*n*	*k*	Best-known solution	Ngueveu et al. (2010)	Mattos-Ribeiro and Laporte (2012)	Chen et al. (2012)	Ke and Feng (2013)	Lysgaard and Wøhlk (2014)	Rivera et al. (2015)	Sze et al. (2017)	Ke (2018)	Nucamendi-Guillén et al. (2018)	Liu and Jiang (2019)	Ramadhan and Imran (2019)	Smiti et al. (2020)	Kyriakakis et al. (2021)	Damião et al. (2021)
CMT1	50	5	2,230.35	•	•	•	•	•	•	•	•	•	•	•	•	•	•
CMT2	75	10	2,391.63		•	•	•	•	•	•	•	•	•		•	•	•
CMT3	100	8	4,045.42	•	•	•	•	•	•	•	•	•	•		•	•	•
CMT4	150	12	4,987.52	•	•	•	•	•	•	•	•	•	•		•	•	•
CMT5	199	17	5,806.02							•	•					•	•
CMT11	120	7	7,314.55				•		•	•	•	•	•		•	•	•
CMT12	100	10	3,558.92	•	•	•	•	•	•	•	•	•	•		•	•	•
GWKC1	240	9	54,672.49	-		-		-			•			-			-
GWKC2	320	10	100,560.00	-		-		-		•				-			-
GWKC3	400	10	170,923.55	-		-		-			•			-			-
GWKC4	480	10	261,993.00	-		-		-		•				-			-
GWKC5	200	5	114,163.64	-		-	•	-		•	•			-		•	-
GWKC6	280	7	139,384.46	-		-		-						-	•		-
GWKC7	360	8	179,388.00	-		-		-		•				-			-
GWKC8	440	10	193,698.00	-		-		-		•				-			-
GWKC9	255	14	4,721.39	-		-		-			•			-			-
GWKC10	323	16	6,578.75	-		-		-						-	•		-
GWKC11	399	18	9,210.45	-		-		-		•				-			-
GWKC12	483	19	12,495.60	-		-		-			•			-			-
GWKC13	252	26	3,605.57	-		-		-						-	•		-
GWKC14	320	29	5,107.02	-		-		-						-	•		-
GWKC15	396	33	6,943.41	-		-		-						-	•		-
GWKC16	480	37	9,183.58	-		-		-						-	•		-
GWKC17	240	22	3,060.50	-		-		-			•			-			-
GWKC18	300	27	4,216.01	-		-		-		•				-			-
GWKC19	360	33	5,502.08	-		-		-		•				-			-
GWKC20	420	38	7,015.83	-		-		-						-	•		-

• Indicates that best-known solution was reported.
- Indicates that the instance was not considered in their experiments.

(2021) reported also the BKS, the other three algorithms reported the BKS for different instances. Particularly, in the case of Smiti et al. (2020), this is the most recent study that provided new BKS in the literature, finding 14 of them and specifically improving 7 out of 20 BKS for the GWKC instances.

5. Conclusions

In this paper, a review of vehicle routing problems with cumulative objectives by means of CCVRP and Cum-VRP is presented. A review search strategy was designed and conducted to find the existing literature for the mentioned problems and their variants. Then, we introduced a classification analysis resuming the distinct features of the reviewed problems and the solution approaches found. From it, interested scholars and practitioners can obtain the current map of problem variants and position their current problem or find out appropriate approaches for their cumulative vehicle routing at hand. In addition, several problem variants, as well as relevant practical applications and cases, have been identified and listed. Secondly, a detailed discussion of the solution contributions concerning the solution methods is presented. In addition, we listed the main problem instance sets used in the literature and identified those used the most by the authors. Finally, an analysis of the best-known results for these instances is presented, showing the most effective solution approaches to deal with small and large-size instances.

From the conducted literature analysis, we can conclude that the mathematical models and exact approaches developed for this type of problems are limited as the size of the instances grow. In particular, the largest instance size solved to optimality is around 40 nodes. This justifies and explains the development of heuristics and metaheuristics to tackle large-size instances. Regarding these procedures, there is a slight preference for developing single-based solution algorithms since these approaches have shown their effectiveness by finding the BKS for almost all legacy instances. In summary, the algorithms based on variable neighborhood search are the ones that have reported the BKS over all instances. It is important to remark that, as recent metaheuristics are presented over time, the best-known solutions have been updated. This improvement is more evident in the GWKC instances in comparison with the CMT ones. Recently, the use of decomposition matheuristics started to have more presence as they provide an attractive combination of metaheuristics and mathematical models by partially solving reduced versions of the problem at hand (relaxed version of the global problem), whereas the metaheuristic algorithm is used to accomplish the remaining restrictions of the problem. The advantage of these approaches is that subproblems are solved to optimality.

Considering the rich VRP literature, some relevant research directions and trends can be incorporated or addressed considering cumulative objectives:

- *Split Deliveries:* This perspective, studied in other VRPs, can be extended in current cumulative problems involving time-sensitive freight delivery as well as in humanitarian environments where partial delivery becomes crucial. This becomes relevant in emergency relief situations, given the need of maximizing the rescue response in affected areas. For instance, in the supply of medical aid, where damaged zones' demands exceed single-vehicle capacity and for which accelerating to start attending severely affected areas is essential to reduce casualties and economic losses (Bodaghi et al., 2020). This might result in the need for splitting to provide at least one part of the total delivery as soon as possible and the rest before the end of the shift (Alarcon Ortega, Schilde, & Doerner, 2020). A relevant application of this type of problem variant can be found in contexts such as the supply of vaccines (Xing et al., 2020), where prompt delivery is essential to maximize the number of people to be immunized. On the other hand, in commercial contexts, this type of delivery approach might increase customer satisfaction as long as the receiving nodes agree with the split delivery policy. A side benefit can be obtained in the reduction of the number of required vehicles to serve all customers. (Archetti & Speranza, 2012).
- *Pick-up and delivery:* In recent years, companies are ever more mindful of the savings that can be made by combining deliveries and pick-ups. The cumulative vehicle routing problem variants considering pick-ups and deliveries would be interesting to be addressed in environments such as meal deliveries, distribution of medical supply, postal deliveries, industrial refuse collection, recycling services, school bus routing, industrial gases delivery, or JIT (just in time) manufacturing Wassan and Nagy (2014). In this regard, simultaneous pick-ups and deliveries can also be considered similarly as in Koç, Laporte, and Tükenmez (2020).
- *Stochastic and Dynamic environments:* In almost all real-world applications, uncertainty and dynamism are inherent characteristics in their involved routing problems. For instance, in these environments, new customers can appear during the realization of planning (Ritzinger, Puchinger, & Hartl, 2016), demand might be uncertain (Zhong et al., 2020), or stochastic service/travel times need to be considered. Still, this research direction has to be further explored in the literature for cumulative vehicle routing problems, even when these parameters are frequently used to describe dynamic environments (Oyola, Arntzen, & Woodruff, 2018). In this regard, incorporating geospatial information such as in Žunić, Hindija, Beširević, Hodžić, and Delalić (2018) can be a relevant addition in cumulative routing problems so to identify and include, for example, critical traffic areas and design policies to reduce the impact of disruptive traffic events (e.g., crashes, failures in road, bottlenecks, protests) in a predefined logistics network.
- *Green Logistics*: Nowadays, the growing vehicle traffic levels in cities and relevant supply chain nodes lead to congestion and environmental pollution, impacting on logistics costs/profits (Simeonova, Wassan, Wassan, & Salhi, 2020; Schulte, Lalla-Ruiz, González-Ramírez, & Voß, 2017). In time-dependent approaches, the cost structure may be viewed as the cost per distance unit across an arc proportional to the vehicle's load when traversing that arc, provided that the sum of the customers' demands is on-board in the vehicle when departing from the depot. Particularly, fuel consumption and CO_2 emissions do not depend entirely on the distance but are also determined by factors such as type of vehicle and engine, speed, street surface, and vehicle load (Lysgaard & Wøhlk, 2014; Meneghetti & Ceschia, 2020; Fernández Gil et al., 2020). Lastly, the accurate consideration of the weight along routes becomes essential in the context of electric vehicles, where energy depends on speed and weight (Tahami, Rabadi, & Haouari, 2020; Murakami, 2017).

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A

Table 8.

Table 8
Acronyms table related to approaches.

ACO	Ant Colony Optimization
ACOMR	Ant Colony Optimization with Multiple Rounds
ALNS	Adaptive Large Neighborhood Search
AVNS	Adaptive Variable Neighborhood Search
Bi	Bi-objective
BSO	Brain Storm Optimization
B&C&P	Branch-and-Cut-and-Price
CCARP	Cumulative Capacitated Arc Routing Problem
CCVRP	Cumulative Capacitated Vehicle Routing Problem
CFAA	Constant Factor Approximation Algorithm
CG	Column Generation
Cum-VRP	Cumulative Vehicle Routing Problem
GA	Genetic Algorithm
GRASP	Greedy Randomized Adaptive Search Procedure
GRASP-MILP	Hybrid algorithm using GRASP and the MILP model
Hard	Time windows constraints must be satisfied
Ho	Homogeneous fleet
Ht	Heterogeneous fleet
IG	Iterated Greedy
ILS	Iterated Local Search
MA	Memetic Algorithm
mC&W	Modified Clarke & Wright
MS-ELS	Multi-Start Evolutionary Local Search
MS-INS	Multi-start algorithm with Intelligent Neighborhood Selection
Multi	Multi-objective approach
Multiple trips	When more than one trip is allowed per vehicle
NNH	Nearest Neighborhood Heuristic
NSGA	Non-dominated Sorting Genetic Algorithm
PIACO	Parallel Improved Ant Colony Optimization
PLS	Perturb-based Local Search
POPMUSIC	Partial Optimization Metaheuristic under Special Intensification Conditions
Priority	Indicates if the approach incorporates priorities associated with customers
PSO	Particle Swarm Optimization
PVA	Profitable Visit Algorithm
RGA	Recursive Granular Algorithm
Route duration	Routes are limited by a predefined parameter (e.g., distance, time, etc.)
RRT	Record-to-Record Travel
SA	Simulate Annealing
Soft	Time windows can be violated but involving a penalty cost
SVNS	Skewed Variable Neighborhood Search
TPH	Two-phase Heuristic
TS	Tabu Search
VND	Variable Neighborhood Descent
VNS	Variable Neighborhood Search

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