Development and Test of Simulation Tools for EIC Beam-beam Interaction

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Requirements for Next Generation of EIC

- **Key EIC machine parameters identified in 2015 Long Range Plan are**

  1) Polarized (~70%) electrons, protons, and light nuclei,

  2) Ion beams from deuterons to the heaviest stable nuclei, variable center of mass energies ~20-100 GeV, upgradable to ~140 GeV,

  3) High collision luminosity ~10^{33}-10^{34} cm^{-2}sec^{-1}, and

  4) Possibly have more than one interaction region.

- **To reach such a high luminosity, both eRHIC and JLEIC designs aimed at**

  → increasing bunch intensities,

  → reducing transverse beam sizes at IPs,

  → increasing collision frequency.

Beam-beam Challenges in EIC

- **Luminosity**

$$L = \frac{N_1 N_2\, f_c}{2\pi\sqrt{\sigma_{x1}^2 + \sigma_{x2}^2}\,\sqrt{\sigma_{y1}^2 + \sigma_{y2}^2}} R$$

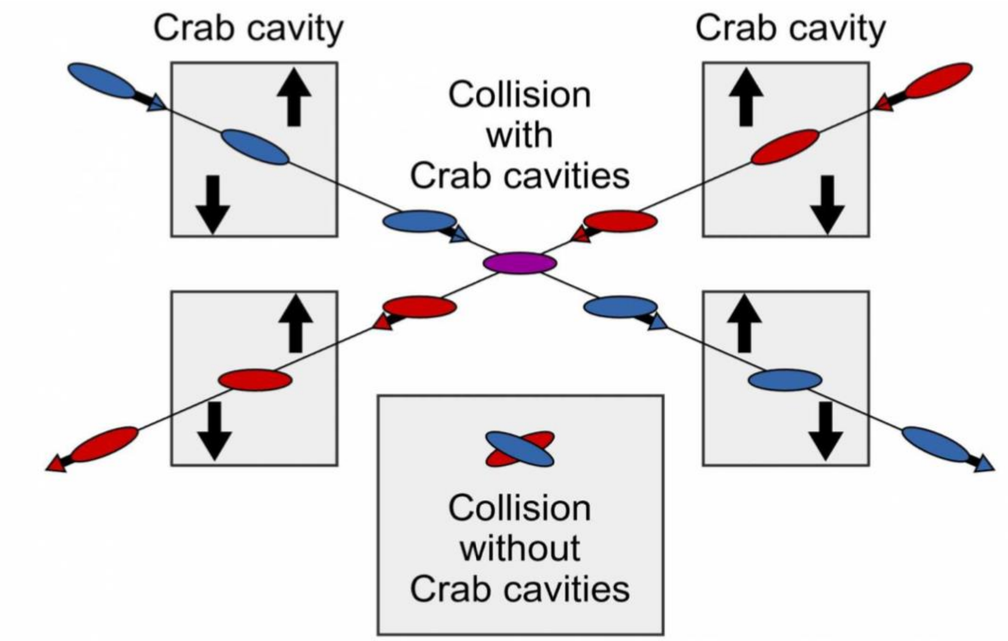
- **Beam-beam parameter**

$$\xi_{x,y} = \frac{\beta_{x,y}^*}{4\pi} \frac{N r_0}{\sigma_{x,y}(\sigma_x + \sigma_y)}$$

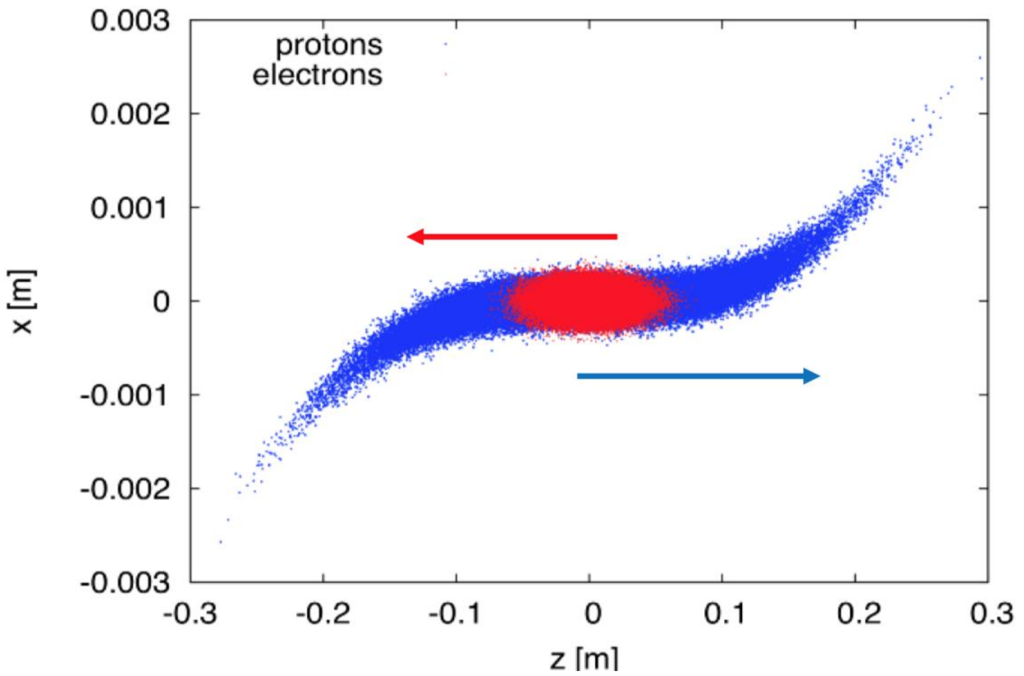
- **Challenges of beam-beam interaction in EIC**

1) **High beam-beam parameters**: proton ring ~ 0.015, electron ring~0.1
2) **large crossing angles**: eRHIC ← 25mrad , JLEIC ← 50 mrad
3) **Crab cavities must to be used** to compensate geometric luminosity loss

Collision with Crab Cavities



394MHz CC used in this example



Due to **long proton bunch length** and **finite wave length of crab cavities**, protons in the bunch head and tail are not well crabbed, which may introduce **offset beam-beam interaction, synchro-betatron resonance, and poor proton beam lifetime and large luminosity degradation rate**.

4 Challenging Items to Be Addressed

Beam-beam interaction have been identified as one of the most important challenges needed to be addressed to reduce the overall design risk in the 2017 NP Community EIC Accelerator R&D panel report.

4 challenging items have been selected for this project:

1) **beam dynamics** study and **numerical simulation** of collision **with crab cavities**,

2) quantitative understanding of **damping decrement** to the beam-beam performance ,

3) impacts on protons with **electron bunch swap-out** in **eRHIC** ring-ring design, and

4) impacts on beam dynamics with **gear-changing beam-beam interaction** in **JLEIC**.

Items 1 and 2 are common to both eRHIC and JLEIC. Item 3 is for eRHIC. Item 4 id for JLEIC.

Priorities in 2017 Jones EIC R&D Report

Items	Panel Priority	Panel Sub-Priority	Line
Task 1: Beam dynamics with crab cavities	High High High	A A -	1 4 32
Task 2: Damping Decrement effect	High		30
Task 3: Electron bunch swap-out effect	High	B	16
Task 4: Gear-change beam-beam effect	High High	B C	20 38

Who we are

- In the project, we join expertise from 4 institutions

  Dr. Yun Luo : Lead-PI, Brookhaven National Laboratory

  Dr. Yves Roblin: Co-PI, Thomas Jefferson National Laboratory

  Dr. Ji Qiang: Co-PI, Lawrence Berkeley Nation Laboratory

  Prof. Yue Hao: Co-PI, Michigan State University
- We also include the following experts

  Dr. He Zhang: Thomas Jefferson National Laboratory

  Dr. Derong Xu: postdoc, Michigan State University
- **This collaboration team with strong backgrounds in :** weak-strong beam-beam simulation, strong-strong beam-beam simulation, particle tracking in accelerators, nonlinear beam dynamics, accelerator operation and experiments, and so on.

How We Address Those Challenges

- To separate the artificial emittance growth in the strong-strong simulation, we developed and implemented **new simulation algorithms** in BeamBeam3D.
- To include lattice nonlinearity, we implemented **nonlinear truncated map and symplectic map tracking** in BeamBeam3D. We installed real RF cavities and IR multipole magnetic field too.
- To study the effects of **bunch replacement** in eRHIC and **gear change beam-beam** in JLEIC, we will **develop new simulation codes**.
- **With those new tools**, we will be able to move forward in investigating the **physics behind emittance growth and luminosity degradation,** and looking for **remedies for them.**

Deliverables & Milestones

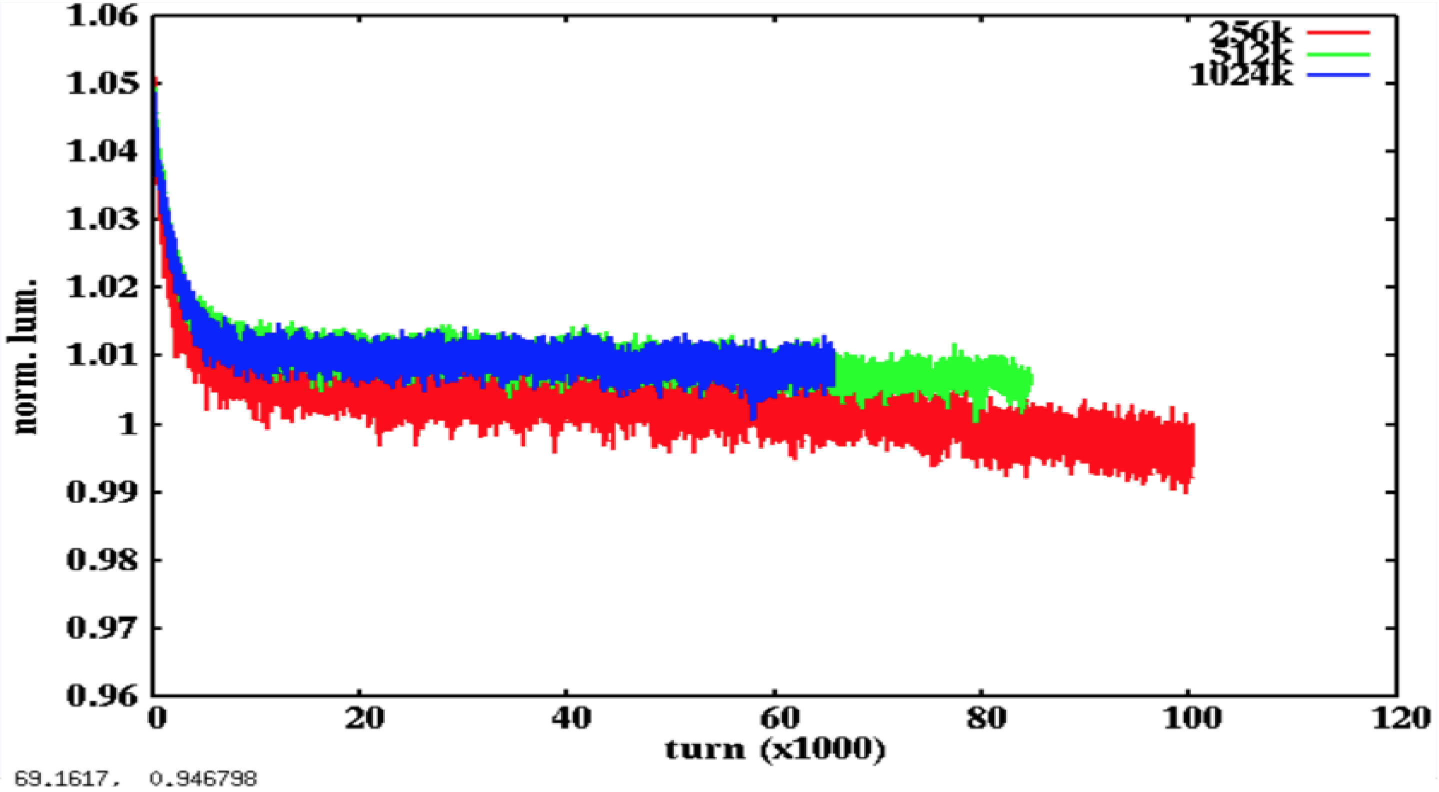
Year 1				Year 2			
Qtr 1	Qtr 2	Qtr 3	Qtr 4	Qtr 1	Qtr 2	Qtr 3	Qtr 4
Task 1.1, 1.2, 1.3 (BNL, MSU, JLab)							
Task 1.4 (LBNL, MSU)				Task 3.1 (LBNL) Task 3.2, 3.3 (BNL)			
Task 2.1, 2.2, 2.3 (LBNL, BNL) Task 2.4 (BNL, JLab)				Task 4.1 (LBNL), Task 4.2, 4.3, 4.4 (JLab)			

This project involves two years with 4 main tasks:

1) In **Year 1** , we will install spectral Poisson solver (**Task 1**) and nonlinear tracking methods (**Task 2**) in BeamBeam3D.
2) In **Year 2** , we will develop new codes to study bunch swap-out in eRHIC (**Task 3**) and gear-change beam-beam interaction in JLEIC **Task 4**).
3) Beam dynamics study spans these two years (**Task 1**).

Predicted Luminosity Degradation from Beam-Beam Simulation Depends on the Number of Macroparticles

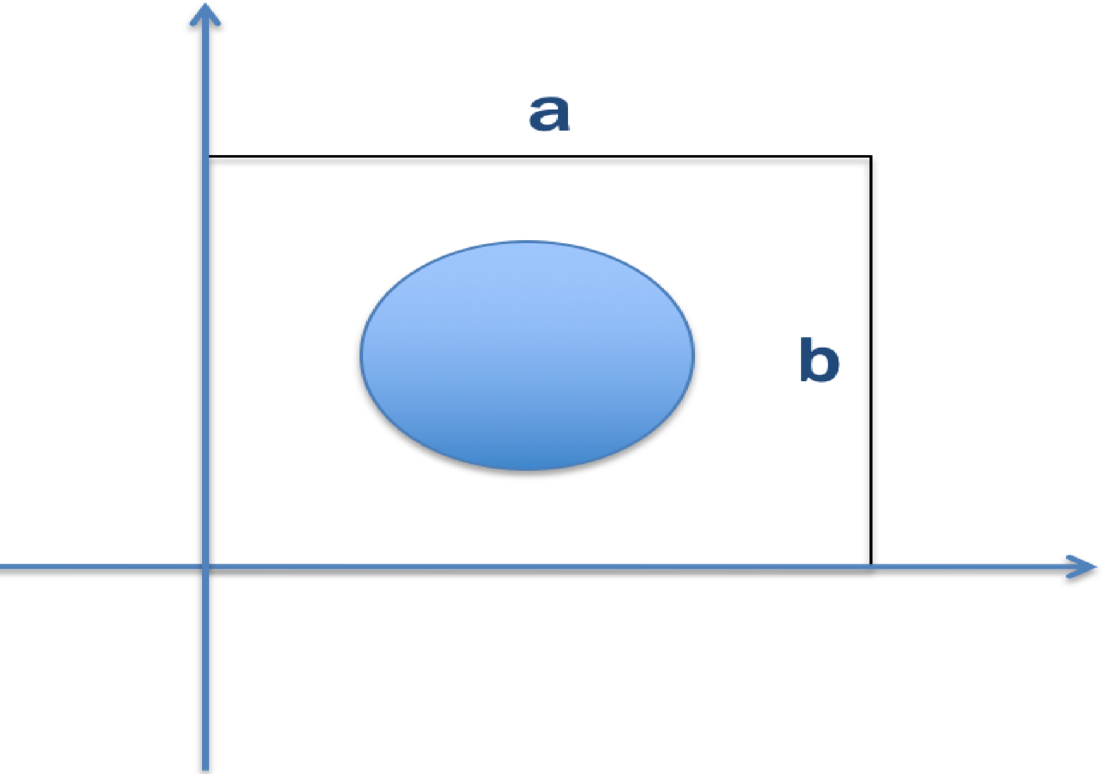
- Strong-strong beam-beam simulation subject to numerical noise driven emittance growth and luminosity degradation
- Increase of macroparticle number helps reduce numerical noise effects



J. Qiang

Reducing the Numerical Noise Effects through a Spectral Method

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = -4\pi\rho,$$



$$\rho(x,y) = \sum_{l=1}^{N_l} \sum_{m=1}^{N_m} \rho^{lm} \sin(\alpha_l x) \sin(\beta_m y)$$

$$\phi(x,y) = \sum_{l=1}^{N_l} \sum_{m=1}^{N_m} \phi^{lm} \sin(\alpha_l x) \sin(\beta_m y),$$

$$\rho^{lm} = \frac{4}{ab} \int_0^a \int_0^b \rho(x,y) \sin(\alpha_l x) \sin(\beta_m y) dx dy$$

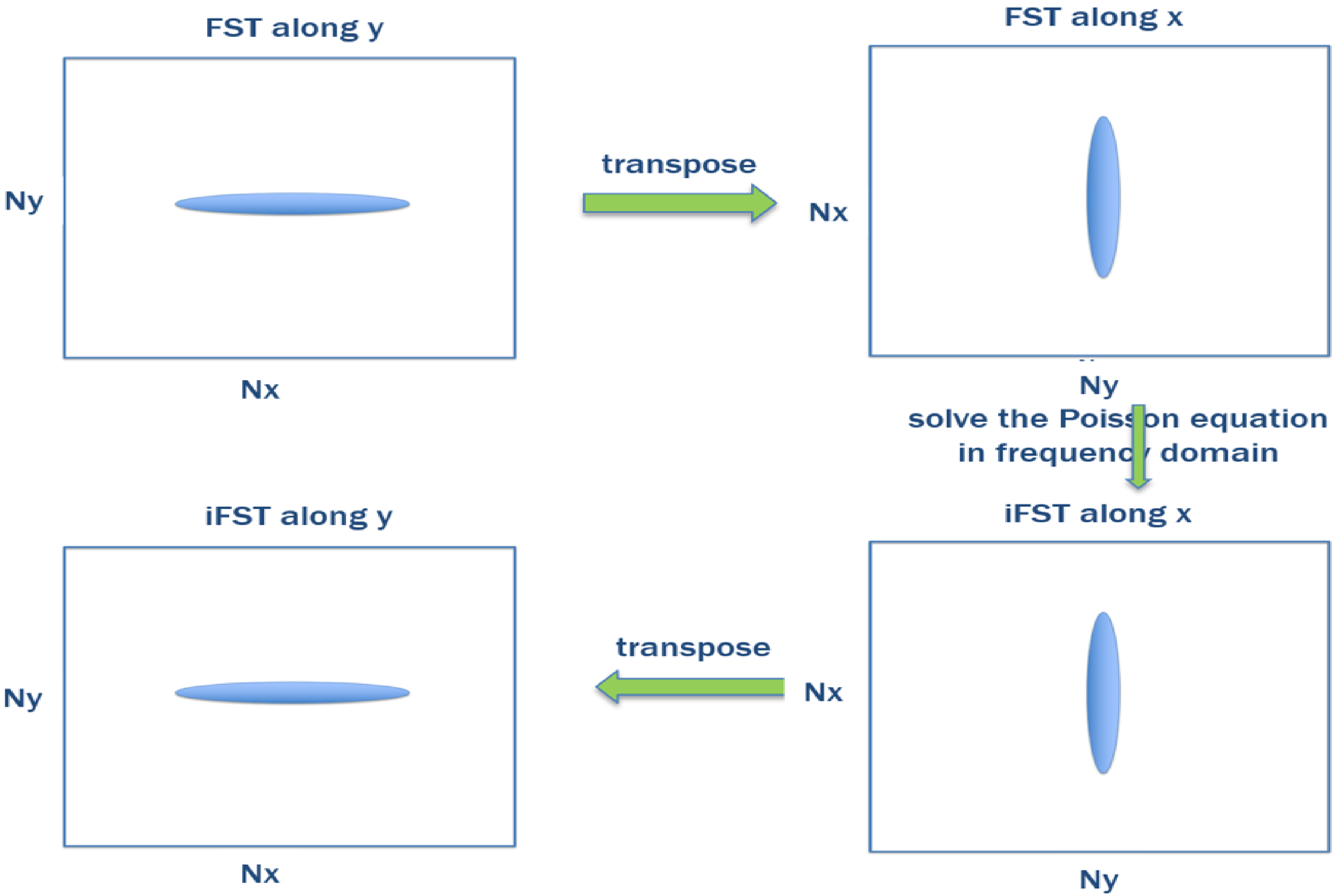
$$\phi^{lm} = \frac{4}{ab} \int_0^a \int_0^b \phi(x,y) \sin(\alpha_l x) \sin(\beta_m y) dx dy,$$

where $\alpha_l = l\pi/a$ and $\beta_m = m\pi/b$.

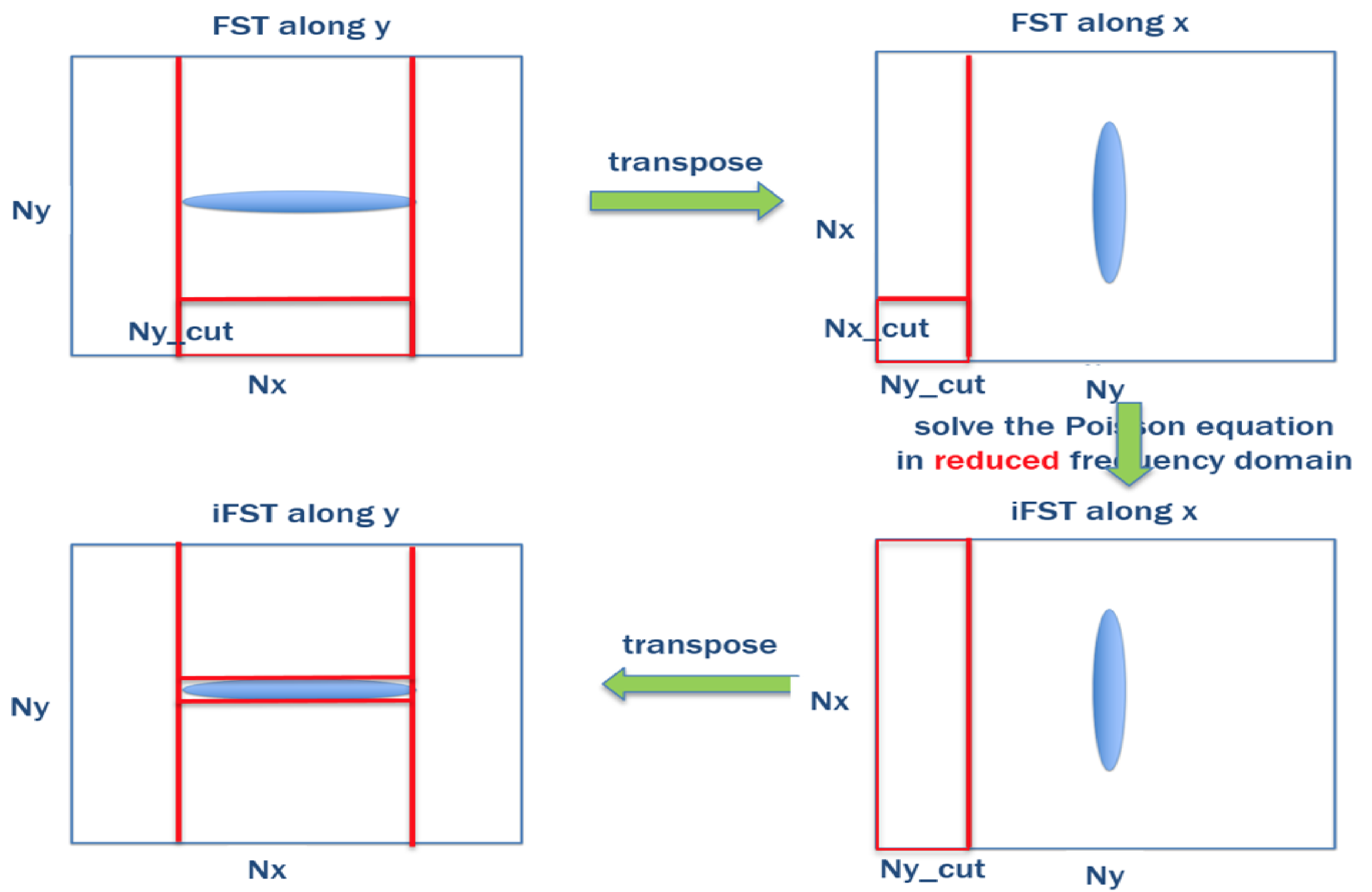
$$\phi^{lm} = \frac{4\pi\rho^{lm}}{\gamma_{lm}^2}$$

where $\gamma_{lm}^2 = \alpha_l^2 + \beta_m^2$.

A Brutal Force Implementation of Spectral Poisson Solver



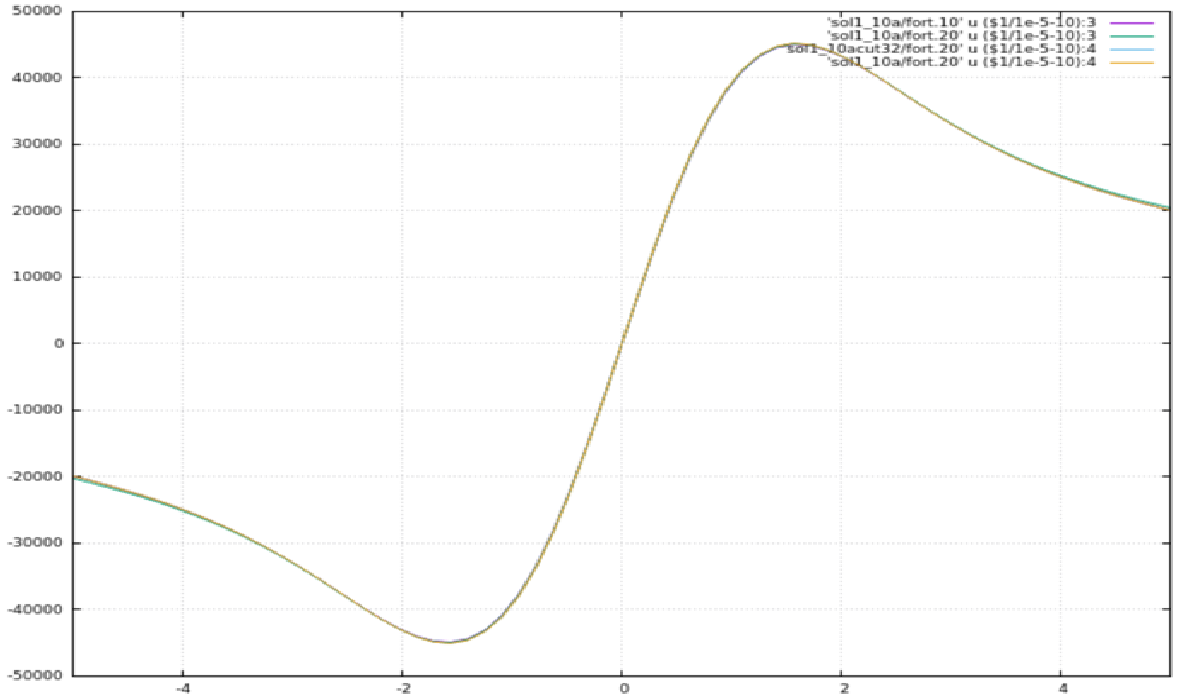
An Efficient Implementation of Spectral Poisson Solver



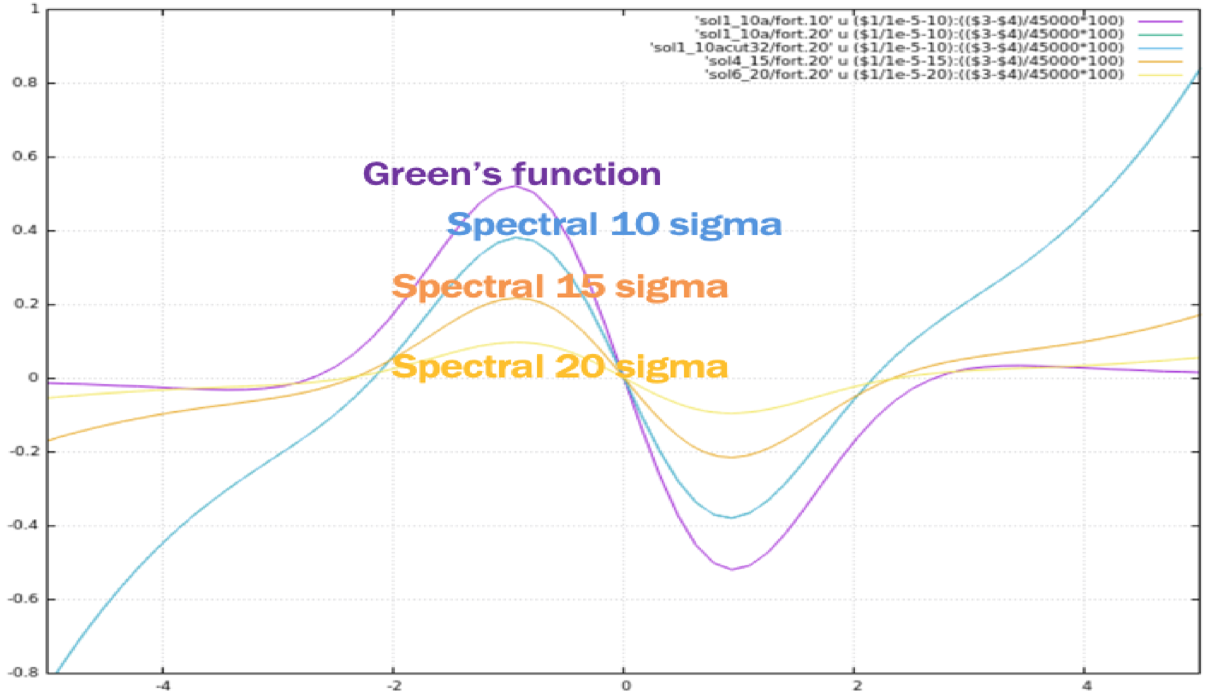
Greater than a factor of 3 saving computational cost in comparison to the brutal force method.

Solution of Electric Fields of a Gaussian Distribution from the Green's Function and the Spectral Methods with Aspect Ratio 1

Beam-Beam Field

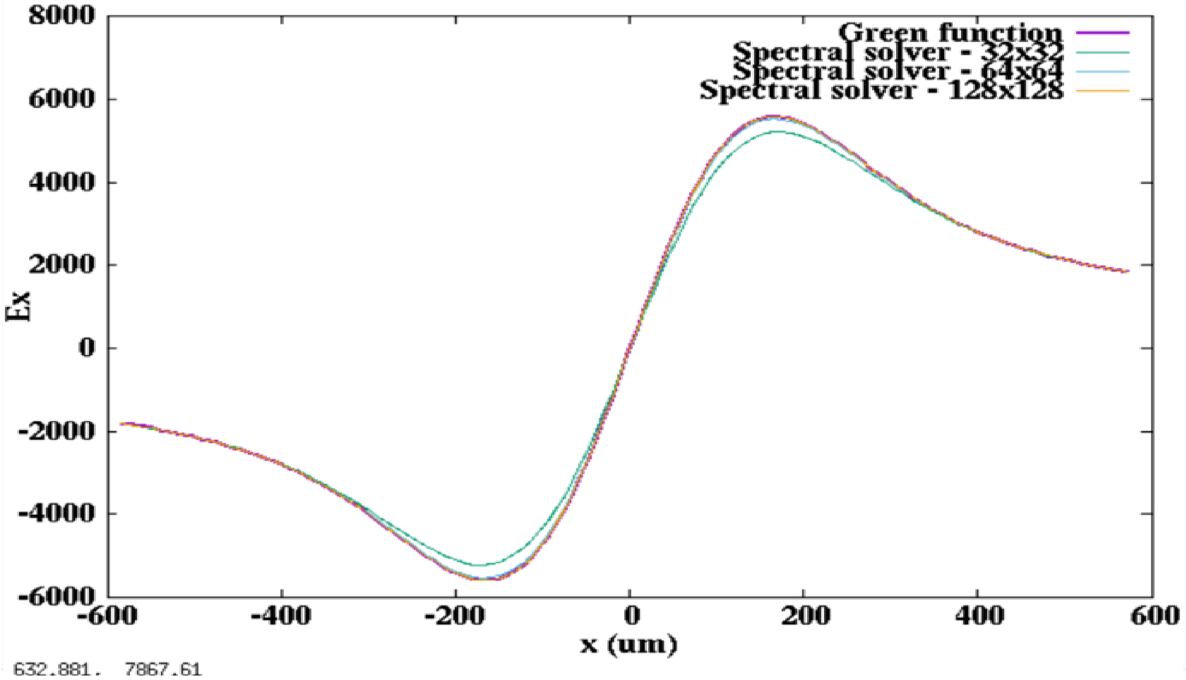


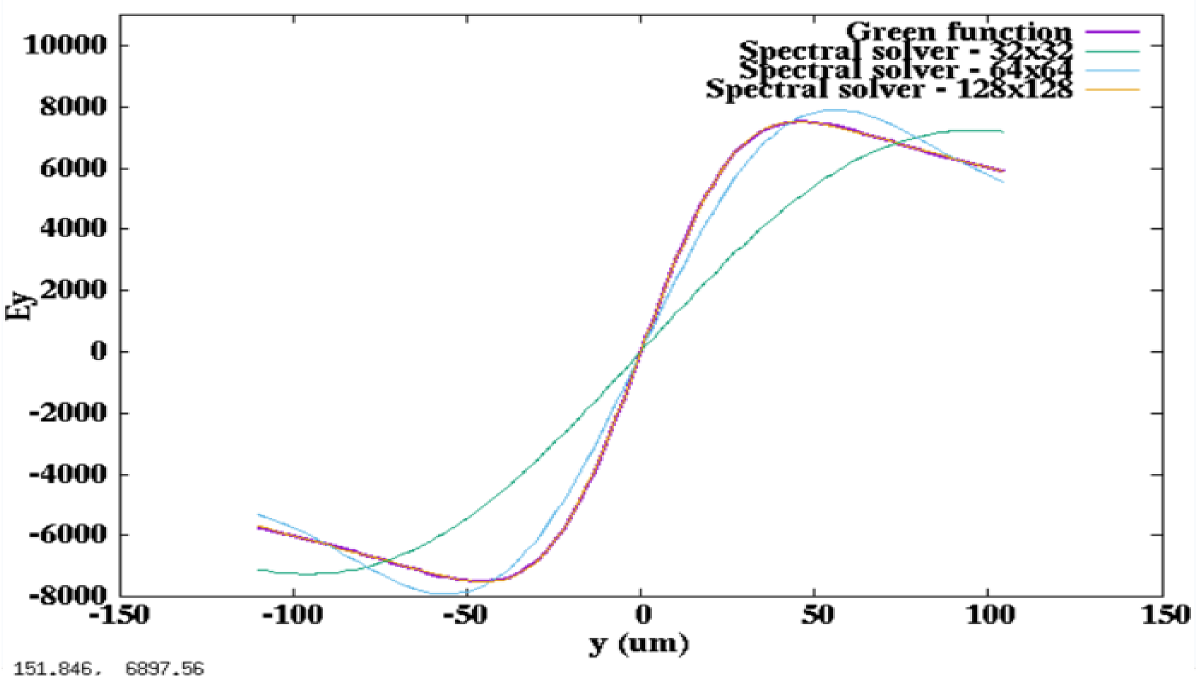
Relative Errors (%)



- Spectral solver agrees well with the Green's function solver.
- The major error of the spectral solver is around the edge of the beam.
- Such error decreases with larger computational domain.

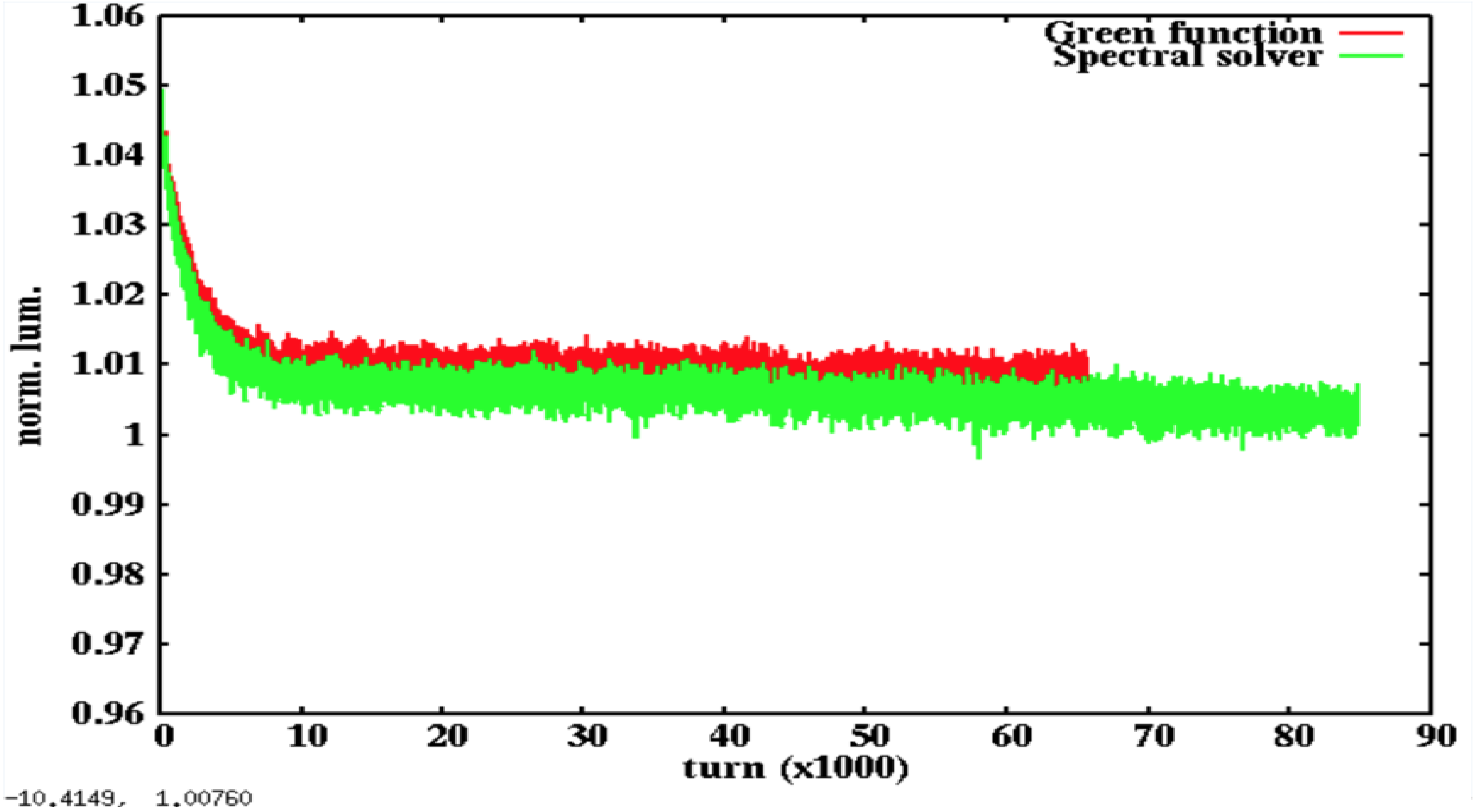
Solution of Electric Fields of a Gaussian Distribution from the Green's Function and the Spectral Methods with Aspect Ratio ~5





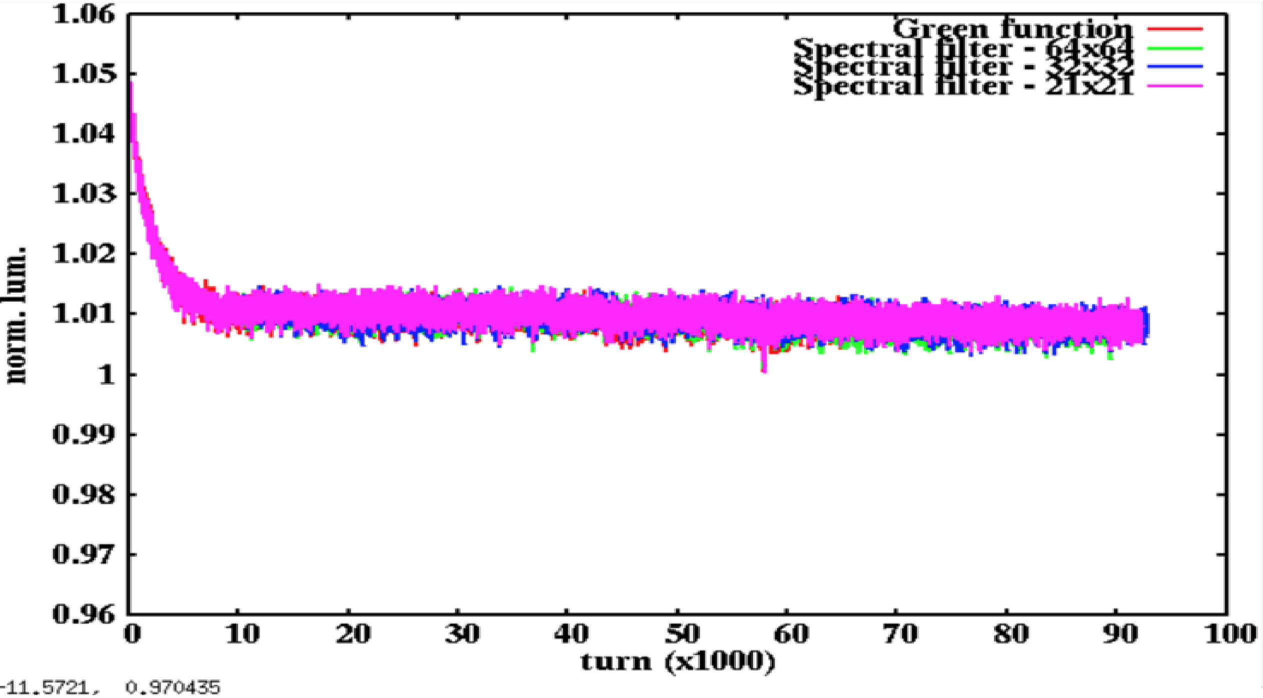
- 128x128 modes need to be used in the spectral solver in order to have good agreement with the Green function solver.
- large number of modes are needed for large aspect ratio beam.

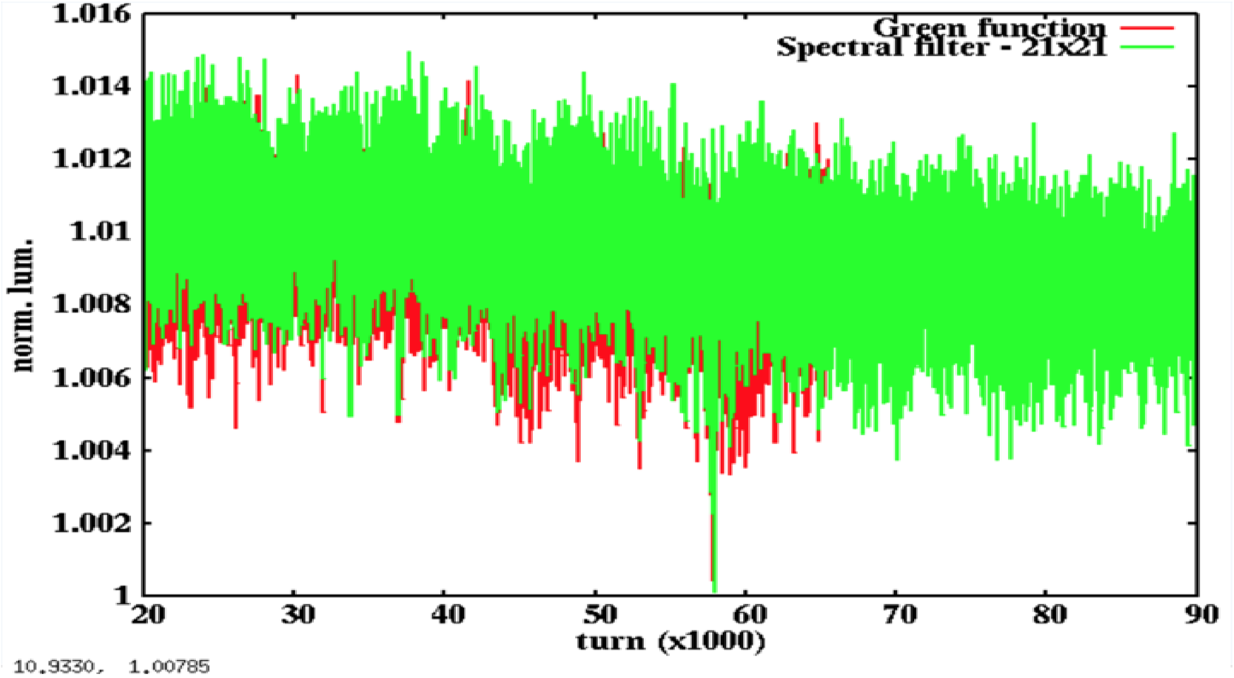
Luminosity Evolution from the Spectral Solver and the Green's Function Solver



- Both solvers agree with each other quite well.
- Slight larger degradation from the spectral solver might be due the use of large number of modes.

Luminosity Evolution Using the Spectral Filter





- Spectral filter agrees with the original solution well.
- Slightly smaller luminosity degradation with smaller number of modes.

Methodology[1,2]

Y. Roblin

Assuming a dynamic system, the initial and final coordinates of which are $(\boldsymbol{q}_i, \boldsymbol{p}_i)$ and $(\boldsymbol{q}_f, \boldsymbol{p}_f)$, the symplecticity is preserved if the coordinates satisfied a nonlinear implicit partial differential equation as follows:

$$(\vec{p}_i, \vec{p}_f) = (\vec{\nabla}_{q_i} F_1, -\vec{\nabla}_{q_f} F_1)$$

$$(\vec{p}_i, \vec{q}_f) = (\vec{\nabla}_{q_i} F_2, \vec{\nabla}_{P_f} F_2)$$

$$(\vec{q}_i, \vec{p}_f) = (-\vec{\nabla}_{p_i} F_3, -\vec{\nabla}_{q_f} F_3)$$

$$(\vec{q}_i, \vec{q}_f) = (-\vec{\nabla}_{p_i} F_4, \vec{\nabla}_{p_f} F_4).$$

Here $F_1(\boldsymbol{q}_i, \boldsymbol{q}_f), F_2(\boldsymbol{q}_i, \boldsymbol{p}_f), F_3(\boldsymbol{p}_i, \boldsymbol{q}_f), F_4(\boldsymbol{p}_i, \boldsymbol{p}_f)$ are the four most commonly used **generating functions** (GF) in mixed variables.

The generating function and the respective PDEs can be constructed from a truncated map $\boldsymbol{M}$ of the dynamic system using **differential algebra** (DA).

Methodology[1,2]

Construct (Taylor expansion of) **the Generating Function** (represented as a DA vector):

We have the truncated map $\boldsymbol{M}$, represented as DA vectors.
Given a DA map $\boldsymbol{N}$, we can calculate the inverse of the map $\boldsymbol{N}^{-1}$. (The linear part of $\boldsymbol{N}$ is required to be inversable.)

For example, construct the 2nd kind of GF $F_2(\boldsymbol{q}_i, \boldsymbol{p}_f)$:
1. Denote the transfer map $\boldsymbol{M} = (\boldsymbol{M}_1, \boldsymbol{M}_2)$. $\boldsymbol{M}_1$ is the part for final positions and $\boldsymbol{M}_2$ the part for final moments. Same for the identity map $\boldsymbol{I} = (\boldsymbol{I}_1, \boldsymbol{I}_2)$.
2. Let $\boldsymbol{N} = (\boldsymbol{I}_1, \boldsymbol{M}_2)$, then $(\boldsymbol{q}_i, \boldsymbol{p}_f) = \boldsymbol{N}(\boldsymbol{q}_i, \boldsymbol{p}_i)$ and $(\boldsymbol{q}_i, \boldsymbol{p}_i) = \boldsymbol{N}^{-1}(\boldsymbol{q}_i, \boldsymbol{p}_f)$.
3. $(\boldsymbol{q}_f, \boldsymbol{p}_i) = \left((\boldsymbol{M}_1, \boldsymbol{I}_2) \circ \boldsymbol{N}^{-1}\right)(\boldsymbol{q}_i, \boldsymbol{p}_f) = \widehat{\boldsymbol{F}}_2(\boldsymbol{q}_i, \boldsymbol{p}_f)$
4. F_2 is the potential of $\widehat{\boldsymbol{F}}_2$. Integrate $\widehat{\boldsymbol{F}}_2$ by any path to get F_2

Methodology[1,2]

Symplectic Tracking:

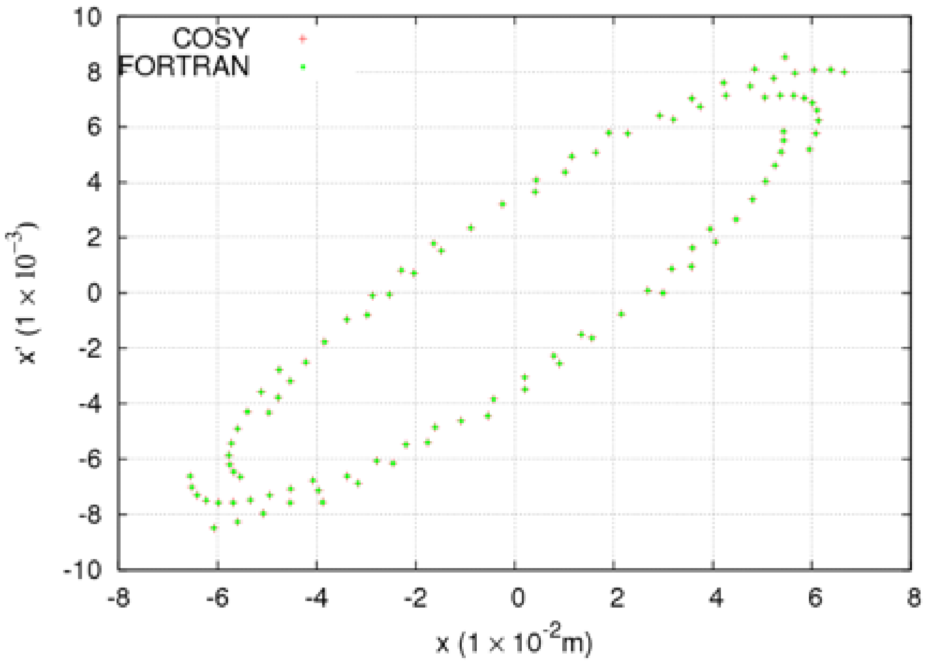
1. Use the truncated map to construct the GF as described aforehead.
2. Construct the PDEs by taking derivatives of the GF.
3. Calculate $\boldsymbol{X}_f = \boldsymbol{M} \circ \boldsymbol{X}_i$, where $\boldsymbol{M}$ is the truncated map, $\boldsymbol{X}_i$ is the initial coordinates.
4. Use $\boldsymbol{X}_i$ and $\boldsymbol{X}_f$ as initial guess to solve the PDEs iteratively. Since $\boldsymbol{M}$ is very close to the real symplectic map, the solution should converge very fast in just a few iterations.

[1] Modern Map Methods in Particle Beam Physics, page 293
M. Berz, Academic Press, 1999, ISBN 0-12-014750-5
[2] Symplectic Tracking in Circular Accelerators with High Order Maps
M. Berz, in: *"Nonlinear Problems in Future Particle Accelerators"* (1991) 288-296, World Scientific

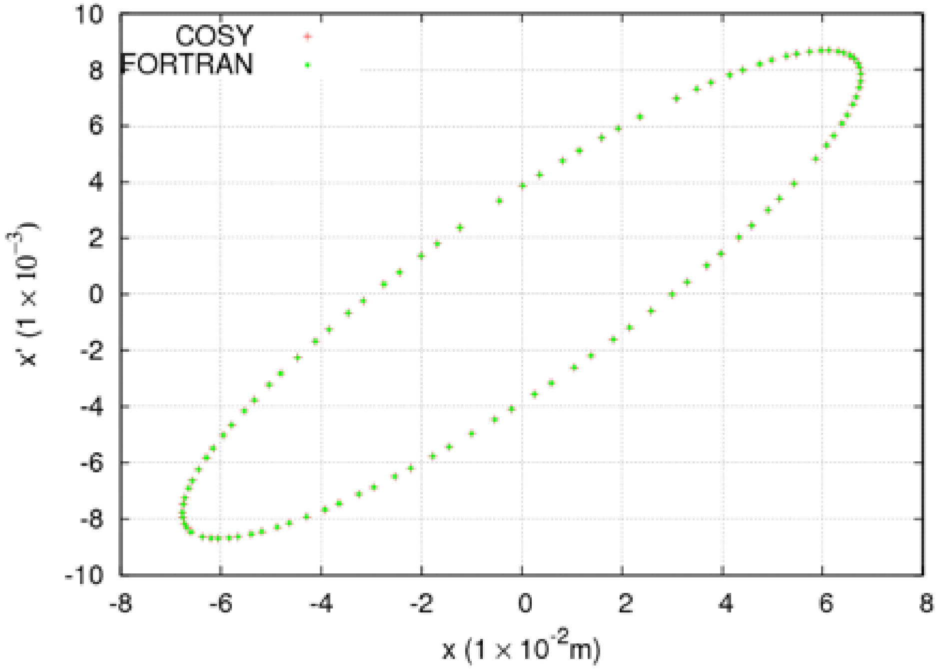
Code Verification

1. Verified for truncated map tracking and symplectic tracking with COSY Infinity using a 4D map with coupling for 1,000,000 turns.
2. Two codes agree very well.

Truncated map

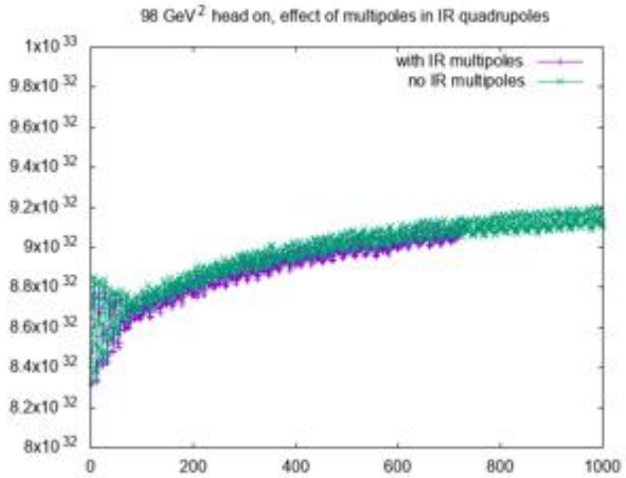


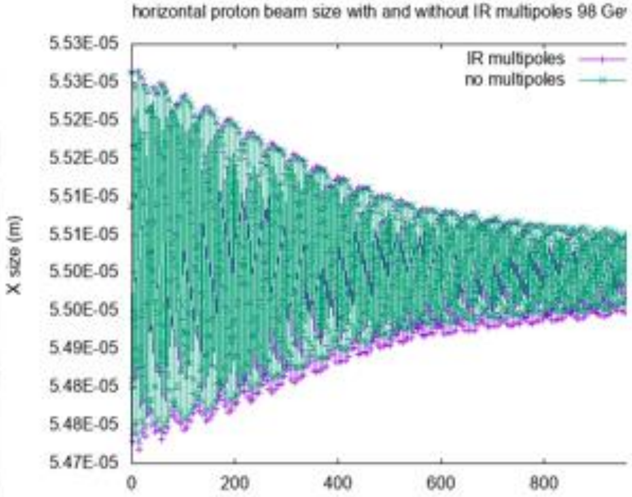
GF2

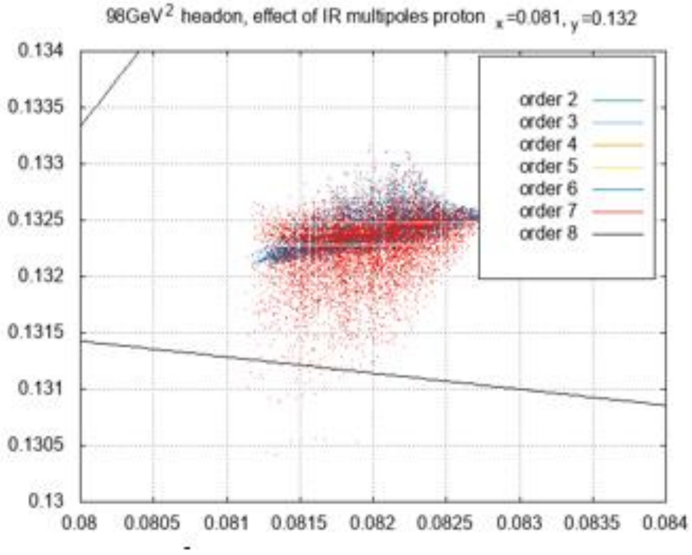


Adding effect of multipoles in IR quads.

- Implemented as multipole kicks $\frac{\pi}{2}$ apart from the IP. Modified BB3D and introduced expansions up to do-decapole. Used multipole values from LHC studies.







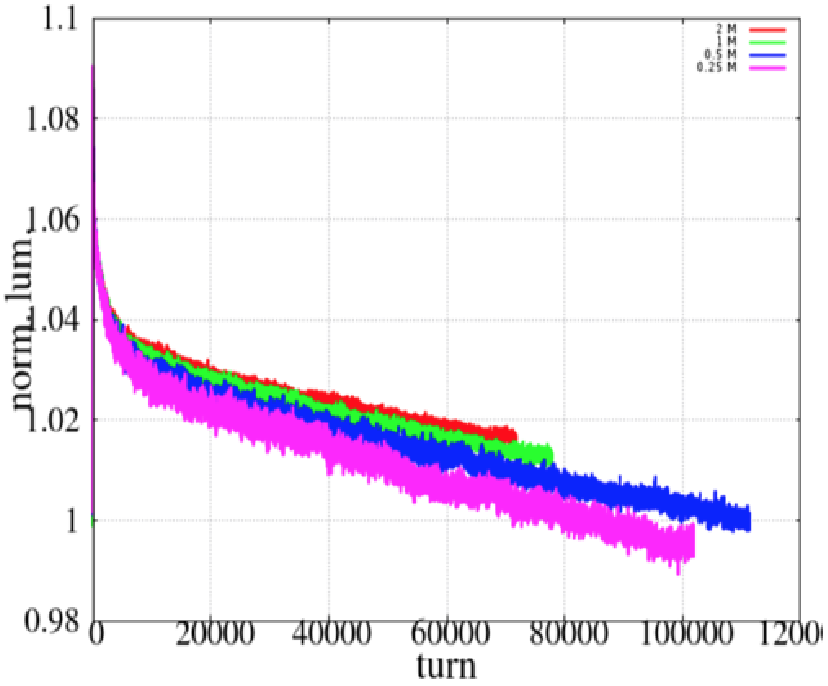
Jefferson Lab

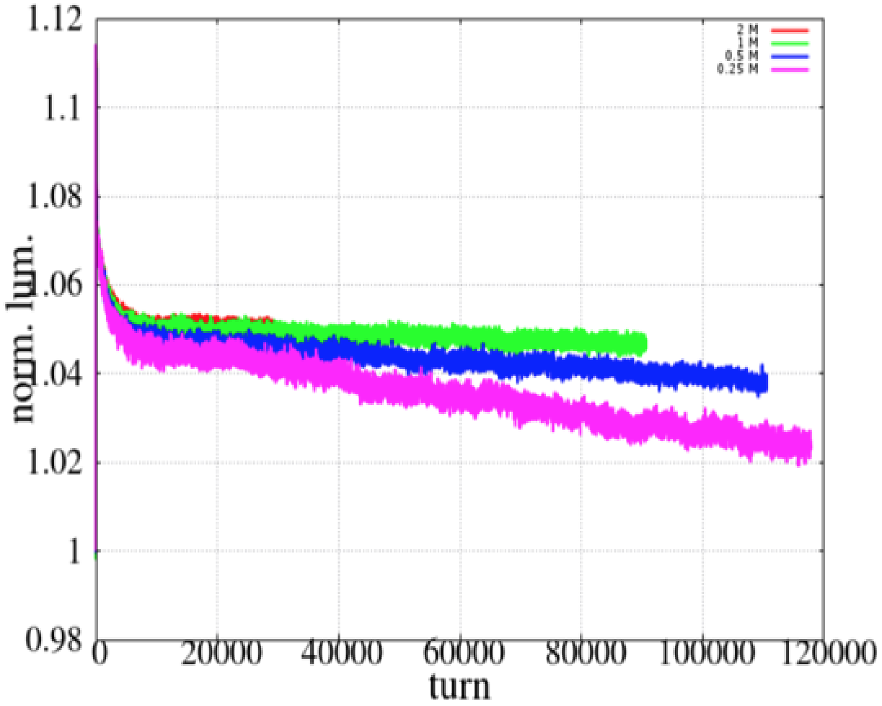
Studies of Crab Crossing Dynamics

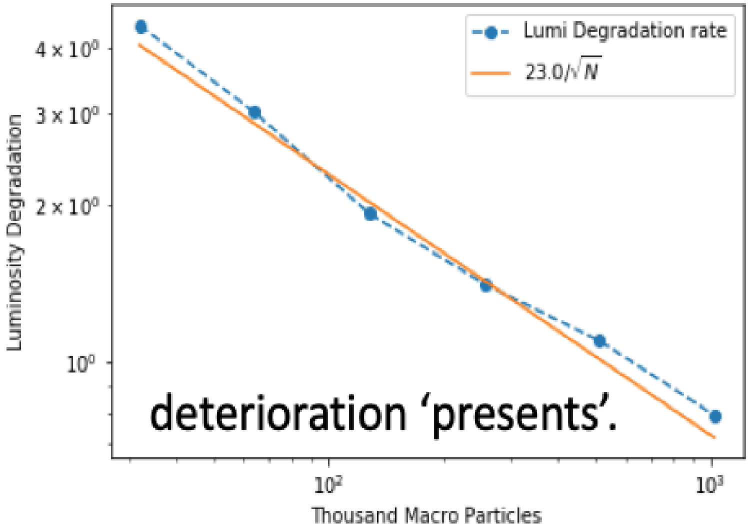
Y. Hao

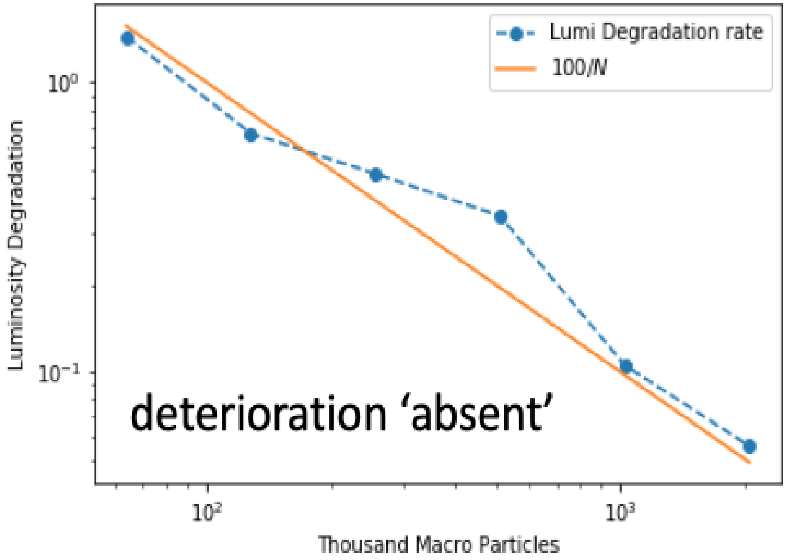
- Key beam dynamics topic for both EIC Designs, which is to answer:
  - Beam-beam parameter 0.015 (p) + 0.1 (e) is achievable under large crossing angle (~25-50 mrad), and long proton bunch ($k_c\sigma_{p,z}$~0.5)
  - Need combination of simulation tools to get clear understandings
    - Self-consistent solver with reduced numerical noise (Task 1.1)
    - Simplified models (Soft-Gaussian) + frequency maps to identify root causes
    - Combination of weak-strong and strong-strong simulation
  - Topics to be studied
    - Deterioration due to possible synchro-beta resonances (Task 1.2, Year 1 and Year 2)
    - Non-ideal settings for crab cavities and effect of cavity noise (Task 1.3/1.4, Year 2)

Finite Bunch Length Effect









This hints us, the some resonance may exist and make the scaling more complicated than a random walk

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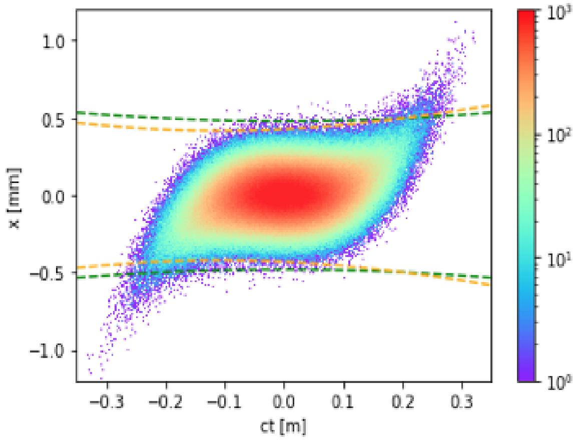
Dependence of Degradation Rate

$$\frac{dL}{dt} \sim f(\nu_{p,x}, \nu_{p,z}, \nu_{e,x}, \xi_e, \xi_p)$$

Resonance Strength



For fixed geometry, crab cavity and bunch length, the luminosity degradation is complicated.

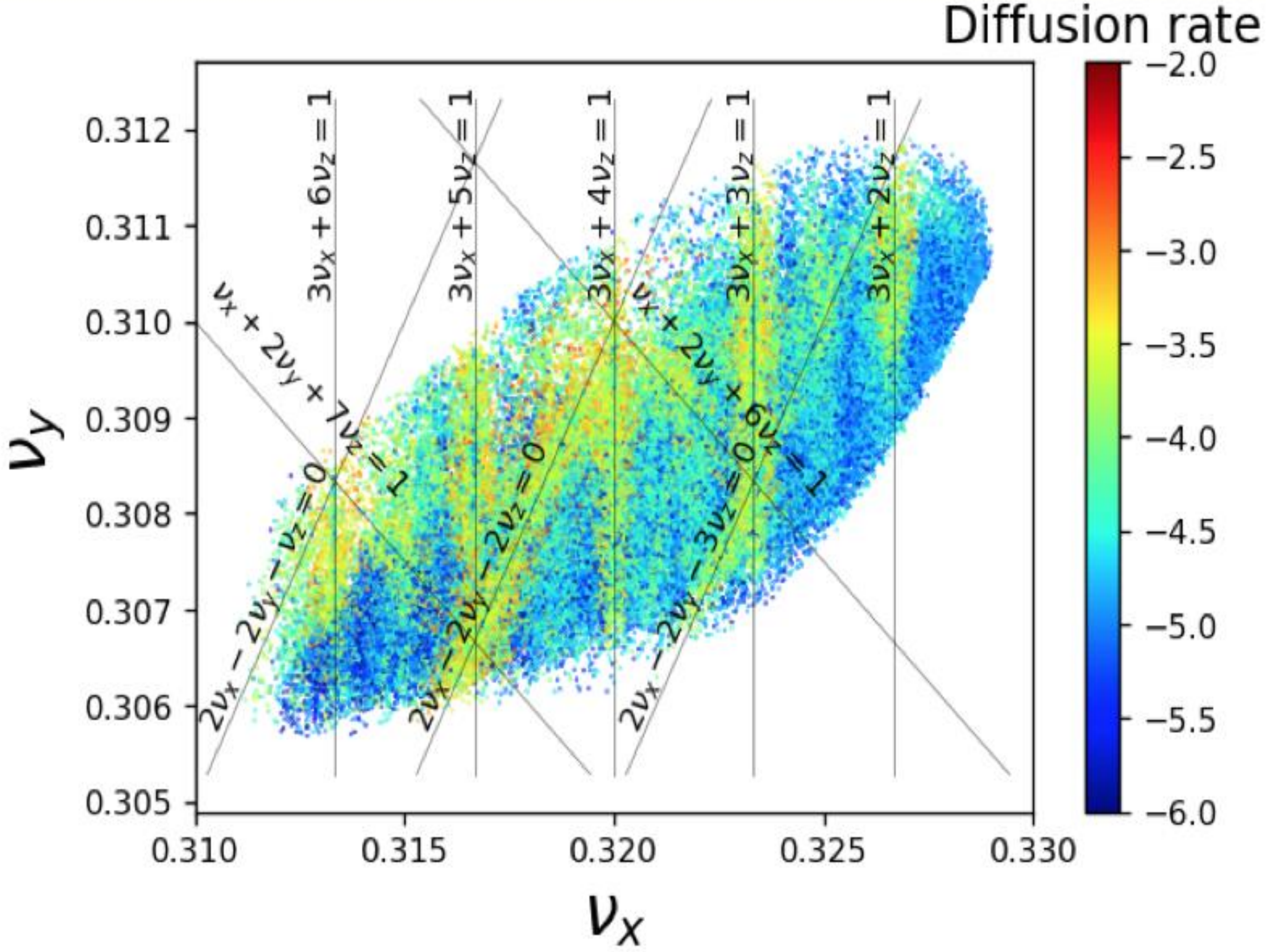
Simulations not only reveal the problem, but also provide the hint of root cause and possible remedies.

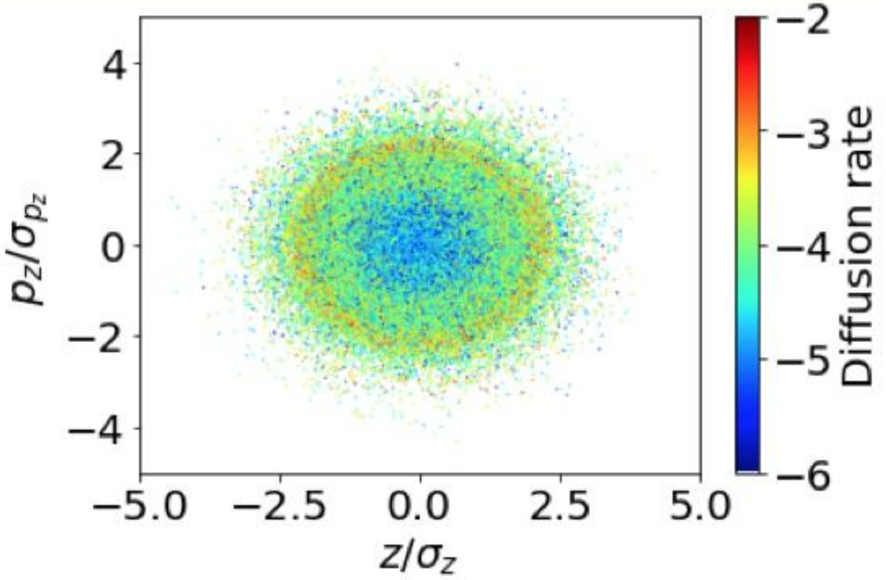
Modifies the equilibrium beam size, $\sigma_{e,x} < \sigma_{p,x}$

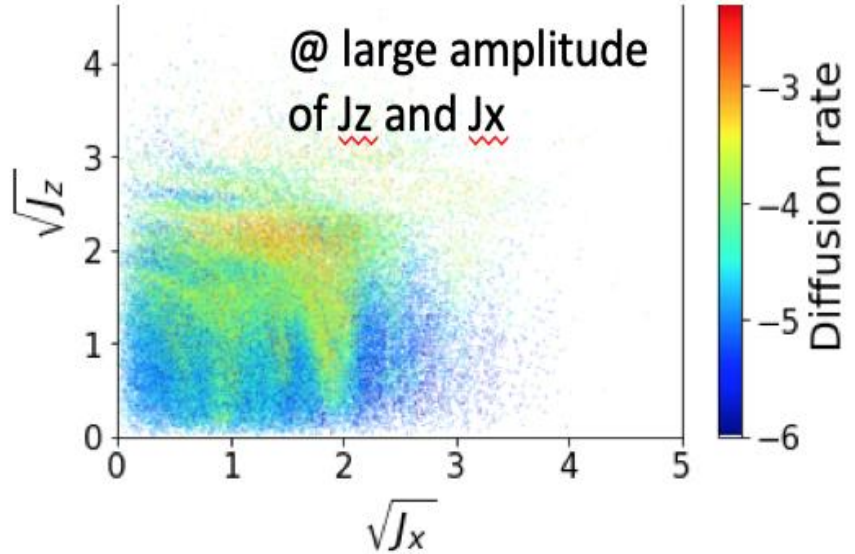
Resonance condition:

$$m\nu_{p,x} + n\nu_{p,z} = k$$

Diffusions



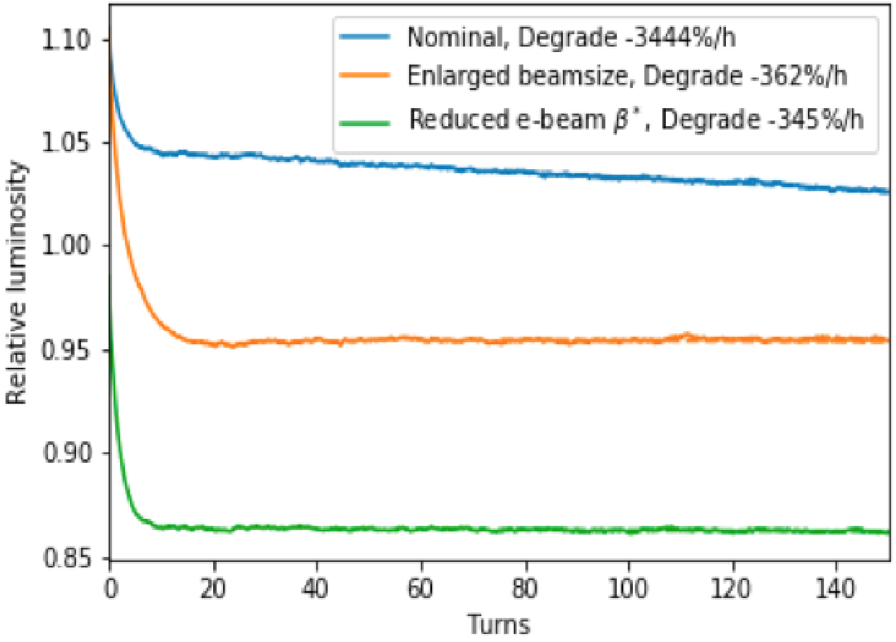




Separation of vertical lines indicates synchro-betatron resonances

Possible Remedies

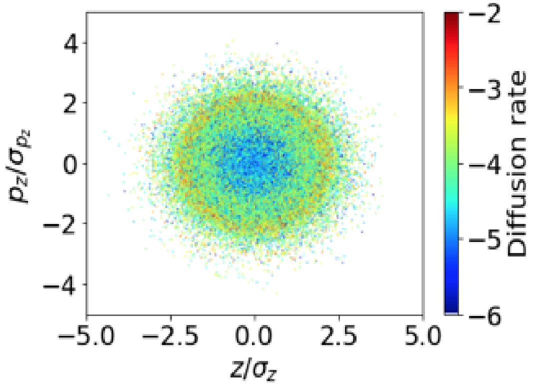
- Possible Remedies
  - Reduce the beam-beam parameter of ion beam (large impact to luminosity)
  - Change the electron beam size to match the 'crab tail' of proton beam better. (smaller impact to luminosity)
  - Still under investigation. Based on design needs.

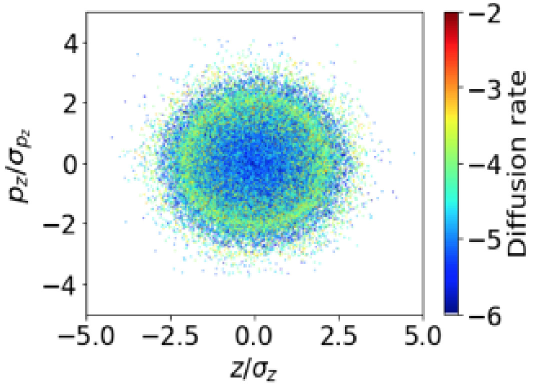


Change e-beam size,

10X reduction in degradation rate.

Works in time scale of 1s, Need careful extrapolation to hour level, with proper simulation tool





Achieved Deliverables and Schedule

Year 1				Year 2			
Qtr 1	Qtr 2	Qtr 3	Qtr 4	Qtr 1	Qtr 2	Qtr 3	Qtr 4
Task 1.1, 1.2, 1.3 (BNL, MSU, JLab)							
Task 1.4 (LBNL, MSU)				Task 3.1 (LBNL) Task 3.2, 3.3 (BNL)			
Task 2.1, 2.2, 2.3 (LBNL, BNL) Task 2.4 (BNL, JLab)				Task 4.1 (LBNL), Task 4.2, 4.3, 4.4 (JLab)			

Status	Task	Description
✓	Task 1.1	Code modifications to BeamBeam3D to reduce numeric noises
✓	Task 1.2	Further study of the beam-beam induced synchro-betatron resonance
✓	Task 2.1	Replace the linear ring map by a nonlinear map to up to a certain order
✓	Task 2.2	Implement high order non-linear field errors in IRs in BB3D
✓	Task 2.3	Implement real RF cavities for additional damping control in BB3D
✓	Task 2.4	Integrate and test all nonlinear tracking implementations inBB3D

Summary of Expenditures by Fiscal Year

BNL

	FY 2018	FY 2019	FY2020
a) Funds allocated	366,000	664,583	
b) Actual costs to date	67,417	346,273	
c) Uncosted commitments	0	0	
d) Uncommitted funds (d=a-b-c)	298,583	318,310	

JLAB

	FY 2018	FY2019	Total
a) Funds allocated	218,000	218,000	436,000
b) Actual costs to date	26,307	187,043	213,350
c) Uncosted commitments		18,812	18,812
d) Uncommitted funds (d=a-b-c)	191,693	205,855	232,162

LBNL

	FY 2019	FY 20XX	FY 20XX
a) Funds allocated	165,000	165,000	
b) Actual costs to date	107,000	140,000	
c) Uncosted commitments	40,000	25,000	
d) Uncommitted funds (d=a-b-c)	18,000	165,000	

MSU

	FY 2019	FY 2020	FY 20XX
a) Funds allocated	120,000	120,000	
b) Actual costs to date	55,000		
c) Uncosted commitments			
d) Uncommitted funds (d=a-b-c)	65,000	120,000	

Unit: $

Outlook for 2020

<table>
<tr><th colspan="4">Year 1</th><th colspan="4">Year 2</th></tr>
<tr><th>Qtr 1</th><th>Qtr 2</th><th>Qtr 3</th><th>Qtr 4</th><th>Qtr 1</th><th>Qtr 2</th><th>Qtr 3</th><th>Qtr 4</th></tr>
<tr><td colspan="8">Task 1.1, 1.2, 1.3 (BNL, MSU, JLab)</td></tr>
<tr><td colspan="4">Task 1.4 (LBNL, MSU)</td><td colspan="4">Task 3.1 (LBNL) Task 3.2, 3.3 (BNL)</td></tr>
<tr><td colspan="4">Task 2.1, 2.2, 2.3 (LBNL, BNL) Task 2.4 (BNL, JLab)</td><td colspan="4">Task 4.1 (LBNL), Task 4.2, 4.3, 4.4 (JLab)</td></tr>
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In FY 2020, or Year of this project, we will deliver:

1) **Task 3: code development to study bunch swap-out effects in eRHIC**
2) **Task 4: code development to study gear-change beam-beam in JLEIC**
3) **Task 1: continue beam dynamics study, Task 1.3, Task 1.4**

Concerns:

Since JLEIC dropped gear-change BB interaction scheme, JLAB PI will send request to Dr. Farkhondeh to rescope Task 4 in this project.

Summary

- We joined expertise from BNL, JLAB, LBNL and MSU to address four challenging items related to beam-beam interaction in next generation EIC. **We reached all milestones and goals in Year 1.**
- We implemented spectral method Poisson solver and nonlinear particle tracking methods in strong-strong simulation code BeamBeam3D.
- We made progress in investigating the physics behind the proton beam emittance growth and luminosity degradation.
- **We are looking forwards to the same success in Year 2 of this project.**