# ECE 5325/6325: Wireless Communication Systems
## Lecture Notes, Spring 2010

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**Lecture 6**

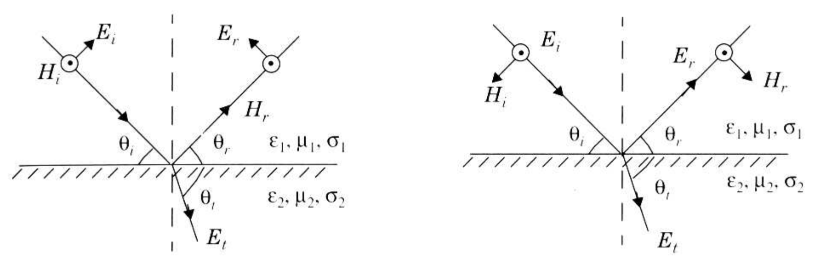
Today: (1) Reflection (2) Two-ray model (3) Cellular Large Scale Path Loss Models

- Reading for today's lecture: 4.5, 4.6, 4.10. For Tue: Haykin/Moher handout (2.9-2.11), get from WebCT.
- Homework: Please number questions as numbered on assignment, and turn in solution pages in order.
- Exam 1 is Tue, Feb 9. Please be prepared.

## 1 Reflection and Transmission

There are electric and magnetic waves that serve to propagate radio energy. The electric waves can be represented as a sum of two orthogonal *polarization* components, for example, vertical and horizontal, or left-hand and right-hand circular. What happens when these two components of the electric field hit the boundary between two different dielectric media?

We talk about the plane of incidence, that is, the plane containing the direction of travel of the waves (incident, reflected, and transmitted), and perpendicular to the surface (plane where the two media meet). See Figure 4.4 on page 115, reproduced in Fig 1.




(a) E-field in the plane of incidence (b) E-field normal to the plane of incidence

**Figure 4.4** Geometry for calculating the reflection coefficients between two dielectrics.

Figure 1: Figure 4.4 from Rappaport.

Notes about the notation from Rappaport:

- "Parallel" refers to the E-field having direction parallel to the plane of incidence (as in Figure 4.4(a)); "perpendicular" means perpendicular (normal) to the plane of incidence (as in Figure 4.4(b)).

- Use subscripts $i$, $r$, and $t$ to refer to the incident, reflected, and tranmitted field.
- $\epsilon_1$, $\epsilon_2$, is the permittivity of medium 1 and 2. (units Farads/meter) (Note: F = sec / $\Omega$)
- $\mu_1$, $\mu_2$ is the permeability of medium 1 and 2. (units Henries/meter) (Note: H = $\Omega$ sec )
- $\sigma_1$, $\sigma_2$ is the conductance of medium 1 and 2 (units Siemens/meter). (Note: S = 1 / $\Omega$)
- The complex dialectric constant is $\epsilon = \epsilon_0 \epsilon_r - j\epsilon'$ where $\epsilon_0 = 8.85 \times 10^{-12}$ F/m is free space permittivity, $j = \sqrt{-1}$, and $\epsilon' = \frac{\sigma}{2\pi f}$, and $\epsilon_r$ is the relative permittivity. Don't get confused by the subscripts in this equation.
- A material is a good conductor when $\sigma_k > f\epsilon_k$.
- The intrinsic impedance of medium $k$ is $\sqrt{\mu_k/\epsilon_k}$.

Then the reflection coefficients are given by

$$\begin{aligned} \Gamma_\parallel \triangleq \frac{E_r}{E_i} &= \frac{\eta_2 \sin\theta_t - \eta_1 \sin\theta_i}{\eta_2 \sin\theta_t + \eta_1 \sin\theta_i} \\ \Gamma_\perp \triangleq \frac{E_r}{E_i} &= \frac{\eta_2 \sin\theta_i - \eta_1 \sin\theta_t}{\eta_2 \sin\theta_t + \eta_1 \sin\theta_i} \end{aligned} \tag{1}$$

where $\theta_t$ is determined by *Snell's Law*:

$$\sqrt{\mu_1\epsilon_1}\sin(90^o - \theta_i) = \sqrt{\mu_2\epsilon_2}\sin(90^o - \theta_t) \tag{2}$$

Also, the angle of incidence is equal to the angle of reflection:

$$\theta_i = \theta_r$$

Finally, the reflected and transmitted field strengths are:

$$\begin{aligned} E_r &= \Gamma E_i \\ E_t &= (1+\Gamma)E_i \end{aligned}$$

where you chose $\Gamma$ based on the polarization of the incident E-field, *i.e.*, use either $\Gamma_\parallel$ or $\Gamma_\perp$.

There is a special case of (1) when the first medium is free space (or approximately, air) and $\mu_1 = \mu_2$. These two conditions are the case for most of the things we care about. In this case you can show (good HW problem!) that

$$\begin{aligned} \Gamma_\parallel &= \frac{-\epsilon_r \sin\theta_i + \sqrt{\epsilon_r - \cos^2\theta_i}}{\epsilon_r \sin\theta_i + \sqrt{\epsilon_r - \cos^2\theta_i}} \\ \Gamma_\perp &= \frac{\sin\theta_i - \sqrt{\epsilon_r - \cos^2\theta_i}}{\sin\theta_i + \sqrt{\epsilon_r - \cos^2\theta_i}} \end{aligned} \tag{3}$$

See Figure 4.6 on page 118 of Rappaport. At some angle $\theta_i$, there is no reflection of the parallel E-field from (3). This angle is called the "Brewster angle", which is given by

$$\sin\theta_B = \sqrt{\frac{\epsilon_1}{\epsilon_1 + \epsilon_2}}$$

When medium 1 is free space, and $\epsilon_2 = \epsilon_0 \epsilon_r$,

$$\sin\theta_B = \frac{1}{\sqrt{1+\epsilon_r}}$$

This is the same as Equation 4.28 in Rappaport.

Note that as $\theta_i \rightarrow 0$, $\Gamma_\parallel \rightarrow 1$ and $\Gamma_\perp \rightarrow -1$.

Also, for perfect conductors (as described in Section 4.5.3), we also have $\Gamma_\parallel = 1$ and $\Gamma_\perp = -1$.

**Example: Reflection from ground**
Find the reflection coefficients for typical ground at an incident angle of 15 degrees at 100 MHz.
**Solution:** Assume free space is medium 1 and that 'typical ground' has $\epsilon_r = 15$. Note $\sin 15^o = 0.259$, and $\cos 15^o = 0.933$, so from (3),

$$\begin{aligned}\Gamma_\parallel &= \frac{-15(0.259) + \sqrt{15 - 0.933}}{15(0.259) + \sqrt{15 - 0.933}} = -0.0176 \\ \Gamma_\perp &= \frac{0.259 - \sqrt{15 - 0.933}}{0.259 + \sqrt{15 - 0.933}} = 0.871\end{aligned}$$

# 2 Two-Ray (Ground Reflection) Model

Section 4.6 in Rappaport develops a theoretical model for propagation slightly better than the free space assumption. This model includes not just one path, but also another path that reflects off of the ground. "The World Is Flat", if you will. The model isn't hard to develop, and provides an important theoretical underpinning to the multiple breakpoint model we covered in lecture 5.

Remember, *powers of multipath DON'T add together*. Only voltages or field strength of multipath actually add together. The voltage on the antenna is proportional to the electric field at the antenna position. So let's talk about adding electric fields.

See Figure 4.7 on page 121 of Rappaport.

## 2.1 Direct Path

Recall that the electric field magnitude decays as $1/d$ in free space. So, similar to how we wrote the received power with a reference distance $d_0$, we write the E-field strength as the E-field strength at a reference distance, multiplied by $d_0/d$, for a path (distance of travel for waves) length $d$. Also, assume the signal is a simple sinusoid at the carrier frequency, $f_c$. So

$$E(d,t) = E_0 \frac{d_0}{d} \cos\left(2\pi f_c \left(t - \frac{d}{c}\right)\right) \tag{4}$$

For the LOS path, given a distance along the ground of $L$, antenna heights $h_t$ and $h_r$ at the TX and RX, respectively, the $d = \sqrt{L^2 + (h_t - h_r)^2}$. So

$$E_{LOS} = E_0 \frac{d_0}{\sqrt{L^2 + (h_t - h_r)^2}} \cos\left(2\pi f_c \left(t - \frac{\sqrt{L^2 + (h_t - h_r)^2}}{c}\right)\right) \tag{5}$$

### 2.2 Reflected Path

Let's assume that $L$ is very long compared to the antenna heights. So, the angle of incidence is approximately 0. In this case the reflection coefficient (assume perpendicular polarization) is -1. The reflected path travels longer than the direct path, for total length $\sqrt{L^2 + (h_t + h_r)^2}$ (one can use the "method of images" to show this). Then

$$E_g = -E_0 \frac{d_0}{\sqrt{L^2 + (h_t + h_r)^2}} \cos\left(2\pi f_c \left(t - \frac{\sqrt{L^2 + (h_t + h_r)^2}}{c}\right)\right) \tag{6}$$

### 2.3 Total Two-Ray E-Field

We are interested in the magnitude of the total E-field, $E_{TOT} = E_{LOS} + E_g$, that is, the quantity that multiplies the $\cos(2\pi f_c t)$ term. Using trig identities, and this approximation:

$$\Delta = \sqrt{L^2 + (h_t + h_r)^2} - \sqrt{L^2 + (h_t - h_r)^2} \approx \frac{2h_t h_r}{L}$$

we can show that

$$|E_{TOT}| \approx 2E_0 \frac{d_0}{d} \sin\left(\frac{2\pi f_c}{c} \frac{2h_t h_r}{L}\right)$$

But at large $L$, the argument of the sin is approximately 0, and the $\sin x \approx x$. This is when $x < 0.3$ radians, which in our equation, means that for

$$L > \frac{20 h_t h_r f_c}{c}$$

we can use this approximation (also noting that $L \approx d$):

$$|E_{TOT}| \approx 2E_0 \frac{d_0}{d} \left(\frac{2\pi f_c}{c} \frac{2h_t h_r}{d}\right) = \frac{\text{const}}{d^2}$$

This means that the power decays as $1/d^4$! See Figure 2.

In summary, when there are two paths, one direct and one ground reflection, the theoretical models show behavior that has two different path loss exponents, $1/d^2$ for $d$ less than a threshold, and $1/d^4$ for $d$ above the threshold. This matches what we've observed from measurements and presented as the empirical "multiple breakpoint" model.

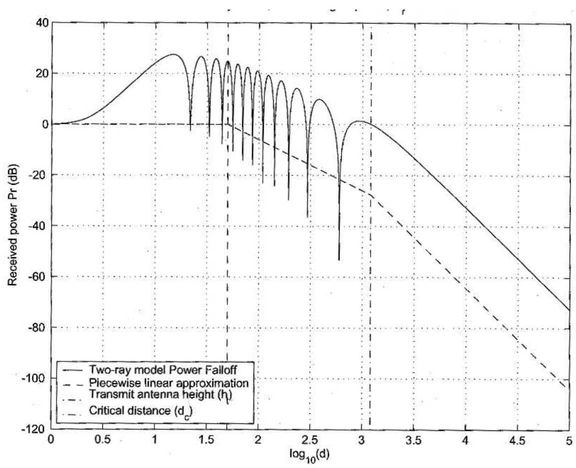



Figure 2: Received power as a function of log distance in two-ray model, Figure 2.5 from Andrea Goldsmith, "Wireless Communications", Cambridge University Press, 2005.

However, a note: this is just a theoretical model. Typical cellular or indoor channels do not have just two paths. One of the 6325 assignments is to consider $T$-ray model, for $T > 2$. For example, if you had a ceiling reflection as a 3rd path. Or a ceiling-floor two-bounce path as a 4th ray. As $T$ goes up, you don't see the $1/d^4$ behavior.

# 3 Cellular Large Scale Path Loss Models

Now, let's consider some empirical models that are used in cellular systems. These types of models are better than free space or the log-distance model, and have been designed to better fit measured data. Note that there are many "modifications" that people continually add to these models to better fit their environments of interest.

## 3.1 Okumura-Hata Models

The median (50th percentile) propagation loss at frequency $f$, distance $d$, and TX and RX antenna heights $h_{te}$ and $h_{re}$, is given in the Okumura model as,

$$L_{50}(dB) = L_F(f,d) + A_{mu}(f,d) - G(h_{te}) - G(h_{re}) - G_{AREA}$$

where all terms are in dB even though Rappaport does not explicitly denote thenm as such, and where $L_F(f, d)$ is the free space path loss in dB at distance $d$ and frequency $f$, $A_{mu}$ is the median loss compared to free space (found from Table 4.23), $G_{AREA}$ is a factor due to the type of environment (open, quasi open, or suburban, as given in Figure 4.24), and

$$
\begin{aligned}
G(h_{te}) &= 20 \log_{10} \frac{h_{te}}{200\text{m}}, \quad \text{for } 30\text{m} < h_{te} < 1000\text{m} \\
G(h_{re}) &= \begin{cases} 10 \log_{10} \frac{h_{re}}{3\text{m}}, & 0\text{m} < h_{re} < 3\text{m} \\ 20 \log_{10} \frac{h_{re}}{3\text{m}}, & 3\text{m} < h_{re} < 10\text{m} \end{cases}
\end{aligned}
$$

**Example: Rappaport Example 4.10, page 153**

The Hata model is largely function-fitting, expressing a formula that captures most of the results of Okumura.

Again, these are based on large sets of measurements. There are many adjustments to this model, particularly for PCS (the COST-231 model for 1900 MHz), for microcells, etc. In general, these models have a standard deviation of about 10 or more dB when compared to the actual measurements. So, it is typically useful to obtain more accurate means of predicting received powers, in particular, software that predicts based on the geometry of a city or area, and measurements which give feedback to the model to allow it to (to some extent) correct its errors).

## Discussion

What are some of the assumptions made in these models that we have not talked about, or in particular, that may be incorrect?