Fixed Intercept and Random Coefficient (FIRC) models

To illustrate the operation of the program, we use the data from the Tennessee's Student/Teacher Achievement Ratio study project (STAR) (Shin & Raudenbush, 2011), which was a statewide effort to study the effect of reduced class size on student academic performance in Tennessee. Windows model execution is illustrated.

We will first look at the effects of reduced of class size using a two-level model with students nested within school.

HLM2 Statistical package input

We will use SPSS file input in our example. There are two data files for the HLM2 FIRC analysis, one at the student level, and one at the school site level.

Level-1 file. The level-1 file, **STAR1.SAV** has math and reading proficiency data as well as the type of class of 5,786 students participated in STAR. The variables are:

- MATH a math test in IRT scale score metric
- CLASSTYP an indicator of class type (1 = small with 13-17 students, 0 = other)

Level-2 file. The level-2 file, **STAR3.sav** has data collected from 79 schools that the students attended. The variable is:

- SIZE school size

Using HLM2, the MDM file **STARHM2.MDM** is created.

Executing analyses based on the MDM File

We first illustrate a) a random intercept and a fixed treatment coefficient model, then b) a fixed intercepts and a fixed treatment coefficient model, finally followed by c) a fixed intercepts and a random treatment coefficient model. We summarize the results at the end of this section.

A random intercept and a fixed treatment coefficient model

The command file, **STARHM2A.HLM**, contains the model specification input responses for the fixed intercepts and a fixed treatment coefficient model. Figure 1 displays the model specified.

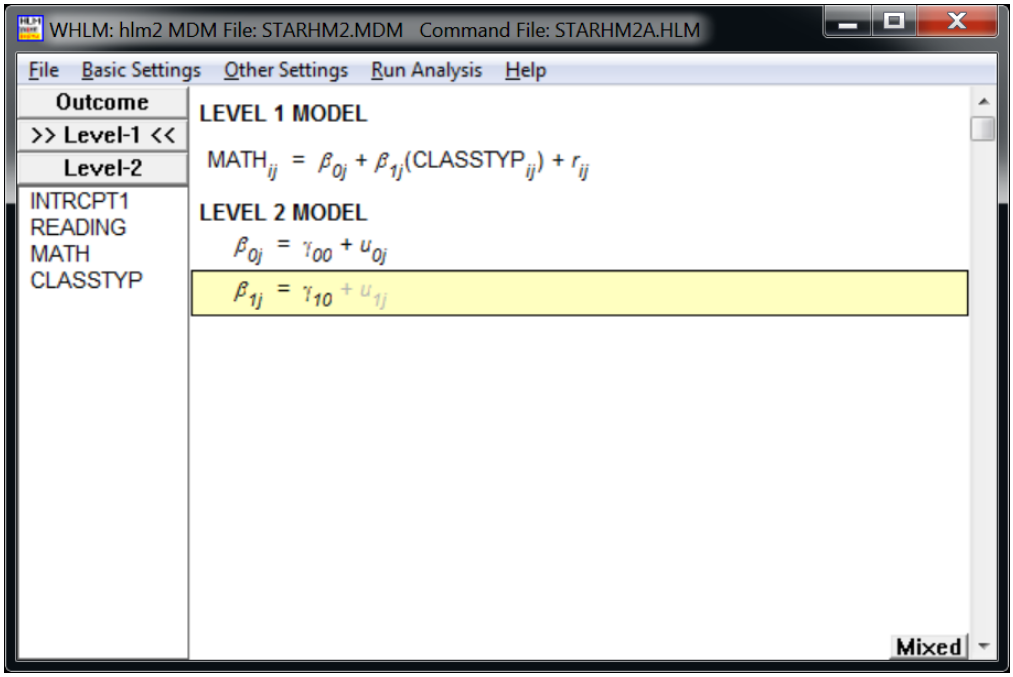

Figure 1 The random intercept and a fixed treatment coefficient model specification for the STARHM2 example

The results of the analysis are given below.

Problem Title: Random Intercept and A Fixed Treatment Coefficient Model

The data source for this run = STARHM2.MDM
The command file for this run =STARHM2A.HLM
Output file name = STARHM2A.HTML
The maximum number of level-1 units = 5786
The maximum number of level-2 units = 79
The maximum number of iterations = 100
Method of estimation: full maximum likelihood

The outcome variable is MATH

Summary of the model specified
Step 2 model

Level-1 Model
$MATH_{ij} = \beta_{0j} + \beta_{1j}*(CLASSTYP_{ij}) + r_{ij}$

Level-2 Model
β0j = γ00 + u0j
β1j = γ10

Mixed Model
$MATH_{ij} = \gamma_{00}$
$+ \gamma_{10}*CLASSTYP_{ij} + u_{0j} + r_{ij}$

Final Results - Iteration 3

Iterations stopped due to small change in likelihood function

σ^2 = 1804.30900

Standard error of σ^2 = 33.77701

τ

INTRCPT1,β_0	458.63366

Standard error of τ

INTRCPT1,β_0	77.25120

Approximate confidence intervals of tau variances
INTRCPT1 : (327.960,641.373)

Random level-1 coefficient	Reliability estimate
INTRCPT1,β_0	0.945

The value of the log-likelihood function at iteration 3 = -3.001734E+004

Final estimation of fixed effects:

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	483.000600	2.506586	192.693	78	<0.001
For CLASSTYP slope, β_1					
INTRCPT2, γ_{10}	9.087321	1.232934	7.370	5706	<0.001

Final estimation of fixed effects
(with robust standard errors)

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	483.000600	2.615873	184.642	78	<0.001
For CLASSTYP slope, β_1					
INTRCPT2, γ_{10}	9.087321	2.340424	3.883	5706	<0.001

Final estimation of variance components

Random Effect	Standard Deviation	Variance Component	*d.f.*	χ^2	*p*-value
INTRCPT1, u_0	21.41573	458.63366	78	1540.50368	<0.001
level-1, *r*	42.47716	1804.30900			

Statistics for the current model
Deviance = 60034.676525
Number of estimated parameters = 4

A fixed intercepts and a fixed treatment coefficient model

The command file, **STARHM2B.HLM**, contains the model specification input responses for the fixed intercepts and a fixed treatment coefficient model. A conventional way to specify such model is to include $J - 1$ school site dummy variables into the model. HLM2 offers a simple step to set up the model.

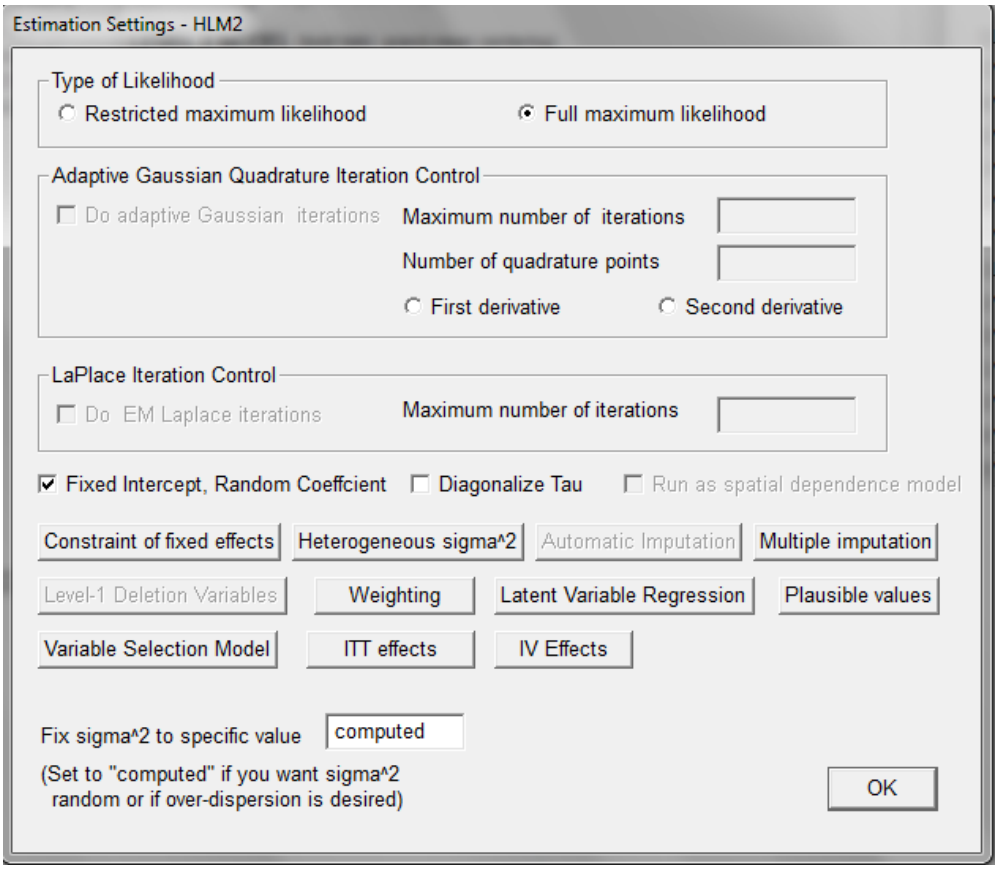

Figure 2 Estimation settings – HLM2 dialog box

After clicking **OK**, the fixed intercepts and fixed treatment coefficient will be displayed, as shown in Figure 3. Note that the level 2 model for β_{0j} is a no-intercept model.

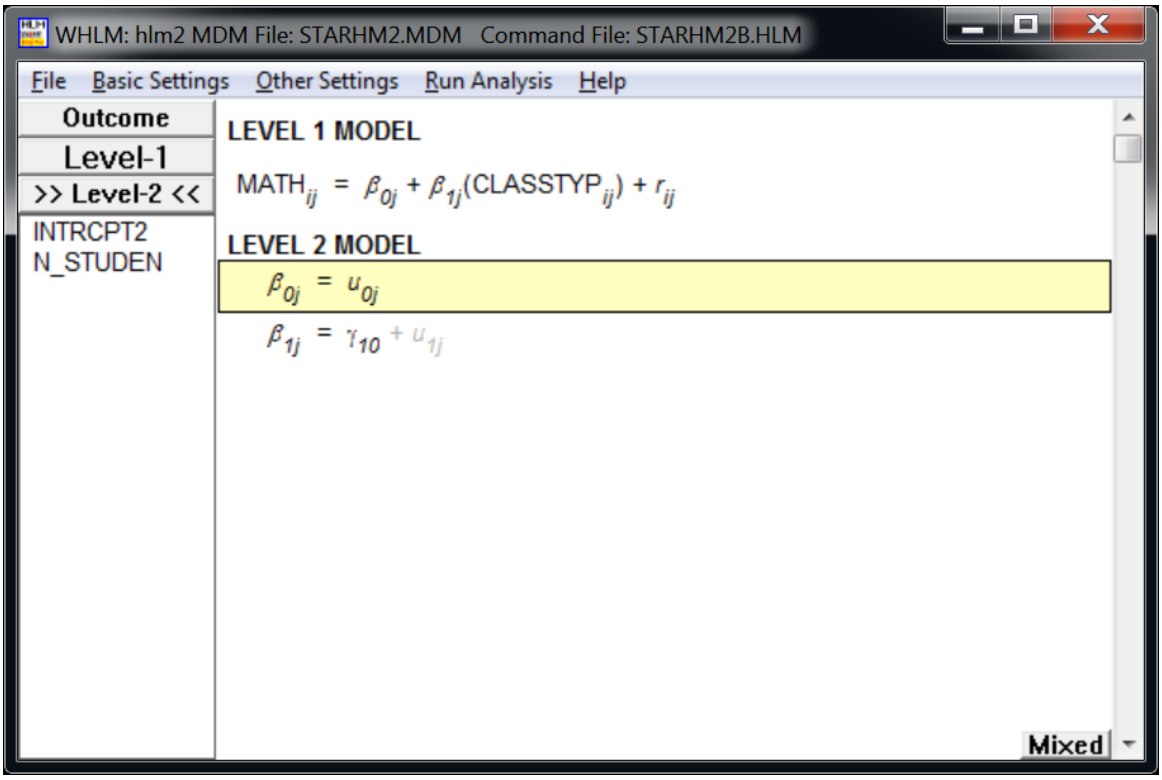

Figure 3 The fixed intercepts and a fixed coefficient model specification for the STARHM2B example

Here is the output:

Specifications for this HLM2 run

Problem Title: Random Intercept and Fixed Coefficient Model

The data source for this run = STARHM2.MDM
The command file for this run = STARHM2B.HLM
Output file name = hlm2.html
The maximum number of level-1 units = 5786
The maximum number of level-2 units = 79
The maximum number of iterations = 100
Method of estimation: full maximum likelihood

The outcome variable is MATH

Summary of the model specified

Step 2 model

Level-1 Model

$MATH_{ij} = \beta_{0j} + \beta_{1j}*(CLASS\_TY_{ij}) + r_{ij}$

Level-2 Model

$\beta_{0j} = u_{0j}$

$\beta_{1j} = \gamma_{10}$

Mixed Model

$MATH_{ij} =$

$+ \gamma_{10}*CLASSTYP_{ij} + u_{0j} + r_{ij}$

Final Results - Iteration 6

Iterations stopped due to small change in likelihood function

$\sigma^2 = 1804.31836$

The value of the log-likelihood function at iteration 6 = -2.973353E+004

Final estimation of fixed effects:

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For CLASSTYP slope, β_1					
INTRCPT2, γ_{10}	9.127153	1.233739	7.398	5706	<0.001

Final estimation of fixed effects (with robust standard errors)

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For CLASSTYP slope, β_1					
INTRCPT2, γ_{10}	9.127153	2.343758	3.894	5706	<0.001

Final estimation of variance components

Random Effect	Standard Deviation	Variance Component	d.f.	χ^2	*p*-value
level-1, *r*	42.47727	1804.31836			

Statistics for the current model
Deviance = 59467.054530
Number of estimated parameters = 2

A fixed intercepts and a random treatment coefficient model

The command file, **STARHM2C.HLM**, contains the model specification input responses for the fixed intercepts and a random treatment coefficient model. Figure 4 displays the model specified.

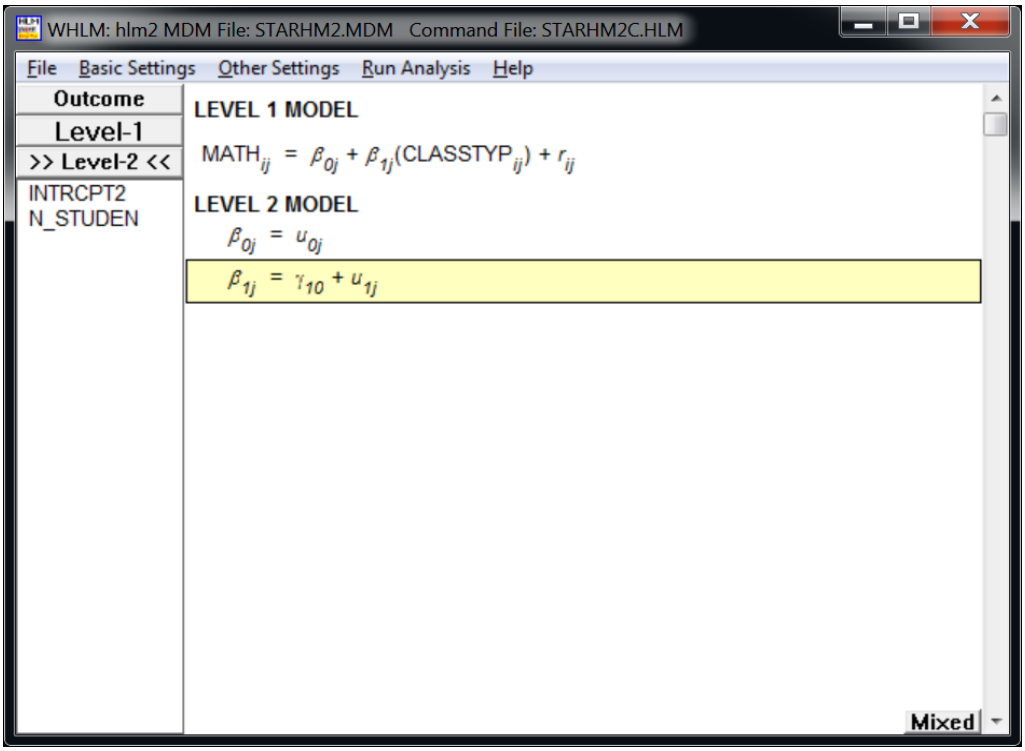

Figure 4 The fixed intercepts and a random treatment coefficient model specification for the STARHM2C example

Here is the output:

Problem Title: Fixed Intercepts and A Random Treatment Coefficient Model

The data source for this run = STARHM2.MDM
The command file for this run = STARHM2C.HLM
Output file name =STARHM2C.HTML
The maximum number of level-1 units = 5786
The maximum number of level-2 units = 79
The maximum number of iterations = 100
Method of estimation: full maximum likelihood

The outcome variable is MATH

Summary of the model specified

Step 2 model

Level-1 Model

$MATH_{ij} = \beta_{0j} + \beta_{1j}*(CLASSTYP_{ij}) + r_{ij}$

Level-2 Model

$\beta_{0j} = u_{0j}$
$\beta_{1j} = \gamma_{10} + u_{1j}$

Mixed Model

$MATH_{ij}$ =
+ γ_{10}*$CLASSTYP_{ij}$ + u_{0j} + u_{1j}*$CLASSTYP_{ij}$ + r_{ij}

Final Results - Iteration 11

Iterations stopped due to small change in likelihood function

σ^2 = 1742.81131

τ
CLASSTYP,β_1 301.75903

τ (as correlations)
CLASSTYP,β_1 1.000

Random level-1 coefficient	Reliability estimate
CLASSTYP,β_1	0.705

The value of the log-likelihood function at iteration 11 = -2.968392E+004

Final estimation of fixed effects:

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For CLASSTYP slope, β_1					
INTRCPT2, γ_{10}	8.538461	2.328306	3.667	78	<0.001

Final estimation of fixed effects (with robust standard errors)

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For CLASSTYP slope, β_1					
INTRCPT2, γ_{10}	8.538461	2.327321	3.669	78	<0.001

Final estimation of variance components

Random Effect	Standard Deviation	Variance Component	d.f.	χ^2	*p*-value
CLASSTYP slope, u_1	17.37121	301.75903	78	280.89921	<0.001
level-1, *r*	41.74699	1742.81131			

Statistics for the current model
Deviance = 59367.836357
Number of estimated parameters = 3

Summary of the Results

Table 1 summarizes the results for the three models.

Model Estimate	A Random Intercept and A Fixed Treatment Coefficient	Fixed Intercepts and a Fixed Treatment Coefficient	Fixed Intercepts and a Random Treatment Coefficient
Average Treatment Effect	9.087321	9.127153	8.538461
Model-Based Standard Error of Average Treatment Effect	2.340424	1.233739	2.328306
Robust Standard Error of Average Treatment Effect1	2.340424	2.343758	2.327321
Variance of Treatment Effect	NA	NA	301.75903 (χ^2 = 280.89921, *df* = 78, *p* < 0.001)

Table 1 Summary of the treatment estimates from the three models

The results of the FIRC models with a minimum of assumptions suggest that there is evidence of cross-site variation in the program impact.