TWO PORT NETWORKS

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Introduction:

A general network having two pairs of terminals, one labeled the "input terminals'' and the other the "output terminals,'' is a very important building block in electronic systems, communication systems, automatic control systems, transmission and distribution systems, or other systems in which an electrical signal or electric energy enters the input terminals, is acted upon by the network, and leaves via the output terminals. A pair of terminals at which a signal may enter or leave a network is also called a ***port,*** and a network like the above having two such pair of terminals is called a ***Two - port network.*** A general two-port network with terminal voltages and currents specified is shown in the figure below. In such networks the relation between the two voltages and the two currents can be described in six different ways resulting in six different systems of Parameters and in this chapter we will consider the most important four systems.

Impedance Parameters: Z parameters (open circuit impedance parameters)

We will assume that the two port networks that we will consider are composed of linear elements and contain no independent sources but dependent sources *are* permissible. We will consider the two-port network as shown in the figure below.

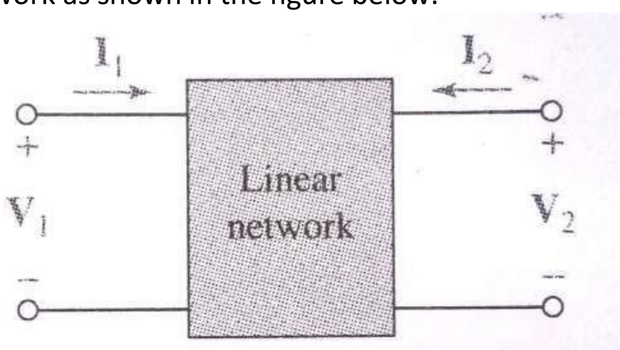


Fig: A general two-port network with terminal voltages and currents specified. The two-port network is composed of linear elements, possibly including dependent sources, but not containing any independent sources.

The voltage and current at the input terminals are $\mathbf{V_1}$ & $\mathbf{I_1}$, and $\mathbf{V_2}$ & $\mathbf{I_2}$ are voltage and current at the output port. The directions of $\mathbf{I_1}$ and $\mathbf{I_2}$ are both customarily selected as *into* the network at the upper conductors (and out at the lower conductors). Since the network is linear and contains no independent sources within it, $\mathbf{V_1}$ may be considered to be the superposition of two components, one caused by $\mathbf{I_1}$ and the other by $\mathbf{I_2}$. When the same argument is applied to $\mathbf{V_2}$, we get the set of equations

$$\mathbf{V_1 = Z_{11}I_1 + Z_{12}I_2}$$
$$\mathbf{V_2 = Z_{21}I_1 + Z_{22}I_2}$$

This set of equations can be expressed in matrix notation as

$$\begin{matrix} V_1 & Z_{11} & Z_{12} & I_1 \\ V_2 & Z_{21} & Z_{22} & I_2 \end{matrix}$$

And in much simpler form as

$$[V] = [Z][I]$$

Where **[V] ,[Z] and [I]**are Voltage, impedance and current matrices. The description of the **Z** parameters, defined in the above equations is obtained by setting each of the currents equal to zero as given below.

$$Z_{11} = V_1/I_1 \mid I_2=0$$
$$Z_{12} = V_1/I_2 \mid I_1=0$$
$$Z_{21} = V_2/I_1 \mid I_2=0$$
$$Z_{22} = V_2/I_2 \mid I_1=0$$

Thus ,Since zero current results from an open-circuit termination, the **Z** parameters are known as the ***Open-circuit Impedance parameters***. And more specifically Z_{11} & Z_{22} are called ***Driving point Impedances*** and Z_{12} & Z_{21} are called ***Reverse and Forward transfer impedances*** respectively.

A basic Z parameter equivalent circuit depicting the above defining equations is shown in the figure below.

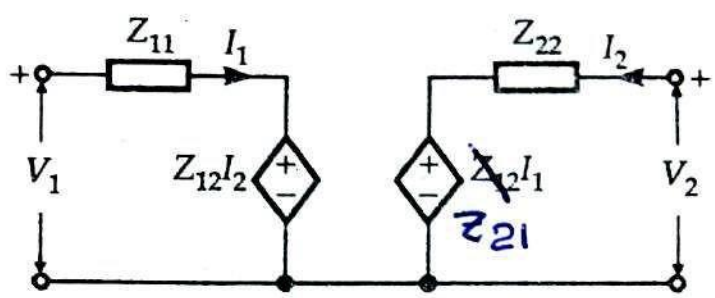


Fig: Z-Parameter equivalent circuit

Admittance parameters: (Y Parameters or Short circuit admittance parameters)

The same general two port network shown for **Z** parameters is applicable here also and is shown below.

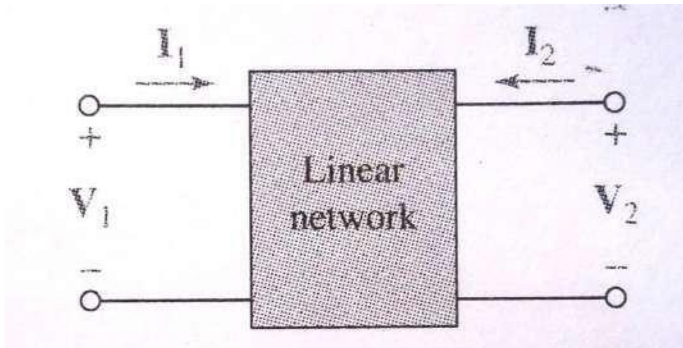


Fig: A general two-port network with terminal voltages and currents specified. The two-port network is composed of linear elements, possibly including dependent sources, but not containing any independent sources.

Since the network is linear and contains no independent sources within, on the same lines of **Z** parameters the defining equations for the Y parameters are given below. $\mathbf{I_1}$ and I_2 may be considered to be the superposition of two components, one caused by $\mathbf{V_1}$ and the other by $\mathbf{V_2}$ and then we get the set of equations defining the **Y** parameters.

$$\mathbf{I_1 = Y_{11}V_1 + Y_{12}V_2}$$
$$\mathbf{I_2 = Y_{21}V_1 + Y_{22}V_2}$$

where the **Y**s are no more than proportionality constants and their dimensions are A/V (Current/Voltage). Hence they are called the **Y** (or admittance) parameters. They are also defined in the matrix form given below.

$$\mathbf{\begin{matrix} I_1 \\ I_2 \end{matrix} \quad \begin{matrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{matrix} \quad \begin{matrix} V_1 \\ V_2 \end{matrix}}$$

And in much simpler form as

$$\mathbf{[I] = [Y][V]}$$

The individual Y parameters are defined on the same lines as Z parameters but by setting either of the voltages $\mathbf{V_1}$ and $\mathbf{V_2}$ as zero as given below.

The most informative way to attach a physical meaning to the **y** parameters is through a direct inspection of defining equations. The conditions which must be applied to the basic defining equations are very important. In the first equation for example; if we let $\mathbf{V_2}$ zero, then $\mathbf{Y_{11}}$ is given by the ratio of $\mathbf{I_1}$ to $\mathbf{V_1}$. We therefore describe $\mathbf{Y_{11}}$ as the admittance measured at the input terminals with the output terminals *short-circuited (*$\mathbf{V_2}$ = 0*)*. Each of the **Y** parameters may be described as a ***current-voltage*** ratio with either $\mathbf{V_1}$ = 0 (the input terminals short circuited) or $\mathbf{V_2}$ = 0 (the output terminals short-circuited):

$$Y_{11} = I_1/V_1 \text{ with } V_2 = 0$$

$$Y_{12} = I_1/V_2 \text{ with } V_1 = 0$$

$$Y_{21} = I_2/V_1 \text{ with } V_2 = 0$$
$$y_{22} = I_2/V_2 \text{ with } V_1 = 0$$

Because each parameter is an ***admittance*** which is obtained by short circuiting either the output or the input port, the **Y** parameters are known as the ***short-circuit admittance parameters.*** The specific name of $\mathbf{Y}_{11}$ is the ***short-circuit input admittance,*** $\mathbf{Y}_{22}$ is the ***short-circuit output admittance,*** and $\mathbf{Y}_{12}$ and $\mathbf{Y}_{21}$ are the ***short-circuit reverse and forward transfer admittances respectively.***

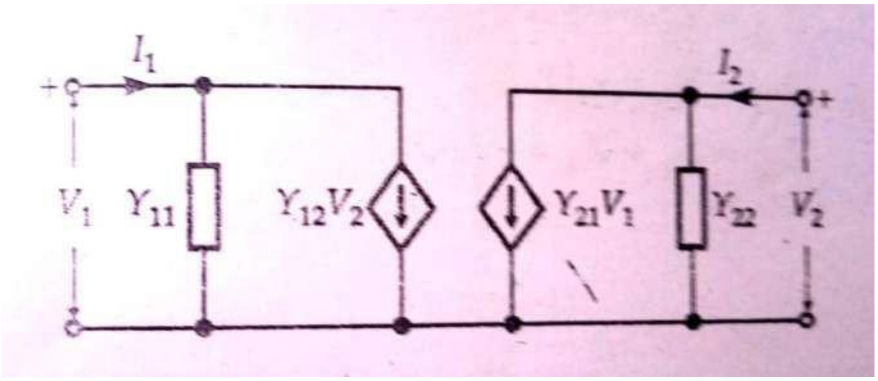


Fig: Y parameter equivalent circuit

Hybrid parameters: (h parameters)

h parameter representation is used widely in modeling of Electronic components and circuits particularly Transistors. Here both short circuit and open circuit conditions are utilized.
The hybrid parameters are defined by writing the pair of equations relating $\mathbf{V}_1$, $\mathbf{I}_1$, $\mathbf{V}_2$, and $\mathbf{I}_2$:

$$V_1 = h_{11}.I_1 + h_{12}.V_2$$
$$I_2 = h_{21}.I_1 + h_{22}.V_2$$

Or in matrix form :

$$\begin{matrix} V_1 \\ I2 \end{matrix} \quad h \quad \begin{matrix} I_1 \\ V_2 \end{matrix}$$

The nature of the parameters is made clear by first setting $\mathbf{V_2 = 0}$. Thus,

$\mathbf{h_{11} = V_1/I_1}$ **with** $\mathbf{V_2 = 0}$ = ***short-circuit input impedance***
$\mathbf{h_{21} = I_2/I_1}$ **with** $\mathbf{V_2 = 0}$ = ***short-circuit forward current gain***

Then letting $\mathbf{I_1 = 0}$, we obtain

$h_{12} = V_1/V_2$ with $I_1=0$ = ***open-circuit reverse voltage gain***
$h_{22} = I_2/V_2$ with $I_1=0$ = ***open-circuit output admittance***

Since the parameters represent an impedance, an admittance, a voltage gain, and a current gain, they are called the "hybrid'' parameters.
The subscript designations for these parameters are often simplified when they are applied to transistors. Thus, $\mathbf{h}_{11}$, $\mathbf{h}_{12}$, $\mathbf{h}_{21}$, and $\mathbf{h}_{22}$ become $\mathbf{h}_i$, $\mathbf{h}_r$, $\mathbf{h}_f$, and $\mathbf{h}_o$, respectively, where the subscripts denote input, reverse, forward, and output.

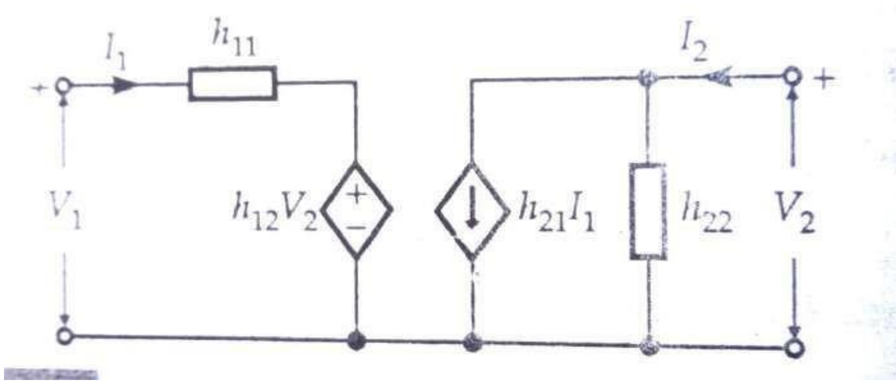


Fig: h parameter equivalent circuit

Transmission parameters:

The last two-port parameters that we will consider are called the **t *parameters,*** the ***ABCD parameters,*** or simply the ***transmission parameters.*** They are defined by the equations

$$\mathbf{V_1 = A.V_2 - B.I_2}$$
$$\mathbf{I_1 = C.V_2 - D.I_2}$$

and in Matrix notation these equations can be written in the form

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

where $\mathbf{V}_1$, $\mathbf{V}_2$, $\mathbf{I}_1$, and $\mathbf{I}_2$ are defined as as shown in the figure below.

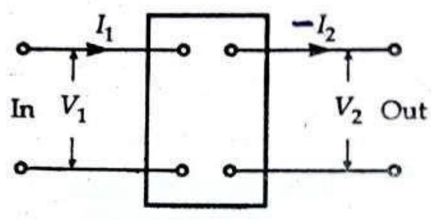


Fig: Two port Network for ABCD parameter representation with Input and output Voltages and currents

The minus signs that appear in the above equations should be associated with the output current, as ($-$**I_2**). Thus, both **I_1** and $-$**I_2** are directed to the right, the direction of energy or signal transmission.

Note that there are no minus signs in the **t** or **ABCD** matrices. Looking again at the above equations we see that the quantities on the left, often thought of as the given or independent variables, are the input voltage and current, **V_1** and **I_1**; the dependent variables, **V_2** and **I_2**, are the output quantities. Thus, the transmission parameters provide a direct relationship between input and output. Their major use arises in transmission-line analysis and in cascaded networks.

The four Transmission parameters are defined and explained below.

*First **A** and **C** are defined with receiving end open circuited i.e. with $I_2 = 0$*

$A = V_1/V_2$ with $I_2 = 0$ = Reverse voltage Ratio
$C = I_1/V_2$ with $I_2 = 0$ = Transfer admittance

*Next **B** and **D** are defined with receiving end short circuited i.e. with $V_2 = 0$*

$B = V_1/-I_2$ with $V_2 = 0$ = Transfer impedance
$D = I_1/-I_2$ with $V_2 = 0$ = Reverse current ratio

Inter relationships between different parameters of two port networks:

Basic Procedure for representing any of the above four two port Network parameters in terms of the other parameters consists of the following steps:

1. *Write down the defining equations corresponding to the parameters in terms of which the other parameters are to be represented.*
2. *Keeping the basic parameters same, rewrite/manipulate these two equations in such a way that the variables V_1, V_2, I_1, and I_2 are arranged corresponding to the defining equations of the first parameters.*
3. *Then by comparing the parameter coefficients of the respective variables V_1, V_2, I_1, and I_2 on the right hand side of the two sets of equations we can get the inter relationship.*

Z Parameters in terms of Y parameters:

Though this relationship can be obtained by the above steps, the following simpler method is used for Z in terms of Y and Y in terms of Z:

Z and Y being the Impedance and admittance parameters (Inverse), in matrix notation they are governed by the following inverse relationship.

$$[Z] = [Y]^{-1}$$

Or:

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1}$$

Thus :

$$Z_{11} = \frac{Y_{22}}{\Delta Y}, Z_{12} = -\frac{Y_{12}}{\Delta Y}$$

$$Z_{21} = -\frac{Y_{21}}{\Delta Y} \text{ and } Z_{22} = \frac{Y_{11}}{\Delta Y}$$

$$\left[\text{Here } \Delta Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = Y_{11}\, Y_{22} - Y_{12}\, Y_{21}\right]$$

Z Parameters in terms of ABCD parameters:

The governing equations are:

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

from the second governing equation [$I_1 = CV_2 - DI_2$] we can write

$$V_2 = \frac{1}{C} \cdot I_1 + \frac{D}{C} \cdot I_2$$

Now substituting this value of V_2 in the first governing equation [$V_1 = AV_2 - BI_2$] we get

$$V_1 = \left[\frac{1}{C} \cdot I_1 + \frac{D}{C} \cdot I_2\right] A - BI_2$$

$$= \frac{A}{C} \cdot I_1 + \frac{AD - BC}{C} \cdot I_2$$

Comparing these two equations for **V_1** and **V_2** with the governing equations of the **Z** parameter network we get **Z** Parameters in terms of ABCD parameters:

$$Z_{11} = \frac{A}{C}, Z_{12} = \frac{AD - BC}{C}$$

$$Z_{21} = \frac{1}{C}, Z_{22} = \frac{D}{C}.$$

Z Parameters in terms of h parameters:

The governing equations of h parameter network are:

$V_1 = h_{11}I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

From the second equation we get

$$V_2 = -\frac{h_{21}}{h_{22}} . I_1 + \frac{1}{h_{22}} . I_2$$

Substituting this value of V_2 in the first equation for V_1 we get:

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$= h_{11} I_1 + h_{12} \left[-\frac{h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} . I_2 \right]$$

$$= \frac{\Delta h}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} . I_2$$

Now comparing these two equations for V_1 and V_2 with the governing equations of the **Z** parameter network we get **Z** Parameters in terms of **h** parameters:

$$Z_{11} = \frac{\Delta h}{h_{22}}, \quad Z_{12} = \frac{h_{12}}{h_{22}}$$

$$Z_{21} = -\frac{h_{21}}{h_{22}}, Z_{22} = \frac{1}{h_{22}}$$

Here $\Delta h = h_{11} h_{22} - h_{12} h_{21}$

Y Parameters in terms of Z parameters:

Y and Z being the admittance and Impedance parameters (Inverse), in matrix notation they are governed by the following inverse relationship.

$$[Y]=[Z]^{-1}$$

Or:

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1}$$

Thus:

$$Y_{11} = \frac{Z_{22}}{\Delta Z}, \quad Y_{12} = -\frac{Z_{12}}{\Delta Z}$$

$$Y_{21} = -\frac{Z_{21}}{\Delta Z}, Y_{22} = \frac{Z_{11}}{\Delta Z}$$

Here $\Delta Z = Z_{11} Z_{22} - Z_{12} Z_{21}$

The other inter relationships also can be obtained on the same lines following the ***basic three steps given*** in the beginning.

Conditions for reciprocity and symmetry in two port networks:

A two port network is said to be ***reciprocal*** if the ratio of the output response variable to the input excitation variable is same when the excitation and response ports are interchanged.
A two port network is said to be ***symmetrical*** if the port voltages and currents remain the same when the input and output ports are interchanged.
In this topic we will get the conditions for ***Reciprocity*** and ***symmetry*** for all the four networks.
The basic procedure for each of the networks consists of the following steps:

Reciprocity:

- First we will get an expression for the ratio of response to the excitation in terms of the **particular parameters** by giving voltage as excitation at the input port and considering the current in the output port as response (by short circuiting the output port i.e setting V_2 as zero). i.e find out (I_2 / V_1)
- Then we will get an expression for the ratio of response to the excitation in terms of the **same parameters** by giving voltage as excitation at the output port and considering the current in the input port as response (by short circuiting the input port i.e. setting V_1 as zero). i.e find out (I_1 / V_2)
- Equating the RHS of these two expressions would be the condition for reciprocity

Symmetry:

- First we need to get expressions related to the input and output ports using the basic Z or Y parameter equations.
- Then the expressions for Z_{11} and Z_{22} (or Y_{11} and Y_{22}) are equated to get the conmdition for reciprocity.

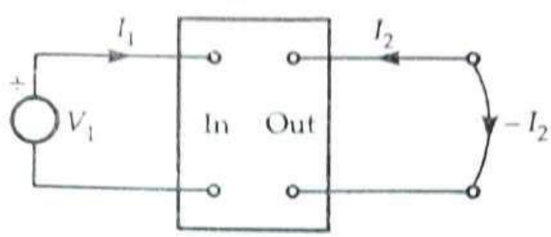
Z parameter representation:

Condition for reciprocity:

Let us take a two port network with **Z** parameter defining equations as given below:

$$\mathbf{V_1 = Z_{11}I_1 + Z_{12}I_2}$$
$$\mathbf{V_2 = Z_{21}I_1 + Z_{22}I_2}$$

First we will get an expression for the ratio of response (I_2) to the excitation (V_1) in terms of the **Z parameters** by giving excitation at the input port and considering the current in the output port as response (by short circuiting the output port i.e. setting V_2 as zero).The corresponding **Z** parameter circuit for this condition is shown in the figure below:



(Pl note the direction of I_2 is negative since when V_2 port is shorted the current flows in the other direction)

Then the Z parameter defining equations are :

$\mathbf{V_1 = Z_{11} . I_1 - Z_{12}. I_2}$ and
$0 = \mathbf{Z_{21} . I_1 - Z_{22}. I_2}$

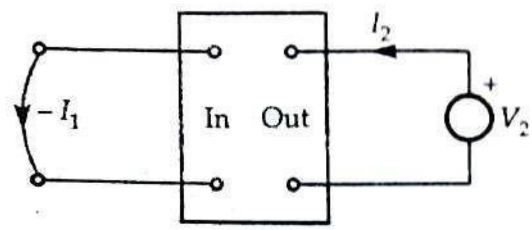
*To get the ratio of response **(I_2)** to the excitation **(V_1)** in terms of the Z parameters **I_1** is to be eliminated fom the above equations.*

So from equation 2 in the above set we will get $\mathbf{I_1 = I_2. Z_{22}/ Z_{21}}$
And substitute this in the first equation to get

$$\mathbf{V_1 = (Z_{11} .I_2. Z_{22}/ Z_{21}) - Z_{12}. I_2 = I_2 [(Z_{11} . Z_{22}/ Z_{21}) - Z_{12}] = I_2 [(Z_{11} . Z_{22} - Z_{12}.Z_{21}) / Z_{21})]}$$

$I_2 = V_1 . Z_{21}/(Z_{11} . Z_{22} - Z_{12}.Z_{21})$

Next, we will get an expression for the ratio of response (**I_1**) to the excitation **(V_2)** in terms of the **Z parameters** by giving excitation **V_2** at the output port and considering the current **I_1** in the input port as response (by short circuiting the input port i.e. setting **V_1** as zero). The corresponding **Z** parameter circuit for this condition is shown in the figure below:



(Pl note the direction of current I_1 is negative since when V_1 port is shorted the current flows in the other direction)

Then the Z parameter defining equations are :

$0 = -Z_{11} . I_1 + Z_{12}. I_2$ and
$V_2 = -Z_{21} . I_1 + Z_{22}. I_2$

*To get the ratio of response (I_1) to the excitation **(V_2)** in terms of the Z parameters **I_2** is to be eliminated fom the above equations.*

So from equation 1 in the above set we will get $I_2 = I_1. Z_{11}/ Z_{12}$
And substitute this in the second equation to get

$V_2 = (Z_{22}.I_1. Z_{11}/ Z_{12}) - Z_{21}. I_1 = I_1 [(Z_{11} . Z_{22}/ Z_{12}) - Z_{21}] = I_1 [(Z_{11} . Z_{22} - Z_{12}.Z_{21}) / Z_{12})]$

$I_1 = V_2 . Z_{12}/(Z_{11} . Z_{22} - Z_{12}.Z_{21})$

Assuming the input excitations **V_1** and **V_2** to be the same, then the condition for the out responses **I_1** and **I_2** to be equal would be

$$Z_{12} = Z_{21}$$

And this is the condition for the reciprocity.

Condition for symmetry:
To get this condition we need to get expressions related to the input and output ports using the basic Z parameter equations.

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

To get the input port impedance I_2 is to be made zero. i.e V_2 should be open.

$$V_1 = Z_{11} . I_1 \quad \text{i.e} \quad Z_{11} = V_1/I_1 \mid I_2=0$$

Similarly to get the output port impedance I_1 is to be made zero. i.e V_1 should be open.

$$V_2 = Z_{22} . I_2 \quad \text{i.e} \quad Z_{22} = V_2/I_2 \mid I_1=0$$

Condition for Symmetry is obtained when the two port voltages are equal i.e. $V_1 = V_2$ and the two port currents are equal i.e. $I_1 = I_2$. Then

$V_1/I_1 = V_2/I_2$ i.e $Z_{11} = Z_{22}$

And hence $Z_{11} = Z_{22}$ is the condition for symmetry in Z parameters .

Y parameter representation:

Condition for reciprocity :

Let us take a two port network with **Y** parameter defining equations as given below:

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

First we will get an expression for the ratio of response **(I_2)** to the excitation **(V_1)** in terms of the **Y parameters** by giving excitation **(V_1)** at the input port and considering the current **(I_2)** in the output port as response (by short circuiting the output port i.e. setting V_2 as zero)

Then the second equation in **Y** parameter defining equations would become

$$I_2 = Y_{21}V_1 + 0 \text{ and } \quad I_2 / V_1 = Y_{21}$$

Then we will get an expression for the ratio of response **(I_1)** to the excitation **(V_2)** in terms of the **Y parameters** by giving excitation **(V_2)** at the output port and considering the current **(I_1)** in the input port as response (by short circuiting the input port i.e setting V_1 as zero)

Then the first equation in Y parameter defining equations would become

$$I_1 = 0 + Y_{12}V_2 \quad \text{and} \quad I_1 / V_2 = Y_{12}$$

Assuming the input excitations V_1 and V_2 to be the same, then the condition for the out responses I_1 and I_2 to be equal would be

$$I_1 / V_2 = I_2 / V_1$$

And hence $Y_{12} = Y_{21}$ is the condition for the reciprocity in the Two port network with Y parameter representation.

Condition for symmetry:

To get this condition we need to get expressions related to the input and output ports (In this case Input and output admittances) using the basic Y parameter equations

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

To get the input port admittance, V_2 is to be made zero. i.e V_2 should be shorted.

$$I_1 = Y_{11} . V_1 \quad \text{i.e} \quad Y_{11} = I_1/V_1 \mid V_2=0$$

Similarly to get the output port admittance V_1 is to be made zero. i.e V_1 should be shorted.

$$I_2 = Y_{22} . V_2 \quad \text{i.e} \quad Y_{22} = I_2/V_2 \mid V_1=0$$

Condition for Symmetry is obtained when the two port voltages are equal i.e. $V_1 = V_2$ and the two port currents are equal i.e. $I_1 = I_2$. Then

$I_1/V_1 = I_2/V_2$

And hence $Y_{11} = Y_{22}$ is the condition for symmetry in Y parameters.

ABCD parameter representation:

Condition for reciprocity :

Let us take a two port network with ABCD parameter defining equations as given below:

$$V_1 = A.V_2 - B.I_2$$
$$I_1 = C.V_2 - D.I_2$$

First we will get an expression for the ratio of response **(I_2)** to the excitation **(V_1)** in terms of the **ABCD parameters** by giving excitation **(V_1)** at the input port and considering the current **(I_2)** in the output port as response (by short circuiting the output port i.e. setting V_2 as zero)
Then the first equation in the **ABCD** parameter defining equations would become

$$V_1 = 0 - B.I_2 = B.I_2$$
$$\text{i.e } I_2 / V_1 = -1/B$$

Then we will interchange the excitation and response i.e. we will get an expression for the ratio of response **(I_1)** to the excitation **(V_2)** by giving excitation **(V_2)** at the output port and considering the current **(I_1)** in the input port as response (by short circuiting the input port i.e. setting **V_1** as zero)

Then the above defining equations would become

$$0 = A.V_2 - B.I_2$$
$$I_1 = C.V_2 - D.I_2$$

Substituting the value of **$I_2 = A.V_2 /B$** from first equation into the second equation we get

$$I_1 = C.V_2 - D.\, A.V_2 /B = V_2 (C - D.\, A /B)$$
$$\text{i.e} \qquad I_1/V_2 = (BC - DA) / B = -(AD - BC)/B$$

Assuming the input excitations **V_1** and **V_2** to be the same , then the condition for the out responses **I_1** and **I_2** to be equal would be

$$I_1 / V_2 = I_2 / V_1$$
$$\text{i.e} \qquad -(AD - BC)/B = -1/B$$
$$\text{i.e } (AD - BC) = 1$$

And hence AD – BC = 1 is the condition for Reciprocity in the Two port network with ABCD parameter representation.

Condition for symmetry:

To get this condition we need to get expressions related to the input and output ports. In this case it is easy to use the Z parameter definitions of Z_{11} and Z_{22} for the input and output ports respectively and get their values in terms of the ABCD parameters as shown below.

$$V_1 = A.V_2 - B.I_2$$
$$I_1 = C.V_2 - D.I_2$$

$$Z_{11} = V_1/I_1 \,\big|\, I_2=0$$

Applying this in both the equations we get

$$Z_{11} = V_1/I_1 \,\big|\, I_2=0 = (A.V_2 - B.I_2)/(C.V_2 - D.I_2) \,\big|\, I_2=0$$
$$= (A.V_2 - B.0)/(C.V_2 - D.0)$$
$$= (A.V_2)/(C.V_2) = A/C$$

$Z_{11} = A/C$

Similarly $Z_{22} = V_2/I_2 \mid I_1=0$

and using this in the second basic equation $I_1 = C.V_2 - D.I_2$

we get $0 = C.V_2 - D.I_2$ **or** $C.V_2 = D.I_2$

$V_2 / I_2 = D/C$

$Z_{22} = D/C$

And the condition for symmetry becomes $Z_{11} = Z_{22}$ **i.e** $A/C = D/C$

Or $A = D$

Hence A = D is the condition for Symmetry in ABCD parameter representation.

h parameter representation:

Condition for reciprocity :

Let us take a two port network with h parameter defining equations as given below:

$$V_1 = h_{11}.I_1 + h_{12}.V_2$$
$$I_2 = h_{21}.I_1 + h_{22}.V_2$$

First we will get an expression for the ratio of response **(I_2)** to the excitation **(V_1)** in terms of the **h parameters** by giving excitation **(V_1)** at the input port and considering the current **(I_2)** in the output port as response (by short circuiting the output port i.e. setting V_2 as zero)

Then the first equation in the **h** parameter defining equations would become

$$V_1 = h_{11}.I_1 + h_{12}.0 = h_{11}.I_1$$

And in the same condition the second equation in the **h** parameter defining equations would become

$$I_2 = h_{21}.I_1 + h_{22}.0 = h_{21}.I_1$$

Dividing the second equation by the first equation we get

$$I_2 / V_1 = (h_{21}.I_1) / (h_{11}.I_1) = h_{21} / h_{11}$$

Now the excitation and the response ports are interchanged and then we will get an expression for the ratio of response **(I_1)** to the excitation **(V_2)** in terms of the **h parameters** by giving excitation **(V_2)** at the output port and considering the current **(I_1)** in the input port as response (by short circuiting the input port i.e. setting V_1 as zero)

Then the first equation in **h** parameter defining equations would become

$$0 = h_{11}.I_1 + h_{12}.V_2 \quad \text{i.e} \quad h_{11}.I_1 = -h_{12}.V_2$$
$$\text{i.e.} \quad I_1 / V_2 = -h_{12} / h_{11}$$

Assuming the input excitations V_1 and V_2 to be the same, then the condition for the out responses I_1 and I_2 to be equal would be

$$I_1 / V_2 = I_2 / V_1$$

i.e = $-h_{12} / h_{11} = h_{21} / h_{11}$

$$\text{i.e.} \quad h_{12} = -h_{21}$$

And hence [$h_{12} = -h_{21}$] is the condition for the reciprocity in the Two port network with h parameter representation.

Condition for symmetry:

To get this condition we need to get expressions related to the input and output ports. In this case also it is easy to use the Z parameter definitions of Z_{11} and Z_{22} for the input and output ports respectively and get their values in terms of the **h** parameters as shown below.

h parameter equations are : $V_1 = h_{11}.I_1 + h_{12}.V_2$

$I_2 = h_{21}.I_1 + h_{22}.V_2$

First let us get Z_{11} :

$$Z_{11} = V_1/I_1 \mid I_2=0$$

$$= h_{11} + h_{12}.V_2 / I_1$$

Applying the condition $I_2=0$ in the equation 2 we get

$$0 = h_{21}.I_1 + h_{22}.V_2 \text{ i.e } -h_{21}.I_1 = h_{22}.V_2$$

$$\text{or } V_2 = I_1(-h_{21} / h_{22})$$

Now substituting the value of $V_2 = I_1(-h_{21} / h_{22})$ in the above first expression for V_1 we get

$$V_1 = h_{11}.I_1 + h_{12}.I_1.(-h_{21} / h_{22})$$

$$\text{Or } V_1/I_1 = (h_{11}.h_{22} - h_{12}.h_{21})/ h_{22} = \Delta h / h_{22}$$

$$\text{Or } Z_{11} = \Delta h / h_{22}$$

Where $\Delta h = (h_{11}.h_{22} - h_{12}.h_{21})$

Now let us get Z_{22} :

$$Z_{22} = V_2/I_2 \mid I_1 = 0$$

Applying the condition $I_1 = 0$ in the second equation we get

$$I_2 = h_{21}.0 + h_{22}.V_2 \text{ i.e } V_2/I_2 = 1/h_{22}$$

$$\text{And } Z_{22} = 1/h_{22}$$

Hence the condition for symmetry $Z_{11} = Z_{22}$ becomes $(\Delta h / h_{22}) = (1/h_{22})$ i.e $\Delta h = 1$

Hence $\Delta h = 1$ is the condition for symmetry in h parameter representation.

Table: Summary of conditions for reciprocity and symmetry for Two port networks in terms of all four parameters.

Parameter	Condition for reciprocity	Condition for symmetry
Z	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
Y	$Y_{12} = Y_{21}$	$Y_{11} = Y_{21}$
h	$h_{12} = -h_{21}$	$\Delta h = 1$
$ABCD$	$AD - BC = 1$	$A = D$

Different types of interconnections of two port networks:

Series Connection:

Though here only two networks are considered, the result can be generalized for any number of two port networks connected in series.

Refer the figure below where two numbers of two port networks **A** and **B** are shown connected in series. All the input and output currents & voltages with directions and polarities are shown.

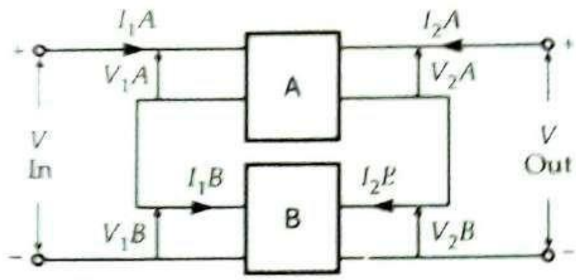


Fig : Series connection of two numbers of Two Port Networks

Open circuit Impedance parameters **(Z)** are used in characterizing the Series connected Two port Networks .The governing equations with **Z** parameters are given below:

For network A :

$$V_{1A} = Z_{11A}\ I_{1A} + Z_{12A}\ I_{2A}$$
$$V_{2A} = Z_{21A}\ I_{1A} + Z_{22A}\ I_{2A}$$

And for network B:

$$V_{1B} = Z_{11B}\ I_{1B} + Z_{12B}\ I_{2B}$$
$$V_{2B} = Z_{21B}\ I_{1B} + Z_{22B}\ I_{2B}$$

Referring to the figure above the various voltage and current relations are:

$$I_1 \equiv I_{1A} \equiv I_{1B}$$
$$I_2 \equiv I_{2A} \equiv I_{2B}$$
$$V_2 = V_{2A} + V_{2B}$$
$$V_1 = V_{1A} + V_{1B}$$

Now substituting the above basic defining equations for the two networks into the above expressions for V_1 and V_2 and using the above current equalities we get:

$$\begin{aligned} V_1 &= V_{1A} + V_{1B} \\ &= (Z_{11A} I_{1A} + Z_{12A} I_{2A}) + Z_{11B} I_{1B} + Z_{12B} I_{2B} \\ &= I_1 (Z_{11A} + Z_{11B}) + I_2 (Z_{12A} + Z_{12B}) \end{aligned}$$

And similarly

$$\begin{aligned} V_2 &= V_{2A} + V_{2B} \\ &= (Z_{21A} I_{1A} + Z_{22A} I_{2A}) + (Z_{21B} I_{1B} + Z_{22B} I_{2B}) \end{aligned}$$
$$V_2 = I_1 (Z_{21A} + Z_{21B}) + I_2 (Z_{22A} + Z_{22B})$$

Thus we get for two numbers of series connected two port networks:

$$V_1 = (Z_{11A} + Z_{11B}) I_1 + (Z_{12A} + Z_{12B}) I_2$$
$$V_2 = (Z_{21A} + Z_{21B}) I_1 + (Z_{22A} + Z_{22B}) I_2$$

Or in matrix form:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11A} + Z_{11B} & Z_{12A} + Z_{12B} \\ Z_{21A} + Z_{21B} & Z_{22A} + Z_{22B} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Thus it can be seen that the Z parameters for the series connected two port networks are the sum of the Z parameters of the individual two port networks.

Cascade connection:

In this case also though here only two networks are considered, the result can be generalized for any number of two port networks connected in cascade.

Refer the figure below where two numbers of two port networks **X** and **Y** are shown connected in cascade. All the input and output currents & voltages with directions and polarities are shown.

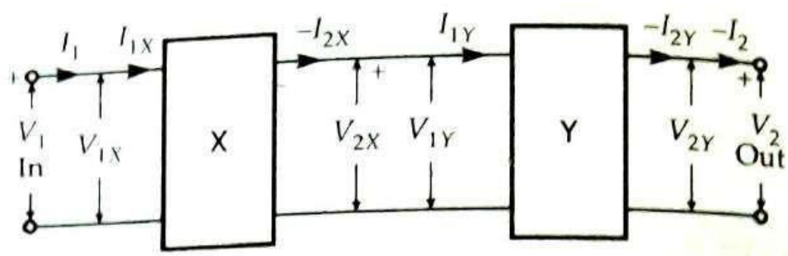


Fig: Two numbers of two port networks connected in cascade

Transmission (ABCD) parameters are easily used in characterizing the cascade connected Two port Networks .The governing equations with transmission parameters are given below:

For network X:

$$V_{1X} = A_X V_{2X} - B_X I_{2X}$$
$$I_{1X} = C_X V_{2X} - D_X I_{2X}$$

And for network Y:

$$V_{1Y} = A_Y V_{2Y} - B_Y I_{2Y}$$
$$I_{1Y} = C_Y V_{2Y} - D_Y I_{2Y}$$

Referring to the figure above the various voltage and current relations are:

$$I_1 = I_{1X} ; -I_{2X} = I_{1Y} ; I_2 = I_{2Y}$$
$$V_1 = V_{1X} ; V_{2X} = V_{1Y} ; V_2 = V_{2Y}$$

Then the overall transmission parameters for the cascaded network in matrix form will become

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_{1X} \\ I_{1X} \end{bmatrix} = \begin{bmatrix} A_X & B_X \\ C_X & D_X \end{bmatrix} \begin{bmatrix} V_{2X} \\ -I_{2X} \end{bmatrix}$$
$$= \begin{bmatrix} A_X & B_X \\ C_X & D_X \end{bmatrix} \begin{bmatrix} V_{1Y} \\ I_{1Y} \end{bmatrix}$$
$$= \begin{bmatrix} A_X & B_X \\ C_X & D_X \end{bmatrix} \begin{bmatrix} A_Y & B_Y \\ C_Y & D_Y \end{bmatrix} \begin{bmatrix} V_{2Y} \\ -I_{2Y} \end{bmatrix}$$
$$= \begin{bmatrix} A_X & B_X \\ C_X & D_X \end{bmatrix} \begin{bmatrix} A_Y & B_Y \\ C_Y & D_Y \end{bmatrix} \begin{bmatrix} V_Y \\ -I_Y \end{bmatrix}$$
$$= \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_Y \\ -I_Y \end{bmatrix}$$

Where

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_X & B_X \\ C_X & D_X \end{bmatrix} \begin{bmatrix} A_Y & B_Y \\ D_Y & D_Y \end{bmatrix}$$

Thus it can be seen that the overall ABCD Parameter matrix of cascaded two Port Networks is the product of the ABCD matrices of the individual networks.

Parallel Connection:

Though here only two networks are considered, the result can be generalized for any number of two port networks connected in parallel.

Refer the figure below where two numbers of two port networks **A** and **B** are shown connected in parallel. All the input and output currents & voltages with directions and polarities are shown.

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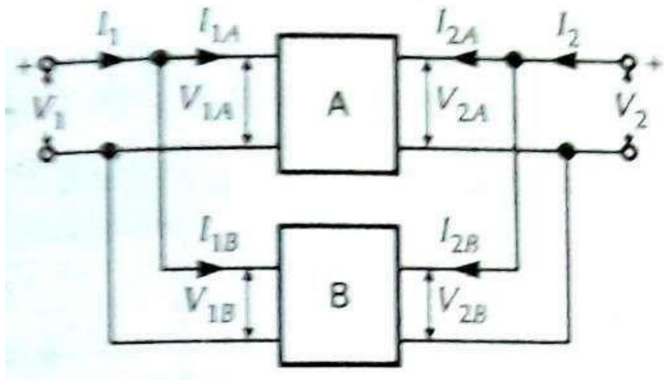


Fig: Parallel connection of two numbers of Two Port Networks

Short circuit admittance (Y) parameters are easily used in characterizing the parallel connected Two port Networks .The governing equations with Y parameters are given below:

For network A:

$$I_{1A} = Y_{11A}\ V_{1A} + Y_{12A}\ V_{2A}$$
$$I_{2A} = Y_{21A}\ V_{1A} + Y_{22A}\ V_{2A}$$

And for network B:

$$I_{1B} = Y_{11B}\ V_{1B} + Y_{12B}\ V_{2B}$$
$$I_{2B} = Y_{21B}\ V_{1B} + Y_{22B}\ V_{2B}$$

Referring to the figure above the various voltage and current relations are:

$$V_1 = V_{1A} = V_{1B}; V_2 = V_{2A} = V_{2B}$$
$$I_1 = I_{1A} + I_{1B}\ ; I_2 = I_{2A} + I_{2B}$$

Thus

$$\begin{aligned} I_1 &= I_{1A} + I_{1B} \\ &= (Y_{11A}V_{1A} + Y_{12A}V_{2A}) + (Y_{11B}V_{1B} + Y_{12B}V_{2B}) \\ &= (Y_{11A} + Y_{11B})\,V_1 + (Y_{12A} + Y_{12B})\,V_2 \end{aligned}$$

$$\begin{aligned} I_2 &= I_{2A} + I_{2B} \\ &= (Y_{21A}V_{1A} + Y_{22A}V_{1B}) + (Y_{21B}V_{1B} + Y_{22B}V_{2B}) \\ &= (Y_{21A} + Y_{21B})\,V_1 + (Y_{22A} + Y_{22B})\,V_2 \end{aligned}$$

Thus we finally obtain the Y parameter equations for the combined network as:

$$I_1 = (Y_{11A} + Y_{11B})V_1 + (Y_{12A} + Y_{12B})V_2$$
$$I_2 = (Y_{21A} + Y_{21B})V_1 + (Y_{22A} + Y_{22B})V_2$$

And in matrix notation it will be:

$$\begin{bmatrix} I_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} Y_{11A} + Y_{11B} & Y_{12A} + Y_{12B} \\ Y_{21A} + Y_{21B} & Y_{22A} + Y_{22B} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Thus it can be seen that the overall Y parameters for the parallel connected two port networks are the sum of the Y parameters of the individual two port networks.

Image impedances in terms of ABCD parameters:

Image impedances Z_{i1} and Z_{i2} of a two port network as shown in the figure below are defined as two values of impedances such that :

a) When port two is terminated with an impedance $\mathbf{Z_{i2}}$, the input impedance as seen from Port one is $\mathbf{Z_{i1}}$ and
b) When port one is terminated with an impedance $\mathbf{Z_{i1}}$, the input impedance as seen from Port two is $\mathbf{Z_{i2}}$

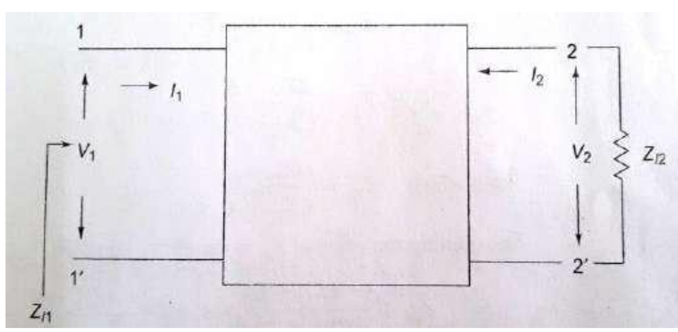


Figure pertining to condition (a) above

Corresponding Relations are : $Z_{i1} = V_1 / I_1$ and $Z_{i2} = V_2 / -I_2$

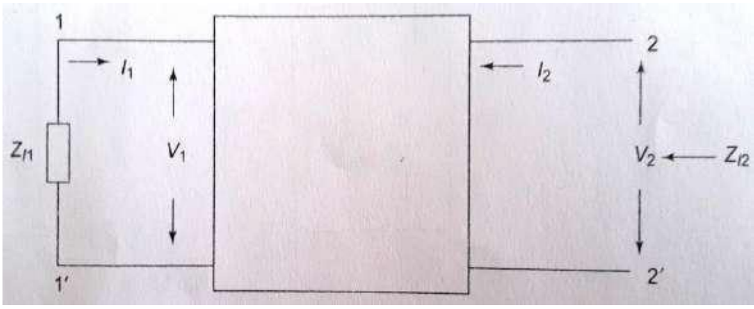


Figure pertining to condition (b) above

Corresponding Relations are : $Z_{i1} = V_1 / - I_1$ and $Z_{i2} = V_2 / I_2$

Such Image impedances in terms of ABCD parameters for a two port network are obtained below:

The basic defining equations for a two port network with ABCD parameters are :

$$\mathbf{V_1 = A.V_2 - B.I_2}$$
$$\mathbf{I_1 = C.V_2 - D.I_2}$$

First let us consider condition (a).

Dividing the first equation with the second equation we get

$$Z_{i1} = \frac{V_1}{I_1} = \frac{AV_2 - BI_2}{CV_2 - DI_2}$$

But we also have $\mathbf{Z_{i2} = V_2 / - I_2}$ and so $\mathbf{V_2 = - Z_{i2} I_2}$. Substituting this value of V_2 in the above we get

$$Z_{i1} = \frac{-AZ_{i2} - B}{-CZ_{i2} - D} = \frac{AZ_{i2} + B}{CZ_{i2} + D}$$

Now let us consider the condition (b):

The basic governing equations [$\mathbf{V_1 = A.V_2 - B.I_2}$] **and** [$\mathbf{I_1 = C.V_2 - D.I_2}$] are manipulated to get

$$V_2 = \frac{DV_1}{AD - BC} - \frac{BI_1}{AD - BC}$$

$$I_2 = \frac{CV_1}{AD - BC} - \frac{AI_1}{AD - BC}$$

$$Z_{i2} = \frac{V_2}{I_2} = \frac{DV_1 - BI_1}{CV_1 - AI_1}$$

But we also have $\mathbf{Z_{i1} = V_1 / - I_1}$ and so $\mathbf{V_1 = - Z_{i1} I_1}$. Substituting this value of $\mathbf{V_1}$ in the above we get :

$$Z_{i2} = \frac{DZ_{i1} + B}{CZ_{i1} + A}$$

Solving the above equations for Z_{i1} and Z_{i2} we get :

$$Z_{i1} = \sqrt{\frac{AB}{CD}}\ ; \quad Z_{i2} = \sqrt{\frac{BD}{AC}}$$

Important formulae, Equations and Relations:

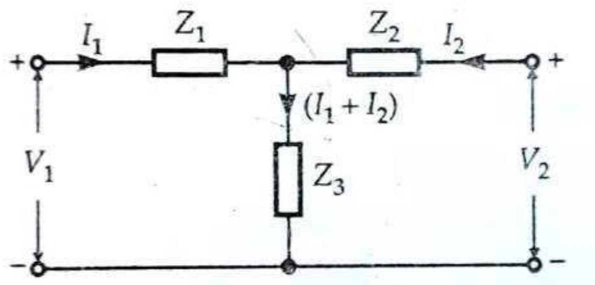
- **Basic Governing equations in terms of the various Parameters:**
  - **Z Paramaters :** $V_1 = Z_{11}I_1 + Z_{12}I_2$
    $V_2 = Z_{21}I_1 + Z_{22}I_2$
  - **Y Parameters:** $I_1 = Y_{11}V_1 + Y_{12}V_2$
    $I_2 = Y_{21}V_1 + Y_{22}V_2$
  - **h Parameters :** $V_1 = h_{11}.\ I_1 + h_{12}.V_2$
    $I_2 = h_{21}.I_1 + h_{22}.V_2$
  - **ABCD Parameters:** $V_1 = A.V_2 - B.I_2$
    $I_1 = C.V_2 - D.I_2$
- **Conditions for Reciprocity and symmetry for Two Port Networks in terms of the various parameters :**

Parameter	Condition for reciprocity	Condition for symmetry
Z	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
Y	$Y_{12} = Y_{21}$	$Y_{11} = Y_{21}$
h	$h_{12} = -h_{21}$	$\Delta h = 1$
ABCD	$AD - BC = 1$	$A = D$

- **Relations of Interconnected two port Networks :**
  - *The overall **Z parameters** for the **series** connected two port networks are the **sum of the Z parameters of the individual two port networks.***
  - *The overall **Y parameters** for the **parallel** connected two port networks are the **sum of the Y parameters of the individual two port networks**.*
  - *The overall **ABCD Parameter matrix** of **cascaded** two Port Networks is the **product of the ABCD matrices of the individual networks.***

Illustrative problems :

Example 1: Find the Z Parameters of the following Two Port Network and draw it's equivalent circuit in terms of Z_1 Z_2 and Z_3 .



Solution: Applying KVL to the above circuit in the two loops ,with the current notation as shown, the loop equations for V_1 and V_2 can be written as :

$$V_1 = I_1 Z_1 + (I_1 + I_2) Z_3$$

or $$V_1 = (Z_1 + Z_3) I_1 + Z_3 I_2 \qquad ...(i)$$

and $$V_2 = I_2 Z_2 + (I_2 + I_2) Z_3$$

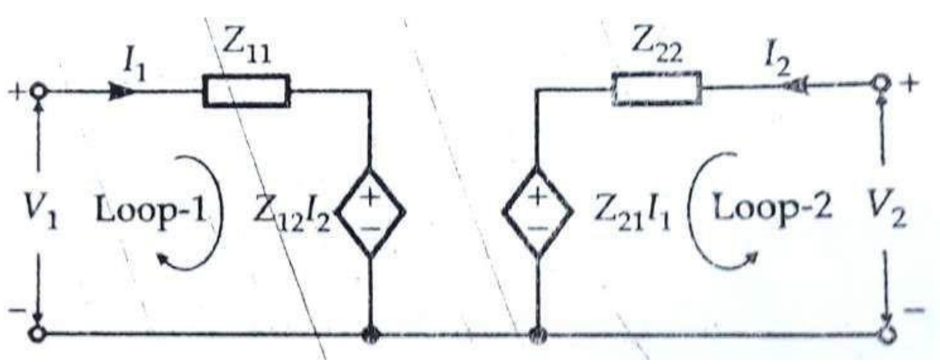
or $$V_2 = Z_3 I_1 + (Z_2 + Z_3) I_2 \qquad ...(ii)$$

Comparing the equations (i) and (ii) above with the standard expressions for the Z parameter equations we get :

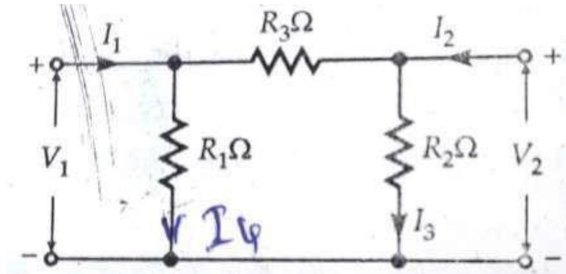
$$Z_{11} = Z_1 + Z_3;\ Z_{12} = Z_3;$$

$$Z_{21} = Z_3\ ;\ Z_{22} = Z_2 + Z_3$$

Equivalent circuit in terms of Z_1 Z_2 and Z_3 is shown below.



Example 2: Determine the Z parameters of the **π** type two port network shown in the figure below.



Solution:

From the basic Z parameter equations We know that

$$Z_{11} = V_1/I_1 \mid I_2=0$$
$$Z_{12} = V_1/I_2 \mid I_1=0$$
$$Z_{21} = V_2/I_1 \mid I_2=0$$
$$Z_{22} = V_2/I_2 \mid I_1=0$$

We will first find out Z_{11} and Z_{21} which are given by the common condition $I_2 = 0$

1. We can observe that $Z_{11} = V_1/I_1$ with $I_2=0$ is the parallel combination of R_1 and $(R_2 + R_3)$.

$$\therefore \quad Z_{11} = R_1 (R_2 + R_3) / (R_1+R_2 + R_3)$$

2. $Z_{21} = V_2/I_1 \mid I_2=0$

By observing the network we find that the current I_1 is dividing into I_3 and I_4 as shown in the figure where I_3 is flowing through R2(and R_3 also since $I_2=0$)

Hence $V_2 = I_3 \, x R_2$

From the principle of current division we find that $I_3 = I_1 . R_1 / (R_1+R_2 + R_3)$

Hence $V_2 = I_3 \, x R_2 = [I_1 . R_1 / (R_1+R_2 + R_3)].R_2 = I_1 . R_1 R_2 / (R_1+R_2 + R_3)$

And $V_2/I_1 = R_1 . R_2 / (R_1+R_2 + R_3)$

$$\therefore \quad Z_{21} = R_1 R_2 / (R_1+R_2 + R_3)$$

Next we will find out Z_{12} and Z_{22} which are given by the common condition $I_1 = 0$

3. $Z_{12} = V_1/I2 \mid I_1=0$

By observing the network we find that the current I_2 is now dividing into I_3 and I_4 as shown in the figure where I_4 is flowing through R_1 (and R_3 also since $I_1 = 0$)

Hence $V_1 = I_4 \, x R_1$

Again from the principle of current division we find that $I_4 = I_2 . R_2 / (R_1+R_2 + R_3)$

Hence $V_1 = I_4 \, x R_1 = [I_2 . R_2 / (R_1+R_2 + R_3)].R_1 = I_2 . R_1 R_2 / (R_1+R_2 + R_3)$

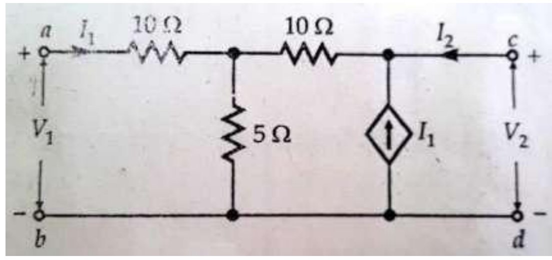
And $V_1/I_2 = R_1 . R_2 / (R_1+R_2 + R_3)$

$$\therefore \quad Z_{12} = R_1 R_2 / (R_1+R_2 + R_3)$$

4. We can again observe that $Z_{22} = V_2/I_2$ with $I_1=0$ is the parallel combination of R_2 and $(R_1 + R_3)$

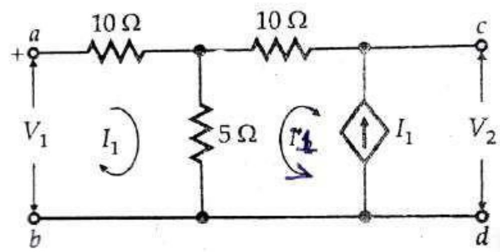
.∴ $Z_{22} = R_2 (R_1 + R_3) / (R_1+R_2 + R_3)$

Example 3 : Determine the Z parameters of the network shown in the figure below.



1). We will first find out Z_{11} and Z_{21} which are given by the common condition $I_2 = 0$ (Output open circuited)

With this condition the circuit is redrawn as shown below.



Since the current source is there in the second loop which is equal to I_1 and I_2 is zero, only current I_1 flows through the right hand side resistance of 10Ω and both currents I_1(both loop currents) pass through the resistance of 5 Ω as shown in the redrawn figure .

Now the equation for loop one is given by :

$V_1 = 10x I_1 + 5 (2 I_1) = 20 I_1$ and $V_1/I_1 = 20Ω$

$$\therefore \; \mathbf{V_1/I_1} \; \big|\mathbf{I_2}{=}0 \; = \; \mathbf{Z}_{11} = 20Ω$$

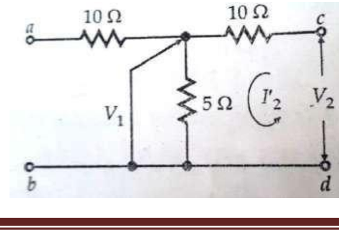
Next the equation for loop two is given by :

$V_2 = 10x I_1 + 5 (2 I_1) = 20 I_1$ and $V_2/I_1 = 20Ω$

$$\therefore \; \mathbf{V_2/I_1} \; \big|\mathbf{I_2}{=}0 \; = \; \mathbf{Z}_{21} = 20Ω$$

2). Next we will find out Z_{12} and Z_{22} which are given by the common condition $I_1 = 0$ (input open circuited)

With this condition the circuit is redrawn as shown below.



Now since the current I_1 is zero ,the current source of I_1 would no longer be there in the output loop and it is removed as shown in the redrawn figure. Further since input current $I_1= 0$,there would be no current in the input side 10Ω and the same current I_2 only flows through common resistance of 5 Ω and output side resistance of 10 Ω .With these conditions incorporated, now we shall rewrite the two loop equations (for input V_1 and output V_2) to get ***Z_{12}* and *Z_{22}***
Equation for loop one is given by :

$V_1 = 5 I_2$ and $V_1/I_2 = 5\Omega$

$$\therefore \ \mathbf{V_1/I_2} \ \Big|_{\mathbf{I_1}=0} = \mathbf{Z}_{12} = 5\Omega$$

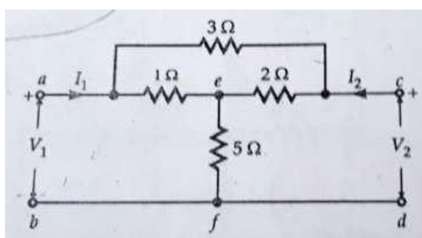
And the equation for loop two is given by:

$V_2 = 10 \times I_2 + 5 \times I_2 = 15 I_2$ and $V_2/I_2 = 15\Omega$

$$\therefore \ \mathbf{V_2/I_2} \ \Big|_{\mathbf{I_1}=0} = \mathbf{Z}_{22} = 15\Omega$$

Finally: $\mathbf{Z}_{11} = 20\Omega$; $\mathbf{Z}_{12} = 5\Omega$; $\mathbf{Z}_{21} = 20\Omega$; $\mathbf{Z}_{22} = 15\Omega$

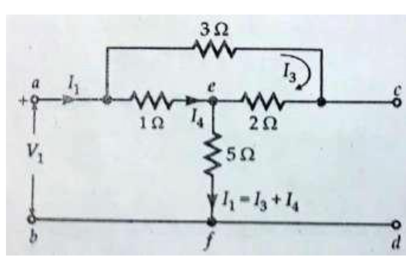
Example 4: Obtain the open circuit parameters of the Bridged T network shown in the figure below.



Open circuit parameters are same as Z parameters.

1)*. *We will first find out Z_{11} and Z_{21} which are given by the common condition $I_2 = 0$ (Output open circuited)

With this condition the circuit is redrawn as shown below.



From the inspection of the figure in this condition it can be seen that (since I_2 is zero) the two resistances i.e the bridged arm of 3Ω and output side resistance of 2Ω are in series and together are in parallel with the input side resistance of 1Ω.

Hence the loop equation for V_1 can be written as:

$V_1 = I_1 x [(3+2) \| 1 + 5] = I_1 x 35/6$ and $V_1/I_1 = 35/6$

$$\therefore \ \mathbf{V_1/I_1 \ |_{I_2=0} \ = \ Z_{11} = \ 35/6\Omega}$$

Next the loop equation for V_2 can be written as :

$V_2 = I_3 x2 + I_1x5$

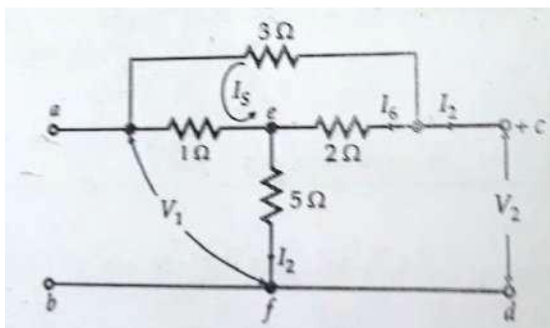
But we know from the principle of current division that the current $I_3 = I_1 x [1/(1+2+3)] = I_1 x 1/6$

Hence $V_2 = I_1 x 1/6 x 2 + I_1x 5 = I_1 x 16/3$ and $V_2 / I_1 = 16/3 \ \Omega$

$$\therefore \ \mathbf{V_2/I_1 \ |_{I_2=0} \ = \ Z_{21} = \ 16/3 \ \Omega}$$

2). ***Next we will find out Z_{12} and Z_{22} which are given by the common condition $I_1 = 0$ (input open circuited)***

With this condition the circuit is redrawn as shown below.



From the inspection of the figure in this condition it can be seen that (since I_1 is zero) the two resistances i.e the bridged arm of 3Ω and input side resistance of 1Ω are in series and together are in parallel with the output side resistance of 2Ω. Further $I_2 = I_5 + I_6$

Hence the loop equation for V_1 can be written as :

$V_1 = I_5 x1 + I_2x5$

But we know from the principle of current division that the current $I_5 = I_2 x [2/(1+2+3)] = I_2 x 1/3$

Hence $V_1 = I_2 x 1/3 x 1 + I_2x 5 = I_2x 16/3$ and $V_1 / I_2 = 16/3 \ \Omega$

$$\therefore \ \mathbf{V_1/I_2 \ |_{I_1=0} \ = \ Z_{12} = \ 16/3 \ \Omega}$$