CAPACITOR

CAPACITANCE OF A CONDUCTOR: when a conductor is given a charge, its electrical potential rises in proportional to the charge given. Thus, if a charge q raises the potential of the conductor V, then

$q \propto V$ or $q = CV$ Where C is a constant depending upon the shape and shize of the conductor, the surrounding medium and the presence of other conductor, the surrounding medium and the presence of other conductor nearby the constant C is known as the capacity of the conductor. $C=q/V$, if $V=1$ $Q=C$

Hence the capacity of the conductor is equal to the charge required to raise the potential of the conductor by 1 volt.

Unit of Capacitance: Units of capacitance are called 'Farad '

1 Farad =1coulomb/volt

$$1\text{ farad} = \frac{3\times10^{9}\text{stat coulomb}}{\frac{1\text{ stat volt}}{300}} = 9\times10^{11}\text{ stat farad}$$

Since farad is very big unit. Its smaller fractions are used in practice.

1 microfarad (1μF) = 10^{-6}F, 1picofarad (1pF)= 10^{-12}F

Dimension of capacitance: $[M^{-1}L^{-2}T^{4}A^{2}]$

DIELECTRIC: It is defined as insulator which do not conduct electricity by induced charges are produced on its faces when placed in a uniform electric field.

DIELECTRIC CONSTANT: It is defined as the ratio of the capacitance of the capacitor with medium between the plates to its capacitance with air between the plates.

ELECTRICAL CAPACITANCE: It is defined as the ability of the conductor to store electric charge.

PRINCIPLE OF CAPACITOR: A capacitor works on the principle that the capacitance of a conductor increases appreciably when the earthed conductor is brought near it. Thus a capacitor has two plates separated by a distance having equal and opposite charges.

When the temperature is raised, the induced electric field decreases and hence the dielectric constant decreases.

Leaf Electroscope: It is an instrument used to detect the presence and nature of charge on a body and also to detect electrical potential difference.

CAPACITOR: A capacitor is an arrangement which can store sufficient quantity of charge. Suppose, on giving a charge q to a capacitor the electric potential of the capacitor becomes V. then the capacitance of the capacitor is $C=q/V$

Thus, the capacitance of a capacitor is defined as the ratio of the charge given to a plate of the capacitor to the potential difference produced b/w the plates. In fact, every conductor can be assumed as a capacitor whose other plate is at infinity.

TYPES OF CAPACITORS

CAPACITANCE OF A SPHERICAL CONDUCTOR: The lines of force emerging from sphere are everywhere normal to the charge surface (Fig. 1), that is, they appear diverging from the Centre along the radii. Therefore in order to determine the potential at any point on the surface of the sphere, or outside the sphere, we can assume the +q to be concentrated at the Centre O of the sphere is

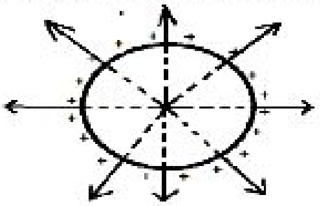

Fig.1.

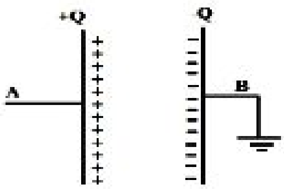

Fig.2.

$V = \frac{1}{4\pi\epsilon_0}\frac{q}{a}$ volt

The capacitance $C= q/V$……………………………………………………………(1)
Substituting the value of V in equation 1 we get

$C= q\Big/\left(\frac{1}{4\pi\epsilon_0}\frac{q}{a}\right)=4\pi\varepsilon_0 a$ for air; $C= 4\pi K\varepsilon_0 a$ for dielectric medium

This is the formula for the capacitance of a spherical conductor. It shows that the capacitance of a spherical capacitor is directly proportional to the radius of the conductor.

PARALLEL PLATE CAPACITOR: Consider a parallel plate capacitor having two plane metallic plates A and B, placed parallel to each other the plates carry equal and opposite charges +Q and −Q respectively as shown in fig 2.

In general, the electric field between the plates due to the charges +Q and −Q remains uniform. If the separation between the plates is much smaller than the size of the plates, the electric field strength between the plates may be assumed uniform.

Suppose A be the area of a parallel plate capacitor and 'd' be the separation between the plates, K dielectric constant of medium between the plates. If σ is the magnitude of surface charge density of plates, then.

$\sigma= Q/A$

The field strength between the plates $E = \frac{\sigma}{K\epsilon_0}$

The potential difference between the plates $V_{AB} = E.d = \frac{\sigma d}{K\epsilon_0}$

Putting the value of σ, we get $V_{AB} = E.d = \frac{(Q/A)d}{K\epsilon_0}$

Capacitance of the capacitor $C = \frac{Q}{V_{AB}}$ or $C = (K\epsilon_0 A)/d$ for air $C = (\epsilon_0 A)/d$

The capacitance of parallel plate capacitor is

(i) Directly proportional to the area of each plate
(ii) Inversely proportional to the distance between the two plates
(iii) The medium between the two plates

CAPACITANCE OF A PARALLEL PLATE CAPACITOR PARTLY FILLED WITH DIELECTRIC:

Suppose a slab of thickness t of some dielectric substance is placed between the two plates of a capacitor as shown in fig.3 let K be the dielectric constant of the slab, d be the distance between the plates and t be the thickness of the dielectric substance kept between the plates then the distance between the plates in air will be (d-t) the intensity of electric field in air between the plates is $E_1 = \frac{q}{\epsilon_0 A}$ and in the dielectric slab is

$E_2 = \frac{q}{K\epsilon_0 A}$ the field E_1 between the plates exists in the distance (d-t) and E_2 in the distance t. hence if the potential difference between the plates be V, then

$$V = V_0 + V_t$$

$$= E_1(d-t) + E_2 t$$

$$= \frac{q(d-t)}{\epsilon_0 A} + \frac{qt}{K\epsilon_0 A}$$

$$V = \frac{q}{\epsilon_0 A}\left[(d-t) + \frac{t}{K}\right]$$

$\therefore$ capacitance of the capacitor is $C = \frac{q}{V}$

$$= \frac{q}{\frac{q}{\epsilon_0 A}\left[(d-t) + \frac{t}{K}\right]}$$

$$= \frac{\epsilon_0 A}{\left[(d-t) + \frac{t}{K}\right]}$$

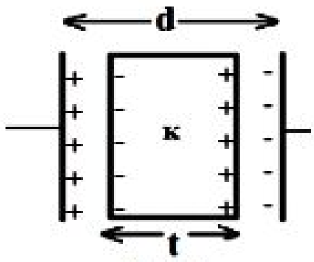

Fig.3

Special cases

(i) if the whole space between the plates be filled with the dielectric substance i.e. t=d then the capacitance will be $C = \frac{\epsilon_0 A}{\left[(d-d)+\frac{t}{K}\right]} = \frac{K\epsilon_0 A}{d}$

(ii) if there be vacuum (or air) in the whole space between the plates (t=0) then the capacitance will be $C = \frac{\epsilon_0 A}{d}$.

(i) If there be a slab of metal ($K=\infty$) of thickness t between the plates, then the capacitance will be $C = \frac{K\epsilon_0 A}{(d-t)}$.

(ii) If the several slabs of dielectric constant K_1, K_2, K_3 ...... and respective thickness t_1, t_2, ... be placed between the two plates then, the capacitance will be

$$C = \frac{\epsilon_0 A}{\left[d-(t_1+t_2+t_3......)+\frac{t_1}{K_1}+\frac{t_2}{K_2}+\frac{t_3}{K_3}....\right]}$$ But $d = (t_1+t_2+t_3......)$

then $C = \frac{\epsilon_0 A}{\left[\frac{t_1}{K_1}+\frac{t_2}{K_2}+\frac{t_3}{K_3}....\right]}$

SPHERICAL CAPACITOR: This capacitor is formed of two concentric hollow metallic spheres A and B as shown in fig. 1 which do not touch each other anywhere. The outer sphere B is connected to the earth. The inner sphere A is connected to a thin metallic rod at the outer end of which is attached a knob. The rod remains emerged above the outer sphere but remains insulated from it. The space between the two spheres is usually filled with some dielectric substance.

Let the radius of sphere A be **a** and that of the sphere B be **b**

The dielectric constant of the medium between the two spheres be K.

Since sphere A is completely surrounded by sphere B. −q charge appears on the inner surface of B and +q charge appears on the outer surface of B. since the sphere B is connected to the earth, the +q charge on its outer surface flows into earth. Thus potential of the sphere A will be due to its own charge q. and due to the charge − q on the inner surface of the sphere B. now the potential on the surface A due to its own charge is $V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{a}$ volt

At every point inside a sphere the potential will be same as on its surface. Therefore inside the sphere B, the potential at every point will be - $\frac{1}{4\pi\varepsilon_0}\frac{q}{b}$ the sphere A is inside sphere B. therefore potential on the sphere A due to the charge on the sphere B is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{b}$$

Resultant potential on sphere A is

$$V=V_1+V_2 = = \frac{1}{4\pi\varepsilon_0}\frac{q}{a} - \frac{1}{4\pi\varepsilon_0}\frac{q}{b}$$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{b-a}{ab}\right)$$

The sphere B is connected to the earth, that is, it is at zero potential. Hence the potential difference between the sphere A and B is

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{b-a}{ab}\right)$$

Therefore, the capacitance of the capacitor is $C=q/V$

$$C = \frac{q}{V} = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$$

** NOTE: If the inner sphere is earthed and the outer sphere is charged, then the capacity of condenser becomes

$$C = 4\pi\varepsilon_{0b} + 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$$

USES OF CAPACITOR

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……………………..

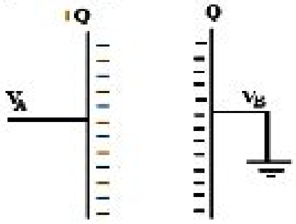
POTENTIAL ENERGY OF A CHARGED CAPACITOR

The energy of a charged capacitor is measured by the total work done in charging the capacitor to a given potential.

When a capacitor is charged by a battery, work is done by the charged battery at the expense of its chemical energy. This energy is stored in the capacitor in the form of electrostatic potential energy.

Let at any instant when charge on capacitor be q, the potential difference between its plates

V= q/C. Now work done in giving an additional infinitesimal charge dq to capacitor



$$dW = V\,dq = (q/C).dq$$

The total work done in giving charge from 0 to Q will be equal to the sum of all such infinitesimally work, which may be obtained by integration. Therefore total work

$$W = \int_0^Q V\,dq = \int_0^Q \frac{q}{C}dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{Q^2}{2C}$$

If V is the final potential difference between capacitor plates, then Q = CV

$$W = (CV)^2/2C = \frac{CV^2}{2} = \frac{QV}{2}$$

This work is stored as electrostatic potential energy i.e. $U = \frac{Q^2}{2C} = \frac{CV^2}{2} = \frac{QV}{2}$

Energy Density: Consider a parallel plate capacitor consisting of plates each of area A,, separated by a distance d. if space between the plates is filled with a medium of dielectric constant K, then capacitance of capacitor $C = \frac{K\in_0 A}{d}$

If σ is the surface charge density of plates, the electric field strength between the plates $E = \sigma/(K\varepsilon_0)$ or $\sigma = K\varepsilon_0 E$

Charge on each plate of capacitor $Q = \sigma A = K\varepsilon_0 EA$

Energy stored by capacitor $U = Q^2/2C = \frac{(K\in_0 EA)^2}{2\,(K\in_0 A/d)}$ **OR** $U = \frac{K\in_0 E^2 Ad}{2}$

Volume of space between capacitor plates = Ad

Energy stored, $U = \frac{K\in_0 E^2 \text{ volume}}{2}$

$$u = \frac{U}{V} = \frac{K\in_0 E^2}{2}$$

Electrostatic Energy stored per unit volume $u_e = \frac{U}{\text{Volume}} = \frac{K\in_0 E^2}{2}$

This is expression for electrostatic energy density in medium of dielectric constant K.

REDISTRIBUTION OF CHARGES

Suppose two insulated conductor A and B of capacitances C_1 and C_2 are given charges q_1 and q_2 respectively and raised to potential V_1 and V_2 respectively. Then $q_1=C_1V_1$ and $q_2=C_2V_2$.

Now, if the two conductors be joined by a thin wire, then the positive charge begins to flow from the conductor of higher potential to the conductor of lower potential till their potential become equal. In other words, on joining the conductors, the charges on them are distributed although the total charge remain conserved (i.e. q_1+q_2)

The combined capacitance will be C_1+C_2 Now if after the redistribution of charges the common potential of the conductors is V, then

$$V = (\text{Total charge})/(\text{Combined capacitance})$$
$$= (q_1+q_2)/(C_1+C_2) = (C_1V_1+C_2V_2)/(C_1+C_2)$$

Suppose, after redistribution, the charge on A is q_1' and on B is q_2'. Then

$q_1' = C_1V$ and $q_2' = C_2V$

Charge flow through the connecting wire is $q_1 \sim q_1'$ or $q_2 \sim q_2'$

$$q_1 \sim q_1' = C_1V_1 - C_1V$$
$$= C_1\left[V_1 - \frac{C_1V_1+C_2V_2}{C_1+C_2}\right]$$
$$= \frac{C_1C_2(V_1-V_2)}{(C_1+C_2)}$$

On connecting two charged conductors, the redistributed charges on them are in the ration of their capacitances.

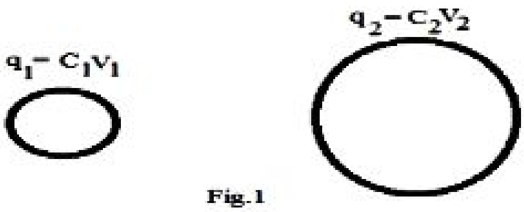


Fig.1

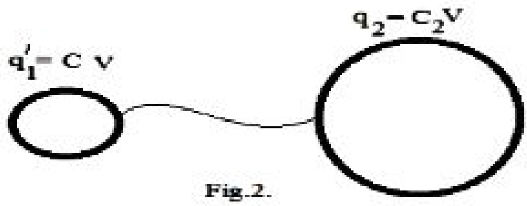


Fig.2.

LOSS OF ENERGY IN REDISTRIBUTION OF CHARGES

When charge flows from a conductor at higher potential to a conductor at lower potential, some work is done in this process. Although the total quantity of charge on the two conductors remains the same, but their total potential energy decreases. Before connecting we have

Potential energy of the first conductor= $\frac{1}{2}C_1V_1^2$

And potential energy of the second conductor = $\frac{1}{2}C_2V_2^2$

Total potential energy of the two conductors is $U = \frac{1}{2}(C_1V_1^2 + C_2V_2^2)$

After connecting the two conductors, their combined capacitance become $(C_1 + C_2)$ and

common potential V. hence after connecting, the potential energy is

$$U' = \frac{(C_1+C_2)V^2}{2} = \frac{(C_1+C_2)}{2}\left(\frac{C_1V_1+C_2V_2}{C_1+C_2}\right)^2$$

The loss in energy is

$$U - U' = \frac{1}{2}(C_1V_1^2 + C_2V_2^2) - \frac{(C_1+C_2)}{2}\left(\frac{C_1V_1+C_2V_2}{C_1+C_2}\right)^2$$

$$U - U' = \frac{C_1C_2(V_1-V_2)^2}{2(C_1+C_2)}$$

Since C_1 and C_2 are both positive and $(V_1-V_2)^2$ being a square term, is also positive. Thus, in redistribution of charges there is always a loss of energy the reason is that in redistribution of energy the charge flow through the connecting wire as a result, some energy lost in the form of heat.

EFFECT OF INTRODUCING DIELECTRIC BETWEEN THE PLATES OF THE CONDENSER

Any medium is made up of molecules or atoms. In atoms, the positive charge is concentrated at the nucleus and electrons (negative charge) revolve around it. In dielectric material the electrons are rigidly bound to the nucleus. When a dielectric material is placed between the plates of the capacitor then the nuclei of its molecules are displaced towards the negative plate and electrons towards the positive plate as shown in fig.A thus one end of each molecule of dielectric becomes negative charge and molecule is said to be polarized. These charges are opposite to the charges of the corresponding plate of the capacitor. Thus an electric field E' is produced within the dielectric which is opposite to the field E due to the charges on the plates. Hence due to the presence of the dielectric the field between the plates is reduced.

The potential difference between the plates is correspondingly reduced (V=Ed) that is, the capacitance of the capacitor increase (C=q/V).
It is found that $C_{med} = K\,C_{air}$, $V_{med} = V_{air}/K$, $E_{med} = E_{air}/K$

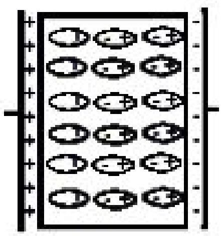


Fig. A

COMBINATION OF CAPACITORS

When there is combination of capacitors in a circuit we can replace the combination with an equivalent capacitor.

An equivalent capacitor, is a single capacitor that has the same capacitance as the actual combination of capacitor.

There are two types of combinations of condenser

CAPACITOR IN SERIES COMBINATION: Capacitor are said to be connected in series between two points if it is possible to proceed from one point to other point along only one path as shown in fig.1

In fig1. Three capacitors of capacitance C_1, C_2, C_3 are connected in series between points A and D.

In series first plate of each capacitor has charge +Q and second plate of each capacitor has charge −Q i.e. charge on each capacitor is Q

Let the potential difference across capacitors C_1,C_2 C_3, be V_1,V_2,V_3 respectively, as the second plate of the first capacitor C1 and the first plate of the second capacitor C_2 are connected together their potential are equal. Let this common potential be V_B.

Similarly the common potential of second plate of C_2 and the first plate of C_3 is V_C.

The second plate of C_3 is connected to earth therefore its potential $V_D=0$.

As charge flow from higher potential to lower potential, therefore $V_A>V_B>V_C>V_D$

For first capacitor $V_1=V_A-V_B=Q/C_1$ (1)

For second capacitor $V_2=V_B-V_C=Q/C_2$ (2)

For third capacitor $V_3=V_C-V_D=Q/C_3$ (3)

Adding equations (1), (2) and (3) we get

$$V_1+V_2+V_3=V_A-V_D=\left[\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\right]Q \quad (4)$$

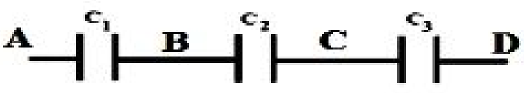


Fig. 1

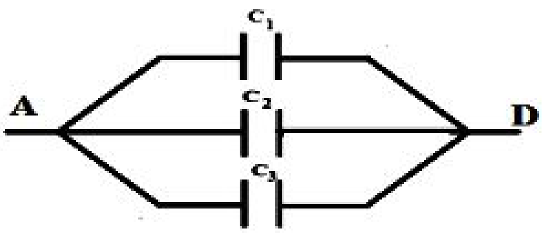


Fig. 2

If V be the potential difference between A and D then. $V_A-V_B=V$ from equation(4) we get

$$V=\left[\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\right]Q \quad (5)$$

If in place of all capacitors, only one capacitor is placed between A and D such that on giving it charge Q, the potential difference between its plates becomes V, then it will be called **equivalent capacitor**. If its capacitance is C, then $V=Q/C$ (6)

Comparing equation (5) and (6) we get.

$$\frac{Q}{C} = \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]Q$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Thus in series arrangement," the reciprocal of equivalent capacitance is equal to the sum of the reciprocals of the individual capacitors.

CAPACITOR IN PARALLEL COMBINATION:

Capacitors are said to be connected in parallel between two points if it is possible to be proceed from one point to another point along different path as shown in fig. 2
In parallel the potential difference across each capacitor is same V. Obviously the potential difference between plates of each capacitor $V_B - V_A = V$
The charge Q given to capacitor divided on capacitors C_1, C_2, C_3

Let q_1, q_2, q_3 be the charges on capacitors C_1, C_2, C_3 respectively then $Q = q_1 + q_2 + q_3$ (a)
$q_1 = C_1V$, $q_2 = C_2V$, $q_3 = C_3V$
Substituting the value of q_1, q_2 and q_3 in (a)
$Q = C_1V + C_2V + C_3V$
$Q = (C_1 + C_2 + C_3)V$ (b)

If, in place of all the three capacitors is replaced by one capacitor of capacitance C between A and B. such that on giving it charge Q, the potential difference between its plates be V, then it will be called equivalent capacitor. If C be the capacitance of equivalent capacitance, then $Q = CV$ (c)
Comparing equation (b) and (c) we get.
$CV = (C_1 + C_2 + C_3)V$
$\mathbf{C = (C_1 + C_2 + C_3)}$

Note: * in parallel combination, V would be the same but the charge would be divided.

*IN SERIES COMBINATION: The potential difference would be divided. However, Q would remain the same

*Negative plate of one capacitor is connected to positive plate of the other and so on.

*IN PARALLEL COMBINATION: The Q would be divided. However, potential difference would remain the same

*positive plate of all the capacitors are connected to one common terminal and the negative plates are connected to other common terminal.

ENERGY STORED IN SERIES AND PARALLEL COMBINATION:

(a) **SERIES COMBINATION:** consider a series combination of n capacitor of capacitance $C_1, C_2, C_3 \ldots\ldots C_n$. if Q is the charge and C is the equivalent capacitance then the electrostatic potential energy of the series combination is given by.

$$U = \frac{Q^2}{2C} \text{ or } U = \frac{Q^2}{2}\left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\ldots\ldots\frac{1}{C_n}\right) \quad \text{OR} \quad U = \frac{Q^2}{2}\frac{1}{C_1}+\frac{Q^2}{2}\frac{1}{C_2}+\frac{Q^2}{2}\frac{1}{C_3}\ldots\ldots\frac{Q^2}{2}\frac{1}{C_n}$$

$$U = U_1 + U_2 + U_3 \ldots\ldots\ldots U_n$$

Where $U_1, U_2, U_3 \ldots\ldots\ldots U_n$ are the energies of the individual capacitors.

(b) **PARALLEL COMBINATION:** consider a parallel combination of n capacitors of capacitances $C_1, C_2, C_3 \ldots\ldots C_n$. If C be the equivalent capacitance and V is the voltage then the electrostatic potential energy of the parallel combination is given by

$$U = \frac{1}{2}CV^2 = \frac{1}{2}V^2(C_1+C_2+C_3\ldots\ldots+C_n)$$

$$U = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + \frac{1}{2}C_3V^2\ldots\ldots + \frac{1}{2}C_nV^2$$

$$U = U_1 + U_2 + U_3 \ldots\ldots\ldots U_n$$