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A model is presented for the glacial transport of basal till material. In Finland, the till blanket typically consists of 1—2 basal till beds, whose thickness is generally 1—3 m. After a lithologic contact in provenance bedrock, the basal till layer is renewed over a distance of 1—10 km in the transport direction. Renewal begins from the bottom of the layer, where the till material eroded from the proximal rocks is gradually replaced by material from the distal rocks. The renewal distance is dependent upon both rock type and area so that short distances are observed above brittle (felsic) rock types and in the central areas of an ice sheet. In the lower parts of the till layer, material has been transported for a shorter distance than in the upper parts. Hence, the till material derived from a dike source forms a ribbon of dike material that slowly rises in the transport direction from the source towards the till surface. The mean abundance of dike material in the till bed reaches its maximum close to the distal contact of the dike, whereafter the abundance asymptotically decreases towards zero. The maximum abundance of dike material in till increases with the width of the dike source. In the case of two basal till layers the material is of more local provenance in the lower layer than in the upper layer.

These geologic observations can be explained with the aid of a 2-dimensional transport model. The mean thickness of basal till layers and their average renewal distance give the estimate 2 m × 10000 m for the dimensions of the model, which are small in comparison to the 1000 km extent of ice sheets. The glacier substratum is assumed to be even and the eroded material is kept as a compact layer during transport at or below the base of the glacier. The model does not describe mixing of till material during erosion and transport, and deposition of till is simply modelled by stopping the basal transport. The main components of the model are the creep of ice, the basal sliding of an ice sheet and erosion of the substratum due to sliding. The ice moves together with the basal debris in a horizontal direction at the combined velocity of creep and basal sliding. The only force in the model enabling the movement of till material in the vertical direction is the action of newly eroded material pushing the previously loosened grains upwards.

The model can be used to predict the source location of different parts of till layers, and also to estimate the amount of till material derived from source zones of varying widths. These applications are demonstrated with the aid of geochemical examples covering the design of sampling and the interpretation of anomalies. The model considerations emphasize the significance of proper sampling, as the dispersion pattern of anomalous source material in till is strongly affected by the length and depth of till samples. The boulder trains on the ablation till surface cannot be explained by the transport model of basal tills, but the transport mechanism of surface boulders is qualitatively examined because of its importance in exploration. The long transport distances commonly observed for boulders are explained by source hills in bedrock, which can feed the boulders to elevated levels in the ice where the glacial flow velocity is high.

Key words: models, glacial transport, till, applications, geochemical methods, boulder trains, Finland

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1. INTRODUCTION

The interpretation of boulder trains and geochemical anomalies in till has had, and still has, important implications for exploration. It is surprising, therefore, that no transport models have been developed in order to facilitate such interpretations. Boulton (1984) presented a theoretic model that describes sediment dispersal by large ice sheets. The model is realized as a complicated computer simulation with a calculation point interval of 25 km. As such, the model cannot be used to compute the dispersion of Finnish basal tills, because the till material is mainly found within 10 km from its bedrock provenance, as demonstrated by Quaternary geologic data.

Geologic observations on the dispersion of glacial drift have been gathered during the past hundred years or more. The lithologic, mineralogic and chemical compositions of the source bedrock and till have been compared in several studies. These studies have led to qualitative understanding of the transport process of till material. It has been possible to estimate the transport distances in different areas and for different grain sizes of till. It has also been noticed that the amount and distribution of source material in till depend on the size and rock type of the source area, on the thickness of till blanket and on whether the samples are taken from the surface or bottom of till bed.

However, the elaboration of geologic data into a transport model has been limited to fitting mathematical curves to the data points. It has been observed that the abundance of source material in till decreases on the distal side of bedrock sources. Lines and exponential curves have been fitted to these declining abundance values, and the parameters of such curves have been compared for various till fractions, source rock types and research areas. The curve most commonly fitted is probably that of a negative exponential function known in Quaternary geology as Krumbein's exponential curve. The parameter that is most often estimated from Krumbein's (1937) curve is called the half-distance value. This distance is measured from the distal source contact to the till site where the abundance of source material in till is reduced to one half of its value at the distal contact.

Geologic studies of glacial dispersion have continued actively, and in recent years the subject seems to have been particularly favoured by the authors of academic theses in Finland. During the last ten years at least 5 master's theses and 3 doctoral theses have been published that are dealing with the subject. Perttunen (1977) studied the glacial transport distances by comparing the lithologic compositions of till and underlying bedrock. Salminen (1980) investigated the transport process in terms of the interpretation of geochemical anomalies in till. Salonen (1986) considered the transport of surface boulders and introduced a new method for boulder studies.

In the following, first the abundant geologic data are reviewed and then some typical cases are described. The geologic data are summarized into statements that must be satisfied by glacial transport models. A simple transport model is then derived starting from glaciologic concepts, and the model is tested against the geologic data. The ranges of the glaciologic model parameters are established, and the behaviour of the model is examined at realistic parameter values. The transport distances and times predicted by the model are compared with those determined geologically. The significance of sampling to the interpretation of transport profiles is studied with the aid of the model. Some examples are given to demonstrate the application of the model to the design of geochemical sampling and the interpretation of boulder trains. Finally, the variation of transport conditions in different parts of the continental ice sheet is discussed, and the limitations and possible future improvements of the model are considered.

2. GEOLOGIC REVIEW

The present study deals mainly with the basal till material, although the ablation tills are also considered in the context of boulder tracing. According to Dreimanis (1976), these types of till can be described as follows. Basal till is deposited underneath a slowly moving glacier, which makes it compact and gives it a strongly developed fabric. The basal till often exhibits fissility, layering and shear structures. Both basal and ablation tills are unsorted, although the basal tills are, on average, richer in fines than the ablation tills. The ablation tills, which deposited from melting ice-sheets, may have become depleted in fines as a result of washout. The ablation till is usually loose, coarse-textured and weakly oriented.

In Finland the till cover is composed predominantly of 1—2 basal till beds, whose thickness usually varies between 1 and 3 meters. In many places the basal till is overlain by a layer of ablation till, mostly less than 1 m thick. In areas of weak glacial erosion in Lapland there are places where up to 5 superimposed till beds are found (Hirvas & Nenonen, 1987). The till cover in Finland rests on bedrock, which is fairly even on average, the local relief being less than 20 m in large areas (Soveri & Kauranne, 1972). The relief is small compared with the more than 1 km thickness of ice sheets, but large in relation to the average 2 m thickness of the till cover.

As emphasized by Lundqvist (1935) and Virkkala (1960), topographic elevations have markedly affected the transport of till material. Compared with the surrounding sediment layers the exposed knobs of bedrock may form source contrasts in terms of both topography and composition (Pulkkinen *et al.*, 1980). The summits of big hills may feed material to higher levels in the ice sheet, where the material is transported more rapidly. As a consequence, the material loosened from hill tops at the initial stage of glaciation can be transported exceptionally faraway. On the other hand, a boulder dislodged from a hill at the end of glaciation may deposit in ablation till very near the hill. Within ice sheets the hill material can also be spread perpendicular to the main flow direction, as the ice flows around hills (Lundqvist, 1935). It is thus difficult to predict the transport distance of the ablation till material that has been eroded from the slopes of hills.

Glacial transport of till manifests itself in different ways, depending on the grain size and the locations of the studied samples. The various till fractions are not transported as such but undergo changes during transport. The transport reduces the coarse material through comminution and grinding with the result that the proportion of fines increases. The till material that was dislodged as stones and boulders, may thus move as coarse fraction for the first part of the transport and as finer fraction towards the end. As a consequence, the coarse till material is found closer to the source than the fine fractions, suggesting that fines were transported for longer distances. This is the most commonly reported relationship between the transport distances of different fractions (Hellaakoski, 1930; Peltoniemi, 1985). Some geochemical studies have also suggested longer transport of the fine fractions (Kauranne, 1959), but the most common observation in geochemical studies is that the fines of till are local in origin compared with the coarse material (Kauranne, 1977; Salminen, 1980; Lehmuspelto, 1987).

Observations on the relative transport distances of different fractions are thus contradictory, which can possibly be attributed to variations in erosion and comminution processes. The coarse material plucked from the source is ground into finer material during transport so that fines are fed into the till even at some distance from the source. In this manner the background portion of fine till fractions is formed. Fine-grained material is also produced directly by abrasion of sources. These abraded fines are added to the background and their dispersion in till reflects the true transport of source material. A geochemical anomaly in the fine till fractions is probably more local if the fines were directly abraded from a source than if they were derived through comminution of the coarse fractions. The proportion of source material in the coarse till fractions declines with distance from the source, first, because of comminution and, second, because the absolute amount of source material is decreasing. Consequently, the composition profiles of the coarse till fractions may give too low estimates for the total transport of till material.

The rate of glacial erosion is affected by the strenght of bedrock as well as by the hardness of the rock material in the glacier sole. When a glacier flows from a hard rock area to a soft one, conditions favour intense erosion. The glacier sole is armed with hard rock particles, which rapidly abrade the soft rock (Sugden & John, 1976). The erosion rate of bedrock depends on the degree of fracturing and the abrasion resistance of source rocks. The glacier easily incorporates rock fragments from loose and fractured hard rock types, whereas softer rocks are more readily eroded by abrasion. In Iceland a marble plate placed beneath a thin glacier (15—40 m) was abraded at about 3 times higher rate than the harder basalt plate (Boulton, 1974). Laboratory tests show that heavy basic rocks are tough and resist abrasion and impacts better than light acid rocks, which are weak (Kauranne, 1970).

Apart from topography, source rock type and till fraction, the transport distance estimates are also affected by the vertical location of samples in the till layer, the thickness of the layer and the position of the layer in the stratigraphy. In general the lower part of a till bed is richer in local rock types than its upper part. As early as 1924 Sauramo reported on this observation, which has since been verified in many geochemical studies (Kauranne, 1959; Nurmi, 1976). It has also been noted (Hirvas *et al.*, 1977) that in areas of thin till cover the proportion of local rock types is high and that the transport distances are thus shorter than in areas of thick overburden. In the case of two till beds, the material is of more local origin in the lower bed than in the upper (Sauramo, 1924; Hirvas *et al.*, 1977; Peltoniemi, 1981). Although many factors affect transport distances and their estimation, studies undertaken at various sites in Scandinavia during a long period have consistently shown that the bulk of till material has been transported less than 10 km from its source.

This transport distance of 10 km implies that till is very local in character, when compared with the continental ice sheet dimension of 1000 km. The local origin of stones and boulders in till was noted in geologic studies already at the beginning of this century both in Finland (Sauramo, 1924; Hellaakoski, 1930) and in Sweden (Lundqvist, 1935). This finding has since been verified by numerous studies, in which the transport of various till fractions has been examined with the aid of stone counts, mineral analyses and geochemical investigations. Virkkala (1971) and Perttunen (1977) in Finland and Gillberg (1965) and Lindén (1975) in Sweden have studied glacial transport by comparison of the lithologic compositions of till and bedrock. Mutanen (1971) and Salonen (1986) have investigated the transport of surface boulders of till. Geochemical studies have mainly concentrated on the transport of fine till fractions, which has also been compared with the transport of coarse till material (Salminen, 1980; Lehmuspelto, 1987).

The geologic studies allow us to generalize that till is local in origin, but there are variations in the degree of localness, depending on the study area and the till fraction. The shortest transport distances (less than 100 m) have been observed for the fine fractions of basal tills in Finnish Lapland (Lehmuspelto, 1987). The ice divide was located in central Lapland, where the basal sliding of ice sheet was slowest and the erosion weakest. Long transport distances of basal till have been reported from southern Sweden (Gillberg, 1965), where the gravel fraction of till was studied. In this fraction the abundance of source material is reduced to half at a distance of 10—20 km from the distal contact of the source area. Accurate estimation of transport distances must be based on a precise definition of the concept. The literature contains numerous suggestions for definitions.

The Krumbein's (1937) half-distance value that was defined in chapter 1, is perhaps the most frequently used measure of transport distance. Puranen and Kivekäs (1979) defined what they called shift distance, which is measured between the proximal contact of the source and the point at which the source material content in till reaches 2/3 of its maximum value. Salminen (1980) defines the transport distance as the distance between the proximal contact of the source and the maximum point of the geochemical anomaly in till. Peltoniemi (1985) defines the renewal distance, which is measured between the proximal contact of the source and the point at which the abundance of source material in till reaches one half of its maximum value. These new measures of transport are all based on the rising proximal part of the abundance curves of source material in till. The new measures differ from each other only in the percentages that define the end point of the measure. An essential difference between Krumbein's half-distance and the other distance estimates is that the former is determined from the declining distal part of the abundance curves and the latter from the rising proximal part of the curves.

There is a basic difference between the proximal and distal parts of the abundance curves. The rising proximal part mainly reflects the dislodgement and transport of material, whereas the decreasing distal part characterizes comminution and transport. The proximal part of the curve is located over the source, where the comminution of newly eroded material has hardly started. Therefore, the transport manifests itself more purely in the proximal part than in the distal part of the curve. In the case of small point-like sources or narrow dike sources the steeply rising proximal part of the abundance curve may be located between sampling points, which means that only the distal part is available for transport estimates. Similarly, when till transport is studied close to the distal contact of extensive source formations, only the distal part of the abundance curve can be used in estimation. The studies published on transport distances often mainly examine the distal part, perhaps due to the above reasons. It is also true, however, that at the contact of two rock types one of the rocks produces the decreasing distal part of the abundance curve, while the other rock type generates the rising proximal part.

The above summary will be elaborated in the following by treating separately the transport of three fractions, i.e. boulders, stones and fines. The studies on boulder transport have the longest history going back more than one hundred years. The sources of the boulders have been searched for in the course of exploration activities and Quaternary geologic studies, and particularly in the former case the source formations are usually small. In studies on boulder trains, often only the absolute number of boulders is reported. In some cases the proportions of the boulders of certain rock types have also been determined per weight, volume or surface units (Lundqvist, 1935). The determination of the proportions is more time-consuming, but is recommended in terms of transport modelling. In general, the boulder studies deal only with the surface boulders, which may represent ablation till but may also be from basal till. This uncertainty hampers the modelling of boulder transport.

During ore exploration work boulders have been found on the surface of till only a few hundred meters from the source on the one hand, and as far as tens of kilometers from the source on the other (Hyvärinen *et al.*, 1973). The proximal boulder of a boulder train is most often found 100—1000 m from the source, and the distance appears to increase with increasing thickness of the till cover (Hirvas, pers. comm., 1987). On average, about 15 % of the surface boulders have been transported less than 1 km, and about 90 % less than 10 km. These percentages are from the comprehensive study of large areal coverage conducted by Salonen (1986, p. 48). At individual sites the values may vary greatly, depending on the till type and the study area, but the majority of surface boulders (diameter exceeding 200 mm) are local and have been transported less than 10 km.

The glacial transport of the stone fraction (20—200 mm) of basal tills has been studied mainly with the aid of stone counts carried out during basic research of Quaternary geology. In older studies, the stones of till were counted in pits dug manually with spades. First the altered surface horizon was penetrated to a depth of 0.5—1 meters, where the counting was started in the fresh basal till. The pit was then deepened until the desired total number of stones was reached. The stones counted in this way represented a depth interval of a few tens of centimeters. In more recent studies, tractor excavators have often been used to dig a pit through the till bed. The proportions of various rock types are then calculated from the excavated till pile, and the mean lithologic composition of the vertical till section is thus estimated. The stone count procedure is seldom described, with the consequence that the length of the stone count sample and the corresponding depth interval are unknown. Further, it is often difficult to determine accurately the till type, the topography of bedrock and the boundaries of source formation.

The glacial transport of stones is illustrated below by graphs with examples selected from the literature. The areal distribution of the example cases is shown in Fig. 1. On the basis of the study site and the source type, the examples have been divided into cases of proximal contact, distal contact and dike sources. Fig. 2 shows that 50 % of the stones in till are replaced by new rocks over a renewal distance of 1—10 km from the proximal contact of a source. Light and brittle granitic rocks seem to be transformed into till at a higher rate than dark and tough mafic rocks, which is consistent with the laboratory tests conducted by Kauranne (1970). According to Fig. 3, the abundance of source rock type is reduced to half at a distance of 1—8 km from the distal contact of the source. The abundance of felsic rock types declines more slowly than that of mafic rocks, suggesting that the granitoids withstand glacial transport better than the basic rocks. Whereas it seems to be easy for the glacier to dislodge stone fraction material from the granitic bedrock, granitoid stones are not readily comminuted during transport.

Shown in Fig. 4 are the transport profiles of stones in till in the case of dike sources of varying thickness. In the profiles of a given area the maximum proportion of dike material in till increases with the dike width. The influence of the source width on the transport profiles has been emphasized by Peltoniemi (1985). The dike material in till is local as it attains the maximum abundance in stone fraction close to the distal contact of the dike. The fine-grained till deriving from dike and point sources is also deposited close to the sources, as indicated by Fig. 5 showing geochemical profiles of the P-fraction of till (grain size below 0.06 mm). The peaks of the geochemical anomalies are located less than one kilometer from the proximal contacts of the sources. Most local of all is the chromium anomaly at Vuonelonoja, reflecting the short transport distances at the ice divide in Lapland (Pulkkinen *et al.,* 1980).

All the examples given in Fig. 5 were studied by the Geochemistry Department of the Geological Survey of Finland, which treats till samples as follows (Gustavsson *et al.*, 1979). The samples are taken with percussion drills provided with a through-flow bit, which can deliver a sample length of 20—30 cm. The sampling depth measured from till surface is known but its position in the till stratigraphy is not. The P-fraction material is sieved from the samples and analysed on an emission spectrometer. The analytical data can also be compiled into vertical till cross-sections instead of profile presentation. This has been done for the zinc anomaly in P-fraction of till in Fig. 6, which is a simplified version of the results reported by Salminen & Hartikainen (1985) from the Ilomantsi area. The figure suggests that the dike material was transported for a shorter distance in the lower parts of the till bed than in the upper parts. In geochemical studies it must be separately established whether the dispersion of an element is due to glacigenic transport or to the more recent hydrogenic transport.

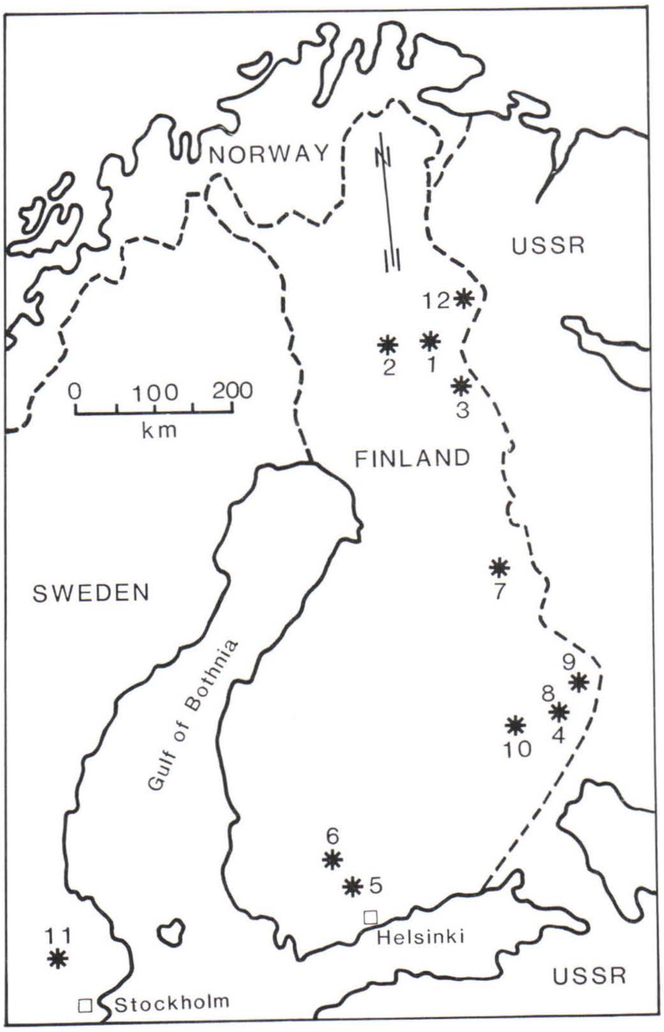

Fig. 1. Location of the example areas. 1 = Akanvaara, 2 = Aska, 3 = Hautajärvi, 4 = Heinävaara, 5 = Hyvinkää, 6 = Hämeenlinna, 7 = Näätäniemi, 8 = Palojärvi, 9 = Parissavaara, 10 = Petäinen, 11 = Uppsala, 12 = Vuonelonoja.

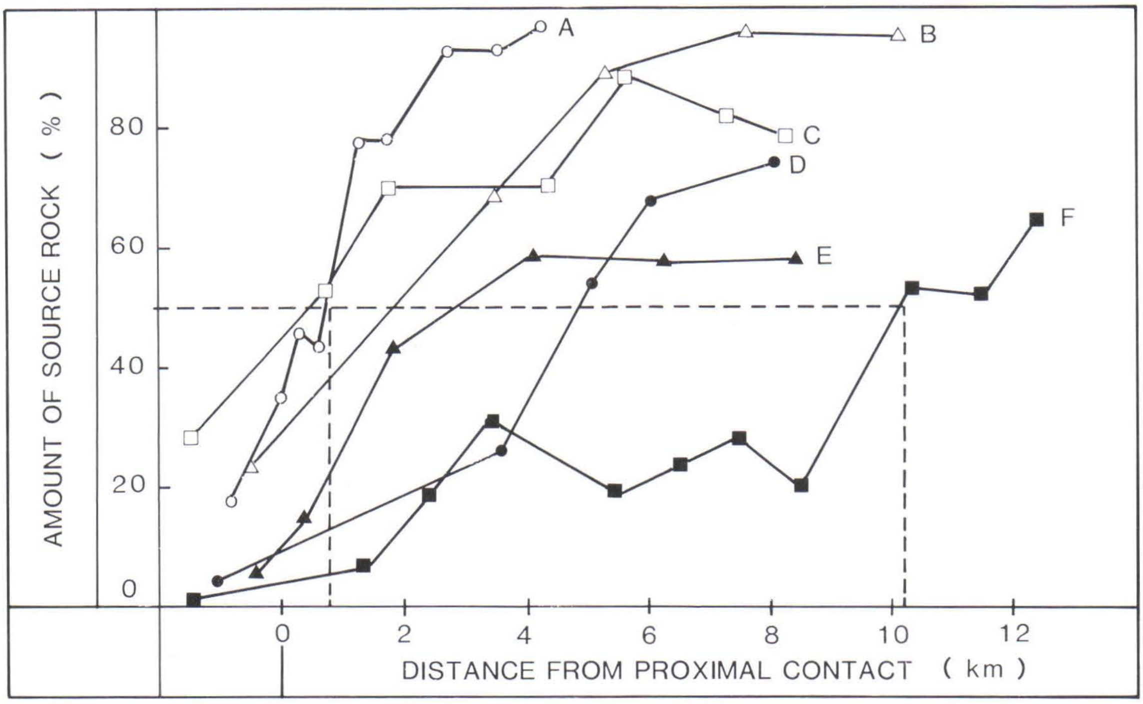

Fig. 2. Examples of the transport of basal till stones at the proximal contact of an extensive source area. The transport direction in the figure is from left to right. The renewal distances are shown with dashed lines. A) Till fraction 20—60 mm. Gneiss granite source in the Heinävaara area (Salminen, 1980). B) Till fraction 20—50 mm. Acid granite source in the Uppsala area (Lindén, 1975). C) Till fraction 20—200 mm. Granitoid source in the Hämeenlinna area (Perttunen, 1977). D) Till fraction ≥ 20 mm. Gabbro source in the Hyvinkää area (Virkkala, 1971). E) Till fraction 20—200 mm. Basic volcanite source in the Hämeenlinna area (Perttunen, 1977). F) Till fraction 20—60 mm (bed III). Basic volcanite source in the Hautajärvi area (Vehkaperä, 1976).

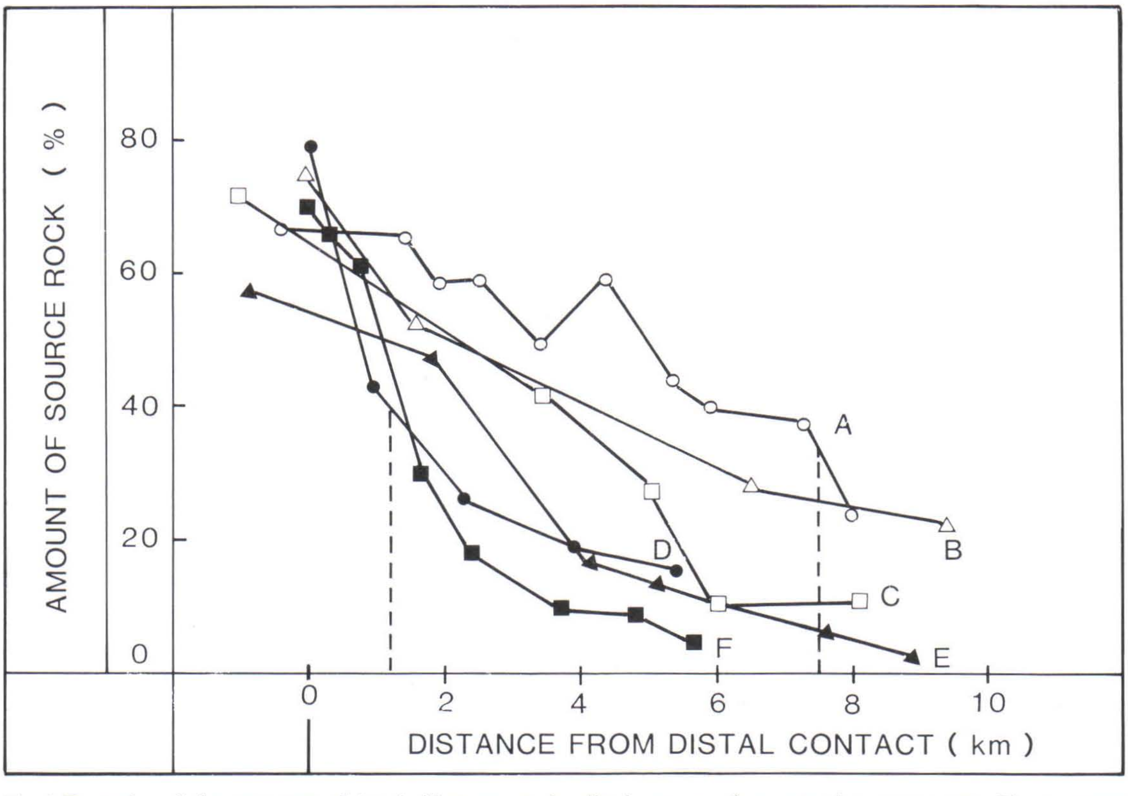

Fig. 3.Examples of the transport of basal till stones at the distal contact of an extensive source area. The transport direction in the figure is from left to right. The half-distance limits are marked with dashed lines. A) Till fraction 20—60 mm. Granitoid source in the Näätäniemi area (Saarnisto *et al.,* 1980). B) Till fraction 20—200 mm (bed III). Granite source in the Aska area (Hirvas *et al.,* 1977). C) Till fraction ≥ 20 mm. Granite source in the Hyvinkää area (Virkkala, 1971). D) Till fraction 20—200 mm (bed III). Gabbro source in the Akanvaara area (Hirvas *et al.,* 1977). E) Till fraction 20—200 mm. Basic volcanite source in the Hämeenlinna area (Perttunen, 1977). F) Till fraction 20—200 mm. Mica schist source in the Heinävaara area (Salminen, 1980).

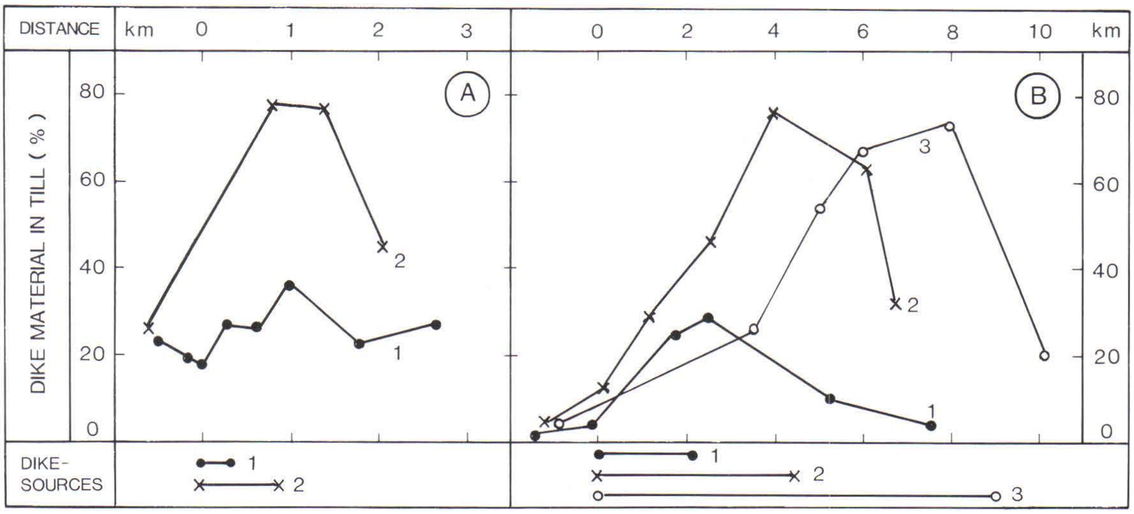

Fig. 4. Examples of the transport of basal till stones from a dike source. The transport direction in the figure is from left to right. A1, A2) Till fraction 8—20 mm. Amphibolite sources in granitoid environment in the Palojärvi area (Salminen, 1980). B1) Till fraction 20—200 mm. Mica schist source in granitoid environment in the Hämeenlinna area (Perttunen, 1977). B2) Till fraction 20—200 mm. Basic volcanite source in granitoid environment in the Hämeenlinna area (Perttunen, 1977). B3) Till fraction ≥ 20 mm. Gabbro source in granitoid environment in the Hyvinkää area (Virkkala, 1971).

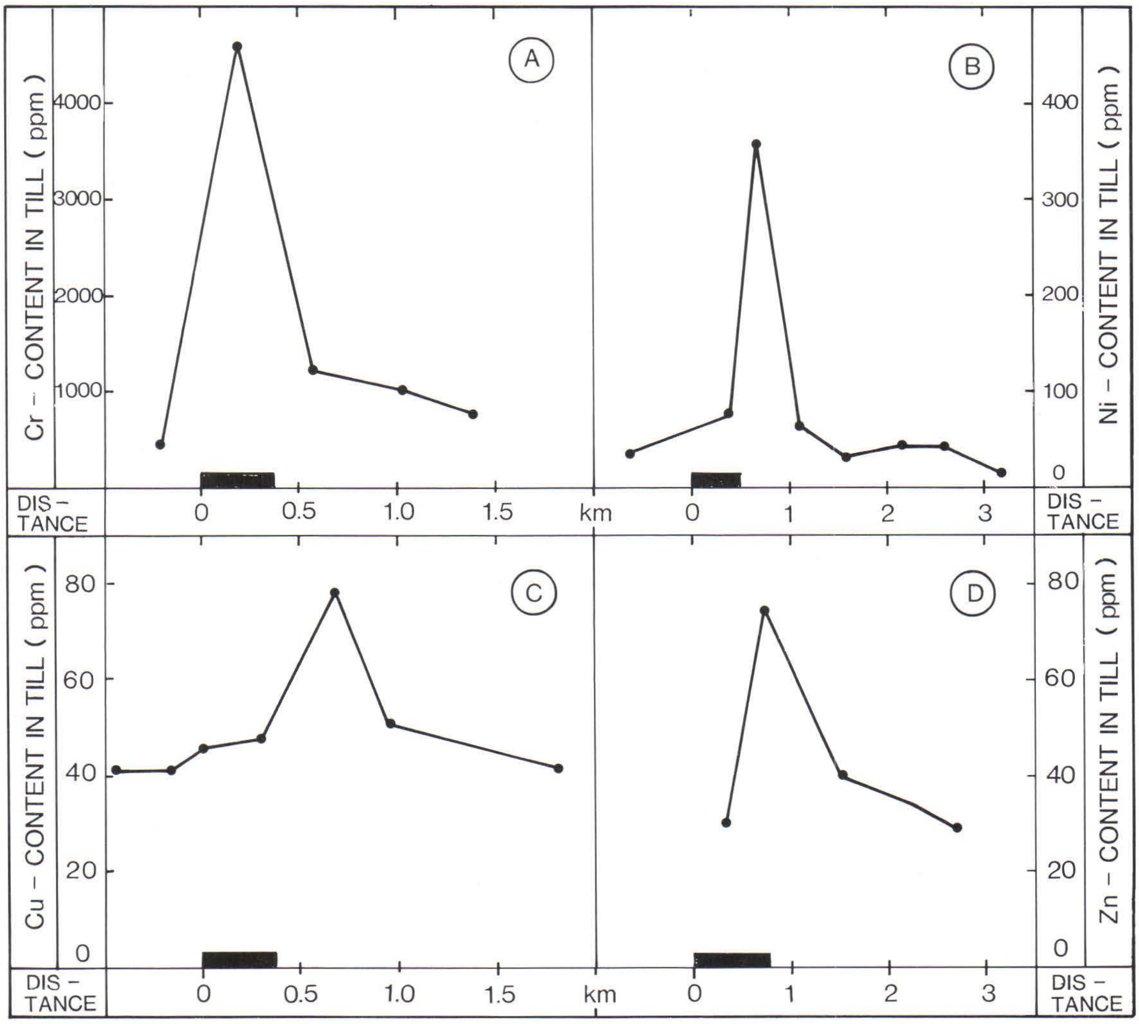

Fig. 5. Examples of the transport of P-fraction material (≤ 0.06 mm) in basal till from point-like and dike sources. The sources are marked in black and the transport direction is from left to right. A) The line averages of Cr-content in till in the Vuonelonoja area, the source being a peridotite body in gneiss granite (Pulkkinen *et al.,* 1980). B) The vertical averages of Ni-content in till in the Petäinen area, where the source zone is composed of the Outokumpu association rocks in mica schist environment (Salminen & Hartikainen, 1985). C) The Cu-content in till in the Palojärvi area. The source is an amphibolite zone in granitoid environment (Salminen, 1980). D) The vertical averages of Zn-content in till in the Parissavaara area, the source being a schist zone in granitoid environment (Salminen & Hartikainen, 1985).

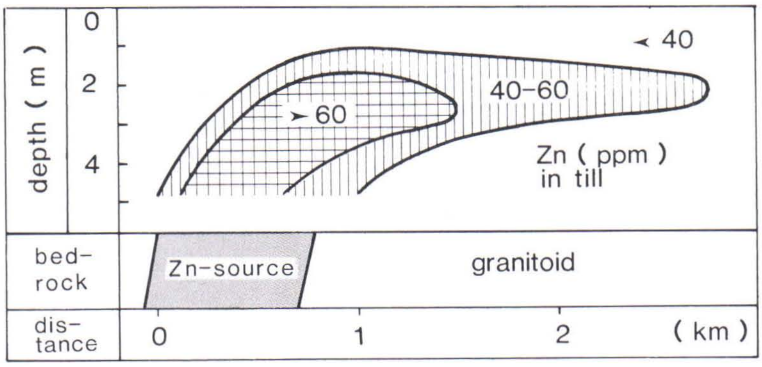

Fig. 6. Vertical cross-section of a zinc anomaly in the P-fraction of till (grain size ≤ 0.06 mm) at Parissavaara, Ilomantsi. Modified from Salminen & Hartikainen (1985).

3. OUTLINING THE MODEL

In the following a framework is outlined for a simple transport model, whose simplicity is based on the use of averages to describe the complex geologic reality. The model grows out of transport profiles of till that repeat themselves in a similar fashion from one area to the next, and whose characteristic features will be condensed below into statements. Let us first consider an example situation, in which a diabase dike is located in the middle of an extensive and flat granitic bedrock area. The dike is 100 m wide and trending perpendicular to the glacial flow direction. The diabase dike is assumed to be very long, thus allowing us to consider the transport as a 2-dimensional case. Let the bedrock be overlain by a typical basal till layer with an average thickness of 2 m. Let us further assume that an ice sheet produced the basal till only by dislodging, breaking and grinding the bedrock material without transporting it. Now granitic till material would only be found over granite, and the diabase material would have formed a till cake, 100 m wide and 2 m high, above the source dike.

However, from the observations cited in the previous chapter, we know that till material has been transported and that the bulk of it is now located at a distance less

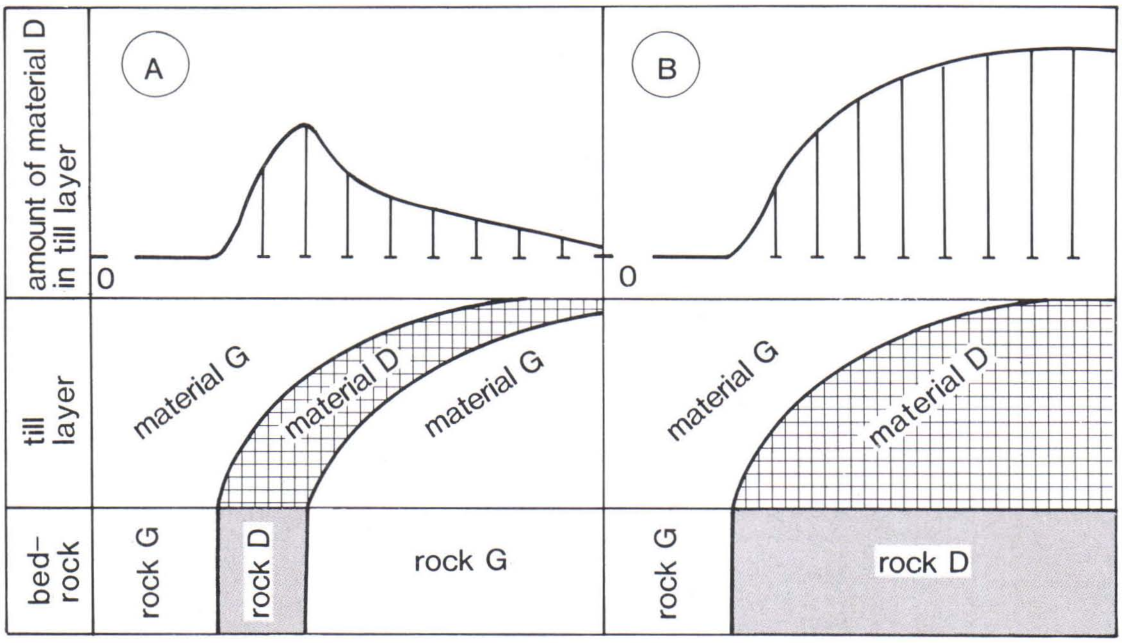

Fig. 7. Schematic diagram of the dispersion of basal till material from A) dike source and B) contact source.

than 10 km from its source. If we distribute the diabase cake evenly over a distance of 10 km, we will have a 2 cm high diabase layer in till. The height of the layer would constitute about one percent of the total thickness of till bed. Hence, within the limits of typical geologic variation, it would be difficult to notice the layer. The cake that developed above the diabase dike or the layer derived from the dike represent extreme cases, and the true distribution of diabase material in basal till lies between them. Numerous Quaternary geologic (Sauramo, 1924; Drake, 1983) and geochemical studies (Halonen, 1967; Nurmi, 1976; Miller, 1984) describe the dispersion of material from dike sources as anomalous ribbons, fans, surfaces or plumes that rise from the source towards the till surface at varying angles. To produce rising anomaly ribbons we need forces that lift the material and forces that transport it. The influence of the transporting forces has probably been greater than that of the intermixing forces, because anomalies have retained their regular shapes.

Sketched in Fig. 7A is an anomalous ribbon of till material rising from the dike source. In accordance with the geologic observations, the figure shows clearly that the anomaly is closer to the source at the base of the till than on its surface. The abundance profile in the figure demonstrates how the proportion of dike material in the vertical till cross-section varies in the transport direction. The abundance begins to rise at the proximal contact of the dike, reaching its maximum at the distal contact and declining thereafter. This behaviour of dike material is in harmony with the observations. Figure 7B illustrates in a similar fashion the case of a contact source formed by the boundary between two rock types. In fact, the contact case can be described with the aid of two extremely wide dikes, and the dike case can be constructed with the aid of two successive contacts. After these basic considerations we can start to define the model more accurately by condensing the observations cited in chapter 2 into the following *statements:*

1. The till cover in Finland rests on bedrock, which is topographically level when compared with the height of continental ice sheets, but uneven compared with the thickness of till cover.
2. The till cover is usually composed of 1—2 basal till beds, whose thickness is generally 1—3 m. In many places the basal till is overlain by a layer of ablation till, which is often less than 1 m thick.
3. The bulk of till has been transported less than 10 km from its source. After a contact source, the coarse material in basal till is renewed over a distance of 1—10 km.
4. The renewal distance of basal till is shorter in the lower parts of the till layer than in the upper parts.
5. The renewal distance of a thin till layer is shorter than that of a thick one.
6. The basal till is renewed more rapidly above brittle (felsic) rock types than above tough (mafic) rocks.
7. In the case of two till beds the renewal distance of the lower bed is shorter than that of the upper bed.
8. The proportion of dike material in basal till reaches its maximum close to the distal contact of the dike source, whereafter it decreases asymptotically towards zero.
9. The maximum proportion of dike material in till increases with the width of dike.
10. The transport distance of dike material is shorter in the lower parts of the till bed than in the upper parts.
11. The transport distance of till at the ice divide in Lapland is shorter than in southern Finland.
12. The majority of boulders on the surface of till cover have their source less than 10 km away.
13. The proximal boulder of the boulder trains is often found on till surface at a distance of 100—1000 m from the source.

The above observations were mostly obtained from investigation lines running in the glacial flow direction approximately perpendicularly over dike or contact sources. Consequently, the modelling will be started with a 2-dimensional case. In the modelling the glacier substratum is assumed to be level. In many areas of Finland the variation in the bedrock topography is considerable when compared with the thickness of till cover, even though the Finnish bedrock is level in relation to the height of continental ice sheets. The modelling deals first with a case, in which only one basal till layer is formed on the bedrock. In the multi-layer case the model can be applied to the lowermost till layer. Appendix A describes the expansion of the model to cover cases of two or more till layers.

Observation *statements* 2 and 3 limit the vertical dimension of transport modelling to a couple of meters and the horizontal dimension to about 10 km. In glaciology, physical models have been presented both for growth of large ice sheets (dimension >1000 km), and for erosion caused by small grains (dimension <1 m). Models with intermediate dimensions of about 10 km have not been presented, as pointed out by geologists in their inspiring dialogue with glaciologists (Hallet, 1981). Since the following model has a horizontal dimension of about 10 km, a geologically reasonable 1 % resolution is obtained with the model even if the distance steps are raised to 100 m. The effect of small roughness and heterogeneity of the source bedrock can thus largely be eliminated by averaging the model parameters over a distance of 100 m.

During glacial transport the till material also undergoes comminution, which transfers material from the coarse grain sizes to the finer fractions. Consequently, the abundance of coarse material from a dike source declines in till more rapidly than does that of the fines. Studies on coarse till fractions thus indicate transport distances that differ from those given by fines (Peltoniemi, 1985). If we want to determine the absolute movement of material in the glacial transport, we have to study unsieved till material. In this way we can establish the total transport of till material without the interference of comminution. By comparing the transport of unsieved material with the apparent transport of different fractions we can study the comminution process as well. The following model describes only the transport of unsieved till material.

After glacial transport has come to an end, the material may still move during postglacial time. A part of the material may go into solution and spread hydromorphically, whereas boulders are often moved as a result of frost heaving, and a whole formation may move by solifluction. Hence, postglacial processes may change the dispersion pattern produced by the glacier and hamper interpretations. In the modelling, however, the postglacial phenomena are omitted and their influence is not discussed until in chapters 9 and 10, which deal with the application of the model. The model describes only the glacigenic (mechanic) transport of basal till material. Therefore, the movement of glacier and the erosion due to it play a key role in the modelling. In the transport model, the reality is simplified by considering the average course of events. The simplification aims to clarify the major features of glacial transport.

4. DERIVING THE MODEL

The movement of an ice sheet can be described with the aid of creep and basal sliding. Creep is due to the gravity forces that tend to spread the glacier under its own weight towards its margins. If the base of glacier is frozen to the substratum, the creep is the only mode of movement in the glacier. Such is the situation in large areas of Greenland and Antarctica, where erosion is weak due to the lack of movement of the cold-based ice sheets. Erosion takes place only if the glacier sole moves in relation to its bed, as in warm-based glaciers. It is not easy to observe the basal sliding under the glacier, and so glaciologists have developed complex mathematical sliding theories (regelation, enhanced plastic flow) to describe the phenomenon. The theories of basal sliding and creep are excellently described in the textbook by Paterson (1981).

The creep velocity depends on the mass balance, thickness, surface slope and temperature of the glacier, as well as on the concentration of debris in the basal ice layers. The variation of horizontal creep velocity $U(z)$ as a function of distance z from the base of the glacier has been studied with the aid of borehole measurements. The velocity is highest on the surface of the glacier and decreases, at first slowly and then more rapidly, towards the bottom of the glacier. The following model assumes that in the basal parts of the glacier the velocity U increases linearly with the distance z, or $U(z) = k \cdot z$, which is a reasonable assumption according to observations. The proportionality constant k is the (engineering) strain rate defined by the partial derivative dU/dz. The basal sliding velocity of the glacier depends on the temperature and water conditions at the glacier base, the friction between the glacier base and the bed, and the roughness of the bed. In the following the velocity of basal sliding is assumed to be constant, and it is denoted with the symbol B. For the total velocity V of glacier motion we then obtain $V(z) = B + U(z) = B + kz$.

The erosion rate of the glacier bed depends on the properties of the glacier and the bed. The rate of erosion increases with the basal sliding velocity and the thickness of the glacier (effective normal pressure on the bed). Erosion rate is also dependent on the coarseness and hardness of the rock or soil types of the substratum. Another significant factor controlling the rate of erosion is the relative difference in hardness of the rock material in the glacier base and the bedrock. In his textbook of glacial geology Drewry (1986) gives a good summary of the detailed physical models proposed for the erosion process. The book also looks closely at the factors affecting erosion. The rate of erosion, which is assumed to be constant in the following transport model, is denoted with the symbol W. To be more accurate, the parameter W refers to the rate at which the thickness of the basal till increases, which is slightly higher than the erosion rate of the bedrock, because the bulk density of till is lower than the density of rock.

Let us consider the situation depicted in Fig. 8A, in which the glacier is flowing on an even substratum from left to right. The glacier bed is composed of two lithologic areas, whose contact is located in the middle of the figure at the origin of the coordinate system. The transport distance increases along the X-axis to the right, and the distance from the bottom of the glacier increases along the Z-axis upwards. Also drawn in the figure is a curve starting at origin and describing the transport path of a single rock particle dislodged from bedrock. Let us examine a differential movement of the particle along the curve at a distance z from the glacier base. During a time inverval, dt, the rock grain moves away from the source, covering a horizontal distance dx and a vertical distance dz. Because the glacier movement velocity at depth z is $V = B + kz$, we get

$$dx = (B + kz)dt \qquad (1)$$

During time dt a new layer of dislodged rock particles is formed under the glacier. This layer is incorporated into the base of the glacier, which forces the earlier eroded grains upwards from the base. The rock particle under consideration is thus moved upwards for a distance

$$dz = W dt \qquad (2)$$

From equations (1) and (2) we obtain

$$dx = (1/W)(B + kz)dz \qquad (3)$$

By integrating equation (3) we get for the path of the grain:

$$x = (B/W)z + \tfrac{1}{2}(k/W)z^2 \qquad (4)$$

or

$$x = z\cdot(B + \tfrac{1}{2}U)/W \qquad (5)$$

Equation (4) shows that the transport path is completely defined when ratios B/W and k/W are known. Inversely, from the observed path it is not possible to determine the parameters separately, but only the above ratios. Hence, the value of one of the

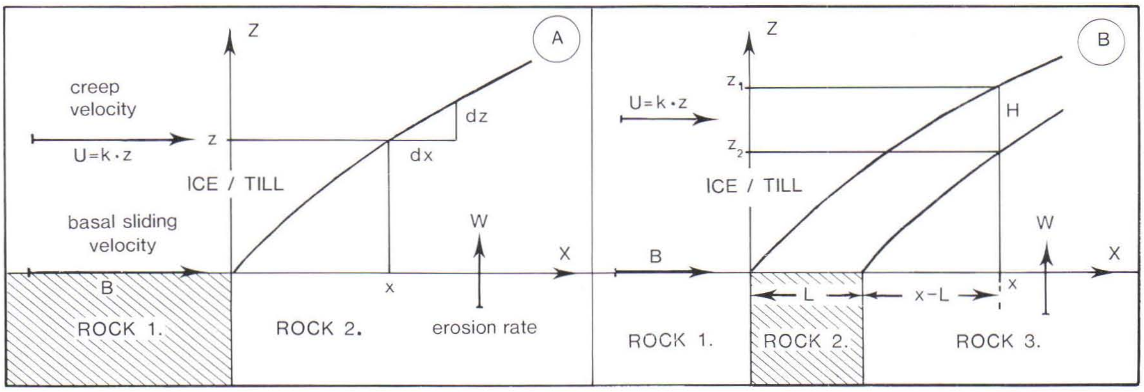

Fig. 8. Symbols used in the transport model of basal tills in the case of A) contact source and B) dike source.

parameters B, W or k can be chosen freely, which then fixes the values of the others. According to equation (5), the shape of the transport path depends on whether basal sliding (B) or creep (U) is dominating. The path approximates a straight line when basal sliding dominates and a parabola when creep dominates the glacial movement. When the equation (4) is solved for the depth z, we get

$$z = f(x) = -S + \sqrt{(S^2 + 2Wx/k)} \tag{6}$$

where

$$S = B/k \tag{7}$$

Equation (4) describes the transport path of one rock particle dislodged from bedrock. Since the particle derives from the contact of two rock types, the curve shows the boundary between the grain paths starting from rock 1 and rock 2. According to the model, the till above the boundary curve is totally composed of rock 1 and the till below the curve contains 100 % of rock 2. In reality, the boundary is less sharp as a result of mixing processes, which are not treated in the present study.

The contact situation described can also be interpreted as a dike case, in which rock 2 forms a dike with semi-infinite width. The case of a narrow dike is formed by two adjacent contacts that produce two successive contact curves as shown in Fig. 8B. These curves distinguish the dike material in till from the material derived from other rocks. Let the dike have a width L, and let us consider the situation at a distance x from the proximal contact of the dike. According to equation (6), the material from this contact is found in the till at the height $z_1 = f(x)$. The distance from the observation point to the distal contact is x—L, and material deriving from this contact is at the height $z_2 = f(x-L)$. Hence, the till material from the depth interval $z_1 - z_2$ originates from the dike. Since $z_2 = 0$ when $x \leqslant L$, the height H of the till column at the point x is

$$\begin{aligned} H &= z_1 && \text{when } x \leqslant L \\ H &= z_1 - z_2 && \text{when } x > L \end{aligned} \tag{8}$$

where z_1 and z_2 are obtained from equation (6) by giving values x and x—L to the transport distance.

When the width L of the dike increases, x—L is reduced as is also z_2, with the consequence that $H = z_1 - z_2$ increases. The absolute height of the till column, and hence the abundance of the dike material at point x, is proportional to the dike width at low values of L. To be accurate, equation (8) holds only when the erosion rate W has the same values for the country rock and the dike. If the erosion rates differ, the transport paths of the particles deriving from the dike and the country rock intersect, causing intermixing in till, particularly around the contacts. In general, local variations in the erosion rate constitute a disturbing factor in the model. The disturbance can be reduced by increasing the cell size in modelling, which stabilizes the erosion rate through averaging over larger units.

5. MODEL PARAMETERS

Textbooks of glaciology (Paterson, 1981) and glacial geology (Drewry, 1986) suggest the following ranges for parameters W, B and k, the accurate setting of whose limits is partly a matter of taste:

$$\begin{aligned} W &= (3\text{—}300)\cdot 10^{-12}\ \text{m/s} = 0.1\text{—}10\ \text{mm/a} \\ B &= (3\text{—}500)\cdot 10^{-8}\ \text{m/s} = 1\text{—}150\ \text{m/a} \\ k &= (0.3\text{—}30)\cdot 10^{-8}\ \text{s}^{-1} \end{aligned} \tag{A}$$

The parameters show intercorrelation, about which there is qualitative and semiquantitative information based on laboratory experiments and glacier observations. The relationships of the parameters are discussed in detail in the above textbooks. The erosion rate W increases with the basal sliding velocity of the glacier. The basal sliding

(B) and the creep (U) velocities both increase as the temperature of the glacier rises. When the concentration of debris at the base of the glacier increases, the strain rate k is reduced. At the same time the friction between the glacier base and the bed increases, causing a decrease in the basal sliding velocity B. We can thus generalize that the values of parameters W, B and k all increase or decrease together with each other.

Since the basal sliding velocity B and the creep velocity U are in the numerator of equation (5) and the erosion rate W is in the nominator, the positive correlation between the parameters tends to stabilize the transport estimates. Let us examine the model behaviour at actual values of the parameters. The following pairs of parameter values $W = 1$ mm/a and $B = 10$ m/a, $W = 4$ mm/a and $B = 20$ m/a, $W = 36$ mm/a and $B = 250$ m/a, have been measured on present-day glaciers of Iceland (Boulton, 1974). When the transport path described by equation (5) rises one meter from the substratum, the lower half of the average till cover in Finland is renewed. By introducing $z = 1$ m, the average strain rate $k = 3 \cdot 10^{-8}$ s^{-1} and the above pairs of parameter values into equation (5) we get the estimates 10.5 , 5.1 and 7.0 km for the renewal distance. All the calculated values are in harmony with the range of renewal distances (1—10 km) that was geologically established (*statement* 3). The calculated transport estimates are reduced only by one half although the parameter values are increased more than tenfold. This demonstrates well the tendency of the model to stabilize transport distances due to the correlation between the parameters.

The stabilization of transport could be a real property of ice sheets, which would explain the regularity of the geologically observed dispersion of till material. The dispersion profiles seem to repeat themselves similar in form even though the observations were made in different environments with diverse procedures. Because of the similarity of dispersion patterns, discrimination between various glacial provinces by means of the model may be difficult. On the other hand, the stabilized transport could make it easier to apply the model to the geochemical interpretation. The interpretation is mainly focused on predicting the source location (transport distance), which is not severely affected even if the model parameters are inaccurate. The stability of transport estimates can be disturbed by factors that are not described by the simple model. The model parameter W depends not only on the properties of the glacier, but also on the topography and the rock types of the glacier bed. The variations in erosion resistance of the substratum do not necessarily affect the glacial flow velocity, which would break the correlation between the model parameters. Therefore, different transport distances of till material can be observed over varying substrata.

Let us now consider the model parameters in terms of time. As starting values for the parameters we shall use the pair $W = 4$ mm/a and $B = 20$ m/a. At this erosion rate the material of a till cover averaging 2 m in thickness would be formed within 500 years, provided the process is continuous. During that time the material would have moved 10 000 meters as a result of basal sliding of the glacier. It has been estimated that the last Fennoscandian ice sheet existed about 20 000—30 000 years (Paterson, 1981; Boulton, 1984), of which 10 000 years could have elapsed before the ice sheet had grown to its full dimensions. Against this background the above time interval of 500 years appears short, implying that the W-value used may be too high. Longer times, e.g. 10 000 years, are obtained for the model transport process by reducing the erosion rate to $W = 0.2$ mm/a. This value is also within the range of the realistic values (A). When we combine a lower velocity of basal sliding $B = 1$ m/a with the longer time , the till material still has moved 10 000 meters owing to basal sliding. The lower B-value would be in the range of values observed below cold-based glaciers (Echelmeyer & Zhongxiang, 1987).

Of the pairs I: $W = 4$ mm/a, $B = 20$ m/a and II: $W = 0.2$ mm/a, $B = 1$ m/a, the latter is the more probable in terms of time. If the glacial activity slows down in parts of the ice sheet or the basal ice stagnates, the glaciation time and the transport time may differ from each other. Then the realistic parameter values would be between those of pairs I and II. Refining the limits of the parameter space can be continued by examining the distribution of basal till material. Both the renewal of till material after a contact source and the decrease of dike source material in till take place in an asymptotic manner (*statement* 8). The second power term in the model equation (4), which describes creep, gives the transport paths some asymptotic character. Hence, the correct form of transport paths can be obtained only when the basal sliding and creep velocities are approximately equal. At the mean strain rate $k = 3 \cdot 10^{-8}$ s^{-1}, we get $U = k \cdot z = 6 \cdot 10^{-8}$ m/s = 2 m/a for the creep velocity at depth $z = 2$ m. Therefore, model curves with the right form can be expected only if the basal sliding velocities stay below 1—10 m/a. This line of approach also suggests that the parameter pair II is closer to the true values than pair I.

When deriving the model it was assumed that the material eroded at the glacier base is incorporated into the base and thus moved by the creep of ice. However, the model can also be used if the eroded material is kept under the glacier and is there deformed by dragging. In principle, the model only requires that the erosion takes place at the interface between a solid substratum and the lowermost layer of dislodged material. In practice, the place of erosion and deformation should have an effect on the model parameters. Different parameter values are expected for situations, in which the dislodged material is within the glacier as basal debris, or under a warm-based glacier as water-saturated till or under a cold-based glacier as ice-laden drift. The creep of ice containing debris is the case most often discussed in textbooks, possibly due to the scarcity of subglacial observations. The subglacial deformation of till has been described in the literature only recently.

Direct observations on the deformation of till are restricted to the margins of thin mountain glaciers, where measurements have been made in excavated tunnels. In Iceland Boulton (1979) studied the deformation of a wet till layer beneath the margin of a retreating, temperate glacier. Probes inserted vertically into the till were bent forward into parabolas as a result of deformation, and about 90% of the total basal movement of the glacier could be ascribed to the deformation of till. Echelmeyer & Zhongxiang (1987) undertook studies on the deformation of ice-laden drift under the margin of a cold-based mountain glacier in China. About 60 % of the overall glacier motion could be attributed to the subglacial deformation of the frozen layer of till-like material. The movement of a thick ice stream in West Antarctica has also been largely attributed to the subglacial deformation of a water-saturated till layer (Alley *et al.,* 1986; Blankenship *et al.,* 1986). This interpretation of the situation beneath about 1-km-thick ice sheet, is not based on direct observations but on seismic reflection studies and on physical model calculations.

The relative importance of basal sliding and creep in transport process can vary during one glaciation cycle. Even this case can in principle be treated with the model by dividing the glaciation into shorter periods characterized by constant parameter values. Since the model and the observations can be considered as first-order approximations of reality, there are no grounds for a more accurate analysis of the parameters. In the following numerical examples the parameters will be given the set of values (P):

$$\begin{aligned} W &= 0.4\ \mathrm{mm/a} \\ B &= 1.0\ \mathrm{m/a} \\ k &= 3\cdot 10^{-8}\ \mathrm{s}^{-1} \end{aligned} \qquad \text{(P)}$$

At these values we get $x = 3.7$ km for the transport distance of the till material located one meter ($z = 1$ m) above the bottom of till bed. In other words, the lowermost half of a 2-m-thick till layer has been renewed after a transport of 3.7 km. According to Quaternary geologic data, the typical renewal distance varies between 1 and 10 km (*statement* 3). The distance calculated from the model lies within this range, when in the set (P) the erosion rate, for example, is kept within 1.5—0.15 mm/a and the other parameters are constant.

6. TESTING THE MODEL

While setting limits for the model parameters it was noticed that, in terms of transport distances, the model was consistent with the Quaternary geologic record (*statements* 3 and 12). In the following we shall examine whether the model behaviour is in accordance with the other observations cited in chapters 2 and 3. The equation for the transport path written in form (5) shows clearly how various factors affect the shape of the path. According to the contact case equation $x = z\cdot(B + \frac{1}{2}U)/W$, the transport distance x of a till grain increases with its distance z from the bottom of the till layer. Therefore, the transport and renewal distances of till are shorter in the lower parts of a till layer than in its upper parts (*statements* 4 and 10). This can be expressed in other words by stating that the renewal distance of a thin till cover is shorter than that of a thick one (*statement* 5) under similar conditions.

According to equation (5), the transport distance x is inversely proportional to the rate W of glacial erosion. At a high erosion rate (brittle substratum) a till grain rises, for example, two meters from the substratum after a shorter transport distance than at a lower rate (tough substratum). Consequently, the basal till is renewed rapidly when the glacier flows into an area that is composed of weak rock or soil types (*statement* 6). As indicated by equation (5), the transport distance of basal till is directly proportional to the velocities B and U of glacier movement. The flow velocities are lower in the central parts of the glacier than at its margins. The short transport distances at the ice divide in Lapland compared with those in southern Finland (*statement* 11) thus have a natural explanation in the model. Equation (5) can also be used to estimate the shape of the transport path. The path approaches a straight line when basal slide (B) predominates, and a parabola when creep (U) is the dominant mode of motion.

The case of a dike source is next considered with the aid of equation (8). The equation allows us to calculate the height H of the material column tansported to the distance x from the proximal dike contact. Let the width of the narrow dike be L, and the glacier parameters be expressed with symbol $S = B/k$. To facilitate the considerations the dike case equations are repeated below

$$\begin{aligned} H &= z_1 \quad \text{when } x \leqslant L \\ H &= z_1 - z_2 \quad \text{when } x > L \\ \text{where} & \\ z_1 &= -S + \sqrt{(S^2 + 2Wx/k)} \\ z_2 &= -S + \sqrt{(S^2 + 2W(x-L)/k)} \end{aligned} \tag{8b}$$

With increasing transport distance, the curves z_1 and z_2 approach each other continuously, as distance x becomes increasingly longer in relation to width L. The difference $H = z_1 - z_2$ is greatest at point $x = L$, where $z_2 = 0$. In other words, the proportion of dike material reaches its maximum in till at the distal contact of the dike, after which the proportion gradually decreases with increasing transport distance (*statement* 8). The maximum amount of dike material in till is obtained by equation

$$H = -S + \sqrt{(S^2 + 2WL/k)} \tag{9}$$

The value of this maximum increases with the width of the dike in accordance with the observations (*statement* 9). Since the maximum depends on the width L, the half-distance estimates based on the maximum also depend on the dike width. When the equation (9) is solved for the dike width L, we get

$$L = (BH + \tfrac{1}{2}kH^2)/W \tag{10}$$

This equation allows the estimation of source width L from the maximum amount H of dike material, if the glacier parameters k, B and W are known.

The above transport model can also be applied to the case of two superimposed till layers, provided that the glacial transport was in the same direction during the formation of each layer. The transport equation for this case is deduced in Appendix A. When a new glacier erodes and transports old till material, which has already been moved once, the transport distances from the original source become longer in the new (upper) till layer. The material of the upper layer may thus be transported up to twice as far from its bedrock sources as the material of the lower layer. This happens according to the model, when the new glacier erodes the whole lower layer and if the model parameters are the same for both glaciers. However, the erosion rate is probably higher when the substratum is composed of till instead of rock. According to equation (5), this shortens the transport distance of the upper till and the total distance will not be doubled. In any case, the new glacier elongates the dispersion of material in the upper layer compared with the lower layer, which is in accordance with the Quaternary geologic observations (*statement* 7). Hence, all the observations on basal till in chapter 3 can be explained by the transport model.

Let us finally consider the transport of surface boulders in the light of the model. The surface boulders can be regarded as the most coarse part of basal till. Hence, after a source contact in the bedrock the boulder fields should be lithologically renewed after a distance of about 1—10 km, as in the case of finer till material. The typical transport distances of boulders are indeed less than 10 km (*statement* 12), which is consistent both with the model and the transport of finer till fractions. In exploration work, however, it has been often noted (*statement* 13), that the proximal boulders of boulder trains are already on the till surface only a few hundred meters from the source. This clearly contradicts the transport behaviour predicted by the model. With the aid of the model and the parameter values (P) we can calculate that, after a transport of 600 m, the material should be located 0.2 meters above the bottom of a till bed and certainly not on the surface of the bed.

Frost may be one reason why the boulders are found on the till surface closer to the source than predicted by the model. Frost has had plenty of time to act since deglaciation. The effect of frost heave on the boulders in till has long been known and its mechanism has been described in textbooks (Okko, 1964). In many places the surface boulders are indeed part of the ablation material, and hence their transport is beyond the scope of the present model. The boulders are dislodged typically from the distal sides of rock knobs. If the knob rises out from basal drift, then boulders are plucked into the glacier clearly above the basal debris. The boulders plucked during final stages of glaciation are transported only for a short time. They are thus deposited close to the source on top of basal till together with ablation material. Except for the anomalous surface boulders, all the observations in chapter 3 are consistent with the model. Therefore, the first round of testing the model does not force us to abandon it.

When deriving equation (5) it was assumed that the glacial flow velocity and the erosion rate are constant during glacial transport. This is a reasonable assumption if the time increment dt is assumed to be so long that the parameter averages within the increments become stabilized. Owing to the assumption, equation (5) is independent of time. With the aid of known erosion rate and till thickness we can estimate the time that the glacier needed to form a till layer. If the renewal distance of the till layer is also known, the mean advance velocity of the glacier can be calculated. Should the calculated and observed velocities be inconsistent, then the assumption on constant parameter values must be questioned. In the previous chapter the model was examined from this angle in the context of parameter determinations.

The model describes the movement as a laminar flow that takes place along horizontal layers. This is compatible with the tightly packed horizontal layering often encountered in basal till. According to the model the youngest material that was last eroded is found at the base of the till layer. Hence, the model creates an inverted stratigraphy within a till layer, if the age of the till material is counted from the moment of its dislodgement instead of the deposition moment. In the model, deposition is simply treated as the peaceful end of transport, although in reality intermixing of material naturally takes place both during transport and deposition. However, it is likely that disturbances due to intermixing are moderate, because of the regularity that has preserved in the fabric and the dispersion of till material. The description of intermixing could be added to the model as either a statistical or a physical diffusion function. On the other hand, the influence of minor intermixing can be eliminated by studying the mean abundances of longer samples. The mean values give the same result that would be obtained by complete mixing of anomalous material into the samples.

7. SIGNIFICANCE OF SAMPLING

The transport model predicts the distribution of source material in the till layer relative to the source location. The material distribution is often presented with the aid of composition and abundance profiles. The profiles give a different picture of material distribution depending much on the depth and the length of samples. If the depths and lengths are not kept constant along investigation lines, the resulting profile is difficult to interpret because the abundance variation may be largely due to sampling instead of real transport. One of the key properties of a model is that it facilitates the planning of appropriate sampling. When examining till composition profiles it is often forgotten that in the vertical direction the events have taken place within a couple of meters, whereas in the horizontal direction the dimension is several kilometers. Consequently, a change of 10 cm in the vertical position of the sampling point may affect the shape of the profile more than a change of 500 m in the horizontal direction. In both cases the magnitude of the change corresponds to 5 % of the model dimensions, e.g. the mean till thickness (2 m) in the vertical and the transport distance (10 km) in the horizontal direction.

In the following the influence of sampling on the transport profiles is illustrated with examples. Let us consider the case of a contact between two rock types on the basis of transport equation (6). The till cover is assumed to be 2 m thick, and the parameter set (P) is used in the calculations. Shown in Fig. 9A is the influence of the sampling depth on the investigation lines running over the source contact, when a sample, 0.5 m long, is taken from four different depths. As shown by the profiles, the abundance of rock 2 starts to increase in till closer to the contact when the sample is taken deeper, which is in agreement with the geologic observations (*statement* 4). Figure 9B illustrates the effect of sample length on the abundance profile. The samples taken are 0.5, 1 and 2 m long, measured from the surface of till bed. The abundance of rock 2 starts to rise closer to the contact and more gradually, when the sample length is increased. It is obvious from the figures that information on the length and location of samples in the till bed is essential to the interpretation of a till profile.

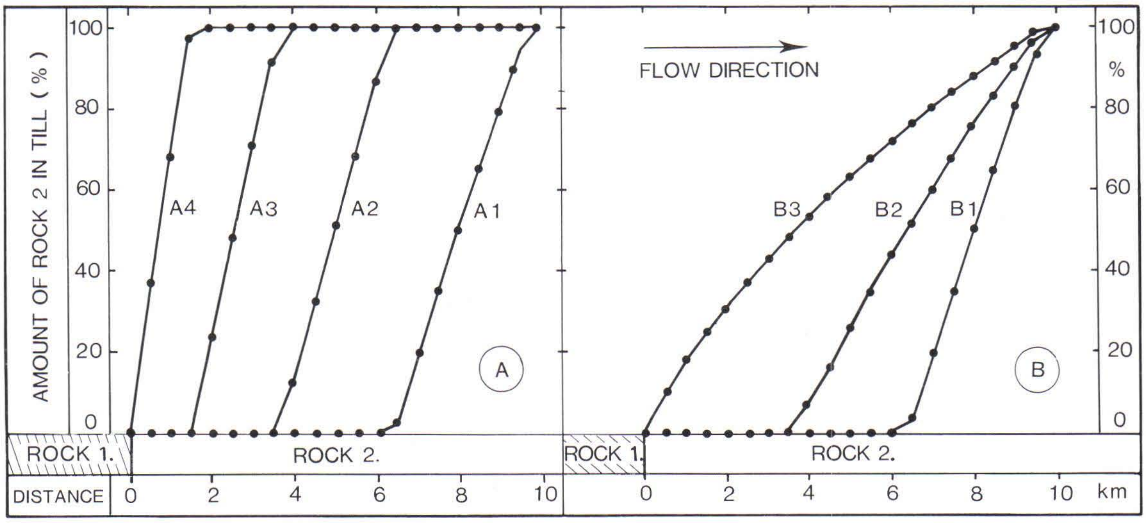

Fig. 9. The influence of (A) sampling depth and (B) sample length on a transport profile running over a lithologic contact, calculated from equation (6) with parameters (P) and assuming a till layer 2 m thick. A) The depth interval of samples measured from the layer surface is 0—0.5 m in profile A1, 0.5—1 m in profile A2, 1—1.5 m in profile A3 and 1.5—2 m in profile A4. B) The sample length measured from the layer surface is 0.5 m in profile B1, 1 m in profile B2, and 2 m in profile B3.

During exploration undertaken by the Geological Survey of Finland till samples are collected for stone counts and geochemical analyses. The samples are taken as mean samples that extend through the whole till layer. Sometimes a sample is divided into two subsamples to obtain separate means for the upper and lower parts of the layer. When the thickness of till layer is also determined, enough information is available to interpret the observations with the aid of the transport model. The use of mean samples has the drawback that we lose information on the vertical variation of till composition. This information would be needed to estimate the source location and width, which in principle is possible using the model (cf. chapter 8). However, the mean samples also have their virtues, for they smooth the effects of till intermixing and bring up the anomaly over larger areas (cf. chapter 9).

In the geochemical mapping undertaken by the Geological Survey of Finland samples are extracted by means of percussion drills equipped with through-flow bits. The bits collect a sample length of 20—30 cm, and the sample location is known only relative to the ground surface. When interpreting the glacial transport of till with the model presented, we should also know the distance of the sample from the base of the till layer. If the sampling is done properly and we get to know the model parameters for different conditions and areas, then the model allows us to estimate the source distance and width based on the observed location of anomalous material within till bed.

8. MODEL PREDICTIONS

In principle, the model presented predicts exactly from where to where each grain is moved by glacial transport. In practice, the model predictions are biased because the simple model cannot describe in detail the complex reality. Perhaps the greatest bias is caused by the roughness of bedrock topography. Therefore, the model should not be taken literally but rather as a conceptual framework to facilitate the planning of sampling and the interpretation of results. In the following, some cases will be treated with the aid of the model to illustrate its restrictions and its use for predictions. In the calculations, the glacier and its substratum are characterized by the parameter set (P). The computation results are shown as various curves, which might prove to be helpful also in practical interpretations. Further, the model is so simple that it can readily be programmed for a pocket calculator, making it easy to test the effect of different parameter values.

Let us consider the relationship between the vertical position of a till grain and the location of its source with the aid of equation (4), as illustrated in Fig. 10. If the grain is located, for example, at a height of 1, 2 or 3 meters from the base of the till layer, its source should be found at a distance of about 4, 10 or 18 kilometers, respectively. If again the till layer is within the height interval of 60—90 cm, its source area is expected at approximately 2—3 km from the observation site. According to the model, the anomalous ribbons of till material are suprisingly thin even when derived from sources that are 1 km wide. This feature of the model is illustrated in Fig. 11. If the source is, say, 100 m wide, the till ribbon it produces is merely 2—3 cm thick, which could well go unobserved in sampling. Widening the source to 800 m increases the height of till material produced only to 20—30 cm. A rule of thumb can easily be derived for the height estimates of till production. If a 2-m-thick till cover is completely renewed over a distance of about 10 km (*statement* 3), then a source, 10 km wide, has produced at its distal contact a 2 meters layer of till. Correspondingly, it can be deduced that a source, 1 km wide, generates a layer thickness of 20 cm, and a source, 100 m wide, a thickness of 2 cm. This is in accordance with the estimates presented in Fig. 11.

In exploration work the target zones of bedrock are often only tens of meters wide. From the above reasoning it follows that such narrow sources should produce anomalous till ribbons whose thickness is a few cm at best. In reality, these thin ribbons are spread out by intermixing during transport and deposition. Thin ribbon-like anomalies could be easily lost when applying the typical sample height of 20 cm, if the till section is covered by only a few samples that are placed at random. The intermixing of till material makes it easier to hit the anomalous layer. The long mean samples through till sections used in exploration guarantee that the anomaly will be hit. At the same time, however, the proportion of the anomalous layer is reduced in the sample, and the abundances may fall below the anomaly threshold. Longer samples in the vertical direction increase the probability of hitting the anomaly, but reduce the intensity

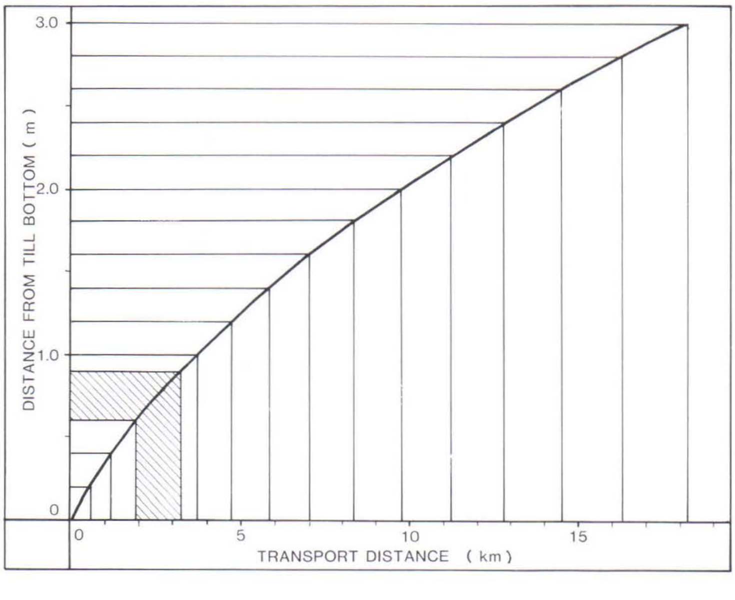

Fig. 10. Relationship between the vertical location of till material and its transport distance calculated from equation (4) with parameters (P).

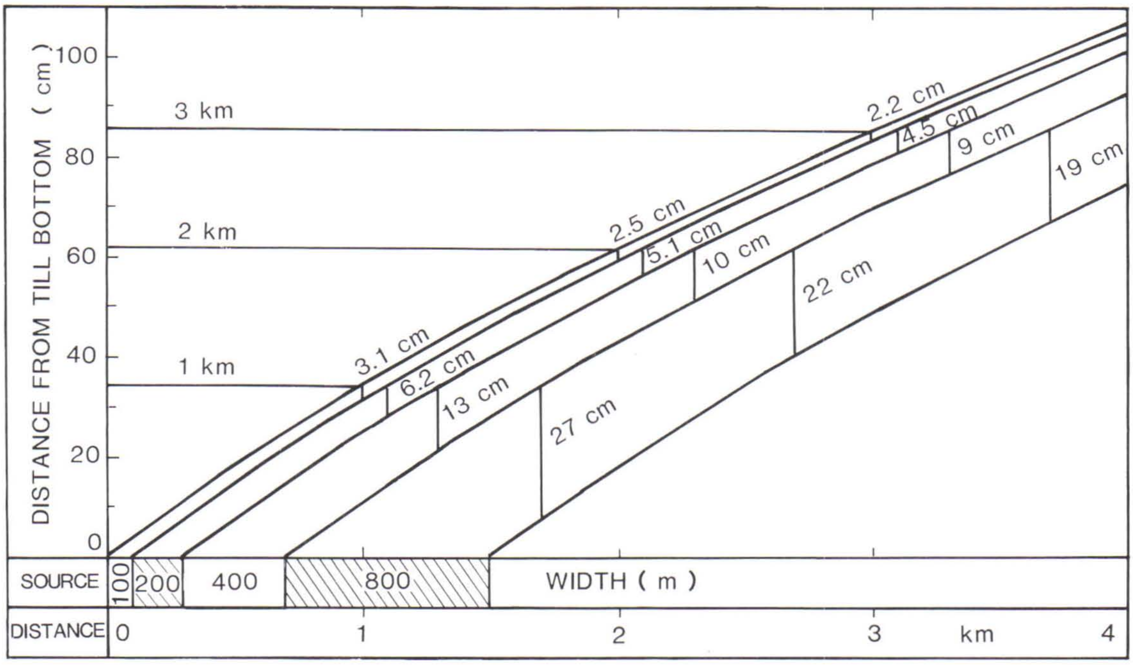

Fig. 11. Relationship between the source width and the height of the till layer derived from the source, calculated by equation (6) with parameters (P).

of the anomaly. Planning is needed to make the sampling optimal under any condition, and this is one of the application areas for the transport model (cf. chapter 9).

Vertical mixing of till material and averaging over vertical till samples have similar effects on transport profiles. In the former case a glacier and in the latter case man distributes the anomalous till material over a larger depth interval. In practice, all samples that are more than 20 cm long can be regarded as mean samples. If a sample is 20 cm long, the 2-cm-high till layer produced by a 100-m-wide dike forms only 10 % of the sample. The proportion of such layer of dike material decreases to 1 %, when estimated as the vertical average of a 2 meters till cover, as depicted in Fig. 12. This figure illustrates how the profiles of vertical averages change their form, when the width of dike source increases from 100 m to 1 km. The maximum abundance of dike material in till is always observed at the distal contact of the dike, and rises only to 17 %, as the width of the source zone increases to 1 km. Similar sets of profiles, as depicted in the figure, could possibly be used in interpretations for the estimation of source widths.

Let us finally examine how the above predictions are affected by the heterogeneity of a source, in which the rock type of a vertical dike is changed during glacial erosion. The country rock of the dike is marked by symbol 0, the dike width is assumed to be 500 m and the magnitude of erosion 2 m, as shown in Fig. 13C. In the initial stage of glaciation, erosion acts on dike rock 1, whose thickness in the preglacial bedrock is 1 m. Towards the end of glaciation, dike rock 2 becomes exposed for erosion. Till material 1 that was eroded at first, occurs as a ribbon in the upper part of the till bed (Fig. 13A). Till material 2 constitutes the lower part of the ribbon, and the whole ribbon is enveloped by till material 0 derived from the host rock. These constituents together form a till bed, 2 m thick. Let us consider the variation of dike material content in till with the aid of vertical averages of the till bed (Fig. 13B).

Dike material 1 produces a mean profile, which does not differ much from the model curves of homogeneous dikes (cf. Fig. 12). The abundance maximum of the profile is, however, more than 4 km away from the proximal contact and not at the distal contact of the dike, as in the case of homogeneous dikes. The maximum abundance of about 5 %, interpreted according to Fig. 12, is due to a 250-m-wide homogeneous dike, while the true dike width is 500 m. The mean profile of dike material 1 would thus lead to erroneous predictions in terms of both the location and the width of the dike. The mean till profile 2 does not encourage to try any predictions at all, as it differs too much in shape from the model curves in Fig. 12. The mean profiles may thus be difficult to interpret, even though they are well-suited to detecting anomalies.

Once an anomaly has been detected it is advisable to determine its height and location in the till section with several samples. When the results are compiled in a cross-section (Fig. 13A), the location and width of the source can be predicted correctly in principle, irrespective of the changes in the source during the erosion. In theory, the vertical cross-section of till allows us to estimate the changes in the size and composition of the source, provided sufficient observational data are available. If the anomalous ribbon only occurs within a distance interval of, say, 4—5 km from the source, it derives from the depth interval of 80—100 cm in the preglacial source (Fig. 13A). In other words, material eroded from a 20-cm-high dike layer is, according to the model, dispersed over a distance of about 1 km even without any mixing of the material.

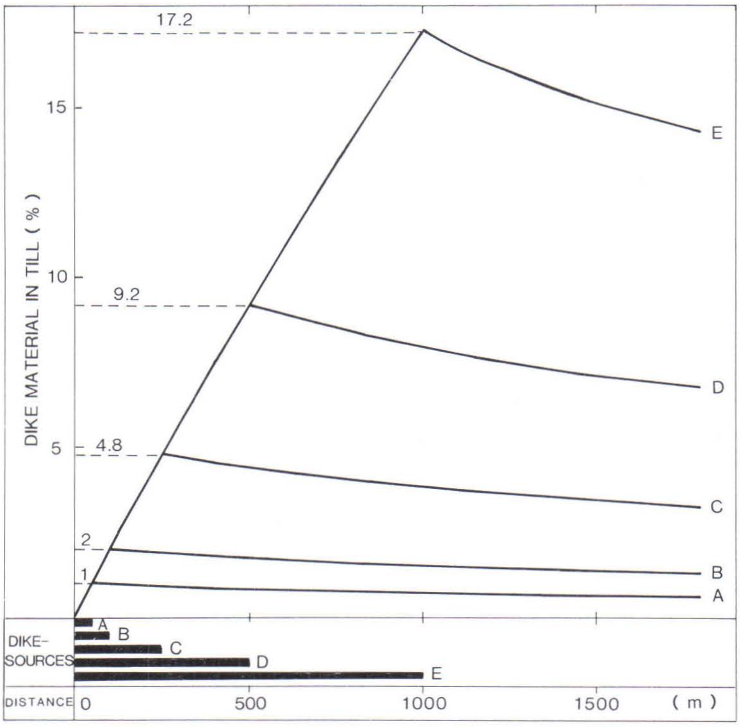

Fig. 12. Vertical averages of dike material in a 2-m-thick till layer calculated from equation (8) at parameter values (P). Profiles A—E refer to dike widths of 50—1000 m.

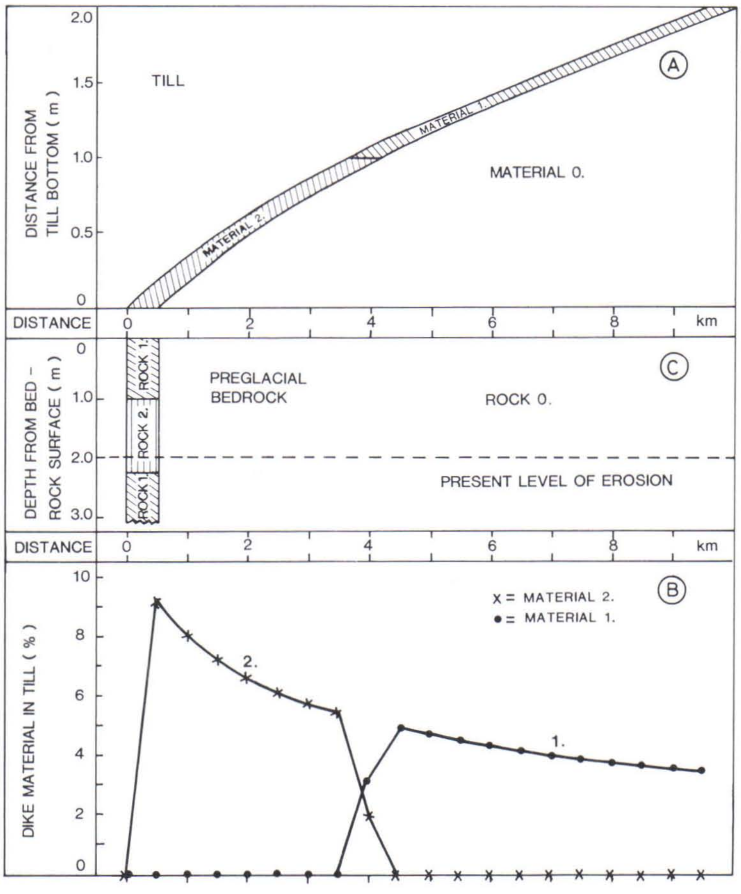

Fig 13. A) Distribution of dike material in a till cross-section and B) profiles 1 and 2 of the vertical mean abundances of dike rocks in till when a dike source changes during erosion as shown by C). Calculated on the basis of transport model and parameters (P).

9. GEOCHEMICAL APPLICATIONS

The previous chapter presented theoretic prediction possibilities based on the model. In the following, attempts are made to apply these possibilities for geochemical sampling design and interpretation. The situation is considered only in terms of the glacial transport described by the model. The hydromorphic anomalies formed in postglacial movements of solutions are beyond the scope of considerations. In order to apply correctly the glacial transport model we must be able to distinguish between glacigenic and hydrogenic anomalies. Nurmi (1976) used chemical extraction techniques in distinguishing anomalies, and Pulkkinen *et al.* (1980) sought to identify anomalies by comparing them with the glacigenic dispersion of magnetite.

When the geochemical till mapping aims at the discovery of mineralized bedrock portions, the task can be divided into two. First, anomalies must be found in till and, second, the location and size of the cause of the anomaly must be established in the bedrock. The cost-effective realization of the first phase requires planning of sampling, which can be made by using the transport model. Apart from the model and its parameters, the planning needs information about the average thickness of the till cover in the study area. Further, the targets and the anomaly threshold must be specified, and the accuracy of analytical facilities must be known. In order to find the targets with minimum costs and effort, the above information can be used to optimize the sampling grid on lines running in the glacial flow direction.

During glacial transport, material is also spread laterally, as modelled by Häkli & Kerola (1966). The lateral dispersion of material is an essential diluting factor for small point-like (ore) sources. In the case of a two-dimensional dike or contact, the significance of lateral dispersion is small, unless we are dealing with complex transport characterized by varying transport directions. If the lateral dispersion is small, the line separation needed to pinpoint an anomaly is directly determined by the size of the targets. In the following we shall examine sampling on lines parallel to glacial transport. The anomaly threshold for the till samples is set at 1 %, which implies that samples must contain more than 1 % target material before the target indication is considered statistically significant.

The compositional variation of geologic material is usually large, and the anomaly threshold of 1 % is on the optimistic side. This does not mean, however, that the samples should contain, say, 1 % gold to be considered anomalous. For a gold anomaly in till it is sufficient if the gold content exceeds 1 % of the concentration in the source. The absolute anomaly threshold in till is thus at 20 ppb if the gold content in the source is 2 ppm. In practice, what is considered as an anomaly also depends on the compositional variation in the background and the accuracy of analytical procedures. The former mainly depends on the study area and the latter on the element analysed and the method of analysis. The 1 % anomaly threshold is assumed to include the uncertainties due to background variations and analytical errors.

In the considerations below we shall still assume that the till layer is 2 meters thick. The targets are defined as dikes, 10 and 100 m in width, trending perpendicular to the direction of glacial flow. The properties of the ice sheet and bedrock are specified by the parameter set (P), implying that the 2-m till cover is renewed over a transport distance of about 10 km (cf. chapter 5). Figure 11 shows that in a till section the material derived from a 100 meters dike is equivalent to a layer about 2 cm thick. Correspondingly, from equation (8) we can calculate that a 10 meters dike produces till material for a layer 2 mm thick. For the dike material to constitute at least 1 % of the till samples (anomaly threshold), the lengths of the samples cannot exceed 100 × 2 mm = 20 cm and 100 × 2 cm = 2 m, when we are searhing for dikes 10 and 100 m wide, respectively.

The maximum sample lengths are also the optimal lengths when we attempt to localize the dike anomalies with a minimum number of samples. The total thickness of the till layer was assumed to be 2 meters, and thus the mean samples intersecting the whole layer have optimal lengths for locating till anomalies caused by dikes 100 m wide. Since the renewal distance of a 2 meters till layer is 10 km, the anomalous dike material in till can, in principle, be located anywhere within the 10 km stretch. Therefore, when till samples 2 m high are taken along the direction of glacial transport at intervals of 10 km, in theory all the anomalies caused by dikes exceeding 100 m in width are found. In practice it may be difficult to take samples 2 m long. If the sample length is reduced to 1 m, the sample spacing in the systematic sampling must be changed.

Figure 10 shows that in the 1-m-thick basal part of the till layer the dike anomaly extends for more than 3 kilometers and in the 1-m-thick surficial part of the layer, for about 6 km. Consequently, if anomalies caused by dikes over 100 m wide are to be detected in till, samples 1 m long must be taken systematically either from the till surface at 6 km intervals or from the base of till at 3 km intervals. When searching for dikes 10 m wide the length of the optimal sample was 20 cm, which is the sample length commonly used by the Geological Survey of Finland in geochemical mapping. Likewise, from Fig. 10 we can deduce that all the anomalies caused by dikes more than 10 m wide will be discovered, in principle, when 20-cm-long samples are taken systematically either from the base of the till at 500 m intervals or from the surface of the till at 1400 m intervals.

Narrow dikes (10—100 m) deliver fairly small amounts of material to till sections, which, when expressed as compact layers, are very thin (2—20 mm). In reality the material is intermixed and a weak anomaly becomes readily discontinuous so that the anomaly is easily missed during systematic sampling. To reduce this possibility it is advisable to take subsamples on both sides of the sampling point proper at, say, 50 m from the point. Since it often costs more to reach the sampling site than to actually take the samples, subsamples are cost-effective. The optimal number of subsamples can be estimated, provided the random variation of till composition and the sampling costs are known.

The above considerations cover the first stage of the geochemical sampling strategy, i.e. the search for and discovery of anomalies in till. In the second stage the detected anomalies are used to predict the location, size and possibly the composition of the source. For that, the vertical location of the anomaly in the till layer must be estimated with additional samples. It is worth digging an exploration pit into the anomalous point to establish the total thickness and the stratigraphic position of the till layer. After that the vertical location of the anomalous till ribbon should be determined within the layer. The following stage of tracing the source can be planned on the basis of the anomaly location. Tracing can be speeded up, provided the anomaly can be located straight from the till cross-section in the field.

Chemical analyses cannot yet be done on the walls of exploration pits directly, but they can be done in the field close to the pits with portable XRF-analysers. In order to locate the anomalous till ribbon more accurately we must take shorter samples in the vertical direction. As a consequence, the relative amount of anomalous material may increase in the sample, and the maximum values of the anomaly could possibly be used to predict the source composition. After the vertical limits of a till anomaly have been established, it is worthwhile comparing the anomaly with its surroundings in terms of various properties. Useful data are the total chemical composition, the lithologic composition, the magnetic and radiometric properties of the anomalous till and the surrounding till. The lithologic composition is determined with stone counts. Radioactivity is established with a gamma spectrometer, and magnetism by measuring the susceptibility directly from the walls of the pit. The vertical location of the anomaly allows us to predict the source width and position with the aid of suitable diagrams (cf. Fig. 11).

If, for example, the stone count shows that the predominant rock type is gabbro in the anomalous till and granite in the background till, a gabbro formation should be looked up at the predicted site of the source on the geologic map. If no outcropping gabbro is found, the positive gravimetric anomaly caused by the gabbro is searched for at the estimated site. If, on the other hand, the anomalous till ribbon is strongly magnetic relative to the background, a magnetic anomaly should characterize the source location. In the search, use can also be made of radiometric properties and combinations of various properties, and thus the target area can be constrained. Since the transport model still is tentative, it is advisable to proceed from the anomalous site in till towards the source by digging exploration pits systematically at intervals of, say, 500 m. With a view to future developments of the model, attempts should be made to locate the anomalous till layer in each pit so that data would accumulate on the continuity of till anomalies.

Discontinuous till anomalies are suggestive of source heterogeneities, varying erosion rates or intermixing of till material. If an anomalous till ribbon disappears completely before the base of till or the source is reached, it is probable that the anomalous material has been eroded away from the surface of the bedrock source. In the case of a heterogeneous formation, the anomalous material may occur below the present-day erosion level. If the anomalous material is interesting enough, it is well worth heading for the estimated source site and drilling there, even if the anomaly in till ends before the source. The above calculations are tentative only, as they are based on a preliminary model whose parameters are not known precisely. The calculations were made using the mean thickness of 2 m for the till cover, but they can readily be transformed to meet other cases. Even if the model parameters have to be changed in future in the course of areal elaborations, the results presented should be indicative at least.

10. BOULDER TRACING

As another example of practical applications let us consider boulder trains and their interpretation. The glacier has usually plucked the boulders from fracture zones or from the lee sides of bedrock hills. In the initial stages of glaciation the small knobs of bedrock could feed many boulders into basal transport. As the bedrock was gradually smoothed by glacial erosion, the abrasion became the dominant mode of erosion instead of plucking, and mainly fine-grained material was taken into transport. The basal entrainment of boulders may have almost ceased at the closing stage of glaciation, as is often shown by a gap between the source formation and the proximal boulder of boulder trains. However, the path to the source can be found also in this gap, when finer till fractions are examined. Some boulders have been entrained into transport from the sides of higher hills which rose up sharply into the ice sheet. These boulders were transported clearly above the basal debris and were deposited as part of the ablation till when the ice eventually melted.

Tills contain on the average less than 10 wt % stone and boulder fraction with a clast size exceeding 2 cm in diameter (Soveri, 1964). The number of boulders (diameter over 20 cm) is thus low, which makes it difficult to establish their distribution within basal till. On account of the large size of boulders it is also difficult to determine accurately their vertical location in the till cross-section. Some of the boulders were destroyed through comminution during glacial transport. As a result, the number of far-travelled boulders is reduced, seemingly shortening the transport distance estimates. The boulders in basal till may rise to the till surface after deglaciation as they are prone to the action of frost because of their large size. Some of the boulders were deposited on the basal till surface together with the ablation material. In terms of the transport model the surface boulders differ from the rest of till material, and it is difficult to predict their transport distances.

If a boulder has been transported together with basal till material, the transport distance of the boulder can be estimated from its vertical location in the till cross-section (cf. chapter 8). A boulder has the indisputable advantage over finer till fractions that it is always an undiluted sample of the source bedrock, irrespective of transport distance. Hence, boulders can be recognized, even when far from the source, at sites where the source material abundance in the fine fractions falls below the background noise. The boulder trains are often discontinuous when compared with the anomalous ribbons of finer till material. Moreover, the boulder trains may come to an end clearly before the source is reached. The anomalous element of the boulders should, therefore, be located in the finer till fractions, where the anomaly is easier to trace.

After deglaciation, frost has heaved boulders from the basal till towards the till surface in many places. This makes the predicted transport distance too long if the distance is estimated by the model from the vertical location of the boulder as described earlier. The prediction thus gives a maximum estimate for the distance range within which the source should be located. The typical depth of frost action is 1—2 m, but in very cold winters with a thin snow cover the frost may reach to a depth of 3 m (Soveri, 1964). The heave mechanism of till boulders is based on the fact that the pore water in till freezes in winter and melts in summer (Okko, 1964). Permafrost alone is not sufficient to heave boulders, but the process requires the alternation of freezing and melting of wet till under the boulders. Alternation like this has been going on in Finland for thousands of years after the last glaciation.

The alternation of frost and thaw started first in Lapland and in Karelia, where the land was not covered by sea after deglaciation. Ostrobothnia emerged later from the sea, and hence frost heave has probably been effective there for a shorter time. This is in accordance with the fact that the stoniest till fields are located in eastern and northern Finland. Frost heave may have raised ore boulders to the surface of basal till very close to the source, which contradicts the predictions of the transport model. In many areas the bulk of larger boulders have come to the surface from the upper 1—2 meters of the till layer. One might say that Nature has carried out vertical averaging by bringing a composite stone and boulder sample to the till surface with the aid of frost. If we can estimate the magnitude of frost heave (the length of average sample), we can use the transport model to calculate the boulder profile generated from basal till by frost.

Let us assume that frost has heaved all the boulders of a 2-m-thick basal till layer to the surface of the layer. If the boulders were derived from a dike source, the resulting boulder profile should resemble the model profiles of Fig. 12. The peak of the profile curve should be located at the distal contact of the dike source and, in principle, the dike width could be estimated from the peak value. If we wish to compare boulder observations with model calculations, we should determine along investigation profiles the rock type proportions of boulders within proper surface elements. However, a cluster of surface boulders is usually an unknown mixture of ablation till boulders and frost-heaved basal till boulders, and hence the modelling of frost action is more or less academic. Although the transport of ablation material is partly beyond the scope of this work, the problem will be preliminarily considered below because of its practical importance.

A small proportion, typically less than 10 % of the surface boulders, have travelled very long distances, often tens of or even a hundred kilometers (Salonen, 1986). These boulders may have been transported repeatedly during several glaciations. One glacier alone can also transport boulders far, if they are entrained high into the ice at the beginning of glaciation, because then the velocity and time of transport are at their maximum. A reason for the small number of far-travelled boulders is that they may break down during transport, thus decreasing in size and finally ceasing to exist as boulders. Therefore, most surface boulders are found at a distance of less than 10 km, and even less than 3 km, from their bedrock sources. The transport model of basal till cannot be applied to predict the provenance of surface boulders. Rough estimates can, however, be made on the basis of the numerous observations compiled by Salonen (1986). Thus, about 50 % of the boulders in a typical till cover derive from a distance of less than 3 km, 30 % have travelled 3—6 km and 20 % more than 6 km.

Surface boulders as well as other ablation material entered glacial transport particularly from topographic elevations. The ice sheet surrounding the elevations entrained finer material from the slopes and summit, and boulders mainly from the lee side of elevations. The material of bedrock hills entered the ice clearly above the layer of basal debris, and it was thus deposited as ablation till. The highest hills feeded material into elevated layers of the ice sheet, where the creep velocity was high and the material could travel far. When hill material entered the ice throughout a thickness corresponding to the relief, the amount of depositing ablation till was proportional to the ruggedness of terrain, as observed in Canada (Shilts, 1976). When the ice flowed around and between hills, it distributed till material also laterally (Lundqvist, 1935).

Figure 14 shows schematically the entrainment and transport of ablation material from a hill source. It has been assumed that the erosion of material is fairly uniform and that the boulders are fed into the transport as continuous trains. The boulder trains at different levels in the ice illustrate the englacial distribution of material at the end of transport. The far-travelled boulders on the right hand side in the trains were plucked from the hill at the initial stage of glacial erosion. Because the creep velocity within the glacier increases upwards, the uppermost boulder trains extend the farthest to the right. After the glacier has eroded away the top of the hill , no more material is entrained from there at the final stage of erosion. Therefore, the uppermost boulder train is the first to come to an end on the left. Downwards the following trains extend increasingly close to the source hill, because rock material can be eroded from the lower slopes of the hill until the end of glaciation.

By summing the boulder trains in Fig. 14 we get a typical transport profile, in which the abundance of hill material in ablation till reaches the maximum close to the source and declines gently with increasing distance from the source. The declining part of the abundance curve may be steepened due to lateral dispersion, because hills behave as point-like sources. The shapes of abundance curves can also be affected by comminution, if the boulders are of some weak rock type. If again the boulders are tough and

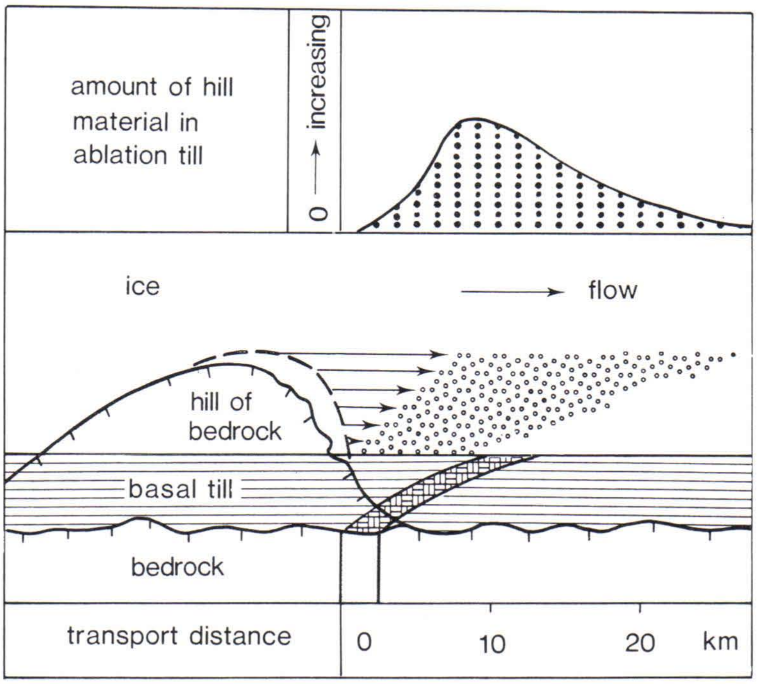

Fig. 14. Schematic presentation of the transport of ablation till material from a hill source (cf. text).

entrained into rapidly flowing ice layers from high hills, then exceptionally long boulder trains are expected. Discontinuous boulder trains result if boulders are plucked from the source sporadically. Short boulder trains may be due to rapid breaking of boulders, or then boulders may have been fed into glacier only for a short time. The lateral migration due to several hills combined with variations in transport conditions may result in boulder trains that are very difficult to interpret.

It is evident from the above that the source location of boulders can be estimated only within a fairly large distance range. However, within this range the estimates can be made more accurate in various ways. By determining the physical properties (density, magnetic properties, resistivity, polarizability) of the boulders, geophysical maps can be used to locate the source formation. Topographically the probable source of surface boulders is found on the distal sides of bedrock hills. The boulders in basal till probably derive from smaller rock knobs or fracture zones in bedrock, which can sometimes be observed under the till cover with the aid of impulse radar (Ulriksen, 1982). A single ore boulder is in itself an indisputable sign that a mineralization occurs 'somewhere'. Prediction of the site of a source located 'somewhere' is, however, realistic only when several anomalous boulders are found. If, in addition, the anomalous element can be localized in the fines of basal till, predictions can be based on the transport model.

11. DISCUSSION

In the central parts of a continental ice sheet the creep of ice is inclined downwards from the ice surface. In the marginal ablation areas of the ice sheet the flow rises towards the glacier surface (Paterson, 1981). In the intervening areas, between the center and the margin, the creep advances more or less parallel to the substratum. The ice flow thus tends to keep the eroded material at the glacier base in the central and intervening areas of the ice sheet, where Finland was located during most of the last glaciation. Therefore, the basal debris is lifted within the ice only by the action of the newly eroded material pushing the previously loosened grains upwards. The lifting velocity depends on the erosion rate of substratum, which is the first component of the transport model for basal tills. In order to erode till material from the substratum, the ice sheet must move by basal sliding. The sliding is the second component of the model, which, apart from eroding, transports material horizontally. The third model component is the creep of ice, which also moves the material horizontally.

When the above components are combined into a two-dimensional transport model as described in chapter 4, it is assumed that the glacier bed is smooth and that the erosion rate and the movement velocities are constant in the study area. These assumptions should hold in areas measuring 10 km, which is the typical renewal distance of basal tills. In real cases, the assumptions do not hold precisely, for the transport velocities change and the substratum undulates from one point to the next. This causes collisions between till particles during transport, and the intermixing of particles is further enhanced by comminution of the material. Thus, on a small scale the constancy assumptions in the model do not hold. Nevertheless, the variations can be smoothed to meet better the situation required by the model by using averages. The model assumes that the intermixing of material is moderate, in which case the deposition can be simply treated as the end of transport.

The degree of intermixing should be studied in vertical till cross-sections by means of samples varying in length. The effect that intermixing has on the results can be reduced by taking samples of sufficient length. If the samples are too long, however, the indications from narrow, explorationally interesting sources can be lost. The ore sources are also often point-like, in which case the two-dimensional transport model should not be applied to them. Sources less than 100 m wide begin to cause difficulties to the model, since the renewal distance for basal till is about 10 km. Modelling of the situation would require a resolution of 1 %, which is seldom reached in geology. The averages associated with increments of 100—1000 m along the distance axis give a proper starting-point for the application of the model.

The unevenness of substratum disturbs the modelling by mixing the till material, and it also may hamper the practical use of transport model. The thickness of till cover may vary by several meters within a distance of a few hundred meters. As a result highly diverse estimates are obtained for the distance of the anomalous till material from the bedrock surface, even at adjacent points. The use of these data in the transport model results in contradictory predictions for the source location. Before the model is applied, the observational data should be smoothed by calculating within each model increment the mean thickness of till cover and the mean depth of the anomaly. When the sample length, number of samples and the distance increment are appropriate, mean profiles applicable to the model interpretations can be computed. However, the optimal sizes of the samples and distance increments are not yet known, and presumably they vary from one area to the next.

The transport distance of till material depends on the glacier velocities and transport time. At the glacier base the motion is slow, resulting in rather short transport distances for the basal till. Upwards in the glacier the creep velocity increases. Therefore, material at a higher elevation in the glacier may travel much farther than the basal debris. In the central parts of ice sheets the substratum material can enter higher levels of the ice only from topographic elevations. The amount of material being fed from hills into the ice above the basal debris is varying, and depends on the relief in the area. The erosion resistance of the glacier bed may also differ in adjacent areas, whereas the time of glacial activities and the glacier velocities depend more systematically on the location of the area in terms of the whole continental ice sheet.

In the areas of ice-divide in northern Finland the till transport distances are short, even though in those areas the continental ice sheet was active for the longest time. In Lapland, the fine till material is often found only a few tens or hundreds of meters from its provenance, while the coarse till fractions, such as stones, have been transported for several kilometers (Lehmuspelto, 1987). This is not easy to explain if we assume that the fines were produced during the transport by comminution from the coarser material. However, the situation can be understood if the fines belong to the basal till and the coarser material to the ablation till. Stones and boulders are particularly abundant on the topographic elevations of the Lappish bedrock, which was fractured and weathered preglacially. Stones from the hills were pressed into the ice at higher levels, where they could travel farther, depositing eventually as a part of ablation till. In contrast, the finer material was abraded at the glacier base, where the motion was slow and the transport distance short.

The short transport distances in Lapland can be attributed to the low velocities of the glacier at the ice divide. Particularly the basal sliding of the ice sheet was slow, as shown by the very minor erosion in Lapland. Assuming 20 000—30 000 years for the duration of the last glaciation in Lapland, we can estimate the glacier velocities there. At transport distances of 250 m (fines) and 2500 m (coarse material) and a duration of glacial activity of 25 000 years, we get 0.01 m/a for the velocity of fines, and 0.1 m/a for the coarse material. These velocities are on the low side even for cold-based ice sheets, but the velocity estimates get higher if the glacier was active in Lapland for only part of the glaciation time.

Central and southern Finland were covered by ice for a shorter period than Lapland. If we assume that the glacier first advanced from the Scandinavian mountain range to southern Finland in 5000 years and finally withdrew back to the mountains also in 5000 years, the last continental ice sheet resided in southern Finland for 10 000—20 000 years. During that time a basal till layer with an average thickness of 2 m was developed, being typically renewed over a distance of 10 km. At the parameter values (P) this took about 5000 years, which is only a fraction of the time available. Once the glacier base had become saturated with basal debris, its friction against the bed may have become high enough to slow down or even stop the basal sliding. Higher up the glacier, however, the transport of englacial material could continue by the creep of ice.

The ablation material in the surface part of till cover always includes some stones and boulders that have clearly travelled farther than the other material. Material like this, which has travelled for tens or even more than a hundred kilometers generally constitutes less than 10 % of the surface stones and boulders (Salonen, 1986). If the rock material in the glacier has reached elevations $z = 3$ or 10 m, the parameter values (P) give $U = k \cdot z = 3$ or 10 m/a, respectively, for the creep velocity. At these velocities englacial material had time to travel 30—200 km in the period of 10 000—20 000 years, during which the continental ice sheet probably covered the southern part of Finland. Therefore, the long-distance transport of surface boulders can be explained by the rock material entrained into the ice from the summits of bedrock hills over 3 m high. Rock material from the rapakivi formations in southernmost Finland travelled within the continental ice sheet to the countries south of the Baltic Sea (Hausen, 1912) and there all the way to Holland (Schuddebeurs, 1981). This implies a transport distance of more than 1000 km.

It can be assumed that the ice sheet advanced from the Scandinavian mountains to southern Finland in 5000 years. It is known that the ice retreated from central Europe back to the Scandinavian mountains in about 10 000 years (Okko, 1964). If the total duration of the glaciation was 20 000—30 000 years, the glacial flow from southern Finland to central Europe might have lasted for about 5000—15000 years. During that time rapakivi boulders could have moved 1000 km at a transport velocity of 60—200 m/a. Such velocities are common in present-day glaciers (Paterson, 1981). At the parameter values (P) of the transport model we get $V = B + k \cdot z = 60$—200 m/a for the velocity, if the material is raised to a height of 60—200 m above the glacier bed. When the Baltic Sea was covered with thick ice and frozen almost to the bottom, the rapakivi formations of Åland rose from the sea bottom into the ice as hills more than 100 m high in places. From these summits rapakivi material was able to enter glacial transport at elevations exceeding 60 m, at which height the material could reach central Europe.

12. FUTURE WORK

Specifications or extensions of the simple model are not urgently needed, because the model is tentative even in its current form. However, in order to make thorough testing of the model, additional information should be needed on the distribution of till material in the transport direction, and particularly in the vertical direction. Vertical direction has not always been considered important when geologic data have been collected, for sample length or depth intervals are seldom reported. The lack of depth data is also due to the fact that data acquisition is difficult without an excavator. The geologic data published do not permit the transport model to be tested more thoroughly nor its parameters to be constrained more accurately than was done above. Even though the geologic observations show many regularities they still do not give the basis for a more sophisticated model.

Further testing of the model will only be feasible if new observational data become available. When such data are collected, the specifications of the model should be taken into account. In terms of data acquisition the following recommendations apply. To begin with, the study target should be a contact between two rock types that clearly differ from each other. The contact should run approximately perpendicular to the glacial flow direction. The lithologic areas should be fairly extensive so that at least the distal rock type covers 10 km in the transport direction. The investigation line should run parallel to the transport, beginning a couple of kilometers before the contact and continuing for at least 10 km after the contact. A suitable point interval on the line would be, say, 500 m. The use of a ground-probing radar is recommended for determining the site of the profile, as the device can be used to record the topography of bedrock and the thickness of till cover.

An excavation, deep enough to intersect the till cover, should be prepared at each observation site. The thickness of till layers and the depth intervals of samples should be recorded. At least two, but preferably four superimposed samples should be taken from each till layer and the sample lengths should cover the whole till cross-section. The intermixing of till should also be studied more comprehensively at individual sites by taking samples of different length. Measurements and analyses should first be done on unsieved samples to avoid the bias in transport distance estimates due to glacial comminution. Samples can be analysed for any component abundances (rock type, mineral, element), provided the source contrast of the component is distinct and its dispersion glacigenic. The best measuring methods for rapid studies are those that can be conducted and interpreted in the field. They create a direct interaction between sampling and data analysis, enhancing the planning of the work and its execution.

There are numerous measuring techniques that can be used in the field. The conventional stone counts can be performed at either a pit or a field base. The mineral compositions can also be determined under microscopes at field bases. The advantage of these methods is that they provide close contact with the samples. Their disadvantages are slowness and the expertise required. In addition, the geologic source contacts are often poorly exposed, which makes their accurate location uncertain. The analytical methods of geochemistry have advanced markedly, making it possible to analyse samples at the base with a portable XRF. The abundance of K, U and Th can also be directly measured on the pit walls with a gamma spectrometer (Österlund, 1982) or on the samples back at base.

Geophysics, too, offers some field methods useful for tracing of certain components, although they have not been used much yet. The distribution of magnetite in till can be determined by measuring the magnetic susceptibility of till samples. The measurements can be done either directly on a pit wall or on samples at the laboratory (Puranen, 1977). By measuring the grain density of till samples (Puranen & Kivekäs, 1979) at a base laboratory, the relative amounts of heavy (mafic) and light (felsic) rock or mineral grains can be estimated. Susceptibility and density measurements can be done rapidly without professional skills. Further, the source contacts can be selected and accurately localized by means of magnetic and gravimetric maps. The major part of Finland is covered by aeromagnetic, aeroradiometric, gravimetric and geochemical maps. Comparison of these voluminous data at selected target areas might also provide information that could be used to test the transport model.

The data collected on test profiles with the methods described above would allow more comprehensive testing of the model. If the outcome of testing supports the model, profiles should be studied in different glacial provinces and over various rock types. In addition to the contact case, transport should be studied from dikes varying in width, and finally from point-like bedrock sources. While the data are being collected, the stratigraphy and topography of the study targets should also be established. This kind of data would provide us with a basis on which to estimate the model parameters typical of each area and lithology. The model cannot predict reliably the provenance of till unless its parameters are determined in controlled situations. More information is also needed on the transport of unsieved till material relative to the transport of different till fractions. Such information would be useful in predictions, because till geochemistry usually studies the finest fraction (grain size below 0.06 mm), and boulder tracing deals with the coarsest till fraction only.

New data could help us to develop and perhaps even extend the model. The model assumes that the erosion rates and the glacier velocities were constant during the period under consideration. The validity of these assumptions can be improved by dividing the period into shorter subperiods, or by dividing the model into submodels. The model also assumes that basal debris (till material) is kept as a compact layer at the base of the glacier. The model is thus applicable only to cases of basal till with one or several layers. In reality some of the material is dispersed higher into the ice by having been fed directly from topographic elevations on the one hand, and by vertical mixing of basal debris on the other. In this way material reaches the more rapidly flowing upper ice layers, where it may travel a greater distance, and is finally deposited as ablation till. The vertical intermixing of material under and in the glacier can be described with diffusion models. By adding to the model a part that describes till intermixing, the model would become conceptually more accurate but at the same time computationally more difficult.

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Appendix A.

TRANSPORT MODEL IN THE CASE OF 2 BASAL TILL LAYERS

Let us consider a situation (Fig. 15) in which two basal till layers, 1 and 2, rest superimposed on bedrock. The till layers were formed during two different glaciations. The flow directions of the glaciers are assumed to be approximately parallel. Let the parameters of the older and younger glacier be (B_1, k_1, W_1) and (B_2, k_2, W_2), respectively, where B stands for the basal sliding velocity, k is the strain rate and W the erosion rate. The till layers are assumed to have been formed as follows. The older glacier eroded and transported material from the bedrock, depositing it as a till layer, D meters thick. The younger glacier abraded and transported the upper part of the old till by thickness D_2, depositing it as a new till layer. Let the thickness of the old, surviving till layer be D_1.

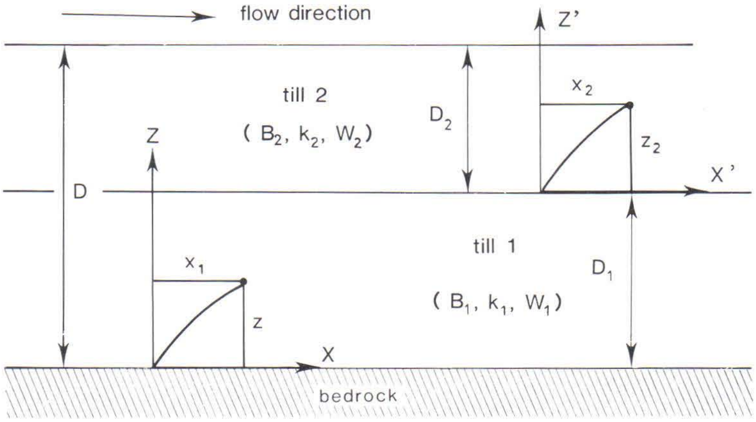

Fig. 15. Symbols for the transport model in the case of two till layers.

According to equation (4), the first glacier transported till material from its bedrock source for a distance

$$x_1 = (B_1 z + \tfrac{1}{2} k_1 z^2)/W_1 \qquad \text{(I)}$$

where z refers to the distance from the bottom of the old till layer, i.e. from the bedrock. Correspondingly, the material of the upper till layer was transported by the second glacier for a distance

$$x_2 = (B_2 z_2 + \tfrac{1}{2} k_2 z_2^2)/W_2 \qquad \text{(II)}$$

where z_2 is the distance from the bottom of the new layer. Since the upper till layer was formed of material of the old layer, the material had already earlier moved in relation to the bedrock sources. The previous movement is obtained by equation (I), if the distance of the new material, z, from the bottom of the old layer is known. This distance is $z = z_2 + D_1$ as shown by Fig. 15.

The total displacement x of the upper layer in relation to the bedrock sources is obtained as the sum of distances (I) and (II): $x = x_1 + x_2$. We want to express the sum equation as a function of the distance z measured from the bedrock. Therefore, the distance z_2 is written in the form $z_2 = z - D_1$, which is then inserted in equation (II) before summing. We get

$$x = x_1 + x_2 = (B_1 z + \tfrac{1}{2} k_1 z^2)/W_1 + [B_2(z - D_1) + \tfrac{1}{2} k_2 (z - D_1)^2]/W_2 \qquad \text{(III)}$$

This equation gives the transport distances for the depth interval $D_1 < z \leqslant D$, or in till layer 2. The transport distances in till layer 1, or for the depth interval $0 < z \leqslant D_1$, are obtained from equation (I). A model of three or more layers could be solved in a similar manner.

As a special case let us finally consider a situation in which a new glaciation totally erodes the old basal till, with the consequence that $D_1 = 0$. Let the parameters of both glaciers be (B_1, k_1, W_1). Now equation (III) is reduced to

$$x = 2 \cdot (B_1 z + \tfrac{1}{2} k_1 z^2)/W_1 \qquad \text{(IV)}$$

By comparing equations (I) and (IV) we see that, irrespective of the depth z, the transport distances are doubled in this type of retransportation. The result given by equation (IV) would also be obtained if the erosion rate in equation (I) were $\frac{1}{2}W_1$. In these circumstances it would therefore be impossible to distinguish between simple and repeated transport.

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