## 2006 ACTM <br> STATE GEOMETRY EXAM

In each of the following you are to choose the best (most correct) answer and mark the corresponding letter on the answer sheet provided. The figures are not necessarily drawn to scale.

1. The length of a rectangle is 3 more than its width. If its perimeter is 24 , what is its area?
A. 27
B. $\quad 191.75$
C. $\quad 33.75$
D. Not enough information
E. None of these
2. In the figure $\boldsymbol{I}, \boldsymbol{m}, \boldsymbol{n}$ are parallel lines with transversals $\boldsymbol{s}$ and $\boldsymbol{t}$. If $\mathrm{AB}=3, \mathrm{BC}=4$, $D E=2 x-3$ and $E F=x+5$ then $D F=$
A. 21
B. $\quad 18.2$
C. $\quad 10.4$
D. 7.8
E. None of these

3. In the figure at the right, with angle measures indicated, what is the measure of $\angle A B E$ ?
A. $\quad 86^{\circ}$
B. $\quad 94^{\circ}$
C. $96^{\circ}$
D. $104^{\circ}$
E. None of these

4. Angle A and angle B are complementary angles and angle A and angle C are supplementary angles. Which of the following is/are true?
I. $m \angle C>m \angle B$
II. $m \angle C-m \angle B=90^{\circ}$
III. $m \angle B+m \angle C=180^{\circ}$
A. I and II only
B. I and III only
C. II and III only
D. I, II and III
E. None of these
5. In the figure $A E=4, A B=B C=C D$ and the area of $\triangle C D E=6$. Find $C E$ to the nearest hundredth.
A. $\quad 9.77$
B. 9.85
C. $\quad 7.14$
D. 7.21
E. None of these

6. $\overline{A B}$ is the perpendicular bisector of $\overline{C D}$ and $\overline{C D}$ is the perpendicular bisector of $\overline{A B}$. Which of the following is/are true?
I. $\quad \mathrm{AC}=\mathrm{BD}$
II. $\overline{\mathrm{AC}} \| \overline{\mathrm{BD}}$
III. $\overrightarrow{\mathrm{AB}}$ bisects $\angle \mathrm{CAD}$
A. I and II only
B. I and III only
C. II and III only
D. I, II, and III
E. None of these

7. In the figure at the right, if $\overline{\mathrm{AB}} \cong \overline{\mathrm{BD}} \cong \overline{\mathrm{CD}}$ and $\overline{\mathrm{AD}} \cong \overline{\mathrm{AC}}$ then $\mathrm{m} \angle \mathrm{ADB}=$
A. $36^{\circ}$
B. $\quad 72^{\circ}$
C. $108^{\circ}$
D. $62.5^{\circ}$
E. Not enough information


Problem 7
8. How many vertices and edges does a prism have if it has 12 faces? Answers are written as (vertices, edges).
A. $(18,28)$
B. $(20,30)$
C. $(22,32)$
D. $(16,26)$
E. None of these
9. The contrapositive of the statement "If $\overline{A B}$ is congruent to $\overline{C D}$ then $A B=C D$ " is
A. If $\overline{A B}$ is not congruent to $\overline{C D}$ then $A B \neq C D$.
B. If $A B=C D$ then $\overline{A B}$ is congruent to $\overline{C D}$.
C. If $A B \neq C D$ then $\overline{A B}$ is congruent to $\overline{C D}$.
D. If $A B \neq C D$ then $\overline{A B}$ is not congruent to $\overline{C D}$.
E. None of these
10. The area of the polygon at the right is
A. $251 / 2$
B. 26
C. $261 / 2$
D. 25
E. None of these


Problem 10
11. The number of lines of symmetry in a regular octagon is
A. 16
B. 4
C. 24
D. 6
E. None of these
12. $\triangle \mathrm{ABC}$ is an isosceles triangle with $\overline{\mathrm{AB}} \cong \overline{\mathrm{AC}}$. Which of the following is/are true?
I. The centroid of the triangle lies on the angle bisector of $\angle \mathrm{BAC}$.
II. The altitude from vertex $A$ is a median of the triangle.
III. The center of the circle that is tangent to the three sides of the triangle lies on the median drawn from $A$.
A. I and II only
B. I and III only
C. II and III only
D. I, II, and III
E. None of these
13. A regular polygon has 20 sides. The measure of an exterior angle of this polygon is
A. $36^{\circ}$
B. $\quad 153^{\circ}$
C. $18^{\circ}$
D. $162^{\circ}$
E. None of these
14. A polygon on a geoboard has an area of 16.5. If it has eleven pegs touching the geoband, how many pegs are in its interior?
A. 12
B. 11
C. 10
D. 14
E. None of these
15. The diagonals of the quadrilateral $A B C D$ are perpendicular. Which of the following could be true?
A. $\quad A B C D$ is a square
B. $A B C D$ is a rhombus
C. $A B C D$ is a kite
D. Either $A, B$, or $C$ might be true
E. None of these are true
16. The diagonals of the quadrilateral $A B C D$ bisect each other. Which of the following is the most accurate?
A. $A B C D$ is a rectangle
B. $A B C D$ is a rhombus
C. $A B C D$ is a parallelogram
D. $A B C D$ is a kite
E. None of these
17. Consider the trapezoid $A B C D$ at the right with $E$ and $F$ midpoints of $\overline{A D}$ and $\overline{B C}$, respectively. If $A B=36, B C=17, C D=15, A D=10$ then $E F=$
A. 25.5
B. 39
C. 19.5
D. 26
E. None of these

18. $\overline{\mathrm{AC}}$ and $\overline{\mathrm{BD}}$ are chords in a circle intersecting at E . If the measure of the $\operatorname{arc} \mathrm{APD}=32^{\circ}$ and the measure of $\operatorname{arc} B Q C=80^{\circ}$ then $\mathrm{m} \angle \mathrm{BEC}$ is
A. $80^{\circ}$
B. $56^{\circ}$
C. $48^{\circ}$
D. $24^{\circ}$
E. None of these

19. A dart board consists of three concentric circles with diameters of 6 inches, one foot, and 18 inches. If a dart is thrown at and hits the board, what is the probability it lands in the shaded region?
A. $\frac{1}{12}$
B. $\frac{1}{3}$
C. $\frac{1}{4}$
D. $\frac{4}{9}$
E. None of these

20. In $\triangle \mathrm{ABC}, \overline{\mathrm{AD}}$ and $\overline{\mathrm{BE}}$ are medians intersecting at F . The ratio of the area of $\triangle \mathrm{DEF}$ to the area of $\triangle A B F$ is
A. $\quad 1$ to 2
B. 2 to 3
C. $\quad 1$ to 4
D. 1 to 3
E. None of these

21. In the figure at the right $\overline{B C}$ and $\overline{D E}$ are perpendicular to $\overline{A D}$. If $A C=10, C E=20$, and $D E=18$, then the area of $\triangle A B C$ to the nearest hundredth is
A. 19.62
B. 24.00
C. $\quad 30.00$
D. $\quad 216.00$
E. None of these

22. In $\triangle A B C, \overline{B D}$ is an altitude, $A B=10, m \angle A B D=60^{\circ}$ and $m \angle B C D=45^{\circ}$. The perimeter of $\triangle A B C$, to the nearest hundredth, is
A. $\quad 35.91$
B. $\quad 24.66$
C. 30.73
D. 26.18
E. None of these


Problem 22
23. What is the area, to the nearest hundredth, of the triangle at the right?
A. 91.04
B. 90.26
C. $\quad 45.13$
D. Not enough information
E. None of these

24. A cylinder has a radius of 3 inches and a height of 4 inches sits on a table. If a cone whose height is 12 inches is placed over the cylinder so that the top of the cylinder touches the inside of the cone and the base of the cone is on the table, what is the volume of the cone?
A. $24 \pi \mathrm{in}^{3}$
B. $64 \pi \mathrm{in}^{3}$
C. $81 \pi \mathrm{in}^{3}$
D. $36 \pi \mathrm{in}^{3}$
E. None of these


Problem 24
25. In the figure $\mathrm{m} \angle \mathrm{FEG}>\mathrm{m} \angle \mathrm{GED}$ and $\overline{\mathrm{EG}} \cong \overline{\mathrm{ED}}$. Which of the following is/are true?
I. $\mathrm{m} \angle \mathrm{GED}>\mathrm{m} \angle E G F$
II. $\mathrm{FG}>\mathrm{FE}$
III. FD $>$ FG.
A. I and II only
B. I and III only
C. II and III only
D. I, II, and III
E. None of these


Name $\qquad$ School $\qquad$
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The following questions will be used only in breaking ties for first, second or third place. Be sure you answer all the questions on the test before attempting these questions. They will be used in the order given to break the ties.

1. In the figure at the right, what is the length of $\overline{\mathrm{AB}}$, to the nearest tenth of an inch?


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2. In the figure at the right $\overline{A B} \| \overline{C D}$ and $\angle A C B \cong \angle C D F$. $C F=5, B F=7, A B=15, D C=8, D E=4$. Compute $A E$.


## Geometry Tie Breaker Questions

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3. The edges of a pyramid with a square base are all equal. A plane parallel to the base passes through the pyramid at the midpoint of the altitude of the pyramid. If an edge of the pyramid is 12 inches, what is the volume of the part of the pyramid that lies below the plane (the shaded portion)?


## KEY

## GEOMETRY 2006

1. C
2. A
3. D
4. A
5. D
6. E
7. C
8. D
9. A
10. B
11. B
12. A
13. D
14. $B$
15. A
16. D
17. B
18. B
19. D
20. A
21. C
22. B
23. C
24. C
25. C

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The following questions will be used only in breaking ties for first, second or third place. Be sure you answer all the questions on the test before attempting these questions. They will be used in the order given to break the ties.

1. In the figure at the right, what is the length of $\overline{\mathrm{AB}}$, to the nearest tenth of an inch?
$C D=54 \sin 32^{\circ}$
$B D=54 \sin 32^{\circ} / \tan 47^{\circ}$
$A D=54 \cos 32^{\circ}$
$A B=A D-B D$
$A B=19.1$ in

2. In the figure at the right $\overline{\mathrm{AB}} \| \overline{\mathrm{CD}}$ and $\angle \mathrm{ACB} \cong \angle \mathrm{CDF}$. $C F=5, B F=7, A B=15, D C=8, D E=4$. Compute $A E$.
$\triangle \mathrm{ABC} \sim \Delta \mathrm{CED}$

$$
\frac{C E}{D E}=\frac{A B}{B C} . \quad C E=\frac{4(15)}{12}=5 .
$$



$$
\frac{A C}{B C}=\frac{C D}{D E} . \quad A C=\frac{8(12)}{4}=24 .
$$

$A E=A C-C E=24-5=19$
3. The edges of a pyramid with a square base are all equal. A plane parallel to the base passes through the pyramid at the midpoint of the altitude of the pyramid. If an edge of the pyramid is 12 inches, what is the volume of the part of the pyramid that lies below the plane (the shaded portion)?

The dimensions of the top square will be $6 \times 6$. The altitude of the pyramid on top is $3 \sqrt{2}$. The altitude of the large pyramid is $6 \sqrt{2}$.
Volume (top pyramid) $=36(3 \sqrt{2}) / 3=36 \sqrt{2} \mathrm{in}^{3}$.
Volume(large pyramid) $=144(6 \sqrt{2}) / 3=432 \sqrt{2} \mathrm{in}^{3}$.
Volume $($ bottom $)=432 \sqrt{2}-36 \sqrt{2}=396 \sqrt{2} \mathrm{in}^{3}$


