INTRODUCTION TO RADAR SIGNAL PROCESSING

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CeSIP
Centre for Signal &
Image Processing

Overview

- History of Radar
- Basic Principles
- Principles of Measurements
- Coherent and Doppler Processing
- Waveforms Design and Pulse Compression
- Closing Remarks
- Reading Material

History – Before Radar

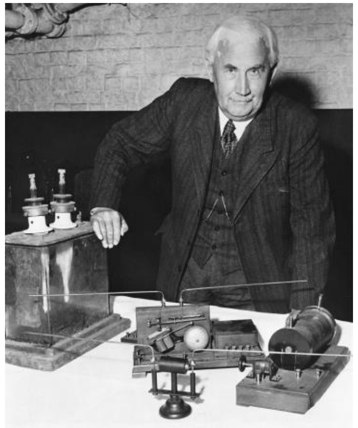
- Between the World Wars, parabolic **sound mirrors**, were used to provide early warming;
- Acoustic mirrors had a **limited** effectiveness, and the **increasing speed** of aircraft in the 1930s meant that they would **already be too close to deal with** by the time they had been detected.
- **Radio** transmitters had already been in use for over a decade for **communications.**

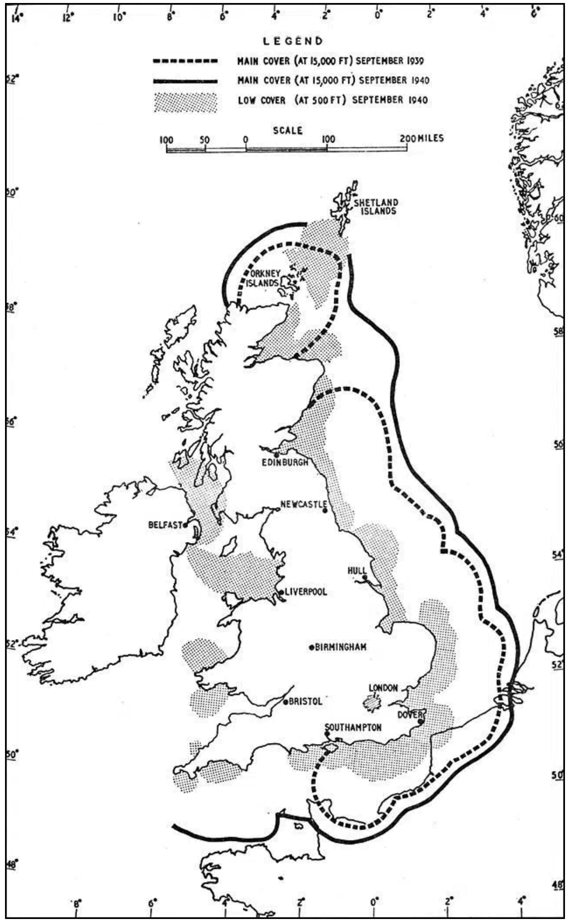
WW1 - WW2

Top: (L) Bombing during the WW1, (R) “Whisper Dishes”
Bottom: (L) WW2 Bombers, (R) Four-horn acoustic locator,1930s

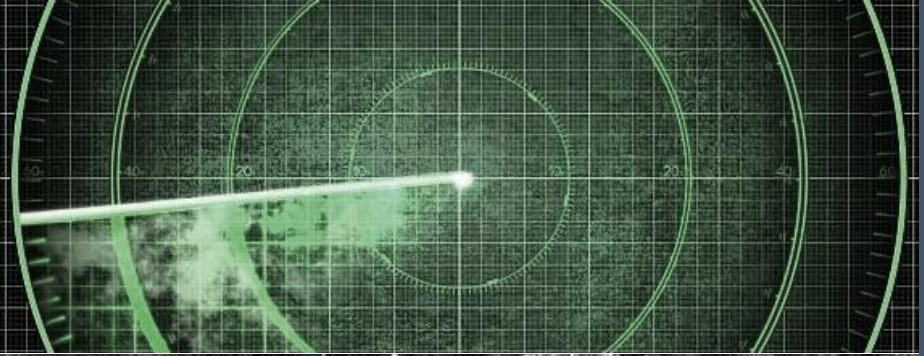
History – Radio Detection

- Radar was first patented and demonstrated in 1904 by the German engineer **Christian Hulsmeyer**;
- **Watson Watt** is generally credited with initiating what would later be called radar;
- In June 17, 1935, a radio-based detection and ranging was first demonstrated in Great Britain;
- The first Radar system used by the British comprised **21** stations placed along the country's eastern coast.





Left: (T) Christian Hu̇lsmeyer, (B) Watson Watt,
Right: Chain Home coverage map

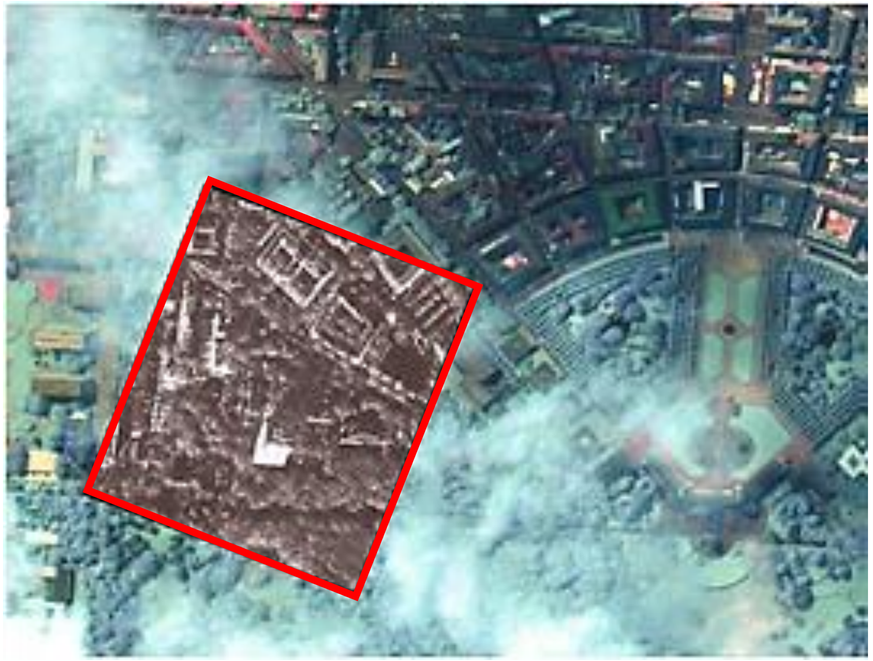


Today – Radar

- Modern Radar are very **diverse**;
  - *Military Radars;*
  - *Imaging Radars;*
  - *Radar Gun;*
  - *Automotive Radars;*
  - *Civil Aviation Radars;*
  - *Weather Radars;*
  - *Ground Penetrating Radars;*

Basic Principles

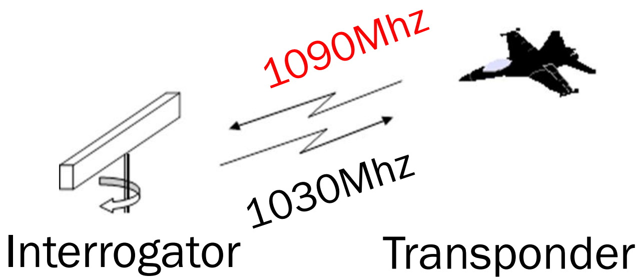
- Radar is an acronym for **RA**dio **D**etection **A**nd **R**anging;
- An object **detection** system that transmits electromagnetic (**EM**) waves and analyses the **echoes** coming from the objects;
- Why use radar?
  - *Radar can operate in any weather conditions (e.g. darkness, fog, rain);*
  - *Radar can perform its function at long and short ranges;*
  - *Radar can provide measurements in high accuracy.*

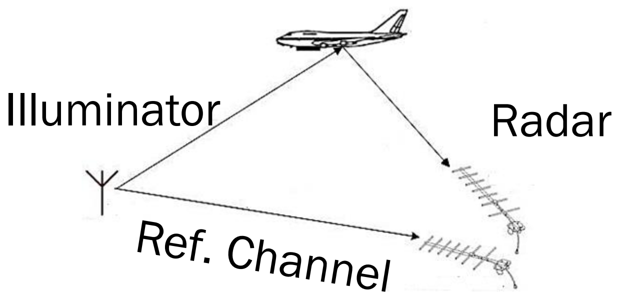


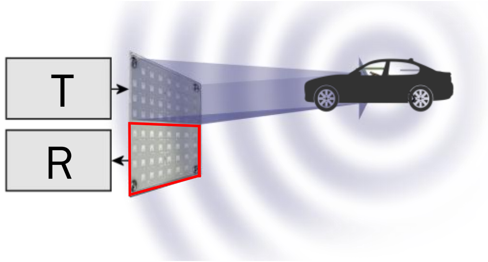
Radar vs. optical image, penetration of clouding, ©Cassidian radar, ©Eurimage, optical.

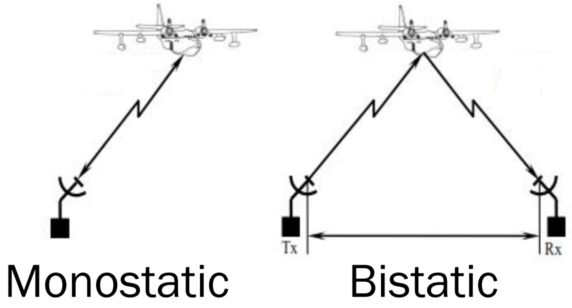
Radar Categorisation

- Operation:
  - ***Primary**: Target monitoring;*
  - *Secondary: Transponder on the target (Fig.);*
- Illuminator:
  - ***Active**: Uses its transmitter to illuminate the target;*
  - *Passive: Exploit illuminators of opportunity (Fig.);*
- Transmission rate:
  - ***Pulsed**: Emit separated pulses;*
  - *Continuous Wave (CW): Constant transmission (Fig.);*
- Geometry:
  - ***Monostatic**: Transmitter and receiver in the same location (Fig. Left);*
  - *Bistatic: Transmitter and receiver in separate locations (Fig. Right).*



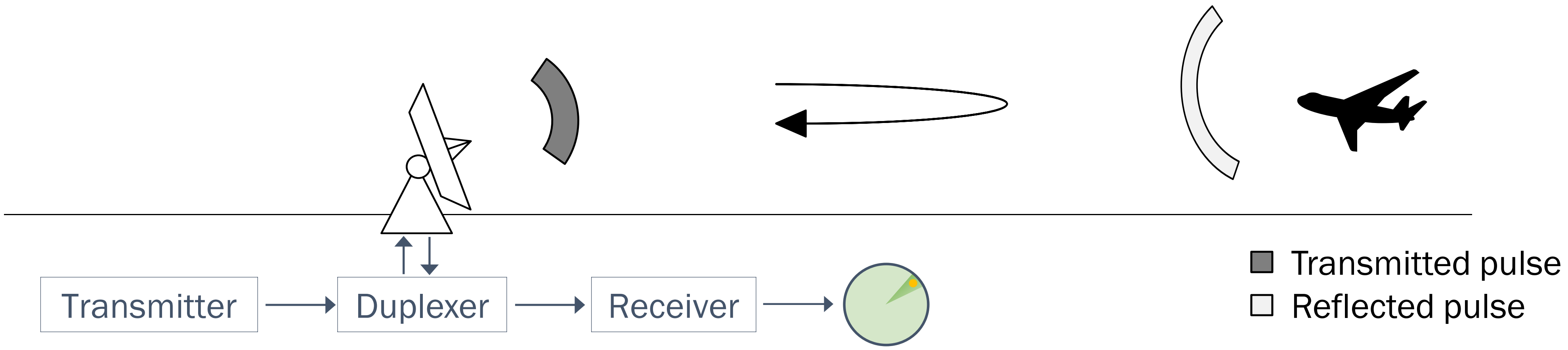






Operating Principles

- The simplest radar operation can be divided into 4 steps:
    1. *The radar is transmitting an EM pulse;*
    2. *The radar switches to listening mode;*
    3. *The pulse is reflected by a target;*
    4. *The radar receives the echoes from the transmitted pulse.*
- Using various properties of the received echo, the radar can extract parameters such as the **range** and **velocity** of the target



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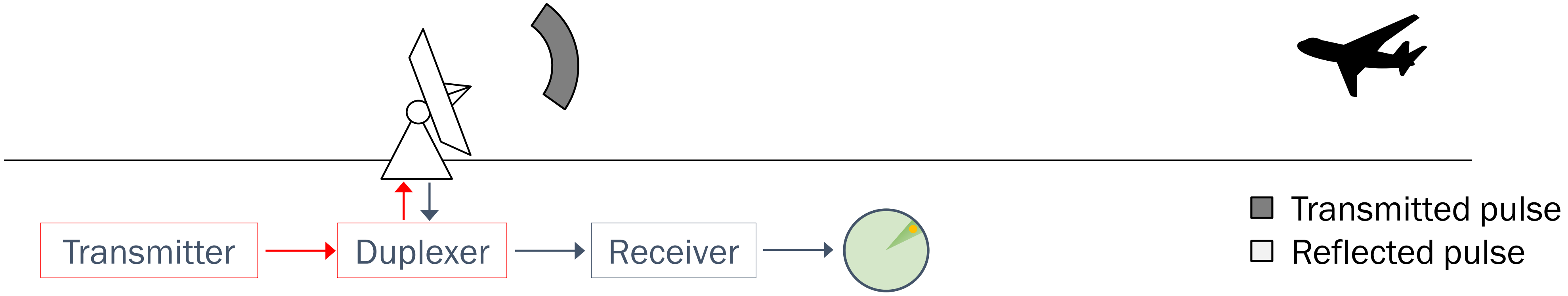

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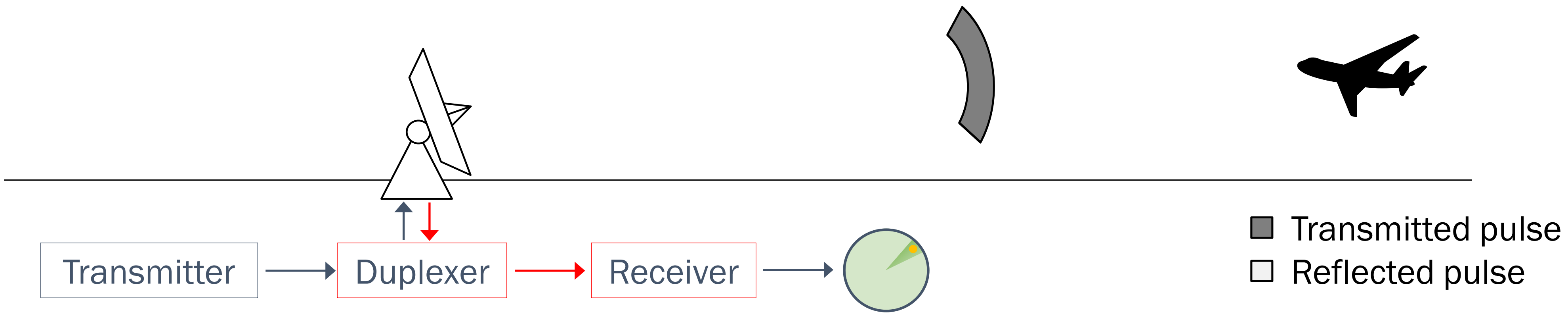

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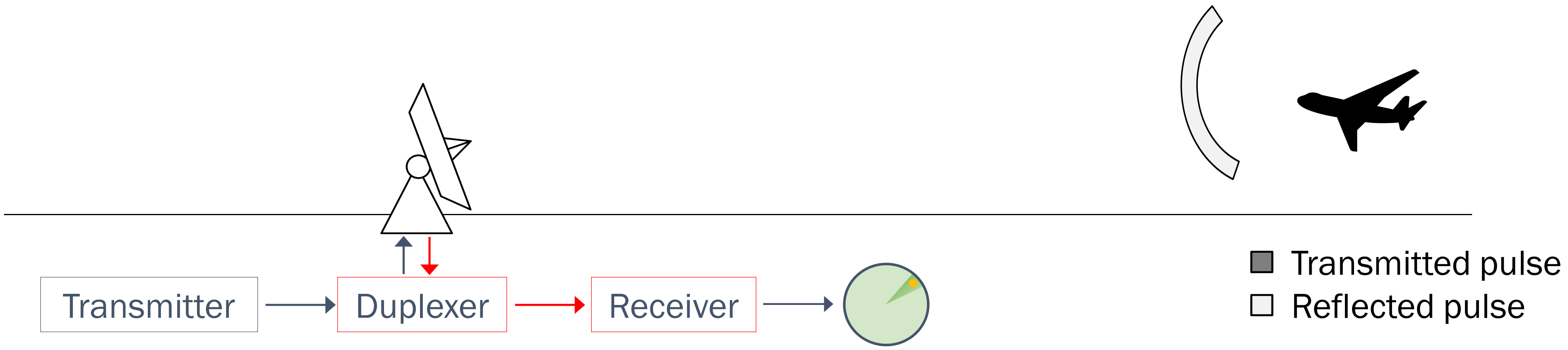

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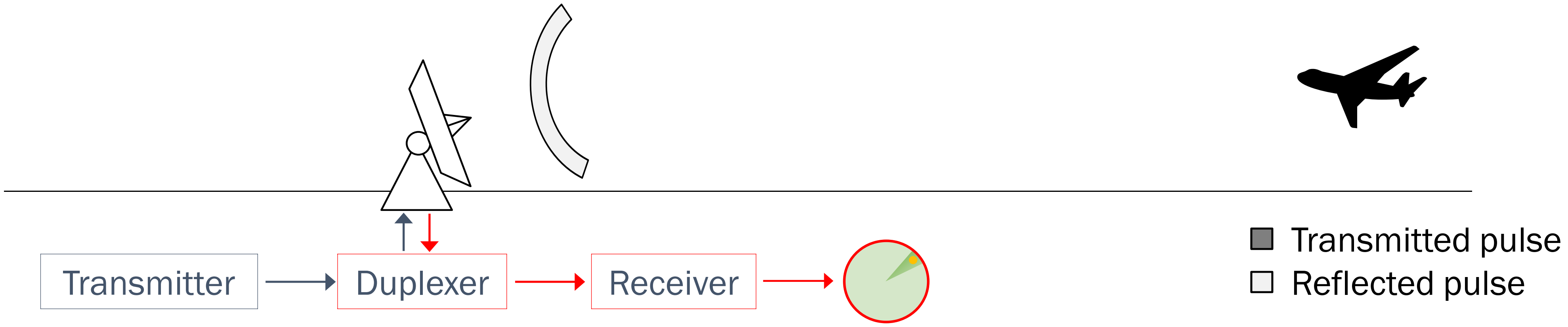


Principles of Measurements

- Radar Equation
- Distance Determination
- Range Resolution
- Direction Determination
- Pulse Repetition Interval
- Maximum Unambiguous Ranges
- Data Matrix and Data Cube

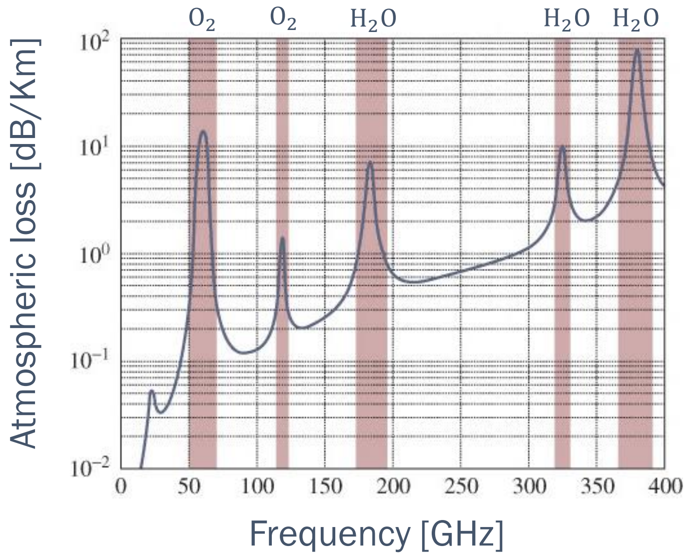
Radar Equation

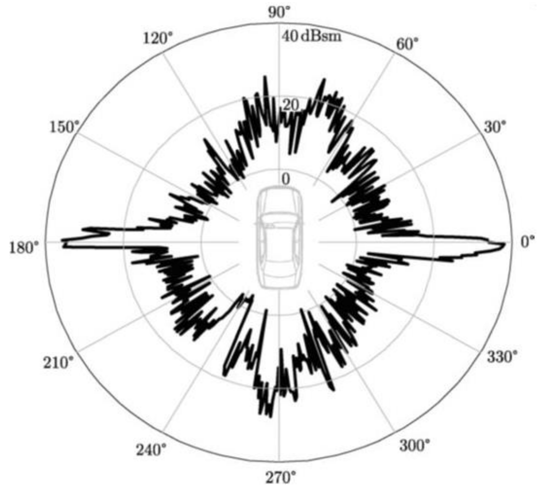
- The radar equation is referring to the power of the echo returning to the radar;

$$P_r = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 L} \rightarrow R = \sqrt[4]{\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 L \, P_r}}$$

P_t : Transmit power;
G : Antenna gain;
λ : Radar operating wavelength;
σ : Target radar cross section (**RCS**);
R : Range from the radar to the target;
L : Other losses (system, propagation).

- Low frequencies are preferable for long-range radar;
- Low RCS targets are harder to detect.



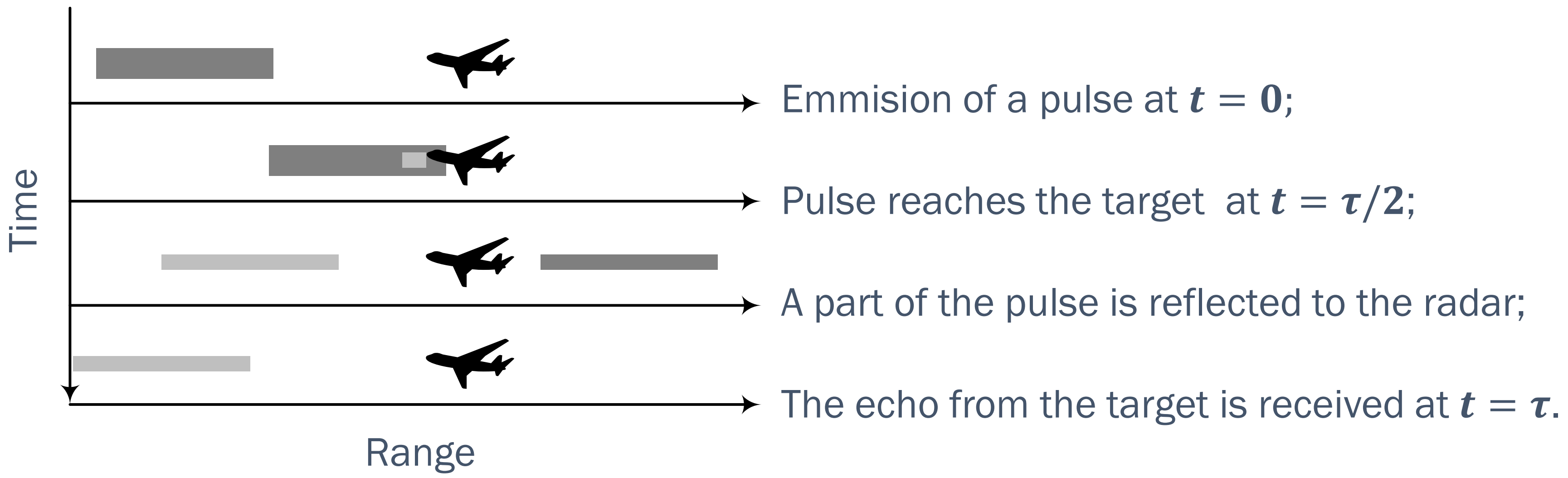


Top: Expected atmospheric path loss as a function of frequency;
Bottom: Mazda 6 RCS, Image courtesy of Hasch et al.

Distance Determination

- To determine the distance between the radar and a target, the delay of the echoed pulse id utilised;
  - *Given that EM waves travel at* $\boldsymbol{c} = 3 \times 10^8 m/s$
  - *If the echo delay is* $\boldsymbol{\tau}$, *the* ***range*** *of the target is:*

$$\boldsymbol{R} = \frac{\tau c}{2}$$



Range Resolution

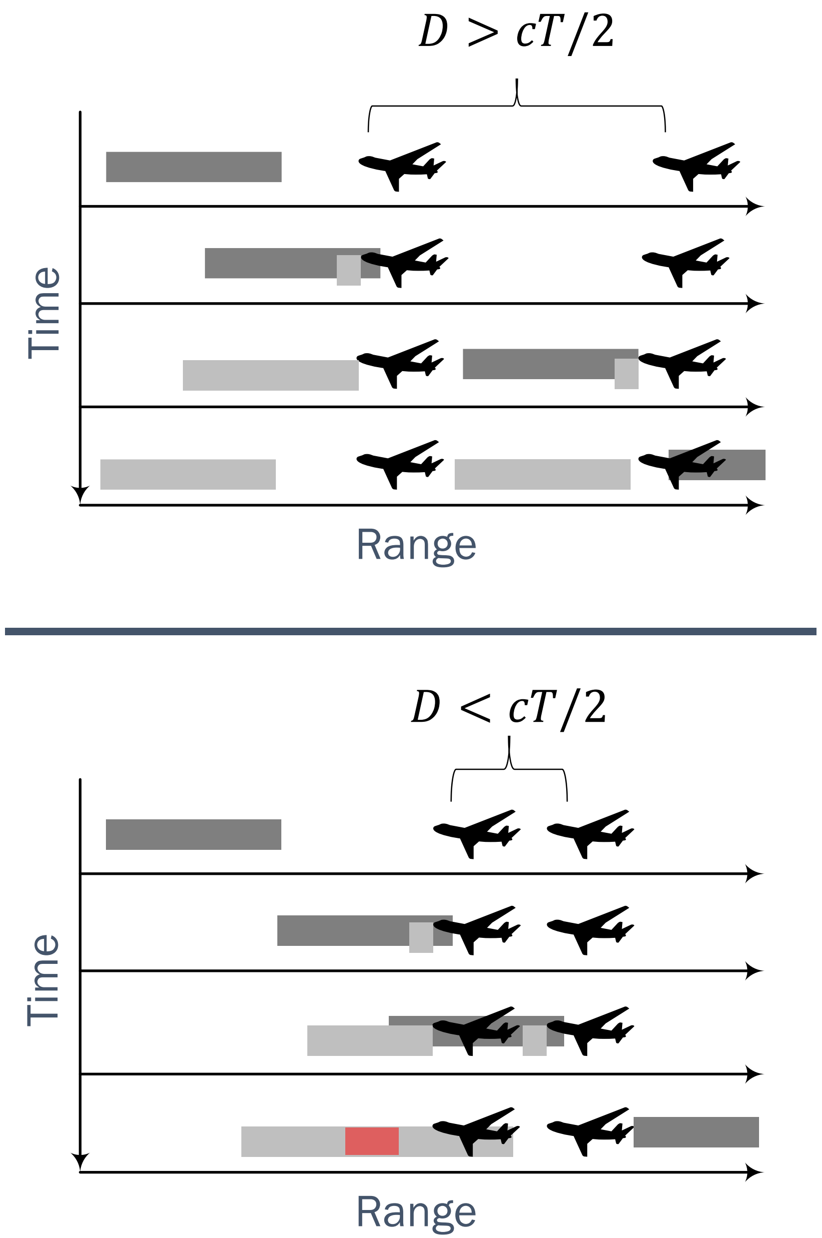
- The **resolution** of radar is its ability to distinguish between targets that are in very **close** proximity.
- The **range resolution** $\boldsymbol{\rho}$ of a radar is:

$$\rho \geq \frac{cT}{2} \approx \frac{c}{2B}$$

$\boldsymbol{T}$: Duration of pulse

$\boldsymbol{B}$: Bandwidth of signal

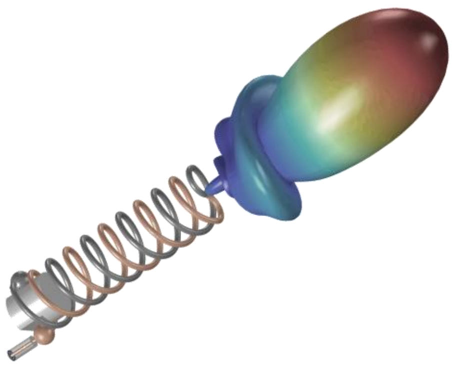
- Sorter pulses will have higher bandwidth, leading to better resolution.

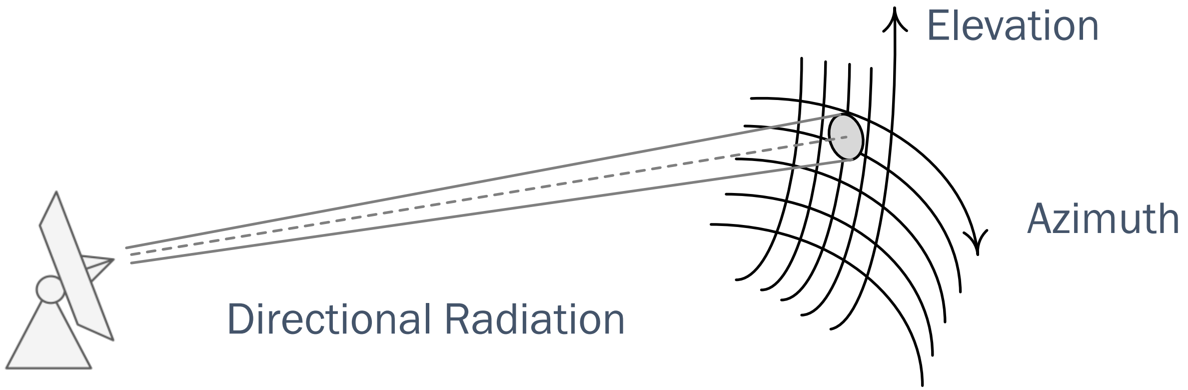


Range resolution issue between targets in close proximity with each other (**T**) Two resolved targets; (**B**) One resolved target. Red part denoted the overlap between the two echoes

Direction Determination

- The target's direction is determined by the **directivity** of the antenna, which represents the ability of the antenna to transmit the energy in a **particular direction.**
- Both the target's **azimuth** and **elevation** angles can be determined by measuring the direction in which the antenna is pointing when the echo signal is received.





Left: Radiation pattern of a Helical Antenna
Right: Illumination in different azimuth and elevation angles using a directional antenna.

Direction Determination (cont.)

- The antenna can be steered in the desired direction **mechanically** or **electronically**.

Example of radar scanning between two azimuth sectors,
Left: Top view;
Right: Radar indicator;

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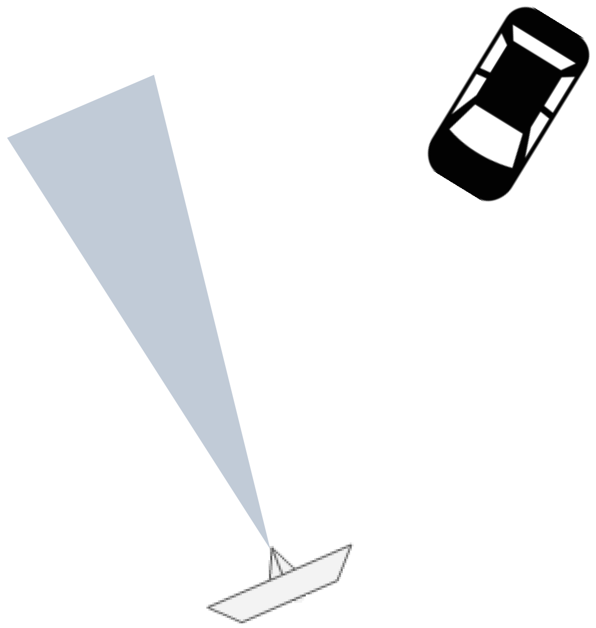

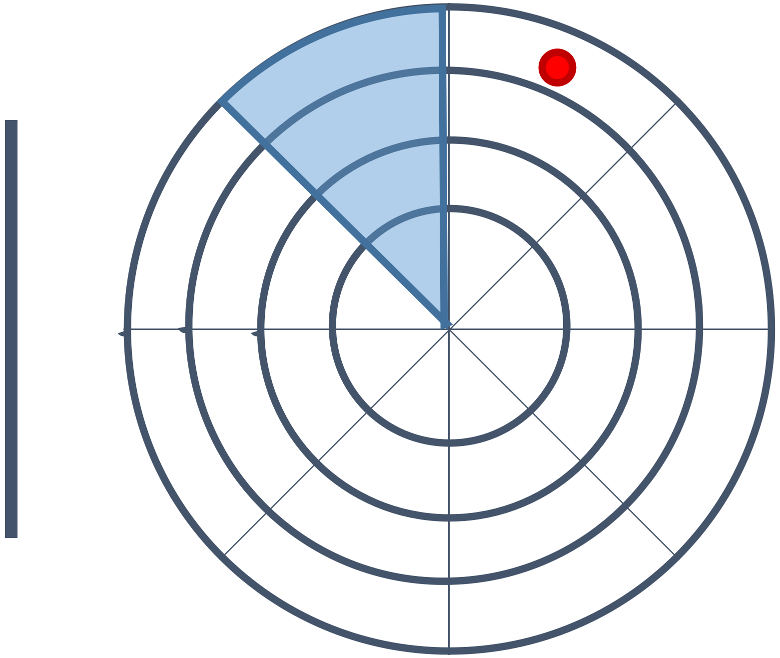

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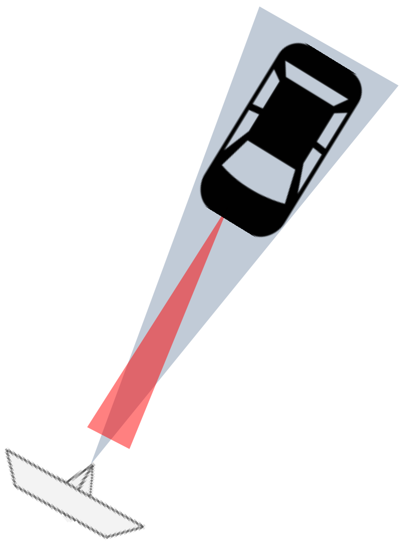
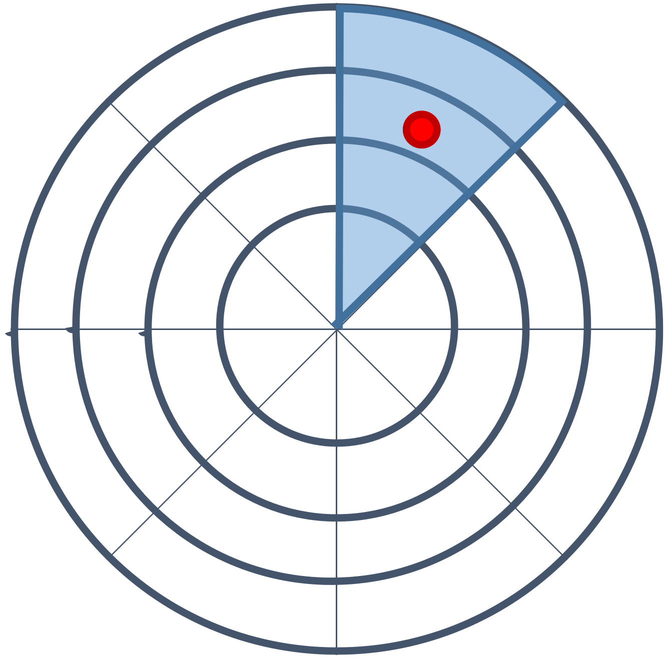
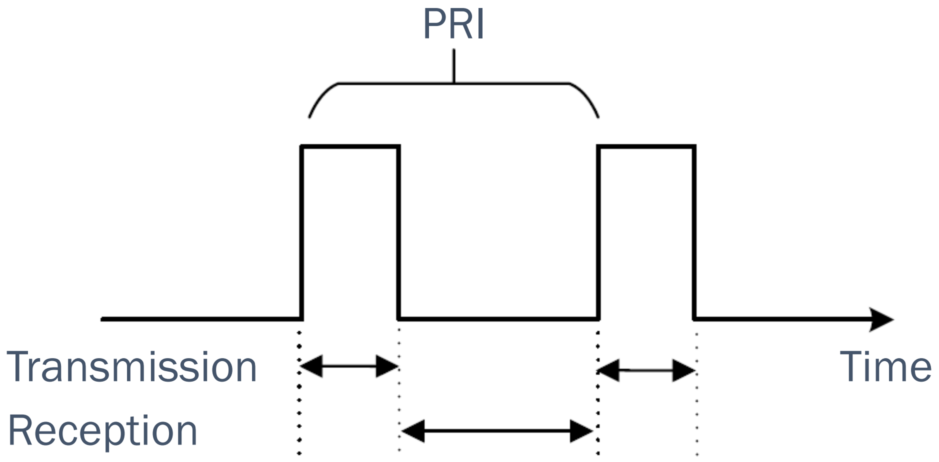




Pulse repetition Interval

- Pulse Repetition Interval (**PRI**) is defined as the time interval between consequent pulses;
- Pulse Repetition Frequency (**PRF**) is given as:
$$\mathrm{PRF} = 1/\mathrm{PRI}$$
- **Duty cycle** is defined as the time proportion of PRI in which the transmission takes place:
$$\text{Duty Cycle} = T/\mathrm{PRI}$$
- If the same antenna is used for transition and reception, the duty cycle gives a measure of how long the radar is "blind".

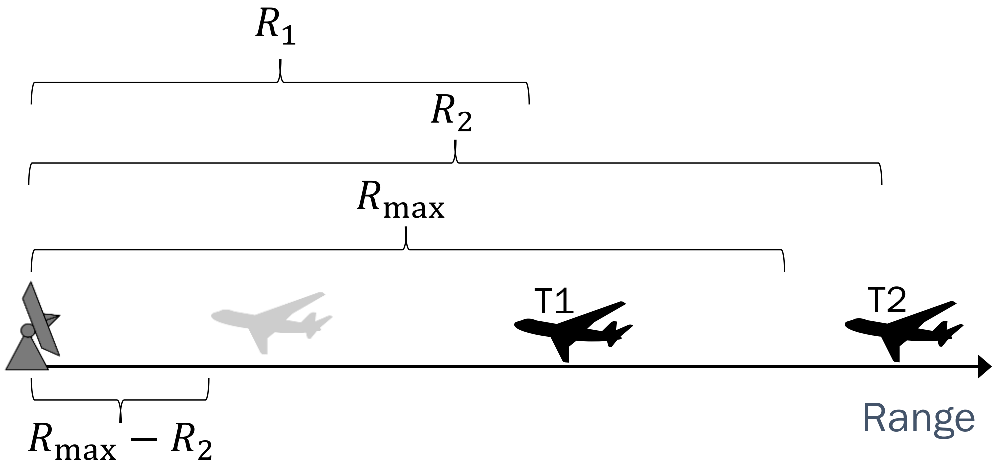


Maximum Unambiguous Range

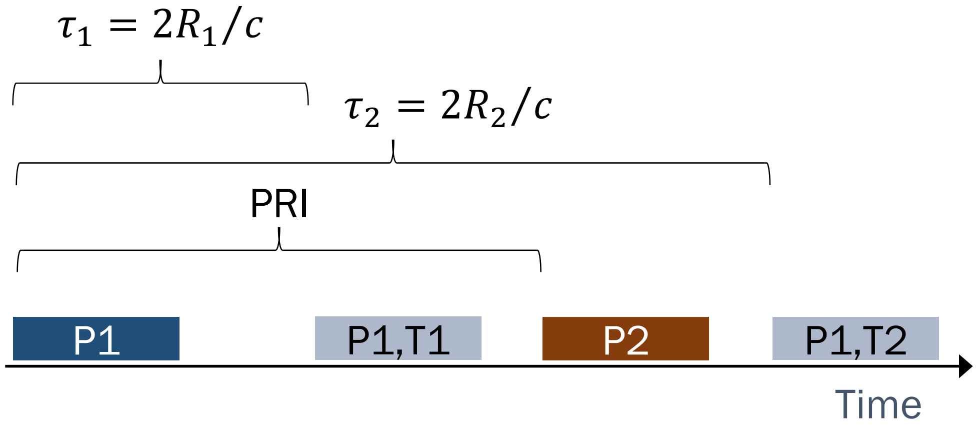
- The maximum unambiguous range defines the maximum distance to locate a target.

$$R_{\max} = \frac{c\mathrm{PRI}}{2} = \frac{c}{2\mathrm{PRF}}$$

- Radar is not able to discriminate between echoes from an older and the current transmission.



Left: Radar and two real targets (dark), one in (T1) and one out (T2) of unambiguous range, second target (T2) appears in closer range (light).



Right: Transmitted (dark) and received pulses (light) at the radar in time, radar confuses the echo from fist pulse to second target (P1,T2) to an echo from second pulse (P2) and a target at a closer range ($R_{\max} - R_2$).

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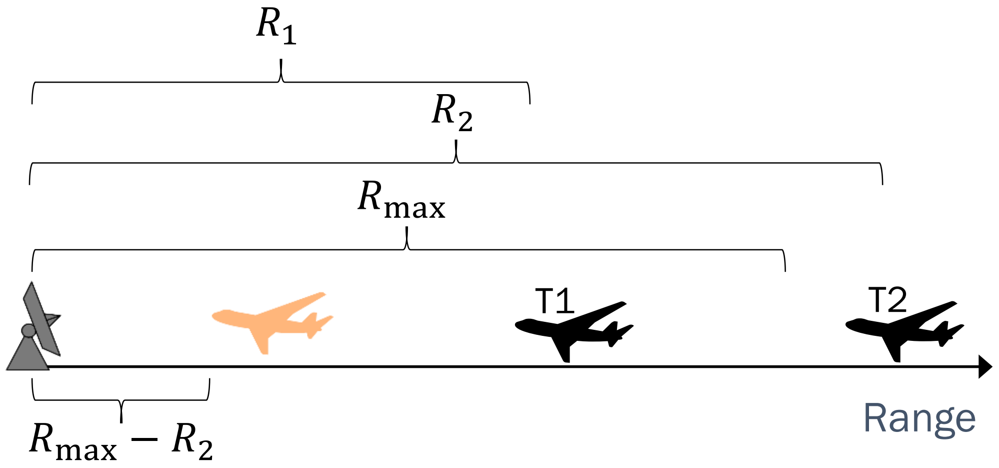

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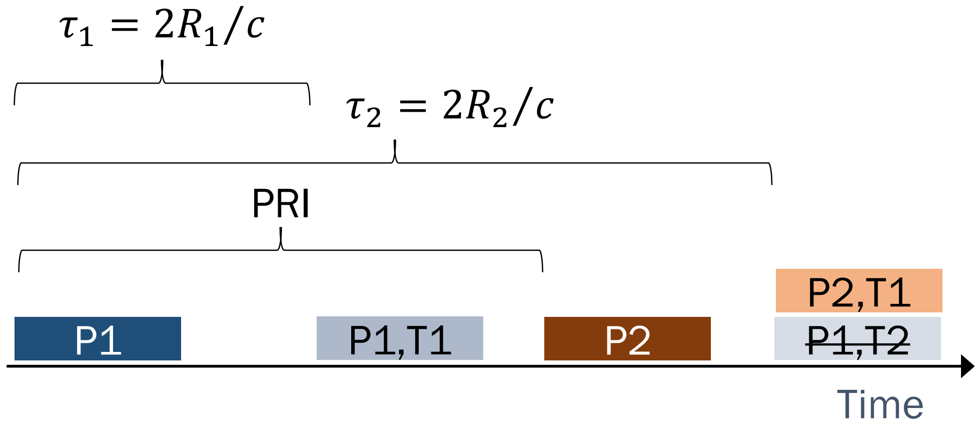

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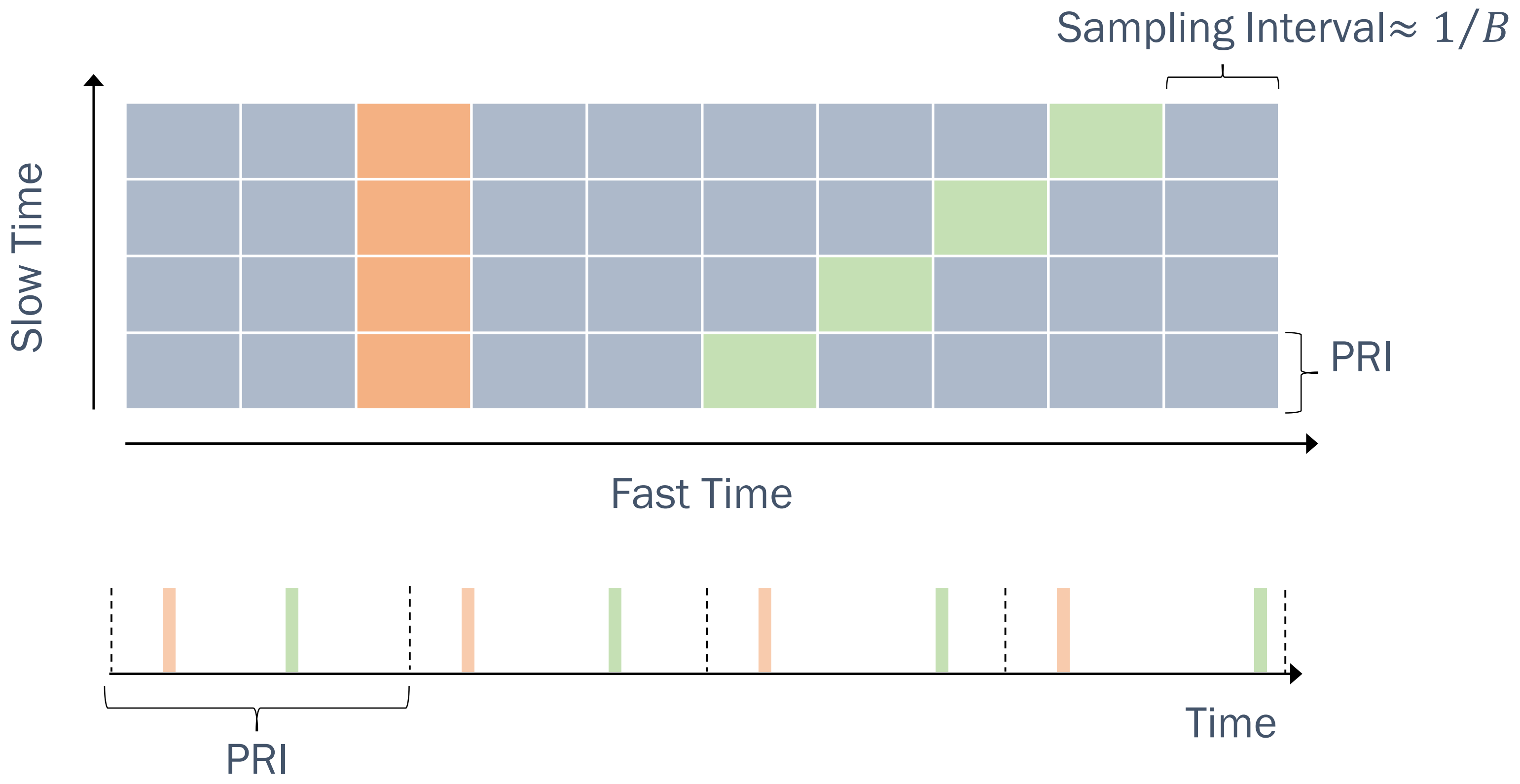
Data matrix

- Radar returns from each PRI are stored in memory for further processing;
- **Fast Time** refers to the different time slots composing a PRI, sampling rate dependent;
- **Slow Time** updates every PRI;



Example of two targets, one staying in the same resolution bin (orange) and one moving in different resolution bins (green);
Top: Data matrix for 10 time resolution bins and 4 PRI;
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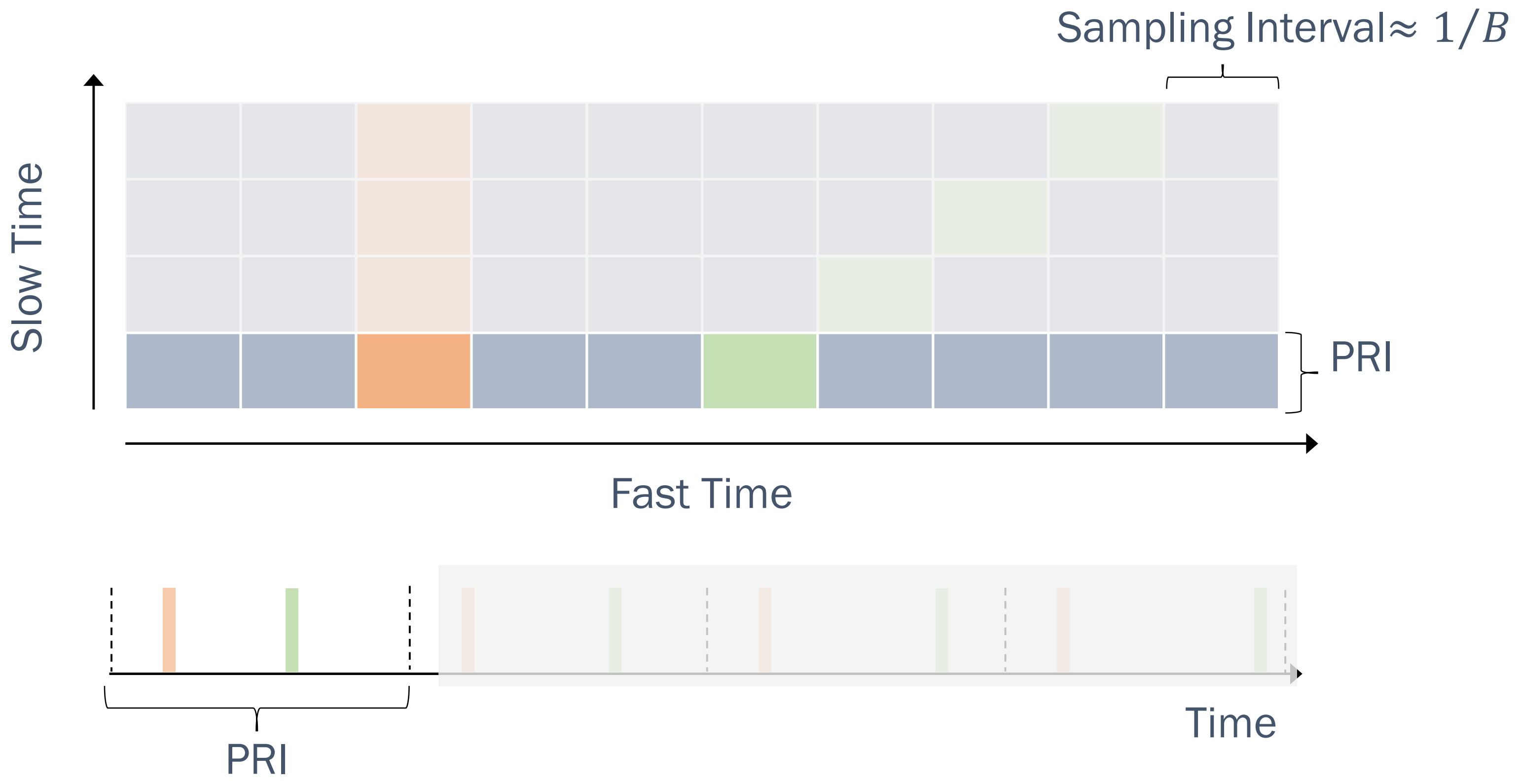

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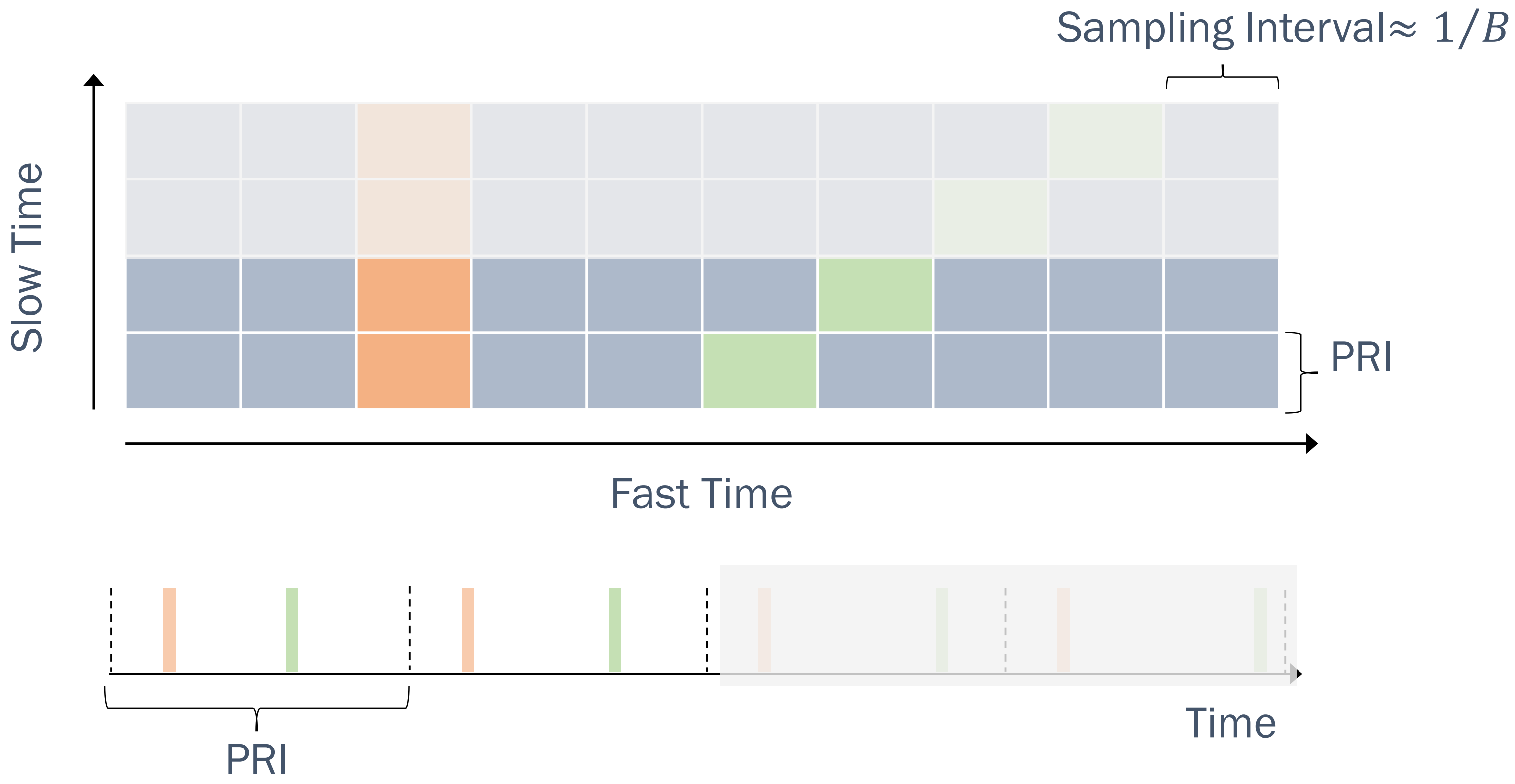

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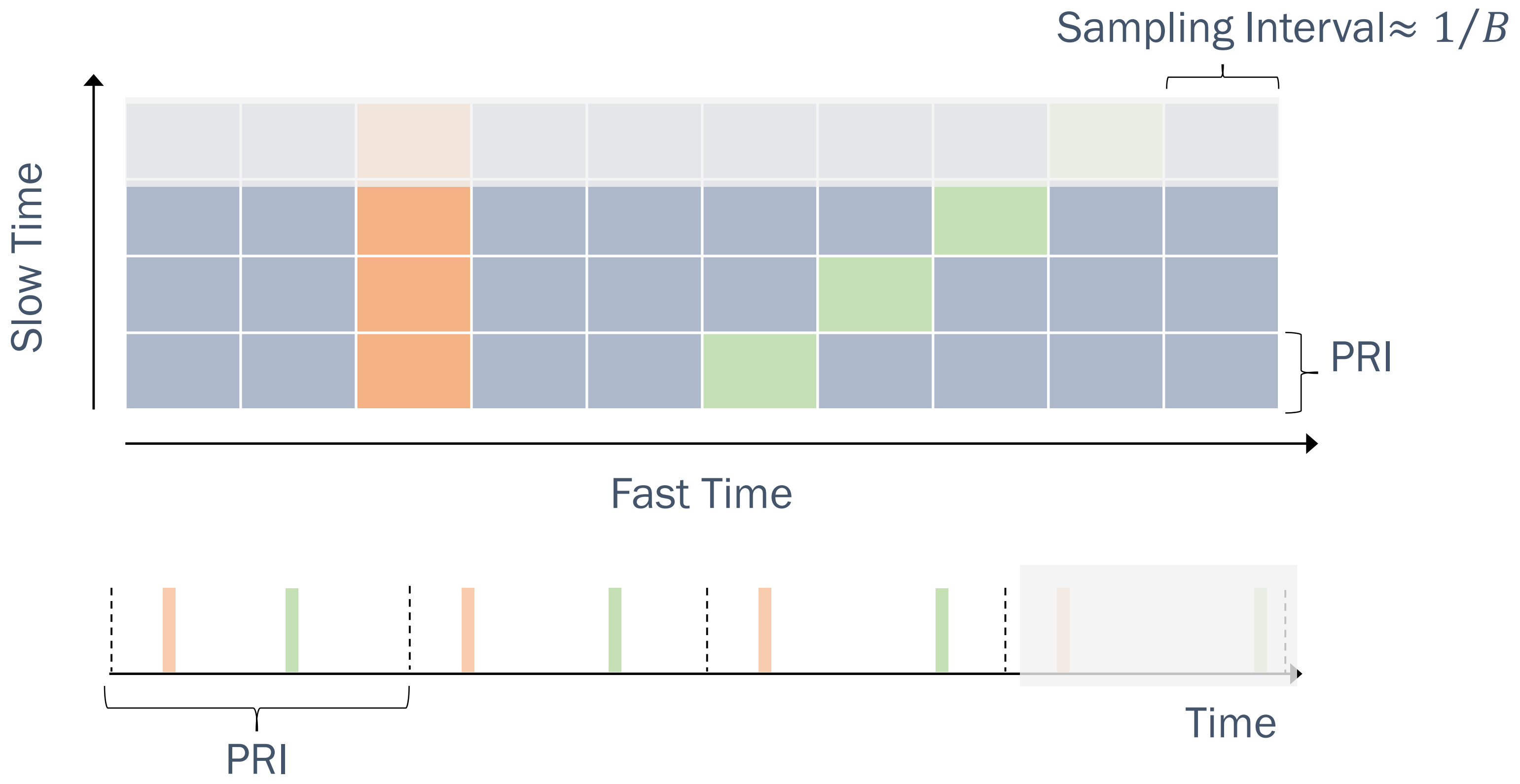

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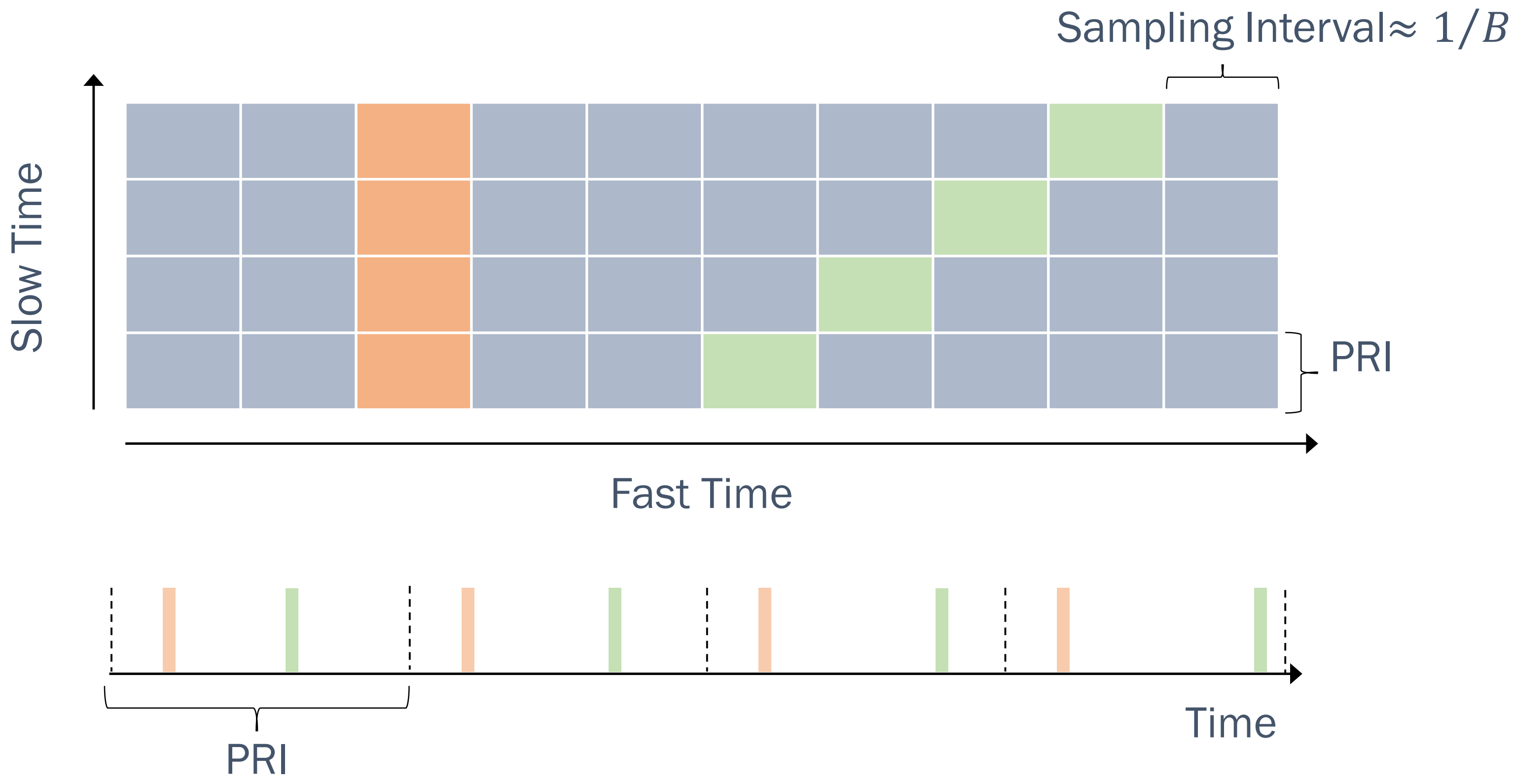

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Data Cube

- Data Cube is an extension to Data Matrix including spatial sampling;
- In cases that the radar uses **multiple** receiving channels, the **data matrices** from each receiver are stacked to form a data cube;

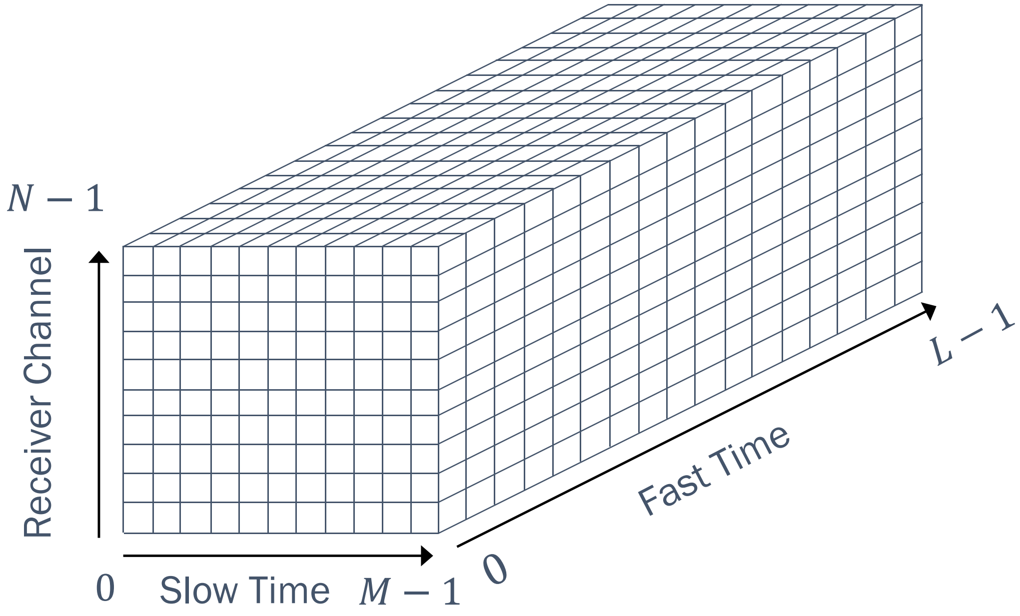
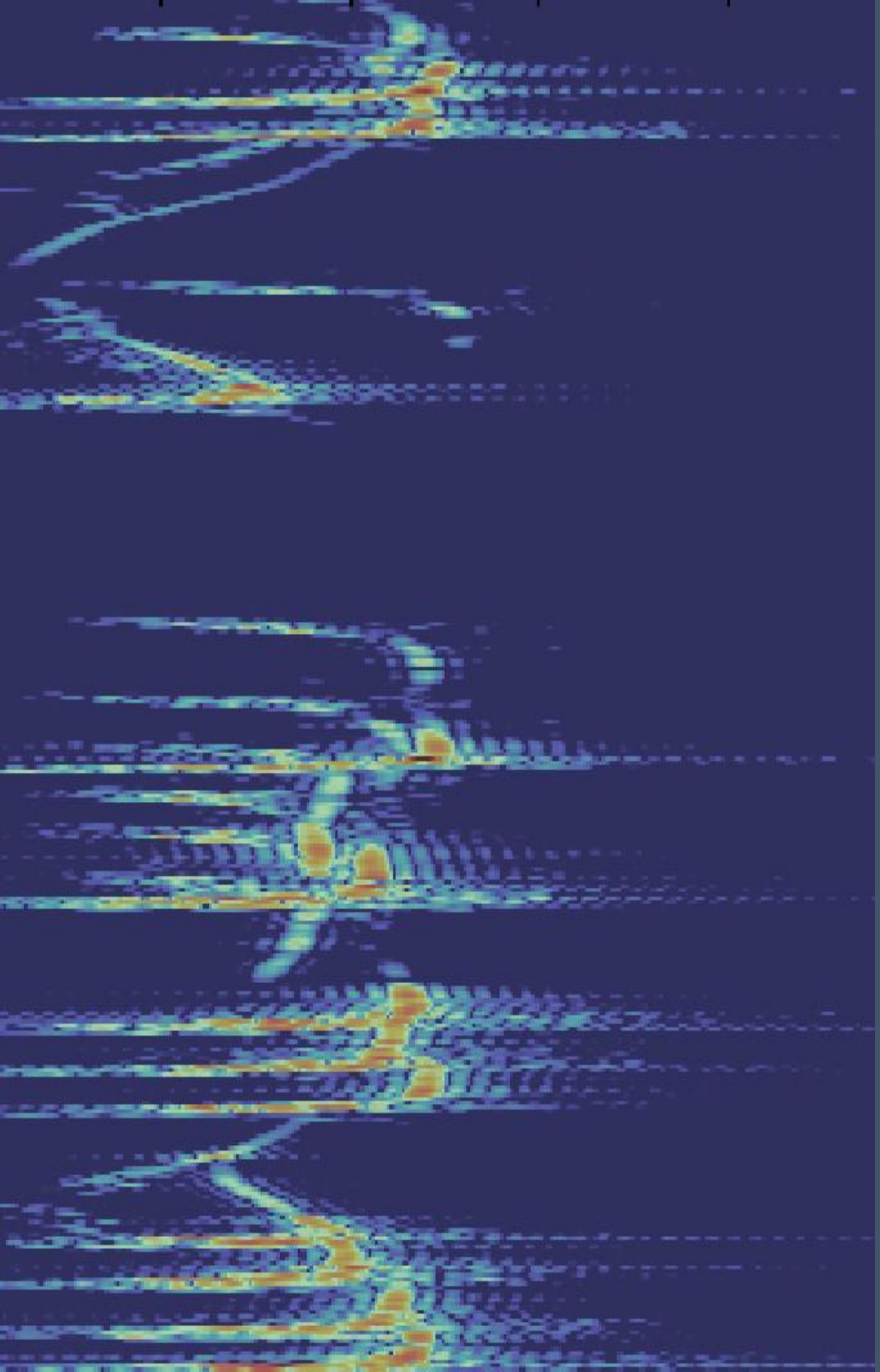


Illustration of a data cube for L time samples in each PRI and M PRI in a system composed of N receiver channels.



Coherent and Doppler processing.

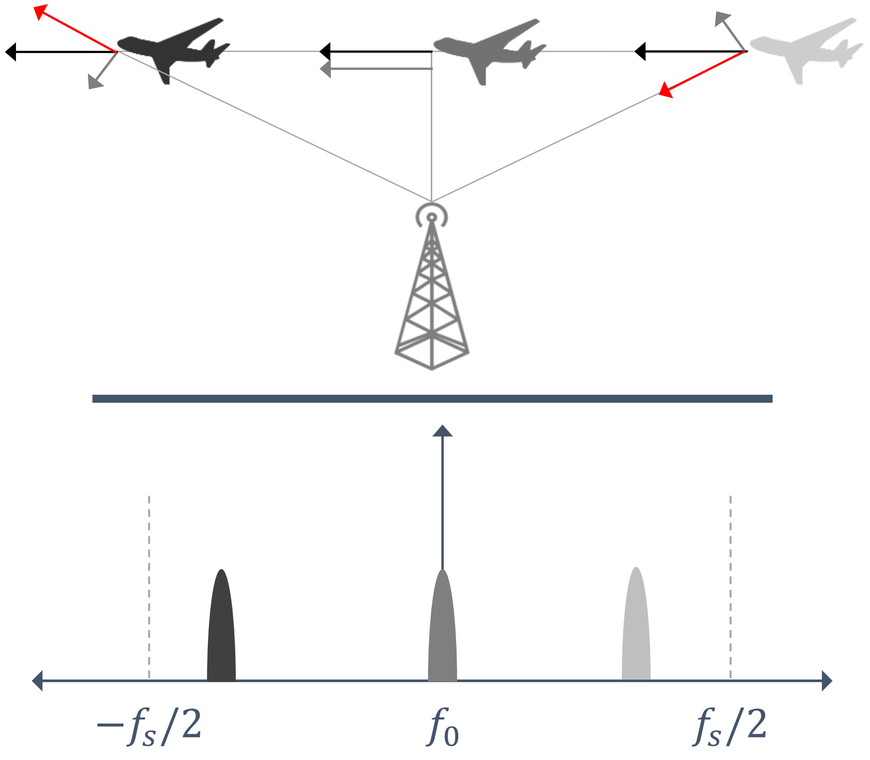
- Spectrum of Continuous Wave Signal;
- Spectrum of Pulsed Signal;
- Range-Doppler Maps;

Spectrum of Continuous Wave Signal

- Consider a continuous wave (CW) radar with operating frequency f_0;
- In the presence of a target moving with radial velocity u_r, due to the **Doppler** phenomenon, the echoed signal will be shifted in frequency by:

$$f_D = \frac{u_r}{c} f_0$$

- **Positive** Doppler shifts ($f_D > 0$) indicate that the target is moving **towards** the radar, while **negative** ($f_D < 0$) **away** from it;



Stationary radar and moving target scenario: (**T**) geometry of the radar target system, (**B**) frequency observed by the radar

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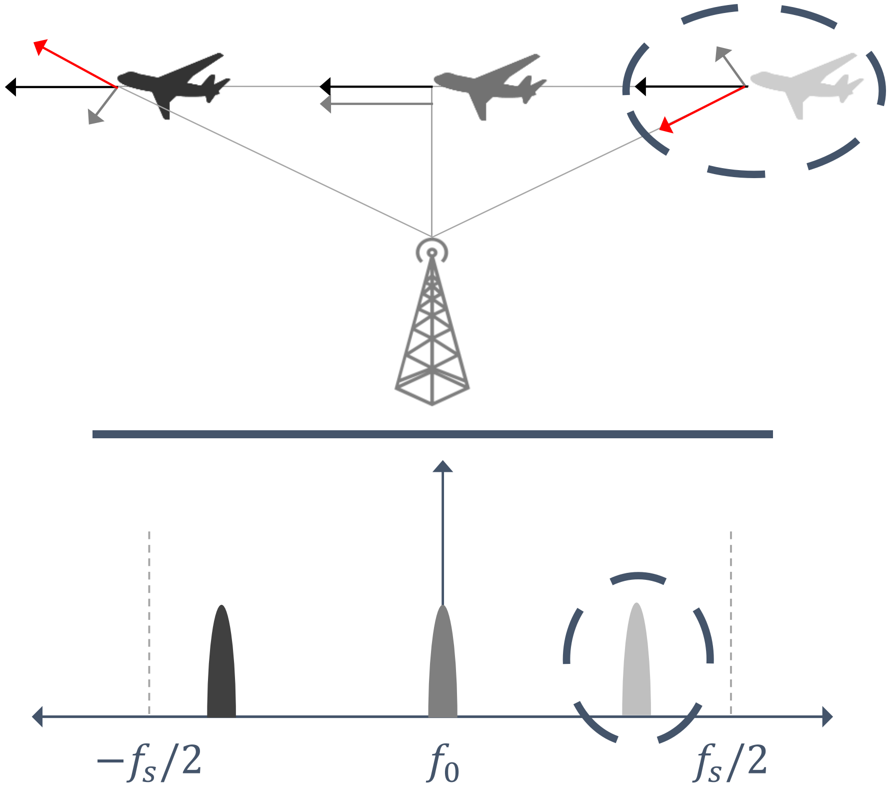

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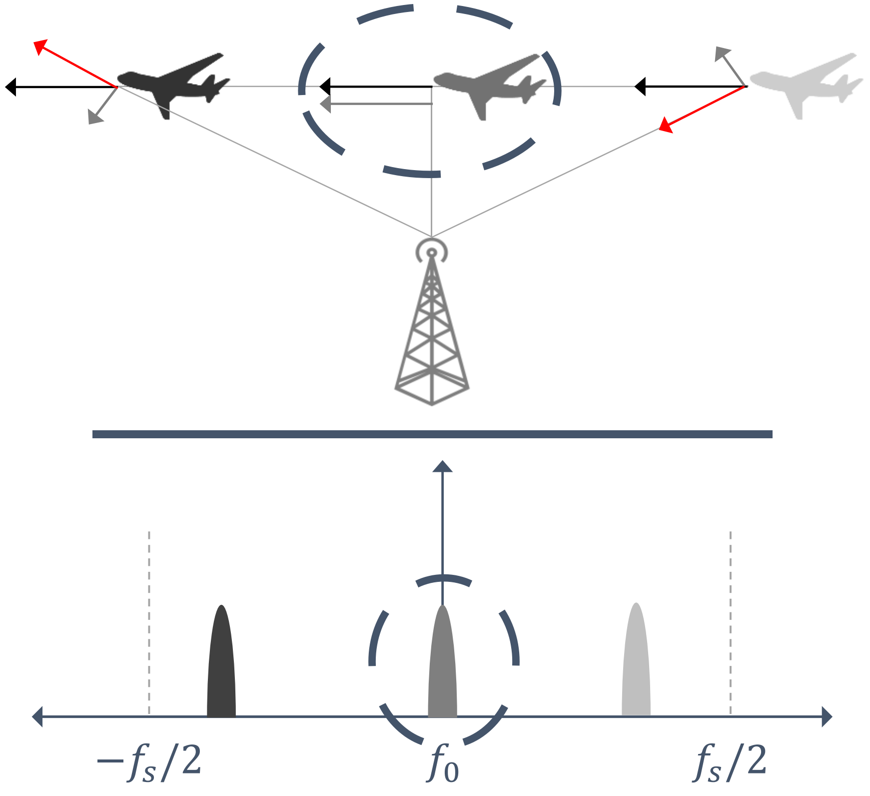

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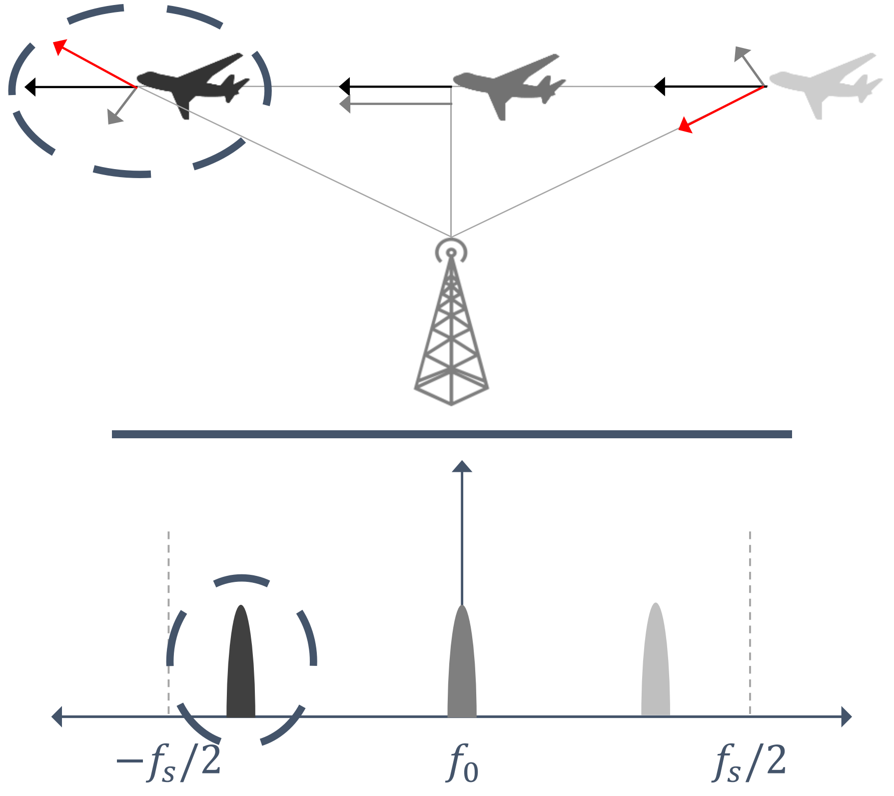

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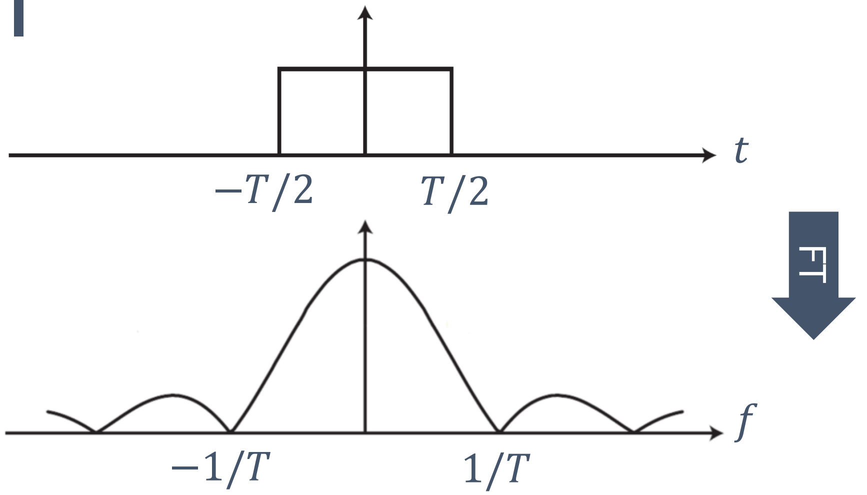
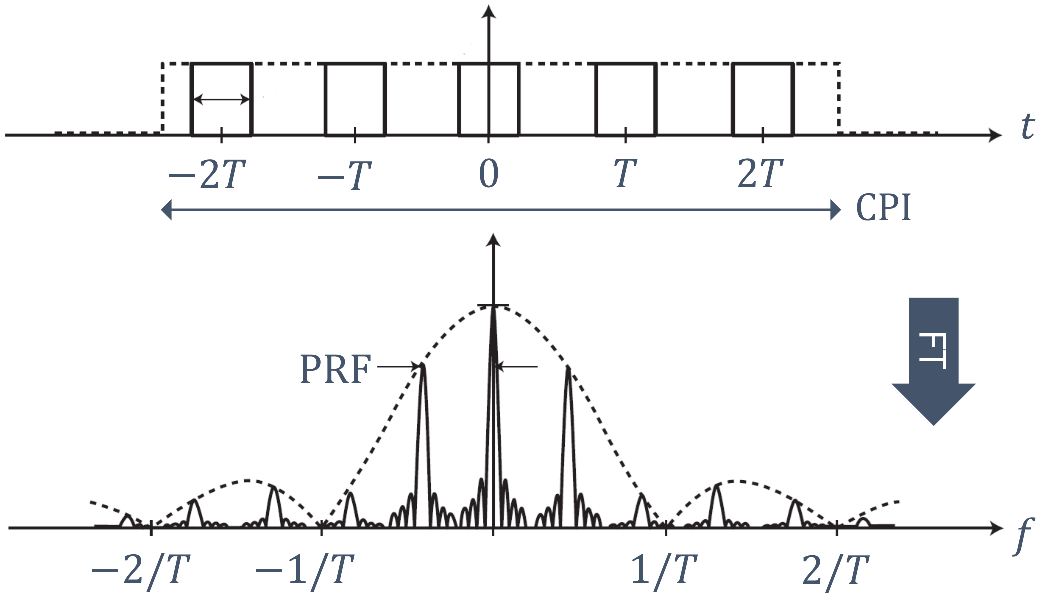
Spectrum of Pulsed Signal

- In most radar systems, the bandwidth of a single pulse may be a few orders of magnitude **greater** than the expected Doppler frequency shift:

$$\frac{1}{T} \gg f_D$$

- Echoes from moving targets **cannot be discriminated** from stationary clatter in spectrum;
- Using consequent pulsed over a coherent pulse interval (**CPI**), the single pulse bandwidth is divided into spectral line of approximate bandwidth $1/\mathrm{CPI}$.





Spectrum of Pulsed Signal

- In most radar systems, the bandwidth of a single pulse may be a few orders of magnitude **greater** than the expected Doppler frequency shift:

$$\frac{1}{T} \gg f_D$$

- Echoes from moving targets **cannot be discriminated** from stationary clatter in spectrum;
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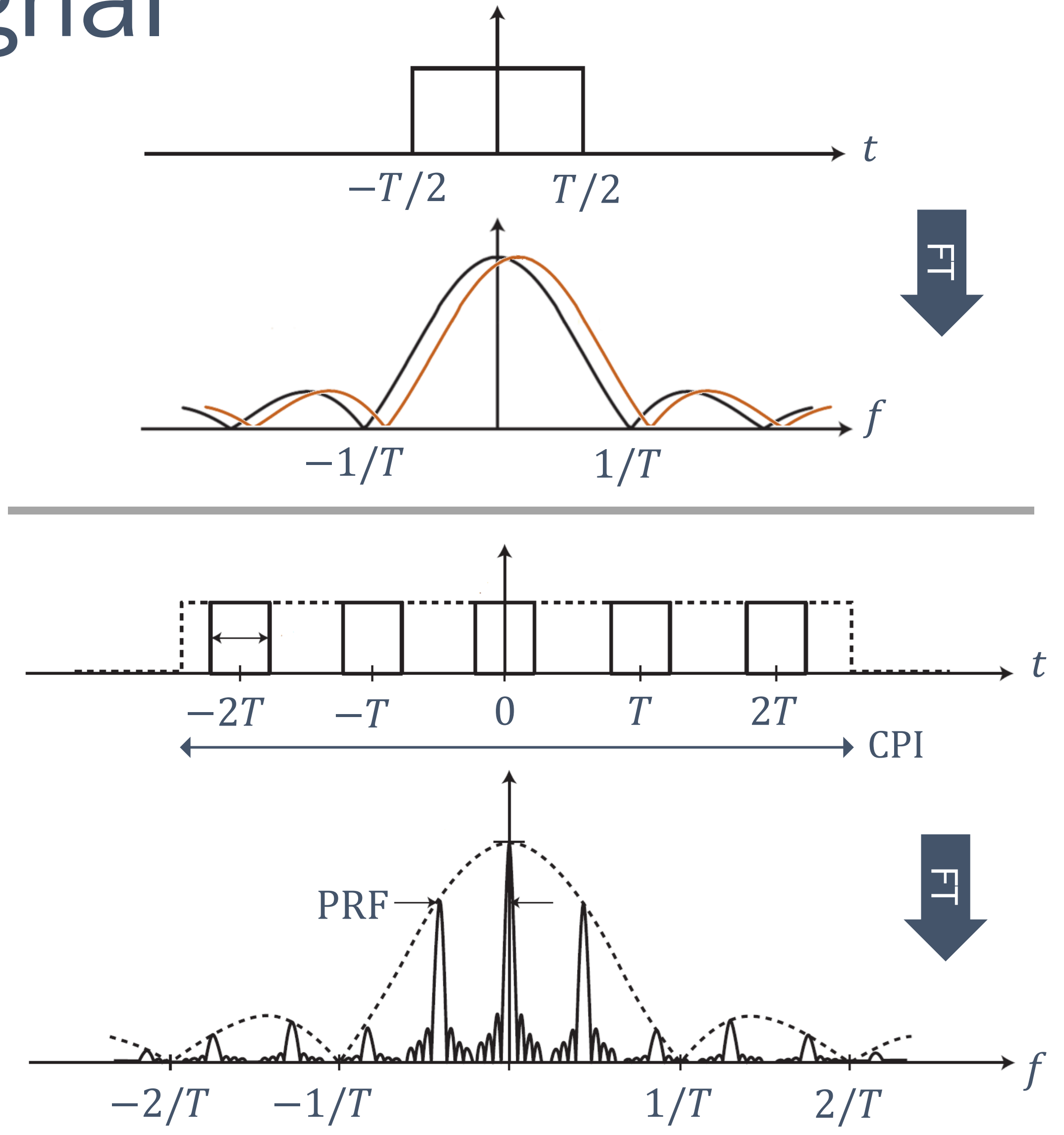

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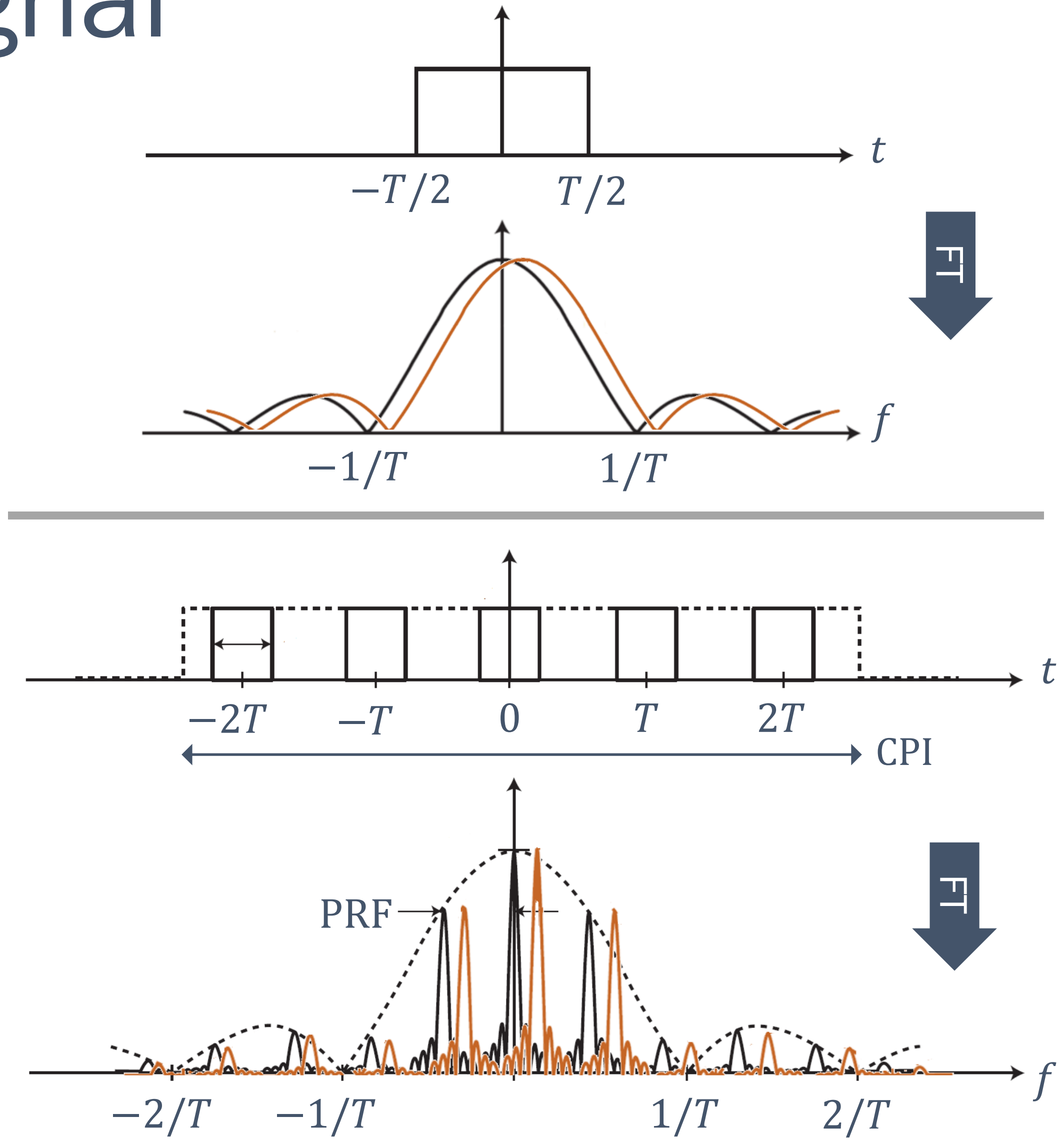
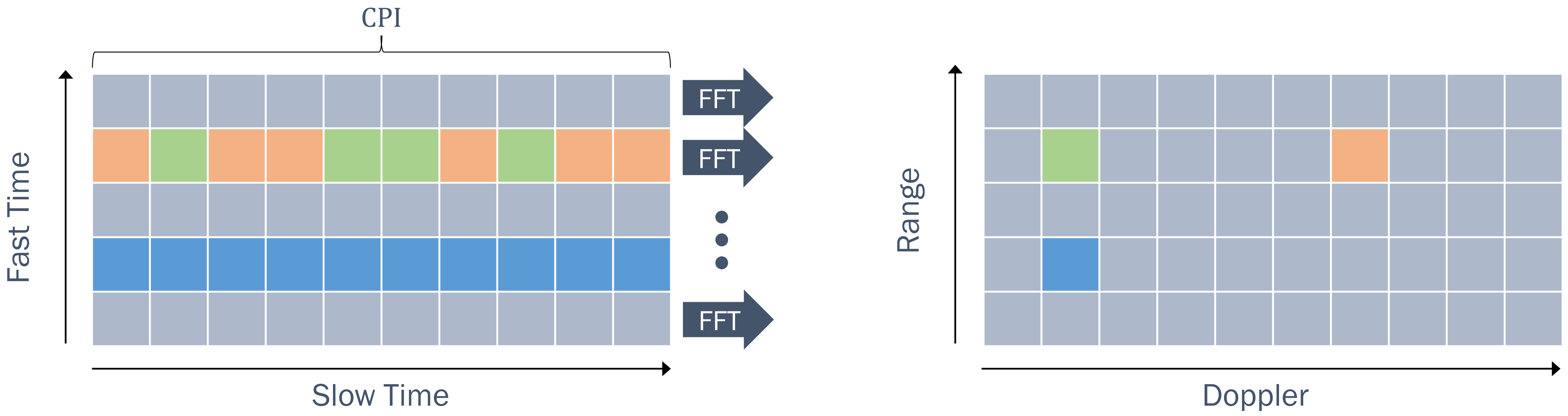
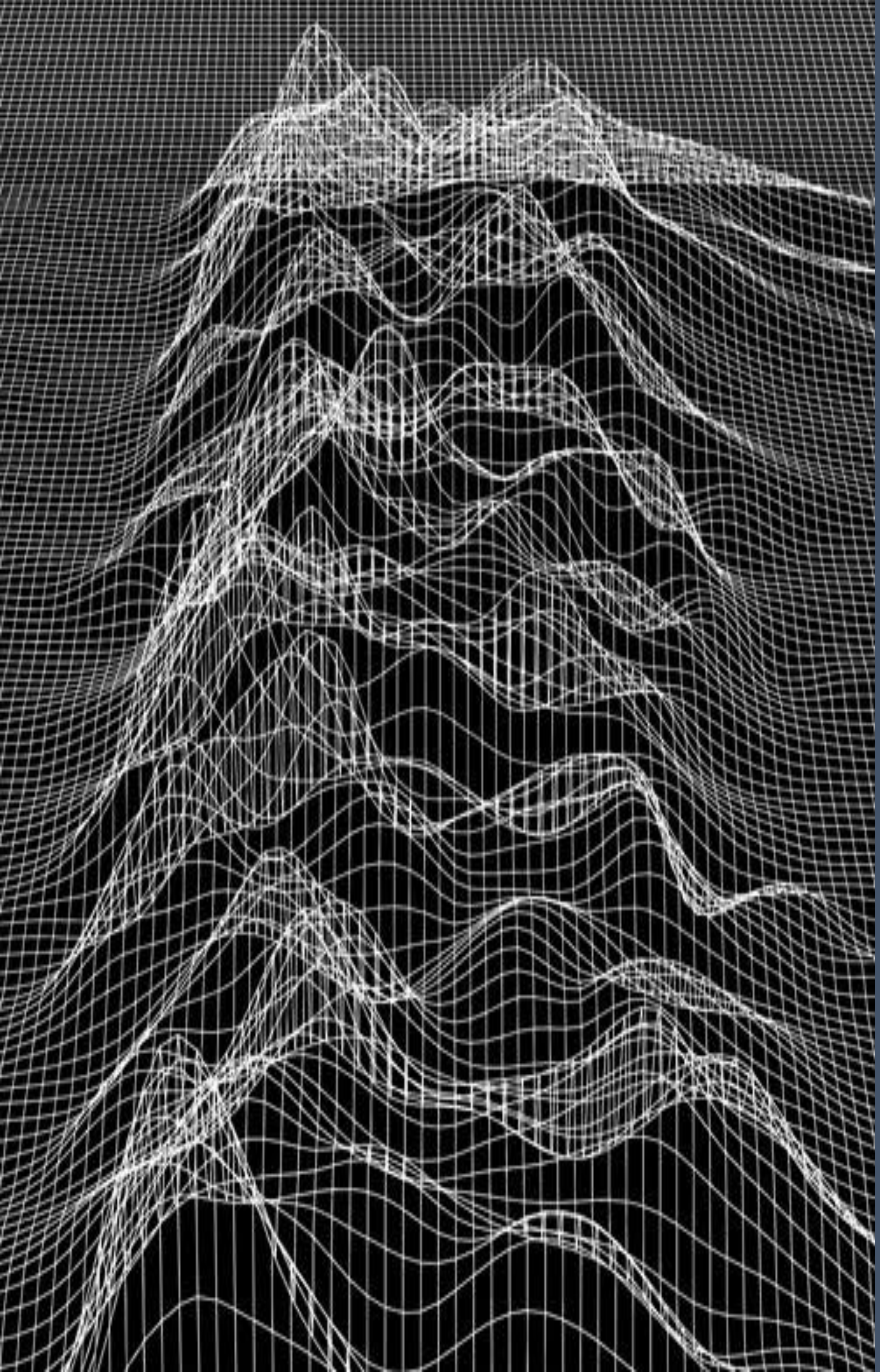


Range-Doppler Maps

- In a moving target the **phase information** appears in each received pulse.
- Different returns can be **separated** in the Doppler domain.
- Range-Doppler map is contracting by converting **Fast time** to **Range** and **Slow time** to **Doppler** by applying Fourier Transform.



Scenario of 3 targets: two in the same range bin and different velocity (green and orange) and one in different range (blue), (**L**) In Data matrix two targets can be separated, (**R**) In Range-Doppler map all 3 targets can be separated.



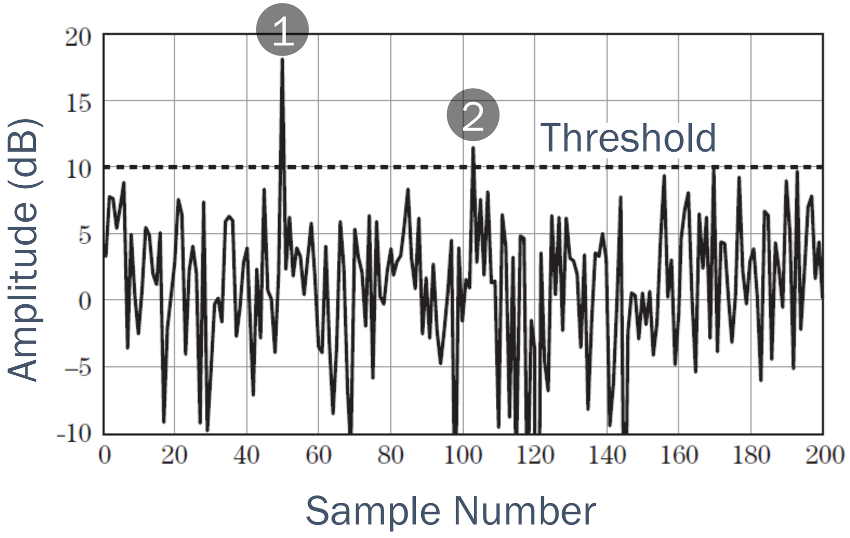
Waveforms Design and Pulse Compression

- Noise and Interference
- Matched Filter
- Pulse compression
- Linear Frequency Modulation
- Ambiguity Function

Noise and Interference

- Noise is a random, unwanted signal characterised by statistical properties;
- Sources of interference can be **internal** (equipment imperfections) or **external** (other RF transmissions), **passive** (clutter) or **active** (jammers);
- The power ratio between the **useful** and **unwanted** signal is defined as **signal-to interfered-plus-noise ratio (SINR):**

$$\mathrm{SINR} = \frac{P_{\mathrm{Signal}}}{P_{\mathrm{Interfernce}} + P_{\mathrm{Noise}}} \rightarrow \mathrm{SNR} = \frac{P_{\mathrm{Signal}}}{P_{\mathrm{Noise}}}$$



Example of a high SNR target (1) and a false detection (2), the radar is not able to discriminate between interference and low SNR targets [Principles of Modern Radar - Mark A Richards].

Matched Filter

- The knowledge of the transmitted signal is utilised to design a **linear filter** that maximises the SNR;
- In the presence of additive Gaussian noise, the optimum filter is a **time reversed** version of the transmitted signal ("matched");

$$h(t) = x^*(\tau_{\max} - t)$$

$h(t)$: Matched filter of $x(t)$;

$\{\cdot\}^*$: Complex conjugate;

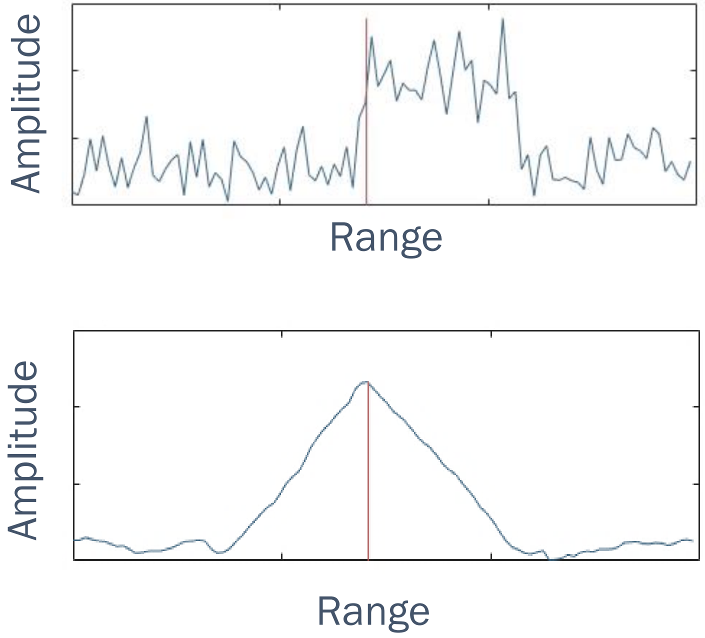
$\tau_{\max}$: Time instant in which the SNR is maximised;

- For noise given by $\mathcal{CN}(0, \sigma^2)$, the maximum SNR is:

$$\mathrm{SNR}_{\max} = \frac{E}{\sigma^2}$$

E: Energy of the pulse.

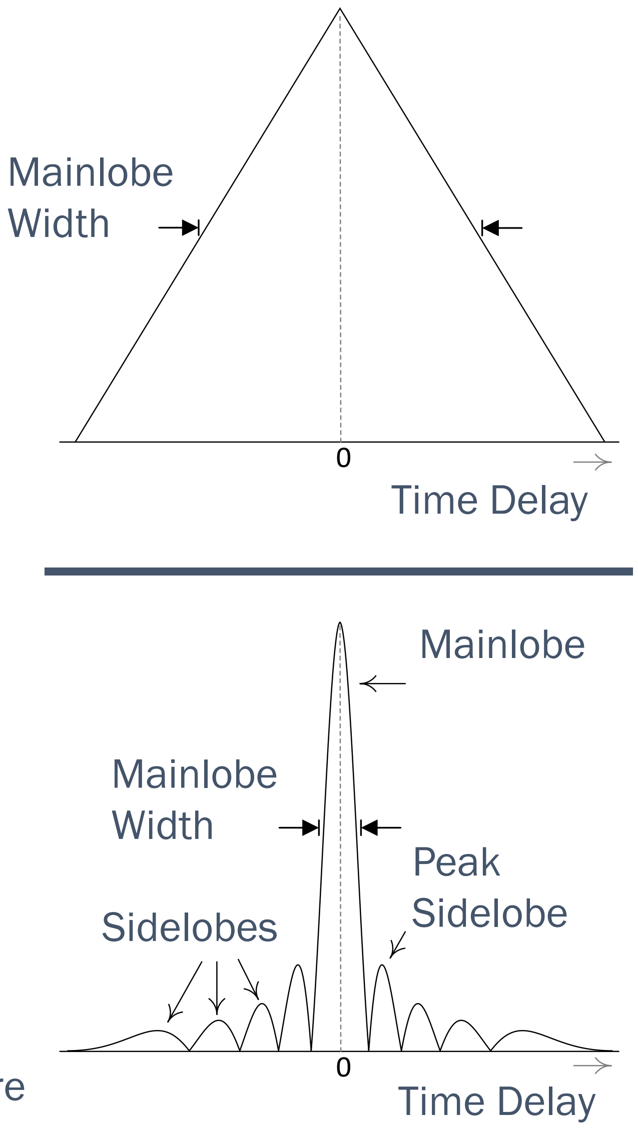
- The output of the matched filter is the auto-correlation of the pulse.



Range profile with a target at the red line (**T**) before and (**B**) after matched filter.

Pulse Compression

- Sort pulses provide good resolution but **not** enough energy for long distances;
- The resolution is (**almost**) proportional to the bandwidth;
- Using pulse compression **long waveforms** (high energy) can achieve the **resolution** of a short pulse by increasing their bandwidth through internal modulation;
- A side effect of pulse compression is the rise of **undesired sidelobes**;



Matched filter output of (**T**) an unmodulated square pulse and (**B**) a linear frequency modulated pulse.

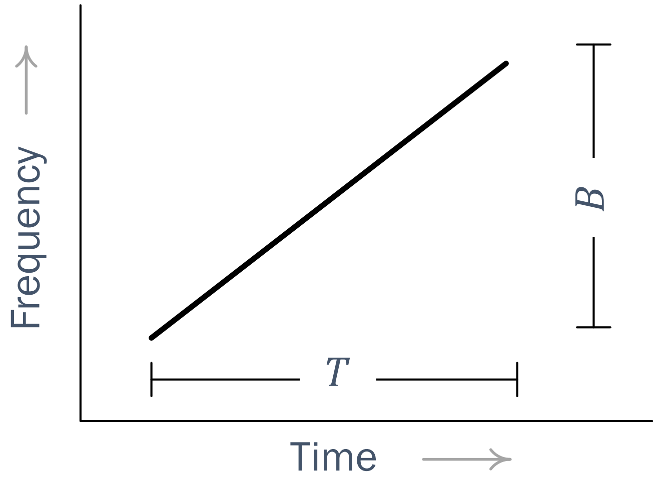
Linear Frequency Modulation

- Pulse compression can be achieved using frequency modulation (**FM**);
- Linear FM (**LFM**) is a very popular choice;
- LFM achieve high resolution while keeping the H/W implementation relative simple;

$$x(t) = e^{j\pi(B/T)t^2}, \quad 0 \le t \le T$$

$$f = \frac{B}{2T}t : \textit{Instantaneous frequency;}$$

- LFM suffer from high sidelobe levels (**SLL**);
- Using **non-linear** FM (NLFM) the SLL can be reduced but are more complex to generate.



Top: Real part of (**L**) an unmodulated pulse and (**R**) a LFM pulse;

Bottom: Time-Frequency profile of a LFM pulse.

Ambiguity Function – Definition

- The ambiguity function (AF) is a 2-D function describing the response of a matched filter when the signal is received with a **delay $\boldsymbol{\tau}$** and a **Doppler shift $\boldsymbol{f_D}$** relative to the expected:

$$A(\tau, f_D) = \left| x(t)x^*(t+\tau)e^{j2\pi f_D t} \right|$$

- The **zero-Doppler cut** of the AF is given by the **autocorrelation** of the pulse:

$$A(\tau, 0\ ) = |x(t)x^*(t+\tau)|$$

- The **zero-Delay cut** of the AF is given by the **Fourier Transform (FT)** of the squared modulus of the pulse:

$$A(0, f_D) = \left| |x(t)|^2 e^{j2\pi f_D t} \right|$$

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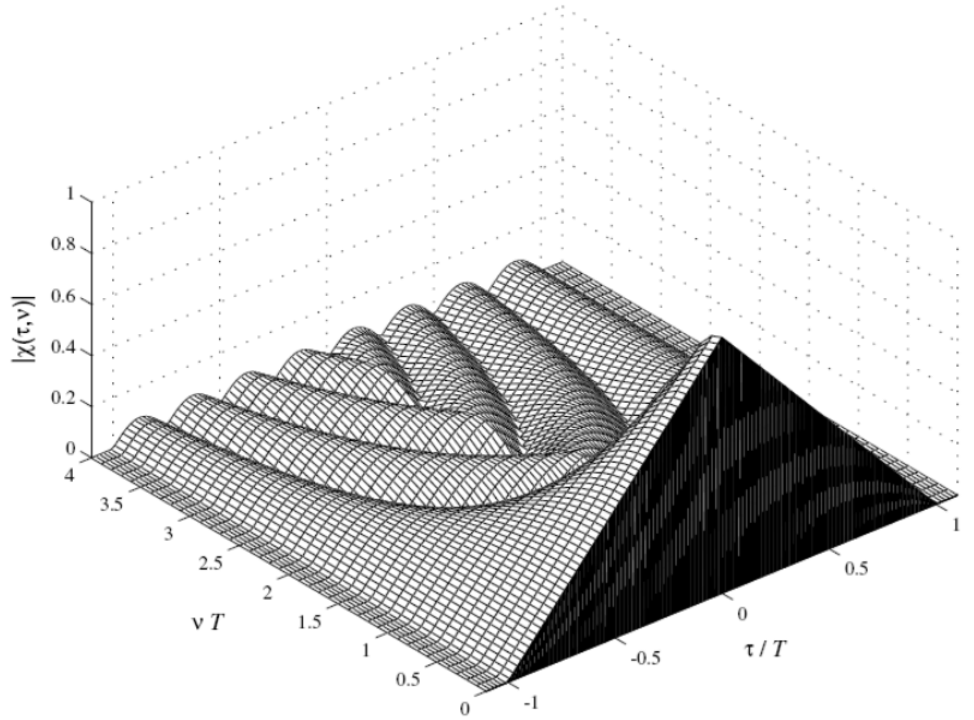
Ambiguity Function – Examples



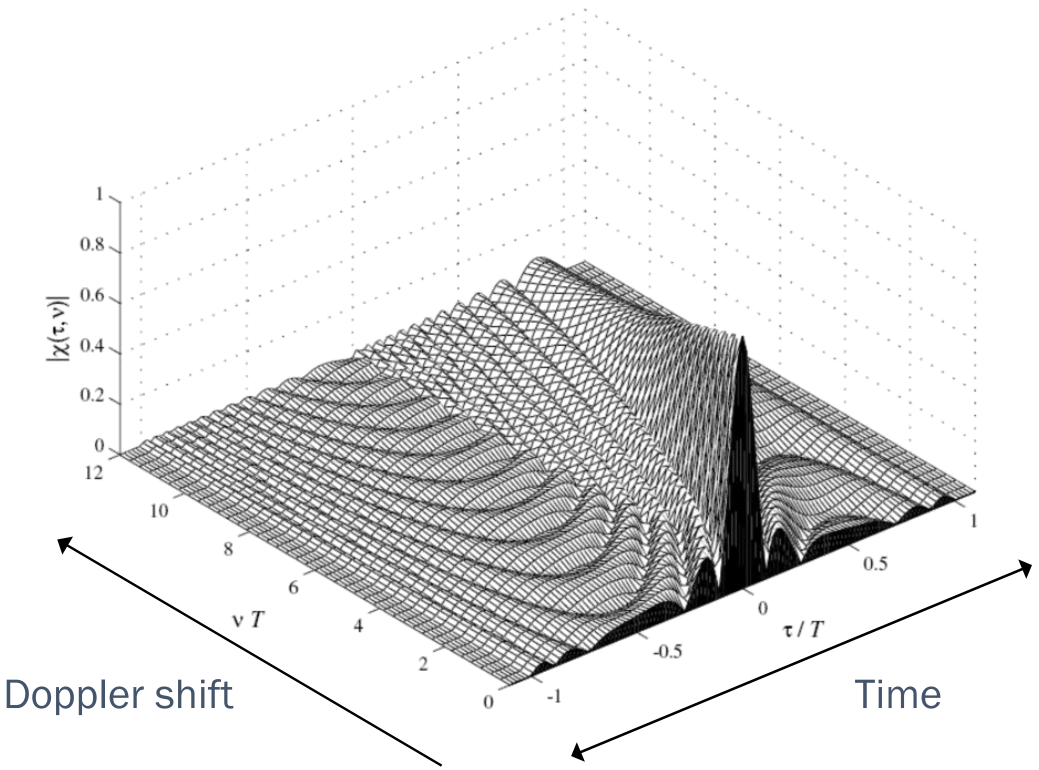
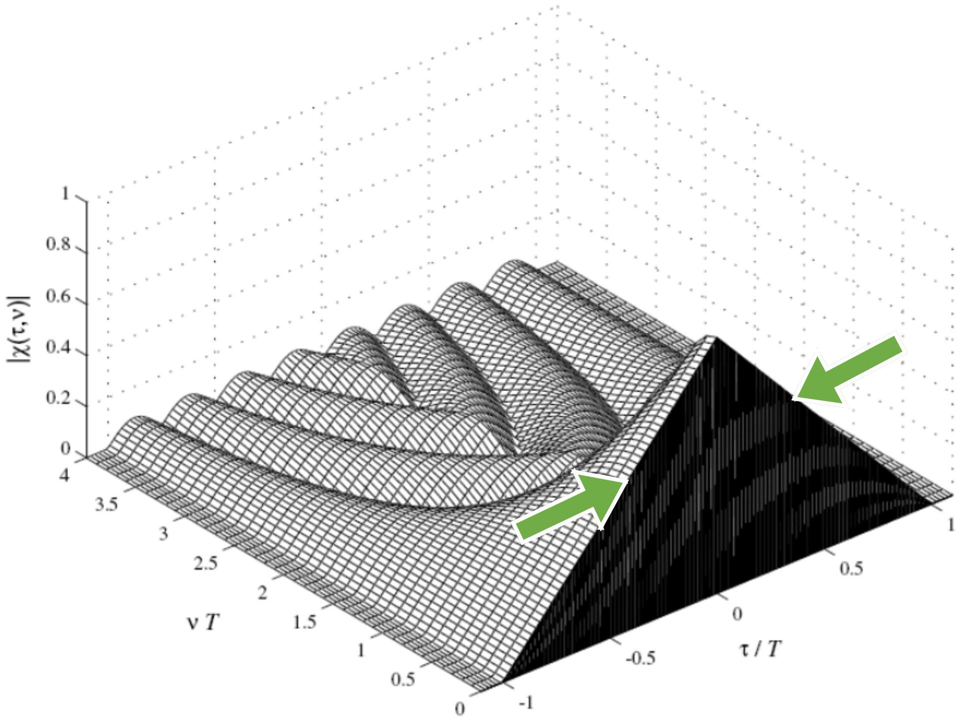
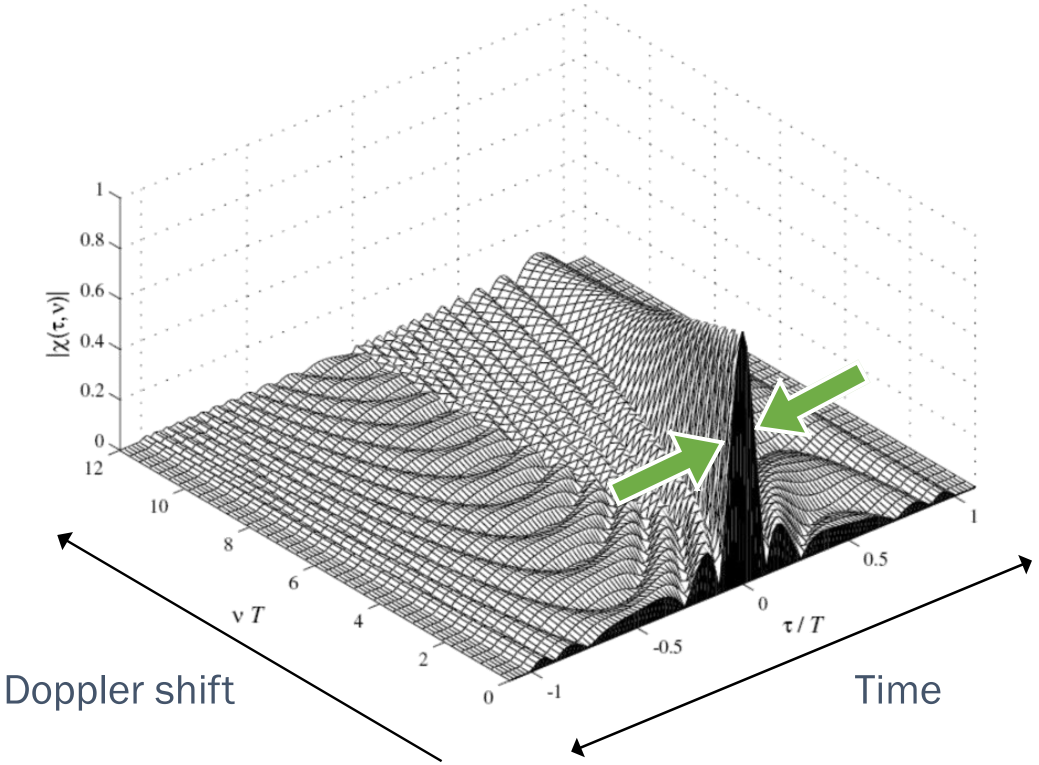


Illustration of the AF for (**L**) an unmodulated pulse, (**R**) a LFM.

Ambiguity Function – Examples





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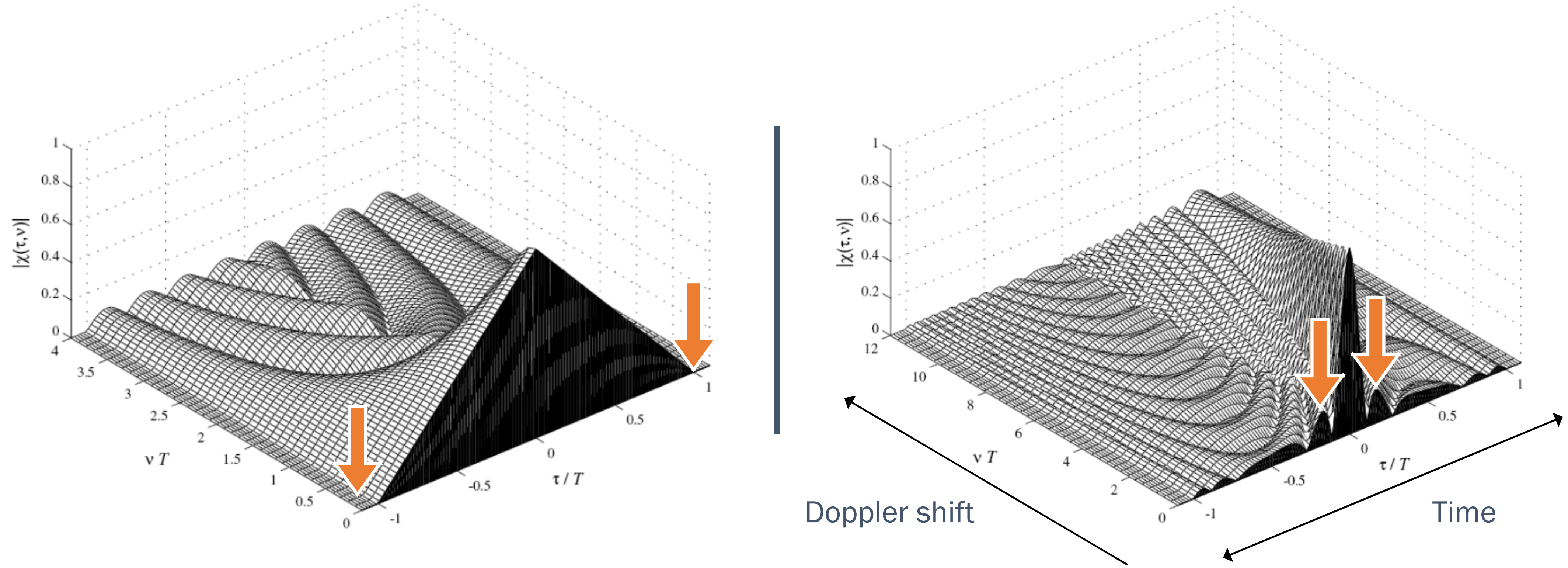
Resolution

Ambiguity Function – Examples



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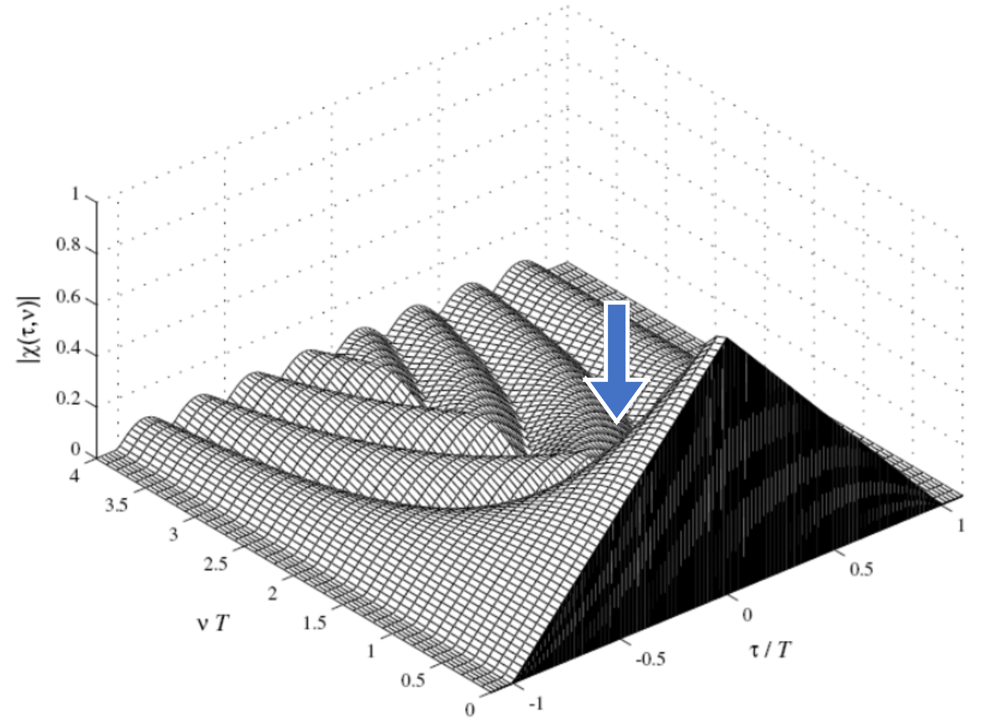
Side Lobe Levels

Ambiguity Function – Examples



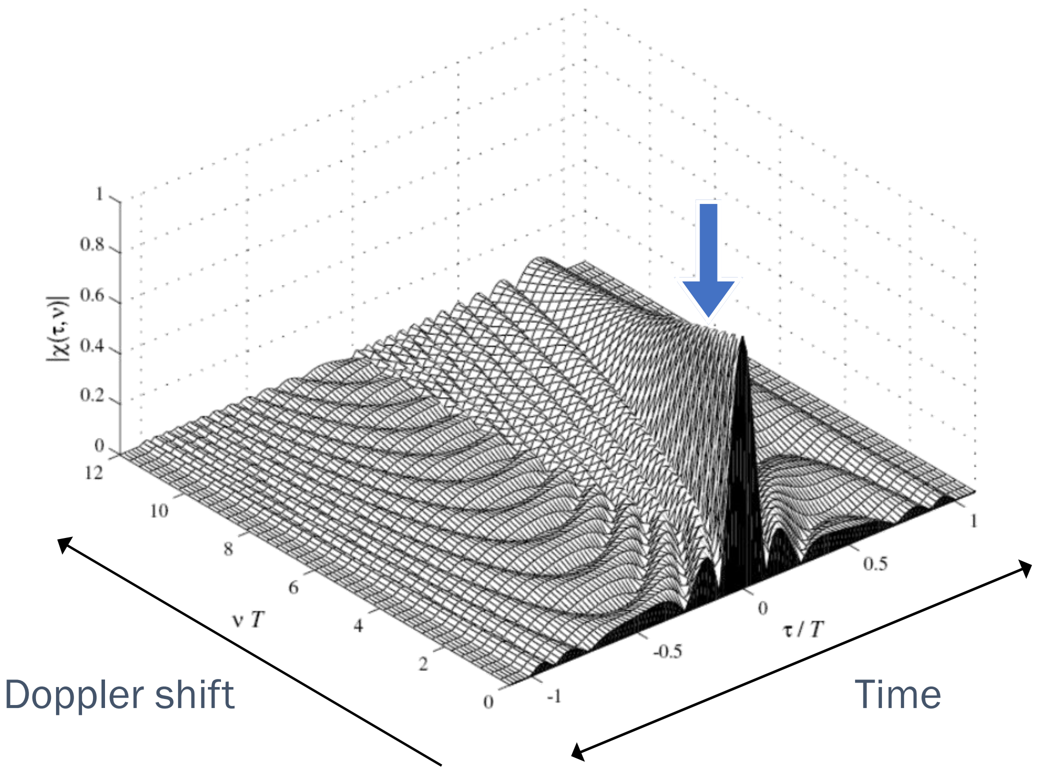


Illustration of the AF for (**L**) an unmodulated pulse, (**R**) a LFM.

Time-Frequency Response

Closing Remarks

- Basic radar principles were discussed;
- Introduction on radar acquisitions and signal processing;
- Introduction on pulse compression and waveform design tools;

Reading Material

- Principles Of Modern Radar: Basic Principles
  – Mark A Richards;
- Radar Signals
  – Nadav Levanon;
- Radar System Analysis and Design Using MATLAB
  – Bassem R. Mahafza.

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THANK YOU