UFS Lecture 13: Passive Modelocking

6 Passive Mode Locking

6.2 Fast Saturable Absorber Mode Locking (cont.)
(6.3 Soliton Mode Locking)
6.4 Dispersion Managed Soliton Formation

7 Mode-Locking using Artificial Fast Sat. Absorbers

7.1 Kerr-Lens Mode-Locking
7.1.1 Review of Paraxial Optics and Laser Resonator Design
7.1.2 Two-Mirror Resonators
7.1.3 Four-Mirror Resonators
7.1.4 The Kerr Lensing Effects

(7.2 Additive Pulse Mode-Locking)

6.2.2 Fast SA mode locking with GDD and SPM

$$T_R \frac{\partial A(T,t)}{\partial T} = \left[g - l_0 + D_f \frac{\partial^2}{\partial t^2} + \gamma |A|^2 + jD_2 \frac{\partial^2}{\partial t^2} - j\delta |A|^2 \right] A(T,t).$$

Steady-state solution is chirped sech-shaped pulse with 4 free parameters:

$$A_s(T,t) \;=\; A_0 \left(\operatorname{sech}\left(\frac{t}{\tau} \right) \right)^{(1+j\beta)} e^{j\psi T/T_R}$$

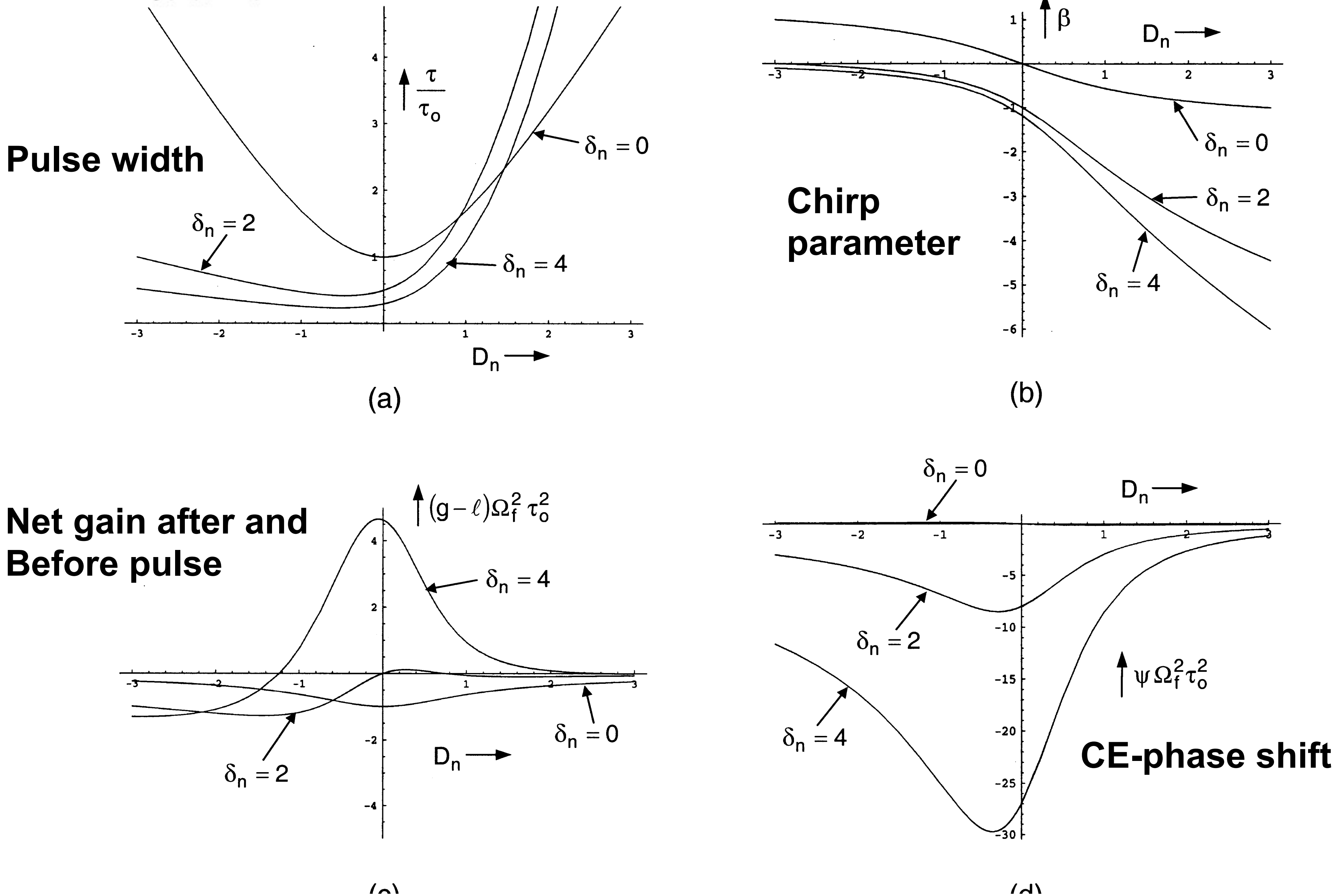
Pulse amplitude: A_0 or Energy: $W = 2 A_0^{\,2} \tau$

Pulse width: τ

Chirp parameter : β

Carrier-Envelope phase shift : ψ

$\tau_0 = 4D_f/(\gamma W)$



Pulse width

Chirp parameter

Net gain after and Before pulse

CE-phase shift

Figure 6.6: (a) Pulsewidth, (b) Chirp parameter, (c) Net gain following the pulse, which is related to stability. (d) Phase shift per pass. [4]

6.4 Dispersion Managed Soliton Formation

in Fiber Lasers

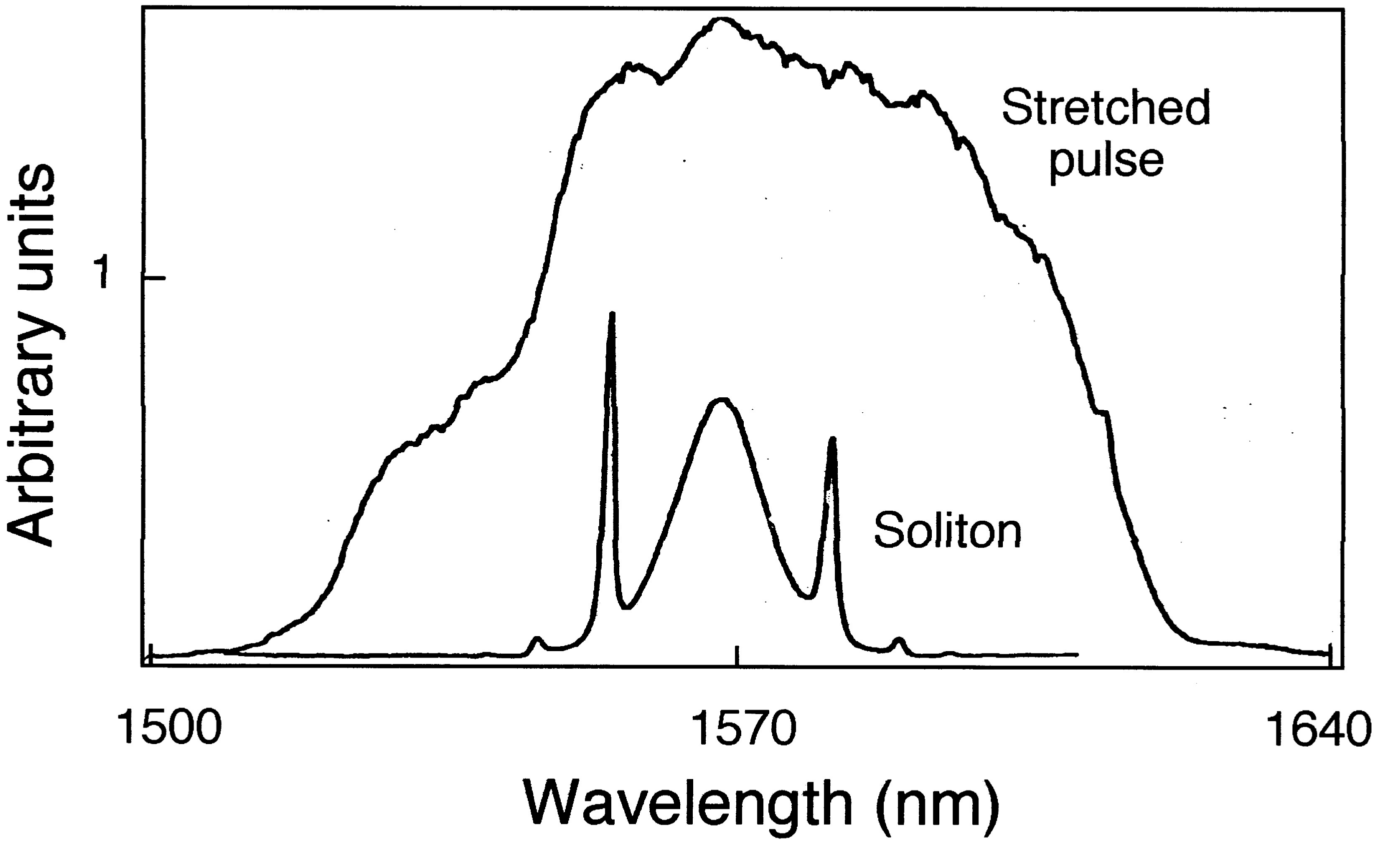


Fig. 6.12: Stretched pulse or dispersion managed soliton mode locking

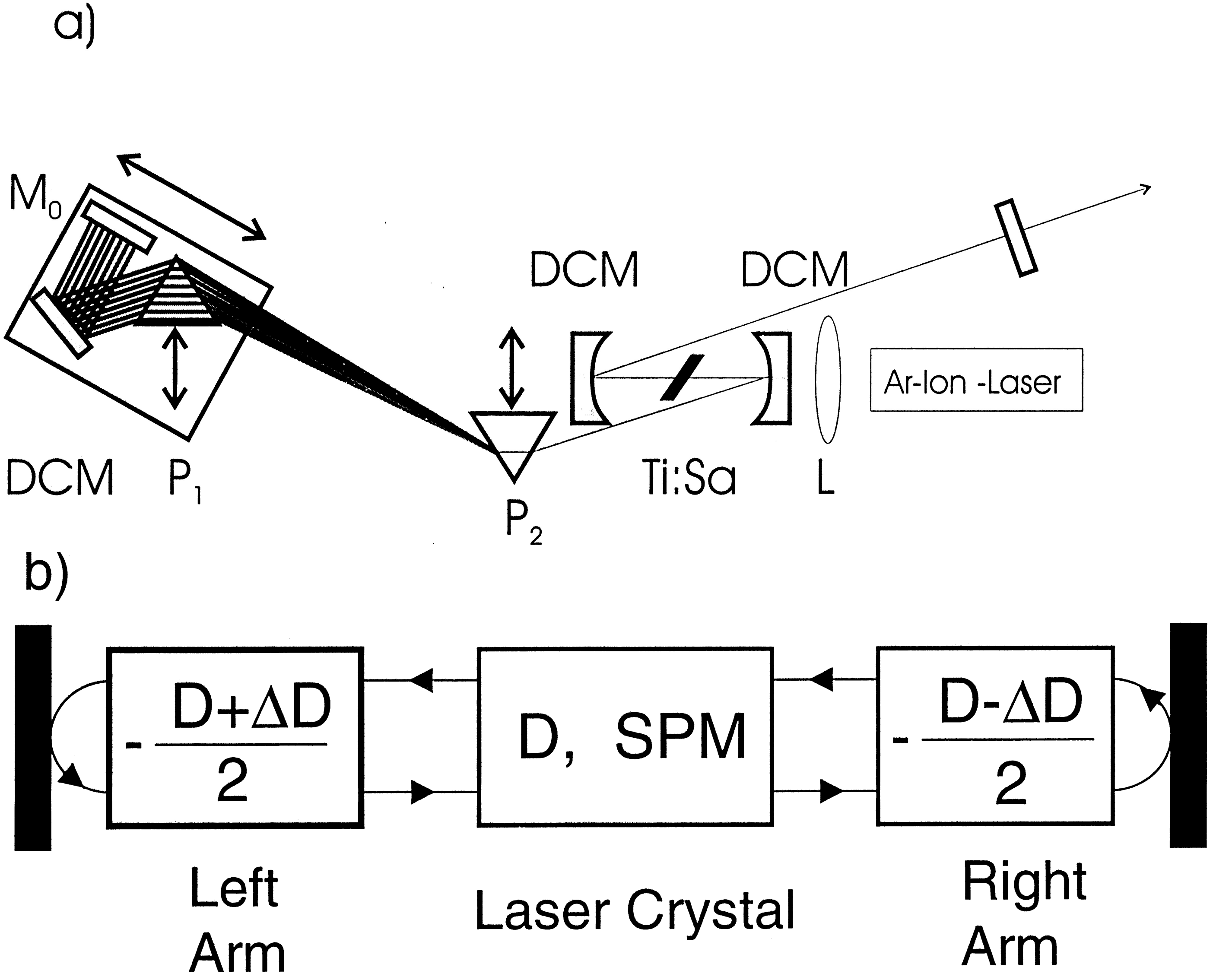
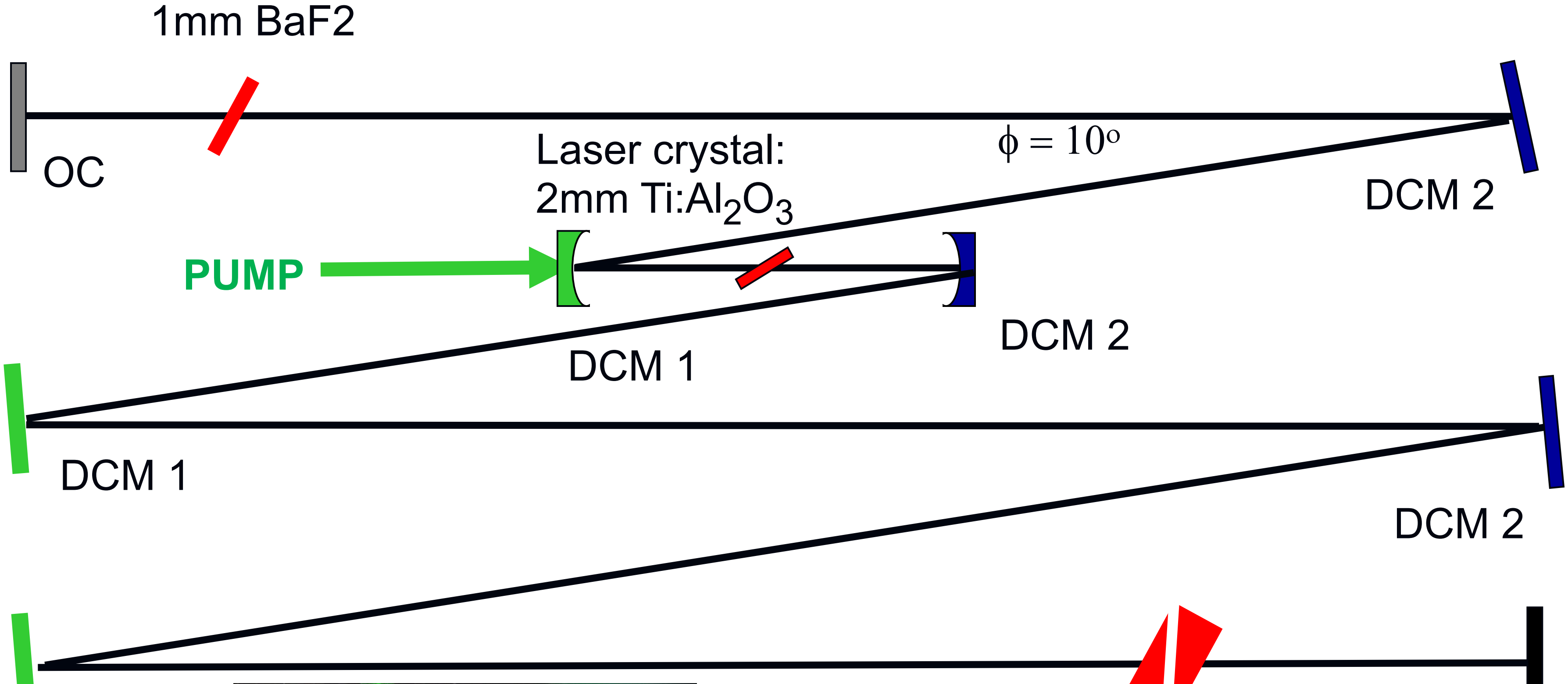


Fig. 6.13: (a) Kerr-lens mode-locked Ti:sapphire laser. (b) Correspondence with dispersion-managed fiber transmission.

Today's Broadband, Prismless Ti:sapphire Lasers



1mm BaF2

OC

Laser crystal:
2mm $Ti:Al_2O_3$

$\phi = 10^o$

DCM 2

PUMP

DCM 2

DCM 1

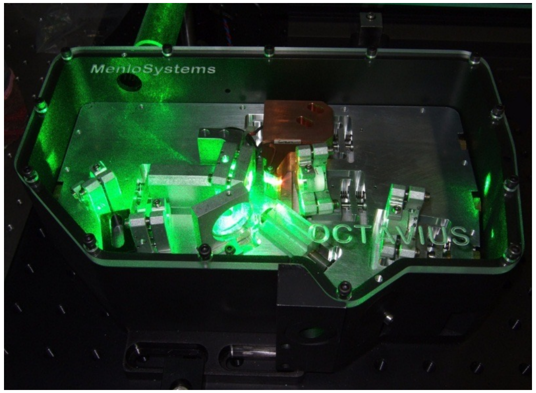
DCM 1

DCM 2

DCM 1

BaF2 - wedges

DCM



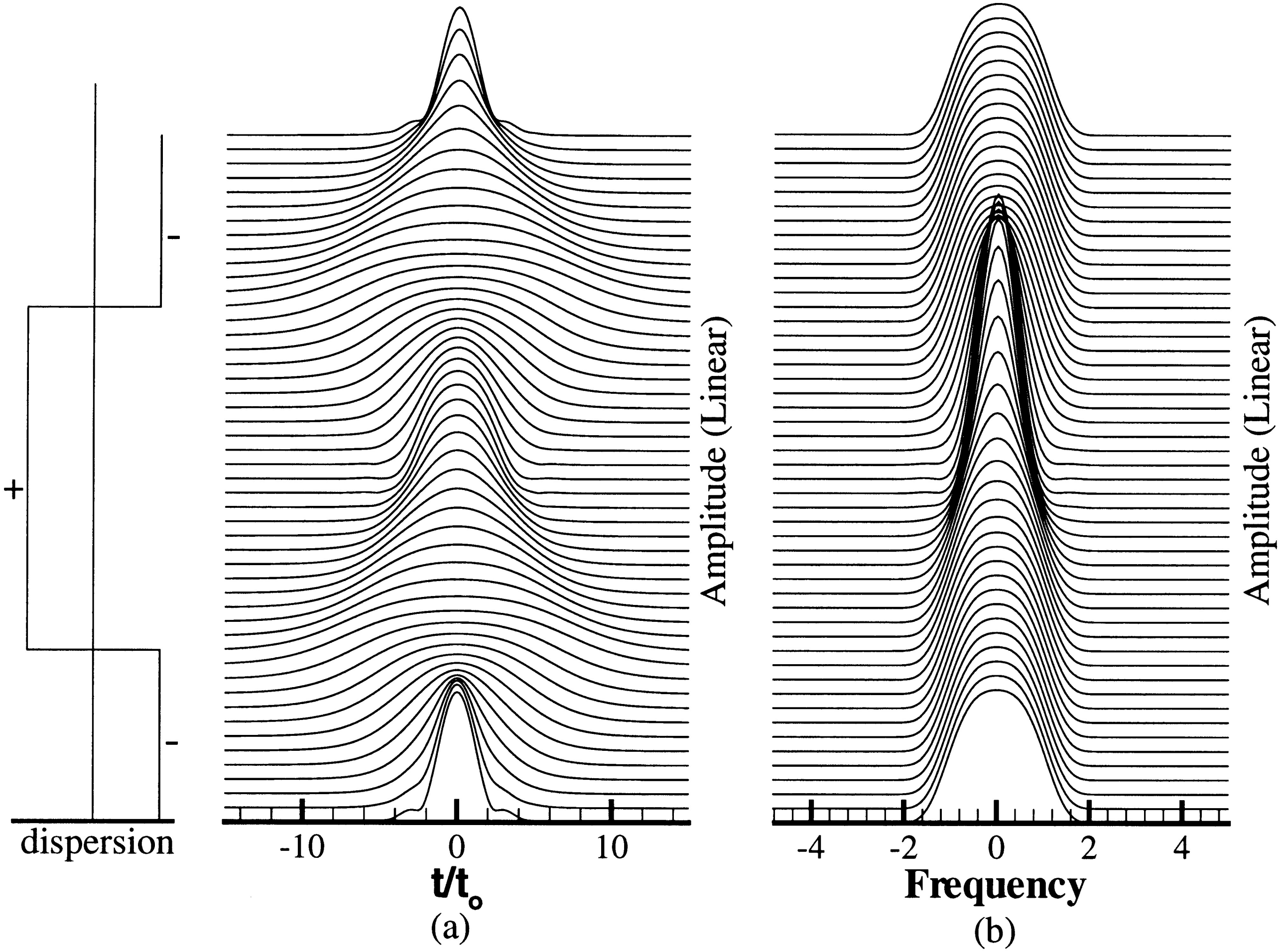


Fig. 6.14: Dispersion managed soliton including saturable absorption and gain filtering

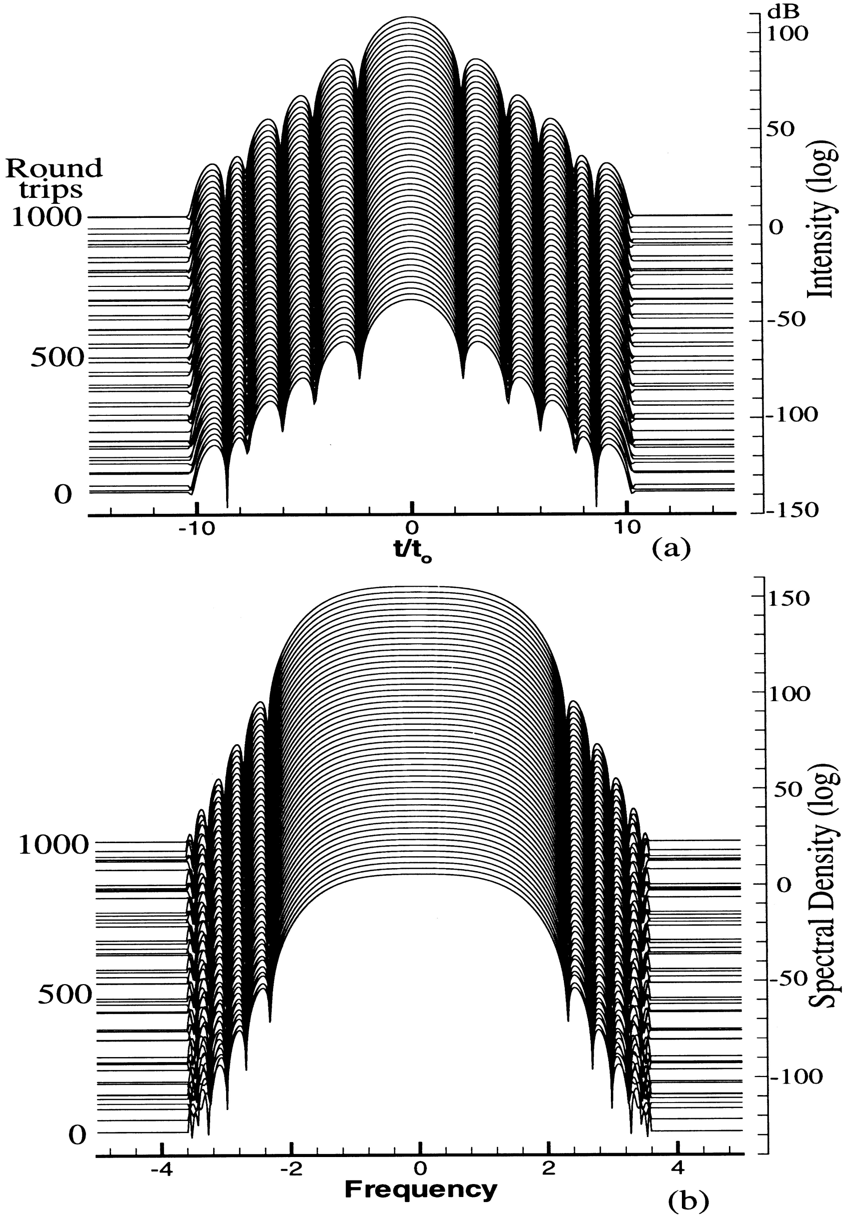


Fig. 6.15: Steady state profile if only dispersion and GDD is involved: Dispersion Managed Soliton

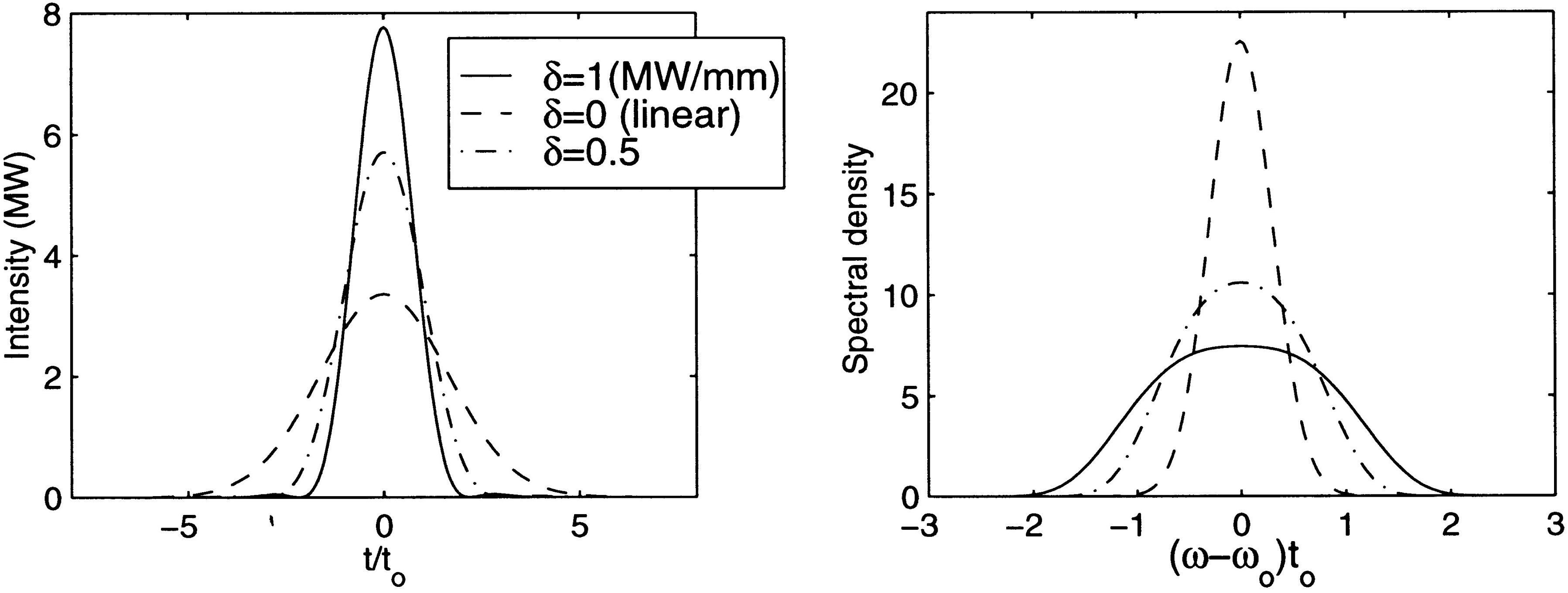
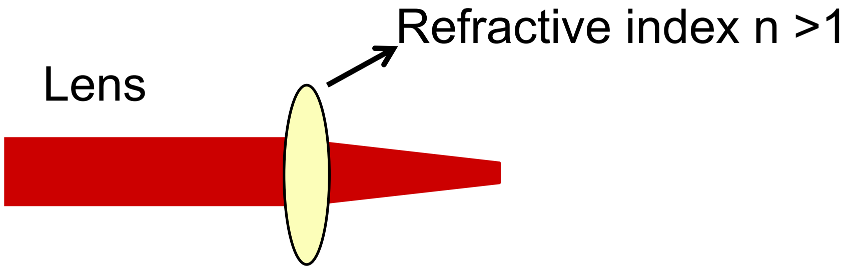
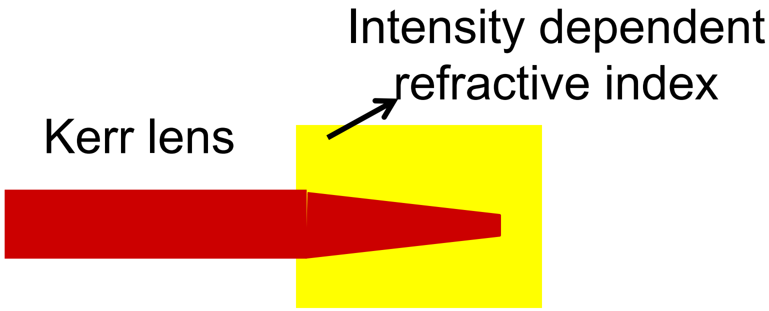


Fig. 6.16: Pulse shortening due to dispersion managed soliton formation

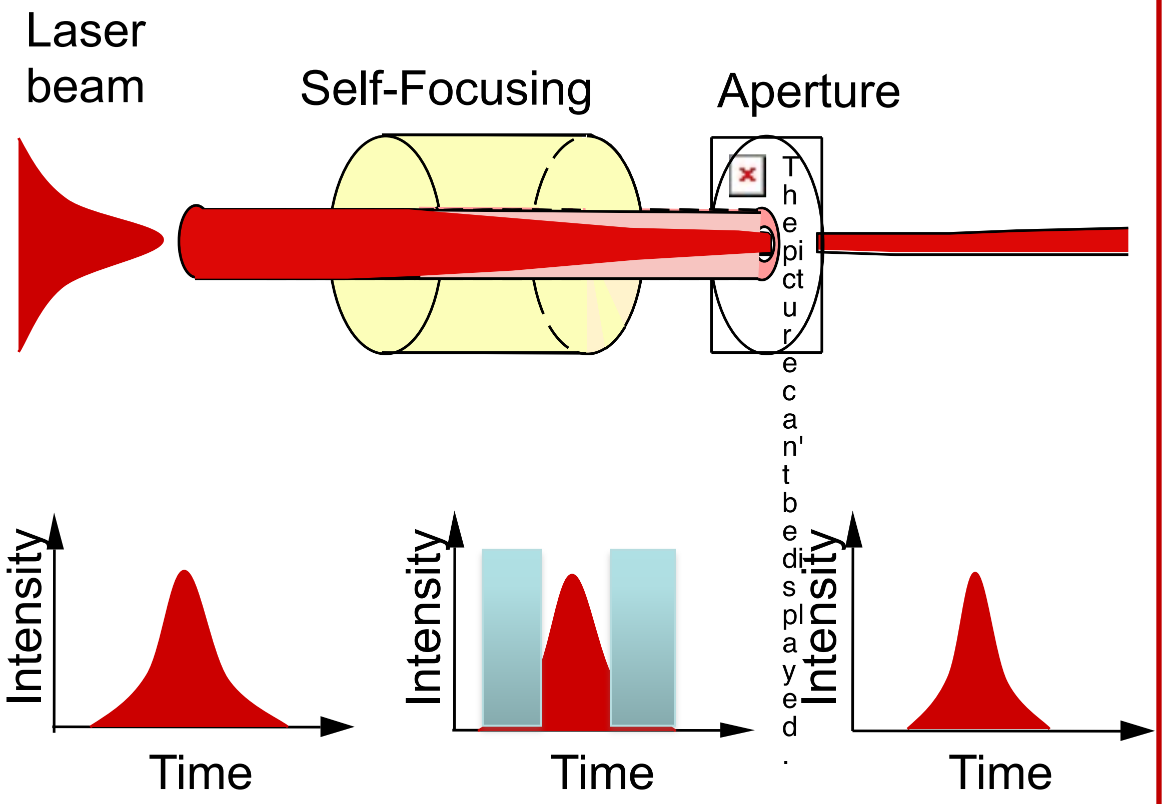
7. Mode locking using artificial fast SA

7.1 Kerr-lens mode locking



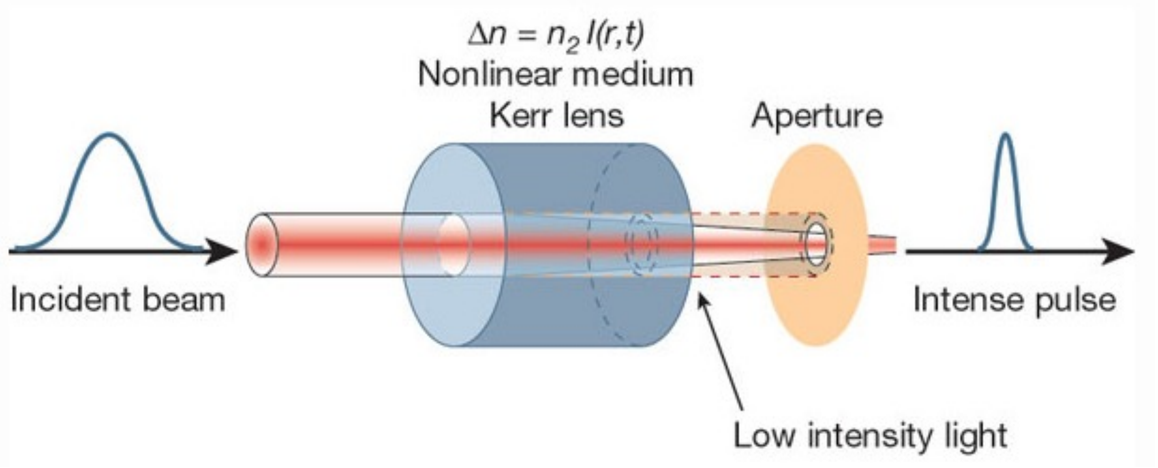


- A spatio-temporal laser pulse propagating through the Kerr medium has a time dependent mode size: pulse peak corresponds to smaller beam size than the wings.
- A hard aperture placed at the right position in the cavity strips of the wings of the pulse, shortening the pulse.
- The combined mechanism is equivalent to a fast saturable absorber.

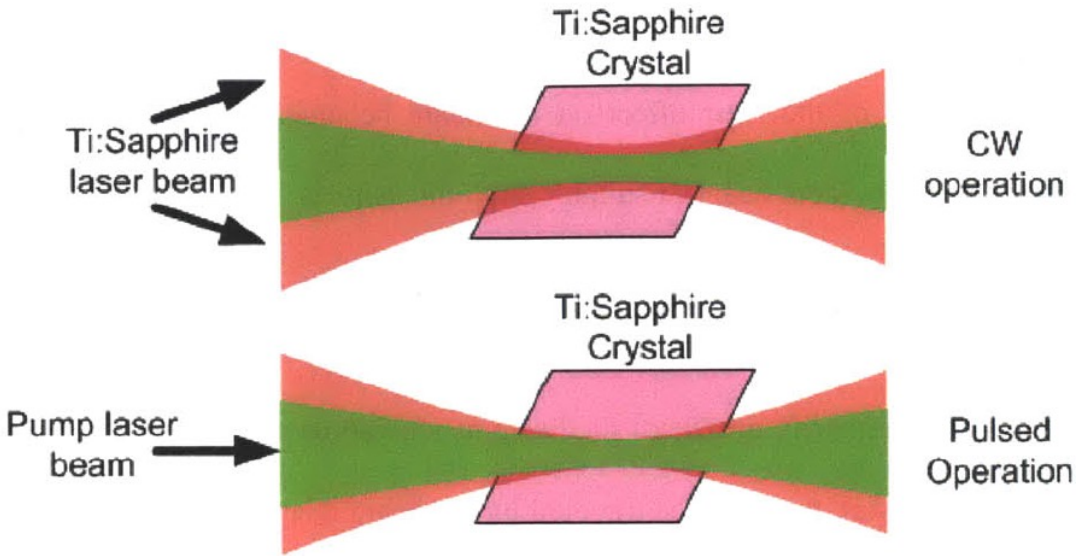


Kerr-lens mode locking: hard aperture versus soft aperture

Hard-aperture Kerr-lens mode-locking: a hard aperture placed at the right position in the cavity attenuates the wings of the pulse, shortening the pulse.



Soft-aperture Kerr-lens mode-locking: gain medium can act both as a Kerr medium and as a soft aperture (i.e. increased gain instead of saturable absorption). In the CW case the overlap between the pump beam and laser beam is poor, and the mode intensity is not high enough to utilize all of the available gain. The additional focusing provided by the high intensity pulse improves the overlap, utilizing more of the available gain.



7.1.1 Review of Paraxial Optics and Laser Resonator Design

$$U(r,z) = \frac{U_o}{q(z)} \exp\left[-jk\frac{r^2}{2q(z)}\right]$$

$$\frac{1}{q(z)} = \frac{1}{R(z)} - j\frac{\lambda}{\pi w^2(z)}$$

q-parameter

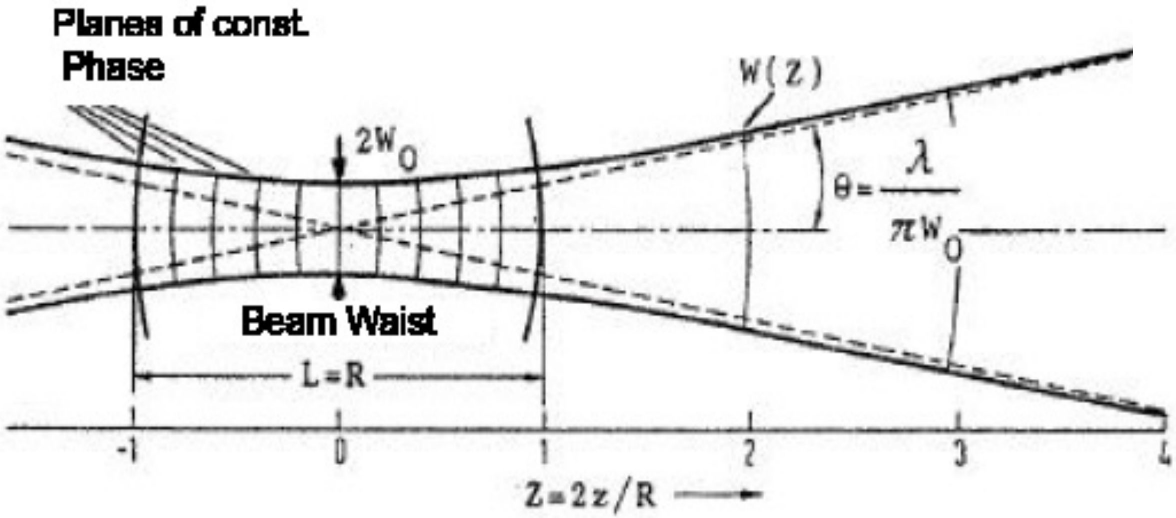


Fig. 7.2: Gaussian beam characteristics

$$I(r,z) = \frac{2P}{\pi w^2(z)} \exp\left[-\frac{2r^2}{w^2(z)}\right]$$

$$w(z) = w_o \left[1 + \left(\frac{\lambda z}{\pi w_o^2}\right)^2\right]^{1/2}$$

$$R(z) = z\left[1 + \left(\frac{\pi w_o^2}{\lambda z}\right)^2\right]$$

Rayleigh Range: $z_R = \frac{\pi w_o^2}{\lambda}$

Confocal Parameter: $b = 2z_R = \frac{2\pi w_o^2}{\lambda}$

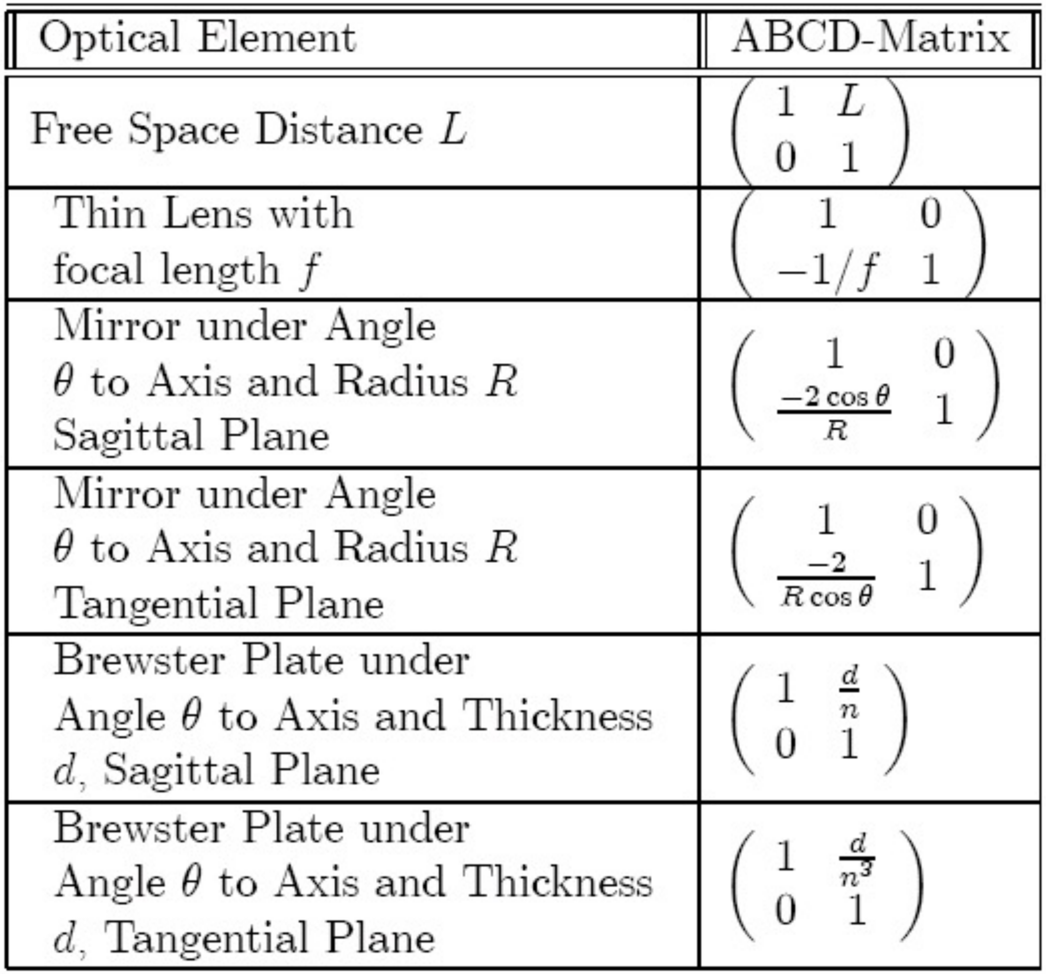
$$\theta = \frac{\lambda}{\pi w_o},$$

Gaussian Beam Propagation → q-parameter Transformation

q_1 → Optical System → q_2

ABCD law:

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D}$$



Optical Element	ABCD-Matrix
Free Space Distance L	$\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$
Thin Lens with focal length f	$\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$
Mirror under Angle θ to Axis and Radius R Sagittal Plane	$\begin{pmatrix} 1 & 0 \\ \frac{-2\cos\theta}{R} & 1 \end{pmatrix}$
Mirror under Angle θ to Axis and Radius R Tangential Plane	$\begin{pmatrix} 1 & 0 \\ \frac{-2}{R\cos\theta} & 1 \end{pmatrix}$
Brewster Plate under Angle θ to Axis and Thickness d, Sagittal Plane	$\begin{pmatrix} 1 & \frac{d}{n} \\ 0 & 1 \end{pmatrix}$
Brewster Plate under Angle θ to Axis and Thickness d, Tangential Plane	$\begin{pmatrix} 1 & \frac{d}{n^3} \\ 0 & 1 \end{pmatrix}$

Table 7.1: ABCD matrices for some optical components

Example: Free Space Propagation

$$q_2 = q_1 + z_2 - z_1,$$

Example: Imaging or focusing of a beam

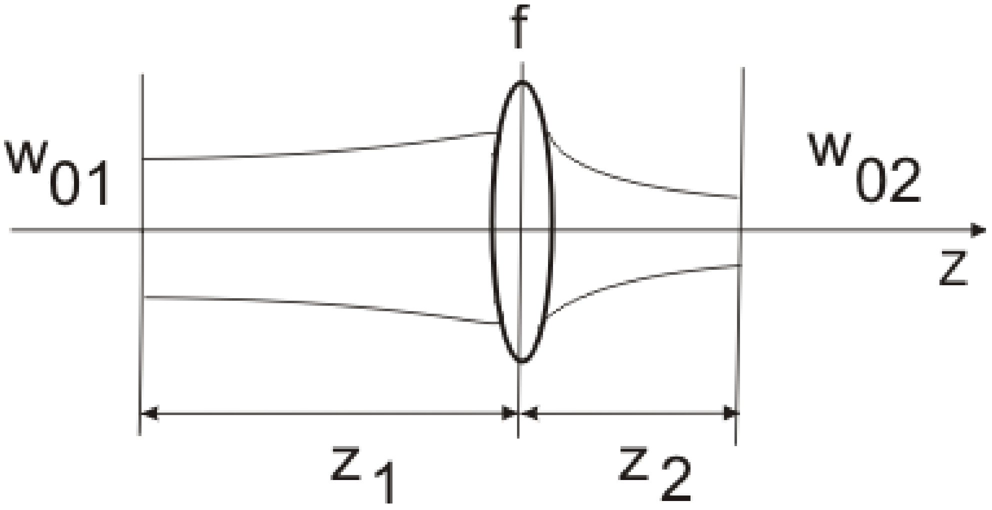


Fig. 7.3: Focusing of a Gaussian beam by a lens.

$$z_2 = f + \frac{(z_1 - f) f^2}{(z_1 - f)^2 + \left(\frac{\pi w_{01}^2}{\lambda}\right)^2}; \qquad \frac{1}{w_{02}^2} = \frac{1}{w_{01}^2}\left(1 - \frac{z_1}{f}\right)^2 + \frac{1}{f^2}\left(\frac{\pi w_{01}}{\lambda}\right)^2$$

7.1.2 Two-Mirror Resonators

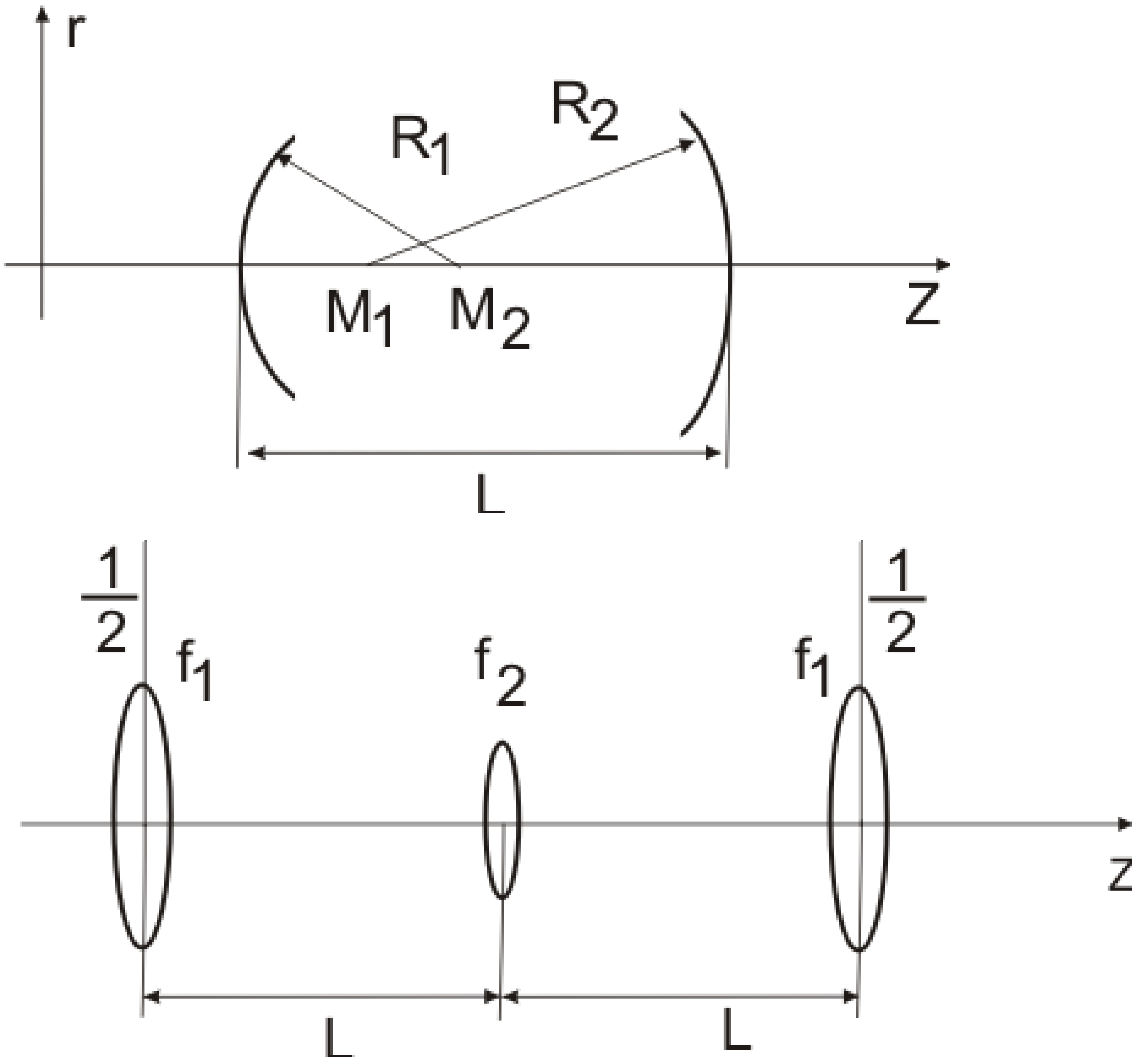


Fig. 7.5: Two-mirror resonator unfolded

$$M=\begin{pmatrix}1 & 0\\ \frac{-1}{2f_1} & 1\end{pmatrix}\begin{pmatrix}1 & L\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ \frac{-1}{f_2} & 1\end{pmatrix}\begin{pmatrix}1 & L\\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\ \frac{-1}{2f_1} & 1\end{pmatrix}$$

$f_1 = R_1/2$, and $f_2 = R_2/2$ $\qquad g_i = 1 - L/R_i, i = 1, 2$

$$M=\begin{pmatrix}(2g_1g_2-1) & 2g_2L\\ 2g_1\left(g_1g_2-1\right)/L & (2g_1g_2-1)\end{pmatrix}=\begin{pmatrix}A & B\\ C & D\end{pmatrix}$$

Lossless Resonator: $\det|M| = 1$

Resonator Stability ⟶ **Eigenvalues within or on unit circle!**

$$\det|M-\lambda\cdot 1| = \det\left|\begin{pmatrix}(2g_1g_2-1)-\lambda & 2g_2L\\ 2g_1\left(g_1g_2-1\right)/L & (2g_1g_2-1)-\lambda\end{pmatrix}\right| = 0$$

$$\lambda^2 - 2\left(2g_1g_2-1\right)\lambda + 1 = 0.$$

$$\begin{aligned}\lambda_{1/2} &= (2g_1g_2-1)\pm\sqrt{(2g_1g_2-1)^2-1},\\ &= \begin{cases}\exp(\pm\theta)\,,\cosh\theta = 2g_1g_2-1, & \text{for } |2g_1g_2-1|>1\\ \exp(\pm j\psi)\,,\cos\psi = 2g_1g_2-1, & \text{for } |2g_1g_2-1|\le 1\end{cases}\end{aligned}$$

$$|2g_1g_2-1|\le 1 \qquad \begin{aligned}\text{stable} &: \ 0\le g_1\cdot g_2 = S\le 1\\ \text{unstable} &: \ g_1g_2\le 0;\ \text{or } g_1g_2\ge 1\end{aligned}$$

Stability Parameter: $S = g_1\cdot g_2 \qquad g_i = (R_i-L)/R_i = -S_i/R_i;$

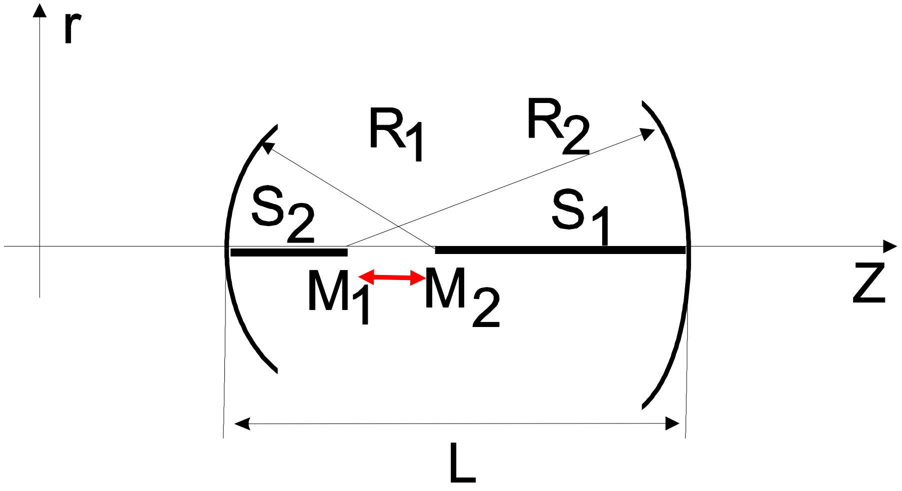


Fig. 7.6: Two-mirror resonator stability

$$\text{stable} : 0 \leq \frac{S_1 S_2}{R_1 R_2} \leq 1.$$

- A resonator is stable, if the mirror radii, laid out along the optical axis, overlap.
- A resonator is unstable, if the radii do not overlap or one lies within the other.

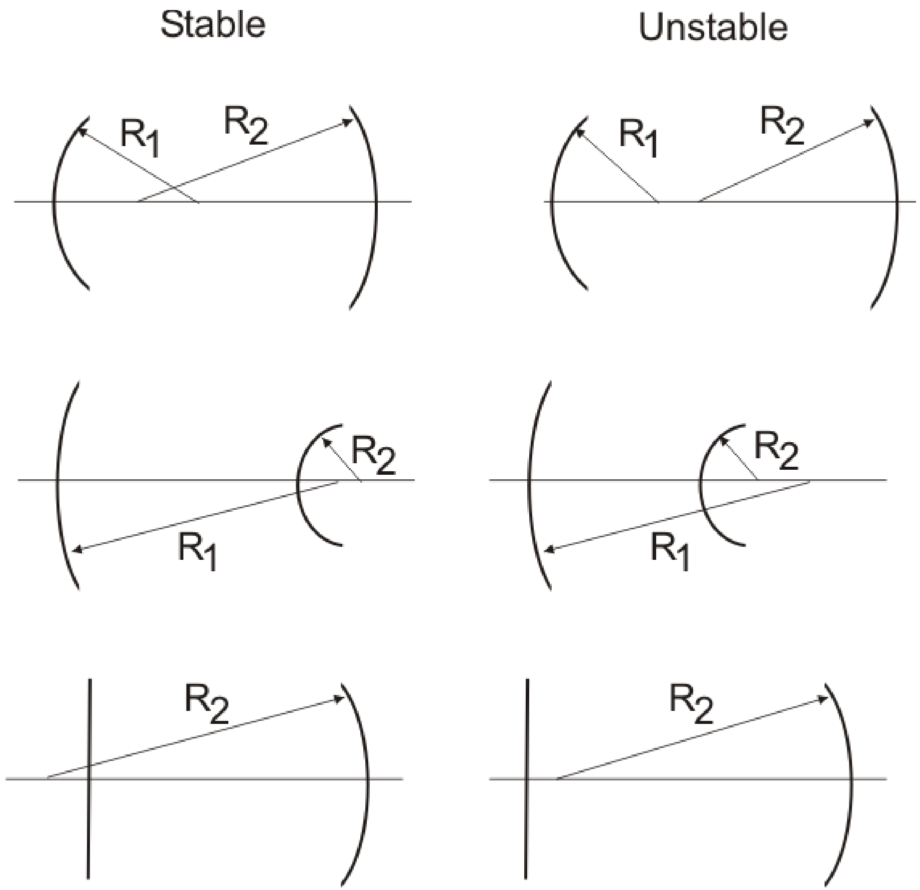


Fig. 7.7: Illustration of stable and unstable resonator configurations

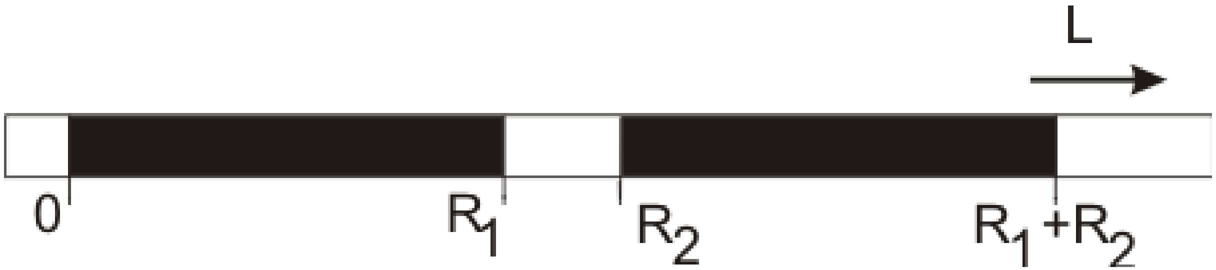


Fig. 7.8: Stable regions (black)

Resonator Mode Characteristics

$$q_1 = \frac{Aq_1 + B}{Cq_1 + D} \longrightarrow \left(\frac{1}{q}\right)^2 + \frac{A-D}{B}\left(\frac{1}{q}\right) + \frac{1-AD}{B^2} = 0.$$

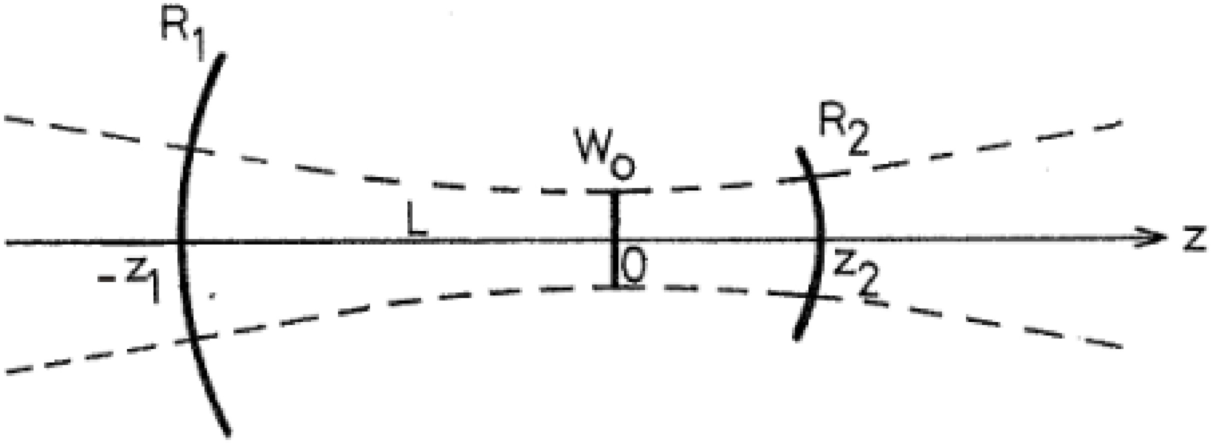


Fig. 7.9: Two-mirror resonator

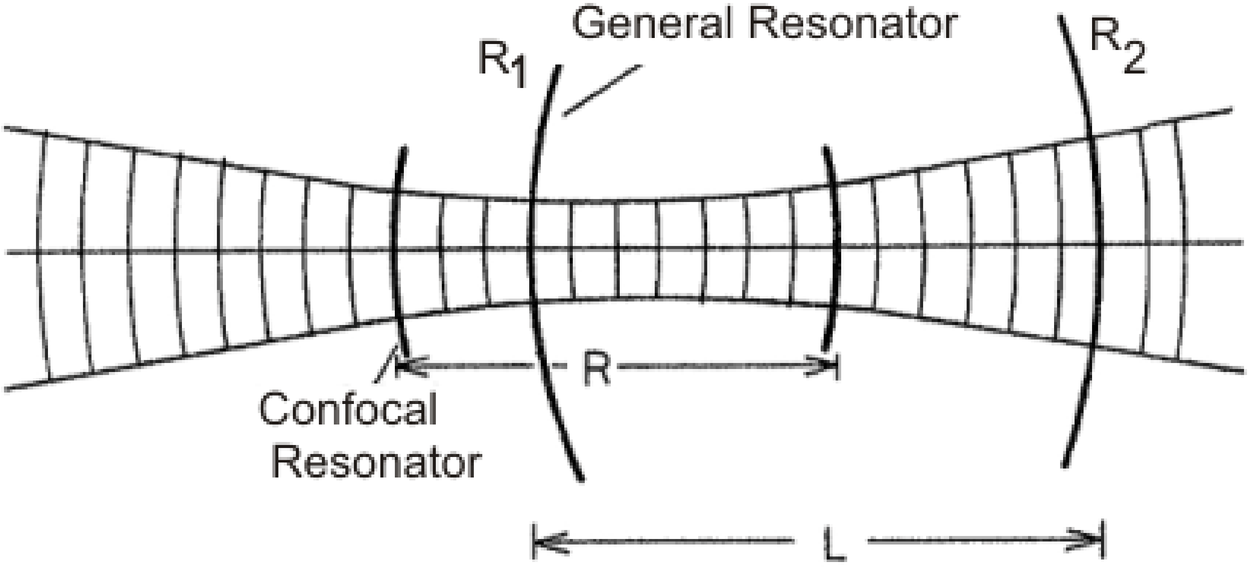


Fig. 7.10: Two-mirror resonator and its relationship with the confocal resonator

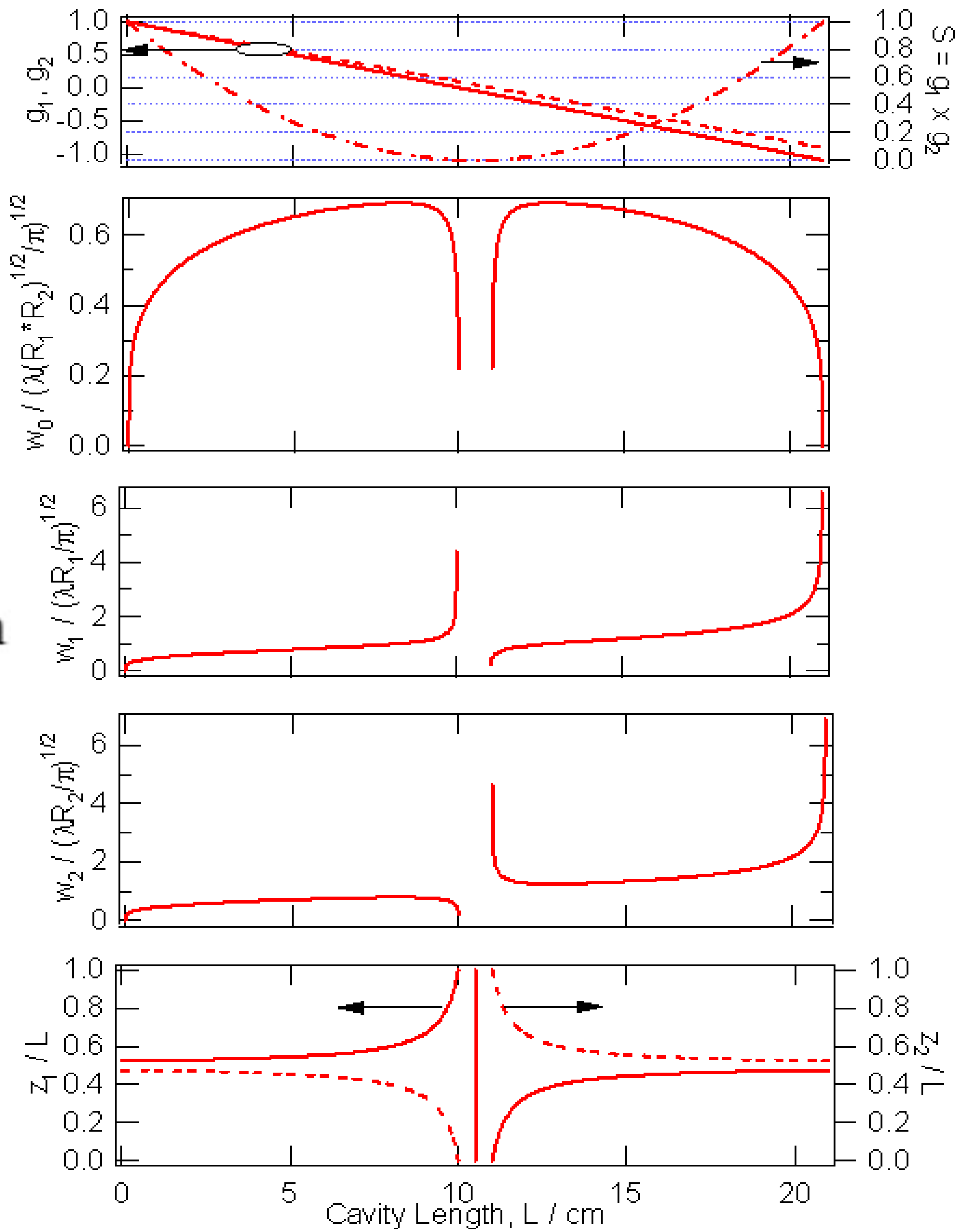


Fig. 7.11: Two mirror resonator characteristics

$R_1 = 10$ cm and $R_2 = 11$ cm

$$d_1 = -\frac{R_1}{2}\frac{1}{1-R_1/(2L_1)},$$

$$R'_1 = -\left(\frac{R_1}{2}\right)^2 \frac{1}{L_1\left[1-R_1/(2L_1)\right]}.$$

$$d_2 = -\frac{R_2}{2}\frac{1}{1-R_2/(2L_2)},$$

$$R'_2 = -\left(\frac{R_2}{2}\right)^2 \frac{1}{L_2\left[1-R_2/(2L_2)\right]}$$

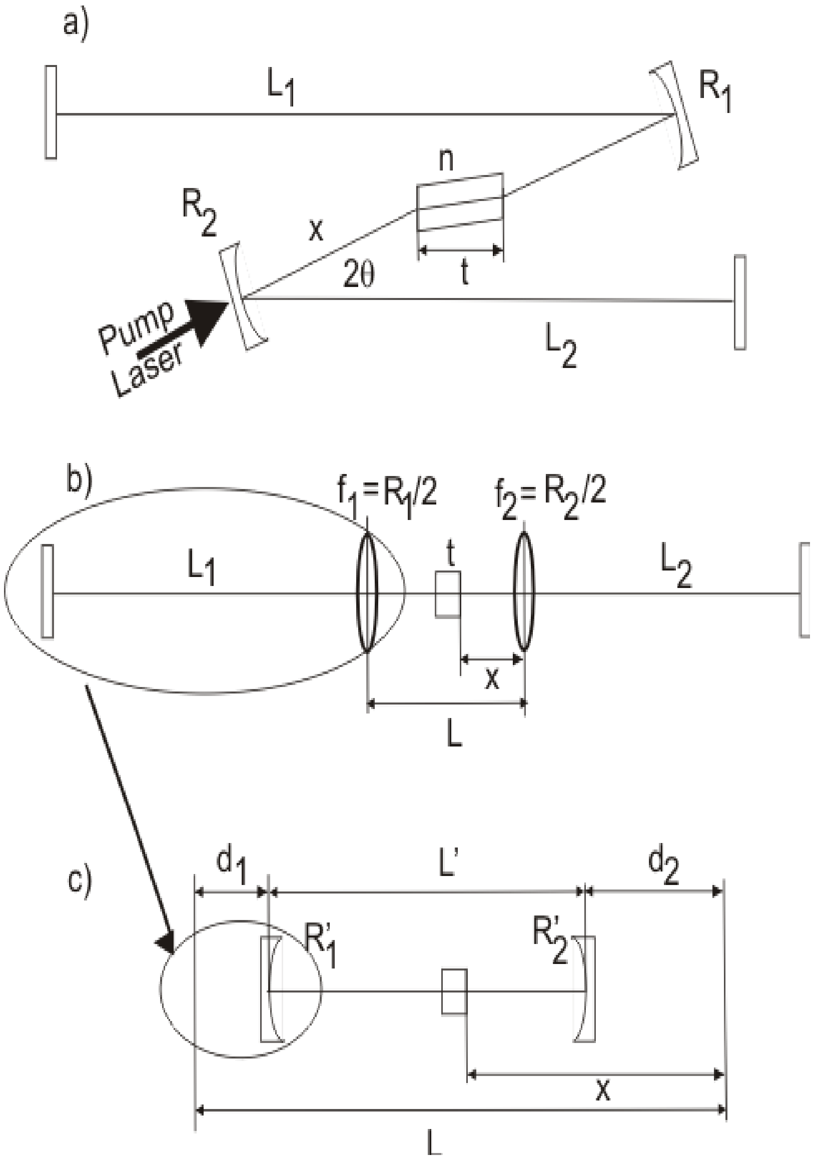


Fig. 7.12: Four mirror resonator

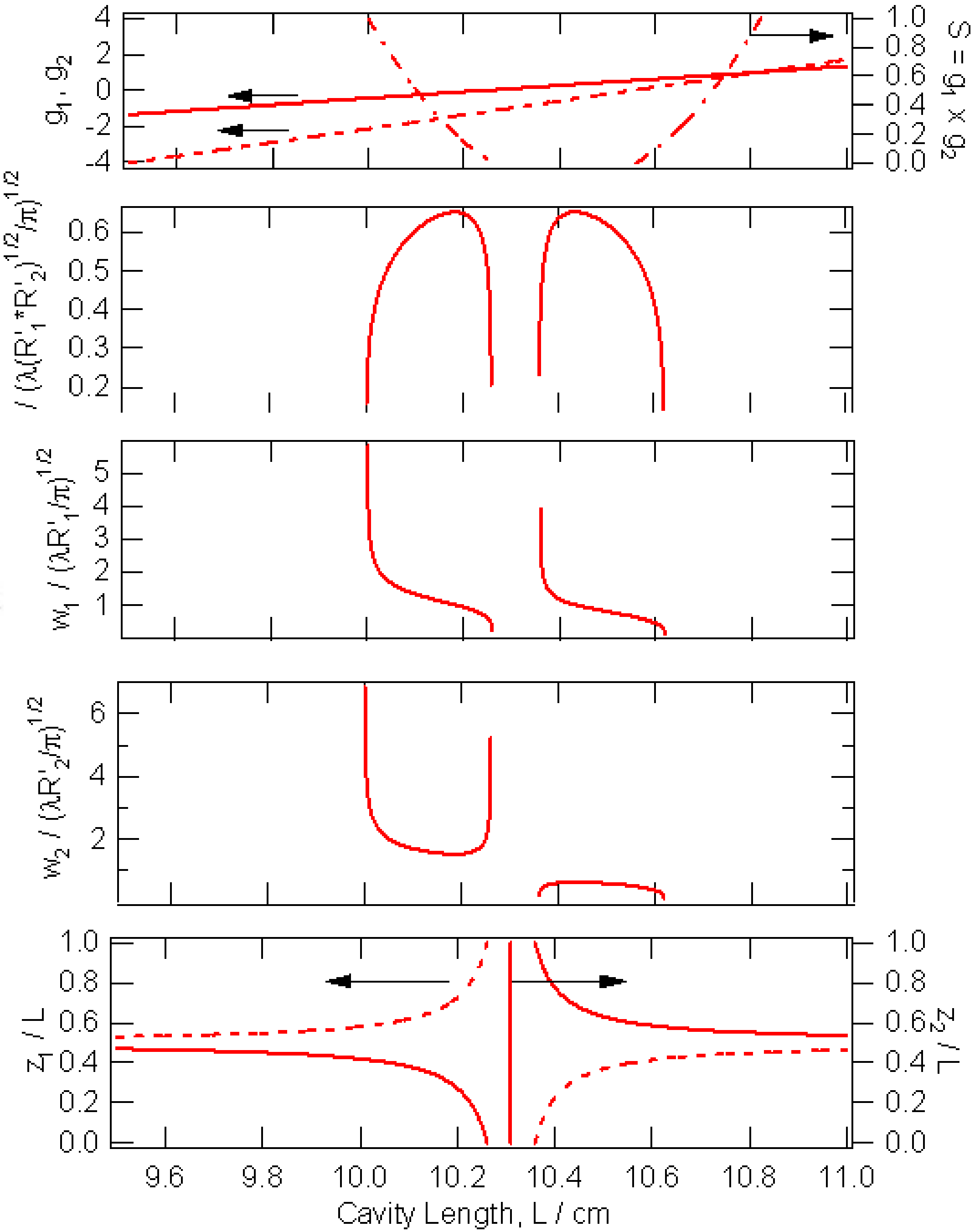


Fig. 7.13: Four mirror resonator

$$R_1 = R_2 = 10 \text{ cm}$$

$$L_1 = 100 \text{ cm and } L_2 = 75 \text{ cm}$$

$$d_1 = -5.26 \text{ cm}$$

$$R_1' = -0.26 \text{ cm}$$

$$d_2 = -5.36 \text{ cm}$$

$$R_2' = -0.36 \text{ cm}$$

$$L' = L - 10.62 \text{ cm}$$

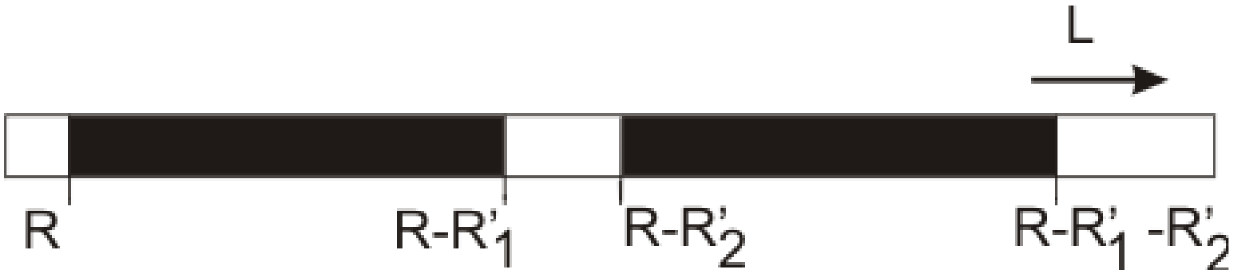


Fig. 7.14: Four mirror resonator stability regions

Astigmatism Compensation:

$$\begin{aligned} f_s &= f/\cos\theta \\ f_t &= f\cdot\cos\theta \end{aligned}$$

If not compensated:

- **no stable cavity, since stability regions for s- and t-planes may not overlap!**
- **foci do not match up**
- **output beam elliptical**

plate thickness

Use Brewster plate at angle:

$$\theta = \arccos\left[\sqrt{1+\left(\frac{N\overset{\downarrow}{t}}{2R}\right)^2}-\frac{Nt}{2R}\right]$$

with $N=\sqrt{n^2+1}\frac{n^2-1}{n^4}$ ***n*: plate refractive index**

7.1.4 The Kerr Lensing Effect

$$n = n_0 + n_2 I \qquad I(r) = \frac{2P}{\pi w^2} \exp\left[-2\left(\frac{r}{w}\right)^2\right]$$

$$n(r) = n_0' \left(1 - \frac{1}{2}\gamma^2 r^2\right), \text{ where}$$

$$n_0' = n_0 + n_2 \frac{2P}{\pi w^2}, \; \gamma = \frac{1}{w^2}\sqrt{\frac{8 n_2 P}{n_0' \pi}}.$$

Thin Lens → Gaussian Duct

Optical Element	ABCD-Matrix
Kerr Medium Normal Incidence	$M_K = \begin{pmatrix} \cos \gamma t & \frac{1}{n_0' \gamma} \sin \gamma t \\ -n_0' \gamma \sin \gamma t & \cos \gamma t \end{pmatrix}$
Kerr Medium Sagittal Plane	$M_{Ks} = \begin{pmatrix} \cos \gamma_s t & \frac{1}{n_0' \gamma_s} \sin \gamma_s t \\ -n_0' \gamma_s \sin \gamma_s t & \cos \gamma_s t \end{pmatrix}$
Kerr Medium Tangential Plane	$M_{Kt} = \begin{pmatrix} \cos \gamma_t t & \frac{1}{n_0'^3 \gamma_t} \sin \gamma_t t \\ -n_0'^3 \gamma_t \sin \gamma_t t & \cos \gamma_t t \end{pmatrix}$

$$\gamma_s = \frac{1}{w_s w_t}\sqrt{\frac{8 n_2 P}{n_0' \pi}}$$

$$\gamma_t = \frac{1}{w_s w_t}\sqrt{\frac{8 n_2 P}{n_0' \pi}}$$

Table 7.2: ABCD matrices for Kerr media

$$\delta_{s,t} = \frac{1}{p}\frac{w_{s,t}(P,z) - w_{s,t}(P=0,z)}{w_{s,t}(P=0,z)}$$

$$p = P/P_{crit},\ \text{with}\ P_{crit} = \lambda_L^2/\left(2\pi n_2 n_0^2\right)$$

$R_1 = R_2 = 10$ cm $L_1 = 104$ cm, $L_2 = 86$ cm, $t = 2$ mm, $n = 1.76$ and $P = 200$ kW.

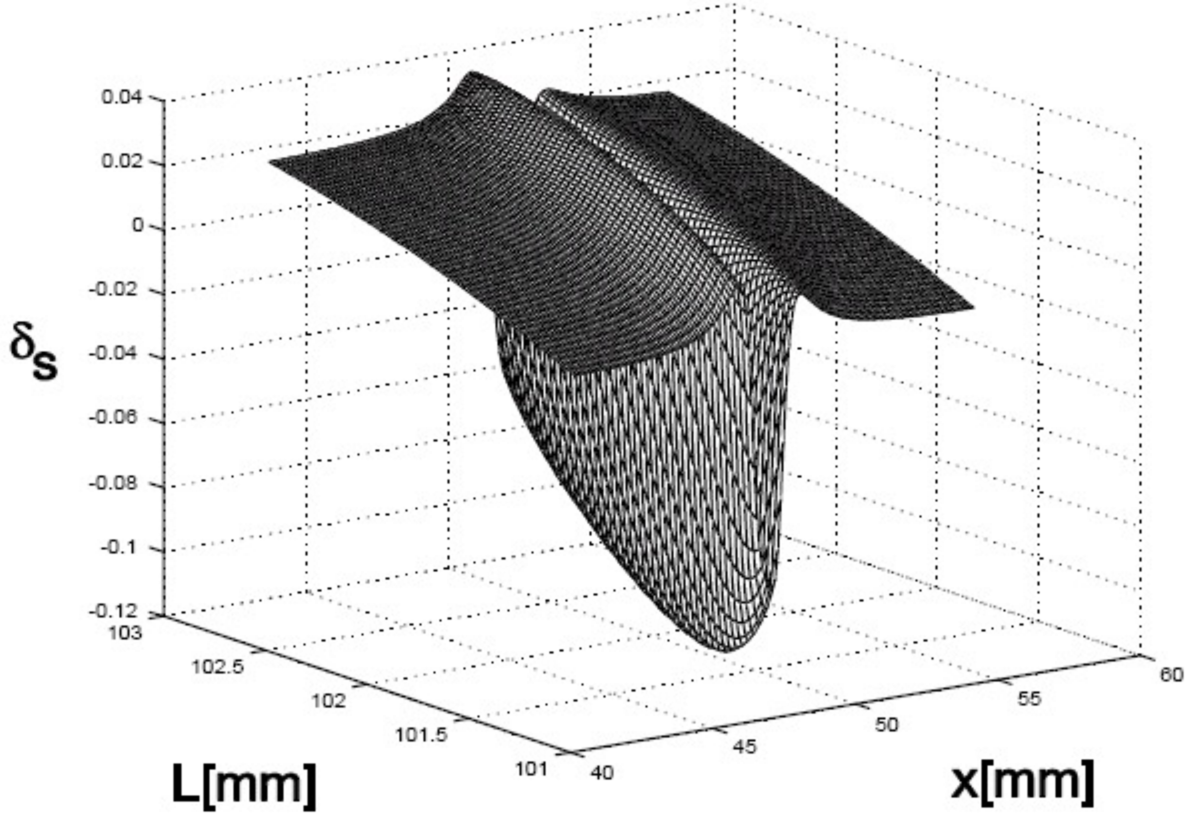


Fig. 7.15: Beam narrowing ratio

$$g \sim \int_0^\infty N(r) * N_P(r) r dr \sim \int_0^\infty \frac{2P_P}{\pi w_P^2} \exp\left[-\frac{2r^2}{w_P^2}\right] \frac{2}{\pi w_L^2} \exp\left[-\frac{2r^2}{w_L^2}\right] r dr$$

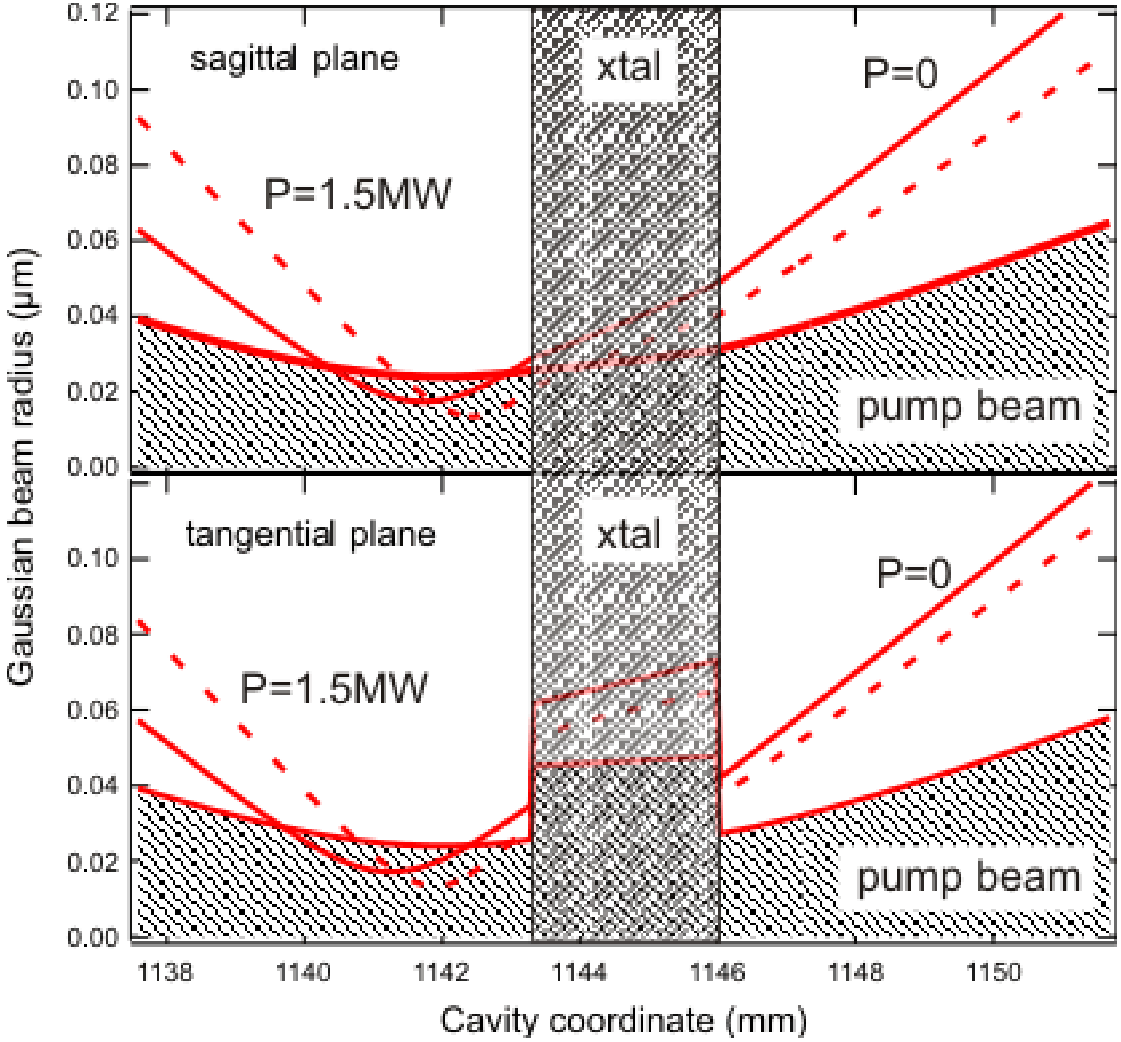


Fig. 7.16: Soft aperture KLM

Evolution of shortest pulse duration