Radiation Intensity:

Radiation intensity is the power radiated in a given direction per unit solid angle, and has unit of watts per square radian (or steradian) (***W/sr***). It is independent of distance (***r***).

a) Solid angle

The measure of a plane angle is a radian. One ***radian*** is defined as the plane angle with its vertex at the center of a circle of radius r that is subtended by an arc whose length is r. A graphical illustration is shown in Figure 4.1(a). Since the circumference of a circle of radius r is $C = 2\pi r$, there are 2π rad ($2\pi r/r$) in a full circle.

The measure of a solid angle is a steradian. One ***steradian*** is defined as the solid angle with its vertex at the center of a sphere of radius ***r*** that is subtended by a spherical surface area equal to that of a square with each side of length ***r***. A graphical illustration is shown in Figure 4.1(b). Since the area of a sphere of radius ***r*** is $A = 4\pi r$, there are 4π ***sr*** ($4\pi r^2/r^2$) in a closed sphere.

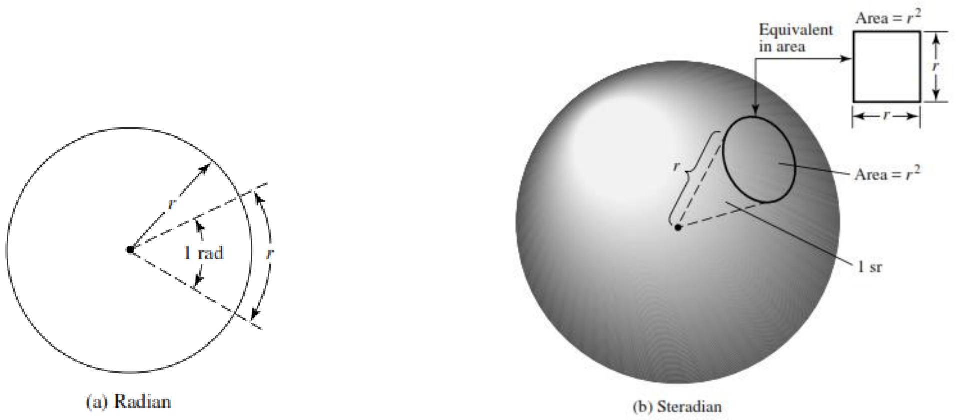


Figure 4.1 Geometrical arrangements for defining a radian and a steradian.

The infinitesimal area ***dA*** on the surface of a sphere of radius ***r***, shown in Figure 4.2, is given by

$$dA = r^2 \sin\theta \, d\theta d\varphi \ (m^2)$$

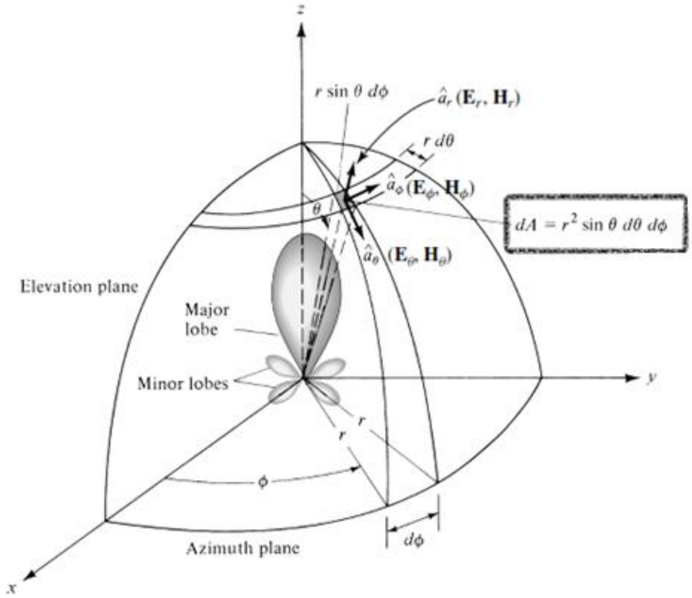


Figure 4.2

Therefore the element of solid angle $d\Omega$ of a sphere can be written as

$$d\Omega = \frac{dA}{r^2} = \sin\theta \, d\theta d\varphi \ (sr)$$

<u>Example: -</u>

For a sphere of radius ***r***, find the solid angle Ω (in square radians or steradians) of a spherical cap on the surface of the sphere over the north-pole region defined by spherical angles of $0 \le \theta \le 30^\circ$, $0 \le \varphi \le 180^\circ$.

<u>Solution: -</u>

$$\Omega = \iint d\Omega = \iint\limits_{0\ 0}^{2\pi\ \pi/6} \sin\theta \, d\theta d\varphi$$

$$= 2\pi \int_0^{\pi/6} \sin\theta \, d\theta = 2\pi[-\cos\theta]_0^{\pi/6} = 2\pi[-0.867 + 1] = 0.83566$$

b) Radiation intensity *U*

$$U = \frac{d\Pi}{d\Omega}, W/sr\ ,$$

and hence,

$$\Pi = \oiint_{4\pi} U\, d\Omega\, , W$$

where, Π, is the radiated power.

There is a direct relation between the radiation intensity *U* and radiation power density *P* (Poynting vector magnitude of far field).

$$P = \frac{d\Pi}{ds} = \frac{d\Pi}{dA}\ , W/m^2$$

Then

$$U = r^2 \cdot P$$

It was already shown that the power density of the far field depends on the distance from the source as $\mathbf{1/r^2}$, since the far field magnitude depends on $\boldsymbol{r}$ as $\mathbf{1/r}$. Thus, the radiation intensity *U depends only on the direction* (θ, φ) *but not on the distance* $\boldsymbol{r}$.

The power pattern is a trace of the function $|U\ (\theta,\varphi)|$ usually normalized to its maximum value. The normalized pattern will be denoted as $\bar{U}(\theta,\varphi)$.

In the far-field zone, the radial field components vanish, and the remaining transverse components of the electric and the magnetic far field are in phase and have magnitudes related by

$$|\mathbf{E}| = \eta\,|\mathbf{H}|.$$

That is why the far-field Poynting vector *P* has only a radial component and it is a real number showing the *radiation density*:

$$P = \frac{1}{2}\eta\,|\mathbf{H}|^2 = \frac{1}{2}\frac{|\mathbf{E}|^2}{\eta}$$

Then, for the *radiation intensity*, we obtain in terms of the electric field

$$U(\theta,\varphi) = \frac{r^2}{2\eta}|\mathbf{E}|^2$$

The above equation leads to a useful relation between the power pattern and the amplitude field pattern:

$$U(\theta,\varphi) = \frac{r^2}{2\eta}\left| E_\theta^2(r,\theta,\varphi) + E_\varphi^2(r,\theta,\varphi) \right| = \frac{1}{2\eta}\left| E_{\theta_p}^2(\theta,\varphi) + E_{\varphi_p}^2(\theta,\varphi) \right|$$

Here, $E_{\theta_p}(\theta,\varphi)$ and $E_{\varphi_p}(\theta,\varphi)$ denote the far-zone field patterns.

Examples:

1) Radiation intensity and pattern of an isotropic radiator:

$$P(r,\theta,\varphi) = \frac{\Pi}{4\pi r^2}$$

$$U(\theta,\varphi) = r^2 \cdot P = \frac{\Pi}{4\pi} = const.$$

$$\Rightarrow \underline{\underline{\bar{U}(\theta,\varphi) = 1}}.$$

The normalized pattern of an isotropic radiator is simply a sphere of a unit radius.

2) Radiation intensity and pattern of an infinitesimal dipole:

From Lecture 3, the far-field term of the electric field is:

$$E_\theta = j\eta\frac{\beta\cdot(I\Delta l)\cdot e^{-j\beta r}}{4\pi r}\cdot\sin\theta \Rightarrow \bar{E}(\theta,\varphi) = \sin\theta,$$

$$U = \frac{r^2}{2\eta}\cdot|\mathbf{E}|^2 = \eta\frac{\beta^2\cdot(I\Delta l)^2}{32\pi^2}\cdot\sin^2\theta,$$

$$\Rightarrow \underline{\underline{\bar{U}(\theta,\varphi) = \sin^2\theta}}.$$

Directivity:

Directivity of an antenna (in a given direction) is the ratio of the radiation intensity in this direction and the radiation intensity averaged over all directions.

The radiation intensity averaged over all directions is equal to the total power radiated by the antenna (Π) divided by 4π. If a direction is not specified, then the direction of maximum radiation is implied.

It can be also defined as the ratio of the radiation intensity (***RI***) of the antenna in a given direction and the ***RI*** of an isotropic radiator fed by the same amount of power:

$$D(\theta,\varphi) = \frac{U(\theta,\varphi)}{U_{av}} = 4\pi\frac{U(\theta,\varphi)}{\Pi}$$

$$D_{max} = 4\pi\frac{U_{max}}{\Pi}$$

The directivity is a dimensionless quantity. The maximum directivity is always≥ 1

<u>Examples:</u>

1) Directivity of an isotropic antenna:

$$U(\theta,\varphi) = U_0 = constant$$

$$\Pi = 4\pi U_0$$

$$D(\theta,\varphi) = 4\pi\frac{U(\theta,\varphi)}{\Pi} = 1$$

$$D_{max} = 1$$

2) Directivity of an infinitesimal dipole:

$U(\theta,\varphi) = \frac{r^2}{2\eta}|E|^2$ but $E_\theta = j\eta\frac{\beta(I\Delta l)e^{-j\beta r}}{4\pi r}\sin\theta \Longrightarrow |E_\theta|^2 = \eta^2\frac{\beta^2(I\Delta l)^2}{(4\pi r)^2}(\sin\theta)^2$

$$U(\theta,\varphi) = \frac{r^2}{2\eta}\left(\eta^2\frac{\beta^2(I\Delta l)^2}{(4\pi r)^2}(\sin\theta)^2\right) = \eta\frac{\beta^2(I\Delta l)^2}{32\pi}(\sin\theta)^2$$

$$\overline{U}(\theta,\varphi) = \sin^2\theta$$

Then we can represent the radiation intensity as $U(\theta,\varphi) = M\cdot\overline{U}(\theta,\varphi)$

The radiated power Π is calculated from the radiation intensity as

$$\Pi = \oiint_{4\pi} U\, d\Omega = M \cdot \int_0^{2\pi}\int_0^{\pi} \sin^2\theta \ \sin\theta \, d\theta d\varphi = M \cdot \frac{8\pi}{3}$$

$$D(\theta,\varphi) = 4\pi \frac{U(\theta,\varphi)}{\Pi} = 4\pi \frac{M \cdot \sin^2\theta}{M \cdot \frac{8\pi}{3}} = \frac{3}{2}\sin^2\theta$$

$$D_{max} = 1.5$$

Exercise:

Calculate the maximum directivity of an antenna with a radiation intensity $U = M \cdot \sin\theta$.

($Answer$: $D_{max} = 4/\pi \approx 1.27$)

Directivity in terms of ***relative radiation intensity*** $\bar{U}(\theta,\varphi)$.

$$U(\theta,\varphi) = M \cdot \bar{U}(\theta,\varphi)$$

$$\Pi = \oiint_{4\pi} U\, d\Omega = M \cdot \int_0^{2\pi}\int_0^{\pi} \bar{U}(\theta,\varphi) \ \sin\theta \, d\theta d\varphi$$

$$D(\theta,\varphi) = 4\pi \frac{U(\theta,\varphi)}{\Pi} = 4\pi\, M \cdot \bar{U}(\theta,\varphi) \Big/ M \cdot \int_0^{2\pi}\int_0^{\pi} \bar{U}(\theta,\varphi) \sin\theta \, d\theta d\varphi$$

$$D_{max} = 4\pi \Big/ \int_0^{2\pi}\int_0^{\pi} \bar{U}(\theta,\varphi) \sin\theta \, d\theta d\varphi$$

Beam solid angle Ω_A:

The ***beam solid angle*** Ω_A of an antenna is the solid angle through which all the power of the antenna would flow if its radiation intensity were constant and equal to the maximum radiation intensity U_0 for all angles within Ω_A.

They reflect the mathematical meaning of the definition above

$$\Pi = \oiint_{4\pi} U\, d\Omega = \oiint_{\Omega_A} U_0\, d\Omega = U_0\, \Omega_A$$

$$\therefore \Omega_A = \frac{\Pi}{U_0} = \oiint_{4\pi} U\, d\Omega \Big/ U_0 = \oiint_{4\pi} \bar{U}\, d\Omega = \int_0^{2\pi}\int_0^{\pi} \bar{U}(\theta,\varphi) \sin\theta\, d\theta d\varphi$$

The relation between the maximum directivity D_{max} and the beam solid angle Ω_A is obvious from previous equation as

$$D_{max} = 4\pi / \Omega_A$$

Antenna gain:

The gain G of an antenna is the ratio of the radiation intensity U in a given direction and the radiation intensity that would be obtained, if the power fed to the antenna were radiated isotropically.

$$G(\theta,\varphi) = 4\pi \frac{U(\theta,\varphi)}{P_{in}}$$

The gain is a dimensionless quantity, which is very similar to the directivity D. When the antenna has no losses, i.e. when $P_{in} = \Pi$, then $G(\theta,\varphi) = D(\theta,\varphi)$. Thus, the gain of the antenna takes into account the losses in the antenna system. It is calculated via the *input power* P_{in}, which is a measurable quantity, unlike the directivity, which is calculated via the radiated power Π.

There are many factors that can worsen the transfer of energy from the transmitter to the antenna (or from the antenna to the receiver):

- mismatch losses,
- losses in the transmission line,
- losses in the antenna: dielectric losses, conduction losses, polarization losses.

The power radiated by the antenna is always less than the power fed to the antenna system, $\Pi \leq Pin$, unless the antenna has integrated active devices. That is why usually $G \leq D$.

According to the IEEE Standards (The Institute of Electrical and Electronics Engineers Standards), the gain does not include losses arising from impedance mismatch and from polarization mismatch.

Therefore, the gain takes into account only the ***dielectric*** and ***conduction*** losses of the antenna system itself.

The radiated power is related to the input power through a coefficient called the ***radiation efficiency*** *e*:

$$\Pi = e \cdot P_{in}, e \leq 1$$

$$G(\theta,\varphi) = e \cdot D(\theta,\varphi)$$

The Maximum gain G_{max} is also related to the maximum directivity D_{max},

$$G_{max} = 4\pi \frac{U_{max}}{P_{in}}$$

$$G_{max} = e \cdot D_{max}$$

Since the gain and directivity are power ratio they can be calculated in decibels as follows:

$$G_{dB} = 10 \log G$$

For the directivity

$$D_{dB} = 10 \log D$$

Antenna efficiency:

The total efficiency of the antenna e_t is used to estimate the total loss of energy at the input terminals of the antenna and within the antenna structure. It includes all mismatch losses and the dielectric/conduction losses (described by the ***radiation efficiency*** e as defined by the IEEE Standards):

$$e_t = e_P \cdot e_r \cdot \underbrace{e_c \cdot e_d}_{e} = e_P \cdot e_r \cdot e$$

Here: e_r is the reflection efficiency (impedance mismatch),

e_p is the polarization mismatch efficiency,

e_c is the conduction efficiency,

e_d is the dielectric efficiency.

The reflection efficiency can be calculated through the reflection coefficient Γ at the antenna input:

$$e_r = 1 - |\Gamma|^2$$

Γ can be either measured or calculated, provided the antenna impedance is known:

$$\Gamma = \frac{Z_{in} - Z_c}{Z_{in} + Z_c}$$

Z_{in} is the antenna input impedance and Z_c is the characteristic impedance of the feed line.

If there are no polarization losses, then the total efficiency is related to the radiation efficiency as

$$e_t = e \cdot (1 - |\Gamma|^2)$$

Voltage **S**tanding **W**ave **R**atio (**VSWR**) is another parameter related to the reflection coefficient, this parameter can be calculated as

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

Beam efficiency:

The ***beam efficiency*** is the ratio of the power radiated in a cone of angle $2\Theta_1$ and the total radiated power. The angle $2\Theta_1$ can be generally any angle, but usually this is the first-null beam width (or minimum).

$$BE = \int_0^{2\pi}\int_0^{\Theta_1} U(\theta,\varphi)\sin\theta\, d\theta d\varphi \Big/ \int_0^{2\pi}\int_0^{\pi} U(\theta,\varphi)\sin\theta\, d\theta d\varphi$$

If Θ_1 is chosen as the angle where the first null or minimum occurs, then the beam efficiency will indicate the amount of power in the ***major*** lobe compared to the total power. A very high beam efficiency (between the nulls or minimums), is necessary for antennas used in radiometry, astronomy, radar, and other applications where received signals through the ***minor*** lobes must be minimized.

Frequency bandwidth (FBW):

This is the range of frequencies, within which the antenna characteristics (input impedance, pattern) conform to certain specifications.

Antenna characteristics, which should conform to certain requirements, might be: input impedance, radiation pattern, beamwidth, polarization, side-lobe level, gain, beam direction and width, radiation efficiency. Separate bandwidths may be introduced: impedance bandwidth, pattern bandwidth, etc.

The **FBW** of ***broadband*** antennas is expressed as the ratio of the upper to the lower frequencies, where the antenna performance is acceptable:

$$FBW = f_{max}/f_{min}$$

Broadband antennas with **FBW** as large as 40:1 have been designed. Such antennas are referred to as *frequency independent antennas*.

For ***narrowband*** antennas, the FBW is expressed as a percentage of the frequency difference over the center frequency:

$$FBW = \frac{f_{max} - f_{min}}{f_0} \times 100\%$$

Usually, $f_0 = (f_{max} + f_{min})/2$ or $f_0 = \sqrt{f_{max} \cdot f_{min}}$

Example:

The radial component of the radiated power density of an antenna is given by $P = A_0 \frac{\sin\theta}{r^2}\ \hat{a}_r, (\frac{W}{m^2})$, where A_0 is the peak value of the power density, θ is the usual spherical coordinate, and $\hat{a}_r$ is the radial unit vector. Determine the total radiated power Π.

Solution:

$$\Pi = \oiint_S \mathbf{P} \cdot ds = \int_0^{2\pi}\int_0^{\pi} A_0 \frac{\sin\theta}{r^2}\ \hat{a}_r \cdot r^2 \sin\theta\ d\theta d\varphi\ \hat{a}_r = A_0\ \pi^2 W$$

Or by using radiation intensity U

$$U = r^2 \cdot P = A_0 \sin\theta\ \ \hat{a}_r, (\frac{W}{sr})$$

$$\Pi = \oiint_{4\pi} U\ d\Omega = \int_0^{2\pi}\int_0^{\pi} A_0 \sin\theta\ \cdot \sin\theta\ d\theta d\varphi = A_0\ \pi^2 W$$

Example:

The normalized radiation intensity of an antenna is represented by

$$U(\theta) = \cos^2(\theta)\cos^2(3\theta), w/sr$$

Find the

a. half-power beamwidth HPBW (in radians and degrees)

b. first-null beamwidth FNBW (in radians and degrees)

Solution:

a- Since the $U(\theta)$represent the power pattern, to find HPBW, set the function equal to half of its maximum value.

$$U(\theta)|_{\theta=\theta_h} = \cos^2(\theta)\cos^2(3\theta)|_{\theta=\theta_h} = 0.5$$

$$\therefore\ \cos(\theta_h) \cdot \cos(3\theta_h) = 0.707$$

$$\theta_h = \cos^{-1}(\frac{0.707}{\cos(3\theta_h)})$$

Since this is an equation with transcendental functions, it can be solved iteratively. After a few iterations, it is found that

$$\theta_h \approx 0.25\, radians = 14.375$$

Since the function $U(\theta)$ is symmetrical about the maximum at $\theta = 0$, then the HPBW is $HPBW = 2\theta_h \approx 0.5\, radians = 28.75$

b- To find the first-null beamwidth (FNBW), set the $U(\theta)$ equal to zero,

$$U(\theta)|_{\theta=\theta_n} = \cos^2(\theta)\cos^2(3\theta)|_{\theta=\theta_n} = 0$$

This lead to two solutions for θ_n.

Either $\cos(\theta_n) = 0 \implies \theta_n = \cos^{-1} 0 = \frac{\pi}{2} radians = 90^\circ$

Or $\cos(3\theta_n) = 0 \implies \theta_n = \frac{1}{3}\cos^{-1} 0 = \frac{\pi}{6} radians = 30^\circ$

The one with the smallest value leads to the FNBW. Again, because of the symmetry of the pattern, the FNBW is

$$FNBW = 2\theta_n = \frac{\pi}{3} radians = 60^\circ$$

Example:

The radial component of the radiated power density of an infinitesimal linear dipole of length $l << \lambda$ is given by $P = A_0 \frac{\sin^2\theta}{r^2}\, \hat{a}_r, (\frac{W}{m^2})$, where A_0 is the peak value of the power density, θ is the usual spherical coordinate, and $\hat{a}_r$ is the radial unit vector. Determine the maximum directivity of the antenna and express the directivity as a function of the directional angles θ and φ.

***Solution*:**

The radiation intensity is given by U

$U = r^2 \cdot P = A_0 \sin^2\theta$, The maximum radiationis directed along $\theta = \pi/2$. Thus, $U_{max} = A_0$, and the total radiated power is given by

$$\Pi = \oiint_{4\pi} U\, d\Omega = \int_0^{2\pi}\int_0^{\pi} A_0 \sin^2\theta \cdot \sin\theta\, d\theta d\varphi = A_0\, \frac{8\pi}{3}\, W$$

we find that the maximum directivity is equal to

$$D_{max} = 4\pi \frac{U_{max}}{\Pi} = 4\pi \frac{A_0}{A_0 \frac{8\pi}{3}} = 1.5$$

And the directivity is given by

$$D(\theta,\varphi) = \frac{U(\theta,\varphi)}{U_{av}} = 4\pi \frac{U(\theta,\varphi)}{\Pi} = 4\pi \frac{A_0 \sin^2\theta}{A_0 \frac{8\pi}{3}} = 1.5 \sin^2\theta$$

Example:

The radiation intensity of the major lobe of an antenna is represented by

$$U = B_0 \cos\theta$$

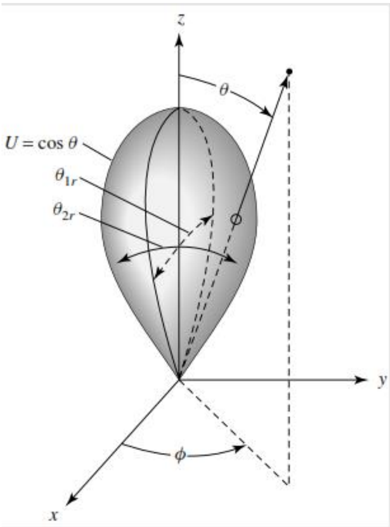
where B_0 is the maximum radiation intensity. The radiation intensity exists only in the upper hemisphere ($0 \le \theta \le \pi/2$, $0 \le \varphi \le 2\pi$), and it is shown in Figure below. Find the

a. beam solid angle.

b. maximum directivity.

Solution:

The half-power point of the pattern occurs at $\theta = 60^\circ$. Thus the beamwidth in the θ direction is 120° or $\Theta_{1r} = \frac{2\pi}{3}$



Since the pattern is independent of the φ coordinate, the beamwidth in the other plane is also equal to $\Theta_{2r} = \frac{2\pi}{3}$

a. Beam solid angle Ω_A:

$$\Omega_A = \int_0^{360^\circ}\int_0^{90^\circ} \bar{U}\, d\Omega = \iint_{0\,0}^{2\pi\ \pi/2} \cos\theta \sin\theta\, d\theta d\varphi$$

$$= \int_0^{2\pi} d\varphi \int_0^{\pi/2} \cos\theta\sin\theta\, d\theta = \pi \int_0^{\pi/2} \sin 2\theta\, d\theta = \pi\ steradian$$

b. maximum directivity D_{max}

$$D_{max} = 4\pi/\,\Omega_A$$

$$D_{max} = 4\pi/\,\pi = 4\ dimensionless = 6.02\ dB$$

<u>Some useful formulas you may be needing them</u>:

- $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
- $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
- $\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$
- $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
- $\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
- $\cos^2\theta = \frac{1+\cos 2\theta}{2}$
- $\sin^2\theta = \frac{1-\cos 2\theta}{2}$
- $\sin^2\theta + \cos^2\theta = 1$
- $\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$
- $\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$
- $\tan\theta = \frac{e^{j\theta} - e^{-j\theta}}{j(e^{j\theta} + e^{-j\theta})}$