Input Impedance:

$$Z_A = R_A + jX_A$$

Here, R_A is the antenna resistance and X_A is the antenna reactance. Generally, the antenna resistance has two terms:

$$R_A = R_r + R_l$$

where, R_r is the radiation resistance, and R_l is the loss resistance.

The antenna impedance is related to the radiated power Π, the dissipated power P_l, and the stored reactive energy, in the following way:

$$P_s = \Pi + P_l + j2\omega(W_m - W_e) = \frac{1}{2} I_0 I_0^* Z_A$$

$$Z_A = \frac{\Pi + P_l + j2\omega(W_m - W_e)}{\frac{1}{2} I_0 I_0^*}$$

Here, I_0 is the current at the antenna terminals; W_m is the average magnetic energy, W_e is the average electric energy stored in the near-field region. When the stored magnetic and electric energy values are equal, a condition of resonance occurs and the reactive part of Z_A vanishes. For a thin dipole antenna, this occurs when the antenna length is close to a multiple of a half wavelength.

Radiation resistance:

The radiation resistance relates the radiated power to the voltage (or current) at the antenna terminals.

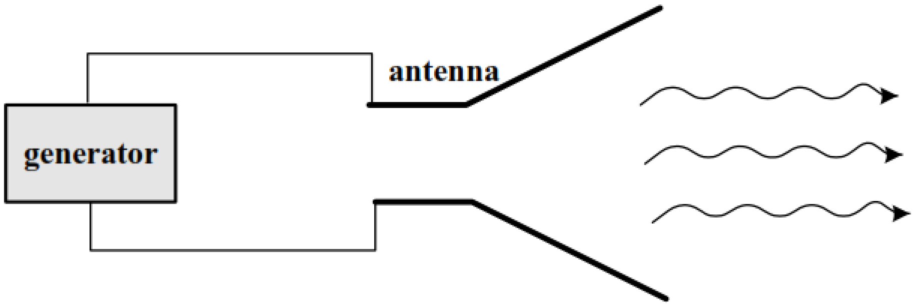
$$R_r = 2\Pi/|I|^2, \Omega$$

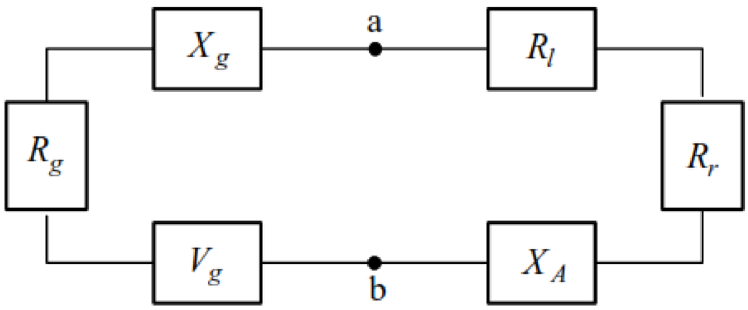
We have already derived the radiated power of an infinitesimal dipole in Lecture 3, as:

$$\Pi = \frac{1}{2} R_r I^2 \Longrightarrow R_r = \frac{2\Pi}{|I|^2}$$

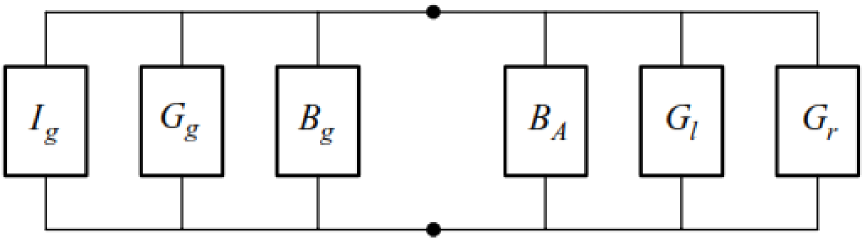
$$R_r^{id} = \frac{2\pi}{3}\eta\left(\frac{\Delta l}{\lambda}\right)^2 . \Omega$$

Equivalent circuits of the transmitting antenna:





(a) Thevenin equivalent



(b) Norton equivalent

In the above model, it is assumed that the generator is connected to the antenna directly. If there is a transmission line between the generator and the antenna, which is usually the case, then $Z_g = R_g + jX_g$ represents the equivalent impedance of the generator transferred to the input terminals of the antenna. Transmission lines themselves often have significant losses.

The impedance transformation by a long transmission line (for lossy transmission line) is given by

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh(\gamma L)}{Z_0 + Z_L \tanh(\gamma L)}$$

Since the propagation constant (γ) is a complex quantity we can write it as:

$$\gamma = \alpha + j\beta$$

where

- α, the real part, is called the attenuation constant
- β, the imaginary part, is called the phase constant

Here, Z_0 is the characteristic impedance of the line, γ is its propagation constant, Z_L is the load impedance, and Z_{in} is the input impedance.

In the case of a loss-free line (lossless transmission line) we get.

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta L)}{Z_0 + jZ_L \tan(\beta L)}$$

where $\gamma = j\beta$.

The term βL is called the electrical length of the transmission line.

Some special cases to consider:

a) Short line: $Z_L = 0, Z_{in} = jZ_0 \tan(\beta L)$

b) Open line: $Z_L = \infty, Z_{in} = -jZ_0 \cot(\beta L)$

c) Quarter wave line: $L = \frac{\lambda}{4}, Z_{in} = \frac{{Z_0}^2}{Z_L}$

d) Half wave line: : $L = \frac{\lambda}{2}, Z_{in} = Z_L$

Example:

An antenna with a radiation resistance of 48 ohms, a loss resistance of 2 ohms, and a reactance of 50 ohms is connected to a generator with open-circuit voltage of 10 V and internal impedance of 50 ohms via a λ/4-long transmission line with characteristic impedance of 100 ohms.

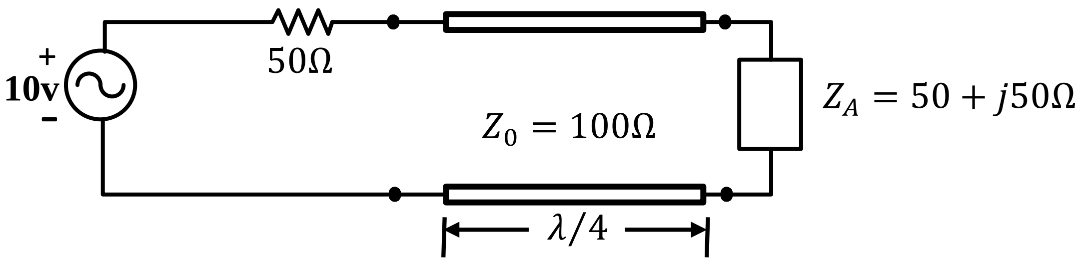
(a) Draw the equivalent circuit

(b) Determine the power supplied by the generator

(c) Determine the power radiated by the antenna

Solution: -

a)



b) $Z_{in} = \frac{{Z_0}^2}{Z_L} = \frac{(100)^2}{50+j50} = 100 - j100\Omega = Z_{Anew}$

$$I_g = \frac{10}{150 - j100} = \frac{10}{150 - j100} = \frac{10}{180.3\angle - 33.7^{\circ}} = 0.05546\angle 33.7^{\circ} A$$

$$P_S = \frac{1}{2} Re\{V_g \cdot I_g^*\} = \frac{1}{2} \times 10 \times 0.05546 \times \cos(33.7^{\circ}) = 0.231\, w$$

c) $P_A = \frac{1}{2}|I_g|^2 Re\{Z_{in}\} = \frac{1}{2}(0.05546)^2 \times 100 = 0.1538w$

$$\Pi = e \cdot P_A$$

$$e = \frac{R_r}{R_r + R_l} = \frac{48}{50} = 0.96$$

$$\Pi = e \cdot P_A = 0.96 \times 0.1538 = 0.148\, w$$

Maximum power is delivered to the antenna when conjugate matching of impedances is achieved:

$$\begin{cases} R_A = R_l + R_r = R_g, \\ X_A = -X_g. \end{cases}$$

Using circuit theory, we can derive the following formulas in the case of matched impedances:

a) power delivered to the antenna

$$P_A = \frac{|V_g|^2}{8(R_r + R_l)} = \frac{|V_g|^2}{8(R_A)}$$

b) power dissipated as heat in the generator

$$P_g = P_A = \frac{|V_g|^2}{8R_g} = \frac{|V_g|^2}{8(R_r + R_l)}$$

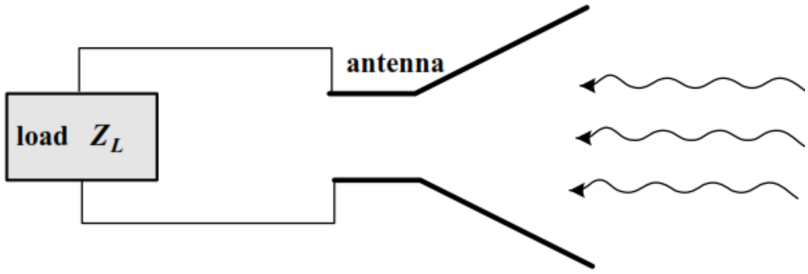
c) radiated power

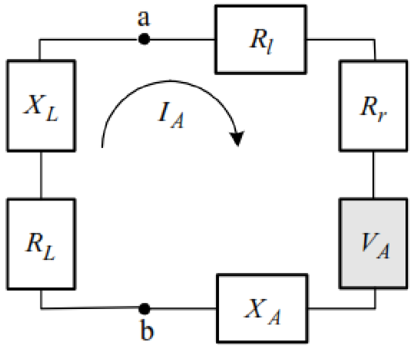
$$\Pi = \frac{1}{2}|I|^2 R_r = \frac{|V_g|^2}{8}\frac{R_r}{(R_r + R_l)^2}$$

d) power dissipated as heat in the antenna

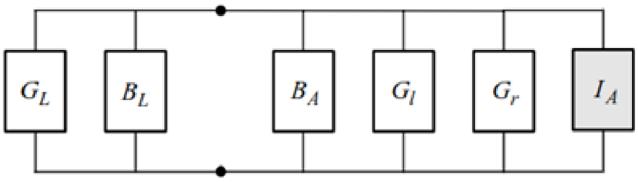
$$P_l = \frac{1}{2}|I|^2 R_l = \frac{|V_g|^2}{8}\frac{R_l}{(R_r + R_l)^2}$$

Equivalent circuits of the receiving antenna:





(a) Thevenin equivalent



(b) Norton equivalent

The incident wave induces voltage V_A at the antenna terminals (measured when the antenna is open circuited). Conjugate impedance matching is required between the antenna and the load (the receiver) to achieve maximum power delivery

$$\left| \begin{array}{l} R_L = R_A = R_l + R_r, \\ X_L = -X_A. \end{array} \right.$$

For the case of conjugate matching, the following power expressions hold:

a) power delivered to the load

$$P_L = \frac{|V_A|^2}{8R_L} = \frac{|V_A|^2}{8R_A}$$

b) Power dissipated as heat in the antenna

$$P_l = \frac{|V_A|^2}{8}\frac{R_l}{(R_r + R_l)^2} = \frac{|V_A|^2}{8}\frac{R_l}{(R_A)^2}$$

c) Scattered (re-radiated) power

$$P_{re} = \frac{|V_A|^2}{8}\frac{R_r}{(R_r + R_l)^2} = \frac{|V_A|^2}{8}\frac{R_r}{(R_A)^2}$$

d) Total captured power

$$P_c = \frac{|V_A|^2}{4\,(R_r + R_l)} = \frac{|V_A|^2}{4R_A}$$

When conjugate matching is achieved, half of the captured power P_c is delivered to the load (the receiver) and half is antenna loss. The antenna losses are heat dissipation P_l and re-radiated (scattered) power P_{re}. When the antenna is non-dissipative half of the power is delivered to the load and the other half is scattered back into space. Thus a receiving antenna is also a scatterer.

The antenna input impedance is frequency dependent. Thus, it is matched to its load in a certain frequency band. It can be influenced by the proximity of objects, too.

Radiation efficiency and antenna losses:

The radiation efficiency e takes into account the conductor and dielectric (heat) losses of the antenna. The conduction and dielectric losses of an antenna are very difficult to compute and in most cases they are measured. Even with measurements, they are difficult to separate and they are usually lumped together to form the e efficiency. **It is the ratio of the power radiated by the antenna and the total power delivered to the antenna terminals** (***in transmitting mode***). The resistance R_l is used to represent the conduction-dielectric losses. In terms of equivalent circuit parameters:

$$e = \frac{R_r}{R_r + R_l}$$

Some useful formulas to calculate conduction losses are given below:

a) dc resistance

$$R_{dc} = \frac{1}{\sigma}\frac{l}{A} \ , \Omega$$

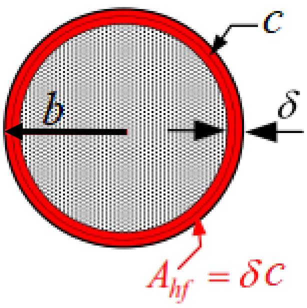
whrer,
σ - specific conductivity, S/m
l - conductor's length, m
A - conductor's cross-section, m^2

b) high-frequency surface resistance R_{hf}

At high frequencies, the current is confined in a thin layer at the conductor's surface (skin effect). This layer is often called the skin layer. Its effective thickness, known as the ***skin depth or penetration depth*** $\boldsymbol{\delta}$, is

$$\delta = \frac{1}{\sqrt{\pi f \sigma \mu}} = \sqrt{2/\ \omega\sigma\mu} \ , m$$

If the skin depth of the metal is very small compared to the smallest diagonal of the cross section of the rod, the current is confined to a thin layer near the conductor surface. Therefore, the high-frequency resistance can be written, based on a *uniform current distribution*, as



$$R_{hf} = \frac{1}{\sigma} \cdot \frac{l}{A_{hf}} = \frac{1}{\sigma} \cdot \frac{l}{\delta c} = \frac{1}{c} \cdot \frac{l}{\sigma \delta} = \frac{l}{c}\sqrt{\frac{\omega\mu}{2\sigma}} = \frac{l}{c} R_s, (ohms)$$

Here the area $A_{hf} = \delta c$ is not the actual area of the conducting rod but is the effective area through which the high-frequency current flows, and

c: is the perimeter (circumference) of the cross section of rod (for a circular wire of radius b)

$$c = 2\pi b$$

R_s: is the conductor surface resistance, $R_s = \sqrt{\frac{\omega\mu}{2\sigma}}, \Omega$

ω: is the angular frequency,

μ: is the permeability,

σ: is the conductivity of the metal.

Finally, the loss resistance is $R_l = 0.5 \cdot R_{hf}$

Example:

A resonant half-wavelength dipole is made of copper (σ = 5.7 $\times 10^7$ S/m) wire. Determine the conduction-dielectric (radiation) efficiency *e* of the dipole antenna, if the operating frequency is *f* = 100 MHz, the radius of the wire *b* is $3\times10^{-4}\,\lambda$, and the radiation resistance of the $\lambda/2$ dipole is R_r =73Ω.

Solution: -

At $f=10^8$ Hz

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{10^8} = 3m$$

$$l = \frac{\lambda}{2} = \frac{3}{2}m$$

$$c = 2\pi b = 2\pi(3\times10^{-4})\lambda = 2\pi(3\times10^{-4})3 = 18\pi\times10^{-4}m$$

$$R_{hf} = \frac{1.5}{18\pi\times10^{-4}}\sqrt{\frac{\pi\times10^8\times4\pi\times10^{-7}}{5.7\ \times10^7}} = 0.698\ ohms$$

$$R_l = 0.5\cdot R_{hf} = 0.5 * 0.698 = 0.349\ ohms$$

$$e = \frac{R_r}{R_r+R_l} = \frac{73}{73+0.349} = 0.995 = 99.52\%$$

$$e = 10\ log_{10}(0.995) = -0.02\ dB$$