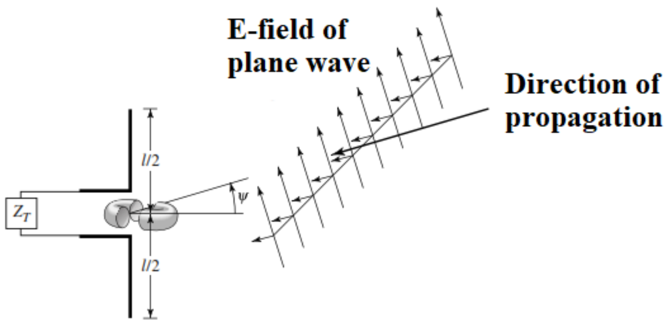
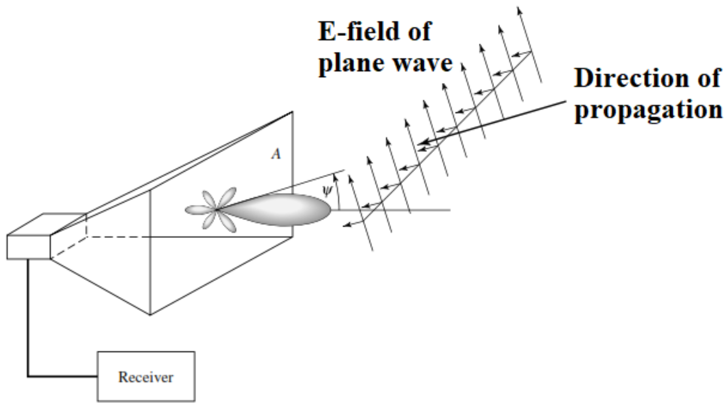
Antenna Effective Length and Effective Areas

An antenna in the receiving mode, whether it is in the form of a wire, horn, aperture,array, dielectric rod, etc., is used to capture (collect) electromagnetic waves and toextract power from them, as shown in Figures 6.1(a) and (b). For each antenna, anequivalent length and a number of equivalent areas can then be defined. These equivalent quantities are used to describe the receiving characteristics of anantenna, whether it be a linear or an aperture type, when a wave is incident uponthe antenna.



(a) Dipole antenna in recieving mode



(b) Aperture antenna in recieving mode

Figure 6.1: Uniform plane wave incident upon dipole and aperture antennas.

1- Effective Length

The effective length of an antenna isa quantity that is used to determine the voltage induced on the open-circuit terminalsof the antenna when a wave impinges upon it. The effective length$l_e$for anantenna is usually a complex vector quantity represented by

$$l_e(\theta,\varphi) = l_\theta(\theta,\varphi)\hat{a}_\theta + l_\varphi(\theta,\varphi)\hat{a}_\varphi$$

The ***effective height*** is a far-fieldquantity and it is related to the *far-zone* field $E_a$radiated by the antenna, with currentI_{in}inits terminals

$$E_a = E_\theta\hat{a}_\theta + E_\varphi\hat{a}_\varphi$$

The effective length represents the antenna in its transmitting and receiving modes, andit is particularly useful in relating the open-circuit voltage $V_{oc}$of receiving antennas.This relationcanbe expressed as

$$V_{oc} = E^i l_e$$

where,

V_{oc} :open-circuit voltage at antenna terminals

E^i : incident electric field

l_e : vector effective length

V_{oc} can be thought of as the voltage induced in a linear antenna of length l_e when $l_e$and $E^i$are linearly polarized. the *effectivelength of a linearly polarized antenna receiving a plane wave in a given direction* is defined as "the ratio of the magnitude of the open-circuit voltage developed at the terminals of the antenna to the magnitude of the electric-field strength in the direction of the antenna polarization.

2- Effective Area (Aperture) A_e

The ***effective antenna aperture*** is the ratio of the available power at the terminals of the antenna to the power flux density of a plane wave incident upon the antenna, which is matched to the antenna in terms of polarization. If there is no specific direction chosen, the direction of maximum radiation intensity is implied.

$$A_e = \frac{P_L}{W_i},$$

where

A_e is the effective aperture, m^2,

P_L is the power delivered from the antenna to the load, W,

W_i is the power flux density (Poynting vector magnitude) of the incident wave, W/m^2.

Using the Thevenin equivalent of a receiving antenna, we can show that equation relates the antenna impedance and its effective aperture as

$$A_e = \frac{|I_A|^2 R_L/2}{W_i} = \frac{|V_A|^2}{2W_i} \cdot \frac{R_L}{[(R_r + R_l + R_L)^2 + (X_A + X_L)^2]}$$

Under condition of conjugate matching, $R_A = R_r + R_l = R_L,\ X_A = -X_L$

$$A_e = \frac{|V_A|^2}{8W_i} \cdot \frac{1}{(R_r + R_l)}$$

For ***aperture type antennas***, the ***effective area*** is smaller than the ***physical aperture area***. Aperture antennas with ***constant amplitude*** and ***phase*** distribution across the aperture have the maximum ***effective area***, which is practically equal to the ***geometrical area***. The ***effective aperture*** of ***wire antennas*** is ***much larger*** than the surface of the wire itself. Sometimes, the ***aperture efficiency*** of an antenna is estimated as ***the ratio of the effective antenna aperture and its physical area***:

$$\varepsilon_{ap} = A_e / A_P$$

Example:

A uniform plane wave is incident upon a very short dipole. Find theeffective area $A_e$and the aperture efficiency, assuming that the radiation resistance isR_r= 80 $(\pi l/\lambda)^2$, andthat the field is linearly polarized along the axis of the dipole. Compare $A_e$withthe physical surface of the wire if$l=\lambda/50$and$d=\lambda/300$, where d is thewire's diameter.

Solution: -

Since the dipole is very short, we can neglect the conduction losses. Wireantennas do not have dielectric losses. Therefore, we assume thatR_l= 0. Underconjugate matching (which is implied unless specified otherwise),

$$A_e = \frac{|V_A|^2}{8W_i} \cdot \frac{1}{(R_r)}$$

The dipole is very short and we can assume that theE-field intensity is thesame along the whole wire. Then, the voltage created by the inducedelectromotive force of the incident wave is

$$V_A = |E| \cdot l$$

The Poynting vector has a magnitude of

$$P = W_i = \frac{|E|^2}{2\eta}$$

Then,

$$A_e = \frac{|E|^2 \cdot l^2}{8|E|^2} \cdot \frac{2\eta}{(R_r)} = \frac{(\lambda/50)^2}{8} \cdot \frac{2\eta}{80\,(\pi l/\lambda)^2}$$

$$A_e = \frac{3\lambda^2}{8\pi} = 0.119 \cdot \lambda^2$$

The physical surface of the dipole is

$$A_P = \pi d l = \pi \cdot \frac{\lambda}{300} \cdot \frac{\lambda}{50} = 2.1 \times 10^{-4} \cdot \lambda^2$$

The aperture efficiency of this dipole is then

$$\varepsilon_{ap} = A_e/A_P = \frac{0.119 \cdot \lambda^2}{2.1 \times 10^{-4} \cdot \lambda^2} = 568.2$$

Relation Between the Directivity D_0 and the Effective Aperture A_e

The simplest derivation of this relation goes through two stages.

Stage 1:

Prove that the ratio D_0/A_e is the same for any antenna. Consider two antennas: A1 and A2. Let A1 be the transmitting antenna, and A2 be the receiving one. Let the distance between the two antennas be R as shown in Figure 6.2.

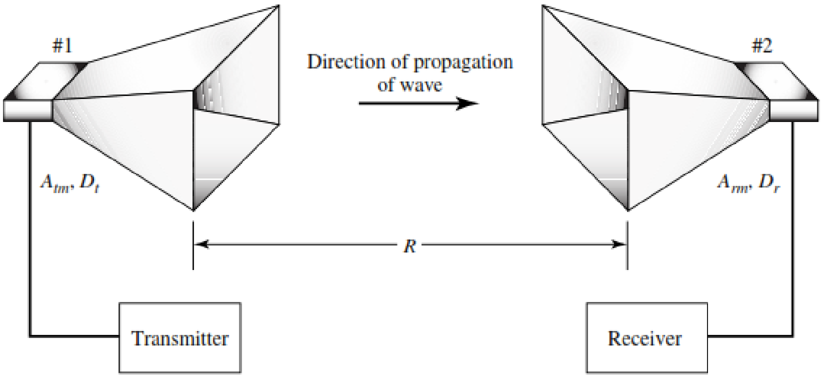


Figure 6.2: Two antennas separated by a distance R.

The power density generated by A1 at A2 is

$$W_1 = \frac{D_1 \Pi_1}{4\pi R^2}$$

Here, Π_1 is the total power radiated by A1 and D_1 is the directivity of A1. The power received by A2 and delivered to its load is

$$P_{L1\to2} = A_{e2} \cdot W_1 = \frac{D_1 \Pi_1}{4\pi R^2} \cdot A_{e2}$$

where A_{e2} is the effective area of A2.

$$D_1 \cdot A_{e2} = 4\pi R^2 \frac{P_{L1\to2}}{\Pi_1}$$

Now, let A1 be the receiving antenna and A2 be the transmitting one. We can derive the following:

$$D_2 \cdot A_{e1} = 4\pi R^2 \frac{P_{L2\to1}}{\Pi_2}$$

If $\Pi_1 = \Pi_2$, then, according to the reciprocity principle in electromagnetics (***The reciprocity in antenna theory states that if antenna #1 is a transmitting antenna and antenna #2 is a receiving antenna, then the ratio of transmitted to received power P_{tra}/P_{rec}, will not change if antenna #1 becomes thereceiving antenna and antenna #2 becomes the transmitting one***).

$P_{L1\to2} = P_{L2\to1}$. therefore,

$$D_1 \cdot A_{e2} = D_2 \cdot A_{e1} \Longrightarrow \frac{D_1}{A_{e1}} = \frac{D_2}{A_{e2}} = \Upsilon$$

We thus proved thatΥ is the same for every antenna.

Stage 2:

Find the ratio$\Upsilon = D_{max}/A_e$for an infinitesimal dipole. The maximum directivity of a very short dipole (infinitesimal dipole) is$D_{max}^{id} = 1.5$. the effective aperture of infinitesimal dipole is $A_e^{id} = \frac{3\lambda^2}{8\pi}$as previously mentioned. Then,

$$\Upsilon = \frac{D_{max}}{A_e} = \frac{1.5}{3\lambda^2} \cdot 8\pi$$

$$\Upsilon = \frac{D_{max}}{A_e} = \frac{4\pi}{\lambda^2}$$

The above equation is true if there are no dissipation, polarization mismatch, and impedance mismatch in the antenna system. If those factors are present, then

$$A_e = (1 - |\Gamma|^2) \cdot |\widehat{\boldsymbol{\rho}}_w \cdot \widehat{\boldsymbol{\rho}}_a|^2 \cdot \left(\frac{\lambda^2}{4\pi}\right) e \cdot D_{max}$$

$$A_e = (1 - |\Gamma|^2) \cdot |\widehat{\boldsymbol{\rho}}_w \cdot \widehat{\boldsymbol{\rho}}_a|^2 \cdot \left(\frac{\lambda^2}{4\pi}\right) G_{max}$$

From these equations, we can obtain a simple relation between the antenna***beam solid angle*** $\Omega_A$and ***effective area***A_e

$$A_e = \frac{\lambda^2}{4\pi} D_{max} = \frac{\lambda^2}{\Omega_A}$$

Other Antenna Equivalent Areas

Before, we have defined the antenna effective area (or effective aperture) as the area, which when multiplied by the incident wave power density, produces the power delivered to the load (the terminals of the antenna) P_L. In a similar manner, we define the ***antenna scattering area*** A_s. It is the area, which when multiplied with the incident wave power density, produces the re-radiated (scattered) power:

$$A_s = \frac{P_{re}}{W_i} = \frac{|I_A|^2 \cdot R_r}{2W_i}, m^2$$

In the case of conjugate matching,

$$A_s = \frac{|V_A|^2}{8W_i} \cdot \frac{R_r}{(R_r + R_l)^2} = \frac{|V_A|^2}{8W_i} \cdot \frac{R_r}{(R_A)^2}, m^2$$

The ***loss area*** A_l is the area, which when multiplied by the incident wave powerdensity, produces the dissipated (as heat) power of the antenna.

$$A_l = \frac{P_l}{W_i} = \frac{|I_A|^2 \cdot R_l}{2W_i}, m^2$$

In the case of conjugate matching,

$$A_l = \frac{|V_A|^2}{8W_i} \cdot \frac{R_l}{(R_r + R_l)^2} = \frac{|V_A|^2}{8W_i} \cdot \frac{R_l}{(R_A)^2}, m^2$$

The ***capture area*** A_c is the area, which when multiplied with the incident wave power density, produces the total power intercepted by the antenna:

$$A_c = \frac{P_c}{W_i} = \frac{|I_A|^2 \cdot (R_r + R_l + R_L)}{2W_i}, m^2$$

In the case of conjugate matching,

$$A_c = \frac{|V_A|^2}{8W_i} \cdot \frac{(R_r + R_l + R_L)}{(R_r + R_l)^2} = \frac{|V_A|^2}{8W_i} \cdot \frac{(R_A + R_L)}{(R_A)^2} = \frac{|V_A|^2}{4W_i} \cdot \frac{1}{R_A}, m^2$$

The capture area A_c is the sum of the effective area A_e, the loss area A_l and the scattering area A_s:

$$A_c = A_e + A_l + A_s$$

When conjugate matching is achieved,

$$A_e = A_l + A_s = 0.5A_c$$

If conjugate matching is achieved for a loss-free antenna, then

$$A_e = A_s = 0.5A_c$$

Example:

A lossless resonant half-wavelength dipole antenna, with input impedance of 73 ohms, is connected to a transmission line whose characteristic impedance is 50 ohms. Assuming that the pattern of the antenna is given approximately by

$$\boldsymbol{U = B_0 \sin^3\theta}$$

Find the maximum absolute gain of this antenna.

Solution:

$$D(\theta,\varphi) = \frac{4\pi\, U(\theta,\varphi)}{\prod}$$

$$\prod = \oiint_{\Omega A} U(\theta,\varphi)\, sin\theta\, d\theta\, d\varphi$$

$$= \oiint_{4\pi} U(\theta,\varphi)\, sin\theta\, d\theta\, d\varphi = \int_0^{2\pi}\int_0^{\pi} U(\theta,\varphi)\, sin\theta\, d\theta\, d\varphi$$

$$\prod = \int_0^{2\pi}\int_0^{\pi} B_0\, sin^3\theta\, sin\theta\, d\theta\, d\varphi = \int_0^{2\pi}\int_0^{\pi} B_0\, sin^4\theta\, d\theta\, d\varphi = B_0\left(\frac{3\pi^2}{4}\right)$$

Note:- $\int_0^{\pi} sin^4\theta\, d\theta = \frac{3\pi}{8}$

$$D(\theta,\varphi) = \frac{4\pi\, U(\theta,\varphi)}{\prod} = \frac{4\pi\, B_0\, sin^3\theta}{B_0\left(\frac{3\pi^2}{4}\right)} = \frac{16}{3\pi} sin^3\theta$$

$$D_{max} = \frac{4\pi\, U_{max}}{\prod} = \frac{16}{3\pi} = 1.697 \;\; when\; \theta = \pi/2$$

Since the antenna is lossless, then the antenna radiation efficiency $e = 1$.

$$G(\theta,\varphi) = \frac{4\pi\, U(\theta,\varphi)}{P_{in}} = e.D(\theta,\varphi)$$

$$G_{max} = \frac{4\pi\, U_{max}}{P_{in}} = e.D_{max}$$

$$G_{abs}(\theta,\varphi) = e_r\; e.D(\theta,\varphi) = (1-|\Gamma|^2)\; e.D(\theta,\varphi)$$

$$G_{abs,max} = e_r \; e.D_{max} = (1 - |\Gamma|^2) \; e.D_{max}$$

$$\Gamma = \frac{Z_{1n} - Z_c}{Z_{in} + Z_c}$$

$$e_r = \left(1 - \left|\frac{73 - 50}{73 + 50}\right|^2\right) = 0.965$$

$$G_{abs,max} = 0.965(1.697) = 1.6376$$

$$G_{abs,max}\,(dB) = 10\, log_{10}(1.6376) = 2.142$$