## Atmospheric Refraction

Speed of radio wave:


## Refractivity

$$
N=(n-1) \cdot 10^{6}
$$

Pressure and water-vapor content decrease rapidly with altitude, while temperature decreases slowly, refractivity and decreases with altitude. Thus, velocity increases with altitude, and rays bend downward. Dominant change in refractivity occurs with along vertical. Key point: not the actual refractivity, but changes (gradient) in refractivity cause rays to bend.

## Earth's Effective Radius Linear Model ( $\mathbf{a}=3440 \mathbf{n m i}=378.137$ km $)$


(a)

(b)

Value for $k$ to get straight-line propagation:

$$
k=\frac{1}{1+a(d n / d h)}
$$

Where $d n / d h$ is the change is refractive index as a function of height. The long-term average refractivity for the U.S. is about. $-39 \mathrm{~N} / \mathrm{km}$. After converting to $n$ and we find $k=$ $4 / 3$. What does this mean?

## Distance to Horizon (antenna height $=$ )

$$
\begin{aligned}
& d=\sqrt{2 k a h}_{a} \text { (Consistent Units) } \\
& d(k m)=4.12 \sqrt{h_{a}}(m)
\end{aligned}
$$

Exponential Model of Refractivity: $N=N_{s} \exp \left(-h / H_{s}\right)$, were $N_{s}=$ refractivity at surface of earth, $h=$ height above sea level, and $H s=$ scale height.


Radar range-angle-diagram used to plot antenna coverage, based on exponential model of refraction with $=313$.

Standard Atmosphere: $N=316 \exp (-Z / 26.5) \quad Z \leq 25$, Where $Z=$ altitude in thousands of feet.

## Radar Measurement Errors Due to Refraction



Calculated angle error (abscissa) due to atmospheric refraction for a standard atmosphere as function of elevation angle and target height (ordinate)

Refraction also caused range errors:

$$
R_{c} \cong 0.42+0.0577 R_{t} \sqrt{\frac{N_{s}}{h}}
$$

Where $R_{c}=$ range correction (m), $R_{t}$ is radar range $(\mathrm{km}), N_{s}$ is surface refractivity in N units, and $h=$ radar altitude in kft.

Measurement of Refractivity: Radiosonde, aircraft, helicopters, small rockets, refractometer

## Anomalous Propagation (AP)

If a strong low-level inversion exists, the radar beam may be refracted such that it strikes the ground some distance from the radar.


Question: What time of day do you expect AP more likely?


## Atmospheric Attenuation

Attenuation of electromagnetic energy in the atmosphere. Solid curve is due to absorption by oxygen. Dashed curve is due to absorption by water vapor.


We modifying radar equations to account for attenuation by multiplying numerator with factor $\exp (-2 \alpha R)$ where $\alpha$ is the one-way the attenuation coefficient in the same units of distance ${ }^{-1}$, and $R$ is the range to the target. For example, the radar equation for distributed targets:

$$
P_{r}=\left(\frac{G^{2} \lambda^{2} P_{t} \theta \phi c \tau}{1024 \ln (2) \pi^{2}}\right) \frac{\eta}{R^{2}}=C \frac{\eta}{R^{2}}
$$

becomes:

$$
P_{r}=\left(\frac{G^{2} \lambda^{2} P_{t} \theta \phi c \tau}{1024 \ln (2) \pi^{2}}\right) \frac{\eta}{R^{2}} e^{-2 \alpha R}=C \frac{\eta}{R^{2}} e^{-2 \alpha R}
$$

The other radar equations are modified the same way. If $\alpha$ is not constant, but a function of distance, then we use $\exp \left(-2 \int \alpha(R) d R\right.$.

Important Note: The figure above is the attenuation and not the attenuation coefficient $\alpha$. What is plotted in the figure is equivalent to $4.34 \alpha$.

Problem. Show that the relationship between attenuation in $\mathrm{dB} / \mathrm{km}$ and attenuation coefficient is

Attenuation in dB per unit distance $=4.34 \times$ attenuation coefficient

Question. Using the figure above, what is the one-way total atmospheric attenuation for a K-band radar at 22 GHz ? over a distance of 20 km ?

Answer. At 22 GHz there is a peak in the water vapor absorption and the attenuation (broken curve) is about $0.2 \mathrm{~dB} / \mathrm{km}$. The attenuation due to oxygen is about $0.01 \mathrm{~dB} / \mathrm{km}$. The attenuation total is then $0.21 \mathrm{~dB} / \mathrm{km}$ or $0.21 \times 20=4.2 \mathrm{~dB}$ or a factor $1 / 2.63=0.38$. Alternatively, $0.21 \mathrm{~dB} / \mathrm{km}$ is equivalent to an attenuation coefficient $\alpha=0.21 / 4.34=$ $0.04839 \mathrm{~km}^{-1}$. The attenuation is then $\exp (-0.04839 \times 20)=0.38$.

Effect of Elevation Angle
Two-way atmospheric attenuation as a function of range and frequency for (a) $0^{\circ}$ elevation angle and (b) $5^{\circ}$ elevation angle.

(b)

