Lecture 13
Dynamic General Equilibrium

Noah Williams

University of Wisconsin - Madison

Economics 702

Dynamic Equilibrium

- We learned how to think about a household that makes dynamic decisions.
- We learned how to think about the intertemporal implications government policy.
- Now we want to introduce investment and capital accumulation.
- With these, and our previous static considerations on the labor market, we put everything together in a Dynamic General Equilibrium.
- We have seen implications of optimal dynamic allocation, now look at equilibrium.
- Will start with two-period model, then extend to infinite horizon

Consumption-Leisure-Savings Decision

- A representative household maximizes $u(c, l) + \beta u(c', l')$
- Its preferences satisfy the usual assumptions.
- It faces two intertemporal budget constraints:

$$\begin{aligned} c + s &= w(h - l) + \pi - T \\ c' &= w'(h - l') + \pi' - T' + (1 + r)s \end{aligned}$$

- As before, we can combine these into PV budget constraint:

$$c + \frac{c'}{1+r} = w(h - l) + \frac{w'(h - l')}{1+r} + \pi + \frac{\pi'}{1+r} - T - \frac{T'}{1+r}$$

The Household's Problem

- We can write the choice problem as a Lagrangian:

$$L = u(c,l) + \beta u(c',l') +$$
$$\lambda\left(w(h-l) + \frac{w'(h-l')}{1+r} + \pi + \frac{\pi'}{1+r} - T - \frac{T'}{1+r} - c - \frac{c'}{1+r}\right)$$

- There are four first order conditions:

$$\begin{aligned} c: \quad u_c(c,l) - \lambda &= 0 \\ l: \quad u_l(c,l) - \lambda w &= 0 \\ c': \quad \beta u_c(c',l') - \frac{\lambda}{1+r} &= 0 \\ l': \quad \beta u_l(c',l') - \frac{\lambda w'}{1+r} &= 0 \end{aligned}$$

Household Problem II

- First order conditions for c and l imply

$$MRS_{l,c} = \frac{u_l(c,l)}{u_c(c,l)} = w$$

- First order conditions for C' and l' imply

$$MRS_{l',c'} = \frac{u_l(c',l')}{u_c(c',l')} = w'$$

- First order conditions for c and c' imply Euler equation:

$$MRS_{c,c'} = \frac{u_c(c,l)}{u_c(c',l')} = \beta(1+r)$$

- Combining these equations gives

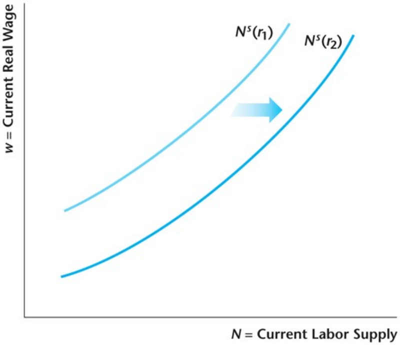
$$\frac{MRS_{l,c}MRS_{c,c'}}{MRS_{l',c'}} = MRS_{l,l'} = \frac{w}{w'}\beta(1+r)$$

Determinants of current labor supply $N = h - l$

$$MRS_{l,l'} = \frac{w}{w'}\beta(1+r)$$

- Higher current wage w raises labor supply.
- Higher future wage w' lowers labor supply.
- Higher interest rate r raises labor supply.
- Higher lifetime wealth reduces labor supply.
- The labor supply curve is the relationship between w and N, and so is upward sloping.
- The other factors shift the labor supply curve.

Figure 11.2 An Increase in the Real Interest Rate Shifts the Current Labor Supply Curve to the Right

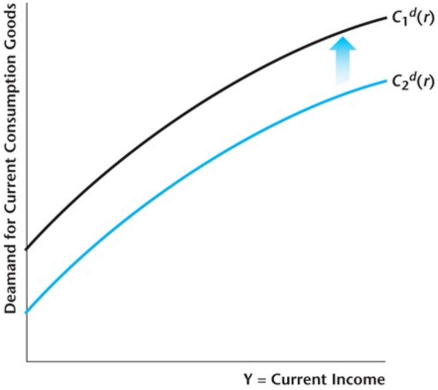


Determinants of current consumption c

$$MRS_{c,c'} = \beta(1+r)$$

- Higher interest rates reduce consumption.
- Higher current (wage or profit) income raises consumption.
- Higher future (wage or profit) income raises consumption.
- The consumption demand curve plots aggregate consumption as a function of current aggregate income, and so is upward sloping.
- The other factors shift the consumption demand curve.

Figure 11.6 An Increase in Lifetime Wealth Shifts the Demand for Consumption Goods Up



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Investment

- We'll treat firm investment slightly differently from how we previously did it, to be closer to the textbook. The implications are nearly identical.
- In each period, the firm has a production function:

$$Y = zF(K, N) \text{ and } Y' = z'F(K', N')$$

- In the first period, the firm chooses how much labor to hire N and how much to invest I (measured in units of the consumption good):

$$\pi = zF(K, N) - wN - I$$

- The investment yields capital in the following period:

$$K' = (1 - \delta)K + I$$

- In the second period, the firm chooses how much labor to hire N' and then sells its un-depreciated capital:

$$\pi' = z'F(K', N') - w'N' + (1-\delta)K'$$

- A representative firm chooses N, N', I, and K' to maximize

$$V = \pi + \frac{\pi'}{1+r},$$

the present value of its profits, where

$$\begin{aligned} \pi &= zF(K, N) - wN - I \\ \pi' &= z'F(K', N') - w'N' + (1-\delta)K' \\ K' &= (1-\delta)K + I \end{aligned}$$

The Firm's Problem

- Write the Lagrangian:

$$L = zF(K,N) - wN - I + \frac{z'F(K',N') - w'N' + (1-\delta)K'}{1+r} + \lambda((1-\delta)K + I - K')$$

- The choice of N involves only static considerations.

$$zF_N(K,N) = w.$$

  Equivalently, N is chosen to maximize current period π.

- The choice of N' is similarly static.

$$\frac{z'F_N(K',N') - w'}{1+r} = 0 \Rightarrow z'F_N(K',N') = w'.$$

  Equivalently, N' is chosen to maximize future π'.

$$
\begin{aligned}
L &= zF(K,N) - wN - I + \frac{z'F(K',N') - w'N' + (1-\delta)K'}{1+r} \\
&\quad +\lambda((1-\delta)K + I - K')
\end{aligned}
$$

- The choice of I and K' is dynamic:

$$
\begin{aligned}
I &: \quad \lambda = 1 \\
K' &: \quad \frac{z'F_K(K',N') + (1-\delta)}{1+r} - \lambda = 0 \\
&\quad \Rightarrow z'F_K(K',N') - \delta = r
\end{aligned}
$$

The net marginal product of capital equals the interest rate.

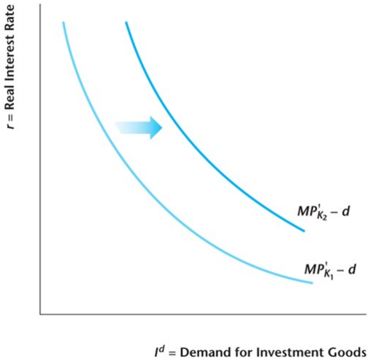
Investment Demand

- Since $K' = (1-\delta)K + I$, investment must satisfy

$$\begin{aligned} z'F_N((1-\delta)K + I, N') &= w' \quad \text{and} \\ z'F_K((1-\delta)K + I, N') - \delta &= r. \end{aligned}$$

- Since $F_{KK} < 0$, I is decreasing in r. This is the investment demand curve.
- Alternatively, since $F_{KN} > 0$, I is increasing in w'.
- An increase in $(1-\delta)K$ reduces I one-for-one.
- An increase in z' raises K', hence I.

Figure 11.10 The Optimal Investment Schedule Shifts to the Right if *K* Decreases or *z'* Is Expected to Increase



More on Investment

- As before, factor prices equal marginal products, but now expected future **net** marginal product of capital determines investment.
- Alternative explanation: Firm trades off the cost of additional capital with the benefit.
  Benefit: addition to future output $= MPK'$.
  Cost: $r + \delta$. Interest cost due to forgone current profit, depreciation costs due to wearing out of capital stock.
- User cost (uc) of capital$= r + \delta$, total cost of use of capital for one period. To determine K' firm equates user cost to expected MPK'.

Changes in Investment/Capital Stock

- Changes in either uc or MPK' affect the firm's capital stock. Decrease in r or δ lowers uc, doesn't change MPK', leads to higher capital stock. To get higher K', increase I.
- Positive change in **expected future** technology z' increases MPK', leading to higher desired K' and so higher I. Increases in labor N' have the same effect, since each unit of K more productive.
- Capital revenue taxation implies $(1-\tau)MPK' = uc$, so can define tax-adjusted user cost $= uc/(1-\tau)$.
- Examples: investment tax credits, depreciation allowances. Same effects as r and δ.
- Complications: firm profit is taxed, not firm revenue. Since depreciation allowances decrease profits, lead to lower taxes. Investment tax credits reduce tax.

More on Investment

- Same capital stock calculations apply for inventories and housing as for physical capital.
- Some capital can be constructed easily, others (new buildings) may take years. So investment needed to increase the capital stock may be spread out over time
- Costs of adjustment: often assume that a company has a fixed business plan and to increase capital has a cost due to reorganization. Larger changes may entail more than proportional increases in costs.
- Explains lags in investment: may be able to double build plant size in a week if pay enough (high cost of adjustment), but more likely will be spread out over time.

Competitive Equilibrium

- Add government with PV Budget Constraint:

$$G + \frac{G'}{1+r} = T + \frac{T'}{1+r}$$

- In a competitive equilibrium, households choose consumption and leisure (c, c', l and l') to maximize utility given wages and interest rates (w, w', and r).
- Firms choose employment and investment (N, N', and I) to maximize value given wages and interest rates (w, w', r).
- The labor market clears in both periods, $N + l = N' + l' = h$.
- The goods market clears in both periods,

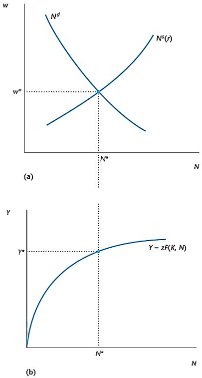
$$zF(K,N) = c + I + G \quad \text{and} \quad z'F(K',N') + (1-\delta)K' = c' + G'$$

- Note that the credit market clears by Walras law.

Labor Market Equilibrium

- Labor supply: increasing in the real wage.
  Substitution effect dominates income effect.
- Labor demand: decreasing in real wage.
  Equate marginal product of labor to the real wage.
- An increase in the interest rate directly and indirectly reduces future wages, raising current labor supply.
  - Direct effect: PDV of wages is $\frac{w'}{1+r}$.
  - Indirect effect:
    - $r = z'F_K(K', N') - d$, decreasing in K'/N'.
    - So higher r reduces K'/N'.
    - $w' = z'F_N(K', N')$, increasing in K'/N'.
    - So lower K'/N' reduces future wages w'.
  - Both work through intertemporal substitution of leisure.

Figure 11.14 Determination of Equilibrium in the Labor Market Given the Real Interest Rate *r*



Goods Market Equilibrium

- Output supply: increasing in real interest rate
  An increase in the interest rate raises current labor supply.
  This increases employment, raising output.
- Output demand: decreasing in real interest rate.
  Higher real interest rates reduce investment.
  Higher real interest rates reduce consumption.

Figure 11.15 Construction of the Output Supply Curve

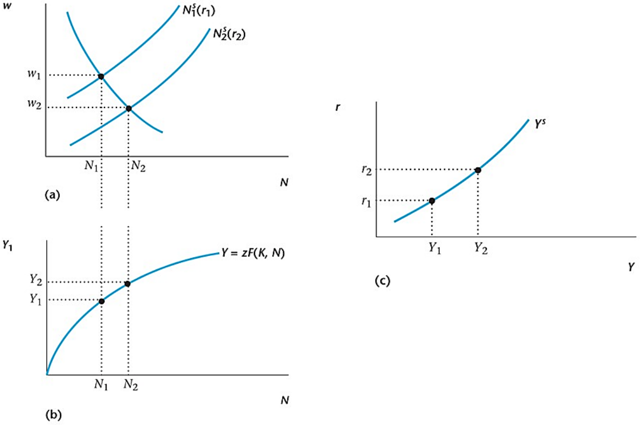


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Figure 9.14 An Increase in Current Total Factor Productivity Shifts the Ys Curve

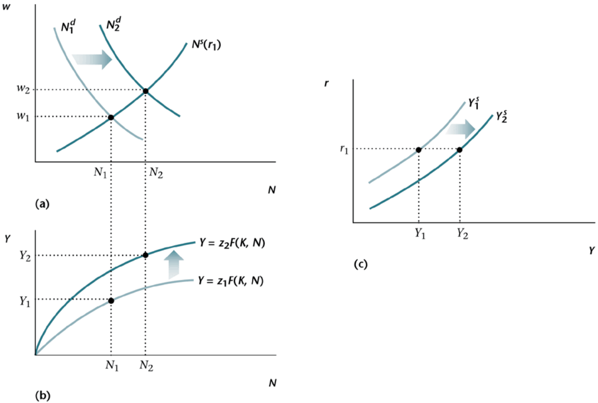

Figure 11.19 Construction of the Output Demand Curve

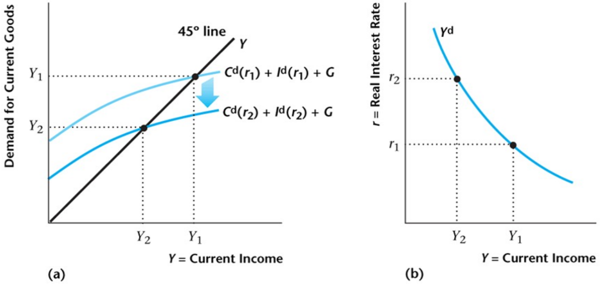

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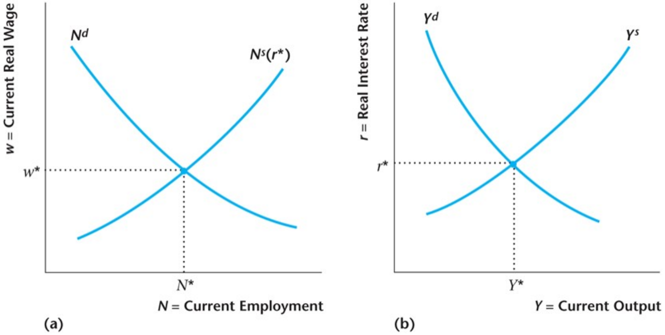
Figure 11.21 The Complete Real Intertemporal Model



Effect of an increase in G on Equilibrium.

- Increase in G raises current output demand.
- Increase in current or future taxes reduces household wealth.
  - Leisure falls and so labor supply increases
  - Consumption demand falls, but by less than G increased.
  - Future labor supply increases, raising investment demand.
- In net, both output demand and supply increase.
- Wages unambiguously fall.
- Do interest rates rise or fall?
  - Wealth effects are small for a temporary change.
  - $G + C$ increases sharply.
  - N increases only slightly.
  - So it is likely that interest rates rise.

Figure 11.20 The Output Demand Curve Shifts to the Right if Current Government Spending Increases

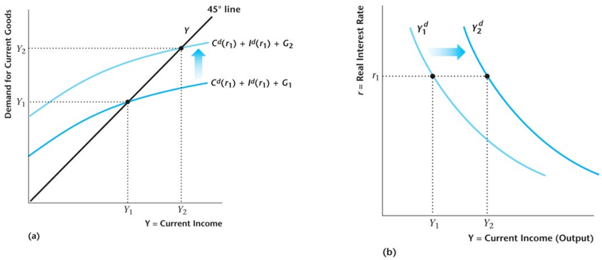


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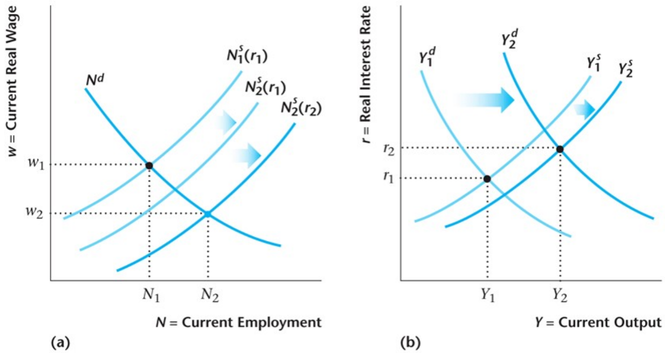
Figure 11.22 A Temporary Increase in Government Purchases



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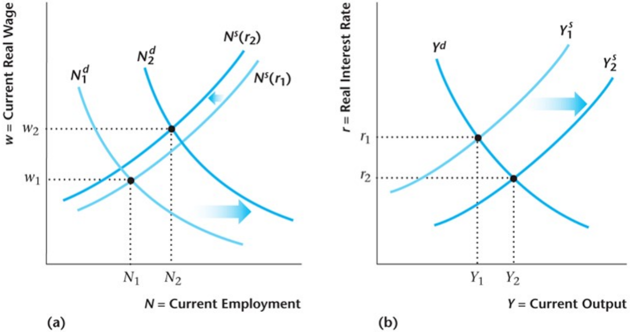
The effect of an anticipated increase in G'

- Increase in current or future taxes reduces household wealth.
    - Leisure falls and so labor supply increases
    - Consumption demand falls.
    - Future labor supply increases, raising investment demand.
- In net, output supply increases but output demand may fall.
- Interest rates fall and output probably rises.
- Wages fall due to the increase in labor supply.
  The effect is partially offset by declining interest rates.
- Can also consider a permanent increase: both G and G'.

Increase in current TFP z

- Labor is more productive, increasing N^d. Output supply curve $Y^s(r)$ shifts out due to direct effect of z and due to increase in labor input.
- No effect on output demand, since TFP unchanged in future, hence no change in $I^d(r)$
- In equilibrium, real interest rate falls, consumption and investment rise, employment rises, real wage rises.
- Productivity shocks are a potential explanation for business cycles, as we'll discuss later.
- Ongoing productivity improvements are the main explanation for long run growth, as we discussed previously.

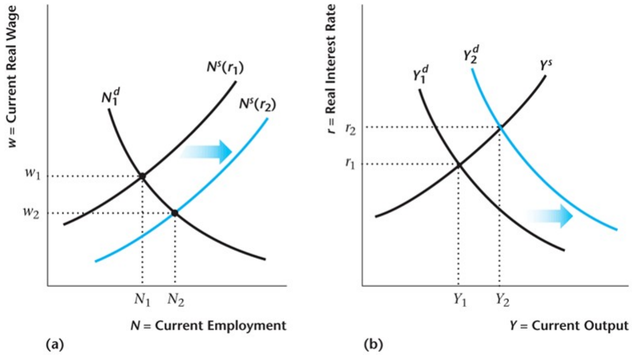
Figure 11.25 The Equilibrium Effects of an Increase in Current Total Factor Productivity



Increase in future TFP z'

- No (direct) effect on labor market (only through change in r), no effect on current production. Hence no effect on $Y^s(r)$.
- Investment demand $I^d(r)$ increases for any r, since $z'F_K(K', N')$ increases. Output demand curve shifts right.
- In equilibrium, real interest rate rises, investment increases, consumption may rise or fall, employment rises, real wage falls, output rises.
- That is, news of future productivity causes boom today. Important in explaining investment boom in the 1990s.

Figure 11.27 The Equilibrium Effects of an Increase in Future Total Factor Productivity



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