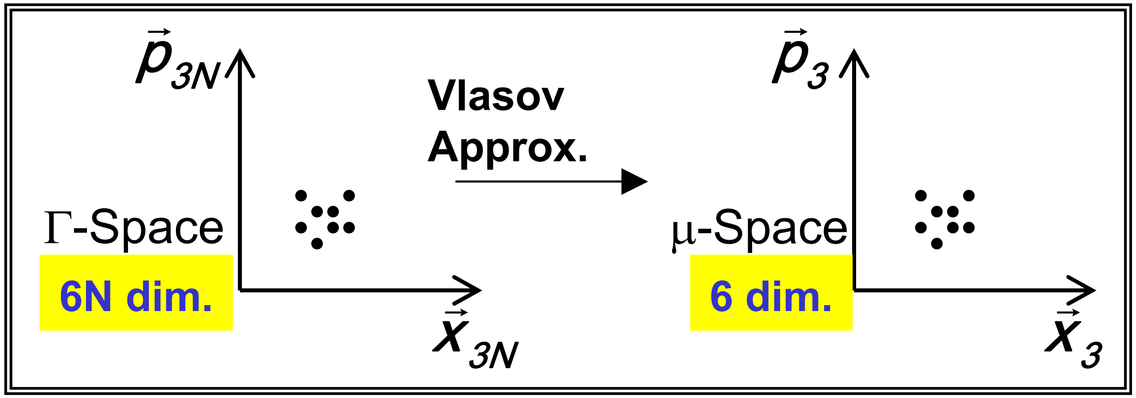
Emittance and Emittance Measurements

S. Bernal

USPAS 08

U. of Maryland, College Park

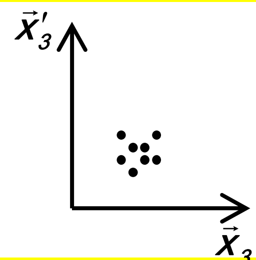
Introduction: Phase and Trace Spaces, Liouville's theorem, etc.



Liouville's Theorem: area conserved in either Γ or μ spaces for Hamiltonian systems

Units of area in *Phase Space* (μ–space) are those of *Angular Momentum*.

$p_x \rightarrow x'$: *Trace Space*



Units of area in *Trace Space* are those of *length*: "metre" (angle measure is dimensionless).

Matrix notation: $\vec{X}_3^{\,T} = [x, y, z]$, $\vec{X}_3'^{\,T} = [x', y', \Delta p/p]$

or $\vec{X}_6^{\,T} \equiv \vec{X}^T = [x, x', y, y', z, \Delta p/p]$

Review of Courant-Snyder Theory

Transverse coordinate for paraxial particle motion in linear lattice obeys :

$$x(s) = Aw(s)\cos\left[\psi(s) + \varphi\right],$$

$$w''(s) + \kappa_0(s)w(s) - \frac{1}{w(s)^3} = 0\,,\;\; w(s) \equiv \beta(s)^{1/2}$$

$\beta(s)$ is a lattice function that depends on $\kappa_0(s)$, but A and φ are different for different particles.

Further, $$\gamma x^2 - 2\alpha x x' + \beta x'^2 = A_x^2$$

α, β, γ: *Courant-Snyder or Twiss* Parameters.

The ellipse equation defines self-similar ellipses of area πA_X^2 at a given plane *s*. The largest ellipse defines the *beam emittance* through

$$\varepsilon_X = A_{Xmax}^2 = \frac{\text{area of largest ellipse}}{\pi}$$

Space charge and emittance (M. Reiser, Chap. 5)

If linear space charge is included, then $\kappa_0(s) \rightarrow \kappa(s)$ and the β function is no longer just a lattice function because it now depends on current also. β becomes a lattice+beam function.

Further, adding space charge does not require changing the emittance definition…Emittance and current are independent parameters in the envelope equation:

$$R'' + k_0^2 R - \frac{K}{R^2} - \frac{\varepsilon^2}{R^3} = 0, \quad a_0 = \sqrt{\frac{\varepsilon}{k_0}}$$

a_0: zero-current 2×rms beam radius

The emittance above is the effective or 4×rms emittance (also called edge emittance) which is equal to the total (100%) emittance for a K-V distribution.

For a Gaussian distribution, the total and RMS emittances are equal when the Gaussian is truncated at 1-σ.

RMS Emittance and Envelope Equation (Pat G. O'Shea)

Linear system: $x'' = -k^2 x,$ **Define:** $a \equiv x_{RMS} = \sqrt{\langle x^2 \rangle},$

$$a' = \frac{\langle xx' \rangle}{a}, \quad a'' = -\frac{\langle xx' \rangle}{a^2} a' + \frac{\langle x'^2 \rangle}{a} + \frac{\langle xx'' \rangle}{a},$$

$$= -\frac{\langle xx' \rangle}{a^2} a' + \frac{\langle x'^2 \rangle}{a} - \frac{k^2 \langle x^2 \rangle}{a},$$

$$= -\frac{\langle xx' \rangle^2}{a^3} + \frac{\langle x'^2 \rangle}{a} - k^2 a,$$

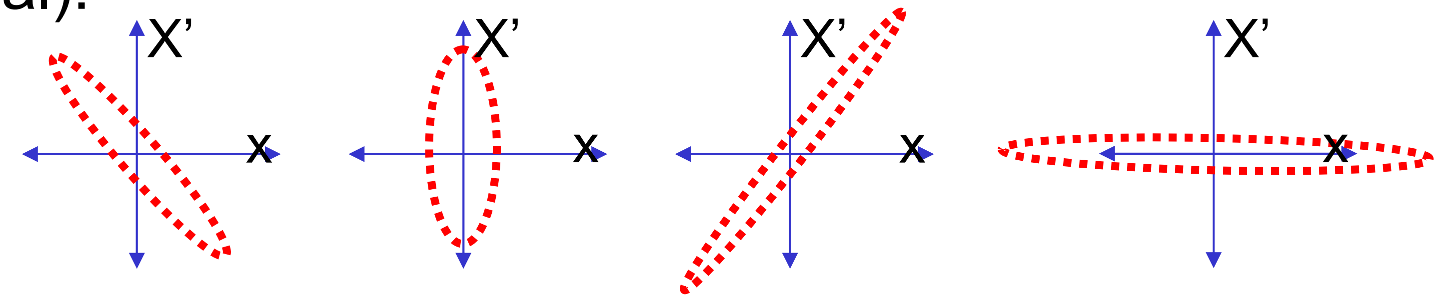
$$a'' = -\frac{1}{a^3}\left[\langle xx' \rangle^2 - \langle x'^2 \rangle \langle x^2 \rangle\right] - k^2 a,$$

$$a'' + k^2 a - \frac{\varepsilon_{RMS}^2}{a^3} = 0,$$

$$\varepsilon_{RMS} = \sqrt{\left(\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2\right)}$$

Emittance Growth (M. Reiser, Chap. 6)

Without acceleration, rms emittance of a mono-energetic beam is conserved under linear forces (*both* external and internal):



The normalized emittance is an invariant:

$$\varepsilon_{nrms} = \beta\gamma\varepsilon_{rms}$$

If initial transverse beam distribution is not uniform, rms emittance grows in a length of the order of $\lambda_p/4$.

If initial beam envelope is not rms-matched, rms emittance grows in a length of the order of λ_β.

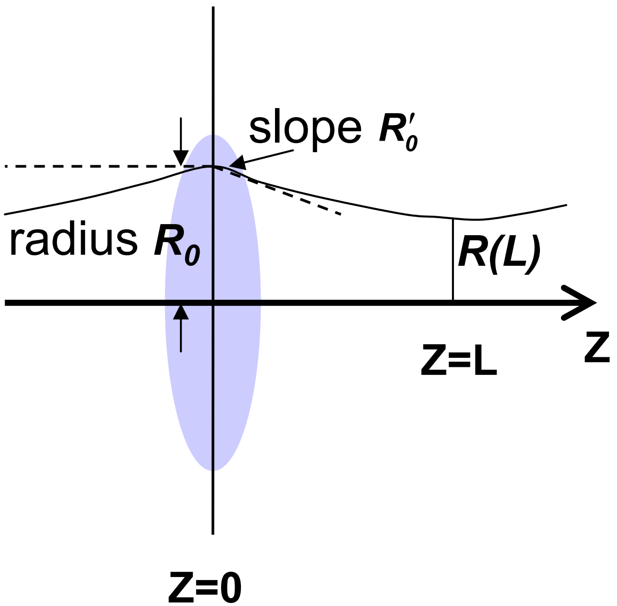
Laminar and Turbulent Flows

Transverse Emittance Measurement Techniques

Technique	Time Resolved ?	Pros	Cons
Basic Optics	No	Simple to implement. Linear space charge OK. Small or large beams.	RMS emittance only. No phase-space.
Pepper-pot	No	Simple to implement. Space charge OK. No beam optics.	Coarse phase-space. Impractical for small beams.
Quad Scan	No	Different schemes. Easy computation.	Space charge limits. No phase-space. Linear optics assumed.
Tomography	No	Detailed phase space info. Linear optics/sp. charge.	More beam manipulation required. Computationally intensive.
Slit-Wire	Yes	Phase space info. Integrated or t-resolved. Space charge OK.	S/N problems. Hard to implement. Comp. intensive.

Basic Optics I

By focusing a beam with a lens and measuring the width at a fixed distance $z=L$ downstream from the lens, one can determine the **emittance**.



Without space charge we have:

$$R(z) = \left[R_0^2 + 2R_0R_0'z + \left(\frac{\varepsilon^2}{R_0^2} + R_0'^2 \right) z^2 \right]^{1/2}.$$

We find:

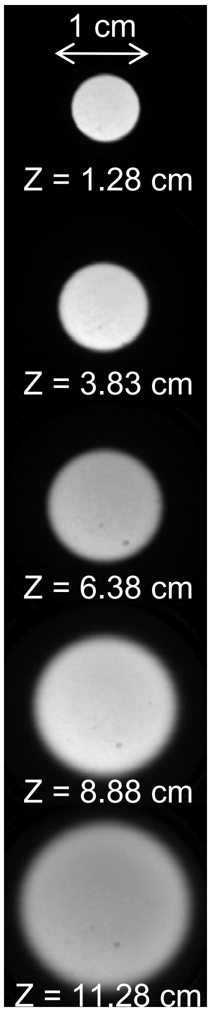
$$\varepsilon = \frac{R_0 R_{\min}}{L},$$

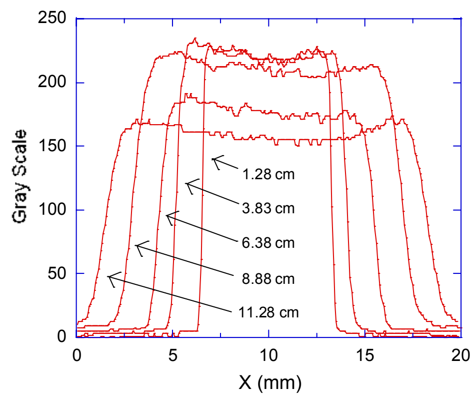
$$R'_{\min} = \frac{\varepsilon}{R_0} \neq 0.$$

Basic Optics II

IREAP

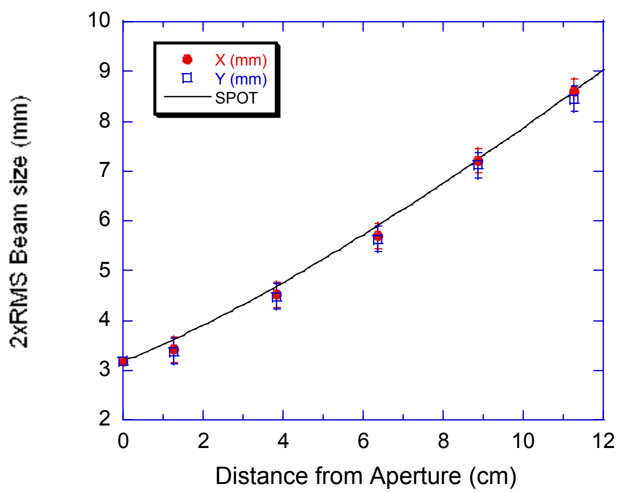
Beam free expansion with space charge: **emittance** can be determined as a fit parameter in envelope calculation.

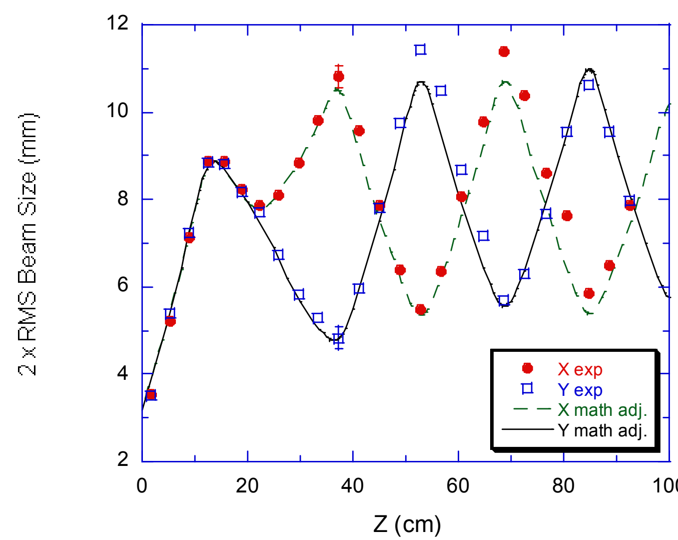




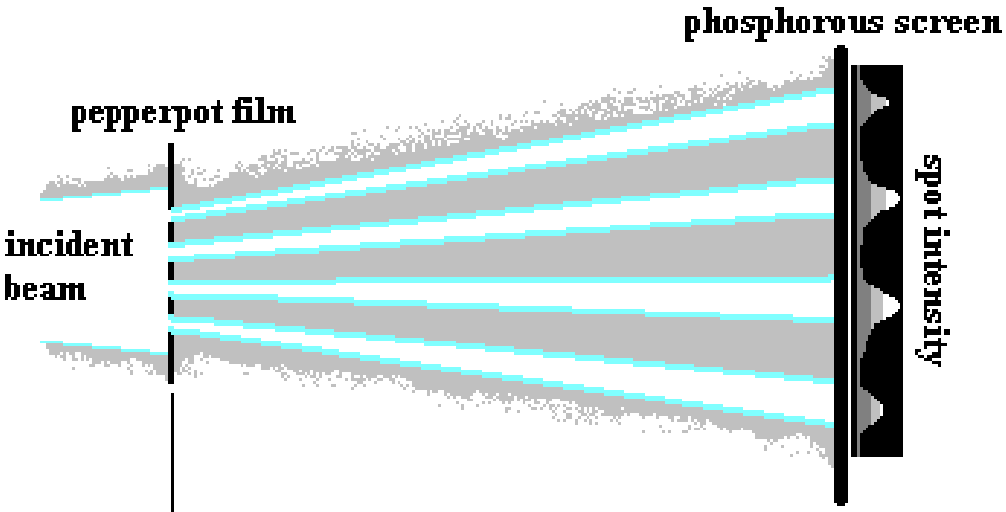
$$R''(z) - \frac{K}{R(z)} - \frac{\varepsilon^2}{R^3(z)} = 0, \qquad \varepsilon = 67\ \mu m.$$

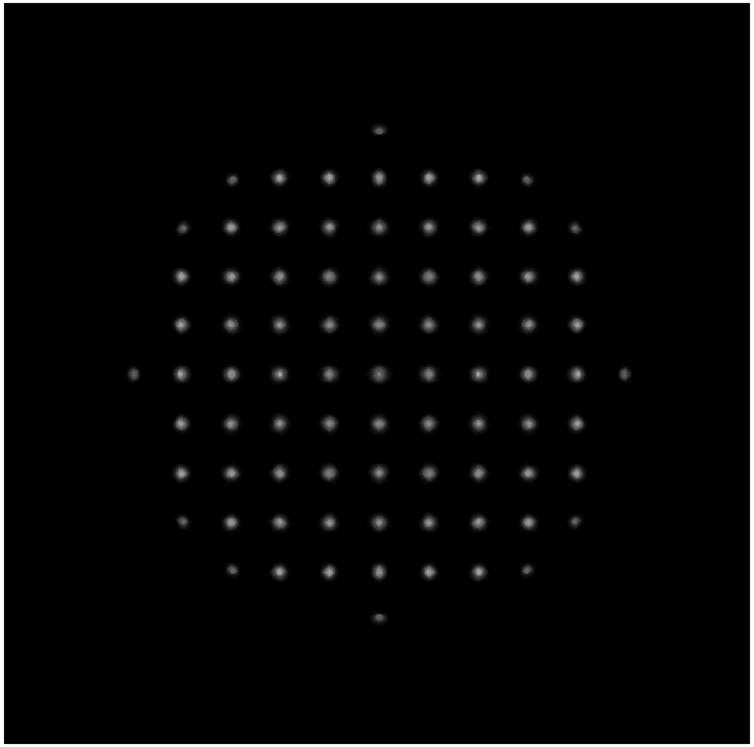
Additional beam optics checks possible with solenoid and quadrupole focusing.

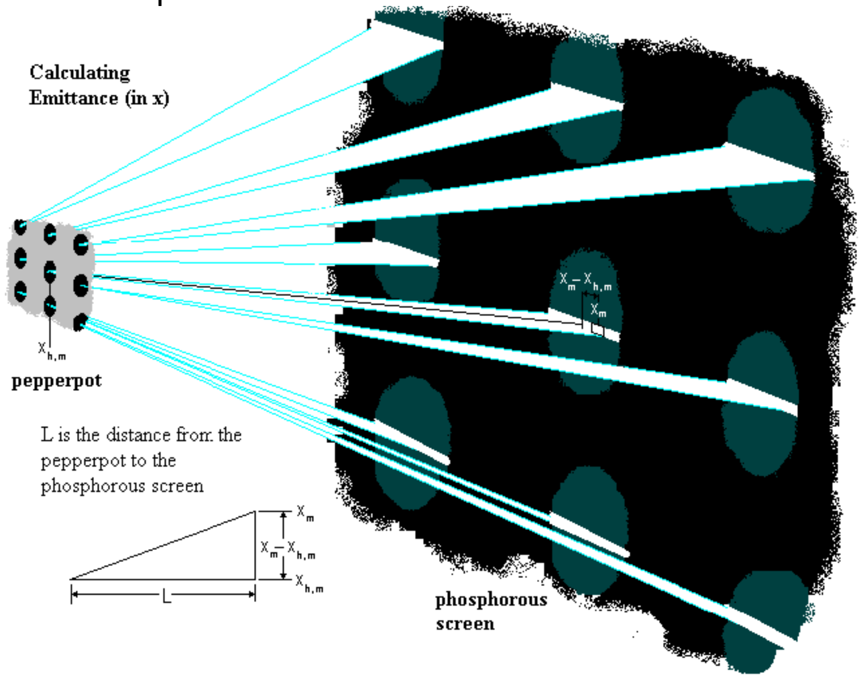




Pepper-Pot Technique

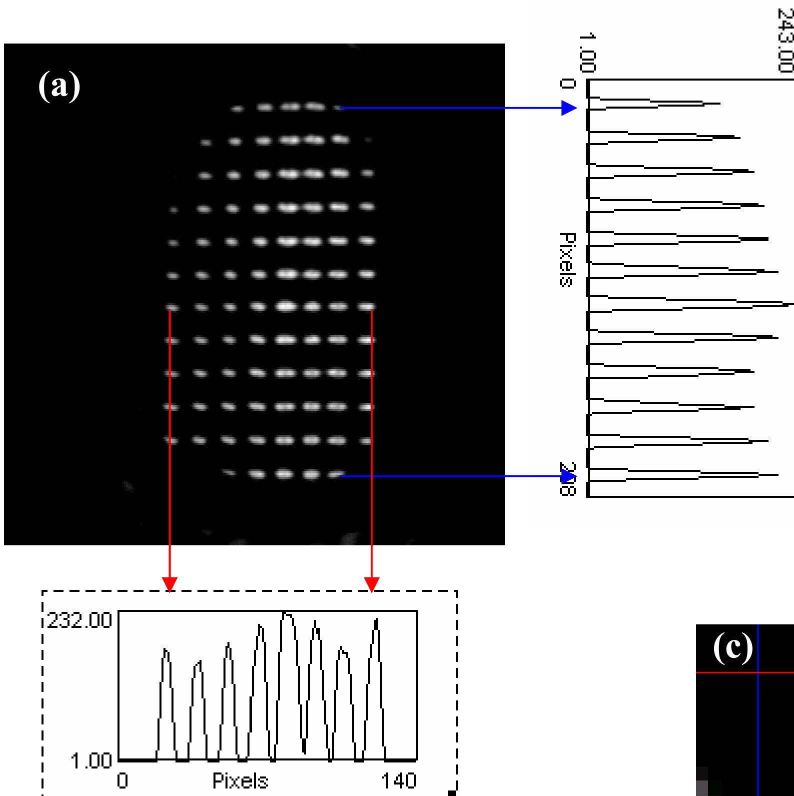


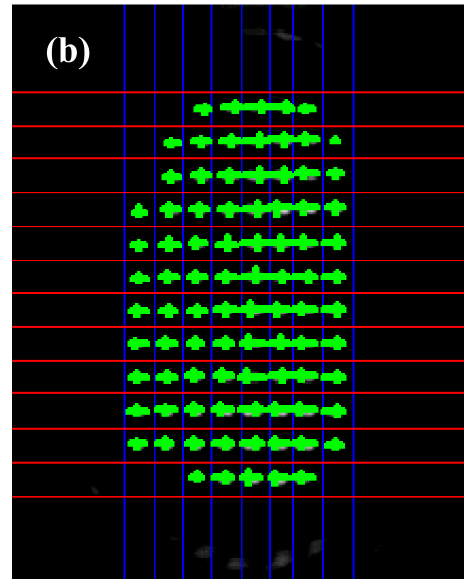




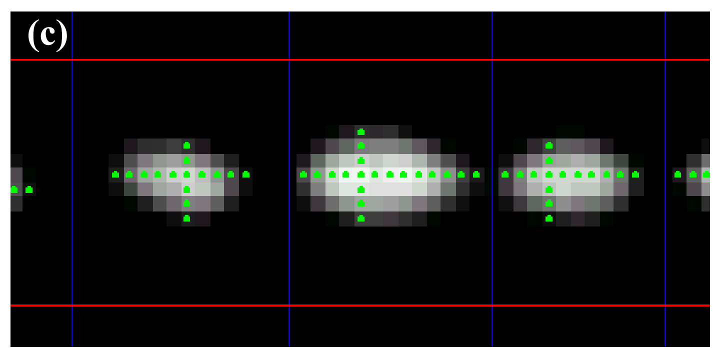
Pepper-Pot Technique

10keV, 100 mA, 70^0 experiment





0.166 mm/pixel



ε_X = 67 μm, ε_Y = 55 μm

Pepper-Pot Technique

IREAP

Calculating emittance and plotting phase space:

$$\langle x \rangle = (1 / I_{tot}) \, \Sigma_m [x_m I_m] \qquad \langle x' \rangle = (1 / I_{tot}) \, \Sigma_m [x'_m I_m]$$

$$\langle x^2 \rangle = (1 / I_{tot}) \, \Sigma_m [(x_m - \langle x \rangle)^2 \, I_m] \qquad \langle x'^2 \rangle = (1 / I_{tot}) \, \Sigma_m [(x'_m - \langle x' \rangle)^2 \, I_m]$$

$$\langle xx' \rangle = (1 / I_{tot}) \, \Sigma_m [(x_m - \langle x \rangle)(x'_m - \langle x' \rangle) \, I_m]$$

The meaning of emittance used for this system is the 4*RMS emittance, defined implicitly (along with normalized emittance) by:

$$\varepsilon^2_{rms} = \langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2 \qquad \varepsilon_{norm} = 4\gamma\beta\varepsilon_{rms}.$$

Phase-space plots are made (on four separate axes) of the following four sets:

$$\{(x_m, y_m)\}_m \qquad \{(x_m, x'_m)\}_m \qquad \{(y_m, y'_m)\}_m \qquad \{(x'_m, y'_m)\}_m.$$

Beam (σ) and Transfer (R) Matrices

Define the beam matrix for, say, *x*-plane:

$$\sigma = \varepsilon \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

So beam ellipse $\gamma x^2 - 2\alpha x x' + \beta x'^2 = \varepsilon$ can be written as

$$x^T \sigma^{-1} x = 1, \quad x^T \equiv [x, x']$$

The condition $\beta\gamma - \alpha^2 = 1$ is written $\det \sigma = \varepsilon$

Consider 2 planes z_0 and z_1, $z_1 > z_0$. Particle coordinates transform as:

$$x_1 = R x_0 \text{ or } x_0 = R^{-1} x_1$$

R is the Transfer matrix.

Then, the beam matrix is transformed according to:

$$\sigma_1 = R \sigma_0 R^T$$

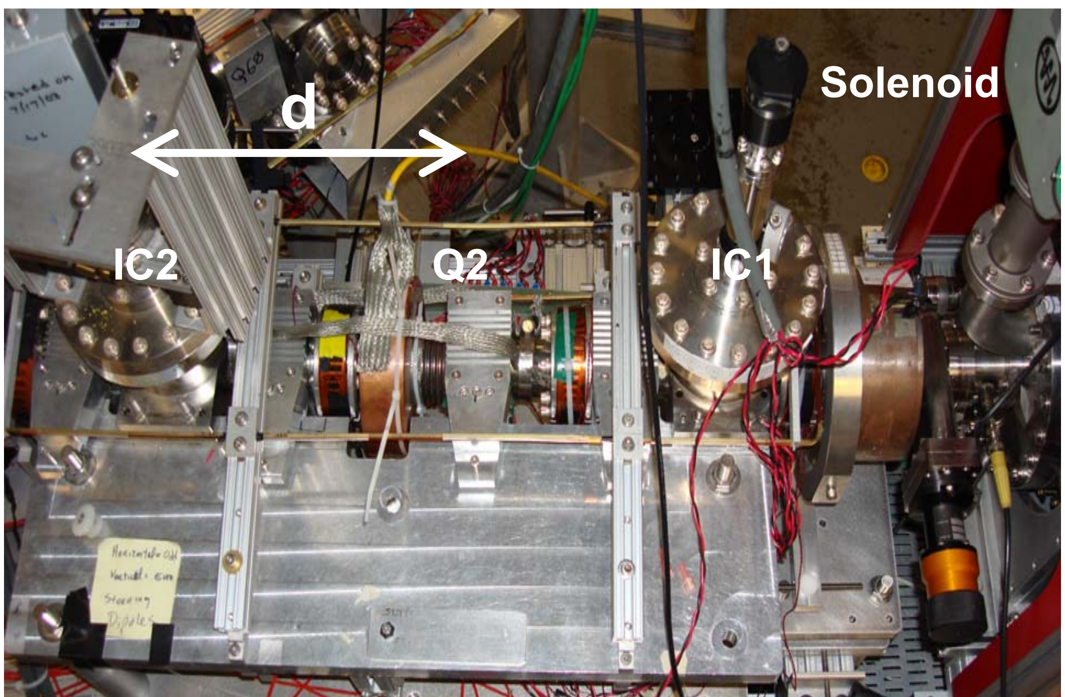
Quadrupole Scan

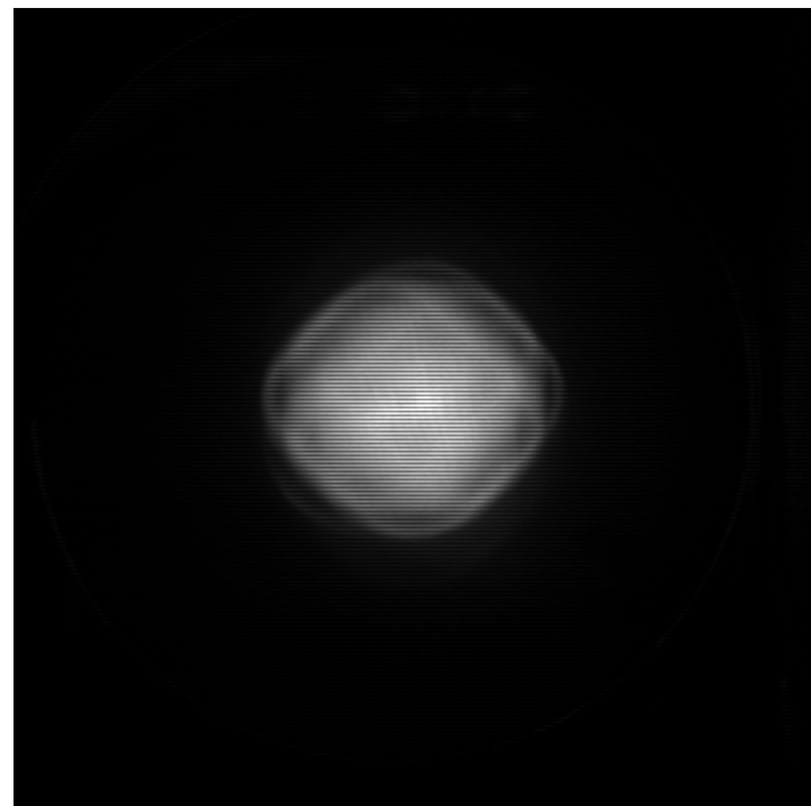
$$\langle x^2 \rangle = A\left(\frac{1}{f^2}\right) - 2AB\left(\frac{1}{f}\right) + (C + AB^2),$$
$$= m_1 I_{quad}^2 - 2m_1 m_2 I_{quad} + (m_3 + m_1 m_2^2),$$

$$\varepsilon = \frac{\sqrt{AC}}{d^2},$$

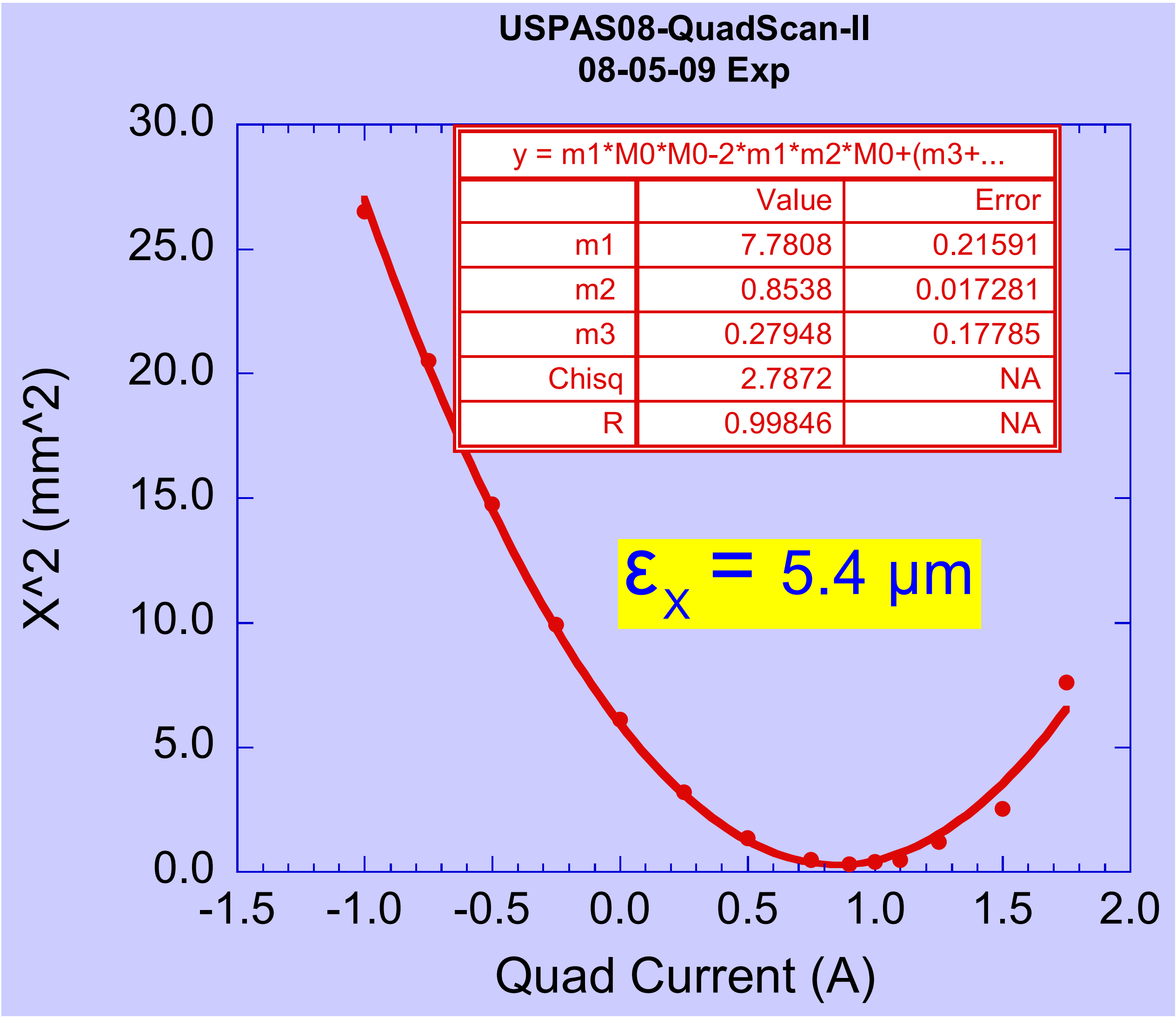
$$\frac{1}{f} = \kappa l_{eff} = \frac{q g_{he} l_{he}}{\gamma m_e \beta c} I_q.$$

$$\alpha = \sqrt{\frac{A}{C}}\left(B + \frac{1}{d}\right), \quad \beta = \sqrt{\frac{A}{C}}, \quad \gamma = \frac{1+\alpha^2}{\beta}.$$





Quadrupole Scan: results for 6 mA beam



References

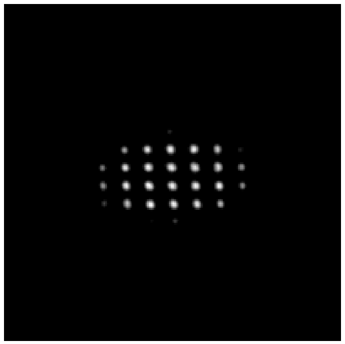
1. Martin Reiser, *Theory and Design of Charged-Particle Beams*, 2nd Ed, Secs. 3.8.2, 5.3.4 (see Table 5.1) and Chapter 6.
2. D. A. Edwards and M. J. Syphers, *An Introduction to the Physics of High Energy Accelerators,* Sec. 3.2.4 (see Table 3.1).
3. Claude Lejeune and Jean Aubert, *Emittance and Brightness: Definitions and Measurements, Advances in Electronics and Electron Physics, Supplement 13A*, Academic Press (1980).
4. Andrew M. Sessler, Am. J. Phys. 60 (8), 760 (1992); J. D. Lawson, "The Emittance Concept", in High Brightness Beams for Accelerator Applications, AIP Conf. Proc. No. 253; M.J. Rhee, Phys. Fluids 29 (1986).
5. Michiko G. Minty and Frank Zimmermann, *Beam Techniques, Beam Control and Manipulation*, USPAS, 1999.
6. S. G. Anderson, J. B. Rosenzweig, G. P. LeSage, and J. K. Crane, "Space-charge effects in high brightness electron beam emittance measurements", PRST-AB, **5**, 014201 (2002).
7. Manuals to computer codes like TRACE3D, TRANSPORT, etc. .

Backup Slides

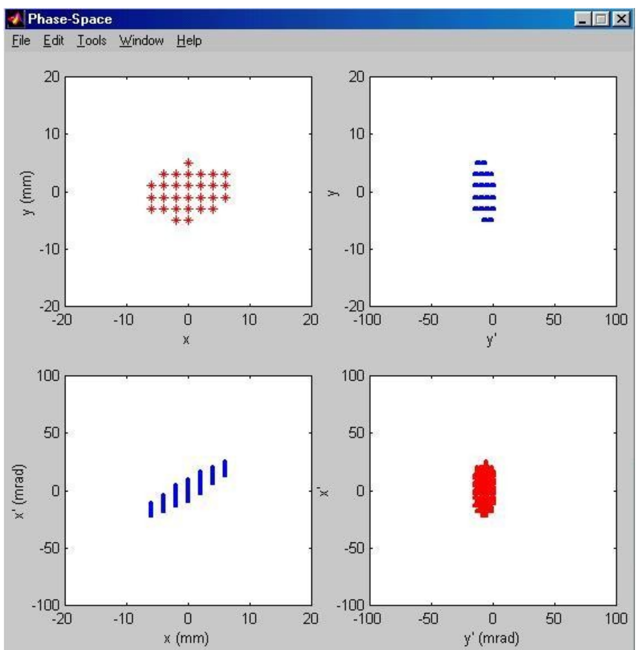
Detectors

Type	Characteristics
Phosphor Screen	**Example: P43 phosphor (green output). Fairly linear. Inexpensive but delicate. Slow time response (up to ms tails.) Used in combination with CCD or MCP/CCD.**
Fast Phosphor Screen	**Example: P22 phosphor (blue output). Fairly linear. Delicate and expensive. Response time in the ns range. Used in combination with CCD or MCP/CCD.**
OTR Screen	**Any mirrored surface. OTR maps beam in real time (fs response) with high spatial resolution. Good for any beam energy. Used in combination with PMT or CCD cameras. Low current beams require long int. times.**
Standard CCD	**Standard, inexpensive (8-bit.) Can be intensified for extra sensitivity. Slow response.** **Cooled CCD higher sensitivity + 16-bit.**
Gated Camera	**0.1 ns time response possible. Expensive.** **Streak cameras: pico-second time resolution.**

Emittance (rms. Norm.) in DC Injection Exps.



24 mA, 10 keV, χ=0.9



Location	S (m)	ε_X (μm)	ε_Y (μm)
Aperture Plate	0	1.50±0.25	1.5±0.25
After 1/4 turn	3.8	1.50	1.80
After 1/2 turn	7.0	2.10	1.40
After 2/3 turn	9.0	1.60	2.50