Emittance Measurements

Accelerator Beam Diagnostics
Uli Raich (CERN)
USPAS and University of New Mexico
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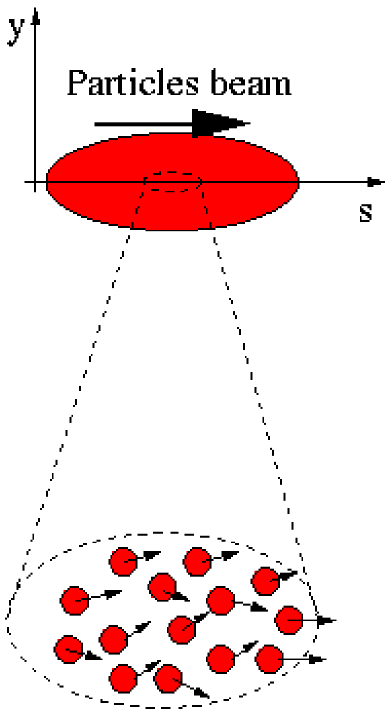
What is "Emittance" ?



A beam is made of many, many particles, each one of these particles is moving with a given velocity. Most of the velocity vector of a single particle is parallel to the direction of the beam as a whole (s). There is however a smaller component of the particles velocity which is perpendicular to it (x or y).

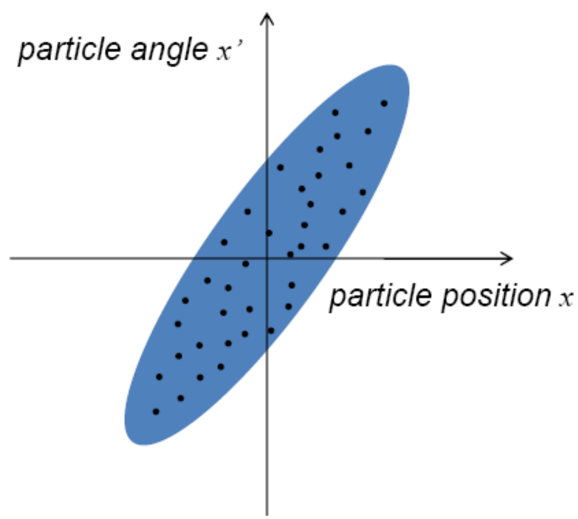
$$\vec{v}_{particle} = v_s \hat{u}_s + v_x \hat{u}_x + v_y \hat{u}_y$$

Transverse Phase Space

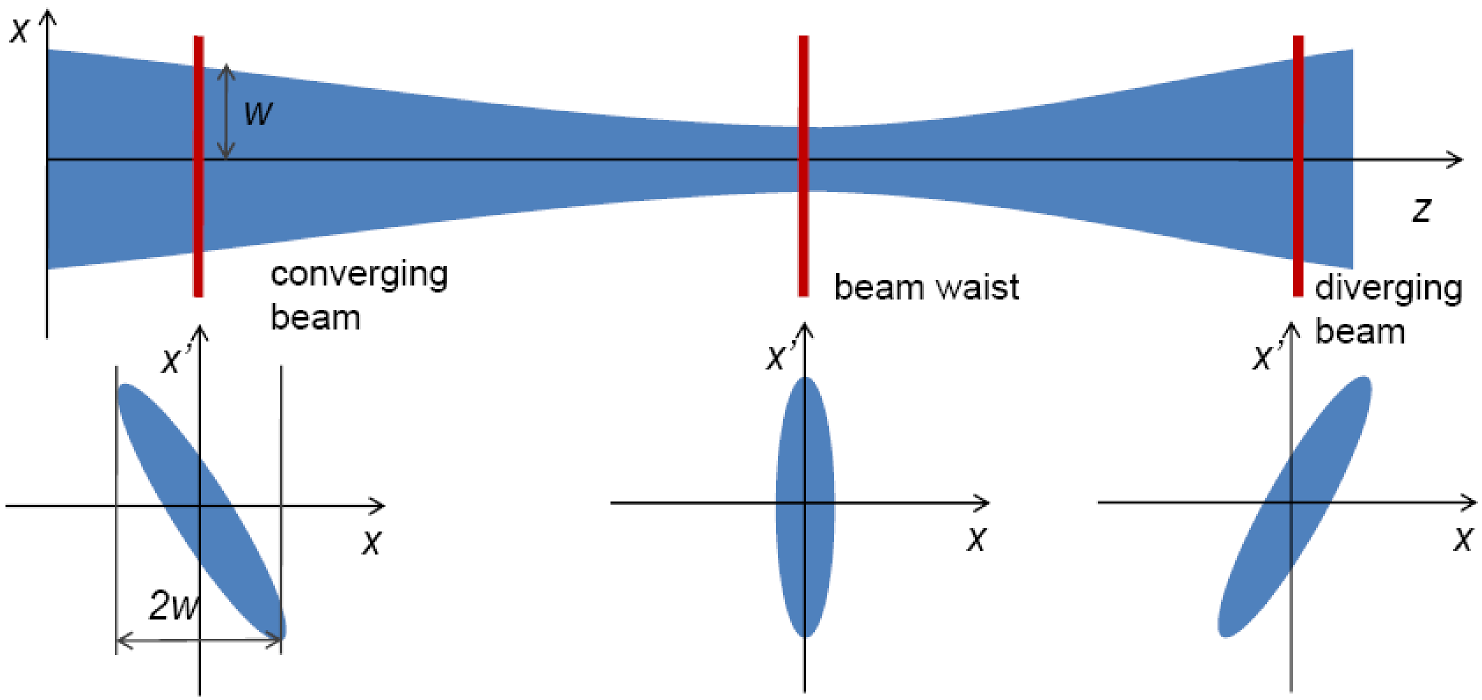
- The emittance describes the beam quality, assuming linear behavior due to second order differential equation.
- It is defined as the area in phase space including the particles (generally an ellipse).

$$\varepsilon = \frac{1}{\pi}\iint_A dx dx'$$

- The measurements are base on beam width and angular width measurements at a single location or multip ... ʌith additional optics calculations.



Variation of the ellipse along the transport line



Along a beamline the orientation and aspect ratio of beam ellipse in x, x' plane varies, but area $\pi\varepsilon$ remains constant

Beam width along z is described with $w(z) = \sqrt{\beta(z)\,\varepsilon}$

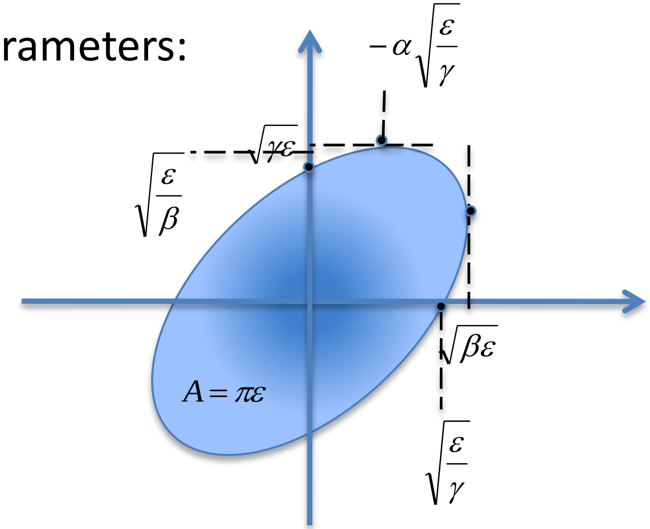
Ellipse Parameters

Beam ellipse and its orientation is defined

by the beam matrix $\begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix}$ for which the emittance is $\varepsilon = \sqrt{\sigma_{11}\sigma_{12} - \sigma_{12}^2}$

which is related to the Twiss or Courant-Snyder parameters:

$$\sigma = \varepsilon \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$



The equation of the beam ellipse is:

$$\varepsilon = \gamma x^2 + 2\alpha x x' + \beta x'^2 \qquad \gamma = \frac{1+\alpha^2}{\beta}$$

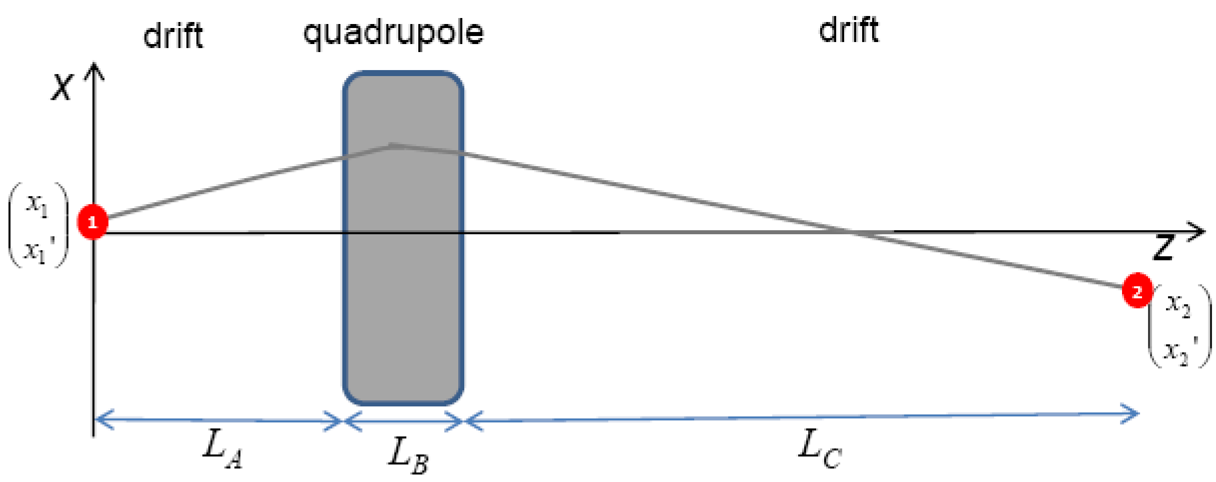
$\sqrt{\beta\varepsilon}$ beam half width
$\sqrt{\gamma\varepsilon}$ beam half divergence
α correlation between x and x’

α> 0 : beam is converging
α < 0 : beam is diverging
α = 0 : beam has minimum or maximum

6-dimensional Phase Space

- Transverse phase space:
  - x,x’ (x-position, angle in horizontal plane)
  - y,y’ (y-positon, angle in vertical plane)
- Longitudinal phase space
  - E, Φ (Energy and phase or time of arrival)

Transport of a single particle along a transfer line



$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = M \cdot \begin{pmatrix} x_1 \\ x_1' \end{pmatrix} = M_C \cdot M_B \cdot M_A \cdot \begin{pmatrix} x_1 \\ x_1' \end{pmatrix} = \begin{pmatrix} 1 & L_C \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\left(\sqrt{k} L_B\right) & 1/\sqrt{k} \sin\left(\sqrt{k} L_B\right) \\ -\sqrt{k} \sin\left(\sqrt{k} L_B\right) & \cos\left(\sqrt{k} L_B\right) \end{pmatrix} \cdot \begin{pmatrix} 1 & L_A \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_1' \end{pmatrix}$$

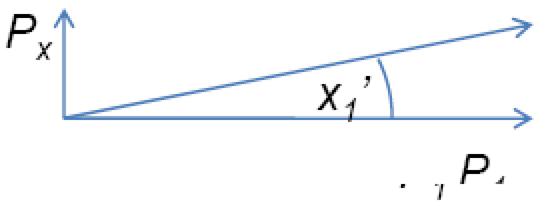
$$M_{Drift} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \qquad M_{Quadrupole} = \begin{pmatrix} \cos\left(\sqrt{k} L\right) & 1/\sqrt{k} \sin\left(\sqrt{k} L\right) \\ -\sqrt{k} \sin\left(\sqrt{k} L\right) & \cos\left(\sqrt{k} L\right) \end{pmatrix}$$

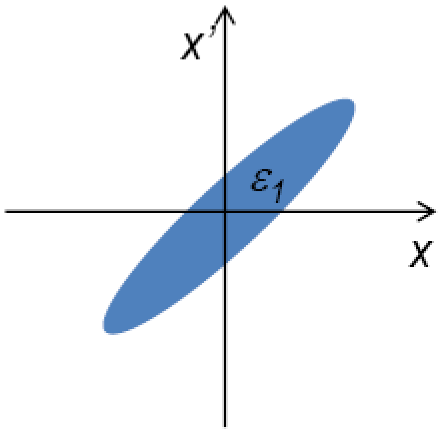
Thin lens approximation:

$$M_{quadrupole} = \begin{pmatrix} 1 & 0 \\ K & 1 \end{pmatrix}$$

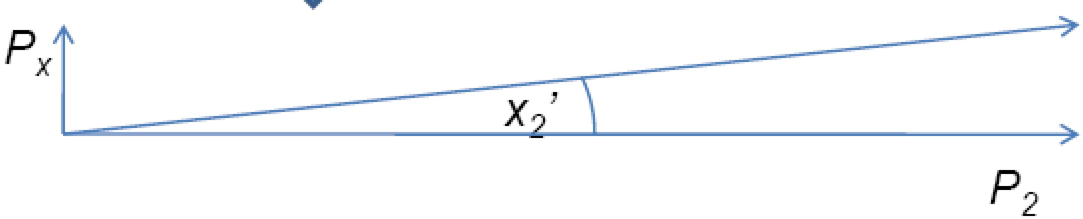
generic names of matrix elements $\quad M = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix}$

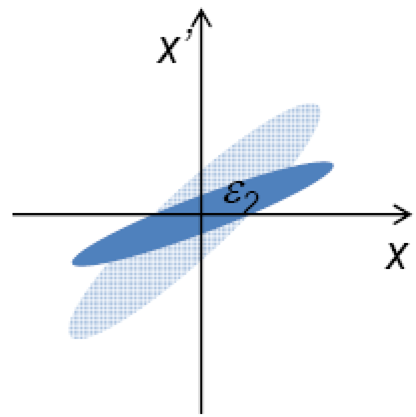
Adiabatic damping





acceleration $\quad \varepsilon_2 = \varepsilon_1 \dfrac{P_1}{P_2}$ with $P = m\beta_{rel}\gamma_{rel}\ c$





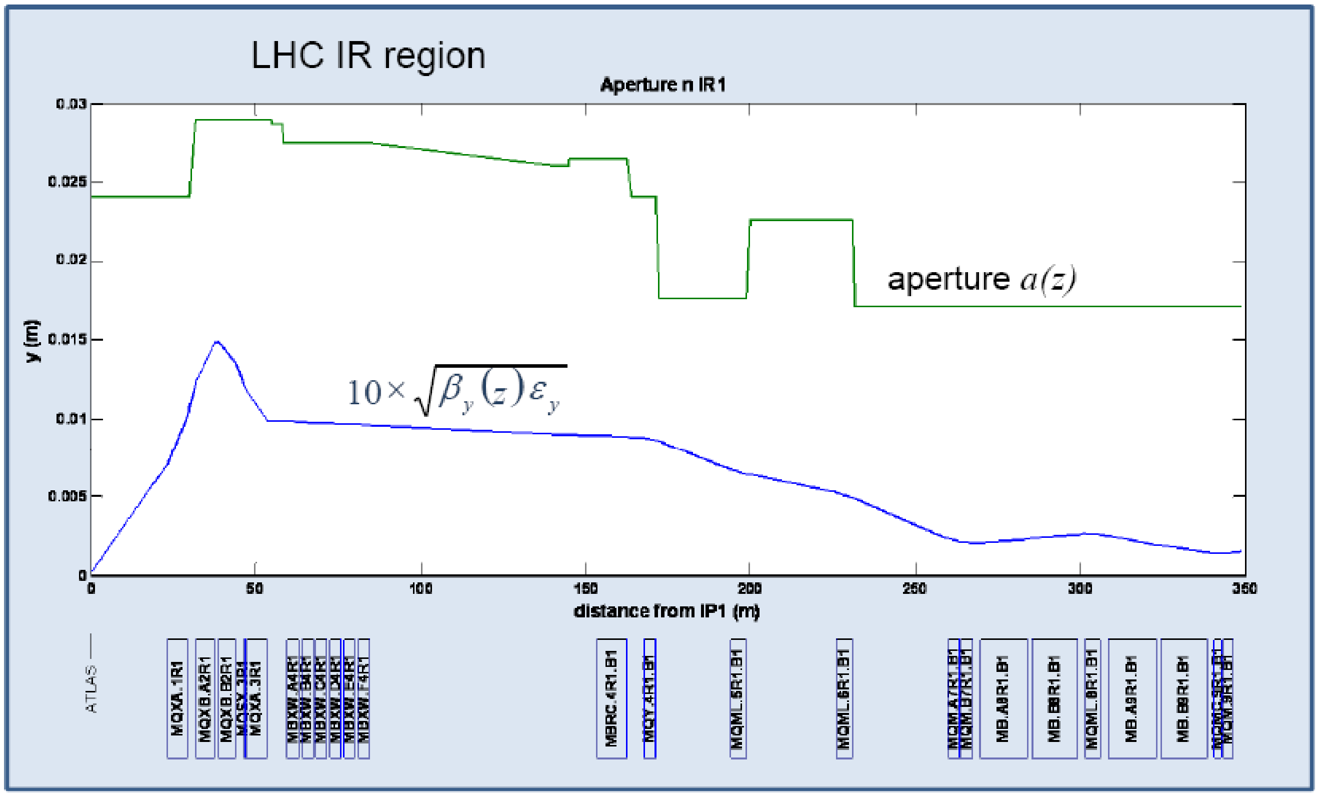
<u>normalised emittance</u> $\varepsilon_N = \beta_{rel}\gamma_{rel}\ \varepsilon$ preserved with acceleration !

To distinguish from normalised emittance ε_N , ε is quoted as "geometric emittance" !

Why measure Emittance?

The emittance tells if a beam fits in the vacuum chamber or not

$$w(z) = \sqrt{\beta(z)\,\varepsilon} \quad < \quad a(z)$$



Why measure Emittance?

Emittance is one of key parameters for overall performance of an accelerator

- *Luminosity of colliders for particle physics*
- *Brightness of synchrotron radiation sources*
- *Wavelength range of free electron lasers*
- *Resolution of fixed target experiments*

Therefore emittance measurement is essential to guide tune-up of accelerator !

How to measure Emittance?

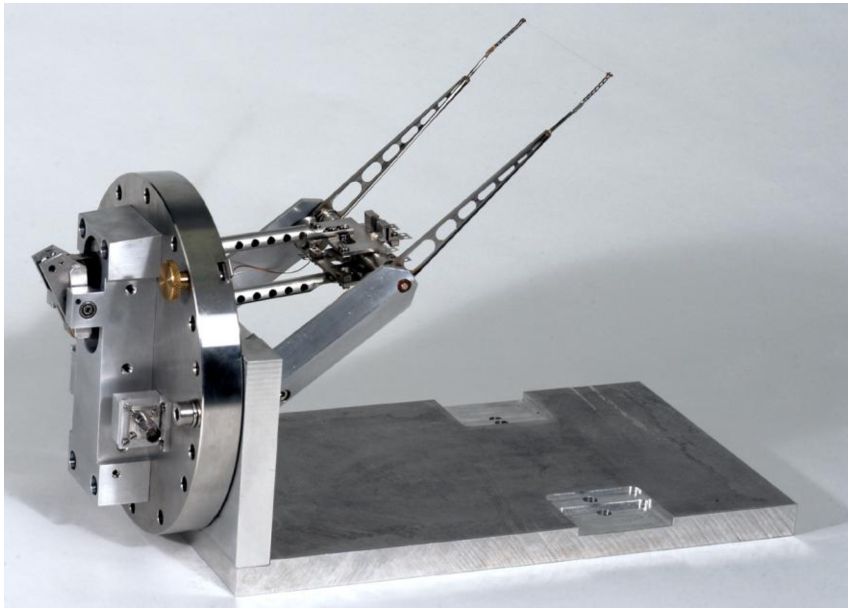
In a Synchrotron the lattice functions are fixed, beam width and emittance are related:

$$\varepsilon_x = \frac{1}{\beta_x(s)}\left[\sigma_x^2 - \left(D(s)\frac{\Delta p}{p}\right)^2\right] \quad \text{and} \quad \varepsilon_y = \frac{\sigma_y^2}{\beta_y(s)}$$

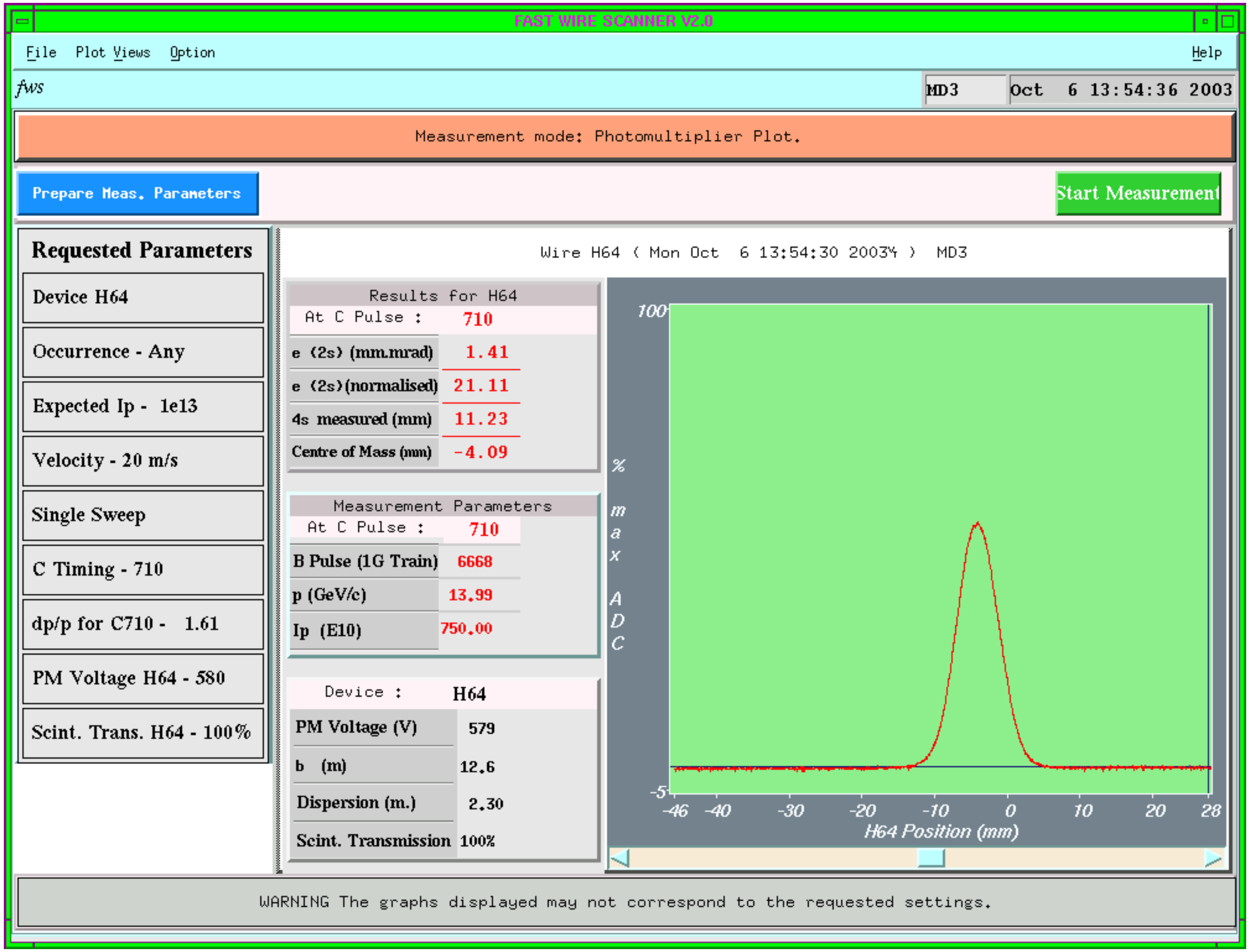
The β function and the Dispersion function D are known or measured with other means

Wire Scanners

A thin wire is quickly moved across the beam
Secondary particle shower is detected outside the vacuum chamber on a scintillator/photo-multiplier assembly
Position and photo-multiplier signal are recorded simultaneously



Wire scanner profile



High speed needed because of heating.

Adiabatic damping

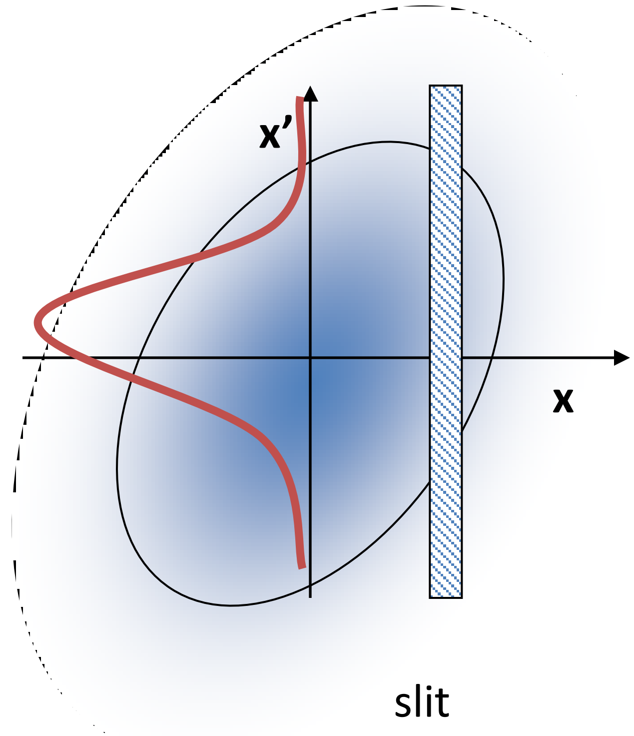
Current increases due to speed increase

Speeds of up to 20m/s $\Rightarrow$200g acceleration

Measure secondary particles outside the vacuum chamber or secondary emission

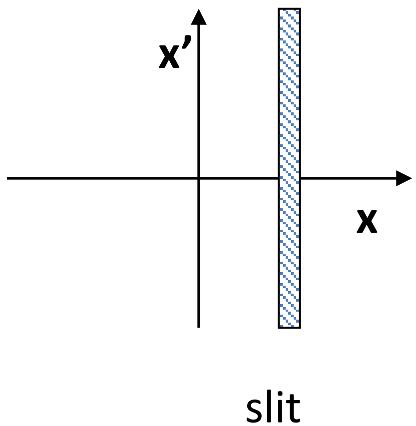
The Slit and Grid method

- If we place a slit into the beam we cut out a small vertical slice of phase space
- Converting the angles into position through a drift space allows to reconstruct the angular distribution at the position defined by the slit

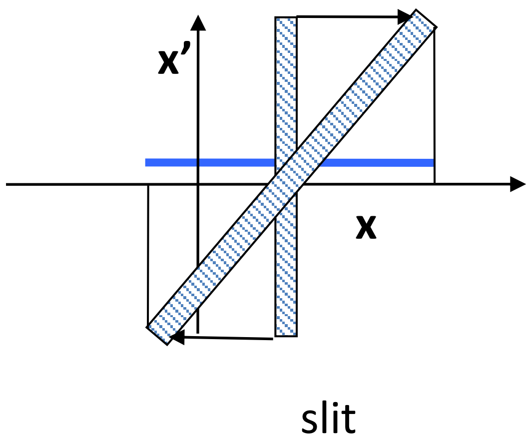


Transforming angular distribution to profile

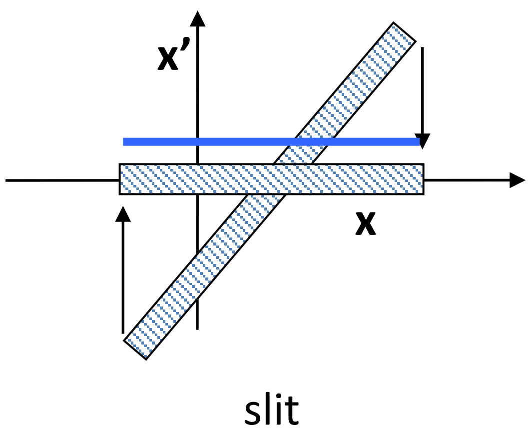
- When moving through a drift space the angles don't change (horizontal move in phase space)
- When moving through a quadrupole the position does not change but the angle does (vertical move in phase space)



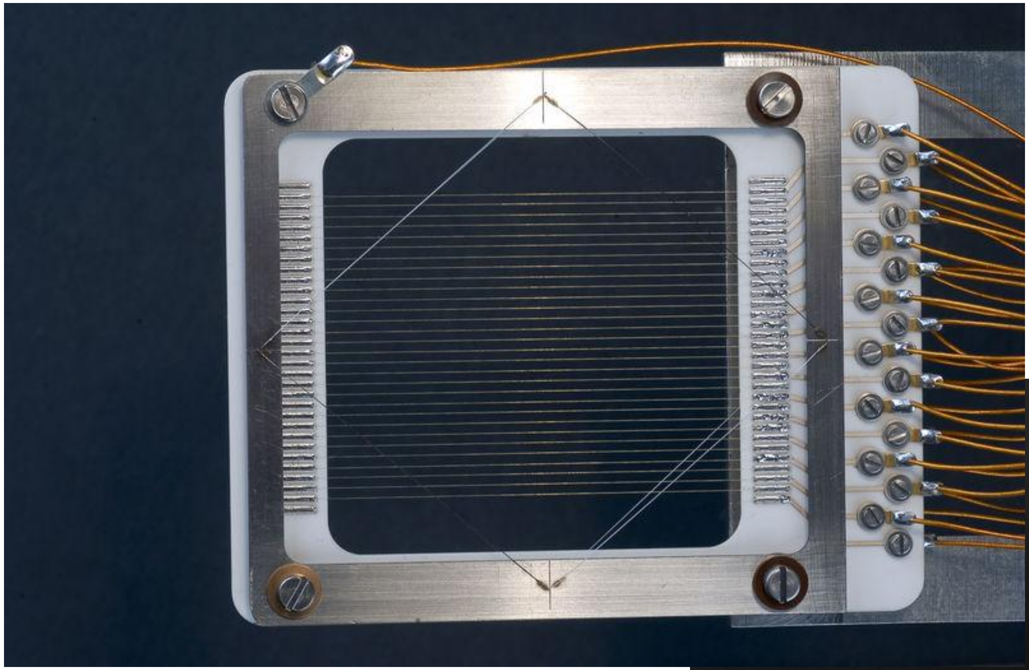
Influence of a drift space

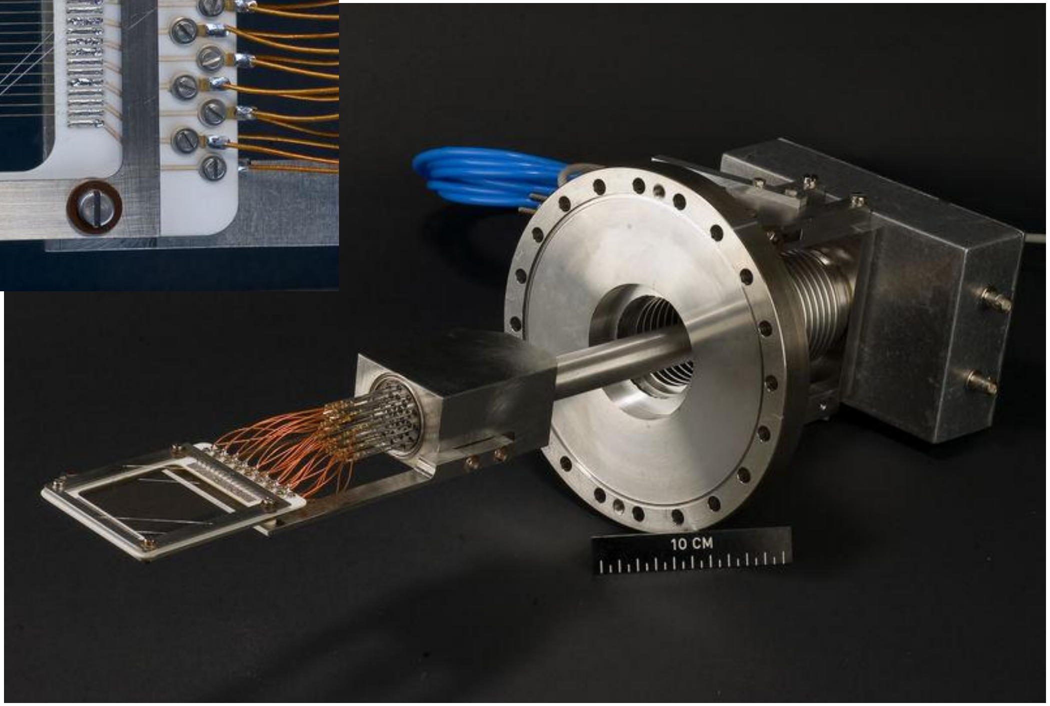


Influence of a quadrupole

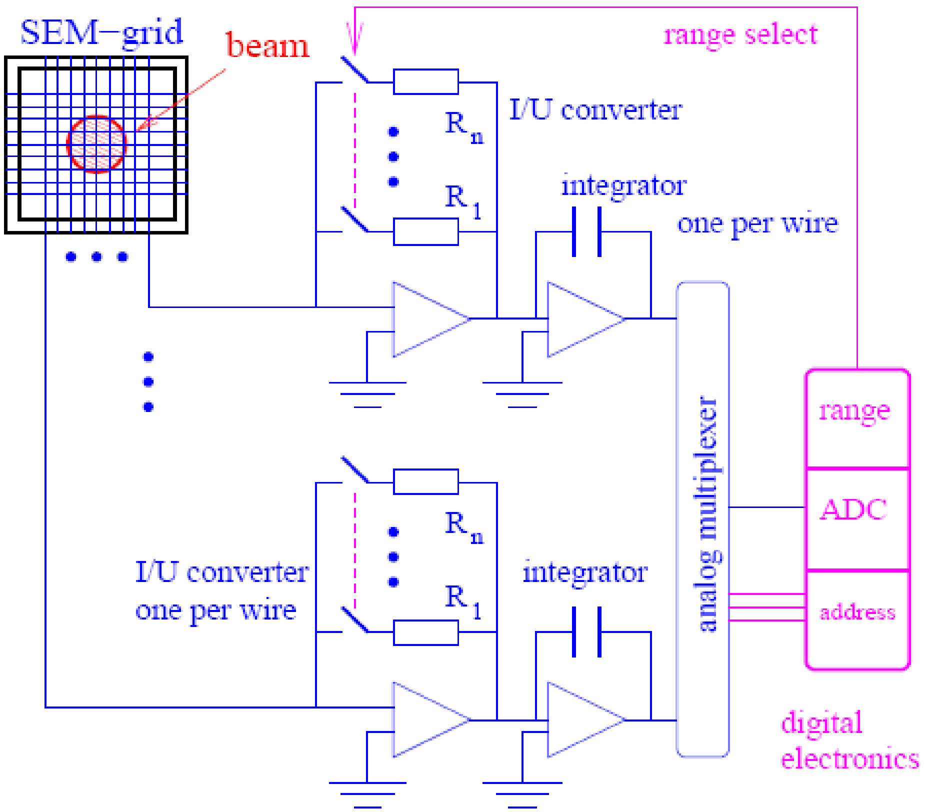


Secondary Emission Grids



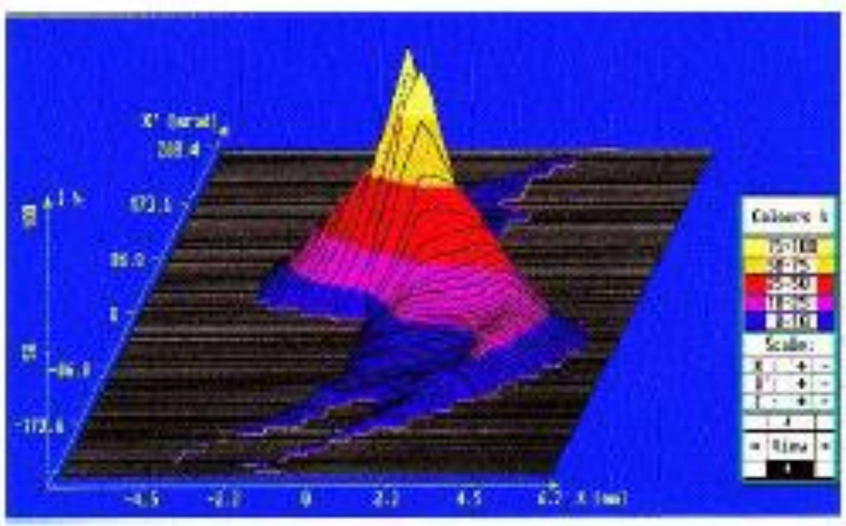


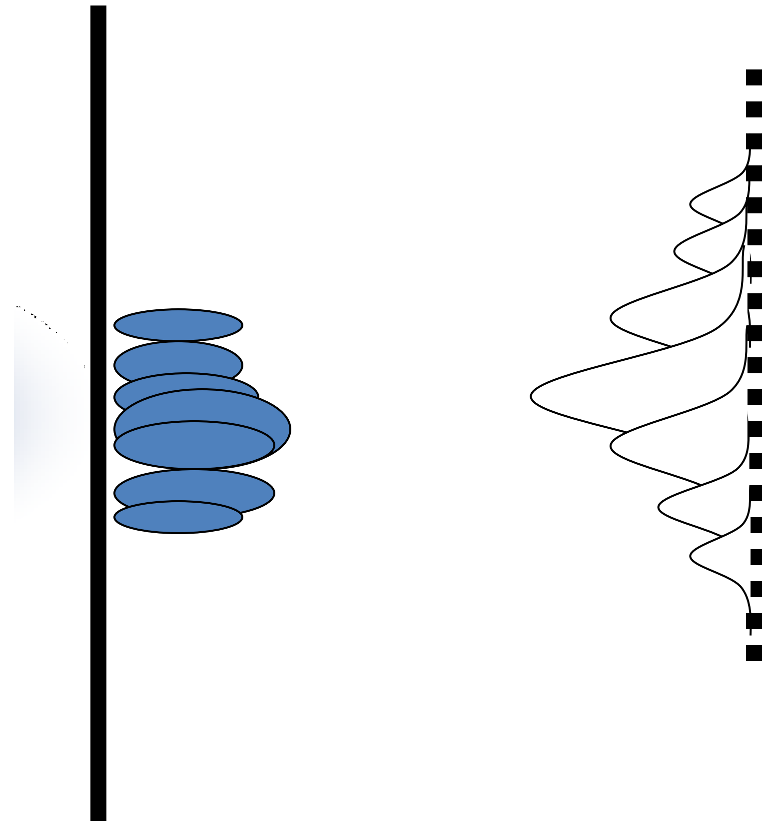
SEMGrid electronics



The Slit and Grid Method

3-dim plot:

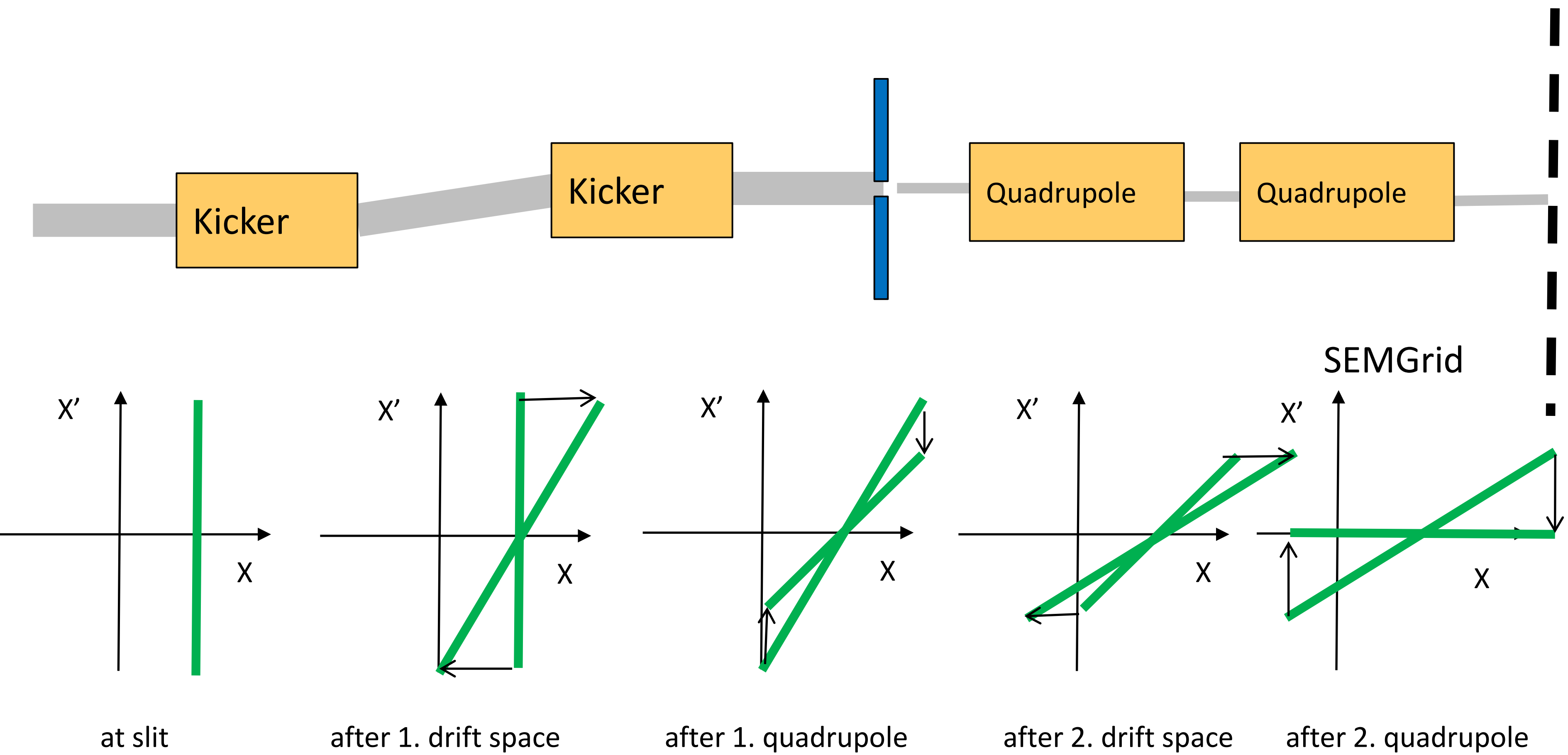




Moving slit emittance measurement

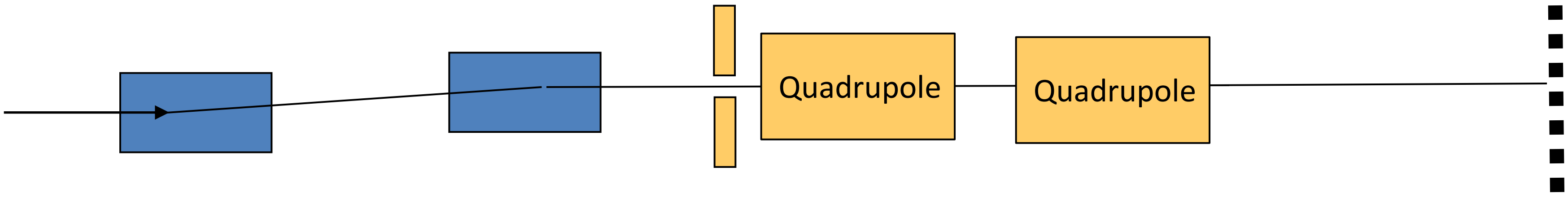
- Position resolution given by slit size and displacement
- Angle resolution depends on resolution of profile measurement device and drift distance
- High position resolution → many slit positions → slow
- Shot to shot differences result in measurement errors

Transverse emittance line



Single pulse emittance measurement

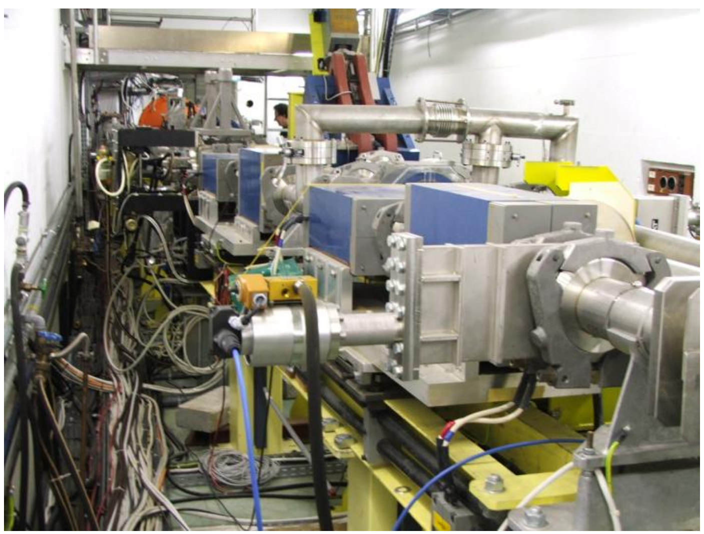
Every 100 ns
a new profile

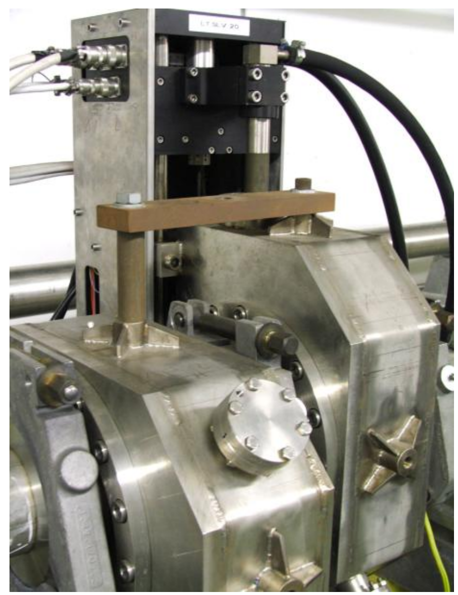


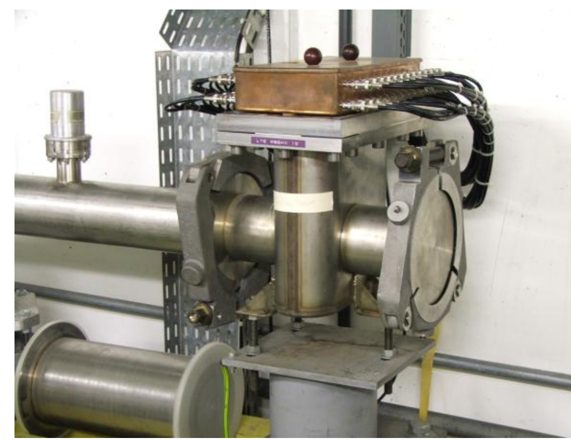
slit

Kickers

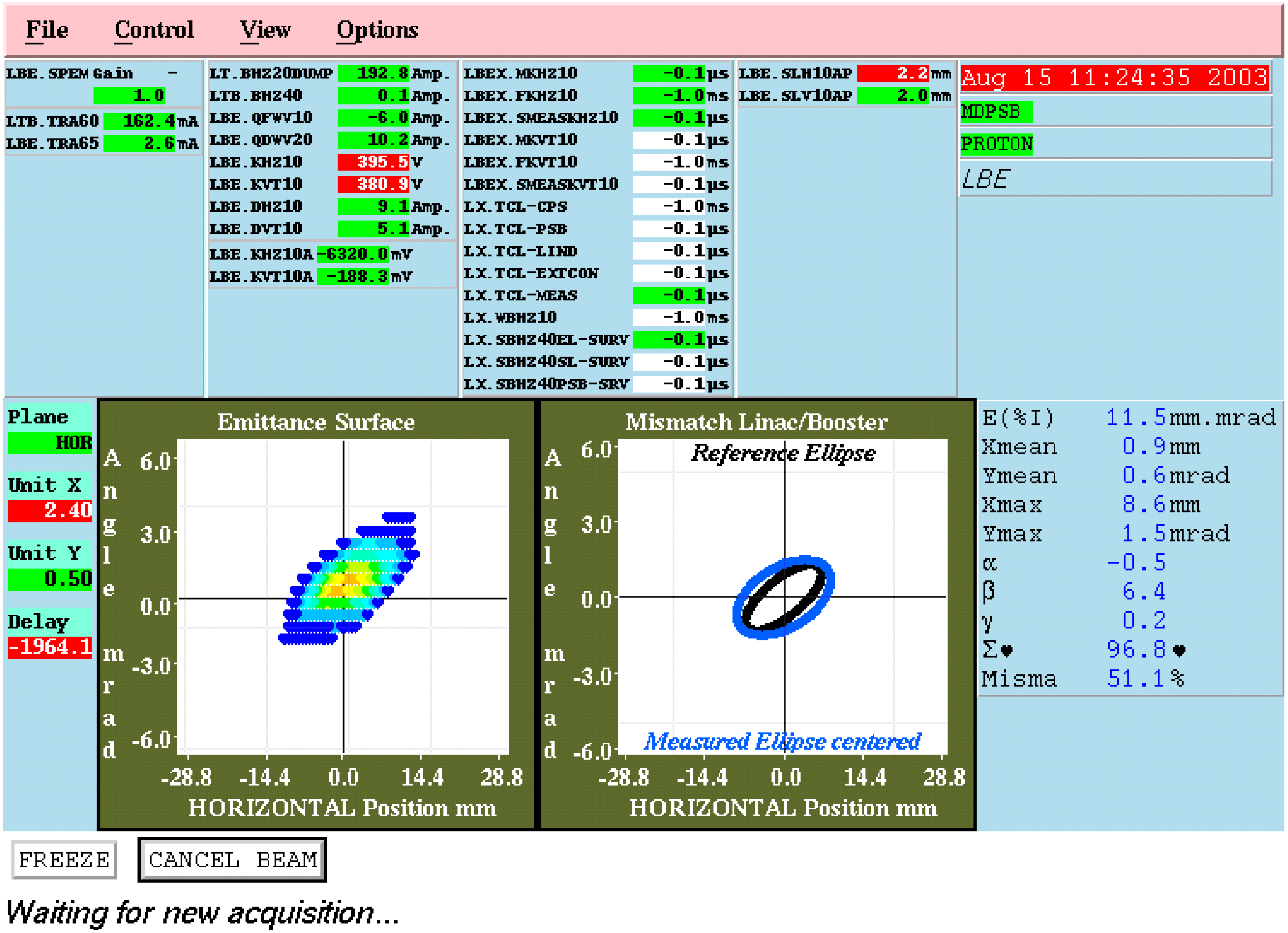
SEMgrid







Result of single pulse emittance measurement

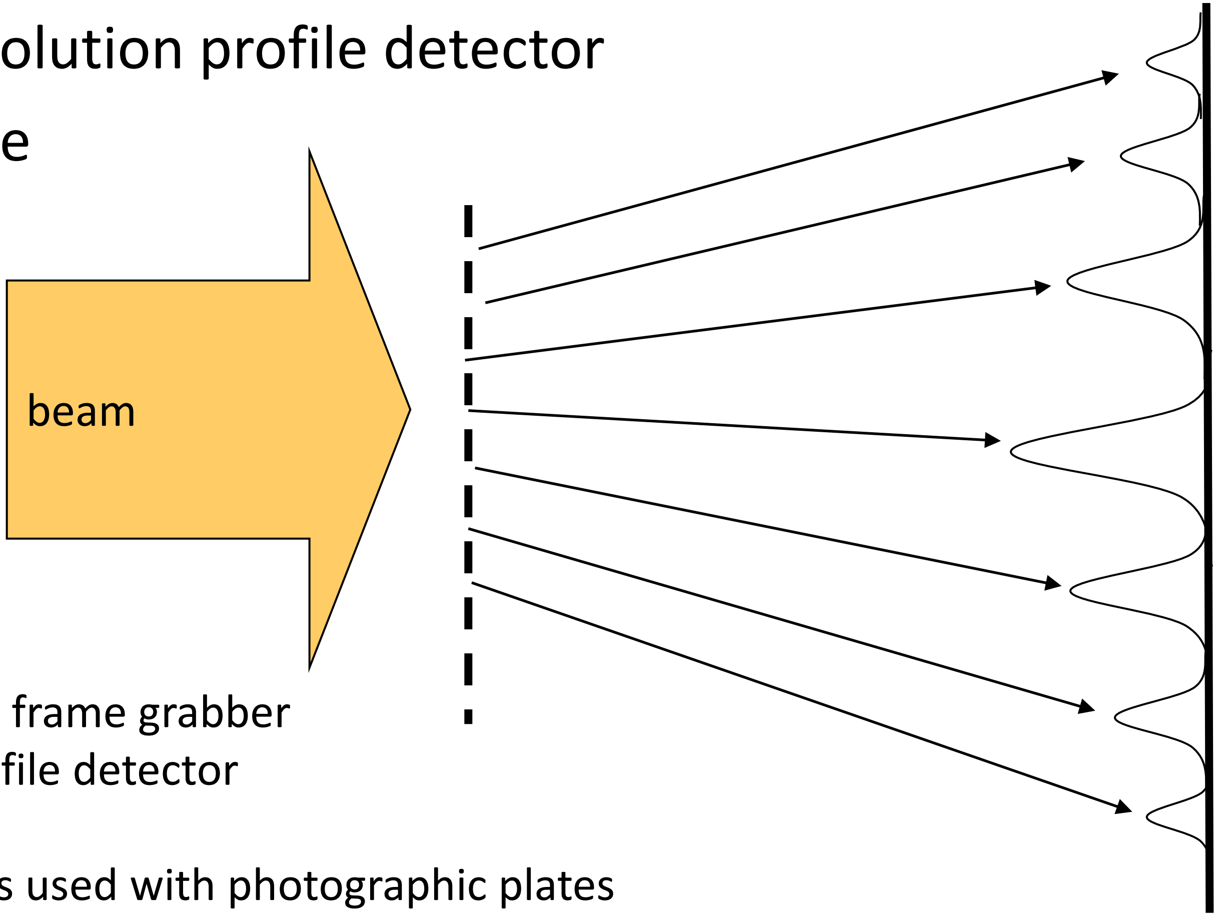


Single Shot Emittance Measurement

- Advantage:
  - Full scan takes 20 μs
  - Shot by shot comparison possible
- Disadvantage:
  - Very costly
  - Needs dedicated measurement line
  - Needs a fast sampling ADC + memory for each wire
- Cheaper alternative:
  - Multi-slit measurement

Multi-slit measurement

- Needs high resolution profile detector
- Must make sure that profiles don't overlap



Scintillator + TV + frame grabber often used as profile detector

Very old idea, was used with photographic plates

Pepperpot

Uses small holes instead of slits

Measures horizontal and vertical emittance in a single shot

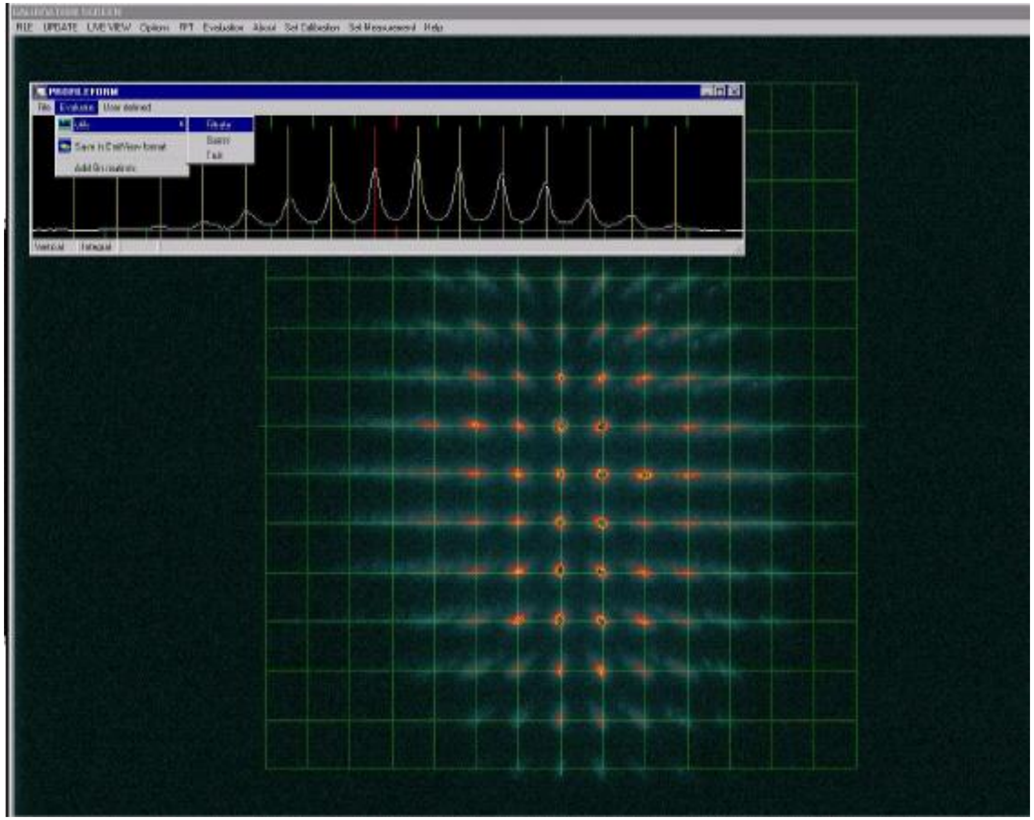
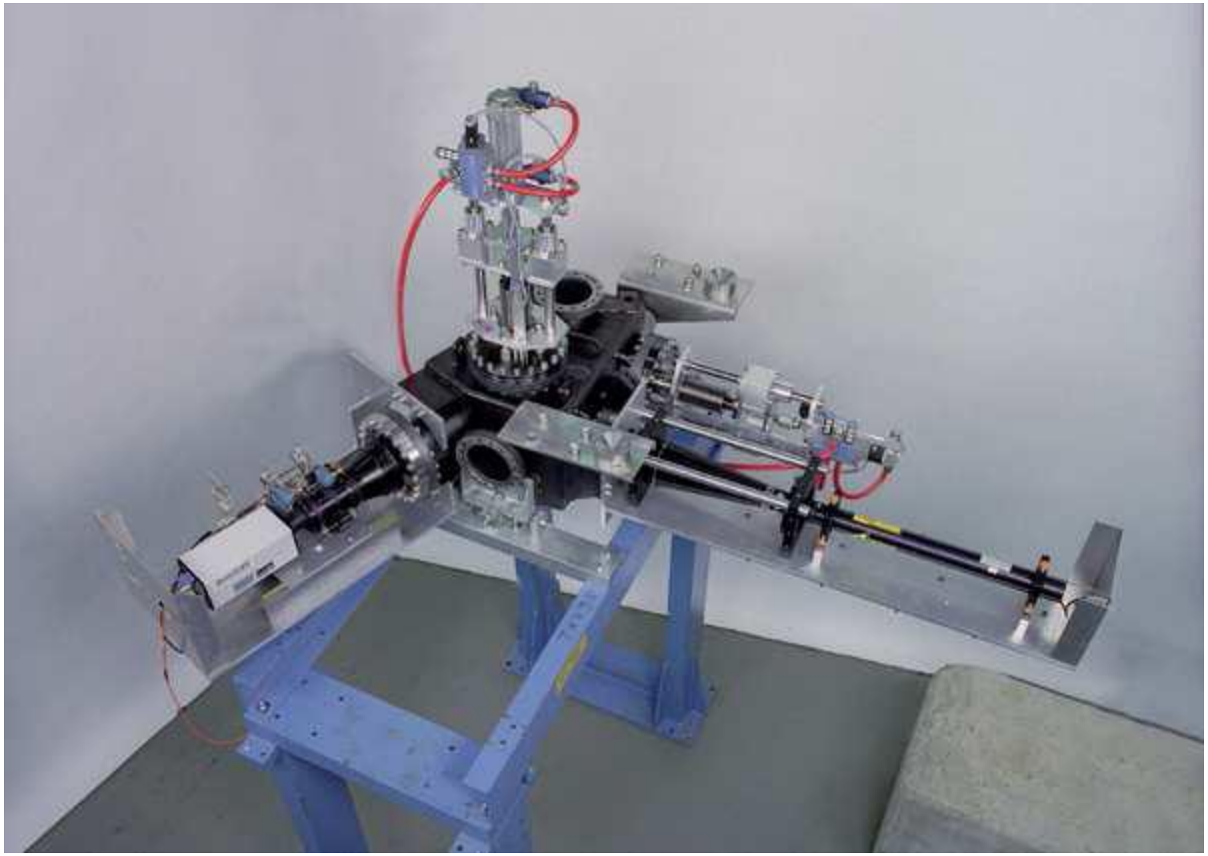


Photo of a Pepperpot Device



Scintillating Screens

Method already applied in cosmic ray experiments

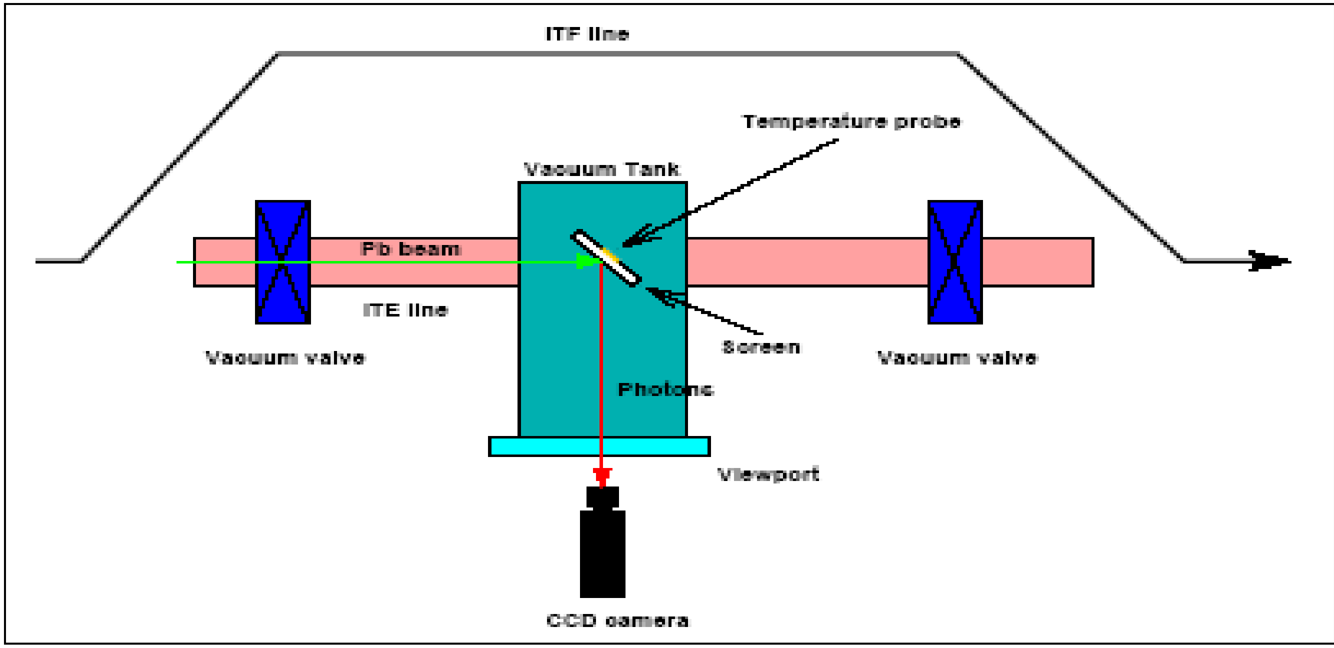
- Very simple
- Very convincing

Needed:

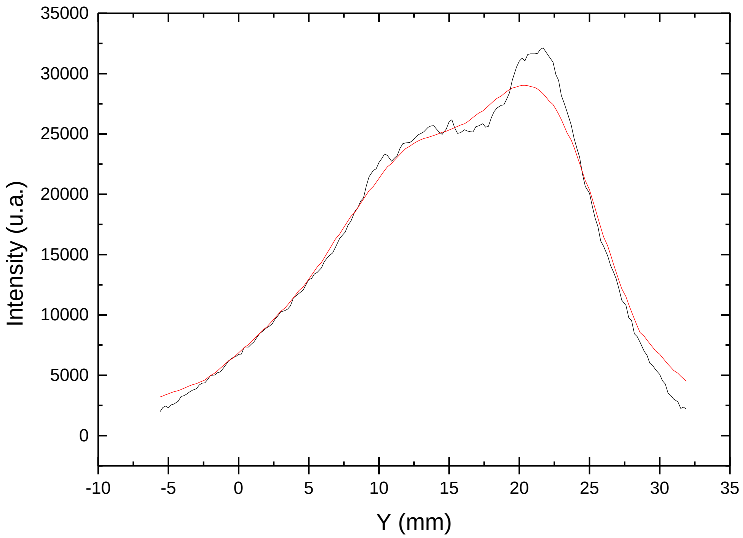
- Scintillating Material
- TV camera
- In/out mechanism

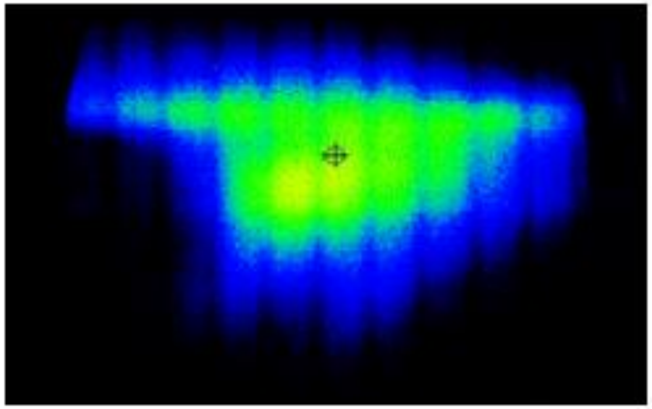
Problems:

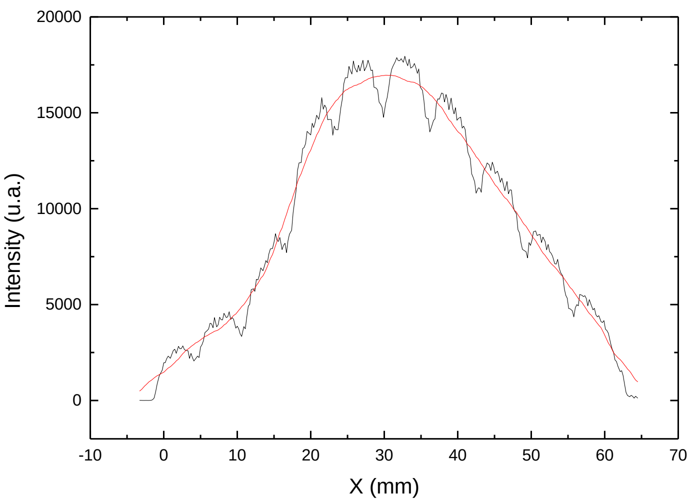
- Radiation resistance of TV camera
- Heating of screen (absorption of beam energy)
- Evacuation of electric charges



Frame grabber







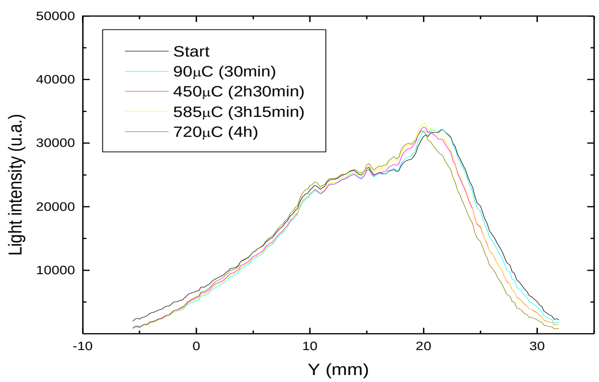
- For further evaluation the video signal is digitized, read-out and treated by program
- In new cameras digitization is done within the camera and digital image information is accessed via Ethernet or USB

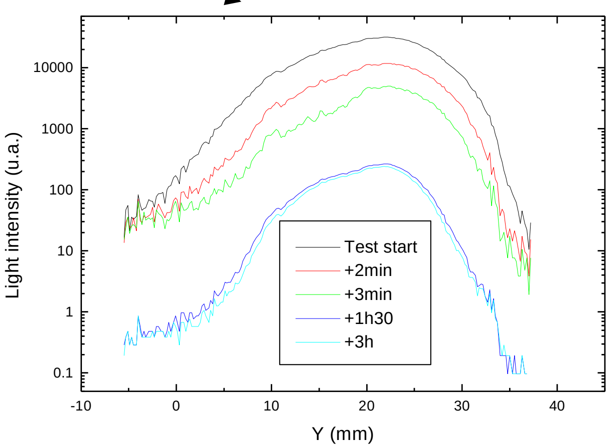
Test for resistance against heat-shock

Material	ρ g/cm³	c_p at 20°C J/gK	k at 100°C W/mK	T_{max} °C	R at 400 °C Ω.cm
Al_2O_3	3.9	0.9	30	1600	10^{12}
ZrO_2	6	0.4	2	1200	10^{3}
BN	2	1.6	35	2400	10^{14}

Better for electrical conductivity (>400°C)

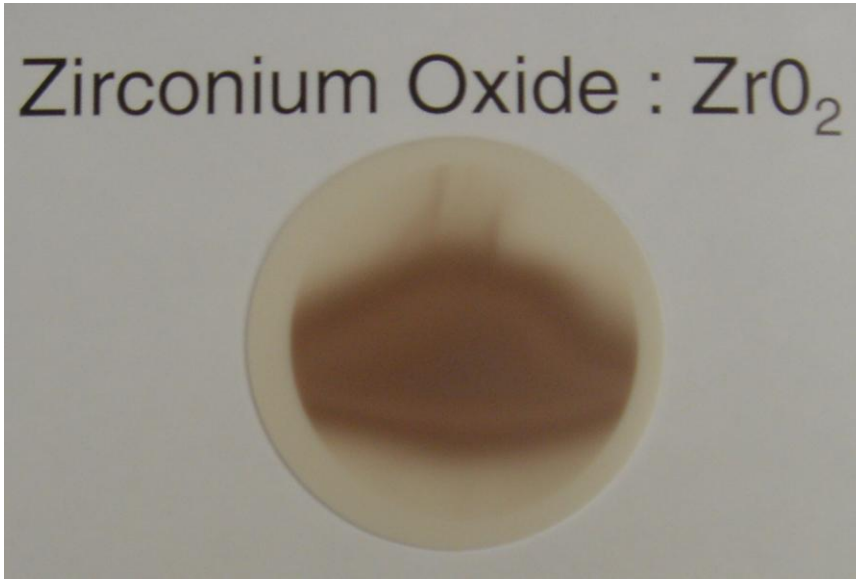
Better for thermal properties
(higher conductivity, higher heat capacity)





Degradation of screen

Degradation clearly visible
However sensitivity stays essentially the same

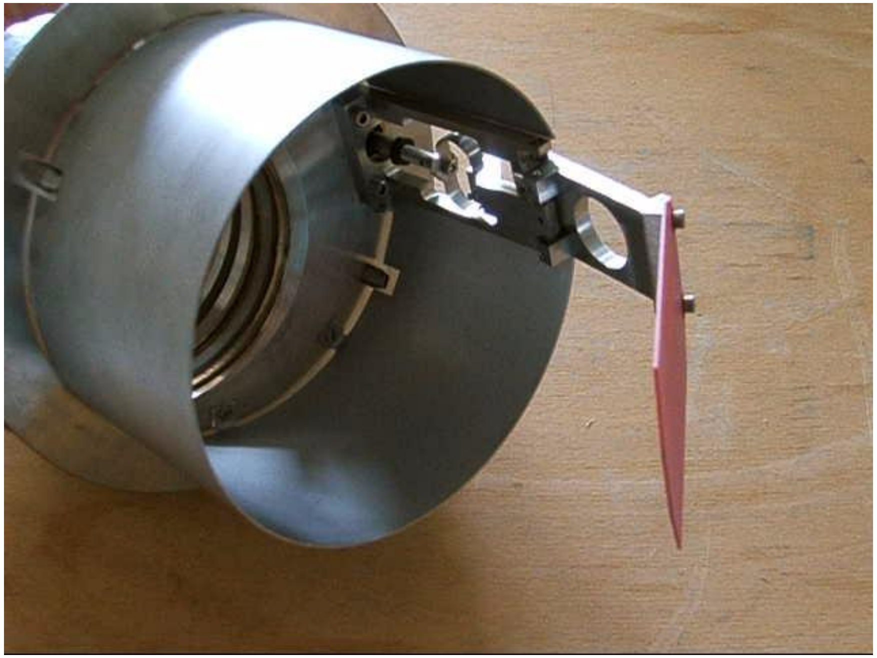


Properties of scintillating material

Abbreviation	Material	Activator	max. emission	decay time
Quartz	SiO_2	none	optical	< 10 ns
	CsI	Tl	550 nm	1 μs
	ZnS	Ag	450 nm	0.2 μs
Chromolux	Al_2O_3	Cr	700 nm	100 ms
	Li glass	Ce	400 nm	0.1 μs
P43	Gd_2O_2S	Tb	545 nm	1 ms
P46	$Y_3Al_5O_{12}$	Ce	530 nm	0.3 μs
P47	$Y_2Si_5O_5$	Ce	400 nm	50 ns

Screen mechanism

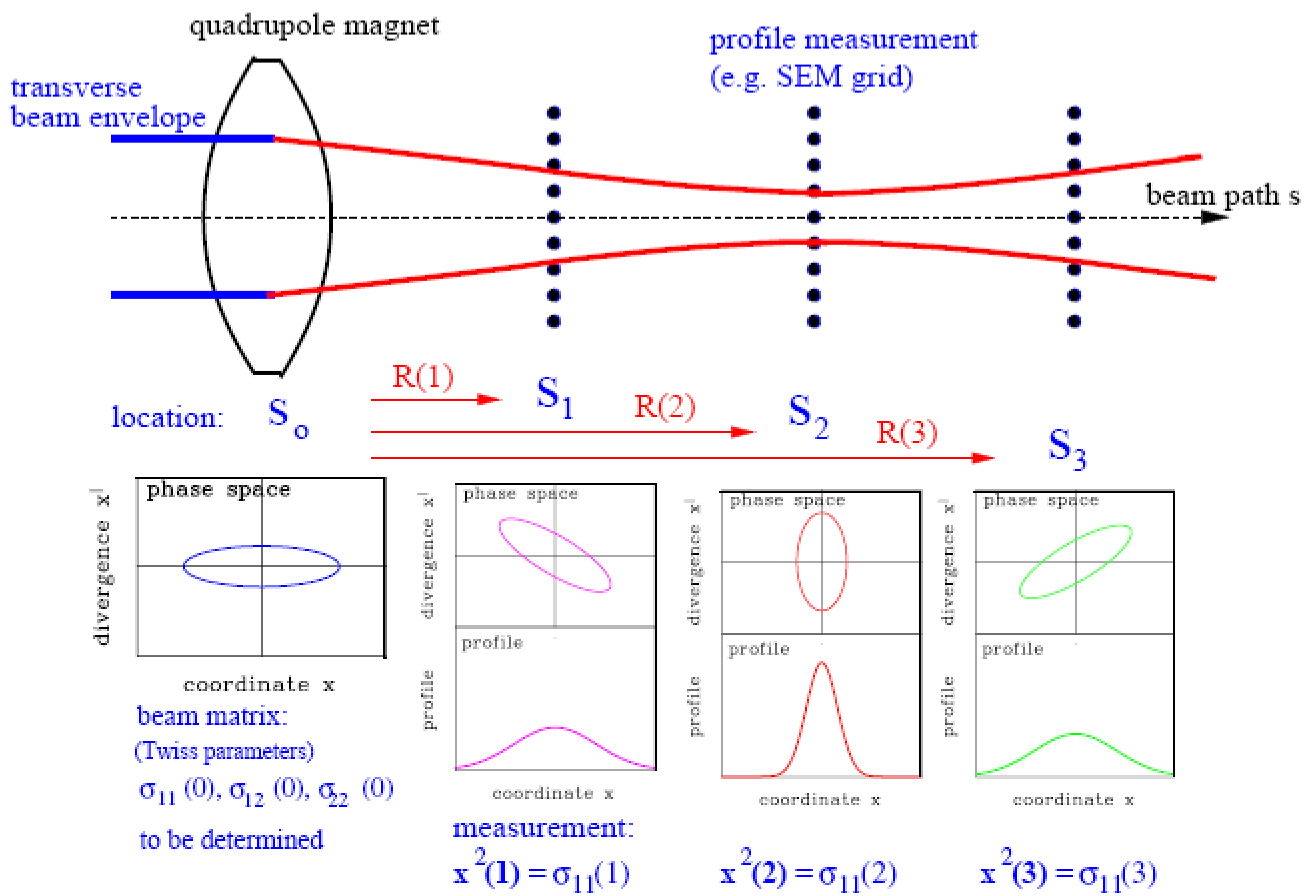
- Screen with graticule



3 Profile Measurement and Quadrupole Scan

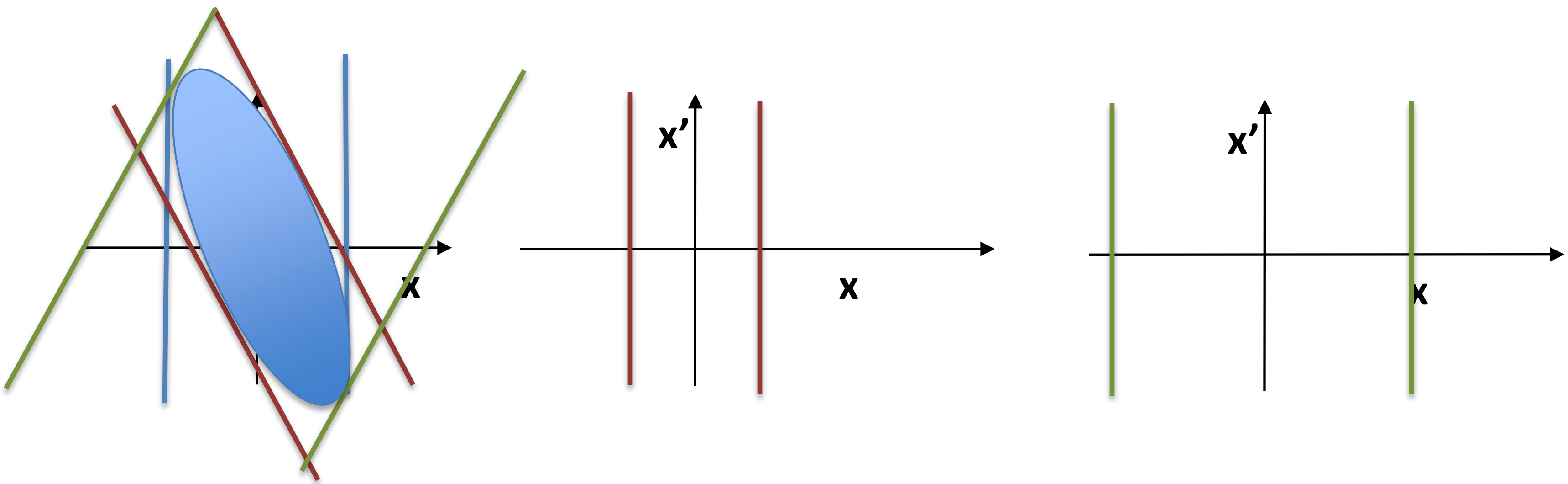
- To determine ε, β, α at a reference point in a beamline one needs at least three *w* measurements with different transfer matrices between the reference point and the *w* measurements location.
- Different transfer matrices can be achieved with different profile monitor locations, different focusing magnet settings or combinations of both.
- Once β, α at one reference point is determined the values of β, α at every point in the beamline can be calculated.
- Three *w* measurements are in principle enough to determine ε, β, α
- In practice better results are obtained with more measurements.
- However, with more than three measurements the problem is over-determined.
- χ^2 formalism gives the best estimate of ε, β, α for a set of *n* measurements w_i i=1-n with transfer matrix elements c_i, s_i.

3-Profile Measurement

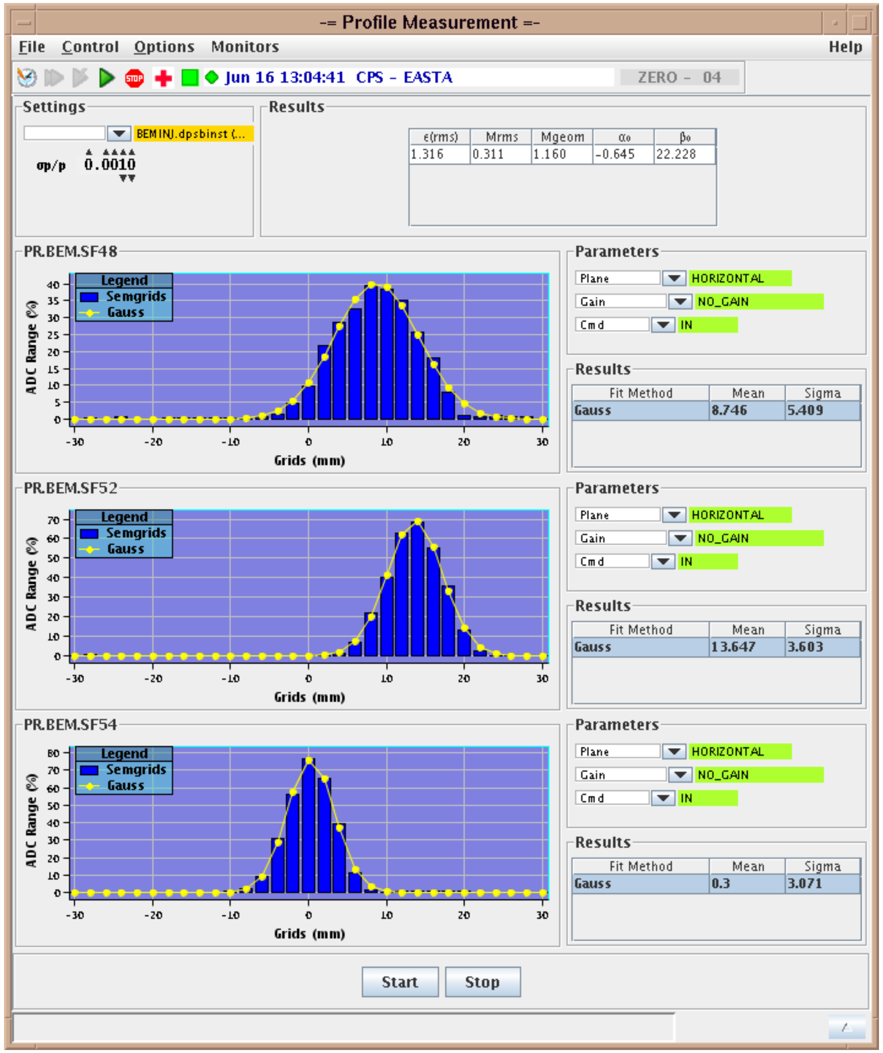


3-Profile Measurement

- Measure 3 profiles at 3 positions around a waist
- Spot width corresponds to vertical lines
- Transform back to the first Profile
- Lines become tangents to the beam ellipse



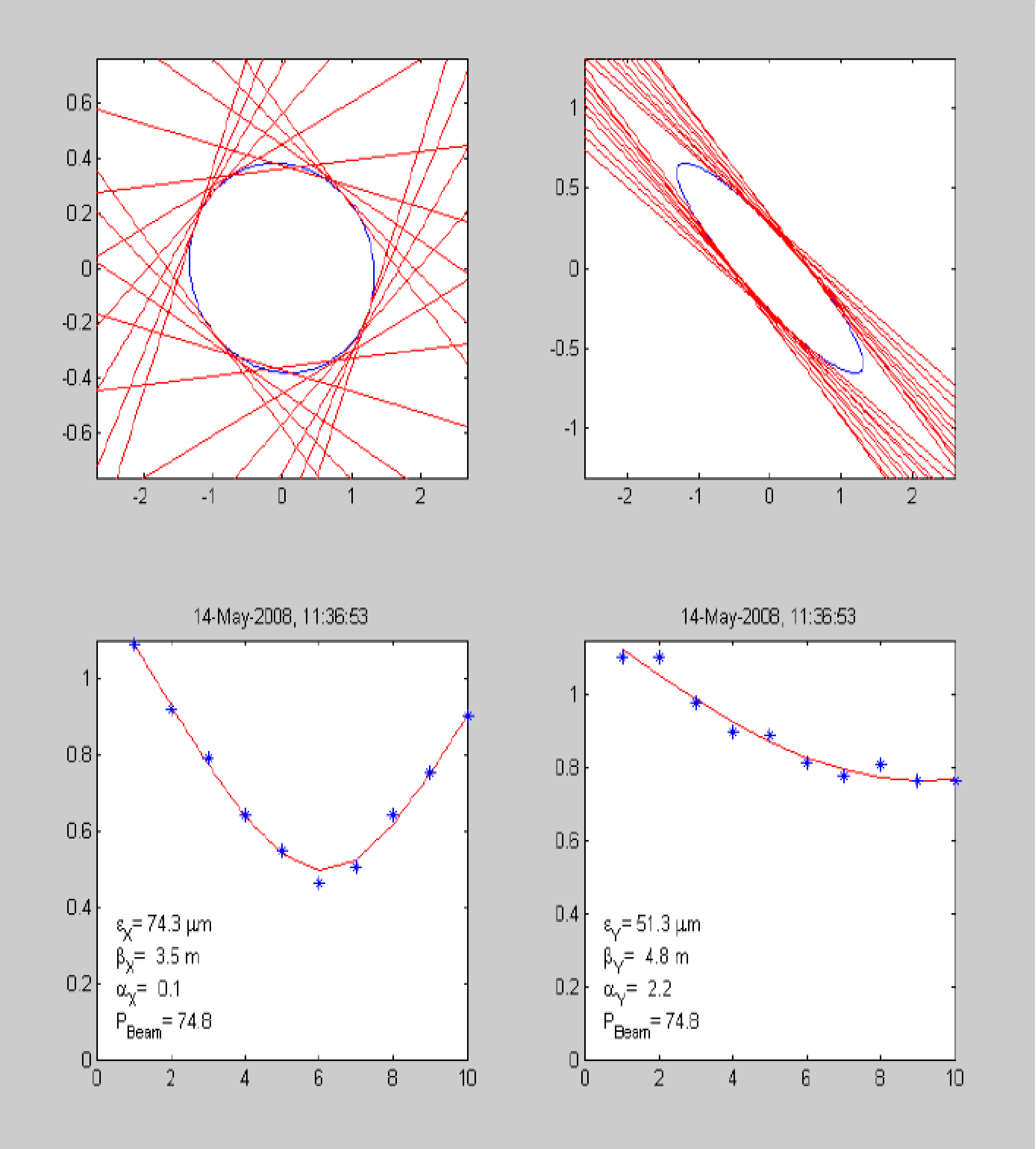
An example from CERN PS



Quadrupole Scan

- Works the same way as the 3-Profile measurement
- The profile is taken at a fixed position (needs a single profile measurement system)
- Vary a quadrupole and measure the profile width for each quadrupole setting

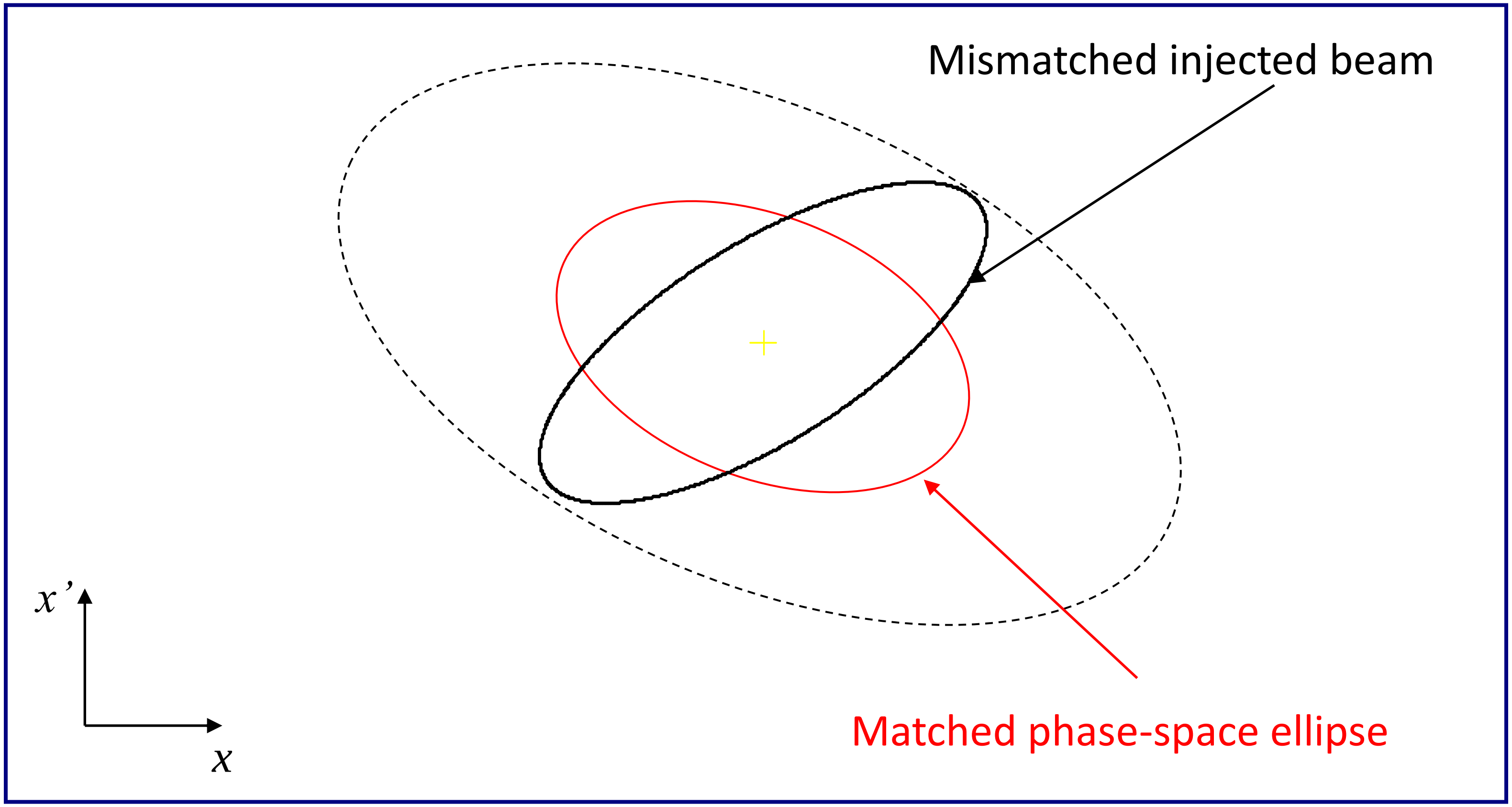
Quadrupole Scan at CTF-3



Optical Mismatch at Injection

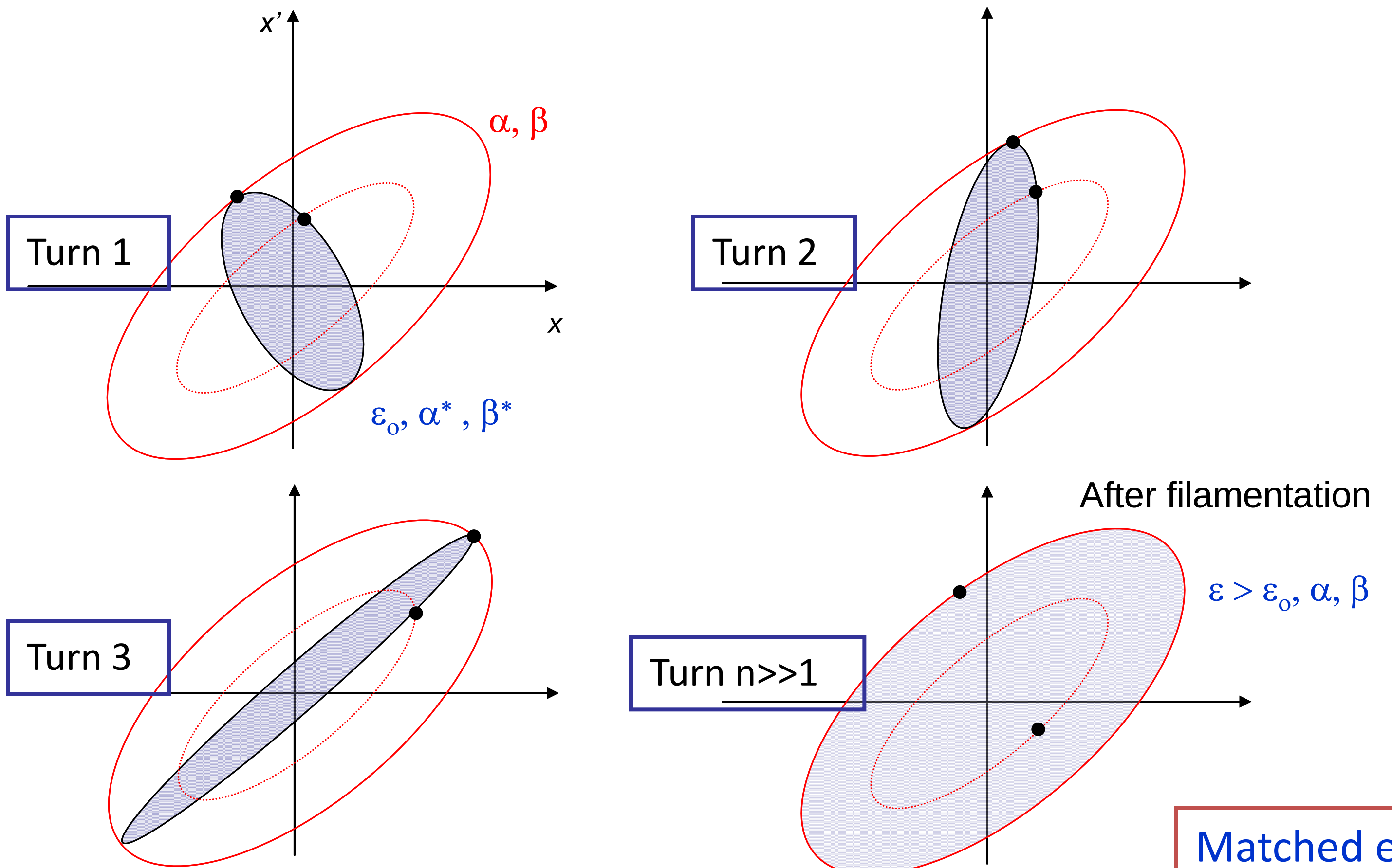
- Can also have an emittance blow-up through optical mismatch
- Individual particles oscillate with conserved CS invariant:

$$a_x = \gamma x2 + 2\alpha xx' + \beta x'2$$



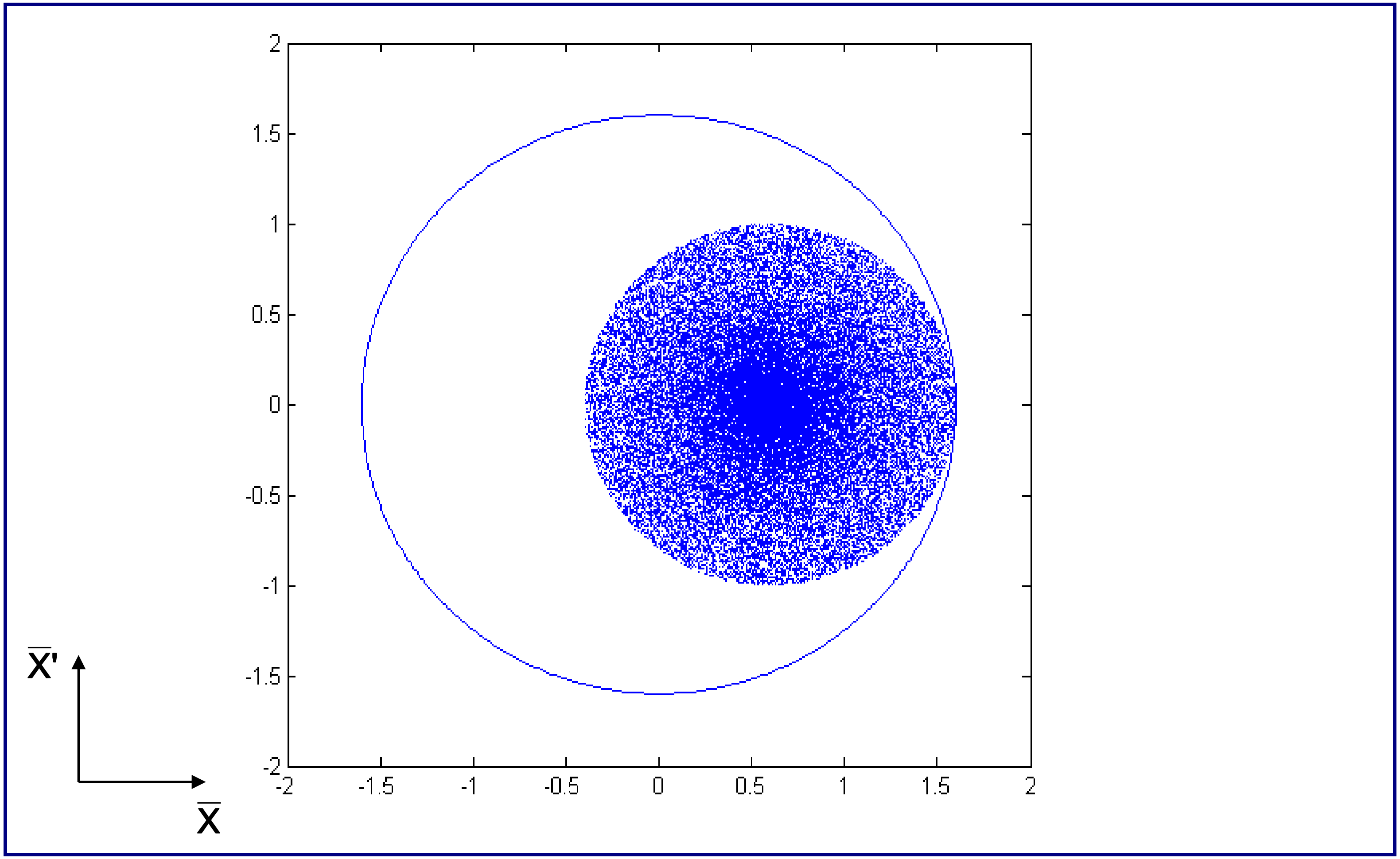
Optical mismatch at injection

- Injected beam of emittance ε, characterised by a different ellipse (α^*, β^*) to matched ellipse (α, β), generates (via filamentation) a large ellipse with original shape (α, β), but larger ε

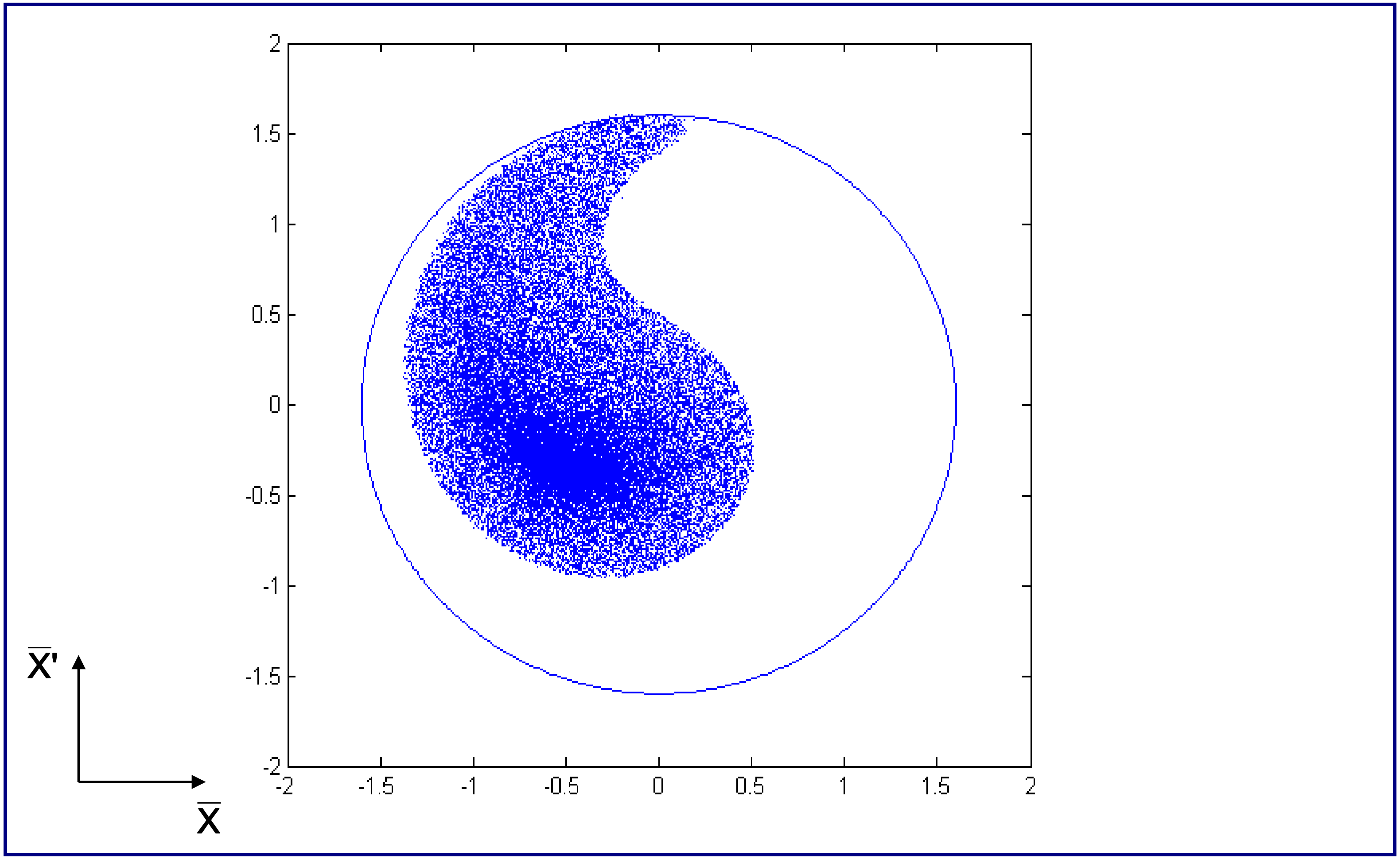


Matched ellipse determines beam shape

Filamentation



Filamentation



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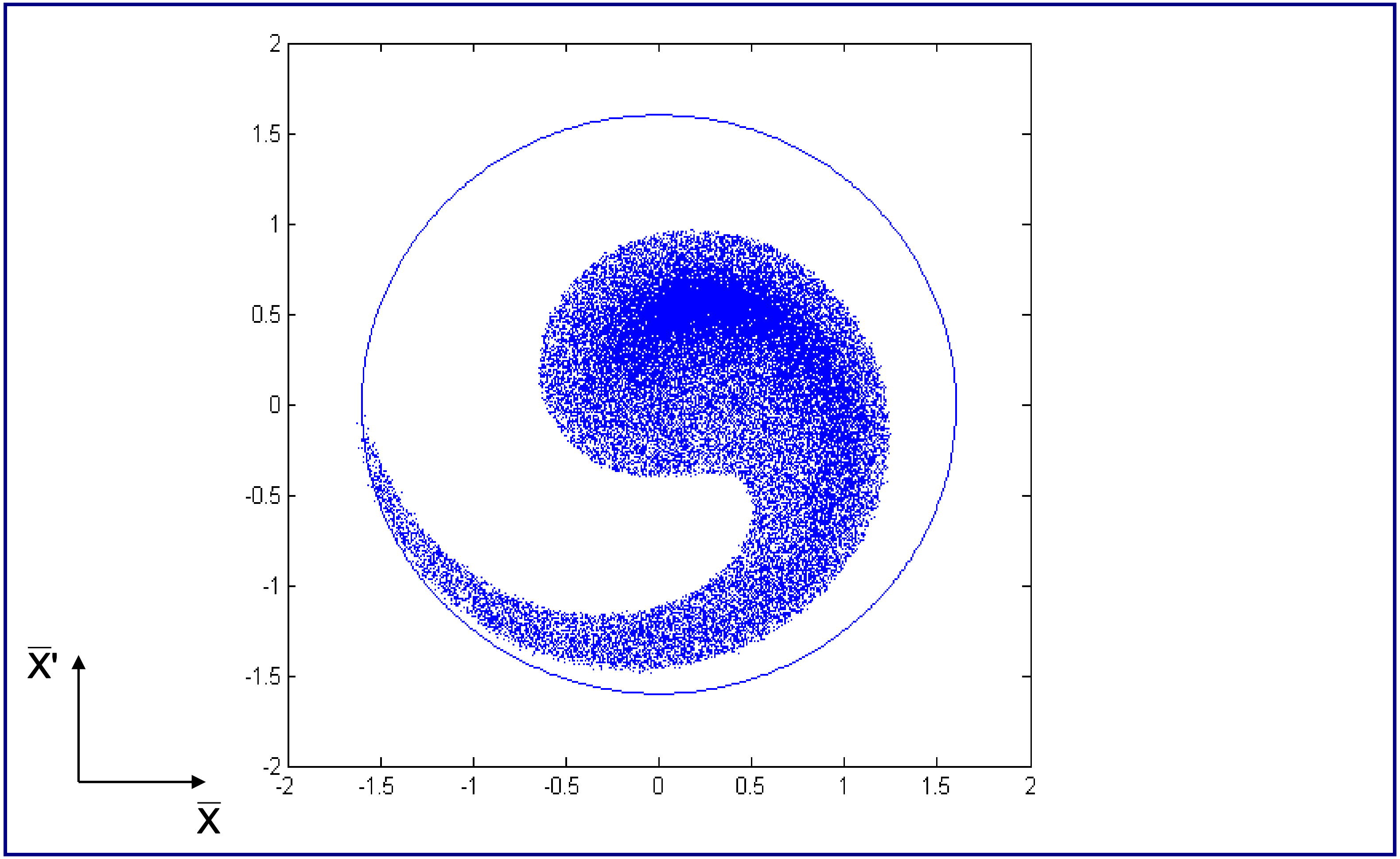
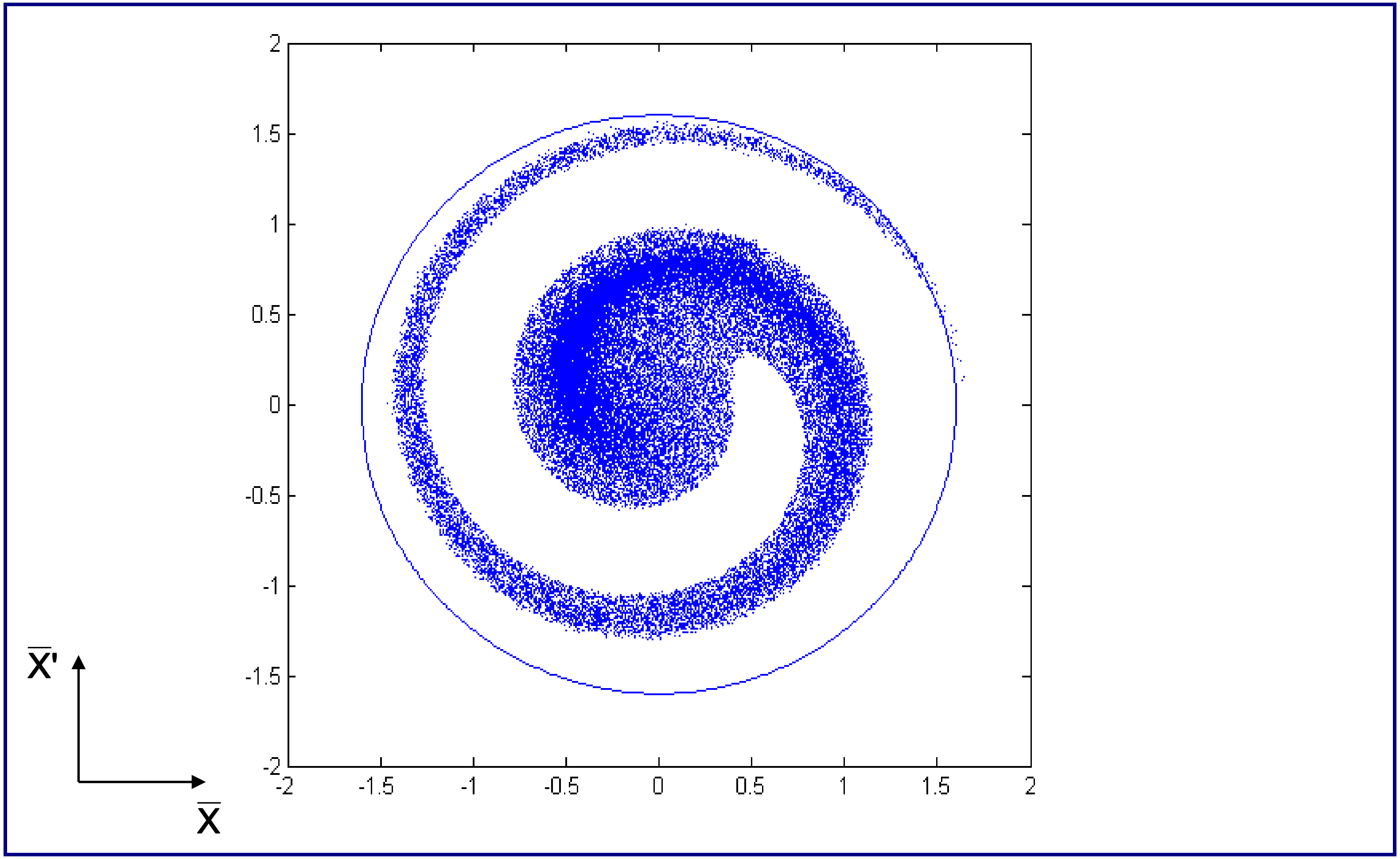
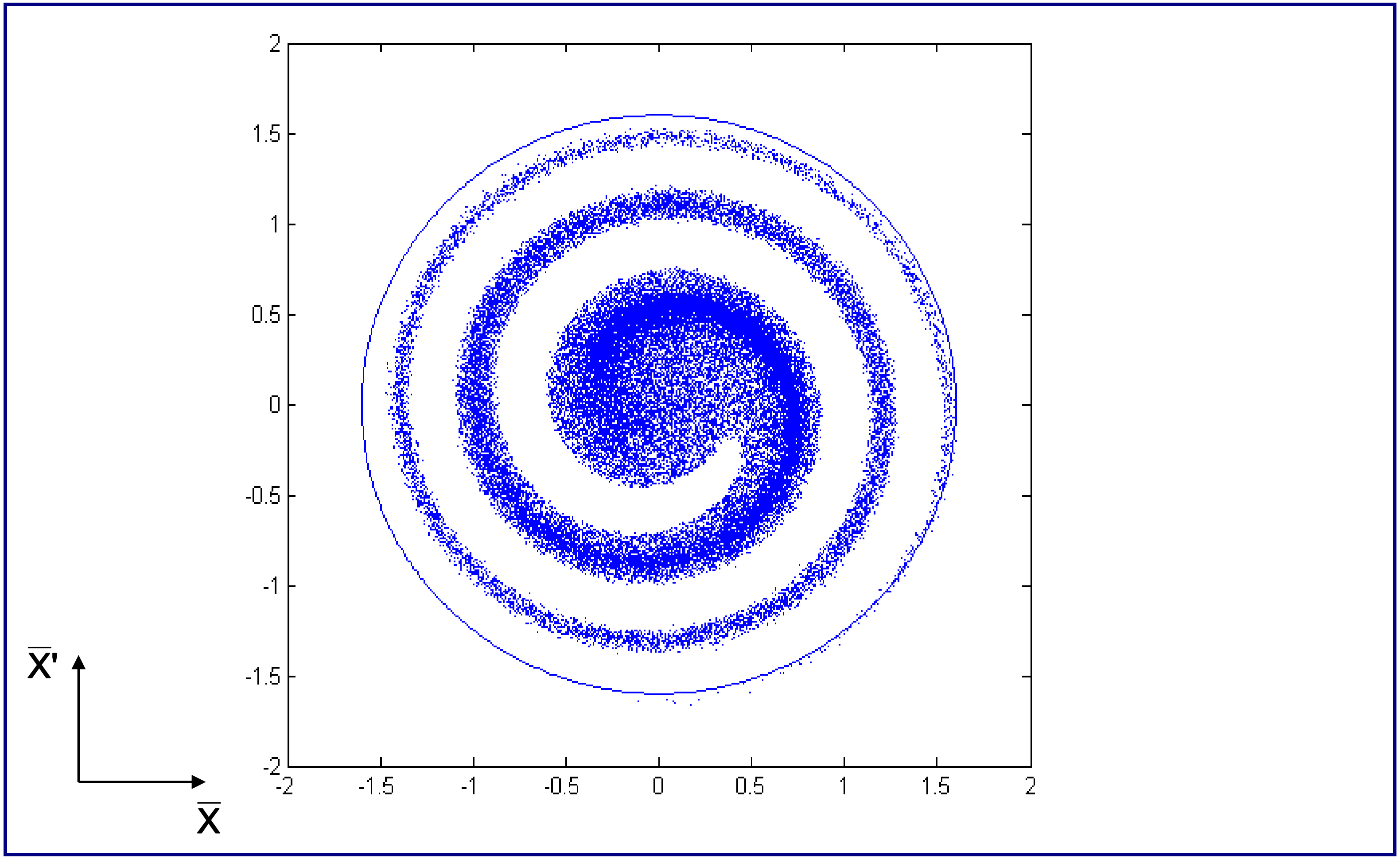


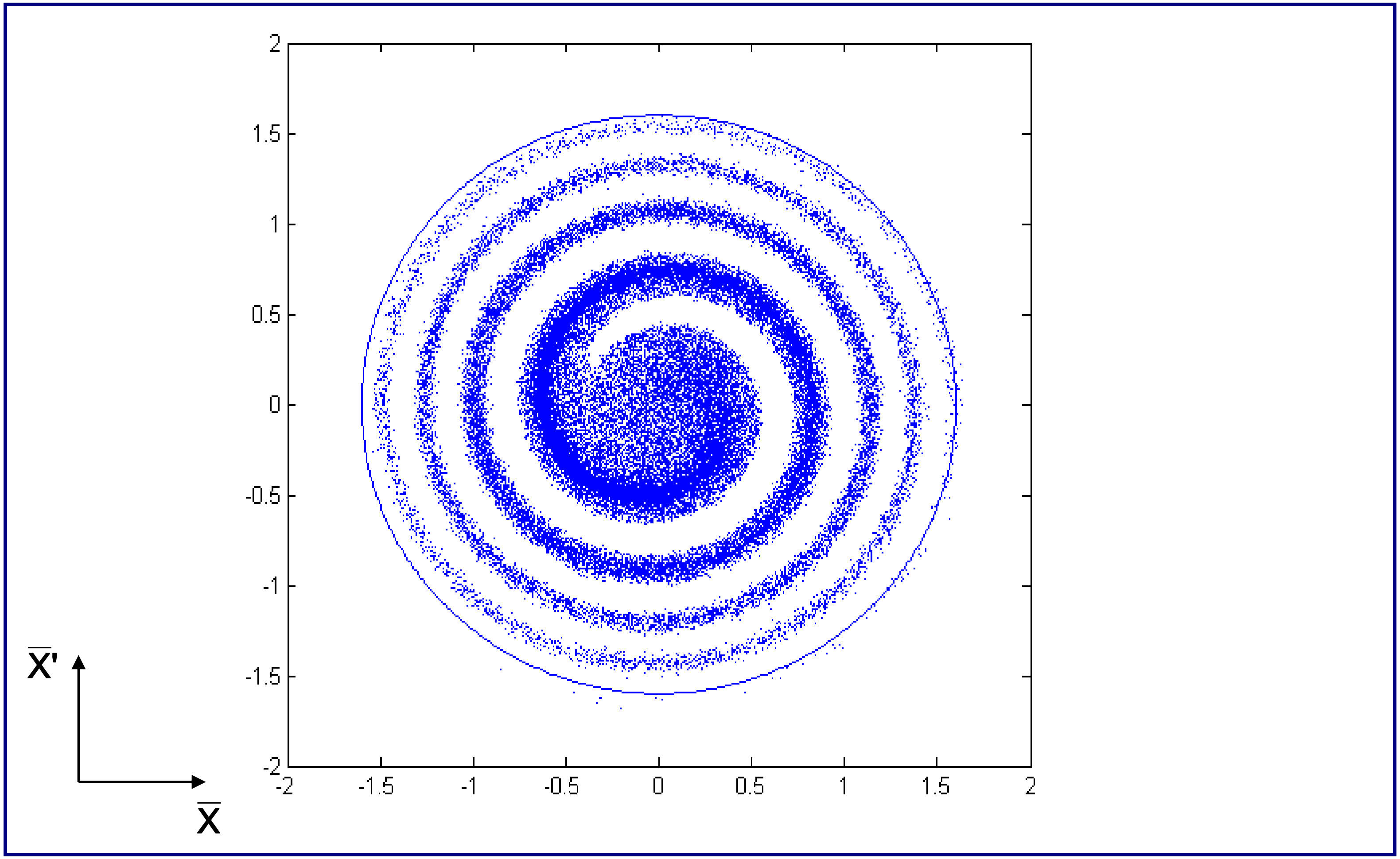
Filamentation



Filamentation



Filamentation



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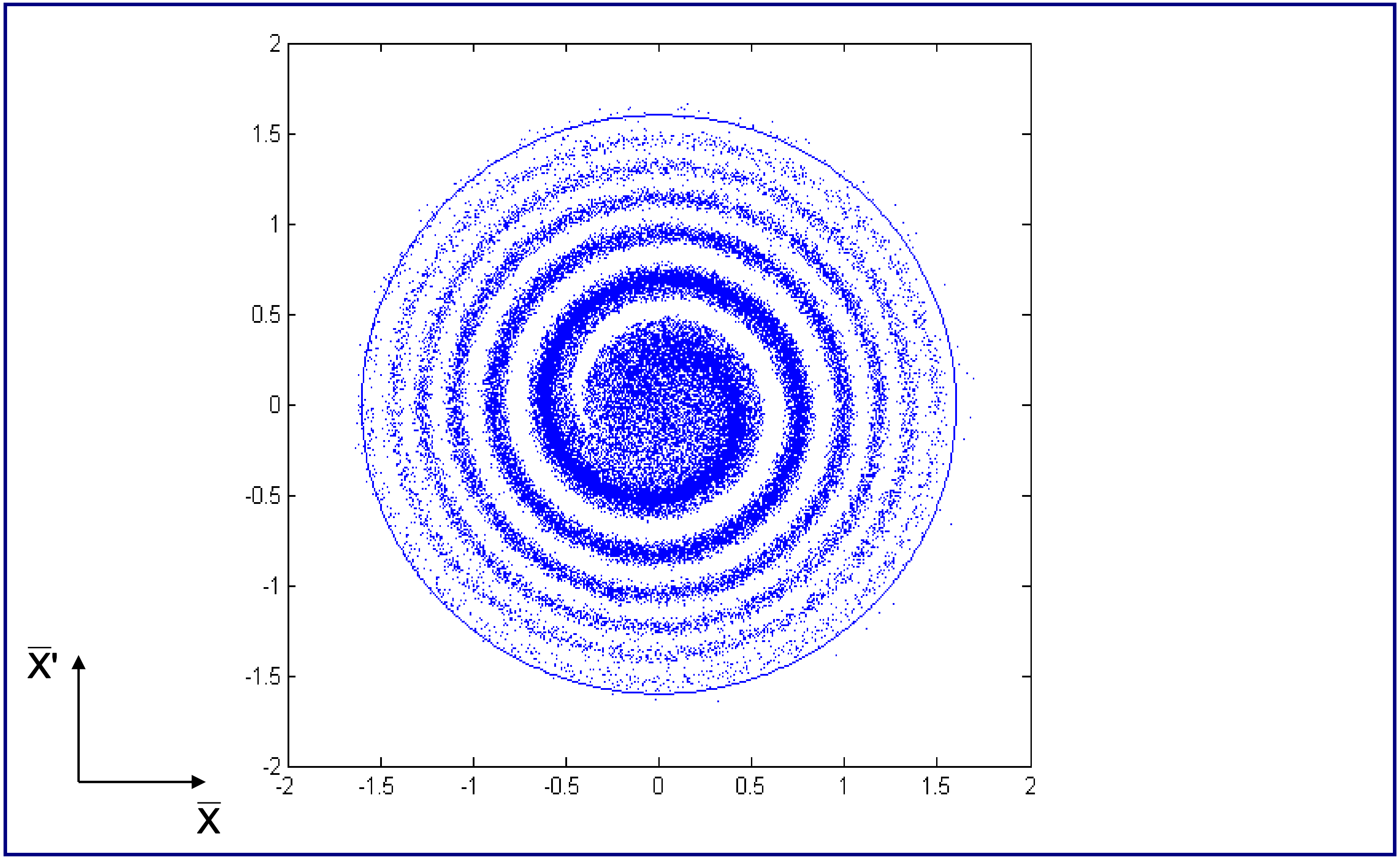

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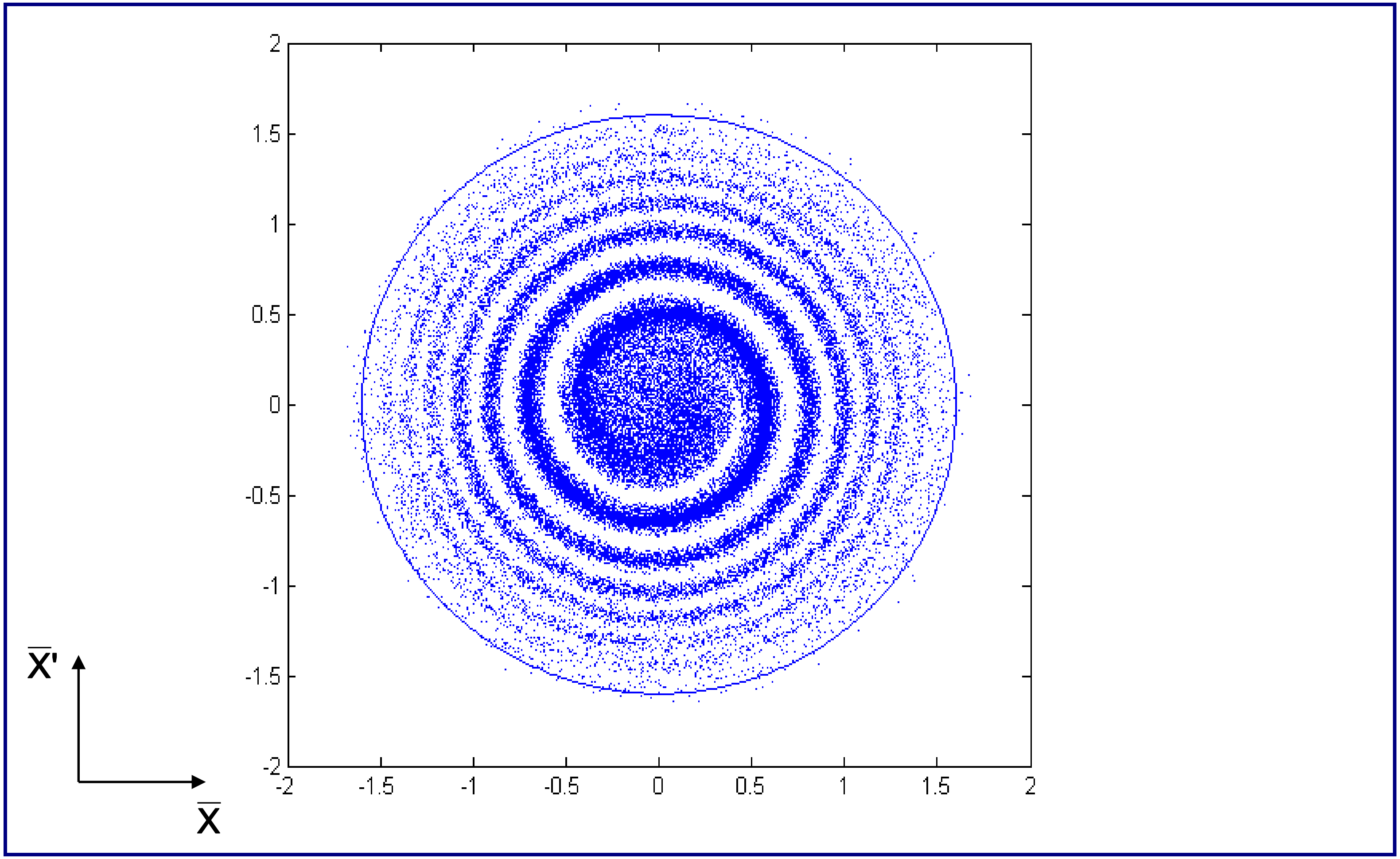

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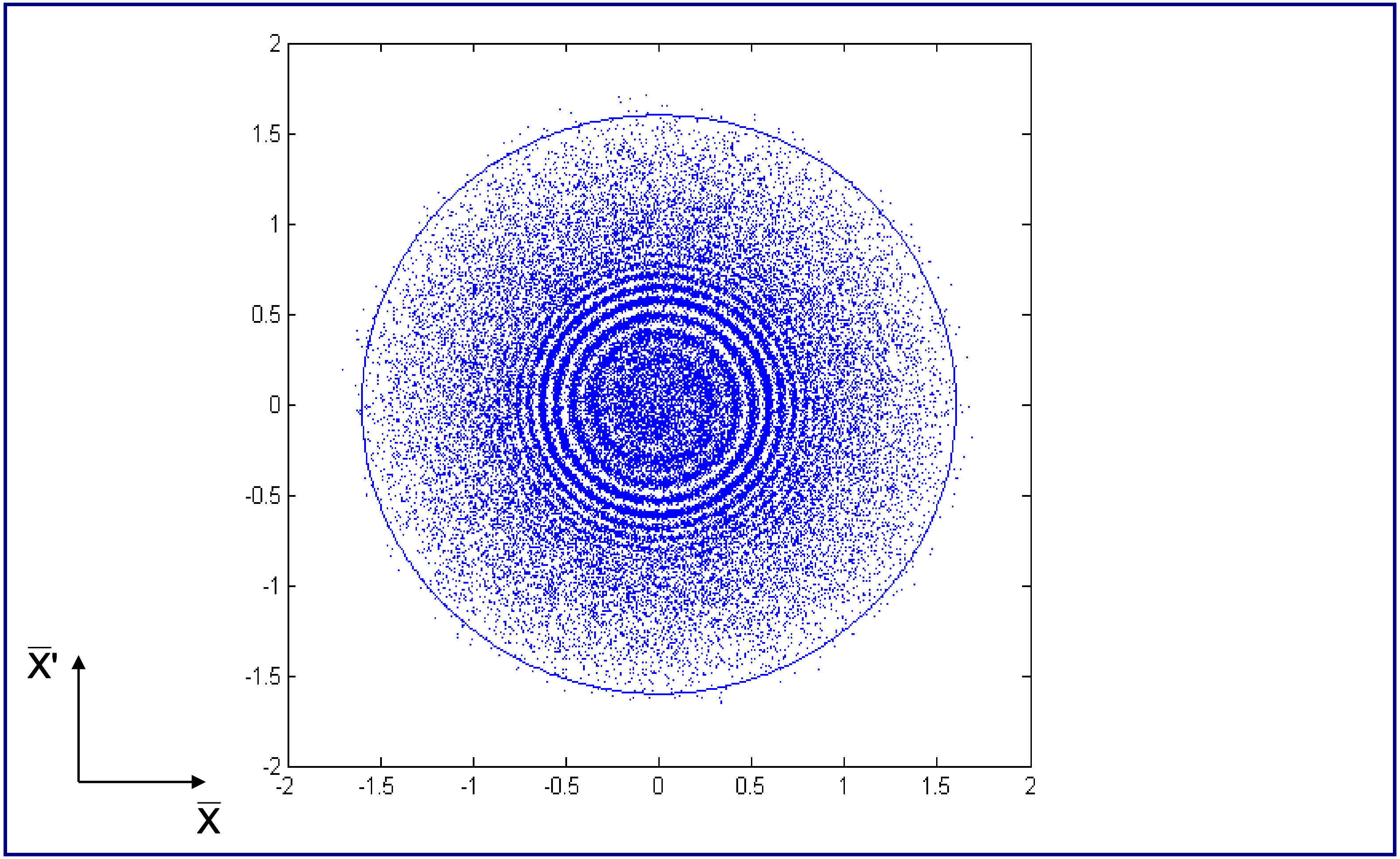

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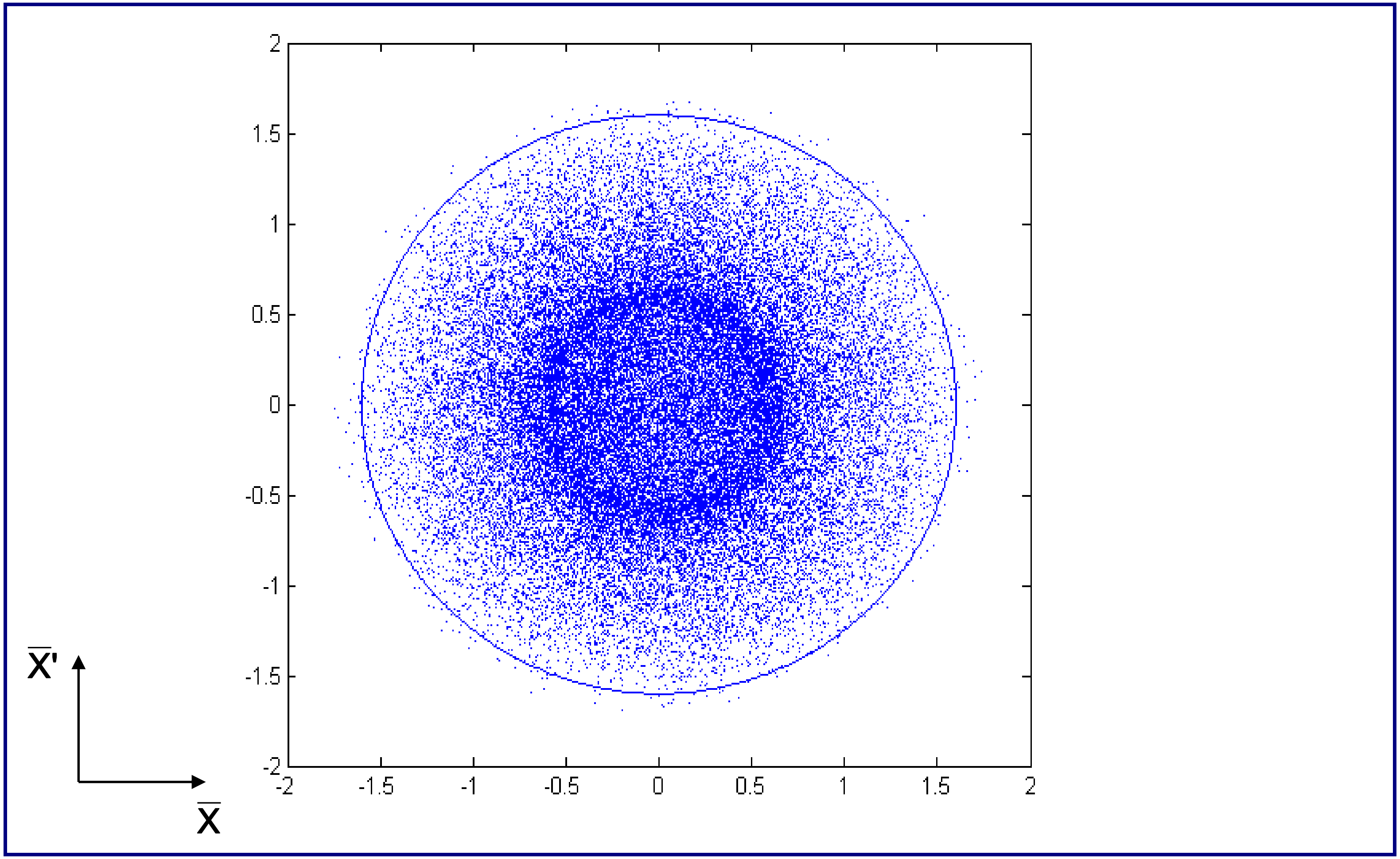
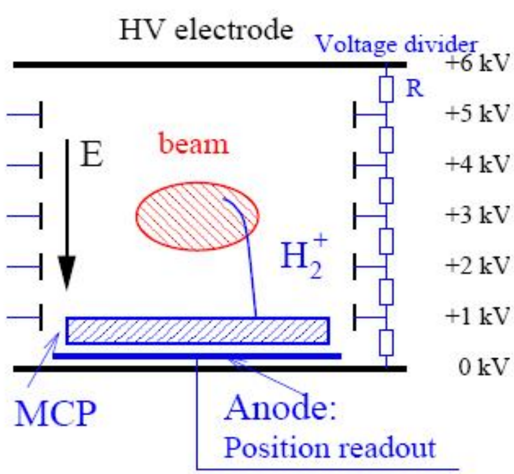


Ionisation Profile Monitor

Uses rest gas in the vacuum
which is ionized.



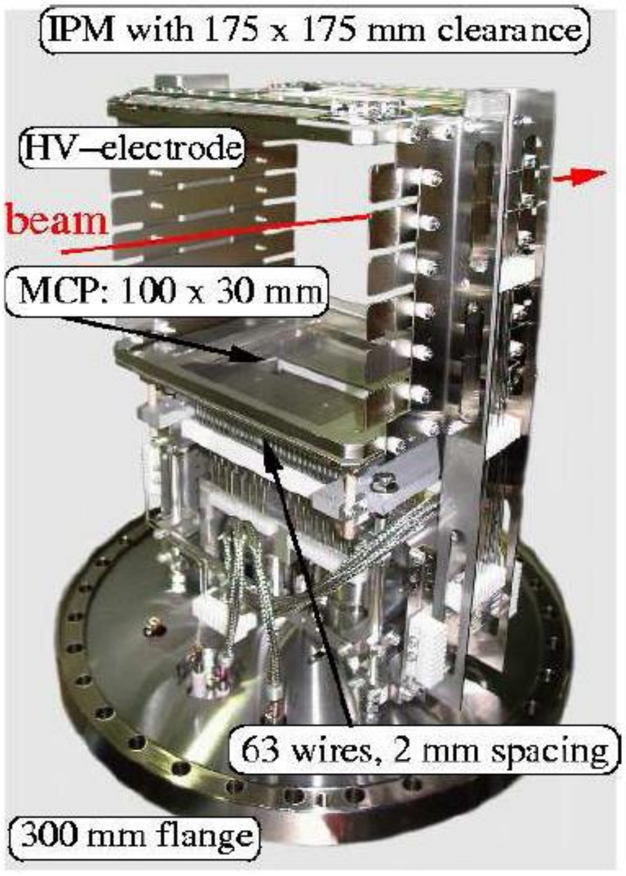
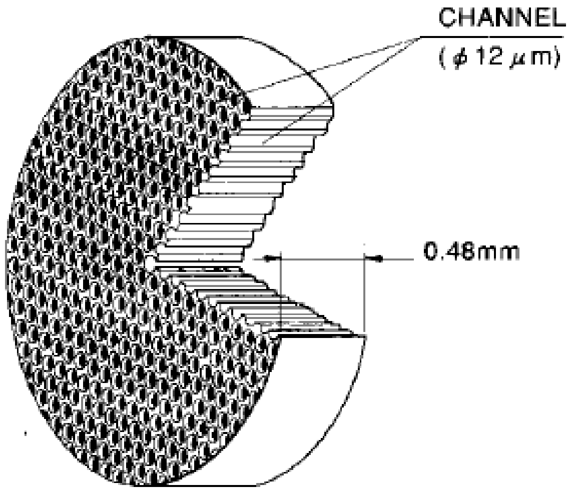
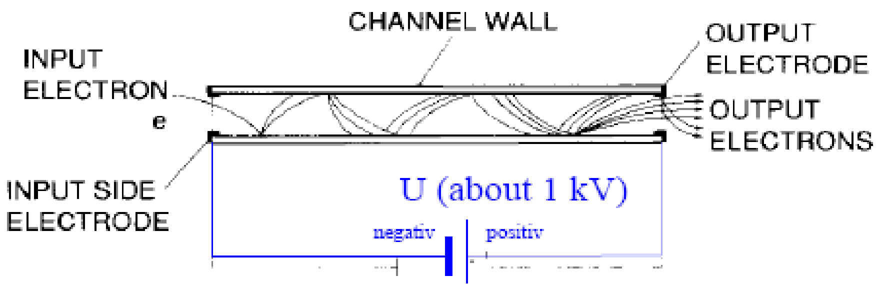
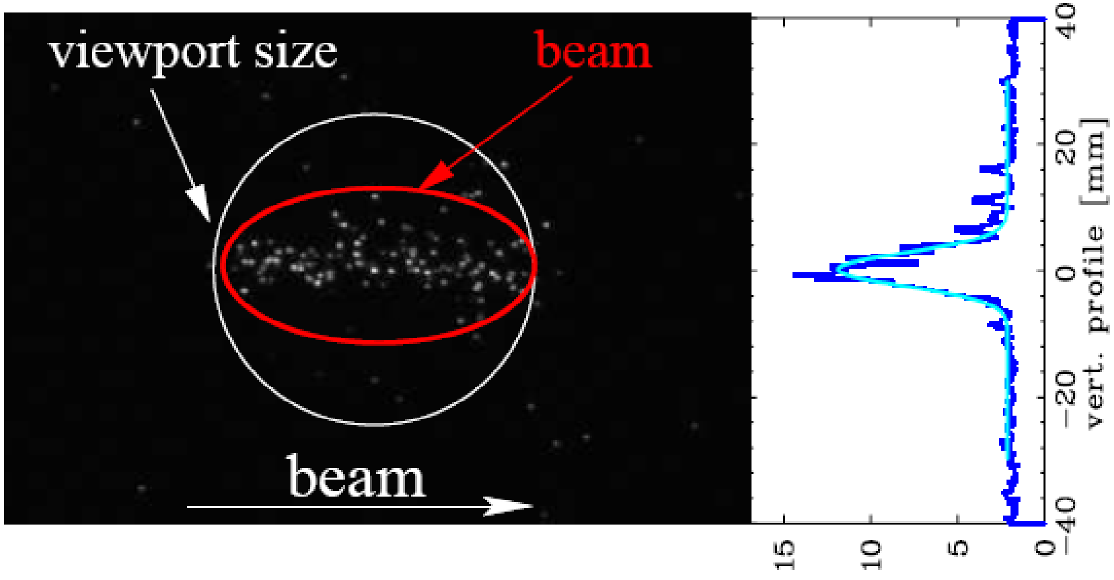


Image Intensifier

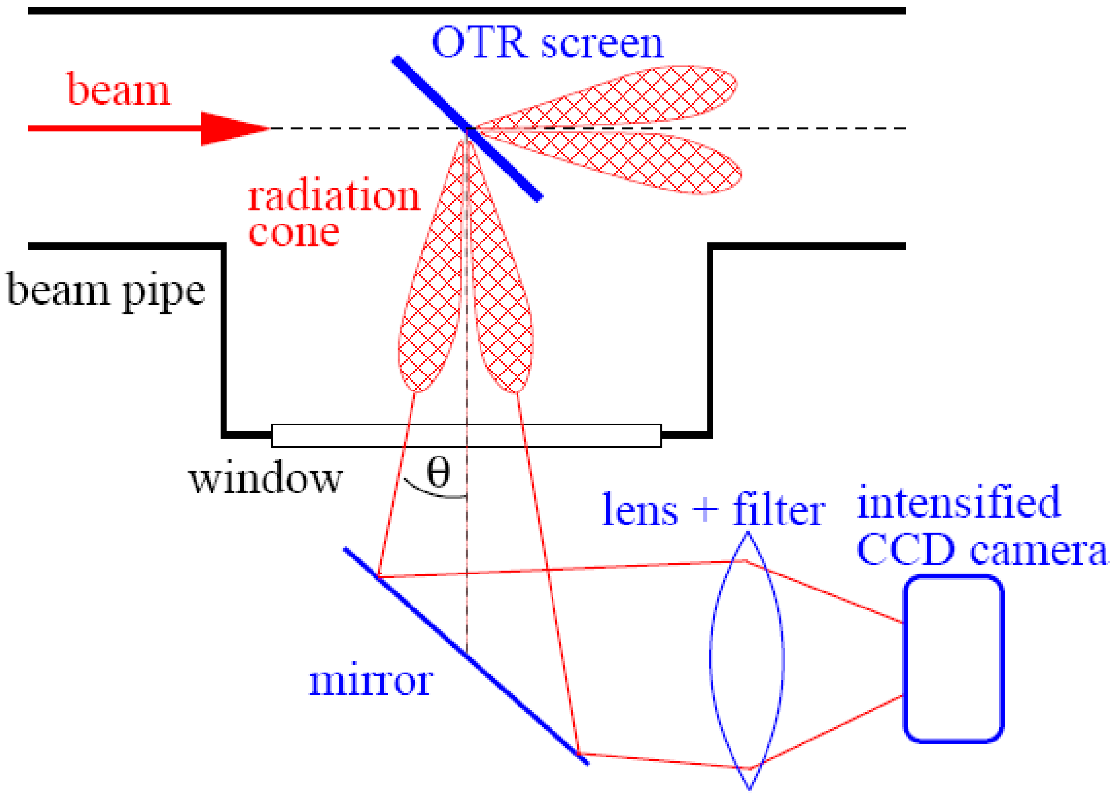




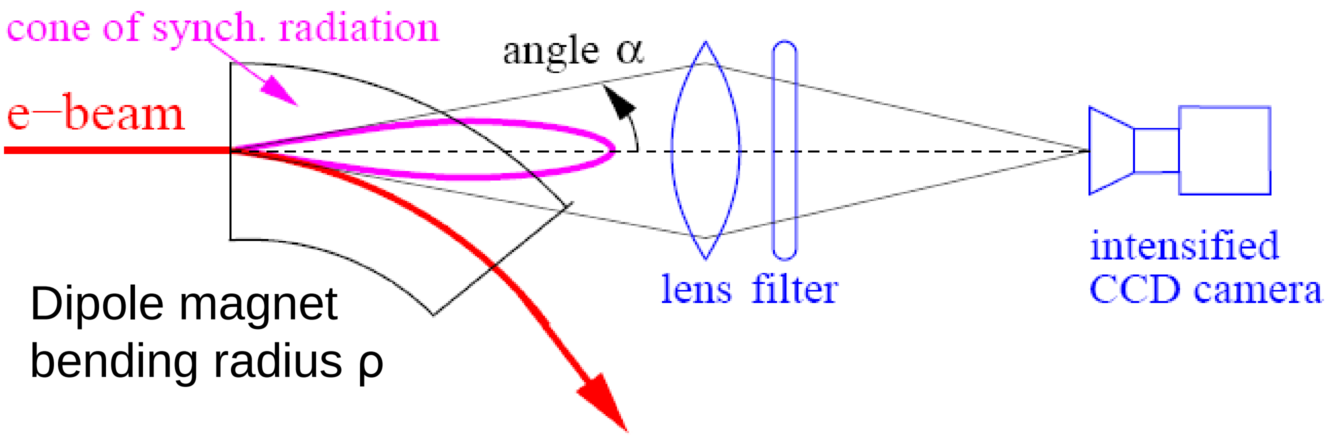
Result from IPM



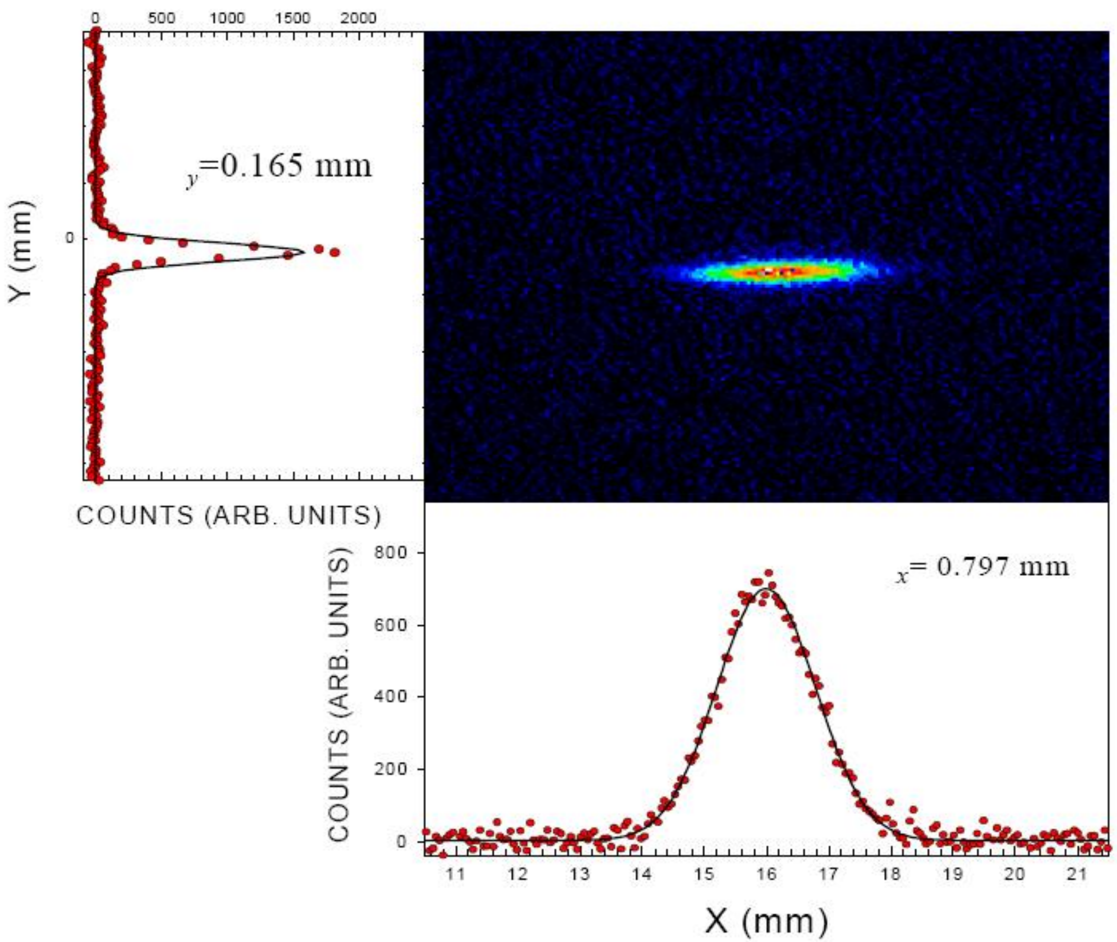
Optical Transition Radiation



Synchrotron Radiation

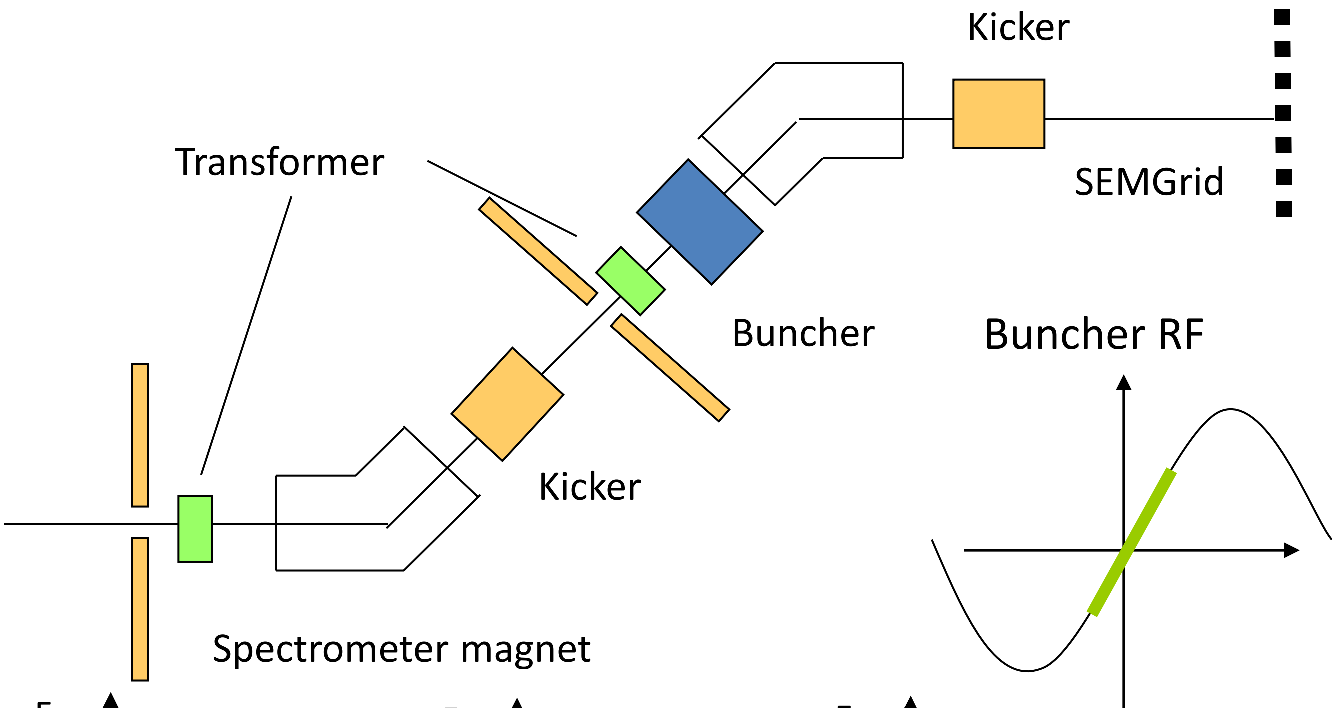


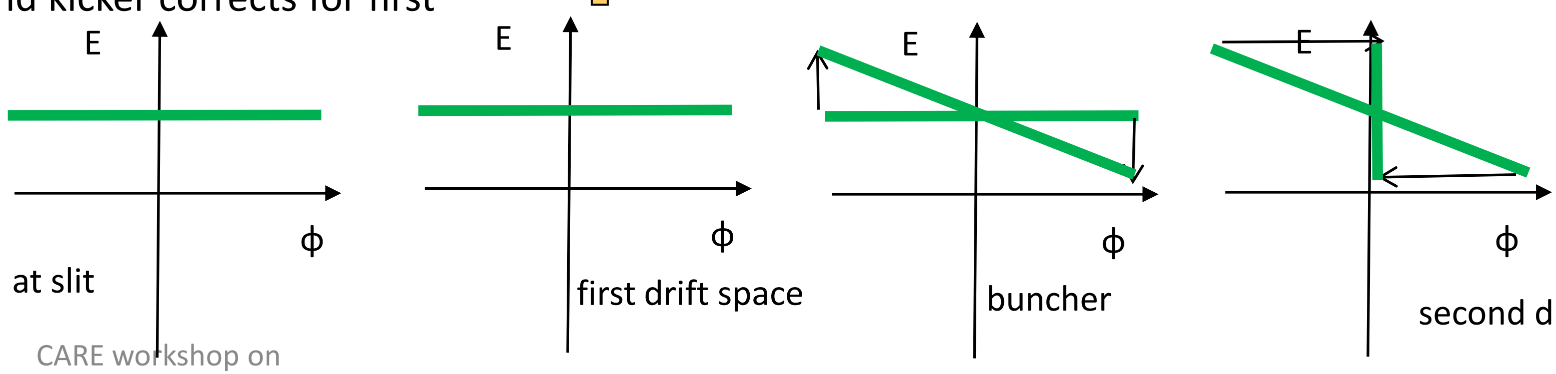
Results from Synchrotron Radiation



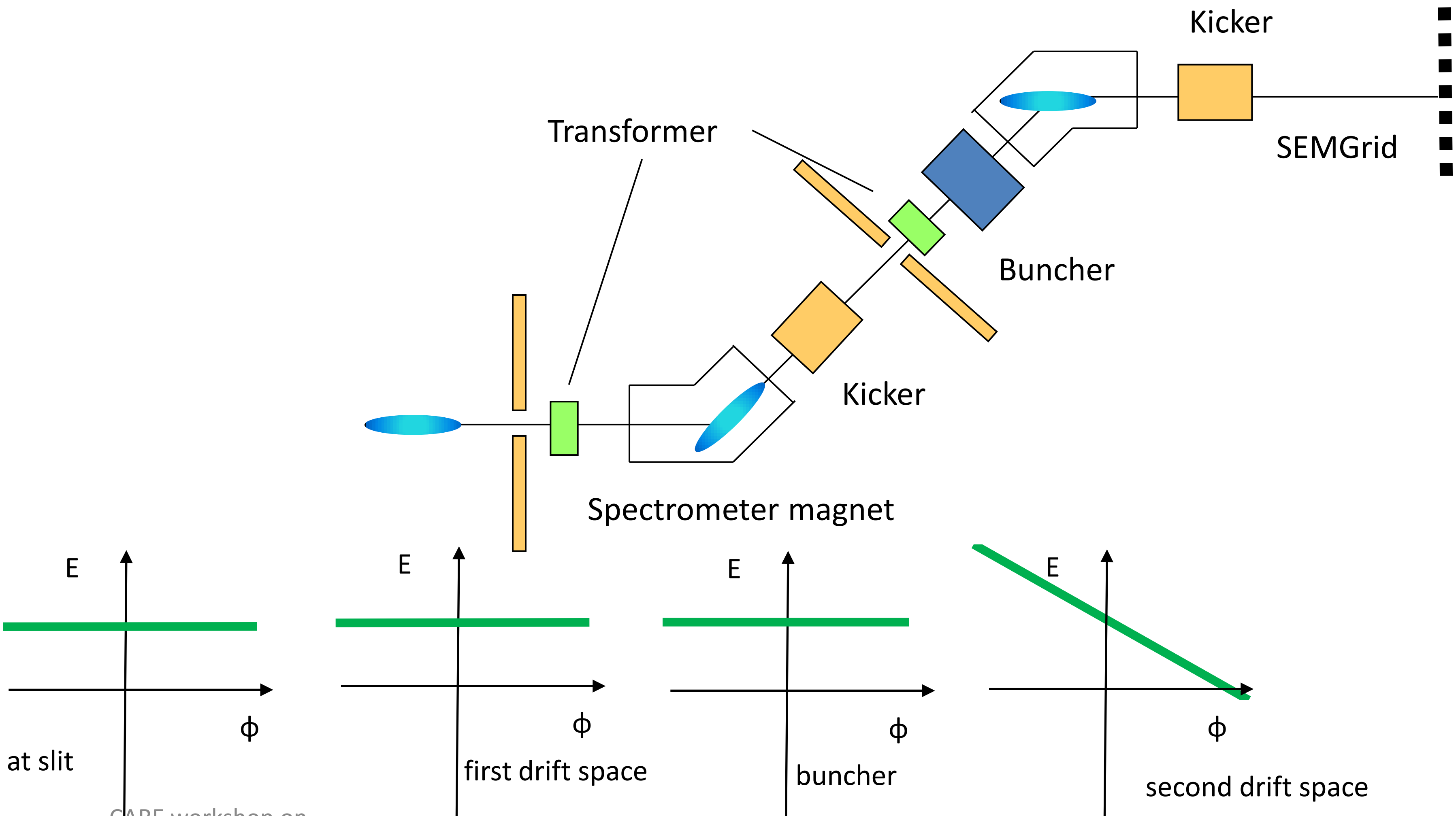
Longitudinal Phase Space Transformation

- Spectrometer produces image of slit on second slit
- second slit selects energy slice
- first kicker sweep phase space over all energies
- buncher rotates energy slice in phase space
- at second spectrometer the phase distribution is transformed into an energy distribution analyzed by the second spectrometer
- second kicker corrects for first kick

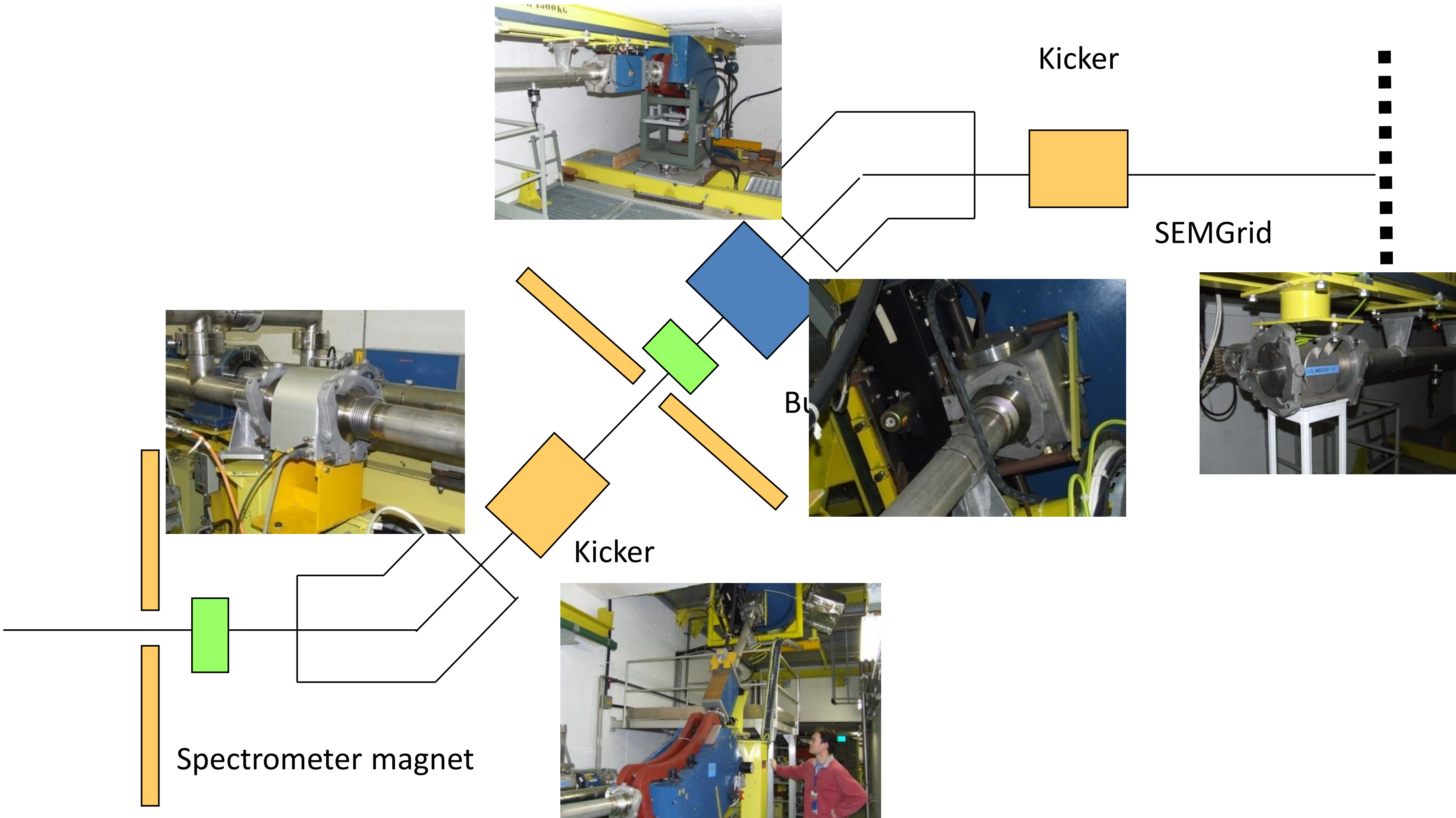




Longitudinal Emittance measurement



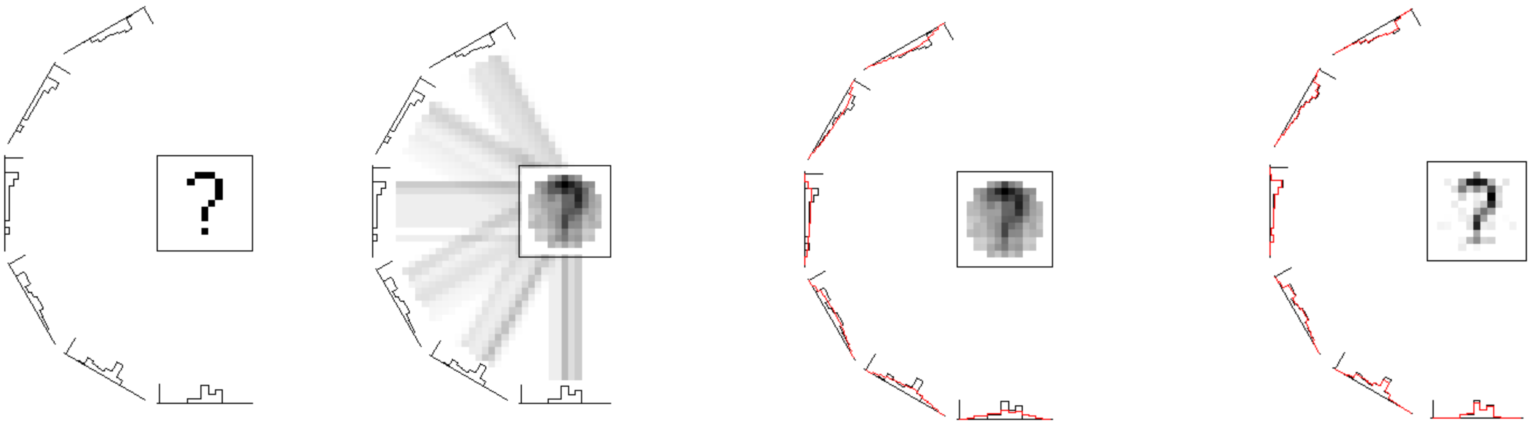
Photos of the line



Computed Tomography (CT)

Principle of Tomography:
- Take many 2-dimensional Images at different angles
- Reconstruct a 3-dimensional picture using mathematical techniques (Algebraic Reconstruction Technique, ART)

The reconstruction



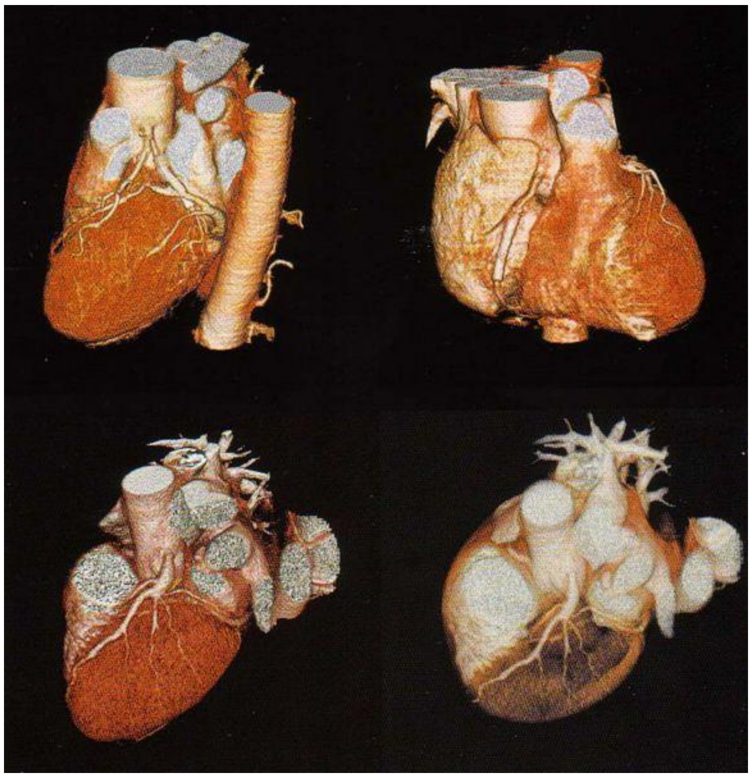
Produce many projections of the object to be reconstructed

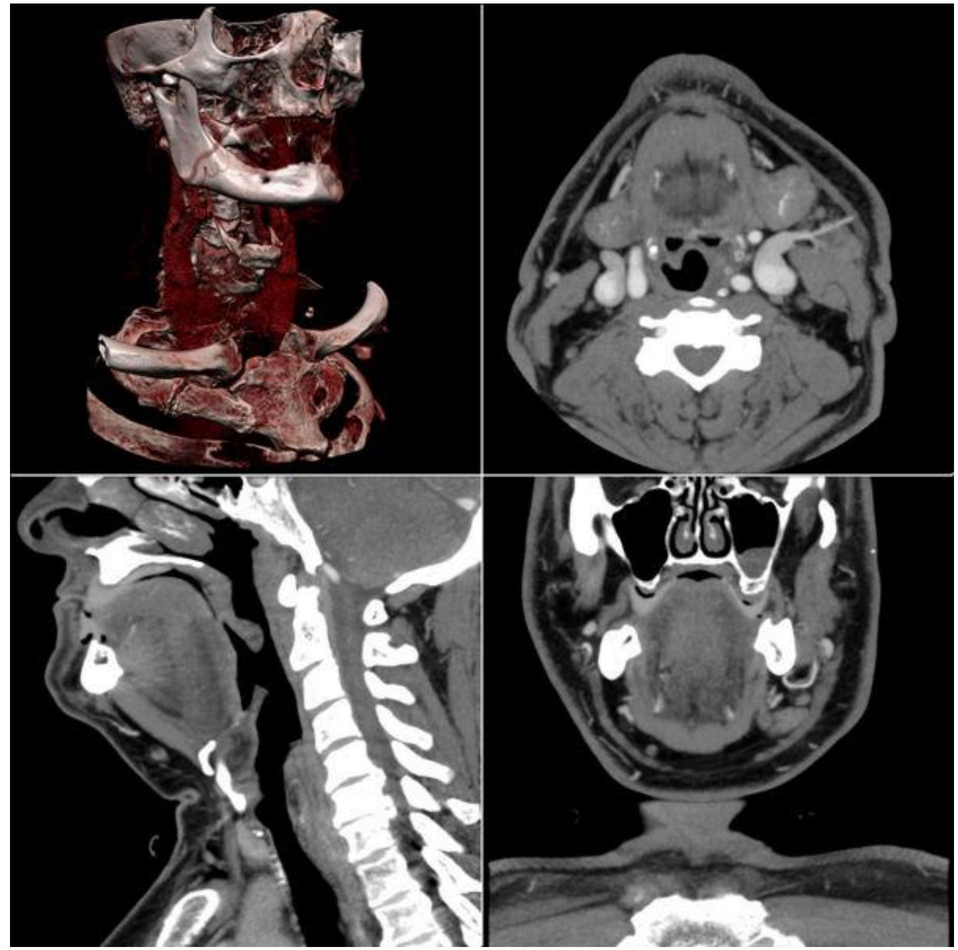
Back project and overlay the “projection rays”

Project the back-projected object and calculate the difference

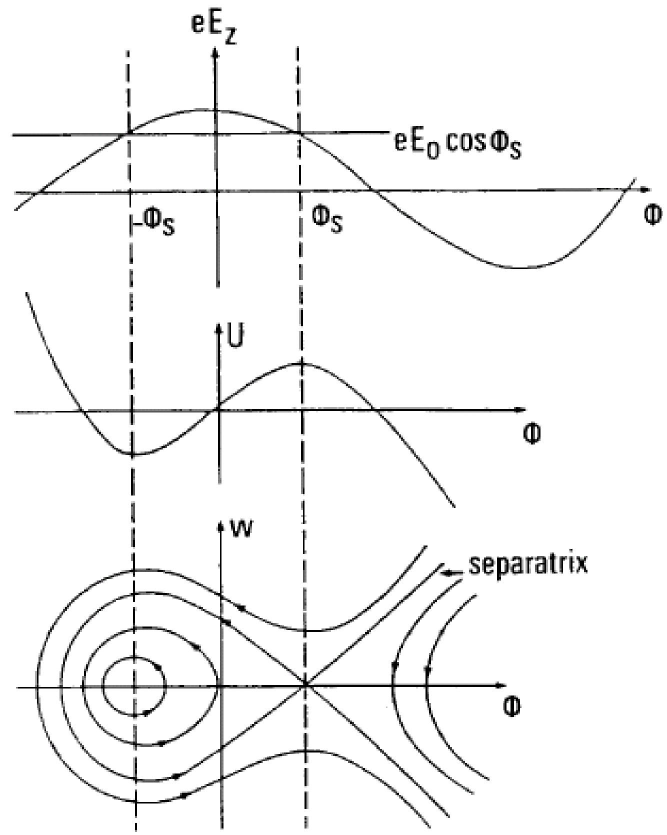
Iteratively back-project the differences to re-construct the original object

Some CT resuluts





Computed Tomography and Accelerators



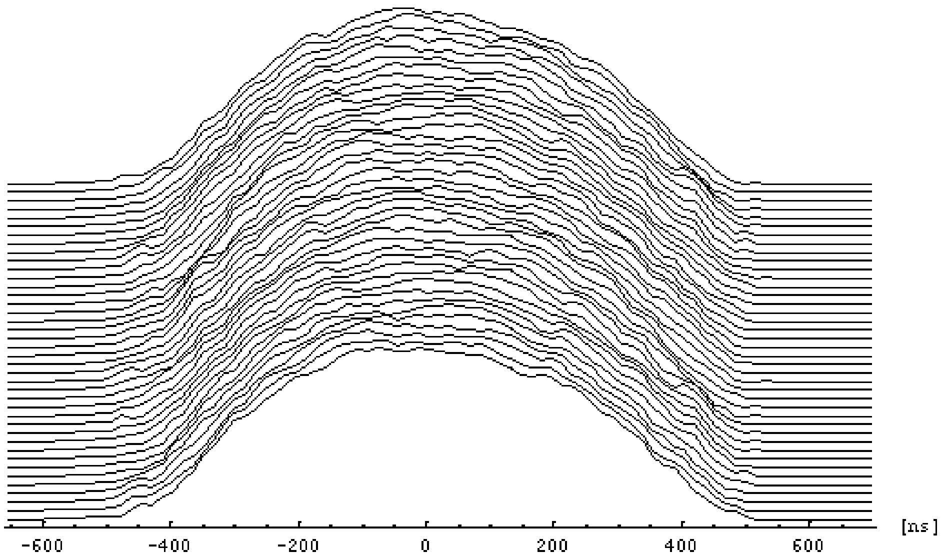
RF voltage

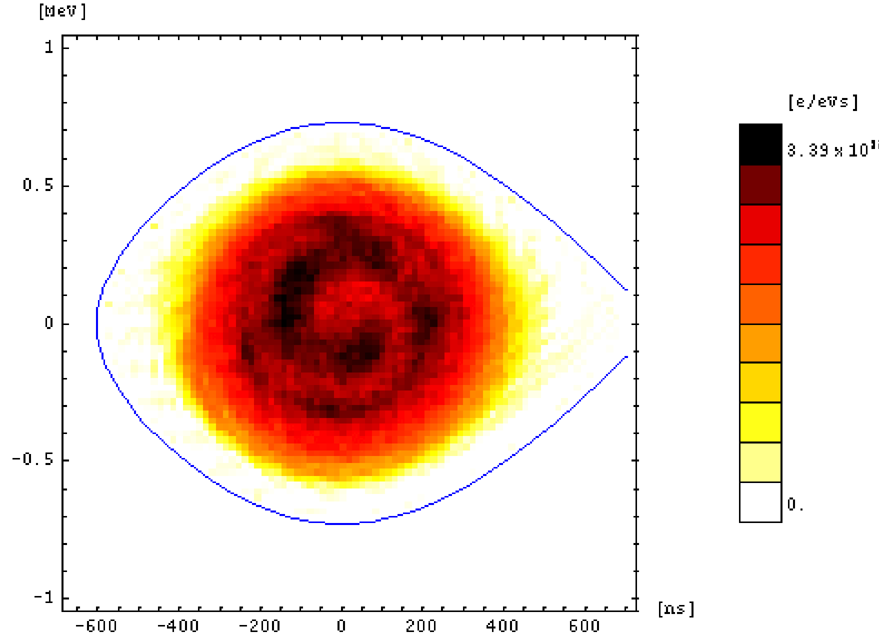
Restoring force for non-synchronous particle

Longitudinal phase space

Projection onto Φ axis corresponds to bunch profile

Reconstructed Longitudinal Phase Space





Bunch Splitting