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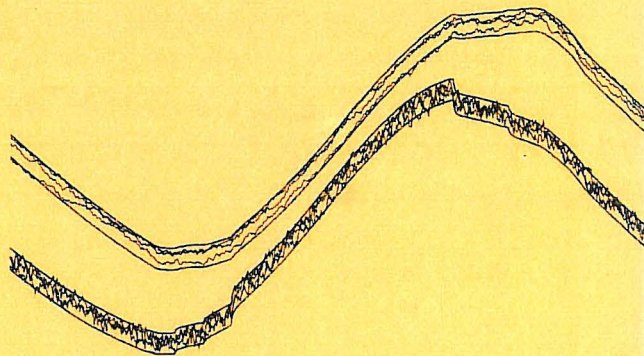
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# Probabilis

Probabilistic Analysis  
Methods for Hybrid  
Ventilation - Preliminary  
Application of Stochastic  
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# PROBABILISTIC ANALYSIS METHODS FOR HYBRID VENTILATION PRELIMINARY APPLICATION OF STOCHASTIC DIFFERENTIAL EQUATIONS

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## ABSTRACT

This paper discusses a general approach for the application of probabilistic analysis methods in the design of ventilation systems. The aims and scope of probabilistic versus deterministic methods are addressed with special emphasis on hybrid ventilation systems. A preliminary application of stochastic differential equations is presented comprising a general heat balance for an arbitrary number of loads and zones in a building to determine the thermal behaviour under random conditions.

## 1 INTRODUCTION

Today most ventilation systems are designed based on deterministic calculations of a few significantly different design cases, where uncertainties and randomness of the parameters are disregarded. In the area of building simulation, the dynamic thermal behaviour for the vast majority of cases is thus assumed deterministic.

The main reason for ignoring randomness is obviously the difficulties encountered in complexity and requirements for computer power, although the significance of the latter consideration decreases rapidly concurrently with the continuous increase in computer performance.

Another reason is the fact that mechanically ventilated heavy buildings are often highly "damped" and shielded toward external loads. This kind of buildings will also control the influence of the internal load effectively by means of the building energy management system and the HVAC system. Thus, the influence of randomness on for instance the internal air temperature becomes quite modest.

However, lighter constructions that are naturally or hybrid ventilated are much more sensitive to stochastic variations in the loads. The function depends thoroughly on natural driving forces characterised by external temperature, wind speed and direction, and the internal load of the building, etc. Due to the fact that this kind of buildings are increasing fast in number either as new built or retrofit stresses the importance of considering the effect of randomness in the design phase.

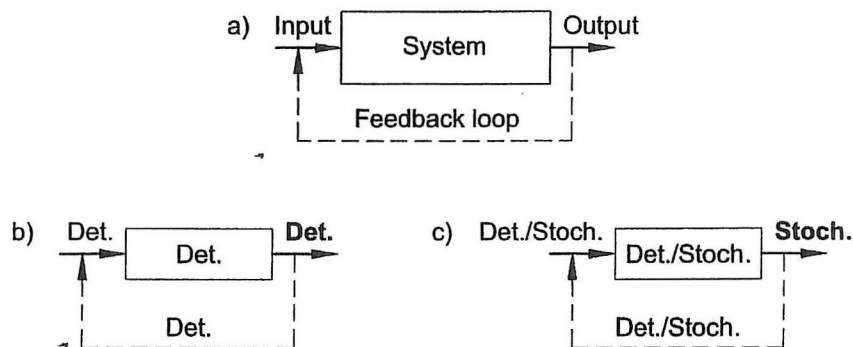
Probabilistic methods may establish a new approach to the design of ventilation systems, which, apart from more realistic modelling, enable designers to estimate the indoor climate, energy consumption and safety on the basis of the entire operating period and not just selected design cases. At the same time designers may include stochastic parameters like inhabitant behaviour, operation, and maintenance to predict the performance of the systems and the level of certainty for fulfilling design requirement under random conditions.

In this paper the aims and scope of probabilistic versus deterministic methods are addressed, and a number of relevant probabilistic methods are mentioned. The theory behind one of the methods, stochastic differential equations (SDE), will be applied on a heat balance for a building part in order to determine the dynamic thermal behaviour under random conditions. Randomness in the input as well as the model coefficients is considered. Results from a preliminary test case will be presented and discussed.

## 2 PROBABILISTIC APPROACH IN VENTILATION DESIGN

### 2.1 Deterministic and stochastic modelling

A model of a ventilated enclosure or building can be represented by the diagram shown in Figure 1.a. The system models the behaviour of the enclosure caused by a given set of input parameters and comprises the physical properties of the building and the ventilation system.



**Figure 1** Graphical representation of model. a) General model. b) Deterministic model (and output); input, system, and feedback loop are all deterministic. c) Stochastic model (and output); at least one component, apart from the output, must be stochastic.

The input consists of the loads applied on the enclosure, external as well as internal. The external loads are given by the outdoor climate, among typical parameters can be mentioned the external air temperature, wind speed and direction, and solar radiation. Among internal load parameters can be mentioned the inhabitants behaviour and heat generation.

The output from the system can be any parameter of interest that can be obtained knowing how the system responds to the input parameters. The output could e.g. be internal air temperature, indoor air quality or energy consumption.

Different control strategies are used in order to maintain the indoor climate of the enclosure. If for instance the internal air temperature exceeds a certain limit, the windows may open automatically. In that case the system reacts to changes in its own state and the model is said to contain feedback. This is represented in Figure 1.a with a feedback loop.

Conventionally, deterministic models have been used to investigate the behaviour of ventilated buildings. A deterministic model is shown in Figure 1.b. where the input, the system, and the feedback loop are described by deterministic quantities solely. Thus, it is assumed that all parameters defining the model are known with 100% certainty.

In reality, some or all of the quantities defining the model are random in nature. A stochastic model is shown in Figure 1.c. where at least one component in the set of parameters defining input, system and feedback loop are modelled stochastic.

The aim of a stochastic system analysis is to enable statistical description of output, given as statistics (e.g. mean value and standard deviation), probabilities, etc. when the input statistics and the corresponding system behaviour are known.

The first step in any stochastic calculation is to identify which parameters, eventually time dependent, to model stochastic, in order to reduce the complexity and the extent of the calculations. In many cases the variability of some parameters are insignificant compared with other parameters.

In some cases the input and the system can be regarded as time independent. In that case, a steady state calculation is performed and the input is modelled by stochastic variables. If, however, a temporal variation of the loads prevail and the thermal capacity of the building is considered, the input (loads) must be described as time dependent stochastic processes. The statistics of the input must then be modelled as functions of time and, similarly, the output from the models. The steady state methods are definitely simpler to use than time dependent methods, however, in return significantly less information is available from the calculations.

## **2.2 Examples of probabilistic methods**

### **Analytical determination of density distribution**

If the joint density function (expressing the mutual dependence between the stochastic parameters) for the input is sufficiently simple, the corresponding output joint density function can be determined analytically. When the joint density function of the output is known it is possible to derive all statistical quantities for the output. However, it is a prerequisite that an explicit analytical expression for the connection between input (loads) and output (temperature, air change rate, energy consumption, etc.) can be established.

The joint density function for the output is found by multiplying the joint density function for the input by the Jacobian determinant of the function describing the system connection between input and output. The method is applied by Hokoi and Matsumoto (1993) for a cooling plant. The advantage of the method is its generality, however, it is limited by the fact that a simple expression for the behaviour of the system must exist, which is unfortunately rare.

### **Monte Carlo Simulation**

By means of Monte Carlo Simulation (Rubinstein, 1981), realisations of the stochastic input parameters are generated according to their joint density functions. The joint density functions are determined either exact or approximated. Then the output is calculated from a deterministic model, which expresses the response of the building model. The procedure is repeated a large number of times and the resulting output data are treated statistically.

Simulation is often used to validate other methods or applied where they are insufficient. Sowa (1998) used the method to calculate the joint density function of the contaminant concentrations in a multizone model when the corresponding probability distributions of the loads are known.

### **Stochastic differential equations**

For a ventilated enclosure or building it is possible to set up a system of first order differential equations, in shape of for instance heat balance equations or contaminant concentration equations, by means of discretisation in a number of volumes each one representing a value of the requested parameter.

If the loads are assumed stochastic processes, it is possible to establish a system of stochastic differential equations (SDE) (Haghighat et al., 1987). The system of equations can then be solved for the requested output parameters by means of standard tools for numerical solution of differential equations, for instance the Runge-Kutta method (Press et al., 1989). A preliminary application of stochastic differential equations has been examined in section 3.



### Stochastic vibration theory

The starting point of stochastic vibration theory (SVT) is Newton's second law of motion for a vibrating system. The loads on the system are described as power spectra where the amount of energy over the frequency domain is specified. In that way, the corresponding power spectra for the output can be found when a transfer function between input and output is established. When the power spectrum for output is determined, the mean value and the standard deviation can be calculated.

This approach has been used by Haghghat et al. (1991) for a stochastic wind-induced air mass vibrating in an opening, purpose provided or component crack. The result is a second order differential equation that governs the behaviour of the air with regard to movement in openings of the building. By means of a power spectrum for the wind-induced pressure the corresponding power spectrum of the volume flow through the opening is calculated.

In conclusion, it can be stated that although analytical determination of the joint density function is an exact method, it is not possible to use in practice for the vast majority of realistic problems. MCS is rather simple to use, but quite demanding in terms of computer power. An obvious application of MCS is validation of other stochastic approaches. SDE seems to be a promising method, especially if a general and robust design tool can be established. SVT is assumed to be most applicable in more isolated applications.

## 3 APPLICATION OF STOCHASTIC DIFFERENTIAL EQUATIONS

In this section a stochastic model for a general heat balance problem is presented. A system of linear Stochastic Differential Equations (SDE) is formulated in section 3.1 describing the temporal variation of the zone temperatures in a building model. The procedure of solving the SDE system can be regarded as an operator transforming the stochastic input quantities to stochastic output quantities. The SDE approach presented here is based on the work conducted by Haghghat et al. (1987).

The input of the SDE system is modelled by a number of independent time varying stochastic processes representing internal and external loads applied on the building. The stochastic output in the present application is the zone temperatures. The solution of the SDE system is obtained either as time varying first and second order statistics, see section 3.2, or as a realisation of the zone temperatures, see section 3.3.

### 3.1 Building model

The model outlined in Figure 2 comprises a general heat balance for an arbitrary number of zones, surfaces and construction parts. There are  $n$  nodes with an unknown temperature (output) and  $m$  nodes with a known temperature; in the following denoted boundary nodes. There are  $k_i$  independent heat flux components applied on the  $i$ 'th node with an unknown temperature. For node  $i$  at the time  $t$  the following heat balance apply

$$C_i \frac{d\theta_i(t)}{dt} = \sum_{\substack{j=1 \\ j \neq i}}^n H_{ij}(t)(\theta_j(t) - \theta_i(t)) + \sum_{j=1}^m H_{ij}^b(t)(\theta_j^b(t) - \theta_i(t)) + \sum_{j=1}^{k_i} \Phi_{ij}(t) \quad , i = 1, 2, \dots, n \quad (1)$$

In the heat balance, (1), all or some of the input parameters,  $\theta_j^b, j = 1, 2, \dots, m, H_{ij}, i, j = 1, 2, \dots, n, H_{ij}^b, i = 1, 2, \dots, n, j = 1, 2, \dots, m$  and  $\Phi_{ij}, i = 1, 2, \dots, n, j = 1, 2, \dots, k_i$ , can be regarded as time varying stochastic processes, the rest being deterministic. Each stochastic input component, denoted by  $z(t)$ , is modelled by a time varying mean value function,  $\bar{Z}(t)$ , superimposed by a fluctuating component comprising of a time varying standard deviation function,  $\sigma_z(t)$ , that is multiplied by a standard white noise process,  $w(t)$

$$z(t) = \bar{Z}(t) + \sigma_z(t)w(t) \quad (2)$$

The white noise process is usually regarded as an appropriate model for rapidly random fluctuating phenomena, when correlation in time becomes small rapidly (Arnold, 1974). According to Tuckwell (1995) the white noise process,  $w(t)$ , is defined as the time derivative of the so called Wiener process,  $W(t)$ , such that

$$dW(t) = w(t)dt \quad (3)$$

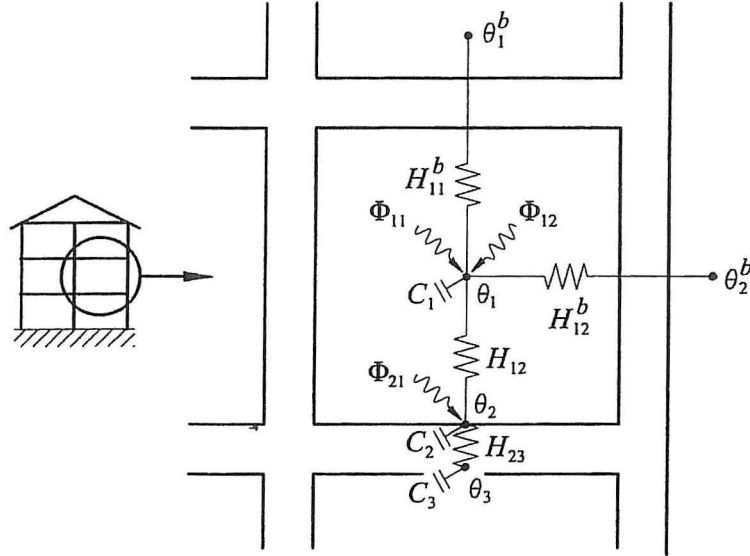


Figure 2 Building calculation model with one internal air temperature for each zone, e.g.  $\theta_i$ , assuming fully mixed conditions. On the surfaces and inside the structures an arbitrary number of nodes are possible, e.g.  $\theta_2$  and  $\theta_3$ . Boundary nodes like the adjacent zone temperature,  $\theta_i^b$ , and the external temperature,  $\theta_2^b$ , prescribe the loads together with convective heat flow supplied to air, e.g.  $\Phi_{11}$  and  $\Phi_{12}$ , and radiation heat flow supplied to surfaces, e.g.  $\Phi_{21}$ . The unknown nodes are interconnected via specific heat losses, e.g.  $H_{12}$  and  $H_{23}$ , and connected to the boundary nodes via another set of specific heat losses, e.g.  $H_{11}^b$  and  $H_{12}^b$ . The specific heat losses may also include heat transfer due to ventilation. The effective thermal heat capacities, e.g.  $C_1$ ,  $C_2$ , and  $C_3$ , are applied as "lumped masses" in the nodes.

By rearranging (1), applying (2) and (3) for each input process, and by removing higher order terms of fluctuating components, the following linear SDE system is obtained

$$d\theta = [\mathbf{X}\theta + \mathbf{x}]dt + \sum_{k=1}^{VAR} [\mathbf{Y}_k\theta + \mathbf{y}_k]dW_k \quad (4)$$

where VAR is the number of stochastic processes. The matrix,  $\mathbf{X}$ , and the vector,  $\mathbf{x}$ , are given as

$$X_{ij} = \begin{cases} -\frac{1}{C_i} \left[ \sum_{\substack{k=1 \\ k \neq i}}^n \bar{H}_{ik} + \sum_{k=1}^m \bar{H}_{ik}^b \right] & \text{for } i = j \\ \frac{1}{C_i} \bar{H}_{ij} & \text{otherwise} \end{cases} \quad i, j = 1, 2, \dots, n$$

$$x_i = \frac{1}{C_i} \left[ \sum_{k=1}^m \bar{H}_{ik}^b \theta_k^b + \sum_{j=1}^{k_i} \bar{\Phi}_{ij} \right] \quad i = 1, 2, \dots, n$$

The matrices,  $\mathbf{Y}_k$ ,  $k = 1, 2, \dots, VAR$  and the vectors  $\mathbf{y}_k$ ,  $k = 1, 2, \dots, VAR$ , depend on the type of each of the VAR input parameters. They are in turn listed below.

For the variables  $\theta_k^b$ ,  $k = 1, 2, \dots, m$

$$Y_{ij} = 0 \quad i, j = 1, 2, \dots, n$$

$$y_i = \frac{1}{C_i} \bar{H}_{ik}^b \sigma_{\theta_k^b} \quad i = 1, 2, \dots, n$$

For the variables  $H_{kl}$ ,  $k, l = 1, 2, \dots, n$

$$Y_{ij} = \begin{cases} -\frac{1}{C_i} \sigma_{H_{kl}} & \text{for } i = j = k \wedge k \neq l \\ \frac{1}{C_i} \sigma_{H_{kl}} & \text{for } i = k \wedge j = l \wedge k \neq l \\ 0 & \text{otherwise} \end{cases} \quad i, j = 1, 2, \dots, n$$

$$y_i = 0 \quad i = 1, 2, \dots, n$$

For the variables  $H_{kl}^b$ ,  $k = 1, 2, \dots, n, l = 1, 2, \dots, m$

$$Y_{ij} = \begin{cases} -\frac{1}{C_i} \sigma_{H_{kl}^b} & \text{for } i = j = k \\ 0 & \text{otherwise} \end{cases} \quad i, j = 1, 2, \dots, n$$

$$y_i = \begin{cases} \frac{1}{C_i} \bar{\theta}_i^b \sigma_{H_{kl}^b} & \text{for } i = k \\ 0 & \text{otherwise} \end{cases} \quad i = 1, 2, \dots, n$$

And for the variables  $\Phi_{kl}$ ,  $k = 1, 2, \dots, n, l = 1, 2, \dots, k_i$

$$Y_i = 0 \quad i, j = 1, 2, \dots, n$$

$$y_i = \begin{cases} \frac{1}{C_i} \sigma_{\Phi_{kl}} & \text{for } i = k \\ 0 & \text{otherwise} \end{cases} \quad i = 1, 2, \dots, n$$

The SDE system, (4), can be solved for the second order statistics, see section 3.2, or for a realisation of the output, see section 3.3.

### 3.2 Moment equations

A convenient description of the stochastic output parameters, i.e. the zone temperatures, are their first and second order statistics given by the time varying expected values,  $E[\theta_i]$ ,  $i = 1, 2, \dots, n$ , and the second order statistical moments,  $E[\theta_i \theta_j]$ ,  $i, j = 1, 2, \dots, n$ . Arnold (1974) shows that the following deterministic differential equations for the first and second order moments of the zone temperatures corresponding to the SDE system (4) can be determined. The two equations (5) and (6) express the statistical moments and the initial conditions for the first and the second order moments, respectively



$$\begin{cases} \frac{dE[\theta]}{dt} = \mathbf{X}E[\theta] + \mathbf{x} \\ E[\theta(t=0)] = E[\theta^0] \end{cases} \quad (5)$$

$$\begin{cases} \frac{dE[\theta\theta^T]}{dt} = \mathbf{X}E[\theta\theta^T] + E[\theta\theta^T]\mathbf{X}^T + \mathbf{x}E[\theta]^T + E[\theta]\mathbf{x}^T \\ + \sum_{k=1}^{VAR} [\mathbf{Y}_k E[\theta\theta^T] \mathbf{Y}_k^T + \mathbf{Y}_k E[\theta] \mathbf{y}_k^T + \mathbf{y}_k E[\theta]^T \mathbf{Y}_k^T + \mathbf{y}_k \mathbf{y}_k^T] \\ E[\theta(t=0)\theta(t=0)^T] = E[\theta^0\theta^{0T}] \end{cases} \quad (6)$$

The equations can be solved for instance by the standard fourth order Runge-Kutta method (Press et al., 1989).

### 3.3 Response realisation

The SDE system (4) has an infinite number of solutions. Every solution corresponds to a realisation of the solution process. The alternative approach of Monte Carlo Simulation uses realisations of the stochastic input processes.

Equation (4) can be rewritten to obtain the following expression

$$d\theta = \mathbf{f}(\theta, t)dt + \mathbf{G}(\theta, t)d\mathbf{W} \quad (7)$$

where the  $\mathbf{f}$  vector,  $\mathbf{G}$  matrix and  $d\mathbf{W}$  vector are given as

$$\mathbf{f}(\theta, t) = \mathbf{X}\theta + \mathbf{x}$$

$$\mathbf{G}(\theta, t) = [\mathbf{Y}_1\theta + \mathbf{y}_1 \cdots \mathbf{Y}_{VAR}\theta + \mathbf{y}_{VAR}]$$

$$d\mathbf{W}^T = [dW_1 \cdots dW_{VAR}]$$

The components of the  $d\mathbf{W}$  vector are generated as realisations of independent normally distributed variables with zero mean and variance  $dt$ . The equation (7) can be solved for instance by a stochastic version of the fourth order Runge-Kutta method, see Arnold (1974).

## 4 TEST CASE

A test case has been chosen in order to demonstrate the SDE approach, see Figure 3. The test case comprises two zones in a typical mechanically ventilated building exposed to internal and external loads. Each zone may represent either one single room or a number of closely related rooms in proper contact. The number of unknown temperatures is six.

The thermal capacity of each zone corresponds to a medium heavy building. The ventilation corresponds to an air change rate of approximately  $3 \text{ h}^{-1}$  expressed in terms of  $H_{vent}$ .

The two main zones are surrounded by three adjacent zones A, B, and C, where C is shared. The mean temperature and the standard deviation for the three adjacent zones etc. are listed in Table 1.

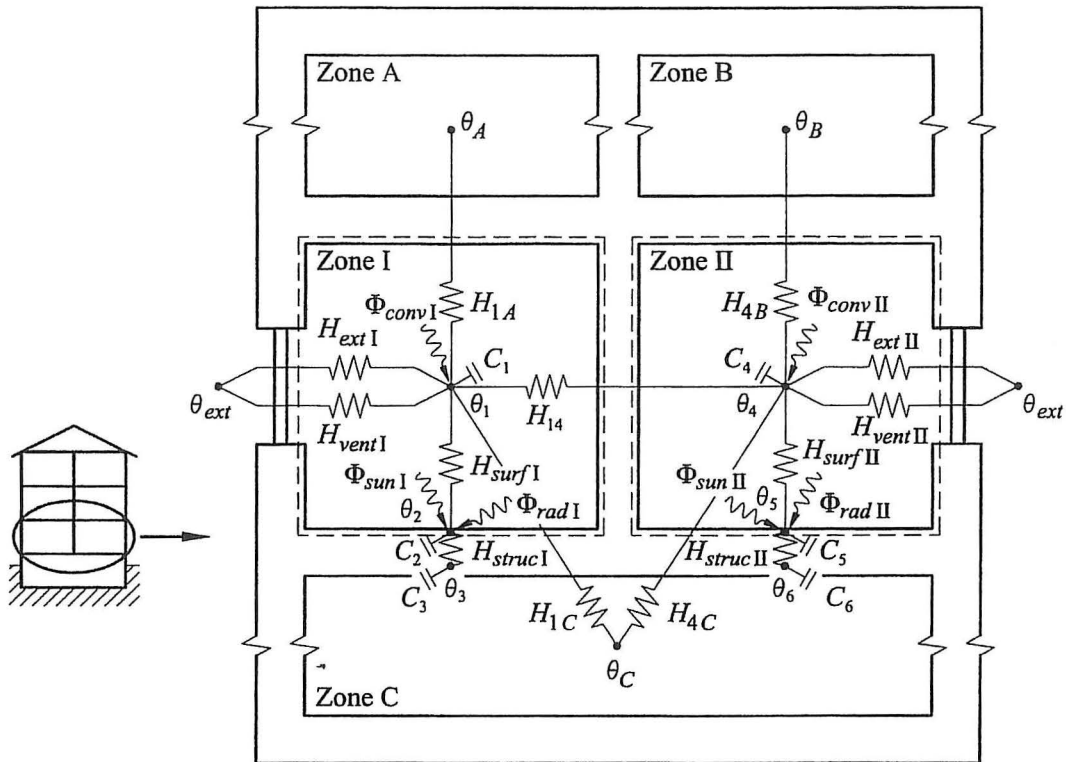


Figure 3 Test case building model: Two zones surrounded by three adjacent zones and the external climate. The external temperature,  $\theta_{ext}$ , is shared by Zone I and Zone II. For each zone, three unknown temperatures are to be determined (internal air, surface layer, and structure), i.e.  $\theta_1 - \theta_3$  for Zone I and  $\theta_4 - \theta_6$  for Zone II. The surface layer accounts for one fourth of the thermal capacity of the room and the “structure” accounts for the rest. By means of the specific heat transfer coefficient for loss between Zone I and Zone II,  $H_{14}$ , mutual energy exchange between the zones is allowed (transmission and ventilation heat loss).

Table 1 Parameters applied in the Test case. Symbols refer to Figure 3

Parameter	Unit	Mean value	Standard deviation	Stochastic ?
$\theta_1 - \theta_6$	°C	Output	Output	Yes
$\theta_{ext}$	°C	Data from Danish DRY	Data from Danish DRY	Yes
$\theta_A$	°C	20	2	Yes
$\theta_B$	°C	26	2	Yes
$\theta_C$	°C	18	1	Yes
$C_1, C_4$	J/K	75 000	0	No
$C_2, C_5$	J/K	1 250 000	0	No
$C_3, C_6$	J/K	3 750 000	0	No
$H_{ventI}, H_{ventII}$	W/K	60	15	Yes
$H_{extI}, H_{extII}$	W/K	15	0	No
$H_{1A}, H_{4B}$	W/K	20	0	No
$H_{1C}, H_{4C}$	W/K	25	0	No
$H_{14}$	W/K	100	30	Yes
$H_{surfI}, H_{surfII}$	W/K	300	0	No
$H_{strucI}, H_{strucII}$	W/K	1 000	0	No
$\Phi_{convI}, \Phi_{convII}, \Phi_{radI}, \Phi_{radII}$	W	Assumed data sets	0.3 times mean value	Yes
$\Phi_{sunI}$ (north), $\Phi_{sunII}$ (south)	W	Data from Danish DRY	Data from Danish DRY	Yes

The calculations are performed during a warm week in summer (mid August), which can be thought of as a kind of design load period.

Two assumed 24-hour load profiles, one for each zone, for the mean values of the internal sensible heat load are applied as step profiles in the simulation. The load profiles are repeated when the simulation exceeds 24 hours. The sensible heat is divided into 50% convective heat,  $\Phi_{conv}$ , and 50% radiative heat,  $\Phi_{rad}$ . The convection heat flow is assumed to influence the internal air, e.g.  $\theta_1$ , and the radiation heat flow is assumed to influence the surface layer, e.g.  $\theta_2$ .

The external climate is "shared" regarding the external temperature, see Figure 3, whereas the solar radiation depends on the orientation of the windows. The Zone I facade is facing north and the Zone II facade is facing south. Mean value processes and standard deviation processes for the external temperature as well as the solar radiation (direct and diffuse) are modelled by means of time series analysis of the Danish Design Reference Year (DRY). The direct and diffuse solar radiation are used to determine the resulting solar gain in each zone,  $\Phi_{sun}$ . The initial mean temperatures are chosen to be 20°C. The initial second order moments are chosen to obtain zero initial standard deviations.

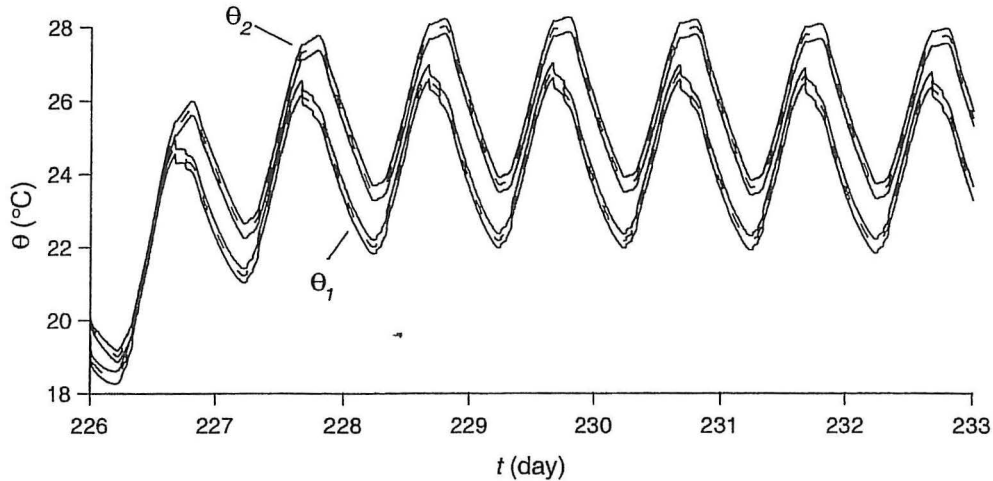


Figure 4 Internal air temperature in Zone I,  $\theta_1$ , and surface layer temperature in Zone I,  $\theta_2$ . The figure shows the mean value (dashed lines) and the 95% confidence interval (solid lines).

Figure 4 shows the mean value process for the internal air and the surface layer temperature of Zone I. In addition the 95% confidence interval is shown corresponding to the mean value function  $\pm 1.96$  times the standard deviation function.

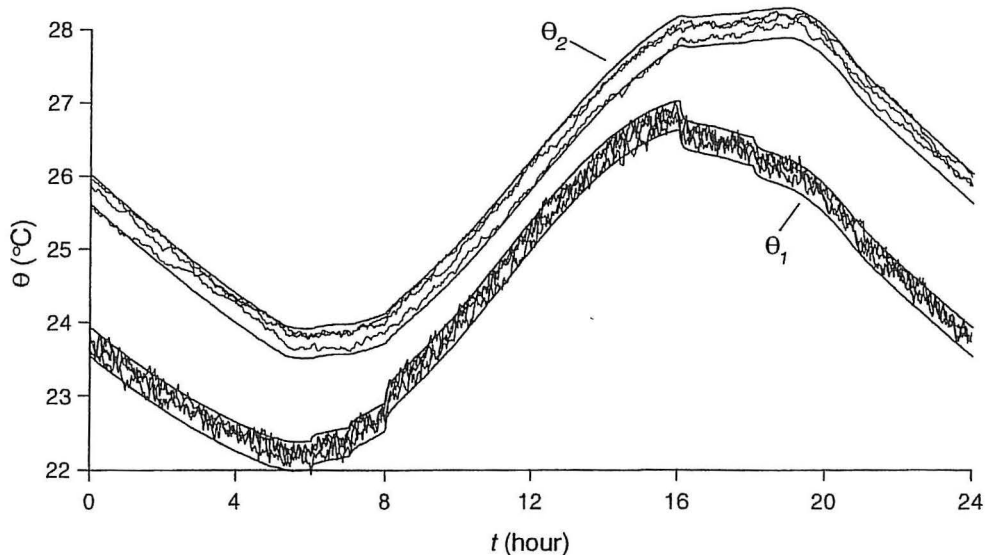


Figure 5 Three stochastic realisations of internal air temperature in Zone I,  $\theta_1$ , and surface layer temperature in Zone I,  $\theta_2$  (thin lines). The figure also shows the 95% confidence intervals (thick lines). In order to illustrate the stochastic fluctuations more clearly the plot is limited to show 24 hours of the week (day 229).

Due to solar radiation and internal long wave radiation, the surface layer temperature exceeds the internal air temperature and, thus, net heat flow from the surface to the room air prevails in this case.



In Figure 5 three realisations of the stochastic output processes are presented together with the 95% confidence interval for the same temperatures as shown in Figure 4. If the confidence intervals of the two temperatures are compared it is found that their sizes are of the same order of magnitude. However, if the fluctuations are observed it is easily seen that the overall stochastic behaviour of the two temperatures differ. The internal air temperature,  $\theta_1$ , obviously fluctuates more rapidly compared with the surface layer temperature,  $\theta_2$ . Expressed in statistical terms, the autocorrelation of  $\theta_2$  is higher than the autocorrelation of  $\theta_1$ , which corresponds well with the physics of the problem. This message is not conveyed by the statistical moment approach.

The random fluctuations in this application are relatively small when compared with the mean values. In this case, the SDE approach is applied on a heat balance in order to examine the thermal behaviour of a relatively heavy building. However, when the approach is applied on a mass balance in order to determine the contaminant concentration or the ventilation capacity in a relatively light and hybrid ventilated building it is expected that randomness will play a more significant role.

## 5 CONCLUSIONS

General characteristics of probabilistic analysis methods in the design of ventilation systems are outlined. The aims and scope of probabilistic versus deterministic methods are addressed with special emphasis on hybrid ventilation systems. Several relevant existing probabilistic methods are mentioned and discussed.

An application of stochastic differential equations is presented and applied on a general heat balance for an arbitrary number of loads and zones in a building to determine the dynamic thermal behaviour under random conditions. Randomness in input as well as model coefficients are considered. A test case is presented comprising two zones in a mechanically ventilated building.

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