The Weak Interaction

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Contents

1 Introduction

The nuclear β-decay caused a great deal of anxiety among physicists. Both α- and γ-rays are emitted with discrete spectra, simply because of energy conservation. The energy of the emitted particle is the same as the energy difference between the initial and final state of the nucleus. It was much more difficult to see what was going on with the β-decay, the emission of electrons from nuclei. Chadwick once reported that the energy spectrum of electrons is continuous. The energy could take any value between 0 and a certain maximum value. This observation was so bizarre that many more experiments followed up. In fact, Otto Han and Lise Meitner, credited for their discovery of nuclear fission, studied the spectrum and claimed that it was discrete. They argued that the spectrum may appear continuous because the electrons can easily lose energy by breamsstrahlung in material. The maximum energy observed is the correct discrete spectrum, and we see lower energies because of the energy loss. The controversy went on over a decade. In the end a definitive experiment was done by Ellis and Wooseley using a very simple idea. Put the β-emitter in a calorimeter. This way, you can measure the total energy deposit. They demonstrated that the total energy was about a half of the maximum energy on average. The spectrum is indeed continuous. The fact that the β-spectrum is continuous was so puzzling to people, even inspiring Niels Bohr to say *"At the present stage of atomic theory, however, we may say that we have no argument, either empirical or theoretical, for upholding the energy principle in the case of β-ray disintegrations"*. He was ready to give up the energy conservation! This quote shows how desperate people were.

The solution to the problem was devised by Pauli. In 1930, he wrote a letter to colleagues attending a meeting at Tubingen. Here is a quote from his letter: `4th December 1930`

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   Dear Radioactive Ladies and Gentlemen, As the bearer of these lines, to whom I graciously
ask you to listen, will explain to you in more detail, how because of the "wrong" statistics
of the N and Li6 nuclei and the continuous beta spectrum, I have hit upon a desperate
remedy to save the "exchange theorem" of statistics and the law of conservation of energy.
Namely, the possibility that there could exist in the nuclei electrically neutral particles,
that I wish to call neutrons, which have spin 1/2 and obey the exclusion principle and
which further differ from light quanta in that they do not travel with the velocity of
light.  The mass of the neutrons should be of the same order of magnitude as the electron
mass and in any event not larger than 0.01 proton masses.  The continuous beta spectrum
would then become understandable by the assumption that in beta decay a neutron is emitted
in addition to the electron such that the sum of the energies of the neutron and the
electron is constant...
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Pauli's "neutron" became our neutrino and the process of β-decay became

$$n \to p + e^- + \overline{\nu_e} \tag{1}$$

2 The Weak Interaction

The weak interaction is responsible for radioactive decays. It is characterised by long lifetimes, and small cross sections. All fermions feel the weak interaction. When present, though, strong and electromagnetic interactions dominate.

Of special note are the neutrinos. Neutrinos feel only the weak interaction, which is what makes them so difficult to study. They are the only particles to experience just one of the fundamental forces.

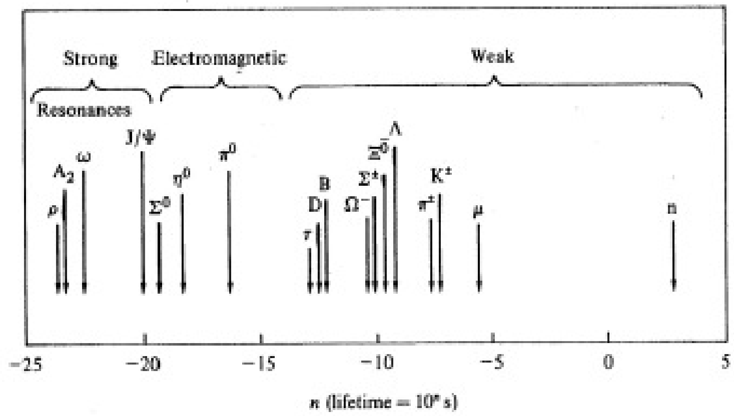

Figure 1: Lifetime of various decays. The strong decays are the fastest, followed by the electromagnetic decays and then the weak decays.

2.1 The 4-point Interaction

The first attempt to construct a theory of the weak interaction was made by Fermi in 1932. In analogy to the electromagnetic interaction, he imagined a 4-point interaction that happened at a single point in space-time. His idea of β-decay is shown in Figure2

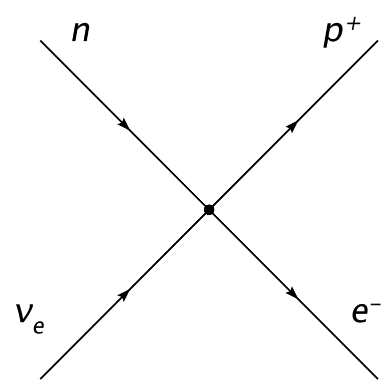

Figure 2: Fermi's 4-point interaction.

In analogy to the electromagnetic interaction, Fermi proposed the following matrix element

$$M = \frac{G_F}{\sqrt{2}}[\overline{u_P}\gamma^\mu u_N][\overline{u_e}\gamma^\nu u_\nu] \tag{2}$$

We take note of some points here

- **Charge changing :** The hadronic current has $\Delta Q = +1$, whereas the lepton current has $\Delta Q = -1$. There is net charge transferred from the hadronic to the lepton current and so we call this a *charged current* interaction.

- **Universality :** There is a coupling factor, G_F, called the Fermi Constant, equal to $1.166 \times 10^{-5}\text{GeV}^{-2}$. Fermi postulated, and is has later been shown to stand up to experiment, that the weak coupling factor is the same for all weak vertices, regardless of the flavour of lepton taking part. This is called *universality* and is an extremely important concept.
- There is no propagator
- The currents have a vector character, purely in analogy to the electromagnetic interaction where it was known that the currents were vector in nature.

The cross section for the interaction $\nu_e + n \rightarrow p + e^-$, as generated from Fermi's 4-point interaction, was calculate shortly after by Bethe. He found that

$$\sigma(n + \nu_e \rightarrow e^- + p) \sim E_\nu(MeV) \times 10^{-43} cm^2 \tag{3}$$

. This is extremely small. You would need about 50 light-years of water to stop one 1 MeV neutrino.

This cross-section also has a problem. It rises linearly with energy ... for ever. This is clearly incorrect and shows that the Fermi model breaks down at high energies. We need a bit of modification to the theory. We need to add a propagator.

2.2 Weak Propagator

We now know that the weak interaction is mediated by two massive gauge bosons : the charged $W^\pm$ and the neutral Z^0. The propagation term for the massive boson is $\frac{1}{M^2_{W,Z} - q^2}$. If we assume that the Fermi theory is the low energy limit of the Weak Interaction, then we can estimate the intrinsic coupling at high energy. In the Fermi limit, the coupling factor appears to $\frac{G_F}{\sqrt{2}}$. At low energies, with $M^2_{W,Z} >> q^2$, the propagator term reduces to just $\frac{1}{M^2_W}$ and we can make the identification

$$\frac{G_F}{\sqrt{2}} = \frac{{g_w}^2}{8M^2_W} \tag{4}$$

. The factor of 8 is just convention, arising from the assignment of the weak vertex factor in Feynman diagrams to be $g_W/2\sqrt{2}$.

g W g

$$Coupling \sim \frac{G_F}{\sqrt{2}} \qquad Coupling :\sim \frac{g^2_w}{8M^2_W}$$

This allows us to compare the intrinsic couplings of the weak interaction with the electromagnetic interaction. The mass of the W boson is $80.4GeV$ and the Fermi constant is $1.166 \times 10^{-5}\text{GeV}^{-2}$.

Plugging this into Equation 4 we get a weak coupling factor of $g_w = 0.65$. Now, remember that the electromagnetic interaction coupling factor is the square root of the fine structure constant, we have

$$\text{EM coupling}: \ \alpha_{EM} = \frac{1}{137} \quad \text{Weak coupling}: \ \alpha_W = \frac{g_w^2}{4\pi} = \frac{1}{30} \tag{5}$$

In fact the weak interaction is, intrinsically, about 4 times *stronger* than the electromagnetic interaction. What makes the interaction so weak is the large mass of the relevant gauge bosons. In fact at very high energies, where $q^2 \sim M_W^2$, the weak interaction is comparable in strength to the electromagnetic interaction.

How about the high energy behaviour? At high energies the mass of the W-boson supresses the total cross section and stops it going to infinity. So the propagator solves that issue as well.

3 Parity Violation

3.1 Parity and The Parity Operator

The parity operation is defined as spatial inversion around the origin :

$$t' \equiv t \quad x' \equiv -x \quad y' \equiv -y \quad z' \equiv -z \tag{6}$$

Consider a Dirac spinor, $\psi(t, x, t, z)$. A parity transformation would transform this spinor to

$$\hat{P}\psi(t, x, y, z) = \psi(t, -x, -y, -z) = \psi'(t', x', y', z') \tag{7}$$

. We can prove that the relevant operator is actually γ^0. That is,

$$\psi'(t', x', y', z') = \psi(t, -x, -y, -z) = \pm\gamma^0\psi(t, x, y, z) \tag{8}$$

.

Consider a Dirac spinor, $\psi(t, x, y, z)$, that obeys the Dirac equation

$$i\gamma^0\frac{\partial\psi}{\partial t} + i\gamma^1\frac{\partial\psi}{\partial x} + i\gamma^2\frac{\partial\psi}{\partial y} + i\gamma^3\frac{\partial\psi}{\partial z} - m\psi = 0 \tag{9}$$

Under the parity transformation : $\psi'(x', y', z', t') = \hat{P}\psi(x, y, z, t) = \gamma^0\psi(x, y, z, t)$. Since $(\gamma^0)^2 = 1$, this implies that

$$\psi(x, y, z, t) = \gamma^0\psi'(x', y', z', t') \tag{10}$$

Substituting this into the Dirac equation we have

$$i\gamma^0\gamma^0\frac{\partial\psi'}{\partial t} + i\gamma^1\gamma^0\frac{\partial\psi'}{\partial x} + i\gamma^2\gamma^0\frac{\partial\psi'}{\partial y} + i\gamma^3\gamma^0\frac{\partial\psi'}{\partial z} - m\gamma^0\psi' = 0 \tag{11}$$

We use the chain rule to express the derivative in terms of the primed coordinate system e.g.

$$\frac{\partial\psi'}{\partial x} = \frac{\partial x'}{\partial x}\frac{\partial\psi'}{\partial x'} = -\frac{\partial\psi'}{\partial x'} \tag{12}$$

since $x' = -x$ under parity. In the Dirac equation,

$$i\gamma^0\gamma^0\frac{\partial\psi'}{\partial t'} - i\gamma^1\gamma^0\frac{\partial\psi'}{\partial x'} - i\gamma^2\gamma^0\frac{\partial\psi'}{\partial y'} - i\gamma^3\gamma^0\frac{\partial\psi'}{\partial z'} - m\gamma^0\psi' = 0 \quad (13)$$

and since γ^0 anticommutes with γ^i for $i = 1, 2, 3$,

$$i\frac{\partial\psi'}{\partial t'} + i\gamma^0\gamma^1\frac{\partial\psi'}{\partial x'} + i\gamma^0\gamma^2\frac{\partial\psi'}{\partial y'} + i\gamma^0\gamma^3\frac{\partial\psi'}{\partial z'} - m\gamma^0\psi' = 0 \quad (14)$$

Multiplying on the left by γ^0, and recalling that $(\gamma^0)^2 = 1$, we then get

$$i\gamma^0\frac{\partial\psi'}{\partial t'} + i\gamma^1\frac{\partial\psi'}{\partial x'} + i\gamma^2\frac{\partial\psi'}{\partial y'} + i\gamma^3\frac{\partial\psi'}{\partial z'} - m\psi' = 0 \quad (15)$$

which is the Dirac equation in the primed coordinates. Hence, under parity transformations the Dirac equation is unchanged (as it should be) *provided* that the bispinors transform as

$$\psi \to \hat{P}\psi = \pm\gamma^0\psi \quad (16)$$

If we apply the parity operator twice then we must return the original wavefunction : $\hat{P}^2 = {\gamma^0}^2 = 1$. The eigenvalues of the parity operator are, therefore, ± 1. Hadrons are eigenstates of $\hat{P}$. The parity of a fermion is opposite that of the anti-fermion, whereas the parity of a boson is the same as its anti-boson. We arbitrarily take particles to have *positive or "even" intrinsic parity*, and the anti-particle (if a fermion) is said to have *negative or "odd" parity.* The parity of a combined system is the product of the parity of its constituent parts.

3.2 Parity Violation

In 1956, T.D. Lee and C.N. Yang were trying to solve a very puzzling problem called the $\tau - \theta$ problem. Two strange mesons, called the τ and the θ, appeared to be identical in every respect : mass, spin, charge etc. The problem was that the τ was observed to decay into three pions $\pi^+\pi^+\pi^-$ or $\pi^+\pi^0\pi^0$. The other one, the θ , decays into two pions $\pi^+\pi^0$. Both are spin zero particles of strangeness one. The analysis of the final state showed that the τ decays into a parity odd state, while the θ into a parity even state. This seems impossible if the two particles were the same. Lee and Yang, after studying this, pointed out in 1956 that maybe these two particles could be the same particle. Of course this would be possible only if the parity is not preserved in these decays. They examined carefully the available evidence for parity conservation, and concluded that there was a lot of evidence for parity conservation in the strong and the electromagnetic interactions, while there was none in the weak interaction. They further proposed various ways the parity (non)conservation could be tested experimentally in the weak interaction.

Almost immediately C.S. Wu devised and carried a beautiful experiment to test the possibility of parity violation in beta decay. She set up a system of $^{Co}60$ atoms which all decayed via β emission to $^{Ni}60$. She aligned them in a magnetic field, so that all their spin vectors lined up and then let them decay, measuring the direction of the outgoing electron. If parity were conserved, she would expect to see electrons emitted isotropically. Why? Have a look at Figure 3

The spin vector of the Cobalt atom, labelled as J in the diagram, points to the left in both this world and the parity transformed mirror world. It is an example of an *axial vector* which doesn't change direction under a co-ordinate inversion (another example is angular momentum : $\mathbf{L} = \mathbf{r} \times \mathbf{p}$. Under a co-ordinate inversion, $\mathbf{r} \to -\mathbf{r}$ and $\mathbf{p} \to -\mathbf{p}$, which leaves the angular momentum unchanged). Suppose an electron were to be omitted in the direction of the spin vector in this world. In the

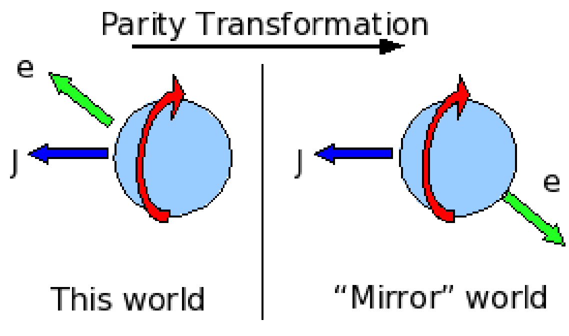

Figure 3: A schematic of Wu's parity conservation experiment.

mirror world the electron will be going in the other direction, opposite the direction of spin. Parity conservation implies that the probability of one interaction happening in this world is the same as the probability of it's mirror image occurring, and so we should see the same numbers of events where the electron were emitted anti-parallel to the spin, as the number of events in which the electron were emitted parallel to the spin vector.

What Wu saw was that electrons were emitted preferentially in the direction of the spin vector - a clear violation of parity conservation. It wasn't small either - *almost all* of the electrons were emitted in only one direction. It seemed as if the violation was *maximal*.

Parity, which had long been believed to be a true and fundamental symmetry of nature, fell in 1957, traumatising many respectable physicists.

3.3 CP Violation

Many desperate physicists tried to save the situation by appealing to CP invariance. We know that parity (P) is violated in the weak interaction, which can be seen from the decay

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \tag{17}$$

in which the neutrino is always emitted with left-handed helicity.

The weak interaction is not invariant under charge conjugation (C) either. For the charge conjugate of the previous decay is

$$\pi^- \rightarrow \mu^- + \overline{\nu_\mu} \tag{18}$$

in which the anti-neutrino still has left-handed helicity. The anti-neutrino in the real world always comes out *right*-handed. However if we combine the two operations we are back in business : CP changes a left-handed neutrino into a right-handed anti-neutrino, which is what is observed in nature. Many people breathed a sigh of relief, deciding that what we should have meant by the "mirror"-image of a right-handed electron was a left-handed positron.

Unhappily for them, CP is also violated. This was first shown by Cronin and Fitch (that's another lecture course) in 1964. It's small, about 0.3% of weak interactions violate CP, but it's there. It means that there is a true violation of mirror symmetry in nature which can't be argued away be redefinitions, and that there is a difference in the laws of nature in our world and in the mirror world. This is lucky for us as it is probably the reason why we now live in a matter-dominated universe.

Name	Symbol	Current	Number of components	Effect under Parity
Scalar	S	$\overline{\psi}\psi$	1	+
Vector	V	$\overline{\psi}\gamma^\mu\psi$	4	(+,-,-,-)
Tensor	T	$\overline{\psi}\sigma^{\mu\nu}\psi$	6	
Axial Vector	A	$\overline{\psi}\gamma^\mu\gamma^5\psi$	4	(+,+,+,+)
Pseudo-Scalar	P	$\overline{\psi}\gamma^5\psi$	1	-

Table 1: All possible bilinear covariant combinations of γmatrices

3.4 Building it into the theory - the V-A Interaction

Alright. So parity is violated - let's not worry about how (in fact, noone really knows yet). How do we go about building this into our model so we can at least describe it? To do this we go back to our currents. The most general matrix element we can write is

$$M \propto [\overline{u_{\psi,f}}\ \hat{O}\ u_{\psi,i}]\frac{1}{M^2 - q^2}[\overline{u_{\phi,f}}\ \hat{O}\ u_{\phi,i}] \tag{19}$$

where $\hat{O}$ is a combination of γ matrices.

It turns out that there are only 5 independent bilinear covariant expressions that you can form out of the γ matrices. They are labelled for how they behave under the Parity operation (see Table 1).

In this table $\sigma^{\mu\nu} = \frac{i}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$.

Now, let's see how each of these currents behaves under a parity transformation. Ignoring the tensor current (which has two indices, rather than one and which therefore will not represent a theory which, at low energies, is a point-contact interaction) and noting that the parity transformation is

$$\psi' = \gamma^0\psi \tag{20}$$

$$\overline{\psi'} = (\psi')^\dagger\gamma^0 = (\gamma^0\psi)^\dagger\gamma^0 = \psi^\dagger\gamma^{0\dagger}\gamma^0 = \psi^\dagger \tag{21}$$

where we have used the property that $\gamma^{0\dagger} = \gamma^0$ and $(\gamma^0)^2 = 1$.

- **Scalar, S :**

$$\overline{\psi}\psi \to \overline{\psi'}\psi' = \psi^\dagger\gamma^0\psi \tag{22}$$

$$= \overline{\psi}\psi \tag{23}$$

$$\tag{24}$$

- **Vector, V :**

$$\overline{\psi}\gamma^\mu\psi \to \overline{\psi'}\gamma^\mu\psi' = \overline{\psi}\gamma^0\gamma^\mu\gamma^0\psi \tag{25}$$

$$= \overline{\psi}\gamma^0\psi(\mu = 0) \tag{26}$$

$$-\overline{\psi}\gamma^\mu\psi(\mu > 0) \tag{27}$$

$$\tag{28}$$

- **Axial Vector, A :**

$$\overline{\psi}\gamma^\mu\gamma^5\psi \rightarrow \overline{\psi'}\gamma^\mu\gamma^5\psi' = \overline{\psi}\gamma^0\gamma^\mu\gamma^5\gamma^0\psi \quad (29)$$
$$\overline{\psi}\gamma^\mu\gamma^5\psi \quad (30)$$
$$(31)$$

- **Pseudo Scalar, P :**

$$\overline{\psi}\gamma^5\psi \rightarrow \overline{\psi'}\gamma^5\psi' = \overline{\psi}\gamma^0\gamma^5\gamma^0\psi \quad (32)$$
$$= -\overline{\psi}\gamma^5\psi \quad (33)$$
$$(34)$$

Unravelling which of these currents was responsible for the weak interaction took quite a lot of experimental and theoretical time. We are looking for a combination for which the charged weak interaction only couples to left-handed chiral particles. The left-handed chiral projection operator is $P_L = \frac{1}{2}(1-\gamma^5)$. Hence the current we want looks something like

$$\overline{\psi}\hat{O}\frac{1}{2}(1-\gamma^5)\phi \quad (35)$$

. To cut a very long story short, experiment showed that the operator $\hat{O}$ was just the vector operator, γ^μ, so the whole interaction was

$$\overline{\psi}\gamma^\mu\frac{1}{2}(1-\gamma^5)\phi \quad (36)$$

. If we expand this we get

$$\frac{1}{2}(\overline{\psi}\gamma^\mu\phi - \overline{\psi}\gamma^\mu\gamma^5\phi) \quad (37)$$

and comparing to the table this makes the vector (V) and axial vector (A) currents responsible for the parity violating nature of the weak interaction.

This is the famous **V-A** interaction. Parity violation comes from the fact that the behaviour of the vector and axial vector currents under a parity transformation are different. As you can see from the table, the vector current flips sign under parity whereas the axial vector doesn't. The interference between these two terms creates the parity violation. One can see this schematically by remembering that what we observe is usually the square of the amplitude. Suppose the amplitude is pure V-A. Then

$$|M|^2 \sim (V-A)(V-A) \quad (38)$$
$$= VV - 2AV + AA \quad (39)$$
$$(40)$$

If we apply a parity transformation then the sign of the V term flips, but the sign of the A term doesn't.

$$\hat{P}\{|M|^2\} \sim \hat{P}\{(V-A)(V-A)\} \quad (41)$$
$$= \hat{P}\{VV - 2AV + AA\} \quad (42)$$
$$= (-V)(-V) + AA - 2A(-V) \quad (43)$$
$$= VV + AA + 2AV \quad (44)$$

Comparing the $|M|^2$ and $\hat{P}\{|M|^2\}$ we see a difference from -2AV to +2AV. Without having the cross term, AV, made up of currents with opposite parity behaviours, one would end up with $|M|^2 = \hat{P}\{|M|^2\}$ and therefore there would be no parity violation.

The V-A interation actually violates parity maximally as both currents have the same strength. Parity isn't just violated in a small percentage of interactions, it's violated in all of them. One can test this by allowing the currents to have different weights

$$\frac{1}{2}\overline{\psi}\gamma^{\mu}(c_V - c_A\gamma^5)\phi \tag{45}$$

Experimentally it is found that $c_V = 1$ and $c_A = 1$.

The weak charged current can therefore be written as

$$j^{CC}_{weak} = \frac{g_w}{\sqrt{2}}\overline{u}\gamma^{\mu}\frac{1}{2}(1-\gamma^5)u \tag{46}$$

3.5 The V-A Interaction and Neutrinos

The inclusion of the left-handed chiral projection operator in the current implies that the charged weak interaction only couples left-handed chiral particles, or right-handed chiral antiparticles.

$$\overline{\psi}\gamma^{\mu}\frac{1}{2}(1-\gamma^5)\phi = (\overline{\psi_L} + \overline{\psi_R})\gamma^{\mu}\phi_L \tag{47}$$

$$= \overline{\psi_L}\gamma^{\mu}\phi_L \tag{48}$$

What does this mean for neutrinos? Well, we know that neutrinos are observed to all have left-handed helicity, and anti-neutrinos all have right-handed helicity. Since neutrinos (even if they do have mass) are ultra-relativistic, this implies that all neutrinos have left-handed chirality, and antineutrinos have right-handed chirality. The neutrinos can only be made in weak interactions and so are all made as left-handed chiral particles. They have no choice.

This is an important but subtle point - neutrinos do not necessarily have *intrinsic* left-handed helicity. They have left-handed chirality because they can only be made by the weak interaction, and the weak interaction only makes left-handed chiral particles or right-handed chiral antiparticles. To a good approximation, since neutrinos are almost massless, helicity and chirality are the same thing, so the neutrino is always generated with left-handed helicity. This does not preclude the possibility of the existance of a neutrino with right-handed helicity. It can be shown, however, that the probability of generating a neutrino with right-handed helicity is proportional to $(\frac{m_\nu}{E_\nu})^2$ and is therefore almost impossible (m_ν is the absolute neutrino mass. We know this is less than about 2 eV. For a neutrino with energy of, say, 10 MeV the probability of emitting a wrong sign neutrino is around 4×10^{-14}).

This argument doesn't preclude the possibility of the existence of a right-handed chiral neutrino either. Unfortunately, if it does exist, it doesn't couple to any of our fundamental forces (with the possible exception of gravity, and even then it is extremely weak) and hence may as well not exist. Hence, the existence of a right-chiral neutrino state is almost a question of philosophy - but not quite. As we will see in our discussion of neutrino oscillations, a right-chiral neutrino state could still have indirect but visible effects in some neutrino oscillation experiments.

Electrons, on the hand, are massive and can come in both left- and right-handed chiral states. However, only the left-handed electrons couple to the charged weak interaction, i.e. to the $W^{\pm}$ boson. It is possible for the Z^0 to couple to right-handed chiral particles as well. As neutrinos are only *created* by the *charged* weak current, this makes no difference to the properties of the neutrino.

4 What you should know

- Properties of the charged current weak interaction, especially coupling factors.
- Parity violation and its importance to specifying the weak interaction. Know why the weak current is V-A (but you don't have to know how to show it).
- The importance of parity violation to neutrinos.

5 Furthur reading

Griffiths - Chapter 10.