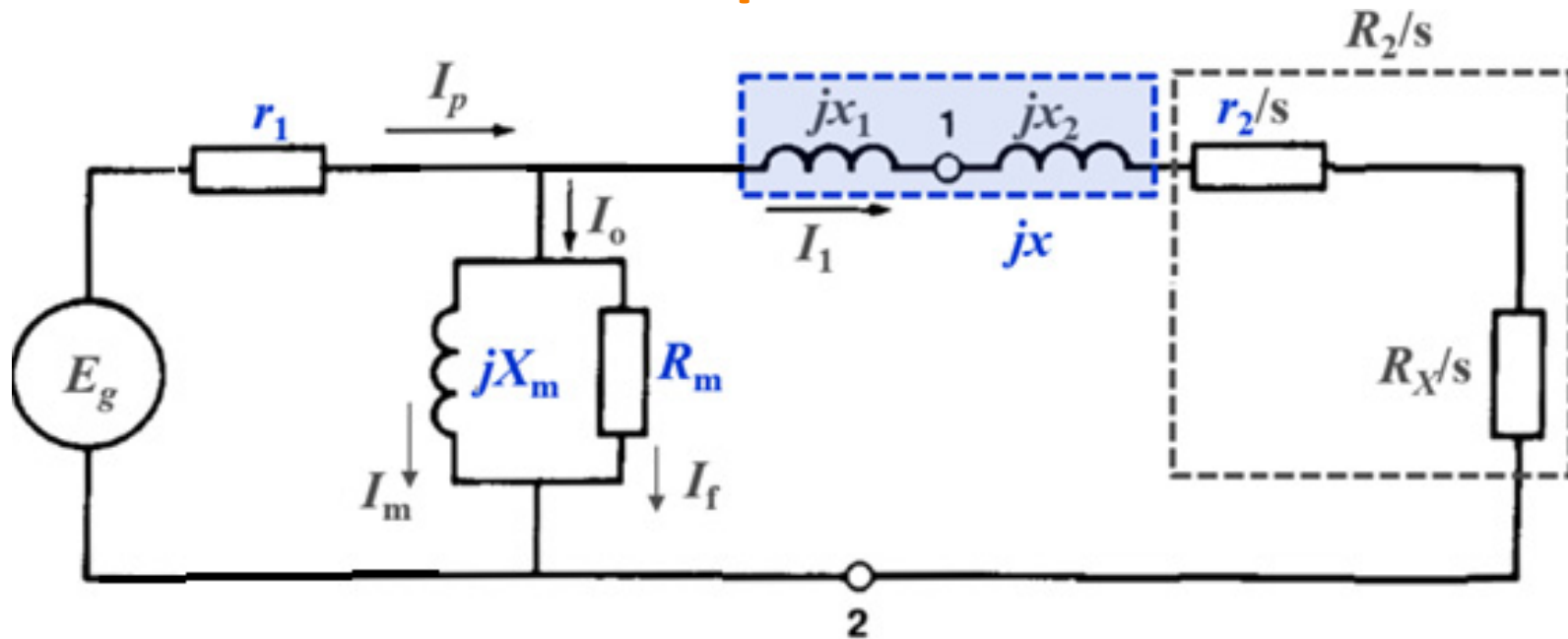


Tests to determine the equivalent circuit



- Estimate r_1 , r_2 , X_m , R_m and x (note: $r_2 + R_X = R_2$ where R_X is the external resistance)
 1. No-load test
 2. Locked-rotor test
- Learn Example 15-1

No-load test

At no-load, slip $s \approx 0 \rightarrow$
 R_2/s is high, $I_1 \ll I_o$

Steps:

1. Measure stator resistance R_{LL} between any two terminals (assuming a Y connection)

$$r_1 = R_{LL} / 2$$

2. Run the motor at no-load using rated line-to-line voltage E_{NL} . Measure no-load current I_{NL} and 3-phase active power P_{NL}

$$|S_{NL}| = \sqrt{3} E_{NL} I_{NL}$$

$$Q_{NL} = \sqrt{|S_{NL}|^2 - P_{NL}^2}$$

$$P_{NL} \approx P_f + 3I_{NL}^2 r_1 \Leftrightarrow P_f \approx P_{NL} - 3I_{NL}^2 r_1 \quad (\text{Ignoring } P_v)$$

$$R_m = \frac{(E_{NL} / \sqrt{3})^2}{P_f / 3} = \frac{E_{NL}^2}{P_{NL} - 3I_{NL}^2 r_1} \quad X_m = \frac{(E_{NL} / \sqrt{3})^2}{(Q_{NL}) / 3} = \frac{E_{NL}^2}{Q_{NL}}$$

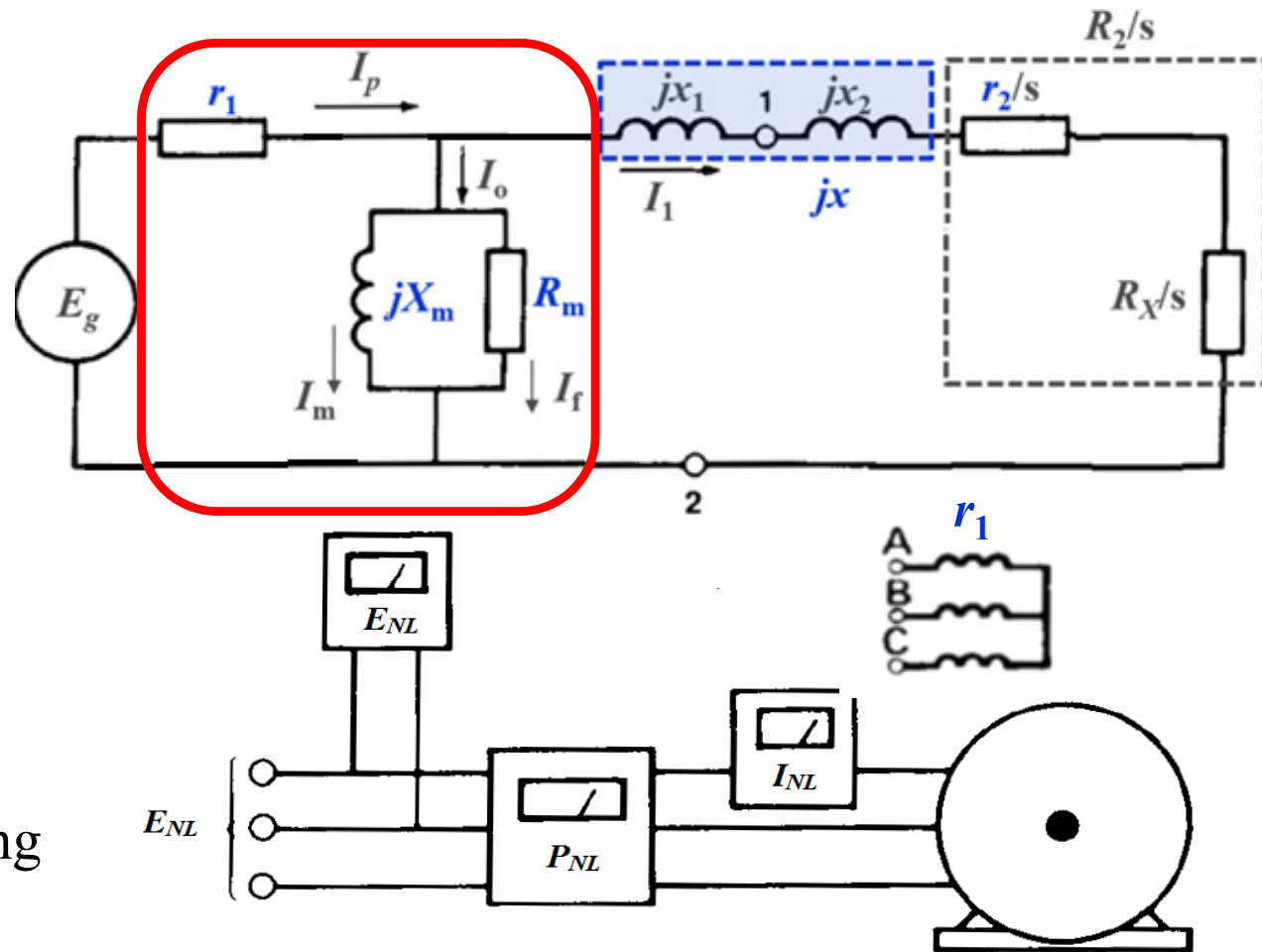


Figure 15.17

A no-load test permits the calculation of X_m and R_m of the magnetizing branch.

Blocked (locked) rotor test

When the rotor is locked without R_X , slip $s=1$, $I_p \gg I_o \rightarrow r_2 = R_2/s$, neglect the magnetizing branch

Steps:

1. Apply reduced 3-phase voltage E_{LR} to the stator so that the stator current $I_p \approx$ the rated value
2. Measure line-to-line voltage E_{LR} , current I_{LR} and 3-phase active power P_{LR}

$$|S_{LR}| = \sqrt{3} E_{LR} I_{LR}$$

$$Q_{LR} = \sqrt{|S_{LR}|^2 - P_{LR}^2}$$

$$x = \frac{Q_{LR}}{3I_{LR}^2}$$

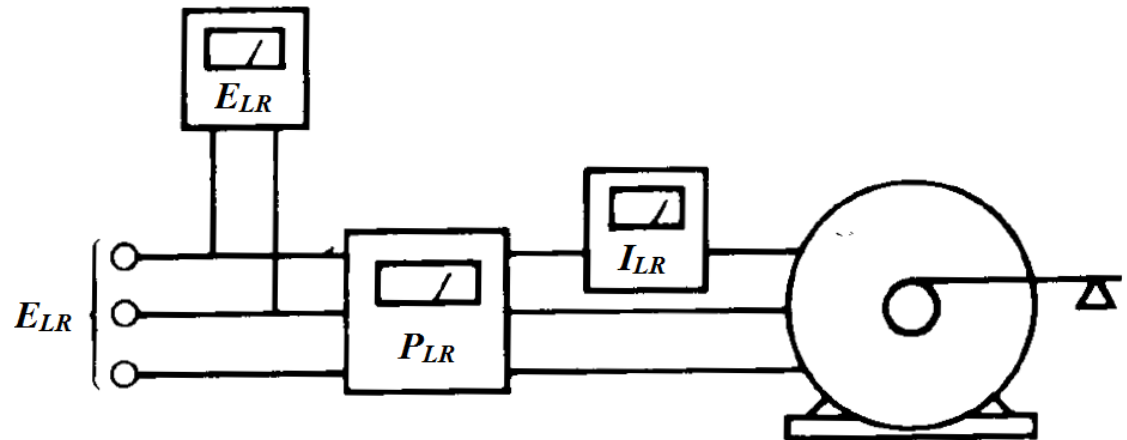
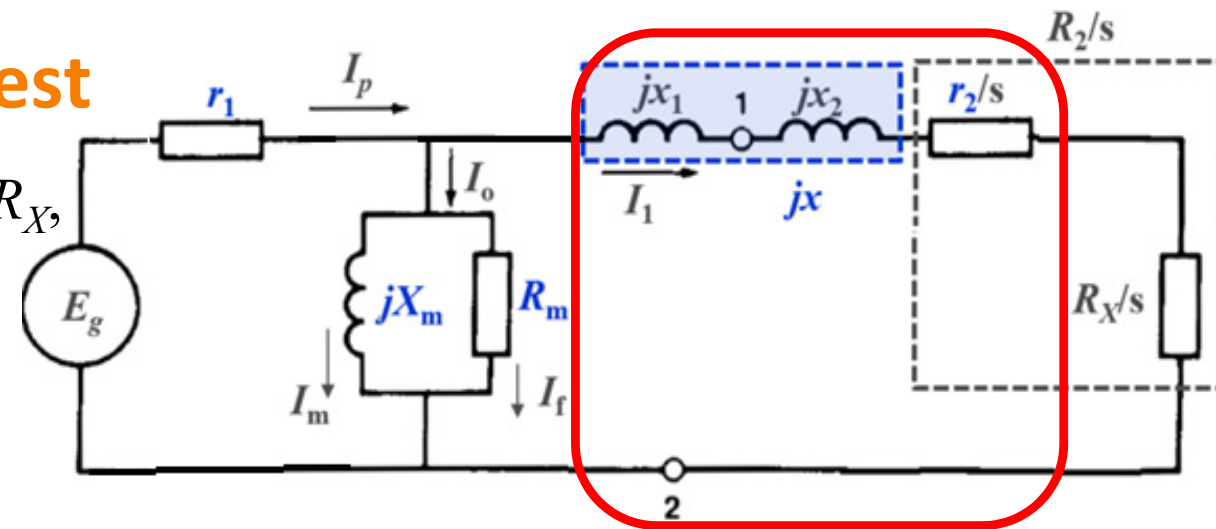
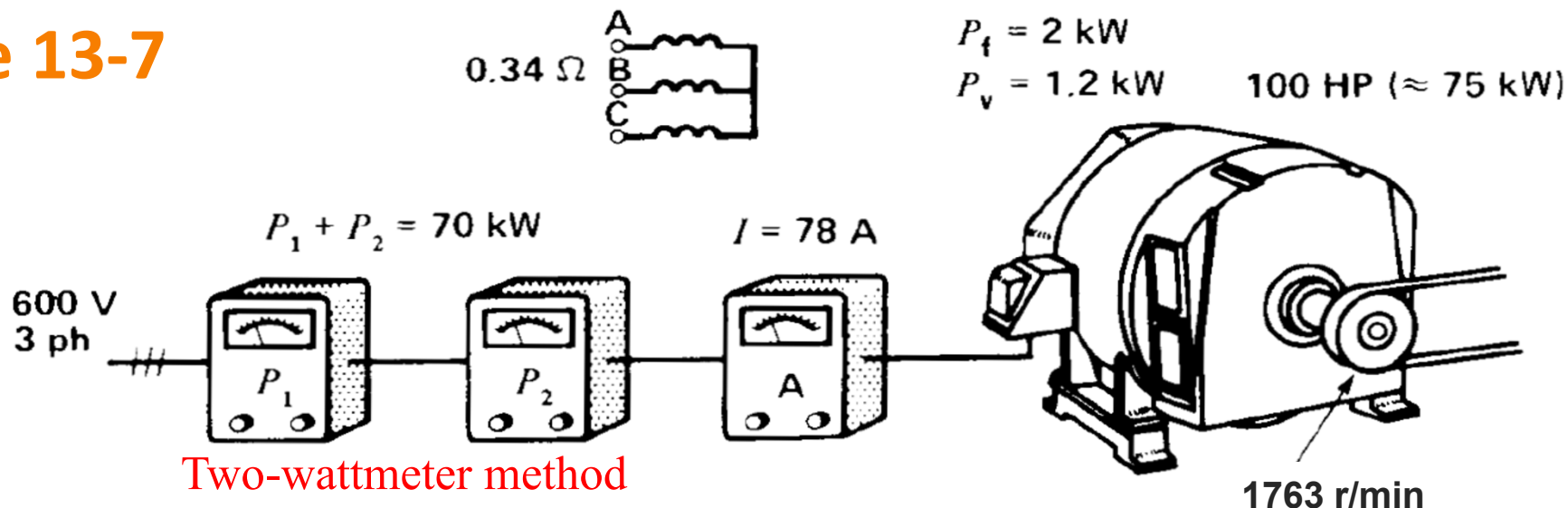


Figure 15.18

A locked-rotor test permits the calculation of the total leakage reactance x and the total resistance $(r_1 + r_2)$. From these results we can determine the equivalent circuit of the induction motor.

$$P_{LR} \approx 3I_{LR}^2 r_1 + 3I_{LR}^2 r_2 \Rightarrow r_2 \approx P_{LR} / (3I_{LR}^2) - r_1$$

Example 13-7



A 3-phase induction motor having a nominal rating of 100 hp ($\sim 75 \text{ kW}$) and a synchronous speed of 1800 r/min is connected to a 3-phase 600 V source.

Resistance between two stator terminals $= 0.34 \Omega$. Calculate

- a. Power supplied to the motor

$$P_e = P_1 + P_2 = 70 \text{ kW}$$

- b. Rotor I^2R losses P_{jr}

$$r_1 = 0.34/2 = 0.17 \Omega$$

$$P_{js} = 3I_1^2 r_1 = 3 \times 78^2 \times 0.17 = 3.1 \text{ kW}$$

$$P_r = P_e - P_{js} - P_f = 70 - 3.1 - 2 = 64.9 \text{ kW}$$

$$s = (n_s - n)/n_s = (1800 - 1763)/1800 = 0.0205$$

$$P_{jr} = sP_r = 0.0205 \times 64.9 = 1.33 \text{ kW}$$

- c. Mechanical power supplied to the load

$$P_m = P_r - P_{jr} = 64.9 - 1.33 = 63.5 \text{ kW}$$

$$P_L = P_m - P_v = 63.5 - 1.2 = 62.3 \text{ kW}$$

$$= 62.3 \times 1.34 = 83.5 \text{ hp}$$

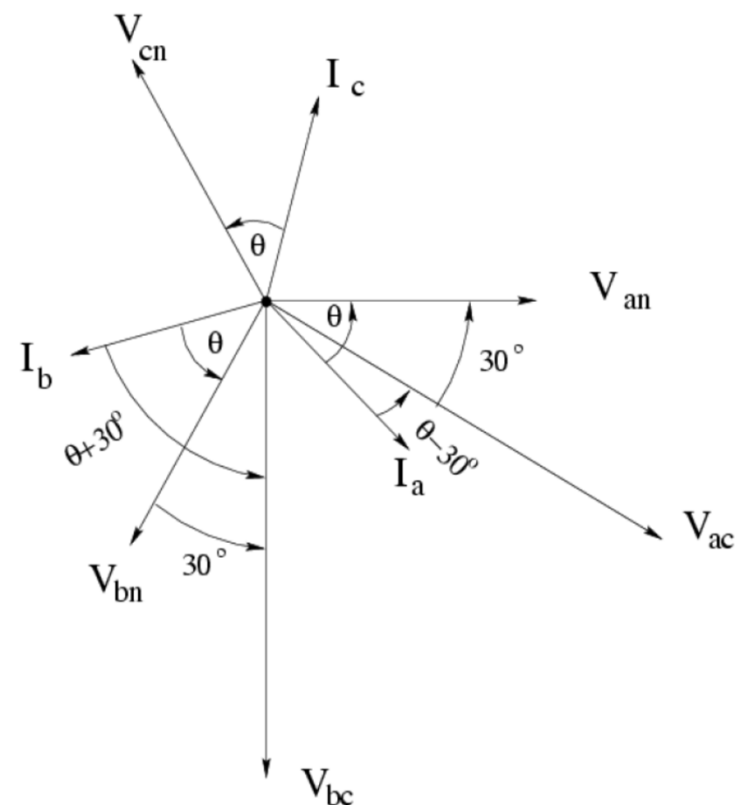
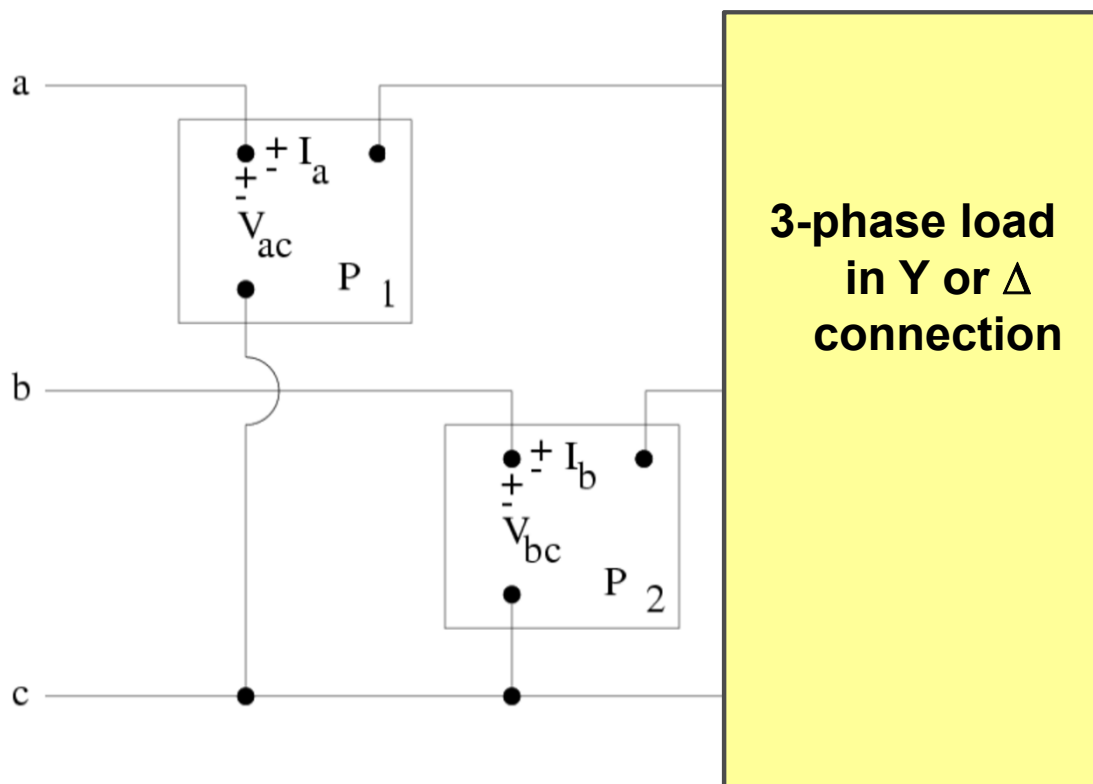
- d. Efficiency

$$\eta = P_L / P_e = 62.3 / 70 = 89\%$$

- e. Torque developed at 1763 r/m

$$T_m = 9.55 P_r / n_s = 9.55 \times 649000 / 1800 = 344 \text{ N}\cdot\text{m}$$

Two-wattmeter method to measure 3-phase active power



$$P_1 = |V_{ac}| |I_a| \cos(\theta - 30^\circ) = E_L I_L \cos(\theta - 30^\circ)$$

$$P_2 = |V_{bc}| |I_b| \cos(\theta + 30^\circ) = E_L I_L \cos(\theta + 30^\circ)$$

$$P_1 + P_2 + j\sqrt{3}(P_1 - P_2) = P_{3\phi} + jQ_{3\phi} = S_{3\phi}$$

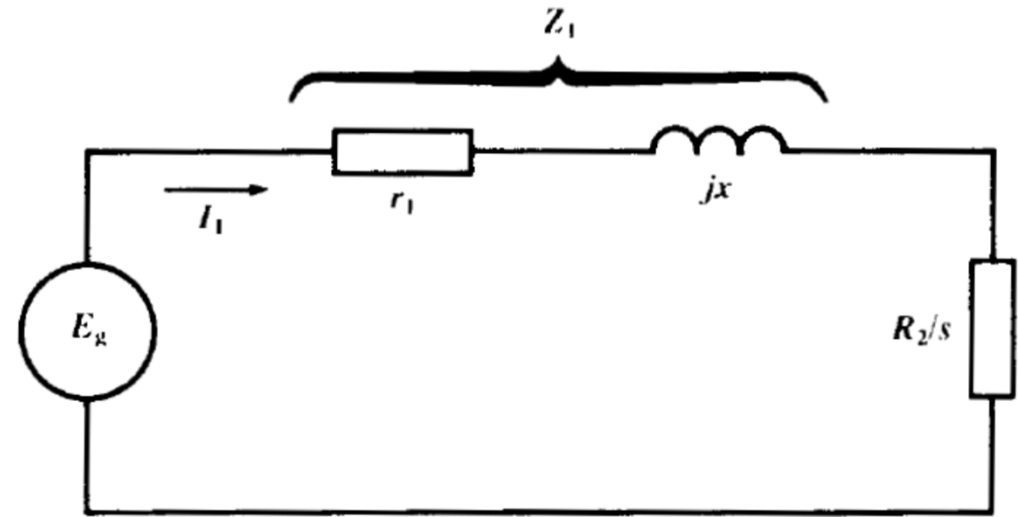
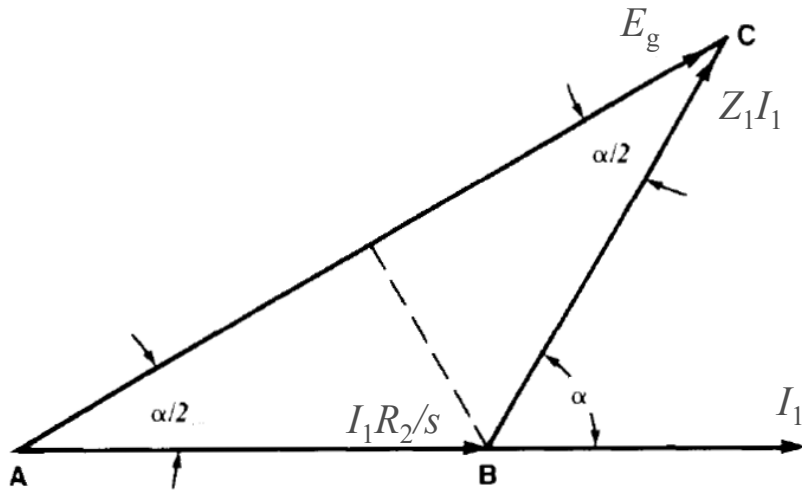
Proof: $E_L I_L [\cos(\theta - 30^\circ) + \cos(\theta + 30^\circ)] + j\sqrt{3} E_L I_L [\cos(\theta - 30^\circ) - \cos(\theta + 30^\circ)]$

$$= E_L I_L 2 \cos \theta \cos 30^\circ + j\sqrt{3} E_L I_L 2 \sin \theta \sin 30^\circ$$

$$= \sqrt{3} E_L I_L \cos \theta + j\sqrt{3} E_L I_L \sin \theta = S_{3\phi}$$

Breakdown (maximum) Torque

- When $|Z_1|=|R_2/s|$, P_r and torque T both reach their maximum values



$$\frac{|E_g|}{2} = \frac{|I_1| R_2}{s} \cos \frac{\alpha}{2} = |I_1 Z_1| \cos \frac{\alpha}{2} \quad \Rightarrow \quad |I_1| = |I_{1b}| \stackrel{\text{def}}{=} \frac{|E_g|}{2 |Z_1| \cos \frac{\alpha}{2}} \quad s = s_b \stackrel{\text{def}}{=} \frac{R_2}{|Z_1|}$$

$$T \leq T_b \stackrel{\text{def}}{=} \frac{9.55 P_r |_{I_1=I_{1b}, s=s_b}}{n_s} = \frac{9.55 |I_{1b}|^2 R_2}{n_s s_b}$$

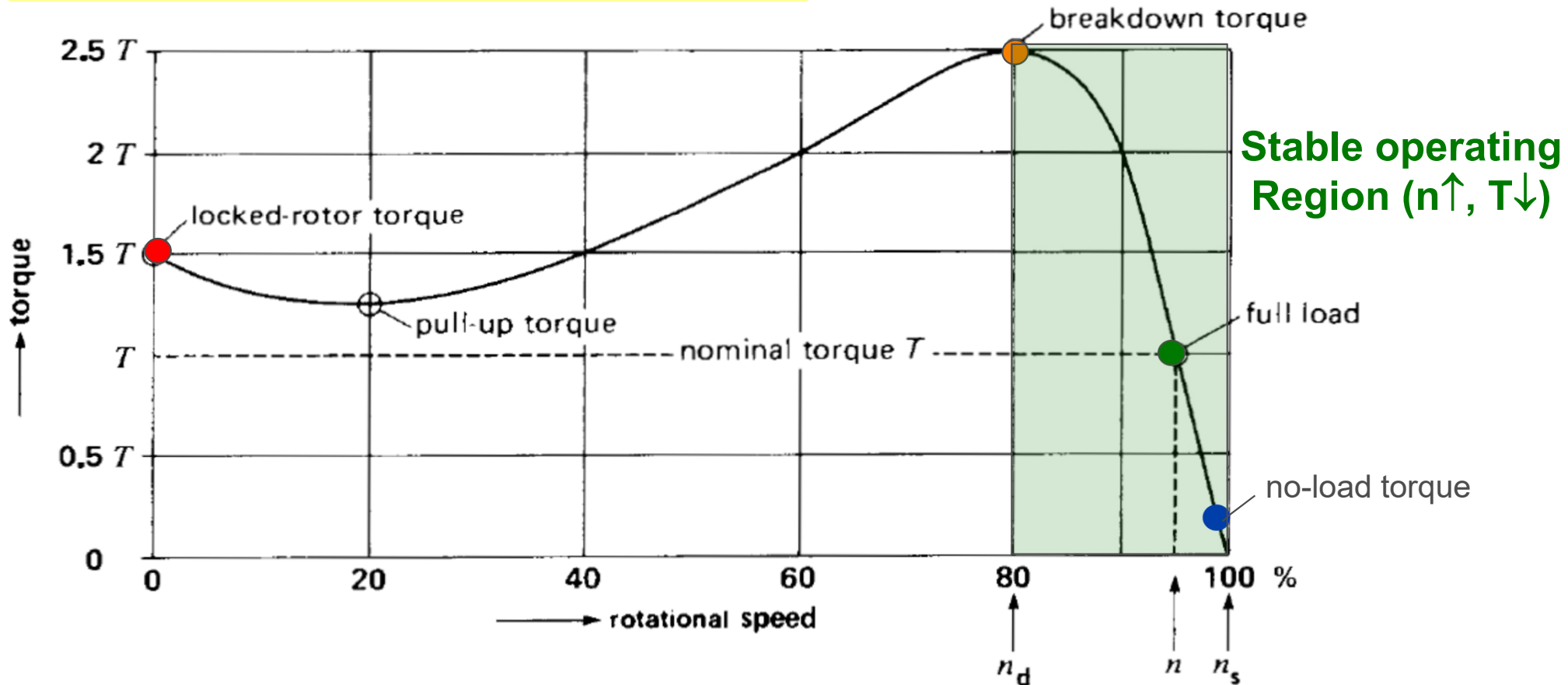
$$T_b = \frac{9.55 |E_g|^2}{4 n_s |Z_1| \cos^2 \frac{\alpha}{2}}$$

Torque-Speed Curve

$$s = \frac{n_s - n}{n_s}, \quad P_r = |I_1|^2 \frac{R_2}{s}, \quad I_1 = \frac{E_g}{Z_1 + \frac{R_2}{s}} = \frac{E_g}{Z_1 + \frac{R_2 n_s}{n_s - n}}$$



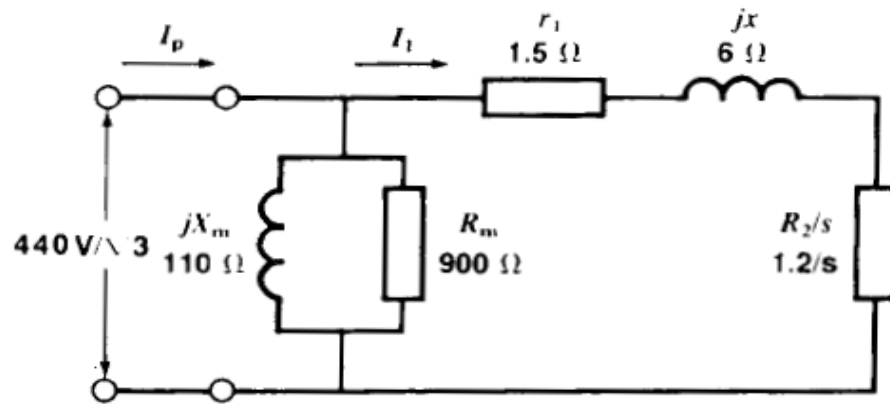
$$T = \frac{9.55 P_r}{n_s} = \frac{9.55 |E_g|^2 R_2}{\left| Z_1 + \frac{R_2 n_s}{n_s - n} \right|^2 (n_s - n)}$$



- The curve is nearly linear between no-load and full-load because s is small and R_2/s is big (Z_1 is ignored)

$$I_1 \approx s E_g / R_2 \quad T \approx \frac{9.55 |E_g|^2}{R_2 n_s^2} (n_s - n) = \frac{9.55 |E_g|^2}{R_2 n_s} s$$

Two practical squirrel-cage induction motors



Motor rating:

5 hp, 60 Hz, 1800 r/min, 440 V, 3-phase
 full-load current: 7 A
 locked-rotor current: 39 A

r_1 = stator resistance 1.5 Ω

r_2 = rotor resistance 1.2 Ω

jx = total leakage reactance 6 Ω

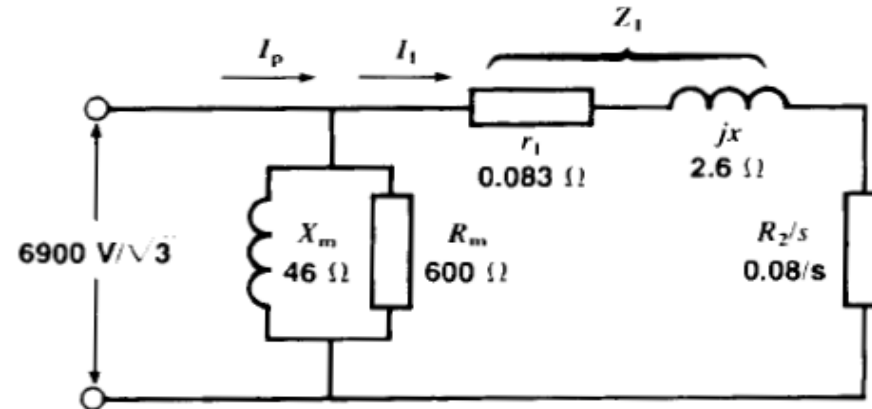
jX_m = magnetizing reactance 110 Ω

R_m = no-load losses resistance 900 Ω

(The no-load losses include the iron losses plus windage and friction losses.)

Figure 15.12

Equivalent circuit of a 5 hp squirrel-cage induction motor. Because there is no external resistor in the rotor, $R_2 = r_2$.



Motor rating:

5000 hp, 60 Hz, 600 r/min, 6900 V, 3-phase
 full-load current: 358 A
 locked-rotor current: 1616 A

r_1 = stator resistance 0.083 Ω

r_2 = rotor resistance 0.080 Ω

jx = total leakage reactance 2.6 Ω

jX_m = magnetizing reactance 46 Ω

R_m = no-load losses resistance 600 Ω

The no-load losses of 26.4 kW (per phase) consist of 15 kW for windage and friction and 11.4 kW for the iron losses.

Figure 15.13

Equivalent circuit of a 5000 hp squirrel-cage induction motor. Although this motor is 1000 times more powerful than the motor in Fig. 15.12, the circuit diagram remains the same.

Torque-Speed Curve: 5 hp motor

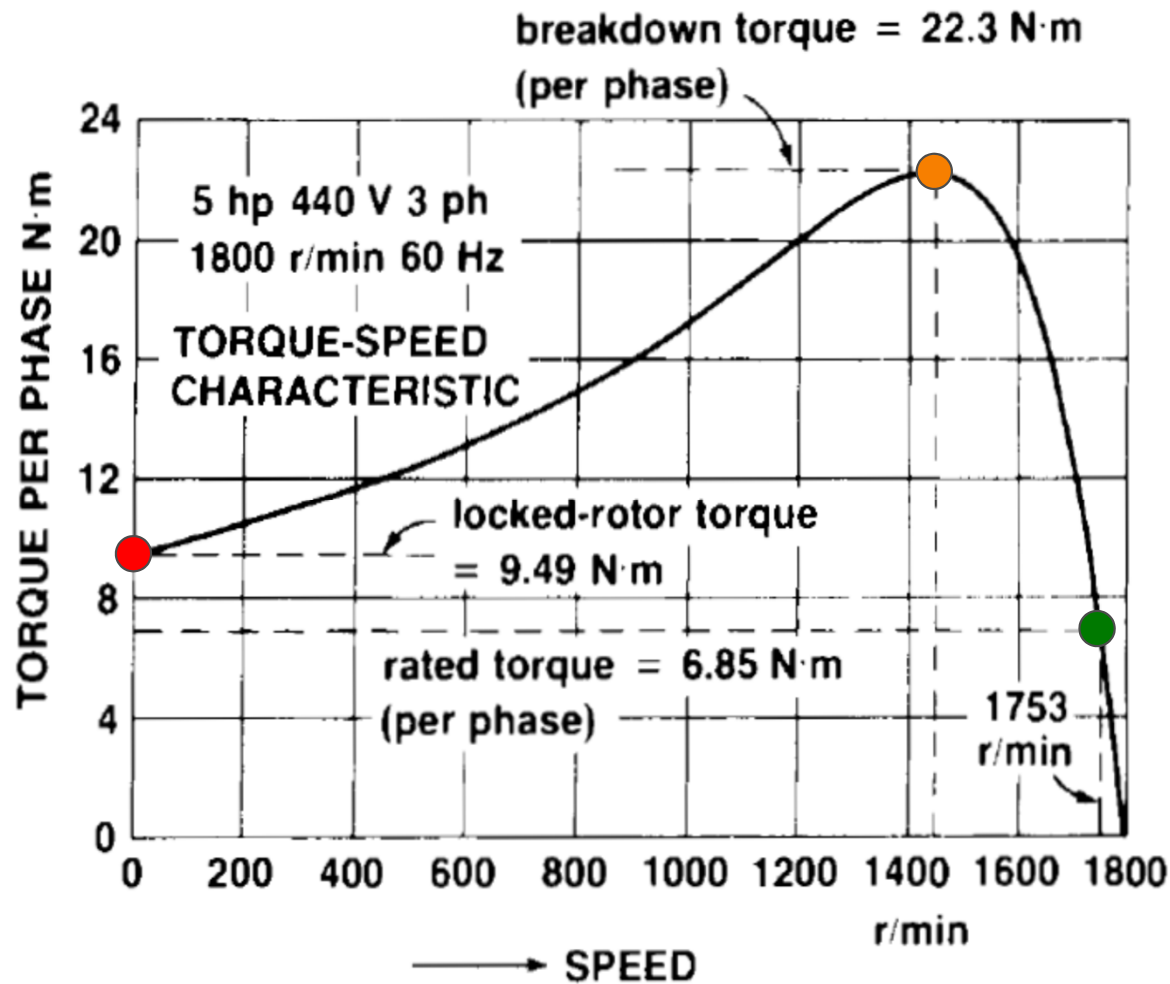


TABLE 15A TORQUE-SPEED CHARACTERISTIC

5 hp, 440 V, 1800 r/min, 60 Hz squirrel-cage induction motor

s	I_1	P_r	T	n
	[A]	[W]	[N·m]	[r/min]
0.0125	2.60	649	3.44	1777
0.025	5.09	1243	6.60	1755
0.026	5.29	1291	6.85	1753
0.05	9.70	2256	12.0	1710
0.1	17.2	3547	18.8	1620
0.2	26.4	4196	22.3	1440
0.4	33.9	3441	18.3	1080
0.6	36.6	2674	14.2	720
0.8	37.9	2150	11.4	360
1	38.6	1788	9.49	0

The rated power of 5 hp is developed at $s = 0.026$.

- When the motor is stalled, i.e. **locked-rotor condition**, the current is 5-6 times the **full-load** current, making I^2R losses 25-36 times higher than normal, so the rotor must never remain locked for more than a few second
- Small motors (15 hp and less) develop their breakdown torque at about 80% of n_s

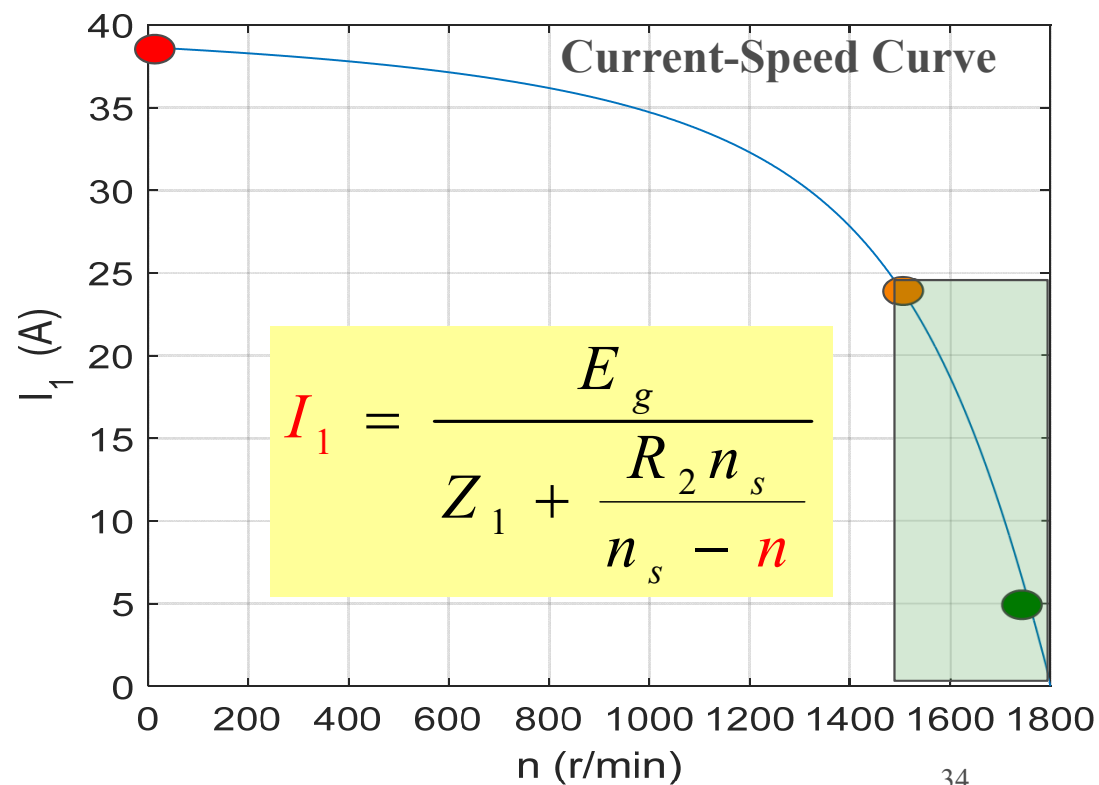
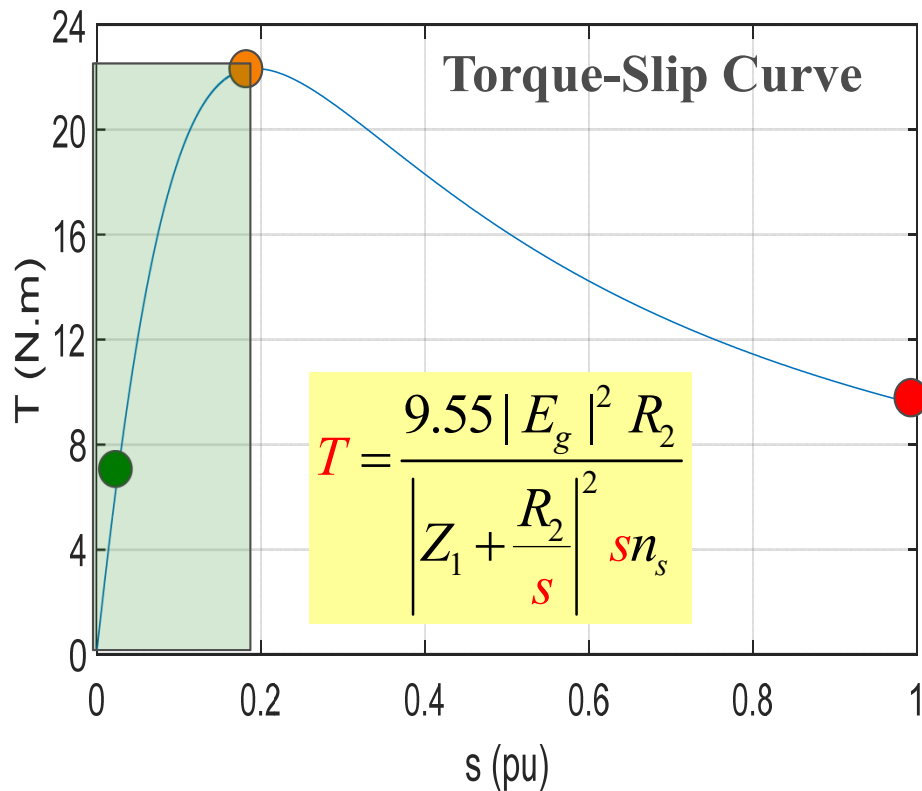
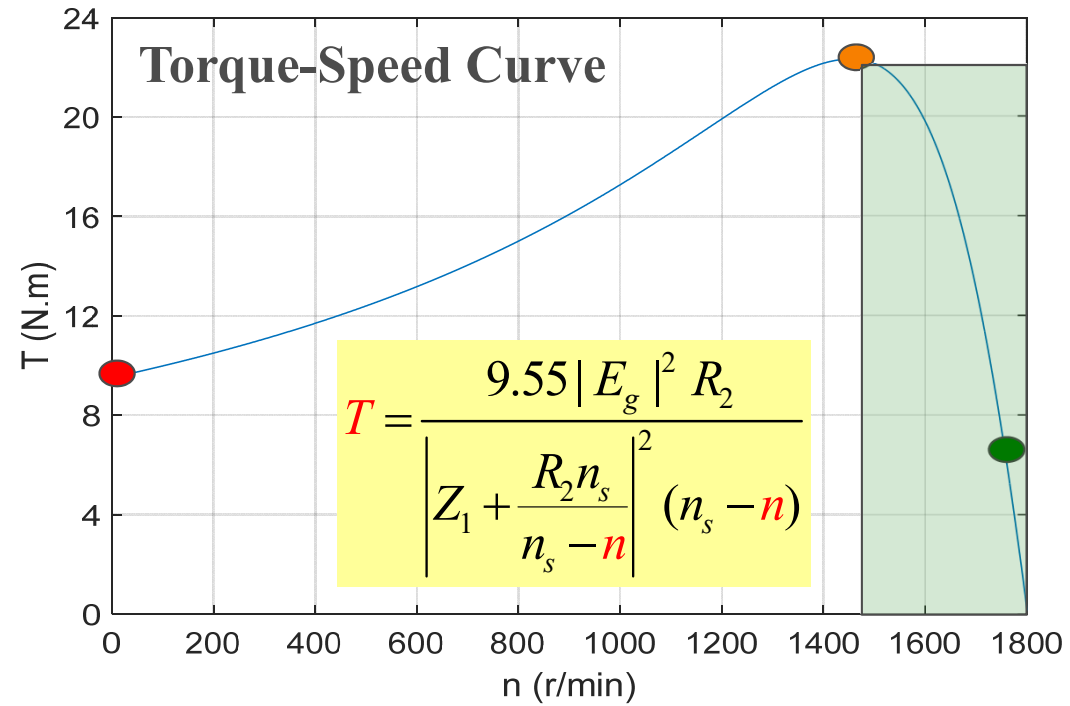
The 5 hp induction motor

$$E_g = 440/1.73 \text{ V}$$

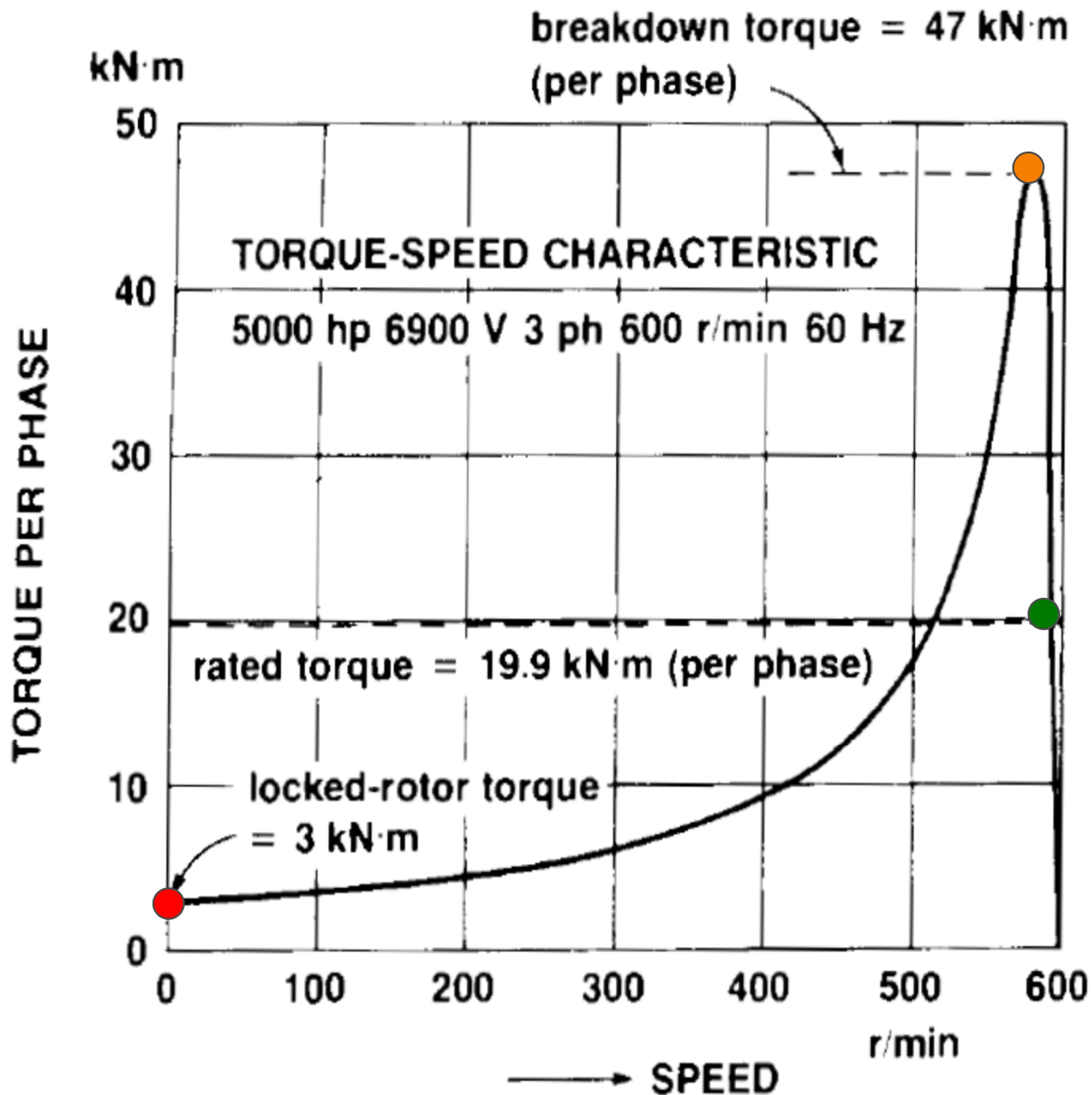
$$R_2 = 1.2 \ \Omega$$

$$Z_1 = 1.5 + 6j \ \Omega$$

$$n_s = 1800 \text{ r/min}$$



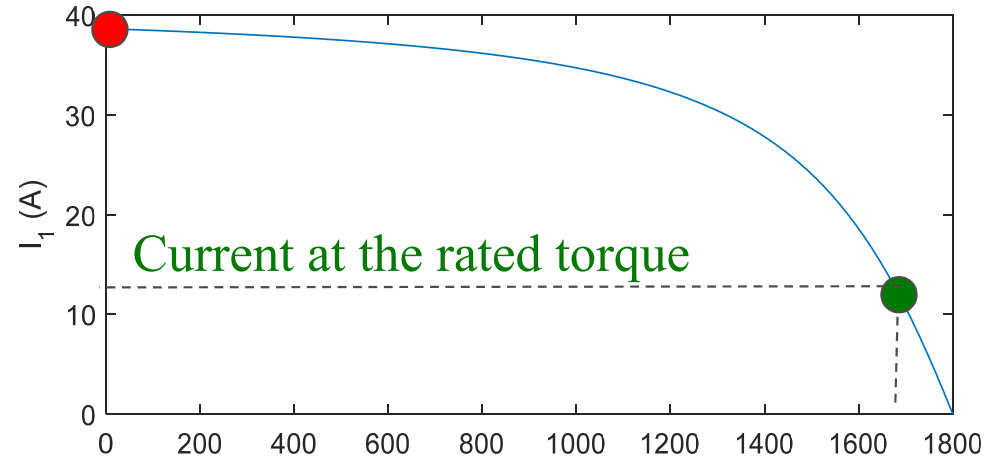
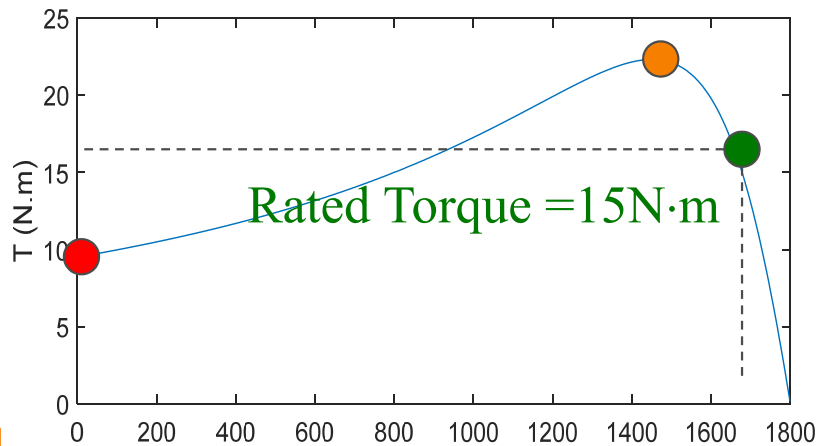
Torque-Speed Curve: 5000 hp motor



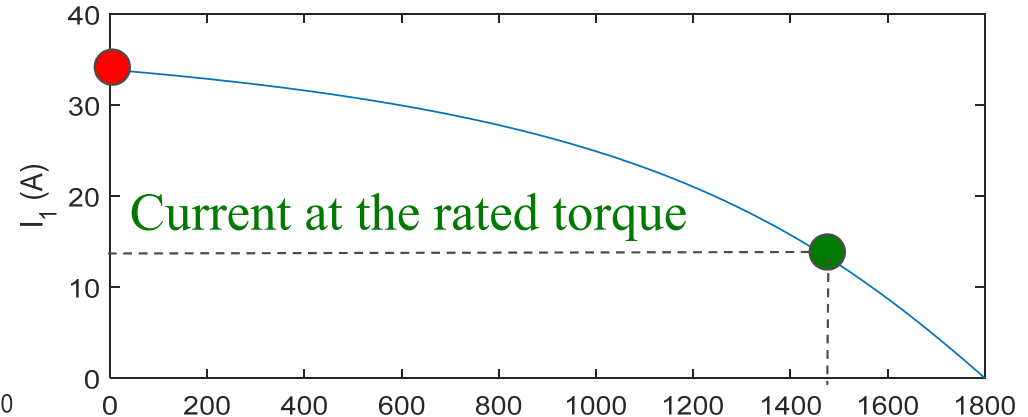
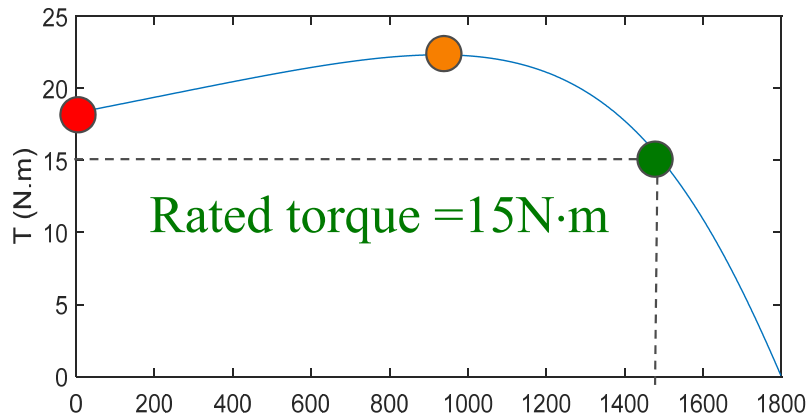
- Big motors (>1500 hp) :
- Relatively low **starting (locked-rotor) torque**
 - **Breakdown torque** at about 98% of n_s
 - **Rated n** is close to n_s

Effect of rotor resistance (5 hp induction motor)

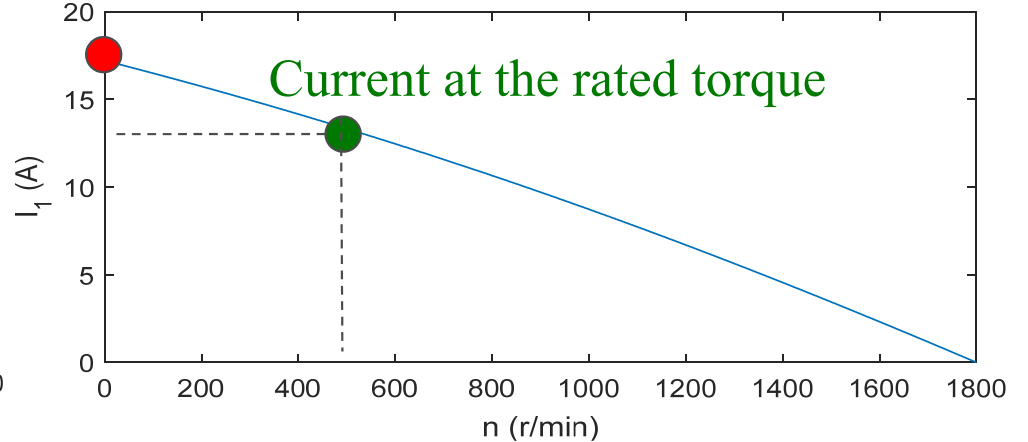
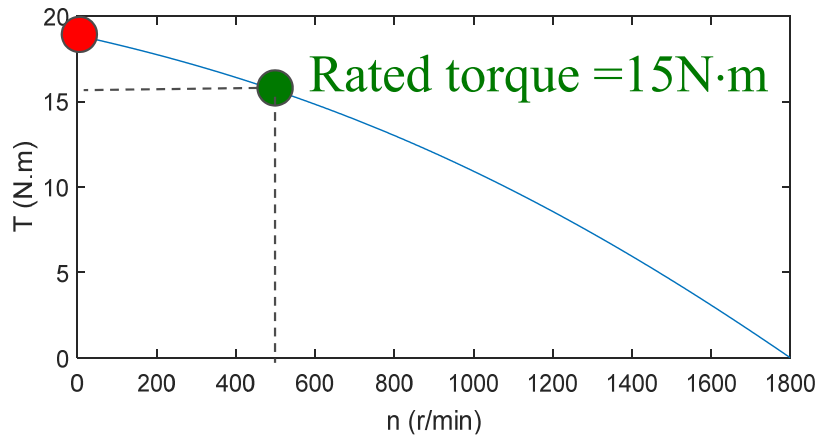
R_2



$2.5R_2$



$10R_2$



Effects of rotor resistance

- When $R_2 \uparrow$, starting (locked-rotor) torque $T_{LR} \uparrow$, starting current $I_{1,LR} \downarrow$, and breakdown torque T_b remains the same
- **Pros & Cons** with a high rotor resistance R_2
 - It produces a high starting torque T_{LR} and a relatively low starting current $I_{1,LR}$
 - However, because the torque-speed curve becomes flat, it produces a rapid fall-off in speed with increasing load around the rated torque, and the motor has high copper losses and low efficiency and tends to overheat
- **Solution**
 - For a squirrel-cage induction motor, design the rotor bars in a special way so that the rotor resistance R_2 is high at starting and low under normal operations
 - If the rotor resistance needs to be varied over a wide range, a wound-rotor motor needs to be used.

Asynchronous generator

Connect the 5 hp, 1800 r/min, 60Hz motor to a 440 V, 3-phase line and drive it at a speed of **1845 r/min**

$$s = (n_s - n) / n_s = (1800 - 1845) / 1800 = -0.025 < 0$$

$$R_2/s = 1.2 / (-0.025) = -48 \Omega < 0$$

The negative resistance indicates the actual power flow from the rotor to the stator

Power flow from the rotor to the stator:

$$|E| = 440 / 1.73 = 254 \text{ V}$$

$$|I_1| = |E| / |-48 + 1.5 + j6| = 254 / 46.88 = 5.42 \text{ A}$$

$$P_r = |I_1|^2 R_2/s = -1410 \text{ W (in fact, rotor} \rightarrow \text{stator)}$$

If we still assume the power flow directions as induction motors (from the stator to rotor) in slide #23, P_r , P_m and P_e are all negative. Alternatively, we may assume P_r , P_m and P_e to be from the rotor to stator as generators:

Mech. power & torque inputs to the shaft:

$$P_{jr} = |I_1|^2 R_2 = 35.2 \text{ W}$$

$$P_m = P_r + P_{jr} = 1410 + 35.2 = 1445 \text{ W}$$

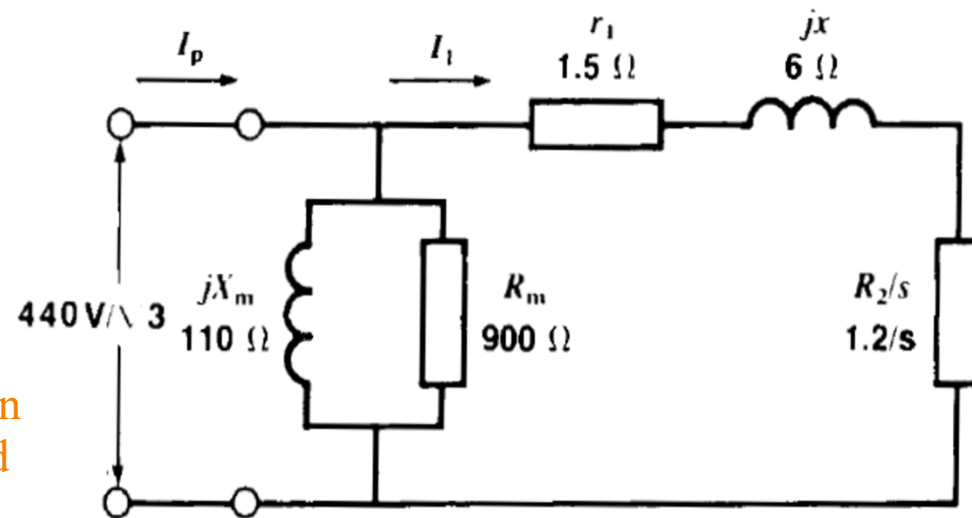
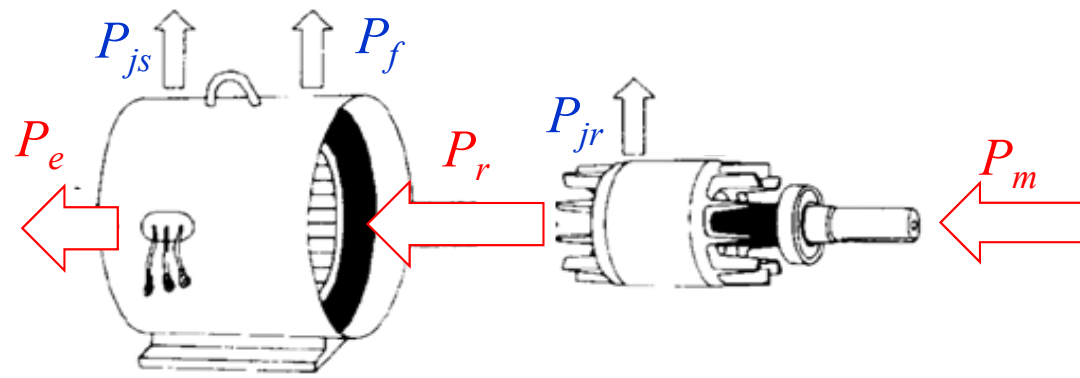
$$T = 3 \times 9.55 \times P_m / n = 22.3 \text{ N}\cdot\text{m}$$

Total active power delivered to the line:

$$P_{js} = |I_1|^2 r_1 = 44.1 \text{ W}, \quad P_f = |E|^2 / R_m = 71.7 \text{ W}$$

$$P_e = P_r - P_{js} - P_f = 1410 - 44.1 - 71.7 = 1294 \text{ W}$$

$$P_{3\phi} = 3P_e = 3882 \text{ W}$$



Reactive power absorbed from the line:

$$Q_{3\phi} = (|I_1|^2 x + |E|^2 / X_m) \times 3 = (176 + 586) \times 3 = 2286 \text{ var}$$

Complex power delivered to the line:

$$S_{3\phi} = P_{3\phi} - Q_{3\phi} = 3882 - j2286 \text{ VA}$$

$$\cos\theta = 86.2\%$$

Efficiency of this asynchronous generator

$$\eta = P_e / P_m = 1294 / 1445 = 89.5\%$$