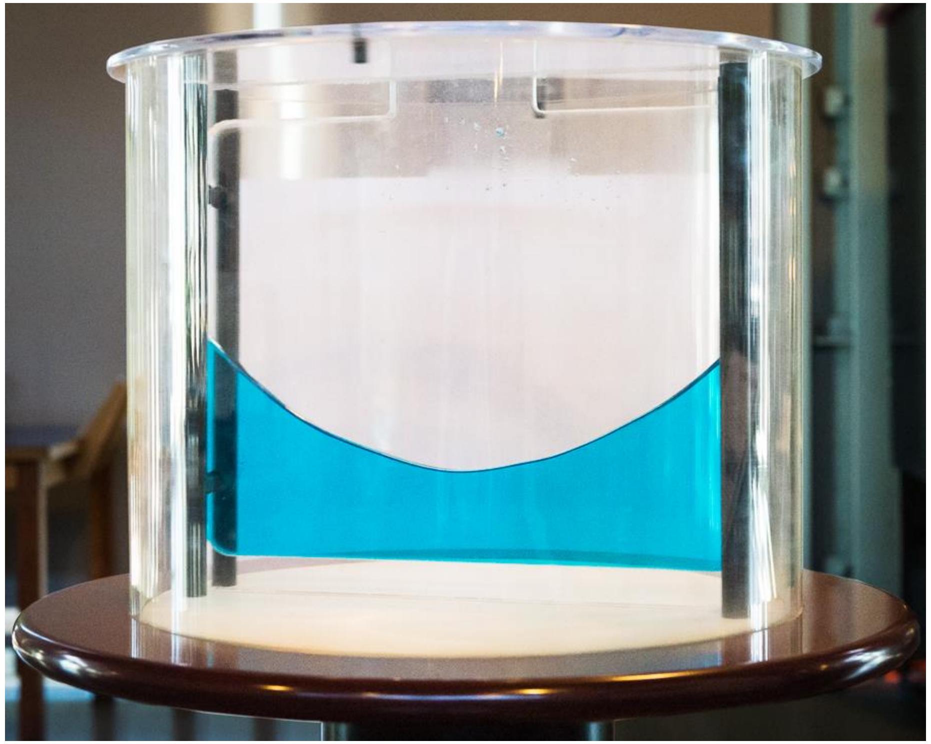
Florida International University, Department of Civil and Environmental Engineering

CWR 3201 Fluid Mechanics, Fall 2018

Fluid Statics



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2.1 INTRODUCTION

Fluid Statics: Study of fluids with no relative motion between fluid particles.

- No shearing stress (no velocity gradients)
- Only normal stress exists (pressure)

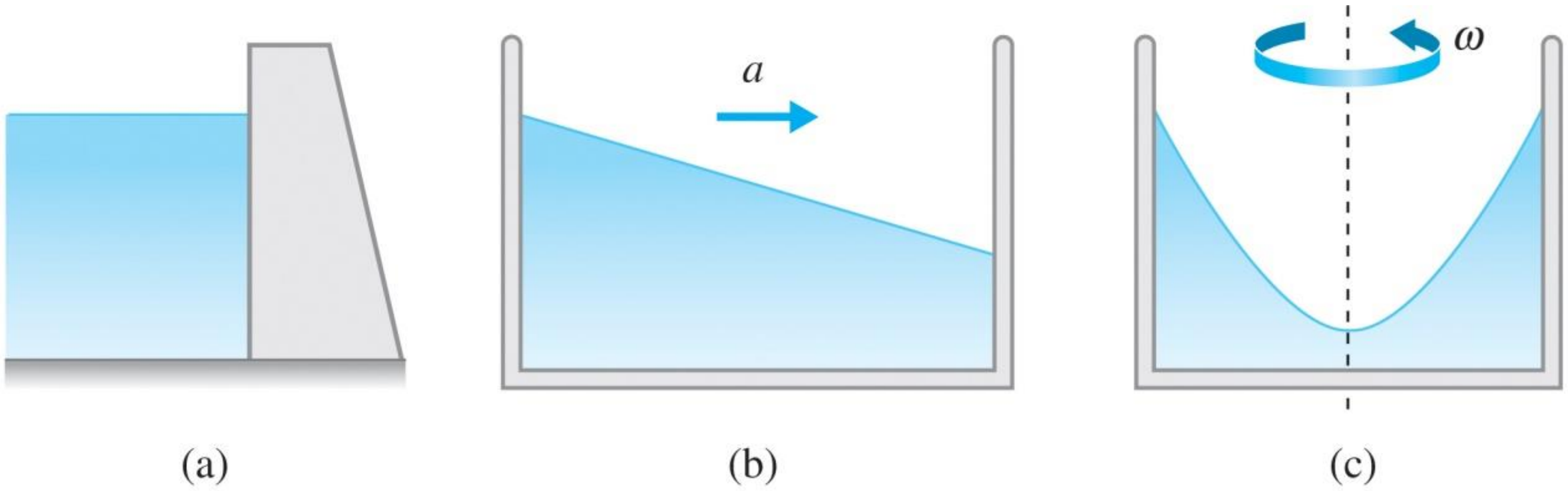
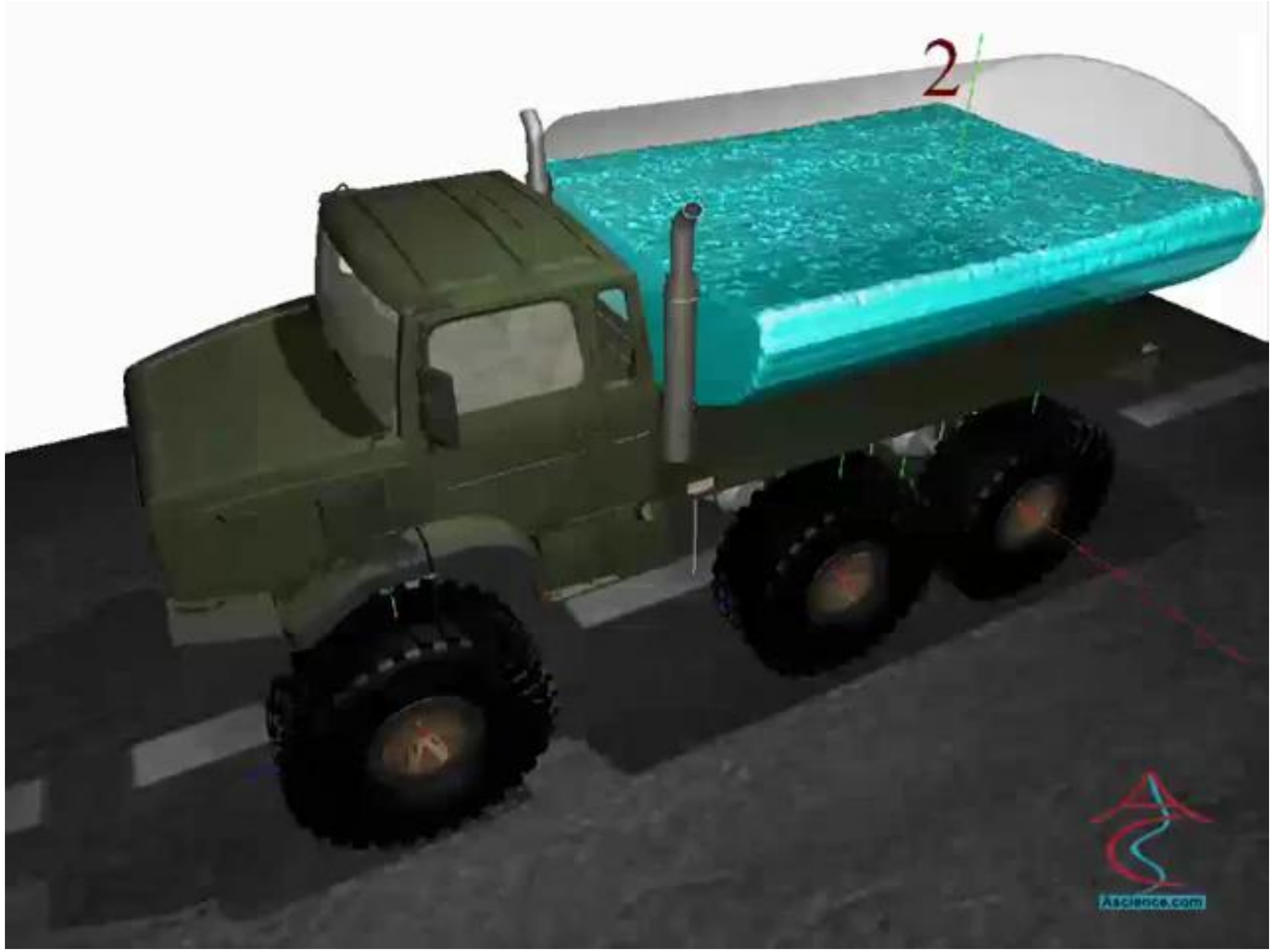


Fig. 2.1 Examples included in fluid statics: (a) liquids at rest; (b) linear acceleration; (c) angular rotation.

MOTIVATION



Source: asciencecom, Youtube
(https://www.youtube.com/watch?v=jqpl4ME6rRY)

MOTIVATION (CONT.)

Youtube (https://www.youtube.com/watch?v=Zip9ft1PgV0)

MOTIVATION (CONT.)

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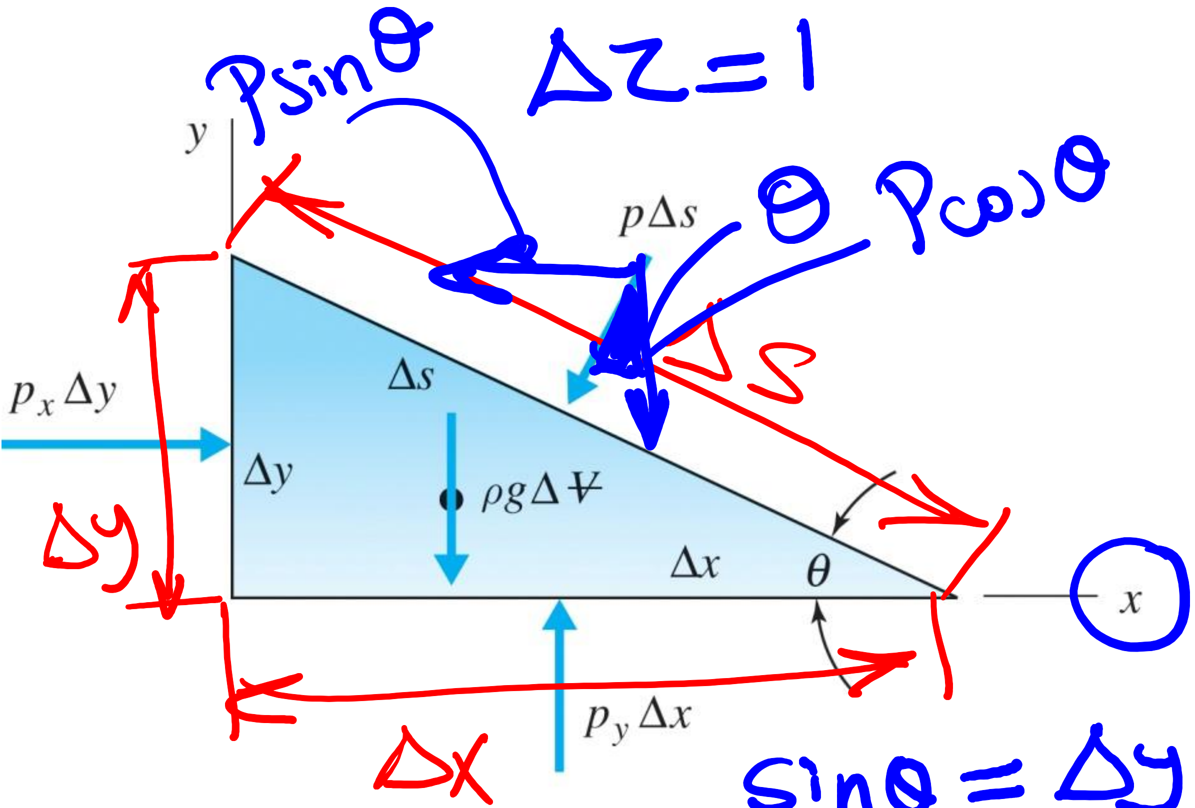
2.2 PRESSURE AT A POINT

- Pressure is an infinitesimal normal compressive force divided by the infinitesimal area over which it acts.



$$\sin\theta = \frac{\Delta y}{\Delta s}$$

Fig. 2.2 Pressure at a point in a fluid.

$$\Delta s = \frac{\Delta y}{\sin\theta}$$

- From Newton's Second Law (for *x*- and *y*-directions):

$$\Sigma F_x = m a_x$$

$$P_x \Delta y - p\sin\theta\, \Delta s = \frac{\rho}{2} \Delta x \Delta y\, a_x$$

$$P_x \cancel{\Delta y} - P \cancel{\sin\theta} \frac{\cancel{\Delta y}}{\cancel{\sin\theta}} = \frac{\rho}{2} \Delta x \cancel{\Delta y}\, a_x$$

$$\boxed{P_x - P = \frac{\rho}{2} \Delta x\, a_x}$$

$$P_y - P = \rho\,(a_y + g)\,\Delta y$$

2.2 PRESSURE AT A POINT

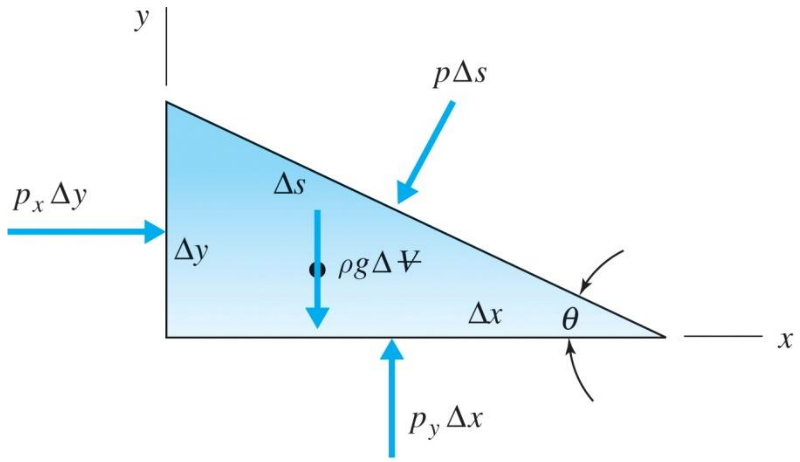


Fig. 2.2 Pressure at a point in a fluid.

- As the element goes to a point (Δx, Δy →0)

$$P_x - P = 0$$

$$P_y - P = 0$$

$$\boxed{P_x = P_y = P}$$

- Pressure in a fluid is **constant** at a point.
- Pressure is a **scalar** function.
- It acts **equally in all directions** at a point for both static and dynamic fluids.

2.3 DERIVATION OF GENERAL FORM OF PRESSURE VARIATION

- Newton's Second Law in "*x*", "*y*" and "z"-directions:

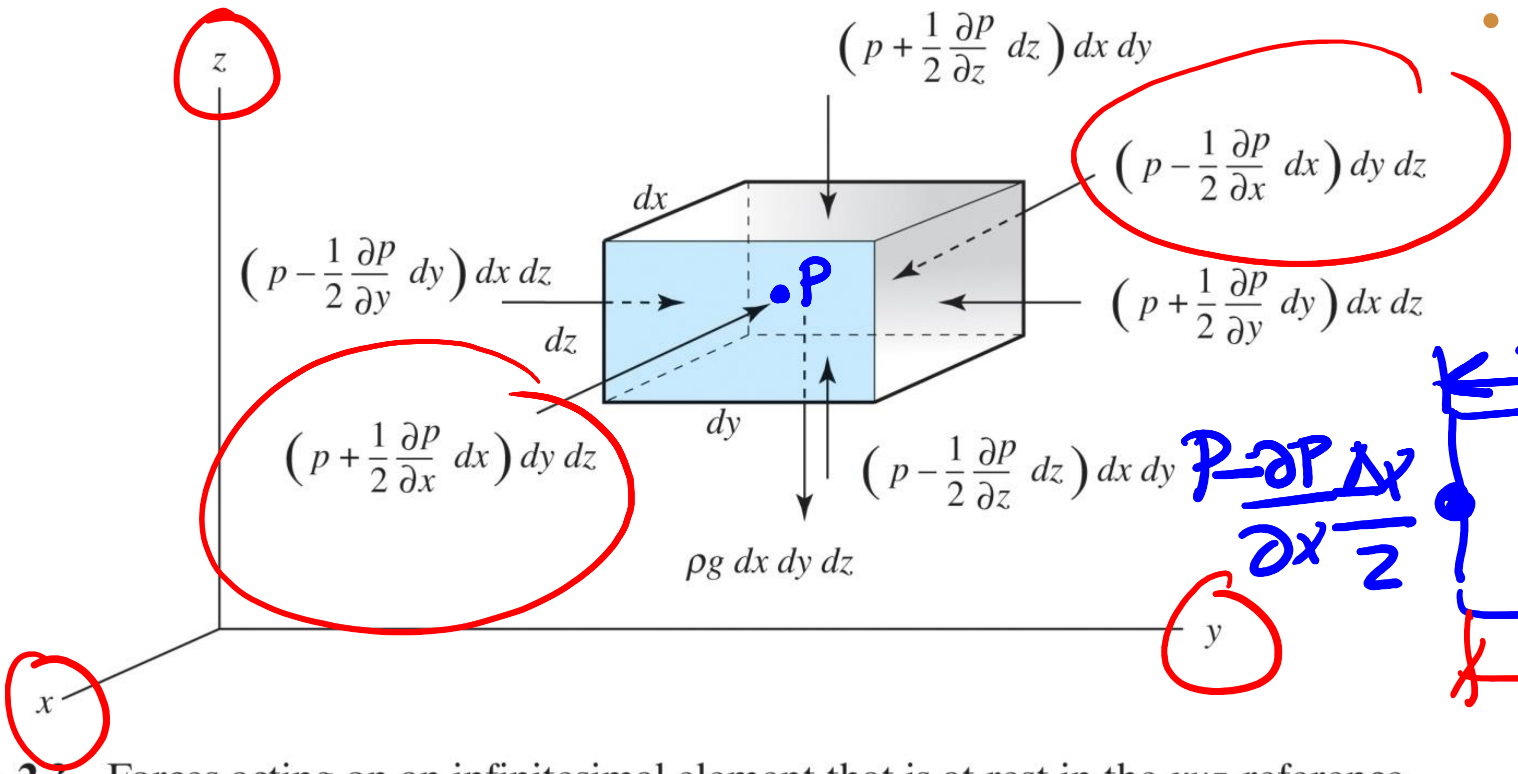
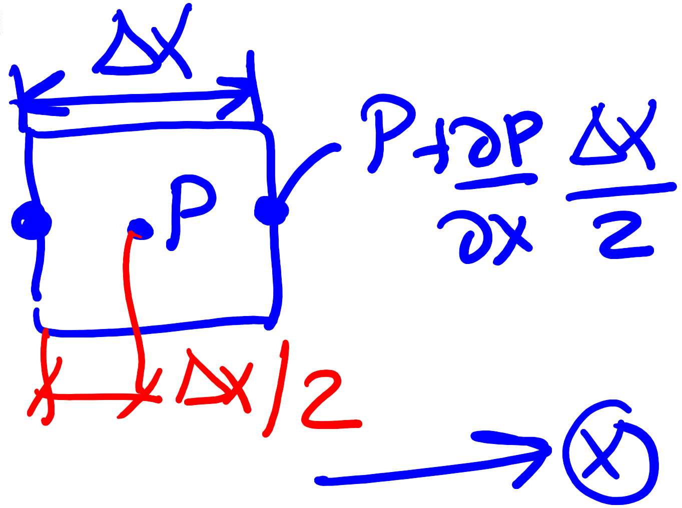


Fig. 2.3 Forces acting on an infinitesimal element that is at rest in the *xyz*-reference frame. The reference frame may be accelerating or rotating.



$$\left(\cancel{P}-\frac{1}{2}\frac{\partial P}{\partial x}dx\right)dydz-\left(\cancel{P}+\frac{1}{2}\frac{\partial P}{\partial x}dx\right)dzdy=\rho\,dxdydz\,a_x$$

$$-\frac{\partial P}{\partial x}\cancel{dxdydz}=\rho\,\cancel{dxdydz}\,a_x \rightarrow \boxed{\frac{\partial P}{\partial x}=-\rho a_x}$$

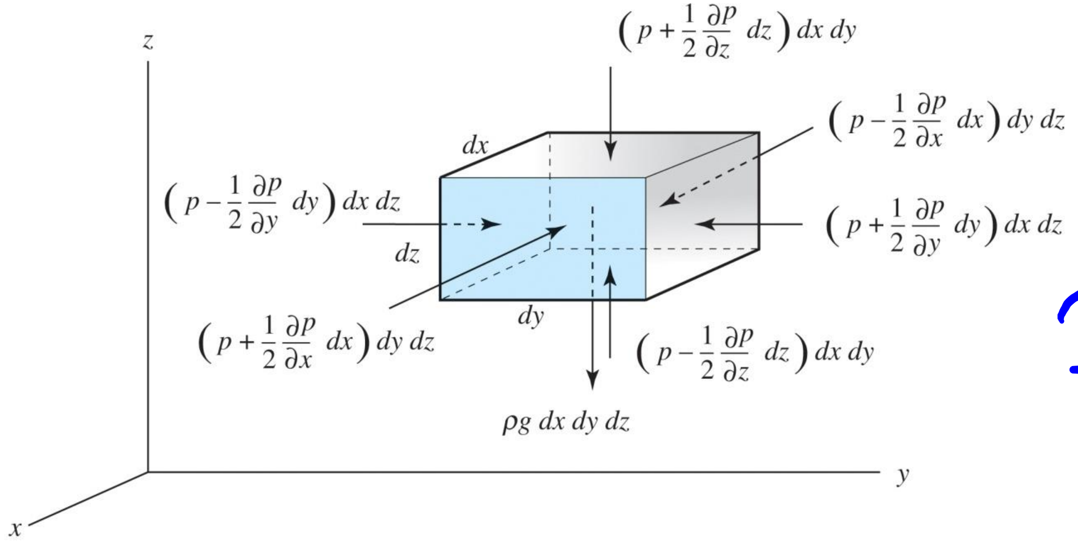


Fig. 2.3 Forces acting on an infinitesimal element that is at rest in the *xyz*-reference frame. The reference frame may be accelerating or rotating.

- Using the Chain rule, the pressure change in any direction can be calculated as:

$$\frac{\partial p}{\partial y} = -\rho a_y$$

$$\frac{\partial p}{\partial z} = -\rho(a_z+g)$$

- Then the pressure differential becomes:

$$dp = \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy + \frac{\partial p}{\partial z}dz$$

$$\boxed{dp = -\rho a_x dx - \rho a_y dy - \rho(a_z+g)dz}$$

2.4 FLUIDS AT REST

- The pressure differential (from the previous slide) is:

$$dp = -\rho a_x dx - \rho a_y dy - \rho(a_z + g)\,dz$$

- At rest, there is no acceleration (a = 0):

$$dp = -\rho g\,dz$$

$$dp = -\gamma\,dz$$

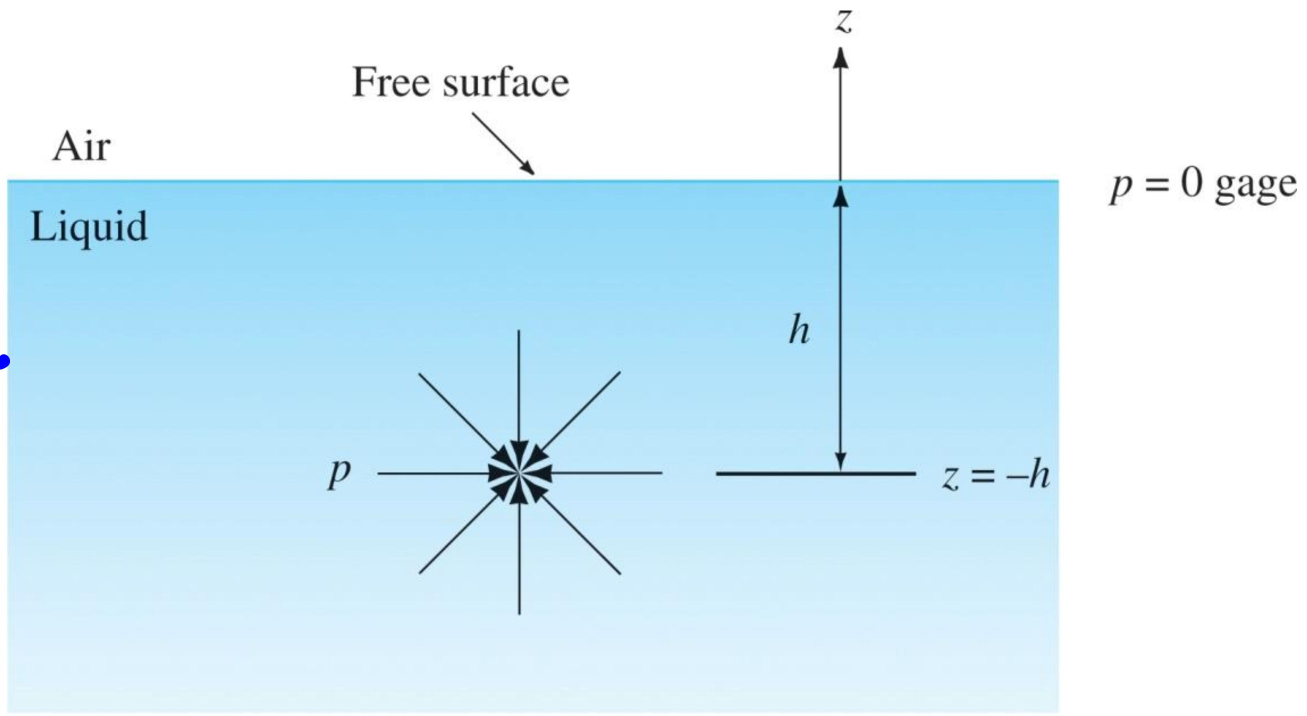


Fig. 2.4 Pressure below a free surface.

No pressure variation in the *x*- and *y*-directions (horizontal plane). Pressure varies in the *z*-direction only (d*p* is negative if d*z* is positive).

Pressure decreases as we move up and increases as we move down.

2.4 FLUIDS AT REST

2.4.1 Pressure in Liquids at Rest

- At a distance h below a free surface, the pressure is:

$$\int_0^P dp = \int_0^{-h} -\gamma \, dz$$

$$P = -\gamma z \Big|_0^{-h} \Rightarrow \boxed{P = \gamma h}$$

p = 0 at h = 0.

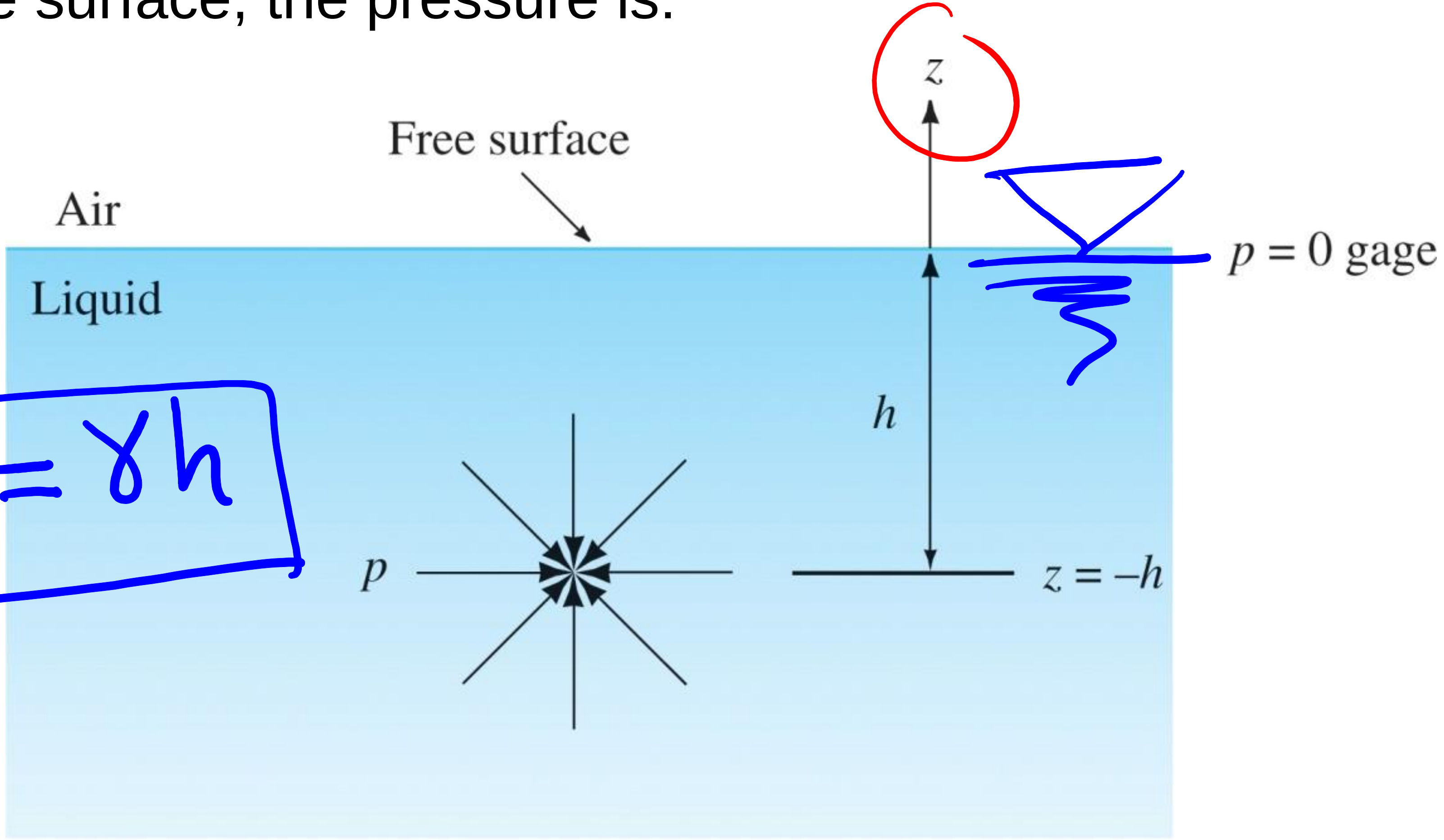


Fig. 2.4 Pressure below a free surface.

2.4.3 Manometers

Manometers are instruments that use columns of liquid to measure pressures.

- (a) displays a U-tube manometer used to measure **relatively small pressures**
- (b): **Large pressures** can be measured using a liquid with large γ_2.
- (c): **Very small pressures** can be measured as small pressure changes in p_1, leading to a relatively large deflection H.

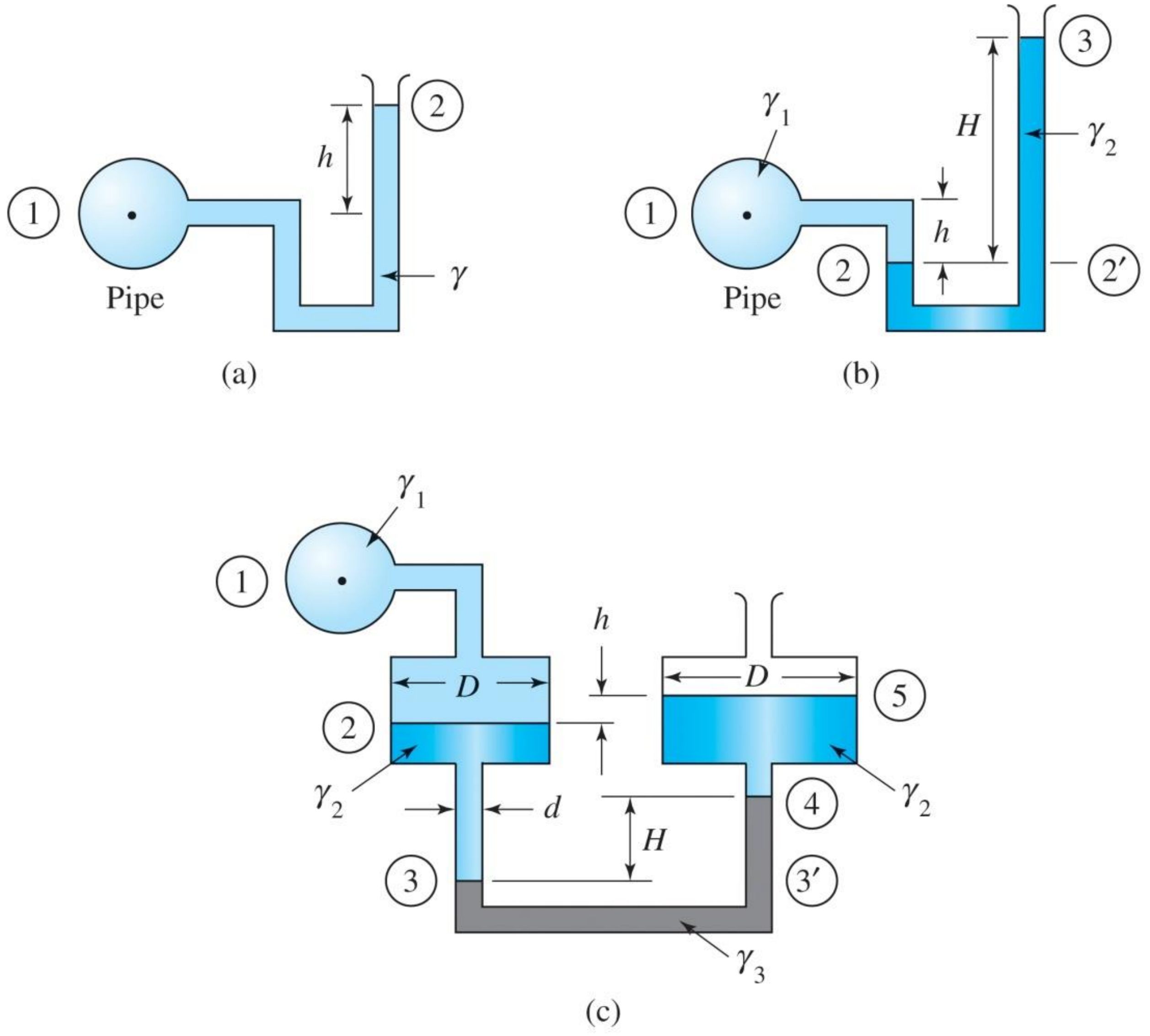


Fig. 2.7 Manometers: (a) U-tube manometer (small pressures); (b) U-tube manometer (large pressures); (c) micromanometer (very small pressure changes).

Example: **P.2.40**. Find the gage pressure in the water pipe shown in Fig. P2.40

$$P + 1000 \times 9.81 \times 0.05 + 1.59 \times 1000 \times 9.81 \times 0.07 = P_M + 0.8 \times 1000 \times 9.81 \times 0.1 \quad \cdots (1)$$

$P = ?$

Also,

$$P_M = 13.6 \times 1000 \times 9.81 \times 0.05 + 0$$

In (1)

$\boxed{P = 5873\ Pa}$ $\boxed{P = 5.87\ kPa}$

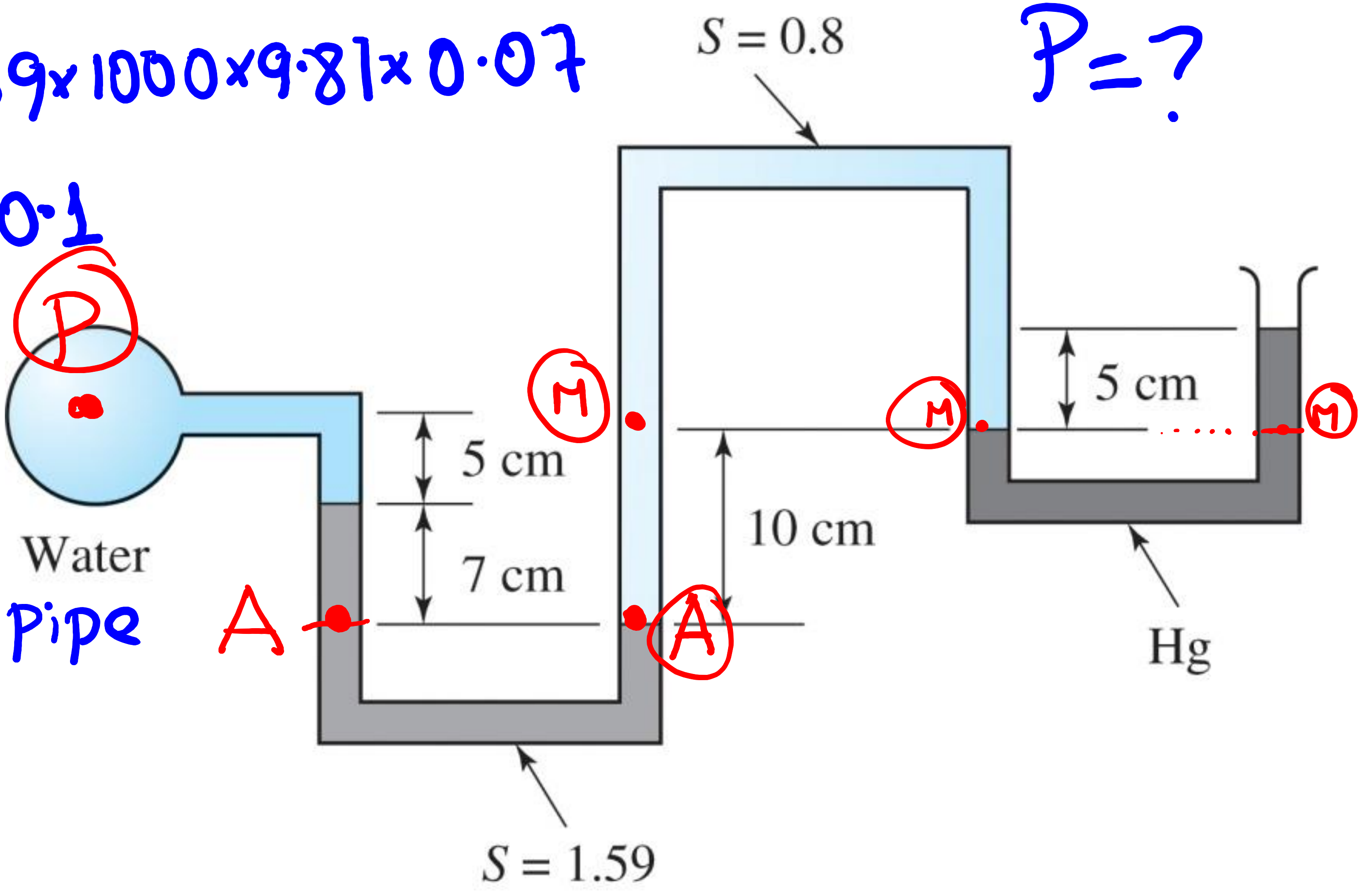


Fig. P2.40

Example: **P.2.41**. For the inclined manometer containing mercury, shown in Fig. P2.41, determine the pressure in pipe *B* if the pressure in pipe *A* is 10 kPa. Pipe *A* has water flowing through it, and oil is flowing in pipe *B*.

$P_B = ?$ $\quad P_A = 10{,}000\ Pa$

$$10{,}000\,Pa + 1000\times 9.81\times 0.07$$
$$= P_B + 0.87\times 1000\times 9.81\times 0.1 + 13.6\times 1000\times 9.81\times \frac{9}{100}\sin 40^\circ$$

$$\boxed{\begin{aligned} P_B &= 2115\ Pa \\ P_B &= 2.11\ KPa \end{aligned}}$$

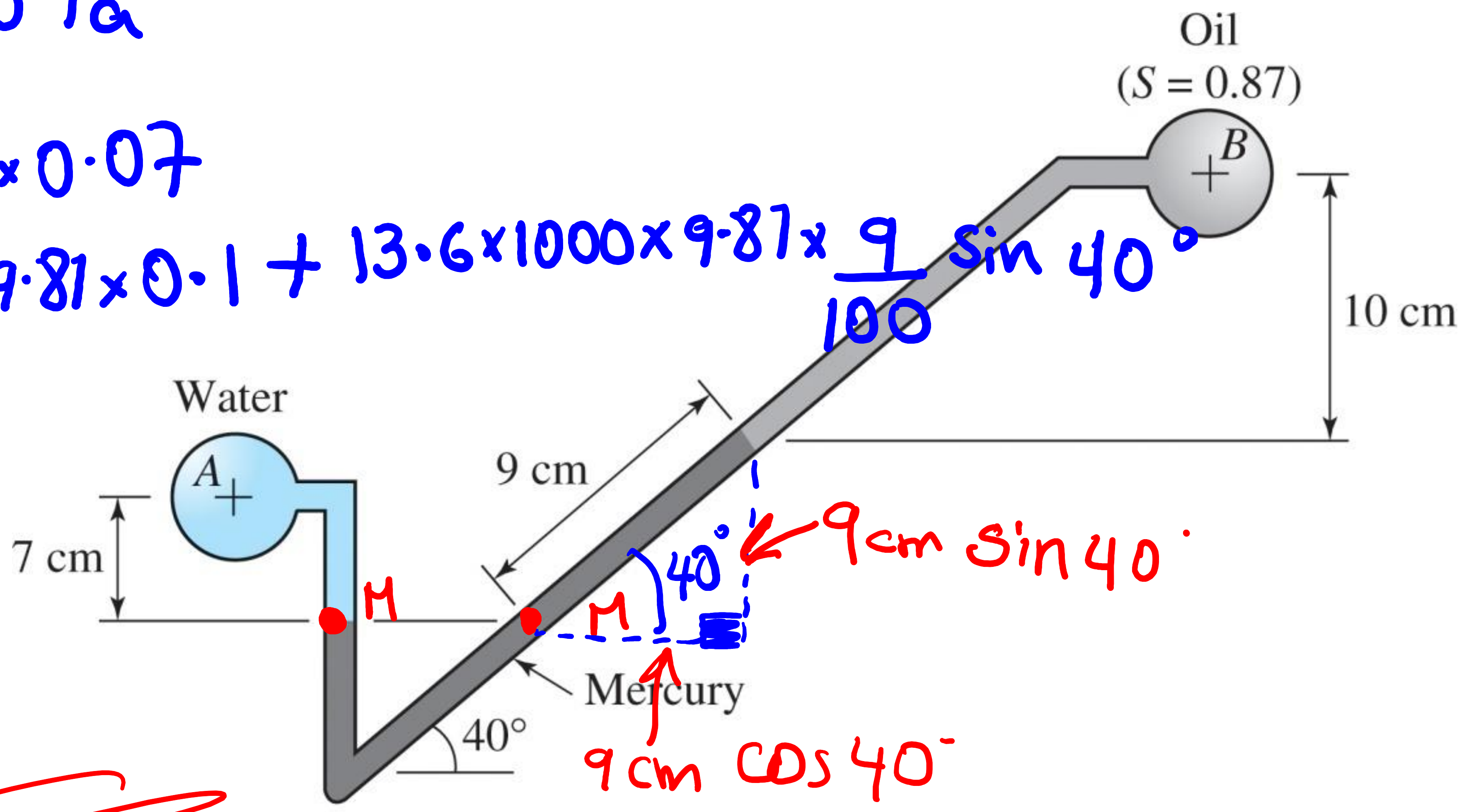
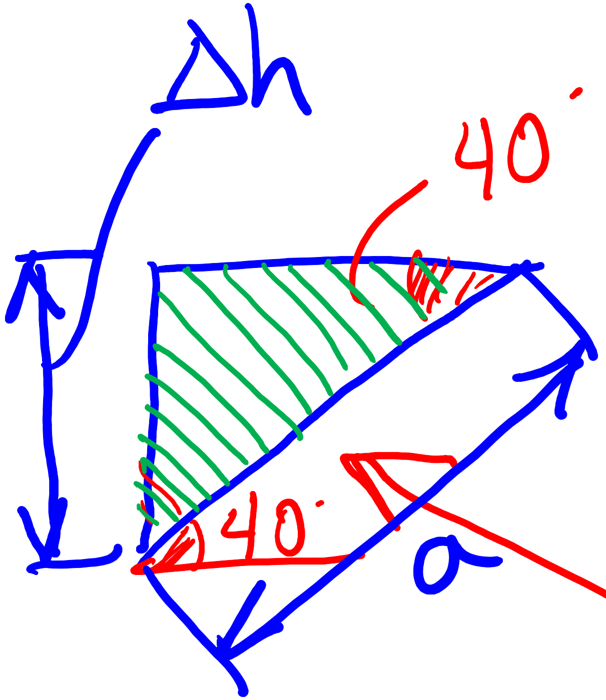


Fig. P2.41

Example: P.2.42. The pressure in pipe B in Problem P2.41 is reduced slightly. Determine the new pressure in pipe B if the pressure in pipe A remains the same and the reading along the inclined leg of the manometer is 11 cm (**Tip: See problems 2.41 and 2.42)**



$$a + 9\,cm + \Delta h = 11\,cm$$

$\sin\alpha = p/r$

$\cos\alpha = q/r$

$\tan\alpha = p/q$

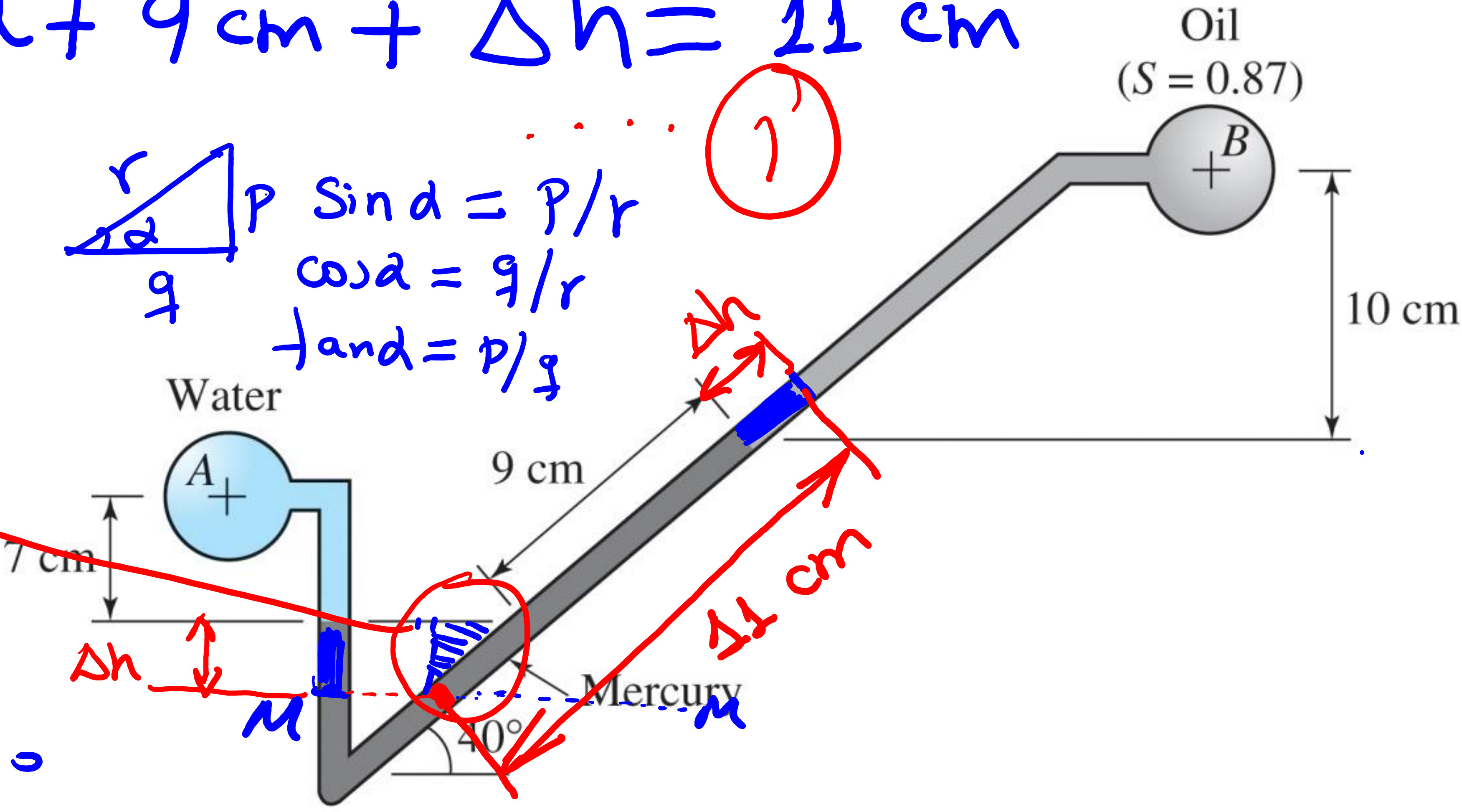


Fig. P2.41

$$\sin 40^\circ = \frac{\Delta h}{a}$$

$$a = \Delta h / \sin 40^\circ$$

In (i) $\frac{\Delta h}{\sin 40^\circ} + 9\,cm + \Delta h = 11\,cm$

$\Delta h = 0.783\,cm$

$$\boxed{\Delta h = \frac{0.783}{100}\,m}$$

$$10{,}000 + 1000 \times 9.81 \times \left(\frac{7 + 0.783}{100}\right) = P_B + 0.87 \times 1000 \times 9.81 \times \left(\frac{10 - 0.783\sin 40^\circ}{100}\right) + 13.6 \times 1000 \times 9.81 \times \frac{11\sin 40^\circ}{100}$$

$$\boxed{\begin{array}{l} P_B = 519.1\,Pa \\ P_B = 0.52\,kPa \end{array}}$$

$\sin\alpha = h/y$

$h = y\sin\alpha$

2.4.4 Forces on Plane Areas

- The total force of a liquid on a plane surface is:

$$\boxed{F = \int_A p\,dA}$$

- After knowing the equation for pressure ($P = \gamma h$):

$$F = \int_A \gamma h\,dA$$

$$F = \int_A \gamma y \sin\alpha\,dA$$

$$\boxed{F = \gamma \sin\alpha \int y\,dA}$$

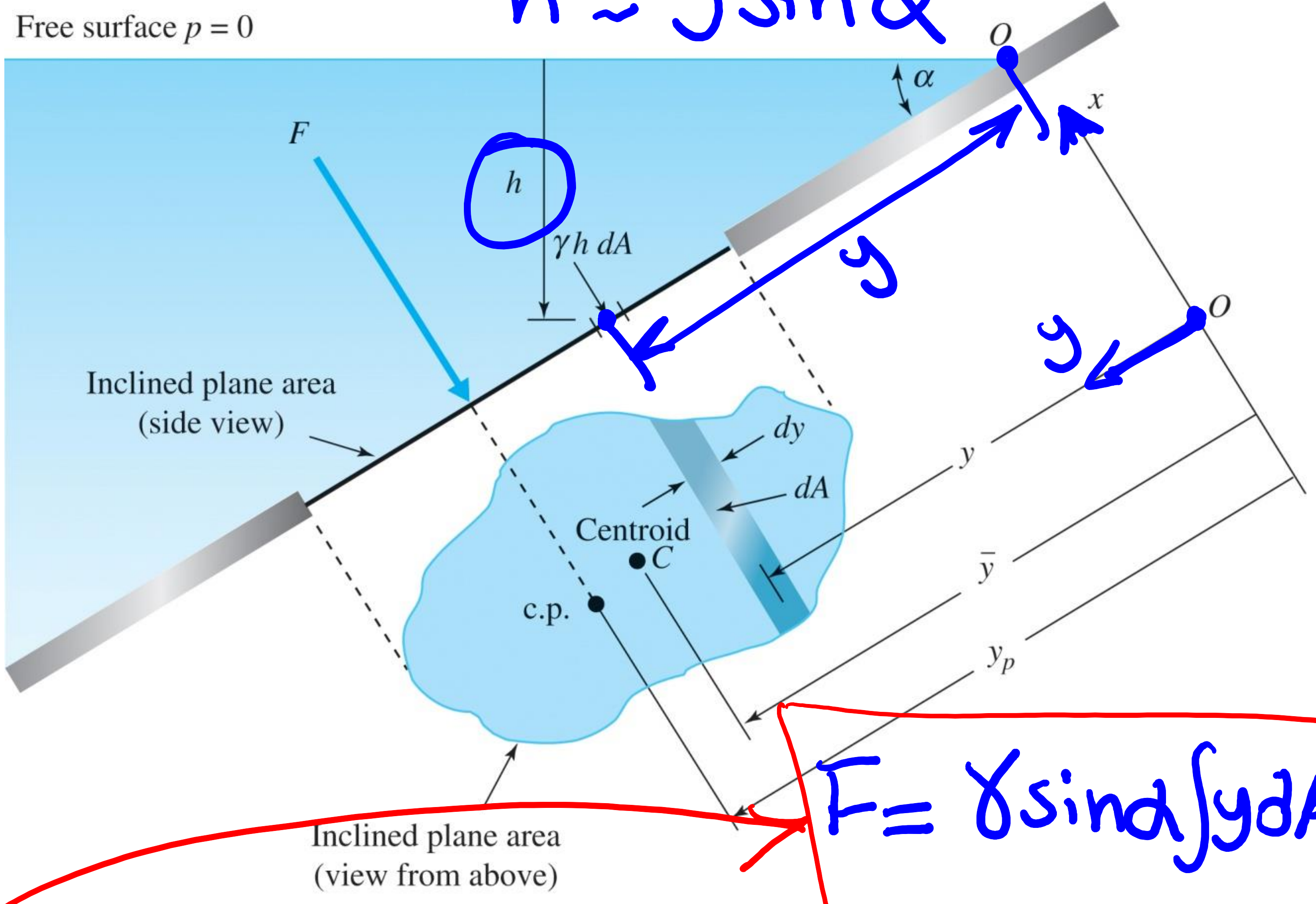


Fig. 2.8 Force on an inclined plane area.

2.4.4 Forces on Plane Areas

$$F = \int_A \gamma h \, dA = \gamma \bar{h} A$$

p_c

$$F = \gamma \bar{y} \sin\alpha \, A$$

$$\bar{h} \cdot A = \int h \, dA$$

$$\bar{h} = \frac{\int h \, dA}{A}$$

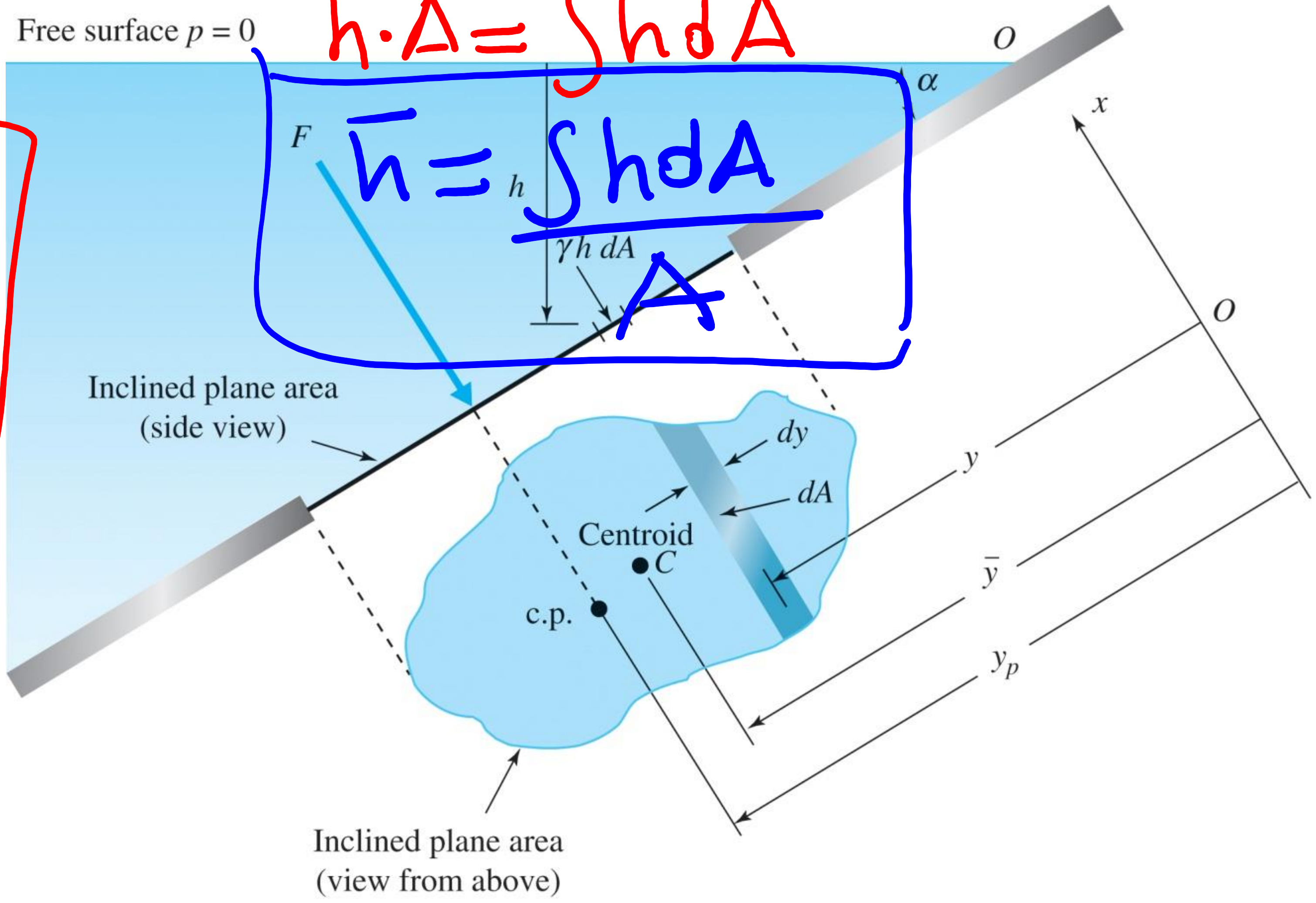


Fig. 2.8 Force on an inclined plane area.

$\bar{h}$: Vertical distance from the free surface to the centroid of the area

p_C: Pressure at the centroid

The **centroid** or geometric center of a plane figure is the arithmetic mean ("average") position of all the points in the shape.

2.4 FLUIDS AT REST

$$x_p \cdot F = \int x P \, dA$$

$$x_p \cdot F = \int x \gamma y \sin\alpha \, dA$$

$$x_p = \frac{\cancel{\gamma \sin\alpha} \int x y \, dA}{\cancel{\gamma \sin\alpha} \int y \, dA}$$

$$\boxed{x_p = \frac{\int x y \, dA}{\int y \, dA}}$$

(c.p.)

- The center of pressure is the point where the resultant force acts:
  - Sum of moments of all infinitesimal pressure forces on an area, A, equals the moment of the resultant force.

$$y_p \cdot F = \int y \cdot \gamma y \sin\alpha \, dA$$

$$\boxed{y_p = \frac{\int y^2 \, dA}{\int y \, dA}}$$

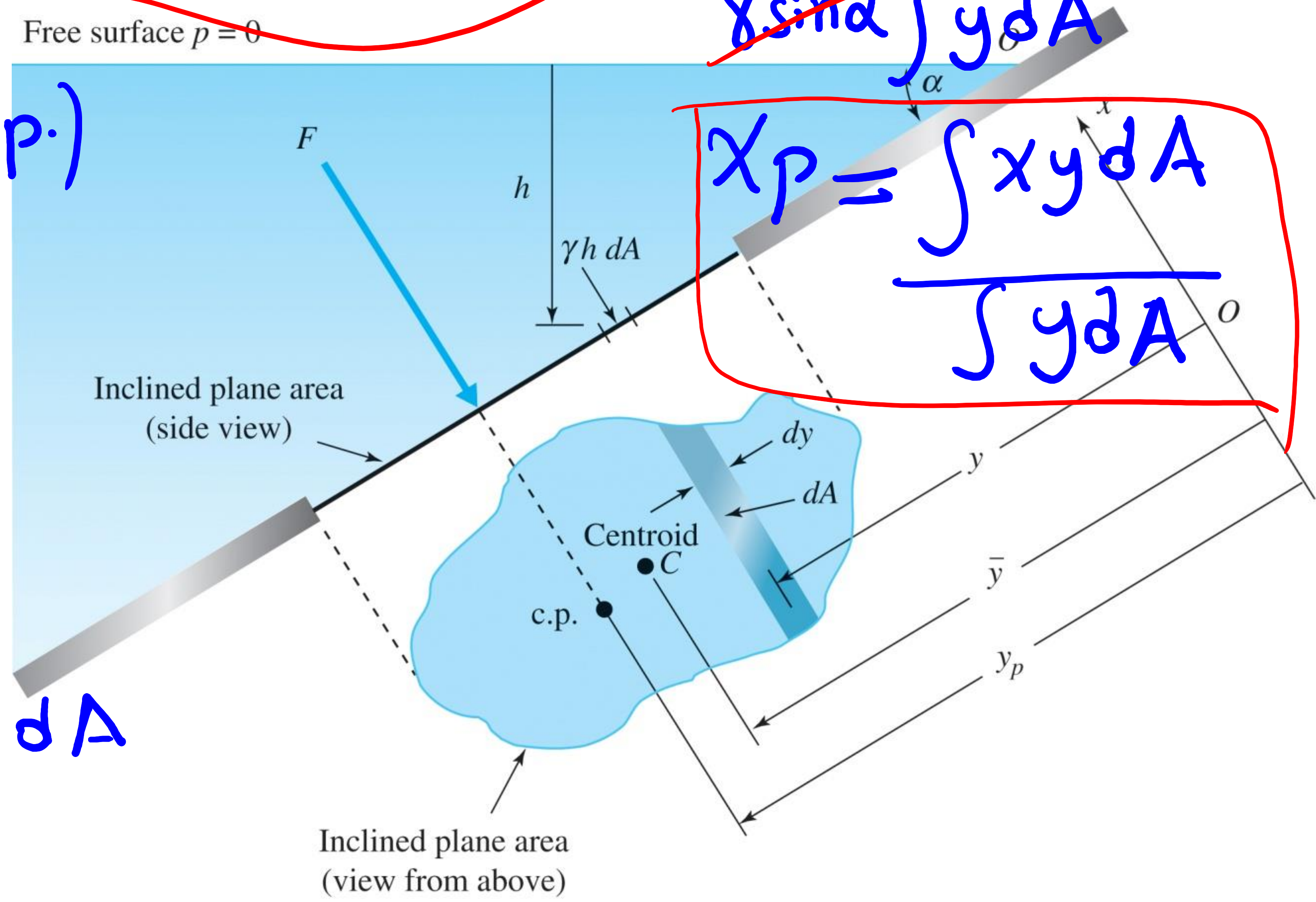


Fig. 2.8 Force on an inclined plane area.

$$x_p = \bar{x} + \frac{\bar{I}_{xy}}{A\bar{y}}$$

$$y_p = \bar{y} + \frac{\bar{I}}{A\bar{y}}$$

$\bar{y}$: Measured parallel to the plane area to the free surface

- The moments of area can be found using:

$$I_x = \int y^2 \, dA$$

$$I_{xy} = \int xy \, dA$$

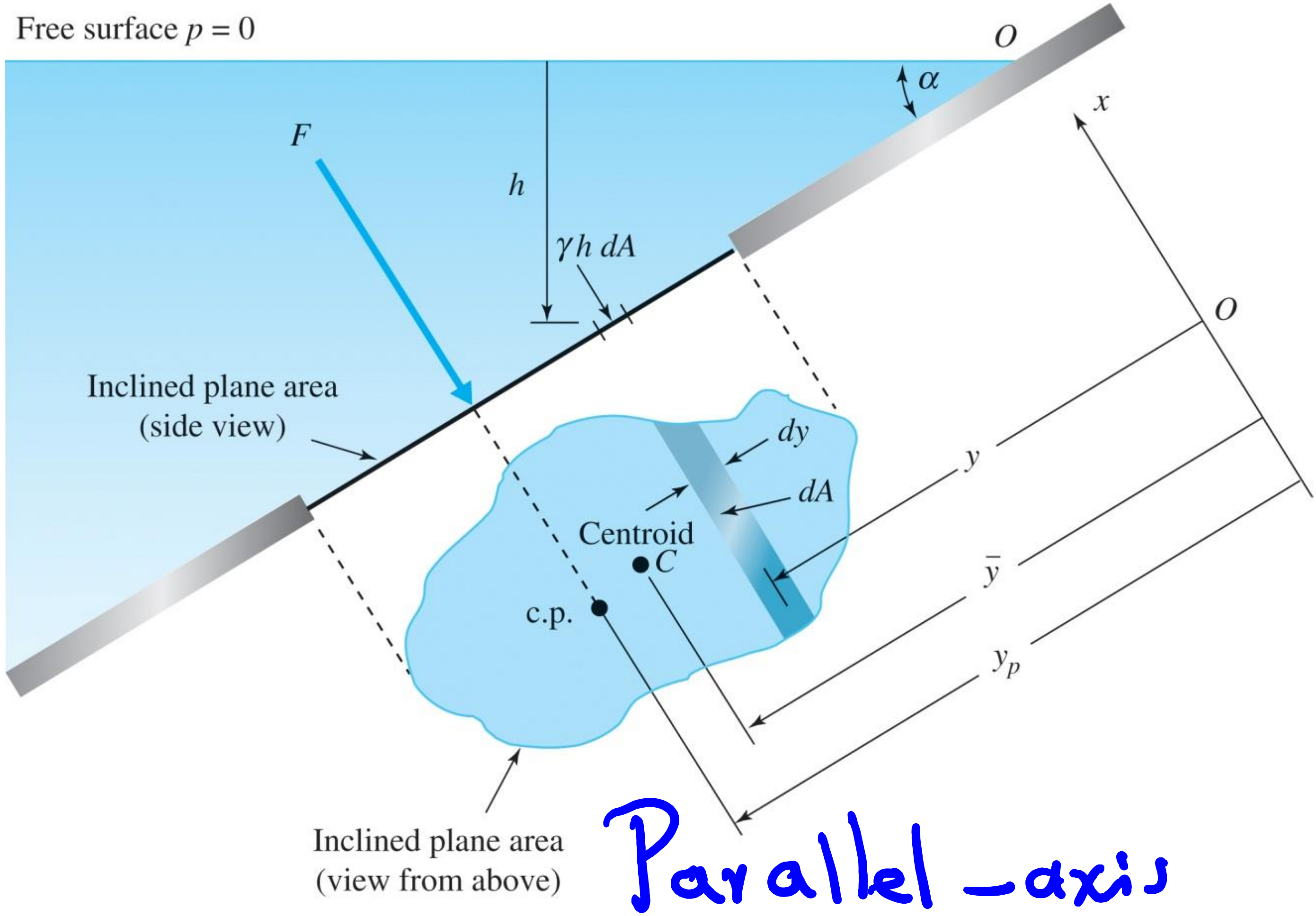


Fig. 2.8 Force on an inclined plane area.

Parallel-axis transfer theorem

$$\boxed{I_x = \bar{I} + A\bar{y}^2}$$

~~See Appendix C for~~ centroids and moments

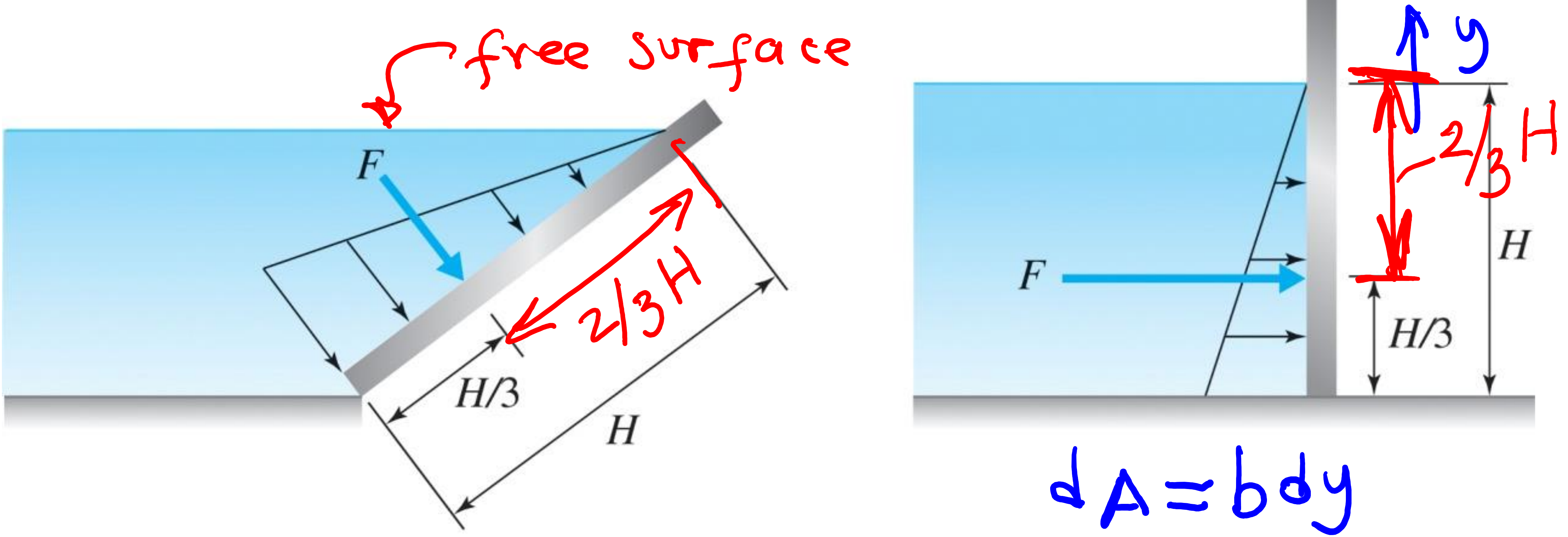


Fig. 2.9 Force on a plane area with top edge in a free surface.

$$y_p = \frac{\int y^2 dA}{\int y dA} = \frac{\int_0^H y^2 \cancel{b}\, dy}{\int_0^H y \cancel{b}\, dy} = \frac{y^3/3 \Big|_0^H}{y^2/2} = \frac{H^3/3}{H^2/2} = \frac{2}{3}H$$

Example: P.2.56. Determine the force *P* needed to hold the 4-m wide gate in the position shown in Fig. P2.56.

$P = ?$

Moments: $P(5) = F(10 - y_p)$ ... 1

$F = \gamma \bar{h} \cdot A$

$A = 5 \times 4 = 20\ m^2$

$\bar{h} = 4 + \frac{4}{2} = 6\ m$

$F = 1000 \times 9.81 \times 6 \times 20$

$\boxed{F = 1.177 \times 10^6\ N}$

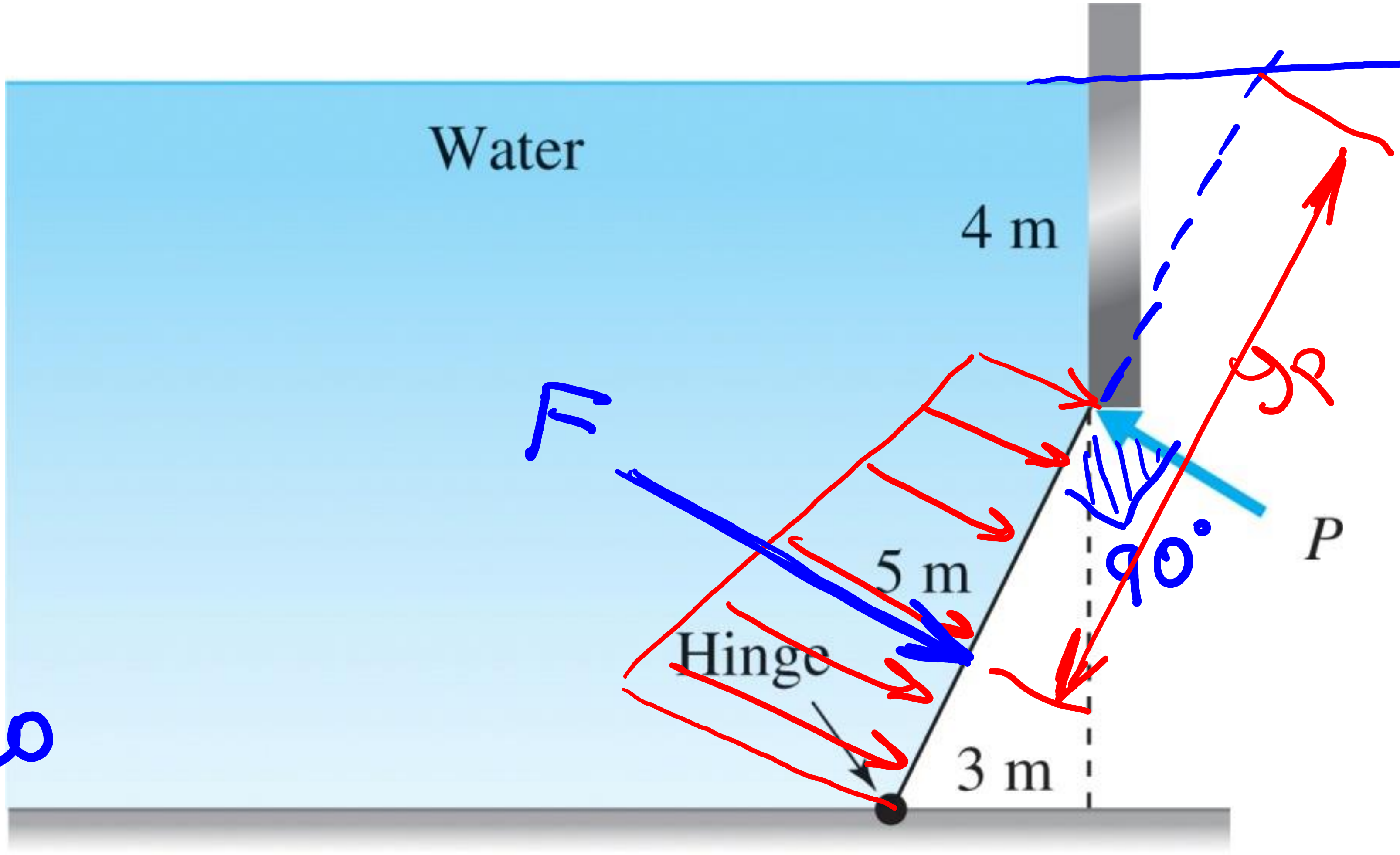


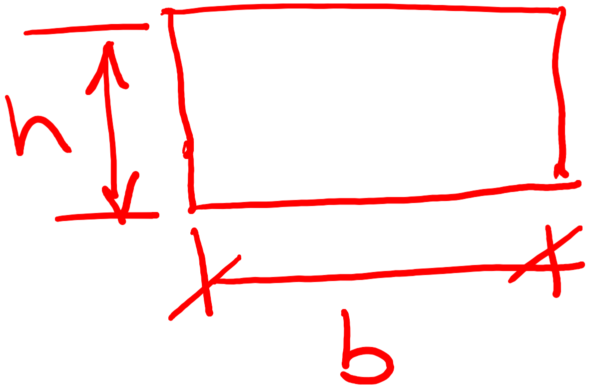
Fig. P2.56

* $y_p = \bar{y} + \dfrac{\bar{I}}{A\bar{y}}$

$\bar{y} = 5 + 2.5 = 7.5\,m$

$A = 20\,m^2$

$\bar{I} = 4 \times 5^3/12$



$I = \dfrac{bh^3}{12}$

$$\boxed{y_p = 7.778\,m}$$

In 1 $P(5) = 1.177 \times 10^6 (10 - 7.778)$

$$\boxed{P = 523{,}000\,N = 523\,kN}$$

Example: P.2.62. At what height *H* will the rigid gate, hinged at a central point as shown in Fig. P2.62, open if *h* is:

a) 0.6 m? b) 0.8 m? c) 1.0 m?

a) $h = 0.6\ m$

$y_p = H + 1.2m$

$y_p = \bar{y} + \frac{\bar{I}}{A\bar{y}}$

$\bar{y} = H + 0.9$

$A = 1.8b$

$\bar{I} = b \times 1.8^3/12$

$* \quad \cancel{H} + 1.2 = \cancel{H} + 0.9 + \frac{\cancel{b} * 1.8^3}{12(1.8\cancel{b})(H+0.9)}$

$\boxed{H = 0\ m}$

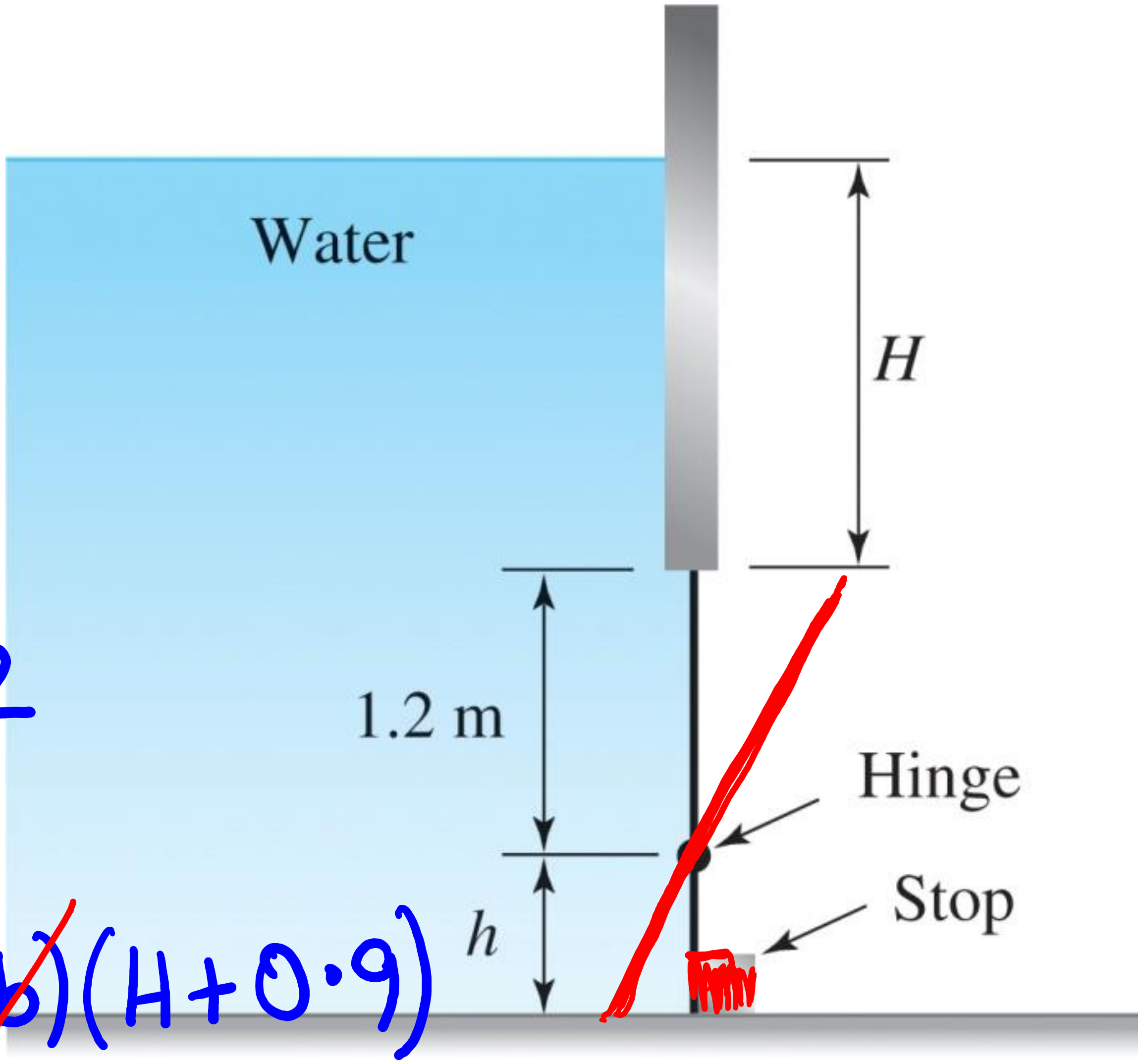
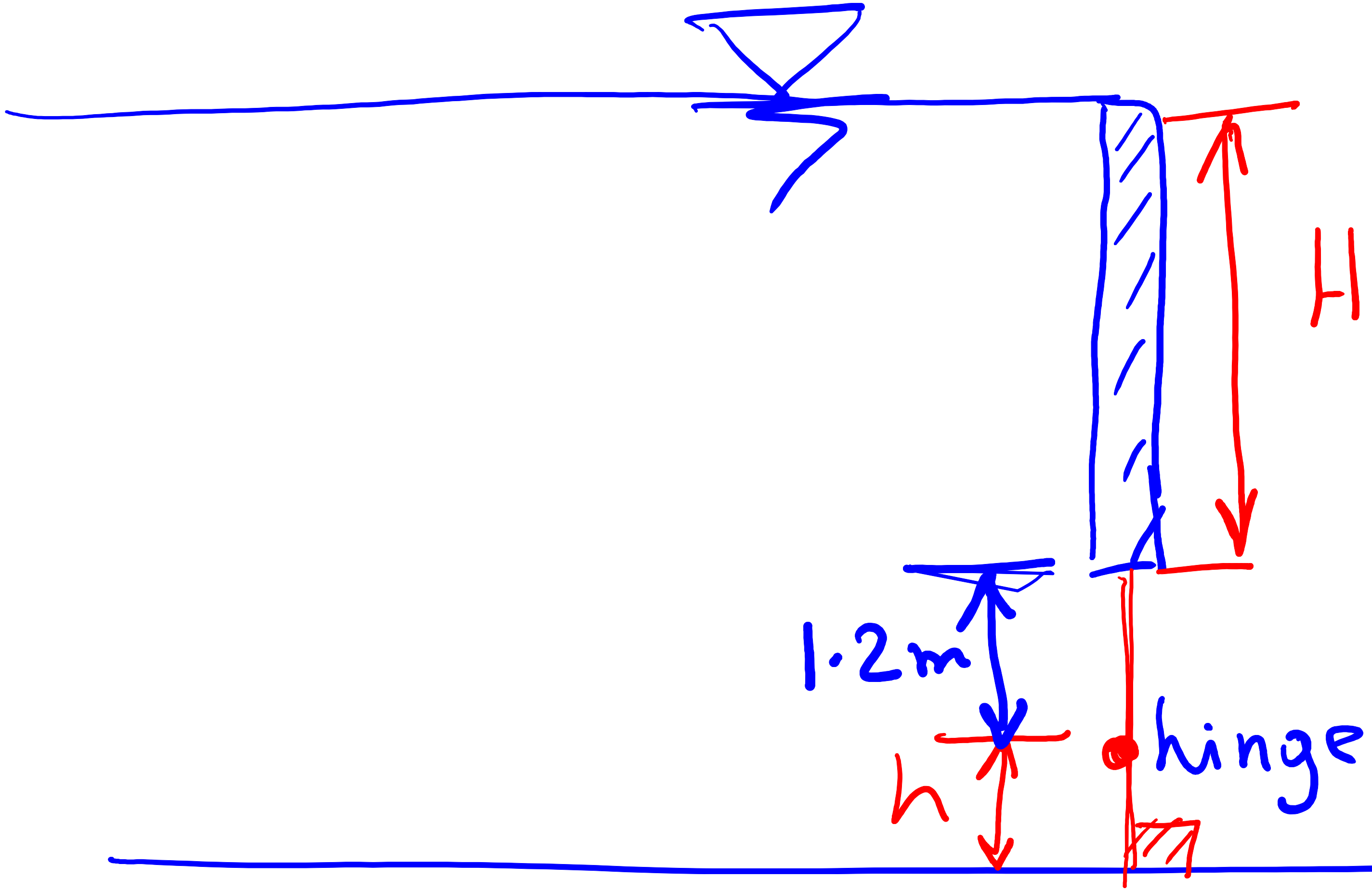


Fig. P2.62

if $H > 0$ m, gate will rotate

c) $h = 1.0$ m



$$y_p = 1.2m + H$$

$$y_p = \bar{y} + \frac{I}{A\bar{y}}$$

$$\bar{y} = H + \left(\frac{1.2 + 1.0}{2}\right)$$

$$= (H + 1.1)$$

$$A = 2.2b \qquad I = b \cdot \frac{h^3}{12} \Rightarrow \frac{b \cdot 2.2^3}{12} =$$

$$\Rightarrow \boxed{H = 2.93m}$$

2.4.5 Forces on Curved Surfaces

https://www.youtube.com/watch?v=zV-JO-l7Mx4

- Direct integration cannot find the force due to the hydrostatic pressure on a curved surface.
- A free-body diagram containing the curved surface and surrounding liquid needs to be identified.

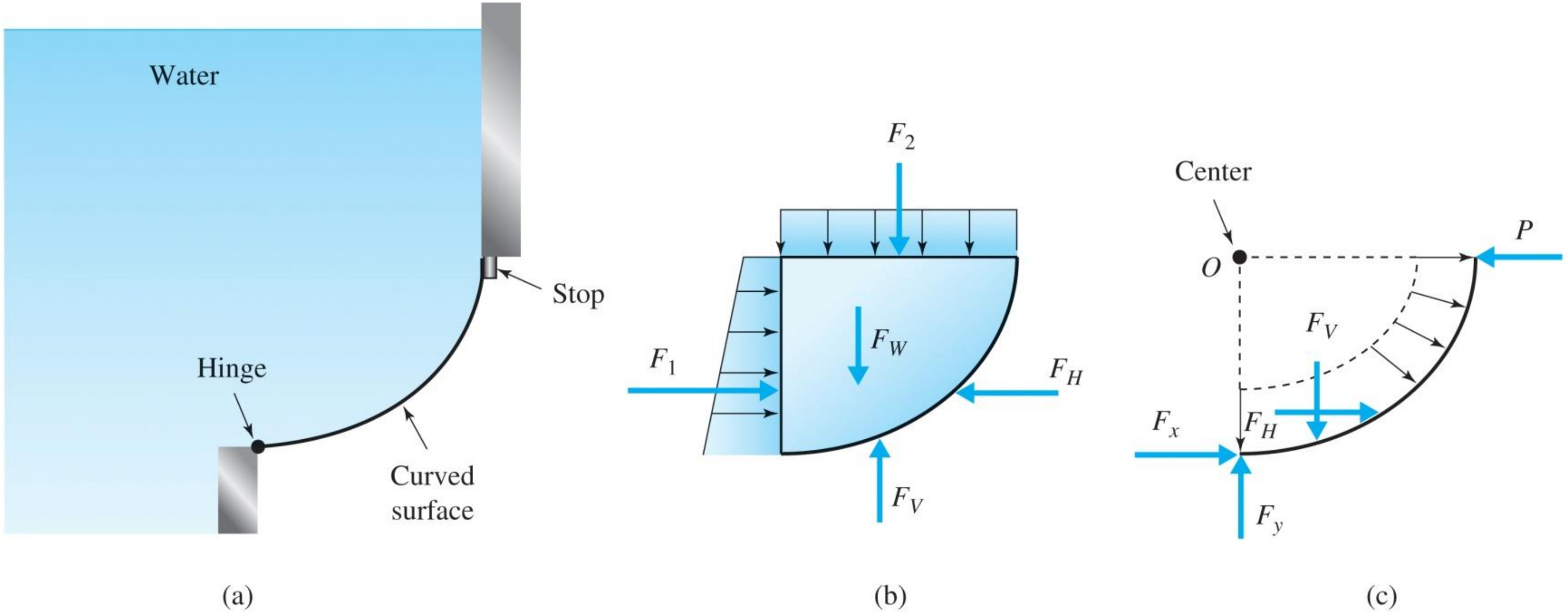


Fig. 2.11 Forces acting on a curved surface: (a) curved surface; (b) free-body diagram of water and gate; (c) free-body diagram of gate only.

Example: **P.2.72**. Find the force *P* required to hold the gate in the position shown in Fig. P.2.72 . The gate is 5-m wide.

$P = ??$ $\quad w = 5m$

$\therefore \Sigma M_{Hinge} = 0$

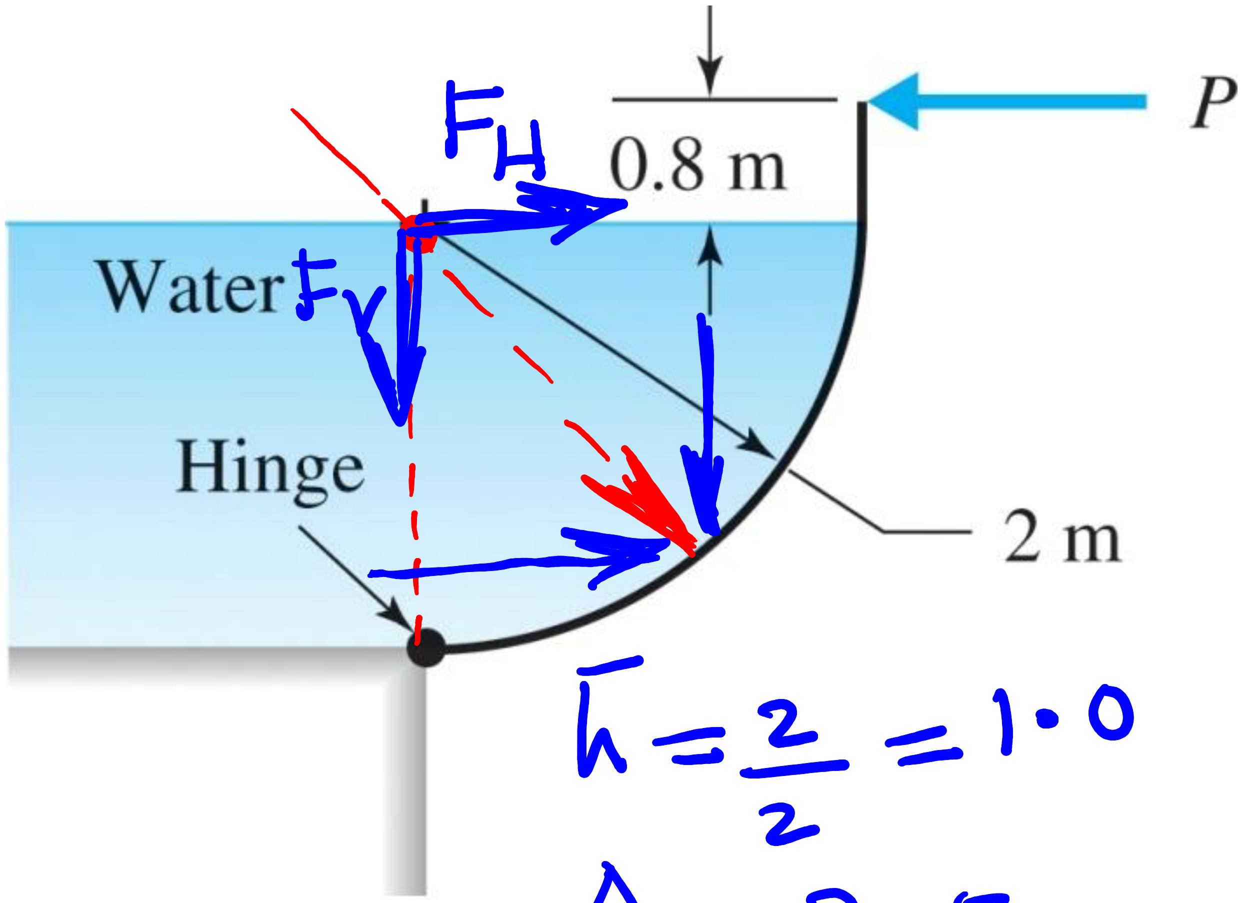
$P(2.8) = F_H \times 2.0 \quad \cdots (1)$

$F_H = \gamma \bar{h} \cdot A = 1000 \times 9.81 \times 1.0 \times 10$

$F_H = 98.1\ kN = 98,100\ N$

In (1)

$$\boxed{P = 70.07\ kN}$$



$\bar{h} = \frac{2}{2} = 1.0$

$A = 2 \times 5 = 10\ m^2$

Fig. P2.72

Example: **P.2.77**. Find the force P if the parabolic gate shown in Fig. P.2.77 is

a) 2-m wide and H = 2 m

b) 4-ft wide and H = 8 ft.

$P = ??$

a) $H = 2m, \; W = 2m$

$\Sigma M_{Hinge} = 0$

$$\boxed{P(2) = F_H\left(\frac{2}{3}\right) + F_V X_p}$$

$* \; F_H = \gamma \bar{h} \cdot A \qquad \bar{h} = 1.0$

$A = 2 \times 2 = 4 m^2$

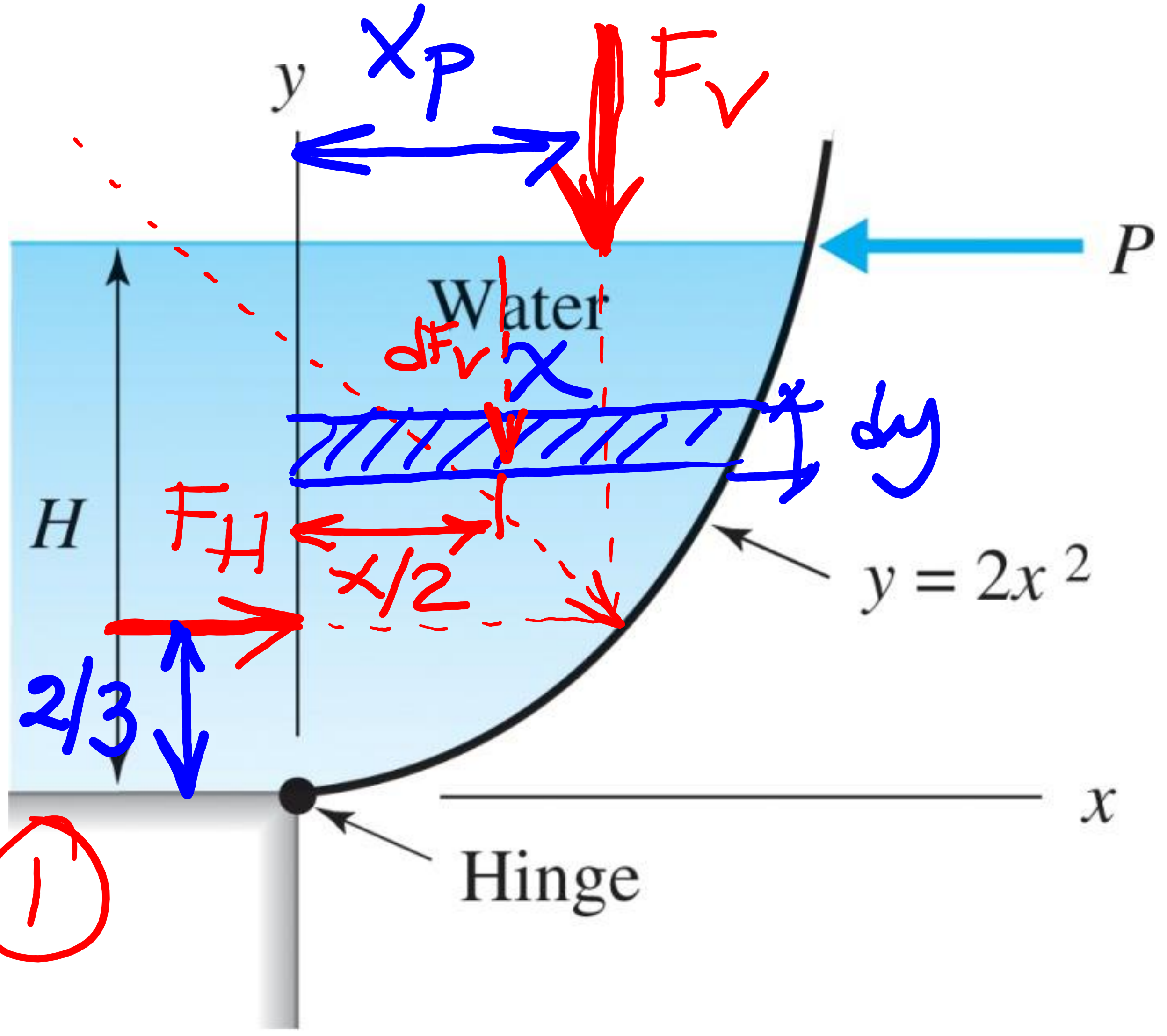


Fig. P2.77

$F_H = 9810 \times 1 \times 4 = 39,240\ N$

volume

$* F_V = \text{weight of water} = \gamma \forall = \gamma \int_{\forall} d\forall$

2m

$d\forall = x \cdot dy \cdot w = 2x\,dy$

$y = 2x^2 \rightarrow dy = 4x\,dx$

$F_V = 9810 \int_0^1 2x(4x)\,dx = 9810 \int_0^1 8x^2\,dx$

$= 9810 \left. \frac{8x^3}{3} \right|_0^1 = 26,160 N$

$$* \; X_p \cdot F_V = \int \frac{X}{2} \cdot dF_V$$

$$X_p \cdot F_V = \int \frac{x}{2} \gamma (8x^2)\, dx = \gamma \int_0^1 4x^3\, dx = \gamma \frac{\cancel{4}x^4}{\cancel{4}} \Big|_0^1$$

$$X_p = \frac{9810}{26160} = 0.375 \text{ m}$$

In 1

$$P = 17,985 \text{ N}$$

$$\boxed{P = 17.99 \text{ KN}}$$

2.4.6 Buoyancy (Archimedes' principle)

https://www.youtube.com/watch?v=2RefIvqaYg8

- **Buoyancy force on an object equals the weight of displaced liquid.**

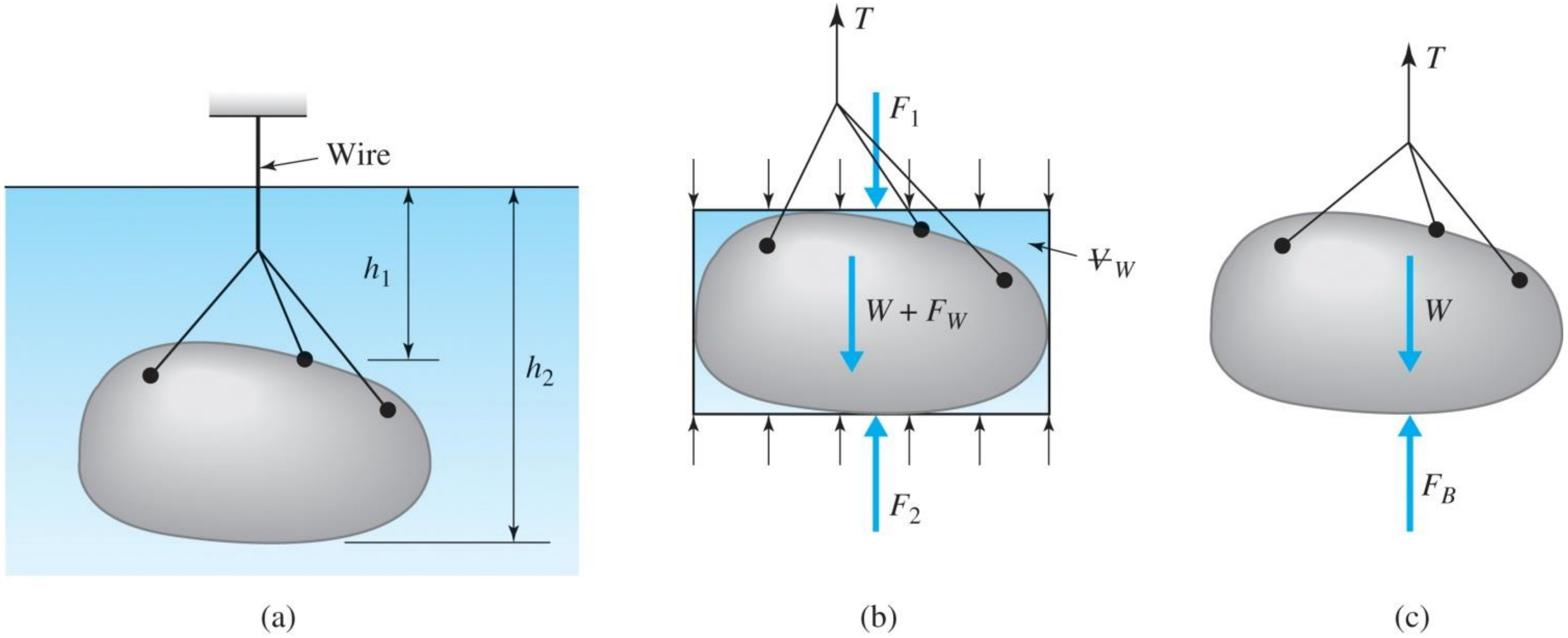


Fig. 2.12 Forces on a submerged body: (a) submerged body; (b) free-body diagram; (c) free body showing the buoyant force F_B.

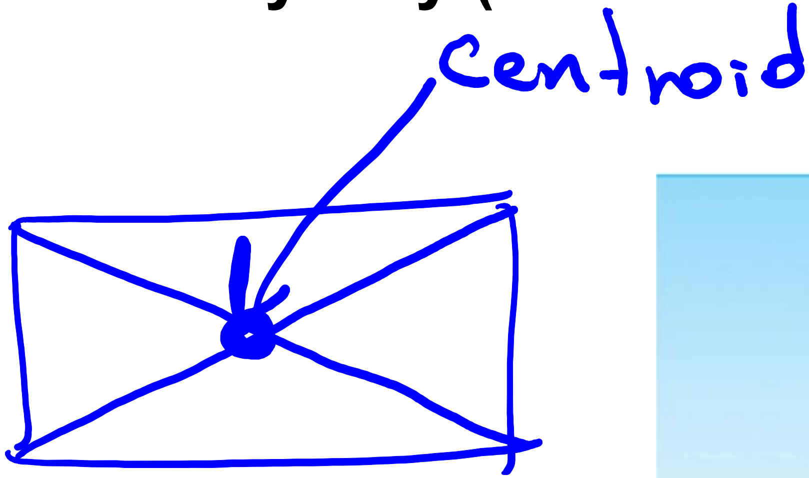
$F_B + T - W = 0$

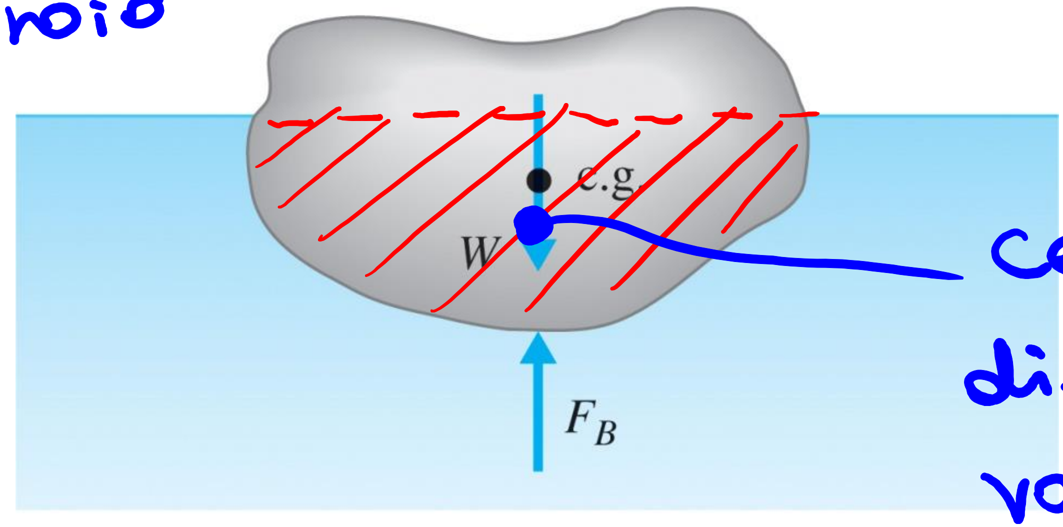
V is the volume of displaced fluid and *W* is the weight of the floating object.

2.4 FLUIDS AT REST

2.4.6 Buoyancy (Archimedes' principle)

$W = F_B$





centroid of the displaced liquid volume

Fig. 2.13 Forces on a floating object.

- The buoyant force acts through the centroid of the displaced liquid volume.
- An application of this would be a hydrometer that is used to measure the specific gravity of liquids.
  - For pure water, this is 1.0

2.4 FLUIDS AT REST

2.4.6 Buoyancy (Hydrometers)

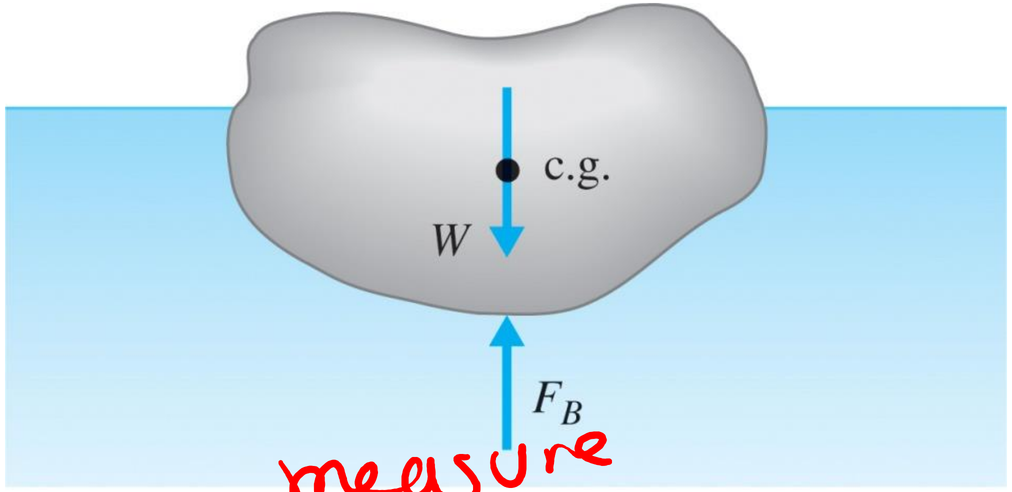


Fig. 2.13 Forces on a floating object.

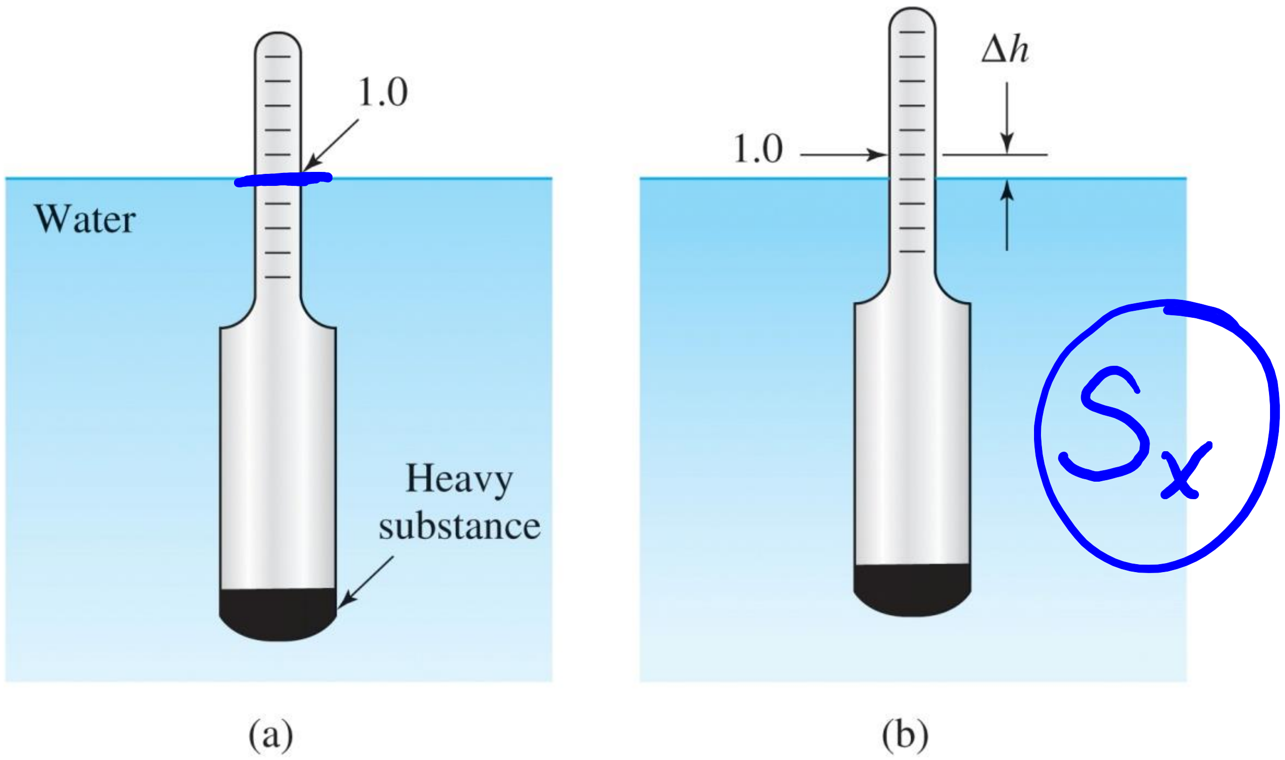


Fig. 2.14 Hydrometer: (a) in water; (b) in an unknown liquid.

measure

$$\Delta h = \frac{V}{A}\left(1 - \frac{1}{S_x}\right)$$

- Where Δh is the displaced height
- A: Cross-sectional area of the stem
- $S_x = \frac{\gamma_x}{\gamma_{water}}$ (specific gravity of unknown liquid)
- For a given hydrometer, V and A are fixed.

Example: **P.2.78**. The 3-m wide barge shown in Fig. P.2.78 weighs 20 kN empty. It is proposed that it carry a 250-kN load. Predict the draft in:

a) Fresh water

b) Salt water (S = 1.03)

a) $S = 1.0$

$W = 3m$

$Weight = 270\ kN$

$F_B = 270{,}000$

$F_B = \gamma_{fluid} \forall_{displaced}$

$S * \rho_w * g \cdot \forall_{displaced}$

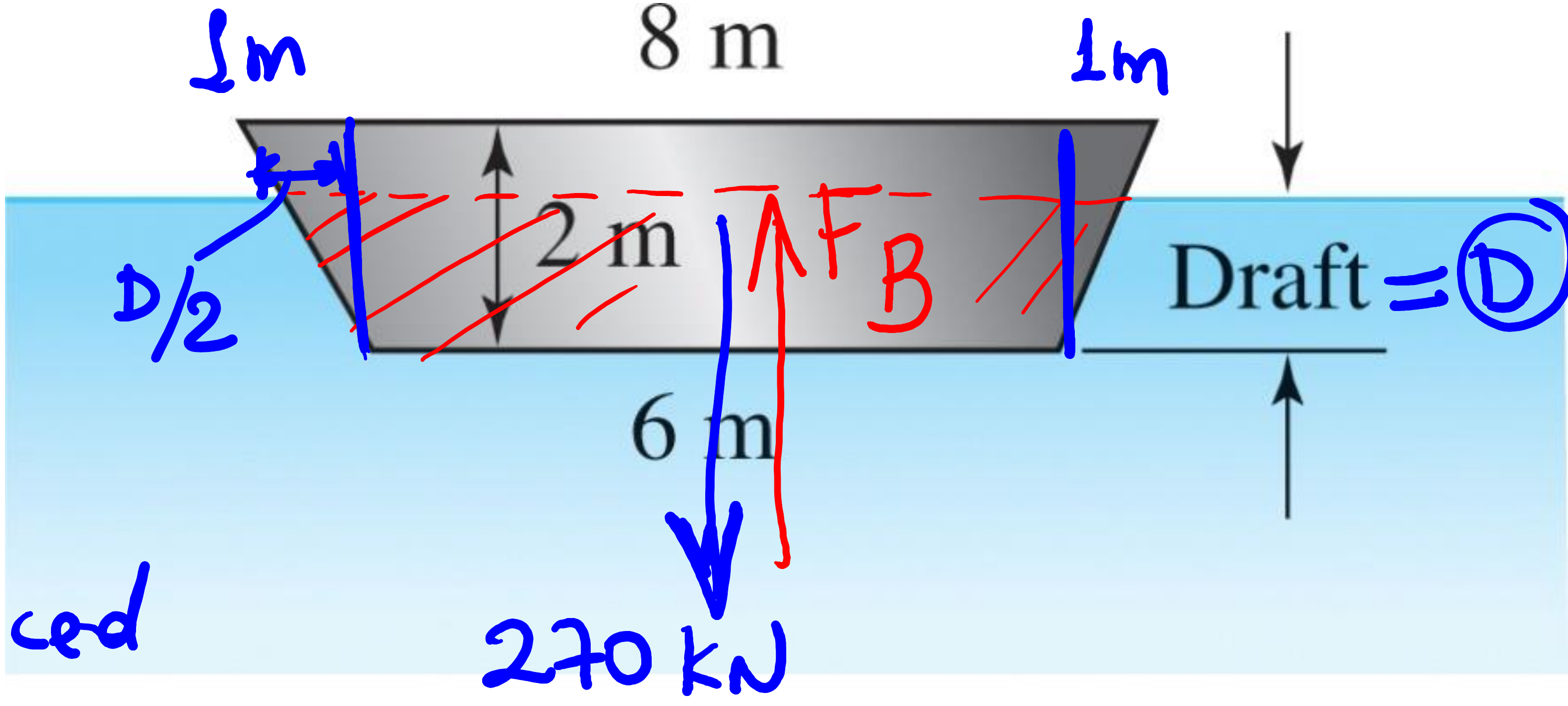
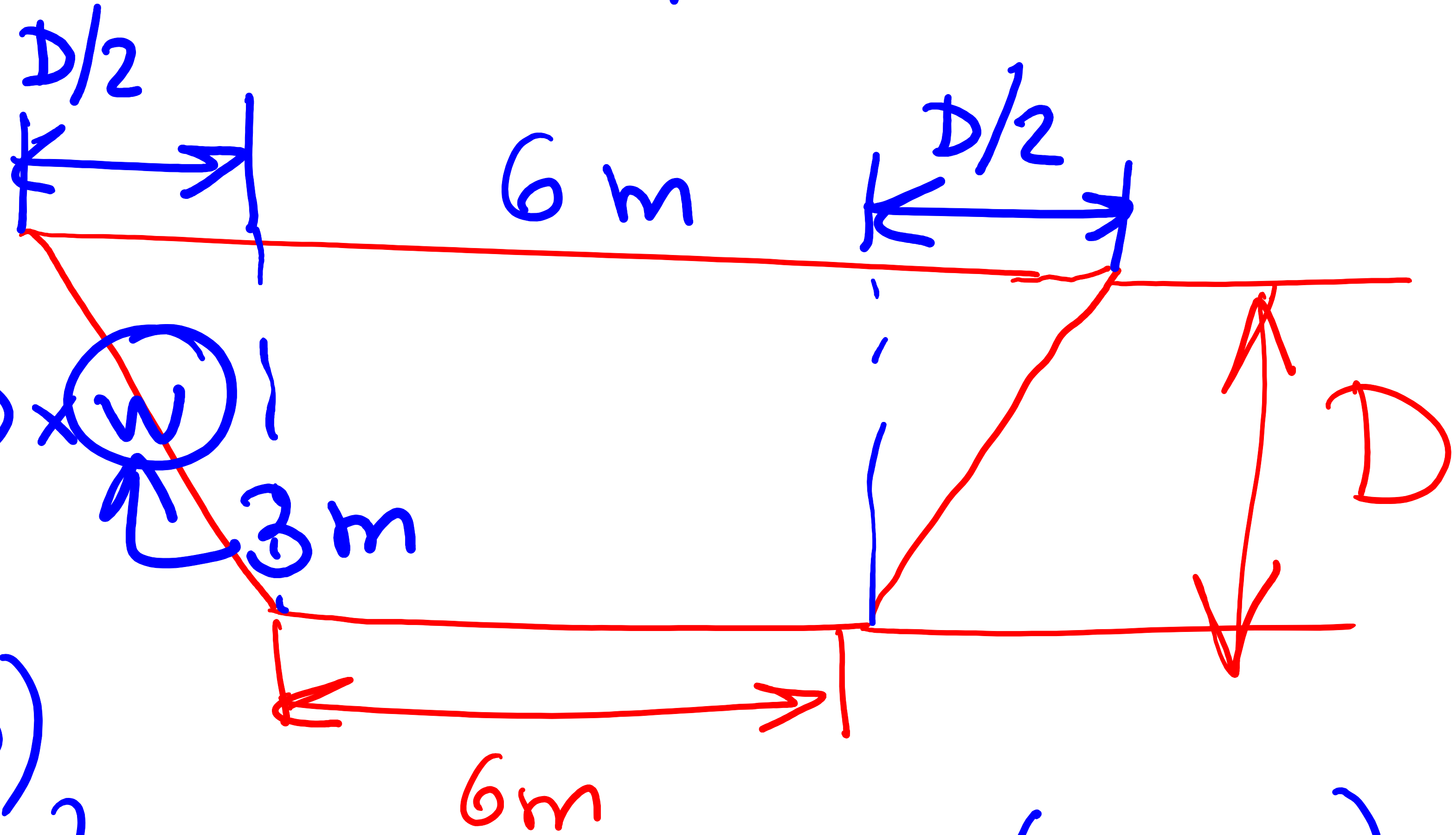


Fig. P2.78

$$1.0 \times 1000 \times 9.81 \times \forall = 270{,}000$$



$$\forall = \frac{(6+6+D)}{2} D \times w$$

$$\forall = \frac{(12+D)}{2}(3D)$$

$$27.52 = 1.5D(12+D)$$

$$D = 1.372m$$

b) $D = 1.336\ m$

2.4.7 Stability

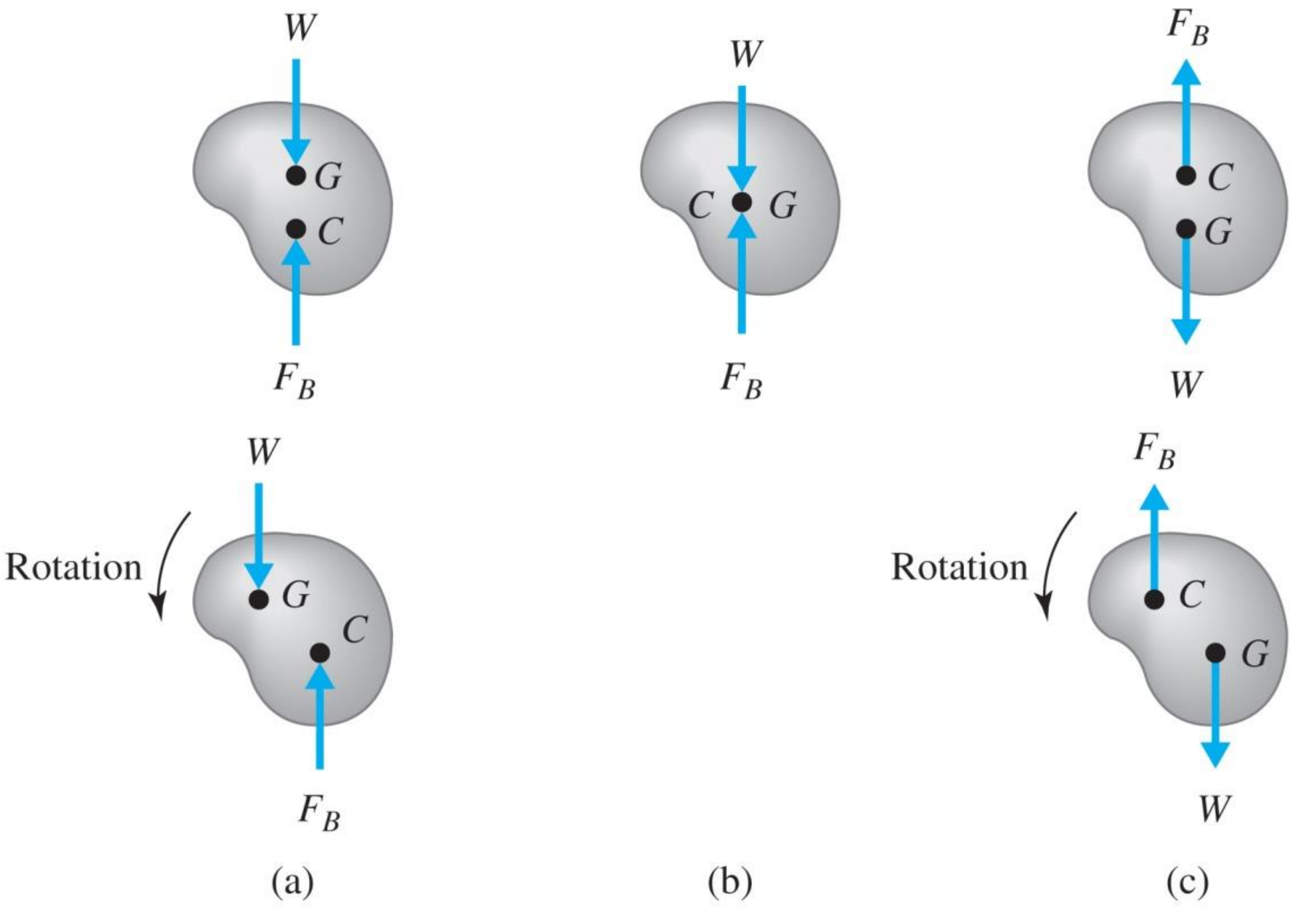


Fig. 2.15 Stability of a submerged body: (a) unstable; (b) neutral; (c) stable.

G above C

opposite

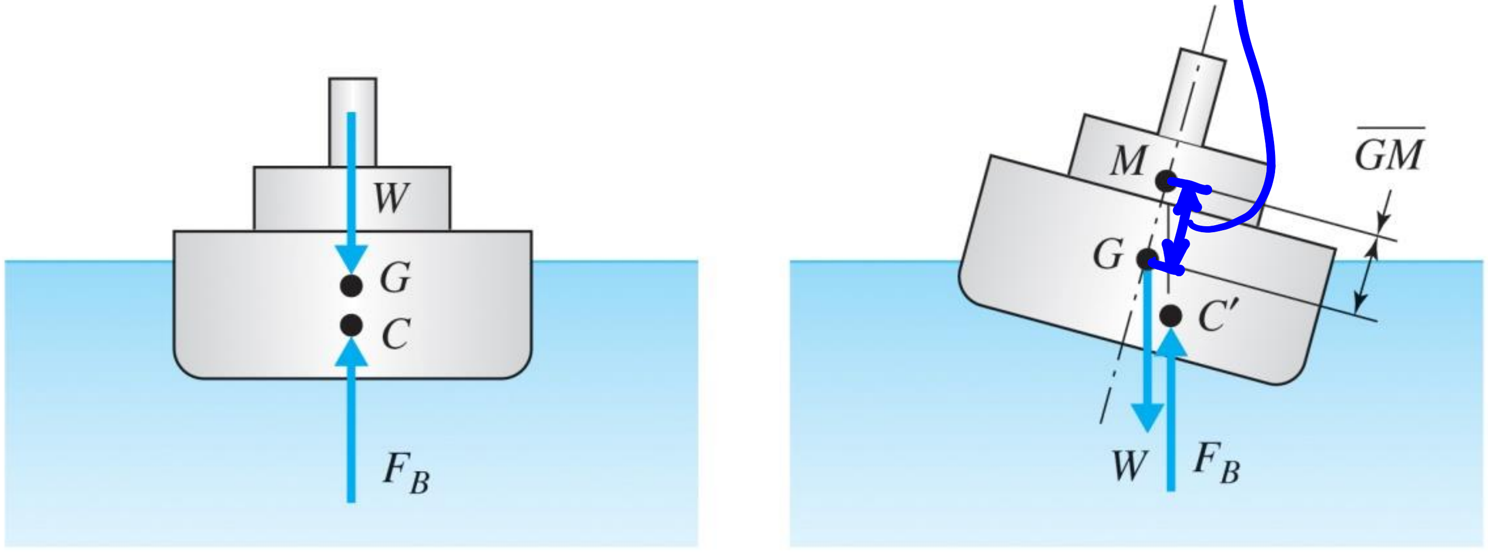
- In (a) the center of gravity of the body is above the centroid C (center of buoyancy), so a small angular rotation leads to a moment that increases rotation: unstable.
- (b) shows neutral stability as the center of gravity and the centroid coincide.
- In (c), as the center of gravity is below the centroid, a small angular rotation provides a restoring moment and the body is stable.

2.4 FLUIDS AT REST

Metacentric height

https://www.youtube.com/watch?v=QUgXf2Rj2YQ

metacentric height



(a) (b)

Fig. 2.16 Stability of a floating body: (a) equilibrium position; (b) rotated position.

- The **metacentric height $\overline{GM}$** is the distance from G to the point of intersection of the buoyant force before rotation with the buoyant force after rotation.
- If $\overline{GM}$ is positive: Stable
- If $\overline{GM}$ is negative: Unstable

$$GM = CM - GC$$

$$\boxed{GM = \frac{\bar{x}}{\tan\alpha} - GC \quad *}$$

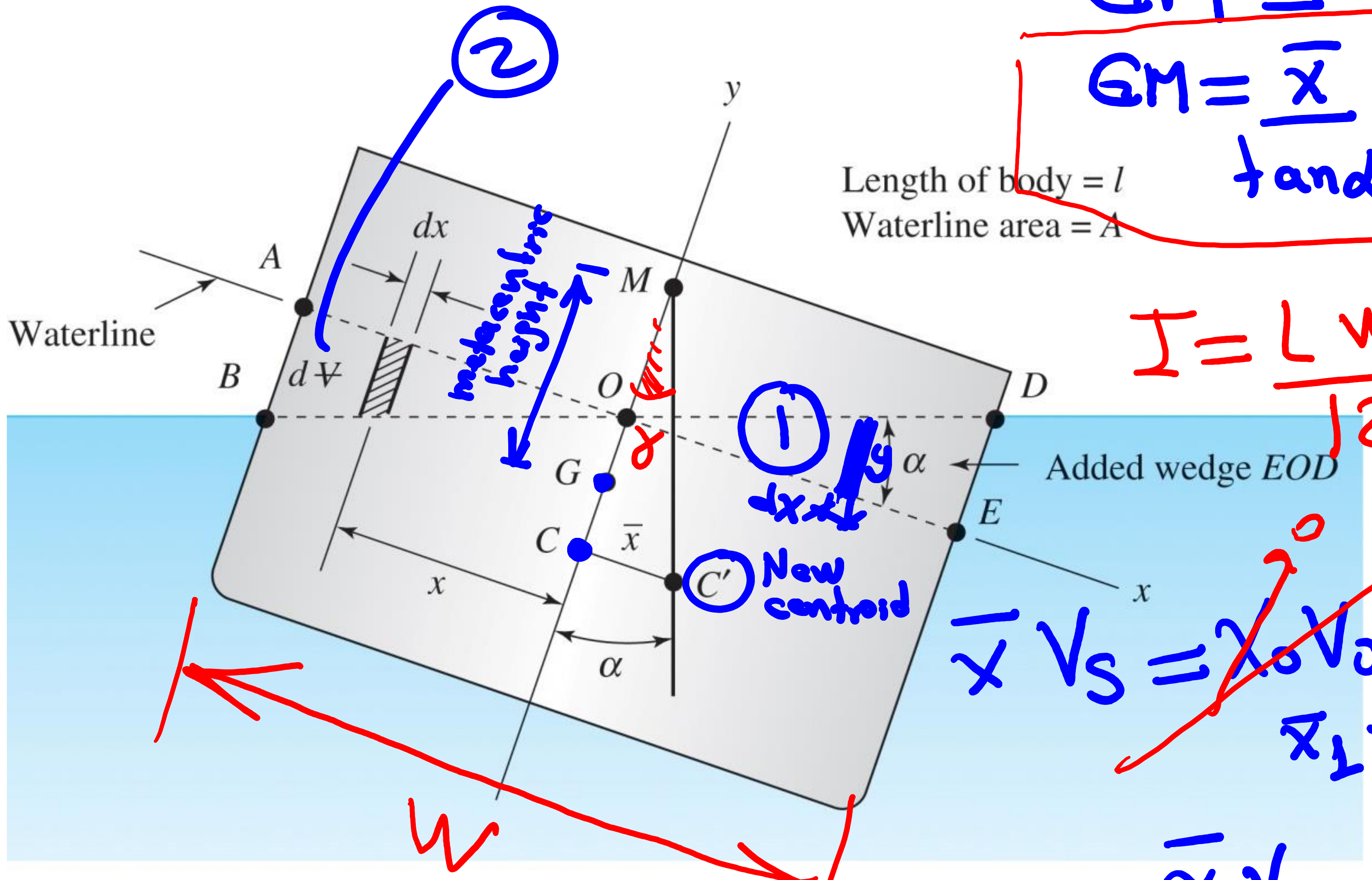
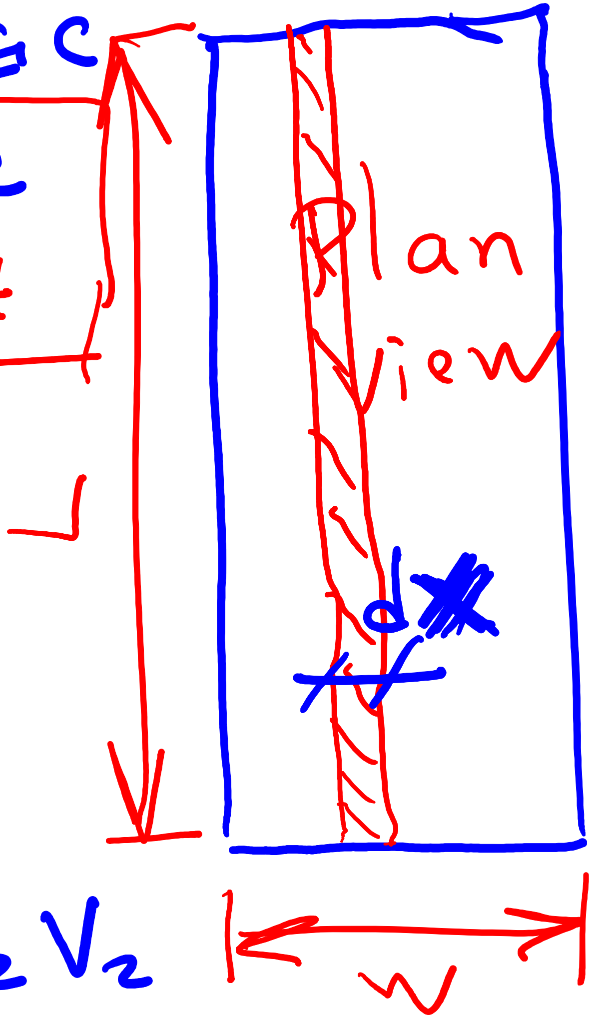


Fig. 2.17 Uniform cross section of a floating body.

$$I = \frac{Lw^3}{12}$$



$$\bar{x}V_s = \cancel{\bar{x}_0 V_0} + \bar{x}_1 V_1 - \bar{x}_2 V_2$$

$$\bar{x}V_s = \int_{\forall_1} x\,dV - \int_{\forall_2} x\,dV \quad \cdots (1)$$

$$\int x^2 \, dA = I$$

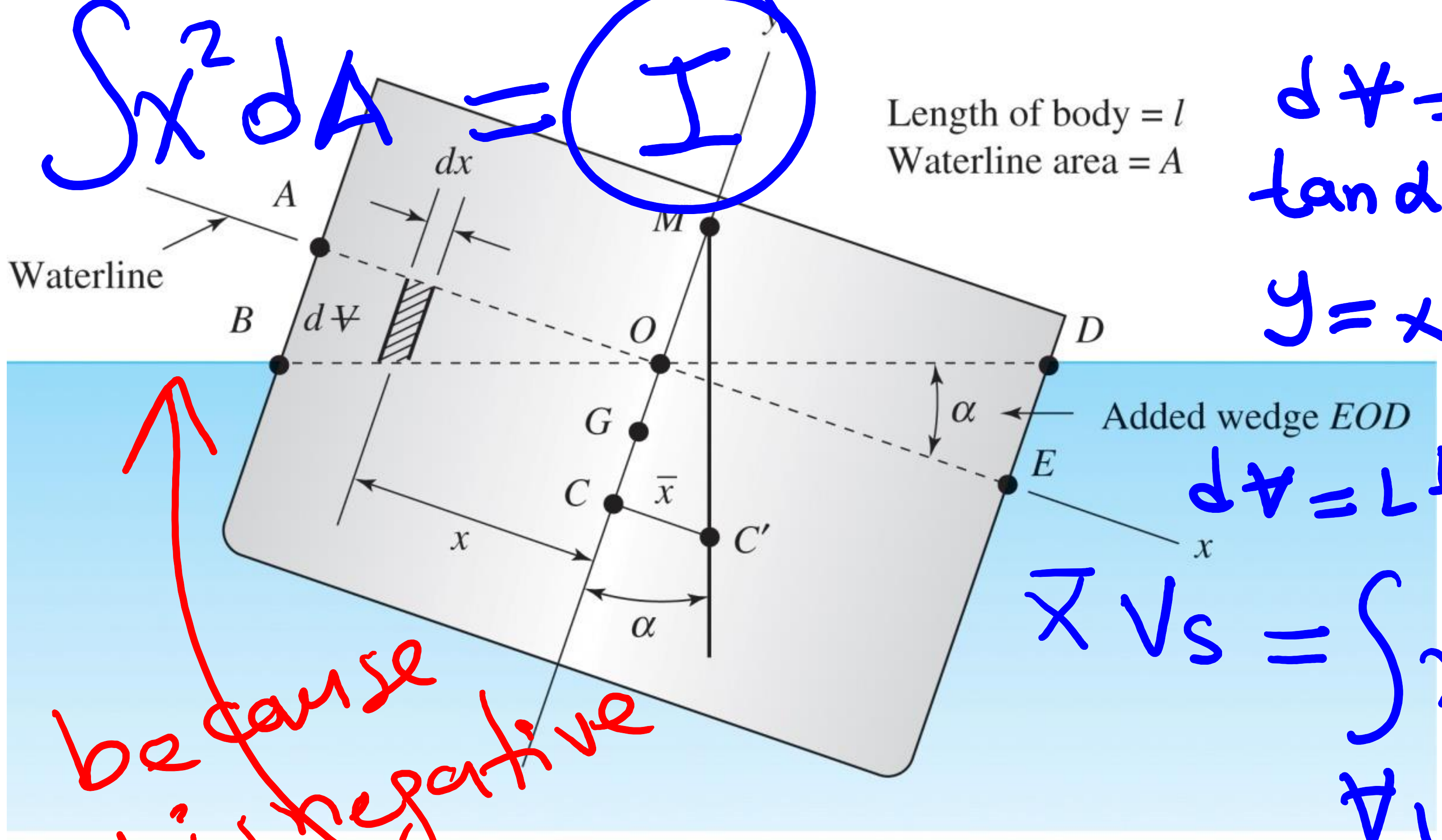


Fig. 2.17 Uniform cross section of a floating body.

$$d\forall = y L \, dx \qquad dA$$

$$\tan\alpha = \frac{y}{x}$$

$$y = x\tan\alpha$$

$$d\forall = L\tan\alpha \, x\,dx = \tan\alpha \cdot x \cdot dA$$

$$\bar{x}\,\forall_s = \int_{\forall_1} x^2 \cdot \tan\alpha \, dA + \int_{\forall_2} x^2 \tan\alpha \, dA$$

because y is negative

$$\bar{x}\,\forall_s = \tan\alpha \left[I_o \right]$$

In (*)

$$GM = \frac{I_o \cancel{\tan\alpha}}{\forall_s \cancel{\tan\alpha}} - GC \rightarrow \boxed{GM = \frac{I_o}{\forall_s} - GC}$$

Example: P.2.94. The barge shown in Fig. P2.94 is loaded such that the center of gravity of the barge and the load is at the waterline. Is the barge stable?

$$GM = \frac{I_o}{\forall_s} - GC$$

$$GM = \frac{\cancel{L} \times 8^3}{12 \times 8 \times 3 \times \cancel{L}} - 1.5$$

$$\boxed{GM = 0.277\ m}$$

> 0 [It is stable]

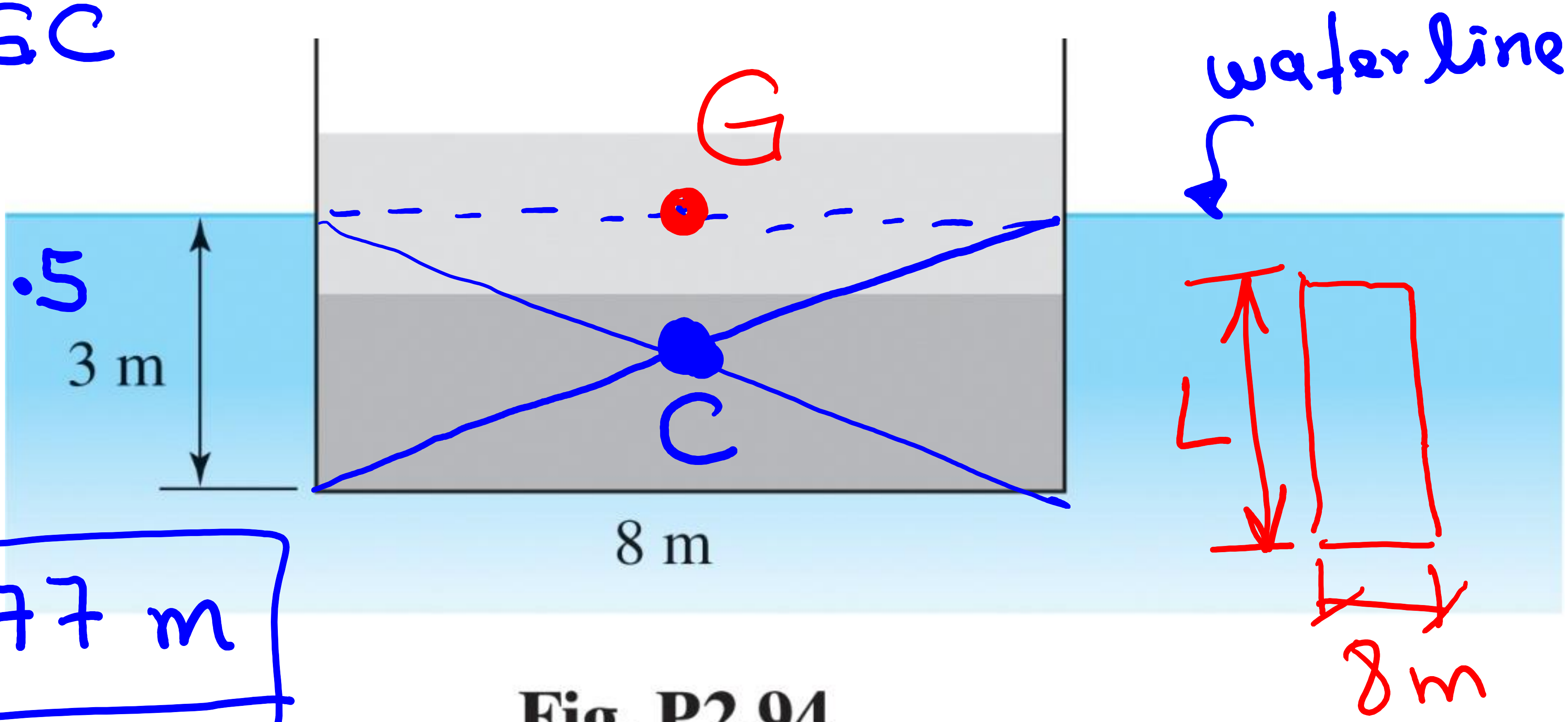


Fig. P2.94

Example: P.2.92. For the object shown in Fig. P2.92, calculate S_A for neutral stability when submerged.

$G = C$

$$\boxed{\bar{y} \cdot A = \Sigma y_i A_i}$$

$A = 32\ cm^2$

$$\bar{y}(32) = 16 \times 4 + 8 \times 8.5 + 8 \times 9.5$$

$$\bar{y} = 6.5\ cm$$

$$W = S \rho_w g \forall$$

$$\bar{y}' \cdot W = \Sigma W_i \bar{y}' \rightarrow \bar{y}' = \frac{\Sigma W_i \bar{y}'}{W}$$

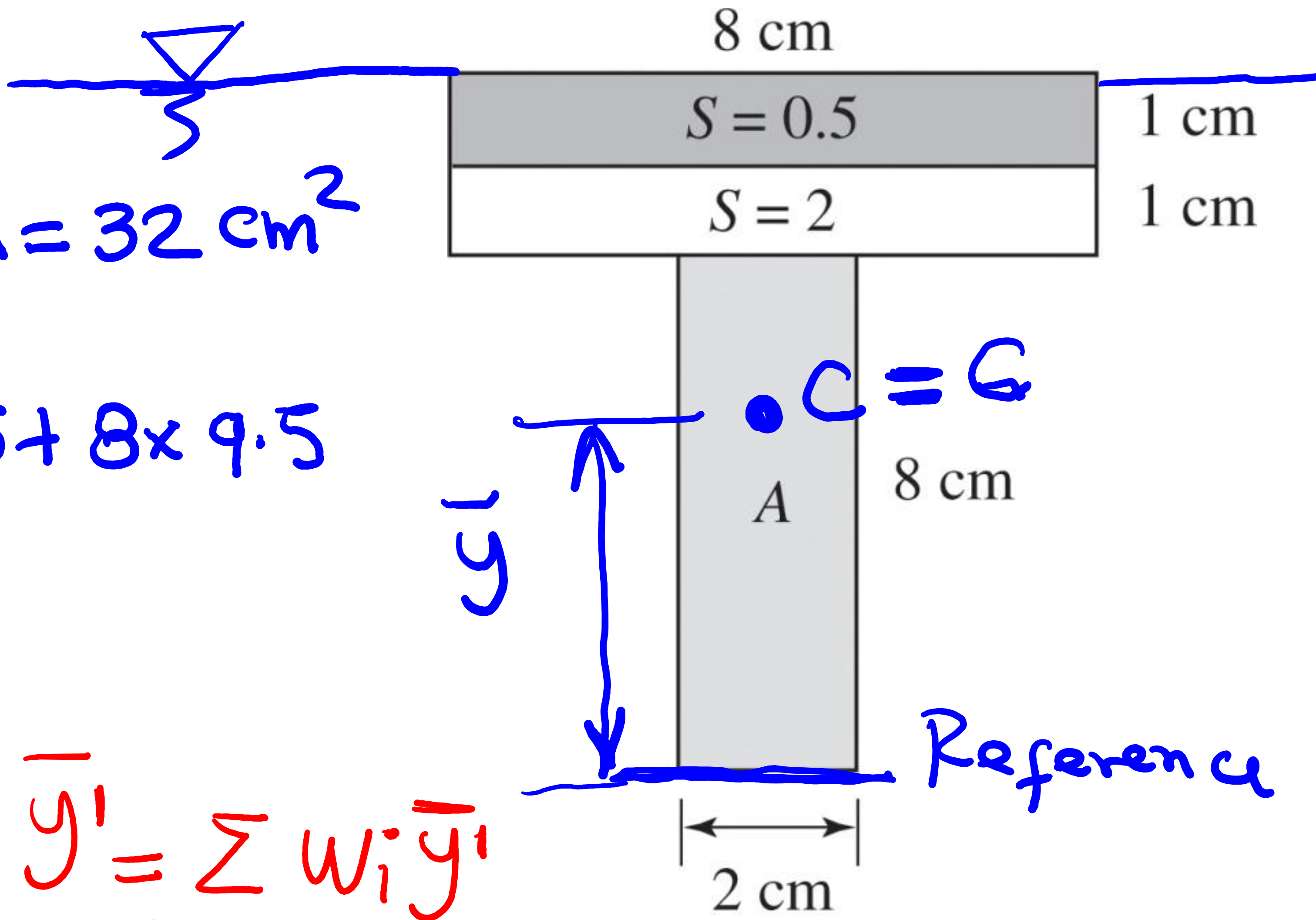


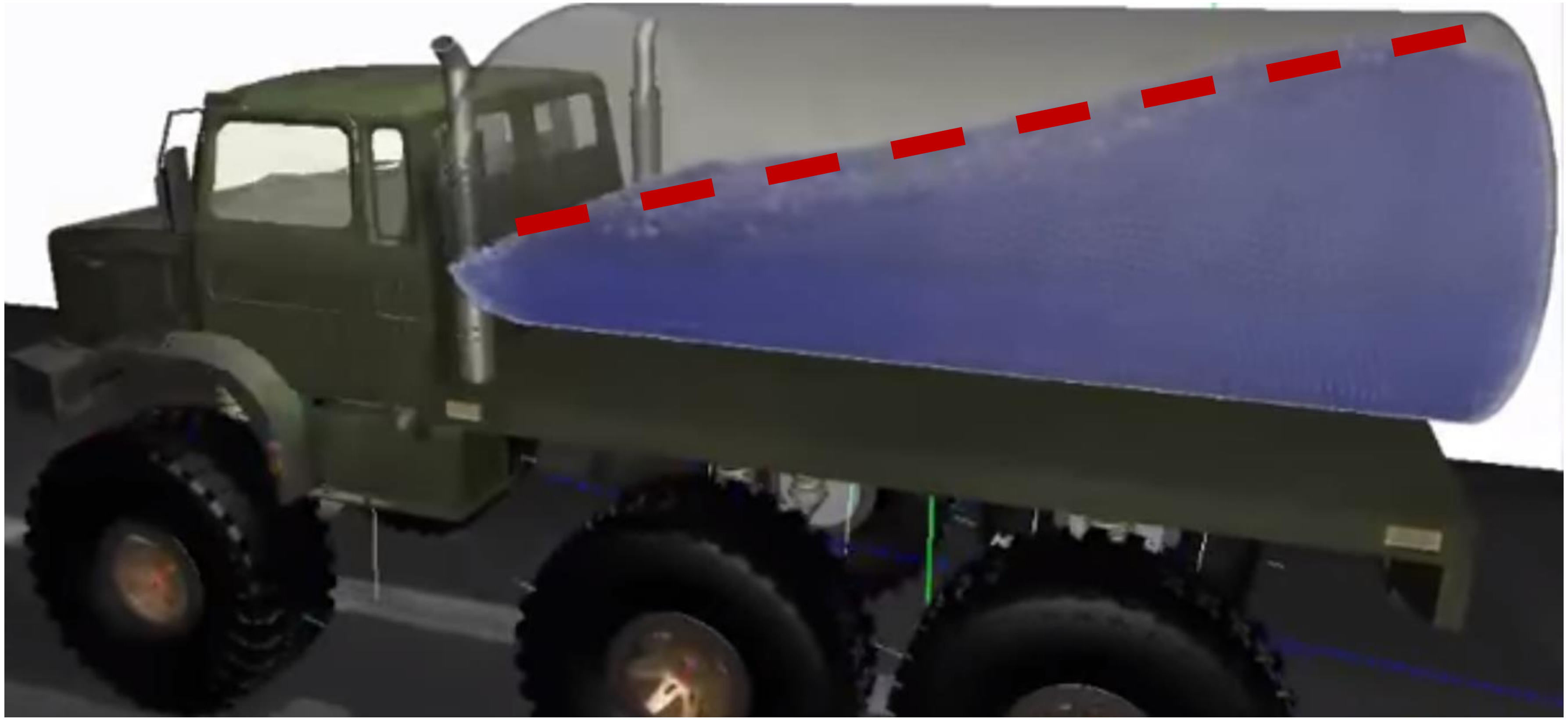
Fig. P2.92

$$6.5 = \frac{S_A \times \cancel{\rho_w g}(16)^{\times 4} + 2\cancel{\rho_w g}(8)^{\times 8.5} + 0.5\cancel{\rho_w g}(8)^{\times 9.5}}{S_A \cancel{\rho_w g}(16) + 2\cancel{\rho_w g}(8) + 0.5\cancel{\rho_w g}(8)}$$

$$6.5(16 S_A + 20) = 64 S_A + 174$$

$$\boxed{S_A = 1.1}$$

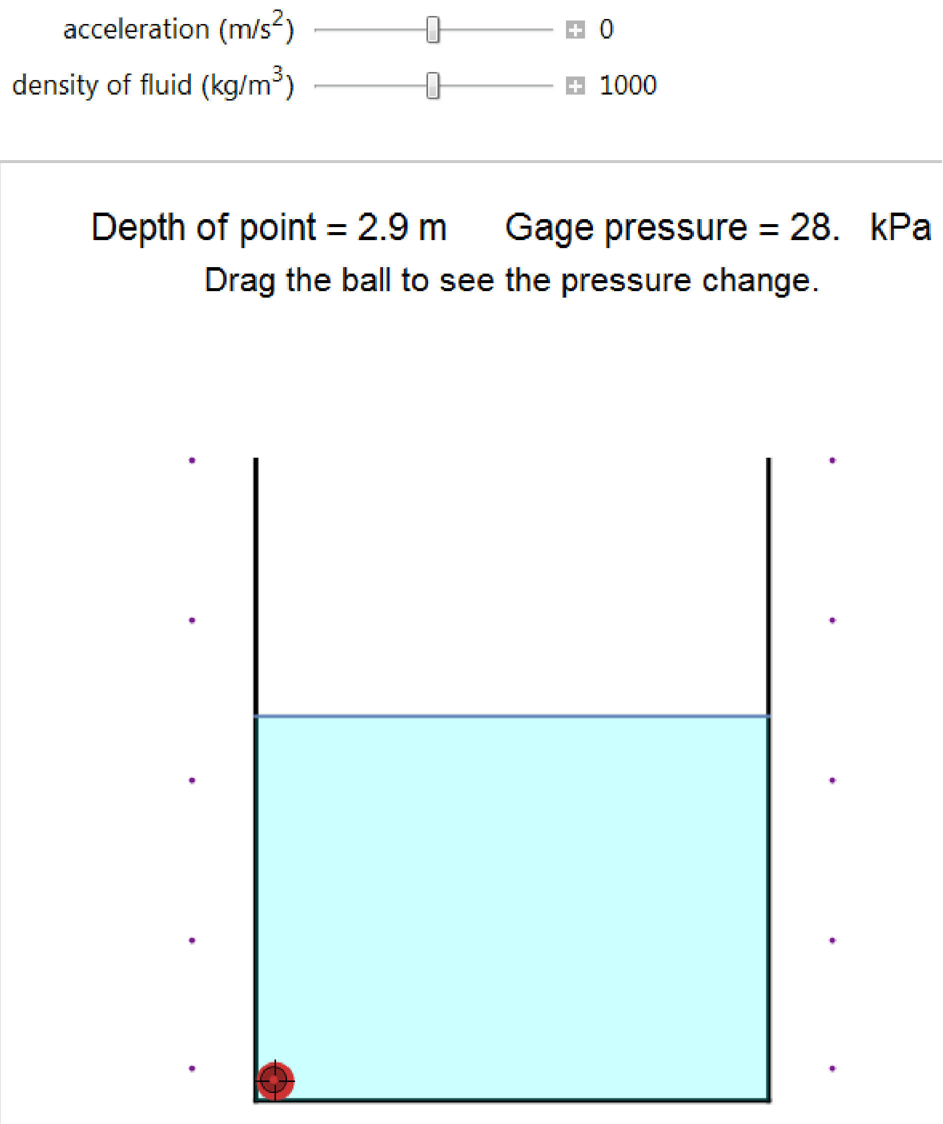
Linearly Accelerating Containers



Source: asciencecom, Youtube (https://www.youtube.com/watch?v=jqpl4ME6rRY)

DEMONSTRATION

Pressure within an Accelerating Container



Source: Jon Barbieri and Peter Hassinger, "Pressure within an Accelerating Container"
http://demonstrations.wolfram.com/PressureWithinAnAcceleratingContainer/

2.5 LINEARLY ACCELERATING CONTAINERS

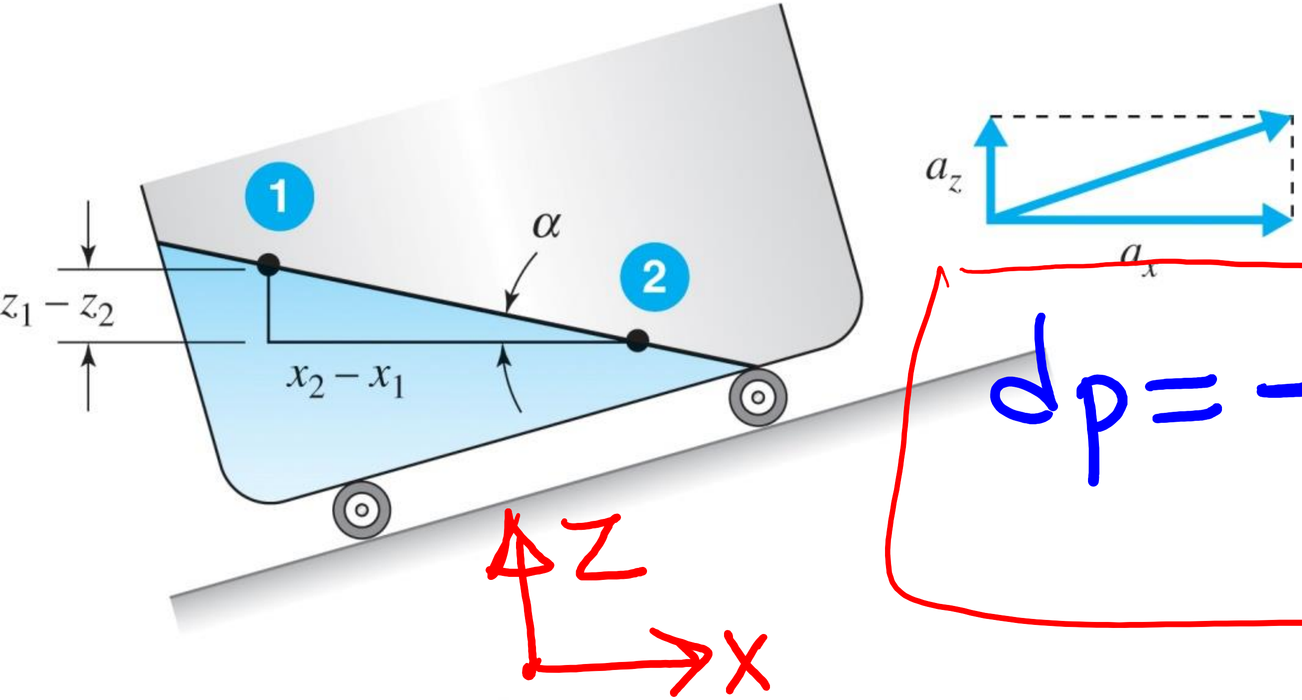


Fig. 2.18 Linearly accelerating tank.

- The derived pressure differential equation is:

$$dp = -\rho a_x dx - \rho a_y dy - \rho(a_z + g) dz$$

- When the fluid is linearly accelerating with horizontal (a_x) and vertical (a_z) components:

$$dp = -\rho a_x dx - \rho(a_z + g) dz$$

2.5 LINEARLY ACCELERATING CONTAINERS

$$\tan\alpha = \frac{z_1 - z_2}{x_2 - x_1} = \frac{-(z_2 - z_1)}{x_2 - x_1} = -\frac{dz}{dx}$$

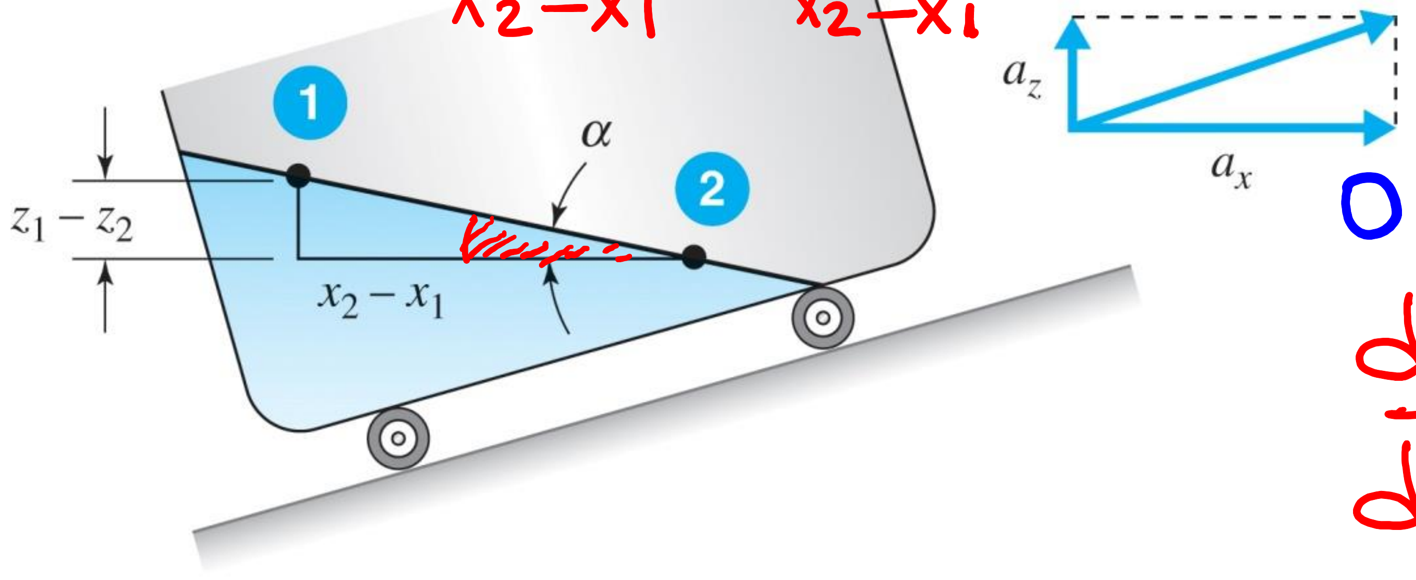


Fig. 2.18 Linearly accelerating tank.

- As points 1 and 2 lie on a constant-pressure line:

$$0 = -\cancel{\rho} a_x dx - \cancel{\rho}(a_z + g) dz$$

$$\frac{dz}{dx} = -\frac{a_x}{a_z + g}$$

$$\cancel{-}\tan\alpha = \cancel{-}\frac{a_x}{a_z + g}$$

α = angle that the constant-pressure line makes with the horizontal.

$$\boxed{\tan\alpha = \frac{a_x}{a_z + g}}$$

Example: P.2.97. The tank shown in Fig. P2.97 is accelerated to the right at 10 m/s^2. Find:

a) P_A, b) P_B, c) P_C

$a_x = 10\ m/s^2$ $\quad$ $L = 1$

Initial Volume of air = 4

$bh = 8$

$^{*}\tan\alpha = \dfrac{10}{9.81} = \dfrac{h}{b}$

$h = 1.019\,b$

$1.019\,b^2 = 8 \rightarrow$ $b = 2.8\ m$, $h = 2.85\ m$

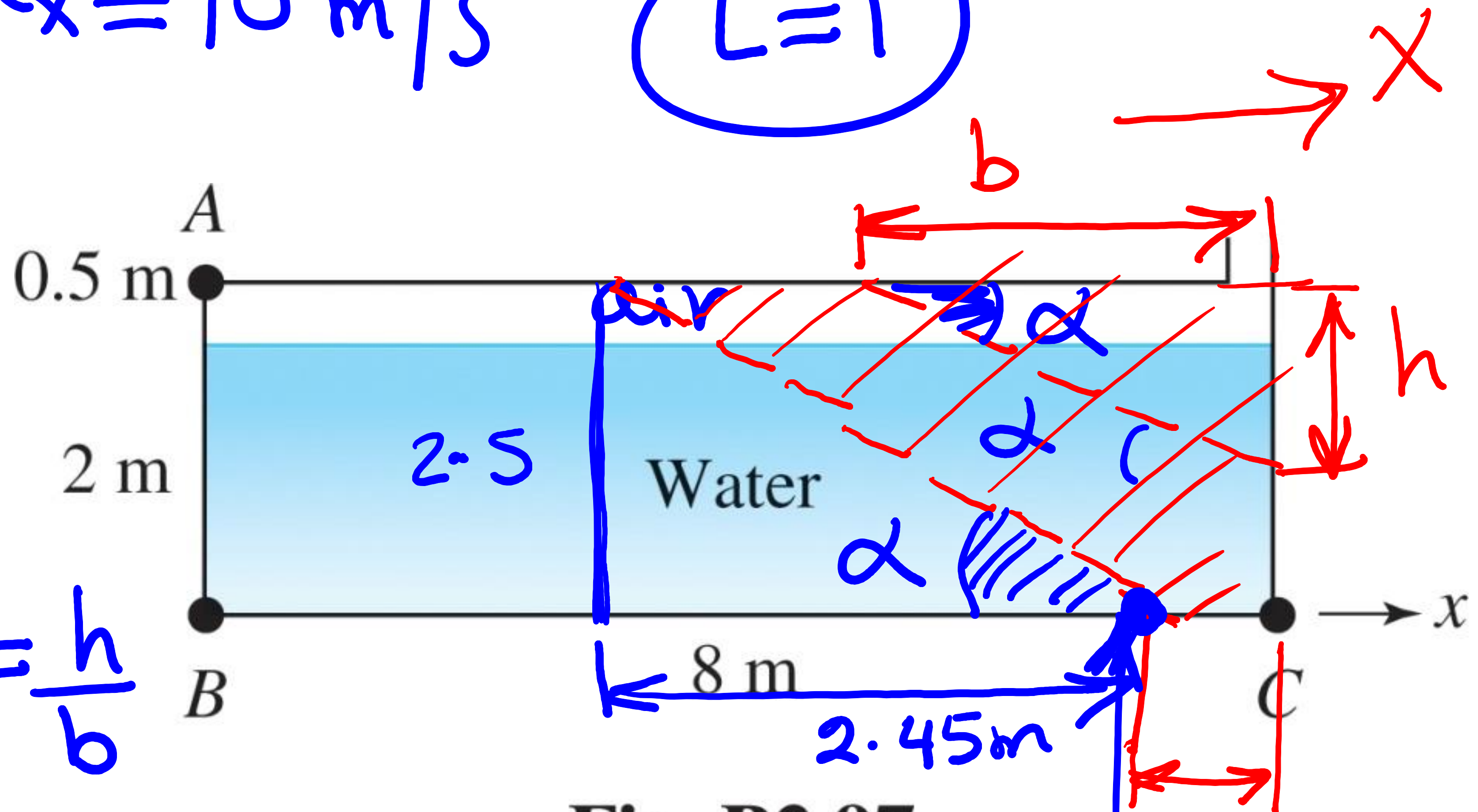


Fig. P2.97

M

* Assumption of location of water surface is incorrect.

$\alpha = 45.5^\circ$

$$4 = \left(\frac{w + w + 2.45}{2}\right) \times 2.5$$

Similarly

$w = 0.374\,m$

$P_A = 51.74$ KPa

* $P_c = 0$

* P_B $dp = -\rho a_x dx - \rho(a_z + g) dz$

BM BM BM

$$P_B - P_M = -\rho(10)(x_B - x_M) - \rho(0 + 9.81)(z_B - z_M)$$

$$P_B = -1000(10)(-7.626) - 0 = 76260\,Pa = 76.2\,KPa$$

Example: **P.2.99**. The tank shown in Fig. P2.99 is filled with water and accelerated. Find the pressure at point *A* if a = 20 m/s^2 and L = 1 m.

$$dp = \underset{AR}{-\rho a_x dx} - \rho (a_z + g)\, \underset{AR}{dz}$$

$$P_A - \cancel{P_R}^{\,0} = -1000(17.32)(X_A - X_R) - 1000(10+9.81)(Z_A - Z_R) \quad \ldots (1)$$

$$\tan\beta = 1/2$$

$$\beta = 26.56^\circ$$

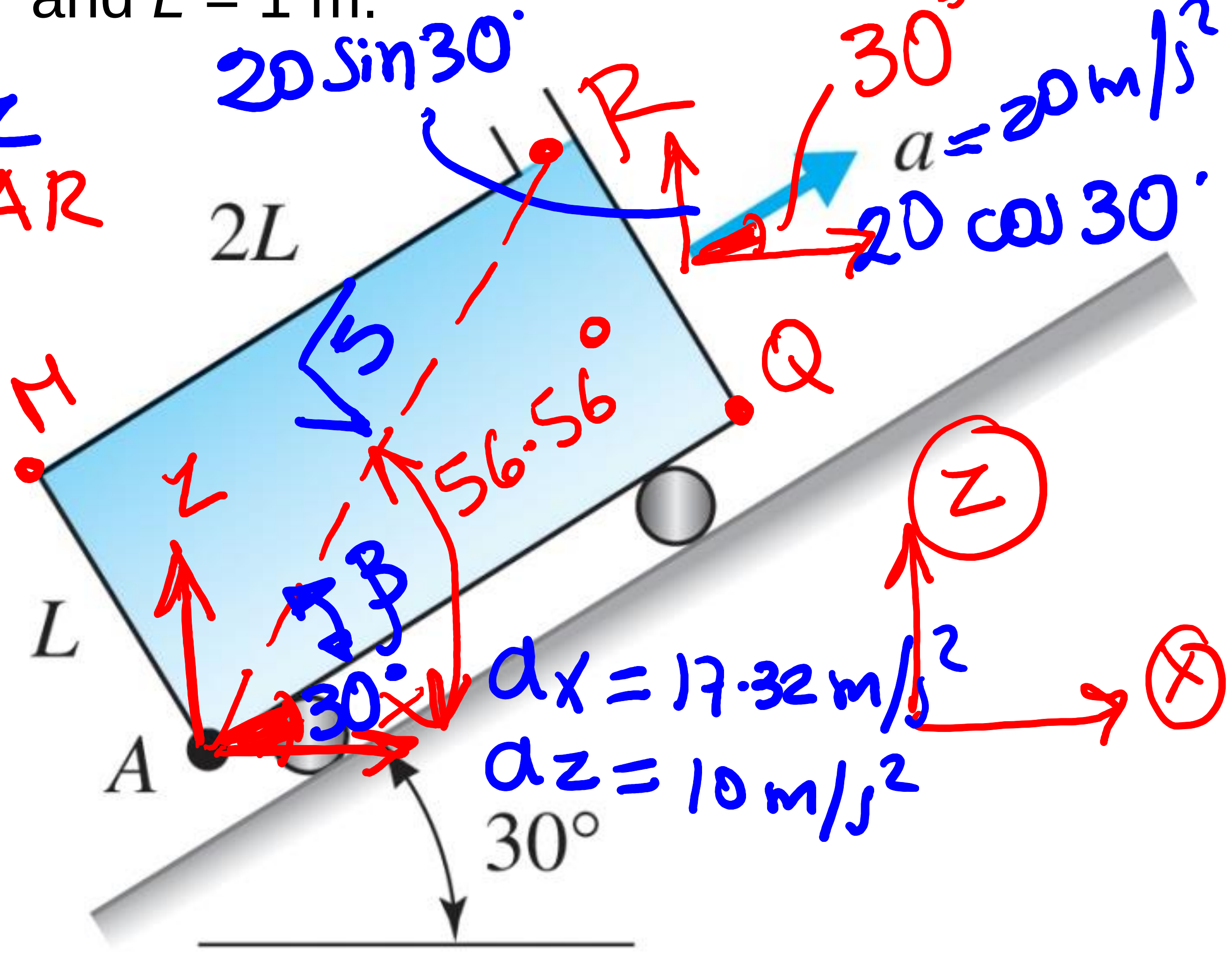


Fig. P2.99

In 1

$$P_A - 0 = -1000(17.32)(0 - \sqrt{5}\cos 56.56^\circ)$$
$$-1000(19.81)(0 - \sqrt{5}\sin 56.56^\circ)$$

$$P_A = 58{,}305\ Pa = \underline{58.3\ kPa}$$

2.6 ROTATING CONTAINERS

https://www.youtube.com/watch?v=RdRnB3jz1Yw

- For a liquid in a rotating container (cross-section shown):

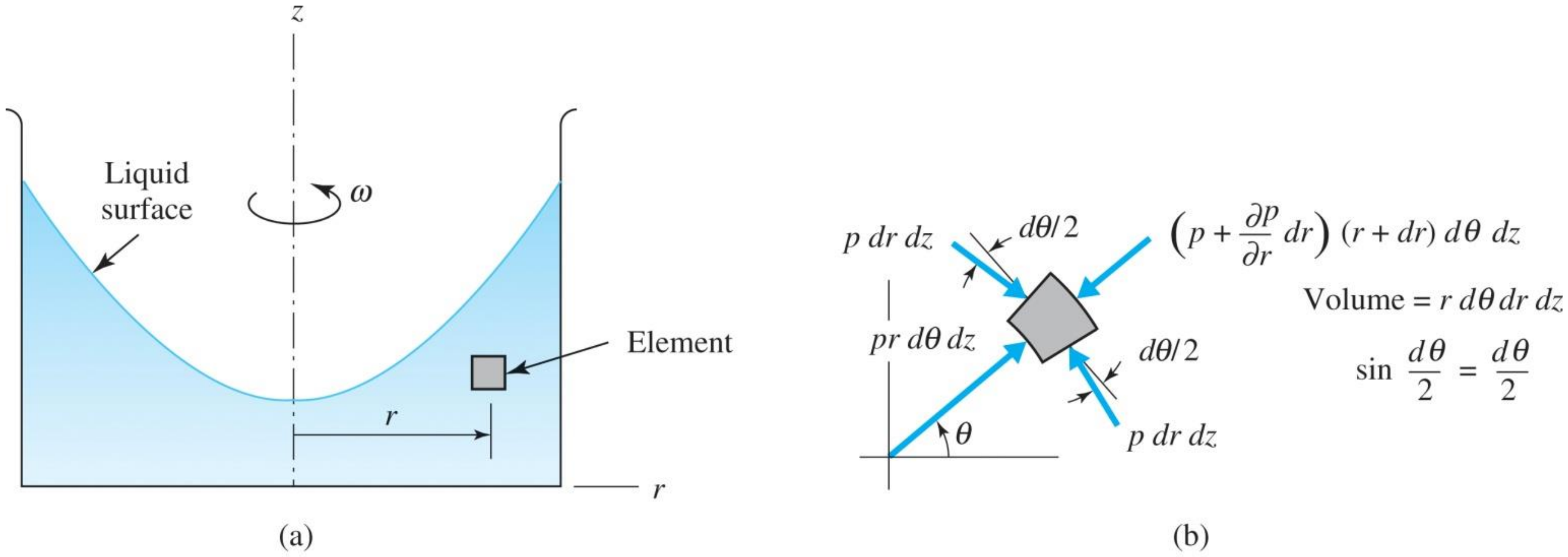


Fig. 2.19 Rotating container: (a) liquid cross section; (b) top view of element.

- In a short time, the liquid reaches static equilibrium with respect to the container and the rotating *rz*-reference frame.
- Horizontal rotation will not affect the pressure distribution in the vertical direction.
- No variation in pressure with respect to the θ-coordinate.

2.6 ROTATING CONTAINERS

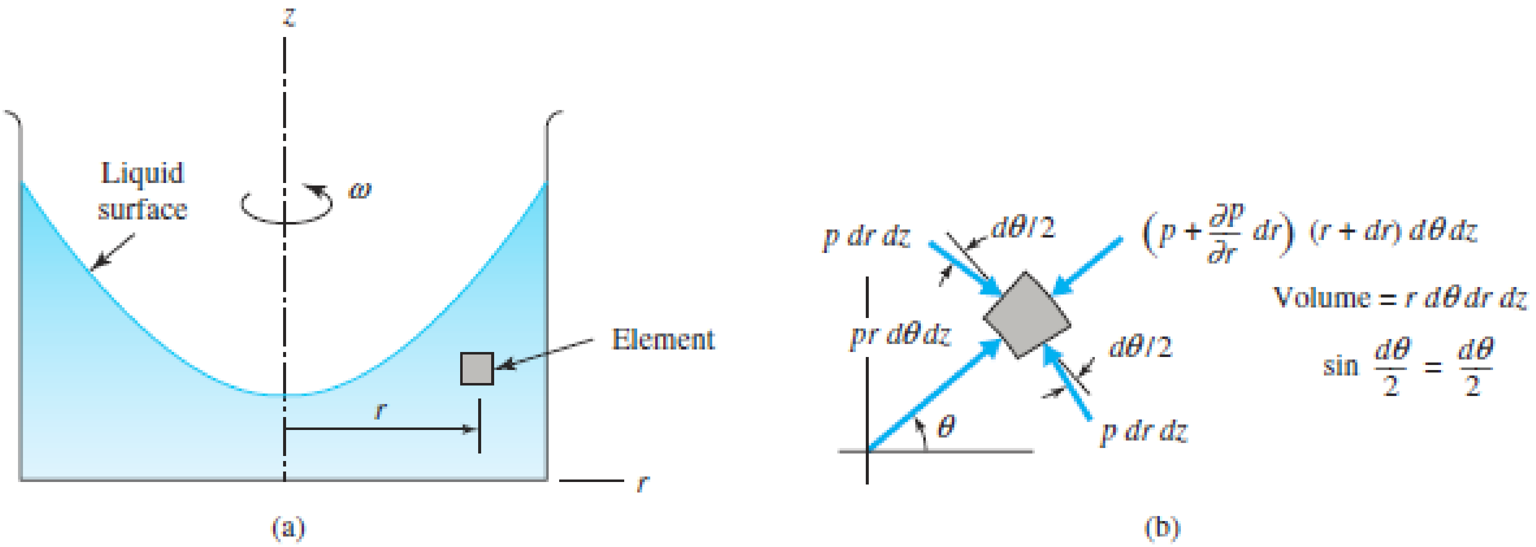


Figure 2.19 Rotating container: (a) liquid cross section; (b) top view of element.

- Between two points (r_1,z_1) and (r_2,z_2) on a rotating container, the static pressure variation is:

$$P_2 - P_1 = \rho \frac{\omega^2}{2}\left(r_2^2 - r_1^2\right) - \rho g (z_2 - z_1)$$

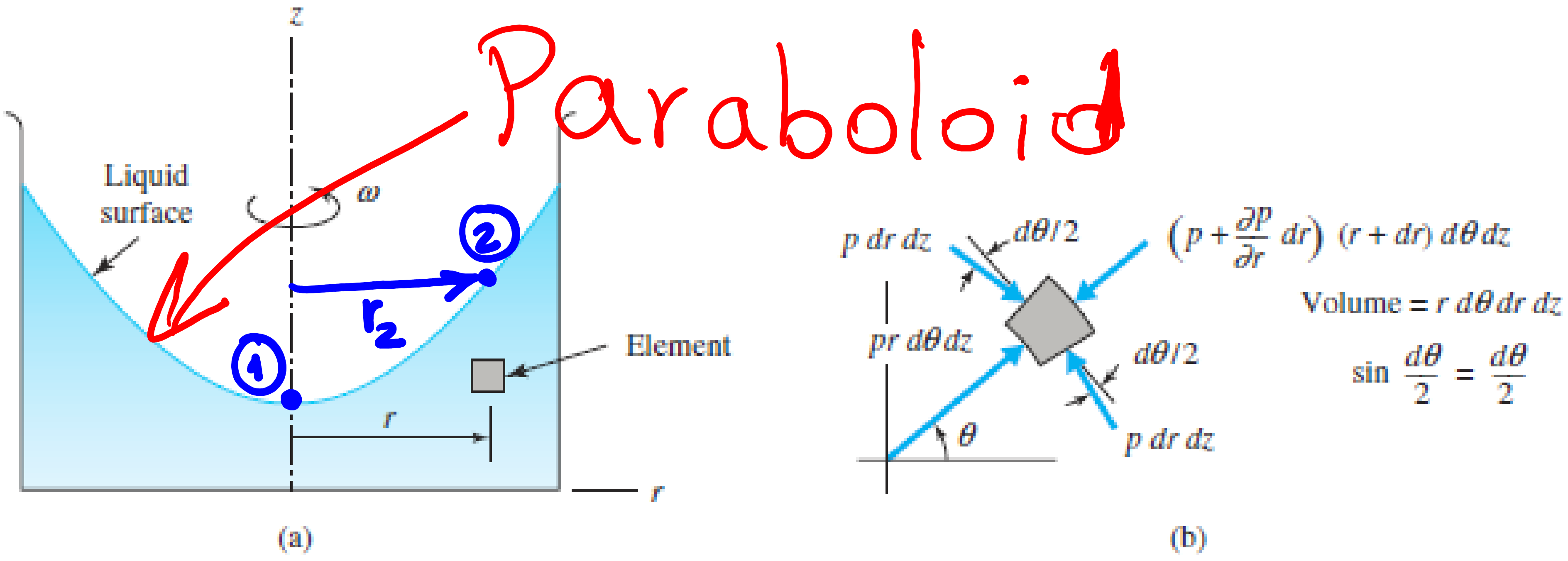


Figure 2.19 Rotating container: (a) liquid cross section; (b) top view of element.

- If two points are on a constant-pressure surface (e.g., free surface) with point 1 on the z-axis [r_1=0]:

$$0 = \cancel{\rho}\frac{\omega^2}{2}(r_2^2 - 0) - \cancel{\rho} g (z_2 - z_1)$$

$$\boxed{\frac{\omega^2 r_2^2}{2} = g(z_2 - z_1)}$$

  - The free surface is a **paraboloid of revolution**.

Example: **P.2.106**. For the cylinder shown in Fig. P2.106, determine the pressure at point *A* for a rotational speed of 5 rad/s.

$P_A = ?$ $\omega = 5$ rad/s

Air volume before and after rotation is the same. CASE (H)

$$\pi \times 0.6^2 \times 0.2 = \frac{1}{2} \pi \times r_2^2 \times h$$

$$r_2^2 h = 0.144 \quad \ldots (1)$$

$$* \frac{\omega^2 r_2^2}{2} = g(\underline{z_2 - z_1}) \quad (h)$$

$$\frac{5^2 r_2^2}{2} = 9.81 h \quad \ldots (2)$$

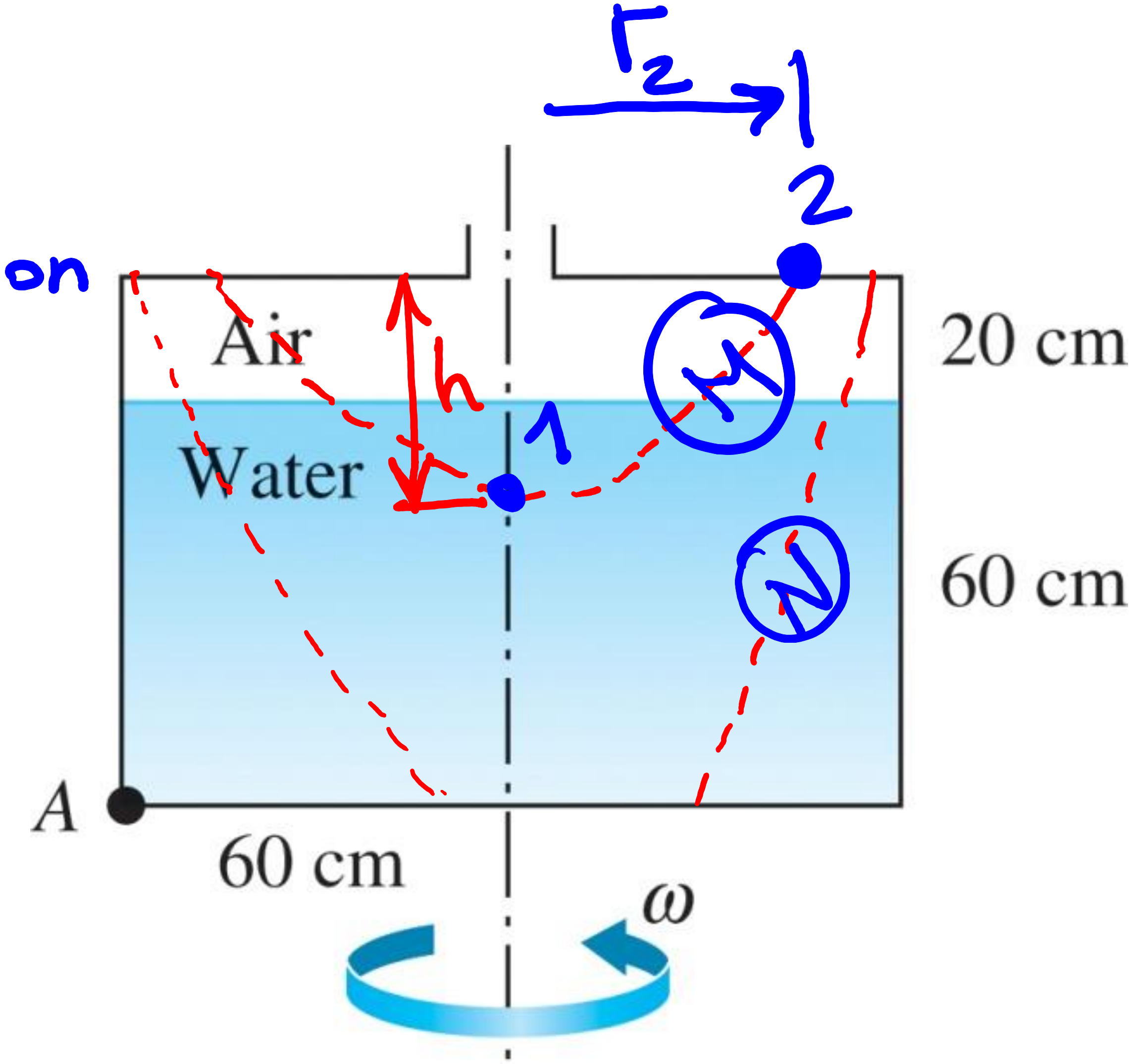


Fig. P2.106

$$\frac{25}{2}\left(\frac{0.144}{h}\right) = 9.81\,h \Rightarrow \underline{h = 0.428\,m}$$

$$r_2 = 0.58\,m$$

Case M holds

$h < 0.8\,m$ and

$r_2 < 0.6\,m$

$$* \; P_A - \cancel{P_1}^{\,0} = \frac{\rho\omega^2}{2}\left(r_A^2 - \cancel{r_1^2}^{\,0}\right) - \rho g\left(Z_A - Z_1\right)$$

$$P_A = 1000 \times \frac{5^2}{2}\left(0.6^2\right) - 1000(9.81)(-0.372)$$

$$P_A = 8149\,Pa = \underline{8.1\,kPa}$$

Example: **P.2.107**. The hole in the cylinder of Problem P2.106 is closed and the air pressurized to 25kPa. Find the pressure at point *A* if the rotational speed is 5 rad/s.

$P_A = 8149\ Pa$ +

$25{,}000\ Pa$

$P_A = 33{,}149\ Pa$

$P = 25{,}000\ Pa$

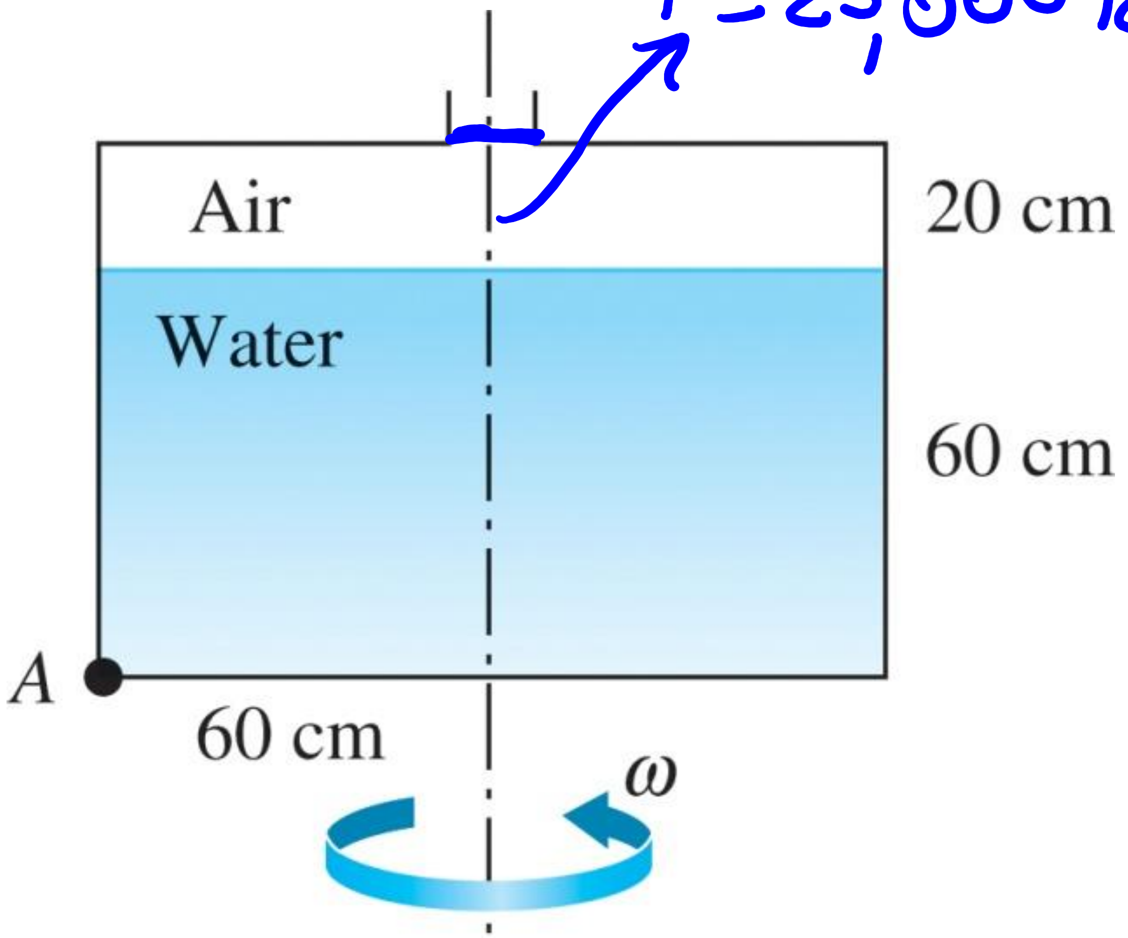


Fig. P2.106