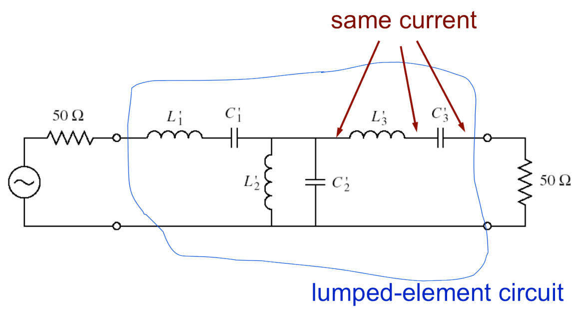
ECE 391

supplemental notes - #1

Lumped vs. Distributed Circuits

Lumped-Element Circuits:

- Physical dimensions of circuit are such that voltage across and current through conductors connecting elements does not vary.
- Current in two-terminal lumped circuit element does not vary (phase change or transit time are neglected)



Lumped vs. Distributed Circuits

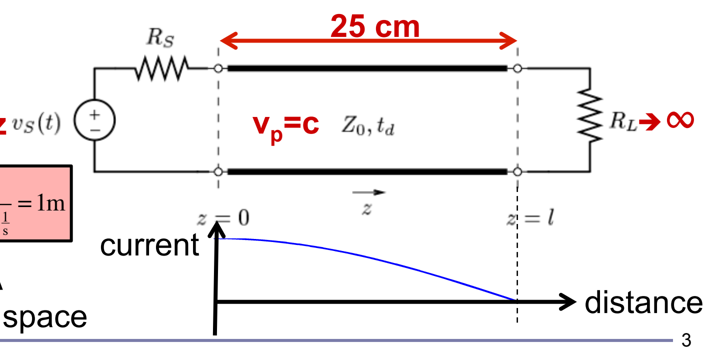
Distributed Circuits:

- Current varies along conductors and elements;
- Voltage across points along conductor or within element varies

➔ phase change or transit time **cannot be neglected**

Example:

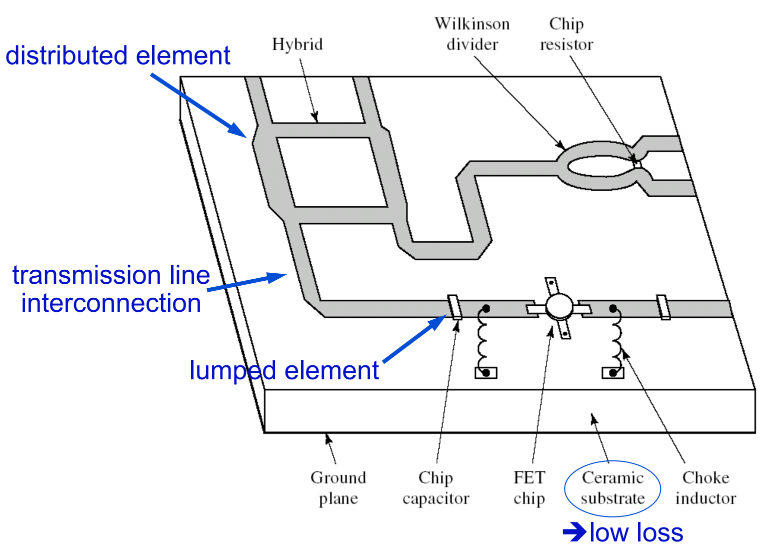
f = 300MHz



$$\lambda = \frac{c}{f} = \frac{3\times10^8 \frac{\text{m}}{\text{s}}}{300\times10^6 \frac{1}{\text{s}}} = 1\text{m}$$

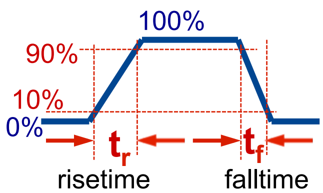
wavelength λ
= 1 period in space

Example: Hybrid Microwave Integrated Circuit (MIC)



When Do We Need to Consider Transmission Lines Efects

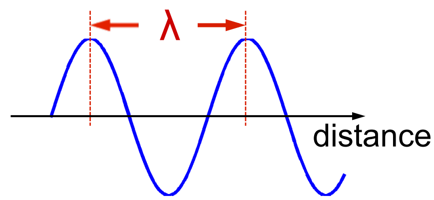
digital domain



flight time (delay time) $t_d = d/v_p$

t_r, t_f vs t_d

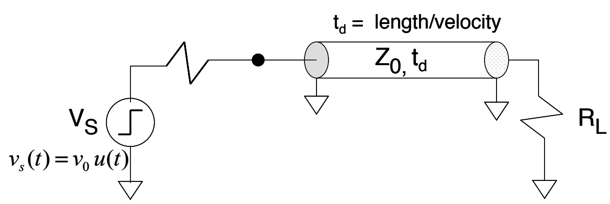
analog domain



maximum dimension d

d vs λ

Lumped vs. Distributed



- **Rule-of-thumb (heuristic)**

  Delay time t_d = length of line / velocity

  Rise time of signal t_r (fall time t_f)

Signal path can be treated as

lumped element if $t_r / t_d > 6$

distributed element if $t_r / t_d < 2.5$

Example

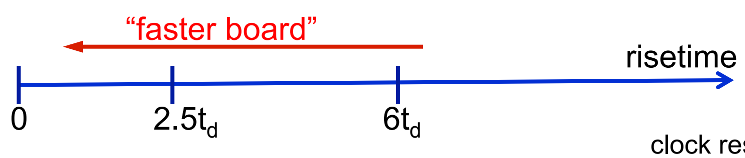
- CMOS Buffer with $t_r = t_f = 0.5$ ns
- FR-4 PCB (velocity ≈ 0.45 c)
- Note: $v_p \approx c/\sqrt{\varepsilon_r}$

Determine maximum distance d so that $t_r/t_d > 6$

$$t_r/t_d = t_r/(d/v_p) > 6 \quad \Rightarrow \quad d < t_r v_p/6$$

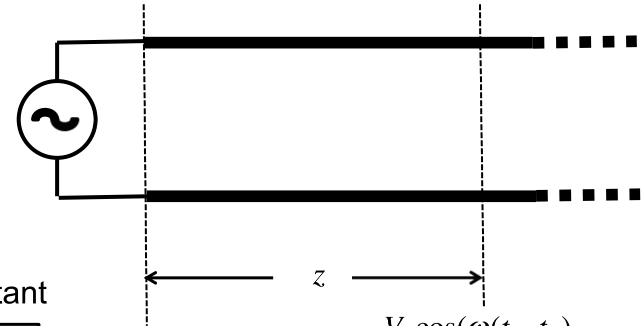
$d < 0.5\text{ns} * (0.45*30\text{cm/ns})/6 = 1.125$ cm ≈ **11 mm**

TL Problem Classification



simulate and check after layout of ckt		no problems go for it!	clock reset signals with critical timing (or edge sensitivity)
simulate and check after layout of ckt	consider quick hand analysis	no problems go for it!	non-critical signals (e.g. data lines with $t_{su} \approx 6\ t_d$)
simulate and check after layout of ckt	no problems expected		status signals; slow, insensitive lines
In this region, other consid. begin to surface (EMI, crosstalk)			

Sinusoidal Signals



$V_0 \cos(2\pi ft)$
$= V_0 \cos(\omega t)$

phase constant

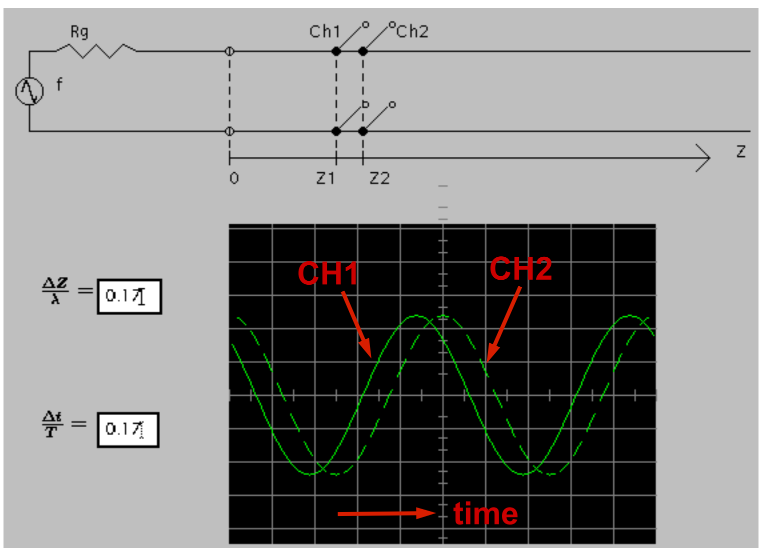
$$\beta = \frac{\omega}{v_p} = \frac{2\pi}{\lambda}$$

$$V_0 \cos(\omega(t - t_d)) = V_0 \cos(\omega t - \omega t_d) = V_0 \cos(\omega t - \omega \frac{z}{v_p}) = V_0 \cos(\omega t - \beta z)$$

In practice: lumped if $t_d < 0.1$ T ($d < 0.1\ \lambda$)

safely lumped if $d < 0.01\lambda$

Illustration



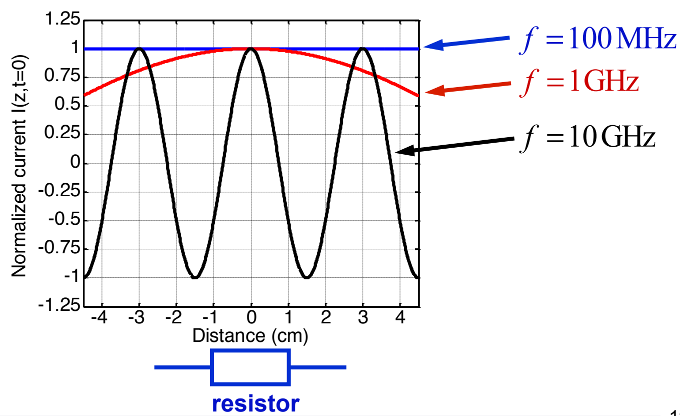
Example: Lumped vs. Distributed

For example, the spatial dependence z of the current

$$i(z,t) = I_0 \cos(\omega t - \beta z)$$

in a conductor can be neglected if $z/\lambda << 1$.

Example:



Concept: Electrical Length

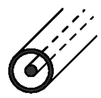
- Electrical length (θ, E) is a measure of the physical length expressed in terms of wavelength λ

$$\theta = E = 2\pi \frac{z}{\lambda} \quad \text{(in radians)}$$

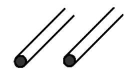
$$\theta = E = 360^0 \frac{z}{\lambda} \quad \text{(in degrees)}$$

$$E = \frac{z}{\lambda} \quad \text{(as fraction of wavelength)}$$

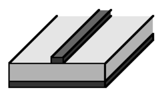
Transmission Line Examples



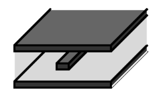
coaxial line



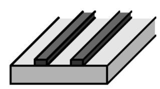
two-wire line (also twisted-pair)



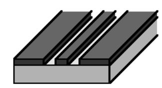
microstrip



stripline



coplanar strip (CPS)

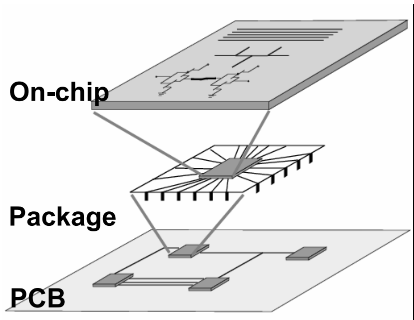


coplanar waveguide (CPW)

Transmission Lines/Interconnects

- What is a transmission line?
  - lumped vs. distributed circuit?
  - how many conductors?
- Types of transmission lines
  - on-chip (→ transmission line?)
  - off chip
  - PCB, packages
  - cables (coax, twisted pair, …)
  - etc.
- Applications of transmission lines
  - interconnections
    - signal transmission
    - power transmission
  - circuit elements/functional components (at high frequencies)
    - filters (e.g. coupled lines, stubs, …)
    - couplers
    - power dividers
    - matching networks
  - …

Interconnect Technologies



⋮

cables, etc.

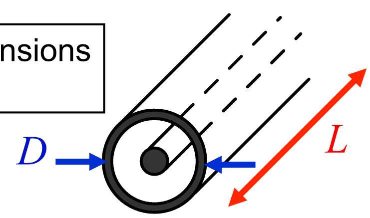
Inter-connection Type	Line Width (μm)	Line Thickness (μm)	Max. Length (cm)
On-Chip	0.5-2	0.5-2	0.3-1.5
Thin-Film	10-25	5-8	20-45
Ceramic	75-100	16-25	20-50
PCB	60-100	8-70	40-70

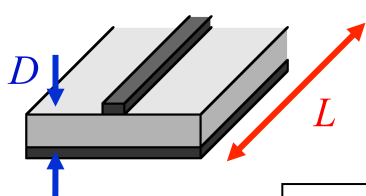
Source: IBM (plus changes)

Transmission Line – Characteristic Dimensions

Cross-sectional Dimensions << wavelength

$D << \lambda$

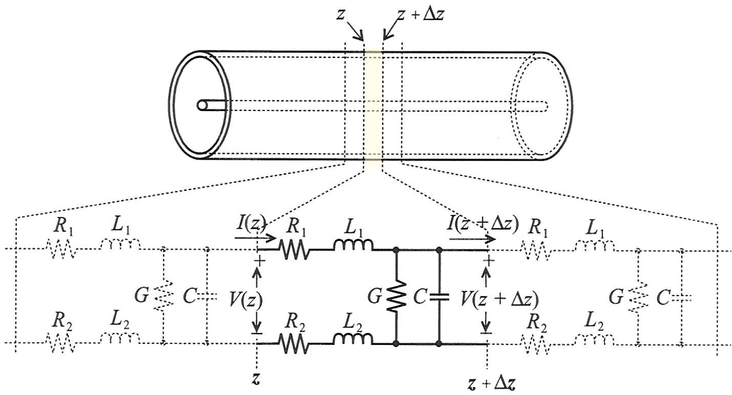




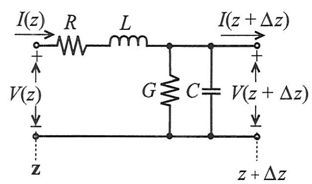
$L < \lambda \cdots L >> \lambda$

Lengths vary from fractions of a wavelength to many wavelengths

Model for Transmission Line



generic equivalent circuit model



Note: here R,L,G,C represent total values per section

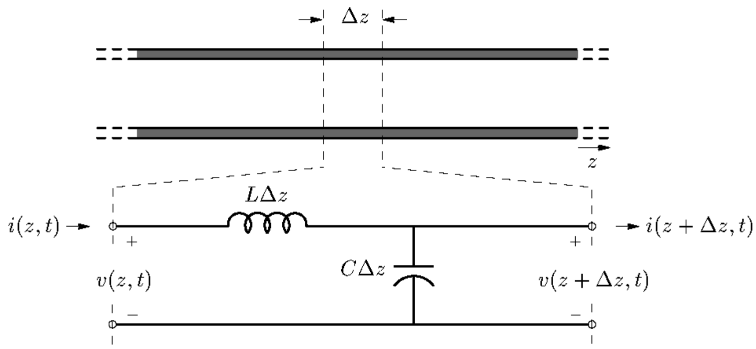
Transmission Line Parameters

Cross-sectional view of typical uniform interconnects:

- Capacitance between conductors, *C* (F/m)
- Inductance of conductor loop, *L* (H/m)
- Resistance of conductors (conductor loss), *R* (Ω/m)
- Shunt conductance (dielectric loss), *G* (S/m)

➡ ***R,L,G,C*** are specified as per-unit-length parameters

Derivation of Transmission Line Equations



$$-\left[v(z+\Delta z,t)-v(z,t)\right]=L\Delta z\frac{\partial i(z,t)}{\partial t}$$

$$-\left[i(z+\Delta z,t)-i(z,t)\right]=C\Delta z\frac{\partial v(z+\Delta z,t)}{\partial t}$$

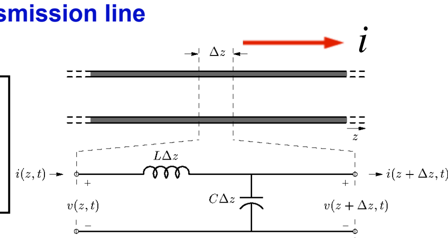
Note: here L,C are per-unit-length parameters

Transmission Line Equations

Lossless transmission line

$$-\left[v(z+\Delta z,t)-v(z,t)\right]=L\Delta z\frac{\partial i(z,t)}{\partial t}$$

$$-\left[i(z+\Delta z,t)-i(z,t)\right]=C\Delta z\frac{\partial v(z+\Delta z,t)}{\partial t}$$



after taking $\Delta z \to 0$

Telegrapher's Equations

$$-\frac{\partial v(z,t)}{\partial z}=L\frac{\partial i(z,t)}{\partial t} \qquad -\frac{\partial i(z,t)}{\partial z}=C\frac{\partial v(z,t)}{\partial t}$$

Wave Equations

$$-\frac{\partial v(z,t)}{\partial z} = L\frac{\partial i(z,t)}{\partial t} \qquad -\frac{\partial i(z,t)}{\partial z} = C\frac{\partial v(z,t)}{\partial t}$$

Wave Equations

$$\frac{\partial^2 v(z,t)}{\partial z^2} = LC\frac{\partial^2 v(z,t)}{\partial t^2} = \frac{1}{v_p^2}\frac{\partial^2 v(z,t)}{\partial t^2}$$

$$\frac{\partial^2 i(z,t)}{\partial z^2} = LC\frac{\partial^2 i(z,t)}{\partial t^2} = \frac{1}{v_p^2}\frac{\partial^2 i(z,t)}{\partial t^2}$$

General Wave Solutions

$$\frac{\partial^2 v}{\partial z^2} = LC\frac{\partial^2 v}{\partial t^2} = \frac{1}{v_p^2}\frac{\partial^2 v}{\partial t^2}$$

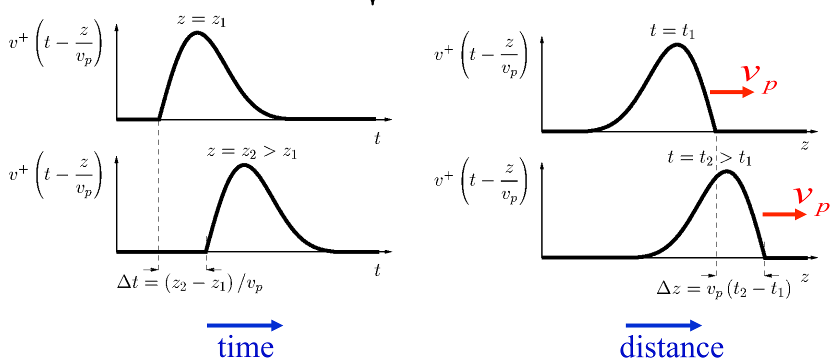
General Solution for Voltage

$$v(z,t) = v^+(z - v_p t) + v^-(z + v_p t)$$
$$= \underbrace{v^+(t - z/v_p)}_{+z \text{ direction}} + \underbrace{v^-(t + z/v_p)}_{-z \text{ direction}}$$

Velocity of Propagation $v_p = \frac{1}{\sqrt{LC}}$ (m/s)

Illustration of Space–Time Variation of Single Traveling Wave

$$v(z,t) = v^+(z - v_p t) + v^-(z + v_p t)$$
$$= v^+(t - z/v_p) + v^-(t + z/v_p)$$



Wave Solutions for Current

$$i(z,t) = i^+(t - z/v_p) + i^-(t + z/v_p)$$

$$-\frac{\partial v(z,t)}{\partial z} = L\frac{\partial i(z,t)}{\partial t} \qquad -\frac{\partial i(z,t)}{\partial z} = C\frac{\partial v(z,t)}{\partial t}$$

$$\frac{1}{v_p}\left\{v^+(t-z/v_p) - v^-(t+z/v_p)\right\} = L\left\{i^+(t-z/v_p) + i^-(t+z/v_p)\right\}$$

$$\frac{1}{v_p}\left\{i^-(t-z/v_p) - i^-(t+z/v_p)\right\} = C\left\{v^+(t-z/v_p) + v^-(t+z/v_p)\right\}$$

$$i(z,t) = \frac{v^+(t - z/v_p)}{Z_0} - \frac{v^-(t + z/v_p)}{Z_0}$$

Characteristic Impedance $Z_0 = v_p L = \frac{1}{v_p C} = \sqrt{\frac{L}{C}}\ (\Omega)$

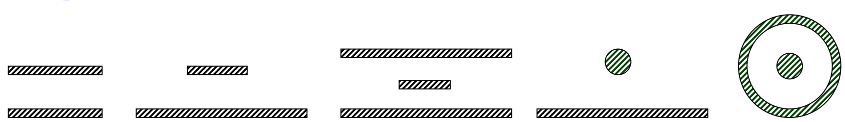
Summary of Transmission Line Parameters

- Capacitance per-unit-length C (F/m)
- Inductance per-unit-length L (H/m)
- Characteristic impedance Z_0 (Ω)
- Velocity of propagation v_p (m/s)
- Per-unit-length delay time $t_p = 1/v_p$ (s/m)
- Delay time (TD) $t_d = l/v_p = l\,t_p$ (sec)

$$Z_0 = \sqrt{\frac{L}{C}} \qquad v_p = \frac{1}{\sqrt{LC}} \qquad t_p = \sqrt{LC} \qquad t_d = l\sqrt{LC}$$

$$L = Z_0/v_p = Z_0\,t_p \qquad C = 1/(Z_0 v_p) = t_p/Z_0$$

Properties of Ideal Transmission Lines



- **C from 2D electro-static solution**
- **L from 2D magneto-static solution**

- **Velocity of propagation**

$$v_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}} = \frac{c_0}{\sqrt{\varepsilon_r}} \qquad c_0 \approx 30\,\text{cm/nsec}$$

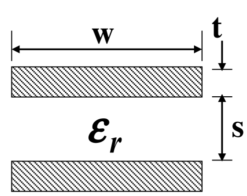
(neglecting magnetic materials)

Propagation Speeds for Typical Dielectrics

Dielectric	Rel. Dielectric Constant ε_r	Propagation speed (cm/nsec)	Delay time per unit length (ps/cm)
Polyimide	2.5 – 3.5	16-19	53 - 62
Silicon dioxide	3.9	15	66
Epoxy glass (PCB)	5.0	13	75
Alumina (ceramic)	9.5	10	103

Example T-Line Structures

Parallel-Plate Line



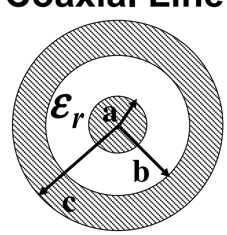
$$C = \varepsilon_0 \varepsilon_r \frac{w}{s}$$

$$L = \mu_0 \frac{s}{w}$$

$$Z_0 = \sqrt{\frac{\mu}{\varepsilon_0 \varepsilon_r}} \frac{s}{w}$$

$$R_{DC} = \frac{2\rho}{wt}$$

Coaxial Line



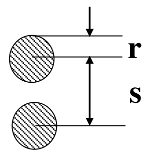
$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln(b/a)}$$

$$L = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$Z_0 = \sqrt{\frac{\mu}{\varepsilon_0 \varepsilon_r}} \frac{\ln(b/a)}{2\pi}$$

$$R_{DC} = \frac{\rho}{\pi a^2} + \frac{\rho}{\pi(c^2 - b^2)}$$

Examples (cont’d)

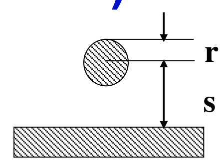


$$R_{DC} = \frac{2\rho}{\pi r^2}$$

$$C = \frac{\pi\varepsilon}{\cosh^{-1}(s/2r)}$$

$$L = \frac{\mu_0}{\pi}\cosh^{-1}(s/2r)$$

$$Z_0 = \frac{1}{\pi}\sqrt{\frac{\mu_0}{\varepsilon}}\cosh^{-1}(s/2r)$$



$$R_{DC} = \frac{\rho}{\pi r^2}$$

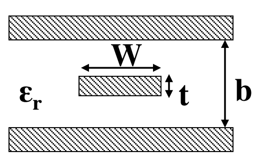
$$C = \frac{2\pi\varepsilon}{\cosh^{-1}(s/r)}$$

$$L = \frac{\mu_0}{2\pi}\cosh^{-1}(s/r)$$

$$Z_0 = \frac{1}{2\pi}\sqrt{\frac{\mu_0}{\varepsilon}}\cosh^{-1}(s/r)$$

Examples (cont’d)

stripline

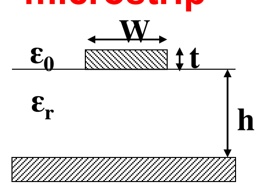


$$Z_0 = \frac{1}{4}\sqrt{\frac{\mu}{\varepsilon}}\ln\left(\frac{1+w/b}{w/b+t/b}\right)$$

$$C = \sqrt{\varepsilon_r}\,\frac{1}{c_0 Z_0}$$

$$L = \sqrt{\varepsilon_r}\,\frac{Z_0}{c_0}$$

microstrip



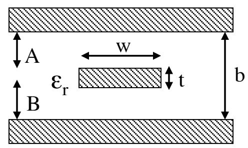
$$Z_0 = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon_0\varepsilon_{eff}}}\ln\left(\frac{4h}{d}\right)$$

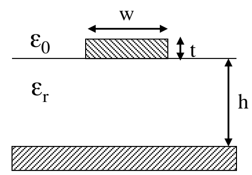
$$d = 0.536w + 0.67t$$

$$\varepsilon_{\text{eff}} \approx 0.475\varepsilon_r + 0.67$$

(very approximate!)

Alternative Formulas





$$Z_{0,sym} = \frac{60\Omega}{\sqrt{\varepsilon_r}} \ln\left(\frac{4b}{0.67\pi(t+0.8w)}\right)$$

for w/b < 0.35 and
t/b < 0.25

$$Z_0 = \frac{87\Omega}{\sqrt{\varepsilon_r + 1.41}} \ln\left(\frac{5.98h}{0.8w+t}\right)$$

for $0.1 < w/h < 2$
and $1 < \varepsilon_r < 15$

$$Z_{0,offset} \cong 2\frac{Z_{0,sym}(2A,w,t,\varepsilon_r) * Z_{0,sym}(2B,w,t,\varepsilon_r)}{Z_{0,sym}(2A,w,t,\varepsilon_r) + Z_{0,sym}(2B,w,t,\varepsilon_r)}$$

more accurate if 2A # 2A+t and 2B # 2B+t