PHYS 320

Lecture 2

Distance and Size of a Celestial Body

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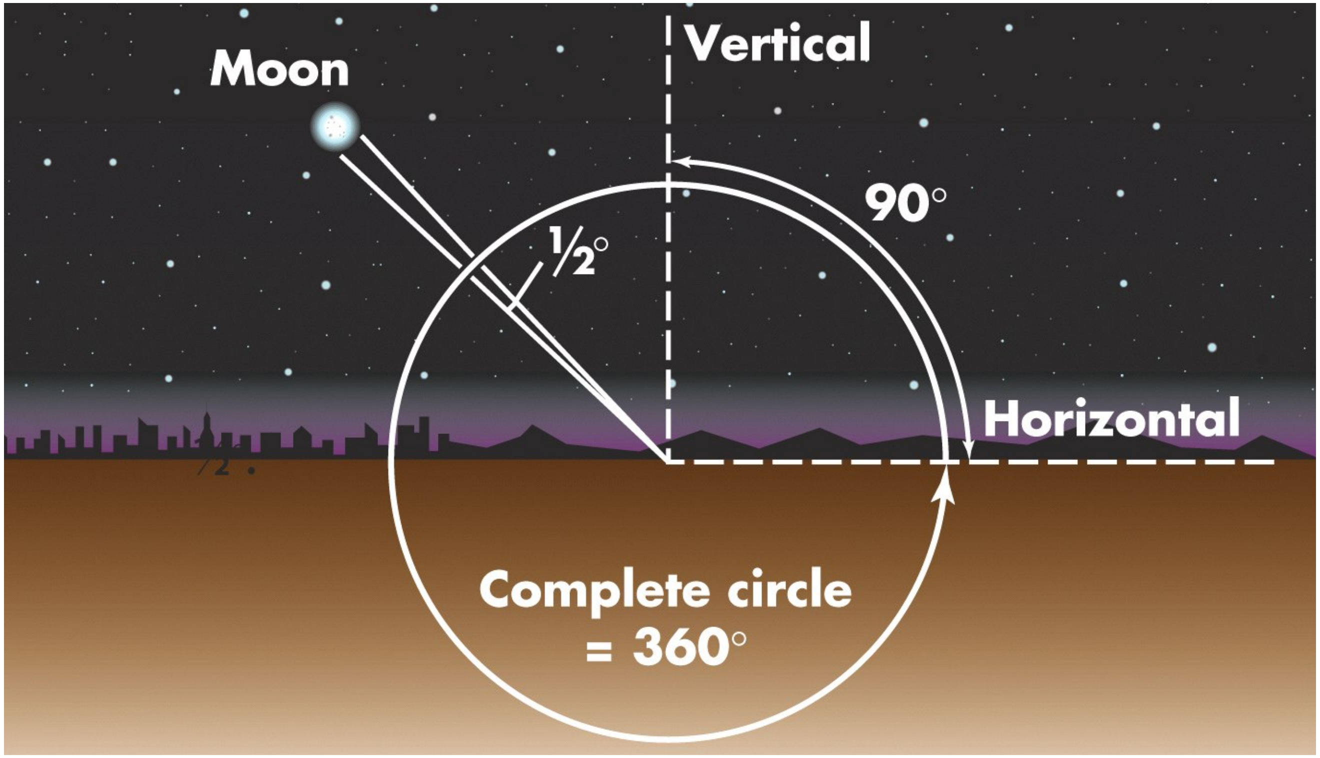
Outline

- Lecture 2 – Distance and Size of a Celestial Body
  - 2.1 Size and Distance of an Object
  - 2.2 Angular Diameter
  - 2.3 Angular Size in Astronomy
  - 2.4 Angular Distance
  - 2.5 Telescope Resolution
  - 2.6 Actual Sizes of Celestial Objects
  - 2.7 Powers-of-10 Notations
  - 2.8 Keywords and Summary


How far and how big?

2.1 Size and Distance of an Object



- Astronomers use **angular measure** to describe the apparent size of a celestial object - what fraction of the sky that object seems to cover.
- If you draw lines from your eye to two edges of the Moon, the angle between the lines is the **angular size** of the Moon.

Angle and Radian

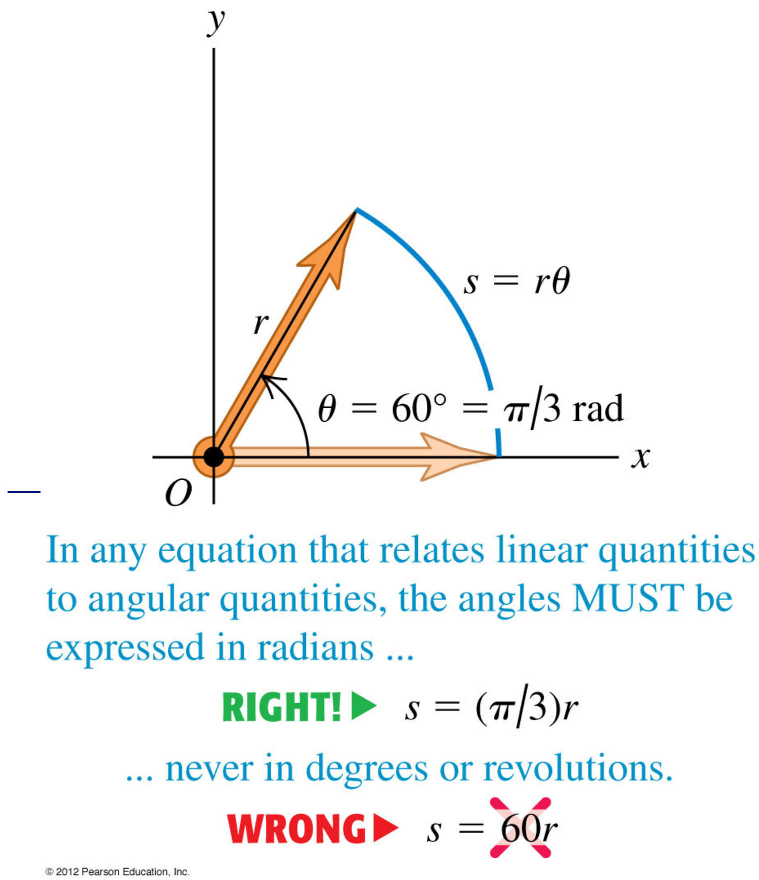
- What is the circumference S ?

$$s = (2\pi)r \qquad 2\pi = \frac{s}{r}$$

- θ can be defined as the arc length s along a circle divided by the radius r:

$$\theta = \frac{s}{r}$$

- θ is a pure number, but commonly is given the artificial unit, radian (“rad”)



Conversions

- Comparing degrees and radians

$$2\pi(rad) = 360^{\circ} \qquad \pi(rad) = 180^{\circ}$$

- Converting from degrees to radians

$$\theta\,(rad) = \frac{\pi}{180^{\circ}}\,\theta\,(degrees)$$

- Converting from radians to degrees

$$\theta(\deg rees) = \frac{180^{\circ}}{\pi}\theta(rad) \qquad 1\,rad = \frac{360^{\circ}}{2\pi} = 57.3^{\circ}$$

Conversion

- A waterwheel turns at 360 revolutions per hour. Express this figure in radians per second.

A) 3.14 rad/s
B) 6.28 rad/s
C) 0.314 rad/s
D) 0.628 rad/s
E) $\pi/5$ rad/s

Basic Units of Angular Measurement in Astronomy

- Degree, arc-minute and arc-second

$$1^\circ(\deg) = 60'(arc\min)$$

$$1'(arc\min) = 60''(arc\sec)$$

$$1^\circ(\deg) = 3600''(arc\sec)$$

- Converting from radians to degrees

$$1\ rad = \frac{180^\circ}{\pi}(rad) = 57.2958^\circ = 3438' = 206265''$$

Ex.4a: the size of the Moon: the **angular diameter** (or **angular size**) of the Moon is 0.5° or the Moon subtends an angle of 0.5°.

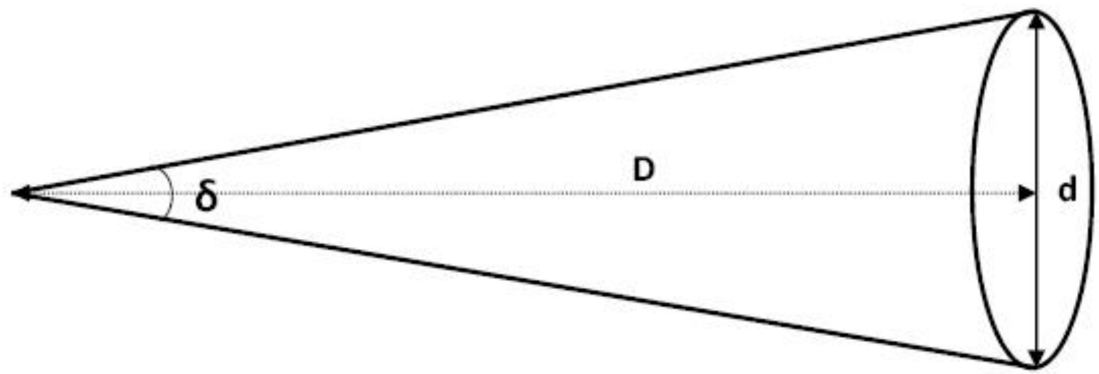
Ex.4b: the angular diameter of the Sun as of 2014 August 25 is about 1900". How does it compare with the Moon?

Ex.4c: the angular size of the Moon is approximately the same as a car of 2-meter long viewed at the distance of 200 meters away.

Ex.4d: 1° is the angular size of your finger an arm's length away.

Q: does it make sense?

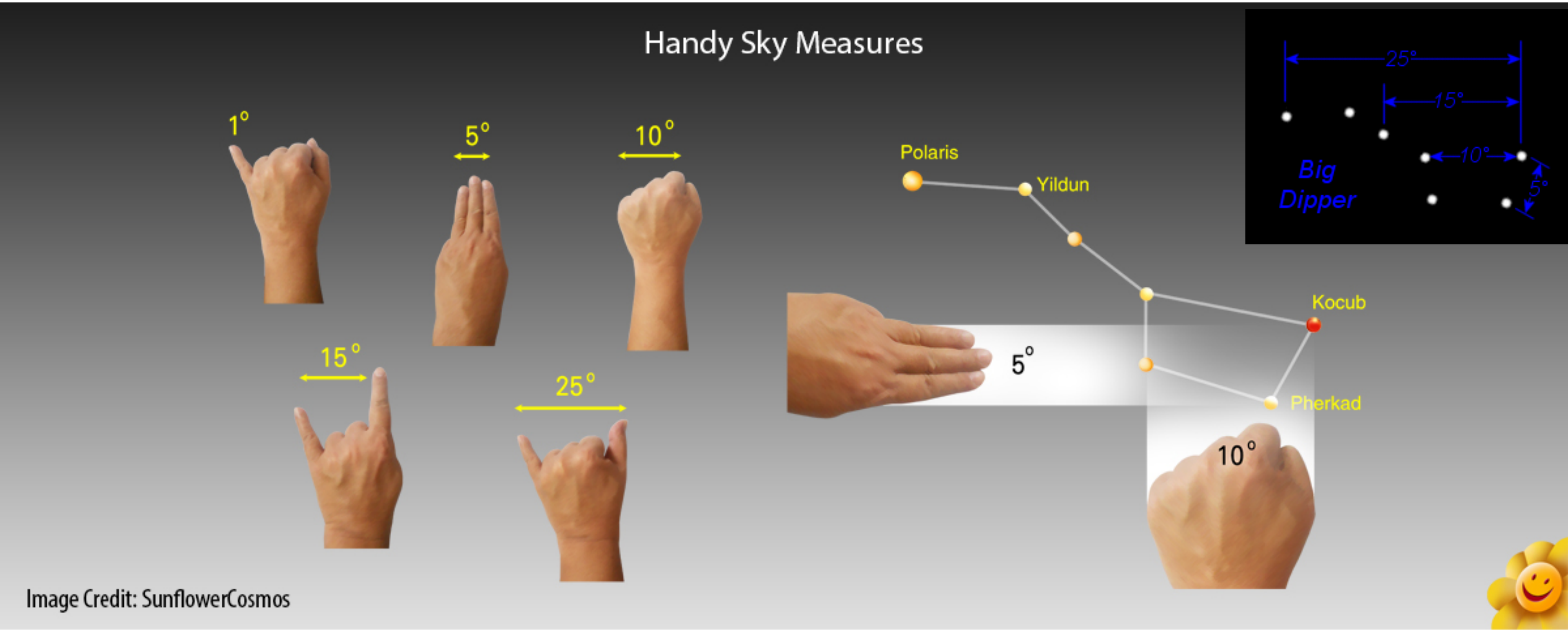
2.2 Angular Diameter



$$\delta = 2\arctan\left(\frac{d}{2D}\right)$$

- *d* is the actual diameter of the object
- *D* is the distance to the object
- δ is the angular diameter in unit of radian

Handy Sky Measures



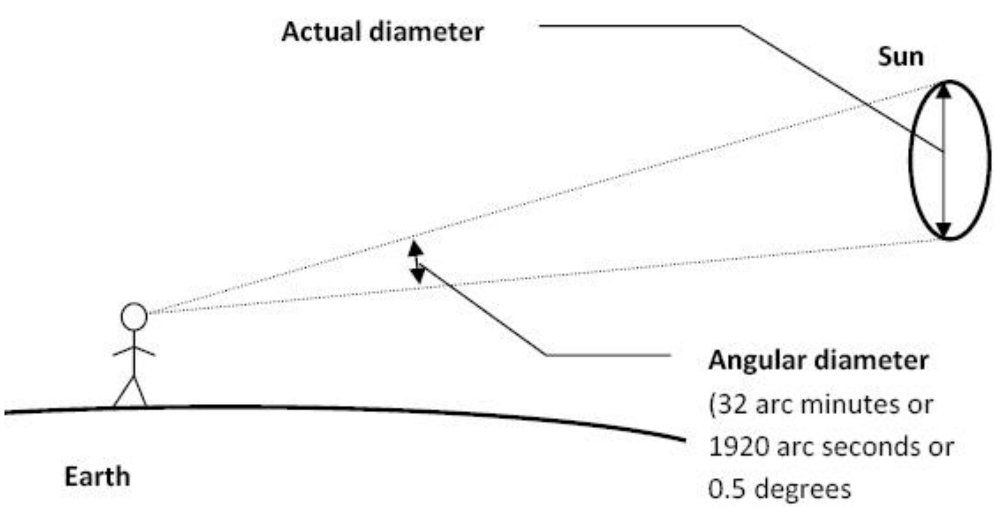
$$\delta = 2\arctan\left(\frac{d}{2D}\right)$$

2.3 Angular Size in Astronomy

$$\delta = 2\arctan\left(\frac{d}{2D}\right)$$

$$when\ D >> d$$

$$\delta \approx \frac{d}{D}$$



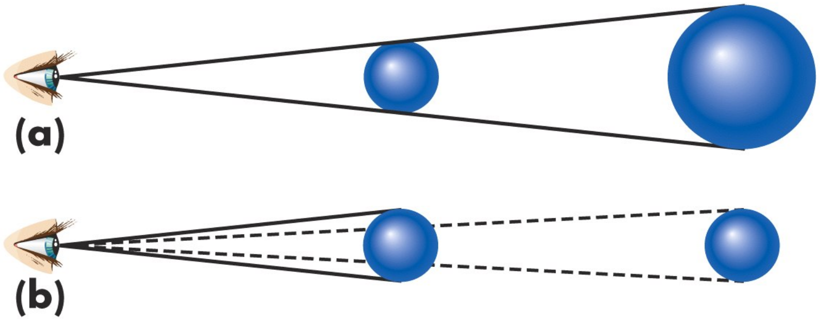
- δ depends upon the actual diameter of the object *d*
- δ also depends upon the actual diameter of the object *D*
- δ is the angular diameter in unit of radian

$$\delta'' \approx 206{,}265\left(\frac{d}{D}\right) \qquad \delta' \approx 3{,}438\left(\frac{d}{D}\right)$$

Use in Astronomy

$$\delta \approx \frac{d}{D}$$

$$\delta'' \approx 206,265\left(\frac{d}{D}\right)$$



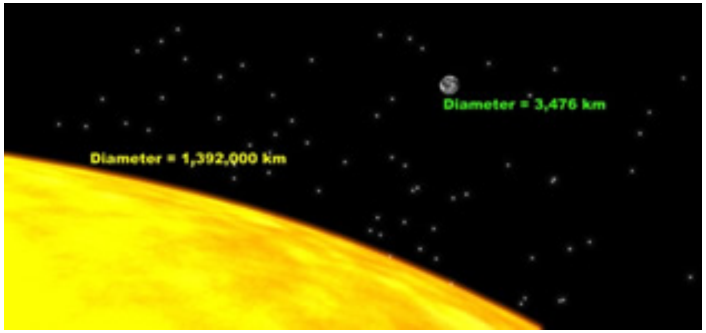
- δ depends upon the actual diameter of the object *d*
- δ also depends upon the actual diameter of the object *D*
- Same object: the further away, the smaller? $\delta \sim 1/D$
- Different objects of the same size, the further away, the larger? $\delta \sim d/D$
- Different objects with different sizes may have the same angular sizes?

Sun and Moon Viewed from Earth

Q: the Sun's distance from Earth is about 400 times the Moon's distance. How large is the Sun compared with the Moon?

$$\delta'' \approx 206{,}265\left(\frac{d}{D}\right)$$

Object	Orbital Radius D (x 10^6 km)	Equatorial Radius R (km)	D/R_o	R/R_o
Sun	---	696,000	---	1.0
Mercury	57.9	2,439	83.2	0.0035
Venus	108.2	6,052	155.5	0.0087
Earth	149.6	6,387	215	0.0092
--Moon		1,738		0.0025
Mars	228.0	3,393	327.6	0.0049
Jupiter	779.3	71,398	1118	0.1026
Saturn	1427	60,000	2050	0.0862
Uranus	2871	25,559	4125	0.0367
Neptune	4497	24,800	6461	0.0356
Pluto	5913	1,140	8496	0.0016



Jupiter's Actual Diameter

Ex.5: On July 26, 2003, Jupiter was 943 million kilometers from Earth and had an angular diameter of 31.2". Using the small-angle formula, determine Jupiter's actual diameter.

$$\delta'' \approx 206,265\left(\frac{d}{D}\right)$$

$$d \approx \frac{\delta D}{206,265}$$

$$= \frac{31.2'' \times 943 \times 10^6 km}{206,265} = 143 \times 10^3 km$$

Q: what's the angular size of the Earth observed from Jupiter?

$$\delta'' \approx 206,265\left(\frac{6,387 \times 2}{943 \times 10^6}\right) = 2.8''$$

Object	Orbital Radius D (x 10^6 km)	Equatorial Radius R (km)	D/R_o	R/R_o
Sun	---	696,000	---	1.0
Mercury	57.9	2,439	83.2	0.0035
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Telescope Resolution

Ex.6: Under excellent conditions, a telescope on earth can see details with an angular size as small as 1″. What is the greatest distance at which you could see details as small as 1.7-m using the telescope?

$$\delta'' \approx 206,265\left(\frac{d}{D}\right)$$

$$D \approx \frac{206,265d}{\delta}$$

$$= \frac{206,265 \times 1.7m}{1''} = 3.5 \times 10^{2}\,km$$

Q: how about a quarter coin?

$$D \approx \frac{206,265d}{\delta}$$

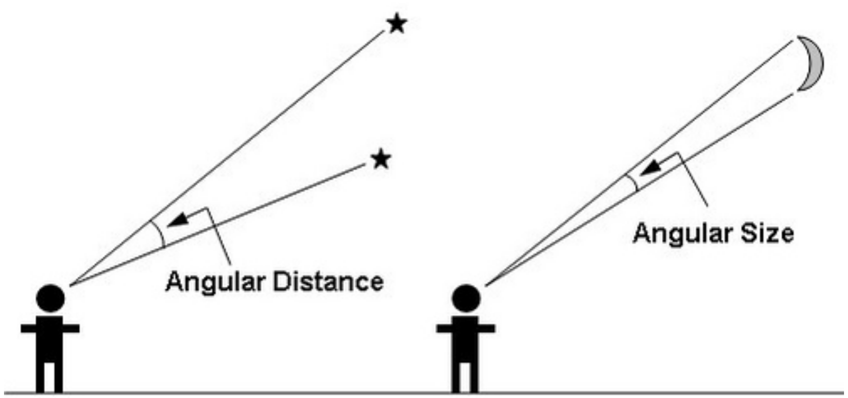
$$= \frac{206,265 \times 24.26mm}{1''} = 5.0m$$

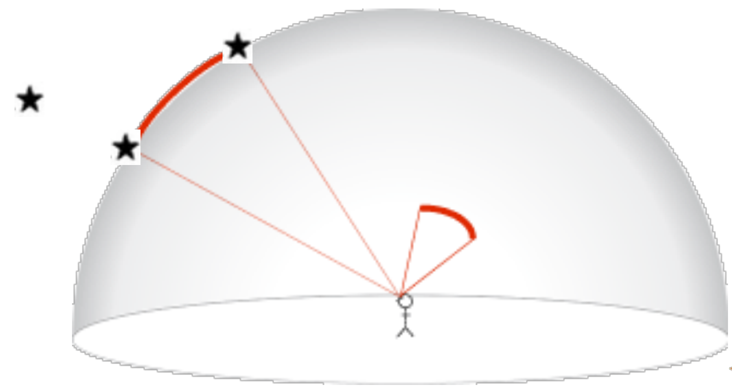
2.4 Angular Distance

- If you draw lines from your eye to each of two stars, the angle between the lines is the **angular distance** between the two stars.
- **Note:** here we refer to the **distance projected to the surface of an imaginary celestial sphere centered at the observer**, as if the two objects were in this same spherical surface.

$$\delta'' \approx 206,265\left(\frac{d}{D}\right)$$

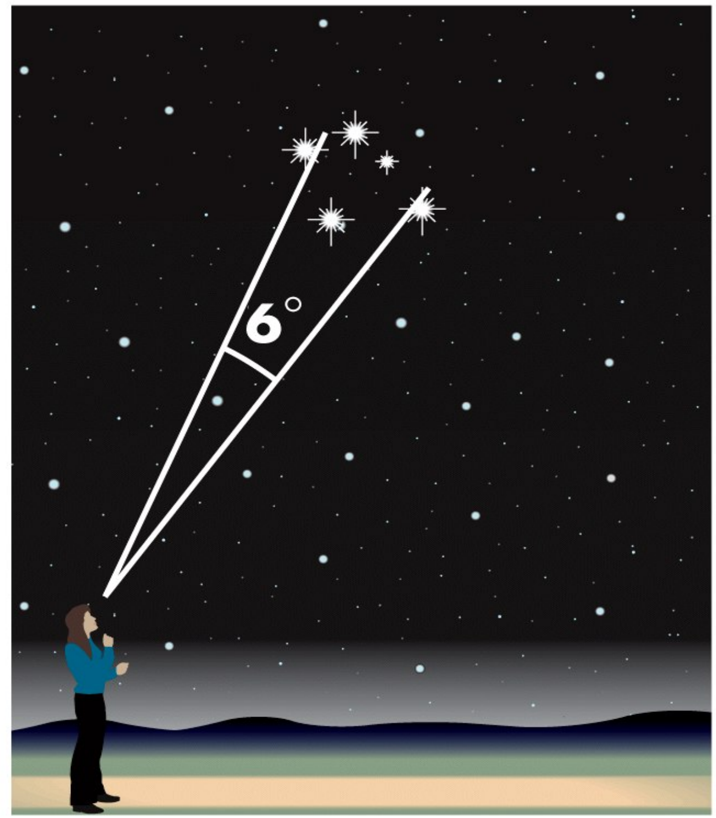
Q: does the radius of this sphere matter?





Angular Distance

Q: if the Sun is at the eastern Horizon and the Moon at the western Horizon, what’s the angular distance between them?



Units of Distance in Astronomy

- Solar Radius ($R_\odot$)
  - 695,508 km = 6.95508×10^8 m
- Astronomical Unit (AU)
  - One AU is the average distance between Earth and the Sun
  - 1.496×10^8 km or 92.96 million miles
  - the distance of light traveled in 500s
- Light Year (ly)
  - One ly is the distance light can travel in one year at a speed of about 3×10^5 km/s or 186,000 miles/s
  - 1 ly = 9.46×10^{12} km or 63,240 AU
- Parsec (pc)
  - the distance at which 1 AU subtends an angle of 1 arc-second

Object	Orbital Radius D (x 10^6 km)	Equatorial Radius R (km)	$D/R_\odot$	$R/R_\odot$
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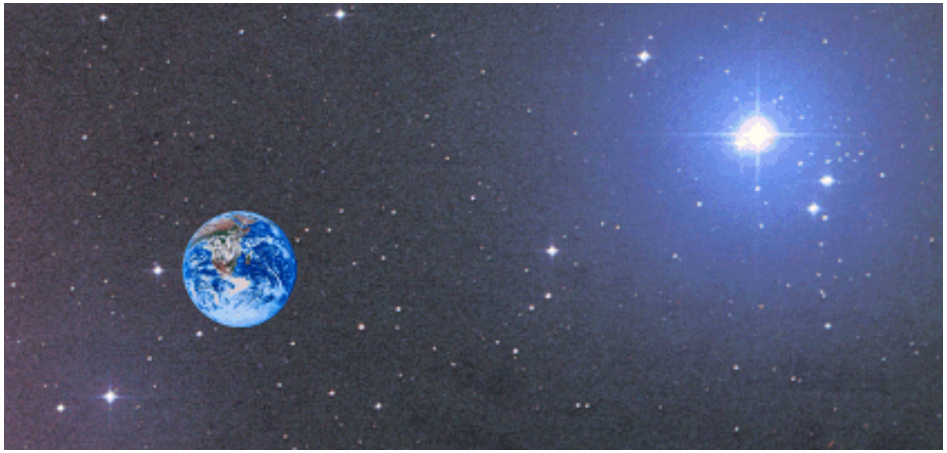
Q: What are the distances of planets to the Sun in units of AU ?

Q: 1 pc = ? Km = ? ly

Proxima Centauri

Ex.7: Proxima Centauri, the second closest star to Earth, is at the distance of 4.2 ly. If its diameter is 0.15 that of the Sun, what is its angular size as observed on Earth?



$$\delta_p'' \approx 206,265\left(\frac{d_p}{D_p}\right)$$

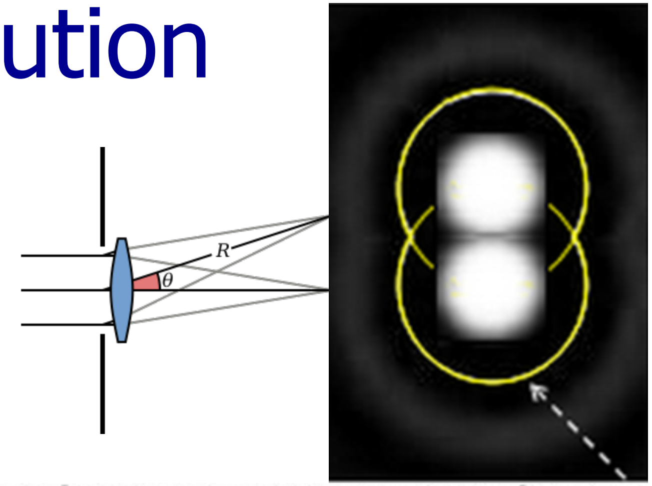
$$\delta_s'' \approx 206,265\left(\frac{d_s}{D_s}\right)$$

$$\frac{\delta_p}{\delta_s} = \left(\frac{d_p}{d_s}\right)\left(\frac{D_s}{D_p}\right) = 0.15\left(\frac{149.6*10^6}{4.2*9.46*10^{12}}\right) = 5.6*10^{-7}$$

$$\delta_p = \delta_s\left(\frac{d_p}{d_s}\right)\left(\frac{D_s}{D_p}\right) = 32'*5.6*10^{-7} = 1920''*5.6*10^{-7} = 0.001''$$

2.5 Telescope Resolution

- **Diffraction**



$$I(\theta) = I_0 \left(\frac{2J_1(ka\sin\theta)}{ka\sin\theta} \right)^2 = I_0 \left(\frac{2J_1(x)}{\ } \right)^2$$

$$x = ka\sin\theta = \frac{2\pi a}{\lambda}\frac{q}{R} \qquad J_1(x) = ($$

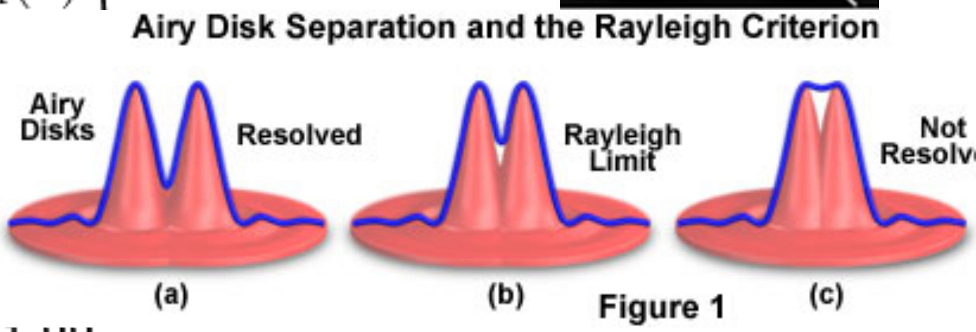
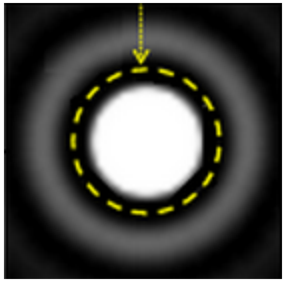


Figure 1

$$\sin\theta \approx \frac{3.83}{ka} = \frac{3.83\lambda}{2\pi a} = 1.22\frac{\lambda}{2a} = 1.22\frac{\lambda}{d}$$

$$q_1 = R\sin\theta \approx 1.22 f\frac{\lambda}{D} \approx 1.22\lambda f\# \qquad \theta_1 = \frac{q_1}{f} = 1.22\frac{\lambda}{D}$$



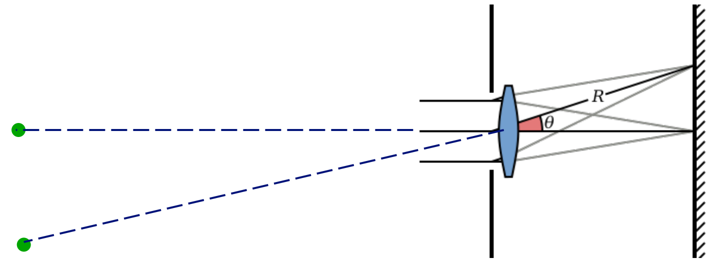
Telescope Resolution

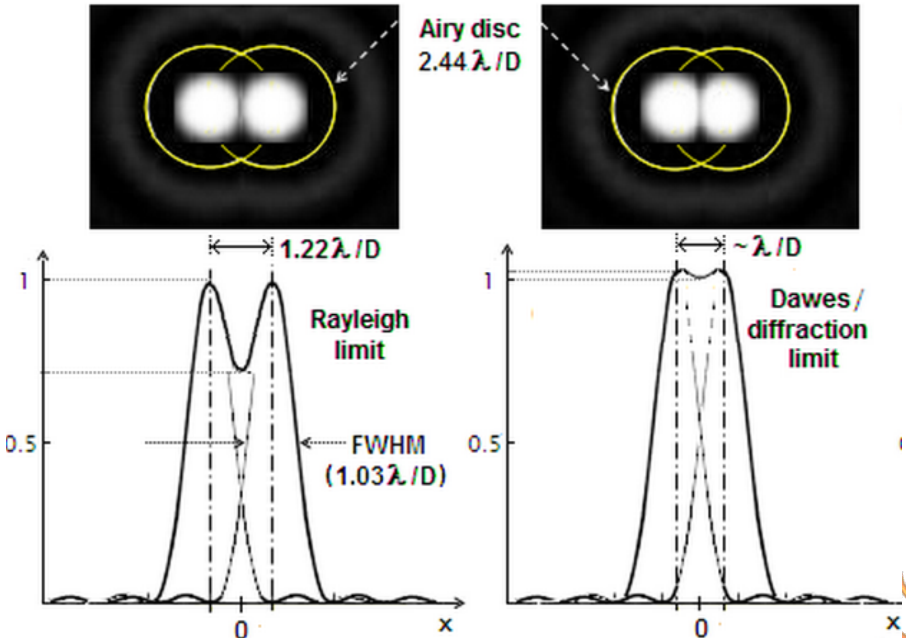
- **Rayleigh criterion:** angle defined as that for which the central peak of one PSF falls upon the first minimum of the other

$$\theta\ (rad) = 1.22\frac{\lambda}{D} \qquad \theta\ ('') = 0.25\frac{\lambda\,(\mu m)}{D\,(m)}$$

- **Sparrow criterion:** angular separation when the combined pattern of the two sources has no minimum between the two centers

$$\theta\ (rad) = \frac{\lambda}{D}$$





Telescope Resolution

- ***Angular resolution:*** *can be quantified as the smallest angle between two point sources for which separate recognizable images are produced*

$$\theta\ (rad) = 1.22\frac{\lambda}{D} \qquad \theta\ (rad) = \frac{\lambda}{D}$$

$1\,rad = 206265''$ $\qquad$ $1'' = 725$ km on the Sun

eye	SDO/HMI	Hinode	NST	ATST	KECK
6mm	14cm	50cm	1.6m	4m	10m
1	5.4×10^2	6.9×10^3	7.1×10^4	4.4×10^5	2.7×10^6
> 60”	0.74”	0.21”	0.06”	0.03”	0.01”
43500 km	534 km	150 km	47 km	19 km	< 8 km

2.6 Actual Size of Celestial Objects

- Advanced techniques, such as adaptive optics, interferometry, space telescopes etc, are often needed to directly measure the angular size of celestial objects. With the knowledge of distance, we can know the linear size of the objects.

$$\delta'' \approx 206,265\left(\frac{d}{D}\right)$$

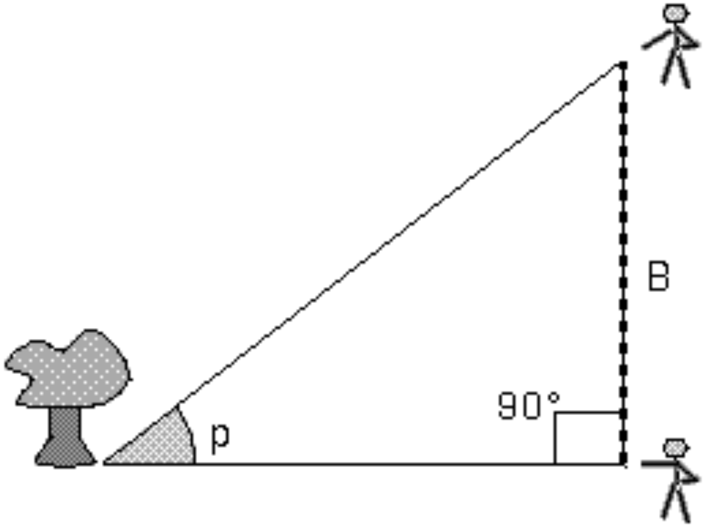
- The rasius of stars may be measured indirectly by luminosity, the radiation energy rate (next week):

$$L = 4\pi R^2 \sigma T^4 \quad (\text{Joule s}^{-1})$$

Q: how can we acquire the knowledge of distance from Earth?

Parallax Measurement

Ex.9: Parallax: measuring the different angular position of a remote object from two different locations on Earth.



If we know one side (B) and one angle (p) of a right triangle, we can derive the length of the other two sides.

The distance **d** is determined by the **baseline length B** and angle **p** (**parallax**) as:

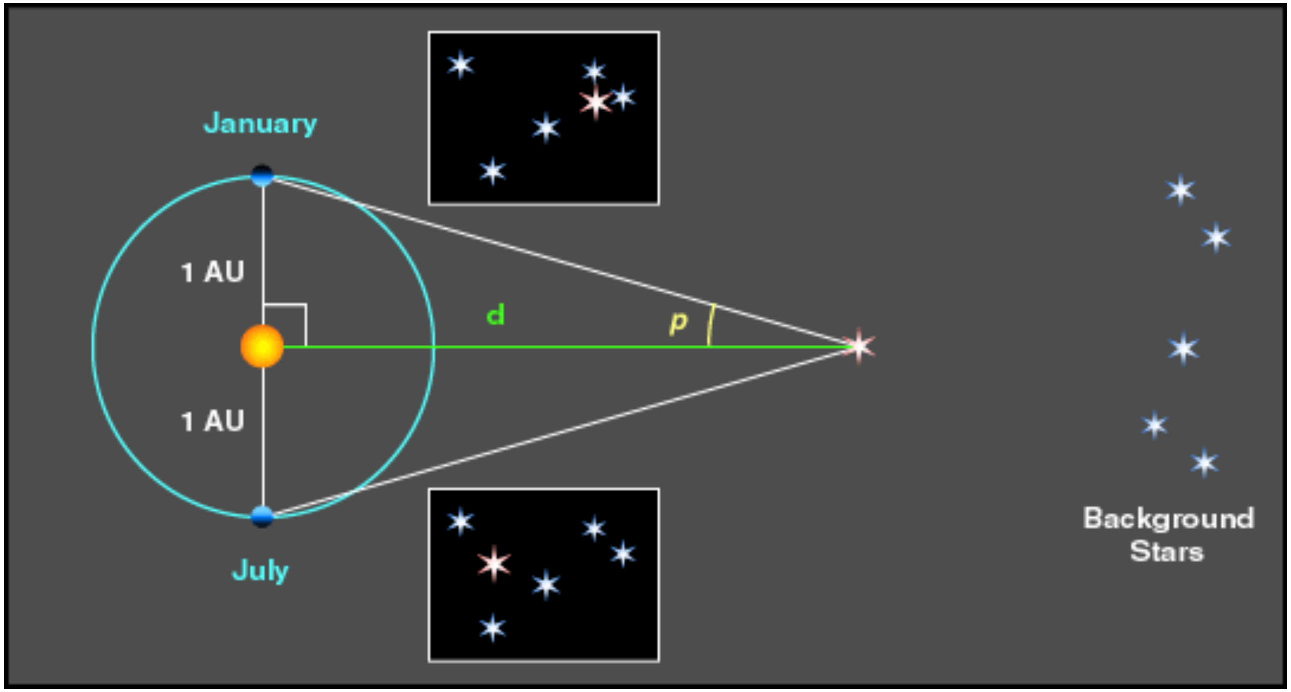
$$d = B/\tan(p)$$

or simply:

$$d = B/p \qquad \text{when } d >> B$$

The scale of solar system was first determined by **trigonometric parallax**. A greater distance (d) can be determined with a longer baseline (B).

The farther away, the smaller the angle p.

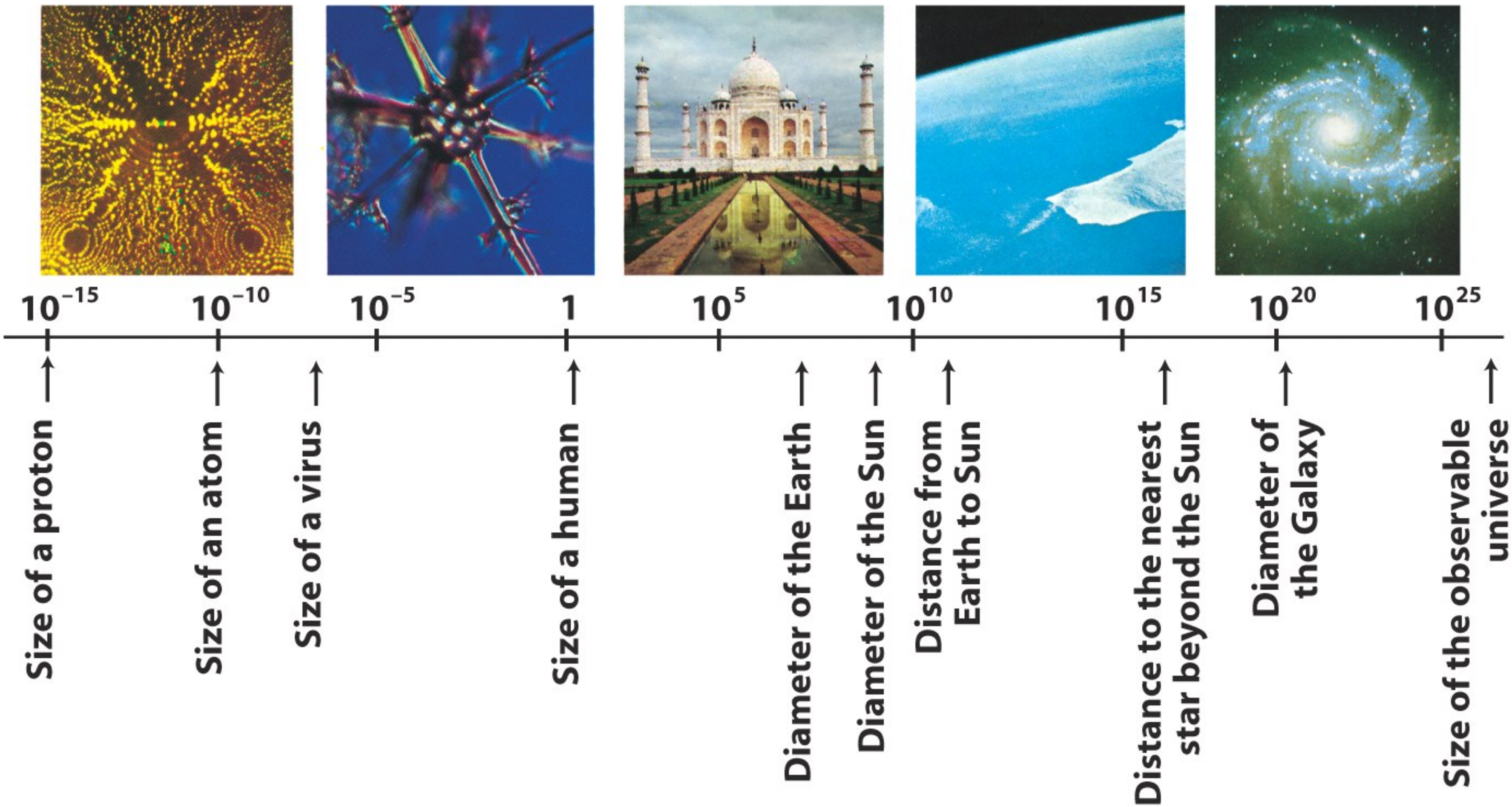


Annual Parallax: observing a celestial object 6 months apart, B becomes the Sun-Earth distance, i.e., 1 AU.

Parsec (**Par**allax arc**sec**), or pc, is an astronomical distance units. 1 pc is the distance of an object whose parallax is 1 arcsec; 1pc = 3.26 ly.

Q: what is the parallax of Proxima Centauri?

2.7 Powers-of-10 Notation



Common Prefixes

Factor		Name	Symbol
(billion)	10^9	Giga-	G
(million)	10^6	Mega-	M
(thousand)	10^3	kilo-	k
(hundredth)	10^{-2}	centi-	c
(thousandth)	10^{-3}	milli-	m
(millionth)	10^{-6}	micro-	μ
(billionth)	10^{-9}	nano-	n

Key words

- angle
- angular diameter (angular size)
- angular measure
- arcminute
- arcsecond
- degree (°)
- distance - astronomical unit (AU)
- exponent
- laws of physics
- model
- powers-of-ten notation
- small angle formula
- theory

Summary

- To understand the universe, astronomers use the laws of physics to construct testable theories and models to explain observations and predict new phenomena.
- Astronomers study planets to learn the formation of the solar system.
- Astronomers study the Sun to learn the structure and evolution of stars and sun-earth connection (climate and space weather).
- Solar system constitutes the Sun and all the celestial bodies that orbit the Sun.
- The thousand-yard model gives a clear picture of sizes and relative distances of all the celestial bodies orbiting the Sun by scaling the solar system.
- Astronomers use angular measurements to denote the apparent size and distance of celestial objects.

Telescope resolving power

- Ground-based telescopes can resolve up to 0.01".
- The Hipparcos satellite launched in 1989 can measure p approaching 0.001", or the stellar distance up to 1000 pc = 1 kpc.
- The center of our Milky Way Galaxy is 8 kpc. So stellar parallax is useful only for neighborhood stars (<30-500 pc).
- VLBA interferometer technology can measure the distance of up to 10 kpc.