PSET 2 Part 1

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CS231A

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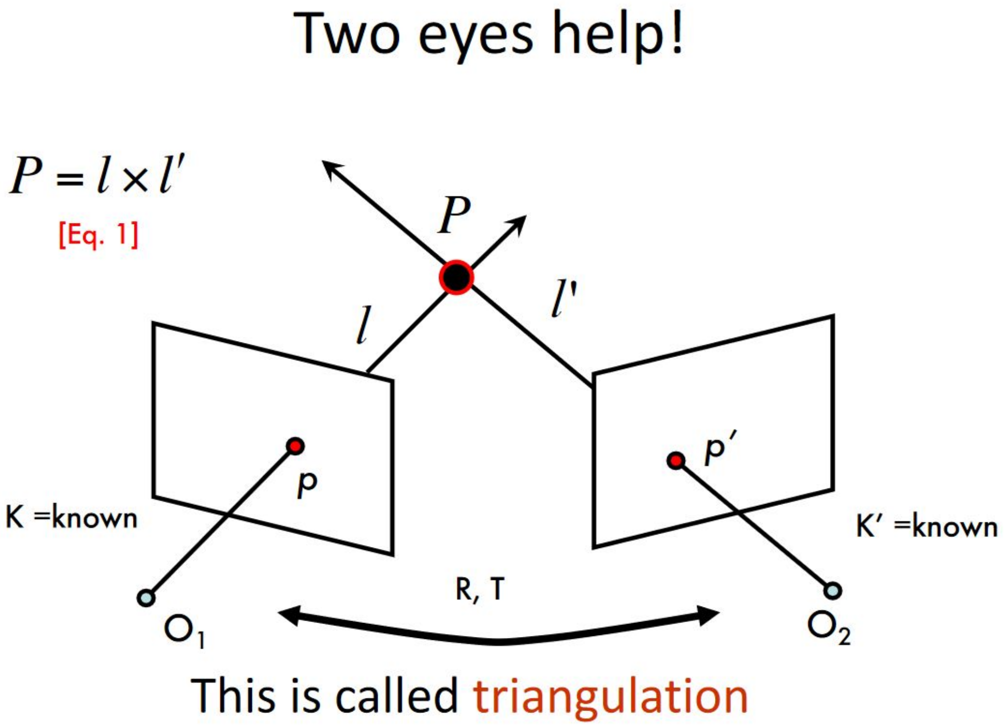
Overview

- Lecture Review
- PSET 2

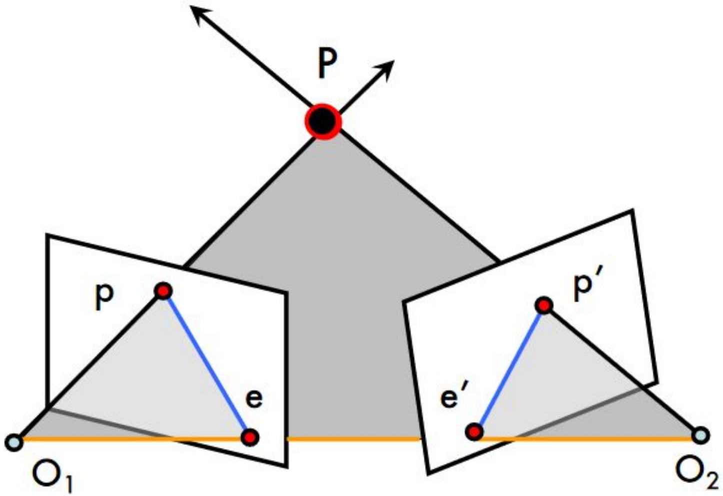
Lecture - Epipolar Geometry

Triangulation - determining a point in 3D space given its projections onto two, or more, images.



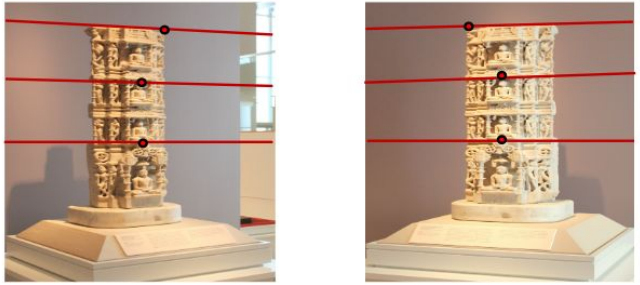
Lecture - Epipolar Geometry

Epipolar geometry



- Epipolar Plane
- Baseline
- Epipolar Lines
- Epipoles e, e′

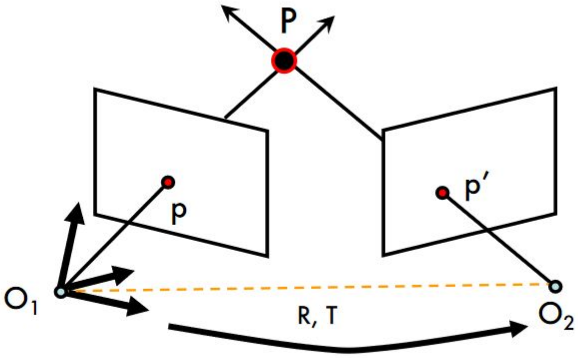
  = intersections of baseline with image planes
  = projections of the other camera center



Lecture - Epipolar Geometry

$$M = [I \quad 0] \qquad M' = [R_{wc'}^T \quad -R_{wc'}^T T_{wc'}]$$

$$p = R_{wc'} p' + T_{wc'}$$



$$p^T \cdot [T \times (R\, p')] = 0 \rightarrow p^T \cdot [T_\times] \cdot R\, p' = 0$$

$$p^T E p' = 0$$

$$\ell' = E^T p$$

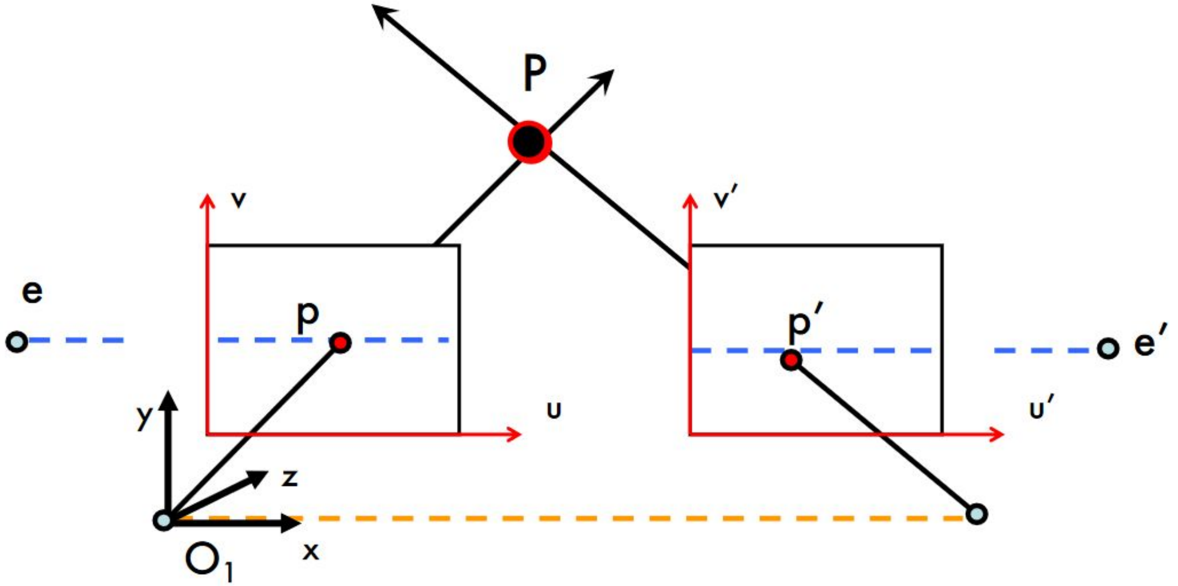
$$Ee' = 0. \text{ Similarly } E^T e = 0.$$

$$M = K\,[I \quad 0] \qquad M' = K'\,[R_{wc'}^T \quad -R_{wc'}^T T_{wc'}]$$

$$p^T K^{-T} [T_\times] R K'^{-1} p' = 0$$

$$F = K^{-T} [T_\times] R K'^{-1}$$

Lecture - Stereo Systems



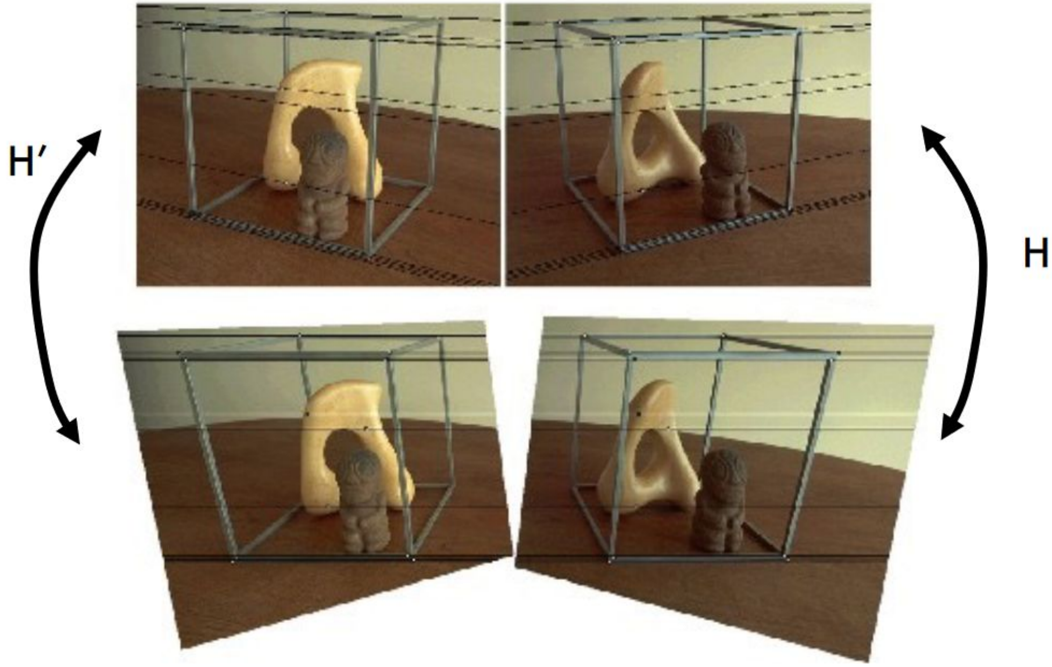
$$\mathbf{T} = [\ T \quad 0 \quad 0\ ]$$
$$\mathbf{R} = \mathbf{I}$$

How are p and p′ related?

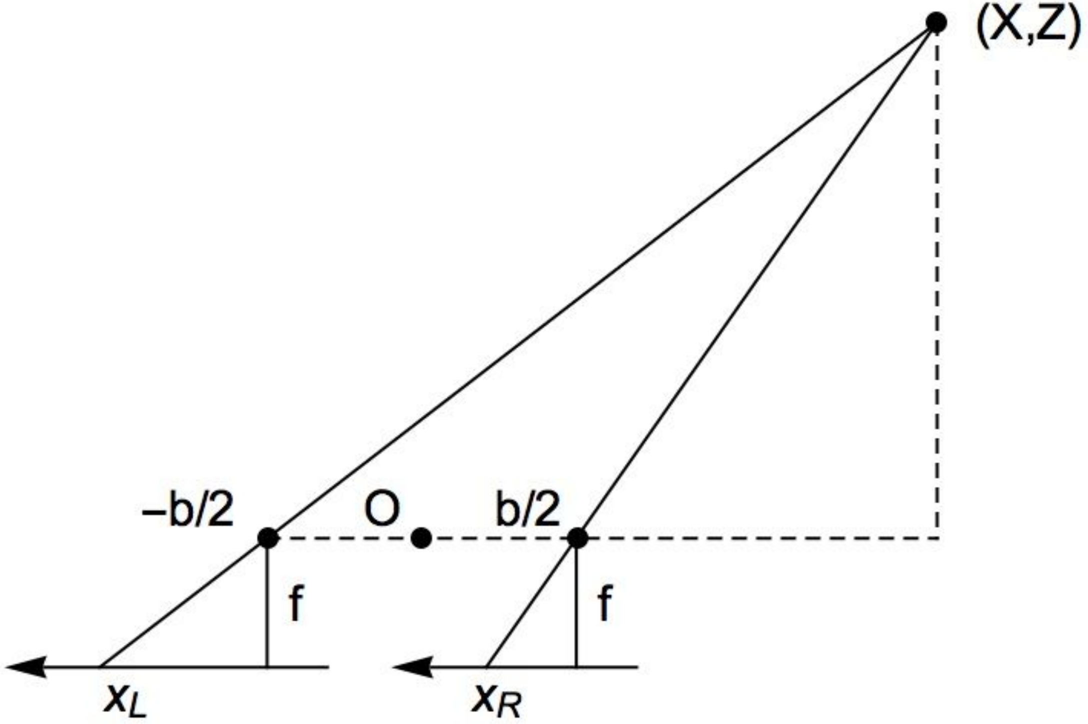
$$\Rightarrow (u \quad v \quad 1)\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix}\begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0 \Rightarrow (u \quad v \quad 1)\begin{pmatrix} 0 \\ -T \\ Tv' \end{pmatrix} = 0 \Rightarrow Tv = Tv'$$
$$\Rightarrow v = v'$$

Lecture - Stereo Systems

Rectification: making two images "parallel"



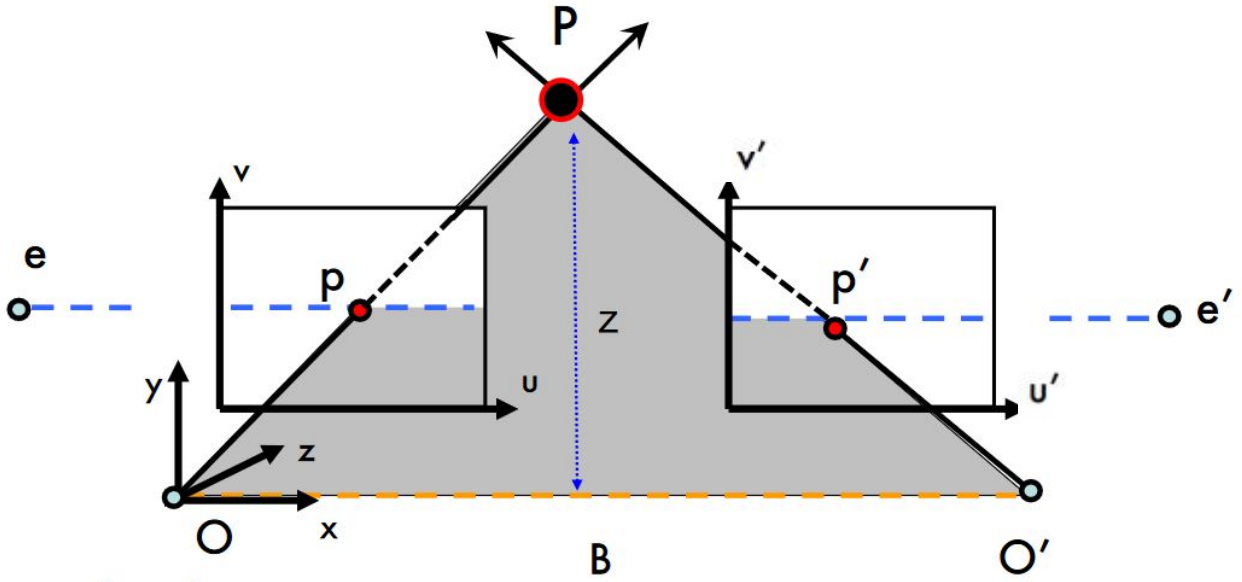
Lecture - Stereo Systems



$$\frac{x_L}{f} = \frac{X + \frac{b}{2}}{Z}, \quad \frac{x_R}{f} = \frac{X - \frac{b}{2}}{Z}$$

$$\delta = x_L - x_R = \frac{bf}{Z}.$$

Lecture - Stereo Systems



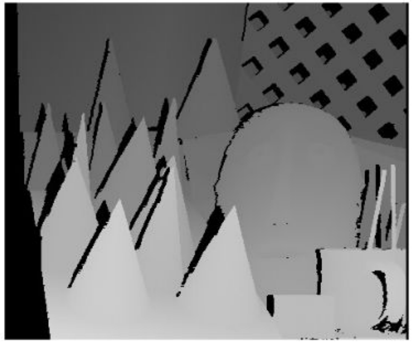
$$p = \begin{bmatrix} p_u \\ p_v \\ 1 \end{bmatrix} \qquad p' = \begin{bmatrix} p'_u \\ p_v \\ 1 \end{bmatrix}$$

disparity = $p_u - p'_u \propto \dfrac{B \cdot f}{z}$ [Eq. 1]

Disparity is inversely proportional to depth z!

Lecture - Stereo Systems

http://vision.middlebury.edu/stereo/



Disparity map / depth map

Lecture - Stereo Systems

Correspondence problem

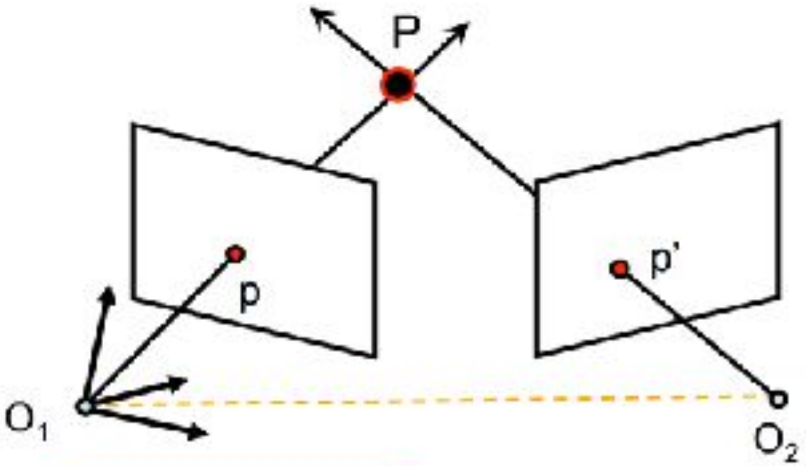
Correlation Methods (1970–)

PSET 2

1. Problem 1 - Fundamental Matrix Estimation From Point Correspondences
2. Problem 2 - Matching Homographies for Image Rectification

Fundamental Matrix

A matrix which maps the relationship of correspondences between stereo images

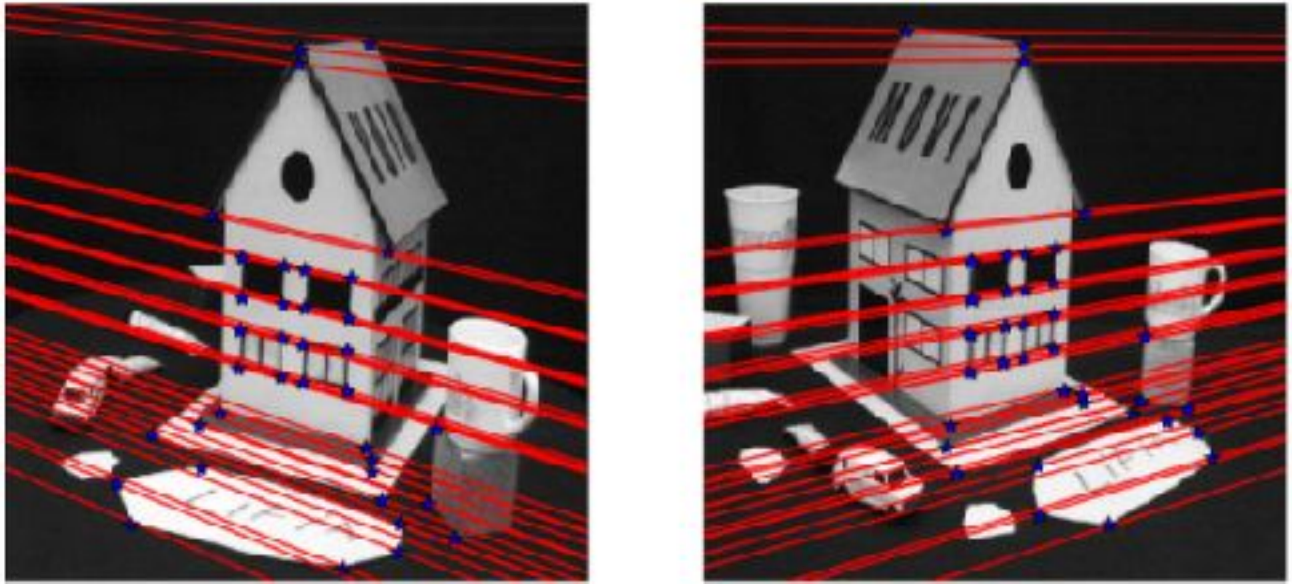


[Eq. 13] $$p^T F p' = 0$$

$$F = K^{-T} \cdot [T_\times] \cdot R K'^{-1}$$

Ex)

Image and correspondences given in the homework



How to compute F?

Eight point algorithm

[Eq. 13] $$p^T F p' = 0 \Rightarrow$$

$$p = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \quad p' = \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix}$$

$$(u, v, 1)\begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix}\begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0$$

$$\Rightarrow (uu', uv', u, vu', vv', v, u', v', 1)\begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = 0$$

[Eq. 14]

Let’s take 8 corresponding points

How to compute F?

Eight point algorithm

Problem?

W is highly unbalanced (not well conditioned)

Estimating F

$$\mathbf{W}\begin{pmatrix} u_1u'_1 & u_1v'_1 & u_1 & v_1u'_1 & v_1v'_1 & v_1 & u'_1 & v'_1 & 1 \\ u_2u'_2 & u_2v'_2 & u_2 & v_2u'_2 & v_2v'_2 & v_2 & u'_2 & v'_2 & 1 \\ u_3u'_3 & u_3v'_3 & u_3 & v_3u'_3 & v_3v'_3 & v_3 & u'_3 & v'_3 & 1 \\ u_4u'_4 & u_4v'_4 & u_4 & v_4u'_4 & v_4v'_4 & v_4 & u'_4 & v'_4 & 1 \\ u_5u'_5 & u_5v'_5 & u_5 & v_5u'_5 & v_5v'_5 & v_5 & u'_5 & v'_5 & 1 \\ u_6u'_6 & u_6v'_6 & u_6 & v_6u'_6 & v_6v'_6 & v_6 & u'_6 & v'_6 & 1 \\ u_7u'_7 & u_7v'_7 & u_7 & v_7u'_7 & v_7v'_7 & v_7 & u'_7 & v'_7 & 1 \\ u_8u'_8 & u_8v'_8 & u_8 & v_8u'_8 & v_8v'_8 & v_8 & u'_8 & v'_8 & 1 \end{pmatrix}\begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = 0 \quad \mathbf{f}$$

[Eqs. 15]

- Homogeneous system $\mathbf{W}\,\mathbf{f} = 0$
- Rank 8 ⟶ A non-zero solution exists (unique)
- If N>8 ⟶ Lsq. solution by SVD! ⟶ $\hat{\mathrm{F}}$

$\|\mathbf{f}\| = 1$

Final step

Reduce rank(F) to 2

$$F = U \begin{bmatrix} s_1 & 0 & 0 \\ 0 & s_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} V^T$$

Where:

$$U \begin{bmatrix} s_1 & 0 & 0 \\ 0 & s_2 & 0 \\ 0 & 0 & s_3 \end{bmatrix} V^T = SVD(\hat{F})$$

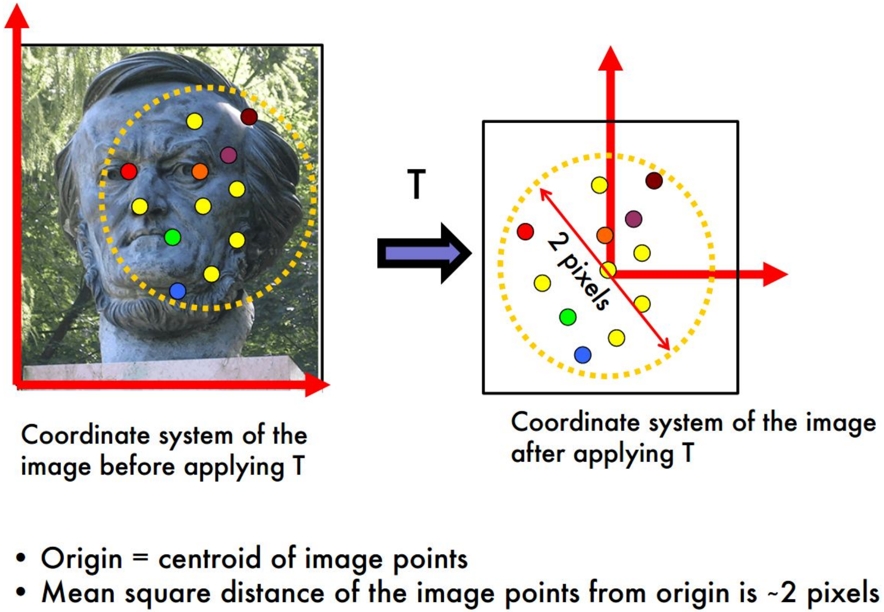
[HZ] pag 281, chapter 11, “Computation of F”

Possible improvement?

Pre-condition our linear system to get more stable result

origin = centroid of the points

mean square distance of the image points from origin is ~2px



The Normalized Eight-Point Algorithm

0. Compute T and T′ for image 1 and 2, respectively

1. Normalize coordinates in images 1 and 2:

$$q_i = T\ p_i \qquad q_i' = T'\ p_i'$$

2. Use the eight-point algorithm to compute $\hat{F}_q$ from the corresponding points q_i and q_i' .

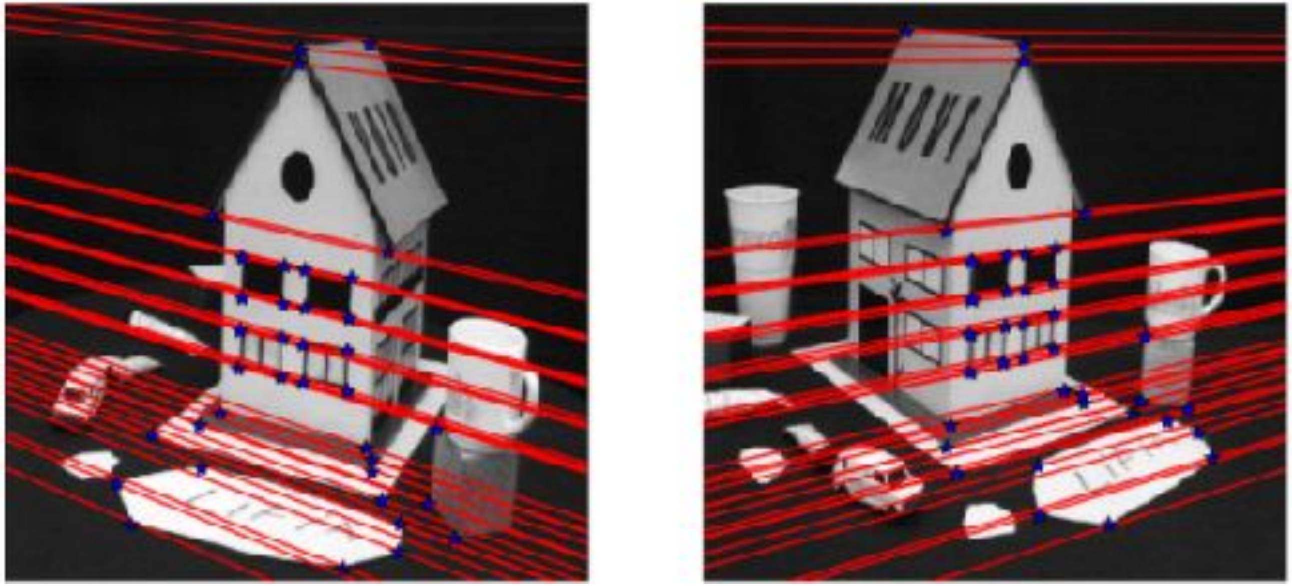
1. Enforce the rank-2 constraint: $\rightarrow \mathrm{F_q}$

such that:

$$\begin{cases} \mathrm{q^T\ F_q\ q' = 0} \\ \mathrm{det(\ F_q) = 0} \end{cases}$$

2. De-normalize F_q: $F = T'^T F_q T$

Epipolar lines



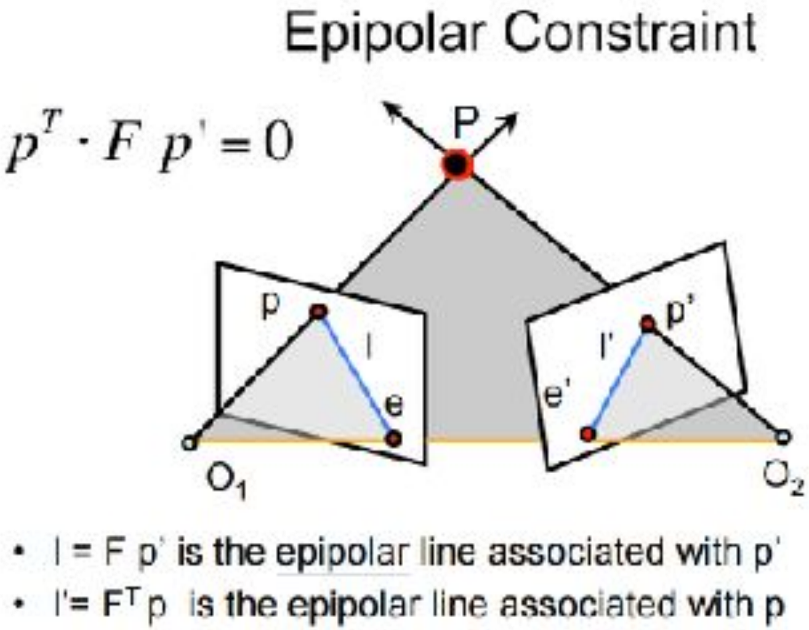
Distance to epipolar lines

Computing epipolar lines from F

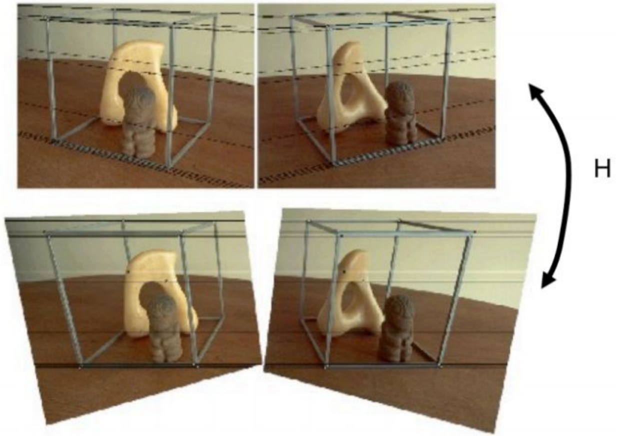
l = Fp’

l’ = F^Tp

$$\text{distance}(ax + by + c = 0, (x_0, y_0)) = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}.$$



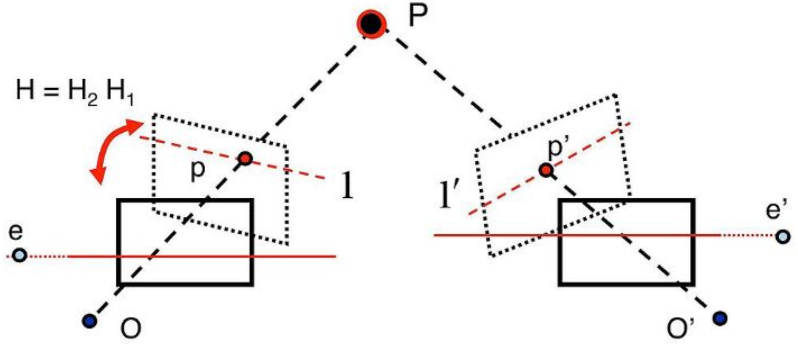
Make two images parallel to each other
⇒ epipole at infinity along the horizontal axis



Make two images parallel to each other
⇒ epipole at infinity along the horizontal axis

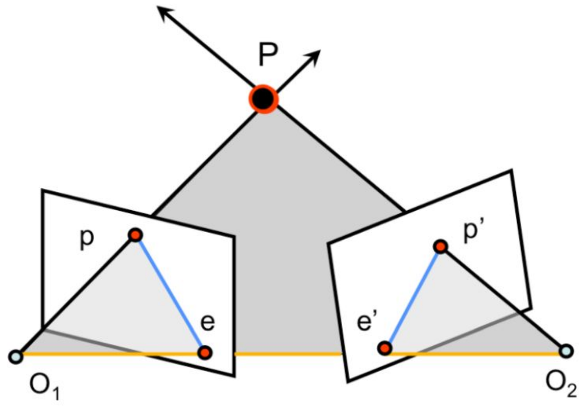
$Ee' = 0$. Similarly $E^T e = 0$.

1. Find epipoles
2. Solve for E



1. Alternative

- Epipolar line l = Fp’
- Epipole lies on epipolar lines l . x = 0
- Epipole is an intersection of all epipolar lines

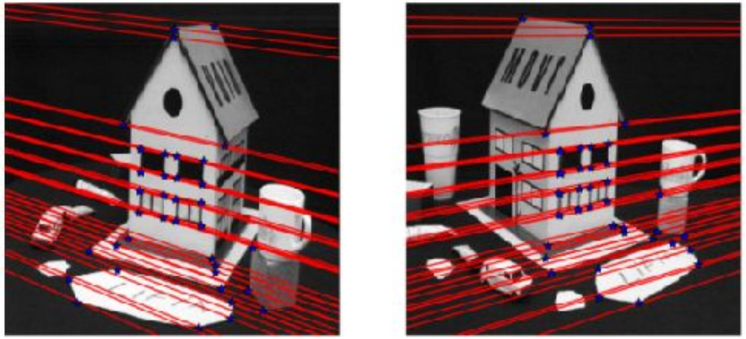


1. Compute epipole

Due to noisy measurement, not all epipolar lines intersect in a single point

⇒ Find a point that minimizes least square error of fitting a point to all the epipolar lines

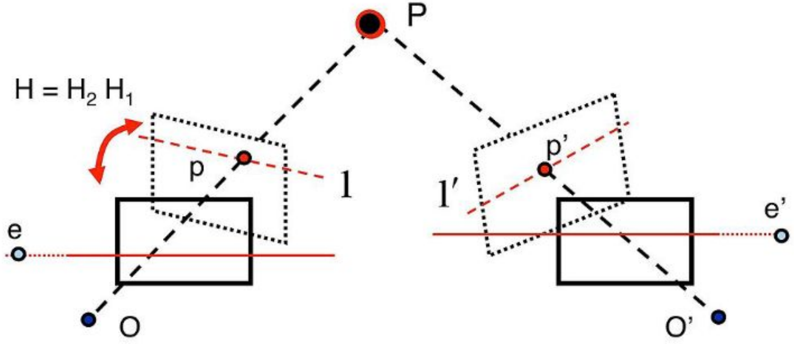
⇒ Solve least square by SVD



$$\begin{bmatrix} \ell_1^T \\ \vdots \\ \ell_n^T \end{bmatrix} e = 0$$

2. Find two homographies that shift epipoles to infinity

a. **Find homography H_2 that maps the second epipole e’ to a horizontal axis at infinity (f, 0, 0)**
b. Find the matching homography H_1 for the first image



Find homography H_2 that maps the second epipole e' to a horizontal axis at infinity (f, 0, 0)

i. Translate the second image s.t. the center is at (0, 0, 1) in homogeneous coord (***T***)

ii. Apply rotation to place the epipole on the horizontal axis (f, 0, 1) (***R***)

iii. Bring epipole at infinity on the horizontal axis (f, 0, 0) (***G***)

$$H_2 = T^{-1}GRT$$

Find homography H_2 that maps the second epipole e' to a horizontal axis at infinity (f, 0, 0)

i. Translate the second image s.t. the center is at (0, 0, 1) in homogeneous coord (***T***)

$$T = \begin{bmatrix} 1 & 0 & -\frac{\text{width}}{2} \\ 0 & 1 & -\frac{\text{height}}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

Find homography H_2 that maps the second epipole e' to a horizontal axis at infinity (f, 0, 0)

ii. Apply rotation to place the epipole on the horizontal axis (f, 0, 1) (***R***)

$$(e'_1, e'_2, 1)$$

The translated

$$R = \begin{bmatrix} \alpha \frac{e'_1}{\sqrt{e'^2_1 + e'^2_2}} & \alpha \frac{e'_2}{\sqrt{e'^2_1 + e'^2_2}} & 0 \\ -\alpha \frac{e'_2}{\sqrt{e'^2_1 + e'^2_2}} & \alpha \frac{e'_1}{\sqrt{e'^2_1 + e'^2_2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

where $\alpha = 1$ if $e'_1 \geq 0$ and $\alpha = -1$ otherwise.

Find homography H_2 that maps the second epipole e’ to a horizontal axis at infinity (f, 0, 0)

iii. Bring epipole (f, 0, 1) at infinity on the horizontal axis (f, 0, 0) (***G***)

$$G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{f} & 0 & 1 \end{bmatrix}$$

Find homography H_2 that maps the second epipole e’ to a horizontal axis at infinity (f, 0, 0)

i. Translate the second image s.t. the center is at (0, 0, 1) in homogeneous coord (***T***)

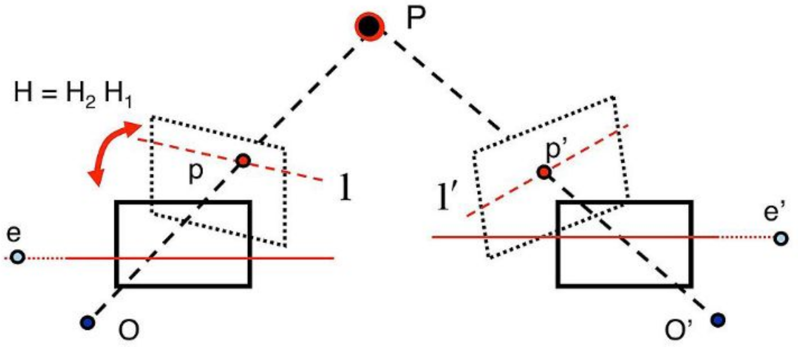
ii. Apply rotation to place the epipole on the horizontal axis (f, 0, 1) (***R***)

iii. Bring epipole at infinity on the horizontal axis (f, 0, 0) (***G***)

$$H_2 = T^{-1}GRT$$

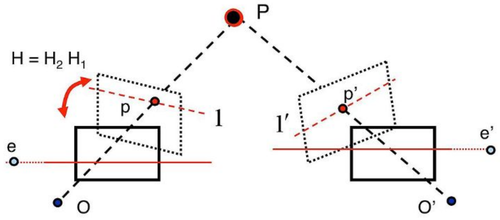
2. Find two homographies that shift epipoles to infinity

a. Find homography H_2 that maps the second epipole e’ to a horizontal axis at infinity (f, 0, 0)
b. Find the matching homography H_1 for the first image



Find the matching homography H_1 for the first image

$$\arg\min_{H_1} \sum_i \|H_1 p_i - H_2 p_i'\|^2$$



Although the derivation is out of the scope of this class,

$$H_A = \begin{bmatrix} a_1 & a_2 & a_3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H_1 = H_A H_2 M$$

$$M = [e]_\times F + ev^T$$

$$v^T = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

Find the matching homography H_1 for the first image

$$\arg\min_{H_1} \sum_i \|H_1 p_i - H_2 p'_i\|^2$$

$$H_1 = H_A H_2 M$$

$$\hat{p}_i = H_2 M p_i$$

$$\hat{p}'_i = H_2 p'_i$$

$$\arg\min_{H_A} \sum_i \|H_A \hat{p}_i - \hat{p}'_i\|^2$$

Find the matching homography H_1 for the first image

$$\hat{p}_i = H_2 M p_i$$
$$\hat{p}'_i = H_2 p'_i$$

$$H_A = \begin{bmatrix} a_1 & a_2 & a_3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\hat{p}_i = (\hat{x}_i, \hat{y}_i, 1) \text{ and } \hat{p}'_i = (\hat{x}'_i, \hat{y}'_i, 1)$$

$$\arg\min_{\mathbf{a}} \sum_i (a_1 \hat{x}_i + a_2 \hat{y}_i + a_3 - \hat{x}'_i)^2$$

Solving least-square $W\mathbf{a} = b$

$$W = \begin{bmatrix} \hat{x}_1 & \hat{y}_1 & 1 \\ & \vdots & \\ \hat{x}_n & \hat{y}_n & 1 \end{bmatrix} \qquad b = \begin{bmatrix} \hat{x}'_1 \\ \vdots \\ \hat{x}'_n \end{bmatrix}$$

Thanks!

Questions