# THE SUPERCONDUCTING CAVITY STABILIZED OSCILLATOR

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## ABSTRACT

Superconducting Cavity Stabilized Oscillators (SCSOs) have produced the most stable clocks to date for integration times between $10^2$ and $10^3$ seconds, achieving a fractional frequency stability of $2 \times 10^{-16}$ for a sampling time of 100 s. The principal contributors to cavity frequency variations are: (a) acceleration effects due to gravity and vibrations (b) temperature variations, (c) variations in the energy stored in the cavity, and (d) noise introduced by the frequency stabilization circuit. We discuss the prospects for improvements in all these areas for both ground-based and space-based SCSOs, which may lead to SCSOs with fractional frequency stabilities below $10^{-17}$. SCSOs of this frequency stability will be useful for testing fundamental physical principles.

## *I. INTRODUCTION*

High stability clocks or oscillators, as discussed in independent papers by Nordtvedt, Damour, and Haugen in these proceedings, are useful for testing fundamental physical principles. For example, an H-maser in a sub-orbital flight verified the Einstein weak equivalence principle to about 0.01% by measuring the gravitational redshift of a rocket borne H-maser relative to a ground-based H-maser[1].

Superconducting Cavity Stabilized Oscillators (SCSOs) utilizing 8.6 GHz $TM_{010}$ Nb cavities with unloaded quality factors (unloaded Qs) of up to $1 \times 10^{11}$ have achieved fractional frequency stabilities[2] of a few parts in $10^{16}$ for sampling times of about 100 s, making them the most precise clocks in this range of sampling times. An SCSO can be classified as a "ruler" clock since its frequency is proportional to the velocity of light divided by a length, the cavity radius (see Eqn 1). An ensemble of three SCSOs has been used to make a null gravitational redshift measurement relative to an H-maser with a 2% accuracy[3] and to set a limit on the secular drift of the fine structure constant[4]. This paper discusses the possibility of improving both the short-term and the long-term fractional frequency stability of both ground-based and space-based SCSOs to below $10^{-17}$ allowing them to make more precise measurements of fundamental physical principles.

## *II. THE DESCRIPTION OF THE SCSO*

This section describes the SCSO, which was developed[5,6] from 1971 until 1978. The SCSO utilizes a superconducting Nb microwave cavity operating at 1.2 K in the $TM_{010}$ mode, at 8.6 GHz, with its interior under ultrahigh vacuum. To insure mechanical stability the walls of the cavity are of about the same thickness as its 1.3 cm radius. At 1.2 K, these cavities have achieved unloaded Qs of up to $1 \times 10^{11}$ after undergoing ultrahigh vacuum firing and chemical polishing[7].

Figure 1 is a schematic of the cavity and its mounting in a vacuum can that is immersed in pumped liquid helium. The cavity is supported from its top, and it is connected with indium sealed vacuum flanges to the pump-out port and to the microwave waveguide. High vacuum conditions for the cavity are maintained by means of a permanent internal vacuum accomplished with a copper pinch-off and by the exterior vacuum can. The cavity temperature is controlled to 1 μK short term (over times up to 1000 s) and to 10 μK long term (for a fraction of a day and longer) with an active temperature control system, which uses a Ge resistance thermometer (GRT) and a heater. To limit trapped magnetic flux in the superconducting cavity and to reduce the magnetic stress on the cavity, magnetic shields reduce the magnetic field at the cavity to less than 10 mG. Both the dewar and the microwave electronics are tilt controlled to reduce the effect of angular variations with respect to the local gravity. The entire apparatus consisting of three SCSO systems is cooled in a top loading liquid helium dewar.

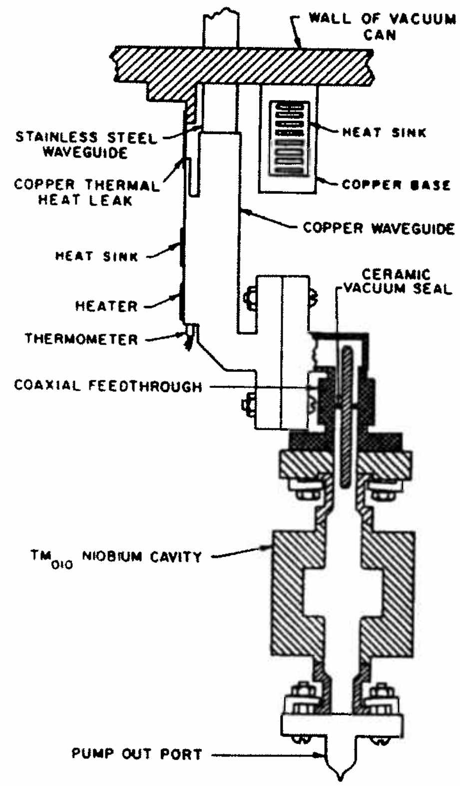


*Figure 1. Schematic of SCSO cavity and its mounting.*

A simplified schematic of the SCSO circuit is shown in Fig. 2. The circuit utilizes the high Q cavity resonance to frequency stabilize a voltage controlled oscillator, a Gunn oscillator. A small portion of the Gunn oscillator power is phase modulated at 1 MHz. The Gunn oscillator is tuned so the carrier of the phase modulated signal is near the cavity resonance. An AM modulated signal is reflected by the cavity and then detected with a square law detector. The sign and amplitude of the detected AM signal represent the deviation of the Gunn oscillator frequency from the cavity frequency. The AM signal, after demodulation, is used to servo the frequency of the Gunn oscillator. The connection of the resonator to the room temperature electronics is made using stainless steel waveguide with copper baffles in order to minimize thermal losses.

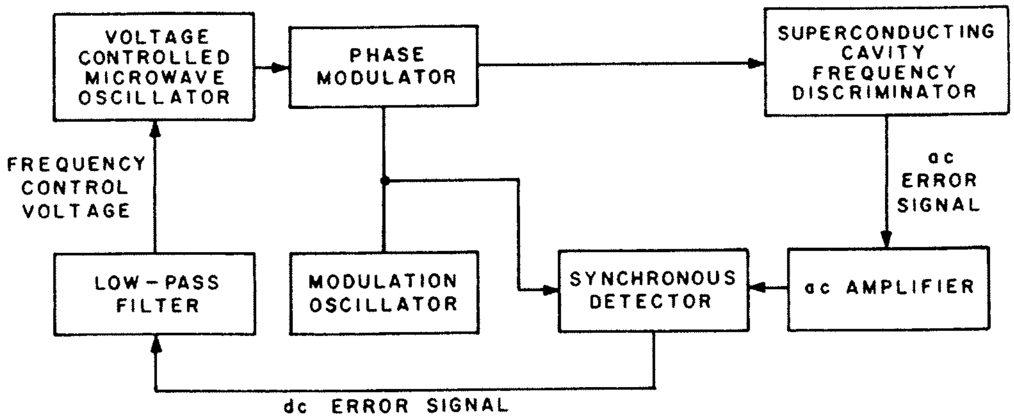


*Figure 2. Simplified block diagram of SCSO*

Figure 3 is a graph of the square root of the Allan variance, $\sigma_y$, as a function of sampling time, $\tau$. The figure contains data from three sources. The circles are data[2] from the ensemble of three SCSOs and the square is a data point at 100 s for SCSO #3 which has the highest unloaded Q ($1 \times 10^{11}$) in the ensemble. This data was taken with a measurement noise bandwidth of 10 Hz. For $\tau < 100$ s, $\sigma_y$ is approximated by $10^{-14}/\tau$. The noise floor of $\sigma_y = 1.3 \times 10^{-16}$ for SCSO #3 is reached at 100 s. For longer sampling times, $\sigma_y$ increases due to long-term frequency drift. The triangles are data derived from the comparison of SCSO #3 with an H-maser[3]. The Allan variance is calculated after the effects of ambient conditions (barometric pressure, earth tides, temperatures) are removed. For sampling times less than 100 s, $\sigma_y$ is limited by the H-maser. The combination of the H-maser and SCSO #3 gives $\sigma_y$ of $1.2 \times 10^{-15}$ for sampling times out to about $10^5$ s. These data indicate that an SCSO with an

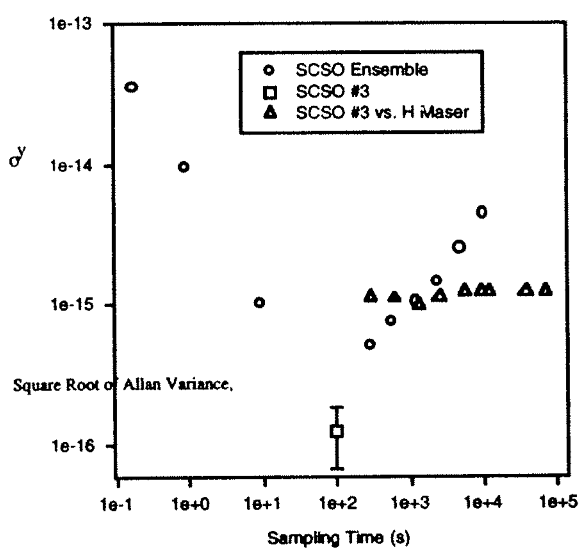


*Figure 3. Allan variance vs. sampling time for an 8.6 GHz SCSO.*

unloaded Q of $1 \times 10^{11}$ has a fractional frequency drift of $\leq 10^{-15}$ per day demonstrating that stress relaxation and creep are not yet evident in the SCSO frequency drift.

## *III. PERFORMANCE OF IMPROVED SCSO*

We consider the strategies for improving both the noise floor and the long-term frequency stability of the SCSOs. The discussion is divided into two sections: stability of the superconducting cavity frequency, and frequency noise introduced by the frequency stabilization circuit. A number of smaller disturbance effects including quantum fluctuations, thermally induced phonons, external pressure variations, motion of flux quanta, particle radiation effects, and low temperature structural changes (creep and stress relaxation) are estimated to cause frequency instabilities below the $10^{-18}$ level, and thus they are not discussed.

### *B. Stability of Cavity Frequency*

The superconducting cavity serves as the frequency determining element of the SCSO. Thus, the cavity frequency and its stability must be well understood and controlled. The cavity is approximately a right circularly cylindrical cavity of radius R and length L, and it is operated in the $TM_{010}$ mode. The frequency, $\nu_o$, of an ideal $TM_{010}$ mode cavity is independent of its length.

$$\nu_o \cong \frac{2.405}{2\pi}\frac{c}{R} \tag{1}$$

The numerical factor is the argument of the zeroth order Bessel function at its first root, and c is the velocity of light. This expression for frequency is only approximately correct since the actual cavity has holes in its end plates to assist in manufacture and to provide a means of coupling to a waveguide. The practical factors that limit the stability of the cavity frequency are discussed below.

**Acceleration**. Vibrations and variations in local gravity will affect the frequency by changing the cavity dimensions. For the vertically mounted $TM_{010}$ mode cavity, the frequency is dependent to first order only on the average cavity radius and by symmetry is only sensitive to second order in the tilt angle from vertical. The frequency sensitivity to changes in gravitational acceleration $\Delta g$ is estimated by assuming that the length of the cavity is stretched by its self weight, and the cavity radius is reduced due to the effect of Poisson's ratio.

$$\frac{\Delta \nu}{\nu_o} \cong -\frac{\Delta R}{R} \cong \frac{1}{3}\cdot\frac{\Delta l}{l} \cong \frac{1}{3}\cdot\frac{l\rho\,(\Delta g)}{Y} \quad\Rightarrow\quad \frac{\Delta \nu}{\nu_o} \cong 4\times 10^{-9}\cdot\frac{\Delta g}{g} \tag{2}$$

where $R$, $l$, $\rho$, and $Y$ are the radius, length, density and Young's modulus of the cavity.

The sensitivity to acceleration variations can be significantly reduced by supporting the cavity from its center, thus through symmetry compensating any contribution to frequency change in the top half with an opposite change in the bottom half. The connection to the waveguide can be made via a choke flange joint, thus leaving the center support as the only mechanical connection to the cavity. The expected improvement from the center support should decrease the frequency sensitivity to acceleration by at least a factor of $10^{-2}$, yielding a sensitivity of $4 \times 10^{-11}$ $\Delta g/g$. For a ground-based SCSO, the principal contribution to $\Delta g$ is due to earth tides, which typically give a peak-to-peak $\Delta g$ of 200 μGal. This yields a fractional frequency variation of $8 \times 10^{-18}$, and a contribution to $\sigma_y$ much less than this value for sampling times much shorter or longer than 6 hours (1/2 of the tidal period). For a space-based SCSO, the principal contribution to $\Delta g$ is drag on the space vehicle. For space vehicle accelerations at the $10^{-7}$ g level, the fractional frequency variations are about $4 \times 10^{-18}$. If the space vehicle were drag compensated, the frequency variations from this source would be much smaller.

**Temperature**. Fluctuations in temperature affect the frequency stability via two main effects on the cavity: thermal expansion and the variation with temperature of the skin depth. The thermal expansion has phonon and electron (superconducting) contributions. The skin depth temperature dependence is described by the BCS theory[8]. One of the authors (Turneaure) has measured the temperature dependence of the 8.6 GHz, TM010 mode Nb cavities, and it can be represented by

$$\frac{\partial(\Delta \nu/\nu_0)}{\partial T} = \frac{1.58 \times 10^{-4}}{T^2} \cdot \exp\left(\frac{-17.12}{T}\right) + 4.32 \times 10^{-11} \cdot T^3 \tag{3a}$$

$$\left.\frac{\partial(\Delta \nu/\nu_0)}{\partial T}\right|_{T=1.2\,K} \cong 1.45 \times 10^{-10}\ K^{-1} \tag{3b}$$

where the first term of Eqn 3a represents the skin depth temperature dependence and the electron thermal expansion, and the second term represents the lattice thermal expansion. In the configuration described in Section II, the short-term temperature stability was 1 μK, and the long-term stability was about 10 μK. Chui and Lipa[9] have used paramagnetic salt thermometers in a four stage thermal isolation system to demonstrate temperature stability better than 1 nK. Their system has also been flown in space as part of the Shuttle Lambda Point Experiment program. We propose to use a simplified version of this system to achieve temperature control to 10 nK or better, therefore reducing the temperature induced frequency fluctuations to approximately the $10^{-18}$ level.

**Stored energy**. Fluctuations in the energy U stored in the cavity will change the electromagnetic radiation pressure on the cavity walls and will vary the non-linear superconducting surface reactance. This variation has been measured by one of the authors (Turneaure), and it can be represented by:

$$\frac{\Delta \nu}{\nu_o} = -(k_{EM} + k_X)U \Rightarrow \frac{\partial(\Delta \nu / \nu_o)}{\partial U} \cong -1.7 \times 10^{-6}\ J^{-1} \tag{4}$$

where the $k$ coefficients quantify the electromagnetic radiation pressure and the non-linear surface reactance. In the configuration described in Section II, the stored energy in the cavity was $6 \times 10^{-8}$ J with a short-term stability on the order of $10^{-3}$, yielding a contribution to $\sigma_y$ of less than $1.0 \times 10^{-16}$. Improved electronics will allow better power control, therefore reducing the frequency instabilities caused by this effect to the $10^{-18}$ level.

### *B. Stabilization Circuit*

Conceptually the scheme for the SCSO electronics will be quite similar to the original design. However, some major changes to the circuit design will be incorporated based on the following principles: (a) use of improved microwave technology, which was not available for the original circuit, (b) a lower modulation frequency of about 10 kHz, (c) a stabilizing power servo, and (d) use of cryogenic components. The Gunn oscillator will be replaced with a varactor-tuned dielectric resonator oscillator (DRO) selected for very low close-in phase noise. A lower modulation frequency will reduce the spurious AM due to frequency dependence of the transmission line wave velocity, attenuation, and spurious reflections. Power stability of better than 0.1% should be easily achievable. We will also investigate the use of a lower noise AM detector, which currently limits $\sigma_y$ to about $10^{-15}/\sqrt{\tau}$ for a cavity stored energy of $6 \times 10^{-8}$ J. We expect to be able to reduce the noise floor of $\sigma_y$ to below $10^{-17}$.

## *IV. CONCLUSIONS*

SCSO clocks have demonstrated a $\sigma_y$ noise floor of $2 \times 10^{-16}$ for a sampling time of 100 s and $\sigma_y$ of $1.2 \times 10^{-15}$ for sampling times out to ~ $10^5$ s. The main disturbance effects are due to noise in the electronics and long-term drift caused by fluctuations in the temperature, the local gravity, and the electromagnetic energy stored in the resonator. Proposed improvements show the promise of achieving fractional frequency stabilities below $10^{-17}$ for ground-based and space-based experiments.

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*REFERENCES*


[1] R. F. C. Vessot and M. W. Levine, Phys. Rev. Lett. **45**, 2081 (1980).
[2] S. R. Stein and J. P. Turneaure, in AIP Conference Proc. No. 44, Hugh C. Wolfe, Series ed. (AIP, N.Y., 1978) pp. 192-213.
[3] J. P Turneaure, C. M. Will, B. F. Farrel, E. M. Mattison, and R. F. C. Vessot, Phys. Rev. D **27**, 1705 (1983)
[4] J. P. Turneaure and S. R. Stein, in *Atomic Masses and Fundamental Constants* 5, J. H. Sanders and A. H. Wapstra, eds. (Plenum, N.Y., 1976) pp. 636-42.
[5] S. R. Stein and J. P. Turneaure, Electronic Letters **8**, 321 (1972).
[6] S.R. Stein and J.P. Turneaure, Proc. IEEE **63**, 1249 (1975)
[7] J. P. Turneaure and N. T. Viet, Appl. Phys. Letters **16**, 333 (1970).
[8] J. P. Turneaure, J. Halbritter, and H. A. Schwettman, J.Superconductivity **4**, 341 (1991).
[9] J. A. Lipa, D. R. Swanson, J. A. Nissen, and T. C. P. Chui, Cryogenics **34**, 341 (1994).