

Principles of groundwater flow

Hydraulic head is the elevation to which water will naturally rise in a well (a.k.a. "static level"). Any well that is not being pumped will do for this, but a well that is created especially for measuring hydraulic head is known as an **observation well**, a **monitoring well** or a **piezometer**. For unconfined aquifers the hydraulic head is the same thing as the water table. For confined aquifers it is not.

Observation wells and piezometers

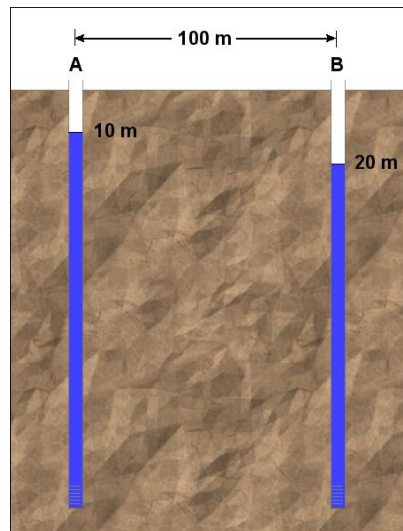
In Chapter 4 Fetter tries to confuse us by expressing head as:

$$h = z + h_p$$

where h is the hydraulic head, z is the point of measurement in the aquifer and h_p is the height of the water column above that point. In fact it's pretty obvious that you have to measure head above a consistent datum (usually sea level), so in fact head (h) is really just the elevation (above sea level) of the standing water level in a well.

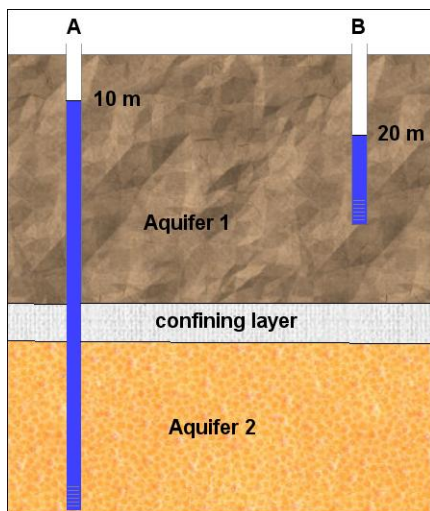
Hydraulic head

The hydraulic gradient between any two points is the slope of the hydraulic head between those points. For the diagram to the right the difference in head between the two wells is 10 m, and the horizontal distance is 100 m, so the hydraulic gradient is $10/100 = 0.1$



Hydraulic gradient

Hydraulic gradient is always expressed as a fraction (e.g. 0.1), not as a per cent, or as an angle.



For the hydraulic gradient to have any meaning the piezometers must be in the same aquifer.

The wells depicted in the diagram to the left cannot be used to estimate hydraulic gradient because they are situated in two different aquifers with a confining layer in between.

Hydraulic gradients refer to single aquifers

The water level in B is the water table at that location, while the water level in A is the **potentiometric surface** for Aquifer 2, at that location.

Hydraulic gradient in two dimensions

Water table or potentiometric surface levels determined from a number of observation wells spaced out over an area can be used to characterize the hydraulic gradient in the two horizontal directions. If we just want to get a linear hydraulic gradient we can use three wells and triangulate as shown on Figure 3.30 in Fetter. If we want to create a more complex surface from a larger number of wells, then we would contour the surface, as in Figure 3.31.

In fact the elevation of the measurement point, (z), (e.g., the elevation of the screen in a well) does become relevant if we want to use a nested set of piezometers (within a metre or two of each other) to estimate the vertical gradient between the various levels.

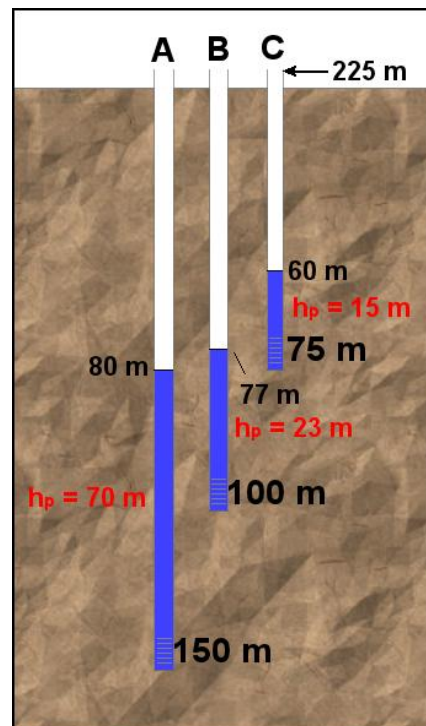
The scenario is illustrated to the right. (The drawing is not to scale. Assume that the three wells are very close together).

Well	z (m)	h_p (m)	h (m)
A	75	70	145
B	125	23	148
C	150	15	165

See the bottom of page 117 in Fetter for a description of the problem. The vertical hydraulic gradients are proportional to the head differences versus the elevation differences between the three piezometers.

The head difference between A and B is 3 m (148-145), while the elevation difference is 50 m (150-100), so the vertical hydraulic gradient is $3/50=0.06$.

The head difference between b and C is 17 m and the elevation difference is 25, so the vertical hydraulic gradient is $17/25=0.68$. The shallow piezometer has a higher head than the deeper ones, so we can conclude that the hydraulic gradient is downward at this location.



Vertical hydraulic gradient

Darcy's law

Darcy's law allows us to estimate groundwater flow rate if we know (or can estimate) the hydraulic gradient and the permeability. In its simplest form Darcy's law is:

$$V = Ki \quad \text{or} \quad V = K \frac{dh}{dl}$$

Where V is the flow velocity, K is the permeability and i is the hydraulic

Darcy's law

gradient (i is the same thing as dh/dl , height difference over length difference). Fetter goes on at some length about the inapplicability of Darcy's law under conditions of turbulent flow, but then finally concludes with the statement that: "Under most natural ground-water conditions the velocity is sufficiently low for Darcy's law to be valid." We'll stick with that.

If we want to use Darcy to estimate flow volume then we would use the equation:

$$Q = AKi \quad \text{or} \quad Q = AK \, dh/dl$$

Where Q is the discharge and A is the cross-sectional area of the aquifer in question.

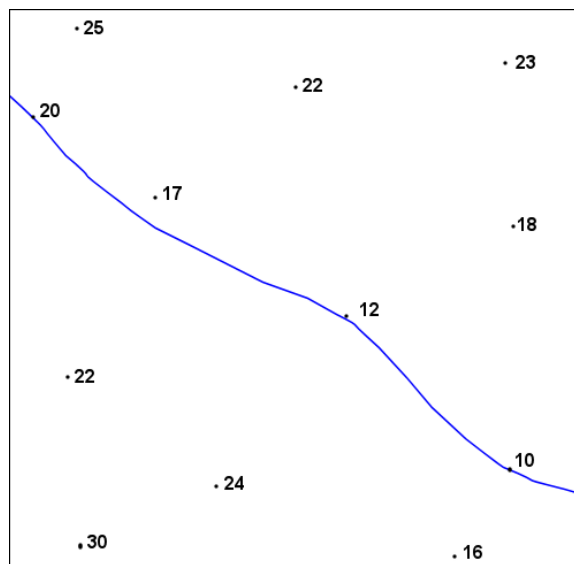
Groundwater will flow down the hydraulic gradient. When we look at flow over the area of an aquifer we are typically interested in two-dimensional flow. In most cases the X and Y dimensions (100s to 10,000s of m) are much greater than the Z dimension (depth: m to 10s of m), and we tend to ignore the vertical component of the flow. When we contour the water table surface on a map, and then draw in flow lines, we are only considering the horizontal component of the flow.

Of course there is a vertical component to groundwater flow and to visualize it we need to look at the problem in cross section.

Flow nets

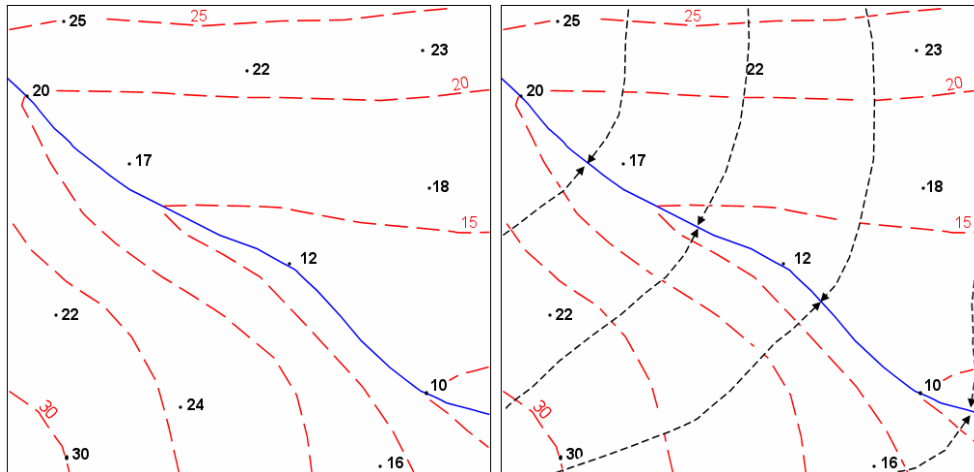
A flow net is a diagrammatic representation of the groundwater flow within an area – either in plan view (map) or cross-section.

Consider the map area shown to the right where the various numbers represent elevation of the water table, either from observation wells or from measurements of the creek elevation.



Flow nets

We can contour the water-table surface as shown below, and then construct flow lines, which are drawn perpendicular to the contours.



Equipotential lines and flow lines in plan view

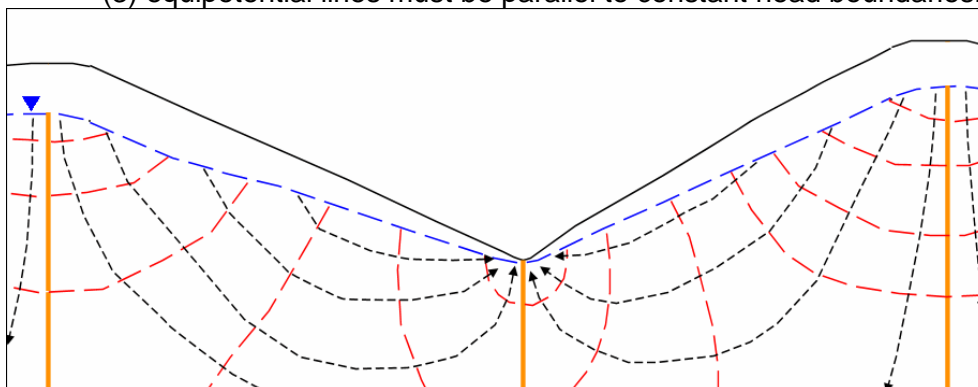
This gives us some idea of directions in which the groundwater is likely to be flowing, not surprisingly – towards the creek, where it is contributing to the baseflow. The water table contour lines are lines of constant hydraulic head. Lines of constant head are also known as **equipotential lines**.

A flow net for cross-section from the SW to the NE corners of this area is depicted in the following diagram. As for the plan-view diagrams, the equipotential lines are dashed-red and flow lines are dotted-black. The water table is the dashed blue line. The heavy orange lines are no-flow boundaries situated at the two drainage basin divides and at the creek. Groundwater does not flow across these boundaries, at least not the relatively shallow groundwater depicted here.

There are some rules for constructing flow nets as follows:

- (1) flow lines and equipotential lines must cross at right angles,
- (2) equipotential lines must meet impermeable boundaries and other no-flow boundaries at right angles
- (3) equipotential lines must be parallel to constant head boundaries.

Rules for constructing flow nets



Equipotential lines and flow lines in cross section

Not all groundwater boundaries are no-flow boundaries. There are four types of boundaries possible:

- 1) No-flow boundaries. These include boundaries between aquifers and “impermeable layers”, and drainage divide boundaries as shown

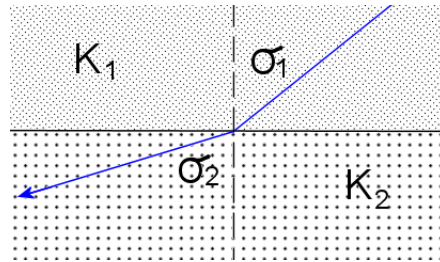
Types of groundwater boundaries

- above. Flow is always parallel to no-flow boundaries.
- 2) Constant-head boundaries, which are the same as equipotential lines. Flow is always perpendicular to constant head boundaries.
 - 3) Water-table boundaries. The water table is a surface of varying head along which the head is known. Flow is normally at an oblique angle to a water-table boundary.
 - 4) Hydraulic conductivity boundaries. These are boundaries between two layers of differing permeability. Flow paths change direction at conductivity boundaries.

The change in direction of a flow path at a hydraulic conductivity boundary can be determined from the formula:

$$K_1/K_2 = \tan\sigma_1 / \tan\sigma_2$$

Where the symbols are defined as shown to the right.



Flow-line refraction at a conductivity boundary

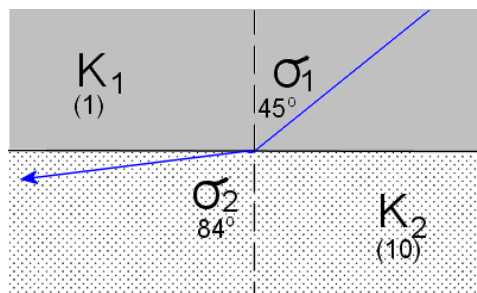
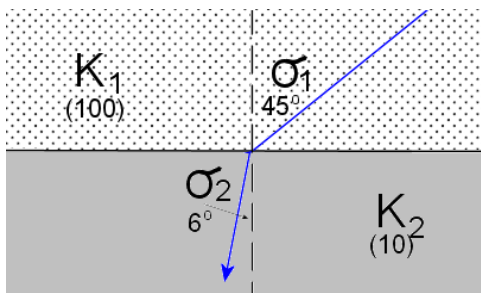
Typically σ_1 will be known and the two K values will be known, so it makes sense to rearrange the formula as follows:

$$\tan\sigma_2 = \tan\sigma_1 / (K_1/K_2)$$

Two examples are given in the table below.

If the conductivity of the second layer is less than that for first layer, the flow lines will steepen. If the conductivity in the second layer is greater than in the first, the flow lines will flatten.

K_1	K_2	σ_1	σ_2
100	10	45	5.7
1	10	45	84.3



Flow-lines for high to low conductivity boundaries and for low to high conductivity boundaries

We will do a number of manual flow net exercises in class, but we will also be using some software to help us understand groundwater flow behaviour and rates. The software is Visual Modflow, which is provided on the CD in Fetter.

Modflow and Visual Modflow

Visual Modflow, which was developed by Waterloo Hydrogeologic in Waterloo Ontario, is an industry-standard program for studying groundwater flow. The version that we have is a student one, not the professional

version, but it will do for our purposes, and learning how to use it will be good experience if you ever do work in Hydrogeology.

Visual Modflow is actually just a “front end” program for the program Modflow, which was developed by the U.S. Geological Survey.