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# "Front" Description in Relativistic Quantum Mechanics

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The problem of introducing a Cartesian position operator canonically conjugate to the momentum operator into relativistic one-particle theories is investigated independent of any particular relativistic wave equation. The known result that such a description is possible for particles with nonvanishing mass is rederived. The general problem of introduction of canonical variables into relativistic theories is formulated and solved. The configurational indices so obtained correspond to directed plane wavefronts rather than point particles.

## 1. INTRODUCTION

SPINNING particles are associated with covariant differential equations within the framework of relativistic quantum mechanics.[1] Thus for spin 1/2 particles one has the Dirac equation, for spin 0 particles one has the Klein-Gordon or Kemmer equation; and for the photon one has the Maxwell equation. To demonstrate that these relativistic "wave equations" do describe relativistic quantum-mechanical particles it is necessary to carry out a particle interpretation of these equations.[2] But in most of these particle interpretations there are puzzling features associated with the configurational space descriptions; the appearance of such puzzling features is perhaps best known in the case of the Dirac equation, but they are nevertheless present in the case of the other equations as well.

These paradoxes always arise from an incorrect identification of the covariant amplitude entering the relativistic equation with the Schrödinger amplitude describing the particle. While the covariant form of the equations exhibit the Lorentz invariance of the theory, it is necessary to be able to reduce the covariant equation to the canonical form involving the Schrödinger amplitude, since it is this amplitude which undergoes the unitary transformations under the various operations belonging to the proper inhomogeneous Lorentz group[3]; and this unitary transformation property is basic[4] to a quantum theory with an underlying Hilbert space (the scalar products in which are to be relativistic invariants). In the case of the Dirac equation this reduction from the covariant amplitude to the Schrödinger amplitude is accomplished by the Foldy-Wouthuysen-Tani transformation.[5] In a subsequent paper Foldy has carried through this reduction in a simple manner for the spin 0 and spin 1 equations also. In all these "reduced" forms solutions with both positive and negative frequencies appear on a symmetrical footing and this is a consequence of the covariance of the relativistic equations with respect to the complex Lorentz group.[6]

Two points are to be noted. First of all, the starting point here is a covariant wave equation and one might ask the question as to whether the final results depend on the particular type of wave equation one started with; this is especially important for higher spin equations. We shall show that the position operators are independent of the choice of the relativistic equation and are very simply related to the structure of the unitary irreducible representations of the Lorentz group. Secondly, the standard methods fail for particles with vanishing mass and finite nonzero spin; it is known, for example, that the photon cannot have a localized state in the sense of being an eigenfunction of the three components of the vector position operator.[8] It is the purpose of this paper to investigate what is the maximal configuration specification that one can provide in such a case; it will turn out that the maximal specification corresponds to a "front" form, i.e., the basic elements correspond to directed planes rather than to points. Here again, one starts with the irreducible representations to which the configuration indices are directly related.

## 2. CANONICAL VARIABLES FOR A RELATIVISTIC PARTICLE

In classical mechanics, with each degree of freedom one associates a pair of dynamical variables $p$, $q$ which

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[1] See, for example, S. S. Schweber, H. A. Bethe, and F. DeHoffman, *Mesons and Fields* (Row, Peterson and Company, New York, 1955) Vol. 1; E. M. Corson, *Tensors, Spinors and Relativistic Wave Equations* (Blackie and Sons, Limited, London, England, 1953).

[2] P. A. M. Dirac, Proc. Roy. Soc. (London) **A155**, 447 (1936); N. Kemmer, *ibid.* **A173**, 91 (1939); R. H. Good, Jr., Phys. Rev. **105**, 1914 (1957).

[3] C. Fronsdal, Phys. Rev. **113**, 1367 (1959).

[4] E. P. Wigner, Ann. Math. **40**, 149 (1939).

[5] L. L. Foldy and S. A. Wouthuysen, Phys. Rev. **78**, 29 (1950); S. Tani, Progr. Theoret. Phys. Kyoto **6**, 267 (1951).

[6] This is an example of the fact that requiring a quantum-mechanical system be describable by "Euclidean" differential equations (i.e., covariant equations not involving the sign of the time components of timelike four-vectors) imposes further restrictions on the system going beyond relativistic invariance alone. All the relativistic wave equations that are usually studied are Euclidean equations and these equations hence remain invariant under the complex Lorentz group. On the other hand, the physical requirement of relativistic invariance does not demand Euclidean invariance. In fact, Euclidean equations cannot be written down for a system with only positive energies; and the appearance of an arbitrary but fixed time direction in every realization of the "particle type" representations of the inhomogeneous Lorentz group is no accident. Similar ideas have been discussed previously[7] by Schwinger and by Nakano.

[7] J. Schwinger, Proc. Natl. Acad. Sci. U. S. **44**, 956 (1958); T. Nakano, Progr. Theoret. Phys. Kyoto **21**, 241 (1959).

[8] T. D. Newton and E. P. Wigner: Revs. Modern Phys. **21**, 400 (1949).

obey the fundamental Poisson bracket relations

$$\{p_i,q_j\}=\delta_{ij},\quad \{p_i,p_j\}=\{q_i,q_j\}=0 \tag{1}$$

at any instant of time, the indices referring to the various degrees of freedom. A triplet of such pairs of canonical variables is associated with a "particle" (without spin), provided the three momenta $p_i$ and the three coordinates $q_i$ transform as the three components of a vector under rotations. Under such transformations the Poisson brackets are preserved and therefore rotations are canonical transformations. For the simplest kind of particles the dynamical variables $J_i$ which constitute the three components of the angular momentum pseudovector are algebraically related to the momenta and coordinates in the form

$$J_i=\epsilon_{ijk}q_jp_k=L_i. \tag{2}$$

It may be, however, that the angular momentum is not equal to this expression but is of the form

$$J_i=L_i+S_i. \tag{3}$$

Then we say that the particle is spinning and the three components $S_i$ constitute the pseudovector spin operator. Since the angular momentum variable is also the infinitesimal generator of rotations, using the expression (1), (2) we obtain

$$\{S_i,p_j\}=\{S_i,q_j\}=0. \tag{4}$$

On using this result together with the fact that the angular momentum operator $J$ is a pseudovector, one obtains the further result

$$\{S_i,S_j\}=\epsilon_{ijk}S_k. \tag{5}$$

Thus the spin variables do not belong to a canonical set, nor are they expressible in terms of the canonical momenta and coordinates.[9] For a "free" spinning particle, the symmetry with respect to translations and rotations in three-dimensional space, i.e., the inhomogenous Euclidean group, requires the momenta $p_i$ and the total angular momenta $J_i$ to be constants of motion. In addition, if the Hamiltonian is independent of spin variables, the spin variables $S_i$ will also be constants of motion. Notice that invariance under the inhomogeneous Euclidean group does not prevent the pseudoscalar variable $S_ip_i$ from entering the Hamiltonian; if it does, then the spin vector is no longer a constant of motion, but the "longitudinal spin" or helicity

$$h=\frac{1}{|p|}S_ip_i=\frac{1}{|p|}J_ip_i \tag{6}$$

is a constant of motion; consequently any supplementary condition involving a specific value of $h$ is preserved in time.

Turning to quantum mechanics, the "particle" is now associated with the same set of dynamical variables which are now represented by noncommuting operators. The Poisson brackets are to be related to commutators and by virtue of the relations (5) and (6) the spin and helicity become quantized; however, the important point to notice is that an elementary quantum-mechanical system is associated with irreducible representations of the inhomogeneous Euclidean group in three dimensions and *not* canonical operators. The Euclidean group consists of the six generators of translations and rotations which obey the commutation relations

$$\begin{aligned}[][T_i,T_j]&=0,\\ [R_i,R_j]&=i\epsilon_{ijk}R_k,\\ [R_i,T_j]&=i\epsilon_{ijk}T_k.\end{aligned} \tag{7}$$

The two operators

$$T^2=T_iT_i;\quad T\cdot R=R\cdot T=T_iR_i \tag{8}$$

commute with all the six generators $T_i$, $R_i$ and are therefore represented by numbers in every irreducible representation, the first number being nonnegative. Coordinate or spin operators cannot be defined over these irreducible representations, since the translation generators are identical with the momentum operators so that $q_i$, for example, would not commute with either of the quantities $T^2$ or $T\cdot R$.

Such a mixing of the various irreducible representations is already brought about by the requirement of relativistic invariance. The irreducible unitary representations of the inhomogeneous Lorentz group have been investigated by Wigner,[4,10] and he has found several classes of representation. We shall be particularly interested in three such classes: class I corresponds to particles with finite mass and finite spin; class II to particles with zero mass and finite spin; and class III to particles with imaginary mass and zero spin.[11] In the first case the manifold of states corresponds to the irreducible representations of the Euclidean group with $\infty>T^2\geqslant0$ and $h=-s,\ -s+1,\ \cdots s$ where $h=(T^2)^{-\frac{1}{2}}T\cdot R$ and $2s$ is a nonnegative integer; the second class has those with $\infty>T^2\geqslant0$ and $h=s$ with $2|s|$ an integer; the third class has $\infty>T^2\geqslant|m^2|>0$ and $h=0$. In all cases the energy is real and nonnegative. The Schrödinger amplitude may hence be written[10] as $\psi(\mathbf{p},\zeta)$ with $\mathbf{p}$ and $\zeta$ corresponding to the three components of momentum and the single helicity index. The Schrödinger equation becomes

$$i\frac{\partial}{\partial t}\psi(\mathbf{p},\zeta)=+(p^2+m^2)^{\frac{1}{2}}\psi(\mathbf{p},\zeta), \tag{9}$$

[9] We mean by "canonical variables" what Schwinger calls "canonical variables of the first kind"; compare J. Schwinger, *Handbuch der Physik* (to be published).

[10] V. Bargmann and E. P. Wigner, Proc. Natl. Acad. Sci. U. S. 34, 211 (1948).

[11] E. C. G. Sudarshan and V. K. Deshpande (to be published).

and the unitary scalar product

$$(\psi,\phi)=\sum_{\xi}\int d^3p\psi^*(\mathbf{p},\zeta)\phi(\mathbf{p},\zeta). \tag{10}$$

From (9) one can see that the energy is positive definite.

### 3. DIRAC EQUATION

Let us now consider the simplest relativistic equation, namely, the Dirac equation which represents particles of spin $\frac{1}{2}$. The covariant differential equation

$$[\gamma^\mu(\partial/\partial x^\mu)+im]\psi=0 \tag{11}$$

can be reduced to the pseudo-Hamiltonian form

$$i\partial\psi/\partial t=(\boldsymbol{\alpha}\cdot\mathbf{p}+\beta m)\psi \tag{12}$$

by multiplication by $i\gamma^0$. It is well known[12] that if one tries to identify $\psi(\mathbf{x},t)$ with a Schrödinger amplitude, so that $\mathbf{x}$ is the representative of the position, then the representative of the velocity is the matrix $\boldsymbol{\alpha}$. This has the consequence that the components of the "velocity" do not commute with each other; and the eigenvalue of any component of the "velocity" is $\pm 1$ (in units of the velocity of light) and, further, the velocity and sign of the energy cannot be simultaneously diagonalized. The identification of the covariant amplitude $\psi$ as a Schrödinger amplitude appeared to be sanctioned by the positive definiteness of the probability density $\psi^*\psi$. Rather than reject the identification of $\psi(x)$ with the Schrödinger amplitude, these unusual features were attributed to be a mysterious feature of relativistic equations. Another such feature was in the lack of time independence of the "orbital angular momentum" $\mathbf{x}\times\mathbf{p}$ for the *free* particle.

The correct position operator and localized amplitudes were worked out by various people[13] but the correct identification of the Schrödinger amplitude was made by Foldy and Wouthuysen[5] who showed that the amplitude

$$\varphi(\mathbf{x},t)=-i\exp\left\{\frac{\beta\boldsymbol{\alpha}\cdot\mathbf{p}}{2p}\tan^{-1}\left(\frac{p}{m}\right)\right\}\psi(\mathbf{x},t) \tag{13}$$

satisfies the Schrödinger-like equation

$$i\frac{\partial\varphi}{\partial t}=\beta(p^2+m^2)^{\frac{1}{2}}\varphi. \tag{14}$$

For $\varphi(\mathbf{x},t)$ the standard identification of position and orbital angular momentum operators[14] leads to no unusual features. Notice that according to (13), $\varphi$ is unitarily related to $\psi$ so that the probability density is unaltered; but the probability current is altered in going from $\psi$ to $\varphi$. This alteration is brought about by dropping all terms which mixed positive and negative energies. We also notice that all the identifications of the dynamical variables of the particle commute with the operator $\beta$ for the sign of the energy. Hence the proper Schrödinger equation is obtained by restricting $\varphi$ in (14) to contain only positive energy solutions: we would then get the true Schrödinger equation

$$i\partial\Phi/\partial t=+(p^2+m^2)^{\frac{1}{2}}\Phi, \tag{15}$$

where $\Phi$ is a two-component amplitude.

With this formulation of the theory we find that the position operator $\mathbf{q}$ has the representative $\mathbf{x}$. In other words, we have a Cartesian position operator with commuting components.[15] Foldy has shown[16] that the Klein-Gordon and Proca fields also can be reduced to the forms (14) and (15) in an analogous manner, so that we can define Cartesian position operators for these systems also. In passing, we also notice that the position operators are canonically conjugate to the momentum operators

$$[x_r,p_s]=i\delta_{rs}. \tag{16}$$

In contrast to the representation introduced by Foldy-Wouthuysen and by Tani (the $C$ representation), another representation (called the $E$ representation) may be introduced[17] in which the analog of (15) is given by

$$i\partial\varphi_E/\partial t=(\boldsymbol{\alpha}\cdot\mathbf{p}/p)(p^2+m^2)^{\frac{1}{2}}\varphi_E. \tag{17}$$

In connection with this amplitude $\varphi_E$ a new $E$-position operator was also introduced [which was defined on positive and negative energy solutions of (17) separately] which had several new features. The components of the $E$-position operator did not commute, but Eq. (16) was satisfied. The "longitudinal component" of the $E$-position operator

$$\mathbf{x}_E^{\text{long.}}=\tfrac{1}{2}(\mathbf{x}_E\cdot\hat{p}+\hat{p}\cdot\mathbf{x}_E)\hat{p}, \tag{18}$$

where

$$\hat{p}=\mathbf{p}/p \tag{19}$$

was identical with the longitudinal component of the $C$-position operator; but the transverse parts were not identical. Hence the $E$-position operator could not be used[18] to specify the "localization indices," i.e., a set

[12] P. A. M. Dirac, *Principles of Quantum Mechanics* (Oxford University Press, New York, 1947), 3rd ed.

[13] See, for instance, M. H. L. Pryce, Proc. Roy. Soc. (London) **A195**, 62 (1948).

[14] Foldy and Wouthuysen distinguish these proper identifications by the prefix "mean"; we prefer to omit this prefix since the Dirac position operator (whose representative is the operation of multiplication of the covariant amplitude by $x$), for example, is *not* an operator defined over the states of the particle (since it mixes the positive energy solutions with negative energy solutions). Of course both the $C$-position operator as well as the $E$-position operator introduced in the following are defined over the (positive energy) states of the particle.

[15] Of course, any unitary transform $q=U(p^2)\mathbf{q}U^{-1}(p^2)$ is also a Cartesian position operator and $U$ can be a function of $p^2$ only if the polar vector transformation property of $\mathbf{q}$ is to be preserved.

[16] L. L. Foldy, Phys. Rev. **102**, 568 (1956). See also K. M. Case, *ibid.* **95**, 1323 (1954).

[17] M. Cini and B. Touschek, Nuovo cimento **1**, 422 (1958); S. K. Bose, A. Gamba, and E. C. G. Sudarshan, Phys. Rev. **113**, 1661 (1959).

[18] The "remedy" suggested by Y. Pac, Progr. Theoret. Phys. Kyoto **22**, 857 (1959), is incorrect since the "mean $E$-position

of three numbers which could be used to specify a coordinate system in Hilbert space.[19] But the $E$-position operator has the nice feature that the $E$-orbital angular momentum is the transverse part of the total angular momentum; and furthermore the $E$-position operator was defined for a "two-component" neutrino (i.e., a zero mass, spin $\frac{1}{2}$ particle with only one helicity) for which the $C$-position operator cannot be defined.

Since $\mathbf{x}_E$ satisfies (16) it follows that

$$[\mathbf{x}_E{}^{\text{long}\cdot},\hat{p}]=0. \tag{20}$$

Hence these quantities constitute configuration indices; these are only three independent indices since the unit vector $\hat{p}$ is completely specified by two angles and together with these $\mathbf{x}_E{}^{\text{long}\cdot}$ provide only a distance

$$\begin{aligned}\rho&=\tfrac{1}{2}(\mathbf{x}_E\cdot\hat{p}+\hat{p}\cdot\mathbf{x}_E),\\ &=\tfrac{1}{2}(\mathbf{x}\cdot\hat{p}+\hat{p}\cdot\mathbf{x}).\end{aligned} \tag{21}$$

The configurational description provided by the set $(\rho,\hat{p})$ is that of a directed wavefront for which $\hat{p}$ indicates the unit normal and $\rho$ is the perpendicular distance from an arbitrarily chosen origin. The wavefront for a free particle advances normal to itself and the speed of advance

$$v=\partial\rho/\partial t=p/(p^2+m^2)^{\frac{1}{2}} \tag{22}$$

is the speed of a particle of momentum $\mathbf{p}$. Since the values of $(\rho,\hat{p})$ are continuous, the eigenstates are not normalizable but are the limits of normalizable functions. If by $|\rho',\theta,\varphi\rangle$ we represent such an "ideal" state, a normalizable state is given by

$$|f\rangle=f(\rho',\theta,\varphi)|\rho',\theta,\varphi\rangle, \tag{23}$$

with

$$\begin{aligned}\langle f|f\rangle&\equiv\int f^*(\rho',\theta,\varphi)f(\rho',\theta,\varphi)\rho'^2d\rho'\,\sin\theta d\theta d\varphi.\\ &=1.\end{aligned} \tag{24}$$

Since these considerations hold for finite mass and zero mass particles (including the two-component neutrino) one expects them to be of more general validity than the Cartesian configuration description. By a reduction similar to the one employed by Foldy[16] we can demonstrate this result for the Klein-Gordon, Proca, and Maxwell equations, but nothing essentially new is obtained in this fashion. Instead we demonstrate the generality of the "front" form of configurational description by relating the configuration indices $(\rho,\hat{p})$ to the Lorentz group.

It is perhaps appropriate to point out that the choice from among a set of unitarily equivalent position operators[15] is an arbitrary one and is equivalent to the assignment of a specific law for the interaction of the system with prescribed external fields which are "known" to be "localized" in a suitable manner.

## 4. CONFIGURATIONAL INDICES AND THE LORENTZ GROUP

We have remarked (in Sec. 2) that the proper Lorentz transformations already mix the irreducible representations of the three-dimensional inhomogeneous Euclidean group. For class I particles the manifold[4,10] responds to all possible real values for the three momentum components. Hence in this case the operator $Q_r=i(\partial/\partial P_r)$ which differentiates the momentum amplitude can be defined. For class II particles there is the helicity restriction so that one cannot differentiate freely with respect to the three components but the operator

$$Q^l=\tfrac{1}{2}[(P_rP_r)^{-\frac{1}{2}}P_sQ_s+Q_s(P_rP_r)^{-\frac{1}{2}}P_s] \tag{25}$$

is defined. Finally for class III particles, since the momentum spectrum excludes a sphere of radius $(-m^2)^{\frac{1}{2}}$, not even $Q^l$ is defined.

Let us now construct the generalized $E$-position operator $\mathbf{q}_E$. For this purpose introduce the operator

$$\mathbf{Q}^{\text{tr}}=(P_rP_r)^{-1}\mathbf{P}\times\mathbf{J} \tag{26}$$

and write

$$\mathbf{q}_E=\mathbf{Q}^{\text{tr}}+Q^l(P_rP_r)^{-\frac{1}{2}}\mathbf{P}. \tag{27}$$

By direct computation one can verify that in the case of the Dirac equation $\mathbf{q}_E$ so defined coincides with the $E$-position operator. We also notice that for a nonrelativistic spinning particle, for which $\mathbf{J}=\mathbf{L}+\mathbf{S}$,

$$\mathbf{q}_E=\mathbf{x}+(P_rP_r)^{-1}\mathbf{P}\times\mathbf{S}, \tag{28}$$

which shows that this operator differs from the standard position operator by "spin" contributions. In the general case the two terms on the right-hand side may not be separately defined[19] but $\mathbf{q}_E$ is defined in all cases. In complete analogy with the Dirac particle case, the components of $\mathbf{q}_E$ do not commute and one has

$$[q_r{}^E,q_s{}^E]=i\epsilon_{rst}\frac{\mathbf{J}\cdot\mathbf{P}}{(P_rP_r)^2}P_i=i\epsilon_{rsi}\frac{\mathbf{S}\cdot\mathbf{P}}{(P_rP_r)^2}P_i \tag{29}$$

so that the lack of commutation is a "spin" effect. Finally the generalized $E$-orbital angular momentum

$$\mathbf{q}_E\times\mathbf{P}=\mathbf{J}-\mathbf{J}\cdot\hat{P}\hat{P} \tag{30}$$

is equal to the transverse part of the total angular momentum. All these operators are defined for both class I and class II particles. Thus the $E$-position operator is a more "natural" position operator.[11]

However, for spinning particles $\mathbf{p}_E$ cannot be used as a set of configuration indices, but we can proceed in exactly the same fashion as in the previous section and

operator" introduced there, namely, $\mathbf{x}-(1/2p^2)(\boldsymbol{\sigma}\times\mathbf{p})$ does not commute with the sign of the energy of the $E$ representation and hence is not defined on positive and negative energy states separately.

[19] It is well known that the transverse part of the "spin" is not gauge-invariant in the case of the photon.

introduce the "front" form[20] with

$$\rho=Q^l=\tfrac{1}{2}(\hat{P}\cdot\mathbf{Q}+\mathbf{Q}\cdot\hat{P}). \tag{31}$$

Thus a uniform description of all observed particles (belonging to class I and class II) is made possible in the "front" form.

The question now arises as to when one can introduce a "point" form in terms of a Cartesian position operator in analogy to the $C$-position operator. From the foregoing demonstration it follows that this is equivalent to the possibility of separating out the spin part [compare Eq. (28)]. For this purpose it suffices to be able to define the pure orbital angular momentum $\mathbf{L}=\mathbf{J}-\mathbf{S}$; this can always be done if $Q_r$ can be defined since $\mathbf{L}=\mathbf{Q}\times\mathbf{P}$. Hence the "spin" can be separated out and the Cartesian configuration indices introduced in the case of class I particles. Thus for class I particles we can use either the "front" form or the "point" form of configurational description but for class II particles only the "front" form is possible. For either class of particles, in the "front" form the helicity may be used along with the configurational indices for a complete specification of the state.

## 5. DISCUSSION

The development in the previous sections shows that if we are willing to identify the simplest relativistic quantum-mechanical entities with relativistic particles, at least for spinning "particles" of zero mass, the notion of position does exhibit unusual features. Notice that this unfamiliar nature has nothing to do directly with the vanishing mass (and consequent infinite Compton wavelength) since for zero mass particles without spin, one can define localized states; and one could do this for a Dirac particle with vanishing mass. It is curious that such particles do not seem to exist and that the two known particles of zero mass, namely, the neutrino and the photon, do not permit configurational description in the "point" form.

It is of particular significance to note that the photon does not admit a "point" description and hence the statement that a photon is at any specific point at a definite instant is meaningless. And the notion of "signal" propagation and signal velocity in relativistic quantum mechanics is much more subtle than has been generally acknowledged. We also notice that while special relativity requires the invariance of dynamical laws under change of the Lorentz frame, physical interpretation requires the choice of a definite (arbitrary, but fixed) time direction. The specification of the proper quantum-mechanical state and more generally the description of a sequence of quantum-mechanical phenomena is then dependent upon this chosen time direction. This chosen time direction enters in a natural fashion into the realization of the representations of the Lorentz group by (infinite-dimensional) unitary matrices. In the physically interesting cases of free particles considerable ingenuity has gone into the construction of covariant relativistic equations to represent free particles of class I and class II. The elegance of formal manifest covariance is thus an irrelevant feature. The question naturally arises whether one can demand that interacting particles be represented by local manifestly covariant differential equations. If there is any *fundamental* reason for such a requirement, the present authors are unaware of it; and the analysis of the quantum-mechanical description initiated in the foregoing sections casts doubts on the existence of any such reason.

From our point of view the choice of the time direction is a necessary prerequisite to any attempt at physical description; and thus the generalization of the notion of localized states to class II particles given by Fronsdal[3] is unacceptable within this framework.

In this paper we have confined our attention to free particles only. The study of interacting relativistic particles is best done in relation to specific relativistic equations. The systematic analysis along these lines is to be presented in another paper in collaboration with K. Bardakci.

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[20] The definitions and notions introduced here are different in principle from those of Dirac. See P. A. M. Dirac, Revs. Modern Phys. **21**, 392 (1949).