Dot & Cross Product
properties

$a \cdot b = |a||b|\cos\theta$

1. Use $\boxed{\text{Theorem 3}}$ to prove the Cauchy-Schwartz Inequality: $|\mathbf{a} \cdot \mathbf{b}| \leq |\mathbf{a}||\mathbf{b}|$

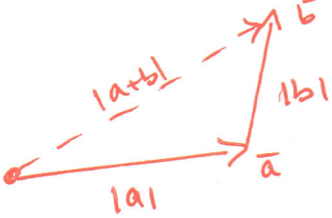
$$|a \cdot b| = |a||b||\cos\theta| \leq |a||b| \cdot 1 = |a||b|$$

since $0 \leq \theta \leq \pi$

$\Rightarrow \quad -1 \leq \cos\theta \leq 1$

$\Rightarrow \quad 0 \leq |\cos\theta| \leq 1$

2. The Triangle Inequality for vectors is $|\mathbf{a} + \mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$.
   (a) Give a geometric interpretation of the Triangle Inequality.



   (b) Use the Cauchy-Schwartz Inequality to prove the Triangle Inequality. [Hint: $|\mathbf{a} + \mathbf{b}|^2 = (\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b})$]

$$|a+b|^2 = (a+b) \cdot (a+b)$$
$$= a \cdot a + a \cdot b + b \cdot a + b \cdot b = |a|^2 + 2(a \cdot b) + |b|^2$$
$$\leq |a|^2 + 2|a||b| + |b|^2 = (|a| + |b|)^2$$

$$\Rightarrow \quad |a+b| \leq |a| + |b|$$

3. Show that $|\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2|\mathbf{b}|^2 - (\mathbf{a} \cdot \mathbf{b})^2$.

$$|a \times b| = |a||b| \sin\theta$$
$$|a \times b|^2 = |a|^2|b|^2 \sin^2\theta$$
$$= |a|^2|b|^2(1 - \cos^2\theta)$$
$$= |a|^2|b|^2 - |a|^2|b|^2\cos^2\theta$$
$$= |a|^2|b|^2 - |a \cdot b|^2$$