Generating Utilization Vectors for the Systematic Evaluation of Schedulability Tests

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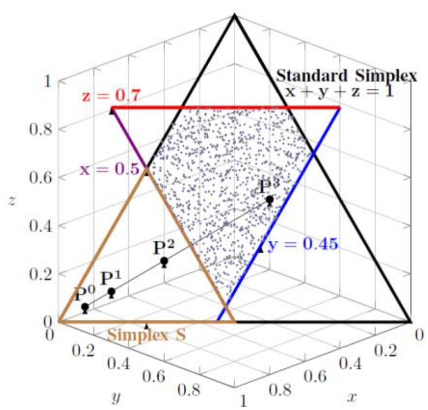
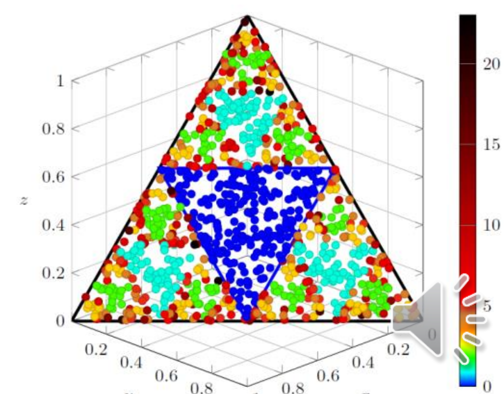


RTSS 2020



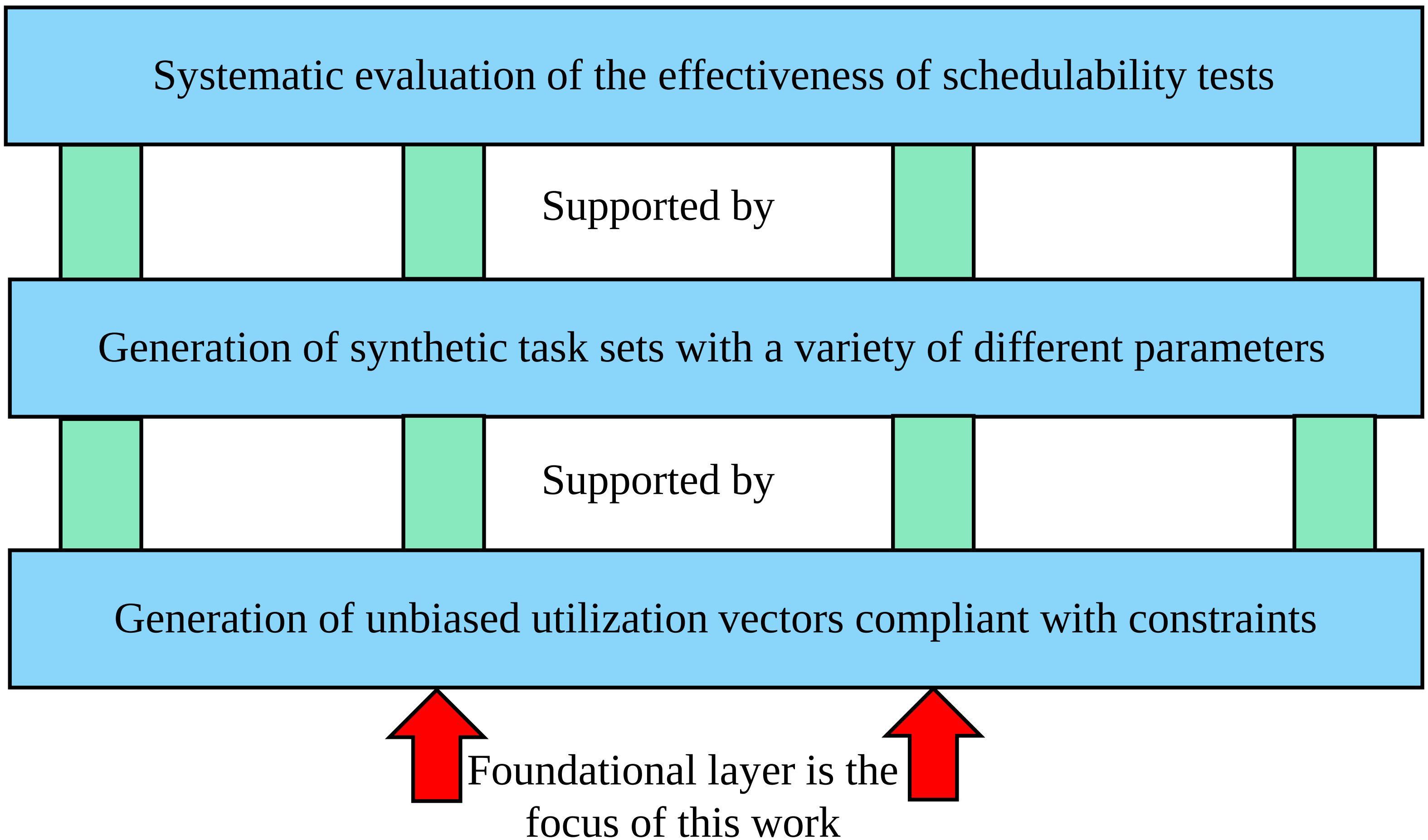
Introducing the DRS Algorithm

- **Dirichlet-Rescale (DRS) algorithm**

$$\mathbf{u} = \mathrm{DRS}(n, U, \mathbf{u}^{\mathbf{max}}, \mathbf{u}^{\mathbf{min}})$$

  - **Returns:**
    A vector of *n* components (utilization values) $\mathbf{u} = (U_1, U_2, \ldots, U_n)$
    such that $\sum_{i=1}^{n} U_i = U$
    $\forall i \; U_i^{max} \geq U_i \geq U_i^{min} \geq 0$
  - **Inputs:**
    n – size of the vector required
    U – total utilization required
    $\mathbf{u}^{\mathbf{max}} = (U_1^{max}, U_2^{max}, \ldots, U_n^{max})$ vector of maximum constraints
    $\mathbf{u}^{\mathbf{min}} = (U_1^{min}, U_2^{min}, \ldots, U_n^{min})$ vector of minimum constraints

Motivation

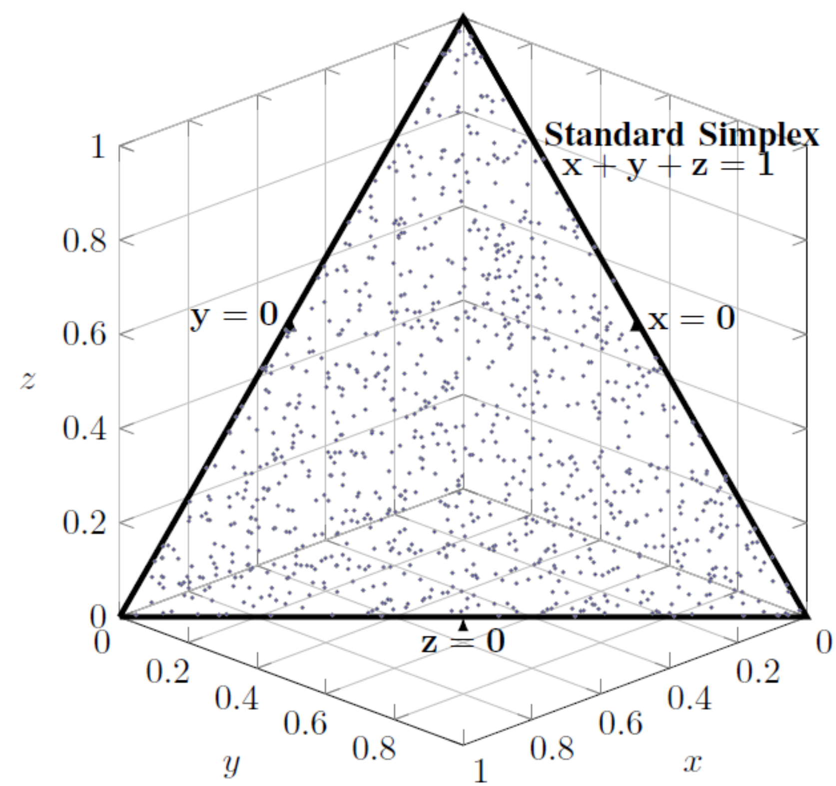

Systematic evaluation of the effectiveness of schedulability tests
Supported by
Generation of synthetic task sets with a variety of different parameters
Supported by
Generation of unbiased utilization vectors compliant with constraints
Foundational layer is the focus of this work

Key criteria for utilization vector generation

- **Uniformity**
  - The vectors of utilization values generated must be unbiased i.e. the vectors must be uniformly distributed within the valid region
    - Bias in the sets of vectors generated can undermine the conclusions drawn from studies into schedulability test effectiveness (Bini and Buttazzo, 2005 [6])
- **Efficiency**
  - Necessary to generate millions of task sets to achieve statistically significant sample sizes in wide-ranging systematic evaluations
    - Typically 1000 task sets per data point for high quality results (Davis, 2016 [11])
- **Flexibility**
  - Capable of handling constraints on individual task utilization values
    - So the utilization vectors can be tailored to the specific requirements of the problem at hand (examples later), while still producing a uniform distribution of vectors within the valid region given by the constraints

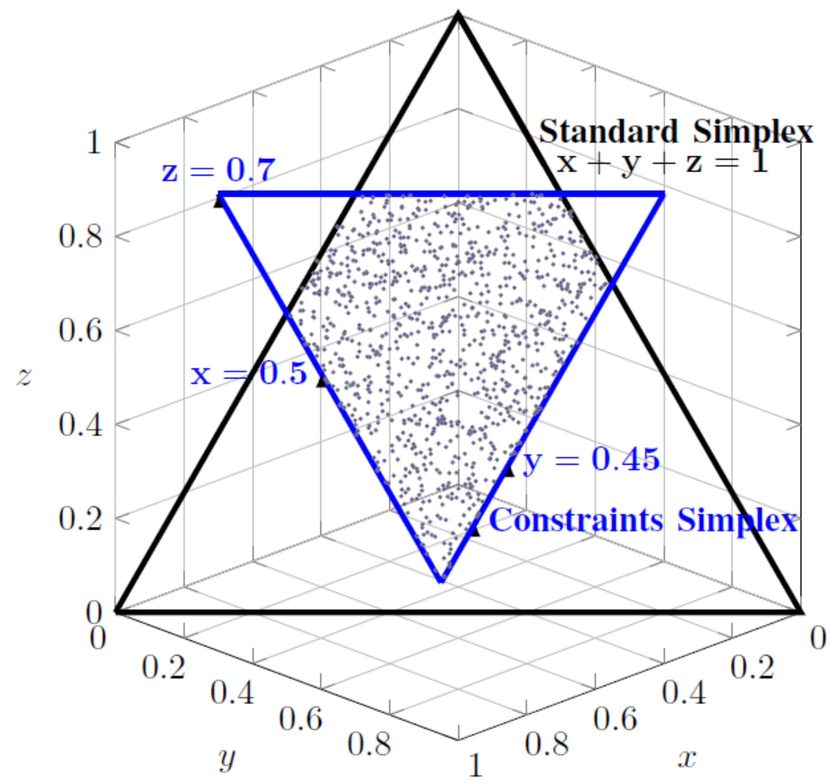
Mathematical background

- **Vectors and Simplices**
  - n-dimensional vectors with components that sum to U
  - Each vector represents a point in n-dimensional space (n=3 for visualization)
  - Canonical form $x+y+z=1$ with $x \geq 0, y \geq 0, z \geq 0$
  - Equation $x+y+z=1$ defines a hyperplane (plane in 3-D space)
  - Combined with inequalities defines a standard n-1 dimensional simplex embedded in n dimensional space (triangle in 3-D space)
  - Vectors required are points uniformly distributed within this simplex



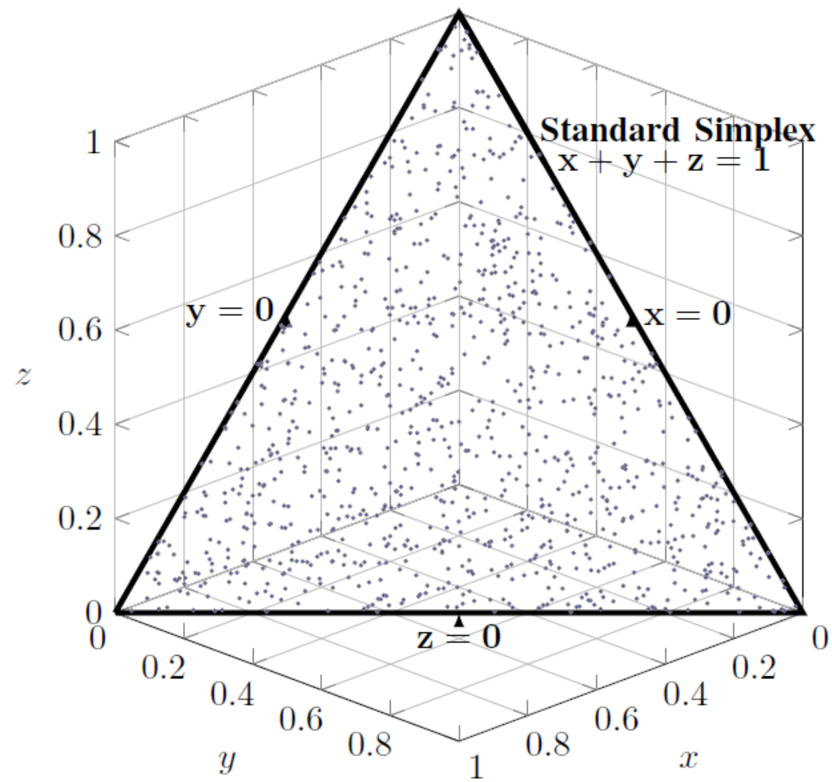
Mathematical background

- **Adding constraints**
  - Maximum constraints form a ***constraints simplex*** on the same hyperplane as the standard simplex ($x+y+z=1$ and $x \leq 0.5$, $y \leq 0.45$, $z \leq 0.7$)
  - Vectors required are points uniformly distributed within the ***valid region*** i.e. within the intersection of the constraints simplex and the standard simplex
  - ***Duality*** between the two simplices – we could generate points in either simplex and use the other as the constraints
  - Minimum constraints can be handled by transforming the problem into a canonical form where all minimum constraints are zero (see the paper)



Related work

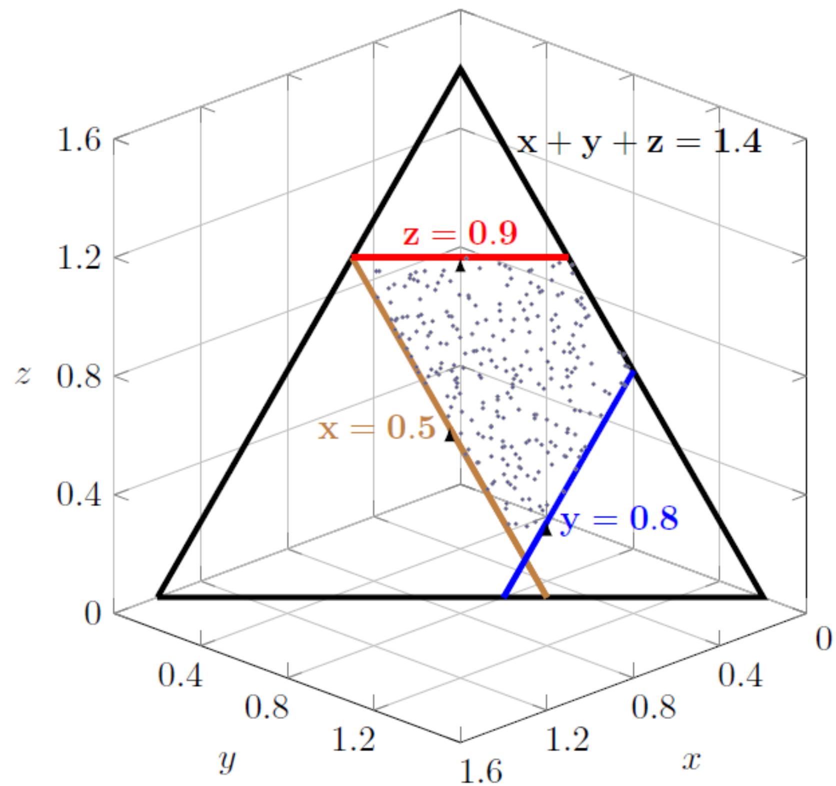
- **UUnifast algorithm**
  - First work on this topic published in the Real Time Systems literature
  - Bini and Buttazzo, 2005 [6]
  - Solves the problem with no maximum or minimum constraints
  - Useful for single processor systems



- **Flat Dirichlet distribution**
  - In the Maths literature, Olkin and Rubin, 1964 [24] published work on the Dirichlet distribution
  - Can also be used to solve the problem with no constraints for single processor systems

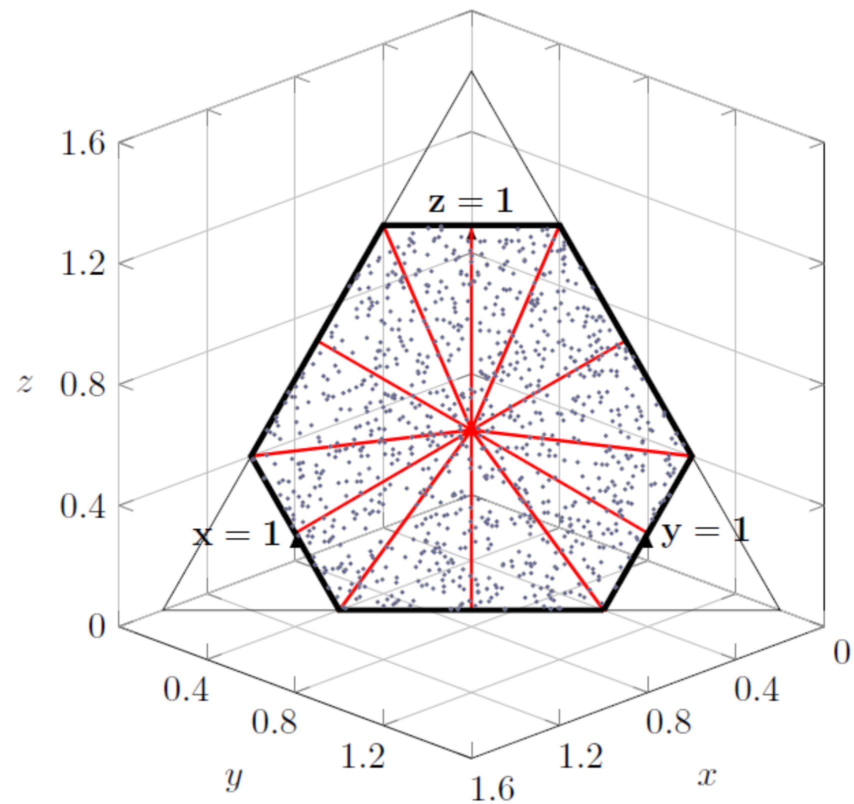
Related work

- **UUnifast-Discard algorithm**
  - Davis and Burns (2010) [14]
  - Developed for multiprocessor systems, where $U > 1$, but $U_i > 1$ is invalid
  - Addresses the problem of maximum (and minimum) constraints
  - Very simple (naïve) approach – uses UUnifast then discards any points that do not comply with the constraints
  - Suffers from the ***curse of dimensionality***: If the constraints on each component halve the volume of the valid region then the proportion of useful points is $1/2^n$ (fine when n=3, not so good when n=50)



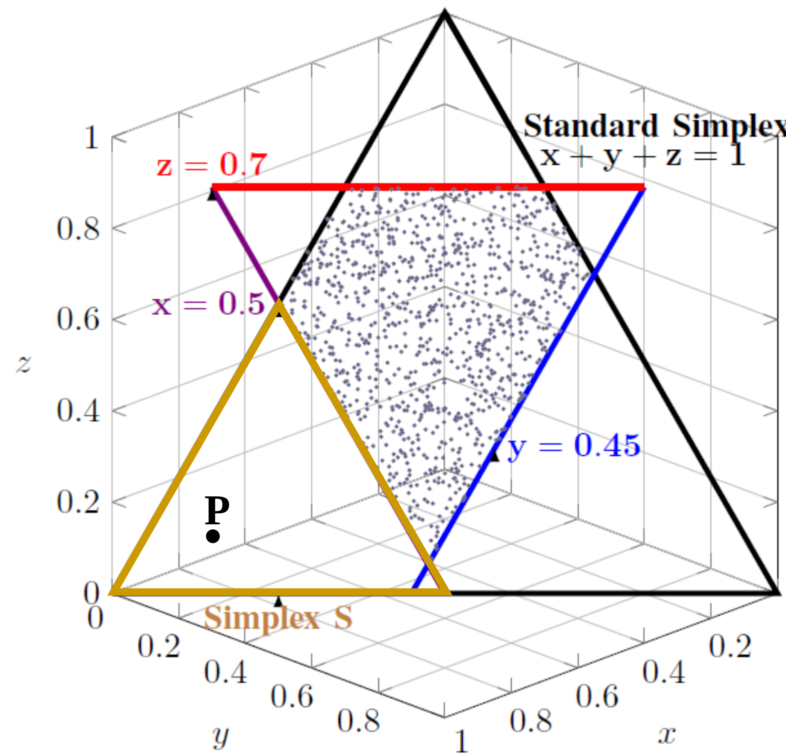
Related work

- **RandFixedSum**
  - Invented by Stafford, 2006 [28] and adapted for task set generation by Emberson et al., 2010 [17]
  - Efficiently addresses the problem of *symmetric* maximum and minimum constraints (i.e. the same constraints for all tasks)
  - De facto standard approach for modelling multiprocessor systems
  - Does not cater for ***asymmetric*** constraints and cannot be adapted to do so because of its reliance on symmetry for its efficiency



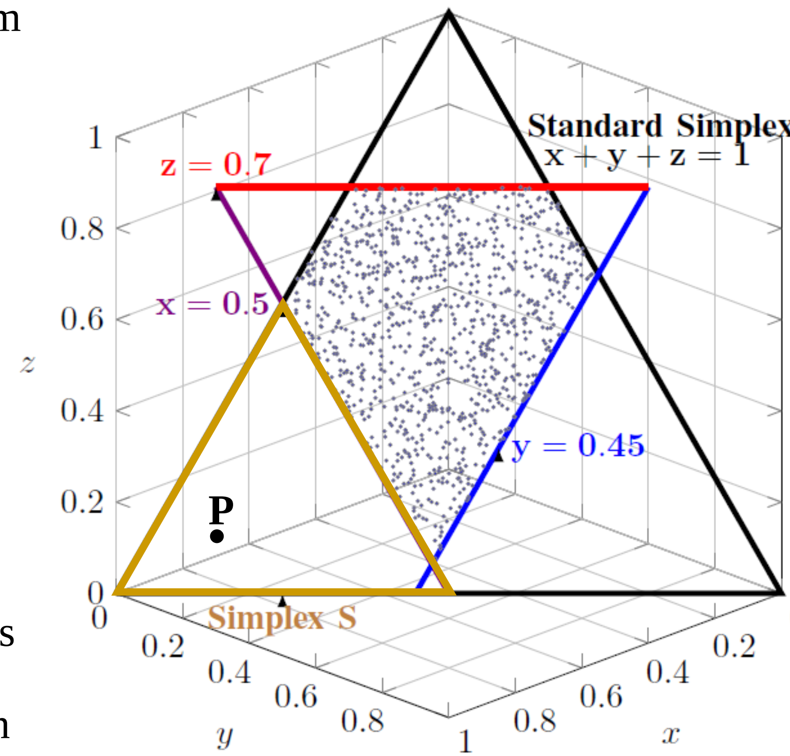
Dirichlet-Rescale (DRS) algorithm

- **DRS algorithm**
  - Addresses the intractability drawbacks: of UUnifast-Discard (discarding points) and of RandFixedSum (would need to generate points in very many different simplices to deal with a valid region that is an irregular shape)
  - Basic concept is to generate a point in the standard simplex then if it is not in the valid region, make a series of transformations shifting the co-ordinates of the point until it is within the valid region
  - Crucially these transformations must preserve the uniform distribution of points



How DRS works

- **DRS Algorithm outline operation**
  1. Transform the problem into a canonical form by removing minimum constraints
  2. Exploits duality to switch the standard and constraints simplices for efficiency
  3. Generate a point **P** on the standard simplex using the Dirichlet distribution
  4. If **P** satisfies the constraints then return **P** (reversing the initial transformation)
  5. Otherwise, defines Simplex S based on the broken constraints (S contains **P**)
  6. Map Simplex S onto the standard simplex via a matrix transformation
  7. This scale and translate transformation alters the coordinates of **P** making it more likely that the point will now be in the valid region
  8. Goto step 4.



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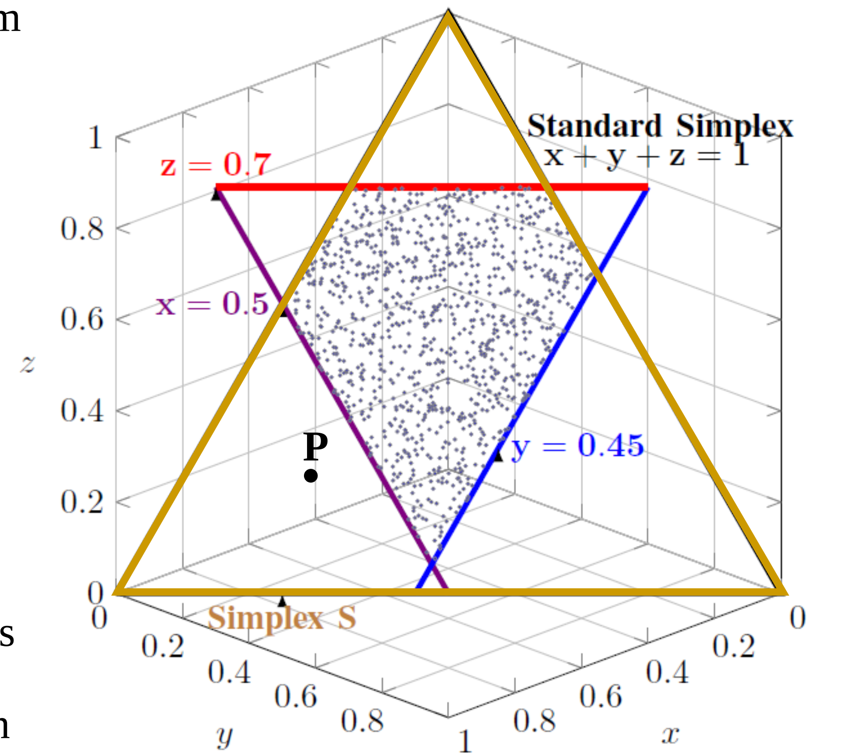

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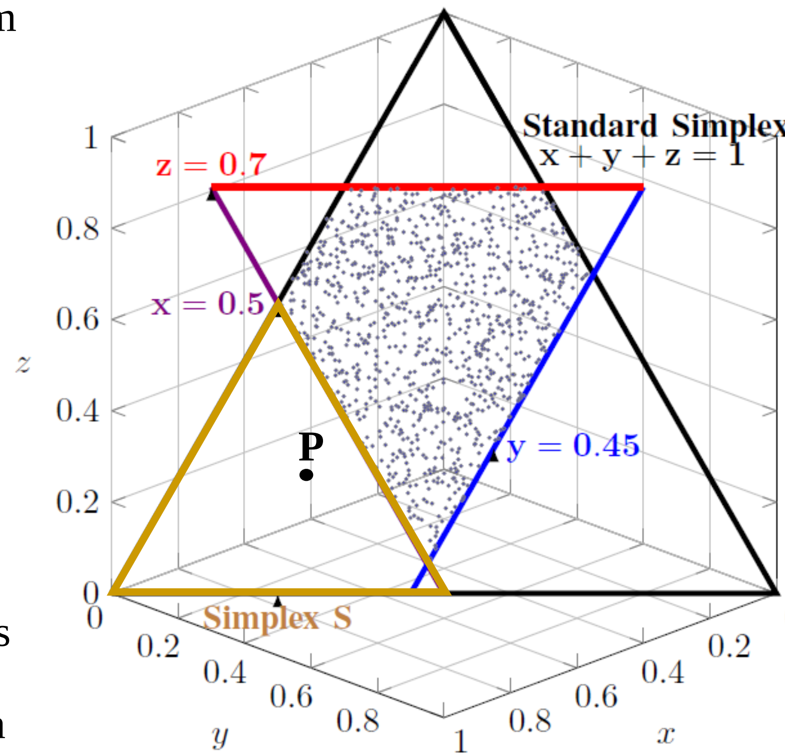

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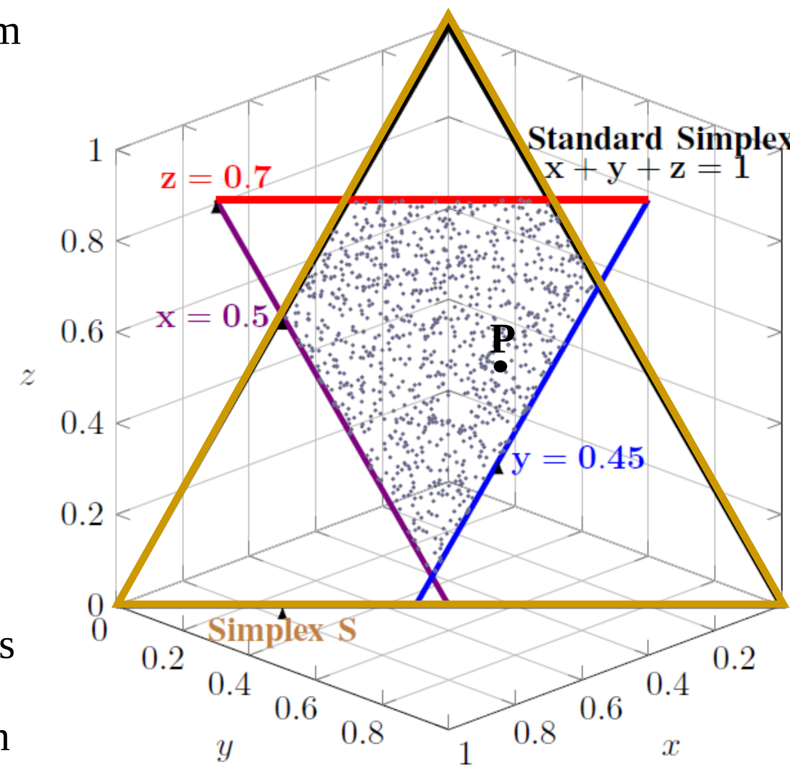
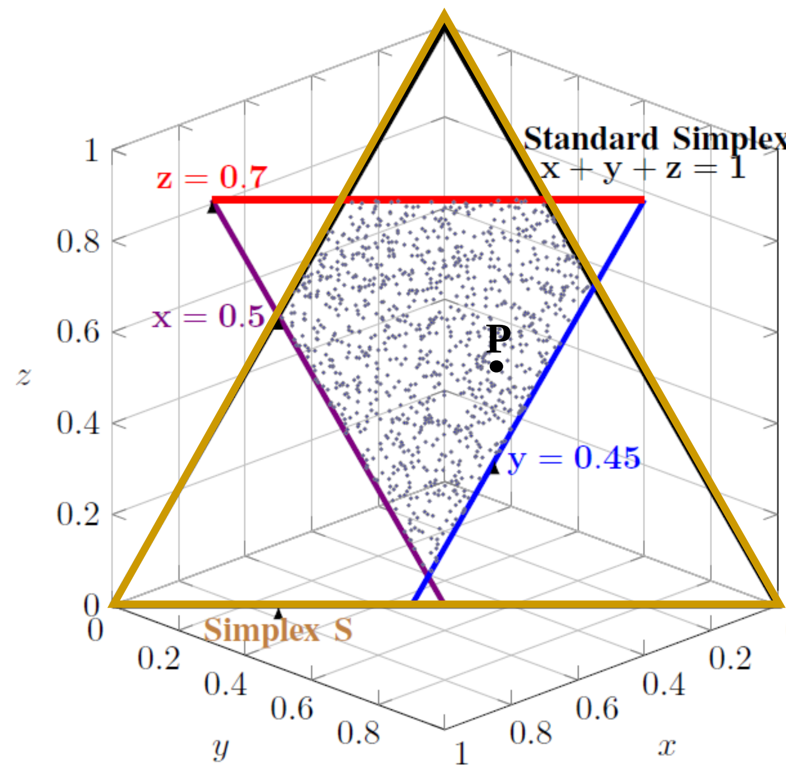


How DRS works

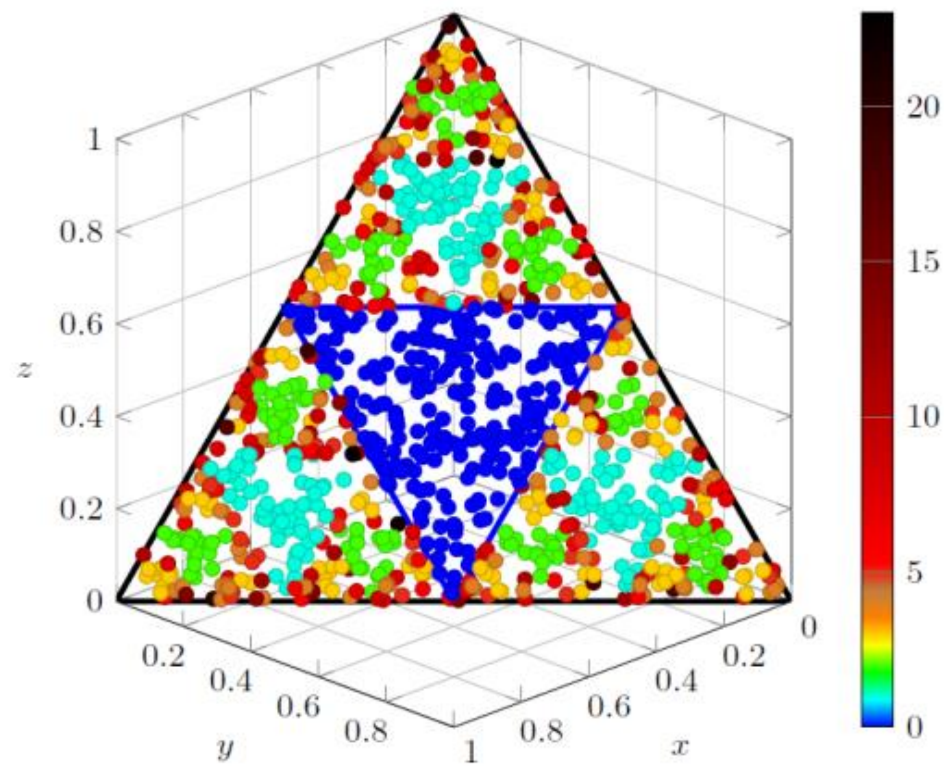
- **Ensuring Uniformity**
  - Distribution of initial points generated over the standard simplex is uniform
  - Hence the distribution of points is also uniform over Simplex **S**
  - The matrix transformation that maps Simplex S onto the standard simplex is an ***Affine*** transformation (i.e. a scale and translate transformation).
  - Therefore the points that are uniformly distributed over Simplex **S** become uniformly distributed over the standard simplex and hence uniformly distributed over the valid region



How DRS works (convergence)

- **Convergence**
  - Let $p = \dfrac{\text{volume(valid region)}}{\text{volume(standard simplex)}}$
  - After q iterations, the minimum converged volume $c \geq 1-(1-p)^q$
  - As $q \to \infty$, $c \to 1$ and so the algorithm converges
- **Illustration of convergence**
  - Heat map color codes the number of rescales needed to converge:
  - Here p = 0.25 and all 1000 initial points converged within 24 rescales

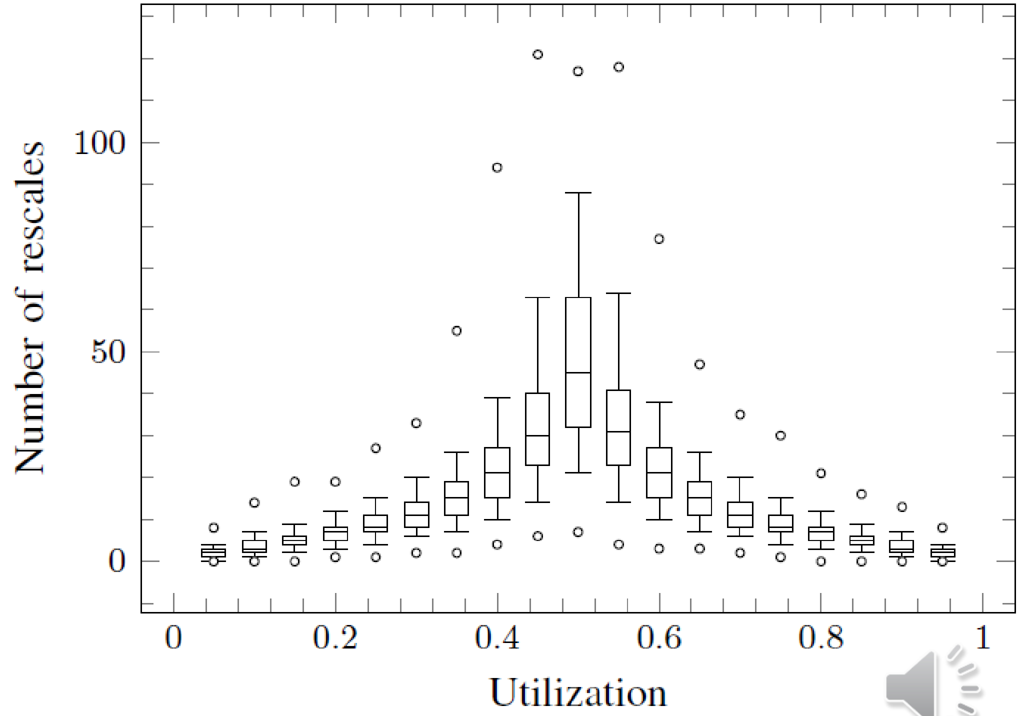
[Note this was done for illustration purposes with the duality optimization disabled, otherwise no rescaling would be necessary since every point generated would be within the smaller constraints simplex (blue)]



DRS Performance

- **Experiment A**
  - n = 50 and 10,000 runs for each U in [0.05, 0.95] in steps of 0.05
  - For each run: $\mathrm{DRS}(n, U, \mathbf{u}^{\max})$ with constraints $\mathbf{u}^{\max} = \mathrm{UUnifast}(n, 1)$

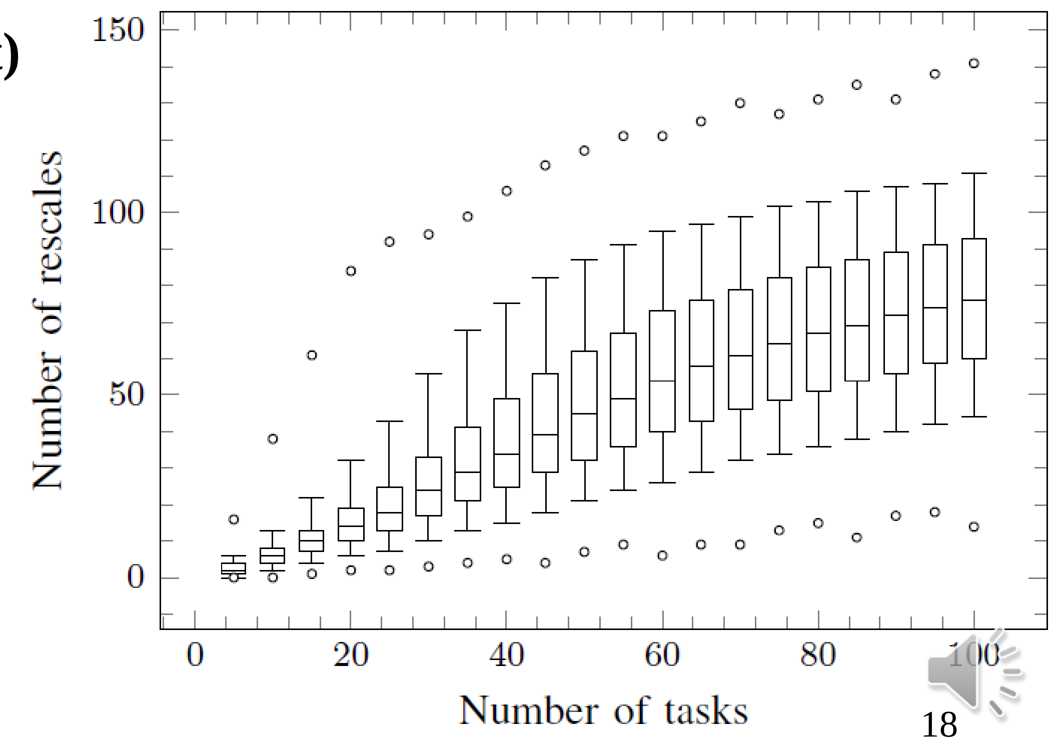
- **Number of Rescales (Box plot)**
  - Worst-case occurs for U = 0.5 when constraints and standard simplex are the same size
  - Max rescales < 200 (upper circle)
    Min rescales (lower circle)
    Mean (middle line of box)
    Percentiles (5%, 25%, 75%, 95%)



DRS Performance

- **Experiment B**
  - Similar to Expt. A, but *U* fixed at 0.5 and *n* varied from 5 to 100 in steps of 5
  - For each run: $\mathrm{DRS}(n, U, \mathbf{u}^{\max})$ with constraints $\mathbf{u}^{\max} = \mathrm{UUnifast}(n, 1)$

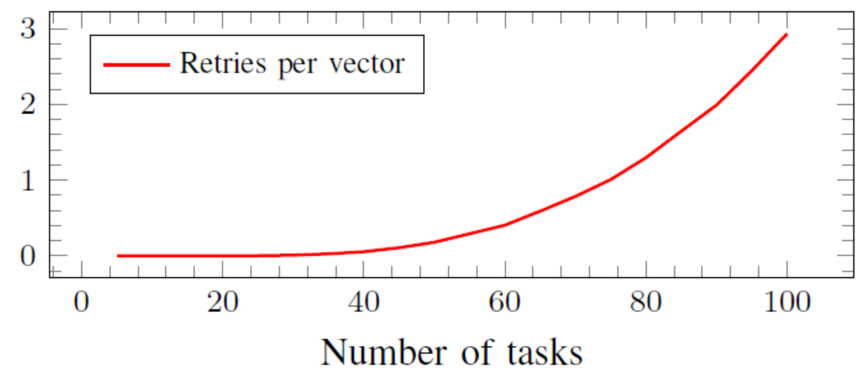
- **Number of Rescales (Box plot)**
  - Number of rescales gradually increases with increasing size of the vectors (number of tasks)
  - Max rescales < 200 (upper circle)
    Min rescales (lower circle)
    Mean (middle line of box)
    Percentiles (5%, 25%, 75%, 95%)



DRS Performance

- **Experiment B (continued)**

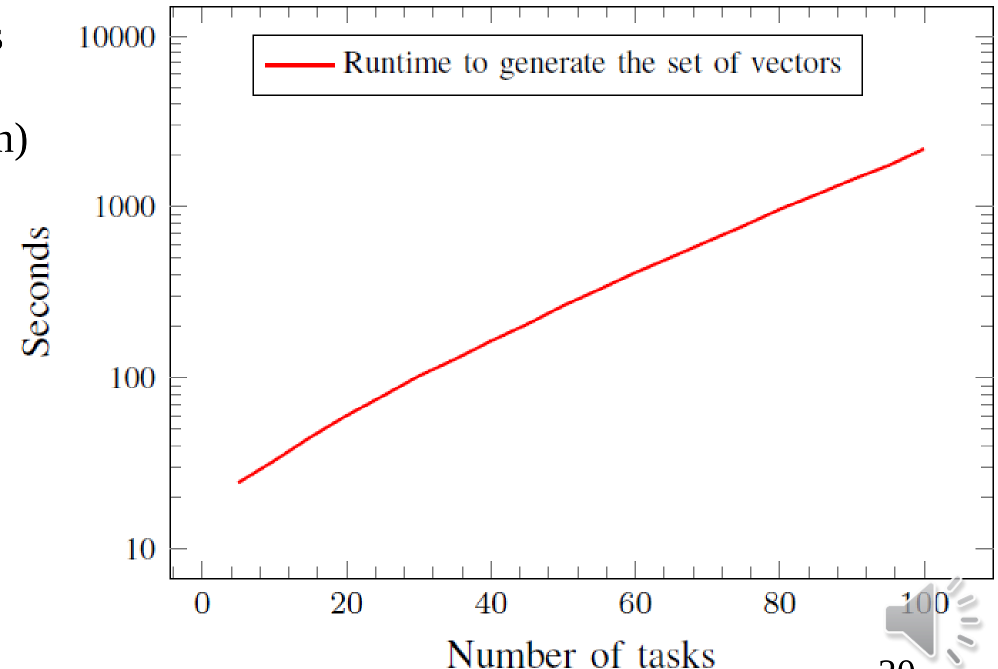
- **Number of Retries**
  - Rescale operations can lead to the accumulation of Floating Point error
  - A retry is done by generating another point if the total error (sum of component values minus required utilization) exceeds 0.01%
  - Number of retries increases with increasing size of the vectors, but remains low for $n \leq 100$



DRS Performance

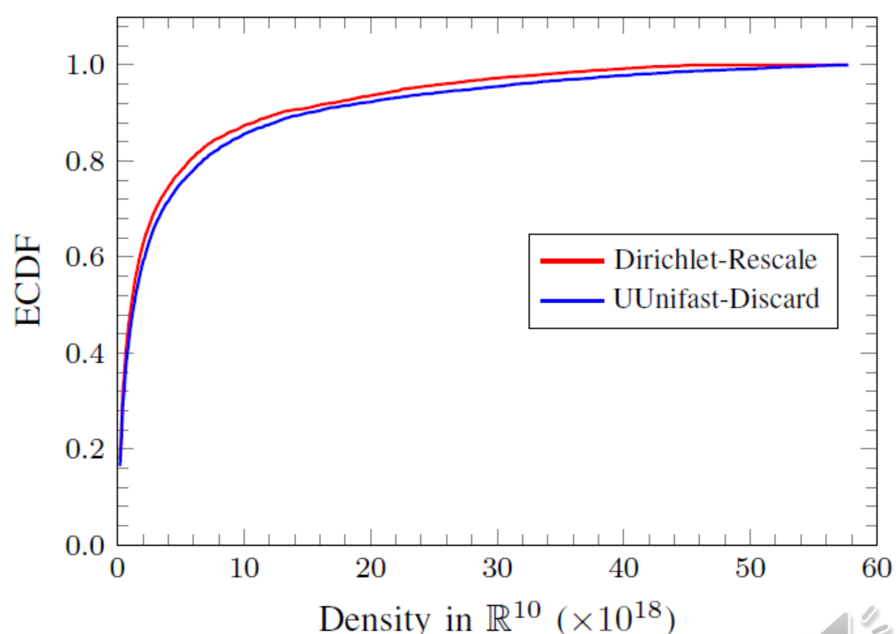
- **Experiment C**
  - Runtime to generate all the vectors needed for a standard "benchmark" schedulability analysis experiment (1000 vectors for each of 18 utilization levels from U = 0.05 to 0.95 in steps of 0.05, 18,000 vectors in all)
- **Runtimes:**
  - Used a Pi 4 to obtain reliable timings
  - Runtimes well approximated by a polynomial of order 3 (cubic function) $R^2 = 0.999$
  - Typical use would be on a laptop or desktop PC (e.g. Dell XPS 13 with Intel™ i7-1065G7 at 3.5GHz
    - ~6 seconds for 10 tasks
    - ~60 seconds for 50 tasks
    - ~6 minutes for 100 tasks

    (approx. 6 times faster than a Pi 4)



DRS Performance

- **Experiment D**
  - Verified the uniformity of the distribution of vectors produced by DRS via comparison with UUnifast-Discard
- **Statistical test**:
  - Examined the density of points produced in 1000 small reference simplices (within the valid region) via the DRS algorithm and UUnifast-Discard
  - Compared the Empirical Cumulative Distribution Functions (ECDF) using a statistical test: Kolmogorov-Smirnov (KS) test
  - KS-statistic = 0.04, p-value = 1.0
  - No evidence that the vectors produced come from different distributions
  - Cannot reject the null hypothesis that the distributions are the same



Use of the DRS algorithm

- **Main use is in the systematic evaluation of schedulability tests**
  - Used to underpin the generation of synthetic task sets with execution times derived from the utilization values
- **Asymmetric constraints**:
  - Occur when execution times have multiple values or are composed from multiple parts:
    - Mixed Criticality Systems (e.g. C(LO), C(HI))
    - Multi-core systems (e.g. processor demand, bus demand, memory demand, etc.),
    - Typical and worst-case execution times
    - Self-suspensions and resource locking
- **No constraints or symmetric constraints:**
  - DRS can be used to replace UUnifast for single processor systems, and RandFixedSum and UUnifast-Discard for multiprocessor systems

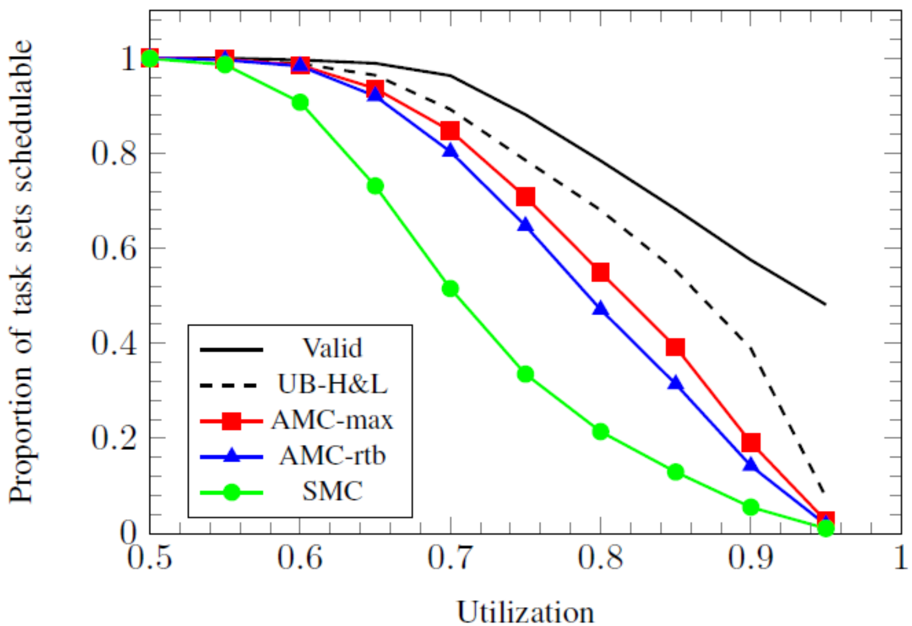
Mixed Criticality Systems Example

- **Schedulability Analysis Experiment**
  - Reproduced from the Adaptive Mixed Criticality (AMC) scheduling paper (Baruah et al., 2011 [4])
- **Using DRS**:
  - Independent control of total $U(LO)$ and $U(HI)$
  - Independent selection of $U_i(LO) \leq U_i(HI)$ and hence $C_i(LO) \leq C_i(HI)$
  - Eliminates generation of invalid (infeasible) task sets
  - $U_i(HI)$ generated by calling $\mathrm{DRS}(n^{HI}, U^{HI}_{HI}, \mathbf{u}^{1})$
  - Maximum constraints set to 1 for LO-criticality tasks and to $U_i(HI)$ for HI-criticality tasks
  - $U_i(LO)$ generated by calling $\mathrm{DRS}(n, U^{LO}, \mathbf{u}^{\max})$

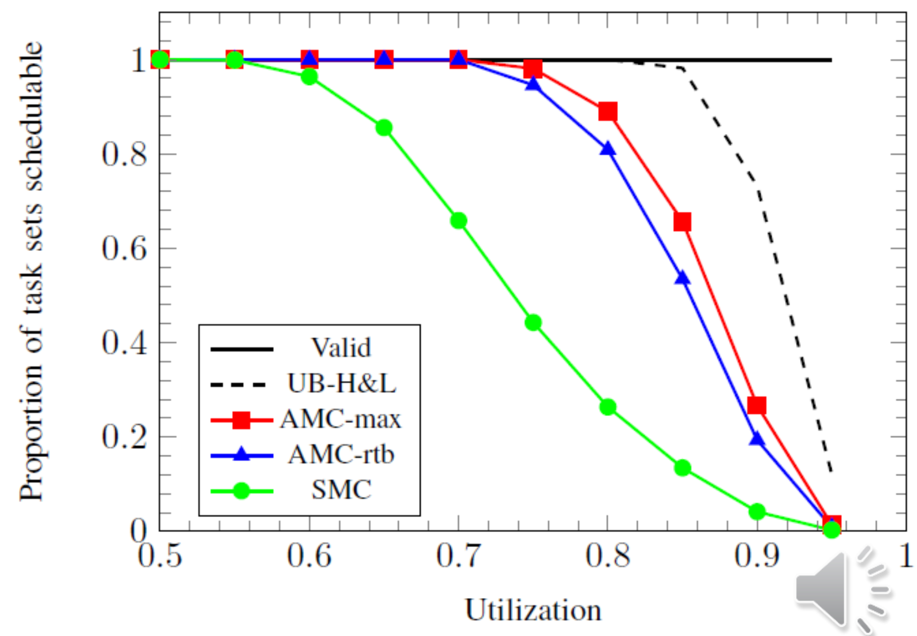
Mixed Criticality Systems Example

- **Schedulability Analysis Experiment**
  - Reproduced from AMC paper [4]
  - DRS highlights sharper transition of AMC and larger improvement over SMC
  - More nuanced and realistic results – could affect decisions on which methods to use

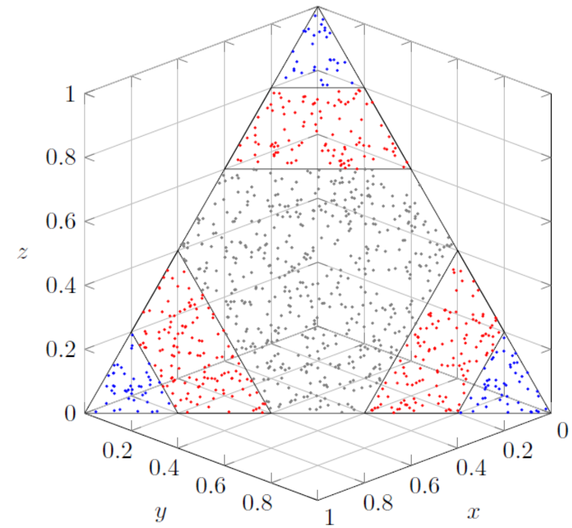
Baruah et al.



DRS used in task set generation



Why use an unbiased distribution of utilization vectors?

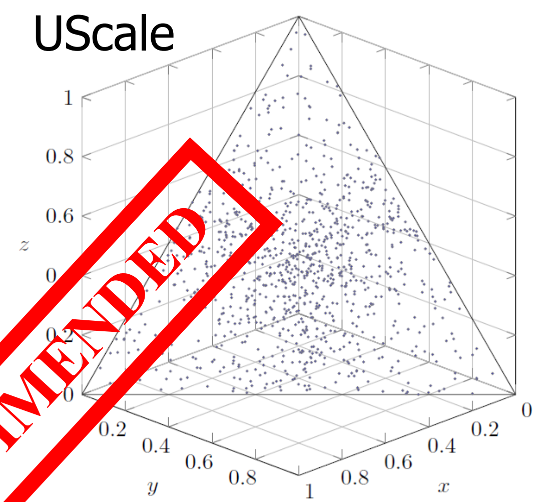


- **What is meant by an unbiased?**
  - Vectors generated are uniformly distributed across the valid region
  - Does not mean the component values themselves are uniformly distributed (common misconception)

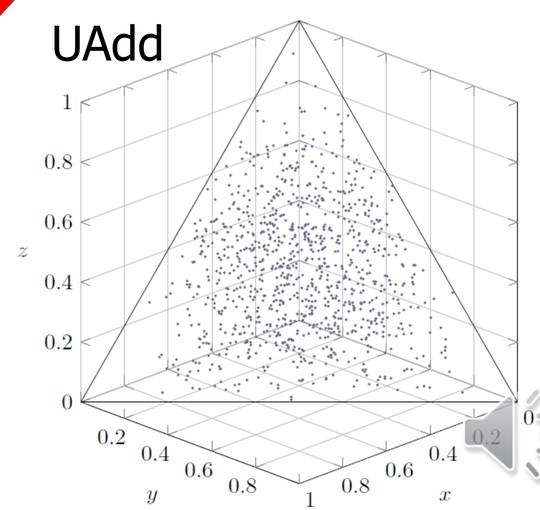
- **Why use a unbiased distribution?**
  - For generic schedulability analysis experiments, using a uniform distribution of utilization vectors means that each possible vector that complies with the constraints has the same chance of being selected
  - The distribution is thus unbiased, provides full and fair coverage of all valid possibilities, and is therefore arguably the appropriate one to use
  - Not using a uniform distribution of vectors risks biasing the results of schedulability analysis experiments

Easy ways of introducing bias…

1. **Confound variables** (n and U)
   - Select U_i from a uniform distribution [0,1] and keep adding tasks until the required total utilization is reached
   - Confounds n and U, so we cannot distinguish the effects of higher task set cardinality from those of higher task set utilization
2. **Simple scaling (UScale)**
   - Select n values for U_i from a uniform distribution [0,1] and then scale them to achieve the required total utilization U
3. **Addition of components (UAdd)**
   - Use UUnifast for each of multiple parts of U_i and then add these values together



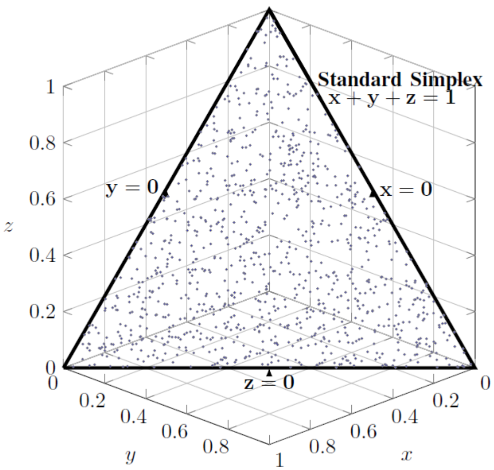
NOT RECOMMENDED

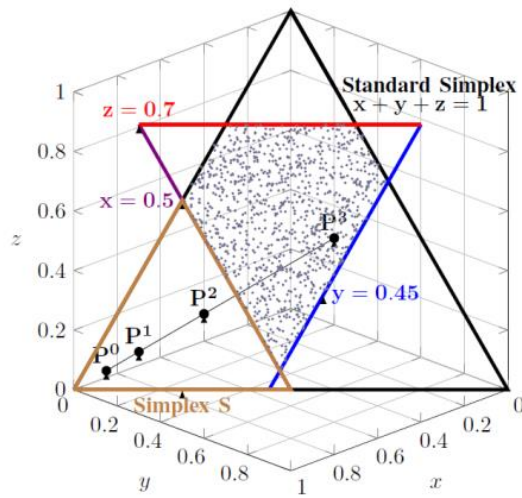


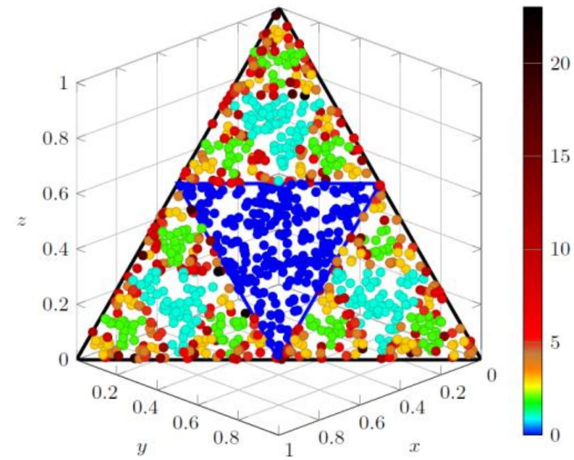
Conclusion:
Why use the DRS algorithm?

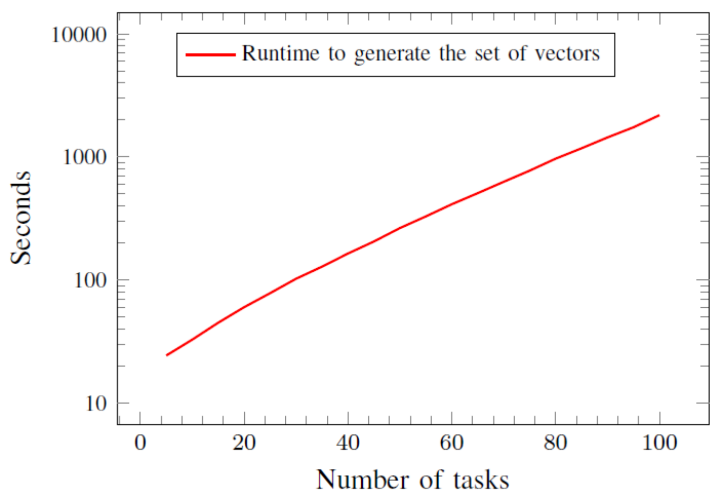
- **Flexible - general purpose algorithm**
  - Supports asymmetric constraints on maximum and minimum utilization for each task
    - Used to obtain unbiased distributions when execution times have multiple values or are composed from multiple parts
    - Useful for tailoring task sets to specific problem requirements, limitations, or domain specific constraints
  - Can also be used to replace UUnifast, UUnifast-Discard, and RandFixedSum
- **High performance**
  - Supports efficient generation of task sets with cardinality up to n = 100 with individual constraints
  - Additional experiments show that DRS supports generation of task sets with cardinality up to n = 200 with a commensurate slowdown in performance
- **Python source code is publicly available**
  - Permanently archived at https://doi.org/10.5281/zenodo.4118059
  - Can be installed via: `pip install drs` (https://pypi.org/project/drs/)
  - Also provide a C library enabling the DRS algorithm in Python to be called directly from C/C++ code

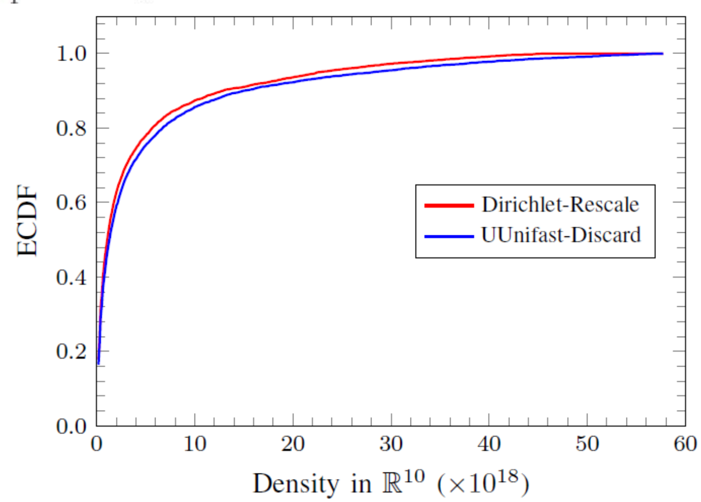
Questions?


Standard Simplex
x + y + z = 1
y = 0
x = 0
z = 0
z
y
x
0
0.2
0.4
0.6
0.8
1


Standard Simplex
x + y + z = 1
z = 0.7
x = 0.5
y = 0.45
P0
P1
P2
P3
Simplex S
z
y
x
0
0.2
0.4
0.6
0.8
1


z
y
x
0
0.2
0.4
0.6
0.8
1
0
5
10
15
20


Runtime to generate the set of vectors
Seconds
Number of tasks
10
100
1000
10000
0
20
40
60
80
100


Dirichlet-Rescale
UUnifast-Discard
ECDF
Density in $\mathbb{R}^{10}$ ($\times 10^{18}$)
0.0
0.2
0.4
0.6
0.8
1.0
0
10
20
30
40
50
60