MODULE 3

PULSE MODULATION

INTRODUCTION

- Many Signals in Modern Communication Systems are digital . Also, analog signals are transmitted digitally.
- Reduced distortion and improvement in signal to noise ratios.
- PAM, PWM , PPM , PCM and DM.
- Data transmission, digital transmission, or digital communications is the physical transfer of data (a digital bit stream or a digitized analogue signal) over a point-to-point or point-to-multipoint communication channel.

Ex: optical fibers, wireless channels, computer buses....

➢ ELEMENTS OF DIGITAL COMMUNICATION SYSTEMS

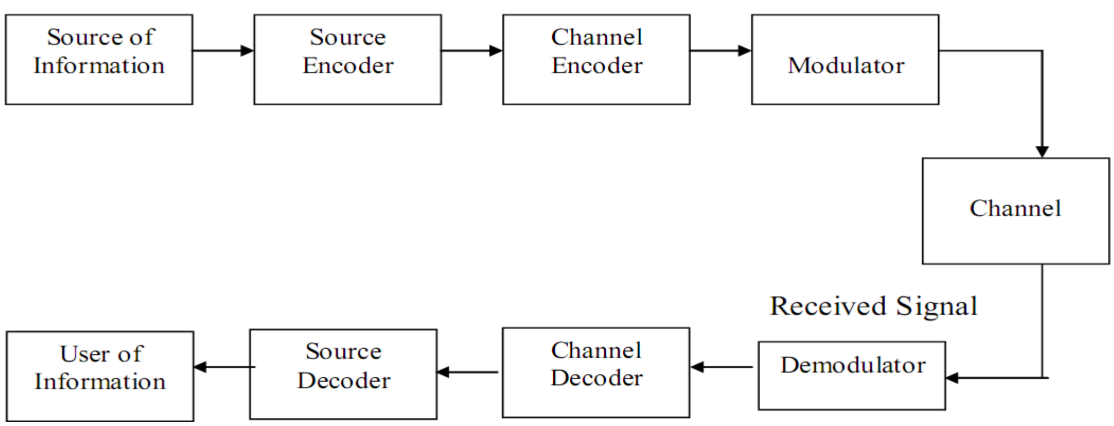


Fig.Block diagram of Digital Communication

1. **Discrete Information Source:** It generates message to be transmitted. Examples are the data from computers, text data or tele type data.

2. **Source Encoder**: It assigns codes to the symbols (samples) generated from discrete information source. The code word having n number of bits. Each distinct sample having

distinct(unique) code word. If code word length is 8 bit(n), we can have 256 distinct symbols(ie.,2^n).

3. **Channel Encoder**: We know that channel is the major source of notice due to that there are more chance of getting errors while propagating through channel. To avoid that channel encoding is required. In that extra bits are added to the binary sequence generated by the source encoder. These extra bits are called as redundant bits. These bits are defined with proper logic. The redundant will be helpful to detect the errors at the receiver bit sequence.

4. **Digital Modulator**: In digital modulator the message signal is digital data and carrier is analog one, in most cases we use sinusoidal waves. Some examples are ASK,FSK,PSK.MRI techniques.

5. **Channel**: It provides the link between transmitter and rceiver. Channel may be wired or wireless channel.

   ❖ Problems associated with channel:

1. Addictive Noise: This noise is occur due to internal solid state devices or resistors used in channel.

2. Ampltude and Phase Distortion: This noise is occurred due to non-linear characteristics of the channel.

3. Attenuation: This is due to internal resistance of the channel.

6. **Demodulator:** This device is used to detect the digital message signal from the modulated signal.

7. **Channel Decoder**: This is used to detect and correct the errors that occur in the digital message signal.

8. **Source Decoder**: This produces the sampling signal from the given digital message signal.

9. **Destination**: The sampled signal is converted into audio signal or video signal or any text signal depending on the signal.

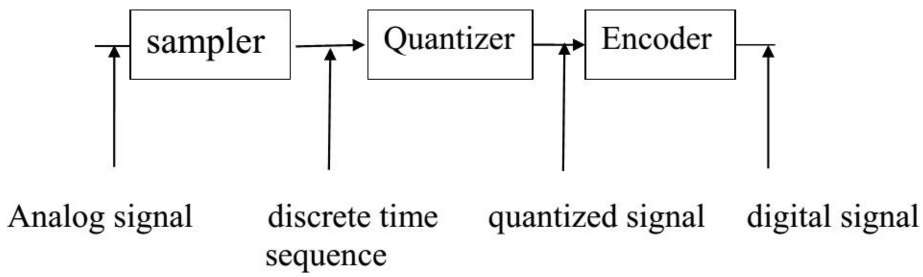


Fig. Basic block diagram of an A/D converter

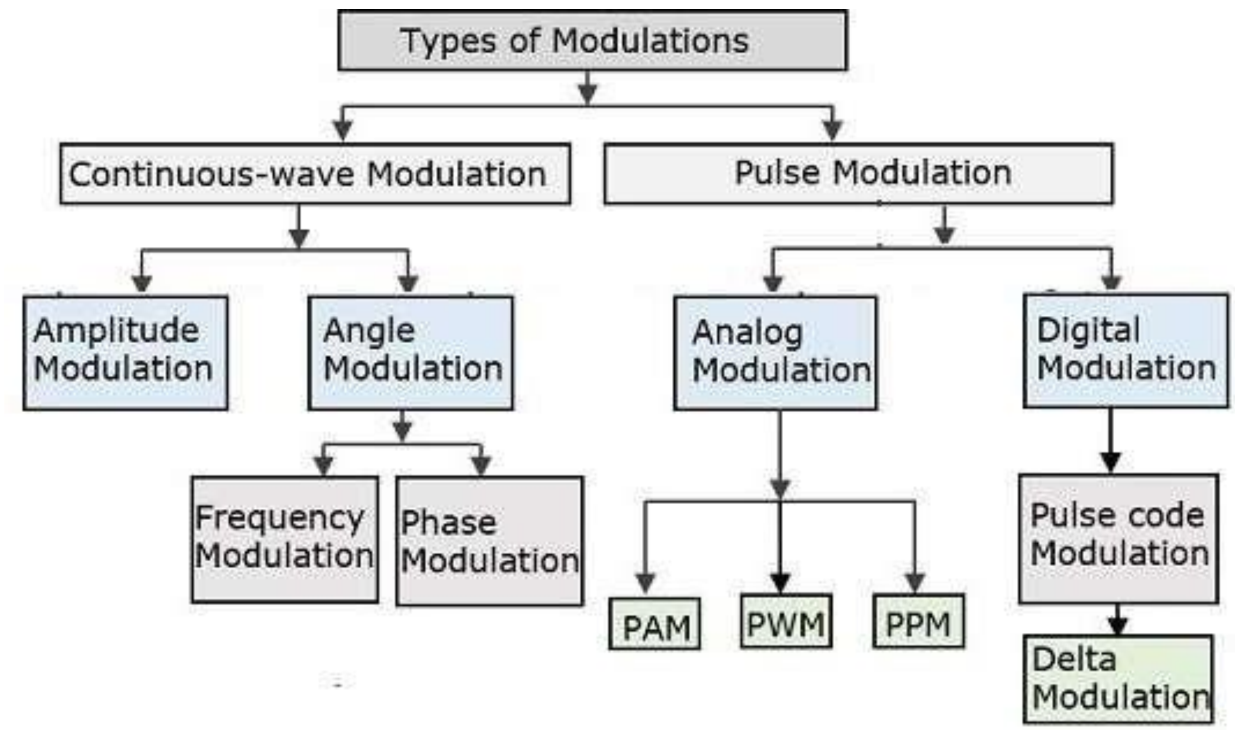
<u>Advantages of digital communication systems</u>

1. Easy way of transmission of signals
2. Connection of more calls through one channel i.e., Multiplexing is possible using Digital Communication.
3. Source Encoding and Channel Encoding can be used to detect errors at the received signal.
4. Using repeaters between source and destination, we can reproduce the original signal with less distortions.
5. Security is the major advantage of digital communication compared to Analog Communication.
6. Transmitting analogue signals digitally allows for greater signal processing capability.
7. Digital communication can be done over large distances through internet and other things.
8. The messages can be stored in the device for longer times, without being damaged.
9. Advancement in communication is achieved through Digital Communication.

Disadvantages of digital communication systems

1. Sampling Error
2. Digital communications require greater bandwidth than analogue to transmit the same information.
3. The detection of digital signals requires the communications system to be synchronized, whereas generally speaking this is not the case with analogue systems.
4. Digital signals are often the approximation of voice signals, ie, we don"t get the exact analogue signal.

➢ **TYPES OF MODULATION – TREE DIAGRAM**



In Continuous Wave modulation schemes some parameter of modulated wave varies continuously with message.

In Analog pulse modulation some parameter of each pulse is modulated by a particular sample value of the message.

Pulse modulation of two types

1. Analog Pulse Modulation
   - Pulse Amplitude Modulation (PAM)
   - Pulse width Modulation (PWM)
   - Pulse Position Modulation (PPM)
2. Digital Pulse Modulation
   - Pulse code Modulation (PCM)
   - Delta Modulation (DM)

1. Analog Pulse Modulation

Analog pulse modulation results when some attribute of a pulse varies continuously in one-to-one correspondence with a sample value. In analog pulse modulation systems, the amplitude, width, or position of a pulse can vary over a continuous range in accordance with the message amplitude at the sampling instant, as shown in Figure 6.2. These lead to the following

Three types of pulse modulation:

1. Pulse Amplitude Modulation (PAM)
2. Pulse Width Modulation (PWM)
3. Pulse Position Modulation (PPM)

PAM: In this scheme high frequency carrier (pulse) is varied in accordance with sampled value of message signal.

PWM: In this width of carrier pulses are varied in accordance with sampled values of message signal. Example: Speed control of DC Motors.

PPM: In this scheme position of high frequency carrier pulse is changed in accordance with the sampled values of message signal.

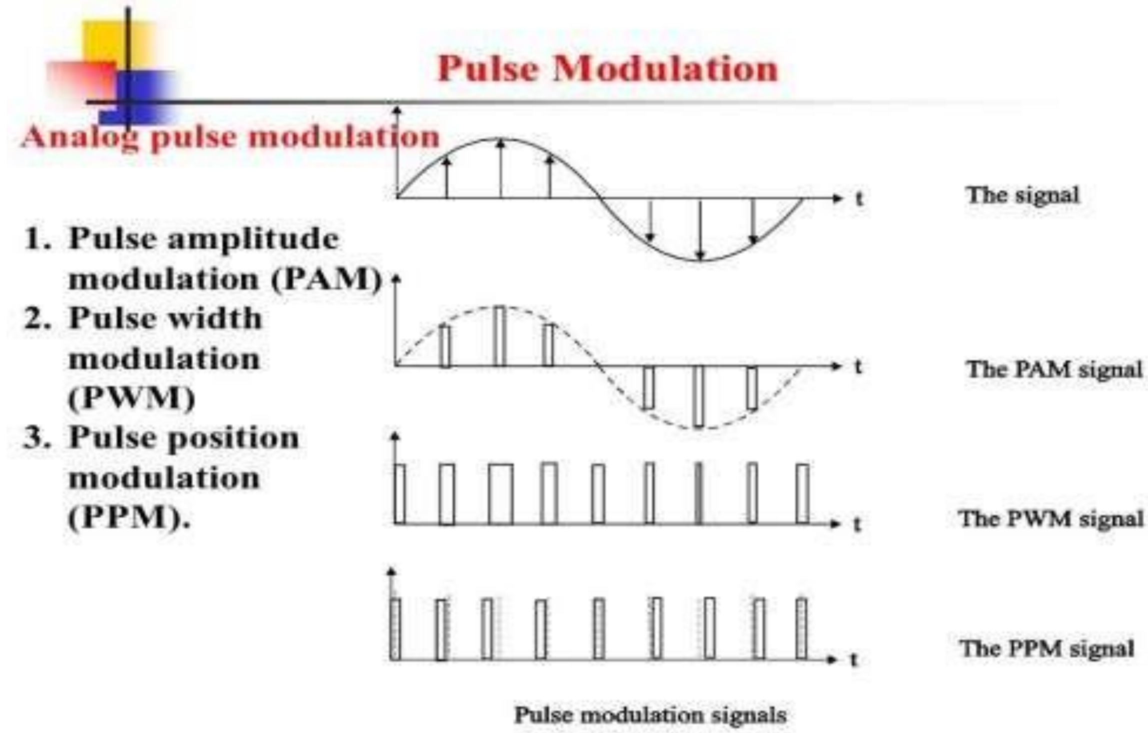


Fig. Representation of Various Analog Pulse Modulations

2. Digital Pulse Modulation

In systems utilizing digital pulse modulation, the transmitted samples take on only discrete values. Two important types of digital pulse modulation are:

1. Delta Modulation (DM)
2. Pulse Code Modulation (PCM)

ANALOG PULSE MODULATION

1. Pulse Amplitude Modulation (PAM):

In pulse amplitude modulation, the amplitude of regular interval of periodic pulses or electromagnetic pulses is varied in proposition to the sample of modulating signal or message signal. This is an analog type of modulation. In the pulse amplitude modulation, the message signal is sampled at regular periodic or time intervals and this each sample is made proportional to the magnitude of the message signal. These sample pulses can be transmitted directly using wired media or we can use a carrier signal for transmitting through wireless.

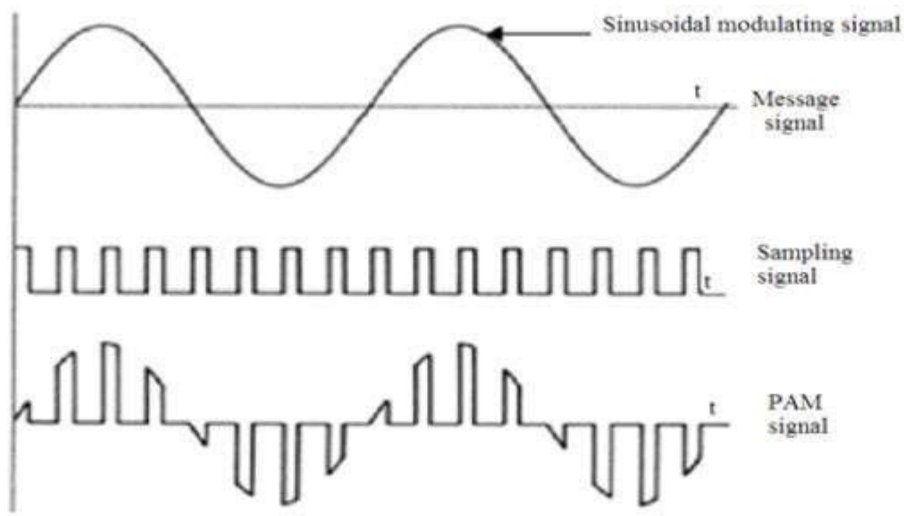


Fig. Pulse Amplitude Modulation Signal

There are two types of sampling techniques for transmitting messages using pulse amplitude modulation, they are

- **FLAT TOP PAM:** The amplitude of each pulse is directly proportional to instantaneous modulating signal amplitude at the time of pulse occurrence and then keeps the amplitude of the pulse for the rest of the half cycle.

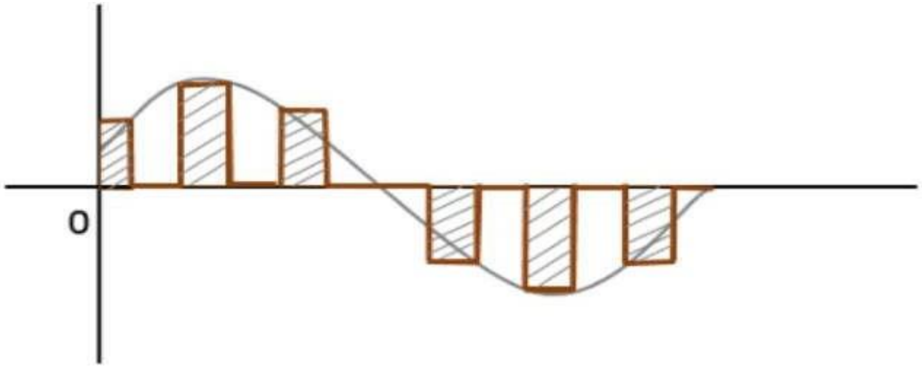


Fig. Flat Top PAM

- **Natural PAM:** The amplitude of each pulse is directly proportional to the instantaneous modulating signal amplitude at the time of pulse occurrence and then follows the amplitude of the modulating signal for the rest of the half cycle.

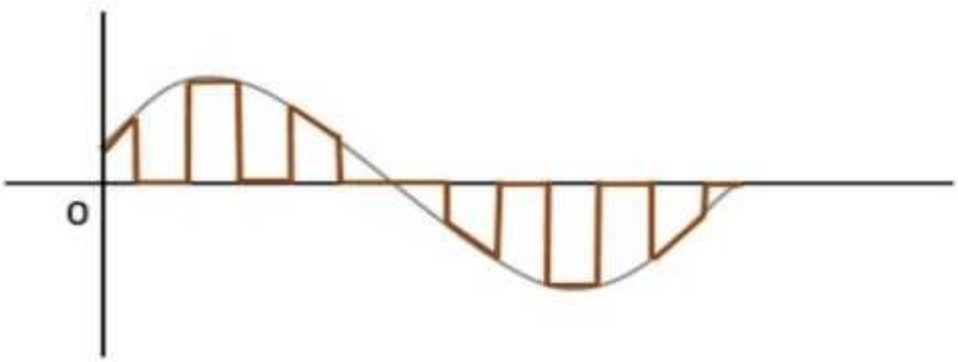


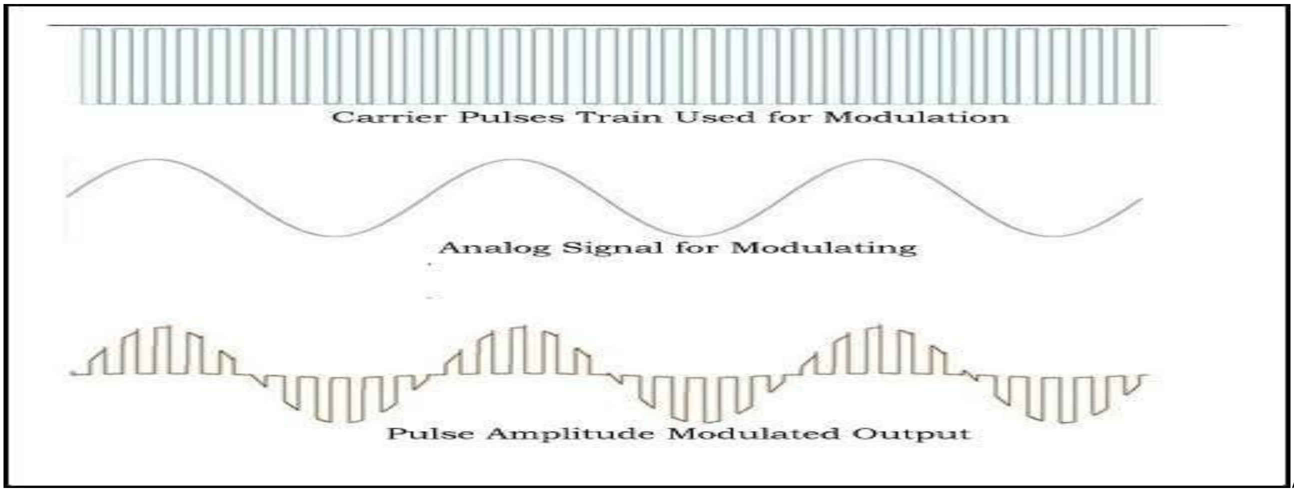
Fig. Natural PAM

Flat top PAM is the best for transmission because we can easily remove the noise and we can also easily recognize the noise. When we compare the difference between the flat top PAM and natural PAM, flat top PAM principle of sampling uses sample and hold circuit. In natural principle of sampling, noise interference is minimum. But in flat top PAM noise interference maximum. Flat top PAM and natural PAM are practical and sampling rate satisfies the sampling criteria.

There are two types of pulse amplitude modulation based on signal polarity

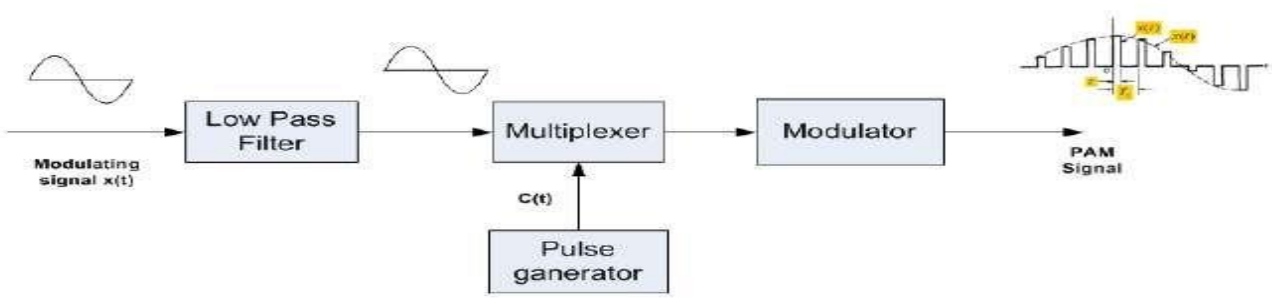
1. Single polarity pulse amplitude modulation
2. Double polarity pulse amplitude modulation

In single polarity pulse amplitude modulation, there is fixed level of DC bias added to the message signal or modulating signal, so the output of modulating signal is always positive. In the double polarity pulse amplitude modulation, the output of modulating signal will have both positive and negative ends.

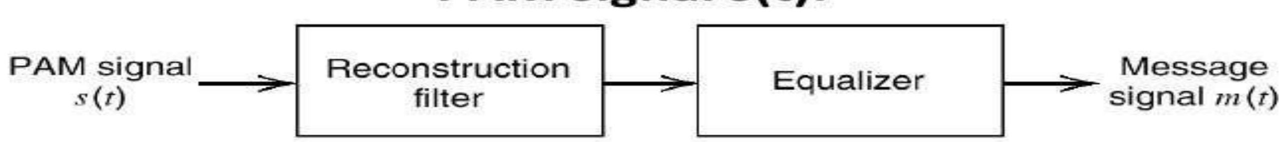


/*

Block diagram of PAM generation



System for recovering message signal m(t) from PAM signal s(t).



Advantages of Pulse Amplitude Modulation (PAM):

- It is the base for all digital modulation techniques and it is simple process for both modulation and demodulation technique.
- No complex circuitry is required for both transmission and reception. Transmitter and receiver circuitry is simple and easy to construct.
- PAM can generate other pulse modulation signals and can carry the message or information at same time.

Disadvantages of Pulse Amplitude Modulation (PAM):

- Bandwidth should be large for transmitting the pulse amplitude modulation signal. Due to Nyquist criteria also high bandwidth is required.
- The frequency varies according to the modulating signal or message signal. Due to these variations in the signal frequency, interferences will be there. So noise will be great. For PAM, noise immunity is less when compared to other modulation techniques. It is almost equal to amplitude modulation.
- Pulse amplitude signal varies, so power required for transmission will be more, peak power is also, even at receiving more power is required to receive the pulse amplitude signal.

Applications of Pulse Amplitude Modulation (PAM):

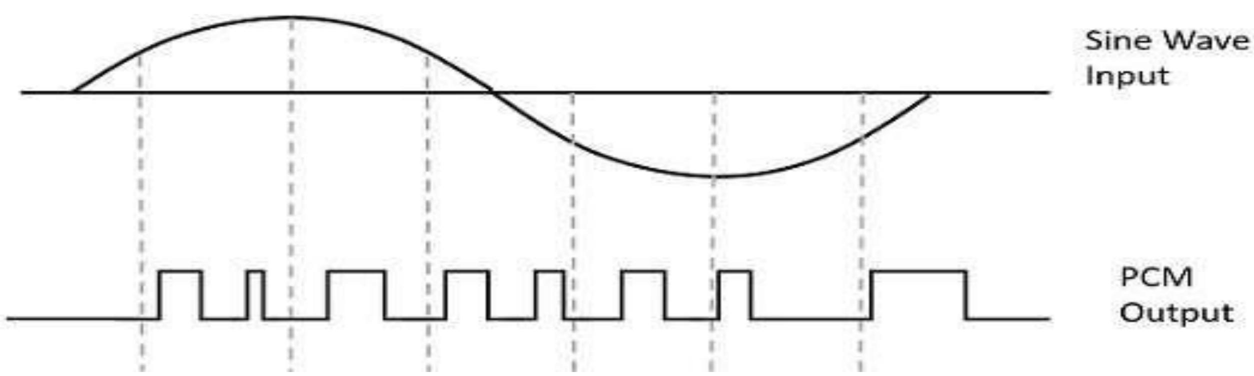
- It is mainly used in Ethernet which is type of computer network communication, we know that we can use Ethernet for connecting two systems and transfer data between the systems. Pulse amplitude modulation is used for Ethernet communications.
- It is also used for photo biology which is a study of photosynthesis.
- Used as electronic driver for LED lighting.
- Used in many micro controllers for generating the control signals etc.

DIGITAL PULSE MODULATION

Modulation is the process of varying one or more parameters of a carrier signal in accordance with the instantaneous values of the message signal.

1. PULSE CODE MODULATION(PCM)

The message signal is the signal which is being transmitted for communication and the carrier signal is a high frequency signal which has no data, but is used for long distance transmission.
There are many modulation techniques, which are classified according to the type of modulation employed. Of them all, the digital modulation technique used is **Pulse Code Modulation (PCM)**.
A signal is pulse code modulated to convert its analog information into a binary sequence, i.e., **1s** and **0s**. The output of a PCM will resemble a binary sequence. The following figure shows an example of PCM output with respect to instantaneous values of a given sine wave.

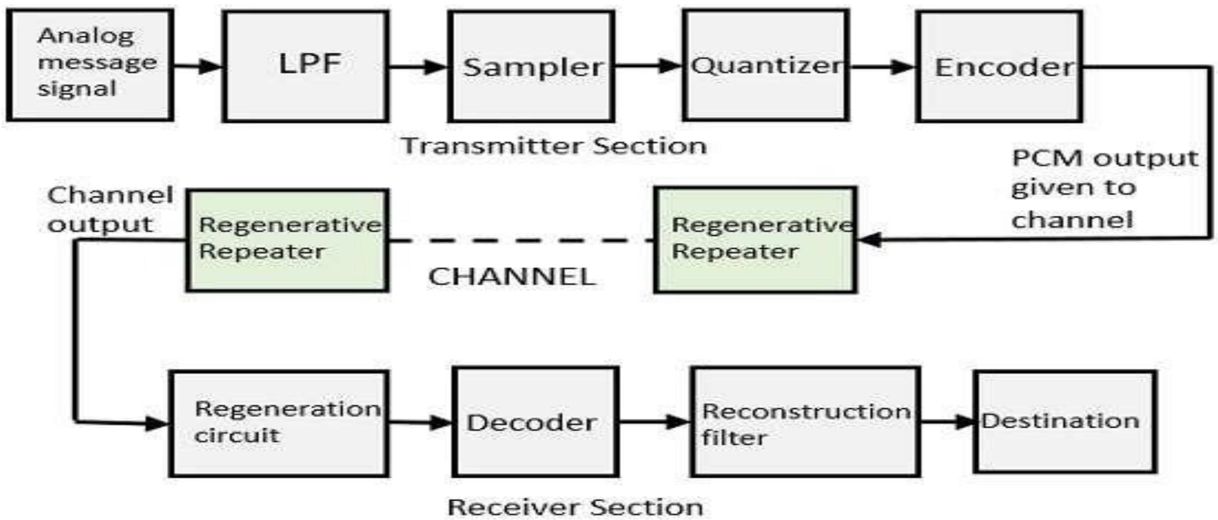


Instead of a pulse train, PCM produces a series of numbers or digits, and hence this process is called as **digital**. Each one of these digits, though in binary code, represent the approximate amplitude of the signal sample at that instant.
In Pulse Code Modulation, the message signal is represented by a sequence of coded pulses. This message signal is achieved by representing the signal in discrete form in both time and amplitude.

Basic Elements of PCM

The transmitter section of a Pulse Code Modulator circuit consists of **Sampling, Quantizing** and **Encoding**, which are performed in the analog-to-digital converter section. The low pass filter prior to sampling prevents aliasing of the message signal.

The basic operations in the receiver section are **regeneration of impaired signals, decoding,** and **reconstruction** of the quantized pulse train. Following is the block diagram of PCM which represents the basic elements of both the transmitter and the receiver sections.



- **Low Pass Filter**

This filter eliminates the high frequency components present in the input analog signal which is greater than the highest frequency of the message signal, to avoid aliasing of the message signal.

- **Sampler**

This is the technique which helps to collect the sample data at instantaneous values of message signal, so as to reconstruct the original signal. The sampling rate must be greater than twice the highest frequency component **W** of the message signal, in accordance with the sampling theorem.

- **Quantizer**

Quantizing is a process of reducing the excessive bits and confining the data. The sampled output when given to Quantizer reduces the redundant bits and compresses the value.

- **Encoder**

Encoder assigns code words to quantized sampled values. This coding techniques uses bits 0 and 1. If number of quantized levels are 16 then each sample is assigned with 4 bit code word.

- **Regenerative repeater:**

The PCM has an ability to control the distortion and noise caused by the transmission of bits along the channel. This ability is accomplished by several regenerative repeaters located at sufficient placing along channel.

Regenerative repeaters have three functions.

1. Equalizing
2. Timing circuits
3. Decision making device

Equalizer shapes the received pulse so as to compensate amplitude and phase distortion caused by the channel.

Timing circuits provides periodic pulse trains.

- Decision making device compares amplitude of equalized pulse plus noise to the pre-defined threshold levels to make decisions whether the pulse is present or not.
- If the pulse is present (i.e. decision is yes), clean new pulse is generated and transmitted through channel to next regenerative pulse. If the pulse is not present (i.e. the decision is no), it generates clean base line to next regenerative repeater, provided the noise too large caused bit error by taking the wrong decision

- **Decoder**

Decoder reboots all the received bits to make more words then it decodes as quantized PAM signals.

- **Reconstruction Filter:**

All coded words are passed through low pass filter so that analog signal can be reconstructed from quantized PAM signal.The cut off frequency of low pass filter is f_m Hz which is equal to band width of message signal.

- **Destination**

It receives the signal from the reconstructive filter output is analog signal.

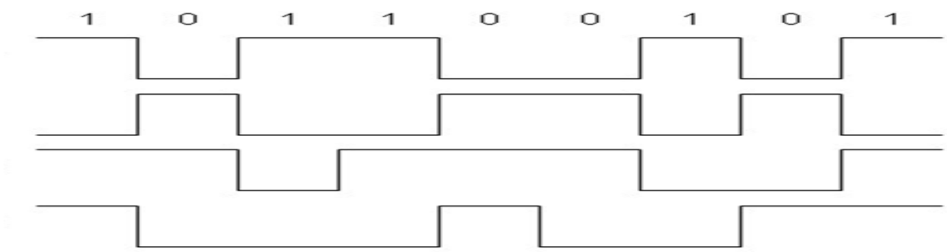


Fig.PCM waveform

<u>Bit rate and bandwidth requirements of PCM</u> :

- The bit rate of a PCM signal can be calculated form the number of bits per sample × the sampling rate. Bit rate $=nb\times fs$ The bandwidth required to transmit this signal depends on the type of line encoding used.
- A digitized signal will always need more bandwidth than the original analog signal. Price we pay for robustness and other features of digital transmission.

<u>Important Relations</u>

- Quantization Noise $(Nq)=\Delta^2/2$
- Signal to Noise ratio $(SQNR)=32.22n$ $or\ SQNR\ in\ dB=1.76+6.02n\cong(1.8+6n)dB$
- $Bit\ rate=No.of\ bits\ per\ sample\times sampling\ rate=nfs$
- Bandwidth for PCM signal =n.fm

  Where,

  n – No. of bits in PCM code

  Fm – signal bandwidth

  fs – sampling rate

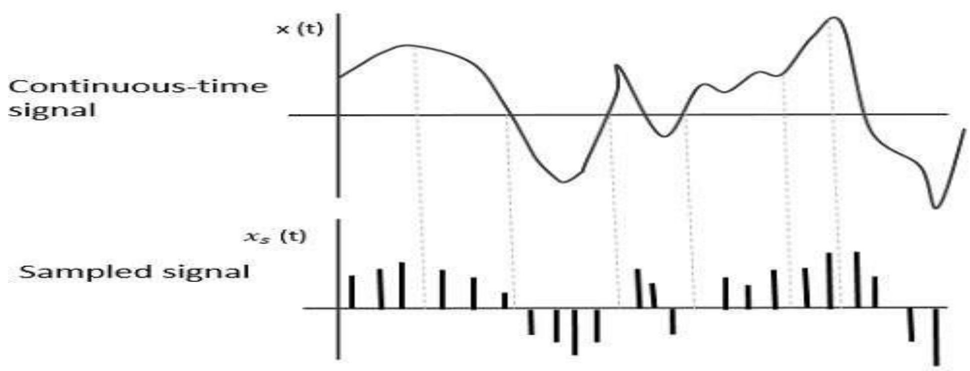
SAMPLING, QUANTIZATION AND CODING

1. Sampling

- **Definition:** Sampling is defined as –The process of measuring the instantaneous values of continuous-time signal in a discrete form.‖
- **Sample** is a piece of data taken from the whole data which is continuous in the time domain.

When a source generates an analog signal and if that has to be digitized, having **1s** and **0s** i.e., High or Low, the signal has to be discretized in time. This discretization of analog signal is called as Sampling.

The following figure indicates a continuous-time signal **x (t)** and a sampled signal **x_s (t)**. When **x (t)** is multiplied by a periodic impulse train, the sampled signal **x_s (t)** is obtained.



Sampling Rate

To discretize the signals, the gap between the samples should be fixed. That gap can be termed as a **sampling period T_s**.

Sampling Frequency $fs=1/Ts$

Where,

Ts is the sampling time

fs is the sampling frequency or the sampling rate

Sampling frequency -is the reciprocal of the sampling period. This sampling frequency, can be simply called as **Sampling rate**. The sampling rate denotes the number of samples taken per second, or for a finite set of values.

For an analog signal to be reconstructed from the digitized signal, the sampling rate should be highly considered. The rate of sampling should be such that the data in the message signal should neither be lost nor it should get over-lapped. Hence, a rate was fixed for this, called as Nyquist rate

Nyquist Rate

Suppose that a signal is band-limited with no frequency components higher than **W** Hertz. That means, **W** is the highest frequency. For such a signal, for effective reproduction of the original signal, sampling rate should be twice the highest frequency.

This means,

$$fs=2W$$

Where,

fs is the sampling rate

W is the highest frequency

This rate of sampling is called as **Nyquist rate**.

A theorem called, Sampling Theorem, was stated on the theory of this Nyquist rate.

Sampling Theorem

The sampling theorem, which is also called as **Nyquist theorem**, delivers the theory of sufficient sample rate in terms of bandwidth for the class of functions that are band limited.

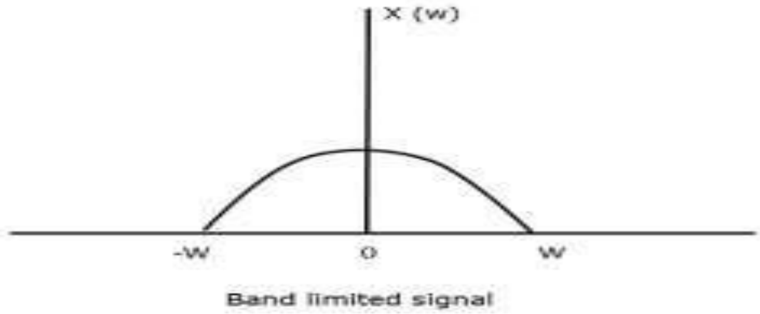
The sampling theorem states that, –a signal can be exactly reproduced if it is sampled at the rate **f_s** which is greater than twice the maximum frequency **W**.

To understand this sampling theorem, let us consider a band-limited signal, i.e., a signal whose value is **non-zero** between some **–W** and **W** Hertz.

Such a signal is represented as $x(f)=0 for |f|>W$

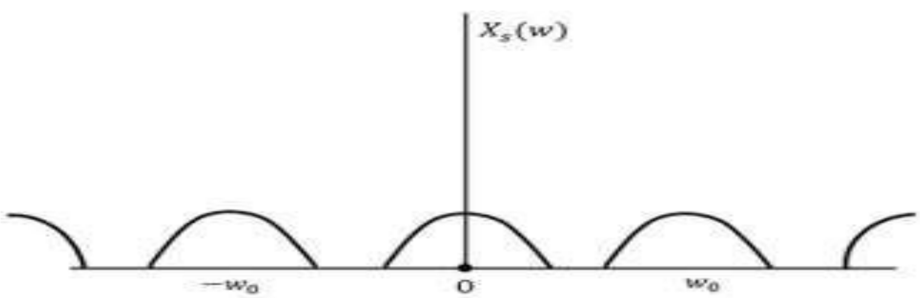
For the continuous-time signal **x (t)**, the band-limited signal in frequency domain, can be represented as shown in the following figure.

.



We need a sampling frequency, a frequency at which there should be no loss of information, even after sampling. For this, we have the Nyquist rate that the sampling frequency should be two times the maximum frequency. It is the critical rate of sampling.

If the signal **x(t)** is sampled above the Nyquist rate, the original signal can be recovered, and if it is sampled below the Nyquist rate, the signal cannot be recovered.
The following figure explains a signal, if sampled at a higher rate than **2w** in the frequency domain.



The above figure shows the Fourier transform of a signal xs(t). Here, the information is reproduced without any loss. There is no mixing up and hence recovery is possible.

The Fourier Transform of the signal
xs(t) is Xs (w)=1Ts∑n=−∞∞X(w−nw0)

Where Ts = **Sampling Period** and w0=2πTs

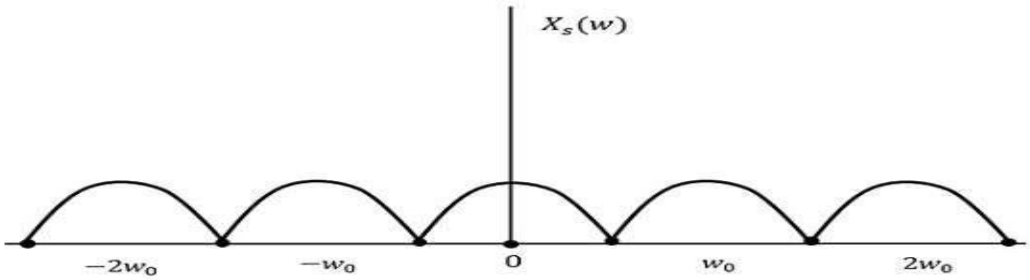
Let us see what happens if the sampling rate is equal to twice the highest
frequency (**2W**) That means,

Fs =2W

Where,

Fs is the sampling frequency

W is the highest frequency

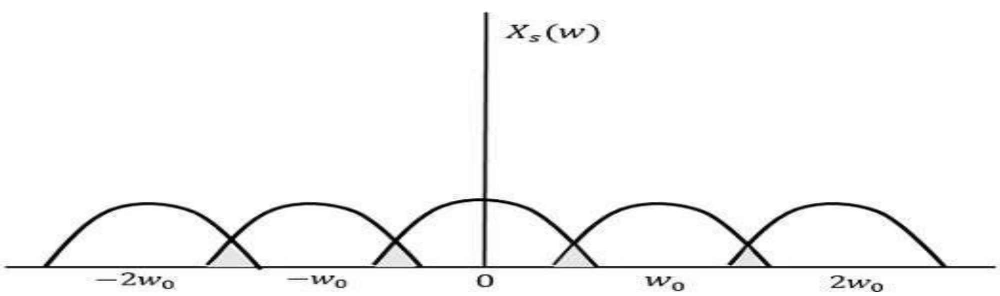


The result will be as shown in the above figure. The information is replaced without any loss. Hence, this is also a good sampling rate.

Now, let us look at the condition,

$$Fs < 2W$$

The resultant pattern will look like the following figure



We can observe from the above pattern that the over-lapping of information is done, which leads to mixing up and loss of information. This unwanted phenomenon of over-lapping is called as Aliasing

Aliasing

Aliasing can be referred to as ―the phenomenon of a high-frequency component in the spectrum of a signal, taking on the identity of a low-frequency component in the spectrum of its sampled version.‖

The corrective measures taken to reduce the effect of Aliasing are −

- In the transmitter section of PCM, a **low pass anti-aliasing filter** is employed, before the sampler, to eliminate the high frequency components, which are unwanted.
- The signal which is sampled after filtering, is sampled at a rate slightly higher than the Nyquist rate.

This choice of having the sampling rate higher than Nyquist rate, also helps in the easier design of the **reconstruction filter** at the receiver.

Scope of Fourier Transform

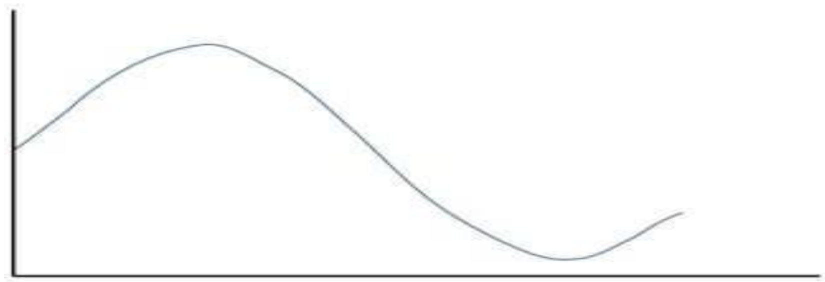
It is generally observed that, we seek the help of Fourier series and Fourier transforms in analyzing the signals and also in proving theorems. It is because −

- The Fourier Transform is the extension of Fourier series for non-periodic signals.
- Fourier transform is a powerful mathematical tool which helps to view the signals in different domains and helps to analyze the signals easily.
- Any signal can be decomposed in terms of sum of sines and cosines using this Fourier

transform. The digitization of analog signals involves the rounding off of the values which are approximately equal to the analog values. The method of sampling chooses a few points on the analog signal and then these points are joined to round off the value to a near stabilized value. Such a process is called as **Quantization**.

Quantizing an Analog Signal

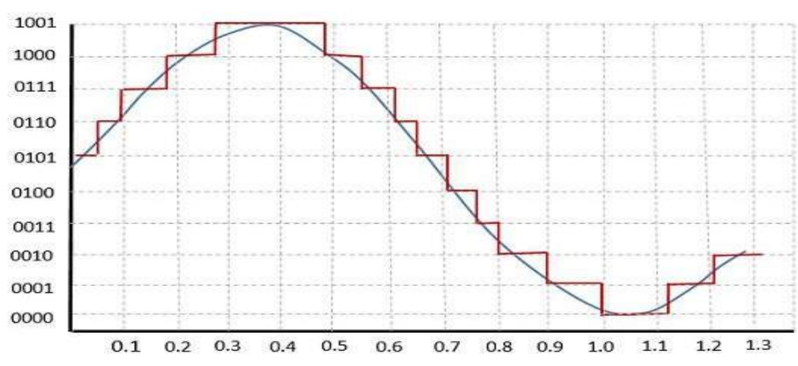
The analog-to-digital converters perform this type of function to create a series of digital values out of the given analog signal. The following figure represents an analog signal. This signal to get converted into digital has to undergo sampling and quantizing



The quantizing of an analog signal is done by discretizing the signal with a number of quantization levels.

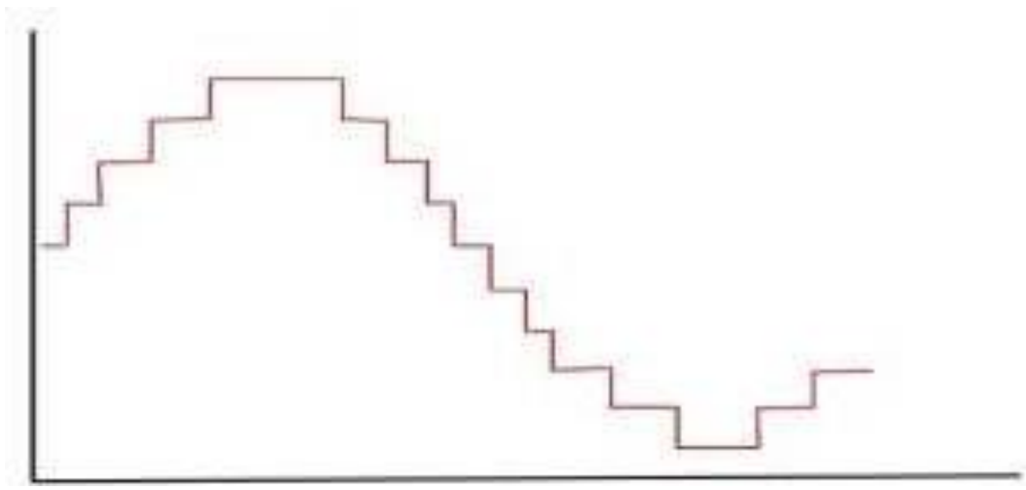
Quantization is representing the sampled values of the amplitude by a finite set of levels, which means converting a continuous-amplitude sample into a discrete-time signal.

The following figure shows how an analog signal gets quantized. The blue line represents analog signal while the brown one represents the quantized signal.



Both sampling and quantization result in the loss of information. The quality of a Quantizer output depends upon the number of quantization levels used. The discrete amplitudes of the quantized output are called as **representation levels** or **reconstruction levels**. The spacing between the two adjacent representation levels is called a **quantum** or **step-size**.

The following figure shows the resultant quantized signal which is the digital form for the given analog signal.



This is also called as **Stair-case** waveform, in accordance with its shape.

Types of Quantization

There are two types of Quantization - Uniform Quantization and Non-uniform Quantization.

1. The type of quantization in which the quantization levels are uniformly spaced is termed as a **Uniform Quantization**.
2. The type of quantization in which the quantization levels are unequal and mostly the relation between them is logarithmic, is termed as a **Non-uniform Quantization**.

There are two types of uniform quantization.

1. Mid-Rise type
2. Mid-Tread type.

The following figures represent the two types of uniform quantization

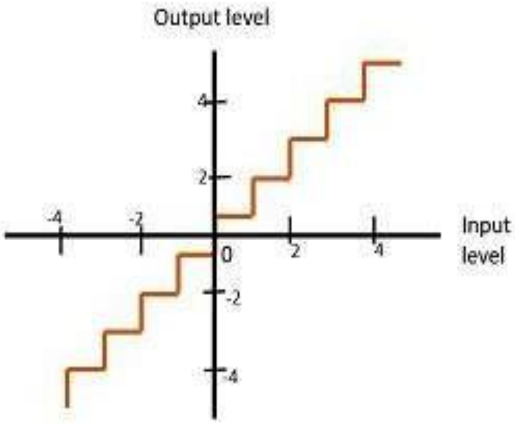


Fig 1 : Mid-Rise type Uniform Quantization

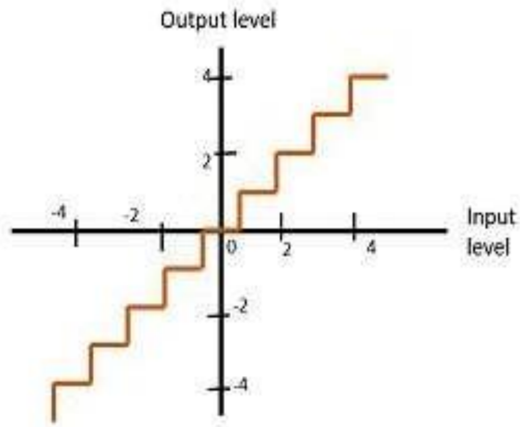


Fig 2 : Mid-Tread type Uniform Quantization

Figure 1 shows the mid-rise type and figure 2 shows the mid-tread type of uniform quantization.

1. The **Mid-Rise** type is so called because the origin lies in the middle of a raising part of the stair- case like graph. The quantization levels in this type are even in number.
2. The **Mid-tread** type is so called because the origin lies in the middle of a tread of the stair-case like graph. The quantization levels in this type are odd in number.

Both the mid-rise and mid-tread type of uniform quantizer are symmetric about the origin.

$\Delta = (max - min)L$

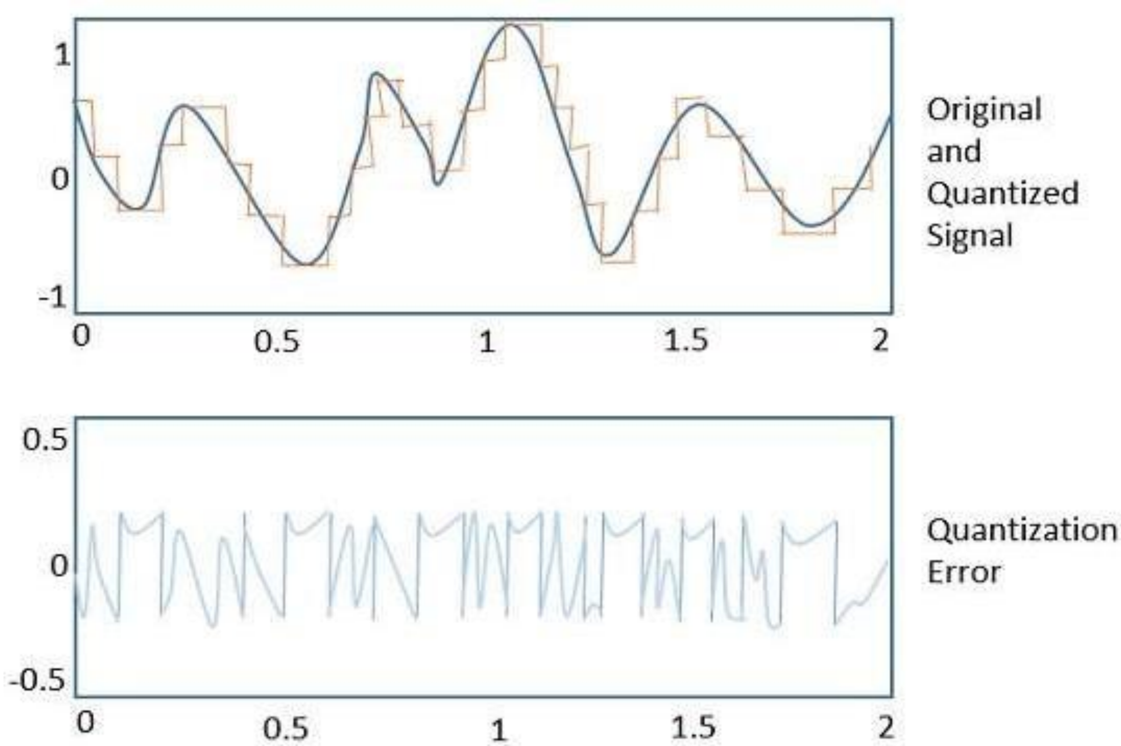
$nb = log2L$

<u>Quantization Error</u>

For any system, during its functioning, there is always a difference in the values of its input and output. The processing of the system results in an error, which is the difference of those values.The difference between an input value and its quantized value is called a **Quantization Error**.

A **Quantizer** is a logarithmic function that performs Quantization (rounding off the value). An analog-to- digital converter (**ADC**) works as a quantizer.

The following figure illustrates an example for a quantization error, indicating the difference between the original signal and the quantized signal.



Quantization Noise

It is a type of quantization error, which usually occurs in analog audio signal, while quantizing it to digital. For example, in music, the signals keep changing continuously, where a regularity is not found in errors. Such errors create a wideband noise called as Quantization Noise.

➢ COMPANDING IN PCM SYSTEMS

The word **Companding** is a combination of Compressing and Expanding, which means that it does both. This is a non-linear technique used in PCM which compresses the data at the transmitter and expands the same data at the receiver. The effects of noise and crosstalk are reduced by using this technique

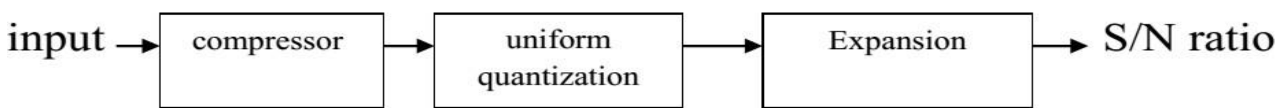


Fig. Companding

Companding means it amplifies the low level signals as well as attenuate high level at the transmitter side. At the receiver side reverse operation done. It attenuates the low level signals and amplifies the high level signals you get the original signal. Non-uniform quantization cannot be applied directly by using companding technique.

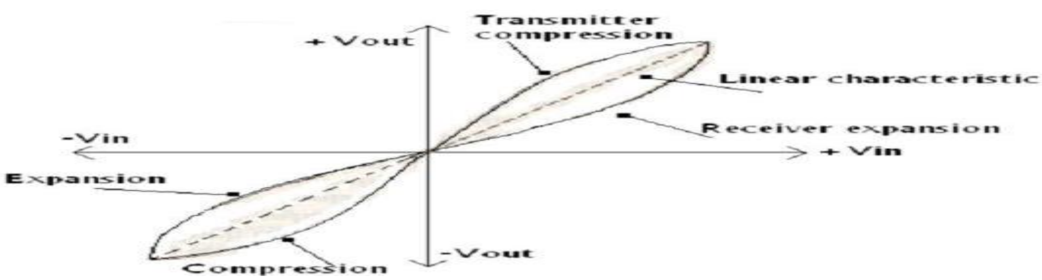


Fig Companding curves for PCM

Companding is used to maintain constant Signal to Noise Ratio throughout dynamic quantization ratio

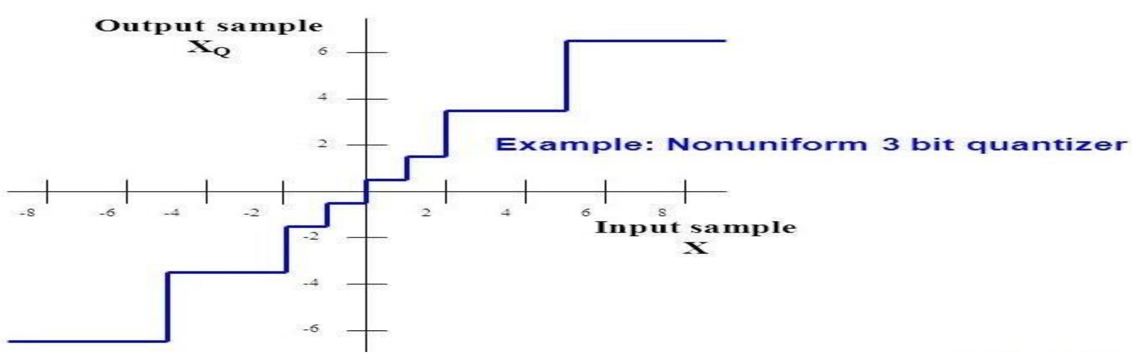


Fig. Non Uniform Quantization

There are two types of Companding techniques. They are –

1. **<u>A-law Companding Technique</u>**

i. Uniform quantization is achieved at **A = 1**, where the characteristic curve is linear and no compression is done.

ii. A-law has mid-rise at the origin. Hence, it contains a non-zero value.

iii. A-law Companding is used for PCM telephone systems.

$$Y = \frac{A|x|}{1+\ln(A)} \quad ; \text{where } 0 \le x \le 1/A$$

$$= \frac{1+\ln A|x|}{1+\ln(A)} \quad ; \quad 1/A \le x \le 1$$

practically A=87.56

if A=1 we get uniform quantization

2. **<u>μ-law Companding Technique</u>**

i. Uniform quantization is achieved at **μ = 0**, where the characteristic curve is linear and no compression is done.

ii. μ-law has mid-tread at the origin. Hence, it contains a zero value.

iii. μ-law companding is used for speech and music signals.

$$Y = \pm\frac{\ln(1+\mu|x|)}{\ln(1+\mu)} \quad ; |x| \le 1$$

Practically μ value is 256

For the samples that are highly correlated, when encoded by PCM technique, leave redundant information behind. To process this redundant information and to have a better output, it is a wise decision to take a predicted sampled value, assumed from its previous output and summarize them with the quantized values. Such a process is called as **Differential PCM (DPCM)** technique.

SAMPLING PROCESS

Due to the increased use of computers in all engineering applications, including signal processing, it is important to spend some more time examining issues of sampling. In this chapter we will look at sampling both in the time domain and the frequency domain.

We have already encountered the sampling theorem and, arguing purely from a trigonometric-identity point of view, have established the Nyquist sampling criterion for sinusoidal signals. However, we have not fully addressed the sampling of more general signals, nor provided a general proof. Nor have we indicated how to reconstruct a signal from its samples. With the tools of Fourier transforms and Fourier series available to us we are now ready to finish the job that was started months ago.

To begin with, suppose we have a signal $x(t)$ which we wish to sample. Let us suppose further that the signal is bandlimited to B Hz. This means that its Fourier transform is nonzero for $-2\pi B < \omega < 2\pi B$. Plot spectrum.

We will model the sampling process as multiplication of $x(t)$ by the "picket fence" function

$$\delta_T(t) = \mathrm{X}\delta(t - nT).$$

We encountered this periodic function when we studied Fourier series. Recall that by its Fourier series representation we can write

$$\delta_T(t) = \frac{1}{T}\sum_n e^{jn\omega_s t}$$

where $\omega_s = \frac{2\pi}{T}$. The frequency $f_s = \omega_s/(2\pi) = 1/T$ is the sampling frequency in samples/sec. Suppose that the sampling frequency is chosen so that $f_s > 2B$, or equivalently, $\omega_s > 4\pi B$.

THE SAMPLING THEOREM

If x(t) is bandlimited to B Hz then it can be recovered from signals taken at a sampling rate $f_s >$ 2B. The recovery formula is

$$x(t) = \sum_n x(nT)g(t - nT)$$

where

$$g(t) = \frac{\sin(\pi f_s t)}{\pi f_s t} = \mathrm{sinc}(\pi f_s t).$$

Show what the formula means: we are interpolating in time between samples using the sinc function.

We will prove this theorem. Because we are actually lacking a few theoretical tools, it will take a bit of work. What makes this interesting is we will end up using in a very essential way most of the transform ideas we have talked about.

$$\overline{X}(\omega) = \frac{1}{T}\sum_n X(\omega - n\omega_s)$$

1. The first step is to notice that the spectrum of the sampled signal,

is periodic and hence has a Fourier series. The period of the function in frequency is ω_s, and the fundamental frequency is

$$p_0 = \frac{2\pi}{\omega_s} = \frac{1}{f_s} = T.$$

By the fourier.Series. we can write

$$\overline{X}(\omega) = \sum_n c_n e^{jn\omega T}$$

where the c_n are the F.S. coefficients

$$c_n = \frac{1}{\omega_s}\int_{\omega_s} \overline{X}(\omega) e^{-jn\omega T} d\omega = \frac{2\pi}{\omega_s}\frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{T} X(\omega) e^{-jn\omega T} d\omega.$$

But the integral is just the inverse F.T., evaluated at $t = -nT$:

$$c_n = \frac{1}{T}\,\frac{2\pi}{\omega_s} x(t)\Big|_{t=-nT} = x(-nT),$$

so

$$\overline{X}(\omega) = \sum_n x(-nt) e^{jn\omega T} = \sum_n x(nt) e^{-jn\omega T}.$$

2. Let $g(t) = \mathrm{sinc}(\pi f_s t)$. Then

$$g(t) \Leftrightarrow T\,\mathrm{rect}\left(\frac{\omega}{2\pi f_s}\right).$$

3. Let

$$y(t) = \sum_n x(nT)g(t - nT)$$

We will show that $y(t) = x(t)$ by showing that $Y(\omega) = X(\omega)$. We can compute the F.T. of $y(t)$ using linearity and the shifting property:

$$Y(\omega) = \sum_n x(nT)T\,\mathrm{rect}\left(\frac{\omega}{2\pi f_s}\right)e^{-j\omega nT} = T\,\mathrm{rect}\left(\frac{\omega}{2\pi f_s}\right)\sum_n x(nT)e^{-j\omega nT}$$

Observe that the summation on the right is the same as the F.S. we derived in step 1:

$$Y(\omega) = T\,\mathrm{rect}\,\frac{\omega}{2\pi f_s}\overline{X}(\omega).$$

Now substituting in the spectrum of the sampled signal (derived above)

$$Y(\omega) = T\,\mathrm{rect}\left(\frac{\omega}{2\pi f_s}\right)\left(\frac{1}{T}\sum_n X(\omega - n\omega_s)\right) = X(\omega)$$

since $x(t)$ is bandlimited to $-\pi f_s < \omega < \pi f_s$ or $-f_s/2 < f < f_s/2$.

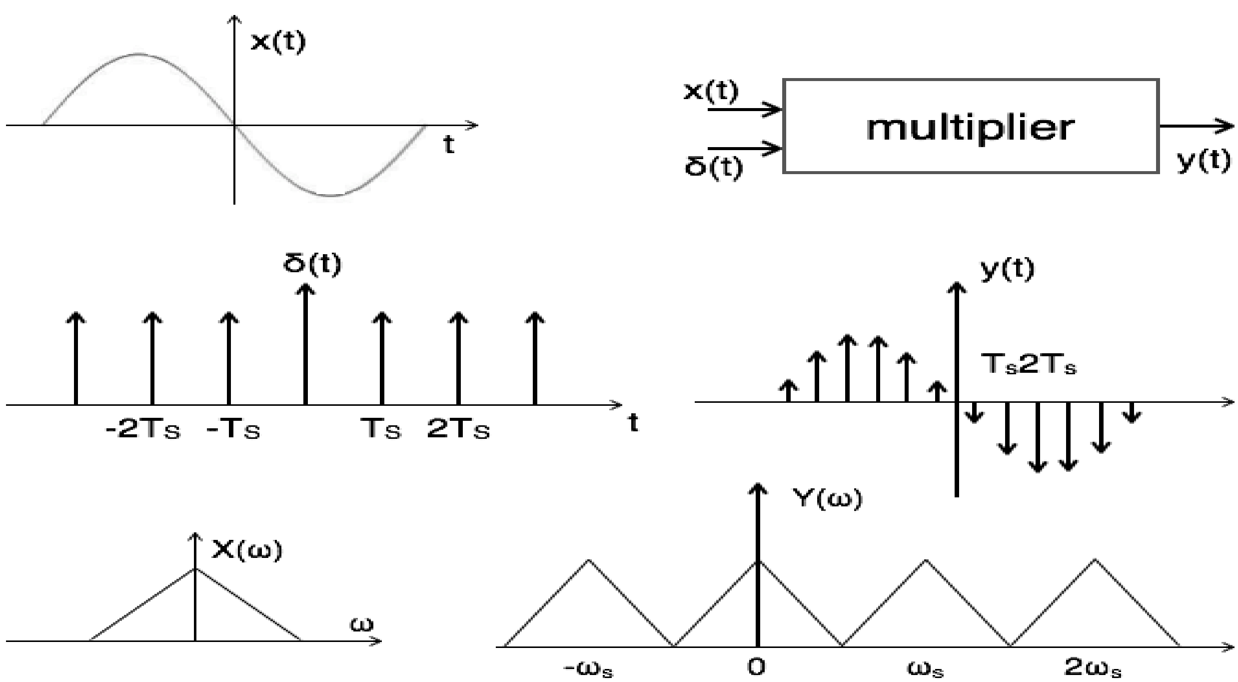


Fig. Sampling

Notice that the reconstruction filter is based upon a sinc function, whose transform is a rect function: we are really just doing the filtering implied by our initial intuition.In practice, of course, we want to sample at a frequency higher than just twice the bandwidth to allow room for filter rolloff

- **QUANTIZATION**

Quantization approximates the sampled value to nearest discrete value from the set of finite discrete levels. Quantizers are of two types.

1. Mid treed quantizer
2. Mid rise quantizer

Quantizing step size, $\Delta=(x_{max}-x_{min})/q$

Q=number of quantized level

$$\Delta=(x_{max}-x_{min})/2^n$$

Where n is number of bits used to represent each level

1. **Mid treed Quantizer**

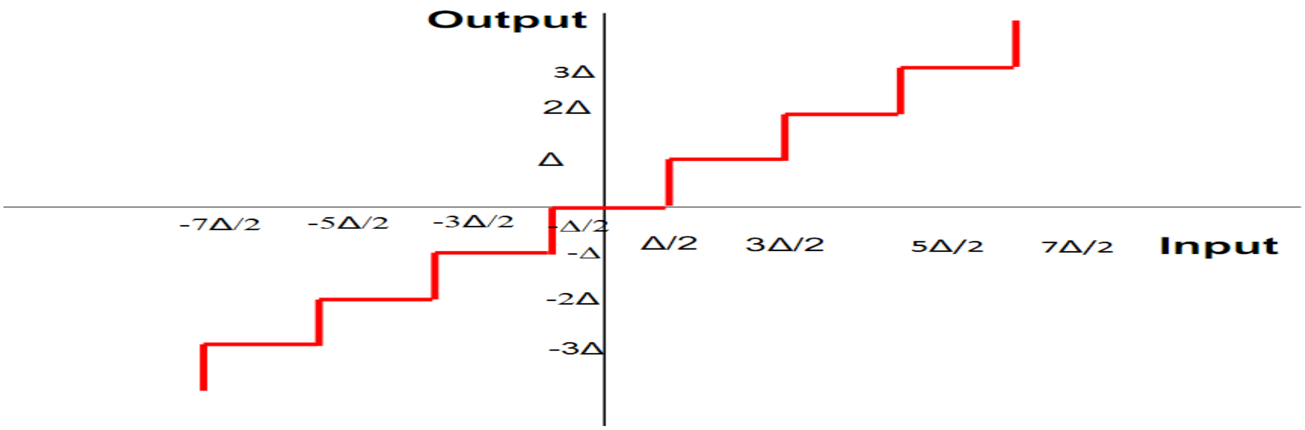


Fig. Mid tred Quantizer

Error is $\pm\ \Delta/2$

Quantization error=quantized value-actual sampled value

$Q_e = x_q(nT_s) - x(nT_s)$.

In mid treed quantization the input values lies between ± Δ/2, ± 3Δ/2, ± 5Δ/2, . . . in that output values are quantized values at ± Δ,± 2Δ,±3Δ,……. Suppose the input (i.e. sampled value) lies between ± Δ/2 which is approximated as zero. If the input values lies between Δ/2 to 3Δ/2 this quantizer approximates sampled value as Δ. Here the origin of treed of stair case lies at midpoint so the name is called mid treed quantizer. In that maximum quantization error is Δ/2 and minimum quantization error is -Δ/2.

2. MidRise Quantizer:-

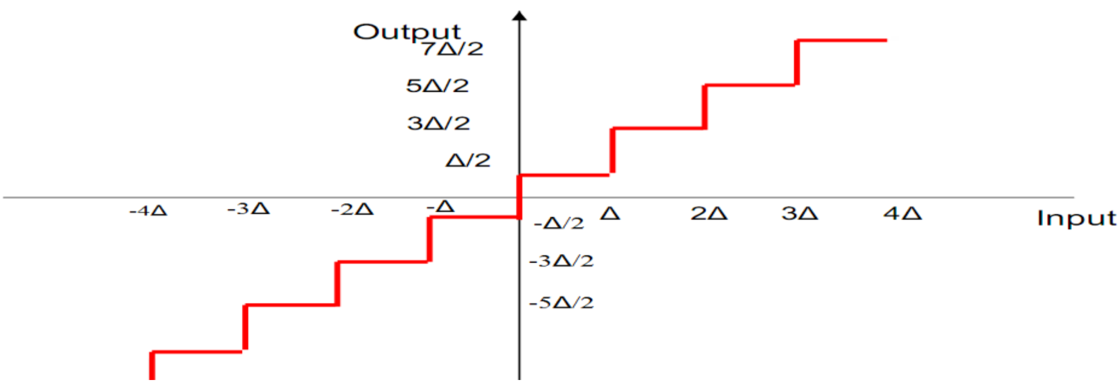


Fig. MidRise Quantizer

In mid rise quantizer the input values are ± Δ,± 2Δ,± 3Δ,….. the quantized values are ± Δ/2,± 3Δ/2,± 5Δ/2,….   The quantization error is ± Δ/2

Quantization error

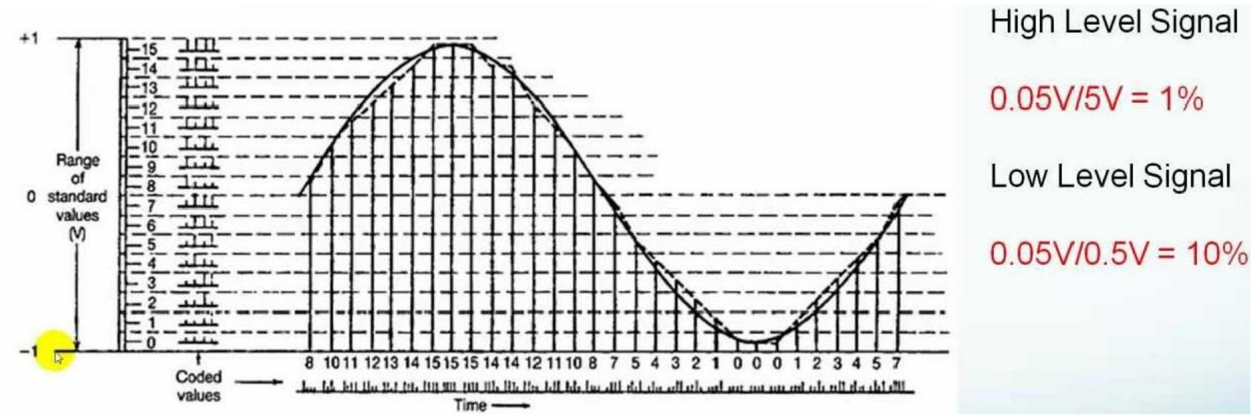


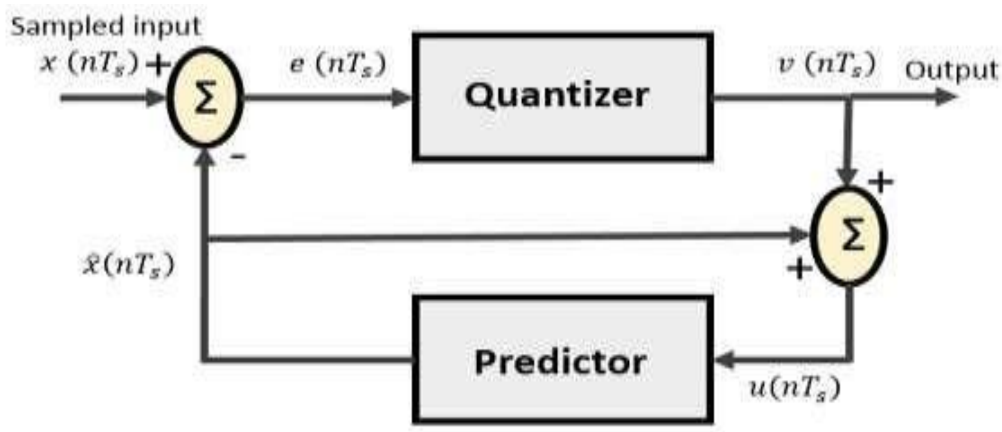
Fig. Quantization effect in PCM

In mid treed quantization the input values lies between ± Δ/2, ± 3Δ/2, ± 5Δ/2, . . . in that output values are quantized values at ± Δ,± 2Δ,±3Δ,……. Suppose the input (i.e. sampled value) lies between ± Δ/2 which is approximated as zero. If the input values lies between Δ/2 to 3Δ/2 this quantizer approximates sampled value as Δ. Here the origin of treed of stair case lies at midpoint so the name is called mid treed quantizer. In that maximum quantization error is Δ/2 and minimum quantization error is -Δ/2.

2. DIFFERENTIAL PCM (DPCM)

DPCM Transmitter

The DPCM Transmitter consists of Quantizer and Predictor with two summer circuits. Following is the block diagram of DPCM transmitter.



The signals at each point are named as −

i. x(nTs) is the sampled input
ii. x^(nTs) is the predicted sample
iii. e(nTs) is the difference of sampled input and predicted output, often called as prediction error
iv. v(nTs) is the quantized output
v. u(nTs) is the predictor input which is actually the summer output of the predictor output and the quantizer output

The predictor produces the assumed samples from the previous outputs of the transmitter circuit. The input to this predictor is the quantized versions of the input signal x(nTs).

Quantizer Output is represented as −

$$v(nTs)=Q[e(nTs)]$$

$$=e(nTs)+q(nTs)$$

Where,

q (nT_s) is the quantization error

Predictor input is the sum of quantizer output and predictor output,

$$u(nTs)=x\hat{}(nTs)+v(nTs)$$

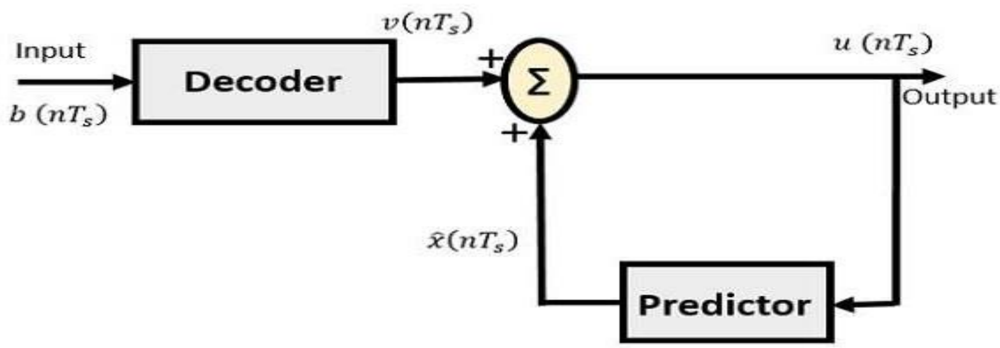
$$u(nTs)=x\hat{}(nTs)+e(nTs)+q(n$$

$$Ts)\ u(nTs)=x(nTs)+q(nTs)$$

The same predictor circuit is used in the decoder to reconstruct the original input

DPCM Receiver

The block diagram of DPCM Receiver consists of a decoder, a predictor, and a summer circuit. Following is the diagram of DPCM Receiver.



The notation of the signals is the same as the previous ones. In the absence of noise, the encoded receiver input will be the same as the encoded transmitter output.As mentioned before, the predictor assumes a value, based on the previous outputs. The input given to the decoder is processed and that output is summed up with the output of the predictor, to obtain a better output.

The sampling rate of a signal should be higher than the Nyquist rate, to achieve better sampling. If this sampling interval in Differential PCM is reduced considerably, the sampleto-sample amplitude difference is very small, as if the difference is **1-bit quantization**, then the step-size will be very small i.e., **Δ** (delta).

Advantages of DPCM

1) Bandwidth Requirement Of DPCM Is Less Compared To PCM
2) Quantization Error Is Reduced Because Of Prediction Filter.
3) Numbers Of Bits Used To Represent .One Sample Value Are Also Reduced Compared To PCM

3. DELTA MODULATION

The sampling rate of a signal should be higher than the Nyquist rate, to achieve better sampling. If this sampling interval in Differential PCM is reduced considerably, the sample-to-sample amplitude difference is very small, as if the difference is **1-bit quantization**, then the step-size will be very small i.e., **Δ** (delta).

The type of modulation, where the sampling rate is much higher and in which the step size after quantization is of a smaller value **Δ**, such a modulation is termed as **delta modulation**.

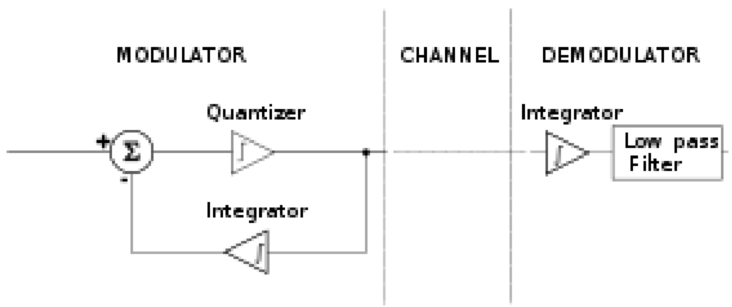


Fig. Block diagram of delta modulator and demodulator

Features of Delta Modulation

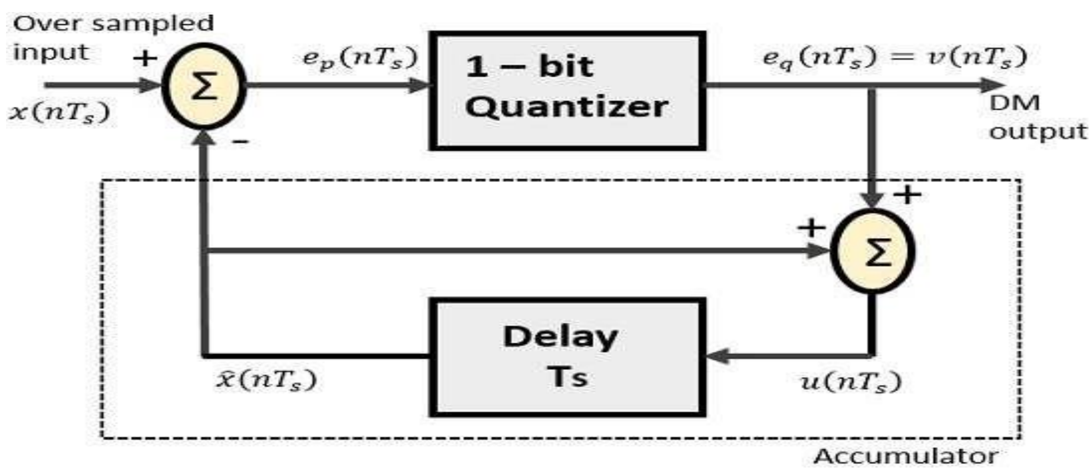
Following are some of the features of delta modulation.

- An over-sampled input is taken to make full use of the signal correlation.
- The quantization design is simple.
- The input sequence is much higher than the Nyquist rate.
- The quality is moderate.
- The design of the modulator and the demodulator is simple.
- The stair-case approximation of output waveform.
- The step-size is very small, i.e., **Δ** (delta).
- The bit rate can be decided by the user.
- This involves simpler implementation.

Delta Modulation is a simplified form of DPCM technique, also viewed as **1-bit DPCM scheme**. As the sampling interval is reduced, the signal correlation will be higher.

- **Delta Modulator**

The Delta Modulator comprises of a 1-bit quantizer and a delay circuit along with two summer circuits. Following is the block diagram of a delta modulator.



The predictor circuit in DPCM is replaced by a simple delay circuit in DM.

From the above diagram, we have the notations as −

- x(nTs) = over sampled input
- ep(nTs) = summer output and quantizer input
- eq(nTs) = quantizer output = v(nTs)
- x^(nTs) = output of delay circuit
- u(nTs) = input of delay circuit

Using these notations, now we shall try to figure out the process of delta modulation.

ep(nTs)=x(nTs)−xˆ(nTs) ------------------(1)

=x(nTs)−u([n−1]Ts)

=x(nTs)−[xˆ[[n−1]Ts]+v[[n−1]Ts]] ---------------(2)

Further,

v(nTs)=eq(nTs)=S∑.[ep(nTs)]------------------ (3)

u(nTs)=xˆ(nTs)+eq(nTs)

Where,

xˆ(nTs) = the previous value of the delay circuit

$$eq(nTs) = \text{quantizer output} = v(nTs)$$

Hence,

$$u(nTs)=u([n-1]Ts)+v(nTs)\text{----------------------} \quad (4)$$

The present input of the delay unit = (The previous output of the delay unit) + (the present quantizer output)

Assuming zero condition of Accumulation,

$$u(nTs)=S\sum j=1n\sum\ [ep(jTs)]$$

Accumulated version of DM output = $\sum j=1nv(jTs)$--------------------(5)

Now, note that

$$x\hat{}(nTs)=u([n-1]Ts)=\sum j=1n-1v(jTs) \text{------------}(6)$$

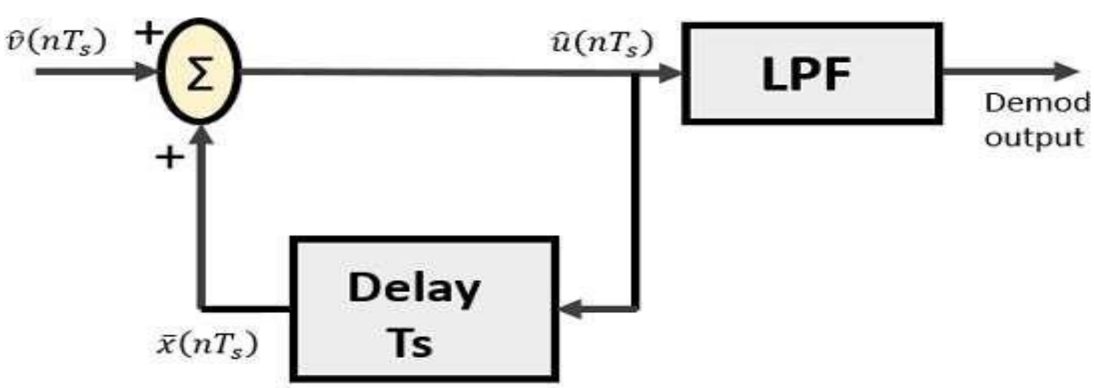
Delay unit output is an Accumulator output lagging by one sample

From equations 5 & 6, we get a possible structure for the demodulator.

A Stair-case approximated waveform will be the output of the delta modulator with the step-size as delta (**Δ**). The output quality of the waveform is moderate

Delta Demodulator

The delta demodulator comprises of a low pass filter, a summer, and a delay circuit. The predictor circuit is eliminated here and hence no assumed input is given to the demodulator. Following is the diagram for delta demodulator.



From the above diagram, we have the notations as −

- v^(nTs) is the input sample

- u^(nTs) is the summer output
- x¯(nTs) is the delayed output

A binary sequence will be given as an input to the demodulator. The stair-case approximated output is given to the LPF.

Low pass filter is used for many reasons, but the prominent reason is noise elimination for out-of-band signals. The step-size error that may occur at the transmitter is called **granular noise**, which is eliminated here. If there is no noise present, then the modulator output equals the demodulator input.

Advantages of DM Over DPCM

- 1-bit quantizer
- Very easy design of the modulator and the demodulator However, there exists some noise in DM.
- Slope Over load distortion (when **Δ** is small)
- Granular noise (when Δ is large)

Advantages of Delta Modulation

- In Delta modulation electronic circuit requirement for modulation at transmitter and for demodulation at receiver is substantially simpler compare to PCM.
- In delta modulation, amplitude of speech signal does not exceed maximum sinusoidal amplitude.
- Signaling rate and bandwidth of DPCM or delta modulation is less than PCM technique.

Disadvantages of Delta Modulation

- If changes in signal is less than the step size, then modulator no longer follow signal. Thus produces train of alternating positive and negative pulses.
- Modulator overloads when slope of signal is too high.
- High bit rate.
- It requires predictor circuit and hence it is very complex.
- Its practical usage is limited.

Delta modulation has two major drawbacks that are

1. **Slope overload distortion**

   This distortion arises because of large dynamic range of input signal.

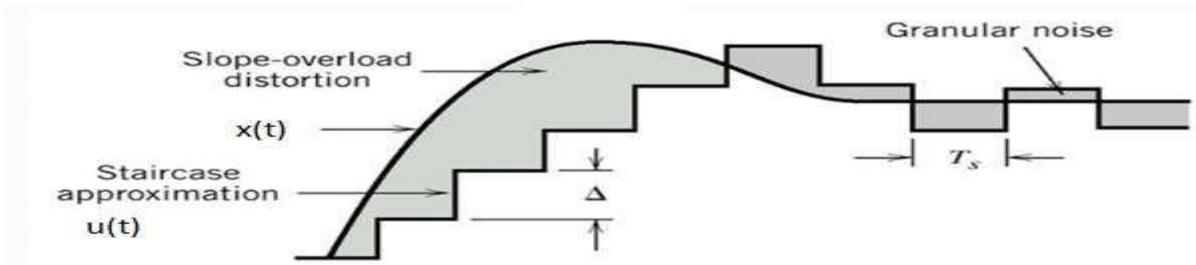


Fig.1: Quantization Errors in Delta Modulation

We can observe from fig.1 , the rate of rise of input signal x(t) is so high that the staircase signal can not approximate it, the step size _Δ‘ becomes too small for staircase signal u(t) to follow the step segment of x(t).Hence, there is a large error between the staircase approximated signal and the original input signal x(t).This error or noise is known as **slope overload distortion** .To reduce this error, the step size must be increased when slope of signal x(t) is high. Since the step size of delta modulator remains fixed, its maximum or minimum slopes occur along straight lines. Therefore, this modulator is known as Linear Delta Modulator (LDM).

2. **Granular noise**

Granular noise occurs when step size is too large compared to small variations in the input signal. This means that for very small variations in the input signal, the staircase signal is changed by large amount because of large step size. The error between the input and approximated signal is called granular noise. The solution to this problem is to make step size small. Adaptive Delta Modulation

To overcome the quantization error due to slope overload distortion and granular noise, the step size (Δ) is made adaptive to variations in input signal x(t). Particularly in the step segment of the x(t) , the step size is increased. Also, if the input is varying slowly, the step size is reduced. Then this method is known as Adaptive Delta Modulation (ADM).

The adaptive delta modulators can take continuous changes in the step size or discrete changes in the step size

4. ADAPTIVE DELTA MODULATION

In digital modulation, we have come across certain problem of determining the step-size, which influences the quality of the output wave.

A larger step-size is needed in the steep slope of modulating signal and a smaller step size is needed where the message has a small slope. The minute details get missed in the process. So, it would be better if we can control the adjustment of step-size, according to our requirement in order to obtain the sampling in a desired fashion. This is the concept of Adaptive Delta Modulation.

The performance of a delta modulator can be improved significantly by making the step size of the modulator assume a time-varying form. In particular, during a steep segment of the input signal the step size is increased. Conversely, when the input signal is varying slowly, the step size is reduced.

In this way, the size is adapted to the level of the input signal. The resulting method is called adaptive delta modulation (ADM).

There are several types of ADM, depending on the type of scheme used for adjusting the step size. In this ADM, a discrete set of values is provided for the step size.

$$\Delta(nT_s) \text{ or } 2\delta(nT_s)$$

is constrained to lie between minimum and maximum values.

The upper limit, δ_{max}, controls the amount of slope-overload distortion. The lower limit, δ_{min}, controls the amount of idle channel noise. Inside these limits, the adaptation rule for $\delta(nT_s)$ is expressed in the general form

$$\delta(nTs) = g(nTs).\ \delta(nTs - Ts) \qquad \text{------- (3.55)}$$

where the time-varying multiplier $g(nT_s)$ depends on the present binary output $b(nT_s)$ of the delta modulator and the M previous values $b(nT_s - T_s), \ldots\ldots b(nT_s - MT_s)$.

This adaptation algorithm is called a constant factor ADM with one-bit memory, where the term "one bit memory" refers to the explicit utilization of the single pervious bit $b(nT_s - T_s)$ because equation (3.55) can be written as,

$$g(nTs) = K \quad \text{if } b(nTs) = b(nTs - Ts)$$
$$g(nTs) = K^{-1} \quad \text{if } b(nTs) = b(nTs - Ts) \qquad \text{------ (3.56)}$$

This algorithm of equation (3.56), with K=1.5 has been found to be well matched to typically speech and image inputs alike, for a wide range of bit rates.

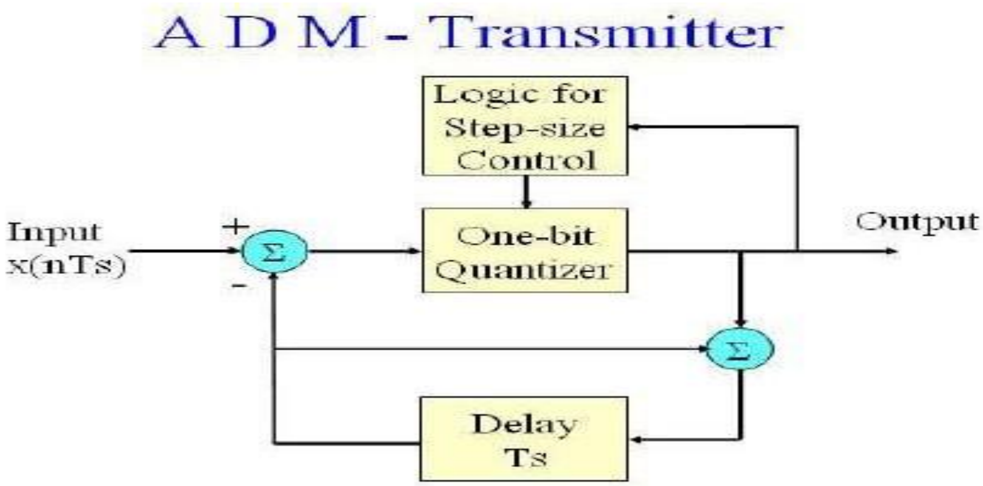


Figure: 3.17a) Block Diagram of ADM Transmitter.

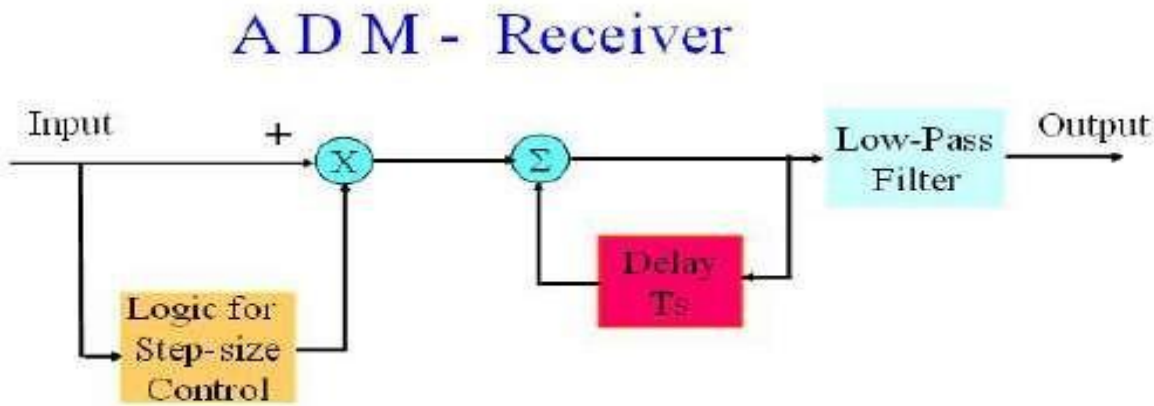


Figure: 3.17 b): Block Diagram of ADM Receiver.

A large step size was required when sampling those parts of the input waveform of steep slope. But a large step size worsened the granularity of the sampled signal when the waveform being sampled was changing slowly. A small step size is preferred in regions where the message has a small slope. This suggests the need for a controllable step size - the control being sensitive to the slope of the sampled signal.

The Implementation of ADM Modulator

The audio signal will pass through a low-pass filter, which can remove all the unwanted signal and only obtain the audio signal. The input signals of the comparator are the audio signal and triangle wave signal, and then the output of the comparator is the square wave signal. The D type flip flop is used as sampling

and then the output signal of the flip flop is the modulated ADM signal. After that the signal will feedback to tunable gain amplifier and level adjuster. In accordance with the different between the input signal x(t) and the reference signal X (t), we can change the magnitude of the gain of the tunable amplifier. If the different of the input signal and the reference signal is very large, then the level adjuster will change the gain of the t unable amplifier so that the value of Δ(t) will become large. On the other hand, if the different of the input signal and the reference signal is very small, then the level adjuster will change the gain of the tunable amplifier so that the value of Δ(t) will become small. With this advantage, when the frequency variation of the input signal is large, then we can increase the value of Δ(t) to prevent the occurrence slope overload. And when the frequency variation of the input signal is small, then we can decrease the value of Δ(t) to reduce the error.

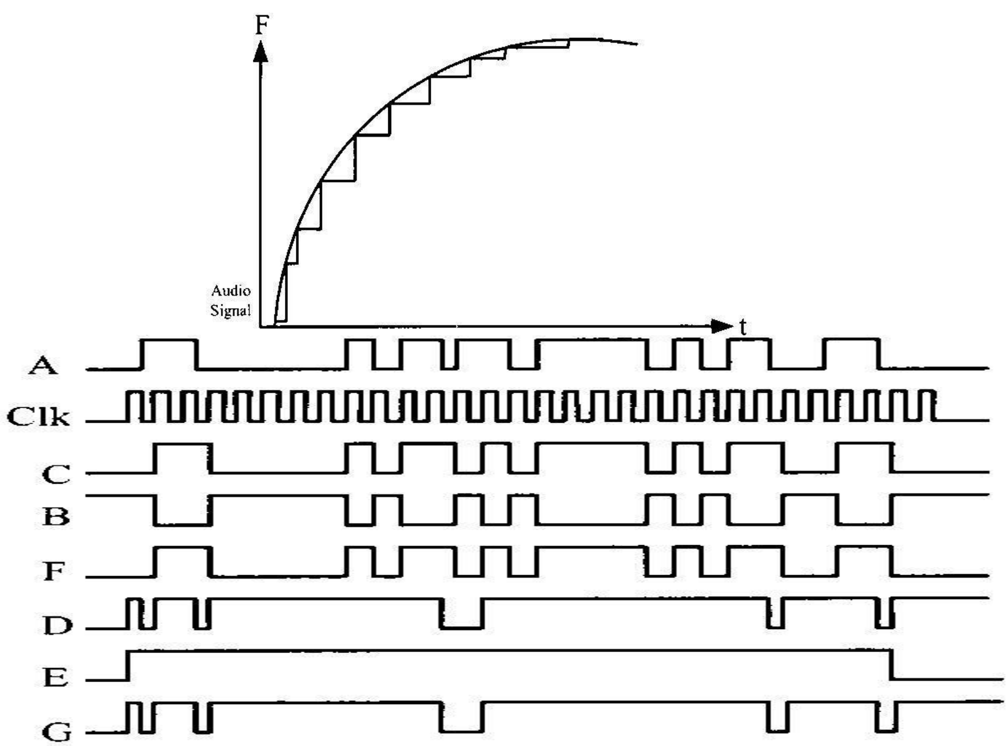


Fig. Waveforms of ADM

COMPARISON OF PCM AND DM SYSTEMS

When the analog signal is sampled, it can be quantized and encoded by any one of the following techniques-

b. Pulse code modulation (PCM)
c. Delta Modulation (DM)
d. Differential pulse code modulation (DPCM)

a. **PCM:** The analog speech waveform is sampled and converted directly into a multi bit digital code by an A/D converter. The code is stored and subsequently recalled for playback

b. **DM:** Only a single bit is stored for each sample. This bit 1 or 0, represents a greater than or less than condition, respectively as compared to the previous sample. An integrator is then used on the output to convert the stored nit stream to an analog signal.

c. **DPCM:** Stores a multibit difference value. A bipolar D/A converter is used for playback to convert the successive difference values to an analog waveform.

.

These techniques convert an analog pulse to its digital equivalent. The digital information is then transmitted over the channel. The major difference among the techniques are given below-

Sr. No.	Parameter	PCM	Delta modulation (DM)	Adaptive Delta Modulation (ADM)	Differential Pulse Code Modulation (DPCM)
1.	Number of bits	It can use 4, 8 or 16 bits per sample.	It uses only one bit for one sample.	Only one bit is used to encode one sample.	Bits can be more than one but are less than PCM.
2.	Levels, step size	The number of levels depend on number of bits. Level size is fixed.	Step size is fixed and cannot be varied.	According to the signal variation, step size varies (Adapted).	Fixed number of levels are used.
3.	Quantization error and distortion	Quantization error depends on number of levels used.	Slope overload distortion and granular noise is present.	Quantization error is present but other errors are absent.	Slope overload distortion and quantization noise is present.
4.	Bandwidth of transmission channel	Highest bandwidth is required since number of bits are high.	Lowest bandwidth is required.	Lowest bandwidth is required.	Bandwidth required is lower than PCM.
5	Feedback.	There is no feedback in transmitter or receiver.	Feedback exists in transmitter.	Feedback exists.	Feedback exists.
6	Complexity of notation	System is complex.	Simple.	Simple.	Simple.
7.	Signal to noise ratio	Good.	Poor.	Better than DM.	Fair.
8.	Area of applications	Audio and video Telephony.	Speech and images.	Speech and images.	Speech and video.

Table 2.4.1 Comparison between PCM, Adaptive Delta Modulation and Differential Pulse Code Modulation

Noise in PCM and DM systems

Signal to Quantization Noise ratio in PCM:

The signal to quantization noise ratio is given as:

$$\boxed{\frac{S}{N_q} = \frac{\text{Normalized signal power}}{\text{Normalized noise power}}}$$

$$= \frac{\text{Normalized signal power}}{\frac{\Delta^2}{12}}$$

The number of quantization value is equal to:

$$q = 2^v$$

Putting this value in eqn (6), we get

$$\Delta = \frac{2X_{max}}{2^v}$$

Substitute this value in eq, we get

$$\frac{S}{N_q} = \frac{\text{Normalized signal power}}{\left[\frac{2X_{max}}{2^v}\right]^2 * \frac{1}{12}}$$

Let the normalized signal power is equal to P then the signal to quantization noise will be given by

$$\boxed{\frac{S}{N_q} = \frac{3P * 2^{2v}}{X_{max}^2}}$$

COMPARISON OF PCM AND DM SYSTEMS

S.No	Parameter	Pulse Code Modulation	Delta Modulation
1	Number of bits	Very high, It can use 4,8 or 16 bits per sample	It uses one bit per sample
2	Quantization levels	It depends on number of bits $q=2^v$	One bit quantizer is used
3	Type of error	Quantization error	Slope overload error and granular noise
4	Signal to Noise Ratio	Very high	Moderate
5	Bandwidth	Highest bandwidth is needed since the number of bits are high	Lowest bandwidth is enough
6	Complexity	Complex system to implement	Simple to implement