MODULE-3

TELECOMMUNICATIONS TRAFFIC: Introduction, Unit of traffic, Congestion, Traffic measurement, Mathematical model, lost call systems, Queuing systems. **SWITCHING SYSTEMS:** Introduction, Single stage networks, Gradings, Link Systems, GOS of Linked systems. [Text-1]

8Hours

Chapter 1: TELECOMMUNICATIONS TRAFFIC:

3.1 Introduction:

- When any industrial plant is to be designed, an initial decision must be made as to its size, in order to obtain the desired throughput.
- In telecommunication system, it is the traffic to be handled. This determines the number of trunks to be provided.

Trunk:

- In teletraffic engineering, trunk is used to describe any entity that will carry one call.
- It may be an international circuit with a length of thousands of kilometers or a few meters of wire between switches in the same telephone exchange.

Trunking: the arrangement of trunks and switches within a telephone exchange is called trunking.

Traffic variation in minutes

- The figure 3.1 shows the average traffic in minutes. It shows the number of calls, in progress on a large telecommunication system (example: telephone exchange) made over a few minutes.
- Here the number of calls varies in a random manner as individual calls begin and end.

Traffic variation during a day

- Figure 3.2 shows the way in which the average traffic varies during the working day at a typical medium size telephone exchange or a transmission route.
- There are very few calls during the night.
- The number of calls rises as people go to work and reaches a maximum by the middle of the morning.
- It falls at the mid-day, as people go to home from working place and it has a further peak in the evening as people make social calls.
- Busy hour: It is a period of one hour, which corresponds to the peak traffic load. In figure 3.2 Busy hour is from 10 am to 11 am.

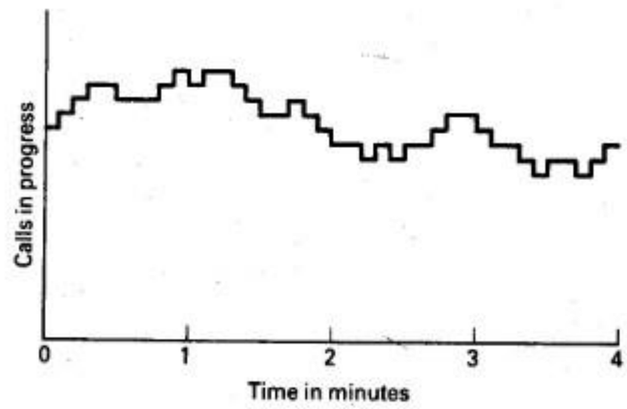
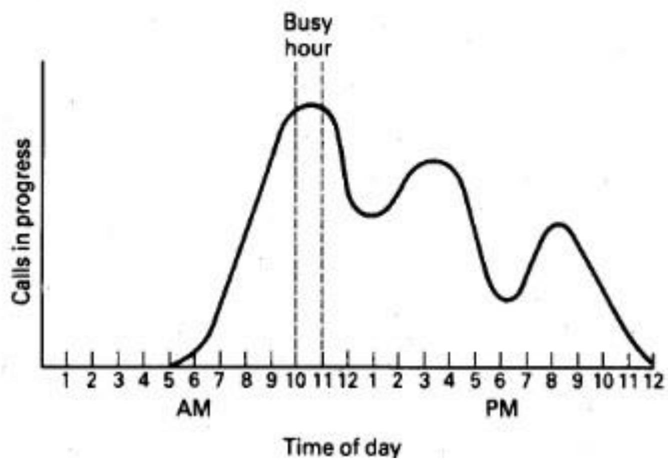


Figure 3.1 Short term traffic variation



1.2 The unit of traffic

- Traffic intensity or simple traffic is defined as the average number of calls in progress.
- The unit of traffic is Erlang [E], named after A.K Erlang the Danish pioneer of traffic.
- On a group of trunks the average number of calls in progress depends on both the number of calls which arrive and their duration.

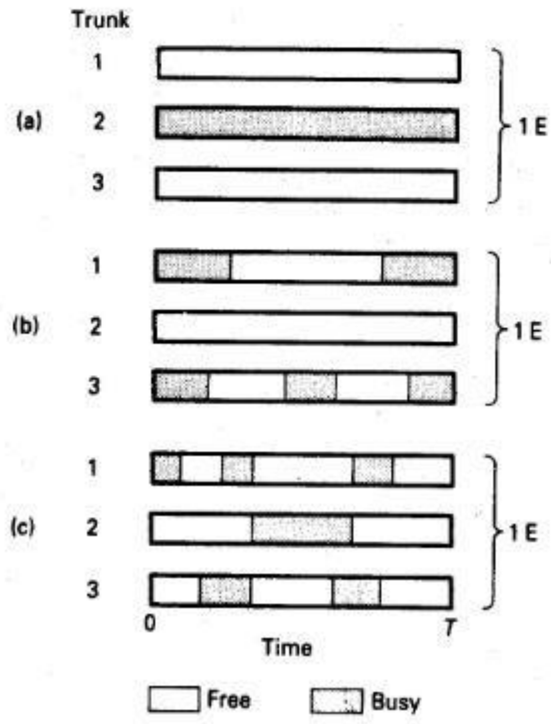


Fig 3.3 Examples of 1 Erlang of traffic carried on three trunks

Holding time:

Duration of call is often called its holding time, because its holds a trunk for that time.

The example in figure 3.3 shows how one Erlang of traffic can result from one trunk being busy all of the time, for each of two trunks being busy for half of time or from each of three trunks being busy for one third of the time as in figure a, b and c.

In North America, traffic expressed in terms of **hundreds of call seconds per hour** (CCS).

1 hour= 3600 seconds

1 Erlang=36CCS

From the definition of the Erlang that the traffic carried by the group of trunk is given by

$$A=\frac{Ch}{T} \text{……………………..(1.1)}$$

A= traffic in Erlangs

C= average number of call arrivals during time T

h= average call holding time.

If T=h, then A=C. Thus number of calls arriving during a period equal to the mean duration of the calls.

Since a single trunk cannot carry more than one call, A<=1, the traffic is a fraction of an Erlang equal to the average propagation of time for which the trunk is busy. This is called the Occupancy of the trunk.

P3.1. At certain exchange a total of 5000 calls are originated during the busy hour. If the average holding time of a call is $2\frac{1}{2}$ minutes, calculate the flow of traffic during this period.

Solution:

A= C * t

$$= \frac{5000 * 2.5}{60}$$

= 208.3 E

P3.2. On an average during the busy hour, a company makes 120 outgoing calls of average duration 2 minutes. It receives 200 incoming calls of average duration 3 minutes. Find the

a). Outgoing traffic

b) Incoming traffic

c). Total traffic

$$A_{OG} = \frac{out\ going\ Calls * h}{t} = \frac{120 * 2}{60} = 4E$$

$$A_{IC} = \frac{Incoming\ Calls * h}{t} = \frac{200 * 3}{60} = 10\ E$$

Total traffic =outgoing traffic + incoming traffic

=4+10

=14E

P3.3: During the busy hour, on average, a customer with a single telephone line makes three calls and receives three calls. The average call duration is 2 minutes. What is the probability that a caller will find the line engaged?

Occupancy of line: (3+3)*2/60 =0.1E = Probability of finding the line engaged.

3.3 Congestion

The situation, where all the trunks in a group of trunks are busy, and so it can accept no further calls, this state is known as congestion.

In message switching system, calls that arrive during congestion wait in a queue until an outgoing trunk becomes free. Thus they are delayed but not lost, such systems are queuing or delay system.

In circuit switching system (example: telephone exchange), all attempts to make a call over a congested group of trunks are unsuccessful, such systems are called as lost-call systems.

In lost call system

Traffic Carried= trffic offered- traffic lost

The proportion of calls that is lost or delayed due to congestion is a measure of the service provided. It is called as grade of service (B). For a lost call system, the grade of service, B can be defined as:

$$B=\frac{Number\ of\ calls\ lost}{Number\ of\ calls\ offered}$$

Hence, also:

$$B=\frac{Trffic\ lost}{Traffic\quad offered}$$

= Proportion of the time which congestion exists

= Probability of congestion

=Probability that a call will be lost due to congestion

Thus, if traffic A Erlangs is offered to a group of trunks having grade of service B, the traffic lost is AB and the traffic carried= A-AB =A(1-B) Erlangs

The larger the grade of service, the worse is the service given. The grade of service is normally specified for the traffic at the busy hour. At other times it is much better.

In practice, busy hour GOS can vary from, 1 in 1000 for cheap trunks inside an exchange to 1 in 100 for interexchange connections and 1 in 10 for expensive international routes.

Dimensioning problem: It is the basic problem of determining the size of a telecommunication system.

P3.4: During the busy hour, 1200calls were offered to a group of trunks and six calls were lost. The average call duration was 3 minutes. Find:

1. **the traffic offered**
2. **the traffic carried**
3. **the traffic lost**
4. **the grade of services**
5. **the total duration of the periods of congestions**

Solutions:

Traffic carried: $A=\frac{C*h}{T}=\frac{1200*3}{60}=60E$

Traffic offered: $A=\frac{C*h}{T}=\frac{1194*3}{60}=59.7E$

Traffic lost: $A = \frac{C * h}{T} = \frac{6*3}{60} = 0.3E$

B=GOS=traffic lost/ traffic carried =0.3/60=0.005

The total duration of periods of congestion=0.005*3600=18 Seconds

3.4 Traffic measurement

- It means that measuring of busy hour traffic is necessary & regularly operating company need to measure and must keep records.
- By definition, measuring the traffic carried amounts to counting calls in progress at regular intervals during the busy hour and averaging results
- In the past, engineers counted the plugs inserted in a manual switchboard or the number of selector-off normal in an automatic exchange by *peg count or switch count* method. Later automatic traffic records were used in automatic exchange.
- In modern stored program controlled system, the central processors generate records of the calls that they set up.

P3.5: Observations were made of the number of busy lines in a group of junctions at intervals of 5 minutes during the busy hour. The results obtained were: 11,13,8,10,14,12,7,,15,17,16,12

It is therefore estimated that the traffic carried, in Erlangs was:

$$\frac{11+13+8+10+14+12+7+9+15+1+16+12}{12} = 12E$$

3.5 A mathematical model

To obtain analytical solutions to teletraffic problems it is necessary to have a mathematical model of the traffic offered to telecommunication system.

A simple model is based on the following assumptions

- Pure chance traffic
- Statistical equilibrium

Pure chance traffic: The assumption of pure chance traffic means that call arrivals and call terminations are independent random events. Sometimes it is also called as Poissonian traffic.

If call arrivals are independent random events, their occurrence is not affected by previous calls. Sometimes traffic is called as memoryless traffic.

This assumption of random call arrivals & termination leads to the following results.

1. The number of call arrivals in a given time has a Poisson distribution i.e.

   $$P(x) = \frac{\mu^x}{x!} e^{-\mu}$$

Where x is the number of call arrivals in a given time T and μ is the mean number of call arrivals in time T. For this reason, pure-chance traffic is also called Possonian traffic.

2. The intervals, T, between call arrivals are the intervals between independent random event. These intervals have a negative exponential distribution, i.e,:
   $P(T \geq t)=e^{-t/T}$
   Where T is the mean interval between call arrivals.
3. Since the arrival of each call and also the intervals between two random events, call durations, T, are also the intervals between two random events and have a negative exponential distribution, i,e.,
   $P(T \geq t)=e^{-t/h}$
   Where h is the mean call duration (holding time)

The intervals, T, between call arrivals are the intervals between independent random events, have a negative exponential distribution i.e.

Since the arrival of each call and its termination are independent random events, call duration, T are also intervals between two random events have a negative exponential distribution i.e.,

Statistical equilibrium: The assumption of Statistical equilibrium means that the generation of traffic is a stationary random process i.e., probabilities do not change during the period being considered. Consequently the mean number of calls in progress remains constant.

Statistical equilibrium is not obtained immediately before the busy hour, when the calling rate is increasing nor at the end of the busy hour, when calling rate is falling.

P3.6: On average one call arrivers every 5 seconds. During a period of 10 seconds, what is the probability that:

1. **No call arrives?**
2. **One call arrives?**
3. **Two call arrives?**
4. **More than two calls arrive?**

$P(x) = \frac{\mu^x}{x!} e^{-\mu}$, Where μ = 2

1. $P(0)= \frac{2^0}{0!} e^{-2} = 0.135$
2. $P(1)= \frac{2^1}{1!} e^{-2} = 0.270$
3. $P(2) = \frac{2^2}{2!} e^{-2} = 0.270$
4. $P(>2)=1-P(0)-P(1)-P(2)$
   $= 1-0.135-0.270-0.270$
   $= 0.325$

P3.7 In a telephone system the average call duration is 2 minutes. A call has already lasted 4 minutes. What is the probability that:

1. The call will last at least another 4 minutes?
2. The call will end within the next 4 minutes?

These probabilities can be assumed to be independent of the time which has already elapsed?

1. P(T)≥ t)=$e^{-t/h}$= e^{-2} 0.135
2. P(T≤ t) =1- P(T ≥ t) = 1-0.135= 0.865

State transition diagram for N trunks

When we have N trunks the number of calls in progress varies randomly as shown in figure 3.4
It is an example of Birth and death process also called as **renewal process**. The number of calls in progress is always between 0 and N. Thus it has N+1 states and its behavior depends on the probability of change from each state to the one above and to the one below it. This is called as **simple Markov chain** as shown in figure 3.4

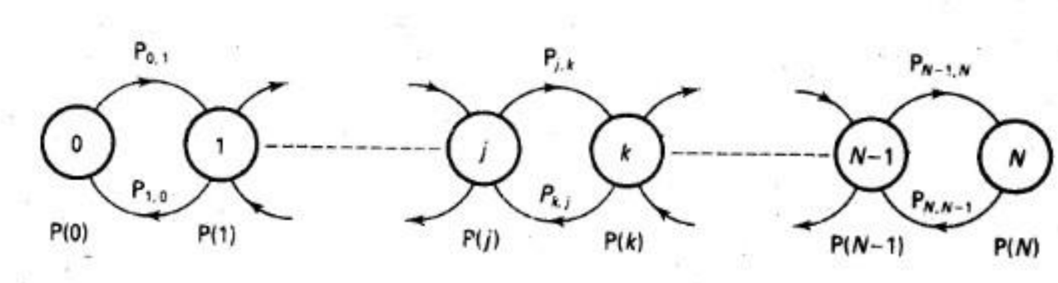


Fig 3.4 state transition diagram for N trunks

In figure P(j) is the probability of state j and P(k) is the probability of the next state higher state k, $P_{j,k}$ is the probability of a state increase to k, given that the present state is j. $P_{k,j}$ is the probability of a decrease to j, given that the present state is k. The probabilities P(0), P(1),………..P(N) are called the state probabilities and the conditional probabilities $P_{j,k}$ $P_{k,j}$, are called the transition probabilities of the Markov chain. If there is statistical equilibrium, these probabilities do not change and the process is said to be a regular Markov chain.

If there is statistical equilibrium, these probabilities do not change and process is said to be a regular Markov chain.

Consider a very small interval of time δt, starting at time t. Since δt is very small, the probability of something happening during it is small. The probability of two or more events during δt, is negligible. The events which can happen during δt are thus as follows.

One call arriving, with probability P(a).

One call ending, with probability P(e).

No change, with probability 1-P(a)-P(e).

W. k. t the traffic carried by a group of trunk is given by

$A = \frac{Ch}{T}$ ---- (1.1),

Then the mean number of calls arriving during the average holding time, h is C=A. Thus the mean number arriving during time One call arriving, with probability δt is $A\delta t / h$ << and represents the probability, P(a), of a call arriving during δt.

$P_{j.k}$=P(a)=Aδt/h.......... (1.2)

If the mean holding time is h and the number of calls in progress is k, one expects an average of k calls to end during a period h. The average number of calls ending during δt is therefore kδt/h. Since δt is very small kδt/h<<1 and represents the probability, P(e), of a call ending during δt.

$P_{k,j}$=P(e)=kδt/h .... (1.3)

If the probability of j calls in progress at time t is P(j), then the probability of a transition from j to k busy trunks during δt is:

P(j→k) = P(j) P(a) = P(j) Aδt/h ....(1.4)

If the probability of k calls in progress at time t is P(k), then the probability of a transition from k to j busy trunks during δt is:

P(k→j) = P(k) P(e) = P(k) kδt/h...........................(1.5)

The assumption of statistical equilibrium requires that P(j→k) =P(k→j). Otherwise the number of calls in progress would steadily increase or decrease. Thus, from equations (1.4) and (1.5)

K P(k) δt/h = A P(j) δt/h

P(k) $=\frac{A}{k}$ P(j) ....(1.6)

Hence: P(1) = $\frac{A}{1}$ P(0)

P(2) = $\frac{A}{2}$ P(1) = $\frac{A^2}{2.1}$ P(0)

P(3) = $\frac{A}{3}$ P(2) = $\frac{A^3}{3*2*1}$ P(0)

And, in general:

P(x) = $\frac{A^x}{x!} P(0)$…………………(1.7)

The assumption of pure chance traffic implies a very large number of sources. Thus, x can have any value between zero and infinity and the sum of their probabilities must be unity. Then

$$1 = \sum_{x=0}^{\infty} P(x) = \sum_{x=0}^{\infty} \frac{A^x}{x!} P(0) = e^A P(0)$$

$P(0) = e^{-A}$

$$P(x) = \frac{A^x}{x!} e^{-A}$$

Thus, if call arrivals have a Poisson distribution, so does the number of calls in progress. This requires an infinite number of trunks to carry the calls. If the number of trunks available is finite, then some calls can be lost or delayed and the distribution is no longer Poissonian.

3.6 Lost call systems

Theory:

Erlang determined the grade of service (i.e. the loss probability) of a lost call system having N trunks when offered traffic A as shown in figure 3.5

The solution depends on the following assumptions

- Pure chance traffic
- Statistical equilibrium
- Full availability
- Calls encounter during congestion are lost.

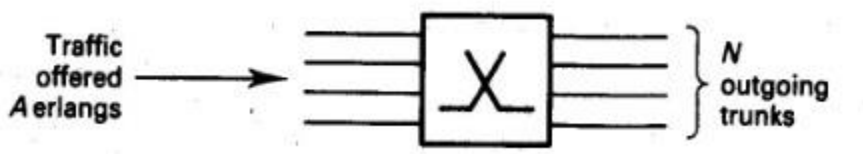


Fig 3.5 Lost call system

The pure chance traffic implies that call arrivals and call terminations are independent random events.

The Statistical equilibrium implies that probabilities do not change.

Full availability means that every call that arrives can be connected to any outgoing trunk which is free. If the incoming calls are connected to the outgoing trunks by switches, switches must have sufficient outlets to provide access to every outgoing trunk.

The lost-call assumption implies that any attempted calls which encounter congestion are immediately cleared from the system.

Here we are assuming that the traffic offered is the total arising from all successful and unsuccessful calls.

If there are x calls in progress, then equation (1.7) gives

$$P(x) = \frac{A^x}{x!}P(0)$$

However, there cannot be a negative number of calls and there cannot be more than N. Thus, we know with certainty that $0 \leq x \leq N$.

$$\sum_{x=0}^{N} P(x) = 1 = \sum_{x=0}^{N} \frac{A^x}{x!} P(0)$$

Hence, $P(0) = \frac{1}{\sum_{x=0}^{N} \frac{A^x}{x!}}$

Substituting in equation (1.7) gives:

$$P(x) = \frac{A^x / x!}{\sum_{k=0}^{N} \frac{A^k}{k!}} \text{ ....(1.8)}$$

This is called first Erlang distribution/Erlang's lost call formula.

P(N) , Since this is the probability of congestion, i.e., the probability of a lost call, which is the grade of service B, This is given the symbol $E_{1,N}(A)$ which denotes the loss probability for a full availability group of N trunks offered traffic A Erlangs.

$$B = E_{1,N}(A) = \frac{A^N / N!}{\sum_{k=0}^{N} \frac{A^k}{k!}} \text{ -------------(1.9)}$$

Equation 1.9 is a special case of equation (1.8)

The grade of service of a loss system with N full availability trunks, offered A *erlangs* of traffic, is given by $E_{1,N}(A)$.

$$B = E_{1,N-1} = \frac{A^{N-1} / (N-1)!}{\sum_{k=0}^{N-1} \frac{A^k}{k!}}$$

$$\sum_{k=0}^{N} \frac{A^k}{k!} = \frac{A^{N-1} / (N-1)!}{E_{1,N-1}(A)} + \frac{A^N}{N!}$$

Substituting in equation (1.9)

$E_{1,N-1}(A)=\frac{AE_{1,N-1(A)}}{N+AE_{1,N-1(A)}}$

Since $E_{1,0}$ =1, this iterative formula enables $E_{1,N(A)}$ to be computed for all values of N. Table shows values $E_{1,,N(A)}$.

P3.8: A group of five trunks is offered 2 E of traffic. Find:

1. **The grade of service**
2. **The probability that only one trunk is busy**
3. **The probability that only one trunk is free**
4. **The probability that at least one trunk is free.**

$$B= E_{1,N}(A) = \frac{A^N/N!}{\sum_{k=0}^{N} \frac{A^k}{k!}}$$

$$B= E_{1,N}(A)= \frac{32/120}{1+\frac{2}{1}+\frac{4}{2}+\frac{8}{6}+\frac{16}{24}+\frac{32}{120}} \quad \text{for N=5, A=2}$$

$$=\frac{0.2667}{7.2667}=0.037$$

From equation (1.8) P(1)=2/7.2667=0.275

P(4)= $\frac{16/24}{7.2667}$=0.0917

P(x<5) =1-P(5)=1-B = 1-0.0037=0.963

P3.9 A group of 20 trunks provides a grade of service of 0.01 when offered 12E of traffic.

1. **How much is the grade of service improved if one extra trunk is added to the group?**
2. **How much does the grade of service deteriorate if one trunk is out of service?**

$$E_{1,21}(12)=\frac{12*E_{1,20(12)}}{21+12E_{1,20(12)}}$$

$$=\frac{12*0.01}{21+12*0.01} = 0.0057$$

$$E_{1,20}(12)=0.01=\frac{12*E_{1,19(12)}}{20+12E_{1,19(12)}}=0.2+0.12E_{1,19}(12)= 12E_{1,19}(12)$$

$E_{1,19}(12)$=0.017

Traffic performance:

If the offered traffic, A, increases, the number of trunks, N, must obviously be increased to provide a given grade of service. However, for the same trunk occupancy the probability of finding all trunks busy is less for a large group to trunks can have a higher occupancy than a small one, i.e. the large group is more efficient. This is shown by figure 3.6. For a grade of grade of service of 0.002(i.e. one lost call in 500). For example, 2E of traffic requires seven trunks and their occupancy is 0.27E. However, 20E requires 32 trunks and their occupancy is 0.61E.

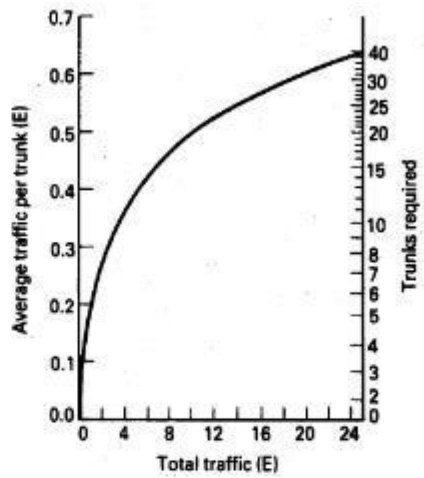


Fig 3.6 Trunk occupancies for full availability groups of various sizes (Grade of services =0.002)

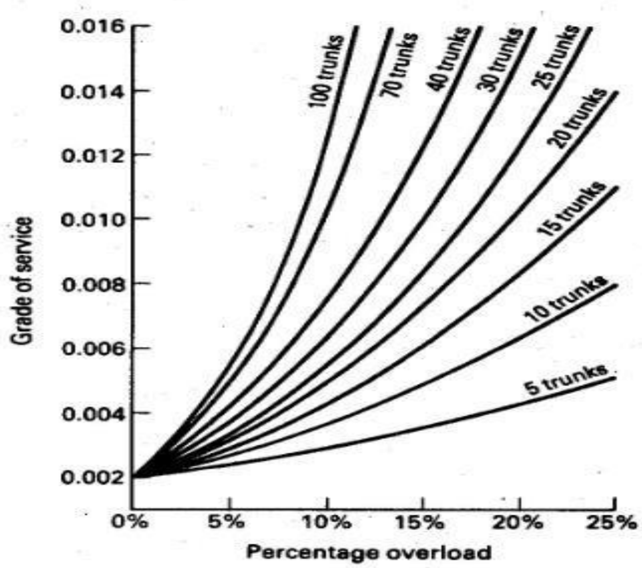


Fig 3.7 effect of overload on grade of service

The penalty paid for the high efficiency of large group of trunks is that the grade of service (GOS) deteriorates more with traffic overloads than for small Groups of trunks. Figure 3.7 shows the variation of grade of service with respect to offered traffic for different sizes of group, which were all dimensioned to provide a GOS of 0.002 at their normal traffic load. For a group of five trunk, a 10% overload increases the GOS by 40%, However, for a group of 100 trunks, it causes the GOS to increase by 550%.

For the above reason the Digital switching system operating companies uses a two criteria. Two grade of service are:

1) Normal criteria: for nominal traffic load.
2) Overload criteria: Larger GOS for normal load and overload

Sequential Selection:

In many switching systems, trunks in a group are selected by means of sequential search. A call is not connected to trunk number2 unless number1 is busy. It is not connected to number 3 unless both numner1 and nunber2 are busy and so on.

Call finding the last choice trunk is busy or lost. As a result, the first trunk has a very high occupancy and the traffic carried by subsequent trunk is less. The last choice trunk is very highly loaded indeed.

The behavior is shown in figure 3.8. The performance of such an arrangement can be analyzed as follows. Let traffic A Erlangs be offered to the group of trunks. From equation (3.9) the GOS of a single- trunk is

$E_{1,1}(A) = A/1+A$

Traffic overflowing from the first trunk to the second is

$A\ E_{1,1}(A) = A^2/1+A$

Therefore traffic carried by the first trunk is

Traffic offered-traffic lost=A- $A^2/1+A$= A/1+A

In general traffic carried by kth trunk = Traffic lost from group of first k-1 trunks – Traffic lost from group of first k trunk

$=A\ [E_{1,K-1}(A) \ - \ E_{1,K}(A)]$

The traffic offered to first trunk is Poissonian. Traffic overflowing to the second trunk is more peaky.

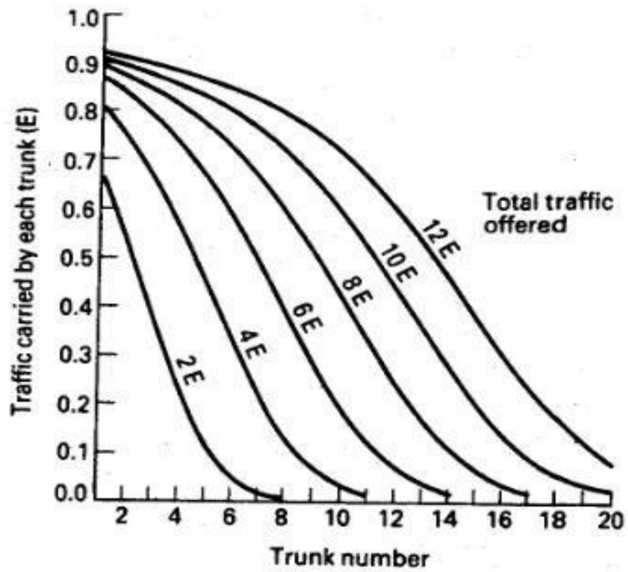


Fig 3.8 Distribution of traffic over trunks of a group with sequential search

P3.11: If sequential is used for the group of trunks five trunks is offered 2E of traffic. Then how much is the traffic carried by

1. **The first choice trunk?**
2. **The last choice trunk?**

1. The traffic carried by the first choice trunk is:

A/1+A =2/(1+2) = 0.67E

$$E_{1,5}(2) = B = E_{1,N}(A) = \frac{32/120}{1+\frac{2}{1}+\frac{4}{2}+\frac{8}{6}+\frac{16}{24}+\frac{32}{120}} \quad \text{for } N=5,\ A=2$$

$$= \frac{0.2667}{7.2667} = 0.037$$

$$E_{1,4}(2) = \frac{16/24}{1+\frac{2}{1}+\frac{4}{2}+\frac{8}{6}+\frac{16}{24}} \quad \text{for } N=4,\ A=2$$

Traffic carried b the last choice trunk =2 (0.095-0.037) =0.12E.

Loss systems in tandem

GOS of several links in tandem is explained below. For connections consisting of two links, having grade of service B1, B2, is offered traffic A Erlangs, then:

Traffic offered to second link = A(1-B1) therefore reaching destination $=A(1-B1)(1-B2)$

$= A (1+B1B2-B1-B2)$

And the overall grade of service is $B=B1+B2-B1B2$

If B1,B2 <<1, as they should be, then B1,B2 is negligible and overall grade of service is simply B=B1+B2

N general for n-trunk connection, we may write

$$B=\sum_{k=1}^{n} B_k$$

Use of traffic tables:

Table 3.1: Traffic capacity table for full- availability groups

Number of trunks	1 lost call in				Number of trunks	1 lost call in			
	50 (0.02)	100 (0.01)	200 (0.005)	1000 (0.001)		50 (0.02)	100 (0.01)	200 (0.005)	1000 (0.001)
	E	E	E	E		E	E	E	E
1	0.020	0.010	0.005	0.001	51	41.2	38.8	36.8	33.4
2	0.22	0.15	0.105	0.046	52	42.1	39.7	37.6	34.2
3	0.60	0.45	0.35	0.19	53	43.1	40.6	38.5	35.0
4	1.1	0.9	0.7	0.44	54	44.0	41.5	39.4	35.8
5	1.7	1.4	1.1	0.8	55	45.0	42.4	40.3	36.7
6	2.3	1.9	1.6	1.1	56	45.9	43.3	41.2	37.5
7	2.9	2.5	2.2	1.6	57	46.9	44.2	42.1	38.3
8	3.6	3.2	2.7	2.1	58	47.8	45.1	43.0	39.1
9	4.3	3.8	3.3	2.6	59	48.7	46.0	43.9	40.0
10	5.1	4.5	4.0	3.1	60	49.7	46.9	44.7	40.8
11	5.8	5.2	4.6	3.6	61	50.6	47.9	45.6	41.6
12	6.6	5.9	5.3	4.2	62	51.6	48.8	46.5	42.5
13	7.4	6.6	6.0	4.8	63	52.5	49.7	47.4	43.4
14	8.2	7.4	6.6	5.4	64	53.4	50.6	48.3	44.1
15	9.0	8.1	7.4	6.1	65	54.4	51.5	49.2	45.0
16	9.8	8.9	8.1	6.7	66	55.3	52.4	50.1	45.8
17	10.7	9.6	8.8	7.4	67	56.3	53.3	51.0	46.6
18	11.5	10.4	9.6	8.0	68	57.2	54.2	51.9	47.5
19	12.3	11.2	10.3	8.7	69	58.2	55.1	52.8	48.3
20	13.2	12.0	11.1	9.4	70	59.1	56.0	53.7	49.2
21	14.0	12.8	11.9	10.1	71	60.1	57.0	54.6	50.1
22	14.9	13.7	12.6	10.8	72	61.0	58.0	55.5	50.9
23	15.7	14.5	13.4	11.5	73	62.0	58.9	56.4	51.8
24	16.6	15.3	14.2	12.2	74	62.9	59.8	57.3	52.6
25	17.5	16.1	15.0	13.0	75	63.9	60.7	58.2	53.5
26	18.4	16.9	15.8	13.7	76	64.8	61.7	59.1	54.3
27	19.3	17.7	16.6	14.4	77	65.8	62.6	60.0	55.2
28	20.2	18.6	17.4	15.2	78	66.7	63.6	60.9	56.1
29	21.1	19.5	18.2	15.9	79	67.7	64.5	61.8	56.9
30	22.0	20.4	19.0	16.7	80	68.6	65.4	62.7	58.7
31	22.9	21.2	19.8	17.4	81	69.6	66.3	63.6	58.7
32	23.8	22.1	20.6	18.2	82	70.5	67.2	64.5	59.5
33	24.7	23.0	21.4	18.9	83	71.5	68.1	65.4	60.4
34	25.6	23.8	22.3	19.7	84	72.4	69.1	66.3	61.3
35	26.5	24.6	23.1	20.5	85	73.4	70.1	67.2	62.1
36	27.4	25.5	23.9	21.3	86	74.4	71.0	68.1	63.0
37	28.3	26.4	24.8	22.1	87	75.4	71.9	69.0	63.9
38	29.3	27.3	25.6	22.9	88	76.3	72.8	69.9	64.8
39	30.1	28.2	26.5	23.7	89	77.2	73.7	70.8	65.6
40	31.0	29.0	27.3	24.5	90	78.2	74.7	71.8	66.6
41	32.0	29.9	28.2	25.3	91	79.2	75.6	72.7	67.4
42	32.9	30.8	29.0	26.1	92	80.1	76.6	73.6	68.3
43	33.8	31.7	29.9	26.9	93	81.0	77.5	74.3	69.1
44	34.7	32.6	30.8	27.7	94	81.9	78.4	75.4	70.0
45	35.6	33.4	31.6	28.5	95	82.9	79.3	76.3	70.9
46	36.6	34.3	32.5	29.3	96	83.8	80.3	77.2	71.8
47	37.5	35.2	33.3	30.1	97	84.8	81.2	78.2	72.6
48	38.4	36.1	34.2	30.9	98	85.7	82.2	79.1	73.5
49	39.4	37.0	35.1	31.7	99	86.7	83.2	80.0	74.4
50	40.3	37.9	35.9	32.5	100	87.6	84.0	80.9	75.3

3.8 Queuing system:

The second Erlang distribution: The queuing system is shown in figure 3.9

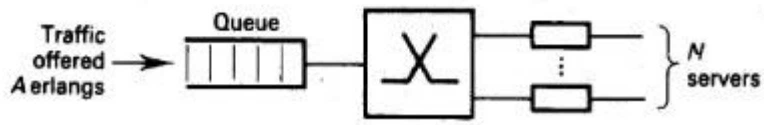


Fig 3.9 Queuing system

Here traffic A is offered to a queuing system with N trunks. Here Erlang determined the probability of encountering delay. Trunks are often called as servers

Erlang solution depends on the following assumptions

- Pure chance traffic
- Statistical equilibrium
- Full availability
- Calls which encounter congestion enter a queue and are stored there until a server becomes free.

It is some time called as M/M/N system

Note: This notation introduced by Kendall

It describes the operating system as X/Y/N Where X is the input process Y is the service time distribution and N is the numbers of servers. The following symbols are used M stands for Markov Process (i.e. random arrivals and terminations).

The pure chance traffic implies that call arrivals and call terminations are independent random events.

The Statistical equilibrium implies that probabilities do not change during the period being considered i.e. $A < N$. If $A \geq N$, calls are entering the system at a greater rate than they leave. As a result, the length of the queue must increase towards infinity. This is not statistical equilibrium.

Let x be the total number of calls in the system. Thus, when $x<N$, then x calls are being served and there is no delay. When x>, all the servers are busy and incoming calls encounter delay; there are N calls being served and x-N calls in the queue.

If $x \leq N$:

There is no queue and the behavior of the system is the same as that of a lost-call system in the absence of congestion. Thus, from equations (1.7)

$$P(x) = \frac{A^x}{x!} P(0)$$

If $x \geq N$: The probability of a call arrival in a very short period of time, δt, from equations (1.2) is given by

$$P(a) = A\delta t/h$$

Where h is the mean service time

Thus the probability of a transition from x-1 to x calls in the system during δt, from equations (1.4) is given by:

P(x-1→x) = P(x-1) P(a) = P(x-1) Aδt/h

Since all servers are busy, only the N calls being served can terminate (instead of x calls in a lost- call system). Therefore, modified equation 1.3 is given by

P(e)= N δt/h

And the probability of a transition from x to x-1 calls is given by:

P(x→x-1) = P(x) P(e) = P(k) N δt/h

For statistical equilibrium

P(x→x-1) =P(x-1→x).

P(x) N δt/h = P(x-1) A δt/h

P(x) $=\frac{A}{N}$P(x-1).............. 1.10

But P(N) $=\frac{A^N}{N!}$P(0)

P(N+1) $=\frac{A}{N}$P(N) $=\frac{A^{N+1}}{N.N!}$P(0)

P(N+2) $=\frac{A}{N}$P(N+1) $=\frac{A^{N+2}}{N^2 N!}$P(0) and so on

In general, for x ≥ N :

$$P(x)= \frac{A^x}{N^{x-N}N!} P(0) = \frac{N^N}{N!} \left(\frac{A}{N}\right)^x P(0) \qquad \text{----------(1.11)}$$

If there is no limit to the possible length of Queue, the x can have any value between zero and infinity,

$$\sum_{x=0}^{\infty} P(x) = 1$$

Thus from equations 1.7 and 1.11

$$1 \quad {}^{N-1} A^x \quad N \quad A^{x\,\infty} \quad A^k$$

$$\frac{1}{P(0)} = \sum_{x=0} \frac{}{x!} + \frac{}{N!}\left(\right) \frac{}{N} \sum_{k=0} \left(\frac{}{N}\right) \text{-------- } (1.12)$$

Where k = x-N. Since $\frac{A}{N} \leq 1$ then

$$\sum_{k=0}^{\infty} \left(\frac{A}{N}\right)^{k} = \left[1-\frac{A}{N}\right]^{-1}$$

$$\frac{1}{P(0)} = \sum_{x=0}^{N-1} \frac{A^x}{x!} + \frac{A^N}{N!}\left[1-\frac{A}{N}\right]^{-1}$$

$$\frac{1}{P(0)} = \left[\frac{N A^N}{N!(N-A)} + \sum_{x=0}^{N-1} \frac{A^x}{x!}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.13)$$

Thus, P(x) is given by equations 1.7 or 1.11 depending on whether x ≤ N or x ≥ N

N is given by equation (1.13). This is called as second Erlang distribution

Probability of delay:

Delay occurs if all servers are busy, i.e. x ≥ N. Now from equation 1.11 the probability that are at least z calls in the system (where z ≥N) is given by

P(x ≥ z)= $\sum_{x=z}^{\infty} P(x)$

$$= \frac{N^N}{N!} P(0) \sum_{x=z}^{\infty} \left(\frac{A}{N}\right)^{x}$$

$$= \frac{N^N}{N!} P(0) \left(\frac{A}{N}\right)^{z} \sum_{k=0}^{\infty} \left(\frac{A}{N}\right)^{k}$$

Where k = x-N

P(x ≥ z)= $\frac{N^N}{N!} P(0) \left(\frac{A}{N}\right)^{z} \left(1-\frac{A}{N}\right)^{-1}$ ------- (1.14)

P(x ≥ z)= $\frac{N^N}{N!} P(0) \left(\frac{A}{N}\right)^{z} \frac{N}{N-A}$

The probability of delay is P_D= P(x≥N)

$$P_D = \frac{A^N}{N!}\frac{N}{N-A}P(0) \text{ ----------} (1.15)$$

$P_D = E_{2,N}(A)$

The probability of delay, for a system with N servers offered traffic A Erlangs, is thus given by equation 1.15 where P(0) is given by equation 1.1. This formula is called as Erlang delay formula.

The probability of delay increases towards 1.0 as an increase towards N. When A>N, the length of the queue grows indefinitely, and is shown in figure 1.10.

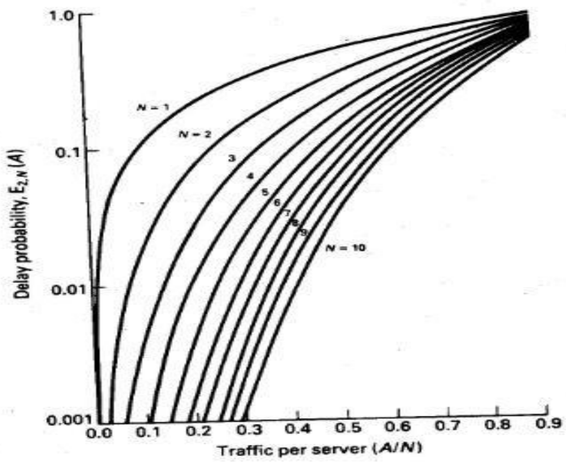


Fig 3.10 Delay probabilities for queuing systems (A= traffic in Erlangs, N=number of servers)

Finite queue capacity:

A practical system cannot contain an infinite queue. Thus, when the queue has become full, calls that arrive subsequently are lost. If the queue can only hold up to Q calls, then $x \leq Q+N$ and equation (1.12) becomes:

$$\frac{1}{P(0)} = \sum_{x=0}^{N-1} \frac{A^x}{x!} + \frac{N^N}{N!} \left(\frac{A}{N}\right)^N \sum_{k=0}^{Q} \left(\frac{A}{N}\right)^k$$

$$\frac{1}{P(0)} = \sum_{x=0}^{N-1} \frac{A^x}{x!} + \frac{A^N}{N!} \frac{1-\left(\frac{A}{N}\right)^{Q+1}}{1-\left(\frac{A}{N}\right)} \text{ - - - - - - - - } (1.16)$$

However, if the loss probability is small, there is negligible error in using equation (1.13).

The loss probability can be estimated by first assuming that the queue capacity is infinite and then calculating $P(x \geq Q+N)$

$$P(x \geq Q+N) = \frac{N^N}{N!} \left(\frac{A}{N}\right)^{Q+N} \frac{N}{N-A} P(0)$$

Now from equation 1.14

$P(x \geq Q+N) =$

$= \left(\frac{A}{N}\right)^Q P_D$..............................(1.17)

Hence, the queue capacity, Q, needed to obtain an adequately low loss probability can be found

Some other useful results

Equations 1.1 to 1.15 lead to further results,

Follows:

1. Mean number of calls in the system:
   i) When there is delay, the mean number of calls is

$$\bar{x}_1 = \frac{A}{N - A} + N$$

   ii) Averaged over all time, the mean number of calls is

$$\bar{x} = \frac{A}{N - A} E_{2,N}(A) + A$$

2. Mean length of Queue:
   a. When there is delay, the mean queue length is

$$\bar{q}_1 = \bar{x}_1 - N = \frac{A}{N - A}$$

   b. Mean length of queue averaged over all time is

$$\bar{q} = \bar{q}_1 P_D = \frac{A}{N - A} E_{2,N}(A)$$

3. Mean delay time when the queue discipline is first in first out (FIFO):
   i) When there is a delay, mean delay is $\bar{T}_1$

$$\bar{T}_1 = \frac{h}{N - A}$$

   Where h is the mean holding time

   ii) Averaged over all time, the mean delay, $\bar{T}_1$ is

$$\bar{T} = E_{2,N}(A)\bar{T}_1$$

$$= E_{2,N}(A)\frac{h}{N-A}$$

4. Distribution of delays (FIFO queue discipline):

Since the holding times have a negative exponential probability distribution, so since the holding times have a negative exponential probability distribution, so do the delays, T_D. Hence:

i). When there is delay, $P(T_D \geq t) = e^{-t/\bar{T}_1}$

ii) Averaged over all time,

$$P(T_D \geq t) = E_{2,N}$$

$(A)e^{-t/\tau}$

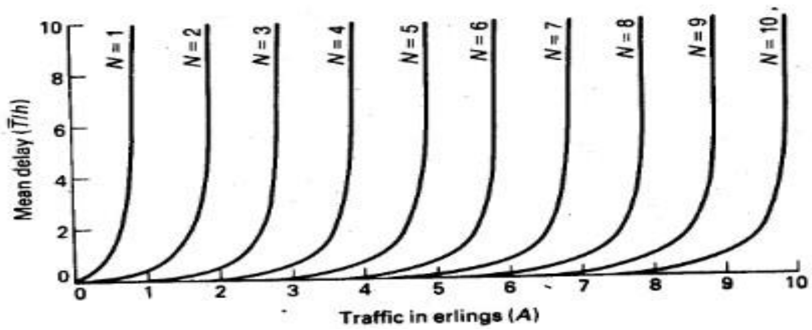


Fig 3.11 Mean delays for queuing systems with FIFO queue discipline (T=mean delay, h=mean service time, N= number of server)

System with single server:

When there is only single server, the probability of it being is simply its occupancy, A and this is the probability of delay i.e. $E_{2,N}(A)$=A. As a result, the expression from previous section yields below results

N=1, $P_D = E_{2,N}(A) = A$

$$P_D = \frac{A^N}{N!}\frac{N}{N-A}P(0)$$

Then $P(0) = P_D / \left[\frac{A^N}{N!}\frac{N}{N-A}\right]$ $P(0) = A / \left[\frac{A}{1!}\frac{1}{1-A}\right]$

P(0)= 1-A

$$\bar{x}_1 = \frac{A}{1-A} + 1 = \frac{1}{1-A}$$

$$\bar{x} = \frac{A}{1-A} * A + A = \frac{A}{1-A}$$

$$\bar{q}_1 = \bar{x}_1 - N = \frac{A}{1-A}$$

$$\bar{q} = q_1 P_D = \frac{A^2}{1-A}$$

$$\bar{T}_1 = \frac{h}{1-A}$$

$$\bar{T} = E_{2,N}(A)\bar{T}_1 = \frac{Ah}{1-A}$$

$$P(x) = \frac{N^N}{N!}\left(\frac{A}{N}\right)^x P(0) = A^x(1-A)$$

$P(x \geq z) = A^z$

Problem 3.12: A PBX has three operators on duty and receives 400 calls during the busy hour. Incoming calls enter a queue and are dealt with in order of arrival. The average time taken by an operator to handle a call is 18 seconds. Calls arrivals are Poissonian and operator service times have a negative exponential distribution.

1. **What percentage of calls have to wait for an operator to answer them?**
2. **What is the average delay, for calls and for those which encounter delay?**
3. **What percentage of calls are delayed for more than 30 seconds?**

1. A= 400 * 18/3600 =2.0E

From equation (4.13)

$$\frac{1}{P(0)} = \left[\frac{N A^N}{N!(N-A)} + \sum_{x=0}^{N-1} \frac{A^x}{x!}\right]$$

$$\frac{1}{P(0)} = \left[\frac{3\ 2^3}{3!(3-2)} + 1 + \frac{2}{1} + \frac{4}{2}\right] = 4 + 1 + 2 + 2 = 9$$

$P_0 = 1/9$

From equation 1.15

$$P_D = \frac{A^N}{N!} \frac{N}{N-A} P(0)$$

$$P_D = \frac{8}{} \frac{3}{3!3-2} \left(\frac{3}{1} + \frac{1}{9}\right) = \frac{4}{9}$$

i.e. 44% of calls have delay on answer

2. When there is delay, the mean delay is:

$$\bar{T}_1 = \frac{h}{N-A} = \frac{18}{(3-2)} = 18 \text{ seconds}$$

Where h is the mean holding time

The delay averaged over all time, the mean delay, $\bar{T}$ is

$$\bar{T} = E_{2,N}(A)\bar{T}_1 = 18 * 4/9 = 8 \text{ seconds}$$

When there is delay, $P(T_D \geq t) = e^{-t/\bar{T}} = e^{-30/18} = 18.9\%$

Averaged over all time,

$P(T_D \geq t) = E_{2,N}(A) e^{-t/\bar{T}} = 18.9 * 0.44 = 8.3\%$

Problem 3.13: A message- switching center sends message on an circuit at the rate of 480 characters per second. The average number of characters per message is 24 and the message lengths have a negative exponential distribution. The input of messages is a Poisson process and they are served in order of arrival.

How many messages can be handled per second if the mean delay averaged over al messages) is not exceed 0.5 second?

For a single server the mean delay is

$$\bar{T} = \frac{Ah}{N - A}$$

$$A = \frac{\bar{T}}{h + \bar{T}}$$

Now h=24/480= 0.05 second

A=0.5/(0.05+0.5)=0.909=Ch

Where C= number of messages per second

C= 0.909/0.05=18.2

Delay tables:

$$\sum_{k=0}^{N} \frac{A^k}{k!} = \frac{A^N / N!}{E_{1,N}(A)}$$

$$\sum_{k=0}^{N} \frac{A^k}{k!} = \frac{A^N}{N!\,E_{1,N}(A)} - \frac{A^N}{N!}$$

Substitute the above equation in P(0)

$$\frac{1}{P(0)} = \left[\frac{N A^N}{N!(N-A)} + \sum_{x=0}^{N-1} \frac{A^x}{x!}\right]$$

$$\frac{1}{P(0)} = \left[\frac{N A^N}{N!(N-A)} + \frac{A^N}{N!\,E_{1,N}(A)} - \frac{A^N}{N!}\right]$$

$$= \frac{A^N}{N!} \frac{N E_{1,N}(A) + (N-A)}{(N-A) E_{1,N}(A)}$$

Substituting in equation 1.15

$$\mathbf{P_D} = \frac{A^N}{N!} \frac{N}{N-A} P(0) \text{ ----------(1.15)}$$

$$E_{2,N}(A) = \frac{A^N}{N!} \frac{N}{N-A} \frac{N!}{A^N} \frac{(N-A)E_{1,N}(A)}{N E_{1,N}(A) + (N-A)}$$

$$=-\frac{N E_{1,N}(A)}{A\,E_{1,N}(A)+(N-A)}$$

$$=\frac{(N-A)\,E_{1,N}(A)}{A\,E_{1,N}(A)+(N-A)}$$

$$E_{2,N}(A) = \frac{N}{(N-A)} E_{1,N}(A)$$

We can calculate the, $E_{2,N}(A)$from $E_{1,N}(A)$ tables.

Queues in tandem:

Queuing system are connected in tandem the delays are cumulative. If first stage has a Poissonian input and a negative exponential distribution of holding times, The second and subsequent stages are also Poissonian. Thus the operation can be considered as independent for calculating their delays.

The delay probability and the mean delay for the complete system are the sum of these for the individual stages.

However, the probability distribution of such several random variables is obtained by convolution of the separate distributions.

This computation is difficult, so it is usual to specify for each stage the probability of delay exceeding a given value and add these probabilities to obtain a measure of the overall GOS.

This will be pessimistic estimate, because the probability of a long delay at more than one stage should be small.

Applications of delay formulae

Delay formulas are useful in two application areas

In telephone exchange and its switching network → is a circuit switching system (or a lost call system) In an exchange with registers when all registers are busy, incoming calls are lost

In queuing system Message (switching system or packet switching system). Here if outgoing trunks are busy, messages or packets enter a queue until an outgoing trunk becomes free.

Chapter2:

SWITCHING SYSTEMS: Introduction, Single stage networks, Gradings, Link Systems, GOS of Linked systems.

3.9 Introduction:

In Module2 we learnt the basic prerequisite of this switching system. The basic function of an exchange is making (switching) a connection between calling and called subscriber.

3.10 Classification of Switching Networks

The classification of switching node is based on inlets and outlets,

Single Stage switching Networks

Two Stage switching Networks

Three Stage switching Networks

Four Stage switching Networks

3.11 Single Stage networks:

Single stage switching network having M inlets and N outlets, consisting of a matrix of crosspoints. These may, for example, be separate relays or electronic devices or the contacts of a crossbar switch. The network could also be constructed by multiplying the banks of M uniselectors or one level of a group of M two motion selectors, each having N outlets. In Future system may use photonic switches, in which optoelectronics devices are used as crosspoints to make connections between optical-fiber trunks. The below figure 3.12 shows electromechanical switches, the circle indicates the side of the switch associated with the control mechanism (e.g. the wipers of a strowger switch or the bridge magnet of a crossbar switch).

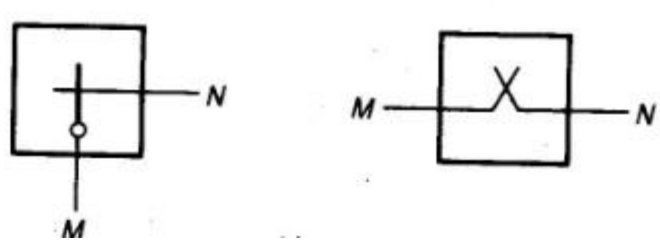
The matrix of crosspoints switch gives full availability; no calls are lost unless all outgoing trunks are congested. The number of simultaneous connections that can be made is either M (if M < N) or N (if N<M). The switch contains MN crosspoints.

For M=N, total number of crosspoints = $C=N^2$ -------- (2.1)

Thus cost (as indicated by the number of crosspoints) increases as the square of the size of the switch. However, efficiency decreases inversely with N.

It is therefore uneconomic to use a single stage network for large numbers of inlets and outlets.

Fig 3.12 switch symbols



For example, a switch with 100 inlets and outlets requires 10,000 crosspoints and only 1% of these can be in use at any time. Switches for making connection between large numbers of trunks are therefore constructed as networks containing several stages of switches.

Matrix of cross point switches issued to make connection between N similar circuits, then each circuit is connected to both an inlet and an outlet. Operation of the crosspoints at coordinates (j, k) to connect inlet j to outlet k thus performs the same function as operating crosspoints (k, j) to outlet j. Consequently, half the crosspoints are redundant and can be eliminated. This results in the triangular crosspoints matrix shown in figure 3.13 The number of crosspoints of required is

$C_1=\frac{1}{2}N(N-1)$ …… (2.2)

These are used in providing ringing tone and ringing current are sent over separate one way trunks depending on the customer's line is calling or being called.

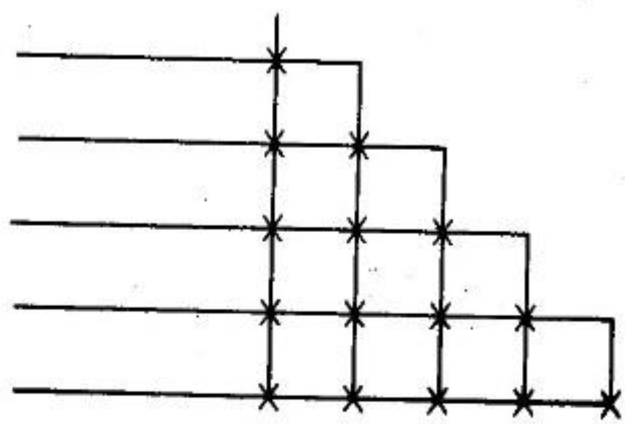


Fig 3.13 triangular cross point matrix for connecting both way trunks

3.12 : Gradings

The number of outgoing trunks connected to incoming trunks is known as availability.

3.12.1 : Principle:

For a route switch or a concentrator it is not necessary for each incoming trunk to have access to every outgoing trunk. It is adequate if each incoming trunk has access to a sufficient number of trunks on each route to give the required grade of service. This is known as limited availability. The number of outgoing trunks to which an incoming trunk can obtain connection is called the availability and corresponds to the outlet capacity of the switches used.

Figure 3.14(a) shows 20 trunks on an outgoing route to which incoming trunks have access bymeans of switches giving an availability of only ten.

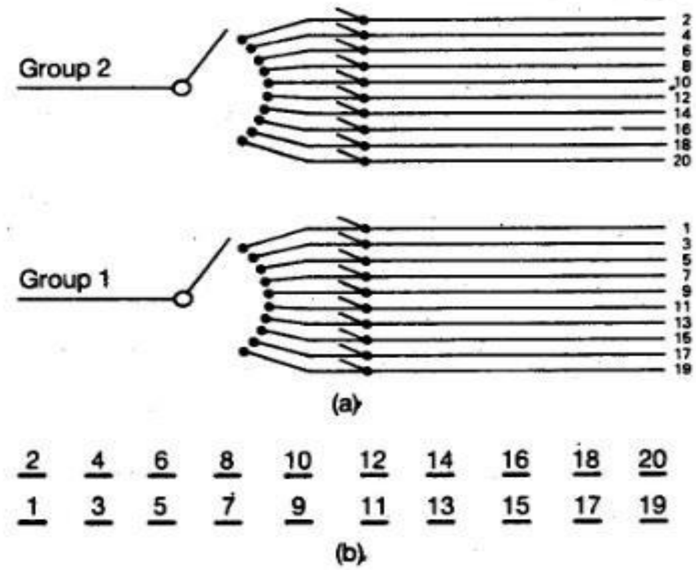
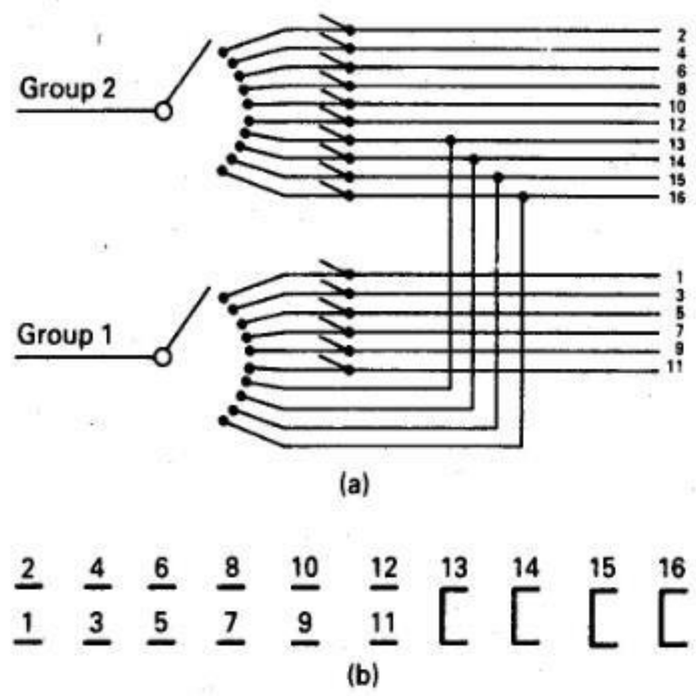


Figure 3.14 Twenty trunks connected in two separate groups to switches of availability 10 (a) Full diagram (b) Grading diagram

In Figure 3.14(a), the outlets of the switches are multiplied together in two separate groups and ten outgoing trunks are allocated to each group.



· Figure 3.15 Sixteen trunks interconnected to two groups of switches of availability 10. (a) Full diagram. (b) Grading diagram.

If the traffic offered to the two groups of incoming trunks is random, peak loads will seldom occur simultaneously in the two groups. Efficiency can therefore be improved through mixing the traffic by interconnecting the multiples of the two groups so that some of the outgoing trunks are available to both groups of switches.

If the switches search sequentially for free outlets, the later-choice outlets carry the least traffic. It is therefore desirable to connect the later-choice trunks to both groups of selectors, as shown in Figure 3.15(a) and grading diagram shown in figure 3.15(b).

In this arrangement, the first six outlets are in two separate full-availability groups; the last four outlets are common to both groups and carry the traffic that overflows when the first six outlets **of** either group are busy.

Design of progressive gradings

In order to form a grading, the switches having access to the outgoing route are multiplied into a number of separate groups, known as *graded groups.*

On early choices each group has access to individual trunks and on late choices trunks are common, as shown in Figure 3 .15. This diagram shows a small grading for only two groups -of switches. For larger numbers of outgoing trunks, gradings may contain four or more groups.

Figure 2.5 shows four-group gradings. Since the traffic decreases with later choices of outlet, the number of groups connected together increases from individual connections on the early choices through partial commons (doubles) to full commons on the late choices.

Switches hunt over the outlets sequentially from a home position.

In designing a grading to provide access to N outgoing trunks from switches having availability ***k,*** the first step is to decide on the number of graded groups ***g.***

Ifall the choices were individual trunks, we would have

$N = gk$.

If all the choices were full commons,

$N = k$.

Since the grading contains a mixture of individuals, partial commons and full commons, then $k < N < gk$.

A reasonable choice for N is $N = \frac{1}{2}gk$ and traffic simulations have shown that the efficiency of such gradings is near the optimum.

The number of groups is thus chosen to be: $N=\frac{1}{2}gk$

$$2N=gk$$

$$g = \frac{2N}{k} \text{ -------(2.3)}$$

Since the grading should be symmetrical, ***g*** must be an even number, so the value of ***g*** given by equation (2.3) is rounded up to the next even integer.

It is now necessary to decide how the g_k trunks entering the grading are to be interconnected to *N* outgoing trunks.

For a two-group grading there is only one solution. If the number of columns of 'singles' is ***s*** and the number of commons is ***c*,** then:

Availability= $k = s + c$

No. of trunks = $N = 2s + c$

$$s = N - k \text{ and } c = 2k - N$$

If the grading has more than two groups, there is no unique solution. It is necessary to choose from the possible solutions the best one, i.e. the grading with the greatest traffic capacity.

- The traffic offered to adjacent outlets will not differ greatly, so they should not be connected to very different sizes of common.
- There should thus be a smooth progression on the choices from individuals to partial commons, from smaller partial commons to larger ones, and from partial commons to full commons.
- The numbers of choices of each type in a group should therefore be as nearly equal as possible.
- This is achieved by minimizing the sum of the successive differences between the number of choices of one type and those of the type following it.

Let g have q factors: $f1 < f2 < \ldots < f_i \ldots < fq$. Where

$f1 = 1$ and $f_q = g$

Let ri be the number of choices having their incoming trunks connected *as* f_i tuples

$$\sum_{i=1}^{q} r_i = k \text{ ... ... (2.4)}$$

Now each f_i tuple contains g/f_i outgoing trunks.

$$\sum_{i=1}^{q} \frac{r_i g}{f_i} = k \dots\dots\dots\dots (2.5)$$

Since there are only two equations and more than two unknowns (if $q > 2$), there are a number of different solutions for $(r_1, \dots\dots\dots r_4)$. These are ·round and, for each, the sum of the successive differences, D, is given by :

$$D = |r_1 - r_2| + |r_2 - r_3| + \dots\dots\dots\dots\dots\dots + |r_{q-1} - r_q| \quad ---(2.6)$$

The best grading is that having the smallest value of D.

Design of Progressive grading:

This is the most commonly used to grading methods

Step1: Find out the number of groups

g=2N/k

Step2: Find out the number of single, doubles and quadruples

Step3: Write simultaneous equations

Step4: single, doubles and quadruples tabulate the results

Sl. No	Single	doubles	Quadruples	Sum of successive difference
1.				
2.				
3.				

Note: The Scheme which gives the maximum successive difference is the best grading.

Problem 3.14

Design a grading for connecting 20 trunks to switches having ten outlets.

Solution:

The number of graded groups, given by equation (2.3) is g = 2*N/k= 40/ 10 = 4, and the factors of *g* are 1, 2 and 4.

Let the number of choices having singles = *s*

the number of choices having doubles = *d*

the number of choices having quadruples = q

substituting in equations (2.4)

$$\sum_{i=1}^{q} r_i = k$$

$$s + d + q = 10 \text{------ } (1)$$

$$4s + 2d + q = 20 \text{ ----------- } (2)$$

From (2) - (1) = $3s + d = 10$

Substituting in equations (2.5)

$$\sum_{i=1}^{q} \frac{r_i g}{f_i} = k$$

$s = 1: d = 7$ and $q = 10 - 8 = 2$

$s = 2: d = 4$ and $q = 10 - 6 = 4$

$s = 3: d = 1$ and $q = 10 - 4 = 6$

$s = 4$: $d < 0$, so this is not possible .

There are thus three possible gradings, which are shown in Figure 3.16. The sums of the successive differences for these gradings are respectively given by:

$D = |r_1 - r_2| + |r_2 - r_3| + \ldots\ldots\ldots\ldots + |r_{q-1} - r_q|$

Case1) $D_1 = |7-1|+|2-7|$ $D_1 = 6 + 5 = 11$

$D_2 = |4 - 2|+|4-4| = 2 + 0 = 2$

$D_3 = |1-3| + |6 - 1| = 2 + 5 = 7$

The second grading (shown in Figure 5.5(b)) is therefore the best.

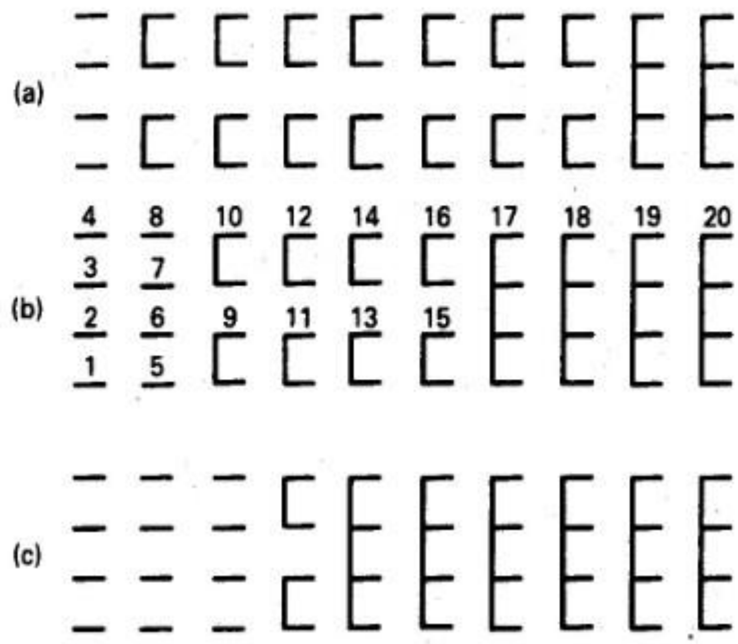


Figure 3.16 Four group grading for 20 trunks (availability 10)

If a growth in traffic makes it necessary to increase the number of trunks connected to a grading, this can be done by reducing the number of commons and p a r t i al commons and increasing the number of individuals. Figure 3.17 shows the grading of Figure 2.5(b) rearranged to provide access to 25 trunks.

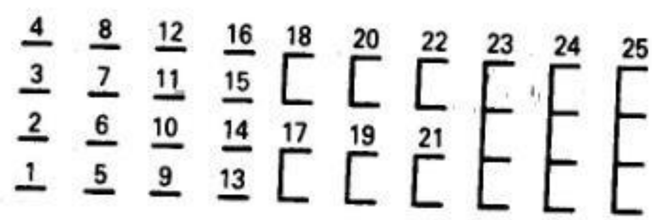


Figure 3.17 a). Grading of Figure 3.16 (b) modified to accommodate 25 trunks.

Problem 3.15:

Find the best formation for a 6 group grading for 32 trunks with availability of 10.

The number of graded groups, given by equation (2.3) is

g = 2*N/ k= 2*32/ 10 =6.4 = 6, and the factors of *g* are 1, 2 3, 6.

Let the number of choices having singles = *s*

the number of choices having doubles = *d*

the number of choices having triples = t

the number of choices having six lines (full commons) = fc

Note: s, d, t, or fc can never exceed 10

$$s + d + t + fc = 10 \text{ availability ------ (1)}$$

$$6s + 3d + 2t + fc = 32 \text{ trunks------------ (2)}$$

From (2) - (1) = $5s + 2d + t = 22$

Note: s, d, t, fc can never exceed 10.

S cannot be more than 4.

Successive difference is calculated by

$$*D = |r_1 - r_2| + |r_2 - r_3| + \ldots\ldots + |r_{q-1} - r_q|$$

$5s + 2d + t = 22$

When

S= 4, 2d+t=2, d cannot be >1 hence d=1 and t=15

S= 3, 2d+t=7, d cannot be >3

S= 2, 2d+t=10, d cannot be >5

S= 1, 2d+t=17, d cannot be >8

S= 0, 2d+t=22, d cannot be >10

SL .No	No. of singles S	No. of double lines d 2d+t=7	No. of triple line (t) 5s+2d+t =22	No. of 6 lines or full commons (fc) S+d+t+fc=10	Successive difference is calculated *
Case 1	4	0	2	4	4+2+2=8
Case 2	4	1	0	5	3+1+5=9
Case 3	3	3	1	3	0+2+2=4
Case 4	3	1	3	2	2+2+1=5
Case 5	3	1	5	1	2+4+4=10

Case 6	3	0	7	0	3+7+7=17
Case 7	S=2	5	2	1	3+3+1=7
Case 8	S=2	D=4	T=4	Fc=0	|2-4|+|4-4|+|0-4|= 2+0+4=6
Case 9	S=2	D=3	T=6	Fc= -1 (invalid)	
Case 10	2	2	8	-2 (invalid)	
Case 11	2	1	10	-3(invalid)	
Case 12	2	0	12	-4(invalid)	
Case 13	1	8	1	0	7+7+1=15
Case 14	S=1	d=7	T=15	Fc= -7 once invalid don't consider that. (invalid)	
Case 15	S=1	D=6	T=11	c= -4 (invalid)	
Case 16	1	5	7	Fc= -4 (invalid)	
Case 17	1	4	9	Fc= -4 (invalid)	
Case 18	1	3	11	Fc= -4 (invalid)	
Case 19	1	2	13	Fc= -4 (invalid)	
Case 20	1	1	15	Fc= -4 (invalid)	
Case 21	1	0	17	Fc= -4 (invalid)	

There is one case where in successive difference is minimum. i.e., 4

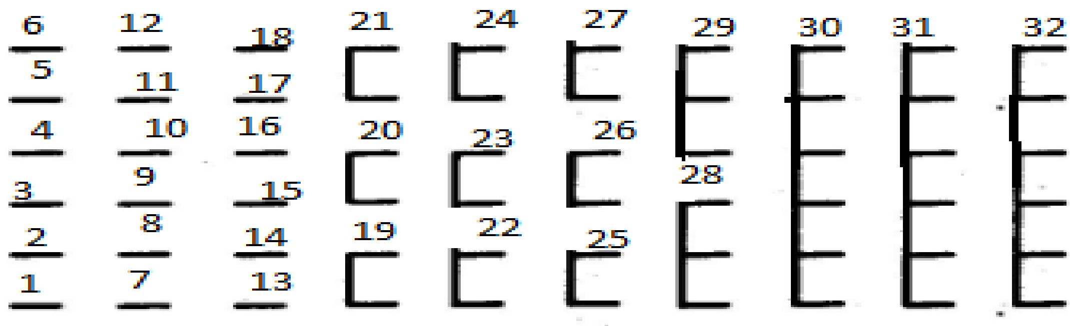


Figure 3.17b) Grading diagram for minimum successive difference

Where in s=3 d=3 and t=1 and fc=3

3.12.2 Other form of Grading

Skipped Grading

In an O'Dell grading, the partial commons are arranged as separate groups, so each is available toonlysomeof the incomingtrunks.

For example, in Figure 3.16(b) the upper of pairs serves only the first two groups. However, the principle of grading is based on the sharing of outgoing trunks between different sets of incoming trunks. Efficiency can be improved if this principle can be applied to the whole of a grading instead of only to parts of it.

This can be done by connecting non-adjacent groups, in addition to adjacent groups, as shown in Figure 3.18. This is known as skipping.

In this grading in addition to communing adjacent groups, non-adjacent groups also are commonly connected. This avoids upper half and lower half of the group to be separated. Traffic is evenly distributed in both the halves.

Figure 3.18a Skipped grading

Homogeneous Grading

Progressive gradings are intended to be used with switches that hunt sequentially from a fixed home position. However if switches do not hunt from a single position, or they select outlets at random, there is no advantage in connecting some outlets to singles and others to partial or full commons. The grading should then be designed to share each trunk between an equal numbers of groups, as shown in figure 3.18b. this is known as Homogeneous Grading.

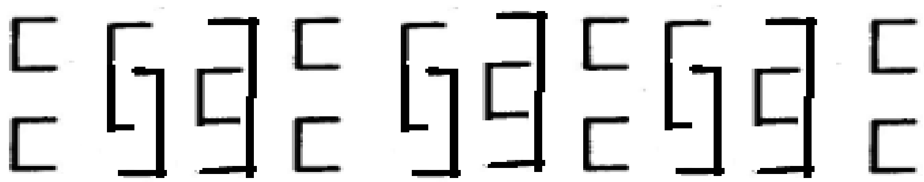


Figure 3.18b Homogeneous grading.

Traffic capacity of grading

In an ideal grading, the interconnections would ensure that each outgoing trunk carried an identical traffic load.

Thus, if total traffic A is carried by N trunks, the occupancy of each trunk is A/N .

It is assumed that each trunk being busy is an independent random event.

Each call has access k to trunks (where k is the availability), and the probability of all k trunks being busy is thus:

$$B = (A/N)^k$$

The number of trunks required to carry A Erlangs with a GOS of B is therefore given by:

$$N = (AB)^{1/k} \qquad (2.7)$$

This is *Erlang's ideal grading formula* and gives a linear relationship between the traffic md the number of trunks required.

Practical gradings do not satisfy the conditions for Erlang's ideal grading. However, they satisfy a linear relationship between traffic capacity A and number of trunks for a given grade of service B.

Figure3.19

An approximate curve of A against N can therefore be derived from Erlang's full-availability theory for N k and extended as a straight line for $N \geq k$.

From equation (2.7) this line is given by:

$$A = A_k + (N - k) B^{1/k}$$

Where Ak is the traffic carried by a full-availability group of k trunks (with GOS = B).

Figure 3.19 shows a family of curves plotted from the above modified Erlang formula.

Problem 3.16: Find the traffic capacity of the two-group grading shown in Figure 3.15 if the required grade of service is 0.01.

For k = 10 and, from Table 4.1, A_k = 4.5 E.

From equation (5.8):

$$A = A_k + (N - k)B^{1/k}$$

$= 4.5 + (16 - 10) \times 0.01^{0.1}$

$= 4.5 + 6 \times 0.631$

= 8.3 E

(Given in table 4.1 that a full-availability group of 16 trunks can handle 8.9 E with 0.01 grade of service.)

3.12.3 Application of Grading:

Gradings have been widely employed in step-by-step systems.

In trunk distribution frames (TDF) between the ranks of selectors provide cross-connections in the form of gradings. Another example of the use of a grading in a link system is in the Bell No.1 ESS system.

The subscribers' concentrator of this system is shown in Figure 3.21 (a). The number of crosspoints required in the primary switches is reduced by omitting them in a systematic manner.

Each primary switch is equivalent to four groups of four-outlet selectors having access to eight trunk s through the homogeneous grading shown in Figure3.20(b).

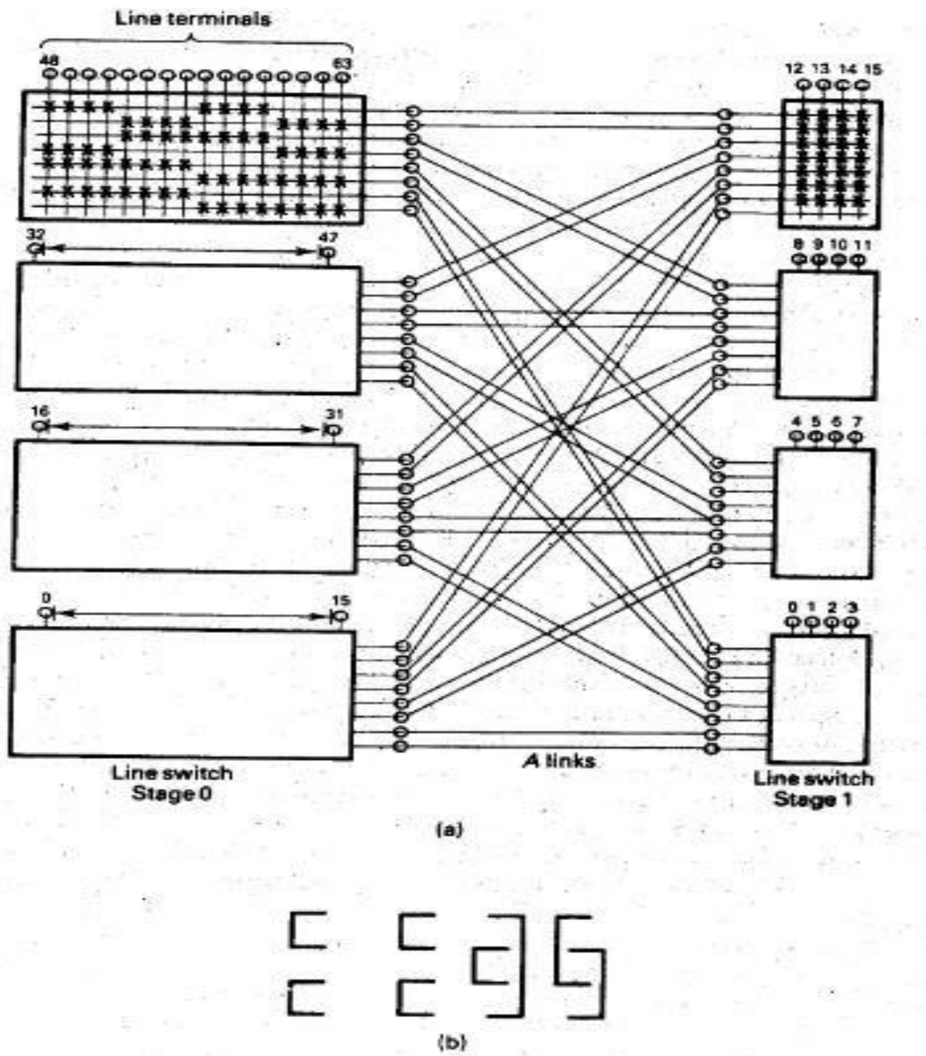


Figure 3.20 Two stage concentrator used in Bell No.1 ESS system. a) Arrangement of trunks,

b). Four group homogeneous grading incorporated at stage [AT & T lab copy right document]

3.13 : Link Systems

General

Two stage Switching network:

In the two-stage network (of previous chapter) there is only one link between each primary switch and each secondary switch. Thus, it may be impossible to make a connection from a given incoming trunk to a selected outgoing trunk because the link is already being used for another connection between that primary switch and that secondary switch. This situation is called *blocking.*

It is also known as a *mismatch*, because free links exist but none of them can be used for the required connection.

If connection must be made to one particular outgoing trunk the probability of blocking is unacceptably high . For this application, it is therefore necessary to use a network with more stages e.g. the four-stage network in order to have a choice of paths through the network.

The two-stage network of Figure 3.21 can be used as a route switch. If it serves ten outgoing routes with ten trunk on each route, then trunk no.1 of each route is connected to secondary switch no. 1, trunk no. 2 is connected to switch no.2, and soon.

Thus, an incoming trunk can obtain connection to the selected outgoing route via any of the links outgoing from its primary switch. The call is only lost if all the paths to free outgoing trunks are blocked.

The probability of this occurring simultaneously for links is obviously much smaller than the probability of a single link being bus Similarly, if the incoming trunks are from several different routes, one trunk from each route is normally terminated on each primary switch.

Step-by-step selection is unsuitable. This is because calls are lost by internal blocking in addition to congestion of the external trunks.

It has been seen that the grade of service of a link system depends on the way it is used. We may classify these uses as follows:

Mode 1: Connection is required to one particular free outgoing trunk. (Since conditional selection is used, an attempt will not be made to set up this connection unless the trunk *is* free.)

Mode 2: Connection is required to a particular outgoing route, but any free trunk on that route may be used.

M ode 3: Connection may be made to any free outgoing trunk.

It will be seen from Figure 3.21 that a concentrator operates in mode 3, a route switch operates in mode 2 and anexpander operates in mode 1.

Two-stage networks

If the two-stage network shown in Figure 3.21 has N incoming and N outgoing trunks and contains primary switches having n inlets and secondary switches having n outlets.

Then no. of primary switches (g) = no. of secondary switches = no. of outlets per primary switch = no. of inlets per secondary switch,

where

$$g = N/n \quad \text{.........} \ (1)$$

The number of crosspoints per primary switch = number of crosspoints per secondary switch = $gn = N$........(2)

The total number of crosspoints ($C2$) in the network = (number of switches) x (crosspoints per switch) i.e. C_2=g N + g N from equation 1 in 2.9

C_2=2g N =2*N/n*N = $2N^2/n$ ---(2.9)

Since there is one link from each primary switch to each secondary switch, the number of links is equal to no. of primary switches x no. of secondary switches, i .e

No. of links = g^2 = $(N/n)^2$ (2.10)

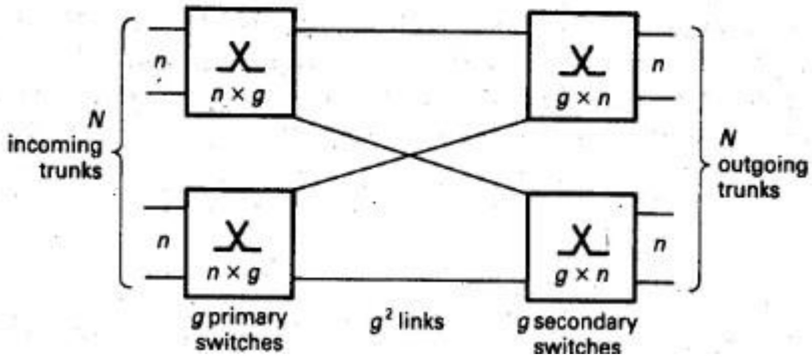


Figure 3.21 Two stage switching network

The number of crosspoints thus varies as l/n, but the numbers of link varies as $1/n^2$. If *n* is made very large to reduce the number of crosspoints, there will be too few links to carry the traffic. Let the number of links be equal to the number of incoming and outgoing trunks, a reasonable choice, since each set of trunks carries the same total traffic.

Then $g^2 = N$...................(*)

Substituting (*) in equation (2.10) gives $N=(N/n)^2$

$N=n^2$

$n=\sqrt{N}$...........................(2.11)

Then the total number of crosspoints (from equation (2.9)) is

$C_2 = 2N^{3/2}$------------------------(2.12)

Equation (2.11) can be only a guide; one should select the nearest integer *to n* that is a factor of *N*.

Also, in practice, designers are often constrained to use switch units of fixed sizes. For example, crossbar switches may be of sizes 10 x 10or 10 x 20.

The Bell No. 1 ESS system uses switches constructed from modules of size 8 x 8 and the British Telecom TXE2 system uses modules of size *5* x 5.

The number of crosspoints per incoming trunk (from equation (2.12)) is $2N^2$

The cost per trunk therefore increases fairly slowly with the number of trunks. For large networks, however, it becomes more economic to use network s with more than two stages.

Incoming Trunks not equal to Outgoing trunks (M≠N)

Consider a concentrator with *M* incoming trunks and *N* outgoing trunks (*M* >*N*).

Let each primary switch have *m* inlets and each secondary switch have *n* outlets. Then

No. of primary switches = M/ m=g (1)

No. of secondary switches = N/n=g (2)

No. of crosspoints per primary switch = m x N/ n (3)

No. of crosspoints per secondary switch = n x M/ m. (4)

The total number of crosspoints is:

The number of links = number of primary switches x number of secondary switches

$$C_2 = g\, x\, \frac{mN}{n} + g\, x\, \frac{nM}{m}$$

$$C_2 = \frac{M}{m}\frac{mN}{n} + \frac{N}{n}\frac{nM}{m}$$

$$C_2 = MN \left[\frac{1}{n} + \frac{1}{m}\right] \quad (5)$$

The number of links = number of primary switches x number of secondary switches

$$g^2 = \frac{MN}{mn}$$

Since the traffic capacity is limited by the number of outgoing trunks, there is little point in providing more than this number of links, so let the number of links *be N*.

$$\frac{MN}{mn} = N$$

And M=mn

n=M/m……(6)

Substituting equation (6) in equation (5)

$$C_2 = MN \left[\frac{m}{M} + \frac{1}{m}\right] \quad (7)$$

In order to minimize C_2 treat *m* as if it were a continuous variable and differentiate with respect to it

$$\frac{dC_2}{dm} = MN\left[\frac{1}{M} - \frac{1}{m^2}\right]$$

$$= 0 \text{ when } m=\sqrt{M} \quad (8)$$

$m = n = \sqrt{M}$

Hence, from equation (8)in equation (6):

$m = n = \sqrt{M}$

Thus, the number of crosspoints is a minimum when the number of inlets per primary switch equals the number of outlets per secondary switch.

Substituting in equation (2.13)

$$C_2 = MN\left[\frac{1}{\sqrt{M}} + \frac{1}{\sqrt{M}}\right]$$

$$= \text{MN}\left[\frac{2}{\sqrt{M}}\right]$$

$$= 2\frac{M}{\sqrt{M}}\text{N}$$

$$= 2M^{\frac{1}{2}}N$$

Incoming Trunks not equal to Outgoing trunks (M≠N)

Consider a concentrator with *M* incoming trunks and *N* outgoing trunks *(M <N).*

Let each primary switch have *m* inlets and each secondary switch have *n* outlets. Then

No. of primary switches = M/ m=g (1)

No. of secondary switches = N/n=g (2)

No. of crosspoints per primary switch = m x N/ n (3)

No. of crosspoints per secondary switch = n x M/ m. (4)

The total number of crosspoints is:

The number of links = number of primary switches x number of secondary switches=C_2

$$C_2 = g\ x\ \frac{mN}{n} + g\ x\ \frac{nM}{m}$$

$$C_2 = \frac{M}{m}\frac{mN}{n} + \frac{N}{n}\frac{nM}{m}$$

$$C_2 = MN\left[\frac{1}{n} + \frac{1}{m}\right] \qquad (5)$$

The number of links = number of primary switches x number of secondary switches

$$g^2 = \frac{MN}{mn}$$

Since the traffic capacity is limited by the number of outgoing trunks, there is little point in providing more than this number of links, so let the number of links *be N*.

$$\frac{MN}{mn} = M$$

And N=mn

m=N/n……(6)

Substituting equation (6) in equation (5)

$$C_2 = MN \left[\frac{n}{N} + \frac{1}{n}\right] \qquad (7)$$

In order to minimize C_2 treat *n* as if it were a continuous variable and differentiate with respect to it

$$\frac{dC_2}{dn} = MN\left[\frac{1}{N} - \frac{1}{n^2}\right]$$

$$MN\left[\frac{1}{N} - \frac{1}{n^2}\right] = 0$$

$$n^2 = N = n = \sqrt{N} = m$$

$= 0$ when $m=\sqrt{N}$ (8)

$m=n=\sqrt{M}$

Hence, from equation (8) in equation (6):

$m=n=\sqrt{M}$

Thus, the number of crosspoints is a minimum when the number of inlets per primary switch equals the number of outlets per secondary switch.

Substituting in equation (2.13)

$$C_2 = MN\left[\frac{1}{\sqrt{N}} + \frac{1}{\sqrt{N}}\right]$$

$$= MN\left[\frac{2}{\sqrt{N}}\right]$$

$$= 2\frac{M}{\sqrt{N}} N$$

1

$$= 2N\bar{2}M$$

Problem 3.17

Design a two-stage switching network for connecting 200 incoming trunks to 200 outgoing trunks.

Now , $\sqrt{200}$ = 14.14.

However, *n* must be a factor of 200, so the nearest practicable values are *n* = 10 and *n* = 20. Two possible networks are shown in Figure 2.12.

No of Crosspoints =2 $N^{3/2}$

= 2 x $(200)^{3/2}$

= 2 x 2.828 x 10^3

= 5656 crosspoints

= almost it contains 6000 crosspoints.

The network of Figure 3.22(a) is suitable for 20 outgoing routes, each having 10 trunks, and that of Figure 3.22(b) is suitable for 10 outgoing routes, each having 20 trunks.

The network in Figure 3.21 has the same number of outgoing trunks as incoming trunks. However, a concentrator has more incoming than outgoing trunks and an expander has more outgoing than incoming trunks

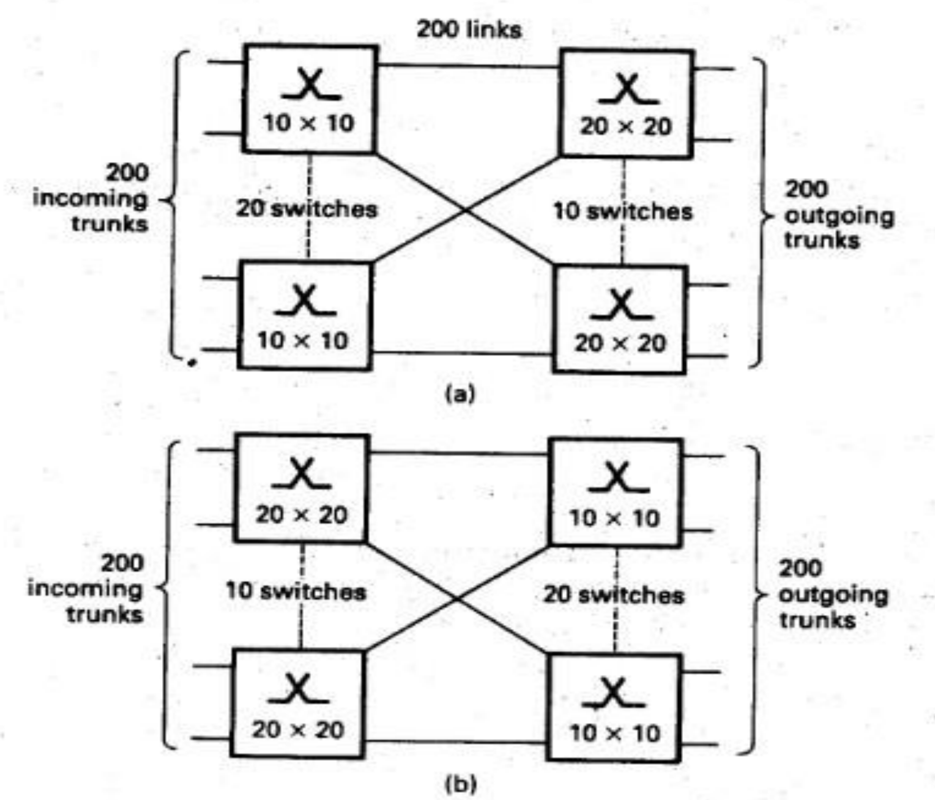


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Figure 3.22 Examples of two-stage networks. (a) For 20 outgoingroutes(10 trunks on each). (b) For 10 outgoing routes (20 trunks on each).

Problem 3.18

Design a two stage network to connect 12 incoming trunks to 9 outgoing trunks and arrive at the optimum solution

Solution:

Incoming trunks M =12

Outgoing trunks N =9

If M>N Hence this is a concentrator

For minimum crosspoints

Aa m= n= $\sqrt{M}$=$\sqrt{12}$ =3.47

But m and n should be integer and factors of 12. Hence two cases are possible

Aa m=n= 3

Aa n=m=4

When m=n= 3

$$C_2 = MN\left[\frac{m}{M} + \frac{1}{m}\right]$$

$$C_2 = 12 * 9\left[\frac{3}{12} + \frac{1}{3}\right]$$

$$= 63$$

When m=n= 4

$$C_2 = MN\left[\frac{m}{M} + \frac{1}{m}\right]$$

$$C_2 = 12 * 9\left[\frac{4}{12} + \frac{1}{4}\right]$$

$$= 63$$

Practical use

Aa n=$\sqrt{N}$ = $\sqrt{9}$ = 3

$m = \frac{M}{\sqrt{n}} = \frac{12}{\sqrt{9}} = \frac{12}{3} = 4$

$$C_2 = MN\left[\frac{m}{M} + \frac{1}{m}\right]$$

$$C_2 = 12 * 9\left[\frac{4}{12} + \frac{1}{4}\right]$$

$$= 63$$

Any of the above design can be used.

Three Stage Networks:

In the three stage networks of figure 3.23 there are

N incoming and N outgoing trunks has primary switches with *n* inlets and tertiary switches with *n* outlets,

then

No. of primary switches (g_1) = No. of tertiary switches (g_3) = N/n.

The secondary switches have *N* /n inlets and outlets.

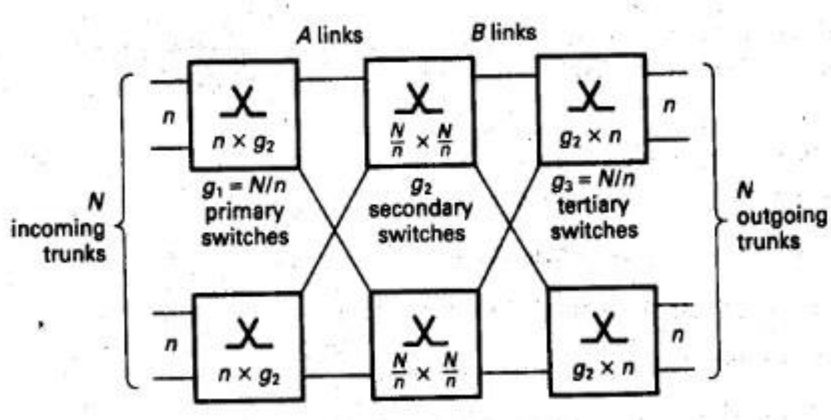


Figure 3.23 Fully interconnected three stage switching network

Let there be N number of primary-secondary links (A links) and N Number of secondary-tertiary links (B are each *N)*

then the number of secondary switches is

$g2 = N \div (N/n)$

= no. of outlets per primary switch = no. of inlets per tertiary switch

No. of crosspoints in primary stage = $n^2(N/n) = nN$

No. of crosspoints in secondary stage = $n\,(N/n)^2 = N^2/n$

No. of crosspoints in tertiary stage = $n^2(N/n) - nN$

and the total number of crosspoints is

$$= nN + \frac{N^2}{n} + nN$$

$$= 2nN + \frac{N^2}{n}$$

$$C_3 = N\left(2n + \frac{N}{n}\right)$$

By differentiating equation (2.17) with respect to n and equating to zero, it can be shown that the number of crosspoints is a minimum when

$$n = \sqrt{N/2}$$

And then $C_3 = 2\sqrt{2}N^{3/2}$

$= \sqrt{2}\, C_2$

$= 2^{3/2} N^{-1/2} \quad C_1$

Design for M>N

If a three-stage concentrator has *M* incoming trunks and *N* outgoing trunks (M > N), its primary switches each have *m* inlets and its tertiary switches each have *n* outlets. Then

No. of primary switches = M/m

No. of tertiary switches = N/n

If there are g_2 secondary switches, then

Crosspoints per primary switch = $m\, g_2$

Crosspoints per secondary switches = $\frac{MN}{mn}$

Crosspoints per tertiary switch = $g_2\, n$

The total number of crosspoints is

$$C_3 = \frac{M}{n} X\, m g_2 + g_2\, X \frac{M}{m}\frac{N}{n} + \frac{N}{n} X g_2 n$$

$$= g_2 \left[M + N + \frac{M}{m}\frac{N}{n}\right]$$

Since $M > N$, *let* no. of A links = no. of B links = N.

$$= g_2 \frac{M}{m} = g_2 \frac{N}{n}$$

Hence, $g_2 = n$ and $m = n\, M / N$.

Substituting in equation (2.20):

$C3 = (M + N)\, n + N^2 / n$

Differentiating with respect to n to find a minimum gives:

I,e. $m = \frac{M}{\sqrt{M+N}}$ $\quad n = \frac{N}{\sqrt{M+N}}$

$C3 = 2N\sqrt{N + M}$

To obtain an expander, M is exchanged with N and m with n.

Problem 3.19

Design a three-stage network for connecting 100 incoming trunks to 100 outgoing trunks:

$\sqrt{\frac{100}{2}} = 7.07$ use $n = 5$ or $n = 10$

1. If $n = 5$, there are:

   20 primary switches of size 5 x 5

   5 secondary switches of size 20 x 20

   20 tertiary switches of size 5 x 5

2. If n=10, there are 10 primary switches , 10 secondary switches and 10 tertiary switches, each of size 10 x 10

No. of crosspoints = $C_3 = 2\sqrt{2}N^{3/2}$

$$C_3 = 2\sqrt{2}100^{3/2}$$

= 2828 cross points

$= 3000$ Cross points

Problem 3.20

Design a three-stage network for 100 incoming trunks and 400 outgoing trunks.

$$\frac{100}{\sqrt{100+400}}$$

$$= 4.47$$

$\frac{400}{\sqrt{100+400}} = 11.89$

1. m = 4 or 5; n = 16 or 20

If m = 5, n = 20, there are:

20 primary switches of size *5* x *5*

5 secondary switches of size 20 x 20

20 tertiary switches of size *5* x 20

2. If *m* = 4, *n* = 16, there are:

25 primary switches of size 4 x 4

4 secondary switches of size 25 x 25

25 tertiary switches of size 4 x 16 ·

$C3 = 2N\sqrt{N + M}$

$= 2 \times \sqrt{100 + 400}$

$= 200 \times 22.3$

$= 4460$

Say =4500

Both networks contain 4500 crosspoints.

However, the first contains more secondary Switches and will therefore cause less blocking.

In a three-stage network, the number of the selected outgoing trunk is given by outlet numbers used on the secondary and tertiary switches.

It is not related to the outlet used in the primary switch, since any secondary switch may be used for connection to a given outgoing trunk.

For each connection, two sets of links must be interrogated for the busy/free condition and matched to choose a pair connected to the same secondary switch.

The control of a three-stage network is thus more complex than that of a two-stage one.

For this reason, electromechanical systems usually use trunkings containing a number of separate two-stage networks in tandem. However systems having electronic central control often employ three-stage switching networks.

A fully interconnected three-stage network (as shown in Figure 3.23) requires a ge number of crosspoints when *N* is large. A reduction can be made in the number of crosspoints (at the expense of an increase in blocking) if the secondary switches have links to only some of the primary and tertiary switches, as shown in Figure *3*.24.

The secondary and tertiary switches are arranged in separate groups (frames) and are fully interconnected only within their groups. Each primary switch has one link to each of bese secondary-tertiary groups. (Alternatively, the primary and secondary switches may be arranged in separate groups, to produce the mirror image of Figure 3.24.)

The number of switches is $3n^2$ and each has n^2 crosspoints, so the total number of crosspoints is:

$$C_3 = 3n^4 = 3N^{4/3} \quad \text{-----------} \ (2.23)$$

And the number of crosspoints per incoming trunk is $3N^{1/3}$

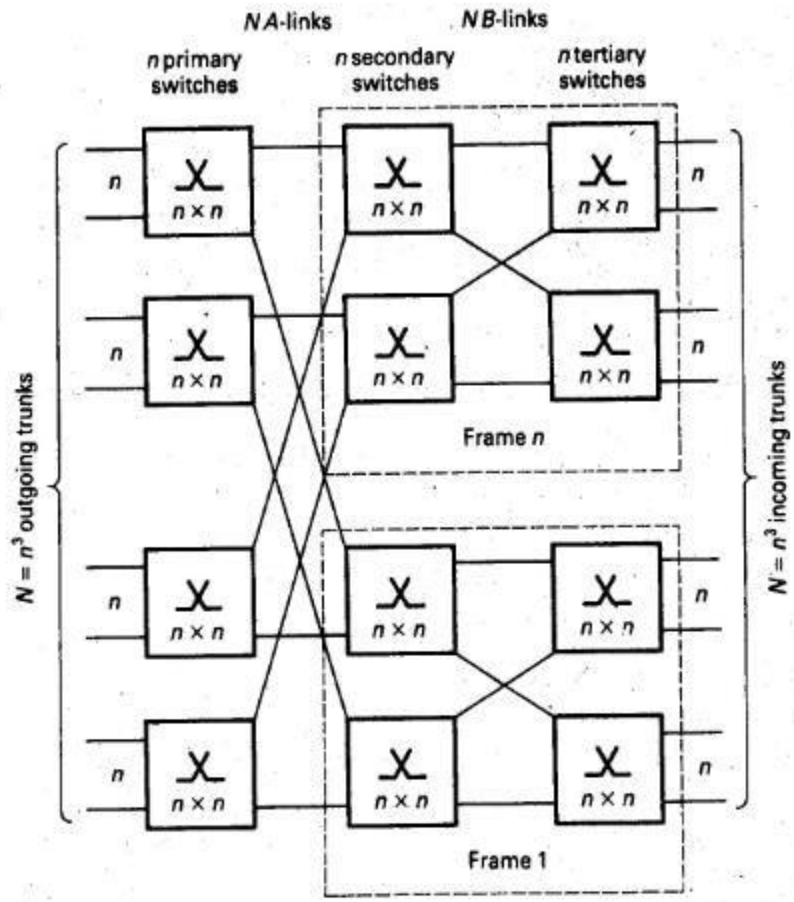


Figure 3.24 Partially interconnected three-stage network.

In Figure 3.23, the third stage added to the two-stage network does not increase the number of outgoing trunks; it increases the mixture of paths available to reach them in order to reduce blocking. The additional stage may therefore be called a *mixing stage.*

In Figure 3.24 the added third stage does not increase the number of paths to an outgoing trunk; it increases the number of outgoing trunks over which the incoming traffic can be distributed. It is therefore called a *distribution stage.*

In Figure 3.23 a primary switch has a link to every secondary switch, so any secondary switch can be used for a connection to a given outgoing trunk. In Figure 3.24, however, there are many more primary switches and each has a link to only one secondary switch of each two-stage frame.

Four-stage networks

A four-stage network can be constructed by considering a complete two-stage network as a single switch and then forming a larger two-stage array from such switches. Figure 3.25 shows a four-stage network for 1000 incoming and 1000 outgoing trunks constructed from two-stage networks (frames) of 100 inlets and 100 outlets using 10 x 10 switches.

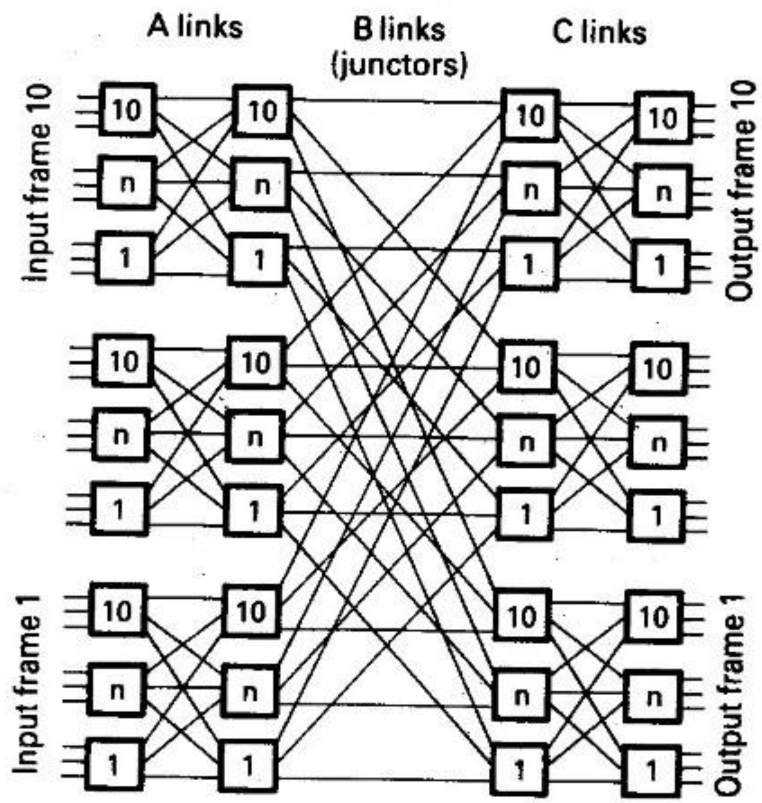


Figure 3.25 Four stage switching network for 1000 incoming trunks and 1000 outgoing trunks using 10 X 10 switches

It is necessary that one trunk (B link) be connected from each secondary switch of an incoming frame to a primary switch of an outgoing frame. These trunks are connected to switches of corresponding numbers on the two frames, thus facilitating marking of the network. Four-stage networks of this type are used in crossbar systems.

If a four-stage network with N incoming and N outgoing trunks is constructed with switches of size $n \times \cdot n$, then $N = n^3$ and the total number of switches is $4n^2$

Thus, the total number of crosspoints is:

$C4 = 4n^2 . n^2$

$= 4 N^{4/3}$ ------------------(2.24)

The number of crosspoints per incoming trunk is $4 N^{1/3}$

It should be noted that the partially interconnected three-stage network of Figure 3.24 corresponds to the four-stage network of previous chapter, truncated at the A links. Adding the fourth stage has not increased the number of trunks, although it has increased the number of crosspoints by one third.

3.14 Grade of Service of Linked systems

General

A Simple theory for calculating the probability of loss in link system, due to C Y Lee concepts explained here

In this method, assumes that trunks and links being busy constitute independent random events.

If two links are connected in tandem, and the probability of one being busy is "p" and if the other being busy is "q" then probabilities of each being free are (1-p) and (1-q) respectively, so the probability of both being free is (1-a)(1-b). Therefor the probability of the path being blocked is 1- (1-a) (1-b).
The occupancy at each stage is the total traffic carried divided by the number links at that stage. However, if the loss is small (as it should be), little error is introduced by using the traffic offered instead of the traffic carried.

In a practical system the assumption of independence may not be valid; because there is usually some degree of dependence between links. This reduces the probability of blocking, because traffic peaks at different stages coincide more often than would happen if they were independent random events.

This overlapping of peaks tends reduce the total time during which blocking occurs. Consequently, Lee's methods over estimates the loss probability. Nevertheless, the method gives reasonably accurate results in most cases. It also has the merit of simplicity. For these reasons, it is widely used.

Two-stage networks

For a two-stage network as shown in Figure 3.22, let the occupancy of the links be a and the occupancy of the outgoing trunks be b. (If the numbers of links and trunks are equal then a = b.)

For mode I (i .e. connection to a particular outgoing trunk) only one link can be used. The probability of this being busy is a and this is the probability of loss.

For example, to provide a GOS of $B_1 = 0.01$. each link and outgoing trunk could only carry 0.01E.

For mode 2 (i.e. connection to an outgoing route with one trunk on each secondary switch) any free link can be used. The probability of loss using a particular link is

= 1 – probability that both link and trunk are free

= 1 – (1 – a)(1 - b)

Let there are g paths available. Assuming that each being blocked is an i ndependent ra ndom event, the probability of simultaneous blocking for all g paths is:

$$B_2 = [1-(1-a)(1-b)]^g$$

$$= [a + (1-a)\, b]^g \qquad (2.25)$$

"Where g is the number of secondary switches

If connection may be made to any outgoing trunk that is free (i.e. mode 3) then it possible to make the connection unless all the outgoing trunks are busy .Thus, if the numbers of incoming trunks, linksand outgoing trunks are equal, no callscan be lost.

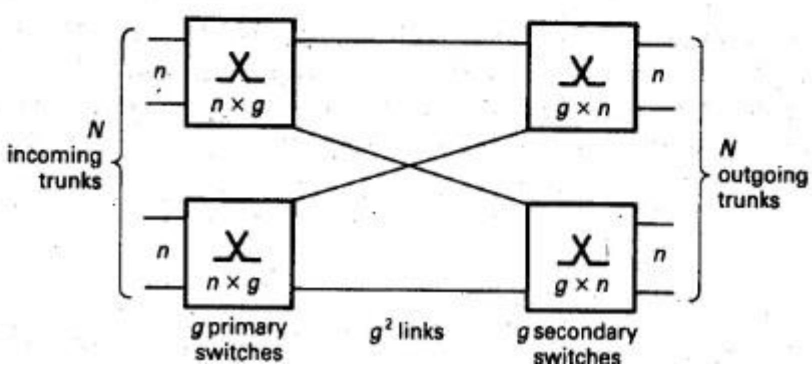
However, this mode of operation is normally used with a concentrator. The number of incoming trunks is then much larger than the number of outgoing trunks. so the grade of service is given by

$$B_3 = E_{1,N}(A)$$

there A is the total traffic offered to the network .

Problem 3.20

Find the grade of service when a total of 30 E is offered to the two-stage switching network of Figure 3.22 and 3.23. the traffic is evenly distributed over the 10 outgoing routes,



Here n=10 and g=10

Incoming trunk=outgoing trunk=100=N

Traffic offered=30E

Switch size= 10x10

The link and trunk occupancies are $a = b$ =A/N= 30/ 100 =0.3 E.

For the two stage network= $B = [1 - (1 - a)(1 - b)]^n$

$$B = [1 - (1 - 0.3)(1 - 0.3)]^{10} = 0.51^{10} = 0.0012$$

1. **Find the traffic capacity of the network if the grade of service is not to exceed 0.01.**

   $B = [1 - (1 - a)(1 - b)]^n$

   If a=b, n=10 and B= 0.01

$B \leq 0.01 = [1 - (1 - a)^2]^{10}$

$1 - (1 - a)^2 \leq 0.01^{0.1} = 0.631$

1

$a \leq 0.39$

Thus offered traffic is given by,

A= ax number of links or outgoing links or outgoing trunks or incoming trunks A= 0.3925x100

$A \leq 39.25$ E

Problem 3.21

1. **Find the grade of service when a total of 30 E is offered to the two-stage switching network. The traffic is evenly distributed over the outgoing routes. Assume the number of trunks to be 200.**

   Also find the traffic capacity of the network if the grade of service is not to exceed 0.01.

Solution:

Here n=40 and g=5

Incoming trunk=outgoing trunk=200=N

Traffic offered=30E

Let Switch size be = 40x40

Number of switches = g=5

The link and trunk occupancies are $a = b$ = A/N= 30/200 = 0.15 E.

For the two stage network= $B = [1 - (1 - a)(1 - b)]^g$

$$B = [1 - (1 - 0.15)(1 - 0.15)]^5 = 0.51^5 = 0.001646$$

$$B = [1 - (1 - a)(1 - b)]^g$$

If a=b, g=5 and B= 0.01

$B \leq 0.01 = [1 - (1 - a)^2]^5$

$1 - (1 - a)^2 \leq 0.01^{0.1}$ 1

$a \leq 0.2242E$

Thus offered traffic is given by ,

A= a x number of links or outgoing links or outgoing trunks or incoming trunks

A= 0.2242x200

$A \leq 44.84$ E

Three-stage networks

For a fully interconnected three-stage network (as shown in Figure 3.23)

Let Occupancy of *A* links be *a*

Occupancy of *B* links be *b*

Occupancy of outgoing trunks be c.

For mode l (i.e. connection to a particular outgoing trunk), the choice of a second switch determines the *A* and *B* links.

Probability that both links are free = (1-a) (l-b)

Probability of blocking = $1 - (1 - a)(1 - b)$

However, there are g_2 secondary switches.

Probability that all g_2 independent paths are simultaneously blocked is

$B_1 = [1 - (1 - a)(1 - b)]^{g_2}$

$= [a + (1 - a)b]^{g_2}$

Thus, for similar occupancies, the three-stage network provides the same GOS connection is to individual trunks as the two-stage network does for connections group of trunks.

For mode 2 (i.e. a connection to any free trunk in a route having one to connected to each tertiary switch):

Probability of blocking for a particular trunk

$= 1 - (1 - B_1)(1 - c)$

$= B_1 + (1 - B_1) c$

Therefore Probability of simultaneous blocking for all g_3 independent paths is $B_2 = [B_1 + c(1 - B_1)]^{g_3}$

Where g_3 is the number of tertiary switches

Problem 3.22

a. Design three stage network interconnecting 100 incoming trunks to 100 outgoing trunks

b. Compare the grades of service provided by the two networks of Example when each operates in mode l and is offered 30 E of traffic:

c. What is the traffic capacity of each network if the required grade of service is 0.01?

Solution: (a):

$\sqrt{\frac{100}{2}} = 7.07$

use $n = 5$ or $n = 10$

1. If $n = 5$, there are:

   20 primary switches of size 5 x 5

   5 secondary switches of size 20 x 20

   20 tertiary switches of size 5 x 5

   If n=10, there are
   10 primary switches of size 10 x 10
   10 secondary switches 10 x 10 and
   10 tertiary switches of size 10 x 10

b). a = b = A/N= 30/ 100 = 0.3 E

(b): $B = [l - (1 - a)(1 - b)]^g$

i.e. g=5=n

$B = [l - (1 - 0.3)(1 - 0.3)]^5 = 0.51^5 = 0.035$ for n=5

For network (b): $B = 0.51^{10} = 0.0012$

$B = [l - (1 - 0.3)(1 - 0.3)]^{10} = 0.012$ for n=10

c) Solution

$B = [l - (1 - a)(1 - b)]^n$ for a=b,

$$[1 - (1 - a)^2)^5 = 0.01$$

$$1 - (1 - a)^2 = O.Ol^{1/5} = 0.398$$

$a = 0.224$

Total traffic capacity = A= a x N

= 100 x 0.224 = 22.4 E

For network (b): N=100, n=5, B=0.01

$B = [l - (1 - a)(1 - b)]^n$ for a=b,

$$[1 - (1 - a)^2]^{10} = 0.01$$

$$1 - (1 - a)^2 = 0.01^{1/10} = 0.631$$

$a = 0.393$

Total traffic capacity = 100 x 0.393 = 39.3 E

Partially interconnected Networks GOS

For a partially interconnected three-stage network, as shown in Figure 3.23, there is only one path between an incoming trunk and an outgoing trunk.

The probability that this is free is $(1 - a)(l - b)$

and the probability of blocking is $1 - (1 - a)(l - b)$.

For a connection to a trunk on an outgoing route with *n* trunks, each connected to a different frame, the probability of loss using a particular trunk is

$$1 - (1 - a)(l - b)(l - c)$$

But there are *n* such trunks available. Assuming that each being busy is an independent random event, the probability of simultaneous blocking for all path s is

$$B_2 = [1 - (1 - a)(l - b)(l - c)]^n \qquad (2.28)$$

Four-stage networks

For a four-stage network, as shown in Figure 3.24 let Occupancy of *A* links be *a*

Occupancy of *B* links be *b*

Occupancy of C links be *c*

Occupancy of outgoing trunks be d

For a connection from a given inlet on an input frame to a particular outlet on an output frame (i.e. mode 1), the call may use any primary switch in the output frame. This switch is connected by a Blink to only one secondary switch in the particular input frame. From this switch there is only one A link to the primary switch of the given incoming trunk.

Probability of this path being free is $(1 - a)(1 - b)(1 - c)$

$\therefore$ Probability of this path being blocked is $1 - (1 - a)(1 - b)(1 - c)$

Probability that all g_2 independent paths are simultaneously blocked is

$$B_1 = (1 - (1 - a)(1 - b)(1 - c))^{g2}$$

Where g_2 is the number of secondary switches in input frame = number of primary switches in output frame.

For a route of n outgoing trunks:

Probability of loss for a particular trunk

$= 1 - (1 - B_1)(1 - d)$

$= B_1 + (1 - B_1) d$

$\therefore$ Probability of simultaneous blocking for all n independent paths is

$B_2 = [B_1 + d(1 - B_1)]^n$

Problem 3.23

Design a grading for connecting 25 trunks to switches having 12 outlets.

Solution:

The number of graded groups,

g = 2*N/k= 50/ 12= 4.1, and the factors of g are 1, 2 and 4.

Let the number of choices having singles = s

the number of choices having doubles = d

the number of choices having quadruples = q

substituting in equations

$s + d + q = 12$ ------- (1)

$4s + 2d + q = 25$ -----------(2)

From (2) - (1) = $3s + d = 13$

Substituting in equations (2.5)

$s = 1: d = 10$ and $q = 12 - 11 = 2$

$s = 2: d = 7$ and $q = 12 - 9 = 3$

$s = 3: d = 4$ and $q = 12 - 7 = 5$

$s = 4: d = 1$, and $q = 12 - 5 = 7$

$s = 5, d < 1$ so this is not possible .

There are thus three possible gradings, which are shown in Figure 2.5. The sums of the successive differences for these gradings arerespectively given by:

$D = |r_1 - r_2| + |r_2 - r_3| + ... + |r_{q-1} - r_q|$

Case1) $D_1 = |1-10| + |10-2|$ $D_1 = 9 + 8 = 17$

$D_2 = |2-7| + |7-3| = 5 + 4 = 9$

$D_3 = |3-4| + |4-5| = 1 + 1 = 2$

$D_3 = |4-1| + |1-7| = 3 + 6 = 9$

Best grading

having Successive Difference is least value is good grading method or allocation.

S=3,d=4 and q=5 then SD=2 (least Successive Difference)