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Effective Penetration Depth and Effective Resistance in Moisture Transfer

M. J. Cunningham

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The concepts of "effective penetration depth" and "effective resistance" have appeared in the literature recently with reference to lumped models of moisture transfer. These quantities are shown to be equivalent. The effective depth for the one-sided case is shown to be $\sqrt{\frac{D}{2\omega}}$ and for the two-sided case is 1/6 the material thickness, with corresponding effective resistances. Significantly, the effective penetration depth is shown to be not dependent upon the surface resistance of the boundary layer and any coatings. A simple lumped model used in defining the concepts gives very close agreement with the exact solution to the diffusion equation for the amplitude and phase of the mean moisture content for a periodic driving potential. This suggests that effective penetration depth and effective resistance can be regarded as universal physical quantities and not merely approximations or artifacts arising out of simplifications.

Nomenclature

a depth (m)
A area (m^2)
d effective penetration depth (m)
D diffusion coefficient $(m^2\,s^{-1})$
h see formula (10) (m^{-1})
l half width (m)
m moisture concentration $(kg\,m^{-3})$
m_c effective cyclic moisture concentration $(kg\,m^{-3})$
m_1 periodic component of the external moisture concentration $(kg\,m^{-3})$
$\overline{m}$ non-periodic component of the moisture concentration $(kg\,m^{-3})$
M_T total cyclic moisture content $(kg\,m^2)$
R effective resistance $(s\,m^{-1})$
R_m material resistance $(s\,m^{-1})$
R_s boundary layer and coating resistance $(s\,m^{-1})$
t time (s)
V volume (m^3)
ϕ phase $(radians)$
ω angular frequency $(radians\,s^{-1})$
ω' see formula (10) (m^{-1})

Subscripts
m material

p periodic
s surface
0 external

Introduction

Lumped modelling is one of the techniques used to aid in the understanding of the moisture performance of structures, e.g. [1,2]. In this approach some aspects of the physical situation to be modelled are simplified partly for this reason, but also, importantly, to provide physical insight by the introduction of concepts that have physical meaning and which aid in the understanding of the details of moisture transfer in complex systems.

Two such concepts have appeared recently in the literature of lumped modelling of moisture transfer in buildings, viz. "effective resistance" see for example Cunningham [1] and "effective penetration depth" see Keresteciogu et al [2]. Effective penetration depth gives an indication of the depth that moisture penetrates into a material under transient and cyclic conditions, while effective resistance is used to give an idea as to the amount of moisture transfer resistance the mean moisture content of a material encounters in transferring into and out of that material.

To paraphrase Keresteciogu et al [2], in the case of effective resistance, workers use "effective" convective mass transfer coefficients and actual thicknesses and surface areas, and in the case of effective penetration depth workers use "actual" convective mass transfer coefficients and surface area and "effective depth" and represent the moisture capacity in issue using a "lump" of this thickness.

This work defines these concepts in terms of a simple lumped model, in which moisture is allowed to transfer from an interior node to an exterior node through a resistance whose value is determined by the effective depth and the resistance of the surface boundary layer and coatings. It is shown that this gives the same amplitude and phase response for the appropriate lumped moisture concentrations as the exact solution found by solving the diffusion equation for appropriate boundary conditions. This agreement is found to be exact in the one-sided case and nearly so in the two-sided case. (In the one-sided case the effective penetration depth is small enough that the material as a whole is effectively of infinite thickness. The two-sided case occurs when the effective penetration depth approaches the half-thickness of the material, so that the internal moisture profile cannot be regarded as made of two separate profiles, one from each side.)

The effective penetration depth for the one-sided case is found to be $\sqrt{\frac{D}{2\omega}}$ with a corresponding effective resistance, while for the two-sided case the effective penetration depth is one sixth of the material thickness, again with a corresponding effective resistance.

There are three important outcomes of this analysis. Firstly, as mentioned above, effective resistance and effective penetration depth are equivalent.

Secondly, it can be seen that the effective penetration depth does not depend upon the size of any boundary layer and coating resistance (but the effective resistance obviously does).

Thirdly, it is shown that effective resistance and effective penetration depth can be regarded as universal physical quantities and not merely approximations or artifacts arising out of simplified considerations. Any previous understanding that lumped modelling in moisture transfer is very approximate can no longer be sustained, allowing the concepts of effective resistance and effective penetration depth to be elevated to that of universal physical quantities. (The caveat must be added that these considerations are made under conditions of a constant diffusion coefficient and isothermal conditions.)

The underlying lumped model

Figure 1 illustrates the lumped model that forms the basis of the analysis in this work. Figure 1(a) is the one-sided case and Figure 1(b) the two-sided case. These are two-node models, where in the two-sided case, Figure 1(b) A and A$'$ are identical points. Node A (and A$'$) represents the external moisture content driving potential, and node B represents a lumped moisture content at depth d or l within the material. In the two-sided case the material is of thickness $2l$. In the one-sided case, the lumped moisture content is thought of as being derived from a uniform moisture concentration spread through the material to a depth of $2d$, while in the two-sided case the lumped moisture content is thought of as being derived from a uniform moisture concentration spread through the entire thickness, $2l$ of the material. In the one-sided case, the two nodes are connected together by two series resistances, one, R_m, representing the resistance of the material, and the other, R_s, representing the resistance of the boundary layer and any surface coating on the material. In the two-sided case, nodes A and B are connected by two parallel resistances $R_m + R_s$.

The total resistance between nodes A and B is

$$R = R_m + R_s \tag{1}$$

for the one-sided case, and

$$R = \frac{1}{2}(R_m + R_s) \tag{2}$$

for the two-sided case, since in this case two parallel paths connect the interior node to the exterior node A or A$'$.

The performance of this model is governed by the differential equation

$$V\frac{dm}{dt} = A\frac{m_0 - m}{R} \tag{3}$$

i.e.

$$2a\frac{dm}{dt} = \frac{m_0 - m}{R} \tag{4}$$

since the volume V through which the moisture is regarded as spread is given by

$$V = 2aA$$

where $a = d$ for the one-sided case and $a = l$ for the two-sided case.

If the driving moisture content is periodic and is given by

$$m_0 = \overline{m} + m_1 \sin \omega t$$

then the steady solution to equation (4) is

$$m = \overline{m} + \frac{m_1}{\sqrt{1 + (2\omega a R)^2}} \sin(\omega t - \phi) \tag{5}$$

where

$$\phi = \tan^{-1} 2\omega a R$$

Definitions

The following definitions are made

Total cyclic moisture content, M_T - the periodic component of the total moisture content per unit area within the material. ($kg\, m^{-2}$)

Effective cyclic moisture concentration, m_c - the total cyclic moisture content divided by the thickness $2a$ through which it is regarded being spread. ($kg\, m^{-3}$) i.e.

$$m_c = \frac{M_T}{2a}$$

Using these definitions, the solution, equation (5) above, can be expressed as

$$m = \overline{m} + \frac{M_T}{2a} = \overline{m} + m_c$$

where

$$m_c = \frac{M_T}{2a} = \frac{m_1}{\sqrt{1 + (2\omega a R)^2}} \sin(\omega t - \phi) \tag{6}$$

Effective penetration depth - that depth d which gives the correct amplitude and phase response for the effective cyclic moisture concentration, m_c, concentrated at depth d when the moisture gains access to the surface through the resistance R associated with that depth viz.

$$R = \frac{d}{D} + R_s \tag{7}$$

R is known as the *effective resistance.*

Descriptively, the effective cyclic moisture concentration should appear to be concentrated at the effective depth. Therefore, in transferring cyclically in and out of the material, it should encounter a material resistance corresponding to the effective depth plus an additional surface resistance.

It is not known *a priori* whether these definitions are consistent, i.e., that such a depth can be found that will correctly reproduce the amplitude and phase response for the effective cyclic moisture concentration concentrated at that depth. Unless consistency can be shown, the concepts of effective penetration depth and effective resistance are of little value. Much of the rest of this paper is devoted to showing that such a depth does exist.

The one-sided case

This case, illustrated in Figure 2, occurs where the effective penetration depth is small enough that the material as a whole is effectively of infinite thickness.

To explore whether the concepts of effective penetration depth and effective resistance as defined above are meaningful, it is necessary to solve the diffusion equation for the case of interest, calculate the total cyclic moisture content, M_T, and hence the effective cyclic moisture concentration, m_c, and ascertain whether the result has an amplitude and phase response that is identical to that given by the underlying lumped model when the definitions of effective penetration depth and effective resistance are used.

This is shown as follows.

For a periodic driving potential

$$m_0 = \overline{m} + m_1 \sin \omega t$$

Carslaw and Jaeger [3] give the steady solution as

$$m = \overline{m} + \frac{m_1 h}{\sqrt{(h+\omega')^2 + \omega'}} e^{-\omega' x} sin(\omega t - \omega' x - \delta) \tag{8}$$

$$= \overline{m} + m_p \tag{9}$$

where

$$h = \frac{1}{R_s D} \; and \; \omega' = \sqrt{\frac{\omega}{2D}} \tag{10}$$

and

$$\delta = \tan^{-1}\left(\frac{\omega'}{h+\omega'}\right)$$

The total cyclic moisture content is calculated by integrating the periodic part of the moisture content throughout the depth of the material, i.e.

$$\begin{aligned} M_T &= \int_0^\infty m_p dx \\ &= \frac{m_1 h}{\sqrt{(h+\omega')^2+\omega'}} \int_0^\infty e^{-\omega' x} sin(\omega t - \omega' x - \delta) dx \\ &= \frac{m_1}{\omega' \sqrt{(1 + (1 + \frac{2\omega'}{h})^2}} \sin(\omega t - \phi) \end{aligned} \tag{11}$$

where

$$\phi = \tan^{-1}(1 + \frac{2\omega'}{h})$$

If we make the identification of apparent penetration depth as

$$d = \frac{1}{2\omega'} = \sqrt{\frac{D}{2\omega}} \tag{12}$$

then, by definition, the corresponding effective resistance will be

$$R = \frac{d}{D} + R_s$$

from equation (7).

If the concepts of effective resistance and penetration depth are to be meaningful, this effective resistance should make its appearance in the exact solution for the total cyclic moisture content, equation (11), in the appropriate manner.

Indeed, from equations (1), (7), (10) and (12) we have

$$\begin{aligned} 1 + \frac{2\omega'}{h} &= 1 + 2R_s D\sqrt{\frac{\omega}{2D}} = 2\omega d(\frac{1}{2\omega d} + R_s) \\ &= 2\omega d(\frac{d}{D} + R_s) = 2\omega d(R_m + R_s) \\ &= 2\omega d R \end{aligned} \tag{13}$$

Equation (12) identifies the effective penetration depth d, which is taken for the one-sided case as the midpoint of the region through which the total cyclic moisture content M_T is spread. Hence the effective cyclic moisture concentration m_c is found by dividing M_T by $2d$, i.e., from equations (11), (12) and (13)

$$m_c = \frac{M_T}{2d} = \frac{m_1}{\sqrt{1 + (2\omega d R)^2}} \sin(\omega t - \phi) \tag{14}$$

where

$$\phi = \tan^{-1} 2\omega d R \tag{15}$$

The lumped model solution, equation (6) can be seen to be identical to this exact solution, confirming that, for the one-sided case, penetration depth and effective resistance are meaningful, indeed exact, quantities and are given by

$$d = \frac{1}{2\omega'} = \sqrt{\frac{D}{2\omega}} \tag{16}$$

and

$$R = \frac{d}{D} + R_s \tag{17}$$

respectively.

Note that the value for d is equal to 1/2 of the exponential distance factor ω' in equation (8) governing the shape of the envelope of the moisture profile from the surface into the material.

The two-sided case

This case, illustrated in Figure 3, occurs when the moisture distribution profiles from each side of the material begin to overlap. This will happen when the frequency is lower or the material thinner, more precisely when $\omega' l$ becomes less than 1. The same procedure is followed as in the one-sided case to show that the appropriate identification of an effective depth and the corresponding effective resistance gives rise to agreement between the lumped and the exact model for predictions of the amplitude and phase of the effective cyclic moisture concentration.

Carslaw and Jaeger [3] give the steady solution in the two-sided case as

$$\begin{aligned} m &= \overline{m} + \frac{h m_1 \Gamma_0}{\Gamma_1} \sin(\omega t + \gamma_0 - \gamma_1) \\ &= \overline{m} + m_p \end{aligned} \quad (18)$$

where

$$\Gamma_0 e^{j\gamma_0} = \cosh \omega' x \, cos \omega' x + j \sinh \omega' x \sin \omega' x \quad (19)$$

and

$$\begin{aligned} \Gamma_1 e^{j\gamma_1} &= \omega' \sinh \omega' l \cos \omega' l - \omega' \cosh \omega' l \sin \omega' l + h \cosh \omega' l \cos \omega' l \\ &\quad + j(\omega' \sinh \omega' l \cos \omega' l + \omega' \cosh \omega' l \sin \omega' l + h \sinh \omega' l \sin \omega' l) \end{aligned} \quad (20)$$

The periodic part of the solution m_p can be written as

$$m_p = \frac{h m_1}{\Gamma_1} \left(\sin(\omega t - \gamma_1) \cosh \omega' x \cos \omega' x + \cos(\omega t - \gamma_1) \sinh \omega' x \sin \omega' x \right)$$

from equation (18) and (19).

The total cyclic moisture content is found by integrating through the material as

$$M_T = \int_{-l}^{l} m_p dx$$

which gives

$$\begin{aligned} M_T &= \frac{h m_1}{\omega' \Gamma_1^2} \left\{ h \left(\sinh \omega' l \cosh \omega' l + \sin \omega' l \cos \omega' l \right) \sin \omega t \right. \\ &\quad \left. - \left(2\omega' (\sinh^2 \omega' l + \sin^2 \omega' l) + h (\sinh \omega' l \cosh \omega' l - \sin \omega' l \cos \omega' l \right) \cos \omega t \right\} \end{aligned} \quad (21)$$

where, from equation (20)

$$\begin{aligned} \Gamma_1^2 &= 2\omega'^2 \left(\sinh^2 \omega' l + \sin^2 \omega' l \right) + h^2 \left((\cosh \omega' l \cos \omega' l)^2 + (\sinh \omega' l \sin \omega' l)^2 \right) \\ &\quad + 2h\omega' \left(\cosh \omega' l \sinh \omega' l - \cos \omega' l \sin \omega' l \right) \end{aligned} \quad (22)$$

The two-sided case occurs when $\omega' l < 1$ so the series approximations for the exponential functions can be used, viz.

$$\begin{aligned} \sin \omega' l &\simeq \omega' l - \frac{(\omega' l)^3}{6}, \quad \cos \omega' l \simeq 1 - \frac{(\omega' l)^2}{2}, \\ \sinh \omega' l &\simeq \omega' l + \frac{(\omega' l)^3}{6}, \quad \cosh \omega' l \simeq 1 + \frac{(\omega' l)^2}{2} \end{aligned}$$

which results in

$$M_T \simeq \frac{2lhm_1}{\sqrt{h^2 + \left(2\omega'^2 l + \frac{2}{3}h(\omega' l)^2\right)^2}} \sin(\omega t - \phi)$$

where

$$\phi = \tan^{-1}\left(\frac{2\omega'^2 l}{h} + \frac{2}{3}(\omega' l)^2\right)$$

The total cyclic moisture content M_T is spread through the material which is of thickness $2l$, so the effective cyclic moisture concentration is given by

$$m_c = \frac{M_T}{2l} \simeq \frac{m_1 \sin(\omega t - \phi)}{\sqrt{1 + \left(\frac{2\omega'^2 l}{h} + \frac{2}{3}(\omega' l)^2\right)^2}} \tag{23}$$

But from equation (10)

$$\begin{aligned} \frac{2\omega'^2 l}{h} + \frac{2}{3}(\omega' l)^2 &= \frac{\omega}{D} R_s D l + \frac{1}{3}\frac{\omega}{D} l^2 \\ &= 2\omega l \, . \, \frac{1}{2}(R_s + \frac{l}{3D}) \end{aligned} \tag{24}$$

The effective penetration depth is now identified as

$$d = \frac{1}{3} l$$

i.e., 1/6 of the material thickness, $2l$. The corresponding effective resistance from equations (2) and (7) will be

$$R = \frac{1}{2}(R_s + \frac{l}{3D})$$

allowing for the two parallel paths to the surface in the two-sided case.

This means that equation (24) becomes

$$\frac{2\omega'^2 l}{h} + \frac{2}{3}(\omega' l)^2 = 2\omega l R$$

Hence, the expression for the effective cyclic moisture concentration, equation (23), becomes

$$m_c \simeq \frac{m_1}{\sqrt{1 + (2\omega l R)^2}} \sin(\omega t - \phi) \tag{25}$$

where

$$\phi = \tan^{-1} 2\omega l R \tag{26}$$

Again, this result can be seen to be the same as that of the lumped model, equation (6), confirming that, for the two-sided case, penetration depth and effective resistance are meaningful quantities and are given by

$$d = \frac{1}{3} l \tag{27}$$

and

$$R = \frac{1}{2}(R_s + \frac{l}{3D}) \tag{28}$$

respectively.

The solution just derived, equations (25), (27) and (28) will be called the low frequency solution to the two-sided case.

On the other hand, if $\omega' l$ is large, then it can be shown, by approximating the hyperbolic functions in equations (21) and (22) with $\frac{1}{2}\exp\omega' l$, that the two-sided case reduces to the one-sided case. In other words, the one-sided solution can be used as an approximate solution to the two-sided case if $\omega' l \gg 1$. This will be called the high frequency solution to the two-sided case.

It remains to investigate the overlap region, $\omega' l \sim 1$, and also to ascertain the accuracy of the low and high frequency solutions to the two-sided case.

To achieve this, the low and high frequency solutions, equations (25-28) and (14-17) respectively, and the exact two-sided solution following from equation (21) are graphed together, see Figure 4. This figure shows the normalised cyclic moisture concentration m_c/m_1 plotted against $\omega' l$ for various values of ω'/h. Inspection of this graph shows that there is a region of considerable overlap in the validity of the low and high frequency solutions. In this range of overlap in the worst case (viz. $\omega' l \simeq 1$, $\frac{\omega'}{h} = 0$) the best approximation is within 7% in magnitude of the exact solution. In this region the phases also have their maximum difference at about 0.1 radian (6 degrees). For all other values of $\omega' l$ and $\frac{\omega'}{h}$ one or other approximation gives values of m_c/m_1 which are virtually indistinguishable from the exact solution. The range of validity of each approximation is clearly seen in Figure 4.

By equating the low and high frequency values for the effective penetration depths, equations (27) and (16) respectively, it can be seen that the two approximations give the same results at $\omega' l = \frac{3}{2}$. The solution to the two-sided case over the entire range of $\omega' l$ can be therefore given as

$$\begin{array}{llll} d = \frac{1}{3}l & R = \frac{1}{2}\left(R_s + \frac{l}{3D}\right) & \omega' l < \frac{3}{2} \\ d = \frac{1}{2\omega'} = \sqrt{\frac{D}{2\omega}} & R = R_s + \frac{d}{D} & \omega' l \geq \frac{3}{2} \end{array}$$

with the effective cyclic moisture concentration being given accurately by equation (25).

Other Definitions

The result obtained for effective resistance in the two-sided case, viz.

$$\begin{aligned} R &= \frac{1}{2}(R_s + \frac{l}{3D}) \\ &= \frac{1}{2}(R_s + \frac{0.33l}{D}) \end{aligned}$$

compares well with the result obtained by the author elsewhere [1] viz.

$$\begin{aligned} R &= \frac{1}{2}\left(R_s + \frac{4l}{\pi^2 D}\right) \\ &= \frac{1}{2}\left(R_s + \frac{0.41l}{D}\right) \end{aligned}$$

which was derived from transient considerations.

Kerestecioglu et al give a different definition for effective penetration depth. They demand that the total moisture content appear to be distributed uniformly over an effective depth at a concentration equal to the surface moisture content, i.e.,

$$\int_0^\infty m\,dx = m_s d$$

This definition, although suggestive, has some problems. Firstly, it is necessary that the cyclic moisture content rather than the total moisture content appear in the definition, otherwise the effective penetration depth becomes dependent upon the level of the mean moisture content $\overline{m}$, which is not physically realistic, i.e., we require

$$\int_0^\infty m_p\,dx = m_{sp} d \tag{29}$$

where

$$m_s = \overline{m} + m_{sp}\sin(\omega t + \phi_s)$$

Secondly, there are problems with the amplitude and phase response in this definition as the following analysis shows.

The result of the integration of equation (29) is given by equation (11), which is equivalent to

$$M_T = \frac{m_1 h}{2\omega'\sqrt{(h+\omega')^2+\omega'}}\left(\sin(\omega t-\delta) - \cos(\omega t - \delta)\right)$$

But from equation (8)

$$m_{sp} = \frac{m_1 h}{\sqrt{(h+\omega')^2+\omega'}}\sin(\omega t - \delta)$$

The total cyclic moisture content M_T is not in phase with the cyclic surface moisture concentration m_{sp}. Hence if the definition equation (29) is used we find

$$d = \frac{1}{2\omega'}\left(1 - \cot(\omega t - \delta)\right)$$

This expression for the effective depth is time dependent, and at some times is negative and at others is even infinite. However, the mean value of the effective penetration depth over one period is

$$d = \frac{1}{2\omega'}$$

which is identical to the result derived in this work.

Discussion

In the two-sided case, as $\omega' l$ becomes smaller, the exponential envelopes of the cyclic moisture content approach each other from each side and begin to overlap, so that the one-sided approximation breaks down. This in turn blocks any further migration of the effective depth towards the centre of the material and freezes the effective depth value at 1/6 of the thickness. However, the total cyclic moisture is now spread throughout the material, and the entire thickness, $2l$, must be used in calculating the effective cyclic moisture concentration. (It may be better to visualize the moisture concentrated at two centres, each at 1/3 of the half thickness of the material.) This is perhaps a slightly less clear physical interpretation to that of the one-sided case where the total cyclic moisture content is thought of as being spread through a slab of thickness twice the effective depth, and the effective cyclic moisture concentration is thought of as being concentrated at the centre of that slab.

The effective depth in both cases is independent of the surface resistance of the boundary layer and any coatings. This is so because, although increasing the surface resistance decreases the amplitude of the moisture swings within the material, it does not change the exponential distance scaling factor $1/\omega'$ which determines the effective thickness in the one-sided case. In the two-sided case the effective depth is fixed at 1/6 the material thickness for reasons explained above. On the other hand, both the effective resistance and the cyclic moisture content do change with the surface resistances for obvious reasons.

An interesting case occurs when the material is relatively thick, the frequency relatively low and the surface resistance small, specifically when

$$R_s \ll \frac{d}{D} = \frac{1}{\sqrt{2\omega D}}$$

and we have the one-sided case. This situation would occur say for thicker walls with low surface resistance exposed to yearly time periods.

In this case, it will be found from formulae (14)-(17) that the effective cyclic moisture concentration has an amplitude of $m_1/\sqrt{2}$ and a phase of $\pi/4$, independently of the type of material and the frequency.

Table 1 contains values of effective resistance and effective penetration depth for the one-sided case for two values of diffusion coefficient, 10^{-9} and $10^{-10}\, m^2\, s^{-1}$ and three periods, 1 year, 1 day and 1 hour.

It can be seen that effective penetration depths are of the order of a millimetre or less, unless the period is very long. In other words, the two-sided case is not usually encountered unless the period is of the order of a year. The surface resistance is taken as $10^7\, s\, m^{-1}$, which implies these materials are uncoated. It can be seen that, except in the case of one year, the effective resistance is not much affected by the nature of the material, which is to be expected if the effective depth is small. If a surface coating of any significant resistance is applied, greater than say $10^8\, s\, m^{-1}$, then it will dominate in the effective resistance of the material even at yearly frequencies.

Material Type	Diffusion Coefficient $m^2 s^{-1}$	Surface Resistance $s\,m^{-1}$	Period					
			1 year		1 day		1 hour	
			d (mm)	R ($s\,m^{-1}$)	d (mm)	R ($s\,m^{-1}$)	d (mm)	R ($s\,m^{-1}$)
Wood	10^{-10}	10^7	15.9	1.69×10^8	0.83	1.83×10^7	0.17	1.17×10^7
Plasterboard	10^{-9}	10^7	50.3	6.03×10^7	2.63	1.26×10^7	0.54	1.05×10^7

Table 1: Effective penetration depth d and effective resistance R (one-sided case).

The predictions in this table agree well with experimental results discussed in Kerestecioglu et al. They find that their experimental data for wood for a 24-hour period is best fitted by an effective penetration depth of 1.4 mm; formula (16) implies that this wood has a diffusion coefficient of $2.9 \times 10^{-10}\ s\,m^{-1}$ which is typical. They also quote data from Kamel et al [4] and here find that a room with gypsum walls, nylon carpet and linen furniture is best fitted by an effective penetration depth of about 2 mm over a 24-hour period. Table 1 gives an effective penetration depth of 2.63 mm for plasterboard with a diffusion coefficient of $10^{-9}\ s\,m^{-1}$ at this period.

The success of this agreement, and the theoretical accuracy of the concepts of effective resistance and effective penetration depth shown here also reflect the unforseen accuracy of the lumped model. This model is accurate at any frequency, suggesting a re-visit of modelling work that had mostly yearly periods in mind (e.g., Cunningham, [1]).

Conclusion

It has been shown that effective resistance and effective penetration depth are different aspects of the same thing. For a constant diffusion coefficient and under isothermal conditions, it has been shown that the effective depth for the one-sided case is $\sqrt{\frac{D}{2\omega}}$ and for the two-sided case is 1/6 the material thickness, with corresponding effective resistances. The effective penetration depth is shown to be not dependent upon the surface resistance of the boundary layer and any coatings. These formulae give good agreement with existing experimental results.

An interesting special case arises typified by uncoated thick walls subjected to yearly moisture cycles. Under conditions discussed, for any material whatsoever, the cyclic moisture concentration has a phase lag of $\pi/4$ and an amplitude of $1/\sqrt{2}$ of the periodic component of the driving moisture concentration.

The concepts arise out of examining the moisture transfer into materials by a simple lumped model in which the effective cyclic moisture concentration is imagined concentrated at the effective depth, and has access to the surface through the corresponding effective resistance. Significantly, it has been shown that this lumped model is exact for the one-sided case, and nearly so in the two-sided case, suggesting the elevation of the concepts of effective resistance and effective penetration depth to universal physical quantities. In this process, the lumped model is also shown to be much more than just a rough approximation, suggesting that a fresh look at the field of lumped modelling may be appropriate.

References

[1] M.J. Cunningham, The Moisture Performance of Framed Structures-A Mathematical Model *Bldg Envir.* **23** 123-135 (1988).

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1 Figure and Table Captions

Figure 1: The underlying lumped models, showing the material moisture concentrations at node B having access to the external nodes A, A$'$, through a material resistance R_m and a surface resistance R_s.

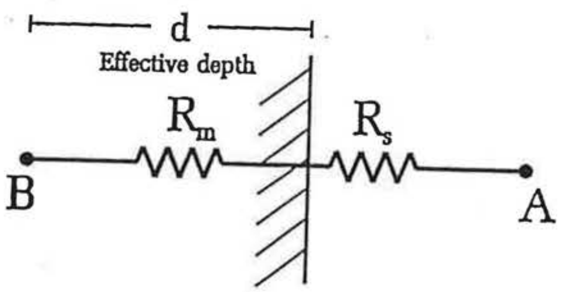
Figure 2: One-sided case. Actual cyclic moisture concentration has the envelope shown, but the moisture appears to be concentrated at an effective depth d, with an effective concentration m_c.

Figure 3: Two-sided case. Actual cyclic moisture concentration has the envelope shown, but the moisture appears to be concentrated at an effective depth d, with an effective concentration m_c.

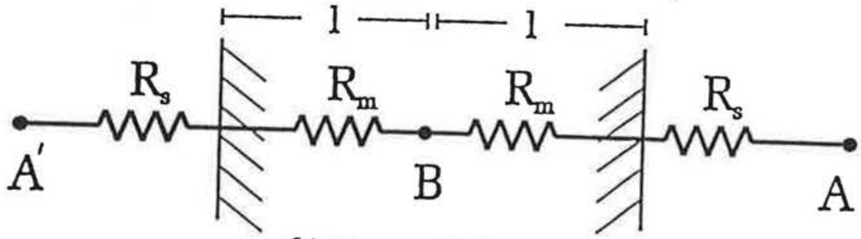
Figure 4: The two-sided model: exact, low and high frequency solutions for effective cyclic moisture concentration.

Table 1: Effective penetration depth d and effective resistance R (one-sided case).

See p. 14 for caption



(a) One – sided case



(b) Two – sided case

Figure 1

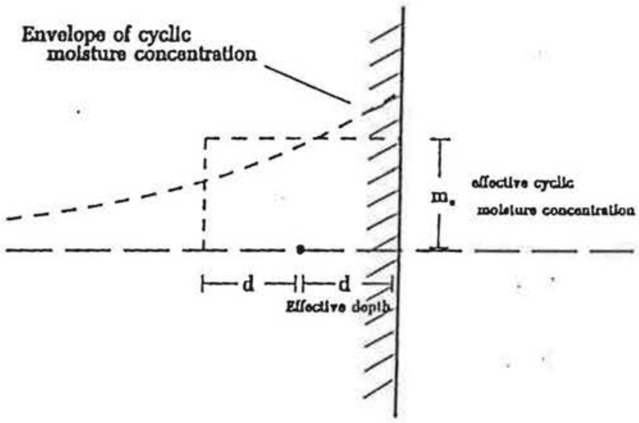


Figure 2: One – sided case

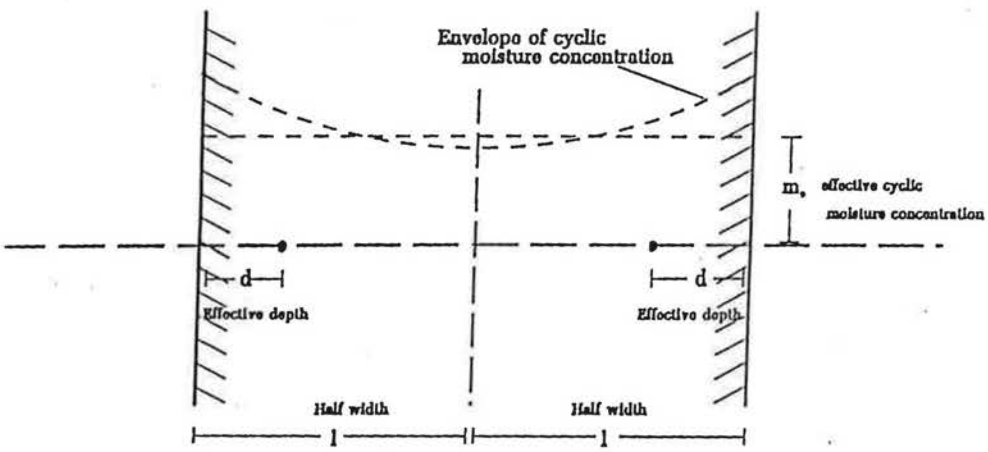


Figure 3

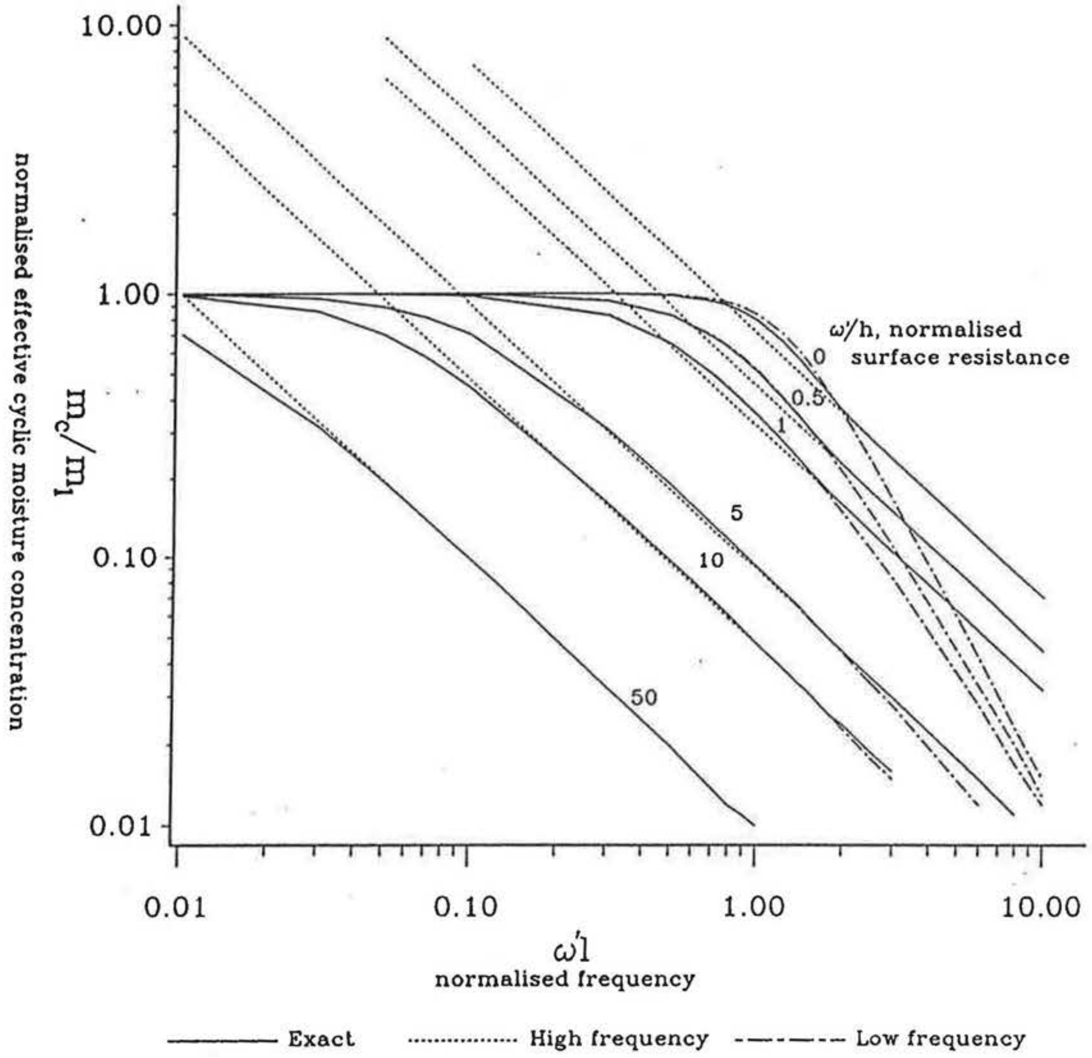

10.00
1.00
0.10
0.01
normalised effective cyclic moisture concentration
m_c/m_1
ω'/h, normalised surface resistance
0
0.5
1
5
10
50
0.01
0.10
1.00
10.00
ω'l
normalised frequency
Exact
High frequency
Low frequency

Figure 4