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Algebraic K-Theory

Victor P. Snaith
Editor

American Mathematical Society

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Providence, Rhode Island

The Fields Institute for Research in Mathematical Sciences

The Fields Institute is named in honour of the Canadian mathematician John Charles Fields (1863–1932). Fields was a visionary who received many honours for his scientific work, including election to the Royal Society of Canada in 1909 and to the Royal Society of London in 1913. Among other accomplishments in the service of the international mathematics community, Fields was responsible for establishing the world's most prestigious prize for mathematics research—the Fields Medal.

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Contents

Preface

The Second Great Lakes Conference on Algebraic K-Theory was hosted by The Fields Institute for Research in Mathematical Sciences in March 1996. It proved to be a mathematically very exciting occasion with many interesting new results being unveiled. The success of the meeting together with a desire to commemorate Bob Thomason, one of K-theory's most influential figures, led to this Proceedings volume.

As editor, I would like to express my thanks and those of the organizers of the conference to the staff of The Fields Institute who made this meeting so rewarding and enjoyable. In particular, I wish to extend special thanks to the President and Scientific Director, John Chadam, for inviting the Great Lakes K-Theory Conference to The Fields Institute, and to the Publications Assistant, Erna Unrau, for making the editing of this volume so easy for me.

Victor Snaith

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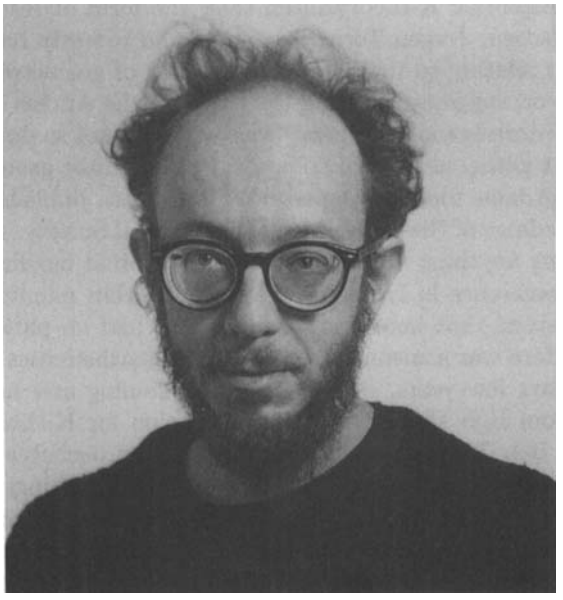

Robert Wayne THOMASON
1952-1995

In the mid 1970's, when Bob Thomason was a star graduate student at Princeton University, algebraic K-theory was in a very tentative state. It had begun as Whitehead-ian algebra-to-capture-geometry, showing lots of low-dimensional promise. Then, with the work of Quillen, algebraic K-theory had suddenly expanded into a grand cosmological entity with its own, extremely fascinating, ontology and epistomology. It was clear that the subject constituted a major mathematical nexus but the evidence was in the form of hard-won hints about how Quillen's K-theory might unify such diverse phenomena as homotopy types of diffeomorphism groups of manifolds, special values of L-functions, polylogarithms, intersection theory of algebraic varieties, Weil cohomology theories and so on. As a result, no one was sure whether K-theory was the domain of topologists, algebraists or some other mathematical species. In many places this hardly mattered, the subject could still easily be shrugged off as a new fad with fine ambitions, lots of conjectured promise, but with no "big theorem" to mark some post-Quillen progress.

Bob Thomason was the one who was to provide such a "big theorem".

It must have been clear all the way up the educational ladder that Bob was exceptionally clever. For example, at eighteen he had won first prize in the North America wide competition entitled "Great Books of the Western World". Consequently, I was unaware that Bob was still a graduate student when I first met him around 1976. André Weil is reputed to have advised that one should learn from "the masters, not the students" – I can only conclude that he did not have Bob Thomason in mind!

My friendship with Bob got off to a rather shaky start. In 1975 topologists' contributions to algebraic K-theory often took the form of results about infinite loopspaces. Ib Madsen, Jørgen Tornehave and I had recently resolved all of Peter May's conjectures relating to the infinite loopspaces of geometric topology. Frank Adams, as an editor, suggested sending the paper to the Annals of Mathematics in Princeton. Midwestern homotopy theory was evidently not so popular in Princeton and the typescript gathered a lot of cobwebs, but an editor gave a copy to Bob to read. Eventually Adams took the paper away in disgust, publishing it a few weeks later in the Proceedings of the Cambridge Philosophical Society. Anyway, the point is that I was feeling anything but cheerful towards Bob at our first meeting, which took place at a conference in Evanston. However, within minutes of meeting Bob it was clear to anyone that here was someone who had no patience for intra Ivy League politics. Here was someone intent on doing mathematics for its own sake!

During the next few years, mostly through listening and not understanding, I learned a lot from Bob about the prevailing vision for K-theory. His intensity notwithstanding, Bob Thomason was always very approachable - focussed on research goals, he invariably had time patiently to explain things to lesser mortals such as myself. Also, he had a knack for putting across the *conceptual* picture. In the late 1970's he, Bill Dwyer, Eric Friedlander and I got involved in studying mod p K-theory which has been inflicted with Bott periodicity. When A is a commutative ring there is a map of graded rings, $\pi_*^S(BA_+) \longrightarrow K_*(A)$, from the stable homotopy of the classifying space of the units of A (with a disjoint point attached) to the algebraic K-theory of A. This map comes from Quillen's group-completion theorem and exists also for homotopy with coefficients. In order to say something about this map I had managed to calculate the stable homotopy when inflicted with Bott periodicity and had therefore inflicted algebraic K-theory with similar periodicities so as to be able to detect the asymptotic residues of this map. I had obtained a few rudimentary results about new elements in K-theory, but Bob immediately saw that my Bott periodicities were really connected with the periodicity in étale cohomology. For some time he had been trying to establish Quillen's prediction that K-theory was related to étale cohomology, after some low dimensional exceptions. Bob already possessed a number of very original constructions in which one did homological algebra with spectra and stable homotopy theory instead of groups and modules. With these techniques he had been trying to prove rather too much about the predicted connection. However, with the aid of our four-author paper, Bob soon had the required theorem - that Bott periodic algebraic K-theory mod p^n of any reasonable scheme had an Atiyah-Hirzebruch spectral sequence starting with étale cohomology. This was exactly a result of the much-sought-after type. It showed that, in high dimensions, the predicted connection between K-theory and cohomology existed.

This connection, together with its concomitant insights, served as the basis for many of Bob best theorems. Now he had a machine through which to funnel results from topological K-theory into algebraic K-theory, a sheet-anchor giving much of

the stability of purpose to his later research. His subsequent string of important results forms the backbone of modern K-theory, particularly in its applications to arithmetic and algebraic geometry. His main theorem was a tour de force. It is very hard going and a proper study takes a long time, lots of work during which one passes through phases which Bob wryly described as the progression from "thrill seeker" to "reckless cheat" to "honest man". Personally, I doubt that I ever made it past the "reckless cheat" phase. However, many other authors have studied the ideas and methods. I am very grateful to Steve Mitchell who, in this volume, has written an "essay which will smooth the way for others" through Thomason's Theorem.

I always felt a certain sadness, and here I am not referring to his untimely death, about Bob's career. Before coming to North America it had never occurred to me that one must not only prove the "right" theorem but one must prove it in the "right" place. The fictions, fads and factions of mathematical society are quite clear to me now, but in those days I never understood why Bob's achievements went so comparatively unrewarded. To his K-theoretic friends, therefore, it came as no great surprise when, in October 1989, Bob moved to a position in the C.N.R.S. at the Unversité de Paris 7. With that move, Bob was mathematically where he felt most at home.

Sadly, at the height of his mathematical powers, Bob died, tragically. He would have been forty-three years old on November 5, 1995. A longtime diabetes sufferer, just before his birthday he went into diabetic shock and died in his Paris apartment. He will be missed immensely but, I like to think, mathematically his name will survive as long as does the subject algebraic K-theory, which he loved so much. And that will be a very long time!

Victor Snaith
February 1997

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Appendix I

The Second Great Lakes Conference on Algebraic K-Theory

Program List of Speakers and Topics

T. Chinburg	Galois Structure of deRham Cohomology
J.-L. Colliot-Thélène	Brauer Groups and Zero-Cycles on Varities over Local Fields
E. Friedlander	Cohomology of Finite Group Schemes over a Field
S. Lichtenbaum	Motives of Curves
R. McCarthy	A Chain Complex for the Spectrum Homology of the Algebraic K-Theory of an Exact Category
V. Voevodsky	Cohomological Operations in Motivic Cohomology
C. Weibel	The Work of Bob Thomasan (1952 - 1995)

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Appendix II

List of Participants

Andersen, Kasper K. S., The Fields Institute, Toronto, Ontario, Canada
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Arlettaz, Dominique, University of Lawsanne, Lausanne, Switzerland
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Cohen, Fred, The University of Rochester, Rochester, NY USA
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Kolster, Manfred, McMaster University, Hamilton, Ontario, Canada
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Weibel, Charles, Rutgers University, New Brunswick NJ USA
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Yui, Noriko, Queen's University, Kingston, Ontario, Canada
Zelov, Vitaly, The Fields Institute, Toronto, Ontario, Canada

Algebraic K-Theory

Victor P. Snaith, Editor

The conference proceedings volume is produced in connection with the second Great Lakes K-theory Conference that was held at The Fields Institute for Research in Mathematical Sciences in March 1996. The volume is dedicated to the late Bob Thomason, one of the leading research mathematicians specializing in algebraic K-theory. In addition to research papers treated directly in the lectures at the conference, this volume contains the following: i) several timely articles inspired by those lectures (particularly by that of V. Voevodsky), ii) an extensive exposition by Steve Mitchell of Thomason's famous result concerning the relationship between algebraic K-theory and étale cohomology, iii) a definitive exposition by J-L. Colliot-Thélène, R. Hoobler, and B. Kahn (explaining and elaborating upon unpublished work of O. Gabber) of Bloch-Ogus-Gersten type resolutions in K-theory and algebraic geometry. This volume will be important both for researchers who want access to details of recent development in K-theory and also to graduate students and researchers seeking good advanced exposition.

Features:

- invaluable access to new, previously unavailable research results
- an ideal reference for some important results in K-theory techniques in algebraic geometry
- material for advanced seminars for graduate students
- a number of papers connecting number theory and K-theory in a way that complements The Fields Institute Monograph Series from the 1993–1994 emphasis year on L-functions

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