# Notices

## OF THE

# AMERICAN MATHEMATICAL SOCIETY

VOLUME 16, NUMBER 3 ISSUE NO. 113 APRIL, 1969

# Notices

# OF THE

# AMERICAN MATHEMATICAL SOCIETY

**Edited by Everett Pitcher and Gordon L. Walker**

## CONTENTS

# MEETINGS

## Calendar of Meetings

**NOTE:** This Calendar lists all of the meetings which have been approved by the Council up to the date at which this issue of the *Notices* was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change. This is particularly true of the meetings to which no numbers have yet been assigned.

| Meeting No. | Date | Place | Deadline for Abstracts* |
|---|---|---|---|
| -- | ------ | ------------- | May 1, 1969** |
| 667 | August 25-29, 1969 (74th Summer Meeting) | Eugene, Oregon | July 1, 1969 |
| 668 | October 25, 1969 | Cambridge, Massachusetts | Sept. 9, 196 |
| | November 21-22, 1969 | Baton Rouge, Louisiana | |
| | November 29, 1969 | Ann Arbor, Michigan | |
| | January 22-26, 1970 (76th Annual Meeting) | Miami, Florida | |
| | August 24-28, 1970 (75th Summer Meeting) | Laramie, Wyoming | |
| | January 21-25, 1971 (77th Annual Meeting) | Atlantic City, New Jersey | |

*The abstracts of papers to be presented in person at the meetings must be received in the Headquarters Offices of the Society in Providence, Rhode Island, on or before these deadlines. The deadlines also apply to news items. The next two deadlines for by-title abstracts will be April 25 and June 24, 1969.

**Deadline for the *Notices* for the June issue; no meeting is scheduled in June.

The *Notices* of the American Mathematical Society is published by the Society in January, February, April, June, August, October, November and December. Price per annual volume is $10.00. Price per copy $3.00. Special price for copies sold at registration desks of meetings of the Society, $1.00 per copy. Subscriptions, orders for back numbers (back issues of the last two years only are available) and inquiries should be addressed to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904. Second-class postage paid at Providence, Rhode Island, and additional mailing offices.

# Six Hundred Sixty-Fourth Meeting
## Americana of New York
## New York, New York
## April 2-5, 1969

The six hundred sixty-fourth meeting of the American Mathematical Society will be held at the Americana of New York in New York City on April 2-5, 1969.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, the following four addresses will be presented: Professor David Shale, University of Pennsylvania, will speak on "Some probabilistic ideas in mathematical physics" at 11:00 a.m. on Friday, April 4; Professor Cathleen Morawetz, New York University, will speak on "Energy flow: wave motion and geometrical optics" at 2:00 p.m. on Friday, April 4; Professor Katsumi Nomizu, Brown University, will speak on "Differential geometry of complex hypersurfaces" at 11:00 a.m. on Saturday, April 5; Professor William Browder, Princeton University, will speak at 2:00 p.m. on Saturday, April 5, on "Invariants of manifolds." All four addresses will be presented in the Georgian Ballroom A.

There will be sessions for ten-minute contributed papers both mornings and afternoons of Friday and Saturday. There will be provision for late papers.

The Council of the Society will meet on Friday, April 4, at 5:00 p.m. in the Versailles Terrace. There will be an intermission for dinner.

### SYMPOSIUM ON MATHEMATICAL ASPECTS OF ELECTRICAL NETWORK THEORY

With the support of the Air Force Office of Scientific Research and the U.S. Army Research Office (Durham), there will be a symposium on Mathematical Aspects of Electrical Network Theory on April 2 and April 3. The AMS-SIAM Committee on Applied Mathematics chose the topic of the symposium and appointed the Organizing Committee which consists of Dr. W. A. Blackwell, Dr. Frank Branin, Dr. Robert Brayton, Professor Frank Harary, and Professor Herbert S. Wilf (chairman). The hour speakers at the symposium will be Professor J. W. T. Youngs, University of California, Santa Cruz; Dr. Robert Brayton, IBM Corporation; Dr. D. C. Youla, Brooklyn Polytechnic Institute; and Professor R. J. Duffin, Carnegie-Mellon University. In addition, there will be twelve half-hour addresses. The symposium will be held in the Regency Ballroom on Wednesday, April 2, and in the Georgian Ballroom A on Thursday, April 3.

### REGISTRATION

The registration desk will be in Loire Suites 4 and 5. It will be open from 9:00 a.m. to 5:00 p.m. on Wednesday through Saturday, April 2-5.

### EXHIBITS

A joint book exhibit will be on display in Vendome Suite 10 from 9:00 a.m. to 5:00 p.m. on each of the four days.

### ACCOMMODATIONS

Persons intending to stay at the Americana should make their own reservations with the hotel. A reservation blank and a listing of room rates can be found on page 452 of the February issue of these *Notices*.

### MAIL ADDRESS

Registrants at the meeting may receive mail addressed in care of the American Mathematical Society, Americana of New York, 801 7th Avenue (and 52nd Street), New York, New York 10019.

## SYMPOSIUM ON MATHEMATICAL ASPECTS OF ELECTRICAL NETWORK THEORY

### WEDNESDAY, 2:00 P.M.

First Session, Regency Ballroom

10:00-10:50

Kirchhoff's laws and map-coloring problems
Professor J.W.T. Youngs, University of California, Santa Cruz

11:00-11:20

A new perturbation theory for nonlinear network synthesis
Professor Roger W. Brockett, Massachusetts Institute of Technology

Second Session, Regency Ballroom

2:00-2:50

Network models
Professor Richard Duffin, Carnegie-Mellon University

3:00-3:20

Successive secants in the solution of nonlinear network equations
Professor Ronald A. Rohrer, University of California, Berkeley

3:30-3:50

Electrical network analysis, network-programming, and matroid theory
Professor George Minty, Indiana University

4:00-4:20

Existence of solution to electrical network problem via homology sequences
Dr. J. Paul Roth, IBM, T. J. Watson Research Center

4:30-4:50

An algebraic-topological overview of network analogies, the vector calculus, and Maxwell's equations
Dr. Franklin H. Branin, Jr., IBM, Kingston, New York

### THURSDAY, 10:00 A.M.

Third Session, Georgian Ballroom A

10:00-10:50

Nonlinear RLC Networks
Dr. Robert Brayton, IBM, T. J. Watson Research Center

11:00-11:20

Equivalent nonlinear electrical networks
Professor P. P. Varaiya, University of California, Berkeley

11:30-11:50

Numerical techniques for fast and accurate transient analysis of nonlinear networks
Professor D. A. Calahan, University of Michigan

Fourth Session, Georgian Ballroom A

2:00-2:50

Some recent developments in the synthesis of multivariable positive-real matrices
Dr. Dante C. Youla, Brooklyn Polytechnic Institute

3:00-3:20

Some properties of a nonlinear model for synchronizing digital transmission networks
Dr. Irvin Sandberg, Bell Laboratories

3:30-3:50

Some formulation aspects of electrical network theory
Professor Myril B. Reed, Colorado State University

4:00-4:20

Graph theory and its applications in circuit theory
Professor S. Park Chan, University of Santa Clara

3:30-4:50

Some applications in electrical network theory of a linear graph theorem by Berge and Ghouila-Houri
Dr. Dan Wolaver, Massachusetts Institute of Technology

## PROGRAM OF THE SESSIONS

The time limit for each contributed paper is 10 minutes. The contributed papers are scheduled at 15 minute intervals. To maintain this schedule, the time limit will be strictly enforced.

### FRIDAY, 9:00 A.M.

Session on Logic and Foundations I, Vendome 11-12

9:00-9:10

(1) Completeness theorems for the propositional calculus of kind W. Preliminary report
Mr. Donald J. Brown, Stevens Institute of Technology (664-33)
(Introduced by Professor Stephen L. Bloom)

9:15-9:25

(2) Intermediate theories with respect to Keisler's ultraproduct ordering
Mr. L. Taylor Ollmann and Professor Anil Nerode*, Cornell University (664-7)

9:30-9:40

(3) A completeness theorem for theories of kind W
Professor Stephen L. Bloom, Stevens Institute of Technology (664-24)

9:45-9:55

(4) Consistency of some non-Fregean theory
Professor Roman Suszko, Stevens Institute of Technology (664-23)
(Introduced by Professor Stephen L. Bloom)

10:00-10:10

(5) Variant forms of quantification of the predicate of use in analysis
Professor Ira Rosenbaum, University of Miami (664-72)

10:15-10:25

(6) Representation theorems for context sensitive languages
Professor Leonard H. Haines, University of California, Berkeley (664-88)
(Introduced by Professor Michael A. Harrison)

### FRIDAY, 9:00 A.M.

Session on Differential equations, Chambord 14-15

9:00-9:10

(7) A theorem on contractive stability
Dr. Anilchandra A. Kayande, University of Rhode Island (664-78)

9:15-9:25

(8) On the continuation problem for a second order differential equation
Professor Herman E. Gollwitzer, University of Tennessee (664-12)

9:30-9:40

(9) A Green's function approach to perturbations of periodic solutions
Dr. Carl Kallina, Mobil Research and Development Corporation, Princeton, New Jersey (664-19)

9:45-9:55

(10) A note on the solution of matrix differential equations by partitioning
Professor Victor Lovass-Nagy* and Professor David L. Powers, Clarkson College of Technology (664-49)

10:00-10:10

(11) On the existence of the wave operators, corresponding to ordinary differential operators of even order, under the inverse conditions. Preliminary report
Dr. John B. Butler, Jr., Portland State College (664-1)

10:15-10:25

(12) Integro-differential equations of Volterra type
Professor M. Rama Mohana Rao* and Professor Chris P. Tsokos, University of Rhode Island (664-92)

10:30-10:40

(13) Functional differential inequalities and stability near sets
Professor Chris P. Tsokos* and Professor M. Rama Mohana Rao, University of Rhode Island (664-93)

10:45-10:55

(14) The existence and stability of nonlinear operator differential equations in Hilbert spaces
Mr. Chia-Ven Pao, University of Pittsburgh (664-103)

FRIDAY, 9:00 A.M.

Session on Analysis I, Regency Foyer

9:00-9:10

(15) The type of an entire function satisfying a linear differential equation
Professor Boo-Sang Lee* and Professor S. M. Shah, University of Kentucky (664-18)

9:15-9:25

(16) On the continuation of analytic sets. Preliminary report
Dr. Bernard Shiffman, Massachusetts Institute of Technology (664-35)

9:30-9:40

(17) Univalent functions with univalent derivatives. II
Professor Swarupchand M. Shah* and Mr. Selden Y. Trimble, University of Kentucky (664-86)

9:45-9:55

(18) Integrals and derivatives of functions in MacLane's class A and of normal functions
Dr. Karl F. Barth and Professor Walter J. Schneider*, Syracuse University (664-95)

10:00-10:10

(19) Additional properties of a new unified class of polynomials
Professor Moses E. Cohen, Michigan Technological University (664-54)

10:15-10:25

(20) A note on certain dual series equations involving Laguerre polynomials
Dr. Hari M. Srivastava, West Virginia University (664-58)

10:30-10:40

(21) Concerning the extended Stieltjes mean $\sigma$-integral and a mean integral of a function with respect to a function pair
Mr. Dean B. Priest* and Professor Russell A. Stokes, University of Mississippi (664-59)

10:45-10:55

(22) The asymptotic behavior of the special function $\mu(x, \beta, \alpha)$
Dr. J. J. Dorning, Dr. B. Nicolaenko* and Dr. James K. Thurber, Brookhaven National Laboratory, Upton, Long Island, New York (664-62)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

FRIDAY, 9:00 A.M.

Session on Number Theory and Graph Theory, Loire Suite 2-3

9:00-9:10

(23) On a class of transcendental numbers
Professor Al Somayajulu, Canisius College (664-15)

9:15-9:25

(24) Continued fractions of algebraic numbers
Mr. John J. McKibben, University of Massachusetts (664-70)
(Introduced by Professor Helen F. Cullen)

9:30-9:40

(25) On Fermat's Last Theorem
Professor J. M. Gandi, York University (664-63)

9:45-9:55

(26) On a maximization problem
Dr. Triloki N. Bhargava, Kent State University, and Mr. Russ A. Smucker* Goshen College (664-94)

10:00-10:10

(27) An elementary estimate for primitive elements in finite fields
Professor Clifton T. Whyburn, East Carolina University (664-13)

10:15-10:25

(28) A characterization of clique graphs
Dr. Fred S. Roberts* and Mr. Joel H. Spencer, RAND Corporation, Santa Monica, California (664-28)

10:30-10:40

(29) Enumeration of spanning trees interacting with subgraphs. Preliminary report
Dr. Peter V. O'Neil, College of William and Mary (664-81)

10:45-10:55

(30) Uniquely partially orderable graphs
Professor Martin S. Aigner, University of North Carolina at Chapel Hill, and Professor Geert C. E. Prins*, Wayne State University (664-31)

FRIDAY, 11:00 A.M.

Invited Address, Georgian Ballroom A

Some probabilistic ideas in mathematical physics
Professor David Shale, University of Pennsylvania

FRIDAY, 2:00 P.M.

Invited Address, Georgian Ballroom A

Energy flow: wave motion and geometrical optics
Professor Cathleen Morawetz, New York University

FRIDAY, 3:15 P.M.

Session on Analysis II, Regency Foyer

3:15-3:25

(31) On the existence and approximation of nonlinear evolution
Professor Edwin H. Rogers, Rensselaer Polytechnic Institute (664-22)

3:30-3:40

(32) On the regularity of the solutions of a class of functional equations
Dr. Halina Swiatak, McGill University (664-41)
(Introduced by Professor Hans W. E. Schiverdtfeger)

3:45-3:55

(33) A global uniqueness theorem for partial differential operators
Dr. David K. Cohoon, Bell Telephone Laboratories, Whippany, New Jersey (664-57)

4:00-4:10

(34) Gentle degenerate perturbations are smooth

Mr. Edward H. Thiel, University of California, Davis (664-102)

4:15-4:25

(35) On weighted averages at a jump discontinuity

Professor Craig Comstock, University of Michigan (664-25)
(Introduced by Professor Ronald G. Douglas)

4:30-4:40

(36) Some applications of the P operator

Professor Charles Fox, Sir George Williams University (664-32)

4:45-4:55

(37) Extreme eigenvalues of Toeplitz matrices. Preliminary report

Mr. Daniel E. Hughes, Washington University (664-30)

5:00-5:10

(38) Inverse limits, entropy, and weak isomorphism for discrete dynamical systems

Professor James R. Brown, Oregon State University (664-65)
(Introduced by Professor Philip M. Anselone)

5:15-5:25

(39) Semilinear Markov processes

Dr. Joseph Horowitz, University of Toledo (664-66)

FRIDAY, 3:15 P.M.

Session on Logic and Foundations II, Vendome 11-12

3:15-3:25

(40) $\omega$-homomorphisms

Mr. Charles H. Applebaum, Rutgers University (664-45)

3:30-3:40

(41) Algorithms relating first-order theories to various combinatorial systems

Professor Paul Axt and Professor Wilson E. Singletary*, Pennsylvania State University (664-76)

3:45-3:55

(42) Nonstandard almost periodic functions on a group

Professor Lawrence D. Kugler, University of Michigan, Flint College (664-48)

4:00-4:10

(43) A finite set of groupoid equations in one variable with unsolvable decision problem. Preliminary report

Professor Peter Perkins, College of the Holy Cross (664-84)

4:15-4:25

(44) An axiomatic set theory

Professor Hidegoro Nakano, Wayne State University (664-3)

FRIDAY, 3:15 P.M.

Session on Applied Mathematics, Chambord 14-15

3:15-3:25

(45) On the validity of the geometrical theory of diffraction by star shaped cylinders

Professor Clifford O. Bloom, University of Michigan (664-39)
(Introduced by Professor Joel A. Smoller)

3:30-3:40

(46) On three dimensional transient wave motions on a running stream

Professor Lokenath Debnath, East Carolina University (664-47)

3:45-3:55

(47) Effects of viscosity on long waves

Professor Shih-liang Wen, Ohio University (664-53)

4:00-4:10

(48) Eigenfunctions corresponding to the band pass kernel with large center frequency

Professor Abdul Jabbar Jerri, Clarkson College of Technology (664-42)

4:15-4:25

(49) Leibniz rule for fractional derivatives and application to infinite series

Mr. Thomas J. Osler, St. Joseph's College (664-21)

4:30-4:40

(50) Ordinary matrix differential equations with stochastic coefficients

Mr. Leon H. Sibul*, Pennsylvania State University, and Professor Craig Comstock, University of Michigan (664-27)

(Introduced by Dr. Frank Stenger)

4:45-4:55

(51) Asymptotic duality over closed convex sets

Professor Adi Ben-Israel, Northwestern University, and Professor Kenneth O. Kortanek*, Cornell University (664-52)

5:00-5:10

(52) Maximum norm stability for parabolic difference schemes in half-space

Professor Stanley Osher, University of California, Berkeley (664-20)

5:15-5:25

(53) Equal and almost equal weight quadrature formulas

Mr. David K. Kahaner, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico (664-6)

FRIDAY, 3:15 P.M.

Session on General Topology, Loire Suite 2-3

3:15-3:25

(54) On stable homeomorphisms of infinite-dimensional manifolds

Professor Raymond Y. Wong, University of California, Santa Barbara (664-36)

(Introduced by Professor Ky Fan)

3:30-3:40

(55) A note on the intersection of free maximal ideals

Professor Stewart M. Robinson, Cleveland State University (664-104)

3:45-3:55

(56) A generalization of continuity

Professor Gustave Rabson, Clarkson College of Technology (664-98)

4:00-4:10

(57) Extended topology: Continuity II

Professor Preston C. Hammer, Pennsylvania State University (664-61)

4:15-4:25

(58) Rings of normal functions

Professor Kenneth Hardy, Carleton University (664-67)

4:30-4:40

(59) Continuous selections on the hyperspace of a continuum

Professor Kazimierz Kuratowski and Professor Sam B. Nadler*, State University of New York at Buffalo (644-69)

4:45-4:55

(60) Some analogues of a theorem of Michael. Preliminary report

Mr. Joe A. Guthrie, Texas Christian University (664-85)

(Introduced by Professor Hisahiro Tamano)

5:00-5:10

(61) Partial order in bitopological spaces

Professor Yong Woon Kim, Wisconsin State University — Eau Claire (664-38)

5:15-5:25

(62) Some properties of mesocompact spaces. Preliminary report.
Dr. Vincent J. Mancuso, St. John's University (644-90)

SATURDAY, 9:00 A.M.

Session on Functional Analysis I, Regency Foyer

9:00-9:10

(63) On exp ih in Banach *-algebras
Professor Barnett W. Glickfeld, University of Washington (664-105)

9:15-9:25

(64) On simple Banach algebras
Mr. Moshe Rosenfeld, University of Washington (664-99)

9:30-9:40

(65) More on the existence of expectation type maps on B*-algebras
Dr. Andre de Korvin* and Dr. R. J. Easton, Indiana State University (644-4)

9:45-9:55

(66) The orbits of unitary groups in Hilbert spaces
Professor Choy-Tak Taam, George Washington University (664-91)

10:00-10:10

(67) Representations by basic sets in the space of square summable functions
Dr. Boshra H. Makar, St. Peter's College (664-56)

10:15-10:25

(68) Local properties of distributions
Professor Milos A. Dostal, University of Montreal (664-10)

10:30-10:40

(69) Gaussian measures and a "Functional Equation" for measures
Dr. Lawrence J. Corwin, Massachusetts Institute of Technology (664-37)

SATURDAY, 9:00 A.M.

Session on Topology, Vendome 11-12

9:00-9:10

(70) The decomposition of 3-manifolds with several boundary components
Dr. Jonathan L. Gross, Princeton University (664-26)

9:15-9:25

(71) The Thom class of a vector bundle and relevant localization theorems in differential geometry
Professor Tadashi Nagano, University of Notre Dame (664-40)

9:30-9:40

(72) Application of Morse theory to some homogeneous spaces
Professor Srinivasa Ramanujam, University of Washington (664-44)

9:45-9:55

(73) $H_2(G')$ for tamely embedded graphs
Mr. D. S. Cochran*, Princeton University, and Professor Richard H. Crowell, Dartmouth College (664-60)

10:00-10:10

(74) Cross sectionally connected 2-spheres are tame
Mr. R. A. Jensen, University of Wisconsin (664-75)
(Introduced by Mr. R. H. Bing)

10:15-10:25

(75) Finite CW-complexes with second homotopy groups infinitely generated as fundamental group modules
Professor Burtis G. Casler, Louisiana State University (664-97)

SATURDAY, 9:00 A.M.

Session on Algebra, Chambord 14-15

9:00-9:10

(76) Some remarks on Baer rings
Professor M. F. Janowitz, University of Massachusetts (664-8)

9:15-9:25

(77) Richart and Baer rings of linear transformations. II
Professor Charles K. Martin, Virginia Polytechnic Institute (664-89)

9:30-9:40

(78) Prime rings satisfying a generalized polynomial identity
Professor Wallace S. Martindale, III, University of Massachusetts (644-11)

9:45-9:55

(79) Higher differentials and ramification
Professor Robert W. Berger, Louisiana State University (664-17)

10:00-10:10

(80) Homological invariants of local rings
Mr. Tor H. Gulliksen, Queen's University (664-106)
(Introduced by Professor Paulo Ribenboim)

10:15-10:25

(81) Representation of cylindric algebras by sheaves. Preliminary report
Professor Stephen D. Comer, Vanderbilt University (664-96)

10:30-10:40

(82) The Galois theory of infinite Dedekind fields. Preliminary report
Professor Louis V. DiBello, St. John Fisher College and University of Rochester (664-79)

10:45-10:55

(83) Algebras defined by patterns of zeros
Professor Robert L. Davis, University of North Carolina at Chapel Hill (664-9)

SATURDAY, 9:00 A.M.

Session on Groups and Lattices, Loire Suite 2-3

9:00-9:10

(84) Extensions of group theoretical properties
Professor Bernhard Amberg, University of Texas (664-73)
(Introduced by Professor George G. Lorentz)

9:15-9:25

(85) Metabelian p-groups which contain a self-centralizing element. Preliminary report
Dr. Marc W. Konvisser, Battelle Memorial Institute (664-29)

9:30-9:40

(86) On finite groups possessing characteristic cyclic series. Preliminary report
Professor John R. Durbin, University of Texas (664-83)

9:45-9:55

(87) The Mathieu groups as automorphism groups of linear codes
Mr. Edward P. Shaughnessy, Lafayette College (664-43)

10:00-10:10

(88) Isomorphism of the automorphism groups and projectivities of primary abelian groups
Professor Kai Faltings, University of Texas (664-101)
(Introduced by Professor Paul R. Meyer)

10:15-10:25

(89) Power-associative quasigroups
Mr. Thomas L. Bartlow, Villanova University (664-46)

10:30-10:40

(90) An abstract version of the Krull principal ideal theorem. Preliminary report
Dr. Theodore J. Benac, United States Naval Academy, and Sister Clair Archambault*, Catholic University of America (664-50)

10:45-10:55

(91) Nonassociative lattices
Miss Helen Skala, Illinois Institute of Technology (664-68)

SATURDAY, 11:00 A.M.

Invited Address, Georgian Ballroom A

Differential geometry of complex hypersurfaces
Professor Katsumi Nomizu, Brown University

SATURDAY, 2:00 P.M.

Invited Address, Georgian Ballroom A

Invariants of manifolds
Professor William Browder, Princeton University.

SATURDAY, 3:15 P.M.

Session on Functional Analysis II, Regency Foyer

3:15-3:25

(92) On certain compact convex sets which are similar to Choquet simplexes. Preliminary report
Mr. John N. McDonald, Rutgers University (664-34)

3:30-3:40

(93) The algebra of finite operators and the topological module of kernels
Professor Thomas R. Chow, Oregon State University (664-16)

3:45-3:55

(94) Extremal structure of closed convex sets
Professor Leonard A. Asimow, University of California, Los Angeles (664-14)

4:00-4:10

(95) On the connectedness of isomorphism classes
Professor Robert A. McGuigan, University of Massachusetts (664-80)

4:15-4:25

(96) Mapping m into c. Preliminary report
Mr. Alfred Tong* and Professor Donald R. Wilken, State University of New York at Albany (664-74)

4:30-4:40

(97) Topologies on sequence spaces
Professor William H. Ruckle, Lehigh University (664-77)

4:45-4:55

(98) Counting functions for FK-spaces
Mr. Grahame Bennett, St. John's College, Cambridge, England, and Lehigh University (664-100)

5:00-5:10

(99) The general Stone-Weierstrass problem
Professor Charles A. Akemann, University of California, Santa Barbara (664-2)

5:15-5:25

(100) Approximation of continuous functions on Lindelöf spaces
Professor Anthony W. Hager, Wesleyan University (664-82)

Session on Geometry, Vendome 11-12

3:15-3:25

(101) A fifth intersection-free infinite periodic minimal surface (IPMS) of cubic symmetry. Preliminary report

Dr. Alan H. Schoen, NASA-Electronics Research Center, Cambridge, Massachusetts (664-64)

(Introduced by Dr. Edwin H. Farr)

3:30-3:40

(102) Convex polyhedra with a center of symmetry

Professor Ethan D. Bolker, Bryn Mawr College (664-55)

3:45-3:55

(103) A new method for infinitesimal rigidity of surfaces $K > 0$

Professor Edgar D. Kann, City University of New York, Queens College (664-107)

4:00-4:10

(104) Conformal transformations of Riemannian manifolds

Professor Morio Obata, Lehigh University (664-87)

4:15-4:25

(105) A homotopy group of algebraic arcs found by smoothed approximations

Professor Paul Cherenack, Indiana University (664-51)

Leonard Gillman
Associate Secretary

Rochester, New York

# Six Hundred Sixty-Fifth Meeting
# Netherland Hilton Hotel
# Cincinnati, Ohio
# April 18-19, 1969

The six hundred sixty-fifth meeting of the American Mathematical Society will be held at the Netherland Hilton Hotel, Cincinnati, Ohio, on April 18-19, 1969. The sessions will be held in the Continental Room on the lobby floor, in the Hall of Mirrors and the Julep Room on the third floor, and in North Hall, South Hall, and Parlors E-F on the fourth floor of the Netherland Hilton Hotel.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, there will be four one-hour addresses. Professor Edward R. Fadell of the University of Wisconsin will speak on Friday, April 18, at 10:45 a.m. His topic will be "Recent developments in fixed-point theory." Professor Pesi R. Masani of Indiana University will address the Society on Friday, April 18, at 1:45 p.m. His subject will be "The role of vector and operator valued measures in functional analysis and probability." Professor William W. Boone of the University of Illinois will speak on Saturday, April 19, at 10:45 a.m. His talk will be entitled "The theory of decision problems in group theory: a survey." Professor François Treves of Purdue University will address the Society on Saturday, April 19, at 1:45 p.m. His topic will be "On local solvability of linear partial differential equations." The two hour addresses scheduled for Friday will be presented in the Continental Room, while those scheduled for Saturday will be presented in the Hall of Mirrors.

By invitation of the same committee there will be two special sessions of selected twenty-minute papers. One of these, to be held on Friday, April 18, at 3:00 p.m. in the Continental Room, has been arranged by Professor Arunas L. Liulevicius of the University of Chicago on the subject of K-Theory and Cohomology Operations; the speakers will be Professors Peter Hoffman, Leif Kristensen, Peter S. Landweber, J. Peter May, and Mark E. Mahowald. The other special session, to be held on Saturday, April 19, at 3:00 p.m. in the Hall of Mirrors, will be on the subject of Algebraic Geometry and has been arranged by Professor Maxwell A. Rosenlicht of the University of California, Berkeley, and Northwestern University; the speakers will be Professors Shreeram Abhyankar, Satoshi Arima, Walter L. Baily, Jr., David Hertzig, and Stephen S. Shatz.

Ten sessions for contributed ten-minute papers have been scheduled. In addition there may be sessions for late papers; detailed information about these sessions will be available at the meeting.

## REGISTRATION

The registration desk will be located in the Fourth Floor Foyer. The desk will be open from 9:00 a.m. to 5:00 p.m. on Friday and from 8:30 a.m. to 3:30 p.m. on Saturday.

## ACCOMMODATIONS

Room reservations should be addressed to the Front Office Manager, Netherland Hilton Hotel, Cincinnati, Ohio 45201. Until April 4, the hotel has guaranteed rates of $14.50 for single rooms and $19.00 for twins and doubles. Free garage parking is provided for registered guests.

## FOOD SERVICE

There are four restaurants within the Netherland Hilton Hotel itself. Other nearby restaurants include Caproni's, 610 Main Street; the Colony Restaurant, 420 Walnut Street; the Gourmet Room in the Terrace Hilton Room in the Terrace Hilton Hotel; the Maisonette, 114 Sixth

Street; and Pigall's, 127 West Fourth Street.

TRAVEL AND LOCAL INFORMATION

The Netherland Hilton Hotel is centrally located in downtown Cincinnati. The bus depot is four blocks from the hotel; the train terminal is eleven blocks away. Sleeping cars to Cincinnati are available on the Penn Central's Cincinnati Limited from New York and Philadelphia, the Chesapeake and Ohio's George Washington from Washington and Newport News, the Norfolk and Western's Pocahontas from Norfolk and Roanoke, and the Louisville and Nashville's Pan-American from New Orleans. Those coming by way of the Greater Cincinnati Airport should take the "Airporter" bus into town; this runs every twenty minutes, costs $1.75 each way, and takes about half an hour. Airlines serving Cincinnati are American, TWA, Allegheny, Mohawk, Delta, Piedmont, Northwest, United and Eastern. The principal highways to Cincinnati are Interstate Routes 71, 74 or 75. The automobile entrance to the hotel is on Race Street, which is one-way southbound.

Among the points of interest to tourists are the Taft House Museum, half a mile east of the Netherland Hilton Hotel, and the Cincinnati Art Museum, one and a half miles northeast of the hotel.

MAIL ADDRESS

Registrants at the meeting may receive mail addressed in care of the American Mathematical Society, Netherland Hilton Hotel, Fifth and Race Streets, Cincinnati, Ohio 45201.

PROGRAM OF THE SESSIONS

The time limit for each contributed paper is 10 minutes. The contributed papers are scheduled at 15 minute intervals. To maintain this schedule, the time limit will be strictly enforced.

FRIDAY, 9:15 A.M.

Session on Algebra I (Algebras), North Hall

9:15-9:25
(1) A derivative theorem for polynomials
Mr. Stephen M. Gagola, State University of New York at Buffalo (665-26)
(Introduced by Professor G. R. Blakley)

9:30-9:40
(2) On free joins of algebras
Professor In Yung Chung, University of Cincinnati (665-23)

9:45-9:55
(3) Deformation and cohomology of coalgebras
Professor Stanislas L. Klasa, Northern Illinois University (665-32)
(Introduced by Professor Henry S. Leonard)

10:00-10:10
(4) Central idempotents in symmetric algebras
Professor Timothy V. Fossum, University of Utah (665-12)

10:15-10:25
(5) The Boolean spectrum and separable algebras
Mr. Andy R. Magid, Northwestern University (665-5)
(Introduced by Professor Daniel Zelinsky)

FRIDAY, 9:15 A.M.

Session on Complex Analysis, South Hall

9:15-9:25
(6) $H^p$ spaces on the classical Cartan domains
Professor Kyong T. Hahn* and Professor Josephine M. Mitchell, Pennsylvania State University (665-31)

9:30-9:40

(7) A short proof of a conformal mapping theorem of Matsumoto
Professor Walter J. Schneider, Syracuse University (665-62)

9:45-9:55

(8) Zeros of partial sums of power series. II
Professor James D. Buckholtz, University of Kentucky (665-30)

10:00-10:10

(9) The second dual of certain spaces of analytic functions
Professor Lee A. Rubel*, University of Illinois, and Professor Allen L. Shields, University of Michigan (665-10)

10:15-10:25

(10) Functions of exponential type not vanishing in a half plane
Professor Narendra K. Govil*, Loyola College of Montreal, and Professor Q. I. Rahman, Université de Montréal (665-42)

FRIDAY, 10:45 A.M.

Invited Address, Continental Room

Recent developments in fixed-point theory
Professor Edward R. Fadell, University of Wisconsin

FRIDAY, 1:45 P.M.

Invited Address, Continental Room

The role of vector and operator valued measures in functional analysis and probability
Professor Pesi R. Masani, Indiana University

FRIDAY, 3:00 P.M.

Special Session on K-theory and Cohomology Operations, Continental Room

3:00-3:20

On the realizability of Steenrod modules
Professor Peter Hoffman, University of Waterloo (665-54)

3:25-3:45

On properties of higher order cohomology operations
Professor Leif Kristensen, Aarhus Universitet and University of Illinois at Chicago Circle (665-65)

3:50-4:10

Symmetric maps between spheres and equivariant K-theory
Professor Peter S. Landweber, Yale University (665-9)

4:15-4:35

Higher order decomposition formulae for $Sq^2$
Professor Mark E. Mahowald, Northwestern University (665-55)

4:40-5:00

The Dyer-Lashof algebra and $H_*(BF)$
Professor J. Peter May, University of Chicago (665-47)

FRIDAY, 3:00 P.M.

Session on Algebra II (Groups), North Hall

3:00-3:10

(11) On groups of exponent four with four generators
Mr. Ricardo B. Quintana, Jr., University of Wisconsin (665-49)

3:15-3:25

(12) On the laws of free nilpotent groups
Professor Frank Levin, Rutgers University (665-46)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

3:30-3:40

(13) On finite solvable linear groups

Professor David L. Winter, Michigan State University (665-6)

3:45-3:55

(14) Blocks of modular representations over nonsplitting fields. Preliminary report

Mr. Wayland M. Hubbart, University of Delaware (655-45)

4:00-4:10

(15) Commutativity of certain algebras. Preliminary report

Professor Allan J. Silberger, Bowdoin College (665-57)

FRIDAY, 3:00 P.M.

Session on Applied Mathematics, South Hall

3:00-3:10

(16) Green's functions for generalized Schroedinger equations. II

Professor John A. Beekman, Ball State University (665-27)

3:15-3:25

(17) Triangular Hermite interpolation in the plane

Professor Paul O. Frederickson, Case Western Reserve University (665-64)

3:30-3:40

(18) Calculation of associated Legendre functions

Dr. Mark M. Lotkin, GE Missile and Space Division, Cherry Hill, New Jersey (665-13)

3:45-3:55

(19) Phase plane analysis for finite difference equations

Dr. Allen R. Strand, Muskingum College (665-51)

4:00-4:10

(20) Some aspects in existence proofs in MHD

Professor Maria Z. v. Krzywoblocki, Michigan State University (665-15)

SATURDAY, 9:00 A.M.

Session on Algebra III (Rings), North Hall

9:00-9:10

(21) Strong inertial coefficient rings

Mr. William C. Brown, Northwestern University (665-20)

9:15-9:25

(22) Some containment relations between classes of ideals and an integral domain

Professor Nick H. Vaughan, North Texas State University (665-16)

9:30-9:40

(23) Commutative endomorphism rings. II

Professor Julius M. Zelmanowitz, University of California, Santa Barbara (665-59)

9:45-9:55

(24) Decomposition theories for modules

Dr. Joe W. Fisher, University of Illinois (665-17)

10:00-10:10

(25) Elliptic singularities of surfaces

Dr. Philip D. Wagreich, University of Pennsylvania and Institute for Advanced Study (665-14)

SATURDAY, 9:00 A.M.

Session on Differential Equations, Parlor E

9:00-9:10

(26) On the stability of nonlinear operator differential equations, and applications

Mr. Chia-Ven Pao* and Dr. William G. Vogt, University of Pittsburgh (665-63)

9:15-9:25

(27) A regularity theorem for weak solutions for first order systems of partial differential operators satisfying an a priori inequality
Mr. David S. Tartakoff, University of California, Berkeley (665-66)

9:30-9:40

(28) Nonexistence of a continuous right inverse for linear partial differential operators with constant coefficients
Dr. David Kent Cohoon, Bell Telephone Laboratories, Whippany, New Jersey (665-28)

9:45-9:55

(29) A two point problem for evolution equations
Mr. Jeffrey M. Cooper, Northwestern University (665-8)

SATURDAY, 9:00 A.M.

Session on Real Analysis, Julep Room

9:00-9:10

(30) Extensions of almost automorphic sequences
Professor Arlington M. Fink, Iowa State University (665-43)

9:15-9:25

(31) Concerning approximate slopes for a given class of functions of bounded variation
Mr. E. B. Hibbs, Jr., University of Texas (665-52)

9:30-9:40

(32) Change of variable in Stieltjes integrals
Mr. S. E. Hayes, University of Texas (665-61)
(Introduced by Professor H. S. Wall)

9:45-9:55

(33) An example in surface area
Professor Casper Goffman, Purdue University (665-44)

10:00-10:10

(34) Orlicz spaces have the weak difference property
Professor Francis W. Carroll* and Dr. Frederick S. Koehl, Ohio State University (665-56)

10:15-10:25

(35) Hilbert space problem four
Professor Gordon Johnson, University of Georgia (665-3)

SATURDAY, 9:00 A.M.

Session on Topology, South Hall

9:00-9:10

(36) Continuous open maps on the closed unit interval $[0,1]$
Professor Shashanka Shekhar Mitra, Wilkes College (665-50)

9:15-9:25

(37) Continuous preimages of the cone over the Cantor set
Professor David P. Bellamy, University of Delaware (665-53)
(Introduced by Professor H. S. Davis)

9:30-9:40

(38) Involution of the 3-sphere which fix 2-spheres
Professor Robert Craggs, University of Illinois (665-7)

9:45-9:55

(39) Approximating continua and domains in an n-cell
Professor Richard J. Tondra, Iowa State University (665-4)

10:00-10:10

(40) On mapping indecomposable continua onto certain chainable indecomposable continua
Professor Jack W. Rogers, Jr., Emory University (665-40)

10:15-10:25

(41) Upper-semi-continuous decompositions of irreducible continua
Professor William R. R. Transue, Auburn University (665-21)

10:30-10:40

(42) Upper-semi-continuous collections of homeomorphic continua filling up an irreducible continuum
Professor Ben Fitzpatrick, Jr., Auburn University (665-22)

SATURDAY, 10:45 A.M.

Invited Address, Hall of Mirrors

The theory of decision processes in group theory: a survey
Professor William W. Boone, University of Illinois

SATURDAY, 1:45 P.M.

Invited Address, Hall of Mirrors

On local solvability of linear partial differential equations
Professor François Treves, Purdue University

SATURDAY, 3:00 P.M.

Special Session on Algebraic Geometry, South Hall

3:00-3:20

Canonical systems on double planes
Professor Satoshi Arima, Waseda University and State University of New York at Buffalo (665-36)

3:25-3:45

Quasirational points of algebraic surfaces
Professor Shreeram Shankar Abhyankar, Purdue University (665-35)

3:50-4:10

On Eisenstein series for an exceptional arithmetic group
Professor Walter L. Baily, Jr., University of Chicago (665-34)

4:15-4:35

Cohomology of Steinberg-Chevalley groups
Professor David Hertzig, Purdue University (665-38)

4:40-5:00

Group schemes and Galois theory
Professor Stephen S. Shatz, University of Pennsylvania (665-37)

SATURDAY, 3:00 P.M.

Session on Algebra IV (Ordered structures), North Hall

3:00-3:10

(43) Semi-simplicity and von Neumann's regularity in categories
Professor Jacqueline K. Klasa, Northern Illinois University (665-33)
(Introduced by Professor Robert McFadden)

3:15-3:25

(44) Nonimbeddable Noether lattices
Professor Kenneth P. Bogart, Dartmouth College (665-39)

3:30-3:40

(45) Normal decomposition in systems without the ascending chain condition
Professor Eugene W. Johnson and Professor John P. Lediaev*, University of Iowa (665-24)

3:45-3:55

(46) Extensions of partial orders on semigroups
Dr. Frank A. Smith, Carnegie-Mellon University (665-60)

4:00-4:10

(47) Subgroup topologies and convergence in product groups
Professor Billy F. Hobbs, Olivet Nazarene College (665-41)

SATURDAY, 3:00 P.M.

Session on Topology and Logic, Julep Room

3:00-3:10

(48) On a problem of Arhangel'skiĭ
Dr. Howard H. Wicke, Sandia Laboratories, Albuquerque, New Mexico (665-48)

3:15-3:25

(49) Pointless axiomatic set theory
Professor Hidegoro Nakano, Wayne State University (665-1)

3:30-3:40

(50) The decision problem for formulas in prenex conjunctive normal form with binary disjunctions
Professor Melven R. Krom, University of California, Davis (665-11)

Paul T. Bateman
Associate Secretary

Urbana, Illinois

# NEWS ITEMS AND ANNOUNCEMENTS

JOHN H. BARRETT MEMORIAL

Friends and colleagues of Professor John H. Barrett have established a memorial fund in his honor. Professor Barrett joined The University of Tennessee faculty in 1961 and became head of the Mathematics Department in 1964. He died January 21, 1969, following an unsuccessful kidney transplant operation.

The fund is being established to perpetuate the memory of Professor Barrett, who made many significant contributions to the development of the mathematics program at The University of Tennessee. The funds will be used for one or both of the following purposes:

(1) To establish an annual award to an outstanding senior in mathematics at The University of Tennessee.

(2) To support a memorial lecture series in Ordinary Differential Equations at The University of Tennessee.

Those wishing to contribute to this fund may do so by sending their donation to John H. Barrett Memorial Fund, c/o Department of Mathematics, The University of Tennessee, Knoxville, Tennessee 37916. Checks should be made payable to The University of Tennessee for the John H. Barrett Memorial Fund.

# Six Hundred Sixty-Sixth Meeting
# University of California
# Santa Cruz, California
# April 26, 1969

The six hundred sixty-sixth meeting of the American Mathematical Society will be held at the University of California, Santa Cruz, in Santa Cruz, California, on Saturday, April 26, 1969.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be two invited hour addresses at this meeting. Professor David Harrison of the University of Oregon will lecture at 11:00 a.m. on Saturday. The title of his talk is "On quadratic forms over general fields." Professor Theodor Ganea of the University of Washington will speak at 2:00 p.m. on Saturday. His lecture is entitled "Numerical homotopy invariants and duality." There will be sessions for contributed papers at 9:30 a.m. and at 3:30 p.m. on Saturday. Late papers may be added to the program. For information concerning late papers, inquire at the registration desk.

All sessions of the meeting will be held in Natural Sciences I.

## REGISTRATION

Registration for the meeting will begin at 8:30 a.m. on Saturday. The registration desk will be located adjacent to Room 3, Science Lecture, in Natural Sciences I.

## ACCOMMODATIONS

There are numerous hotels and motels in the Santa Cruz area, including the following:

DREAM INN, 175 West Cliff Drive
Rates from $15.50 up

PASATIEMPO INN
555 Los Gatos, Highway 17
Rates from $10.00 up

A complete list of motels and hotels near Santa Cruz can be obtained from the Santa Cruz Convention and Visitors Bureau, Santa Cruz, California 95060. Reservations should be made directly with the preferred motel or hotel.

## MEALS

Cafeteria-style luncheon will be available at the University on Saturday at a cost of approximately $0.85.

## TRAVEL

The University of California, Santa Cruz, can be reached by automobile, following Route 17 into Santa Cruz. This leads to Route 1 (Mission Street). Follow Mission Street to Bay Street, and turn right up the hill to the main entrance of the University. Then follow the circular drive through the campus to Natural Sciences I. Parking is available nearby.

Persons coming to the meeting from cities outside the northern California area can obtain air transportation to San Jose, Oakland, or San Francisco, and rent a car to drive to Santa Cruz. The Oakland and San Francisco airports are approximately 60 miles from Santa Cruz, while the San Jose Municipal Airport is about 30 miles from Santa Cruz.

## PROGRAM OF THE SESSIONS

The time limit for each contributed paper is 10 minutes. The papers are scheduled at 15 minute intervals in order that listeners can circulate among the sessions. To maintain the schedule, the time limit will be strictly enforced.

### SATURDAY, 9:30 A.M.

General Session, Room 1, Science Lecture

9:30-9:40

(1) On invariant functions for positive operators
Mr. Humphrey Fong, Ohio State University and Stanford University (666-4)
(Introduced by Professor Louis Sucheston)

9:45-9:55

(2) Symmetric operators with twice continuously differentiable spectral functions
Professor Richard C. Gilbert, California State College, Fullerton (666-7)

10:00-10:10

(3) A connection between Hilbert's integral transform, the symmetries of a square, and quaternions
Dr. R. W. Preisendorfer, University of California, San Diego (666-5)

10:15-10:25

(4) Generalized elementary symmetric functions and quaternion matrices
Professor John de Pillis*, Brookhaven National Laboratory, Upton, New York, and Dr. Joel L. Brenner, University of Arizona (666-6)

10:30-10:40

(5) On the convexity of a function of the permanent of a matrix
Dr. Paul J. Nikolai, Aerospace Research Laboratories, Wright-Patterson AFB, Ohio, and University of California, Santa Barbara (666-11)

### SATURDAY, 9:30 A.M.

First Session on Analysis, Room 125, Natural Sciences I

9:30-9:40

(6) Convex surfaces with prescribed level curves
Professor M. A. Dostal, University of Montreal (666-3)

9:45-9:55

(7) Rational points preserving homeomorphism of real line by entire functions
Professor Daihachiro Sato, University of Saskatchewan (666-29)

10:00-10:10

(8) On mean approximation of holomorphic functions by rational functions with simple poles
Professor David Bell, Rice University (666-1)

10:15-10:25

(9) On the Nőrlund summability of a class of Fourier series
Professor Badri N. Sahney, University of Calgary (666-12)

10:30-10:40

(19) The absolute Nőrlund summability factors of a Fourier series
Mr. Devendra S. Goel, University of Calgary (666-20)
(Introduced by Professor Badri N. Sahney)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

SATURDAY, 9:30 A.M.

First Session on Algebra, Room 185, Natural Sciences I

9:30-9:40

(11) Multiplicity type and subalgebra structure in infinitary universal algebras
Professor Matthew I. Gould, Vanderbilt University (666-28)

9:45-9:55

(12) Compatible operations on binary relations
Mr. Martin K. McCrea, University of California, Davis (666-16)

10:00-10:10

(13) Right zero composition of N semigroups. Preliminary report
Mr. R. P. Dickinson, Jr., University of California, Davis (666-17)
(Introduced by Professor Takayuki Tamura)

10:15-10:25

(14) Commutative semigroup and power joined subsemigroups
Professor Takayuki Tamura, University of California, Davis (666-15)

10:30-10:40

(15) On commutative, power joined semigroups
Professor Richard G. Levin*, Western Washington State College, and
Professor Takayuki Tamura, University of California, Davis (666-14)

SATURDAY, 11:00 A.M.

Invited Address, Room 3, Science Lecture

On quadratic forms over general fields
Professor David Harrison, University of Oregon

SATURDAY, 2:00 P.M.

Invited Address, Room 3, Science Lecture

Numerical homotopy invariants and duality
Professor Theodor Ganea, University of Washington

SATURDAY, 3:30 P.M.

Second Session on Algebra and Theory of Numbers, Room 1, Science Lecture

3:30-3:40

(16) Stacked bases for modules over principal ideal domains
Professor Joel M. Cohen and Professor Herman Gluck*, University of Pennsylvania (666-18)

3:45-3:55

(17) Some definite polynomials which are not sums of squares of real polynomials
Professor Raphael M. Robinson, University of California, Berkeley (666-13)

4:00-4:10

(18) Unsolvable diophantine problems
Dr. Julia B. Robinson, University of California, Berkeley (666-10)

4:15-4:25

(19) Some Diophantine equations which are interrelated. Preliminary report
Professor Philip A. Leonard, Arizona State University (666-25)

SATURDAY, 3:30 P.M.

Second Session on Analysis, Room 125, Natural Sciences I

3:30-3:40

(20) Smooth extensions and extractions in Banach spaces
Professor Peter Renz, Reed College (666-21)

3:45-3:55

(21) A generalization of Markuševič's duality principle

Mr. James B. Cooper, University of California, Santa Barbara (663-153)

(Introduced by Professor J. A. Ernest)

4:00-4:10

(22) Almost periodic compact hulls of K-algebra pairs. Preliminary report

Mr. Michael H. Powell, University of California, Santa Barbara (666-26)

4:15-4:25

(23) On order convergence and topological convergence in a lattice ordered vector space

Professor Charles T. Tucker, University of Houston (666-22)

SATURDAY, 3:30 P.M.

Session on Geometry, Room 185, Natural Sciences I

3:30-3:40

(24) Consequences of Kelly's lemma in reconstructing graphs

Professor R. L. Hemminger, Vanderbilt University (666-30)

3:45-3:55

(25) Gale diagrams of convex polytopes

Dr. Peter McMullen, Western Washington State College (666-19)

(Introduced by Professor John R. Reay)

4:00-4:10

(26) Isoperimetric problems on polytopes. Preliminary report

Professor John R. Reay, Western Washington State College (666-9)

4:15-4:25

(27) A geometric approach to the Heine-Borel theorem. Preliminary report

Mr. Jason Frand*, California State College at Los Angeles, Professor Raymond Killgrove, California State College at Los Angeles, and Professor Henry G. Bray, San Diego State College (666-23)

4:30-4:40

(28) Representation of fractional dimensions and a related expression for the half dimension

Mr. K. Demys, Santa Barbara, California (666-24)

R. S. Pierce
Associate Secretary

Seattle, Washington

# PRELIMINARY ANNOUNCEMENT OF MEETING

## The Seventy-Fourth Summer Meeting University of Oregon, August 26-29, 1969

The seventy-fourth summer meeting of the American Mathematical Society will be held at the University of Oregon, Eugene, Oregon, from Tuesday, August 26, through Friday, August 29, 1969. All sessions will be held on the campus of the university.

There will be two sets of Colloquium Lectures, each consisting of four lectures. Professor Raoul Bott of Harvard University will lecture on Tuesday, August 26, at 1:30 p.m., and on Wednesday, Thursday, and Friday at 8:30 p.m. Professor Harish-Chandra of the Institute for Advanced Study will lecture on Tuesday, August 26, at 2:45 p.m., and on Wednesday, Thursday, and Friday at 9:45 a.m. These lectures will be delivered in the Ballroom of the Erb Memorial Union, and in Room 150 of the Science Building.

The meeting will also include two or three hour addresses and several sessions for contributed 10-minute papers. Abstracts of contributed papers should be sent to the American Mathematical Society, Post Office Box 6248, Providence, Rhode Island 02904, so as to arrive before the deadline date of July 1, 1969. There will be no limit on the number of contributed papers. However, no provision will be made for late papers.

This meeting will be held in conjunction with the meetings of the Mathematical Association of America, the Society for Industrial and Applied Mathematics, Pi Mu Epsilon, and Mu Alpha Theta. The Mathematical Association of America will meet from Monday, August 25, through Wednesday, August 27. The Earle Raymond Hedrick lectures, sponsored by the Association, will be given by Professor Errett Bishop of the University of California, San Diego. The Society for Industrial and Applied Mathematics will present Professor George Carrier of Harvard University as the von Neumann lecturer.

### COUNCIL AND BUSINESS MEETING

The Council of the Society will meet at 5:00 p.m. on Wednesday, August 27, in Room 101 of the Erb Memorial Union. The Business Meeting of the Society will be held on Thursday, August 28, at 4:00 p.m.

At the Business Meeting of January 25, 1969, five resolutions were introduced. They are quoted in full on page 480 of these *Notices*. They may be considered by the Council, by the Executive Committee, and by the Trustees, and with recommendations from one or more of these bodies, may appear as old business in the Business Meeting of August 28, 1969.

The above statement is not the notification to the full membership required by Article X, Section 1, of the By-laws but is rather an advance notice that such formal notification may appear later.

### REGISTRATION

The registration desk will be in the Lounge of the Erb Memorial Union. It will be open on Sunday from 2:00 p.m. to 8:00 p.m. on Monday from 9:00 a.m. to 5:00 p.m.; on Tuesday, Wednesday, and Thursday from 9:00 a.m. to 5:00 p.m.; and on Friday from 9:00 a.m. to 1:00 p.m.

The registration fees for the meeting are as follows:

| | |
|---|---|
| Member | \$3.00 |
| Member's family | .50 |

for the first such registration and no charge for additional registrations.

| | |
|---|---|
| Students | No charge |
| Others | \$7.50 |

At the January 1969 annual meeting, the Joint Committee on Employment Opportunities voted not to have an Employment Register at the Eugene, Oregon, meeting.

### EXHIBITS

Book exhibits and exhibits of educational media will be displayed in or near the Erb Memorial Union

R. S. Pierce
Seattle, Washington Associate Secretary

# NATIONAL REGISTER REPORT

The National Register of Scientific and Technical Personnel has issued a report on the results of the 1968 questionnaire. Following is a table on the Characteristics of Mathematicians, Computer Scientists, and Statisticians in the United States. An examination of the salary survey of the Society which was published in the October 1968 issue of the *Notices* shows that there is a close correlation between the results of the two studies in those portions of the reports where such a comparison can be made. For instance, the AMS salary survey showed a median salary (academic year) of $15,900 at the level of full professor, and the National Register report shows a median salary of $16,000; the AMS survey reported the median salary of an associate professor as $12,500, the National Register reports $12,000; the salary of an assistant professor was reported as $10,100 and for an instructor $8,100 by the AMS, while the National Register reported the median salary at these levels as $10,000 and $7,900.

| | Mathematicians | | Computer Scientists | | Statisticians | |
|---|---|---|---|---|---|---|
| Characteristics | Number | Median Salary | Number | Median Salary | Number | Median Salary |
| Highest Degree | | | | | | |
| Ph. D. | 6,929 | $14,000 | 469 | $18,100 | 929 | $16,000 |
| Professional Medical | 3 | | 1 | | 2 | |
| Master's | 12,094 | 11,400 | 2,736 | 14,400 | 1,147 | 14,000 |
| Bachelor's | 5,147 | 15,000 | 3,513 | 13,800 | 531 | 14,200 |
| Less than a Bachelor's | 29 | | 3 | | 3 | |
| No Report | 275 | 15,000 | 250 | 13,500 | 27 | |
| Type of Employer | | | | | | |
| Educational Institutions | 12,837 | (A) 10,200 | 921 | (A) 12,000 | 889 | (A) 12,500 |
| | | (C) 12,000 | | (C) 12,800 | | (C) 14,800 |
| Federal Government | 1,354 | 15,300 | 516 | 13,700 | 588 | 16,200 |
| Other Government | 308 | 13,100 | 102 | 12,700 | 149 | 12,500 |
| Military | 481 | | 141 | | 36 | |
| Nonprofit Organization | 681 | 18,000 | 475 | 15,100 | 132 | 14,800 |
| Industry and Business | 7,289 | 16,800 | 4,513 | 14,400 | 692 | 15,300 |
| Self-employed | 222 | 21,000 | 45 | | 21 | |
| Other | 219 | 14,000 | 62 | 13,300 | 24 | |
| Not Employed | 786 | | 147 | | 71 | |
| No Report | 300 | 13,200 | 50 | | 37 | |
| Work Activity | | | | | | |
| Research & Development | 5,147 | 14,400 | 2,661 | 13,800 | 743 | 14,400 |
| Basic Research | 2,014 | 12,500 | 159 | 13,500 | 169 | 14,500 |
| Applied Research | 2,556 | 15,000 | 1,496 | 14,000 | 542 | 14,400 |
| Management or Administration | 5,122 | 18,500 | 1,555 | 17,100 | 534 | 18,000 |
| Management or Administration of Research & Development | 2,145 | 20,000 | 825 | 18,000 | 284 | 18,500 |
| Teaching | 9,491 | (A) 10,000 | 212 | (A) 12,000 | 482 | (A) 12,000 |
| | | (C) 10,200 | | (C) 14,300 | | (C) 14,600 |
| Production and Inspection | 184 | 12,200 | 88 | 13,000 | 85 | 13,300 |
| Consulting | 1,065 | 17,000 | 440 | 14,400 | 269 | 15,300 |
| Exploration, Forecasting, and Reporting | 926 | 13,000 | 1,231 | 12,500 | 286 | 13,000 |
| Other | 745 | 14,000 | 427 | 13,000 | 71 | 14,400 |
| Not Employed | 786 | | 147 | | 71 | |
| No Report | 1,011 | 14,000 | 211 | 14,200 | 98 | 15,800 |

| Characteristics | Mathematicians | | Computer Scientists | | Statisticians | |
|---|---|---|---|---|---|---|
| | Number | Median Salary | Number | Median Salary | Number | Median Salary |
| Age | (Median age 36) | | (Median age 33) | | (Median age 39) | |
| 24 or under | 277 | 8,500 | 118 | 10,500 | 11 | |
| 25-29 | 5,267 | 9,800 | 1,710 | 11,900 | 380 | 11,000 |
| 30-34 | 5,541 | 12,000 | 2,121 | 13,800 | 534 | 12,700 |
| 35-39 | 4,317 | 14,500 | 1,512 | 15,700 | 452 | 15,000 |
| 40-44 | 3,413 | 16,200 | 875 | 17,400 | 374 | 16,000 |
| 45-49 | 2,085 | 17,000 | 395 | 17,000 | 310 | 18,000 |
| 50-54 | 1,434 | 16,000 | 147 | 17,400 | 278 | 17,400 |
| 55-59 | 1,031 | 15,500 | 60 | 18,000 | 182 | 17,700 |
| 60-64 | 679 | 14,800 | 18 | | 75 | 18,000 |
| 65-69 | 311 | 14,000 | 7 | | 33 | 15,800 |
| 70 or over | 94 | 11,000 | 3 | | 9 | |
| No Report | 28 | | 6 | | 1 | |
| Salaries of university & college teachers | | | | | | |
| Professor | | | | | | |
| Academic year | 1,166 | 16,000 | 23 | | 99 | 17,800 |
| Calendar year | 277 | 18,500 | 21 | | 61 | 20,000 |
| Associate professor | | | | | | |
| Academic year | 1,177 | 12,000 | 37 | 14,000 | 93 | 13,000 |
| Calendar year | 257 | 14,200 | 26 | 17,500 | 50 | 16,000 |
| Assistant professor | | | | | | |
| Academic year | 2,131 | 10,000 | 47 | 10,700 | 117 | 11,000 |
| Calendar year | 378 | 11,000 | 24 | | 64 | 13,000 |
| Instructor | | | | | | |
| Academic year | 1,011 | 7,900 | 12 | | 16 | |
| Calendar year | 192 | 8,600 | 16 | | 11 | |
| Salary distribution | | | | | | |
| Lower decile | | 8,000 | | 10,300 | | 10,200 |
| Lower quartile | | 10,000 | | 12,000 | | 12,000 |
| Median | | 13,000 | | 14,100 | | 14,900 |
| Upper quartile | | 17,500 | | 17,000 | | 18,200 |
| Upper decile | | 22,300 | | 20,300 | | 22,000 |

# SPECIAL REPORT
## on the Business Meeting at the Annual Meeting in New Orleans

The Secretary wishes to report, in more detail than will appear in the Bulletin, on two items developing from the Business Meeting of January 25, 1969. First, Professor Lee Lorch introduced a resolution that the Executive Committee be requested to take steps to remove the April 1969 meeting from Chicago. No reason was presented as any part of the motion. The motion was seconded and passed. The number of persons in the room early in the meeting exceeded four hundred. Later in the meeting it was somewhat larger. The number of members present was never accurately determined. The membership of the Society, ascertained from a mailing list a few days later, is 13,578. Of these 11,481 have United States mailing addresses, 555 have Canadian mailing addresses and 1,542 have mailing addresses in other parts of the world. The Secretary provides these figures in answer to a request made during the meeting.

In response to the request, the Executive Committee considered the question for a second time. It had previously considered it on request in December 1968 and had voted against any move. The Executive Committee voted on February 2, 1969, to move the meeting to Cincinnati on the same dates of April 18-19, 1969. The vote was four in favor, one opposed, one abstaining, and one absent. Four affirmative votes are required in the By-laws to yield an affirmative vote on a resolution.

The February issue of the *Notices* was already with the printer but it was still possible on January 27, 1969, to footnote several entries in the issue to alert members to the fact that the information in the text was no longer correct. There was then a special mailing to the membership giving them the correct information.

The membership should be aware of some of the expenses of the request at the Business Meeting. Even if the Executive Committee had voted at its meeting of February 2 to remain in Chicago, the added cost to the membership, consisting principally of printing, stuffing, and mailing to notify the membership would have been about $1716. The cost when the decision was to move to Cincinnati was only a little larger, namely about $1979. The difference lies chiefly in the cost of inspecting the site.

The Executive Committee solicited opinions from the membership on the correctness of their action. Selected opinions are printed elsewhere.

Professor Edward L. Dubinsky made an introductory statement and introduced five resolutions as follows:

As a professional organization of academicians, the members of the American Mathematical Society have the right and duty to take corporate action expressing their proper concern with conditions which affect the quality of civilized living and the evolving development of higher education. Specifically, the Society should adopt and support the following five resolutions which we respectfully propose for consideration at the next business meeting.

1. Resolved, that since scientific discovery by its nature requires complete open channels of information, it follows that classified research is a contradiction in terms. Members should consider most seriously participation in any investigation under a contract restricting full exchange of information with learned men everywhere, and as a society we recommend that members seek to disengage themselves from such activity.
2. Resolved, that the American Mathematical Society urges each of its members to use his talents in ways that promote peace and to refrain from activities whose primary purpose is to promote warlike efforts.
3. Resolved, that a committee be appointed to study the causes and course of the current worldwide upheaval in relationships among faculty, students, and administration in higher education, with particular reference to the situation at San Francisco State College. This committee shall

report to the members, with recommendations for suitable action, in the *Notices* of the Society.

4. Resolved, that the *Notices* shall be open for letters and articles discussing issues which concern the members as scholars and citizens generally as well as mathematicians particularly.

5. Whereas the shortage of mathematicians in North American Universities is different and greater among black and brown Americans than among whites, and whereas this situation is not improving, be it resolved that the AMS appoint a committee composed of black and third world mathematicians to study this problem and other problems concerning black and third world mathematicians, and report their conclusions and recommendations to the Society.

The five resolutions can and probably will be studied by the Council in connection with its meeting of April 4 or August 27, and by the Executive Committee (a Committee of the Council and responsible to the Council) and the Trustees, meeting separately or jointly in June. Individual members may wish to address their views to the Council or the Executive Committee. This is best done by submitting them in writing to the Secretary. Please note that this is not a poll and that letters stating only approval or disapproval are of little value.

Everett Pitcher
Secretary

# NEWS ITEMS AND ANNOUNCEMENTS

## SUMMER SCHOOL ON FUNDAMENTAL ASPECTS AND CURRENT DEVELOPMENTS IN COMPUTER SCIENCE

An international summer school on Fundamental Aspects and Current Developments in Computer Science will be held at the Technical University of Denmark, near Copenhagen, on August 11-23, 1969. The school will be held under the auspices of the Technical University of Denmark and sponsored by the NATO Science Committee. The organizers are J.-J. Duby, Scientific Center IBM, France, and H. J. Helms, Technical University of Denmark. The program will include a series of lectures by J. M. Foster, R. E. Griswold, M. L. Minsky, P. Naur, and H. Weber. Additional lectures are to be arranged.

Further information and a pamphlet with an application form may be obtained by writing to Professor Hans Jørgen Helms, Northern Europe University Computing Center, Technical University of Denmark, 2800 Lyngby, Denmark.

## CONFERENCE ON ASPECTS OF ORDINARY DIFFERENTIAL EQUATIONS

The Department of Mathematics of The University of Tennessee plans a regional conference on Boundary Value Problems and Oscillation Theory for Ordinary Differential Equations, June 9-13, 1969. Professor F. V. Atkinson will give a series of lectures and a limited number of contributed papers will be scheduled. There will also be several invited colloquium lectures. It is expected that travel and subsistence allowances will be available for a limited number of participants; however, the entire conference is contingent upon NSF support.

Further information may be obtained by writing to Professor J. S. Bradley, Department of Mathematics, The University of Tennessee, Knoxville, Tennessee 37916.

# INTERNATIONAL CONGRESS OF MATHEMATICIANS

Nice, France, September 1-10, 1970

The Secretariat of the International Congress of Mathematicians
Collège de France
11, place Marcelin-Berthelot
75-Paris 5$^{o}$ - France

If you intend to participate in the International Congress of Mathematicians (Nice, September 1-10, 1970), please fill in the form below and send it to the Secretariat of the Congress before April 30, 1969. This will not commit you in any way. However, the Second Communication, as well as all further information, will be sent only to those who have either filled in and mailed this form or have asked the Secretariat to include them in its list of addresses. Please fill in the form either by hand printing or with a typewriter.

---

(A) HOTELS. Daily cost: one room with breakfast, service, tax included. These rates hold for 1968.

| | Number of rooms | One person without bath | Two persons without bath | Two persons with bath |
|---|---|---|---|---|
| A | | | | 90-150F |
| B | | | | 90-120F |
| C | | 26-53F | 40-45F | 50- 65F |
| D | | 20-25F | 27-36F | 40- 49F |

(B) UNIVERSITY ROOMS. Number of rooms (single) at 9F per day without breakfast. ______

(C) REGISTRATION FEES. Mathematician, 200F; Other, 100F

Title__________________________Last name_________________________First name____________________________

Address__________________________________________________________________Country______________________

Accompanied by______________________________Last name______________________First name_________________

Mode of transportation (please circle): Train Plane Car

Date of arrival______________________________Date of departure______________________________________

Which language do you prefer?________________________________

Do you intend to present a communication? (please circle) Yes No

In which sections do you prefer to participate?_______________________________________________________

(Signature)__________________________________________

# LETTERS TO THE EDITOR

Editor, the *Notices*

The following documents some relevant history of Federal support of research.

About three years ago, certain obnoxious trends became apparent and developed at an alarmingly increasing rate. There was Congressional and Presidential pressure on the DOD and NSF to spread the support of research on the basis of "geographic distribution," in itself quite commendable, but which in practice turned out to be partly pork barrel, and partly was used as a means of extending the DOD's influence into the universities, thus gradually changing the original idea of supporting "pure research." Testimony at Congressional hearings on Research and Development is very illuminating (On S. Res. 110, July 11, 17, ,8, 1967, Part 3, see especially p. 655 et sequ.).

Project Themis.

A number of articles in SCIENCE Magazine have reported this trend. Project Themis is an example, in which, as reported by SCIENCE (7 April 1967) "there is a frank fusion of two sets of goals, one having to do with the universities, one having to do with DOD." And Donald MacArthur, deputy director of the Directorate of Defense Research and Engineering which is running Themis, stated: "... the scientific content of these programs (those of Themis) must be oriented toward areas of science and technology in which a strong mutual interest is shared by the Department of Defense and the University... Thus a further objective of these programs will be to foster closer relationships between the university scientists and engineers who are in daily contact with real military problems." The Themis Project was vigorously opposed by the local chapter of AAUP, which stated in a memo: "Our academic institutions have a vital but different concern in the free development and criticism of scientific knowledge, social institutions and artistic expression... For academic institutions to achieve their goals, it is essential that they function in an atmosphere of independence as complete as is humanly possible... We are seriously concerned at the encroachment by the Department of Defense and other military agencies in financing academic research..."

Although the Themis project has been slightly hampered by recent budget cuts, it is still going strong. As reported in SCIENCE (18 October 1968): "Although the DOD will not be able to start the 50 new university centers under Project Themis which it had planned for 1969, it will probably have enough money to begin half that number of new centers."

Waging "Peacefare."

At another time (17 November 1967) SCIENCE reported that "a study group appointed by the National Academy of Sciences has advised the Department of Defense to increase its support and use of research in the social and behavioral sciences. The Department 'must now wage not only warfare by peacefare as well,' the panel states in its report. 'Pacification, assistance and the battle of ideas are major segments of the DOD responsibility. The social and behavioral sciences constitute the unique resource for support of these new requirements and must be vigorously pursued if our operations are to be effective.' "

Center for Naval Analysis

Last year, in what I regard as a major development in the subversion of universities by the DOD, Rochester University entered into a contract of $8,819,000 with .the Center for Naval Analyses (similar to IDA), in which the contract allocates 5 percent of the budget to the support of 'fundamental unclassified research' at the Rochester Campus. The main part of the contract effort covers a program on problems in naval warfare in the broadest sense, including operational and logistic aspects. Thus support of pure unclassified research becomes here tied for the first time as a percentage of straight services for war making. The AAUP chapter at Rochester has protested this contract vivorously, in a memorandum dated 9 May 1968.

Political-economic blackmail.

Under the pressure of the Vietnam war, one also began to feel some political -economic blackmail in connection with Government contracts or grants. As reported in SCIENCE (5 April 1968), James Simons was fired by IDA on 29 March because of his refusal to engage in military-related research--a refusal which grew out of his opposition to the Vietnam war, even though Simons had indicated his willingness to work on IDA's nonmilitary projects. (Simons is now chairman of the math department at Stonybrook, by the way.) Of course the Simons case is not strictly speaking a university case, but it gave strong hints on the way the wind was blowing. The case of Smale's grant did provide an example within a university.

Universities treated as service centers.

Even more importantly SCIENCE (21 June 1968) reported action by the Senate "to whip rebellious colleges and students back in line" by "denying NASA grants to institutions that bar Armed Forces recruiters from their campuses." Columbia was mentioned specifically because of the resolution of the College Faculty taken earlier this year after Hershey's obnoxious directives to draft boards. As far as I am concerned, there is absolutely no reason why different agencies of the government should act in consort and collusion of this type. It is perfectly conceivable that at different times, one agency of the government may be involved in obnoxious policies while another is involved in desirable policies from the point of view of a university, which may wish to deal with one and not the other.

The Humphrey Statement.

One has to resist the view expressed so revoltingly by Vice President Humphrey, when he said:

"I know many times I read in the press there is a little rebellion on some campuses about government research projects, projects in universities. I don't know whether I ought to say this or not, but I'm a rather free-wheeling man. I feel if you don't want the money, there is another place for it. I sort of feel that if the university wants to exclude itself from the life of the nation, then it will most likely find itself living a rather barren life...I hope that our universities and our Government can work together. I hope that there will not be a breach because if there is it will not be the Government that suffers, because the Government can set up its own laboratories. I don't think that is very smart. I think that the Government ought to work with the private sector...But if a nation is denied that then it has to have some way to protect itself."

(Excerpts from a statement made at the meeting of the Panel on Science and Technology, House Committee on Science and Astronautics, 24 January 1968.)

I believe that far from excluding themselves from "the life of the nation," our universities should exclude themselves from the death of the nation. Humphrey's remarks are all the more frightening since they occur in a context when many influential persons view the universities as service centers for the "government," and tend to treat professors as civil servants. Such views are expressed on the Senate floor increasingly frequently, especially in connection with proposed legislation to introduce more and more restrictions on government grants, DOD or NSF.

Serge Lang

# APRIL MEETING IN THE WEST: Some Reactions of the Membership to the Change in Its Location

In notifying the membership of the change of location of the April meeting from Chicago to Cincinnati, two forms of letter were used. The 12,036 members in the United States and Canada received a letter of information and a slip reading:

> In changing the location of the April meeting, the Executive Committee responded to an expression of opinion of the membership as represented at a single business meeting. The Executive Committee would welcome opinions from the members whether their action was indeed the correct one.

and giving the address of the Secretary. Members outside the United States and Canada received aerogrammes of information in which the slip could not be inserted without added postage, a matter of more than $200.

By February 20, the date of this article, about 320 communications signed by 367 individuals had been received. The rate of arrival suggests that the flow will continue.

The letters range from one word or one line to three pages. Those favoring the change tended to say so very briefly while those opposing it presented arguments at length. Those favoring the change so far slightly outnumber those opposing it. There is, of course, a great deal of duplication of thoughts in the letters. Those opposing the change tend to present a greater variety of reasons and comments. The statements below are extracted from the letters according to the principle that the ideas presented in the letters should be represented but not often repeated. That is, it is a selection of the ideas and reactions in the letters, as complete in their variety as can be made in small compass, and is not a small scale model of the input to date.

* * *

I am in total agreement with the action of the executive committee in moving the Chicago meeting to Cincinnati.

* * *

Wrong!

* * *

I heartily endorse the action of the Executive Committee in changing the location of the April meeting from Chicago to Cincinnati. A small protest is better than none.

* * *

I strongly disagree with the action of the Executive Committee with regard to the Chicago meeting.

* * *

...it is my very strong opinion that the Executive Committee acted wisely and responsibly when they obeyed the obvious mandate from the members to cancel the plans to hold April 18-19, 1969, meeting in Chicago.

* * *

I share the doubts of the Executive Committee as to the wisdom of yielding to an unrepresentative minority in moving the April 1969 meeting from Chicago.

* * *

...the Executive Committee deserves the thanks of the membership of the AMS for responding to the resolution and effecting the change of venue under what must have been difficult circumstances.

* * *

In the earliest possible edition of the *Notices* please advise as to the procedure for impeaching Executive Committee members.

* * *

I am willing to presume that the mathematical competence of the members of the Executive Committee renders that body capable of sound professional decisions. I am however not willing to assume that the members of the Executive Committee are by virtue of their position preeminently qualified to render social judgements, which by implication are attributable to the membership of the society.

* * *

...if this relocation was intended as a commitment to a political and social goal..., then there should have been a mail ballot of the entire membership.

* * *

The separate mailing which was required to announce this change appears to me to be an unjustifiable expense.

* * *

In view of the action taken at the Business Meeting the Executive Committee had little other choice than to change the site of the meeting. However, I am strongly opposed to the action taken at the business Meeting.

* * *

...a great hardship has been created for many mathematicians in the Chicago area and surroundings, most especially for students, who may have to cancel their plans for attendance.

* * *

...if a sizeable fraction of the membership feels sufficiently emotional about meeting in Chicago to the extent that they would not participate in a meeting in that city, then the purposes of holding a meeting in Chicago are sufficiently damaged to the extent that such a meeting loses its importance to the society and its membership.

* * *

...the brutality of the Chicago police last summer was disgraceful and cannot be permitted to go unpunished. As long as a significant number of people feel this way, no meeting in Chicago will be well-attended.

* * *

I approve of moving the meeting from Chicago for a totally irrelevant reason. I have visited Chicago many times and always become depressed for several days as a result. It is Americas dreariest city.

* * *

...it may require more courage to refrain from moving meetings from Chicago than the contrary.

* * *

I particularly resent the spectacle of those who have had their fling in New Orleans acting to restrict mathematics away from a given area.

* * *

Isn't the Executive Committee a little worried that future actions might be reversed and subject the Society to lawsuits?

* * *

I would greatly appreciate it if you would kindly withdraw my paper: ... (submitted for possible presentation at meeting No. 665 before I learned that the meeting was moved from Chicago)...

* * *

...I have found it necessary to resign from the Society because of the purely political action it took in moving the April meeting to Cincinnati.

* * *

I would not attend a meeting in Chicago at this time.

* * *

Had I attended the business meeting in New Orleans I would have voted against the motion to move the meeting.

* * *

That the A.M.S. should take this forthright step makes me proud to be among its members.

* * *

I feel embarassed at the thought of being represented by an organization which wants to 'punish' the city of Chicago in this childish way.

* * *

I am shocked that the Society could permit a handful of extremists, motivated by purely political considerations, to force such a decision. Since the Society is apparently no longer a non-political organization, I must regretfully submit my resignation.

* * *

In my opinion the membership has reacted in an adolescent manner in changing the location of the April meeting. If by this action they have hoped to teach Mayor Daley and the Chicago police force a lesson, in truth they have failed. They will hurt only the hotel owners, their employees, and taxicab drivers.

* * *

As a matter of principle, I am very much opposed to any political involvement of the Society, especially in matters in which opinion throughout America is deeply divided. ...As a Canadian, I do not want to be involved, even indirectly, in USA political discussions, or boycott decisions.

* * *

It was my impression before becoming a member of the Society that its sole purpose was to stimulate research in mathematics.

* * *

The sole motivation of this action is political. Political considerations should play no role in the life of the Society, and might lead to its fragmentation and destruction. The Society includes among its members people of all conceivable shades of political opinion, united only by interest in mathematics.

* * *

One of the enjoyable aspects of [the AMS] is to associate with people of different ethnical backgrounds, religions, races, nationalities, etc. with a common interest, and that interest is mathematics and mathematics alone.

* * *

Must every organization nowadays engage in political activities? It is worth remembering that the American Mathematical Society has traditionally harbored members with a very broad range of political beliefs and is the better for it. Do the officers wish to compromise this aspect of the Society?

* * *

...this action was "kid stuff" and highly degrading for such an august body as the mathematicians.

* * *

I can only marvel at the naiveté of those members of the society who feel that mathematics and mathematicians should be above the political and racial struggles which exist in the United States.

* * *

I consider changing the meeting away from Chicago a childish business, and mere grandstanding for people who like to relieve themselves in public.

* * *

I don't like to be used. If the radicals want to express an opinion why don't they speak for themselves instead of subverting the Society. They profess abhorrence to the machine politics by which Mayor Daley runs Chicago, but their actions betray a kindred mentality.

* * *

I agree that the AMS should stick strictly to mathematics, and hope that the condition of the country will soon improve to the point where we won't have to protest in this way.

* * *

The Society should limit its political activities to those situations which directly affect the research and educational activities of the Society and its members.

* * *

I believe that what took place in Chicago last summer is a threat to democracy and a denigration of the democratic principle and hence our expression of disapproval is both called for and meaningful.

* * *

The Executive Committee's action in changing the location of the April meeting of the Society does not seem to be justified in terms of the aims of the Society. It is difficult, if not impossible, to read into the Chicago affair either any threat or any benefit to the field of mathematics.

* * *

...does this mean we must now go found a new society actually devoted to mathematics?

* * *

Mathematicians object when politics enters mathematics so they should see that mathematics keeps out of politics also.

* * *

I would have expected the mathematical community to be more rational and sophisticated than to go along with the hysterical anti-Chicagoism which followed the events of last August.

* * *

When I vote for officers in the Society must I consider their political posture as well as their mathematical competence?

* * *

By the same logic, I would expect a resolution against participation of AMS members in meetings in the USSR and the various Soviet Bloc countries which participated in the invasion of Czechoslovakia.

* * *

My contact (not a hotel man) said that we spend very little, tip very little, drink mainly only beer at the bar, and sit around for hours scribbling on the napkins. If there is anything to this, we might harm Chicago more by moving all the meetings there.

* * *

Everett Pitcher
Secretary

# PERSONAL ITEMS

Professor R. O. ABERNATHY of Tennessee A and I State University has been appointed to a professorship at South Carolina State College.

Professor M. F. ATIYAH of Oxford University, Oxford, England, has been awarded the Royal Society Medal for his contributions to the field of algebraic geometry and to the study of differential equations.

Professor A. P. CALDERON of the University of Chicago has been named the Louis Block Professor of Mathematics in the Division of the Physical Sciences at the University.

Dr. J. A. COCHRAN of the Bell Telephone Laboratories, Whippany, New Jersey, has been appointed to a visiting professorship at Stanford University for the academic year 1968-1969.

Professor R. H. CROWELL of Dartmouth College has been appointed to a visiting professorship at Princeton University for the academic year 1968-1969.

Mr. J. J. DEELY of the Sandia Corporation, Albuquerque, New Mexico, has been appointed Senior Lecturer at the University of Canterbury, Christchurch, New Zealand.

Dr. R. D. DRIVER of Sandia Corporation, Albuquerque, New Mexico, has been appointed to an associate professorship at the University of Rhode Island.

Professor G. F. FEEMAN of Williams College has been appointed to a professorship at Oakland University.

Dr. BRUCE GILCHRIST of Chappaqua, New York, has been appointed Executive Director of the American Federation of Information Processing Societies, New York, New York.

Mr. D. A. HURLIMAN of Princeton University has accepted a position as a Systems Analyst with Applied Data Research Incorporated, Princeton, New Jersey.

Professor KONRAD JÖRGENS of the University of Heidelberg, Germany, has been appointed to a professorship at the University of Munich, Germany.

Professor W. M. KAHAN of the University of Toronto has been appointed to a professorship in the Computer Science Department of the University of California, Berkeley.

Professor W. M. LAMBERT, JR., of Loyola University of Los Angeles has been appointed to an associate professorship at the University of Detroit.

Professor P. R. MEYER of the Herbert H. Lehman College of the City University of New York has been appointed to a visiting associate professorship at the University of Texas at Austin.

Professor BENJAMIN MUCHENHOUPT of Rutgers University has been appointed a Visiting Member at the Institute for Advanced Study for the academic year 1968-1969.

Professor J. H. VAN VLECK of Harvard University was elected to Foreign Membership in the Royal Society.

Dr. SHIMSHON ZIMERING of the Institute Battelle, Geneva, Switzerland, has been appointed to an associate professorship at Ohio State University.

PROMOTIONS

To Professor. California State Polytechnic College, Pomona: SIMON GREEN; Hebrew University, Jerusalem, Israel: R. J. AUMANN, S. R. FOGUEL; University of Missouri-Rolla: L. E. PURSELL; Technological University, Delft, Netherlands: L. A. M. VERBEEK.

To Associate Professor. Purdue University-Fort Wayne: R. E. PIPPERT.

INSTRUCTORSHIPS

University of Massachusetts: GEORGE POLLINO; Polytechnic Institute of Brooklyn: J. S. PAPADAKIS.

DEATHS

Professor EDWARD HALPERN of the University of Michigan died on January 31, 1969, at the age of 50. He was a member of the Society for 19 years.

Dr. HANS RADEMACHER of the University of Pennsylvania died on February 7, 1969, at the age of 76. He was a member of the Society for 35 years.

# MEMORANDA TO MEMBERS

## MEMOIRS

At the request of the editors of the MEMOIRS, the following information has been prepared. The usual procedure is to have the MEMOIRS printed by the photo-offset process from copy that has been fully prepared by the authors. An author is responsible for the preparation of copy which can be done in several ways: (1) The author may have his copy typed by a commercial typing service at approximately $2.80 per finished page; (2) the Society can prepare copy at a charge of $8 per Varityped page or $3 per type-written page; or (3) the author can have copy prepared by a good technical typist who should be provided with a Guide for Authors of MEMOIRS. The Editorial Department of the Society can do an initial proofreading of the copy for an additional $2 a page, and errors discovered in proofreading are corrected before the proof is sent to the author. Under this system, publication usually takes about two months.

The MEMOIRS can be typeset rather than typewritten with the Editorial Department handling the full procedure. Typesetting is between $18 and $22 a page, and editorial cost is $5 or $6 a page; costs depend on complexity of material. Charges to the author are based on actual costs to the Society. Some savings can be effected if the author takes full responsibility for proofreading, approximately $2 a page. A further saving of about $8 a page results if a foreign, but non-English speaking, typesetter is used, although publication may be delayed. Depending on the production schedule of a printer and the length of time an author needs to read proof, publication might take from eight to ten months.

It is to be noted that papers published in the MEMOIR Series are not subject to publication charges and that an author receives 50 free copies of his book. If the author wishes to furnish the Editorial Department with a list of people to whom copies should be sent, that department will mail the copies directly for him.

## SEMINAR ON MATHEMATICAL PROBLEMS IN THE GEOPHYSICAL SCIENCES

In the February 1969 issue of the *Notices*, it was announced that the AMS 1969 Summer Seminar on Mathematical Problems in the Geophysical Sciences had been canceled. It has now been announced that the seminar, which is being cosponsored by the Society for Industrial and Applied Mathematics and the American Meteorological Society, is being postponed with the present plans being to hold the seminar on the campus of Rensselaer Polytechnic Institute in the summer of 1970.

# ACTIVITIES OF OTHER ASSOCIATIONS

## SOCIETY FOR INDUSTRIAL AND APPLIED MATHEMATICS
Washington, D.C.
June 10-12, 1969

The 1969 National Meeting of SIAM will be held at The Shoreham Hotel, Washington, D.C., on June 10-12, 1969. The program will feature symposia on certain aspects of Special Functions and Wave Propagation. Those invited to participate in the symposia include G. E. Backus, B. C. Carlson, C. W. Clenshaw, A. Erdelyi, L. Felsen, H. W. Gould, H. P. Greenspan, D. A. Ludwig, W. Miller, F. W. J. Olver, and G. Whitman. Sessions for contributed papers on the subjects of the symposia will be held, and there will be a panel discussion on the Handbook of Mathematical Functions which was published by the National Bureau of Standards in 1964.

Further information on the meeting and registration material may be obtained by writing to the General Chairman, SIAM 1969 National Meeting, 33 South 17th Street, Philadelphia, Pennsylvania 19103.

## JOINT CONFERENCE ON MATHEMATICAL AND COMPUTER AIDS TO DESIGN

ACM, SIAM, and IEEE will sponsor a Joint Conference on Mathematical and Computer Aids to Design on October 26-30, 1969, in Anaheim, California. There will be sessions for both invited papers and contributed papers. Those planning to present contributed papers should submit a working title by May 1 to SIAM-1969 Joint MCAD Conference, 33 South 17th Street, Philadelphia, Pennsylvania 19103. An abstract form with full information will be returned. A one page abstract must be submitted by July 1, 1969; the abstract will appear in the Conference Digest which will be available at the meeting. Only papers which have not been presented or published previously should be submitted. Some selection of papers may have to be made to keep the program within desired limits. The prime consideration to be applied in the selection process is that the paper makes a contribution to mathematical and computer aspects of design. Information relating to the technical program may be obtained by writing to Dr. J. F. Traub, Program Chairman, Computing Science Research Center, Bell Telephone Laboratories, Inc., Murray Hill, New Jersey 07974.

# SUMMER INSTITUTES AND GRADUATE COURSES

The following is a list of graduate courses, seminars, and institutes in mathematics being offered in the summer of 1969 for graduate students and college teachers of mathematics. This list is in addition to the list found on pages 385 through 395 of the February issue of these *Notices*.

## Graduate Courses

### ARKANSAS

UNIVERSITY OF ARKANSAS
Fayetteville, Arkansas 72701
Application deadline: June 1
Information: Dr. James E. Scroggs, Department of Mathematics

June 2-July 11
Advanced Calculus I
Foundations of Geometry I
Introduction to Algebraic Theory

June 2-August 22
Finite Dimensional Vector Spaces
Abstract Algebra
Real Variables I
Point Set Topology
Topics in Analysis

July 14-August 22
Advanced Calculus II
Foundations of Geometry II
Linear Algebra

### CALIFORNIA

UNIVERSITY OF CALIFORNIA
Berkeley, California 94708
Application deadline: April 5
Information: Graduate Admissions

June 19-September 6
Introduction to Probability and Statistics at an Advanced Level (Statistics 200B), 4 units
Laboratory Course in Probability and Statistics (Statistics 200M), 1 unit
Information Theory (Statistics 262), 4 units
Special Seminars (Statistics 298B), 2-6 units

### FLORIDA

UNIVERSITY OF FLORIDA
Gainesville, Florida 32601
Application deadline: May 2
Information: Professor George Strecker, Chairman, Graduate Selection Committee, Department of Mathematics

June 16-August 23
Intermediate Differential Equations
Vector Analysis
Complex Variables
Introduction to Partial Differential Equations
Tensor Analysis
Special Topics in Mathematics

### GEORGIA

EMORY UNIVERSITY
Atlanta, Georgia 30322
Application deadline: June 1
Information: Professor Mary F. Neff, Department of Mathematics

June 16-August 14
Math 411 - Foundations of Geometry
Math 424 - Measure Theory
Math 471 - Topology

### ILLINOIS

NORTHWESTERN UNIVERSITY
Evanston, Illinois 60201
Information: Department of Mathematics

June 24-August 16
D08 Foundations of Mathematics
E10 Analysis Seminar
E11 Algebra Seminar
E12 Topology and Geometry Seminar
E14 Probability Seminar

### KANSAS

KANSAS STATE TEACHERS COLLEGE
Emporia, Kansas 66801
Application deadline: June 1
Information: Professor Marion P. Emerson, Head, Department of Mathematics

June 9-August 15
MA 430 Mathematics Programming
MA 431 Probability & Statistics
MA 504 Induction for Elementary Teachers
MA 510 Modern Math in Elementary School
MA 521 Projective Geometry
MA 522 Non-Euclidean Geometry
MA 525 Abstract Algebra
MA 527 Groups, Rings, Fields
MA 531 Differential Equations
MA 535 Advanced Calculus I
MA 536 Advanced Calculus II
MA 541 Mathematical Statistics
MA 565 Matrix Theory
MA 600 Modern Math in Secondary School
MA 610 Seminar

MA 620 Differential Geometry
MA 647 Research Project
MA 650 Thesis, Masters
MA 657 Thesis, Education Specialist
MA 480 Independent Study

WICHITA STATE UNIVERSITY
Wichita, Kansas 67208
Application deadline: June 1
Information: Professor William M. Perel, Department of Mathematics

June 9-August 1
Modern Algebra
Computer Science
Topology
Selected Topics in Statistics

## KENTUCKY

UNIVERSITY OF KENTUCKY
Lexington, Kentucky 40506
Application deadline: May 1
Information: Dr. R. H. Cox, Department of Mathematics

June 9-August 6
MA 533 Partial Differential Equations
MA 565 Linear Algebra
MA 570 Calculus of Several Variables
MA 757 Seminar in Topology
MA 774 Selected Topics in Analysis
MA 778 Seminar in Algebra

## MICHIGAN

UNIVERSITY OF MICHIGAN
Ann Arbor, Michigan 48104
Application deadline: One month prior to beginning of term
Information: Professor Maxwell O. Reade, Department of Mathematics, 3214 Angel Hall

May 5-June 27
Introduction to Differential Geometry
Topics in Analysis
Topics in Algebra
Topics in Topology

June 30-August 23
Introduction to the Foundations of Mathematics
Topics in Modern Mathematics for Teachers
Introduction to Topology

## NEBRASKA

UNIVERSITY OF NEBRASKA
810 Oldfather Hall
Lincoln, Nebraska 68508
Application deadline: June 11; July 21
Information: Chairman, Department of Mathematics

June 9-July 15
326 Introduction to Theory of Rings
329 Topics in Algebra
347 Advanced Topics in Differential Equations
353 Advanced Topics in Analysis

July 17-August 22
324 Commutative Algebra
329 Topics in Algebra
353 Topics in Analysis

## NEW YORK

NEW YORK UNIVERSITY
Courant Institute of Mathematical Sciences
251 Mercer Street
New York, New York 10012
Application deadline: May 1
Information: Chairman, Graduate Department of Mathematics

June 16-July 25
G63.1003 Non-Euclidean and Related Geometries
G63.1111 Linear Algebra I
G63.2391 Special Topics in Geometry
G63.2671 Special Topics in Analysis

July 28-September 5
G63.1006 Introduction to Abstract Algebra
G63.1052 Elementary Numerical Methods
G63.2122 Linear Algebra II
G63.2152 Special Topics in Algebra
G63.2852 Special Topics in Applied Mathematics

STATE UNIVERSITY OF NEW YORK
4246 Ridge Lea Road
Buffalo, New York 14226
Application deadline: April 15 for admission as graduate student, no deadline otherwise.
Information: Director of Graduate Studies, Department of Mathematics

May 26-July 18*
519-20 Algebra
527-28 Topology
531-32 Analysis

July 14-August 22
819 Selected Topics in Algebra
827 Selected Topics in Topology
831 Selected Topics in Analysis
835 Selected Topics in Geometry

*These courses are offered Monday through Thursday, 2 hrs. a day, and cover the basic first year graduate curriculum.

SYRACUSE UNIVERSITY
Syracuse, New York 13210
Information: The Graduate School, 200 Slocum Hall

June 9-July 18
Functions of a Real Variable
Functions of a Complex Variable
Modern Algebra
Introduction to Point Set Topology
Fundamentals of Analysis

July 21-August 29
Modern Algebra
Fundamentals of Analysis

## NORTH CAROLINA

EAST CAROLINA UNIVERSITY
Greenville, North Carolina 27834
Application deadline: May 9; June 17
Information: Dean of Graduate School

June 9-July 16
MATH 345G Non-Euclidean Geometry
MATH 365G Theory of Numbers
MATH 451 Nonlinear Differential Equations
MATH 470 Modern Algebra

July 17-August 22
MATH 346G Non-Euclidean Geometry
MATH 371G Theory of Equations
MATH 450 Linear Differential Equations
MATH 471 Modern Algebra

UNIVERSITY OF NORTH CAROLINA
Chapel Hill, North Carolina 27514
Application deadline: May 25; July 5
Information: Director, Graduate Studies, Department of Mathematics

June 6-July 15
Advanced Calculus I
Elementary Differential Equations
Elementary Theory of Numbers
Elements of Modern Algebra
Matrix Theory
Topics in Applied Mathematics
Journal Seminar

July 17-August 22
Linear Algebra
Advanced Calculus II
Functions of a Complex Variable with Applications
Partial Differential Equations
Elementary Topology I

## SOUTH CAROLINA

THE CITADEL
Charleston, South Carolina 29409
Application deadline: June 2
Information: Dr. Morris King, Director of Graduate Council

June 9-July 18
Modern Algebra

July 21-August 22
Geometry from Advanced Standpoint

## TENNESSEE

UNIVERSITY OF TENNESSEE
Knoxville, Tennessee 37916
Information: Department of Mathematics

June 17-August 25
Algebra (Continuation from Spring Quarter)
Complex Variables (Continuation from Spring Quarter)
Credit Seminar in Algebra
Credit Seminar in Applied Math
Credit Seminar in Topology
Credit Seminar in Computer Science
Credit Seminar in Analysis (Continuation from Spring Quarter)
Credit Seminar in Analysis

VANDERBILT UNIVERSITY
Nashville, Tennessee 37203
Application deadline: Start of Summer Session
Information: E. B. Shanks, Mathematics Department

June 9-July 15; July 17-August 29
Complex Variables
Intermediate Differential Equations
Seminar in Algebra

## TEXAS

TEXAS A & M UNIVERSITY
College Station, Texas 77843
Application deadline: 4 weeks before semester begins
Information: The Registrar

June 2
Math 601 Higher Mathematics for Engineering & Physics
Math 602 Higher Mathematics for Engineering & Physics
Math 609 Numerical Analysis
Math 617 Theory of Functions of a Complex Variable
Math 685 Problems
Math 691 Research

June 14
Math 601 Higher Mathematics for Engineering & Physics
Math 610 Numerical Methods in Differential Equations
Math 622 Laplace Transforms
Math 685 Problems
Math 691 Research

UNIVERSITY OF HOUSTON
Cullen Boulevard
Houston, Texas 77004
Application deadline: April 15

June 2-August 22
Mth 639 Selected Topics in Analysis
Mth 671 Dimension Theory
Mth 682 Selected Topics in Algebra
Mth 684 Cohomology

## VIRGINIA

MADISON COLLEGE
Harrisonburg, Virginia 22801
Application deadline: May 15
Information: Professor J. Emmert Ikenberry, Head of Mathematics Department

June 18-August 11
Math. 510 Advanced Calculus I
Math. 525 Numerical Analysis
Math. 530 Abstract Algebra
Math. 550 Theory of Numbers
Math. 620 Topics in Geometry

## WISCONSIN

MARQUETTE UNIVERSITY
Milwaukee, Wisconsin 53233
Application deadline: May 6
Information: Dean, Graduate School

June 18-July 30 (Registration: June 16,17)
Abstract Algebra I
Topics in Abstract Analysis: Introduction to Hilbert Space

## CANADA

UNIVERSITY OF BRITISH COLUMBIA
Vancouver 8, B. C., Canada
Application deadline: May 1
Information: Dr. A. Frei, Department of Mathematics

June 23-August 15
Nonlinear Differential Equations - Fred G. Brauer
Algebra - Alternative Rings - M. Slater
Topology - Vector Bundles, K-theory - Peter Hilton

# Summer Institutes

## KANSAS

KANSAS STATE TEACHERS COLLEGE
Emporia, Kansas 66801

National Science Foundation Sequential Institute
Dates: June 9-August 15
All participants have been selected.

## NORTH CAROLINA

UNIVERSITY OF NORTH CAROLINA
Chapel Hill, North Carolina 27514

Summer Institute in Mathematics for Teachers in Two-Year Colleges
Dates: June 6-July 15
Sponsoring Agent: National Science Foundation
Subjects Covered: Analysis and Probability
Requirements for Admission: Prerequisites are an introductory course in the foundations of one-variable calculus, some knowledge of analytic geometry of three-space, and an elementary course in linear algebra.
Deadline Date: February 15
Information: Professor Edward A. Cameron, Director, NSF Mathematics Institute, Department of Mathematics, University of North Carolina, Chapel Hill, North Carolina 27514.

## OHIO

KENT STATE UNIVERSITY
Kent, Ohio 44240

Seminar in Analysis, Seminar in Topology
Dates: June 15-July 23
Seminar in Algebra, Seminar in Topology
Dates: July 25-August 29

Seminar in Algebra, Seminar in Topology
Dates: July 25-August 29
Deadline Date: May 17
Information: Professor Richard K. Brown, Chairman, Department of Mathematics, Kent State University, Kent, Ohio 44240.

## TENNESSEE

VANDERBILT UNIVERSITY
Nashville, Tennessee 37203

Summer Institute for College Mathematics Teachers
Dates: June 9-July 19
Sponsoring Agent: National Science Foundation
Subjects Covered: Point set topology and linear algebra
Information: Professor James R. Wesson, Box 1595, Vanderbilt University, Nashville, Tennessee 37203.

## CANADA

UNIVERSITY OF CALGARY
Calgary, Alberta, Canada

Calgary International Conference on Combinatorial Structures and their Applications
Dates: June 2-June 14
Subjects Covered: Combinatorics
Deadline Date: May 1
Information: Dr. E. C. Milner, Department of Mathematics, The University of Calgary, Calgary, Alberta, Canada.

For information on additional <u>Summer Institutes for College Teachers</u> in the field of mathematics, sponsored by the National Science Foundation in 1969, write to the following:

CALIFORNIA

Dr. L. H. Lange
Department of Mathematics
San Jose State College
San Jose, California 95114

COLORADO

Dr. Daniel E. Bailey
Department of Psychology

or

Dr. Claude McMillan
Institute of Computing Science
University of Colorado
Boulder, Colorado 80304

GEORGIA

Dr. B. J. Ball
Department of Mathematics
University of Georgia
Athens, Georgia 30602

ILLINOIS

Dr. Charles E. Morris
Department of Mathematics
Illinois State University
Normal, Illinois 61761

Dr. Wilson M. Zaring
Department of Mathematics
University of Illinois
Urbana, Illinois 61822

MINNESOTA

Dr. Bernard W. Lindgren
Department of Statistics
University of Minnesota
Minneapolis, Minnesota 55455

MISSOURI

Dr. Ralph E. Lee
Department of Mathematics
Institute of Computer Science
University of Missouri
Rolla, Missouri 65401

NEW JERSEY

Dr. Joshua Barlaz
Department of Mathematics
Rutgers - The State University
New Brunswick, New Jersey 08903

OHIO

Dr. Robert R. Stoll
Department of Mathematics
Oberlin College
Oberlin, Ohio 44074

OREGON

Dr. A. F. Moursund
Department of Mathematics
University of Oregon
Eugene, Oregon 97403

PENNSYLVANIA

Dr. Bruce H. Barnes
Department of Computer Science
Pennsylvania State University
University Park, Pennsylvania 16802

# NEWS ITEMS AND ANNOUNCEMENTS

## FOREIGN SCHOLARS AVAILABLE FOR APPOINTMENT

The Committee on International Exchange of Persons, Conference Board of Associated Research Councils, will issue in March 1969 a list of foreign scholars available under the provisions of the Fulbright-Hays Act for appointments in American colleges and universities during the academic year 1969-1970. This list, compiled annually, includes information about scholars nominated by the binational Educational Commissions and Foundations abroad for Fulbright-Hays travel grants. These grants cover costs of round-trip transportation from the home country to the United States, provided that arrangements can be made for a lecturing or a research appointment with appropriate stipend at an American institution of higher learning. Information regarding the procedures for extending invitations and the conditions of appointment is also provided.

A copy of the list and additional information about individual scholars may be obtained from Miss Grace E. L. Haskins, Program Officer, Committee on International Exchange of Persons, 2101 Constitution Avenue, N. W., Washington, D. C. 20418.

## MATHEMATICAL RESEARCH CENTER
Université de Montréal

The Université de Montréal has recently set up a Mathematical Research Center, the main object of which is to carry out fundamental research in a number of domains of mathematics. In the choice of these domains, the Center will give priority to applied mathematics and to mathematics leading more directly to applications: functional analysis, differential equations, partial differential equations, probability theory, statistics, numerical analysis, computer system and automata, mathematical physics (particularly fluid and plasma dynamics), and continuum mechanics. The Center will begin its research activities in September 1969. It will be developed progressively, and it is anticipated that in 1972 it will include some thirty members. The Center members are divided into three categories: (1) permanent members, (2) associate members whose initial commitment is at most for three years, and (3) invited researchers who will join the Center for at most one year. It is expected that the Mathematical Research Center will collaborate closely with the Department of Mathematics, the Computing Department, and the Department of Physics of the University. The libraries of these departments, as well as the Computing Center of the university, are at the disposal of the members of the Mathematical Research Center. The Center is financed by the Université de Montréal which receives, for this purpose, special grants from the National Research Council of Canada and the Defense Research Board.

Further information may be obtained by writing to Centre de recherches mathématiques, Université de Montréal, Case postale 6128, Montreal 101, Canada.

## THE MITTAG-LEFFLER INSTITUTE

Professor Lennart A. E. Carleson, recently of Uppsala University, has been appointed director of the Mittag-Leffler Institute in Djursholm, Sweden. This institute was founded in 1916 when G. Mittag-Leffler, who had earlier founded Acta Mathematica, gave to the Swedish Academy of Sciences his mansion, his mathematical library, and a sum of $200,000. The library contains one of the most complete collections of old and modern books and journals in mathematics. Mittag-Leffler became the first director of the institute and served in this capacity until he died in 1927 at the age of 81. He was succeeded by T. Carleman who was the director until his death in 1949. Due to lack of funds, no new director was appointed in 1949, and the activities of the institute have been limited since that time. Over the intervening years, the institute funds have grown considerably, and recently the Swedish Academy of Sciences has received a sum of $200,000 from the Wallenberg Foundation to build apartments on the premises of the institute for visiting mathematicians. The Swedish and Finnish governments have undertaken the support of the activities of the institute, and it is expected that work at the institute will begin in the fall of 1969. The first two years will be devoted to work in harmonic analysis (in a wide sense) and will be planned in close collaboration with the harmonic analysis group in Paris.

## CLARKSON COLLEGE PH.D. PROGRAM

The Clarkson College of Technology has been authorized by its Board of Trustees to offer a doctor of philosophy degree in mathematics, the new doctoral program to be launched in September 1969. The school now offers Ph.D. programs in chemical engineering, chemistry, physics, and engineering science. The program calls for the completion of a minimum of 90 credit hours corresponding to a minimum of three academic years of full time study, two of which must be in residence at Clarkson. Dissertation work will comprise a minimum of 40 credit hours, seminar work a minimum of six credit hours, and course work a minimum of 39 credit hours. In addition, a reading proficiency in foreign languages selected from French, German, and Russian must be demonstrated.

## CONFERENCE ON THE NUMERICAL SOLUTION OF DIFFERENTIAL EQUATIONS

On June 23-27, 1969, the Department of Mathematics of the University of Dundee will hold a conference on the Numerical Solution of Ordinary and Partial Differential Equations. Invited papers will be presented by several eminent workers in the field of numerical analysis, and time has been allocated for submitted papers. The proceedings will be published. Further information may be obtained by writing to Dr. J. Ll. Morris, Conference Secretary, Department of Mathematics, University of Dundee, Dundee, Scotland.

## SEVENTH BRAZILIAN COLLOQUIUM OF MATHEMATICS

The Seventh Brazilian Colloquium of Mathematics will be held in Pocos de Caldas, Minas Gerais, Brazil, July 6-26, 1969. The program will consist of postdoctoral and graduate courses, one-hour survey lectures, sessions for research announcements, and a panel on mathematics education. Professor Jack Hale, Brown University, and Professor Paul R. Halmos, University of Hawaii, have accepted invitations to give postdoctoral courses on differential equations and invariant subspaces, respectively. Information regarding participation in the meeting may be obtained by writing to Instituto de Matemática Pura e Aplicada, Rua Luiz de Camoes 68, Rio de Janeiro 58, GB, Brazil.

## RECHERCHE COOPERATIVE SUR PROGRAMME N° 25 DU C.N.R.S.

On April 24-26, 1969, the Recherche Coopérative sur Programme N° 25 will be held at the Université de Strasbourg. The following invited papers will be presented: I. Intégration dans les espaces de dimension infinie and II. Relations de commutation canoniques et fonction théta by P. Cartier; Application d'un théorème des points fixes by J. J. Loeffel; Intégrale de Cauchy-Fantappié et cohomologie à croissance lente by A. Martineau; Système de covariance by L. Michel; Statistical mechanics of lattice systems by D. W. Robinson; and Sur les symétries brisées by D. Ruelle. Further information on this conference may be obtained by writing to Secrétariat de la R.C.P. N° 25, Département de Mathématique, Université de Strasbourg, Rue René Descartes, 67-Strasbourg, France.

## ACM SYMPOSIUM ON THEORY OF COMPUTING

The Association for Computing Machinery's Special Interest Committee for Automata and Computability Theory will sponsor a symposium on May 5-7, 1969, at the Marina del Rey Hotel, Marina del Rey, California (near the Los Angeles International Airport). There will be approximately thirty talks dealing with current research in formal languages, extended grammars, programming theory, machine-based and abstract computational complexity, and generalized automata theory.

Further information on the symposium can be obtained by writing to Patrick C. Fischer, Department of Applied Analysis and Computer Science, University of Waterloo, Waterloo, Ontario, Canada.

## SYMPOSIUM ON APPROXIMATION OF FUNCTIONS

The University of Cincinnati will sponsor a Symposium on Approximation of Functions, April 11-12, 1969. The principal speakers will be R. P. Boas, Jr., R. C. Buck, P. J. Davis, H. S. Shapiro, I. J. Schoenberg, and J. L. Walsh.

Further information may be obtained by writing to Professor Edward P. Merkes, Department of Mathematics, University of Cincinnati, Cincinnati, Ohio 45221.

## SUMMER SCHOOL ON GROUP REPRESENTATIONS AND QUANTUM THEORY

The National Committee for Mathematics of the Royal Irish Academy is holding a Summer School in Group Representations and Quantum Theory in Trinity College, Dublin, July 7-18, 1969. Courses will be given by Dr. D. J. H. Garling, Cambridge, on "Representations of locally compact groups" and by Professor L. O'Raifeartaigh, Dublin Institute for Advanced Studies, on "Unitary representations of Lie groups in quantum theory." Introductory lectures and other courses are planned, and it is intended that the lectures will be intelligible to mathematicians and physicists at the graduate level. Opportunities will be provided for seminars and informal discussions, and some social activities will be arranged.

Further information may be obtained by writing to Dr. T. T. West, Director, School of Mathematics, Trinity College, Dublin 2, Ireland. Envelopes should be marked "Summer School."

## NATIONAL STUDY OF MATHEMATICS REQUIREMENTS FOR SCIENTISTS AND ENGINEERS

The National Study of Mathematics Requirements for Scientists and Engineers, which has been concerned with gathering data on mathematics course requirements for certain disciplines, has published a final report on this project. The Board of Advisors of NSMRSE was interested in obtaining recommendations for courses in mathematics that would best serve the needs of students in such diverse fields as genetics, organic chemistry, solid state physics, electrical engineering, and the newer fields of biophysics and bioengineering. Approximately 10,000 scientists and engineers were selected for the study. The conclusions and recommendations that resulted from this study are contained in the Final Report (order number ED022079) which can be obtained from the Government Printing Office.

## NSF REPORT ON SCIENTIST AND ENGINEER EMPLOYMENT IN THE UNITED STATES, 1950-1966

More than 1,412,000 natural scientists and engineers were employed in the United States in January 1966, over two and a half times the number employed 16 years earlier. Private industry was the largest employer with 71 percent of the total in 1966. Federal, state, and local governments employed almost 16 percent; universities and colleges, nearly 13 percent; and nonprofit institutions, about one percent. Between 1950 and 1966, the number of scientists and engineers engaged in research and development rose by 242 percent. By broad occupation, scientists increased more rapidly than engineers, growing by 185 percent from 146,300 to 416,800, whereas engineers increased by 146 percent from 404,600 to 996,000. Among the scientists, more than one-fourth were chemists in 1966. Medical scientists had the most rapid growth over the period, more than double the rate for all scientific occupations. Employment of scientists and engineers increased most rapidly in the nonprofit sector, rising by 359 percent. Government employment (excluding that of public educational institutions) showed the lowest growth, 106 percent.

Employment of mathematicians grew much faster than that of scientists as a whole, increasing by nearly 300 percent, from 13,500 to 51,800. In 1966, approximately one-third of the total employment of mathematicians was in universities and colleges. This sector also accounted for 35 percent of the overall gain, rising from 4,100 in 1950 to 17,500 in 1966. The fastest relative growth was experienced in the ordnance industry, which grew from 100 in 1950 to 3,100 in 1966. Nonprofit organizations and aircraft were also well above average with growth rates of 18.4 and 15.0 percent respectively.

The data, contained in Employment of Scientists and Engineers in the United States, 1950-1966 (NSF 68-30), are based on estimates prepared for NSF by the Bureau of Labor Statistics of the U. S. Department of Labor. This report presents for the first time a consistent historical series on employment of natural scientists and engineers in the various sectors of the economy and relates this to major economic and social developments underlying the trends shown by these statistics. Copies are available from the Superintendent of Documents, U. S. Government Printing Office, Washington, D. C. 20402, for 70 cents a copy.

## NEW INFORMATION RETRIEVAL SYSTEMS FOR STATISTICAL DISTRIBUTIONS

The mathematics department of Temple University and the statistics department of the University of North Carolina are cooperating in setting up an information retrieval system for statistical distributions which will be located in the near future at Chapel Hill, North Carolina. Professor Samuel Kotz of Temple University and Professor N. L. Johnson of the University of North Carolina have collected more than 2,000 papers, reports, and reprints dealing with statistical distributions, and these are now being put into computer form. This system will be able to supply institutions and individuals with information on any distribution or specific characteristics of the distribution, such as mathematical properties and estimation procedures, by means of a computer-implemented filing scheme.

For the use of this service, institutions and individuals will pay an annual fee; and members will be supplied with a list of available reprints classified according to the filing scheme used, plus semi-annual supplements to the original lists. Reprints and abstracts of any of the publications will also be available to members at a nominal charge, and to others at a higher charge.

## ITALIAN NATIONAL RESEARCH COUNCIL FELLOWSHIPS

The Italian National Research Council will award 15 fellowships to citizens of other countries who intend to do research in mathematics at Italian universities during the academic year 1969-1970. Each fellowship will carry a monthly stipend of 180,000 Italian lire (approximately $288) for a maximum of 12 months. In addition, recipients will be given a partial reimbursement of their travel expenses. Applications, written in English, French or Italian, should be addressed to Consiglio Nazionale delle Ricerche, Servizio Affari Scientifici e Tecnologici, Ufficio Attivita di Ricerca, Sezione Borse di Studio, Piazzale delle Scienze 7, Rome, Italy. Applications should include the following information: (a) date and place of birth, citizenship, and residence; (b) nature of the proposed research; (c) names of Italian mathematicians with whom the applicant would like to associate and collaborate; (d) knowledge of Italian (if any) or other foreign languages; (e) address. A brief curriculum vitae and two letters of reference should accompany the application. Applications with all the supporting documents must arrive in the offices of the Consiglio Nazionale delle Ricerche before April 15, 1969.

# ABSTRACTS OF CONTRIBUTED PAPERS

## Abstracts for the Meeting in New York
## April 2-5, 1969

664-1. JOHN B. BUTLER, JR., Portland State College, Portland, Oregon 97207. <u>On the existence of the wave operators, corresponding to ordinary differential operators of even order, under the inverse conditions</u>. Preliminary report.

Let $H^i$, $i = 0, 1$, be selfadjoint operators on $\mathcal{L}_2(-\infty, \infty)$ with purely continuous spectrum on $[0, \infty)$ determined by formal differential operators $L^i = (d/dx)^n + q^i(x)$, $i = 0, 1$, where $n = 2v$, $-\infty < x < \infty$, $q^0(x) = 0$, $q^1(x)$ is continuous on $(-\infty, \infty)$, $q^1(x) = 0$, $x \leqq 0$, $q^1 \in \mathcal{L}_2$. The spectral measures $E^i(\Delta)$ corresponding to $H^i$ have the form $E^i(\Delta)u = \int_\Delta s^i_j(x, \iota)\varphi^i_k(\iota)d\rho^i_{jh}(\iota)$ where $\varphi^i_j(\iota) = \int_{-\infty}^{\infty} s^i_j(t, \iota)u(t)dt$, $\Delta \subseteq [0, \infty)$, $u \in \mathcal{L}_2$. Assume that $[s^i_j, s^i_k](x) = \epsilon_{jk}$; $j, k = 1, \dots, n$; $i = 0, 1$; $s^1_j = s^0_j$, $x \leqq 0$. Let $W_\pm(H^1, H^0)$ denote the wave operators; let $T^i : \mathcal{L}_2 \to \mathcal{L}_2(\rho^i)$ be mappings such that $u \mapsto T^i u = \{\varphi^i_j\}$; let $\hat{W}_\pm : \mathcal{L}_2(\rho^0) \to \mathcal{L}_2(\rho^1)$ be mappings such that $\varphi^0_j \mapsto V^{TI}_{jk}(\iota_\pm)\varphi^0_k$ where $V^{TI}_{jk}(\iota_\pm) = \lim_{\lambda \to \iota \pm i\delta +} V^{TI}_{jk}(\lambda)$, $V^{TI}_{jk}(\lambda) = \delta_{jk} - [(H^0 - \lambda 1)^{-1}Q^1 s^1_j, s^0_i](0)(\epsilon_{ik})$, $Q^1 u = q^1(x)u$, $u \in \mathcal{L}_2$. The matrix $V_{jk}(\lambda)$ is called the Weinstein matrix. Earlier the writer gave conditions, denoted here as (a), sufficient to insure existence of solutions of the inverse problem (Abstract 653-20, these *Notices* 15 (1968), 83). <u>Theorem</u>. If $H^i$, $i = 0, 1$, satisfy (a) then the mappings $\hat{W}_\pm$ exist and the square diagram corresponding to the sequences of mappings $\mathcal{L}_2(-\infty, \infty) \xrightarrow{T^0} \mathcal{L}_2(\rho^0) \xrightarrow{\hat{W}_\pm} \mathcal{L}_2(\rho^1)$ and $\mathcal{L}_2(-\infty, \infty) \xrightarrow{\hat{W}_\pm} \mathcal{L}_2(-\infty, \infty) \xrightarrow{T^1} \mathcal{L}_2(\rho^1)$ commutes. (Received May 23, 1968.)

664-2. CHARLES A. AKEMANN, University of California, Santa Barbara, California 93106. <u>The general Stone-Weierstrass problem</u>.

The Gelfand-Stone correspondence between an abelian C*-algebra and the algebra of continuous functions on its maximal ideal space is thirty years old. Yet in the non-abelian case some very basic questions remain unanswered. One cannot expect to find a well-developed theory (like that for locally-compact spaces) which can be instantly applied to the general case. In this paper we shall develop some tools and terminology for the general case which will be analogous to the topology of the maximal ideal space for an abelian C*-algebra. This structure will then be applied to the problem of generalizing the Stone-Weierstrass theorem to non-abelian C*-algebras. A typical result is the following: <u>Theorem</u>. If B and $\mathfrak{U}$ are C*-algebras with $\mathcal{B} \subset \mathfrak{U}$ and with $\mathcal{B}$ separating the closed left ideals of $\mathfrak{U}$, then $\mathcal{B} = \mathfrak{U}$. (Received October 21, 1968.)

664-3. HIDEGORO NAKANO, Wayne State University, Detroit, Michigan 48202. <u>An axiomatic set theory</u>.

A space S is called a set space when for any $x, y \in S$ we have $x \in y$ or $x \bar{\in} y$, and not both. A class of set spaces subject to some axioms is called an axiomatic set theory. There are many different kinds of axiomatic set theories. In this paper, we define the simplest among them and discuss the cardinal numbers and the ordinal numbers in it. Of course, we can consider the continuum hypothesis in this axiomatic set theory and the independence is trivial by the Existence Theorem. (Received October 18, 1968.)

664-4. ANDRE de KORVIN and R. J. EASTON, Indiana State University, Terre Haute, Indiana 47809. More on the existence of expectation type maps on B*-algebras.

Let N and M be B*-algebras with identity such that $N \subset M$. Theorem. Let N and M be algebras of operators and suppose $\{\rho_\alpha\}$ is a complete set of states on M. Suppose each $\rho_\alpha$ diagonalizes M, is normal on $Z_N$, and there exists a constant $k_\alpha$ such that $\rho_\alpha(x^*xy^*y) \leqq k_\alpha \rho_\alpha(x^*x)\rho_\alpha(y^*y)$, $x,y \in N$. Furthermore suppose that for each $x \in N$, $C_N(x) \cap Z_N \neq \emptyset$. Then there exist expectation like maps $\Psi_\alpha : M \to N_{e_\alpha}$ such that $\rho_\alpha(x^*ax) = \rho_\alpha(\Psi_\alpha(x)a)$, $x \in M$, $a \in N$, where $e_\alpha$ is any projection in $Z_N$ such that $\rho_\alpha$ is faithful on $N_{e_\alpha}$. The $\Psi_\alpha$'s satisfy $\Psi_\alpha(xy) = x\Psi_\alpha(y)x^*$, $x \in N$, $y \in M$. If $Z_N$ satisfies the condition (for each $\alpha$, whenever $\{q_{\alpha\beta}\}$ is an increasing net of projections in $Z_N$, with $\rho_\alpha$ faithful on each $N_{q_{\alpha\beta}}$, then $\sup_\beta q_{\alpha\beta}$ is again a projection in $Z_N$), then the above is true where $e_\alpha$ is the carrier of $\rho_\alpha$. Finally, in the case where the $e_\alpha$'s are the carriers of the $\rho_\alpha$'s, if $\sup e_\alpha = I$, then $\{\Psi_\alpha\}$ form a complete set on N. (Received October 23, 1968.)

664-5. WITHDRAWN.

664-6. DAVID K. KAHANER, Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87544. Equal and almost equal weight quadrature formulas.

A weight function p(x) is a positive continuous function on the open interval $(-1,1)$ with $\int_{-1+0}^{1-0} p(x)dx = W < \infty$. Consider the numerical quadrature formula $\int_{-1}^{1} p(x)f(x)dx \approx W^{(\nu)} \sum_{i=1}^{\nu} f(x_i^{(\nu)})$ with $x_i^{(\nu)}$ distinct in $[-1,1]$ which is exact when f is a polynomial of degree m or less. For $p(x) \equiv 1$, Bernstein has shown that $m < \pi\sqrt{2\nu}$. We present an elementary proof of Theorem 1. If $p(x) \leqq K$, $|x| < 1$, then $\limsup_{\nu\to\infty} \nu/m = \infty$. Theorem 2. Let $[t] \equiv$ greatest integer $\leqq t$. If $(\nu - m - 1)/[m/2] < M$ $(\nu\to\infty)$, then $p(x) \geqq W/((M+2)\pi\sqrt{1-x^2})$, $|x| < 1$. Finally we present some results about quadrature formulas $\sum w_i^{(\nu)} f(x_i^{(\nu)})$ with almost equal weights $w_i^{(\nu)}$. (Received November 18, 1968.)

664-7. L. TAYLOR OLLMANN and ANIL NERODE, Cornell University, Ithaca, New York 14850. Intermediate theories with respect to Keisler's ultra product ordering.

No complete first order theory of an equivalence relation is maximal with respect to Keisler's partial ordering $\triangleleft$. Note that any such theory having a model with arbitrarily large finite equivalence classes has the finite cover property and is thus not minimal. Thus there exist theories which are neither minimal nor maximal with respect to $\triangleleft$. For definitions of terms and the problem, see H. Jerome Keisler, Ultraproducts which are not saturated, J. Symbolic Logic 32 (1967), 23-46. (Received November 22, 1968.)

664-8. M. F. JANOWITZ, University of Massachusetts, Amherst, Massachusetts 01002. Some remarks on Baer rings.

Let L denote the lattice of left annihilators of elements of the Baer ring A. If e,f are idempotents in A, write Ae $\perp$ Af in case there exist idempotents $e_0$, $f_0$ such that Ae = $Ae_0$, Af = $Af_0$, and $e_0f_0 = f_0e_0 = 0$. For M $\subseteq$ A, let L(M) denote the left annihilator of M, and R(M) its right annihilator. Theorem. Let Af = LR(x), eA = RL(x) with Ae $\perp$ Af. There then exists an idempotent g such that Ae $\vee$ Ag = Af $\vee$ Ag = Ae $\vee$ Af, Ae $\wedge$ Ag = Af $\wedge$ Ag = 0, and (Af, Ag) as well as (Ag, Af) forms both a modular and a dual modular pair. If, in fact, e is algebraically equivalent to f, then g may be chosen so that in addition (Ae, Ag) and (Ag, Ae) each form a modular and a dual modular pair. In this case the isomorphism of L(0, Ae) onto L(0, Af) induced by the algebraic equivalence is also induced lattice theoretically by the mapping Ad $\rightarrow$ (Ad $\vee$ Ag) $\wedge$ Af for Ad $\subseteq$ Ae. Corollary. In the projection lattice of a Baer *-ring, orthogonal projective elements are in fact perspective in the interval from 0 to their join. The obvious analogue for the last assertion of the theorem is valid for orthogonal *-equivalent projections. (Received December 6, 1968.)

664-9. ROBERT L. DAVIS, University of North Carolina, Chapel Hill, North Carolina 27514. Algebras defined by patterns of zeros.

A curious by-product of Rota's unifying study of combinatorial theory is that for certain prescribed sets of entries the sets of all real matrices with 0's in those entries, being realizations of "incidence algebras", are closed under multiplication. More remarkably, the only matrix sets defined by such patterns of prescribed 0's which are multiplicatively closed arise in just this way, as can be deduced from David Smith's axiomatic characterization of incidence algebras. This is also one special consequence of the following theorem, whose proof and its consequences reduce such problems to what seems their most natural setting and leads to a significant generalization of the notion of incidence algebra. Let X be any set, T any subset of X $\times$ X, and V(T) the vector space of all functions with support in T and values in a field of characteristic zero. Then the convolution $f*g(x,y) = \sum f(x,z)g(z,y)$, sum over all z $\in$ X, is a well-defined operation under which V(T) is closed if and only if T is a locally finite transitive relation. (Received January 2, 1969.)

664-10. M. A. DOSTAL, University of Montreal, Montreal, Quebec, Canada. Local properties of distributions.

E. M. Stein proved an interesting theorem of Paley-Wiener type for bounded functions from $L^2(R^n)$ (Ann. of Math. 65 (1957), 582-592). His condition for vanishing of the Fourier transform of a given function f outside a convex symmetric compact $\Omega$ is formulated in terms of some analytic properties of the means of f over the dual convex set $\Omega$*. The generalization to the Schwartz distributions is not straightforward and was given by T. Kakita (J. Math. Soc. Japan 14 (1962), 351-357). We study the analogous question where "vanishing" is replaced by "$C^\infty$-smoothness". The conditions obtained differ even formally from those mentioned above and are formulated and proved for the more general spaces of distributions (Beurling distributions). (Received November 25, 1968.)

664-11. WALLACE S. MARTINDALE, III, University of Massachusetts, Amherst, Massachusetts 01003. Prime rings satisfying a generalized polynomial identity.

We simultaneously generalize a theorem of Posner on prime rings satisfying a polynomial identity [Proc. Amer. Math. Soc. 11 (1960), 180-183] and a theorem of Amitsur on primitive rings satisfying a generalized polynomial identity [Trans. Amer. Math. Soc. 114 (1965), 210-225, Theorem 10]. Both proofs are replaced by a single independent proof. Let R be a prime ring, and suppose (for the sake of simplicity in this abstract) that $1 \in R$. Let Q be Utumi's complete ring of right quotients of R, let C be the center of Q, and set $S = RC$. Theorem. If S satisfies a generalized polynomial identity over C, then S contains a minimal right ideal eS and eSe is a finite-dimensional division algebra over C. (Received December 9, 1968.)

664-12. HERMAN E. GOLLWITZER, University of Tennessee, Knoxville, Tennessee 37916. On the continuation problem for a second order differential equation.

Consider the second order equation (*) $y'' + q(t)f(y) = 0$, where q is positive, continuous, and locally of bounded variation on $(-\infty, \infty)$. f is continuous for all y and satisfies $F(u) = \int_0^u f(s)ds \geq 0$ for all u. Let $a \in (-\infty, \infty)$ be fixed and write q as $q(t) = q(a) + q_+(t) - q_-(t)$ for all t, where $q_+, q_-$ are the positive and negative variations of q on $(-\infty, \infty)$, respectively. The following estimate seems to be important in the study of solutions y(t) of (*). Theorem. For all t, $E(a)\exp(-\int_a^t q(s)^{-1}dq_-(s)) \leq E(t) \leq E(a)\exp(\int_a^t q(s)^{-1}dq_+(s))$, where $E(t) = y'(t)^2/2 + q(t)F(y(t))$. Corollary. All solutions of (*) exist on $(-\infty, \infty)$. Boundedness and stability results can also be deduced from this estimate, and similar estimates hold if q is merely positive and locally of bounded variation on $(-\infty, \infty)$. The equations $y'' + q(t)f(y)g(y') = 0$, $(r(t)y')' + q(t)f(y) = 0$ admit similar estimates. (Received January 6, 1969.)

664-13. CLIFTON T. WHYBURN, East Carolina University, Greenville, North Carolina 27834. An elementary estimate for primitive elements in finite fields.

By a technique used by A. Brauer and T. L. Reynolds to generalize a theorem of Vinogradov, a partial corollary of a theorem of Brauer and Reynolds is obtained. Theorem. Suppose r,s,n are positive integers, $r < s$, and p is a prime. Let $\{f(i)\}_{i=1}^m$ be a set of positive numbers less than or equal p, such that $\prod_{i=1}^m f(i) > p^{nr}$, and $\{e(i)\}_{i=0}^s$ be a set of integers with $e(0) = 1 \leq e(1) \leq \ldots \leq e(s) = m$. Identify $GF(p^n)$ with $Z/p[\theta]$. The system $\sum_{j=1}^s g_{ij}x_j = 0$, $1 \leq i \leq r$, has a solution in $GF(p^n)$ such that $x_j = \sum_{k=e(j-1)}^{e(j)} a_k \theta^{k-e(j-1)}$, and if $a_k$ is identified with a representative of its residue class (mod p), $|a_k| < f(k)$. The theorem is employed to obtain an elementary estimate for a primitive element of $GF(p^n)$. (Received January 23, 1969.)

664-14. LEONARD A. ASIMOW, University of California, Los Angeles, California 90024. Extremal structure of closed convex sets.

Various extremal properties of closed convex subsets of locally convex linear spaces are investigated with the purpose of establishing Krein-Milman type theorems as the main goal. We say the closed convex set X has the Krein-Milman property if X is the closed convex hull of its extreme points and extreme rays. Theorem. The following are equivalent: (a) Every closed convex subset of X has the Krein-Milman property. (b) Every closed convex subset of X contains an extreme point. (c) Every closed convex subset Y of X contains a compact set whose complement in Y is convex. (d) Every closed convex subset Y of X contains a compact convex subset whose complement in Y is convex. The equivalence of (a) and (b) for X a (norm) closed bounded subset of a Banach space has been observed by Lindenstrauss as a consequence of the Bishop-Phelps theorem. If X is a closed convex subset of a separable dual Banach space then (by applying a theorem of Bessaga and Pelczyński) "compact" in (c) and (d) can be replaced by "closed and bounded." In fact $x \in X$ is contained in a compact (resp. closed and bounded) subset of X with convex complement if and only if x is contained in a compact (resp. closed and bounded) convex subset of X with convex complement. (Received December 30, 1968.)

664-15. AL SOMAYAJULU, Canisius College, Buffalo, New York 14208. On a class of transcendental numbers.

The object of this paper is to exhibit an entire transcendental function which takes a class of algebraic numbers into transcendental numbers. Theorem 1. There is no Z ($\neq 0$) such that $Z^2$ is rational and $Z I_{\nu+1}(Z)/I_\nu(Z)$ is algebraic for any rational $\nu$. $I_\nu(Z)$ is a Bessel function with purely imaginary argument defined by $I_\nu(Z) = \sum_{n=0} ((Z/2)^{\nu+2n}/n!\Gamma(\nu + n + 1))$. Theorem 2. There is no Z ($\neq 0$) such that $Z^m$ ($m > 2$) is rational and each member of the set $\{Z^{\nu+m-1} I_{\nu+i}(Z)/I_\nu(Z): 1 \leqq i \leqq m/2\}$ is algebraic for any rational $\nu$ such that $\nu \geqq 1 - m/2$. Corollary 1. $\pi^{1/2}\tanh Z$ is transcendental if $Z^2$ is rational. Corollary 2. Setting $\nu = -1/2$ in Theorem 1, we get Siegel's result that the continued fraction $\langle 1,2,3,4,\ldots \rangle$ is transcendental. (Received December 2, 1968.)

664-16. THOMAS R. CHOW, Oregon State University, Corvallis, Oregon 97331. The algebra of finite operators and the topological module of kernels.

Let E be a locally convex Hausdorff vector space; $\Gamma_E$ the set of all continuous seminorms on E which determines the topological of E. Let $\Omega$ be the set of all functions from $\Gamma_E$ into itself. Let L(E,E) be the set of all continuous linear maps from E into itself. Define, for every $\sigma \in \Omega$, $L_\sigma(E,E) = \{T | T \in L(E,E) \sup_{(p)(x) \leq 1} p(Tx) < \infty \ \forall p \in \Gamma_E\}$. Then, for every $p \in \Gamma_E$, $\|T\|_{\sigma(p),p} = \sup_{\sigma(p)(x) \leqq 1} p(Tx)$ defines a seminorm on $L_\sigma(E,E)$. Let $\mathcal{T}_\sigma$ be the coarsest locally convex topology on $L_\sigma(E,E)$ so that $\|\cdot\|_{\sigma(p),p}$ is continuous for every p in $\Gamma_E$. Since there is a natural ordering on $\Omega$, the direct limit may be defined. Let $\mathcal{L}(E) = \varinjlim L_\sigma(E)$ and $\mathcal{T} = \varinjlim \mathcal{T}_\sigma$. Theorem. $\mathcal{L}(E)$ is a topological algebra. Let $E' \otimes^- E$ be the inductive topological tensor product of E and its dual E'. Then Theorem. $E' \overline{\otimes} E$ is a topological module over $\mathcal{L}(E)$. (Received January 6, 1969.)

664-17. ROBERT W. BERGER, Louisiana State University, Baton Rouge, Louisiana 70803. Higher differentials and ramification.

Let $R \to S$ be a ringhomomorphism of commutative local rings, M the maximal ideal of S. A differentiation of S over R of order n is a family $\{d^i | i = 0,1,\ldots n\}$ or $\{d^i | i = 0,1,2,\ldots\}$ if $n = \infty$ of R-linear maps $d_n^i$ from S into a ring $D_n(S,R)$ which satisfy the rules $d_n^i(xy) = \sum_{k=0}^i d_n^k x d_n^{i-k} y$ for all $x, y \in S$, $i = 0,1,\ldots, n$, being universal with that property. In $m \geqq n$ there is a natural homomorphism

$\tilde{\eta}_n^m : D_n(S,R)/M \cdot D_n(S,R) \to D_m(S,R)/M \cdot D_m(S,R)$. Let stab $(S,R) = \min\{n \mid \ker \tilde{\eta}_1^n = \ker \tilde{\eta}_1^\infty\}$. Theorem. If R,S are discrete rank 1 valuation rings, S dominating R, such that the residue field extension is separably generated, then stab $(S,R) = e(S,R)$; $e(S,R)$ is the usual ramification index of S over R. Similar results, involving also the exponent of inseparability of the residue field extension, hold if the residue field extension is arbitrary, generalizing results of the author for fields published in Sitzungsberichte der Heidelberger Akademie der Wissenschaften, Math-Naturw. Klasse, 3. Abh. 1966. (Received January 16, 1969.)

664-18. B. S. LEE and S. M. SHAH, University of Kentucky, Lexington, Kentucky 40506. The type of an entire function satisfying a linear differential equation.

Let $f(z)$ be a transcendental entire function satisfying the linear differential equation $P_0(z)w^{(k)}(z) + \ldots + P_k(z)w(z) = P_{k+1}(z)$ where $P_j(z) = a_j z^d + O(z^{d-1})$, $j = 0,1,\ldots, k$, $P_{k+1}(z) = a_{k+1}z^{d_{k+1}} + O(z^{d_{k+1}-1})$ are polynomials and $a_0 \neq 0$. It is known [S. M. Shah, J. Math. Mech. 18 (1968), 131-136; Proc. Amer. Math. Soc. 19 (1968), 1017-1022] that f is a bounded index and hence of exponential type. In this paper the local index $N_\xi$ has been defined and related to the index N. Theorem 1. Let q be the least nonnegative integer such that $q > d_{k+1}$, $(k + q)!|a_0| > \sum_{j=1}^{k} (k + q - j)!|a_j|$. Then $\lim\sup_{r\to\infty} \log M(r,f)/r \leq k + q$. Let $S_n = \{\xi \mid N_\xi = n\}$, $\sigma_n = \{r \mid |\xi| = r, \xi \in S_n\}$. Theorem 2. Let f be a transcendental entire function of bounded index. If $m_l(\sigma_n)$ denotes the logarithmic measure of $\sigma_n$ and if $m_l(\sigma_n) = \infty$ for some $n > 0$, then f is of order one. (Received January 10, 1969.)

664-19. CARL KALLINA, Mobil Research and Development Corporation, Princeton, New Jersey 08540. A Green's function approach to perturbations of periodic solutions.

Consider the linear differential system (1) $\dot{y} = A(t)y + \epsilon q(t,y,\epsilon)$, where y, q are column n vectors, q is continuous in $(t,y,\epsilon)$ and has continuous second partials with respect to y, $\epsilon$ for all t, $0 \leq y \leq R$, $R > 0$, and $0 \leq \epsilon \leq \epsilon_0$, $\epsilon_0 > 0$. Assume $A(t) \in C^1$ and both A and q are periodic in t of period T. Associated with (1) is (2) $\dot{y} = A(t)y$. The paper considers the classical problem of proving the existence of T-periodic solutions of (1) when (2) has nontrivial T-periodic solutions. The purpose of the paper is to show how the results of D. C. Lewis (Ann. of Math. 63 (1956), 535-548) concerning generalized Green's matrices can be incorporated into the treatment of this problem given by J. K. Hale (Oscillations in nonlinear systems, McGraw-Hill, New York, 1963, Chapters 6 and 11). It is felt that the approach presented here brings into sharper focus the essentially algebraic nature of Hale's treatment. (Received January 13, 1969.)

664-20. STANLEY OSHER, University of California, Berkeley, California 94720. Maximum norm stability for parabolic difference schemes in half-space.

Consider a P. D. operator of order 2m: $P(D) = \sum_{r_1+r_2+\cdots+r_s=2m} A_r(D_x^{r_1}/D_{x_1})(D_x^{r_2}/D_{x_2}) \cdots (D^{r_s}/D_{x_s})$, $A_r$ complex-valued constant $n \times n$ matrices. $I\, D/Dt - P(D)$ is uniformly parabolic. Take the initial-boundary value problem in the region $x_1 > 0$, $t > 0$, $-\infty < x_2,\ldots,x_s < \infty$. We approximate it by diff. approximations of the form $(\sum_{j=-a}^{b} B_j S^j)u(x,t + k,h) = (\sum_{j=-a}^{b} C_j S^j)u(x,t,h)$ in this region. S shifts x to x + h. We approximate the boundary conditions $\sum_{j=-a}^{l} P_j^{(i)} S^j u(x,t + k,h)]_{x_1=0} =$

$\Sigma_{j=-a}^{l} Q_j^{(i)} S^j u(x,t,h) ]_{x_1=0}$, $i = -1, -2, \dots, -a_1$. Again all the matrices are constant. Sufficient conditions for stability of these approximations were obtained. In particular, a certain normal mode analysis was used in a manner analogous to the author's previous work on hyperbolic difference schemes. (Received January 13, 1969.)

664-21. THOMAS J. OSLER, 441 South Park Drive, Collingswood, New Jersey 08108. Leibniz rule for fractional derivatives and application to infinite series.

The formula for the fractional derivative of order $\alpha$ with respect to g(z) of the product u(z)v(z) is given by $D_g^{\alpha} uv = \Sigma_{n=-\infty}^{\infty} \Gamma(\alpha + 1) D_g^{\alpha-\gamma-n} u D_g^{\gamma+n} v / \Gamma(\alpha - \gamma - n + 1)\Gamma(\gamma + n + 1)$. The special case $\gamma = 0$ and $\alpha = 0,1,2,\dots$ is the elementary Leibniz rule. The author derives this formula, which to the best of his knowledge is new, from a Cauchy integral formula for fractional derivatives using contour integration. A region of convergence in the z plane is described in terms of the position of the singularities of u(z) and v(z). This formula is then used to generate several interesting infinite series expansions relating special functions of mathematical physics by assigning specific values to the functions u(z), v(z), g(z) and the parameters $\alpha$, and $\gamma$. It is remarkable that the proof of the formula is so simple, that it is easier than the derivations usually found in the literature for series now obtainable from it. Examples include such well-known results as the gamma function representation of F(a,b; c; 1) and Dougall's formula. Other expansions appear to be new. (Received January 13, 1969.)

664-22. E. H. ROGERS, Rensselaer Polytechnic Institute, Troy, New York 12181. On the existence and approximation of nonlinear evolution.

Solutions of well-posed initial value problems are studied via mappings $S(\tau,t)$ carrying states at time $\tau$ to states at time t $(0 \leqq \tau \leqq t \leqq T)$ such that for $0 \leqq \tau \leqq \tau' \leqq t \leqq T$ (a) dom $S(\tau,t) \supseteq S(0,\tau)P$ where P = dom $S(0,\tau)$, (b) $S(\tau,\tau)$ is the identity on $S(0,\tau)P$, (c) $S(\tau,t) = S(\tau', t)S(\tau,\tau')$ on $S(0,\tau)P$. Every such evolution operator S is shown to be representable by a semigroup of operators. The semigroup formulation is used in consideration of evolution in a Banach space to prove that suitably defined stability and consistency properties of a sequence of operator semigroups $N_n$ implies their convergence to an operator semigroup M. Also, conditions of equistability and selfconsistency of $\{N_n\}$ are used to show the existence of an M. In an application Cauchy-Euler approximants are used to prove existence of solutions of the functional problem $y'(t) = \varphi(t,y)$, $0 \leqq t \leqq 1$, with y(t) prescribed when $-a \leqq t \leqq 0$. Here $y(t) \in F$, a Banach space, and the operator $\varphi$ maps a subset of $[0,1] \times X$ to F, where X is the space (sup norm) of continuous maps of $[-a, 1]$ to F. The demands on $\varphi$ are predictably continuity, boundedness and a Lipschitz condition with respect to y, plus $\varphi(t,y) = \varphi(t,z)$ whenever $y(s) = z(s)$, $s \leqq t$. (Received January 14, 1969.)

664-23. ROMAN SUSZKO, Stevens Institute of Technology, Hoboken, New Jersey 07030. Consistency of some non-Fregean theory.

Wittgenstein's ontology may be formalized as a theory of kind W (see Non-Fregean logic and theories, submitted to J. Symbolic Logic and Abstract 664-24 below) based on axioms: (A) the universe of situations S which the sentential variables run over is a complete atomic Boolean algebra with indiscrete topology, the quantifiers being generalized meet and join operations, (B) the states of affairs (SA is unary primitive nonlogical connective) are independent in S and such that $\bigvee pSAp = 1$ and every meet of certain SA and of complements of all other SA is an atom, (C) the meet of all facts $\bigwedge pp$, the real world, is an atom, and (D) there exist at least two and, hence, infinitely many SA. (I) A,B,C, non-D and SA1 follow from F: $\forall p(p \equiv 1 \vee p \equiv 0)$. (II) A,B,C,D is consistent. (Received January 14, 1969.)

664-24. STEPHEN L. BLOOM, Stevens Institute of Technology, Hoboken, New Jersey 07030.
A completeness theorem for theories of kind W.

A language L of kind W is 2-sorted, having both sentential variables (which are also formulas) and nominal (individual) variables. Besides the usual connectives, identity predicate and quantifiers, L has an identity connective $\equiv$ (if $\alpha$ and $\beta$ are formulas, so is $\alpha \equiv \beta$) which is not truth functional. L also has a symbol for the Bernays unifier operator. A consequence operation Cn is defined on L by the rule modus ponens and the logical axioms: standard axioms (for the propositional and quantificational calculus); identity and invariance axioms; unifier axioms. A theory of kind W is a triple $\langle L, Cn, \Phi \rangle$ where $\Phi$ is a set of sentences of L. Certain theories of kind W were studied by Suszko; see Abstract 664-23 above. The notion of an interpretation of L is defined. Completeness Theorem. A set $\Phi$ of sentences is consistent iff there is an interpretation in which $\Phi$ is true. Many corollaries follow. For example, Frege's claim that statements are names of one of two truth values, the true and the false, may be formalized in L by (F): $\forall p(p \equiv \underline{0} \vee p \equiv \underline{1})$, where p is a sentential variable. Theorem. Both (F) and its negation are consistent with the logical axioms of L. (Received January 14, 1969.)

664-25. CRAIG COMSTOCK, University of Michigan, Ann Arbor, Michigan 48104. On weighted averages at a jump discontinuity.

It is well known that, using the Dirichlet kernel, there is an integral transform of a function f(x) which, in general, converges to the average $[f(x^+) + f(x^-)]/2$. Recently Muldoon [Ph.D. Thesis, University of Alberta, 1966] showed how to obtain a weighted average $f(x^+)/3 + 2f(x^-)/3$. We show how to obtain weighted averages of the form $\alpha f(x^+) + \beta f(x^-)$ where $\alpha + \beta = 1$ and $\alpha = (1 + \cos \pi/m)^{-1}/2$ for integer $m > 2$. The kernel is related to various Bessel functions of order $1/m$ which have specific properties at both $\pm$ infinity. (Received January 17, 1969.)

664-26. JONATHAN L. GROSS, Princeton University, Princeton, New Jersey 08540.
The decomposition of 3-manifolds with several boundary components.

The manifolds considered here are connected, orientable, and compact, and the maps are piecewise linear. If M and M' are disjoint 3-manifolds with nonvacuous boundary, one forms a multidisk sum of M and M' by pasting some disks on bd(M), at most one on each component of bd(M), to some disks on bd(M'), at most one on each component of bd(M'). If bd(M) has r components and bd(M') has s components and $r \leq s$, then the number of distinct (up to homeomorphism) multidisk sums of M and M' may be as large as $\sum_{j=1}^{r} \binom{r}{j}\binom{s}{j}$. A 3-manifold P with nonvacuous boundary is called m-prime if whenever P is homeomorphic to any multidisk sum of two 3-manifolds, one of them is a

3-cell. Theorem 1. Every irreducible 3-manifold with nonvacuous boundary has an essentially unique decomposition into an iterated multidisk sum of m-prime 3-manifolds. Theorem 2. The problem of classifying the 3-manifolds with several boundary components reduces to the problem of classifying the m-prime 3-manifolds with connected boundary. (Received January 20, 1969.)

664-27. LEON H. SIBUL, Pennsylvania State University, Ordnance Research Laboratory, P.O.Box 30, State College, Pennsylvania 16801, and CRAIG COMSTOCK, University of Michigan, Ann Arbor, Michigan 48104. Ordinary matrix differential equations with stochastic coefficients.

A matrix differential equation (1) $\dot{y}(t,\omega) = a(t,\omega)y(t,\omega) + x(t,\omega)$ [$\omega \in \Omega$ on complete probability space $(\Omega,F,P)$ and $t \in T$] is studied. It is assumed that the coefficient matrix $a(t,\omega)$ is a sum of a deterministic matrix $\beta(t)$ and stochastic matrix $\alpha(t,\omega)$, and that the inverse operator for (2) $\dot{y}(t,\omega) = \beta(t)y(t,\omega)$ is known. Using these assumptions, (1) becomes a Volterra integral equation with stochastic kernel. This integral equation is solved by Neumann series expansion. A theorem giving sufficient conditions for almost sure convergence of the Neumann series expansion is proved. Expressions are found for the expectation and covariance matrix of $y(t,\omega)$ in terms of the Green's matrix of (2), statistics of the resolvent kernel of the Volterra integral equation, and appropriate statistics of the forcing vector $x(t,\omega)$. The resolvent kernel is a function of stochastic coefficient matrix $\alpha(t,\omega)$ and Green's matrix of differential equation (2). It is shown that, in general, it is incorrect to average (1) first and then solve for the average, as is done in hierarchy methods. The results of this paper have wide application in control system theory, electrical network theory and other areas of applied mathematics. (Received January 22, 1969.)

664-28. FRED S. ROBERTS and JOEL H. SPENCER, 1700 Main Street, Santa Monica, California 90406. A characterization of clique graphs.

Let G be a finite graph. A clique of G is a maximal complete subgraph. Let H be the graph whose vertices are the cliques of G and so that 2 distinct cliques are adjacent iff they intersect. Then we say H is the clique graph of G. Problem. When is a graph H a clique graph of some graph? This problem is partially solved in Hamelink [J. Combinatorial Theory 5 (1968), 192-197]. Let $\mathcal{K}$ be a family of complete subgraphs of H. $\mathcal{K}$ satisfies property $\mathcal{J}$ if whenever elements $L_1, L_2, \ldots, L_p$ in $\mathcal{K}$ pairwise intersect, there is a point in the total intersection. $\mathcal{K}$ satisfies property $\mathcal{J}_m$ if the above holds whenever $p = m$. Theorem 1. H is a clique graph iff there is a family $\mathcal{K}$ of complete subgraphs of H so that (1) if x and y are adjacent points of H, then $\{x,y\}$ is contained in an element of $\mathcal{K}$; and (2) $\mathcal{K}$ satisfies property $\mathcal{J}$. The clique number of H, $\omega(H)$, is the size of the largest clique. Theorem 2. If $\omega(H) \leq 3$, then H is a clique graph iff the family of all cliques of H satisfies property $\mathcal{J}_3$. One application of Theorem 1 is in the proof of reduction theorems of the form: H is a clique graph iff certain smaller or simpler graphs are clique graphs. (Received January 23, 1969.)

664-29. MARC W. KONVISSER, Battelle Memorial Institute, Columbus, Ohio 43201. Metabelian p-groups which contain a self-centralizing element. Preliminary report.

An element x of a group G is called self-centralizing in G if the only elements of G which commute with x are powers of x. Let G be a metabelian p-group, $p \neq 2$, M a normal subgroup of G, and the subsets $Y_i$ of M defined by $Y_0 = 1$ and $Y_i = \{m \mid m \in M \text{ and } [m,x] \in Y_{i-1}\}$. Under these assumptions we prove the following: Theorem 1. $Y_i$ is a subgroup, $Y_i \triangleleft Y_{i+1}$, and $Y_{i+1}/Y_i$ is cyclic for $i = 1,2,\ldots$ . Theorem 2. $|Y_{i+1}/Y_i| \leqq |Y_i/Y_{i-1}|$ and if equality holds, $Y_i = \langle [y_{i+1},x], Y_{i-1} \rangle$ where $Y_{i+1} = \langle y_{i+1}, Y_i \rangle$. We also prove Corollary 1. The number of generators of such a group G does not exceed the class of G. (Received January 30, 1969.)

664-30. DANIEL E. HUGHES, Washington University, St. Louis, Missouri 63130. Extreme eigenvalues of Toeplitz matrices. Preliminary report.

Let f be a real continuous function defined on $(-\pi,\pi)$. Suppose f attains its maximum at exactly one point which may be taken to be 0. Suppose further that at 0, $f(x) = f(0) - s|x|^w L(x) + o(|x|^w L(x))$ where $0 < w,s$ and L is an even positive function which is slowly varying near 0. Let $C_n(f) = (1/2\pi)\int_{-\pi}^{\pi} f(x)\exp[i(k-j)]dx$, $k, j = 0,\ldots, n$, be the nth section of the associated Toeplitz matrix. H. Widom (Trans. Amer. Math. Soc. 106 (1963)) obtained a formula for the asymptotic behavior of the extreme eigenvalues of these matrices. H. Kesten (J. Anal. Math 10 (1965)) obtained the analogous result where the function f attains its maximum at m distinct points and has the form described above at each of these points with $L \equiv 1$. The present paper gives a proof of Kesten's result in the general case which also differs from Kesten's in that it uses an extension of the perturbation theory described by I. I. Hirschman, Jr. (Pacific J. Math. vol. 14, No. 1). (Received January 24, 1969.)

664-31. MARTIN S. AIGNER, University of North Carolina, Chapel Hill, North Carolina 27514, and GEERT C. E. PRINS, Wayne State University, Detroit, Michigan 48202. Uniquely partially orderable graphs.

With a binary relation R on a finite set S one associates an undirected graph G(R) defined by (a) V(G) = S, (b) for all $u, v \in S$, $(u,v) \in E(G)$ if and only if uRv or vRu. Let R be a relation of a particular type $\alpha$, then we call G(R) unique w.r.t. relations of type $\alpha$ if R' of type $\alpha$ and G(R') = G(R) implies R' = R or $R' = \breve{R}$. Conforming to these definitions one defines (strict) partially orderable, uniquely partially orderable, semi-orderable, and uniquely semi-orderable (PO, UPO, SO, USO) graphs. The following two results are obtained: (1) A SO-graph is USO if and only if $G \cong K_{m,n}$ or $G^c$ (complement of G) is connected. (2) A sufficient condition for a PO-graph G to be UPO is that both G and $G^c$ be connected and that $G^c$ contain no isomorph of $K_{1,3}$ as a full subgraph. This second result can be extended slightly. (Received January 27, 1969.)

664-32. CHARLES FOX, Sir George Williams University, Montreal 107, Quebec, Canada. Some applications of the P operator.

The P operator was introduced by E. L. Post in 1930 in order to obtain a real inversion formula for the Laplace Transform. In this paper I show that the P operator can be used to solve a large class of integral equations. Some of these equations can be solved by Fourier Transform and other methods but there are cases in which the P method of solution appears to be the only one available. I also investigate some general properties of the P operator. (Received January 27, 1969.)

664-33. DONALD J. BROWN, Stevens Institute of Technology, Hoboken, New Jersey 07030. Completeness theorems for the propositional calculus of kind W. Preliminary report.

A necessary and sufficient condition (*) is given for a propositional calculus to have a completeness theorem, where the family of interpretations in the completeness theorem is a class of similar logical matrices. Conversely it is shown that for every class of similar logical matrices there exists a propositional calculus which is complete with respect to the given class. The propositional calculus of kind W is shown to satisfy condition (*) and a family of logical matrices is given with respect to which this calculus is complete. This family of interpretations is then compared with the family given by Bloom in A completeness theorem for theories of kind W (to appear). (Received January 27, 1969.)

664-34. JOHN N. McDONALD, Rutgers University, New Brunswick, New Jersey 08903. On certain compact convex sets which are similar to Choquet simplexes. Preliminary report.

An investigation is made of compact convex sets satisfying the following: Definition. A compact convex subset X of a locally convex, $T_2$ vector space over the reals is said to have the equal support property if any two maximal measures on X which have the same resultant have the same closed support. Example. Let B be a function algebra on the compact metric space Y $\ni$ if $m_1$ and $m_2$ are regular Borel measures with supports in the Silov boundary and with $m_1|B = m_2|B$, then $m_1 = m_2$. Suppose A is a subalgebra of B of the form, constants + ideal in B, which separates the points of Y. Then, if (Re A)* denotes the weak* dual of Re A, the set $\{h \in (\text{Re } A)^* \mid h \geqq 0 \text{ and } h(1) = 1\}$ has the equal support property. (Received January 29, 1969.)

664-35. BERNARD SHIFFMAN, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. On the continuation of analytic sets. Preliminary report.

Let S denote the unit circle in the complex plane, and let $T^n = S \times \ldots \times S \subset C^n$. Theorem 1. Let $W \subset C^n$ be a neighborhood of $T^n$, and let A be a (closed) complex analytic curve in $W - T^n$ such that $(\bar{z}_1^{-1}, \ldots, \bar{z}_n^{-1}) \in A$ whenever $z \in A$. Then $\mathrm{Cl}(A) \cap W$ is a complex analytic curve in W. Corollary. If f and g are meromorphic functions on a connected Riemann surface X such that $|f(x)|$ and $|g(x)|$ both approach 1 as $x \to \partial X$, then f and g are algebraically dependent. Another corollary of Theorem 1 is the following result of H. Tornehave (1964): If A is a complex analytic curve in the unit n-disk such that $\mathrm{Cl}(A) - A \subset T^n$, then A is algebraic. Theorem 2. Let W be a domain in $C^n$, and let A be a pure k-dimensional complex analytic subvariety of $W - R^n$, $k \geqq 1$. If $k = 1$, assume that $(\bar{z}_1, \ldots, \bar{z}_n) \in A$

whenever $z \in A$. If A has finite (2 k-dimensional) volume, then $Cl(A) \cap W$ is a complex analytic subvariety of W. The proofs of these theorems use methods of the author [Michigan Math. J. 15 (1968), 111-120] and of E. Bishop [Michigan Math. J. 11 (1964), 289-304]. (Received January 30, 1969.)

664-36. RAYMOND Y. WONG, University of California, Santa Barbara, California 93106. On stable homeomorphisms of infinite-dimensional manifolds.

Let B be a Banach space. A connected metric space X is an B-manifold if each point of X has a neighborhood homeomorphic to B. Let $\ell_2$ denote the separable Hilbert space of all square summable real sequences and let Q denote the Hilbert cube $[0,1]^{\aleph_0}$ under metric $d(x,y) = \sum_{i=1}^{\infty} |x_i - y_i|/2^i$. A connected metric space is a Q-manifold if each point of X has a (closed) neighborhood homeomorphic to Q. An homeomorphism of X into X (space homeomorphism) is stable (in the sense of Brown-Gluck) if it is the finite composition of space homeomorphisms each of which is the identity on some nonvoid open subset of X. Theorem. If X is either an $\ell_2$-manifold or an Q-manifold, then each homeomorphism of X onto X is stable. Previously R. D. Anderson [Michigan J. Math 14 (1967)] has shown that each homeomorphism of Q onto Q is stable and Wong [Trans. Amer. Math. Soc. 128 (1967), 148-154] has shown that each homeomorphism of $\ell_2$ onto $\ell_2$ is stable. Thus our theorem generalizes both Anderson's and Wong's results. In the case of an Q-manifold, we also give a different but simpler argument for Anderson's Theorem. (Received January 20, 1969.)

664-37. LAWRENCE J. CORWIN, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. Gaussian measures and a "Functional Equation" for measures.

Let G be a locally compact Abelian group such that doubling every element is an automorphism, and let $\xi$ be the automorphism of $G \times G$ which takes $(x,y)$ into $(x + y, x - y)$. Call a measure $\mu$ on G Gaussian iff $\exists$ a measure $\nu$ on G such that $\mu \times \mu(E) = \nu \times \nu(\xi(E))$ for all Borel sets $E \subseteq G \times G$. In the case where $\mu$ is a finite positive measure on $\mathbb{R}$, the Gaussian measures are exactly the Gaussian probability distributions with mean 0. A characterization of all Gaussian measures which are absolutely continuous (with respect to Haar measure) is given, and all Gaussian measures on vector groups and on compact groups are determined. (Gaussian measures on 2-adic vector spaces were considered by Mumford, Invent. Math. 3 (1967), 75-135.) (Received January 31, 1969.)

664-38. YONG WOON KIM, Wisconsin State University, Eau Claire, Wisconsin 54701. Partial order in bitopological spaces.

The author defined p-compactness (p- stands for pairwise) and obtained a few standard results in bitopological spaces (it appears in Debrecen). Extending the above results, one defines partial order in a bitopological space and generalizes a few classical theorems in bitopological sense. Definition. A partial order on the family of all bitopological spaces is defined as follows $(X,L_1,L_2) \geq (X,S_1,S_2)$ iff $L_1 \supset S_1$ and $L_2 \supset S_2$. Theorem. If $(X,L_1,L_2)$ is p-Hausdorff and p-compact then for $L_i' \not\subset L_i \not\subset L_i''$ (1) $(X,L_i',L_j)$ is not p-Hausdorff and (2) $(X,L_i'',L_j)$ is not p-compact, $i,j = 1,2$, $i \neq j$. Theorem. If $f : (X,L_1,L_2) \to (Y,S_1,S_2)$ is one to one onto, p-continuous, Y is p-Hausdorff and X is p-compact, then f is p-homeomorphism. (Received January 31, 1969.)

664-39. CLIFFORD O. BLOOM, University of Michigan, Ann Arbor, Michigan 48104. On the validity of the geometrical theory of diffraction by star shaped cylinders.

Let $U_C(x,x';k)$ be the solution of the following scattering problem: $(\Delta + k^2)U = \delta(x,x')$, $x, x' \in D$; $U = 0$, $x \in C$; $\rho^{1/2}|\partial U/\partial\rho - ikU| \to 0$ as $|x| \to \infty$, $y \in D$, $\rho = |x - y|$. Assume that (i) D is the exterior of a star shaped curve C, (ii) only two points p and q of C lie on the boundary between the "illuminated" and "dark" portions of C, and that (iii) C coincides with a circle $C_0$ near p and q. Making use of a priori estimates obtained by Morawetz & Ludwig (Comm. Pure Appl. Math. 21 (1968), 187-203) we prove that $U_C(x,x';k) = U_{C_0}(x,x';k)(1 + O(\exp[-\sigma k^{1/3}]))$ as $k \to \infty$, uniformly with respect to x, $x \in S - R$. Here $\sigma$ is positive, independent of k and of x. S is the "deep shadow" of C, and R is a certain closed, bounded subset of S, determined by the geometry of C in S. Our result confirms J. B. Keller's geometrical theory of diffraction (Geometrical theory of diffraction, J. Opt. Soc. Amer. 52 (1962), 116-130), and appears to be the first rigorous asymptotic solution of the reduced wave equation in the shadow of a nonconvex body. (Received January 31, 1969.)

664-40. TADASKI NAGANO, University of Notre Dame, Notre Dame, Indiana 46566. The Thom class of a vector bundle and relevant localization theorems in differential geometry.

The Thom class of an oriented differentiable vector bundle E is expressed with a differential form U in the sense of the relative de Rham theory. Since U has the support contained in a "small" neighborhood of the zero-section, for a given smooth section v of E the Euler class of E can be expressed as the sum of "the indices" of v times the Euler class of the connected components of the vanishing points set which is assumed to be a smooth submanifold and on which v is assumed to be "nondegenerate". The property above of U allows several applications. (Received February 3, 1969.)

664-41. HALINA SWIATAK, McGill University, Montréal, Québec, Canada. On the regularity of the solutions of a class of functional equations.

The problem of regularity of the continuous and locally integrable solutions of the functional equations (1) $\sum_{i=1}^{k} a_i(x,t)f(\varphi_i(x,t)) = b(x,t)$, wherein $x \in R^n$, $t \in \Omega \subset R^r$, $\varphi_i(x,t) = (\varphi_{i1}(x,t),\ldots,\varphi_{in}(x,t))$, $n > 1$, $r \geqq 1$, is considered. If the functions $a_i(x,t)$, $b(x,t)$, and $\varphi_i(x,t)$ satisfy suitable regularity conditions and if there exist an $\alpha \in \Omega$ and a multi-index p such that $\varphi_i(x,\alpha) \equiv x$ for $i = 1,\ldots,k$ and the distributional differential equation $D_t^p(\sum_{i=1}^{k} a_i(x,t)T(\varphi_i(x,t))) = 0$ $(T \in \mathcal{D}')$ is of constant strength for $t = \alpha$, hypoelliptic for some $x_0$, then every continuous solution f of equation (1) has to be a function of class $C^\infty$. Some additional assumptions allow to prove that all the locally integrable solutions f are equal almost everywhere to some solutions of class $C^\infty$ (or that they are functions of class $C^\infty$). (Received February 3, 1969.)

664-42. ABDUL JABBAR JERRI, Clarkson College, Potsdam, New York 13676. Eigenfunctions corresponding to the band pass kernel with large center frequency.

It is known that the eigenfunctions of the finite integral equation with band limited difference kernel are the prolate spheroidal wave functions. When the band pass difference kernel was considered, no such well-known functions were found, except for the limiting case of zero band width. In this report we show that for large values of the center frequency the eigenfunctions corresponding to the band pass kernel are related to the prolate spheroidal functions. (Received February 4, 1969.)

664-43. EDWARD P. SHAUGHNESSY, Lafayette College, Easton, Pennsylvania 18042.
The Mathieu groups as automorphism groups of linear codes.

Let G be the automorphism group of the extended $(\iota + 1, (\iota + 1)/2)$ quadratic residue code over GF(q) where $\iota$ is a prime of the form $2m + 1$, m prime. It is well known that G contains $PSL_2(\iota)$. When G properly contains $PSL_2(\iota)$, the generators of the stability group of a point can be given. Utilizing this result, one shows that $M_{12}$ appears as the automorphism group of the (12,6) code over GF(3), and $M_{24}$ appears as the automorphism group of the (24,12) code over GF(2). The other Mathieu groups are the stability groups of points. (Received February 3, 1969.)

664-44. SRINIVASA RAMANUJAM, University of Washington, Seattle, Washington 98105.
Application of Morse theory to some homogeneous spaces.

One method of studying the topology of the symmetric spaces $U(n)/O(n)$, $Sp(n)/U(n)$, $SO(2n)/U(n)$, $U(2n)/Sp(n)$, the real, complex and quaternionic Grassmann manifolds is to use Frankel's trace function. Another approach is to think of the symmetric spaces as imbedded in Euclidean spaces and to use the length function. Thus, R. Bott [Morse theory, Bonn University Lecture notes], studies the complex flag manifold. This length function can be used on several homogeneous spaces (the symmetric spaces mentioned above are special cases of these homogeneous spaces). With the help of P. A. Smith's theory of periodic maps it can be shown that the Morse inequalities become equalities. (Received February 3, 1969.)

664-45. CHARLES H. APPLEBAUM, Rutgers University, New Brunswick, New Jersey 08903.
$\omega$-homomorphisms.

Let $\epsilon = \{0,1,2,\ldots\}$. An $\omega$-group G is a pair $\langle \alpha, p \rangle$, where (i) $\alpha$ is a subset of $\epsilon$, (ii) p is a group operation on $\alpha \times \alpha$ which can be extended to a partial recursive function of two variables, (iii) the function which maps each element of $\alpha$ to its inverse under p has a partial recursive extension. The problem of finding a natural analogue of a group homomorphism was posed by Hassett (Some theorems on regressive isols and isolic groups, Doctoral Thesis, Rutgers University, 1966, unpublished). It is proposed that the following definition is such a suitable analogue. For two $\omega$-groups $G_1 = \langle \alpha_1, p_1 \rangle$ and $G_2 = \langle \alpha_2, p_2 \rangle$, $G_1$ is $\omega$-homomorphic to $G_2$, if there exists a homomorphism $\varphi$ from $G_1$ onto $G_2$ such that (a) $\varphi$ has a partial recursive extension, and (b) there is a partial recursive one-to-one function p(x) such that $\alpha_2 \subset \delta p$, and $p(x_2) \in \varphi^{-1}(x_2)$, for $x_2 \in \alpha_2$. (Received February 3, 1969.)

664-46. THOMAS L. BARTLOW, Villanova University, Villanova, Pennsylvania 19085.
Power-associative quasigroups.

If Q is a quasigroup and $x \in Q$ define $x^0$ such that $xx^0 = x$, $x^n = xx^{n-1}$, $x^{-1}$ such that

$xx^{-1} = x^0$, $x^{-n} = (x^n)^{-1}$ where $n \geqq 0$. Definition. A quasigroup is power-associative if each of its elements generates an associative subquasigroup. Theorem. A quasigroup is power-associative iff it satisfies either of the laws $(x^m x^n)x^p = x^m(x^n x^p)$ or $x^m n^n = x^{m+n}$ for all integers m,n,p. For certain classes of quasigroups it suffices for the exponents to be nonnegative. Theorem. Every quasigroup with more than two elements has an isotope which is not power-associative. Definition. A loop is extensively power-associative if each of its loop isotopes is power-associative. Theorem. Extensively power-associative loops can be characterized by a single law. Theorem. In a weak inverse extensively power-associative loop the alternative laws are equivalent. Other conditions satisfied by extensively power-associative loops are given. The arguments involve manipulation of laws. (Received February 4, 1969.)

664-47. LOKENATH DEBNATH, East Carolina University, Greenville, North Carolina 27834. On three-dimensional transient wave motions on a running stream.

This paper considers the three-dimensional transient wave motions in a fluid produced by a moving oscillatory disturbance of frequency $\omega$ acting on the free surface of the fluid. The problem is treated as an initial value problem and is solved by the joint Laplace-Fourier transform method combined with the asymptotic treatment for the double Fourier inversion integrals. The explicit representation of the steady state as well as the transient solution of the problem is presented. It is shown that the ultimate steady state is set up in the limit. A brief discussion on the principal behaviour of the wave motions is made. (Received February 3, 1969.)

664-48. LAWRENCE D. KUGLER, University of Michigan, Flint, Michigan 48503. Nonstandard almost periodic functions on a group.

The notion of almost periodicity of a complex-valued function on a group G was defined by von Neumann in terms of conditional compactness in the algebra of bounded functions on G. The following is a nonstandard characterization of almost periodicity. Theorem. Let $E(f) = \{t \in {}^*G \mid$ for all $x, y \in {}^*G$, $f(xty) - f(xy)$ is infinitesimal$\}$ where $^*G$ is an enlargement of G in the sense of A. Robinson. Then f is almost periodic if and only if for every infinite positive integer N there exist $r_1,\ldots, r_N \in {}^*G$ such that $\bigcup_{i=1}^N r_i E(f) = \bigcup_{i=1}^N E(f) r_i = {}^*G$. Under the topology induced on a natural way by f on $b_f G = {}^*G/E(f)$, $b_f G$ is a compact topological group. Theorem. The algebra of all complex-valued continuous functions on $b_f G$ is isometrically isomorphic to the algebra $A_f = \{ g \mid g$ is almost periodic on G and $E(g) \supset E(f)\}$. (Received February 3, 1969.)

664-49. VICTOR LOVASS-NAGY and DAVID L. POWERS, Clarkson College of Technology, Potsdam, New York 13676. A note on the solution of matrix differential equations by partitioning.

Consider the differential equation (*) $x + Ax = g(t)$ where $A = [A_{ij}]$ and $A_{ij}$ are matrices of complex numbers (i,j = 1,2). It is known that if all $A_{ij}$ are square matrices expressible as polynomials in the same diagonalizable matrix, the solution of (*) can be obtained in terms of the eigenvalues and eigenvectors of this matrix. The authors of this paper discuss the cases where (a) $A_{ij}$ are all commutative square matrices but not expressible as polynomials in the same diagonalizable matrix; (b) $A_{ij}$ are all square matrices but they do not all commute; (c) some of the submatrices are not square. Solutions of (*) are obtained in terms of the submatrices $A_{ij}$ by generalization of Sylvester's Theorem. (Received February 4, 1969.)

664-50. THEODORE J. BENAC, U. S. Naval Academy, Annapolis, Maryland 21402, and SISTER CLAIRE ARCHAMBAULT, 801 Varnum Street, N. E., Washington, D. C. 20017. An abstract version of the Krull principal ideal theorem. Preliminary report.

In 1962 R. P. Dilworth introduced Noether lattices and proved an abstract version of the Krull Principal Ideal Theorem [Abstract commutative ideal theory, Pacific J. Math. 12 (1962), 481-498]. In the present study, by the suitable definition of right principal elements and right Noetherian reticulated semigroups, the conditions of modularity and commutativity used by Dilworth are removed and the main result holds. Principal Element Theorem. Let P be a minimal prime containing the right (join) principal element M of a local right Noetherian reticulated semigroup. Then the rank of P is at most one. (Received February 3, 1969.)

664-51. PAUL CHERENACK, Indiana University, Bloomington, Indiana 47401. A homotopy group of algebraic arcs found by smoothed approximations.

The notion of homotopy group on a real algebraic variety where the paths and homotopy satisfy certain analytic conditions is developed. Let V be a real algebraic variety of dimension $\geqq 3$ and $P \in V$. The basic paths are taken in $T(V,P) = \{f | f : I \to V$ analytic and nonsingular except possibly at P, and algebraic$\}$. The homotopy is defined to be fully analytic and algebraic, not piecewise analytic. The homotopy H is required not to have bad singularities, called 0/1 dimensional singularities. The main difficulty, which is solved by Wallace [Canadian J. Math., 1958] in the case of curves, is the transitivity of the homotopy relation. The cylinder on the cone, a variety in $E^4$, illustrates the nature of the homotopy group thus defined. The group has a torsion part. (Received December 30, 1968.)

664-52. ADI BEN-ISRAEL, Northwestern University, Evanston, Illinois 60201, and KENNETH O. KORTANEK, Cornell University, Ithaca, New York 14850. Asymptotic duality over closed convex sets.

For any set K in $R^n$ and any $\alpha \in R$ let $K^*_\alpha = \{y \in R^n : (y,K) \geqq \alpha\}$. The following pair of problems (I.C) $\sup (c,x)$ s.t. $Ax = b$, $x \in C$, (II.$C^*_0$) $\inf (b,y)$ s.t. $A^T y - c \in C^*_0$, with given $A \in R^{m\times n}$, $b \in R^m$, $c \in R^n$ and a closed convex cone C in $R^n$, were given an asymptotic duality theory in [A. Ben-Israel, A. Charnes and K. O. Kortanek, Bull. Amer. Math. Soc. (to appear)]. This theory is extended here to problem pairs of the type (I) $\sup (c,x)$ s.t. $Ax = b$, $x \in K$, (II) $\inf \{(b,y) + \eta\}$ s.t. $A^T y - c \in \eta K^*_{-1}$, $\eta \geqq 0$, where K is a closed convex set in $R^n$ containing 0 and A, b, c as above. Applications to convex programming and to approximation problems are given. (Received February 5, 1969.)

664-53. SHIH-LIANG WEN, Ohio University, Athens, Ohio 45701. Effects of viscosity on long waves.

The approach of the present study begins with the Navier-Stokes equations. A system of equations governing the motion of nonlinear long waves is derived by an expansion scheme similar to

Friedrichs'. The equations are integrated with respect to the vertical coordinate. The parabolic system is thus reduced to a hyperbolic system for vertically averaged values. The linearized problem for a flat bottom is solved by Fourier's method. Nonlinear solutions are obtained by applying the theory of relatively undistorted waves. It is found that shock formation is delayed by the viscous effect. Various conditions are discussed in determining the viscous, nonlinear and radial decay effects on the solution for a shockless expansion wave-front propagating over large distances. (Received February 5, 1969.)

664-54. MOSES E. COHEN, Michigan Technological University, Houghton, Michigan 49931. Additional properties of a new unified class of polynomials.

In A new class of polynomials relevant to electron collision problems [to appear in Math. Z. (2) 108], we presented $\theta_n(x)$ and its properties. In (Abstract 663-550, these *Notices* 16 (1969), 247), further special cases and properties were reported. Here, some new special cases of $\theta_n(x)$ are recorded, including $H^{(1)}_{n+1/2}(x)$, $H^{(2)}_{n+1/2}(x)$, $H^{(1)}_{n+1/2}(x)H^{(1)}_{n+1/2}(-x)$, $H^{(2)}_{n+1/2}(x)H^{(2)}_{n+1/2}(-x)$ the Hankel functions of the first and second kind, $W_{0,n+1/2}(x)$, $W_{0,n+1/2}(x)W_{0,n+1/2}(-x)$ the Whittaker function, $K_{n+1/2}(x)$, $K_{n+1/2}(x)K_{n+1/2}(-x)$ the modified Bessel function of the third kind and $Y_{n+1/2}(x)$ Neuman's polynomial. A multiplication formula is given whose special cases yield new results involving Jacobi polynomials. A differential equation for $\theta_n(x)$ is presented. Integrals containing Jacobi polynomials are also evaluated. A result of interest is a fourth order linear differential equation satisfied by $P_n^{(\alpha,\beta)}(x)P_n^{(\beta,\alpha)}(x)$ and a third order equation satisfied by $\{C_n^v(x)\}^2$, the product of Jacobi and the square of the Gegenbauer polynomial respectively. (Received January 31, 1969.)

664-55. ETHAN D. BOLKER, Bryn Mawr College, Bryn Mawr, Pennsylvania 19010. Convex polyhedra with a center of symmetry.

Theorem 1 (Suggested by G. D. Chakerian). A convex, three-dimensional polyhedron K is centrally symmetric (c.s.) if and only if sums of pairs of opposite faces (intersections of K with pairs of parallel support planes) are c.s. This generalizes Alexandroff's Theorem 2 on zonohedra: A convex polyhedron with c.s. faces is c.s. The proof of Theorem 1 exploits an idea Shephard used in Canad. J. Math. 19 (1967), 1206-1213, to reprove Theorem 2. Higher-dimensional analogues of Theorem 2 are well known (Shephard, op. cit.). Similar analogues of Theorem 1 are conjectured. (Received February 5, 1969.)

664-56. BOSHRA H. MAKAR, Saint Peter's College, Jersey City, New Jersey 07306. Representations by basic sets in the space of square summable functions.

If $\{z^{(n)}\}$ is a complete orthonormal system in $L^2$, then the sequence $\{p_n\}$ where $p_n = \Sigma_i p_{ni} z^{(i)}$ is said to be basic if the matrix $[p_{ni}]$ is row-finite and admits a unique row-finite reciprocal $[q_{ni}]$. The author shows that the introduction of the function $F(n) = \max_{i,j} \| \Sigma_{r=i}^{j} q_{nr} p_r \|$ leads to a representation theory for elements of $L^2$ in terms of such $p_n$. The author shows also how some theorems about representations in $L^2$ can yield corresponding theorems about representations of analytic functions of a complex variable z through the use of some inequalities which he gives. In the terminology of Whittaker's theory (J. M. Whittaker, Sur les series de base de polynômes quelconques, Gauthier-Villars, Paris, 1949) one typical result is the following: Theorem. If $[p_{ni}]$ is such that $D_n^{1/n} \to 1$ as $n \to \infty$, then $\{p_n(z)\}$ is effective on the unit circle if and only if $\{p_n\}$ as a basic set in $L^2$ is effective for every element f in $L^2$ for which $\limsup_{n\to\infty} |(f, z^{(n)})|^{1/n} < 1$. (Received February 5, 1969.)

664-57. DAVID K. COHOON, Bell Telephone Laboratories, Whippany, New Jersey 07981. A global uniqueness theorem for partial differential operators.

By using the ideas of Hörmander the author was able to characterize the systems L of linear partial differential operators in N dependent and n independent variables with constant coefficients corresponding to which there exists a nontrivial u in $C^{\infty}(\mathcal{R}^n, \mathcal{C}^N)$ such that $Lu(x) = \underline{0}$ for all x in $\mathcal{R}^n$ and such that u(x) vanishes outside of a prescribed open prism V with bounded cross section. In particular if the axis of symmetry of V is parallel to the $x_n$-axis, then L has the above mentioned property if and only if det(L) is given by $\det(L) = bD_n^m + \sum_{|\alpha|< m} a_\alpha D^\alpha$, where b is a nonzero complex number and m is positive. (Received February 5, 1969.)

664-58. HARI M. SRIVASTAVA, West Virginia University, Morgantown, West Virginia 26506. A note on certain dual series equations involving Laguerre polynomials. Preliminary report.

In a recent paper [H. M. Srivastava, Dual series relations involving generalized Laguerre polynomials (in course of publication)] the author has investigated a solution of the dual equations (1) $\sum_{n=0}^{\infty}\{A_n/\Gamma(\alpha + n + 1)\}L_n^{(\nu)}(x) = f(x)$, $0 \leq x < y$, (2) $\sum_{n=0}^{\infty}\{A_n/\Gamma(\beta + n + 1)\}L_n^{(\sigma)}(x) = g(x)$, $y < x < \infty$; where $L_n^{(\alpha)}(x)$ is the Laguerre polynomial, f(x) and g(x) are prescribed functions of x, and in general, $\alpha, \beta, \nu, \sigma > -1$. By considering separately the equations when (i) $g(x) \equiv 0$, (ii) $f(x) \equiv 0$, and expressing the coefficients $\{A_n\}$ in terms of a sequence of integrals involving an unknown function h, it was shown that the problem in each case can be reduced to that of solving an Abel integral equation for h. The object of the present note is to exhibit the fact that the multiplying factor technique, developed by B. Noble [Proc. Cambridge Philos. Soc. 59 (1963), 363-371] for solving dual equations involving series of Jacobi polynomials and used subsequently by John S. Lowndes [Pacific J. Math. 25 (1968), 123-127] and Richard Askey [J. Math. Anal. Appl. 24 (1968), 677-685] for solving essentially the same special case of the dual equations (1) and (2), applies well to the more general problem posed here. (Received February 7, 1969.)

664-59. DEAN B. PRIEST and RUSSELL A. STOKES, University of Mississippi, University, Mississippi 38677. Concerning the extended Stieltjes mean $\sigma$-integral and a mean integral of a function with respect to a function pair.

The mean integral of a function with respect to a function pair, A, I, and M are as in Abstract 69T-B18, these *Notices* 16 (1969), 319. Let $m(\sigma)\int_A f\, d\mu$ denote the natural extension of the Stieltjes mean $\sigma$-integral of f with respect to $\mu$ on A as considered by Porcelli (Illinois J. Math. 2 (1958), 124-128) and others. Theorem 1. If f is a quasicontinuous function from A, each of g and h is a function of bounded variation in Hardy's sense in M and g[m,I] and h[I,n] are continuous for (m,n) in A, then there exist functions $\nu$ and $\mu$ from A of bounded variation in Vitali's sense such that $m - \int_A f\, dg\, dh = m(\sigma)\int_A f\mu\, d\nu - m(\sigma)\int_A f\nu\, d\mu$. Theorem 2. If f is a quasicontinuous from A, g (resp. h) is of bounded variation in Hardy's sense from A, g[I,k] (resp. h[I,k]) is continuous for each k in [b,d] and h (resp. g) is of bounded variation from [a,c], then there exist functions $\nu$ and $\mu$ from A of bounded variation in Vitali's sense such that m $-\int_A f\, dg\, dh = m(\sigma)\int_A f\, d\mu = m(\sigma)\int_A f\, d\mu$. Similar conclusions hold for the interchange of g and h in the former result and of I and k and [a,c] and [b,d] respectively in the latter. An integration by parts type formula then follows for m $-\int_A f\, dg\, dh$. (Received February 7, 1969.)

664-60. D. S. COCHRAN, Princeton University, Princeton, New Jersey 08540, and RICHARD H. CROWELL, Dartmouth College, Hanover, New Hampshire 03755. $H_2(G')$ for tamely embedded graphs.

For any group G, denote by $H_q(G)$ the qth homology group of G with integer coefficients and let G' be the commutator subgroup. Let P be any tamely embedded graph in $S^3$, and set $G = \pi_1(S^3 - P)$. Then $P = P_1 \cup \ldots \cup P_m$ (disjoint), where each $P_i$, i = 1,..., m, is a nonsplittable graph and m is minimal. For each i = 1,..., m, let $\mu_i$ be the number of components and $r_i$ the 1-dimensional Betti number of $P_i$. Also let $E^i_q$ denote the qth elementary ideal of $G_i = \pi_1(S^3 - P_i)$. Then, Theorem. $H_2(G') = 0$ iff $E^i_{r_i - \mu_i + 1} \neq 0$ for all i = 1,..., m. It is of independent interest and follows immediately that Corollary. If L is a nonsplittable link and $G = \pi_1(S^3 - L)$, then $H_2(G') = 0$ iff $\Delta \neq 0$ where $\Delta$ is the Alexander polynomial of L. For a knot (link with one component), $\Delta$ is never zero and the fact that $H_2(G') = 0$ is not new. (Received February 7, 1969.)

664-61. PRESTON C. HAMMER, Pennsylvania State University, University Park, Pennsylvania 16802. Extended topology: Continuity II.

A set of five axioms may be used to characterize a Kuratowski closure function among functions mapping the power set of a space into itself. In this paper, it is shown that a relevant theory of continuity can be established when all these axioms are dropped leaving arbitrary set-valued functions. The resulting continuous mappings may be called implication preserving maps. An amazingly detailed theory is presented displaying topological continuity as a comparatively weak form of continuity. This paper takes continuity outside its realm of general neighborhood spaces but is insufficiently general for some applications. (Received February 7, 1969.)

664-62. J. J. DORNING, B. NICOLAENKO, and JAMES K. THURBER, Brookhaven National Laboratory, Upton, L.I., New York 11973. The asymptotic behavior of the special function $\mu(x,\beta,\alpha)$.

By means of a generalization of an integral identity due to Ramanujan and an application of an important result of W. B. Ford on the asymptotic behavior of functions of exponential type defined by a Maclaurin expansion, we obtain the asymptotic behavior of $\mu(x,\beta,\alpha) = \int_0^\infty x^{\alpha+t} t^\beta / \Gamma(\beta+1)\Gamma(\alpha+t+1)\, dt$ (see Bateman Manuscript Project [Erdélyi et al, Higher transcendental functions, vol. III, McGraw-Hill, New York, 1955, pp. 217-224]) for complex values of x, $\beta$ and $\alpha$. This is an extension of earlier results of the authors on the special functions $\nu(x,\alpha)$ and $\nu(x)$ where we corrected some serious errors in the Bateman manuscript (Abstract 663-512, these *Notices* 16 (1969), 235). (Received February 7, 1969.)

664-63. J. M. GANDHI, York University, Toronto, Ontario, Canada. On Fermat's Last Theorem

Let (1) $x^p + y^p = z^p$ where p is a prime > 2 and $z \equiv 0 \pmod p$ (called Case II), x,y,z are relatively prime to each other. We prove the following: If (1) has integral solution in Case II then for each of the following cases (A) for every factor r of x or y, (B) for every factor r of x - y, (C) for every factor r of x + y other than p, it is necessary that $r^{p-1} \equiv 1 \pmod{p^2}$. From these theorems it immediately follows that if (1) has integral solution in Case II then $2^{p-1} \equiv 1 \pmod{p^2}$ and $3^{p-1} \equiv 1 \pmod{p^2}$. Three different proofs are given for those theorems. Attempts are made to derive the following: If (1) has integral solution in Case II then $2^{p-1} \equiv 1 \pmod{p^{p+1}}$, which is impossible thus implying the impossibility of Fermat's Last Theorem in Case II. (Received February 11, 1969.)

664-64. ALAN H. SCHOEN, NASA/Electronics Research Center, Cambridge, Massachusetts 02139. A fifth intersection-free infinite periodic minimal surface (IPMS) of cubic symmetry. Preliminary report.

Of the countably infinite number of IPMS with a cubic Bravais lattice, three with no self-intersections have been known [(i) H. A. Schwarz, Gesammelte Werke, Vol. I, 1890; (ii) E. R. Neovius, Univ. of Helsingfors, 1883]. A fourth [(iii) Abstract 658-30, these *Notices* 15 (1968), 727], containing no straight lines, is associate to the two [adjoint] Schwarz surfaces. A fifth example, and also the Neovius surface, may be obtained as "complements" to the two Schwarz surfaces, respectively, as follows: embed, in either Schwarz surface, that pair of dual regular maps [(iii)] whose edges are symmetry axes of the surface. From the edges of every intersecting pair of Petrie polygons [(iv) H. S. M. Coxeter, Regular polytopes] having a common symmetry plane $\pi$, construct two congruent skew polygons P and P', separated by $\pi$, with alternate vertices at edge midpoints and vertices, respectively, of the two Petrie polygons. Span P and P' by minimal surfaces M and M'. The complementary IPMS $\equiv \bigcup [M,M']$. The Neovius surface is found to have no associate surface free of self-intersections. The new surface probably has neither a locally finite adjoint surface nor any intersection-free associate surface. Each Schwarz surface belongs to the same space group as its complement. (Received February 11, 1969.)

664-65. JAMES R. BROWN, Oregon State University, Corvallis, Oregon 97330. Inverse limits, entropy, and weak isomorphism for discrete dynamical systems.

By a discrete dynamical system is meant a quadruple $\Phi = (X, \mathcal{B}, \mu, \varphi)$, where $(X, \mathcal{B}, \mu)$ is a normalized measure space and $\varphi : X \to X$ is a measure-preserving transformation. The entropy of the system $\Phi$ is denoted $h(\Phi)$. The inverse limit $\Phi = \text{inv lim } (\Phi_n, \psi_n)$ is defined categorically, where $\Phi_n$ is a dynamical system and $\psi_n : \Phi_{n+1} \to \Phi_n$ is a measure-preserving transformation for each n = 1,2,... . The inverse limit is determined uniquely up to isomorphism, and always exists for Lebesgue systems. We have $h(\Phi) = \lim h(\Phi_n)$, and $\Phi$ is (1) ergodic, (2) mixing (of any order), (3) has zero entropy, or (4) has completely positive entropy iff each $\Phi_n$ has the same property. Applications include systems with quasi-discrete spectrum [L. M. Abramov, Izv. Akad. Nauk SSSR, Ser. Mat. 26 (1962), 513-530], quasi-periodic group automorphisms [T. Seethoff, Ph.D. Dissertation, Oregon State Univ., 1968], and the natural extensions of V. A. Rohlin [Izv. Akad. Nauk SSSR, Ser. Mat. 25 (1961), 499-530]. A variation of Rohlin's construction is introduced to "lift" a weak isomorphism of two dynamical systems $\Phi_1$ and $\Phi_2$ to an isomorphism of systems $\hat{\Phi}_1$ and $\hat{\Phi}_2$, where $\Phi_i$ is a factor of $\hat{\Phi}_i$ with the same entropy (i = 1,2). (Received February 7, 1969.)

664-66. JOSEPH HOROWITZ, University of Toledo, Toledo, Ohio 43606. Semilinear Markov processes.

A strong Markov process $X = (\Omega, x_t, P_x)$ with state space $E = [0,a)$, where $a \leq \infty$, is semilinear if, for each $\omega \in \Omega$, there is a closed set $Z(\omega) \subset R$ and that $Z(\omega) \cap (-\infty,0]$ consists of exactly one point, and, for $t \geqq 0$, $x_t(\omega) = t - \sup\{s \leqq t : s \in Z(\omega)\}$. These processes arise in the study of local times for more general Markov processes, and certain special cases are well known in renewal theory; cf. Doob, Trans. Amer. Math. Soc. 63 (1948), 422-438. Let $\mathfrak{X}$ be the class of such processes. Then Krylov and Jushkevitch (Trudy Moscow Math. Soc. 13 (1965), 114-135) have shown an essentially biunique correspondence between $\mathfrak{X}$ and pairs $\{\alpha, h(x)\}$ where $\alpha \geqq 0$ is constant and $h(x)$, $0 < x \in E$, is positive, nonincreasing, right continuous, and $\int_0^x h(y)dy < \infty$, $x \in E$. In the present work we find explicitly the generator and transition semigroup of the process $X \in \mathfrak{X}$ corresponding to the given pair $\{\alpha, h(x)\}$ and also give a new proof that such an X exists. It is shown that many of the properties of Doob's renewal processes carry over to semilinear processes, and that the present theory includes Kingman's theory of regenerative events (Z. Wahrscheinlichkeitstheorie 2 (1964), 180-224). (Received February 7, 1969.)

664-67. KENNETH HARDY, Carleton University, Ottawa 1, Canada. Rings of normal functions.

An extended real-valued function $\varphi$ on a space X is called normal if $(\varphi_*)^* = \varphi$, where $\varphi_*$ and $\varphi^*$ represent the limit inferior and limit superior of $\varphi$ respectively. It is shown that two normal functions are equal if they agree on a dense set and that every normal function on a dense subspace has a unique, normal extension to the whole space. For a completely regular space X, the set DN(X) of all normal functions which assume finite values on a dense set in X can be given the structure of an algebra. BN(X) and LN(X) are the subalgebras of bounded and locally bounded normal functions. Let K denote the Stone space of the complete Boolean algebra of regular open sets in X. Then BN(X) is isomorphic with C*(K) and LN(X) is isomorphic with C(E), where E is the dense subspace of K consisting of fixed regular open ultrafilters. Connections are established between the rings BN(X), DN(X) and the maximal ring of quotients of C(X). (Received February 11, 1969.)

664-68. HELEN SKALA, 1927 S. Racine, Chicago, Illinois 60608. Nonassociative lattices.

An asymetric order on a set A is a binary relation $\leqq$ such that $a \leqq a$ for each a in A and $a \leqq b$, $b \leqq a$ implies $a = b$. If any two elements a and b of A have a greatest lower bound, denoted by $a \cap b$ (that is, $a \cap b \leqq a$, $a \cap b \leqq b$ and whenever $c \leqq a$, $c \leqq b$, then $c \leqq a \cap b$), and a least upper bound, denoted by $a \cup b$, A is called a nonassociative lattice. The operations $\cup$ and $\cap$ are commutative, absorptive and alternative; they are associative if and only if $\leqq$ is transitive. An element d is said to be distributive if d, with any two elements of A, satisfies the distributive laws (with regard to $\cup$ and $\cap$). Theorem. The set of distributive elements of a nonassociative lattice constitute an associative sublattice. (Received February 13, 1969.)

664-69. KAZIMIERZ KURATOWSKI and SAM B. NADLER, State University of New York at Buffalo, Amherst, New York 14226. Continuous selections on the hyperspace of a continuum.

A continuous selection $f: 2^X \to X$ is a continuous function such that f(A) belongs to A for all $A \in X$ ($2^X$ is the space of all nonempty compact subsets of X metrized with the Hausdorff metric). Theorem. If X is a metric continuum then there exists a continuous selection $f: 2^X \to X$ if and only if X is an arc. This answers to some extent a question raised by E. Michael (Topologies on spaces of subsets, Trans. Amer. Math. Soc. 71 (1951), 152-182.) (Received February 13, 1969.)

664-70. JOHN J. McKIBBEN, University of Massachusetts, Amherst, Massachusetts. Continued fractions of algebraic numbers.

The "simplicity" or "complexity" of a real irrational $\xi$ has often been measured in terms of the rate of growth of the partial quotients $a_0, a_1, \ldots$ of $\xi$. In this paper we suggest that in the case when $\xi$ is computable, a better measure may be found in the rate of growth of the number of necessary internal states of a machine for computing $a_n$. Real algebraic numbers of degree $\geq 3$ are studied from this point of view, examples of relevant computing machines are given, and it is conjectured that these machines are, in a certain sense, best possible. (Received February 12, 1969.)

664-71. WITHDRAWN.

664-72. IRA ROSENBAUM, University of Miami, Coral Gables, Florida. Variant forms of quantification of the predicate of use in analysis.

A. de Morgan's treatment of 'the quantification of the predicate' seems to have neglected the case of monadic predicates. This, and related matter, is considered in the present note. With P a monadic predicate of individuals of given type, and A a class of such individuals, the two statements (1) $P^{:}(A)$, read P is true of every element of A, and (2) $P_{.}(A)$, read P is true of some element of A, are defined, respectively, by (1') $(Ax)(x \in A. \to . P(x))$ and (2') $(Ex)(x \in A. \& . P(x))$. In terms of these statements, the definitions of $\mathrm{Ult}_M P(x)$ and $\mathrm{Pers}_M P(x)$ (Abstract 69T-B31, these *Notices* 16 (1969), 412) become, respectively, (1'') $(E\delta)P^{:}(N_\delta(M))$ and (2'') $(A\delta)P_{.}(N_\delta(M))$. Evidently, also, one has

$-(P^{:}(A)). \leftrightarrow .(-P)_{:}(A); -(P_{:}(A)). \leftrightarrow .(-P)^{:}(A); P_{:}(A) \vee Q_{:}(A). \leftrightarrow .(P \vee Q)_{:}(A); P^{:}(A).\&Q^{:}(A). \leftrightarrow .(P\&Q)^{:}(A); P_{:}(A) \vee P_{:}(B). \leftrightarrow .P_{:}(A \cup B); P^{:}(A).\&P^{:}(B): \rightarrow : P^{:}(A \cap B)$; etc. Also useful, with R a dyadic predicate, and A and B classes, are the statements: (3) $A^{:}R^{:}B$, read every element of A has the relation R to every element of B, and defined by (3') $(Ax)(Ay)(x \in A.\&.y \in B: \rightarrow : xRy)$, and (4) $aR^{:}B$, read a has the relation R to every element of B, and defined by (4') $(u'a)^{:}R^{:}B$, where u'a is the class whose only element is a; and (5) $A^{:}Rb. \leftrightarrow : A^{:}R^{:}(u'b)$; etc. (Received February 13, 1969.)

664-73. BERNHARD AMBERG, University of Texas, Austin, Texas 78712. Extensions of group theoretical properties.

In the following let all group theoretical properties be factor inherited. If $\mathfrak{f}$ and $\mathfrak{e}$ are group theoretical properties, the question arises in which cases the requirement that every extension of a $\mathfrak{f}$-group by an $\mathfrak{e}$-group is an $\mathfrak{e}$-group is equivalent to apparently weaker requirements. Typical Theorem. If $\mathfrak{f}$ and $\mathfrak{e}$ are properties of finite groups such that every finite elementary abelian p-group is a $\mathfrak{f}$-group whenever the cyclic group of order p is a $\mathfrak{f}$-group, then every extension of a $\mathfrak{f}$-group by an $\mathfrak{e}$-group is an $\mathfrak{e}$-group if and only if every product of a normal $\mathfrak{f}$-subgroup and an $\mathfrak{e}$-subgroup is an $\mathfrak{e}$-group. If $\mathfrak{e}$ is a saturated property of finite groups or if $\mathfrak{f}$ and $\mathfrak{e}$ are properties of finite soluble groups, this theorem may be improved considerably. Furthermore, if $\mathfrak{e}$ is the property of being a hyper-$\theta$- group (see Rend. Mat. Padova 41 (1968), 97-118), then the following holds. Theorem. If $\mathfrak{f}$ and hyper-$\theta$ are properties of artinian soluble groups, then every extension of a $\mathfrak{f}$-group by a hyper-$\theta$-group is a hyper-$\theta$-group if and only if every splitting extension of a $\mathfrak{f}$-group by a hyper-$\theta$-group is a hyper-$\theta$-group. The requirements that hyper-$\theta$ is a saturated property of artinian groups and that hyper-$\theta$ is product inherited are also considered. (Received February 13, 1969.)

664-74. ALFRED TONG and DONALD WILKEN, State University of New York at Albany, Albany, New York 12203. Mapping m into c. Preliminary report.

Theorem. Let X be a compact Hausdorff space. Suppose there exists a bounded continuous Borel measure on X. Then there exists a sequence of bounded Borel measures $\mu_n$ on X such that $\mu_n \rightarrow 0$ (weakly) and such that $\|\mu\| \not\rightarrow 0$. Corollary. Let X be a compact Hausdorff space satisfying the above hypothesis. Then there exists a noncompact (bounded) operator $T : C(X) \rightarrow c_0$. Corollary. Let m be the space of all bounded sequences of real numbers. Let $B(m, c_0)$ be the space of all bounded operators from m to $c_0$. Let $K(m, c_0)$ be the subspace of all compact operators from m to $c_0$. Then there does not exist a bounded projection of $B(m, c_0)$ onto $K(m, c_0)$. The preceding corollary answers a question of Arterburn and Whitely, Projections in the space of bounded linear operators (Pacific J. Math. (3) 15 (1965)). (Received February 12, 1969.)

664-75. R. A. JENSEN, University of Wisconsin, Madison, Wisconsin 53706. Cross sectionally connected 2-spheres are tame.

W. T. Eaton (Abstract 656-28, these *Notices* 15 (1968), 510) and Norman Hosay have independently proved that a 2-sphere S in $E^3$ is tame if each horizontal cross section of S is either a simple closed curved or a point. Loveland (Abstract 69T-G8, these *Notices* 16 (1969),327) has shown that if each horizontal cross section of S is connected and locally connected then S is tame.

This paper uses the techniques of Hosay to prove the following Theorem. If each horizontal cross section of a 2-sphere S in $E^3$ is connected then S is tame. This Theorem answers a question raised by Bing (Topology seminar, Ann. of Math. Studies 60, Princeton Univ. Press, Princeton, N.J., p. 82).
(Received February 13, 1969.)

664-76. PAUL AXT and WILSON E. SINGLETARY, Pennsylvania State University, University Park, Pennsylvania 16802. Algorithms relating first-order theories to various combinatorial systems.

An algorithm is presented which associates with each recursively enumerable set A a finitely axiomatizable first order theory $\mathcal{T}_A$ of the same tt-degree. The algorithm is constructed by first using a proof of a theorem of Feferman [J. Symbolic Logic (1957)] which effectively associates with each recursively enumerable set A a recursively enumerable set of axioms for a theory $\mathcal{S}_A$ of the same tt-degree as A. A proof of a theorem of Craig [J. Symbolic Logic (1957)] effectively provides a recursive set of axioms for $\mathcal{S}_A$. Then Hanf's proof [590-39] effectively provides a finitely axiomatizable theory $\mathcal{T}_A$ of the same tt-degree as $\mathcal{S}_A$. The algorithm thus presents $\mathcal{T}_A$ by means of a recursive set of axioms. Given any axiomatized first-order theory, one can effectively produce a recursive function which enumerates its theorems. Hence, applying this result, one has for each of several types of combinatorial systems and decision problems: there exist algorithms associating with an arbitrary combinatorial system $\mathcal{S}$ of a given type a finitely axiomatizable theory $\mathcal{T}$ having the same tt-degree as the given decision problem for $\mathcal{S}$ [cf. Singletary, Bull. Amer. Math. Soc. (1967)]. And there are algorithms giving the reverse association. (Received February 13, 1969.)

664-77. WILLIAM H. RUCKLE, Lehigh University, Bethlehem, Pennsylvania 18015. Topologies on sequence spaces.

A study is made of two means to topologize a space of sequences. The first method rests upon the duality of every sequence space S with the sequence space $\varphi$ (finitely $\neq 0$) by means of the form $((a_j), (b_j)) = \Sigma_j a_j b_j (a_j) \in S, (b_j) \in \varphi$. The second method is a generalization of the Kothe-Toeplitz duality theory. The Kothe-dual $S^\alpha$ of a sequence space S consists of all $(b_j)$ such that $(a_j b_j) \in \iota^1$ (absolutely convergent series) for each $(a_j) \in S$. Other spaces may take the role of $\iota^1$ in the above definition. A means to construct a topology on S by means of this generalized dual is determined. Finally, a particularly suitable type of space (the sum space) to play the role of $\iota^1$ is defined.
(Received February 12, 1969.)

664-78. ANILCHANDRA A. KAYANDE, University of Rhode Island, Kingston, Rhode Island 02881. A theorem on contractive stability.

Let J denote the interval $[t_0, t_0 + T)$ in the nonnegative real line, $T > 0$, and $R^n$ denote the Euclidean real n-space. $\bar{J} = [t_0, t_0 + T]$. Consider the system $dx/dt = f(t,x)$, where f is continuous on $J \times R^n$ into $R^n$ and satisfies Lipschitz condition in x. Let $|\cdot|$ denote a continuous functional on $R^n$ which need not be a true norm. Let $x(t;t_0, x_0)$ denote a trajectory of the system evaluated at t, taking the value $x_0$ at $t_0$. Definition. The system is said to be contractively stable with respect to $(\alpha, \beta, \gamma, t_0, T, |\cdot|)$, $\beta < \alpha < \gamma$, if $|x_0| \leqq \alpha$ implies that (i) $|x(t;t_0,x_0)| < \gamma$ for $t \in J$ and (ii) $|x(t_0 + T; t_0, x_0)| < \beta$. This definition is equivalent to the one given by Weiss and Infante [Proc.

Nat. Acad. Sci. U.S.A. 54 (1965), 44-48]. Theorem. The necessary and sufficient condition for the system to be contractively stable is that there exist a function $V = V(t,x)$ continuous on $J \times R^n$ into $R^1$ satisfying local Lipschitz condition in x, such that (a) $\lim \sup_{h\to 0^+} [V(t + h, x + h f(t,x)) - V(t,x)]/h \leqq 0$, for $t \in J$ and $|x| \leqq \gamma$; (b) $\sup_{|x|<\alpha} V(t_0,x) < \min_{|x|=\gamma} V(t,x)$, $t \in J$; and (c) $\sup_{|x|\leqq\alpha} V(t_0,x) < \min_{\beta\leqq|x|\leqq\gamma} V(t_0 + T, x)$. (Received February 12, 1969.)

664-79. LOUIS V. DiBELLO, 166 Fair Oaks Avenue, Rochester, New York 14618. The Galois theory of infinite Dedekind fields. Preliminary report.

A Dedekind field is an algebraic extension of the rationals $\underset{\sim}{Q}$ whose ring of integers is a Dedekind domain. Theorem 1. There exist Dedekind fields which are infinite and normal over $\underset{\sim}{Q}$. Theorem 2. If F is a finite extension of $\underset{\sim}{Q}$, K a normal extension of F, assume that K is Dedekind. Then $\mathrm{Gal}\,(K/F) \cong G_0 \times \prod_{i=1} \underset{\sim}{Z}_{p_i^{n_i}}$ where $G_0$ is a finite group and the $p_i$ are distinct primes. ($\underset{\sim}{Z}_{p_i^{n_i}}$ is the cyclic group of order $p_i^{n_i}$; the isomorphism is topological, where the right-hand side is topologized by the Tychonoff product topology with all the factors discrete.) Some consequences ot Theorem 2 are discussed. (Received February 13, 1969.)

664-80. ROBERT A. McGUIGAN, University of Massachusetts, Amherst, Massachusetts 01002. On the connectedness of isomorphism classes.

Let X and Y be topologically isomorphic normed linear spaces. Define $D(X,Y) = \log \inf\|T\|\,\|T^{-1}\|$, where the infimum is taken over all isomorphisms T of X onto Y. If X is a normed linear space, [X] denotes the class of all normed spaces topologically isomorphic to X, with isometric spaces identified. [X] is called the isomorphism class of X. Banach (Operations lineaires, p. 242) observed that D defines a pseudometric topology on [X]; this pseudometric space is denoted by ([X],D). A topological space S is locally pathwise connected if, for every x in S and every neighborhood V of x, there is an open set U such that $x \in U$, $U \subset V$, and for every two points y, z in U there is a path joining y to z which does not leave V. Theorem. For every normed linear space X, ([X],D) is both pathwise connected and locally pathwise connected. Corollary. For every X, ([X],D) is connected and locally connected. Corollary. If X is finite dimensional, then ([X],D) is a continuous image of the closed interval [0,1]. The theorem is proved by considering certain convex sets in X. (Received February 12, 1969.)

664-81. PETER V. O'NEIL, College of William and Mary, Williamsburg, Virginia 23185. Enumeration of spanning trees interacting with subgraphs. Preliminary report.

Let H be a subgraph of a graph G. This paper considers methods of calculating $T_H(G)$, the number of trees spanning G and containing H when H is acyclic, and $T(G/H)$, the number of trees spanning G and intersecting H in a nonvoid, nonspanning tree. Sample applications. (1) Let $G_t$ denote the complete graph with t vertices. Let A and B be disjoint sets of vertices of $G_t$ with a and b elements respectively. Let $a \geqq 2$ and $t > a + b$. Let S be the graph formed by removing all branches from A to B, and H any tree spanning exactly A in $G_t$. Then $T_H(S) = (t - a - b)a(t - a)^{b-1}t^{t-a-b-1}$. When b = 0, $T_H(S) = at^{t-a-1}$ is the number of spanning trees of $G_t$ containing a given tree spanning a vertices (see Moon, "Enumerating labelled trees" in Graph theory and theoretical physics, Academic

Press, New York). (2) If $2 \leq k \leq t$, then $T(G_t/G_k) = t^{t-k-1}[t^{k-1} - (t-k)^{k-1} - k^{k-1}]$. (3) If A and B are as in (1), and H is the complete bipartite graph with branches from A to B, then $T(G_t/H) = t^{t-a-b-1}[t^{a+b-1} - (t-b)^{a-1}(t-a)^{b-1} - a^{b-1}b^{a-1}(a+b)]$. (Received February 12, 1969.)

664-82. ANTHONY W. HAGER, Wesleyan University, Middletown, Connecticut 06457. Approximation of continuous functions on Lindelöf spaces.

In what follows X is a space with the Lindelöf property, C(X) its ring of real-valued continuous functions, C*(X) the subring of bounded functions, and A a subring of C*(X) which contains rational constants and separates points and closed sets in X; for $B \subset C(X)$, ul B denotes the collection of uniform limits of sequences from B. We establish Theorem 1. $C(X) = \mathrm{ul}\{f/g : f,g \in \mathrm{ul}\ A$ and $g(x) \neq 0$ for each $x \in X\}$, and from this, Theorem 2. Each f in C(X) is the pointwise limit of a sequence from A; if A is also a lattice, and $f \geq 0$, the sequence from A may be chosen increasing. Without much trouble, these follow. Corollary 1. If A is a lattice, then A is "sequentially dense" in $C_c(X)$ ( $\equiv$ C(X) equipped with the compact-open topology). Corollary 2. The "sequential density character" of $C_c(X)$ does not exceed the weight of X ( $\equiv$ the least cardinal of an open base). Corollary 3. If X has more than one point, the cardinality of C(X) is (weight X)$^{\aleph_0}$. Examples show that when X is not Lindelöf, all results can fail. (Received February 13, 1969.)

664-83. JOHN R. DURBIN, University of Texas, Austin, Texas 78712. On finite groups possessing characteristic cyclic series. Preliminary report.

By a characteristic cyclic series (c.c.s.) of a group G is meant a finite series $1 = G_0 < G_1 < \dots < G_n = G$ with each $G_i$ characteristic in G and each $G_{i+1}/G_i$ cyclic. Theorem 1. A finite group G possesses a c.c.s. iff for each nontrivial $x \in G$ there exists a pair of characteristic subgroups $K < H$ in G such that $x \in H \setminus K$ and H/K is cyclic. Theorem 2. If n is a positive integer, then each group of order n possesses a c.c.s. iff n is square-free. Theorem 3. If G is finite Abelian and $|G| > 4$, then G possesses a c.c.s. iff G has supersolvable automorphism group iff G is a direct sum of cyclic groups of distinct prime power orders. Thus a finite Abelian group G with $|G| > 4$ has supersolvable holomorph iff it has supersolvable automorphism group, and the finite Abelian groups with supersolvable automorphism group differ only slightly from those with solvable automorphism group (determined by Shoda, Math. Ann. 100 (1928), 674-686). Similar results with weaker finiteness conditions are also considered. (Received February 13, 1969.)

664-84. PETER PERKINS, College of the Holy Cross, Worcester, Massachusetts 01610. A finite set of groupoid equations in one variable with unsolvable decision problem. Preliminary report.

This result was suggested by T. Evans and improves the author's original result (Notre Dame J. Formal Logic 8 (1967), 175-185). Let $P = \{a,b : U_i = V_i, 1 \leq i \leq n\}$ be a semigroup presentation in which no $U_i$ or $V_i$ is a single letter and which has unsolvable word problem (M. Hall, J. Symbolic Logic 14 (1949), 115-118). For each word W in a,b associate a groupoid term $\overline{W}$ in a single variable x as follows: $\overline{a}(x) = ((x + x) + x)$, $\overline{b}(x) = (x + (x + x))$, $\overline{Wa}(x) = \overline{W}(\overline{a}(x))$, $\overline{Wb}(x) = \overline{W}(\overline{b}(x))$. Let $E(P) = \{\overline{U}_i(x) = \overline{V}_i(x) | 1 \leq i \leq n\}$. If E(P) has solvable decision problem (for equations) then P has solvable word problem. (Received February 13, 1969.)

664-85. JOE A. GUTHRIE, Texas Christian University, Fort Worth, Texas 76129. Some analogues of a theorem of Michael. Preliminary report.

An important property of $\aleph_0$-spaces is that $C(X,Y)$ is $\aleph_0$ if both X and Y are $\aleph_0$ (E. Michael, J. Math. Mech. 15 (1966), 983-1002). Analogous theorems are given for the spaces considered here. In the following $Z = \{z, z_1, z_2, \ldots\}$ is the symbol for a convergent sequence with its accumulation point z, and $Z_n = Z - \{z_i | i < n\}$. A collection $\mathcal{P}$ of subsets of a space is a cs-pseudobase if for every Z and open nbd U of Z there exists a $P \in \mathcal{P}$ such that $Z_n \subset P \subset U$ for some n. Let $C_{cs}(X,Y)$ denote the space of maps from X to Y with the topology whose subbasic open sets are of the form $\{f | f(Z) \subset U\}$, where U is open in Y. It is noted that subbasic open sets of the form $\{f | f(Z_n) \subset U$ for some n depending on $f\}$ generate the topology of pointwise convergence. A regular space with a countable ($\sigma$-locally finite) cs-pseudobase is called a cs-$\aleph_0$ (cs - $\sigma$)-space. Theorem 1. A cs-$\aleph_0$ (cs-$\sigma$)-space which is a q-space (E. Michael, Israel J. Math. 2 (1964), 173-176) is a separable metric (metric) space. Theorem 2. If X is a cs-$\aleph_0$-space and Y is a cs-$\aleph_0$ (cs-$\sigma$)-space, then $C_{cs}(X,Y)$ is a cs-$\aleph_0$ (cs-$\sigma$)-space. It is noted that if X is a separable metric space and Y is a cs-$\aleph_0$ (cs-$\sigma$)-space, then $C(X,Y)$ is a cs-$\aleph_0$ (cs-$\sigma$)-space. (Received February 13, 1969.)

664-86. SWARUPCHAND M. SHAH and SELDEN Y. TRIMBLE, University of Kentucky, Lexington, Kentucky 40506. Univalent functions with univalent derivatives. II.

This paper is in continuation of our earlier paper [Abstract 663-335, these *Notices* 16 (1969), 182]. Theorem. Let $f(z) = \Sigma_0^\infty a_n z^n$ be an entire function such that $|a_n/a_{n+1}|$ is ultimately a non-decreasing function of n. Let $\nu(r)$ denote the central index. Write $\gamma = \lim\sup_{r\to\infty} \nu(r)/r$, $\delta = \lim\inf_{r\to\infty} \nu(r)/r$. Let $\Lambda$ and $\lambda$ denote respectively the order and the lower order of f. Let $\rho_n$ denote the radius of univalence of $f^{(n)}$ about the origin. Then (i) $\lim\inf_{n\to\infty} (\log \rho_n)/\log n = (1-\Lambda)/\Lambda \leq (1-\lambda)/\lambda = \lim\sup_{n\to\infty} (\log \rho_n)/\log n$, (ii) $(\log 2)/\gamma \leq \lim\inf_{n\to\infty} \rho_n \leq 4/\gamma$, and (iii) $(\log 2)/\delta \leq \lim\sup_{n\to\infty} \rho_n \leq 4/\delta$. Some more results involving $(\rho_N \rho_{N+1} \cdots \rho_n)^{\Lambda/n}(n^{\Lambda-1})$ have also been proved. Corrections in Bull. Amer. Math. Soc. 75 (1969), 153-157: Omit Remark (i), p. 154. Replace R by 4R in (i),Theorem 2, p. 155. (Received February 13, 1969.)

664-87. MORIO OBATA, Lehigh University, Bethlehem, Pennsylvania 18015. Conformal transformations of Riemannian manifolds.

Conjecture. Is a connected compact Riemannian n-manifold, n > 2, admitting an essential conformal vector field conformorphic (= conformally diffeomorphic) to a Euclidean n-sphere? By an essential conformal vector field is meant one such that there is no conformally related Riemannian metric on the manifold with respect to which it is isometric. Theorem. If a connected Riemannian n-manifold, n > 2, admits a complete conformal vector field with singular points at each of which its divergence does not vanish, then the manifold is conformorphic either to a Euclidean n-sphere or to a Euclidean n-space. In the latter case the vector field is homothetic with respect to the Euclidean metric conformal to the original Riemannian metric. It is noted that a conformal vector field with the property in the theorem is always essential. The proof is based on study of the stable and unstable manifolds of the vector field at each singular point. (Received February 12, 1969.)

664-88. LEONARD H. HAINES, University of California, EE and CS Department, Berkeley, California 94720. Representation theorems for context sensitive languages.

Let $G = (V, \Sigma, P, \sigma)$ be a CS (context sensitive) grammar. Call G LCS (left CS) iff each production of P has the form $x\alpha \to xy$ where x in $(V - \Sigma)^*$, $\alpha$ in $V - \Sigma$ and y in $VV^*$. Call $T = (V,\Sigma,P)$ a left transform (LT) iff G is LCS and each production of P preserves length. For any $W \subset (V - \Sigma)^*$ define $T(W) = \{z \text{ in } \Sigma\Sigma^* : w \overset{*}{\Rightarrow} z \text{ for some } w \text{ in } W\}$. Let L be any CS language. Theorem 1. $\exists$ LT T and a context free language $K \ni L = T(K)$. Corollary. L is generated by an LCS grammar. Theorem 2. $\exists$ an LT $T_1$, and RT $T_2$ and a regular set $R \ni L = T_2(T_1(R))$. Call L a left language iff $L = T(R)$ for some LT T and regular set R. Corollary. Each CS language is deterministic iff each left language is deterministic. A cog-slipping automaton is a 6-tuple $A = (K, \Sigma, \Delta, \delta, q_0, F)$ where K and $\Delta$ are finite sets, $\Sigma \subset \Delta$, $F \subset K$, $q_0$ in $K - F$ and $\delta: K \times \Delta \to K \times \Delta$. Let $\overset{*}{\vdash}$ be the transitive closure of $\vdash$ which partially orders $\Delta^* \times K \times \Delta^*$ so that $(x,q,y) \vdash (x',q',y')$ iff either $[\delta(q,y_1) = (q',y_1')$ where $y = y_1y'$ and $x' = xy_1']$ or $[x' = \epsilon, q' = q_0$ and $y' = xy]$. Define $S(A) = \{x \text{ in } \Sigma^* : (\epsilon, q_0, x) \overset{*}{\vdash} (y,q,\epsilon)$ for some y in $\Delta^*$ and q in $F\}$. Theorem 3. L is a left language iff $L = S(A)$ for some cog-slipping automaton A. (Research sponsored by the National Science Foundation under Grant GP-6945.)
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664-89. CHARLES K. MARTIN, Virginia Polytechnic Institute, Blacksburg, Virginia 24061. Rickart and Baer rings of linear transformations. II.

A ring is Rickart if every left (right) annihilator of a one-element set is a principal left (right) ideal generated by an idempotent. Clearly Baer rings are Rickart. We consider certain subrings of the ring L(V) of all linear transformations on a vector space of arbitrary dimension. In particular, let R be a subring of L(V) and let L be the lattice of subspaces invariant under R. If R contains all elements of L(V) which leave each element of L invariant, then R is said to be a d-ring. Necessary and sufficient conditions are obtained in order for a d-ring to be Rickart (Baer). Principally two types of conditions are considered: conditions related to the structure of the lattice of invariant subspaces; conditions related to the form of the matrices or their graph in terms of a suitable basis. (Received February 12, 1969.)

664-90. VINCENT J. MANCUSO, 161-34 28th Avenue, Flushing, New York 11358. Some properties of mesocompact spaces. Preliminary report.

A family F of subsets of a space X is called compact finite if every compact subset of X meets at most finitely many members of F. A Hausdorff space is called mesocompact if every open cover has a compact finite open refinement. Mesocompactness lies strictly between paracompactness and metacompactness. (J. R. Boone, Some characterizations of paracompactness in k-spaces, to appear). Theorem 1. If $f: X \to Y$ is perfect and Y is mesocompact, then X is mesocompact. Corollary 1. The product of a mesocompact space and a compact space is mesocompact. The product of mesocompact spaces need not be mesocompact. However, we have Corollary 2. The product of a mesocompact space and a locally compact mesocompact space is mesocompact. Biquotient maps (E. Michael, Abstract 653-78, these *Notices* 15 (1968), 100) are also studied in relation to mesocompactness. Question. Is a mesocompact space normal or regular? (Received February 14, 1969.)

664-91. C. T. TAAM, George Washington University, Washington, D. C. 20006. The orbits of unitary groups in Hilbert spaces.

The purpose of this paper is to investigate the dynamical aspects of the orbits of one-parameter unitary groups of operators in Hilbert spaces. It is shown that, among other results, the orbit of a point under the action of such a group $U_t$ can be decomposed into mutually orthogonal components of four different types: each component of the first type is almost periodic; each component of the second type is wandering and weakly convergent; each component of the third type is wandering and weakly divergent, and the remaining components are all Poisson-stable but non-almost-periodic. Also the orbit of a point under the action of $U_t$ is almost periodic if and only if the point lies in the closed linear subspace spanned by the eigenvectors of the infinitesimal generator of $U_t$. (Research supported by the U. S. Army Research Office - Durham, Under Contract no. DAHC-04-67-C-0056.) (Received November 12, 1968.)

664-92. M. RAMA MOHANA RAO and CHRIS P. TSOKOS, University of Rhode Island, Kingston, Rhode Island 02881. Integro-differential equations of Volterra type.

The aim of this paper is to study the stability properties of solutions of integro-differential equations of Volterra type by reducing it into a scalar integro-differential equation. The existence of maximal solution and a basic result on integro-differential inequality are developed, adapting the result of W. Mlak [Note on maximal solution of differential equations, Contrib. Differential Equations, Vol. 1, 1963, pp. 461-465]. Using this theory of integro-differential inequalities, we obtain sufficient conditions for uniform asymptotic stability of the trivial solution of integro-differential systems. This study includes C. Corduneanu [Quelques problemes qualitatifs de la theorie des equations integro-differentielles, Colloq. Math. 18 (1967), 77-87] results as a special case. (Received February 14, 1969.)

664-93. CHRIS P. TSOKOS and M. RAMA MOHANA RAO, University of Rhode Island, Kingston, Rhode Island 02881. Functional differential inequalities and stability near sets.

Stability of sets has been discussed by many. Quite recently T. Yoshizawa [Stability theory of Lyapunov's second method, Math. Soc. Japan, 1966] studied the stability of sets which include as a special case in stability, the sense of Lyapunov, orbital stability and ultimate boundedness of solutions. The aim of this paper is to use the comparison principle to study the stability and boundedness behavior of solutions of functional differential equations near sets. Our results include those of T. Yoshizawa [Funkcial Ekvac 5 (1963), 31-69] for ordinary differential equations. (Received February 14, 1969.)

664-94. TRIBOKI N. BHARGAVE, Kent State University, Kent, Ohio 44240, and RUSS A. SMUCKER, Goshen College, Goshen, Indiana. On a maximization problem.

Let N and p be positive integers, $2 \leq p \leq N$, and let $\Pi = \Pi(N,p)$ denote the set of all ordered p-tuples $\pi = (x_1, x_2, \ldots, x_p)$ for which $x_i$, $i = 1,2,\ldots, p$, $\in I^+$, the set of positive integers, and $\sum_{i=1}^{p} x_i = N$. Further let $\sigma_q : \Pi \to I^+$ be defined by $\sigma_q(\pi) = \sum_{i=1}^{p-q} x_i x_{i+1} \ldots x_{i+q}$, $1 \leq q \leq p - 1$, $\pi \in \Pi$, and let $\mu = \max_{\pi \in \Pi} \sigma_q(\pi)$. An explicit expression for $\mu$ in terms of the fixed parameters N, p, and q, is obtained by finding a partition $\bar{\pi} \in \Pi$ such that $\mu = \sigma_q(\bar{\pi})$. A slight generalization of this problem consists in letting N and $x_i$'s, i = 1,2,..., p, to be nonnegative real numbers; an explicit solution for this case is also obtained. (Received February 14, 1969.)

664-95. KARL F. BARTH and WALTER J. SCHNEIDER, Syracuse University, Syracuse, New York 13210. Integrals and derivatives of functions in MacLane's class A and of normal functions.

G. R. MacLane has raised the question, "If $f(z) \in \mathcal{A}$, are $f'(z)$ and $\int f(z)dz \in \mathcal{A}$?" Positive results. Theorem 1. If $f \in \mathcal{A}$ and satisfies $\int_0^1 (1 - r)m(r,f)dr < \infty$, then $f^{(n)}(z) \in \mathcal{A}$ for $n \geqq 1$. Theorem 2. If $f \in \mathcal{A}$ and satisfies $\int_0^1 (1 - r)\log^+(1 - r)^{-1} m(r,f)dr < \infty$, then $\int f(z)dz \in \mathcal{A}$. Corollary. If f is holomorphic and normal in $\{|z| < 1\}$, then $f^{(n)}(z) \in \mathcal{A}$ for each $n \geqq 1$ and $\int f(z)dz \in \mathcal{A}$. Remarks. In the case n = 1 the Corollary is a trivial consequence of some results of W. K. Hayman. The results can be extended to meromorphic functions satisfying $N(r, \infty) = O(1)$. Counterexamples. By a variation of techniques used in the authors' paper Exponentiation of functions in MacLane's class A, J. Reine Angew. Math. (to appear), functions f and g are constructed with the following properties: $f \in \mathcal{A}$ but $f' \notin \mathcal{A}$ and $g \in \mathcal{A}$ but $\int g dz \notin \mathcal{A}$. (Received February 14, 1969.)

664-96. STEPHEN D. COMER, Vanderbilt University, Nashville, Tennessee 37203. Representation of cylindric algebras by sheaves. Preliminary report.

A sheaf of $CA_\alpha$'s is a triple $(X,K,\pi)$ where (i) X and K are topological spaces, (ii) $\pi : K \to X$ is a local homeomorphism onto X, (iii) $\pi^{-1}(x) = K_x$ is a $CA_\alpha$ for each $x \in X$, and (iv) the natural $CA_\alpha$ operations induced on K are continuous. Many of the ring theoretic results concerning representations by sheaves found in Part I of R. S. Pierce's monograph [Modules over commutative regular rings, Memoir Amer. Math. Soc., No. 70] carry over to $CA_\alpha$'s. The following are samples. Theorem. Every $CA_\alpha$ is isomorphic to the $CA_\alpha$ of all sections of a reduced sheaf of $CA_\alpha$'s. Moreover, there are contravariant functors between the category of all $CA_\alpha$'s and the category of reduced sheaves establishing their equivalence. This theorem gives an extension to $CA_\alpha$'s of a sheaf theoretic version of the Stone representation theorem for BA's. An ideal of a $CA_\alpha$ is regular if it is the extension of its restriction to the BA of zero-dimensional elements. Theorem. For a reduced sheaf of $CA_\alpha$'s $(X,K,\pi)$, there is an isomorphism between the lattice of all open subsets of X and the lattice of regular ideals of $\Gamma(X,K)$, the $CA_\alpha$ of sections of $(X,K,\pi)$. (Received February 14, 1969.)

664-97. BURTIS G. CASLER, Louisiana State University, Baton Rouge, Louisiana. Finite CW-complexes with second homotopy groups infinitely generated as fundamental group modules.

Suppose H is a group with presentation $\{w_i, i = 1,2,\ldots : w_i \in (i) = 1, 1 < \epsilon(i), \text{finite}\}$. Then H is a group of type (A). Suppose G is a finitely presented group such that G contains a subgroup H of type (A) and if $W_i$ is a generator of H, then $W_i \notin [G,G]$. Then G is of type (B). Theorem. Suppose K is a CW-complex with finite 3-skeleton and $\pi_1(K)$ is of type B. Then $\pi_2(K)$ is infinitely generated as a $Z(\pi_1(K))$ module. An example of a group of type (A) is $PC = \{ab : a^2\}$. Further PC is isomorphic to the fundamental group of the wedge of the real projective plane and a circle. (Received February 14, 1969.)

664-98. GUSTAVE RABSON, Clarkson College of Technology, Potsdam, New York 13676. A generalization of continuity.

Let $\mathscr{S}$ and $\mathscr{T}$ be arbitrary families of subsets of the sets X and Y respectively. We denote by $C(\mathscr{S},\mathscr{T})$ the class of functions, f, with the property that for each $T \in \mathscr{T}$, $f^{-1}(T) \in \mathscr{S}$. By choosing the families $\mathscr{S}$ and $\mathscr{T}$ appropriately this definition embraces continuous functions, functions continuous at a given point, monotonic functions, measurable functions and semicontinuous functions. We say that $\mathscr{S}$ is compact for X if any covering of X by elements of $\mathscr{S}$ may be reduced to a finite covering. Several generalizations of theorems about semicontinuous functions will be presented. For example the theorem that a lower semicontinuous function on a compact set achieves its minimum value generalizes to the following: Let $\mathscr{S}$ be compact for X and $f \in C(\mathscr{S},\mathscr{T})$. Suppose furthermore that if any finite subfamily of $\mathscr{T}$ covers f(X) then f(X) is covered by one of the members of this subfamily. Then there is an $x_0$ in X such that for any T in $\mathscr{T}$, $f(x_0) \in T \Rightarrow f(x) \subset T$. Similar generalizations are given for the theorems that under appropriate conditions the supremum, the infimum of a finite family, the sum and the product of lower semicontinuous functions are lower semicontinuous. (Received February 14, 1969.)

664-99. MOSHE ROSENFELD, University of Washington, Seattle, Washington 98105. On simple Banach algebras.

Let A be a Banach algebra with an identity. A is simple if it does not contain proper two-sided ideals. A sufficient condition for A to be simple is given by the following Theorem. If $\forall x \in A$, $a \neq 0$, $\exists \bar{x} \in A$ such that $\sigma(x + \bar{x}) \cap \sigma(\bar{x}) = \emptyset$, then A is simple. A B-algebra B, with an identity, in which there exists an element x such that $\sigma(x + y) \cap \sigma(y) \neq \emptyset$ $\forall y \in B$ while x is not contained in a proper two-sided ideal, is constructed. Though B is not simple, we believe that our condition is not necessary. (Received February 14, 1969.)

664-100. GRAHAME BENNETT, Lehigh University, Bethlehem, Pennsylvania 18015. Counting functions for FK-spaces.

m will denote the space of all bounded sequences of complex numbers with $\|x\| = \sup_n |x_n|$. For any sequence $x = (x_m)$ and any positive integer n, let $P_n(x)$ denote the sequence $(x_1, x_2, \ldots, x_n, 0, 0, \ldots)$ and let $\psi(n,x)$ denote the number of nonzero elements in the set $\{x_1, x_2, \ldots, x_n\}$. For each sequence $(\Omega_n)$ of nonnegative integers increasing to infinity, let $U(\Omega_n) = \{x \in m : \psi(n,x) \leq \Omega_n, n = 1,2,\ldots\}$. Then $(\Omega_n)$ is said to be a counting function for the FK-space E if and only if $U(\Omega_n) \subset E$ and $P_n(x) \to x$ uniformly ( in the topology of E) on $\|\ \|$-bounded subsets of $U(\Omega_n)$. Necessary and sufficient conditions are established for an FK-space to have a counting function and these are then used to improve some theorems due to Lorentz. See Acta Math. 80 (1948), 167-190, and Canad. J. Math. 3 (1953), 236-256. (Received February 14, 1969.)

664-101. KAI FALTINGS, University of Texas, Austin, Texas 78712. Isomorphisms of the automorphism groups and projectivities of primary abelian groups.

A group A is a (P)-group iff A is a reduced, abelian p-group such that (a) if A is bounded, it has three independent elements of maximal order; (b) A has an infinite, countable direct summand or rank (A) $\neq 2^{\aleph_0}$. Let $\Gamma A$ be the automorphism group, $LA$ the lattice of all subgroups of A. Theorem. Let A and B be (P)-groups with p > 3 and let $\varphi : \Gamma A \to \Gamma B$ be an isomorphism. Then there exists a unique projectivity or duality $\varphi^{\pi}$ of LA onto LB, such that for all $\gamma \in \Gamma A$ and all direct summands D of A, $\gamma$ stabilizes D iff $\gamma^{\varphi}$ stabilizes $D\varphi^{\pi}$. Moreover, if A, B and C are (P)-groups with p > 3 and $\varphi : \Gamma A \to \Gamma B$ and $\psi : \Gamma B \to \Gamma C$ are isomorphisms, then $(\varphi\psi)^{\pi} = \varphi^{\pi}\psi^{\pi}$. Theorem. Let A be a (P)-group with p > 3, let $\Pi$ be the automorphism group of $\Gamma A$ and let $\Delta$ be the normal subgroup of $\Pi$ consisting of all $\varphi \in \Pi$ which induce the identity automorphism on $\Gamma A / z\Gamma A$. Then, if K is the group of all auto-projectivities and auto-dualities of LA, $\Pi$ is a splitting extension of $\Delta$ by K. (Received February 14, 1969.)

664-102. EDWARD H. THEIL, University of California, Davis, California 95616. Gentle degenerate perturbations are smooth.

This paper discusses the relationships between the notion of a gentle perturbation of a selfadjoint operator $H_0$, introduced originally by K. O. Friedrichs, and that of an $H_0$-smooth perturbation, first discussed by T. Kato. In case the perturbation is degenerate, we have the theorem of the title. If the perturbation is nonnegative, partial results are obtained. We use only the axioms of gentleness introduced by Friedrichs and Rejto, and thereby avoid any additional assumptions on the perturbation (e.g., that it is an integral operator with Hölder-continuous kernel in the spectral representation space of $H_0$). (Received February 14, 1969.)

664-103. CHIA-VEN PAO, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. The existence and stability of nonlinear operator differential equations in Hilbert spaces.

The existence and the stability problem of the operator differential equation $dx(t)/dt = Ax(t)$ $(t \geqq 0)$, where A is a nonlinear operator with domain D(A) and range R(A) both in a complex Hilbert space H, are investigated by using the nonlinear semigroup property. Under the condition $R(I - A) = H$, A generates a nonlinear contraction (resp. negative contraction) semigroup iff A is dissipative, that is, - A is monotone (resp. strictly dissipative) from which the existence, uniqueness and stability or asymptotic stability of solutions are insured. By the introduction of an equivalent inner product inducing a topologically equivalent Hilbert space, the inner product of H with respect to which A is dissipative can be replaced by an equivalent inner product without affecting the existence and the stability of a solution. This fact makes possible the development of a stability theory by the construction of a "Lyapunov functional" by means of a sesquilinear functional. (Received February 14, 1969.)

664-104. STEWART M. ROBINSON, Cleveland State University, Cleveland, Ohio 44110. A note on the intersection of free maximal ideals.

In a previous note (S. M. Robinson, The intersection of the free maximal ideals in a complete space, Proc. Amer. Math. Soc. 17 (1966), 468-469) it is shown that if X admits a complete uniform structure, the intersection of the free maximal ideals in C(X) is $C_K(X)$, the ring of functions with compact support. In this note, several necessary and sufficient conditions are given for a space X to enjoy this property. (Received February 14, 1969.)

664-105. BARNETT W. GLICKFELD, University of Washington, Seattle, Washington 98105. On exp ih in Banach *-algebras.

Let A be a commutative complex Banach algebra with an identity and a continuous involution *. Theorem. If u is a unitary ($uu^* = 1$) element of A which lies in the principal component of the invertible elements of A, then there is an h in A so that h is Hermitian ($h = h^*$) and exp ih = u. The short proof proceeds via Lorch's result that any period of exp may be decomposed into a finite sum of terms $\pm 2\pi ij$, where j is an idempotent. (Received February 14, 1969.)

664-106. TOR H. GULLIKSEN, Queen's University, Kingston, Ontario, Canada. Homological invariants of rings.

Let R, m be a local noetherian ring. The exponents $\epsilon_q(R) = \epsilon_q$ in the product formula for the Poincaré series of R, $P(R) = \Pi_{i=0}^{\infty}(1 + Z^{2i+1})^{\epsilon_{2i}}(1 - Z^{2i+2})^{-\epsilon_{2i+1}}$, are generalized and studied using differential graded algebras (see T. H. Gulliksen, A proof of the existence of minimal R-algebra resolutions, Acta Math. 120 (1968)). Let $\pi(R) = 2p$ if R/m has characteristic $p \neq 0$, $= \infty$ otherwise. Theorem 1. If R is a homomorphic image of a regular local ring and p is a prime ideal in R, then $\epsilon_q(R_p) \leq \epsilon_q(R)$ for $q < \pi(R) - 1$. Theorem 2. Let a be an ideal in R of finite homological dimension, and let E be the Koszul complex generated over R by a minimal set of generators in a. Then a is generated by an R-sequence if and only if $H_1(E)$ is a free R/a-module. Theorem 3. $\epsilon_3(R) = 0$ if and only if R is a local complete intersection. (Received February 14, 1969.)

664-107. EDGAR D. KANN, 31-26 Dwight Avenue, Far Rockaway, New York 11691. A new method for infinitesimal rigidity of surfaces $K > 0$.

Let z be a bending field defined on a surface S immersed in euclidean 3-space. Let r be a position vector and n the unit surface normal of S. Both r and z are assumed C''. It is well known that there exists a unique vector field y called the rotation field of z such that $dz = y \times dr$. If the only rotation fields which exist on S are const. then S is called infinitesimally rigid. The main tool is the Lemma. Let S be a surface with boundary (and $K > 0$) such that the spherical image of S lies on an open hemisphere. Then $y \cdot n \neq 0$ on the boundary of S implies $y \cdot n \neq 0$ in the interior of S. The Lemma is applied to convex surfaces with spherical image not in a hemisphere by using the Darboux transformation. Infinitesimal rigidity is proved for various standard cases, including among others complete, closed convex surfaces with fixed boundaries and convex surfaces with boundaries in contact with a plane (Rembs boundaries). A unifying feature is that all the boundary conditions are treated in the form $y \cdot n = $ const. If the spherical image of S plus boundary does lie in a hemisphere, all results hold even if the surface is not convex in the large. (Received February 14, 1969.)

# Abstracts for the Meeting in Cincinnati, Ohio
# April 18-19, 1969

665-1. HIDEGORO NAKANO, Wayne State University, Detroit, Michigan 48202. Pointless axiomatic set theory.

In the previous paper, An axiomatic set theory, Abstract 664-3, this issue of these *Notices* we defined an axiomatic set theory in which the elements x of the set space S with $S^x = \emptyset$ play an important role. In this paper we construct an axiom system on a set space which has no such element. In this axiomatic set theory the continuum hypothesis fails always. (Received October 23, 1968.)

665-2. WITHDRAWN.

665-3. GORDON JOHNSON, University of Georgia, Athens, Georgia 30601. Hilbert space problem four.

Problem four in A Hilbert space problem book (P. R. Halmos, D. Van Nostrand, New York, 1967) reads as follows: Construct, for every infinite-dimensional Hilbert space a simple continuous curve with the property that every two nonoverlapping chords of it are orthogonal. Along with the solution to the problem the following comment appears, "it is a curious empirical fact that the example is psychologically unique: everyone who tries it seems to come up with the same answer". In this note an elementary solution to the problem is given. (Received November 25, 1968.)

665-4. RICHARD J. TONDRA, Iowa State University, Ames, Iowa 50010. Approximating continua and domains in an n-cell.

Let $E^n$ denote an n-cell, $n \geqq 2$, and if $X \subset E^n$, let iX denote the interior of X in $E^n$. Theorem. There exists a continuum $C \subset E^n$ with the following properties: (i) C is homeomorphic to the 1-point compactification of a connected n-manifold with boundary. (ii) If D is a proper domain (open, connected subset) of $E^n$, then $D = \bigcup_{k=1}^{\infty} C_k$, where for all k, $C_k \subset iC_{k+1}$ and $C_k$ is homeomorphic to C. (iii) If A is a proper continuum in $E^n$, then $A = \bigcap_{k=1}^{\infty} C_k$, where for all k, $C_{k+1} \subset iC_k$ and $C_k$ is homeomorphic to C. (Received December 24, 1968.)

665-5. ANDY R. MAGID, Northwestern University, Evanston, Illinois 60201. The Boolean spectrum and separable algebras.

This paper applies Pierce's representation of an arbitrary commutative ring as sections of a sheaf over the spectrum X of the Boolean ring of idempotents of R to the theory of central separable R-algebras. Theorem. There is a sheaf of abelian groups on X whose global sections are the Brauer group of R and whose stalks are the Brauer groups of the stalks of the sheaf on X representing R. Corollary. The Brauer group of a Boolean ring is trivial. Theorem. The following assertions are true of R if they hold at each stalk of the representing shear: (1) Every central separable algebra has a weakly Galois splitting ring. (2) An isomorphism of central separable subalgebras of a central separable algebra extends to an inner automorphism of the algebra. Both thus hold when R is regular in the sense of von Neumann; more generally, (2) holds if the maximal spectrum of R is totally disconnected. (Received December 23, 1968.)

665-6. DAVID L. WINTER, Michigan State University, East Lansing, Michigan 48823. On finite solvable linear groups.

Let G be a finite solvable group, p a prime. Let G have a faithful irreducible representation of degree n over the complex number field. Then G has an abelian normal p-Sylow subgroup unless, for some positive integer m, $n = mp$ or $n = mq^s$ where $q^s$ is a positive power of a prime with $q^s \equiv \pm 1 \pmod p$. The theorem is "best possible" in the sense that for each of the exceptional values of n mentioned there is a finite solvable group which has a faithful irreducible representation of degree n over the field of complex numbers and which does not have a normal p-Sylow subgroup. (Received December 30, 1968.)

665-7. ROBERT CRAGGS, University of Illinois, Urbana, Illinois 61801. Involutions of the 3-sphere which fix 2-spheres.

Let $\mathfrak{J}$ denote the space of involutions of the 3-sphere $\Sigma$ whose fixed point sets are 2-spheres, and let $\mathscr{S}$ denote the subspace of involutions whose fixed point sets are tame 2-spheres. For a 2-sphere S in $\Sigma$ let $\mathfrak{J}(S)$ denote the involutions of $\Sigma$ whose fixed point sets are S. The chief result of the paper is Theorem 1. $\mathfrak{J}$ is pathwise and locally pathwise connected. Theorem 1 is established with the aid of the following three theorems and a result previously announced by the author (see Theorem 1 in Abstract 634-40, these *Notices* 13 (1966), 370). Theorem 2. $\mathscr{S}$ is dense in $\mathfrak{J}$. Theorem 3. Suppose $f \in \mathfrak{J}(S)$ and $\epsilon > 0$. There is a $\delta > 0$ such that if $g \in \mathfrak{J}(R)$ with $d(f,g) < \delta$, then there is an $\epsilon$-homeomorphism of S onto R. Theorem 4. Suppose S is a 2-sphere in $\Sigma$ and $\epsilon > 0$. There is a $\delta > 0$ such that if C is a 3-cell in $\Sigma$ whose boundary is homeomorphically within $\delta$ of S and if h is a $\delta$-homeomorphism of C onto itself which is the identity on Bd(C), then there is an $\epsilon$-isotopy of C which begins at the identity, is the identity on Bd(C), and pushes h back to the identity. (Received January 9, 1969.)

665-8. JEFFERY M. COOPER, Northwestern University, Evanston, Illinois 60201. A two point problem for evolution equations.

Let H be a separable Hilbert space with scalar product (f,g) and norm $|f|$. For each $t \in [0,T]$, $0 < T < \infty$, let A(t) be an invertible, unbounded selfadjoint operator with domain D(A(t)) dense in H. For each $t \in [0,T]$ there is an orthogonal decomposition $H = H_+(t) \oplus H_-(t)$ which reduces A(t) and such that $A_+(t) = A(t)|D(A(t)) \cap H_+(t)$ and $A_-(t) = -A(t)|D(A(t)) \cap H_-(t)$ are positive selfadjoint operators in $H_+(t)$ and $H_-(t)$. Define the invertible operator B(t) on D(A(t)) by $B(t) = A_+(t) + A_-(t)$. Now suppose (i) $t \to (A^{-1}(t)f,g) \in C^1[0,T]$ for all $f, g \in H$; (ii) $|d(A^{-1}(t)f,f)/dt| \leq 2\alpha|f|^2$, $\alpha < 1$, for all $f \in H$; and (iii) suppose that B(t) also satisfies (i). Let $P_+$ and $P_-$ be the orthogonal projections on $H_+(0)$ and $H_-(T)$ respectively. Theorem. Given $f \in L^2(0,T;H)$, $u_0 \in D(A_+^{1/2}(0))$, and $u_T \in D(A_-^{1/2}(T))$, there is one and only one function $u \in L^2(0,T; D(A(t))) \cap H^1(0,T;H)$ satisfying $A(t)u(t) + u'(t) = f(t)$ in $\mathcal{D}'(0,T;H)$, $P_+u(0) = u_0$, and $P_-u(T) = u_T$. The method employed is that of Lions (Equations différentielles opérationelles et problèmes aux limites, Die Grundlehren der math. Wissenschaften, Band 111, Springer-Verlag, Berlin, 1961, p. 127). (Received January 13, 1969.)

665-9. PETER S. LANDWEBER, Yale University, New Haven, Connecticut. Symmetric maps between spheres and equivariant K-theory.

A map $f: (S^n)^m \to S^n$ from the m-fold product of the n-sphere $S^n$ into $S^n$ is symmetric if it is invariant under all permutations in the symmetric group G = S(m), and its type is the degree of the map $x \to f(x,e,\ldots,e)$ on $S^n$. Theorem. If $n = 2t + 1$ is odd and $f: (S^n)^m \to S^n$ is symmetric of type q, then q is devisible by $m^t$ (and also by $(m - 1)^t,\ldots,2^t$ since a symmetric map of m variables restricts to symmetric maps of any smaller number of variables). A suitable generalization of the Hopf construction, applied to the symmetric map f, yields an equivariant map $g: S' \to S''$ between spheres on which G acts linearly. Moreover the restriction of g to the fixed point sets is a map g' of degree mq between n-spheres. This situation is studied by applying $K_G$-theory, enriched by the presence of Adams operations, with the conclusion that $m^{t+1}$ divides the degree of g'. The final computations are carried out in the representation ring of the symmetric group. (Received January 22, 1969.)

665-10. LEE A. RUBEL, University of Illinois, Urbana, Illinois 61801, and ALLEN L. SHIELDS, University of Michigan, Ann Arbor, Michigan 48104. The second dual of certain spaces of analytic functions.

Let $\varphi$ be a positive, real-valued, decreasing and continuous function on $0 \leq r \leq 1$ with $\varphi(1) = 0$. Let $E_0$ be the Banach space of analytic functions f on the unit disc D for which $f(z)\varphi(|z|) \to 0$ as $|z| \to 1$, with norm $\|f\| = \sup\{|f(z)|\varphi(|z|): z \in D\}$. Let E be the Banach space of analytic functions f on D such that $f(z)\varphi(|z|)$ is bounded in D, under the same norm. Theorem. $(E_0)^{**} = E$. Certain identifications underlie this assertion--the dual of $E_0$ is identified with a quotient space of the space of Borel measures in D that are absolutely continuous with respect to planar Lebesgue measure, and the dual of this space is identified with a subspace of $L^\infty(D)$. The function f is identified with the function $f(z)\varphi(|z|)$. The main tool is the smearing of measures, using a balayage technique. (Received December 23, 1968.)

665-11. MELVIN R. KROM, University of California, Davis, California 95616. The decision problem for formulas in prenex conjunctive normal form with binary disjunctions.

In the concluding section of [S. Ju. Maslov, An inverse method of establishing deducibilities in the classical predicate calculus, Dokl. Akad. Nauk. SSSR, 159 (1964), 17-20 = Soviet Math. Doklady 5 (1964), 1420-1424] there is a positive solution to the decision problem for satisfiability of formulas of the form $\exists x_1 \ldots \exists x_h \forall y_1 \ldots \forall y_k \exists z_1 \ldots \exists z_m \bigwedge_{i=1}^{n}(\alpha_i \vee \beta_i)$ in any first-order predicate calculus without identity and without function symbols, where h, k, m, n are positive integers, $\alpha_i, \beta_i$ are signed atomic formulas and $\wedge$, $\vee$ are conjunction and disjunction symbols, respectively. We show that the decision problem is unsolvable for formulas that are like those considered by Maslov except that they have prefixes of the form $\forall x \exists y_1 \ldots \exists y_k \forall z$. The proof is based on an unsolvable halting problem for a tag system. (Received January 27, 1969.)

665-12. TIMOTHY V. FOSSUM, University of Utah, Salt Lake City, Utah 84112. Central idempotents in symmetric algebras.

Let A be a K-algebra, K a field. The algebra A is a symmetric algebra if there exists an isomorphism $\varphi : A \to A^* = \mathrm{Hom}_K(A,K)$ as (A,A)-bimodules. Every symmetric algebra is a Fröbenius algebra. Set $cf(A) = \{\zeta \in A^* : \zeta(ab) = \zeta(ba)$ for all $a, b \in A\}$ and $qch(A) = \{\zeta \in cf(A) : \zeta(J) = 0\}$ where J is the Jacobson radical of A. Let K be a field, A a symmetric K-algebra with (A,A)-isomorphism $\varphi : A \to A^*$. Let $(a_i)$ and $(b_i)$ be dual bases for A. Finally assume K is a splitting field for A. Theorem. Let e be a central idempotent of A. Then $\varphi(e) \in qch(A)$ if and only if $eJe = 0$. Theorem. If A is semisimple and if $\chi$ is the character of a simple A-module M such that $\chi(1) \neq 0$, then $e = \chi(1)(\Sigma_j \chi(a_j)\chi(b_j))^{-1}\Sigma_i \chi(a_i)b_i$ is the block idempotent in A such that $eM = M$. Theorem. Assume K is an algebraic number field, and let R be the ring of algebraic integers in K. Assume $\Sigma_i Ra_i = \Sigma_i Rb_i = S$ is a subring of A. If $\Sigma_j \chi(a_j)\chi(b_j)$ is an integer then $\chi(1)^{-1}\Sigma_j \chi(a_j)\chi(b_j)$ is an integer. (Received January 27, 1969.)

665-13. MARK M. LOTKIN, 115 Hedgerow Drive, Cherry Hill, New Jersey 08034. Calculation of associated Legendre functions.

A fast and accurate method is described for the calculation of associated Legendre functions $P_n^m(u)$ of degree n, order m, for $0 \leq u \leq 1$, for the case when all the values of m and n with m = 0, 1,..., n; n = 0, 1,..., n* are to be considered. The derivatives $dP_n^m/du$ also are discussed. The need for such calculations arises in many practical applications, such as, for example, in the potential theory. Analytical expressions are provided for the behavior of the $P_n^m(u)$ and $dP_n^m/du$ at $u = 1$. (Received January 29, 1969.)

665-14. PHILIP D. WAGREICH, Institute for Advanced Study, Princeton, New Jersey 08540. Elliptic singularities of surfaces.

Suppose V is an analytic surface and $v \in V$ is an isolated singular point. Let $\pi : V' \to V$ be a resolution of the singularity and suppose $\pi^{-1}(v)$ is the union of r irreducible curves, $X_1,\ldots,X_r$. The fundamental divisor relative to $\pi$ is the unique positive divisor Z on V' such that (1) $Z \cdot X_i \leq 0$, $i = 1,\ldots,r$; (2) for any other positive divisor Z' satisfying $Z' \cdot X_i \leq 0$, $i = 1,\ldots,r$, we have $Z \leq Z'$. Theorem 1. The multiplicity of the local ring $O_v$ is greater than $-Z^2$. We say v is an elliptic singular point if the arithmetic genus $p_a(Z) = (Z^2 + Z \cdot K)/2 + 1$ is equal to 1. Theorem 2. The elliptic double points can be classified up to topological equivalence. There are, in a natural way, a finite number of (at most) countable families $\mathfrak{F}_i$. The same techniques would be sufficient to classify singularities of higher multiplicity and genus. (Received January 29, 1969.)

665-15. MARIA Z. v. KRZYWOBLOCKI, Michigan State University, East Lansing, Michigan 48823. Some aspects in existence proofs in MHD.

Blank, Friedrichs, and Grad derived existence theorems for pre-Maxwell equations using geometric, call topological, aspects of integral theorems of Gauss and Stokes which does not convince the writer that the word topology is justified. Primarily work refers to electro-magneto statics. The writers pass to fluid magnetics assuming that velocity is given a priori or may be determined by both magnetic hydro systems without proof that this is mathematically feasible and how it could be done. In 1963 the writer proposed successive iterations for existence of total possibly analogously remodelled magneto hydro systems omitting geometric considerations. Proceedings were shortened leaving only beginning and end. Blank's reviewing work indicated the lack of relations to basic value problems of fluid magnetic and how to pose conditions on successive iterates so as to yield solutions of such problems. In the present work the writer indicates that answers to Blank's remarks are known, that existence is more important than geometry, quotes important theorems from three writers, lists Lichtenstein others and his hydro existence theorems to show that the proposal of the three writers is difficult, if possible to prove now, without known iterative consequences: very small magnitudes of variables. (Received January 31, 1969.)

665-16. NICK H. VAUGHAN, North Texas State University, Denton, Texas 76203. *Some containment relations between classes of ideals and an integral domain.*

Let D denote an integral domain with $1 \neq 0$ and quotient field K. The set of primary ideals of D will be denoted by $\mathcal{Q}$ and the set of semiprimary ideals of D (i.e. ideals with prime radical) will be denoted by $\mathcal{S}$. If $\pi$ is a general ring property, then we shall call an ideal A of D a $\pi$-ideal provided there exists a $\pi$-domain J such that $D \subset J \subset K$ and $A = AJ \cap D$. The classes of Krull, Dedekind, almost Dedekind, Prufer, PID, and integrally closed ideals will be denoted by $\mathcal{K}$, $\mathcal{D}$, $\mathcal{A}$, $\mathcal{P}$, $\mathcal{PJD}$, and $\mathcal{J}$, respectively. Necessary and sufficient conditions are given in order that $\mathcal{D} \subset \mathcal{S}$, $\mathcal{J} \subset \mathcal{Q}$, $\mathcal{J} \subset \mathcal{S}$, $\mathcal{Q} = \mathcal{J}$, and $\mathcal{S} = \mathcal{J}$. In addition we prove Theorem 1. D is an almost Dedekind domain if and only if $\mathcal{Q} \subset \mathcal{A}$ and proper prime ideals of D are maximal. Theorem 2. D is an almost Dedekind domain if and only if $\mathcal{Q} \subset \mathcal{K}$ and proper prime ideals of D are maximal. Theorem 3. If D is an integral domain, then $\mathcal{PJD} = \mathcal{D}$. (Received January 31, 1969.)

665-17. JOE W. FISHER, University of Illinois, Urbana, Illinois 61801. *Decomposition theories for modules.*

Let R be an arbitrary associative ring and let all R-modules be left R-modules. The tertiary radical of an R-module M, denoted t(M), is defined to be $\{r \in R :$ there exists an essential submodule E of M with $rE = 0\}$. A submodule N of M is called tertiary if each r in R, which annihilates a nonzero submodule of M/N, lies in t(M/N). An R-module S is said to be t-stable if $S \neq 0$ and for each nonzero submodule N of S, t(N) = t(S). An ideal $\mathcal{A}$ in R is called an associated ideal of M if there exists a t-stable submodule S of M such that $\mathcal{A} = t(S)$. Denote the set of associated ideals of M by T(M). An R-module M is said to be t-worthy if each factor module M'' of M satisfies the following conditions: (a) each nonzero submodule of M'' contains a t-stable submodule, and (b) T(M'') is finite. A finite set $\{N_i : i \in I\}$ of submodules of an R-module M is a tertiary decomposition of N in M if (1) $\bigcap_{i \in I} N_i = N$ and for no $i \in I$ is $\bigcap_{j \neq i} N_j \subseteq N_i$, (2) the $N_i$, $i \in I$, are tertiary submodules of M, and (3) $t(M/N_i) \neq t(M/N_j)$ for $i \neq j$. Theorem 1. A necessary and sufficient condition that each submodule of an R-module M have a tertiary decomposition in M is that M be t-worthy. Theorem 2. If $\{N_i : i \in I\}$ and $\{M_j : j \in J\}$ are two tertiary decompositions of N in M, then $\{t(M/N_i) : i \in I\} = \{t(M/M_j) : j \in J\}$. (Received February 4, 1969.)

665-18. WITHDRAWN.

665-19. WITHDRAWN.

665-20. WILLIAM C. BROWN, Northwestern University, Evanston, Illinois 60202. Strong inertial coefficient rings.

Let R be a commutative ring with identity and Jacobson radical p. Let z be a ring homomorphism of R/p into R such that jz = 1 (j: R → R/p the natural projection). The pair (R,z) is a strong inertial coefficient ring (S.I.C.R.) if given any R algebra A, finitely generated as an R module, such that A modulo its Jacobson radical N is separable over R, then there exists an R/p algebra homomorphism z': A → A/N such that j'z' = 1 (j': A → A/N). S.I.C.R.'s yield a proper subset of inertial coefficient rings (E. Ingraham, Inertial subalgebras of algebras over commutative rings, Trans. Amer. Math. Soc. 124 (1966)) and are closed under direct sums, homomorphic images and, with slightly weaker hypotheses, under finitely generated separable extensions. Following the methods of G. Azumaya (On maximally central algebras, Nagoya Math. J. 2 (1951)), any Hensel ring R with a map z as above forms a S.I.C.R. If R is a local domain, the converse of this theorem is also true. (Received February 6, 1969.)

665-21. WILLIAM R. R. TRANSUE, Auburn University, Auburn, Alabama 36830. Upper-semi-continuous decompositions of irreducible continua.

In Abstract 660-28, these *Notices* 15 (1968), 1015, W. S. Mahavier raises the question: Is there an upper-semi-continuous decomposition G of the compact metric irreducible continuum M such that each element of G is a simple closed curve and G is an arc with respect to its elements? This question is answered in the negative. (Received February 7, 1969.)

665-22. BEN FITZPATRICK, JR., Auburn University, Auburn, Alabama 36830. Upper-semi-continuous collections of homeomorphic continua filling up an irreducible continuum.

There exist a compact, locally connected, nondegenerate metric continuum K, not an arc, and an upper-semi-continuous decomposition G of an irreducible compact metric continuum M such that each element of G is homeomorphic to K and the decomposition space is an arc. Transue [Abstract 665-21, above] shows that K can not be a simple closed curve. K may, however, contain simple closed curves, and the collection G may be taken so that no element of G has interior in M. (Received February 7, 1969.)

665-23. IN YUNG CHUNG, University of Cincinnati, Cincinnati, Ohio 45221. On free joins of algebras.

Let R be a commutative ring with 1, and all R-algebras and algebra homomorphisms are assumed to be unitary. An (a commutative) R-algebra A is called a free (commutative) join of a family $(A_\alpha)_{\alpha \in I}$ of subalgebras of A if for any (commutative) R-algebra B and any family $(f_\alpha)_{\alpha \in I}$, $f_\alpha : A_\alpha \to B$, of algebra homomorphisms, there exists a unique algebra homomorphism $f: A \to B$ extending each $f_\alpha$, $\alpha \in I$. Let M be an R-module such that $M = \Sigma_{\alpha \in I} M_\alpha$ (direct), where each $M_\alpha$ is a submodule of M. Theorem. Let an (a commutative) R-algebra A containing M as a submodule be a free (commutative) join of a family $(A_\alpha)_{\alpha \in I}$, where $A_\alpha$ is a subalgebra of A generated by $M_\alpha$ for each $\alpha \in I$. If each $A_\alpha = \sum_n A_{\alpha,n}$ (direct), n = 1,2,..., is a graded algebra such that $A_{\alpha,1} = M_\alpha$, then for any finite pairwise different $\alpha_1,\ldots,\alpha_k \in I$, the linearization $f: M_{\alpha_1} \otimes \ldots \otimes M_{\alpha_k} \to M_{\alpha_1} \ldots M_{\alpha_k}$ (in A) of the multilinear mapping $M_{\alpha_1} \times \ldots \times M_{\alpha_k} \to M_{\alpha_1} \ldots M_{\alpha_k}$ (in A) defined by $(x_1,\ldots,x_k) \to x_1 \ldots x_k$, $x_i \in M_{\alpha_i}$ is an R-isomorphism. Corollary. Let T(M) and S(M) be a tensor algebra and a symmetric algebra of M respectively. Then for any pairwise different $\alpha_1,\ldots,\alpha_k \in I$, $M_{\alpha_1} \ldots M_{\alpha_k}$ (in T(M)) $\cong M_{\alpha_1} \ldots M_{\alpha_k}$ (in S(M)) as R-module. (Received February 7, 1969.)

665-24. EUGENE W. JOHNSON and JOHN P. LEDIAEV, University of Iowa, Iowa City, Iowa 52240. Normal decomposition in systems without the ascending chain condition.

Let L denote a commutative multiplicative lattice. We say that L satisfies the successive residual condition if, for each element A in L, chains of the form $A: C_1 \leq A: (C_1C_2) \leq \ldots \leq A: (C_1C_2 \ldots C_n) \leq \ldots$ are necessarily finite. An element C ($C \neq I$) is the residual component of A by B (or simply a residual component of A) if there exists a positive integer n such that $C = A: B^n = A: B^{n+1} = \ldots$. If each element A in L has only a finite number of distinct residual components, we will say L satisfies the residual component condition. We will say that L satisfies the RCC condition if for each A in L the elements of the form A: C, where C is a residual component of A, satisfy the ascending chain condition. L satisfies the radical condition if every element contains a power of its radical. The BC condition holds in L if for all A,B in L the equality $(A: B^n) \wedge (A \vee B^n) = A$ holds for sufficiently large n. The following three statements are equivalent: (A) Every element of L has a normal decomposition. (B) L satisfies the residual component condition, the BC condition, and the radical condition. (C) L satisfies the BC condition, the successive residual condition, and the RRC condition. (Received February 11, 1969.)

665-25. WITHDRAWN.

665-26. STEPHEN M. GAGOLA, State University of New York at Buffalo, Amherst, New York 14226. A derivative theorem for polynomials.

Let p be a positive integer. Let $L(p) = \{0,1,\ldots,p-1, p\}$. Let C be the field of complex numbers. Let s(k,j,p) be the kth elementary symmetric polynomial function of the p arguments belonging to the set $L(p) - \{j\}$. By convention $s(0,j,p) = 1$ for each $j \in L(p)$. For example $s(1,0,3) = 1 + 2 + 3 = 6$ and $s(2,1,3) = (0)(2) + (0)(3) + (2)(3) = 6$. Theorem. Let $f: C \to C$ be a polynomial function of degree at most p. Then, for every $b \in L(p)$, its bth derivative is $f^{(b)}(z) = \sum [(-1)^{b+j} b!/(p-j)!j!] \cdot s(p-b, j,p) f(z+j)$. The sum in the statement of the theorem is over $j \in L(p)$. If p = 3, then the polynomial function f is at most cubic. Its first three derivatives are $f'(z) = -11 f(z)/6 + 3f(z+1) - 3f(z+2)/2 + f(z+3)/3$. $f''(z) = 2f(z) - 5f(z+1) + 4f(z+2) - f(z+3)$. $f'''(z) = -f(z) + 3f(z+1) - 3f(z+2) + f(z+3)$. The proof follows from a combinatorial identity which is too cumbersome to state here. The theorem generalizes readily to polynomial maps between Banach spaces. (Received February 11, 1969.)

665-27. JOHN A. BEEKMAN, Ball State University, Muncie, Indiana 47306. Green's functions for generalized Schroedinger equations. II.

Let $\{X(w), s \leq w \leq t\}$ be a Gaussian Markov stochastic process with continuous sample functions, and such that $X(s) = x$, $X(t) = y$, with probability one. For appropriate $F[X]$'s, "complex sequential" integrals are defined in terms of finite-dimensional Riemann integrals, and denoted by $E^s_\lambda\{F[X]|X(s) = x, X(t) = y\} \equiv I$, $\text{Re}\ \lambda \geqq 0$, $\lambda \neq 0$. I is proved to be related to the Fourier transform of $E\{G[X;\lambda]|X(s) = 0\}$ for an appropriate $G[X;\lambda]$, and also to a Wiener integral conditioned by $X(0) = 0$, $X(1) = 0$. The latter result would be helpful in Monte Carlo approximations. Appropriate I's satisfy generalized Schroedinger equations and Dirac delta function conditions. Such equations (and hence the allied I's) are approximated by finite difference schemes. Five potentials for the Wiener and Ornstein-Uhlenbeck processes are considered. (Received February 11, 1969.)

665-28. DAVID KENT COHOON, Bell Telephone Laboratories, Whippany, New Jersey 07981. Nonexistence of a continuous right inverse for linear partial differential operators with constant coefficients.

Partial differential operators $P(D)$ with constant coefficients are considered to be linear transformations of $C^\infty(\Omega)$ onto itself, where $\Omega$ is a $P(D)$-convex open subset of $R^n$, where n denotes the number of dependent variables of $P(D)$. It is shown that if the dimension of the space spanned by the characteristics of $P(D)$ is $n - 2$ or less, then $P(D)$ can have no continuous right inverse on $C^\infty(\Omega)$ for any open subset $\Omega$ of $R^n$. Furthermore, if the dimension of the vector space V spanned by the characteristic directions of $P(D)$ is $n - 1$ and $P(D)$ is not hyperbolic in a direction N that is orthogonal to the hyperplane V, then $P(D)$ has no continuous right inverse in $C^\infty(\Omega)$ for any open subset $\Omega$ of $R^n$. (Received February 11, 1969.)

665-29. WITHDRAWN.

665-30. JAMES D. BUCKHOLTZ, University of Kentucky, Lexington, Kentucky 40506. Zeros of partial sums of power series. II.

Let $B_0(z) = 1$, $B_1(z;z_0) = z - z_0$, and define the polynomial sequence $B_n(z;z_0,\ldots,z_{n-1})$ recursively by $B_{n+1}(z;z_0,\ldots,z_n) = zB_n(z;z_1,\ldots,z_n) - z_0B_n(z_0;z_1,\ldots,z_n)$, $n = 1,2,3,\ldots$ . Set $H_n = \max|B_n(0;z_0,\ldots,z_{n-1})|$, where the maximum is taken over all sequences $\{z_j\}_0^{n-1}$ whose terms lie on the unit circle. Let $P = \lim\sup H_n^{1/n}$ $(n \to \infty)$. Theorem 1. If $f(z) = \sum a_n z^n$ has convergence radius 1 or less and $\epsilon > 0$, then all the zeros of infinitely many partial sums of f lie in the disc $|z| < P + \epsilon$. Theorem 2. There is an analytic function $F(z) = \sum A_n z^n$ with convergence radius 1 such that every partial sum of F has a zero on the circle $|z| = P$. Theorems 1 and 2 settle a question raised by J. Clunie and P. Erdős [W. K. Hayman, Research problems in function theory, Athlone Press, University of London, London, 1967, Problem 717]. (Received February 11, 1969.)

665-31. KYONG T. HAHN and JOSEPHINE M. MITCHELL, Pennsylvania State University, University Park, Pennsylvania. $H^p$ spaces on the classical Cartan domains.

Let D denote one of the 4 types of the classical Cartan domains. The domain D possesses an invariant metric with respect to the group of holomorphic automorphisms of D and an invariant Laplacian $\Delta_D$. The Hardy class $H^p$ for $p > 0$ is defined to be the class of functions f which are holomorphic in D and $f_r(z) \equiv f(rz)$ are bounded in the $L^p$-norm for all r, $0 \leqq r < 1$. Following W. Rudin [Trans. Amer. Math. Soc. 78 (1955), 46-66], the authors define a class $\widetilde{H}^p(D)$ of functions f which are holomorphic and $|f|^p \leq h$ on D, where h is the Poisson integral of a Lebesgue integrable function on the Bergman-Šilov boundary b of D. It is shown that the two classes $H^p$ and $\tilde{H}^p$ ($p > 0$) are equivalent for the classical Cartan domains D. The methods used here depend on Harnack's inequality and theorem, the theory of plurisubharmonic functions and the potential theory of classical Cartan domains. The authors further generalize the results obtained by S. S. Walters in [Proc. Amer. Math. Soc. 6 (1950), 800-805]. The most of the results obtained for the classical Cartan domains may be extended to the bounded symmetric domains. (Received February 12, 1969.)

665-32. STANISLAS L. KLASA, Northern Illinois University, De Kalb, Illinois 60115. Deformation and cohomology of coalgebras.

An 8 year old construction of M. Gerstenhaber of the graded Lie algebra (GLA) associated with an (associative) algebra and given also for Lie algebras by Nijenhuis-Richardson is presented here for (coassociative) coalgebras. Let V be a module over a commutative ring k. Define the graded module $L = (L^n)_{n \in \mathbb{N}}$ by $L^n = \mathrm{Hom}_k\ (V, \otimes^{n+1} V)$. Define on L a bilinear product $* : L^n \times L^p \to L^{n+p}$ by $(f*g)(x) = \sum_{\iota} \sum_{i=0}^{n} (-1)^{ip} x_{0,\iota} \otimes x_{1,\iota} \otimes \ldots g^i(x) \otimes \ldots \otimes x_{n,\iota}$; $f \in L^n$, $g \in L^p$; the commutator of * defines on L a GLA structure. As in the classical cases we deduce from it a cohomology of coalgebras. For it we have the Theorem. The set of all coassociative coalgebra structures on V is $\{\delta \in L^1;\ \delta * \delta = 0\}$. If V is f.g. free module we obtain in particular its description as an algebraic subset of $L^1$. If $\delta$ is a coassociative comultiplication on V the coboundary operator is the adjoint ad $\delta$. As in the Gerstenhaber and Nijenhuis-Richardson theories the elements of the group $H^1(C,C)$ can be looked as infinitesimal deformation of $C = (V, \delta)$. Remark. The cohomology obtained is with coefficients in C; replacing C by any C-comodule M we can use 'the same' formula in the general case. (Received February 12, 1969.)

665-33. JACQUELINE L. KLASA, Northern Illinois University, De Kalb, Illinois 60115. Semisimplicity and von Neumann's regularity in categories.

P. Dubreil's exposè (Sém. Dubreil-Pisot, 1964-1965, # 23) mentioned a theorem of Valuce concerning regularity of certain semigroups of endomorphisms. Using the technique of J. Klasa (Equivalences de Green dans les catégories bien filtrantes, C. R. Acad. Sci. Paris, 266, p. 901-903 and Sem. Dubreil-Pisot, 1966-1967, # 17) we generalize it to the following: For a well filtering category, that is a category for which each morphism $\alpha$ can be written (not necessarily uniquely) as $\alpha = \theta\sigma$ where $\sigma$ and $\theta$ are epi- and monomorphisms, the semigroup of endomorphisms of any semi-simple object is regular. Conversely there exist certain well filtering categories (that we characterize) for which an object with a regular semigroup of endomorphisms has to be semisimple. It is also possible to characterize in these well filtering categories, projective, injective and free objects. (Received February 12, 1969.)

665-34. WALTER L. BAILY, JR., University of Chicago, Chicago, Illinois 60637. On Eisenstein series for an exceptional arithmetic group.

One constructs a certain arithmetic subgroup of the real form of $E_7$ acting on a bounded symmetric domain of 27 dimensions, and considers the Eisenstein series attached to this group acting on the realization of this domain as a tube domain. One proves that these have rational

Fourier coefficients with a certain Euler product expansion. This raises some natural questions as to the interpretation of these Fourier coefficients. Further details will appear in Bull. Amer. Math. Soc. and in subsequent publications. (Received February 12, 1969.)

665-35. SHREERAM SHANKAR ABHYANKAR, Purdue University, Lafayette, Indiana 47907.
Quasirational points of algebraic surfaces.

Let R be an algebraic surface over an algebraically closed ground field of characteristic zero. Let P be a point of R. P is said to be a quasirational point of R if for every birational map $f: T \to R$ (without fundamental points on T) it is true that every irreducible curve in $f^{-1}(P)$ is rational. The following sufficient condition for quasirationality is obtained. Assume that R can be projected, locally near P, onto a surface S so that the image Q of P is a simple point of S, the local Galois group is a cyclic group of some order d, the branch locus on S at Q is analytically irreducible and has only one characteristic pair (m,n), and the integers d and mn are coprime. Then P is a quasirational point of R. (Received February 12, 1969.)

665-36. SATOSHI ARIMA, State University of New York at Buffalo, Amherst, New York 14226.
Canonical systems on double planes.

k is an algebraically closed subfield of the field C of complex numbers. A ruled surface over k is a nonsingular projective algebraic surface which is birationally equivalent over k to the product of the projective line $P_1$ and a curve B. A nonsingular surface S is a double ruled surface over k if there is a rational mapping over k of S to a ruled surface $P_1 \times B$ of degree 2; S is a double plane over k, if $B = P_1$. A double ruled surface has a pencil of lines, or of elliptic curves, or of nonsingular hyperelliptic curves on it. Theorem. Let S be a nonsingular projective surface of geometric genus $p \geqq 2$, and $\pi: S \to B$ be a pencil of hyperelliptic curves on S, k being a field of definition for S, B and $\pi$. (Therefore S is a double ruled surface over k, i.e. $\exists f: S \to P_1 \times B$, deg f = 2.) Then the rational mapping $\Phi_K: S \to S_K$ associated with the canonical system |K| is as follows: (i) $S \xrightarrow{f} P_1 \times B \to S_K$, $S_K$ is a ruled surface over C, or (ii) $S_K$ is a curve of geometric genus 0, or (iii) $S \xrightarrow{\pi} B \to S_K$. Corollary. If S is a double plane over k of geometric genus $p \geqq 2$, then the image of the rational mapping $S \to S_K$ associated with the canonical system |K| is birationally equivalent over k to the projective space of dimension 1 or 2. (Received February 12, 1969.)

665-37. STEPHEN S. SHATZ, University of Pennsylvania, Philadelphia, Pennsylvania 19104.
Group schemes and Galois theory.

We study the problem of constructing a fundamental group scheme for a given connected scheme. When our base scheme is the spectrum of a field, we obtain a profinite group scheme in a canonical way. Its connected component of identity is associated to the maximal separable extension, so that the quotient is the ordinary profinite Galois group of the field. This construction gives a Galois theory for fields in a form suitable for generalization. It includes Jacobson's theory and Sweedler's results, obtained by different methods. A start is made toward the general case by these techniques, and connections are established between cohomological phenomena over a field and its fundamental group scheme. (Received February 12, 1969.)

665-38. DAVID HERTZIG, Purdue University, Lafayette, Indiana 47907. Cohomology of Steinberg-Chevalley groups.

Steinberg has given a presentation of the universal Chevalley groups G and of their universal central extensions $\Gamma$ over a field K in terms of generators and relations. There is a natural action of these groups on the corresponding Lie algebras $\mathfrak{G}$. The author has developed a technique for the computation of the 1-cohomology $H^1(\Gamma,\mathfrak{G})$ and $H^1(G,\mathfrak{G})$ which depends on determining the cocycles on one-parameter subgroups and then using the relations to piece together the cocycles on the whole group. With few exceptions (very small fields and low rank) the 1-cohomology turns out to be $\mathcal{D}(K) \oplus \mathfrak{Z}(\mathfrak{G})$ where $\mathcal{D}(K)$ is the module of derivations of K and $\mathfrak{Z}(\mathfrak{G})$ is the center of $\mathfrak{G}$. In particular, if K is a finite field the 1-cohomology is almost always trivial. (Received February 12, 1969.)

665-39. KENNETH P. BOGART, Dartmouth College, Hanover, New Hampshire 03755. Nonimbeddable Noether lattices.

A multiplicative lattice is said to have the trivial multiplication if the product of any two nonunity elements is zero. A lattice with the trivial multiplication is a Noether lattice if and only if it is a finite-dimensional modular lattice in which every element except the unity is a join of atoms. This result is used to prove the existence of Noether lattices which cannot be imbedded in the lattice of ideals of any Noetherian ring. It is also used to prove that if M is a proper maximal element of a Noether lattice, then the quotient sublattice A/AM is a complemented modular lattice for all A in L. (Received February 13, 1969.)

665-40. JACK W. ROGERS, JR., Emory University, Atlanta, Georgia 30322. On mapping indecomposable continua onto certain chainable indecomposable continua.

For each $n > 1$, let $w_n$ denote the map from $[0,1]$ onto $[0,1]$ such that (1) if $0 \leq i \leq n$, then $w_n(i/n)$ is 0 if i is even and 1 if i is odd, and (2) $w_n$ is linear on the interval $[i/n, (i + 1)/n]$ for each i ($0 \leq i < n$). Let V denote the collection of all continua (compact connected metric spaces) homeomorphic to a limit of an inverse sequence $\{I_i, f_i\}$, where for each i, $I_i = [0,1]$ and $f_i = w_n$ for some positive integer n. The elements of V are particularly simple chainable indecomposable continua, which somewhat resemble solenoids. But we show, contrary to the case for solenoids, Theorem. Each element of V can be mapped onto any element of V. Indeed, a stronger statement is true. Theorem. Each element of V is a continuous image of every indecomposable continuum. (Received February 13, 1969.)

665-41. BILLY F. HOBBS, Olivet Nazarene College, Kankakee, Illinois 60901. Subgroup topologies and convergence in product groups.

Let H be any discrete abelian group, and let $G = H^N$ be the product group of countably many copies of H. For each coordinate m in N and each $h \neq 0$ in H, a nondiscrete Hausdorff group topology t(m,h) is constructed on G so that t(m,h) is also a subgroup topology (i.e., there exists a t(m,h)-base at 0 in G consisting of subgroups), and so that there is a sequence $(x^k)_{k \in N}$ in G t(m,h)-convergent to 0 in G, while the (projected) sequence $(x^k_m)_{k \in N}$ in the mth coordinate space H converges to h in H (in the discrete topology). Also, a nondiscrete Hausdorff group topology t strictly stronger than the product topology is constructed on G so that t is a subgroup topology, and so that there is a sequence

$(x^k)_{k \in N}$ in G, where $x_i^k = 0$ for all $i < k$ ($k \in N$), that is not t-convergent to 0 in G. (Received February 13, 1969.)

665-42. NARENDRA K. GOVIL, and Q. I. RAHMAN, Loyola College of Montreal, Montreal-262, Quebec, Canada. Functions of exponential type not vanishing in a half plane.

Generalizing a theorem of Professor R. P. Boas, Jr. [Illinois J. Math. 1 (1957), 94-97] we prove the following: Theorem. Let f(z) be an entire function of exponential type $\tau$ having all its zeros in Im $z \leqq k \leqq 0$. If $h_f(\pi/2) = 0$, $h_{f'}(\pi/2) \leqq -c_1 < 0$ and also $h_{g'}(\pi/2) \leqq -c_1 < 0$ where $g(z) = e^{i\tau z}\,\overline{f(\bar{z})}$ then $|f(x)| \leqq 1$, for real x implies $|f'(x)| \leqq \tau/(1 + e^{c_1|k|})$, $-\infty < x < \infty$. The estimate is sharp. Here $h_f(\theta)$ stands for Phragmen-Lindelöff indicator function of f(z). (Received February 13, 1969).

665-43. ARLINGTON M. FINK, Iowa State University, Ames, Iowa 50010. Extensions of almost automorphic sequences.

We ask when an almost automorphic sequence can be extended to an almost automorphic function on the reals. We give a general description of such extensions and this leads to Theorem. Let $F(t + 1, x) = F(t,x)$ for all real t and n-vectors x, with F continuous. If solutions to initial value problems of $x' = F(t,x)$ are unique, then a solution $\varphi$ defined on R is compact almost automorphic if and only if $\varphi(n)$ is an almost automorphic sequence. (Received February 13, 1969.)

665-44. CASPER GOFFMAN, Purdue University, Lafayette, Indiana 47907. An example in surface area.

For any continuous mapping f from 2 space into n space, $n \geqq 2$, with Lebesgue area A(f), if the classical integral formula is defined on a set E and has value I(f; E), then $A(f) \geqq I(f; E)$. An example of a mapping from the 3 cube into itself is given for which this does not hold. (Received February 13, 1969.)

665-45. WAYLAND M. HUBBART, University of Delaware, Newark, Delaware 19711. Blocks of modular representations over nonsplitting fields. Preliminary report.

Let F be an algebraic number field and let T be an irreducible F representation of a finite group G. Let $\overline{F}$ be a residue field of F taken with respect to a prime p. If T remains irreducible under extension of the ground field to a p adic completion of F, then it is possible to associate T with an $\overline{F}$ block of G. Theorem. The number of $\overline{F}$ blocks of highest defect is equal to the number of p regular $\overline{F}$ conjugacy classes of highest defect. The proof of the theorem uses a lifting argument together with a permutation lemma of Brauer. (Received February 13, 1969.)

665-46. FRANK LEVIN, Rutgers University, New Brunswick, New Jersey 08903. On the laws of free nilpotent groups.

Let $F_k(N_c)$ denote a free nilpotent group of class $c \geqq 3$ and rank k. As is known, $F_{c-1}(N_c)$ satisfies only the nilpotent law while $F_{c-2}(N_c)$ satisfies other laws. We exhibit here a basis for the laws of $F_{c-2}(N_c)$ which are not laws of $F_{c-1}(N_c)$ which consists of a single law (which is not a product of more than one nontrivial law), for odd $c \geqq 5$. For even $c \geqq 4$, the law will determine the $c/2$ powers of these laws. (Received February 13, 1969.)

665-47. J. PETER MAY, University of Chicago, Chicago, Illinois 60637. The Dyer-Lashof algebra and $H_*(BF)$.

The mod p homology $H_*(B)$ of an infinite loop space B is a Hopf algebra over both the Steenrod algebra A and the Dyer-Lashof algebra R; the latter is defined in terms of certain homology analogs of the Steenrod operations; commutation formulas relating the A and R operations are known. R is a Hopf algebra and A-coalgebra; its structure and that of its dual have been determined. There is a notion (analogous to that of unstable A-algebra) of allowable AR-Hopf algebra. Let $QX = \varinjlim \Omega^n S^n X$. If X is connected, $H_*(QX)$ is the completely free allowable AR-Hopf algebra generated by $\widetilde{H}_*(X)$. $H_*(QS^0)$ is also explicitly computed as an AR-Hopf algebra; it contains R as a sub A-coalgebra and is generated by R and $H_0(QS^0)$. Let $\widetilde{F} = \varinjlim \mathrm{Hom}(S^n, S^n)$. $\widetilde{F} = QS^0$ as a space and is a topological monoid under composition of maps. $\widetilde{F}$ operates on B and $H_*(B)$ is a Hopf algebra over $H_*(\widetilde{F})$; there are commutation formulas relating the $H_*(\widetilde{F})$ operations to the A and R operations. These formulas, applied to $B = QS^0$, determine $H_*(\widetilde{F})$ as an algebra under the composition product. Boardman and Vogt have shown that F (maps of degree $\pm 1$ in $\widetilde{F}$) and BF are infinite loop spaces under Whitney sum, which, on F, is homotopic to the composition product. This fact is enough to allow explicit algebraic computation of $H_*(F)$ as an AR-Hopf algebra, and this in turn allows explicit computation of $H_*(BF)$ as an AR-Hopf algebra (for all primes p). (Received February 13, 1969.)

665-48. HOWARD H. WICKE, Sandia Laboratories, Albuquerque, New Mexico 87115. On a problem of Arhangel'skiĭ.

Arhangel'skiĭ defined a space of point-countable type as a space which can be covered by a family of bicompact subsets of countable character. (A subset A is of countable character if and only if there is a sequence $D_1, D_2, \ldots$ of open sets including A such that any open set including A includes some $D_n$.) [Trudy Moskov. Mat. Obšč. 13 (1965), 3-55 = Trans. Moscow Math. Soc. 1965, 1-62]. The following theorem is based on joint work of the author and Dr. J. M. Worrell, Jr. Theorem 1. Suppose X is a Hausdorff space of point-countable type. Then X is the range of an open continuous mapping $\varphi$ such that: (1) The domain Y of $\varphi$ is a Hausdorff paracompact p-space. (2) The weight of Y is the weight of X. (3) Y is a subspace of the product space of a zero-dimensional metrizable space and X. A consequence of this is Theorem 2. Suppose X is a Hausdorff space. Then X is of point-countable type if and only if X is the range of an open continuous (in general, many-valued) range-bicompact mapping of a metrizable space. Arhangel'skiĭ proved this (loc. cit.) under the additional hypothesis that X is a Tychonoff space and asked whether it is valid for a wider class of spaces. (Received February 13, 1969.)

665-49. RICARDO B. QUINTANA, JR., University of Wisconsin, Madison, Wisconsin 53706. On groups of exponent four with four generators.

Theorem. Let G be the freest group of exponent four on four generators. Then $G_8$, the eighth term in the descending central series, is contained in the intersection of the derived groups $N(a)'$, where $N(a)$ is the smallest normal subgroup of G containing a and $a \in G - \varphi(G)$. The method of proof is to imbed G homomorphically into the group of endomorphisms of N(a)/N(a)! for each such a, as follows: let ZG denote the group ring over the integers and let $S_4(G)$ denote the ideal of ZG generated by the elements $1 + x + x^2 + x^3$, $x \in G$, together with a - 1. We show that if a,x,y,z are generators of G, then in ZG all products in x - 1, y - 1, z - 1, of degree 7 are congruent to zero modulo $S_4(G)$ and, moreover, that 7 is the least degree with this property. This establishes the index of nilpotency of ZG modulo $S_4(G)$ as 7 and proves the result. (Received February 13, 1969.)

665-50. SHASHANKA SHEKHAR MITRA, Wilkes College, Wilkes-Barre, Pennsylvania 18703. Continuous open maps on the closed unit interval [0,1].

It is proved in this paper that the existence of a continuous open map from the closed unit interval [0,1] to a nondegenerate $T_2$ space X implies that [0,1] and X are homeomorphic. It has also been shown that every continuous open map from [0,1] onto X is necessarily a piecewise homeomorphism in the sense that there exists a partition $0 = a_0 < a_1 < \dots < a_n = 1$ of [0,1] such that $f|[a_i, a_i + 1]$ is a homeomorphism onto X. (Received February 13, 1969.)

665-51. ALLEN R. STRAND, Muskingum College, New Concord, Ohio 43762. Phase plane analysis for finite difference equations.

Phase space analysis in Euclidean 2-space is developed for a first order, autonomous system of finite difference equations with constant coefficients. Theorems showing the uniqueness or periodicity of a phase path, which is a sequence of points in the Euclidean plane, are given. For graphical clarity the continuous flow of a particular system of differential equations is employed such that the path of the difference system and the continuous flow coincide at all integer values of the independent variable of each system. (Received February 13, 1969.)

665-52. E. B. HIBBS, JR., University of Texas, Austin, Texas 78712. Concerning approximate slopes for a given class of functions of bounded variation.

Let F be the collection such that f is an element of F iff f is a real, single-valued function of x, defined and of bounded variation on I = [a,b] with total variation V and length L over I. (1) For $V \neq 0$ there exists a unique x' such that $L = (x' - a) + [(b - x')^2 + V^2]^{1/2}$. Let S be the set to which s belongs iff for each [p,q] in I $|f(q) - f(p)/q - p| \leq s$ for some f in F (re: Lip 1). (2) For $x' \neq b$, (V/b - x') is the glb of S. Let F' be the subcollection of F such that each f in F' is also differentiable on I. (3) If f is in F' and y is in [(V/b - a), (V/b - x')] there exists an x in I such that f'(x) = y (re: Darboux theorem). Let S' be the subset of S corresponding to F'. (4) (V/b - x') is the glb of S' and is not in S'. (Received February 13, 1969.)

665-53. DAVID P. BELLAMY, University of Delaware, Newark, Delaware 19711. Continuous preimages of the cone over the Cantor set.

Definitions. A topological space is almost connected im kleinen at $x \in X$ iff for every open set U containing x, there is a continuum $W \subseteq U$ such that $Int(W) \neq \emptyset$. If X is a compact Hausdorff space, and $A \subseteq X$, T(A) is that set of points which have no closed connected neighborhood missing A. (These concepts are due to H. S. Davis.) C will denote the cone over the Cantor set and $v \in C$ its vertex. Proposition 1. If a compact Hausdorff continuum X fails to be almost connected im kleinen at some point x, there exists a continuous surjection $f: X \to C$. A counterexample to the converse of this result is given. Proposition 2. If S is a compact Hausdorff space and $f: S \to C$ is a continuous surjection, then $f(Tf^{-1}(v)) = C$. (Received February 13, 1969.)

665-54. PETER HOFFMAN, University of Waterloo, Waterloo, Ontario, Canada. On the realizability of Steenrod modules.

A Steenrod module M is a graded module over the mod p Steenrod algebra, where p is any prime. Define the rank of M to be its dimension as a vector space over the field with p elements. Such a module is realizable if there is a space whose mod p cohomology is isomorphic to M by some isomorphism which may change degree. Using a stable version of the K-theoretic method of Adams and Atiyah in their proof of the Hopf invariant result, it is shown that for any p there exist only finitely many isomorphism classes of modules of any given finite rank, each of which can be realized by a space with p-torsion free integral cohomology, and none of which is a direct sum of a pair of such modules of smaller rank. Using a spectrum constructed by Brown and Peterson, a method for realizing certain such modules is described. (Received February 5, 1969.)

665-55. MARK E. MAHOWALD, Northwestern University, Evanston, Illinois 60201. Higher order decomposition formulae for $Sq^2$.

Adams has proved that for each $i \geqq 4$ there is a representation of $Sq^{2^i}$ as a secondary cohomology operation. Indeed, if $Sq^{2^j}$, $j < i$, are all killed then certain secondary operations $\varphi_{j,k}$ are defined and $Sq^{2^i} = \Sigma a_{jk}\varphi_{jk}$ where $a_{jk}$ are in the Steenrod algebra. If, instead of killing all $Sq^{2^j}$, $j < i$, something less is done, then it should be possible to represent $Sq^{2^j}$ as tertiary and even higher order operations. The following can be proved. Theorem. For each integer $k < i$, $i \neq 4$ and $i > 2$ there is a formula $Sq^{2^i} = \Sigma a_j \Psi_j$ where $\Psi_j$ are k-order operations. It should be remarked that in general $\Psi_j$ are not basic operations in that they appear in $\mathrm{Tor}_A^{k,t}(Z_2, Z_2)$ as do the Adams operations $\varphi_{i,j}$. (Received February 13, 1969.)

665-56. FRANCIS W. CARROLL, 231 W. 18th, Columbus, Ohio 43210, and FREDERICK S. KOEHL, 1971 Neil, Columbus, Ohio 43210. Orlicz spaces have the weak difference property.

Woyczyski [Indag. Math. 30 (1968), 95-100] proved: If G is a compact ([1] monothetic group) and if $\varphi$ is a Young function ([2] which satisfies a $\Delta_2$-condition), then the Orlicz space $L_\varphi$ has the weak difference property. This result is improved by replacing [1] by [1'] (group for which $L^1(G)$ has the weak difference property, in particular, for any compact abelian group); [2] is not needed. In view of [1'] and the fact that $L^1$ contains $L_\varphi$, it suffices to show that if g is in $L^1$ and has right differences, $\Delta_h g$, in $L_\varphi$, then g is in $L_\varphi$. g can be assumed to have integral zero. Let $L(h) = \|\Delta_h g\|_{M\varphi}$ be the Luxemburg norm of $\Delta_h g$. Then L is subadditive, and is measurable from Tonelli's theorem. A standard technique shows that L is bounded on G, i.e. for some N and all $h \in G$, $\int \varphi(1/N|g(xh) - g(x)|)dx \leqq 1$. But $|g(x)| = |\int(g(xh) - g(x))dh|$. Then by Jensen's integral inequality, g has norm at most N, and therefore lies in $L_\varphi$. (Received February 14, 1969.)

665-57. ALLAN J. SILBERGER, Bowdoin College, Brunswick, Maine 04011. Commutativity of certain algebras. Preliminary report.

Let $G = SL(2,\mathbb{Z}/(p^n))$, $n > 0$. Let B be the (upper) unipotent subgroup of G. B is isomorphic to the additive group $\mathbb{Z}/(p^n)$. Let $\mu$ be a primitive character of B. Let $A_\mu = \{\Psi: G \to C \mid \Psi(bgb') = \mu(bb')\Psi(g)$, for all $b, b' \in B$, $g \in G\}$. $A_\mu$ is an algebra under convolution on G. Theorem. $A_\mu$ is commutative. See I. M. Gelfand and M. I. Graev, Categories of group representations and the problem of classifying irreducible representations, Soviet Math. Dokl. 3 (1962), 1378-1381, for the case n = 1 or Appendix B of the author's thesis at Johns Hopkins for the general case. The author has also proved this theorem by a direct computation. The minimal idempotents in $A_\mu$ entail functional equations involving Kloosterman's sums. (Received February 14, 1969.)

665-58. WITHDRAWN.

665-59. JULIUS M. ZELMANOWITZ, University of California, Santa Barbara, California 93105. Commutative endomorphism rings. II.

In what follows R is a not-necessarily-commutative ring without zero divisors and M is a left R-module. We define L(M) to be the two-sided ideal of $E(M) = \mathrm{Hom}_R(M,M)$ generated by all homomorphisms of the form $x \mapsto f(x)m$ for some $f \in \mathrm{Hom}_R(M,R)$ and some $m \in M$. Let I(M) be the two-sided ideal of R generated by $\{f(m): f \in \mathrm{Hom}_R(M,R), m \in M\}$. Lemma. If L(M) is nonzero and commutative, then I(M) is contained in the center of R. Theorem 1. If L(M) is nonzero and commutative, then R is a commutative integral domain. Theorem 2. Suppose M is a nontrivial torsionless left R-module with E(M) commutative. Then R is commutative, E(M) is an integral domain, and M is isomorphic to an ideal of R. The above results generalize some of those announced in Abstract 655-30, these *Notices* 15 (1968), 474. (Received February 14, 1969.)

665-60. FRANK A. SMITH, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213. Extensions of partial orders on semigroups.

Let $(S, +, \leq)$ be a partially ordered abelian semigroup with neutral element 0. Theorem 1. If $a, b \in S$, we may extend $\leq$ to an order $\leq^*$ such that $b \leq^* a$ if and only if whenever there is a sequence $x_0, x_1, \ldots, x_n \in S$ such that $x_0 \leq b + x_1$, $a + x_1 \leq b + x_2, \ldots, a + x_n \leq x_0$, we have $x_0 = b + x_i = a + x_i$ for $1 \leq i \leq n$. We also prove the following theorems. Theorem 2. If $(S, +, \leq)$ is a partially ordered idempotent semigroup with neutral element 0, we may extend $\leq$ to a total order $\leq^*$ if and only if $p + q \in \{p,q\}$ for all $p, q \in S$. Theorem 3. If $(S, +, \leq)$ is a partially ordered cancellable abelian semigroup with neutral element 0, we may extend $\leq$ to a total order if and only if $na = nb$ for some $n > 0$ implies $a = b$. (Received February 14, 1969.)

665-61. S. E. HAYES, University of Texas, Austin, Texas 78712. Change of variable in Stieltjes integrals.

All integrals are Riemann-Stieltjes integrals of real functions, defined by a refinement limit. Let $\theta$ be a function whose domain is the interval $[\alpha,\beta]$ and whose range is a subset of the interval $[a,b]$. Theorem 1. The following two statements are equivalent to each other: (i) For any functions f and g such that $\int_a^b fdg$ and $\int_\alpha^\beta f[\theta]dg[\theta]$ exist, $\int_{\theta(\alpha)}^{\theta(\beta)} fdg = \int_\alpha^\beta f[\theta]dg[\theta]$. (ii) For any two numbers

$\xi_1$ and $\xi_2$ in $[\alpha,\beta]$ and any subinterval $[c,d]$ of $[a,b]$ lying between $\theta(\xi_1)$ and $\theta(\xi_2)$, there exists a number $\zeta$ between $\xi_1$ and $\xi_2$ such that $\theta(\xi)$ is in $[c,d]$. Theorem 2. The following two statements are equivalent to each other: (i) For any functions f and g such that $\int_a^b f dg$ exists, $\int_\alpha^\beta f[\theta]dg[\theta]$ also exists. (ii) The function $\theta$ is continuous and there exists a positive integer n such that $\theta$ does not cross any horizontal strip more than n times. Theorem 3. The following two statements are equivalent to each other: (i) For any function f such that the Riemann integral $\int_a^b f$ exists, $\int_\alpha^\beta f[\theta]d\theta$ also exists. (ii) $\theta$ is continuous and of bounded variation. The author has obtained several other theorems similar in form to those above. (Received February 14, 1969.)

665-62. WALTER J. SCHNEIDER, Syracuse University, Syracuse, New York 13210. A short proof of a conformal mapping theorem of Matsumoto.

Many authors have considered the problem of how small a set S on C ( = $\{|z| = 1\}$) can be taken onto how large a set L on a Jordan curve J by the natural extension of a conformal map which takes $|z| < 1$ onto Int J. The strongest known result [Matsumoto 1964] states that S can be a closed set of capacity zero and that L can have positive area. A very short proof of this can be obtained by (i) letting R be a Cantor set of positive area and constructing a sequence $T_n$ of closed Jordan arcs (mutually disjoint from R and each other) which converges to R and such that no rectifiable arc which misses $\bigcup_{n=1}^{\infty} T_n$ can terminate at any point of R (such a construction is obviously possible since the complement of R is a plane domain and as such can be exhausted by a sequence of Jordan configurations), (ii) running a Jordan curve J through $R \cup [\bigcup_{n=1}^{\infty} T_n]$ (this is possible by the Kline-Moore Threading Theorem), (iii) applying a theorem of Beurling which states that a closed set E ($\subset$ C) of end points of radii, whose images under a conformal map are nonrectifiable, is of capacity zero. (Received February 14, 1969.)

665-63. CHIA-VEN PAO and WILLIAM G. VOGT, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. On the stability of nonlinear operator differential equations, and applications.

Consider the nonlinear operator differential equation (i.e., equation of evolution) (*) $dx(t)/dt = Ax(t) + f(x(t))$ ($t \geqq 0$) where A is a linear (unbounded) operator with domain and range both in a real Hilbert space H and f is a (nonlinear) function defined on H into H. The object of this paper is to investigate the existence, the uniqueness and the stability or asymptotic stability of solutions to (*) by using nonlinear semigroup properties. Criteria on A and on f for the generation of a contraction or negative contraction semigroup are established from which the existence, uniqueness, stability and asymptotic stability of solutions of (*) are insured. Applications are given to the second order partial differential equation of the form $\partial u/\partial t = \Sigma_{i,j=1}^{n} (\partial/\partial\xi_i)(a_{ij}(\xi))\partial u/\partial\xi_j + c(\xi)u + f(u)$, $\xi \in \Omega \subset R^n$. Criteria in terms of the coefficients $a_{ij}(\xi)$, $c(\xi)$ and of the function f are obtained. (Received February 14, 1969.)

665-64. PAUL O. FREDERICKSON, Case Western Reserve University, Cleveland, Ohio 44106. Triangular hermite interpolation in the plane.

The problem of Hermite interpolation in the plane has had thorough study for the case of rectangular partitions. The corresponding problem for case of triangular partitions, however, has had very little study, even though it would seem to have certain advantages in some situations. We propose a solution for the case of the most regular sort of triangular partition, one in which either (a) the triangles are all equilateral, with the exception of partial triangles along the boundaries as needed, or (b) the partition is an affine image of one of the first kind. The solution for the first order case is a differentiable function $\bar{f}$ which is a polynomial of degree four on each triangle, such that $\bar{f}$ and $D\bar{f}$ are as prescribed at each vertex. Two basic polynomials are given which, when translated and rotated, serve as a basis for the restriction of $\bar{f}$ to any triangle. These two polynomials are the unique polynomials of degree four with this property. (Received February 14, 1969.)

665-65. LEIF KRISTENSEN, University of Illinois, Chicago, Illinois 60680. On properties of higher order cohomology operations.

Let $K_{(2)}(n)$ be a 2-stage space with stable k-invariant. The action of the Steenrod Algebra $\mathcal{A}$ in $H^*(K_2(n);Z_2)$ can be described in terms of Massey products in $\mathcal{A}$. If A, B, C are matrices with entries in $\mathcal{A}$ with $AB = 0$, $BC = 0$ the Massey product $\langle A, B, C\rangle$ can be formed. The reason for this is that $\mathcal{A}$ is isomorphic to the cohomology of a graded differential "algebra" $\mathcal{O}$ (left distributivity missing) of cochain operations. These Massey products have other applications. Here two more shall be mentioned. Differentials in Adams spectral sequence can be expressed in terms of Massey products (of various length) in $\mathcal{A}$. The same is true for the algebra structure of the set of stable operations in $H_{(2)}$, the cohomology theory based on the spectrum $\{K_{(2)}(n)\}$, (see a forthcoming paper by Kristensen and Madsen [K-M]). Massey products of length three have now been computed. Roughly this computation goes as follows: Let A, B, C be matrices such that $\langle A,B,C\rangle$ is defined. In the above mentioned paper [K-M] a Cartan formula of the form $\langle A, B, C\rangle(xy) = \sum M'(x)m''(y) + \sum m'(x)M''(y) + P_1(x,y) + P_2(x,y)$ was proved. Here M' and M'' are Massey products and $m'$, $m' \in \mathcal{A}$. $P_1$ and $P_2$ are elements in $\mathcal{A} \otimes \mathcal{A}$ only one of which was known. Now both these terms have been computed. (Received February 14, 1969.)

665-66. DAVID S. TARTAKOFF, University of California, Berkeley, California 94720. A regularity theorem for weak solutions for first order systems of partial differential operators satisfying an a priori inequality.

Let $L = L(x,D)$ be a first order $p \times p$ system of partial differential operators with $C_0^\infty(R^n)$ coefficients, let $\Omega$ be a bounded open set in $R^n$ whose boundary $\Gamma$ is smooth and noncharacteristic for L, let $L^*$ denote the formal adjoint of L and let B be defined by smooth, homogeneous boundary conditions of constant rank, $B \subset C^\infty(\bar{\Omega})$. For $r = 0,1,\ldots$, let $W_r(\Omega) = \{u(x): D^\alpha u \in L^2(\bar{\Omega}) \text{ if } |\alpha| \leq r\}$, for $u(x)$ in $L^2(\Omega)$ let $u^0(x)$ in $L^2(R^n)$ be defined by $u(x)$ in $\Omega$ and zero elsewhere, and define $\|u\|^2_{k,\Omega} = \int |(u^0)\hat{}(\xi)|^2(1 + |\xi|^2)^k d\xi$ for $k = -1, 0$. Define $B^* \subset C^\infty(\bar{\Omega})$ as the orthogonal complement of $\beta(x)B$ in $L^2(\Omega)$, where $\beta(x)$ is the coefficient of normal differentiation in L for x in $\Gamma$. Let r be a given nonnegative integer and let $q\|u\|^2_{0,\Omega} \leq \|Lu\|^2_{0,\Omega} + C\|u\|^2_{-1,\Omega}$ for all u in B and sufficiently large q depending on r. If $(u,L^*v) = (f,v)$ for all v in $B^*$ with u in $L^2(\Omega)$ and f in $W_r(\Omega)$, then u belongs to $W_r(\Omega)$ as well. $(\ ,\ )$ denotes the inner product in $L^2(\Omega)$. (Received February 12, 1969.)

# Abstracts for the Meeting in Santa Cruz, California
# April 26, 1969

666-1. DAVID BELL, Rice University, Houston, Texas 77001. On mean approximation of holomorphic functions by rational functions with simple poles.

Given a bounded open set V of the complex plane whose boundary is sufficiently smooth, consider two components $W_1$ and $W_2$ of C-clos V, the complement of the closure of V, to be related if $\int_{\partial W_1} \log \rho(z,W_2)ds = -\infty$, where $\partial W_1$ means the boundary of $W_1$, and $\rho(z,W_2) = \inf\{|z - w| : w \in W_2\}$. Call two components equivalent if they are equivalent under the equivalence relation generated by the above. Theorem. If the boundary of V satisfies certain smoothness conditions, and $S \subset C - V$, then the set of all rational functions whose poles are simple and are contained in S, is dense in the holomorphic $L_p$-functions on V for $1 \leq p < 2$ if: Given a component X of C-clos V, there exists an equivalent component Y satisfying at least one of the following conditions: (i) Y is unbounded, (ii) $\int_{\partial Y} \log \rho(z,S)ds = -\infty$, (iii) $\sum \rho(z,S) = \infty$ $(z \in Y \cap S)$. This, as well as some converse statements, may be deduced from the author's paper (to appear) in the J. Approximation Theory, 1968. (Received February 6, 1969.)

666-2. WITHDRAWN.

666-3. M. A. DOSTAL, University of Montreal, Montreal, Canada. Convex surfaces with prescribed level curves.

The following lemma on convex functions is established: Given (a) a sequence $\{K_i\}_{i\geq 1}$ of compact convex sets in $\mathbb{R}^n$ such that $K_1 \neq \emptyset$, $K_i \subset \text{int } K_{i+1}$ $(\forall i)$ and (b) three sequences of positive numbers, $\{c_i\}$, $\{d_i\}$, $\{u_i\}$, tending to infinity, then there exists a subsequence $\{v_i\} = \{u_{k_i}\}$ and a convex function p defined on the open set $\Omega = \bigcup_{i\geq 1} K_i$ such that ($\alpha$) $v_i \geq c_i$, $K_i = \{z \in \mathbb{R}^n : p(z) \leq v_i\}$ $(\forall i)$; ($\beta$) $\min_\Omega p(z) = k_1$; ($\gamma$) for each point $Z = [z,p(z)]$ of the corresponding convex surface P (= the graph of p) one can find a supporting hyperplane h(Z) of P, passing through Z and such that if $\nu(z) = [C(z), -1]$ is the normal of h(Z), then $\inf\{|C(z)| : z \in K_{i+1} - K\} > d_i$. Furthermore, p is evidently a proper mapping and $\lim_{z\to\Omega} p(z) = +\infty$. This lemma turns out to be useful when we want to find a "good" description of the topology of $\hat{\mathcal{D}}(\Omega)$ (Fourier transforms of the Schwartz space $\mathcal{D}(\Omega)$). (Received November 25, 1969.)

666-4. HUMPHREY FONG, Stanford University, Stanford, California 94305. On invariant functions for positive operators.

Let $(X, \mathcal{A}, \mu)$ be a probability space and let T be a positive linear operator on $L_1(X, \mathcal{A}, \mu)$. Sucheston [Z. Wahr. vol. 8, p. 1-11; 353-356] showed that if $\sup |T^n|_1 < \infty$, then X = Y + Z: there is a function $e \in L_\infty$, $e > 0$ on Y, and $T^*e = e$; the ratio ergodic theorem holds on Y and need not hold on Z. Here an example is given to show that Hopf's decomposition also is not valid on Z. For a bounded sequence of real numbers $(x_n)$, let $m(x_n)$ and $M(x_n)$ denote the minimal and maximal values of Banach limits on $(x_n)$. For each n, $A \in \mathcal{A}$, let $\pi_n(A) = \int_A T^n 1\, d\mu$. We now assume that

$\sup |T^n|_1 < \infty$ and that X = Y. Theorem 1. X = P + N: $A \subset P$ and $\mu(A) > 0$ implies $M[\pi_n(A)] > 0$; N is the disjoint union of sets $X_i$ such that $M[\pi_n(X_i)] = 0$ for each i; there is a nonnegative function $f \in L_1$, f > 0 on P, and Tf = f. Theorem 2. The following conditions are equivalent: (o) There is a function $f \in L_1$ with f > 0 and Tf = f. (i) $\mu(A) > 0$ implies $M[\pi_n(A)] > 0$, $A \in \mathcal{A}$. Theorem 3. If (o) holds, then $m[\pi_n(A)] = M[\pi_n(A)]$ for each $A \in \mathcal{A}$, conversely, if $m[\pi_n(A)] = M[\pi_n(A)]$ for each $A \in \mathcal{A}$ and if T is conservative, then (o) holds. (Received December 24, 1968.)

666-5. R. W. PREISENDORFER, University of California, San Diego, California 92037.
A connection between Hilbert's integral transform, the symmetries of a square, and quaternions.

The Hilbert integral operator $J = -(1/\pi)\int_{-\infty}^{\infty}[\;](x-y)^{-1}dy$ has the well-known property that two applications of J on f yield - f. In operator notation, $J^2 = -I$, where I is the identity operator. Thus J is an operator-theoretic counterpart to $i = \sqrt{-1}$. The analogy between J and i is deepened by considering the composition of J and the reverse operator R where (fR)(x) = f(- x) for all f and x. First observe that $R^2 = I$. Then note that J and R anticommute: RJ = - JR. Hence J and R generate a group {I, J, - I, - J, R, JR, - R, RJ}, the Hilbert dihedral group. Each element of this group may be associated with a symmetry of the square in the complex plane, whose vertices are at (1, - 1, i, - i), by pairing J with a $90^\circ$ counterlockwise rotation of the square, and R with a $180^\circ$ rotation of the square about the imaginary axis. Next, using iR and J, and noting that $J^2 = (iR)^2 = -I$, we find that iR and J generate a group {I, J, - I, - J, iR, iJR, - iR, iRJ}, the Hilbert quaternion group. If the classical quaternions of Hamilton are written as 1, i, j, and k, then these pair respectively with our I, iR, J, and iRJ. (Received January 10, 1969.)

666-6. JOHN DE PILLIS, Brookhaven National Laboratory, Upton, New York 11973, and JOEL L. BRENNER, University of Arizona, Tucson, Arizona 85721. Generalized elementary symmetric functions and quaternion matrices.

Let $P = (A_{ij})$ be an $mn \times mn$ matrix partitioned into the $m^2$ $n \times n$ matrices $A_{ij}$. Choose p,q so that $1 \leq p \leq m$, $1 \leq q \leq n$. If entries of P are complex, then a complex-valued, partition-dependent function $E_{p,q}(\cdot)$ is defined for each P; by choosing m = p = 1, $E_{1,q}(\cdot)$ reduces to $E_q(\cdot)$, the usual qth elementary symmetric function on $n \times n$ complex matrices. Analogously, if P is quaternion-entried, then a nonnegatively valued partition-dependent function $\mathcal{E}_{p,q}(\cdot)$ is defined which, in turn, yields a definition for elementary symmetric functions $\mathcal{E}_q(\cdot)$ on $n \times n$ quaternion-entried matrices (set m = p = 1 and define $\mathcal{E}_q(\cdot)$ to be $\mathcal{E}_{1,q}(\cdot)$). If, moreover, q = n, then $\mathcal{E}_q(\cdot)$ further reduces to the Dieudonné determinant which has been defined for matrices over noncommutative fields. Theorem. Let $P = (A_{ij})$ be quaternion positive semidefinite (i.e., $P = A^*A$). Then $\mathcal{E}_{p,q}((A_{ij})) \leq E_p(\mathcal{E}_q(A_{ij}))$ for all p,q, $1 \leq p \leq m$, $1 \leq q \leq n$. Equality obtains iff P is block diagonal; i.e. iff $A_{ij} = 0$ whenever $i \neq j$. This result had been proved for the complex case by the first author. (Received January 14, 1969.)

666-7. RICHARD C. GILBERT, California State College, Fullerton, California 92631.
Symmetric operators with twice continuously differentiable spectral functions.

Let A be a simple closed symmetric operator with deficiency indices (1,1) in a Hilbert space H. Let $g_0$ be an element of norm 1 in a deficiency subspace of A. Let $A_0$ be a selfadjoint extension of A in H with spectral function $E_0(t)$. Suppose that $(E_0(t)g_0,g_0)$ is a twice continuously differentiable function whose derivative is everywhere positive. Suppose that the Krein function $\theta(\lambda)$ corresponding to a selfadjoint extension or dilation $A^+$ of A has the form $\theta(\lambda) = a + b\lambda + \int_{-\infty}^{\infty}(t - \lambda)^{-1}d\rho(t)$, where $b \geq 0$, a is real, $\rho(t)$ is nondecreasing, bounded and twice continuously differentiable. Then by use of the Stieltjes inversion formula one can prove an expansion theorem for $A^+$, namely $([E^+(\beta) - E^+(\alpha)]f,h) = \int_\alpha^\beta \sum_{i,j=1}^2 C_i(f;t)[C_j(h;t)]^- d_{ij}(t)dt$, where $C_i(f;t)$ is a linear functional in f which is determined by $A_0$, $(d_{ij}(t))$ is a nonnegative matrix function determined by $A^+$, $E^+(t)$ is the spectral function of $A^+$, and $[\ ]^-$ stands for complex conjugate. Using the expansion theorem one can show that if $\rho'(t) > 0$ for all t, then $A^+$ is unitarily equivalent to the multiplication operator in $L^2(-\infty,\infty) \oplus L^2(-\infty,\infty)$. If $\rho'(t) \equiv 0$, then $A^+$ is unitarily equivalent to the multiplication operator in $L^2(-\infty.\infty)$. (Received February 3, 1969.)

666-8. WITHDRAWN.

666-9. JOHN R. REAY, Western Washington State College, Bellingham, Washington 98225. Isoperimetric problems on polytopes. Preliminary report.

The purpose of this note is to call wider attention to a broad family of open problems concerning the numbers of faces of certain families of polytopes. Let $M(d,f_i,f_j)$ [respectively, $m(d,f_if_j)$] be the maximum [respectively, minimum] number of j-dimensional faces (called j-faces) that may occur in a d-dimensional polytope with exactly $f_i$ i-faces. We may assume $i < j$ (by duality) and $f_i \geq \binom{d+1}{i+1}$. Note that the upper and lower bound conjectures concern the case i = 0. The complete solution if d = 3 and partial results when $d \geq 4$ are given. Applications for cases where $i \neq 0$ are given. Sample problems. Generally, determine functions M and m. Which $M(d,f_i,f_j)$ are not allowable problems (e.g. $(d,f_i) = (3,7)$)? What analogues of the Dehn-Sommerville equations exist? (Received February 12, 1969.)

666-10. JULIA B. ROBINSON, University of California, Berkeley, California 94720. Unsolvable diophantine problems.

It is shown that there is no general method of telling whether an arbitrary polynomial $P(x_1,\ldots,x_k)$ with integer coefficients is ever a power of 2 for $x_1,\ldots,x_k$ natural numbers. The proof is based on (*): If $\mathfrak{M}$ is an infinite set of natural numbers and H is a diophantine function such that $(m,H(m)) = 1$ and $2^{H(m)} \equiv 1 \pmod m$ for all $m \in \mathfrak{M}$, then every recursively enumerable set is diophantine in $\mathfrak{M}$. In particular, taking $\mathfrak{M} = \{2^{2^n} - 1\}$ and $H(m) = m + 1$, (*) shows that every recursively enumerable set is diophantine in $\{2^{2^n}\}$. Also, a number is of the form $2^{2^n}$ if and only if it is a power of 2 and of the form $1 + 3(u^2 + v^2)$. Hence every recursively enumerable set is diophantine in $\{2^n\}$. The opening result follows easily. Also taking $\mathfrak{M}$ as any infinite set of primes and $H(m) = m - 1$, (*) shows that every recursively enumerable set is diophantine in any infinite set of primes. This paper will appear in the Proc. Amer. Math. Soc. (Received February 12, 1969.)

666-11. PAUL J. NIKOLAI, University of California, Santa Barbara, California 93106. On the convexity of a function of the permanent of a matrix.

Let $A = [a_{ij}]$ denote an n-square positive semidefinite hermitian matrix. With $S_n$ the full symmetric group let $\mathrm{per}(A) \equiv \sum_{\sigma \in S_n} \Pi_{i=1}^{n} a_{i\sigma(i)}$ denote the permanent of A and let f(t) denote the function $(\mathrm{per}(A^t))^{1/t}$. M. Marcus and M. Newman [Inequalities for the permanent function, Ann. of Math. 75 (1962), 47-62] show that f(t) is either constant or increasing but leave open a conjecture that f(t) be convex-concave relative to t = 0, i.e. convex for $t < 0$ and concave for $t > 0$. By representing f(t) as a tth power mean it is shown that in general f(t) is not convex-concave relative to any value of t but that t log f(t) is convex. This and related results will appear in the Canad. J. Math. (Received February 11, 1969.)

666-12. BADRI N. SAHNEY, University of Calgary, Calgary 44, Alberta, Canada. On the Norlund summability of a class of Fourier series.

C. T. Rajagopal (Proc. Comp. Philos. Soc. 59 (1963), 47-53) has given a set of sufficient conditions for the Nörlund summability of Fourier series. From his two alternative forms, he had derived two sets of particular results on harmonic summability and Cesàro summability. O. P. Varshney (Communication Analyse Math. (1967), 1552-1558) has given a set of necessary and sufficient conditions for the summability of Fourier series. The following theorem is proved: Theorem. Let the sequence $p_n$ be such that p(u) is monotonic decreasing and strictly positive for $u \geqq 0$ and $P(U) = \int_0^U p(n)dn$, $p_n \equiv p(n)$, and let $\Phi(t) = \int_0^t |\varphi(u)| du = O(t/\psi(1/t))$ as $t \to +0$ and $\psi(t)$ is positive, nondecreasing with t, then a necessary and sufficient condition to ensure the Nörlund summability of Fourier series is $\int_1^n P(n)/n\psi(n)\, dx = O(P(n))$. Here the particular case of $\psi(u) = \log u$ is Varshney's result, while the cases $\psi(u) = \log u$, $p_n = 1/(n + 1)$ and $\psi(u) = 1$, $p_n = n^{\alpha}$ for $0 < \alpha < 1$ are the results on harmonic and Cesàro summabilities of Fourier series, respectively. (Received February 11, 1969.)

666-13. RAPHAEL M. ROBINSON, University of California, Berkeley, California 94720. Some definite polynomials which are not sums of squares of real polynomials.

A real polynomial is called positive definite if it does not assume any negative values. In 1888, Hilbert showed that there exist positive definite polynomials in two or more variables which are not expressible as sums of squares of real polynomials. He did not give explicit examples, and following his method exactly would necessarily lead to quite complicated polynomials. But some modifications of the details makes it possible to obtain simple examples. It is shown in this paper that the sextic in two variables, $x^2(x^2 - 1)^2 + y^2(y^2 - 1)^2 - (x^2 - 1)(y^2 - 1)(x^2 + y^2 - 1)$, and the quartic in three variables, $x^2(x - 1)^2 + y^2(y - 1)^2 + z^2(z - 1)^2 + 2xyz(x + y + z - 2)$, are positive definite but are not expressible as sums of squares of real polynomials. However, by a general theorem of Artin (1926), these polynomials must be expressible as sums of squares of rational functions with rational coefficients, and such representations are found. (Received February 7, 1969.)

666-14. RICHARD G. LEVIN, Western Washington State College, Bellingham, Washington 98225, and TAKAYUKI TAMURA, University of California, Davis, California 95616. On commutative, power joined semigroups.

A semigroup $\mathscr{S}$ is called power joined if for all a, b $\in \mathscr{S}$, $\exists$ positive integers m, n such that $a^m = b^n$. A commutative semigroup $\mathscr{S}$ is called archimedean if for all a, b $\in \mathscr{S}$, $\exists$ positive integers m, n and u, v $\in \mathscr{S}$ such that $a^m = bu$ and $b^n = av$. Theorem. The following statements are equivalent. (1) $\mathscr{S}$ is a commutative, power joined semigroup. (2) $\mathscr{S}$ is a commutative, archimedean semigroup whose group homomorphic images are periodic. (3) $\mathscr{S}$ is a commutative semigroup in which for every pair a, b $\in \mathscr{S}$, there exist positive integers l, m, n, s, t, p such that $a^l = a^m b^n$ and $b^s = b^t a^p$. Corollary. A commutative archimedean torsion-free semigroup $\mathscr{S}$ is power joined if and only if the structure group $G_a$ of $\mathscr{S}$ relative to some element a is periodic. The last corollary improves the theorem in Levin's earlier paper which will be published soon in Pacific J. Math. (Received February 12, 1969.)

666-15. TAKAYUKI TAMURA, University of California Davis, California 95616. Commutative semigroup and power joined subsemigroups.

Theorem. S is a commutative archimedean semigroup which is a finite disjoint union of power joined subsemigroups if and only if S has one of the following types: (1) S itself is power joined. (2) S is isomorphic to the direct product of an abelian torsion group and an additive rational group. (3) S is the ideal extension of an abelian group ot type (2) by a commutative nil-semigroup. Corollary. A commutative semigroup is a finite disjoint union of power-joined semigroups which are isomorphic to each other if and only if it is a semilattice of isomorphic power joined semigroups. This corollary holds even if "a finite" is replaced by "two", and "isomorphic" is eliminated. (Received February 12, 1969.)

666-16. MARTIN K. McCREA, University of California, Davis, California 95616. Compatible operations on binary relations.

Let $\mathfrak{F}$ be a set of transformations on a set X. A relation $\rho$ on X is said to be compatible with respect to $\mathfrak{F}$ if af $\rho$ bf whenever a $\rho$ b and f $\in \mathfrak{F}$. This paper extends the work of T. Tamura in The theory of binary relations (Trans. Amer. Math. Soc. 120, (1965), 343-368) to the compatibility operation. The compatibility operation F with respect to a given $\mathfrak{F}$ is defined by $\rho F = \{(af, bf) | a \,\rho\, b, f \in \hat{\mathfrak{F}}\}$ where $\hat{\mathfrak{F}}$ is the semigroup generated by $\mathfrak{F} \cup \{1\}$ where 1 is the identity transformation on X. Definition. A set $\mathfrak{F}$ of transformations of a set X is called doubly transitive if for each x,y,z,w $\in$ X, $x \neq y$, there exists an f $\in \mathfrak{F}$ such that xf = z, yf = w. Theorem. Let G be a groupoid, $\mathcal{C}$ the set of all multiplications of G, and $\mathfrak{F}$ a set of transformations of G. Then the congruence operation compatible with respect to $\mathfrak{F}$ is given by FN where N is the congruence operation if one of the following conditions hold: (a) $\hat{\mathcal{C}} \subseteq \hat{\mathfrak{F}}$, (b) $\hat{\mathcal{C}}\hat{\mathfrak{F}} \subseteq \hat{\mathfrak{F}}\hat{\mathcal{C}}$, (c) $\hat{\mathfrak{F}}\hat{\mathcal{C}}$ is doubly transitive, (d) $\mathfrak{F}$ is a set of translations, (e) $\mathfrak{F}$ is a set of endomorphisms. An application to congruences of semirings is also obtained. (Received February 12, 1969.)

666-17. R. P. DICKINSON, JR., Lawrence Radiation Laboratory, University of California, Livermore, California 94550. Right zero composition of $\eta$ semigroups. Preliminary report.

Two semigroups A and B have a right zero composition S, if S is a right zero union of the semigroups A and B. A semigroup which is commutative, archimedean, cancellative, and nonpotent is called an $\eta$ semigroup. The translations of an $\eta$ semigroup form a separative commutative semigroup in which one component is a group and the remaining components are $\eta$ semigroups. Let $I_1$ be the set of all inner translations on an $\eta$ semigroup $\eta_1$, and let $T_1(\eta_1)$ be the archimedean component of the translations on $\eta_1$ containing $I_1$. Theorem. A necessary and sufficient condition for two $\eta$ semigroups, $\eta_1$ and $\eta_2$, to have a right zero composition is as follows: (1) There exists an isomorphism $\varphi$ from $\eta_2$ into $T_1(\eta_1)$ (embedding). (2) The set union $\varphi(\eta_2) \cup I_1$, which is a subsemigroup of $T_1(\eta_1)$, has $\varphi(\eta_2)$ as an ideal, where $\varphi(\eta_2)$ is the image of $\eta_2$ (Received February 12, 1969.)

666-18. JOEL M. COHEN and HERMAN R. GLUCK, University of Pennsylvania, Philadelphia, Pennsylvania 19104. Stacked bases for modules over principal ideal domains.

Given free modules $A \subset B$ over a PID, we ask: Are there bases for A and B such that each basis element of A is a multiple of a basis element of B? Such bases, if they exist, are called stacked bases for A and B. An obvious necessary condition for the existence of stacked bases is that B/A be a direct sum of cyclic modules. Stacked Bases Theorem. Free modules $A \subset B$ over a PID have stacked bases if and only if B/A is a direct sum of cyclic modules. The Stacked Bases Theorem was conjectured by Kaplansky in 1954. (Received February 5, 1969.)

666-19. PETER McMULLEN, Western Washington State College, Bellingham, Washington 98225. Gale diagrams of convex polytopes.

In a recent paper (Diagrams for centrally symmetric polytopes, Mathematika 15 (1968), 123-138) the author and G. C. Shephard described a new, geometrical formulation of the Gale diagram of a polytope. Here those results are extended, and it is shown that Gale diagrams defined geometrically correspond most naturally to projective equivalence classes of polytopes. The original polytope can also easily be reconstructed from its Gale diagram as follows. Theorem. Let $\bar{V} = \{\bar{v}_1,\ldots,\bar{v}_n\}$ be a Gale diagram of a polytope P. Then $P^* = \{(\eta_1,\ldots,\eta_n) \in E^n \mid \eta_i \geqq 0 \ (i = 1,\ldots,n), \sum_{i=1}^n \eta_i = 1, \sum_{i=1}^n \eta_i \bar{v}_i = 0\}$ is projectively equivalent to any polar dual of P. Various results can be deduced from this formulation, among which is the following: A projective symmetry of a polytope P is a projective transformation which takes P into itself. Theorem. Let P be a polytope. Then there is a polytope Q, projectively equivalent to P, such that to every projective symmetry of Q corresponds a symmetry of Q. (Received February 12, 1969.)

666-20. DEVENDRA S. GOEL, University of Calgary, Calgary, Alberta, Canada. The absolute Nőrlund summability factors of a Fourier series.

Let $\sum a_n$ be a given infinite series with the sequence of partial sums $\{s_n\}$. Let the sequence $\{t_n\}$ be defined by $t_n = (1/P_n)\sum_{k=0}^n p_{n-k}s_k$. If the series $\sum |t_n - t_{n-1}|$ is convergent we say that the series $\sum a_n$ is absolutely Nőrlund summable or summable $|N, p_n|$. Let the Fourier series of f(t) be given by $\sum(a_n \cos nt + b_n \sin nt) = \sum A_n(t)$. We write $\varphi(t) = \{f(x+t) + f(x-t)\}/2$. The following theorem is proved: Theorem. If $\varphi(t)$ is a function of bounded variation in $(0,\pi)$ then the factored Fourier series $\sum A_n(t)\lambda_n$ is summable $|N,p_n|$, where $\{\lambda_n\}$ is positive monotonic nonincreasing sequence and the sequence $\{p_n\}$ satisfies the following conditions: (1) $p_n \geqq 0$, $P_n = p_0 + p_1 + \ldots + p_n \to \infty$ as $n \to \infty$, (2) $\sum|\Delta v_n| = \sum|v_n - v_{n-1}| < A$, $v_n = (n+1)p_n/P_n$ and, (3) $P_n \sum_{m=n}^{a} \lambda_m/(m+2)P_m < C$ for all n. (Received February 11, 1969.)

666-21. PETER RENZ, Reed College, Portland, Oregon 97202. Smooth extensions and extractions in Banach spaces.

The theorems below are stated for the linear spaces where the results are most complete. Partial results and some of the lemmas are valid in separable spaces, spaces admitting $C^\infty$ (or $C^p$) partitions of unity, and spaces with a Schauder base. The starting points for these results are the work of Corson (Symposium on infinite dimensional topology, Ann of Math. Studies, to appear) and Bessaga (Bull. Acad. Polon. Sci. Ser. Math. Astron. Phys. 14 (1966), 27-31). Extension Theorem. Let B be $\ell p(\omega)$ or $c_0(\omega)$. Let $K \subset B$ be compact and $f: K \to B$ be a homeomorphism. We show that f admits an extension f* which is a homeomorphism of B with itself having the property that f* restricted to $B \setminus K$ is a $C^\infty$ diffeomorphism of $B \setminus K$ onto $B \setminus f(K)$. This result and Bessaga's technique lead to the following: Extraction Theorem. Let M be a $C^\infty$ paracompact $\ell p(\omega)$-manifold with p even. Let $K \subset M$ be locally compact and closed and $U \supset K$ be open in M. There is a $C^\infty$ diffeomorphism of M onto $M \setminus K$ which is the identity off of U. (Received February 13, 1969.)

666-22. CHARLES T. TUCKER, University of Houston, Houston, Texas 77004. On order convergence and topological convergence in a lattice ordered vector space.

Suppose G is a $\sigma$-complete lattice ordered vector space with origin $\theta$, H is a lattice ordered vector subspace of G, M is the vector subspace of G which is generated by the set of supremums of countable subsets of H, x is an element of H with the property that if $g_1 \leqq g_2 \leqq g_3 \leqq \ldots$ is a sequence of elements of G such that $x = \bigvee g_i$ then there exists a y in G such that $y \leqq \sum_{i=1}^{n} g_i \wedge \theta$ for every positive integer n, and $\tau$ is a linear topology on G for which the order interval $[-x,x]$ is bounded. Then the order limit of a sequence of points of H is also the topological limit of a sequence of points of M. (Received February 13, 1969.)

666-23. JASON FRAND and R. B. KILLGROVE, California State College, Los Angeles, California 90032, HENRY G. BRAY, San Diego State College, San Diego, California. A geometric approach to the Heine-Borel Theorem. Preliminary report.

Let X be a nonempty set of points, $\mathcal{B}$ a collection of subsets of X such that (1) for all a,b if a,b in X and $a \neq b$ then there are A,B of $\mathcal{B} \ni a \in A$, $b \in B$, $A \cap B = \emptyset$, (2) for all A, B, a if A, B of $\mathcal{B}$ and $a \in A \cap B$, then there is C of $\mathcal{B} \ni a \in C$, $C \subset A \cap B$, (3) every finite union of elements of $\mathcal{B}$ is contained in some element. of $\mathcal{B}$, (4) for each A of $\mathcal{B}$, for each collection $\gamma$ of elements of $\mathcal{B}$ which cover A* (defined below), there is a finite number of elements of $\gamma$ which cover A*. By A* is meant all points p such that every element of $\mathcal{B}$ with point p intersects A nonemptily. This point set geometry gives a base for topological space $(\tau, X)$ wherein $A^*$ is closure of A. Then the compact sets of $(\tau, X)$ are exactly those which are closed and contained in some element of $\mathcal{B}$. The above arose in using Forder's axioms 01-06, and Pasch for a plane, Borsuk & Szmielew's open triangular disks for a base, and the idea that 'bounded' sets are those contained in such disks, to prove Heine-Borel in a way analogous to that for metric spaces. Then (1), (2), and (4) can be proved but (3) needs parallel postulate or redefining bounded as contained in finite union of disks, Non-Desarguesian Moulton planes satisfy 01-06 and parallel postulate. Apparent open topological question: If the HB theorem is true in this sense for some base, must (1), (3), or (4) be true? (Received February 13, 1969.)

666-24. K. DEMYS, 844 San Ysidro Lane, Santa Barbara, California 93103. Representation of fractional dimensions and a related expression for the half dimension.

Fractional dimensions have more or less unwittingly been used in the processes of fractional integration or differentiation and in manipulations of noninteger exponents. They have hitherto not been investigated geometrically, one difficulty being that, e.g., the dimension $n/m$, $|n| < |m|$, cannot even constitute an axis since it itself is not yet a line. However, by a basic theorem of structure, with $n/m > 0$ it must contain an infinity of points. Such dimensions are representable by plane waves cutting the real axis in a nondense infinite point set. For d, the distance between any two such points (i.e. the half wavelength), $d = f(n/m)^{-1}$ and $d \to 0$ as $n \to m$. If $n/m < 0$, the wave similarly cuts the imaginary axis. If $(-3) < n/m < (-2)$, all 3 quaternion axes are cut; if $1 < n/m < 2$, the waves cut concentric circles on the XY-plane; concentrical spherical shells in real 3-space if $2 < n/m < 3$, etc. (In all cases the wave's amplitude, measuring the magnitude of a fractional dimensional quantity, is parallel to the P-axis, whose unit p is such that $|p| = 1$, $p^{2k} = 0$, $k = 0,1,2,3,\ldots$.) A new expression for $z^{1/2} = (a + bi)^{1/2}$, $a,b \geqq 0$, but not both zero, is $z^{1/2} = \lim_{n\to\infty} \sum_{k=0}^{\infty} z^k \binom{n}{2k} / z^k \binom{n}{2k+1}$, the square root thus becoming the ratio of two infinite integers. The computer program is simple. (Received February 13, 1969.)

666-25. PHILIP A. LEONARD, Arizona State University, Tempe, Arizona 85281. Some Diophantine equations which are interrelated. Preliminary report.

Let p be an odd prime. A study of the group of rational points on the curve $y^2 = x^3 - p^2x$ leads to the four Diophantine equations (i) $p^2x^4 + 4y^4 = z^2$, (ii) $x^4 + 4y^4 = pz^2$, (iii) $x^4 + y^4 = 2pz^2$, (iv) $px^4 + y^4 = 2z^2$, and reveals some connections among them. For example: The solutions of (i) can be obtained from those of (ii), (iii) and (iv); if any of the four equations has a nontrivial solution, it has infinitely many; if any two of (ii), (iii) and (iv) have a nontrivial solution, the remaining one does also. (Received February 13, 1969.)

666-26. MICHAEL H. POWELL, University of California, Santa Barbara, California 93106. Almost periodic compact hulls of K-algebra pairs. Preliminary report.

K is the real or complex field. A K-algebra pair is a pair $(A, \mathcal{Q})$ where A is a K-algebra and $\mathcal{Q}$ is a suitably restricted covering of A. Four rules are introduced which assign to each pair $(A, \mathcal{Q})$ a suitably restricted set of locally convex topologies on A. The simplest of these rules yields a notion of a locally convex K-algebra pair (l.c.p.). With respect to any of these rules the notion of a compact hull of a l.c.p. can be introduced and the existence of such compact hulls proved by non-constructive means. Notions are then defined of weakly almost periodic (w.a.p.) and almost periodic (a.p.) linear forms on an l.c.p. Using the double limit property, it is shown that the dual space of the w.a.p. linear forms with Arens multiplication and the weak topology becomes one kind of compact hull of an l.c.p. Using Ascoli's Theorem, it is shown that the dual space of the a.p. linear forms with Arens multiplication yields two more compact hulls of an l.c.p. according as it is given the weak or compact-open topology. (Received February 13, 1969.)

666-27. WITHDRAWN.

666-28. MATTHEW I. GOULD, Vanderbilt University, Nashville, Tennessee 37203. Multiplicity type and subalgebra structure in infinitary universal algebras.

For definitions and previous results, see the author's abstracts in these *Notices* 13 (1966), 860; 14 (1967), 64; and 14 (1967), 128. Full details on finitary multiplicity types can be found in Pacific J. Math. (3) 26 (1968), 469-485. We now consider algebras with infinitary operations; the definition of multiplicity type is extended in the obvious way. Call a multiplicity type $\mu$ standard if either $\mu$ has a last entry, or $|\sup\{\alpha | \mu_\alpha \neq 0\}|$ is a regular cardinal. The subalgebra-ordering of standard multiplicity types is characterized, and normal forms for standard multiplicity types are found. These normal forms have the properties: (1) Every standard multiplicity type is equivalent to a unique normal form. (2) The subalgebra-ordering among compatible normal forms is the pointwise ordering. (3) Every normal form is maximal in the pointwise ordering of all multiplicity types equivalent to it. One result of particular interest is the fact that for $\alpha \geq \omega$, $2^{\bar{\alpha}}$ $\alpha$-ary operations can be replaced by one $\alpha$-ary operation in such a way that subalgebra structure is preserved. (Received February 14, 1969.)

666-29. DAIHACHIRO SATO, University of Saskatchewan, Regina Campus, Regina, Saskatchewan, Canada. Rational points preserving homeomorphism of real line by entire functions.

Let A and B be both countable dense subsets of the set of real numbers R and $f_n$ ($n = 1,2,3,\ldots$) is a sequence of functions of R to R. Then there exists a real entire function $\varphi$ of R onto R with the properties (i) $\varphi(A) = B$, (ii) $\varphi^{-1}(B) = A$, and (iii) $\varphi$ does not coincide with any $f_n$ in any open interval. Corollary. There exists a transcendental entire function f(z) such that the number f(x) is (real) rational or irrational according as x is (real) rational or irrational. Reference. P. Erdös, Some unsolved problems, Michigan Math. J. 4 (1967), 291-300. (Received February 14, 1969.)

666-30. ROBERT L. HEMMINGER, Box 1713 Station B, Vanderbilt University, Nashville, Tennessee 37203. Consequences of Kelly's Lemma in reconstructing graphs.

In the first paper on the Ulam Conjecture P. J. Kelly [A congruence theorem for trees, Pacific J. Math. 7 (1957), 961-968] proved a lemma that has been very useful in obtaining information about a graph G from the collection $\{G_v\}_{v \in V(G)}$ ($G_v$ is the graph with $V(G_v) = V(G) - \{v\}$ and $E(G_v) = \{e \in E(G): e \text{ is not incident with } v\}$). This paper points out the idea common to most of these applications of Kelly's Lemma by using the lemma to prove a theorem from which one gets the above-mentioned information as immediate corollaries. (Received February 14, 1969.)

# ABSTRACTS PRESENTED BY TITLE

During the interval from January 6 through February 11, 1969, the papers printed below were accepted by the American Mathematical Society for presentation by title. The abstracts are grouped according to subjects chosen by the author from categories listed on the abstract form. The miscellaneous group includes all abstracts for which the authors did not indicate a category.

One abstract presented by title may be accepted per person per issue of these *Notices*. Joint authors are treated as a separate category; thus, in addition to abstracts from two authors individually, one joint abstract by them may be accepted for a particular issue.

## Algebra & Theory of Numbers

69T-A39. JOHN DAUNS and KARL H. HOFMANN, Tulane University, New Orleans, Louisiana 70118. Spectral theory of algebras and adjunction of identity.

Let A be a C*-algebra, $R \equiv$ centroid A, $\epsilon$ : center $A \equiv Z \to R$. Let $F : \text{Prim } A \equiv B \to \text{Prim } R \equiv Y$ by $F(b) = \{r \in R \mid rA \subseteq b\}$; set $B' = \text{Prim}(C \times A)$ and $\pi_1, \pi_2 : B' \times Y \to B', Y$ the projections. All ideals of the semidirect product $R \times A$ are identified. Every primitive ideal is of the form $b + F(b)$; $R(1 - e) + b$ where $e + b = 1 \in A/b$; or $A + p$, $Z \subseteq p \in Y$. Let $D = \{(\epsilon(z), -z) \mid z \in Z\}$ and $A = (A + D)/D \subset (R \times A)/D \equiv \tilde{A}_1$. Theorem. $b \to (b, F(b)) : B \to$ graph $F \equiv G \subset B' \times Y$ is a homeomorphism in the hull-kernel topologies. There is an embedding Prim $\tilde{A}_1 \equiv \tilde{B}_1 \subset G \cup Y$. Theorem. For any C*-algebra A the complete regularization of B is given purely algebraically by $b \to \varphi(b) = \bigcap \{q \in B \mid F(q) = F(b)\}$. Theorem. Let $\varphi : B' \to M'$ be the complete regularization. The complete regularization of B is $\varphi \times 1 | G$, or $F : B \to F(B)$, or $\pi_2 | G : G \to F(B)$; while that of $\tilde{B}_1$ is $\varphi \times 1 | \tilde{B}_1 \to \tilde{M}_1$. Corollary (Dixmier). B and $\tilde{B}_1$ have the same Stone-Čech compactifications $\beta B \cong \beta \tilde{B}_1 = \check{M}_1$. This paper will appear shortly in Math. Annalen. (Received January 2, 1969.)

69T-A40. RUDOLF WILLE, Mathematisches Institut, der Universitat Bonn, 53 Bonn, Wegelerstr. 10, W. Germany. On a problem of G. Grätzer.

G. Grätzer introduced in his paper Two Mal'cev type theorems in universal algebra (preprint) the following definitions: A Mal'cev type condition of equational classes is of the form $(P_n)$ "there exists a natural number n, and polynomial symbols $p_0, \ldots, p_{m_n - 1}$ satisfying a set of identities $I_n$", where $(P_{n+1})$ is weaker than $(P_n)$ and the form of $I_n$ is independent of the type of algebras considered. The following theorem partly solves a problem which G. Grätzer stated in his paper: Theorem. Let U be an inequality of congruences formulated with meet, join, and relation-theoretic product. Then one has an algorithm for developing Mal'cev type conditions $(U_n^m)$ such that the congruences of algebras of an equational class $\underline{K}$ satisfy U if and only if $(U_{n_m}^m)$ holds for every natural number m and some $n_m$ (dependent of m); especially, $(U_{n_m}^m)$ implies $(U_{n_{m-1}}^{m-1})$, but there is no m' (independent of U) such that $(U_{n_{m'}}^{m'})$ implies $(U_{n_m}^m)$ for each m. (Received January 6, 1969.) (Author introduced by Dr. Wolfram Schwabhauser.)

69T-A41. CHUNG HARNG SU, University of Wyoming, Laramie, Wyoming 82070. A generalized theorem on vector spaces.

Let V be a vector space over a field F. Let $L_1$ and $L_2$ be two subspaces of V. It is well known that $L_1 \cup L_2$ is a vector space if and only if either $L_1 \subseteq L_2$ or $L_2 \subseteq L_1$. In this note we extend this result. Lemma. Let V be a vector space over a field F of order infinite; let L, $L_1, L_2, \ldots, L_m$ be subspaces of V such that $L \subseteq L_1 \cup L_2 \cup \ldots \cup L_m$; then $L \subseteq L_i$ for some $i \leq m$. Theorem. Let $L_1, L_2, \ldots, L_m$ be subspaces of a vector space V over a field F of order infinite; then $L_1 \cup L_2 \cup \ldots \cup L_m$ is a vector space if and only if there is some $i \leq m$ such that $L_j \subseteq L_i$ for $j = 1,2,\ldots, m$. Corollary 1. Let $L_1, L_2, \ldots, L_m$ be proper subspaces of a vector space V over a field F of order infinite; then $L_1 \cup L_2 \cup \ldots \cup L_m$ is a proper subset of V. Corollary 2. Let $L_1, L_2, \ldots, L_m$ be m proper subspaces of a vector space V over a field of order k; then $L_1 \cup L_2 \cup \ldots \cup L_m$ is a proper subset of V provided that m is less than k. (Received December 19, 1968.) (Author introduced by Professor Virindra M. Sehgal.)

69T-42. JOHN DeCICCO, Illinois Institute of Technology, Chicago, Illinois 60616, and ROBERT V. ANDERSON, Chicago State College, Chicago, Illinois 60621. A study of the inner and outer products in some varieties of quadratic rings R.

Various types of quadratic rings R are determined by properties of the conjugate operation T of the ring R. A quadratic ring R is a vector space relative to its invariant ring X of scalars for which multiplication is not necessarily associative. The skew-conjugate set Y of pure elements v is a vector space Y, relative to X, with, in general, nonassociative scalar multiplication, iff the conjugate operation T of R is linear. The inner product $(z_1, z_2)$, of every two elements of a quadratic ring R, whose conjugate operation T preserves sums, is given by $(z_1, z_2) = (1/2)N[(z_1 + z_2) - N(z_1) - N(z_2)]$. In a homogeneous quadratic ring R, both the inner and the outer products are linear homogeneous. If R is a quadratic ring such that T preserves sums, then R admits a direct involutorial automorphism T iff T preserves the outer product of every pair of elements in R. Under these conditions R admits a reverse involutorial automorphism T iff the outer product of every pair of elements in R is a pure element of R. A homogeneous quadratic ring R, such that T preserves sums, obeys the translation property of the norm iff it admits a reverse involutorial automorphism T such that the outer product $v_1 \times v_2$, of every two pure elements is orthogonal to both $v_1$ and $v_2$. (Received January 13, 1969.)

69T-A43. JOSEPH NEGGERS, University of Alabama, University, Alabama 35486. On extending monomorphisms.

Let X be a partial algebra. Let $f: X \to X$ be a monomorphism. Then there is a partial algebra $X^*$ similar to X and a surjective monomorphism $f^*: X^* \to X^*$ such that $X \subset X^*$ and such that $f^*|X = f$. Furthermore, if X is a finitary algebra, $f^*$ is an automorphism and $X^*$ is a finitary algebra. If Y is any partial algebra similar to X such that for some monomorphisms $i: X \to Y$, $g: Y \to Y$, $gi = if$, then there is a monomorphism $i^*: X^* \to Y$ such that $gi^* = i^*f^*$ and such that $i^*|X = i$. This result can be applied to show, e.g., if X is a partial algebra and if Mon(X) is the semigroup of monomorphisms $f: X \to X$, then there is a family $\{T_\alpha\}_{\alpha \in \Lambda}$ of subsemigroups of Mon(X) such that (i) each $T_\alpha$ can be embedded in a group; (ii) if $f \in Mon(X) \setminus T_\alpha$, then the subsemigroup of Mon(X) generated by $T_\alpha \cup \{f\}$ cannot be embedded in a group; (iii) $Mon(X) = \bigcup T_\alpha$; (iv) if T is any commutative subsemigroup of Mon(X), then $T \subseteq T_\alpha$ for some $\alpha$. (Received January 20, 1969.)

69T-A44. P. B. BHATTACHARYA and S. K. JAIN, University of Delhi, Delhi-7, India. Group rings having nilpotent group of units.

Bateman and Coleman, Proc. Amer. Math. Soc. 19 (1968), 448-449, have determined conditions under which the group of units in the group algebra of a finite group is nilpotent. Theorem (b) in their paper depends upon a lemma: Let R be a ring with identity. Suppose that R has a nilpotent ideal N such that R/N is commutative. Then the group of units in R is nilpotent. That the lemma is not true can be shown by a counterexample: Let R be the ring of all $2 \times 2$ upper triangular matrices over integers modulo 3 and N be its radical. Then R is a ring with identity, N nilpotent and R/N commutative. But units $a = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ and $b = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ are such that $(a,b) = a^{-1}b^{-1}ab = a$ and therefore, $(a,b,b,\dots, b) = a$, showing that the group of units in R cannot be nilpotent. However, the statement of their theorem is true and the authors provide another proof. It is also shown that if R is any artinian ring with identity and G is a finite group such that the group of units in the group ring RG is nilpotent, then RG satisfies an identity $(x_1x_2 - x_2x_1)^n = 0$. As a consequence, the authors derive results which give both theorems of Bateman and Coleman as special cases. The proofs depend upon the well-known structure theorems for rings. (Received January 20, 1969.) (Introduced by Professor Rama Shankar Varma.)

69T-A45. C. K. GUPTA, University of Manitoba, Winnipeg 19, Manitoba. A faithful matrix representation for the free abelian-by-class 2-by-abelian groups. Preliminary report.

Let F be the free abelian-by-class 2 and class 2-by-abelian group of finite or countable infinite rank. Then F is faithfully represented by a group of $4 \times 4$ matrices over a certain commutative ring. (Received January 23, 1969.) (Author introduced by Professor George A. Grätzer.)

69T-A46. JOHN JONES, JR., AF Institute of Technology, Wright-Patterson AFB, Ohio. Solutions of certain matrix equations.

The matrices A, B, C, D are n by n having real elements. Let $f_\gamma(\lambda)$ be any polynomial of degree $n \geq 1$ in $\lambda$ having real coefficients with the 2n by 2n matrices R, $f_\gamma(R)$ defined by $R = \left(\begin{array}{c|c} -B & D \\ \hline -C & A \end{array}\right)$, $f_\gamma(R) = \left(\begin{array}{c|c} U & M \\ \hline V & N \end{array}\right)$, where the n by n matrices U,V,M,N are polynomials of the matrices A, B, C, D. The following results extend those of W. E. Roth [Proc. Amer. Math. Soc. 1 (1950), 586-590] and J. F. Potter [J. SIAM Appl. Math. 14 (1966), 496-502]. Theorem 1. Let R, $f_\gamma(R)$ be defined as above, with either $U^{-1}$ or $M^{-1}$ existing. Then X is a solution of the matrix equations (*) $XDX + AX + XB + C = 0$ if and only if X is a solution of the pair of equations (**) $XM + N = 0$, $XU + V = 0$. Theorem 2. Let $f_\gamma(\lambda)$ be any real polynomial of degree $n \geq 1$ having real coefficients, where $f_\gamma(R)$, R are defined above, with either $U^{-1}$, $M^{-1}$, $N^{-1}$ existing. Let X be a common solution of at least one of the pairs of equations (***) $XU + V = 0$, $XM + N = 0$, or (****) $U - MX = 0$, $V - NX = 0$. Then X is also a solution of the following matrix equation $XMX + Xf_\gamma(-B - DX) - f_\gamma(XD + A)X + V = 0$. (Received January 15, 1969.)

69T-A47. GEORGE A. GRÄTZER, H. LAKSER, and J. PŁONKA, University of Manitoba, Winnipeg 19, Manitoba. Joins and direct products of equational classes of algebras.

For equational classes $K_0$ and $K_1$ of (universal) algebras, $K_0 \vee K_1$ is the smallest equational class containing $K_0$ and $K_1$; $K_0 \times K_1$ is up to isomorphism the class $\{\mathfrak{A}_0 \times \mathfrak{A}_1 | \mathfrak{A}_i \in K_i\}$; $K_0$ and $K_1$ are independent if there is a binary polynomial p satisfying $p(x_0, x_1) = x_i$ on $K_i$ $(i = 0,1)$. M(K) is the condition that all algebras in K have modular congruence lattices. K is trivial if it contains one element algebras only. Theorem 1. If $K_0$ and $K_1$ are independent, then $K_0 \vee K_1 = K_0 \times K_1$; furthermore, if $M(K_0 \vee K_1)$ then every $\mathfrak{B} \in K_0 \vee K_1$ has a unique factorization $\mathfrak{B} = \mathfrak{A}_0 \times \mathfrak{A}_1$, $\mathfrak{A}_i \in K_i$. Theorem 2. If $K_0 \wedge K_1$ is trivial, and $M(K_0 \vee K_1)$, then $K_0 \vee K_1 = K_0 \times K_1$ iff $K_0$ and $K_1$ are independent. Examples show that $K_0 \vee K_1 = K_0 \times K_1$, $M(K_0 \vee K_1)$ do not imply that $K_0 \wedge K_1$ is trivial. Theorem 3. If $K_0$ and $K_1$ are independent, then any Mal'cev type condition (see Abstract 663-134, these *Notices* 16 (1969), 124) that holds for $K_0$ and $K_1$ holds for $K_0 \vee K_1$. A result related to the first statement of Theorem 1 was independently observed by P. Kelenson. (Received January 27, 1969.)

69T-A48. ROBERT GILMER, Florida State University, Tallahassee, Florida 32306. Integral domains with Noetherian subrings.

Let D be an integral domain with identity having prime subring $\Pi$. Theorem 1. The following conditions are equivalent: (1) Each subring of D is Noetherian. (2) Each subring of D with identity is Noetherian. (3) Either D has characteristic 0 and the quotient field of D is a finite algebraic extension of the rational field Q, or D has characteristic $p > 0$ and the quotient field K of D is either algebraic over $\Pi$ or K is a finite algebraic extension of a simple transcendental extension $\Pi(X)$ of $\Pi$. Theorem 2. Suppose that n is a fixed positive integer. These conditions are equivalent: (1) Each subring of D has dimension $\leq n$. (2) Each subring of D with identity has dimension $\leq n$. (3) Either D has characteristic 0 and tr.d. $(K/Q) \leq n - 1$, where K is the quotient field of D, or D has characteristic 0 and tr.d. $(K/\Pi) \leq n$. (Received January 27, 1969.)

69T-A49. JIN BAI KIM, West Virginia University, Morgantown, West Virginia 26506. The rank of the product of two matrices.

Let F be a field, and V(F) be a vector space over F. Let L(V) be the ring of all linear transformations of V(F). With each element A of L(V) we associate two subspaces of V(F) : (1) the range space R(A) of A, consisting of all xA with x in V(F), (2) the null space N(A) of A, consisting of all y in V(F) such that yA = 0. $\rho(A)$ denotes the rank of the linear transformation A. Theorem. If A and B are two linear transformations of a finite-dimensional vector space V(F), then $\rho(AB) = \rho(A) - \dim(R(A) \cap N(B))$. (Received January 16, 1969.)

69T-A50. ALAN L. KOSTINSKY, University of California, Berkeley, California 94720. Finitely generated projective lattices. Preliminary report.

Lattices are regarded as algebras, with the usual sum and product. It is known that a lattice B is projective (in the category of all lattices and lattice homomorphisms) iff there are a free lattice A, an epimorphism f of A onto B, and a monomorphism g of B into A such that f composed with g is the identity on B. Theorem. A finitely generated lattice is projective iff it is isomorphic to a sublattice of a free lattice. The corresponding theorem for finite lattices is stated in a paper to be published by Ralph McKenzie. The method used is basically due to McKenzie and Bjarni Jónsson. (Received February 3, 1969.)

69T-A51. ROBERT M. VANCKO, University of Manitoba, Winnipeg 19, Manitoba, Canada. Two theorems on local independence. Preliminary report.

Let $\mathfrak{A} = \langle A,F \rangle$ be a universal algebra. A subset I of A is independent if every mapping of I into A can be extended to a homomorphism of [I], the subalgebra generated by I, into $\mathfrak{A}$. I is locally independent if every mapping of I into [I] can be extended to an endomorphism of [I]. S. Swierczkowski proved that a finite set A admits a nontrivial algebra $\mathfrak{A}$ generated by 3 elements, such that every 3 elements of $\mathfrak{A}$ are independent, if and only if $|A| = 4$. Theorem 1. A finite set A admits an algebra $\mathfrak{A}$, generated by 3 elements, such that in $\mathfrak{A}$ every 3 elements are locally independent, if and only if $|A| = 4$, or $|A|$ is divisible by 3 or 8. Theorem 2. A finite set A admits an algebra $\mathfrak{A}$, generated by k elements ($k \geqq 4$), such that in $\mathfrak{A}$ every k elements are locally independent, if and only if $|A|$ is divisible by k. (Received February 5, 1969.)

69T-A52. LEROY B. BEASLEY, University of British Columbia, Vancouver 8, B. C., Canada. Linear transformations on matrices. Preliminary report.

Let $M_n(F)$ denote the set of all $n \times n$ matrices over the algebraically closed field F. We investigate those linear transformations on $M_n(F)$ which (i) preserve the minimum polynomial of each $A \in M_n(F)$; (ii) preserve the third elementary symmetric function, $E_3$, for each $A \in M_n(F)$; and (iii) those which map all rank K matrices into a set of rank K matrices. We show that those satisfying (i) are similarity transformations; those satisfying (ii) are of the form $T : A \to UAV$ or $T : A \to UA^T V$ where $UV = e^{i\theta} I_n$ and $3\theta \equiv (0) \pmod{2\pi}$. If $n > 2K^2 + K$, or T maps no matrix of rank less than K into a matrix of rank K, or T is nonsingular, or no matrix of rank less than 2K is in the kernel of T, then those satisfying (iii) are shown to be of the form $T : A \to UAV$, where U and V are nonsingular $n \times n$ matrices. (Received February 11, 1969.)

69T-A53. KENNETH R. DRIESSEL and MICHAEL G. STONE, University of Colorado, Boulder, Colorado 80302. On endomorphism semigroups of relational structures.

Let A be any nonempty set. For $E \subseteq {}^A A$ and m a cardinal let $K_m(E) = \{f \in {}^A A : \forall X \subseteq A$ (if card $X < m$, then $\exists g \in E\ (g \upharpoonright X = f \upharpoonright X))\}$ and let $\rho_m(E)$ be the condition $E = \text{End}\ \mathfrak{A} = \{f \in {}^A A : f(R_i) \subseteq R_i$ for all $i \in I\}$ for some relational structure $\mathfrak{A} = \langle A,R_i\rangle_{i \in I}$ satisfying rank $R_i < m$ for all $i \in I$. (Compare B. Jónsson, J. Plonka, Colloq. Math. 19 (1968), 1-8, M. G. Stone, Abstract 663-404, these *Notices* 16 (1969), 203.) Note that $K_m$ is a closure operator on ${}^A A$; for infinite m, $K_m$ is a topological closure operator which makes ${}^A A$ a topological semigroup. Theorem 1. $\rho_m(E)$ iff $\text{Id}_A \in E$, E is closed under composition and $K_m(E) = E$. Corollary 1. If A is finite, then every semigroup of functions on A which contains the identity function is the endomorphism semigroup of some finitary relational structure on A. Corollary 2. Every semigroup E of functions on

A which contains the identity function on A satisfies $\rho_m(E)$ for some m. The condition $\rho_m(E)$ may be modified by replacing "End $\mathfrak{A}$" by "Epi $\mathfrak{A}$", "Mon $\mathfrak{A}$" or "Aut $\mathfrak{A}$"; characterizations of the set of all E satisfying these modified conditions follow easily from Theorem 1. D. Monk has observed that Theorem 1 also remains true if "relational structure" is replaced by "partial algebra" in $\rho_m(E)$. (Received February 4, 1969.)

69T-A54. GEORGE A. GRÄTZER, J. PŁONKA, and A. SEKANINA, University of Manitoba, Winnipeg 19, Manitoba, Canada. On the number of polynomials of a universal algebra. I

Let $P_n(\mathfrak{A})$ denote the number of n-ary polynomials (over $\mathfrak{A}$) depending on every variable, in case of $p_1(\mathfrak{A})$, excluding x as a polynomial; $p_0(\mathfrak{A})$ is the number of constant unary polynomials. A sequence $\mathfrak{p} = \langle p_0, p_1, \ldots, p_n, \ldots \rangle$ is representable if $p_n = p_n(\mathfrak{A})$ for all n, for some algebra $\mathfrak{A}$. Theorem 1. $\mathfrak{p}$ is representable if $p_0 \neq 0$. Theorem 2. $\mathfrak{p}$ is representable if $p_0 = 0$ and $p_n > 0$ for $n > 0$. Theorem 3. $\mathfrak{p}$ is representable if $p_0 = 0$, and $p_1 > 0$ and (a) for all $n > 0$, $p_{2n+1} > 0$, and 2n divides $p_{2n}$, or (b) n divides $p_n$ for all $n > 0$. Note that the problem proposed here is (modulo a simple binomial formula) equivalent to Problem 42 of Universal algebra (Van Nostrand, Princeton, N. J., 1968), hence Theorems 1-3 can be viewed as partial solutions to the same. (Received February 6, 1969.)

69T-A55. STEPHEN H. BROWN, North Carolina State University, Raleigh, North Carolina 27607. On rational completeness.

Findlay and Lambek [A generalized ring of quotients. I and II, Canad. J. Math. 1 (1958), 77-84, 155-167] define a module M to be rationally complete in case every fractional homomorphism from any module B into M is extendible to a full homomorphism of B into M. They further show that this is equivalent to the requirement that M have no proper rational extension as defined in the above paper. Let $\bar{A}$ denote the maximal rational extension of the R-module $A_R$ and $\hat{A}$ its injective hull. In the notation of Findlay and Lambek, $\bar{A} = \{x \in \hat{A} | (A : x) \leq R^{\#}(\hat{A})\}$, where $R^{\#} = R \oplus Z$. Let $S(S^{\#})$ be the set of annihilator right ideals of the elements of $\hat{A}$ in $R(R^{\#})$. Lemma. If $A_R$ is simple, $x \neq 0 \in \bar{A}$ iff $(x)^r_R$ is maximal in S, iff $(x)^r_{R^{\#}}$ is maximal in $S^{\#}$. Theorem. If R is a ring such that all right ideals are left ideals, then every simple R-module is rationally complete. This condition is not necessary since every completely reducible ring has this property. Furthermore, the matrix ring $\begin{bmatrix} Z_4 & 2Z_4 \\ Z_4 & Z_4 \end{bmatrix}$ is a ring such that no simple module is rationally complete. Theorem. If R is any ring, there exists an R-module $T_R$ such that for any other module $M_R$, $M_R \oplus T$ is rationally complete. From this, it follows that if R is a ring such that every rationally complete module is injective, then R is completely reducible. (Received February 5, 1969.)

69T-A56. CRAIG R. PLATT, University of Manitoba, Winnipeg 19, Manitoba, Canada. Limit ranks of classes of universal algebras. Preliminary report.

Let X be an operator on classes of algebras (that is, if H is a class of algebras, so is X(H)). For ordinals α, define $X^\alpha(H)$ recursively: $X^0(H) = H$, $X^{\beta+1}(H) = X(X^\beta(H))$, and if $\beta$ is a limit ordinal, $X^\beta(H) = \bigcup \{X^\gamma(H) : \gamma < \beta\}$. Define X-rank (H) to be the least ordinal $\varkappa$ such that $X^\varkappa(H) = X^{\varkappa+1}(H)$, if such exists, and ∞ otherwise. For a class H of similar algebras define $\underrightarrow{L}(H)$ [respectively, $\underrightarrow{L}^{1-1}(H)$, $\underrightarrow{L}(H)$, $\underrightarrow{L}^{onto}(H)$] to be the class of all isomorphic copies of direct limits [respectively, 1-1 direct limits, inverse limits, onto inverse limits] of algebras in H. Let $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ be ordinals and $\alpha_1 \leqq \alpha_2$, $\alpha_3 \leqq \alpha_4$. Then there is a class K of algebras with $\underset{\rightarrow}{L}^{1\text{-}1}$-rank (K) = $\alpha_1$, $\underset{\rightarrow}{L}$-rank (K) = $\alpha_2$, $\underset{\leftarrow}{L}^{\text{onto}}$-rank (K) = $\alpha_3$, $\underset{\leftarrow}{L}$-rank (K) = $\alpha_4$. Assuming the axiom of choice, K can be chosen to consist of lattices, semigroups, commutative groupoids, or algebras of any prescribed similarity type, provided it has at least two unary operators or one n-ary operator for some $n \geqq 2$. If we further assume there exists no measurable cardinal (in the sense of e.g., P. J. Cohen, Set theory and the continuum hypothesis, Benjamin, New York, p. 81) then we may replace any or all of $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ by $\infty$, (where we define $\infty \leqq \infty$ and $\alpha \leqq \infty$ for all ordinals $\alpha$). These results extend those of Abstract 68T-A13, these *Notices* 15 (1968), 785. In proving the result for lattices we construct a large discrete category of bounded lattices of type $\langle \vee, \wedge, 0, 1 \rangle$. (Received February 6, 1969.)

69T-A57. WITHDRAWN.

69T-A58. ROBERT C. SHOCK, University of North Carolina, Chapel Hill, North Carolina 27514. Finite dimensional rings. Preliminary report.

A ring R is right finite dimensional provided that R does not contain an infinite direct sum of nonzero right ideals. Proposition 1. R is right finite dimensional if and only if the matrix ring R over R is right finite dimensional if and only if the polynomial ring over R is right finite dimensional. Proposition 2. If G is a finitely generated abelian group and R is right finite dimensional, then the group ring A = RG is right finite dimensional. (Received February 11, 1969.)

## Analysis

69T-B51. SYED M. MAZHAR, Aligarh Mulslim University, Aligarh, India. On the local property of summability $|C, r+1|_k$ of the rth derived series of Fourier series. I.

In this note the following theorems have been proved: Theorem 1. If f(t) is a periodic function with period $2\pi$ and integrable (L) over $(0, 2\pi)$, then the summability $|C, r+1|_k$, $k \geqq 1$, of the rth derived series of the Fourier series of f(t) is not necessarily a local property of the generating function. Theorem 2. Let f(t) be a periodic function with period $2\pi$ and integrable (L) over $(0, 2\pi)$. Then the summability $|C, r+1|_k$ of the rth derived series of the Fourier series depends only on the behaviour of the generating function f(t) in the immediate neighbourhood of the point t = x, if when r is even $\sum |A_n(x)|^k/n < \infty$ and when r is odd, $\sum |B_n(x)|^k/n < \infty$, where $f(x) \sim \sum_0^\infty A_n(x)$ and $\sum B_n(x)$ is the conjugate series of the Fourier series of f(x). These theorems include for k = 1 results of Bhatt (Indian J. Math. 9 (1967), 17-24) and for r = 0 certain result of Flett (Proc. London Math. Soc. 8 (1958), 357-387). (Received January 8, 1969.)

69T-B52. V. I. GAVRILOV, Indian Institute of Technology, Powai, Bombay-76, India. The behaviour of meromorphic functions in the neighborhood of an essential singularity.

In the paper (V. I. Gavrilov, Izv. Akad. Nauk USSR Ser. Math. 30, 4, (1966), 767-788) the distribution of the values of meromorphic (holomorphic) functions in the neighborhood of an essential singularity was studied in terms of $M_h$ - sequences ($\mu_h$-sequences). It is shown that any $\mu_h$-sequence for a holomorphic function f(z) is a $M_h$-sequence and conversely any $M_h$-sequence for f(z) contains a subsequence which is a $\mu_h$-sequence. The concept of a $\mu_h$-sequence is extended so as to be applicable to meromorphic as well as holomorphic functions and it is shown that the above statement remains valid for $\mu_h$-sequences (in the new sense) and $M_h$-sequences. (Received December 2, 1968.) (Author introduced by Professor P. C. Jain.)

69T-B53. IRA ROSENBAUM, University of Miami, Coral Gables, Florida. Contextual definition of 'limit point, at M, of f(x)' in terms of statements of the form: Persistently, at M,P(x).

To the definitions of a previous Note on some types of quantified statements of use in analysis may be added the following contextual definitions of statements involving the phrase 'limit point, at M, of f(x)', a phrase symbolized below by $LP_M f(x)$: (1) $L \in LP_M f(x) . \leftrightarrow . (A\gamma)Pers_M(-\gamma < f(x) - L < \gamma)$, (2) $+\infty \in LP_M f(x) . \leftrightarrow . (A\gamma)Pers_M(f(x) > \gamma)$, and (3) $-\infty \in LP_M f(x) . \leftrightarrow . (A\gamma)Pers_M(f(x) < -\gamma)$. Also, in the extended reals, defining (4) $L \in LB_*(H) . \leftrightarrow . (Aq)(q \in H . \rightarrow . L \leq q)$, (5) $L \in UB_*(H) . \leftrightarrow . (Aq)(q \in H. \rightarrow . q \leq L)$, (6) $L = \max_*(H) . \leftrightarrow : L \in UB_*(H) . \& . L \in H$; (7) $L = \min_*(H) . \leftrightarrow : L \in LB_*(H) . \& . L \in H$; one finds, as expected, (8) $\text{Lim sup}_M f(x) = \max_*(LP_M f(x))$ and (9) $\text{Lim inf}_M f(x) = \min_*(LP_M f(x))$. The results (8) and (9) supplement those obtainable from the definitions of the previously cited Note for the finite case, given next: (10) $\text{Lim sup}_M f(x) = GLB(k: Ult_M(f(x) < k))$, and (11) $\text{Lim inf}_M f(x) = LUB(k: Ult_M(f(x) > k))$. (Received November 27, 1968.)

69T-B54. WILLIAM STENGER, Georgetown University, Washington, D. C. 20007. On the eigenvalues of perturbed operators.

Let A be a selfadjoint operator whose spectrum begins with isolated eigenvalues $\lambda_1 \leq \lambda_2 \leq \ldots$. Let D be defined by $Du = \sum_{j=1}^{r} a_j(u,p_j)p_j$, all $a_j > 0$. The spectrum of A + D begins with isolated eigenvalues $\lambda_1^r \leq \lambda_2^r \leq \ldots$ satisfying $\lambda_i \leq \lambda_i^r \leq \lambda_{i+r}$ $(i = 1,2,\ldots)$. Let $R_\lambda = (A - \lambda I)^{-1}$, let $\rho(\lambda)$ be the rank, and let $\pi(\lambda)$ be the index of the quadratic form obtained from $\{\delta_{jk} + a_j(R_\lambda p_j, p_k)\}$ (j, k = 1, 2,..., r). Let $m(\lambda_i) = \min\{j|\lambda_j = \lambda_i\}$ and $M(\lambda_i) = \max\{j|\lambda_j = \lambda_i\}$. Theorem 1. $\lambda_i = \lambda_i^r$ if and only if for any sufficiently small $\epsilon > 0$, $r + i - M(\lambda_i) \leq \pi(\lambda_i + \epsilon)$. Theorem 2. $\lambda_i^r = \lambda_{i+r}$ if and only if for any sufficiently small $\epsilon > 0$, $r + 1 - m(\lambda_{i+r}) \leq \pi(\lambda_{i+r} - \epsilon) \leq r + i - m(\lambda_{i+r})$ and $m(\lambda_{i+r}) - i \leq \rho(\lambda_{i+r} - \epsilon) - \pi(\lambda_{i+r} - \epsilon) \leq m(\lambda_{i+r}) - 1$. (Received November 21, 1968.)

69T-B55. JACOB BURBEA, Stanford University, Stanford, California 94305. The Szegö kernel function in the ring.

Garabedian [Trans. Amer. Math. Soc. 67 (1949), 1-35] showed the monotonicity property of the Szegö kernel $\hat{K}_D(z,\bar{t})$ of a domain $D \subset C^1$. By introducing the conformal invariant metric $d\hat{s}_D^2 = \hat{K}_D^2(z,\bar{z})|dz|^2$, it is shown that the quantity $\hat{I}_D(z,\bar{z}) = (2/\hat{K}_D^2)\partial^2 \log \hat{K}_D/\partial z \partial \bar{z}$ is positive and conformally invariant. Further $\hat{I}_D(z,\bar{z}) = -2^{-1}C(z)$, where C(z) is the Gaussian curvature of the metric with the line element $d\hat{s}_D^2$. The quantity $J_D(z,\bar{t}) = K_D(z,\bar{t})/\hat{K}_D^2(z,\bar{t})$ is conformally invariant, here $K_D(z,\bar{t})$ is the Bergman kernel of D. Let $t \in \partial D$, it holds $\lim_{z \to t} \text{Re}(z + t)\hat{K}_D(z,\bar{z}) = (4\pi)^{-1}$, $\lim_{z \to t} \hat{I}_D(z,\bar{z}) = 8\pi^2$ and $\lim_{z \to t} J_D(z,\bar{z}) = 4\pi$, where $|\arg(z - t)| \leq \alpha < \pi/2$. Especially, if $D = \{z: 0 < r < |z| < 1\}$, then $\hat{K}_D^2(z,t) = (4\pi^2 z\bar{t})^{-1}\{\wp(\log z\bar{t}, w_1, w_2) - e_3\}$, where $\wp$ is the Weierstrass $\wp$- function with the half periods $w_1 = \pi i$, $w_2 = \log r$. The only root of $\hat{K}_D(z,\bar{t}) = 0$ is $z\bar{t} = -r$. Defining $I_1(\rho) = \hat{I}_D(z,\bar{z})$, $I_2(\rho) = J_D(z,\bar{z})$, for $\rho = |z|$, $r \leq \rho \leq 1$, then $I_k(\rho)$, k = 1,2, are positive non-constant functions of $\rho$ such that $I_k(\rho) = I_k(r/\rho)$. Compare Zarankiewicz [Z. Angew. Math. Mech. 14 (1934), 97-104]. We have $I_1(\rho) = 8\pi^2 + 2\pi^2 (g_2 - 12e_3^2)/(\wp - e_3)$, $I_2(\rho) = 4\pi + 4\pi(\eta_2/w_2 + e_3)/(\wp - e_3)$. Note that $I_1(1) = I_1(r) = 8\pi^2$, $I_2(1) = I_2(r) = 4\pi$. Also, $I_1(\rho)[(I_2(\rho)]$ has only one maximum (minimum) point $\rho_m$ in $[r,1]$ and $\rho_m = \sqrt{r}$. This yeilds a new method for determining the modulus r of any doubly-connected domain D. (Received December 23, 1968.)

69T-B56. STERLING K. BERBERIAN, University of Texas, Austin, Texas 78712. Trace and the convex hull of the spectrum in a von Neumann algebra of finite class.

It is shown that if A is a von Neumann algebra of finite class, with center-valued normalized trace function $a \to \mathrm{Tr}\, a$ [J. Dixmier, Les anneaux d'opérateurs de classe finie, Ann. École Norm. Sup. 66 (1959), 209-261], then conv $\sigma(\mathrm{Tr}\, a) \subset$ conv $\sigma(a)$ for all a in A, where conv denotes convex hull, and $\sigma$ denotes spectrum. It follows that the trace of a generalized nilpotent element of A is zero. The case of one-dimensional center was treated by B. Fuglede and R. V. Kadison [Determinant theory in finite factors, Ann. of Math. 55 (1952), 520-530]. The present essentially algebraic proof avoids determinant theory and is valid for AW*-algebras of finite class possessing a trace.
(Received December 30, 1968.)

69T-B57. W. T. KILEY, Brown University, Providence, Rhode Island 02912. Automorphism groups on compact Riemann surfaces.

For $g \geqq 2$, let N(g) be the order of the largest group of automorphisms on a surface of genus g. Then $N(g) \geqq 8(g + 3)$ if $g = 1 \pmod 4$; $N(g) \geqq 8(g + 7)$ if $g = 5 \pmod{12}$; and $N(g) \geqq 8(g + 15)$ if $g = 33 \pmod{48}$. Further each of these lower bounds equals N(g) for infinitely many such g.
(Received January 7, 1969.) (Author introduced by Professor Robert D. Accola.)

69T-B58. HARI M. SRIVASTAVA, West Virginia University, Morgantown, West Virginia 26506. Dual series relations involving generalised Laguerre polynomials. Preliminary report.

In the present paper the author considers the problem of determining the sequence $\{A_n\}$ such that (*) $\sum_{n=0}^{\infty}\{A_n/\Gamma(n + \alpha + 1)\}L_n^{(\nu)}(x) = f_1(x)$, $0 \leqq x < y$, and (**) $\sum_{n=0}^{\infty} \{A_n/\Gamma(n + \beta + 1)\}L_n^{(\sigma)}(x) = f_2(x)$, $y < x \leqq \infty$; where $L_n^{(\nu)}(x)$ is the generalized Laguerre polynomial defined by $(1 - t)^{-1-\nu}\exp\{- xt/(1 - t)\} = \sum_{n=0}^{\infty} L_n^{(\nu)}(x)t^n$, the functions $f_1(x)$ and $f_2(x)$ are prescribed, and $\alpha, \beta, \nu, \sigma > -1$. The recent observations of K. N. Srivastava [Pacific J. Math. 19 (1966), 529-533; see also its review in Zentral. Math. (1968) by H. M. Srivastava] are exhibited as a special case of the results derived here, and the scope of their further extensions is also discussed systematically.
(Received January 10, 1969.)

69T-B59. EDWARD M. LANDESMAN and ALAN C. LAZER, University of California, Los Angeles, California 90024. Nonlinear perturbations of elliptic boundary value problems at resonance.

Let $D \subset R^n$ be a bounded domain. Let $L = (\partial/\partial x_i) a^{ij}\, \partial/\partial x_j$ ($a^{ij} = a^{ji}$ real, bounded, and measurable on D) be a uniformly elliptic selfadjoint operator. Let $H_0$ be real $L^2(D)$ and $H_1^0$ be the completion of the real inner product space of $C^1$ test functions with compact support contained in D with norm $\|u\|_1 = (\int_D [u^2 + \sum_{i=1}^n (\partial u/\partial x_i)^2] dx)^{1/2}$. Let $w \in H_1^0$ be a nontrivial weak solution of (*) $Lu + \alpha u = 0$ such that any other $H_1^0$ weak solution of (*) is of the form $cw$. Let $g \in C(R,R)$ and assume $\lim_{t\to\infty} g(t) = g(\infty)$ and $\lim_{t\to-\infty} g(t) = g(-\infty)$ exist with $g(-\infty) \leqq g(t) \leqq g(\infty)$ for all $t \in R$. Theorem. Let $h \in H_0$, $D^+ = \{x \in D | w(x) > 0\}$, $D^- = \{x \in D | w(x) < 0\}$ and $\langle\ ,\ \rangle_0$ be the usual $L^2(D)$ inner product. The inequalities $g(-\infty)\int_{D^+}|w|dx - g(\infty)\int_{D^-}|w|dx \leqq \langle h,w\rangle_0 \leqq g(\infty)\int_{D^+}|w|dx - g(-\infty)\int_{D^-}|w|dx$ are necessary and the strict inequalities $g(-\infty)\int_{D^+}|w|dx - g(\infty)\int_{D^-}|w|dx < \langle h,w\rangle_0 < g(\infty)\int_{D^+}|w|dx - g(-\infty)\int_{D^-}|w|dx$ are sufficient for the existence of a weak $H_1^0$ solution of $Lu + \alpha u + g(u) = h$. (Received January 20, 1969.)

69T-B60. WITHDRAWN.

69T-B61. JOHN WERMER, Brown University, Providence, Rhode Island 02912. Invariant subspaces for some algebras.

Let $\mu$ be a finite positive measure with compact carrier $X$ in the z-plane. $R_0(X)$ denotes the algebra of rational functions with poles off $X$. $H^2(\mu)$ denotes the closure of $R_0(X)$ in $L^2(\mu)$. $R_0(X)$ operates on $H^2(\mu)$ by multiplication. We call $\mu$ "good" if there exists a nontrivial closed subspace of $H^2(\mu)$ invariant under the action of $R_0(X)$. Choose countably many disjoint disks $|z - a_j| < r_j$, $j = 1,2,\ldots$, contained in $|z| < 1$, with $\sum_j r_j < \infty$. Put $\Gamma$ = the union of the boundaries of the disks together with $|z| = 1$ and use ds to denote the measure: arc length on $\Gamma$. Note that $H^2(ds) \neq L^2(ds)$. Theorem. ds is good. This is obtained from Lemma. There exists $F$ in $H^2(ds)$ with $|F| = 1$ a.e.-- ds and $F$ is not a constant. For related information, see the abstract by J. Brennan [Abstract 67T-660, these *Notices* 14 (1967), 854]. (Received January 22, 1969.)

69T-B62. JOSEPH WALSH and WALTER J. SCHNEIDER, University of Maryland, College Park, Maryland 20742. On the shape of level loci of harmonic measure.

If $\Gamma_r$ is a Jordan curve of the w-plane whose interior D contains $w = 0$, we define the circularity of $\Gamma_r$ (with respect to $w = 0$) as $\varkappa(\Gamma_r) = [\min|w|, w \text{ on } \Gamma_r]/[\max|w|, w \text{ on } \Gamma_r]$. It is then known [J. L. Walsh, Amer. Math. Monthly 44 (1937)] for the level loci $\Gamma_r$ of Green's function for the interior of $\Gamma_1$ that $\varkappa(\Gamma_r)$ increases as r decreases, $0 < r < 1$, and approaches unity as r approaches zero; here $\Gamma_r$ is the image of $|z| = r$ when $|z| < 1$ is mapped onto D. This note extends that result to level loci of harmonic measure: If D is an annular region of the w-plane, the image of $\rho < |z| < 1$, $\Gamma_r$ the image of $|z| = r$, $\rho < r < 1$, and if $\varkappa(\Gamma_\rho) = 1$, then $\varkappa(\Gamma_r)$ increases as r decreases and approaches unity as r approaches $\rho$. There is a similar result for ellipticity of a curve (comparison of shape with that of an ellipse) with respect to two given foci, also a result on the shape of infinite strips, and another for the shape of an infinite region each of whose boundaries lies in a sector. (Received January 23, 1969.)

69T-B63. FAUSTO A. TORANZOS, Universidad de Buenos Aires, Perú 222, Buenos Aires, Argentina. Contractions in a generalized metric space.

Given a family $\underset{\sim}{F} = \{(X_\alpha, d_\alpha): \alpha \in A\}$ of pairwise disjoint metric spaces, the disjoint union of $\underset{\sim}{F}$ is the pair $(X,d)$ such that $X = \bigcup_{\alpha\in A} X_\alpha$ and $d: X \times X \to \bar{R}$ (extended real line) defined $d(x,y) = d_\alpha(x,y)$ if x and y belong to $X_\alpha$, and $d(x,y) = +\infty$ if x and y belong to different spaces. Proposition.

A disjoint union of metric spaces is a generalized metric space (g.m.s.) and conversely, any g.m.s. can be expressed in a unique way as a disjoint union of metric spaces. Each of the metric spaces mentioned in the above result is a metric component of the g.m.s. in question. Let f be a mapping of A into A and B be a subset of A. Then B is semistable under f if $B \cap f(B) \neq \emptyset$. Theorem. Let f be a contraction mapping on the complete g.m.s. (X,d). Then f has exactly one fixed point in each semistable metric component of (X,d). This theorem generalizes previous results of W. A. J. Luxemburg (Indag. Math. 20 (1958), 540-546) and Diaz & Margolis (Bull. Amer. Math. Soc. 74 (1968), 305-309) as well as the classical Banach's contraction mapping theorem. (Received January 27, 1969.)

69T-B64. R. S. KHAN, Aligarh Muslim University, Aligarh, (U.P.), India. A note on a theorem of ASKEY. II.

Theorem 1. Let $\{\lambda(n)\}$ be a positive monotonic decreasing sequence such that $\sum_{k=n}^{\infty}\lambda(k) = O(n\lambda(n))$, $n \to \infty$. Suppose $S_n = \sum_{k=1}^{n} a_k$ and $\sum_1^{\infty}\lambda(n)\cdot(na_n)^p < \infty$, $a_k \geqq 0$, then $\sum_1^{\infty}\lambda(n)S_n^p < \infty$ and $\sum_1^{\infty}\lambda(n)S_n^p \leqq A\sum_1^{\infty}\lambda(n)\cdot(na_n)^p$, where $p \geqq 1$. If $0 < p < 1$, then the above result is true provided $\{n^{-j}a_n\}$ is monotonic decreasing for some $j > 0$. Theorem 2. Let $\{\lambda(n)\}$ be a positive monotonic sequence and suppose there exists a number $\delta$, $0 < \delta < 1$, such that $\{n^{\delta}\lambda(n)\}$ is monotonic increasing. If $\sum_1^{\infty}\lambda(n)\cdot(na_n)^p < \infty$, then $\sum_1^{\infty}\lambda(n)S_n^p < \infty$ and $\sum_1^{\infty}\lambda(n)S_n^p \leqq A\sum_1^{\infty}\lambda(n)\cdot(na_n)^p$, where $S_n = \sum_{k=n}^{\infty} a_k$, $a_k \geqq 0$ and $p \geqq 1$. If $0 < p < 1$, then this result remains true provided $\{n^{-j}a_n\}$ is monotonic decreasing for some $j > 0$. These theorems generalize results of Hardy and Littlewood (J. für Math. 157 (1927), 141-158) for $p \geqq 1$ and those of A. A. Konyushkov (Mat. Sbornik 44 (86) (1958), 53-84) for $0 < p < 1$. Applying these results, a theorem has been proved which includes, as a special case, a theorem of R. Askey (Acta. Sci. Math. 28 (1967), 169-171) and also a theorem of the author (under publication in Acta Sci. Math.). (Received January 27, 1969.) (Author introduced by Dr. Syed M. Mazhar.)

69T-B65. JAMES V. HEROD, Georgia Institute of Technology, Atlanta, Georgia 30332. A Gronwall inequality for linear Stieltjes integrals.

Let S be an interval of numbers containing zero and J be a linear function from the class of functions of bounded variation on each finite interval of S so that if f is such a function then $J[f]$ is a real-valued function on $S \times S$ and (1) $J[f](x,y) + J[f](y,z) = J[f](x,z)$ for y between x and z, (2) if $f(y) \geqq 0$ for y between x and z then $J[f](x,z) \geqq 0$, and (3) if $x \geqq 0$ and in S then $J[0_x](x,x^+) < 1$ and $J[1_x](x^-,x) < 1$ and if $x \leqq 0$ and in S then $J[1_x](x^+,x) < 1$ and $J[0_x](x,x^-) < 1$. Theorem. There is a function m from $S \times S$ to the real numbers having the following properties: (i) $m(x,y) \geqq 1$ for each $\{x,y\}$ in $S \times S$, (ii) $m(x,y)m(y,z) = m(x,z)$ for y between x and z, (iii) $m(0,x) = 1 + J[m(0,\cdot)](0,x)$ for each x in S, and (iv) if f is of bounded variation on each finite interval of S, P is a number, and $f(x) \leqq P + J[f](p,x)$ for each x in S, then $f(x) \leqq Pm(0,x)$ for each x in S. (Received January 29, 1969.)

69T-B66. HEINRICH W. GUGGENHEIMER, Polytechnic Institute of Brooklyn, Brooklyn, New York 11201. An application of Floquet theory.

The nonlinear differential equation $x'' + Qx - cx^{-3} = 0$, Q periodic, c constant $\neq 0$, has a periodic solution if and only if the solutions of $x'' + Qx = 0$ are all stable. The periodic solution is unique unless

all solutions of $x'' + Qx = 0$ are periodic; in the latter case, all solutions of the nonlinear equations are periodic. The proof is by an interpretation of Floquet theory in unimodular centroaffine differentia geometry. The method extends to $y'' - (Q'/Q)y' + Qy - Q^2y^{-3} = 0$ and some similar equations.
(Received February 5, 1969.)

69T-B67. JACK B. BROWN, Auburn University, Auburn, Alabama 36830. A construction theorem for real connectivity functions.

Suppose (i) $M_1, M_2, M_3, \ldots$ is a monotonic increasing sequence of Cantor subsets of the interval $I = [0,1]$ such that if j is a positive integer and C is a component of $I - M_j$, then $Cl(C) \cap M_{j+1}$ is a Cantor set, and $M_1 \cup M_2 \cup M_3 \cup \ldots$ is dense in I, and (ii) $f_1, f_2, f_3, \ldots$ is a sequence of functions from I into I such that $f_1$ is continuous on I and constant on each component $I - M_1$, and if n is an integer greater than 1, $f_n$ agrees with $f_{n-1}$ on $M_{n-1}$, is constant on each component of $I - M_n$, and is continuous on the closure of each component of $I - M_{n-1}$. Then if f is a function from I into I such that for each x in I, f(x) is the sequential limit of some subsequence of $f_1(x), f_2(x), f_3(x), \ldots$, f is a connectivity function. (Received February 5, 1969.)

69T-B68. FRANK R. DEUTSCH and PETER D. MORRIS, Pennsylvania State University, McAllister Building, University Park, Pennsylvania 16802. On simultaneous approximation and interpolation which preserves the norm.

Let X be a real normed linear space, M a dense subspace of X, and $\Gamma = \{x_1^*, \ldots, x_n^*\} \subset X^*$. The triple $(X, M, \Gamma)$ is said to have property SAIN if: For each $x \in X$ and $\epsilon > 0$ there exists a $y \in M$ such that (1) $\|x - y\| < \epsilon$, (2) $x_i^*(y) = x_i^*(x)$ $(i = 1, \ldots, n)$, and (3) $\|y\| = \|x\|$. Yamabe (Osaka Math. J. 2 (1950), 15-17) showed that the conditions (1) and (2) alone are satisfied by any such triple. Wolibner (Colloq. Math. 2 (1951), 136-137) proved that if $X = C[a,b]$, M = "the polynomials", and the $x_i^*$ are "point evaluations", then $(X, M, \Gamma)$ has property SAIN. This is generalized by Theorem 1. If T is compact Hausdorff, $t_1, \ldots, t_n \in T$, $x_i^*$ denotes point evaluation at $t_i$ $(i = 1, \ldots, n)$, and if M is a dense subalgebra (or a dense linear sublattice containing constants) of C(T), then $(C(T), M, \{x_1^*, \ldots, x_n^*\})$ has property SAIN. This result strengthens the Stone-Weierstrass theorem. Theorem 2. Let X be a Hilbert space, M a dense subspace, and $\Gamma = \{x_1^*, \ldots, x_n^*\} \subset X^*$. Then $(X, M, \Gamma)$ has property SAIN if and only if each $x_i^*$ attains its norm on the unit ball in M. In the case $n = 1$, Theorem 2 is valid in any strictly convex reflexive space. Other results of a general nature, and in the case when $X = L_p$, $1 \leq p < \infty$, are given. (Received February 5, 1969.)

69T-B69. DONALD BRATTON, 2000 W. Hemlock Street, Oxnard, California 93030. Spectra in topological algebras.

An algebra A over the complex numbers is called a topological algebra when it is supplied with a topology $\mathcal{J}$ such that (1) $\mathcal{J}$ is compatible with the underlying vector space of A and is locally convex, and (2) if x is bounded in A and y converges to zero in A, then $xy \to 0$ and $yx \to 0$. An element $a \in A$ is called regular in A when there exists a bounded filter $\mathfrak{F}$ on A such that $xa \to 1$ and $ax \to 1$ as x runs over $\mathfrak{F}$. The equation $x - y = (xa - 1)y + x(1 - ay)$ proves certain properties of regularity. Namely, if $\mathfrak{F}$ is a bounded filter on A such that $xa \to 1$ as x runs over $\mathfrak{F}$ and $\mathcal{G}$ is a bounded filter on A such that $ay \to 1$ as y runs over $\mathcal{G}$, then a is regular and $\mathfrak{F}$ and $\mathcal{G}$ are equivalent filters. If a and b are regular, then ab and ba are regular, and conversely. The equations $(1 - ba)(1 + bxa) = 1 + b((1 - ab)x - 1)a$ and $(1 + bxa)(1 - ba) = 1 + b(x(1 - ab) - 1)a$ show that $1 - ba$ is regular if and only if $1 - ab$ is regular. A number $\lambda$ is called a spectral value of $a \in A$ when $a - \lambda$ is not regular. The set of spectral values of a is called the spectrum of a and one defines the spectral radius r(a) in the usual manner. $r(ab) = r(ba)$ and $r(x^n) = r(x)^n$. The usual properties of spectra hold good. (Received February 7, 1969.)

69T-B70. LEONARD Y.-H. YAP, Rice University, Houston, Texas 77001. Ideals in $L^1(G) \cap L^p(G)$.

Let G be a locally compact abelian group and $1 \leqq p < \infty$. Denote $L^1(G) \cap L^p(G)$ by $B_p(G)$ and if $f \in B_p(G)$, define $\|f\|_{(p)} = \|f\|_1 + \|f\|_p$. Then $B_p(G)$ is a Banach algebra with respect to $\|\cdot\|_{(p)}$ (multiplication in $B_p(G)$ is the usual convolution) and in general $B_p(G)$ has no bounded approximate unit. Theorem. The algebra $B_p(G)$ satisfies Ditkin's condition (see Loomis, An introduction to abstract harmonic analysis). Theorem. Let I be a closed ideal in $B_p(G)$. Then I contains every element f in kernel (hull(I)) such that boundary (hull (f)) $\cap$ hull(I) contains no nonvoid perfect set. C. R. Warner (Trans. Amer. Math. Soc. (1966), 408-423) obtained these results for the case $p = 2$. We also obtained some other properties of $B_p(G)$. (Received February 6, 1969.)

69T-B71. CHING CHOU, State University of New York, Buffalo, New York 14226. Nonuniqueness of left invariant means on a locally compact group.

Let G be a locally compact group and C(G) the space of bounded real-valued continuous functions on G with the sup norm. $\varphi \in C(G)^*$ is called a left invariant mean if $\|\varphi\| = 1$, $\varphi(f) \geqq 0$ whenever $f \geqq 0$, and $\varphi({}_x f) = \varphi(f)$, for $f \in C(G)$ and $x \in G$. The set of left invariant means on C(G) is denoted by $M\ell(C(G))$. G is called amenable if $M\ell(C(G))$ is not empty. Theorem. Let G be a locally compact amenable group. Then card $M\ell(C(G)) = 1$ or $\geqq 2^c$. It is one if and only if G is compact. (c is the cardinality of the continuum.) (Received February 7, 1969.)

69T-B72. ANDRE DE KORVIN, Indiana State University, Terre Haute, Indiana 47809. Stable expectations.

Let N and M be two von Neumann algebras with $N \subset M$. Let G be any subgroup of unitaries of M. All unitaries of M will be denoted by U(M). $N^c$ will denote the relative commutant of N in M. A linear map $\Psi$ of M into M will be called G-stable if $\Psi(X) = \Psi(U^{-1}XU)$ for all U in G. Theorem 1. Let Tr be a faithful, semifinite trace on M. Assume that the restriction of Tr to N is semifinite. Then there exists a normal, faithful, $U(N^c)$-stable expectation $\Psi$ of M on N such that $Tr(A\Psi(X)) = Tr(AX)$ for all X in M and all A in N such that $Tr|A| < \infty$. Theorem 2. A normal U(N)-stable expectation of M on $N^c$ is faithful. (Received January 8, 1969.)

69T-B73. JOAQUIN BUSTOZ, University of Cincinnati, Cincinnati, Ohio 45221. On the unbounded core theorem. Preliminary report.

It has been shown [Abstract 68T-421, these *Notices* 15 (1968), 626] that the bounded core theorem is a consequence of a theorem due to R. R. Phelps. The following theorem analogous to that of Phelps yields the unbounded core theorem. Let A and B be linear spaces of complex-valued functions defined on the sets X and Y respectively. Let A and B include the constant functions and T be a linear operator from A to B with $T1 = 1$. T is said to be real positive if $\mathrm{Re}\, f \geqq 0$, $f \in A$, implies $\mathrm{Re}\, Tf \geqq 0$. T is strictly real positive if $\mathrm{Re}\, f \geqq 0$ is equivalent to $\mathrm{Re}\, Tf \geqq 0$. Theorem. $Tf(Y) \subseteq \mathrm{conv}\, f(X)$ for all $f \in A$ if and only if $T1 = 1$ and T is real positive. $\mathrm{Conv}\, Tf(Y) = \mathrm{conv}\, f(X)$ for every $f \in A$ if and only if $T1 = 1$ and T is strictly real positive. (Received February 7, 1969.)

69T-B74. RAY E. WORTH, Georgia State College, Atlanta, Georgia 30303. Linear independence in semilinear spaces. Preliminary report.

See Abstract 650-25, these *Notices* 14 (1967), 923, for pertinent definitions. Each finite linear combination $\sum \alpha_i x_i$ can be reduced, i.e. replaced by $\sum \beta_i x_i$, where $\beta_i x_i + \beta_j x_j = 0$ implies $\beta_i \beta_j = 0$, and we assume this has been done to each combination. $\{x_1, \ldots, x_n\}$ is linearly independent provided $\sum \alpha_i x_i = \sum \beta_i x_i$ implies $\alpha_i = \beta_i$ for each i. Note this allows $-x$ to be included with x. We get the natural extensions to basis. Not all semilinear spaces have a basis. Theorem 1. If S is a topological semilinear space, A is a lin. indep. subset of S and the span of A has a nonempty topological interior, then A is maximal. Theorem 2. If S is a Banach semilinear space with $S \neq B(S)$ and S has a basis U, then U is not a subset of B(S) if and only if B(S) is a group. Corollary. If under the hypotheses of Theorem 2 U is not a subset of B(S), then $U - B(S)$ is a single point. (Received February 11, 1969.)

69T-B75. V. C. NAIR, Regional College of Education, Ajmer, India, and Ohio State University, Columbus, Ohio 43210. On the Laplace transform. I.

In this paper, two theorems are proved on the Laplace transform and they are used to evaluate multiple integrals and to solve integral equations involving the H function. Various interesting special cases are considered among which are the theorems proved earlier by Saxena [Some theorems in operational calculus and infinite integrals involving Bessel function and G functions, Proc. Nat. Inst. Sci. India, Part A, 27 (1961), 38-61] and Gupta [On certain transforms based on unilateral and bilateral operational calculus in one or two variables, thesis approved by University of Rajastan, 1962, p. 245]. (Received February 3, 1969.)

## Applied Mathematics

69T-C13. JOHN De CICCO and ROBERT DARTSCH, Illinois Institute of Technology, Chicago, Illinois 60616. Some special classes of general physical systems $S_k$ in euclidean space $E_3$.

A general physical system $S_k$ is composed of all the integral solutions C of a triplet of ordinary differential equations in which t', z'', and y''' are expressed explicitly in terms of t, x,y,z,y',z',v, and y''. This aggregate is composed of either $\infty^6$, or $\infty^5$ trajectories C. If the general field of force F is independent of the speed v, then this system may be reduced to a pair of ordinary differential equations which pair is an extended form of the type (G). For a directional and, in particular, a positional force field, a physical system $S_k$ consists of $\infty^5$ trajectories C of the type (G). Finally, the necessary and sufficient conditions are found in the case where there are exactly $\infty^5$ trajectories C, and in the situation for which a physical system $S_k$ is of the type (G). (Received November 12, 1968.)

69T-C14. ADI BEN-ISRAEL, The Technologocai Institute, Northwestern University, Evanston, Illinois 60201. A duality theorem of real or complex linear programming.

Notations. For any $A \in C^{m \times n}$, $b \in C^m$, $c \in C^n$ and a polyhedral cone $S \subset C^n$, let (I.S.) maximize $\{Re(c,x): Ax = b, x \in S\}$, (II. S*) minimize $\{Re(b,y): A^H y - c \in S^*\}$. Theorem. Exactly one of the following holds: (a) Both (I. S.) and (II. S*) are consistent, have optimal solutions, and $\max \{Re(c,x): Ax = b, x \in S\} = \min \{Re(b,y): A^H y - c \in S^*\}$. (b) (I.S.) inconsistent, (II.S) unbounded. (c) (I.S.) unbounded, (II. S*) inconsistent. (d) Both (I.S) and (II. S*) are inconsistent. Corollary. The classical duality theorem of real linear programming for A, b, c real and $S = R^n_+$. Corollary. Levinson's duality theorem of complex linear programming (J. Math. Anal. Appl. 14 (1966), 44-62) for $S = \{x \in C^n: |\arg x| \leq \alpha\}$ and $\alpha \in R^n$ satisfying $0 \leq \alpha \leq \pi/2$. (Received November 27, 1968.)

69T-C15. KENNETH S. MILLER and M. M. ROCHWARGER, Riverside Research Institute, 632 West 125th Street, New York, New York 10027. On estimating spectral moments in the presence of colored noise.

Let $\{q^{(1)}(t)\}$, the signal, be a complex Gaussian process corrupted by additive Gaussian noise $\{q^{(2)}(t)\}$. Observations on $p(t)q(t)$ and $p(t)q^{(2)}(t)$ are assumed to be available where $p(t)$ is a smooth weighting function and $q = q^{(1)} + q^{(2)}$. Using the Fourier transform of the samples of $p(t)q(t)$ and $p(t)q^{(2)}(t)$, estimators are derived for estimating the mean frequency and spectral width of the unknown power spectrum of the unweighted signal process. Asymptotic formulas for the means and variances of these statistics are computed in general, and explicitly for nontrivial examples. The consistency of the moment estimators and their dependence on the frequency resolution, signal-to-noise ratio, and spectral width, are discussed in detail. (Received December 23, 1968.)

69T-C16. WILLIAM J. GORDON, Research Laboratories, General Motors Corporation, Warren, Michigan 48090. Least squares approximation of bivariate functions.

Theorem. Let $F(x,y)$ be piecewise continuous over the Cartesian product domain $\mathcal{R} = I \times I'$. Let $\{\varphi_i(x)\}$ be an orthonormal basis for the inner product space $\Phi_K$ of functions defined on I; and let $\{\psi_j(y)\}$ be an orthonormal basis for the inner product space $\Psi_{K'}$ of functions defined on I', i.e. $\int_I \varphi_i \varphi_k dx = \delta_{ik}$ (i,k = 1(1)K) and $\int_{I'} \psi_j \psi_l dy = \delta_{jl}$ (j, l = 1(1)K'). Among all bivariate functions $\tilde{f}$ of the form $\tilde{f} = \sum g_i(y)\varphi_i(x) + \sum h_j(x)\psi_j(y)$ with arbitrary piecewise continuous univariate functions $g_i(y)$ (i = 1(1)K) and $h_j(x)$ (j = 1(1)K'), the function $f(x,y) = \sum[\int_I F\varphi_i dx]\varphi_i(x) + \sum[\int_{I'} F\psi_j dy]\psi_j(y) - \sum\sum[\int\int_{\mathcal{R}} F\varphi_i\psi_j dxdy]\varphi_i(x)\psi_j(y)$ is the unique function which minimizes the $\mathcal{L}_2$ norm: $\|F - \tilde{f}\| = \{\int\int_{\mathcal{R}}(F - \tilde{f})^2 dxdy\}^{1/2}$. The value of this minimum is $\|F - f\|^2 = \int\int_{\mathcal{R}} F^2 dxdy - \sum \int_{I'}[\int_I F\varphi_i dx]^2 dy - \sum\int_I[\int_{I'} F\psi_j dy]^2 dx + \sum\sum(\int\int_{\mathcal{R}} F\varphi_i\psi_j dxdy)^2$. Corollary. If each of the K univariate functions $\int_I F\varphi_i dx$ (since F is a bivariate function, $\int_I F\varphi_i dx$ is a parametric integral with parameter y) in f(x,y) is replaced by its least squares approximation from $\Psi_{K'}$ and if each of the K' univariate functions $\int_{I'} F\psi_j dy$ is replaced by its least squares approximation from $\Phi_K$, then the function f(x,y) reduces to $f^* = \sum\sum[\int\int_{\mathcal{R}} F\varphi_i\psi_j dxdy]\varphi_i(x)\psi_j(y)$ and $f^*$ is the unique least squares approximation to F from among the restricted class of $K \cdot K'$ parameter functions $\tilde{f}^* = \sum\sum a_{ij}\varphi_i(x)\psi_j(y)$. (Received January 8, 1969.)

69T-C17. RICHARD J. DUFFIN, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213. Linearizing geometric programs.

A geometric program concerns minimizing a function subject to constraint functions, all functions being of posynomial form. In this paper the posynomial functions are condensed to monomial form by use of the inequality reducing a weighted arithmetic mean to a weighted geometric mean. The geometric mean is a monomial and by a logarithmic transformation if becomes a linear function. This observation shows that the condensed program is equivalent to a linear program. Moreover by suitable choice of the weights it is found that the minimum of the condensed program is the same as the minimum of the original programs. This fact together with the duality theorem of linear programming proves that the maximum of the dual geometric program is equal to the minimum of the primal geometric program. With this result as a basis a new approach to the duality properties of geometric programs is carried through. In particular it is shown that a "duality gap" cannot occur in geometric programming. (Received January 13, 1969.)

69T-C18. VED PERKASH MADAN, University of Alberta, Edmonton, Alberta, Canada. On longitudinal vibrations of a semi-infinite Voigt-Kelvin rod with nonuniform viscoelastic properties.

The case of a longitudinal motion of harmonic vibrations in a semi-infinite rod of Voigt-Kelvin material with nonuniform viscoelastic properties is examined. The physical parameters E and $\lambda$ are now a function of x. Closed form solutions are obtained for displacement and the amplitude by solving a system of two nonlinear ordinary differential equations characterizing the motion. (Received January 21, 1969.)

69T-C19. S. NANDA, Queen's University, Kingston,Ontario, Canada. Topology of the Minkowski space.

A topology is defined on the Minkowski space of special relativity by requiring that it is the finest with respect to which the induced topology on every space axis is Euclidean. It is proved, using connectedness arguments and Zeeman's theorem (J. Math. Phys. 5 (1964)) that the following conjecture of Zeeman is true (Topology 6 (1967)): "The group of homeomorphisms consists of the inhomogeneous Lorentz group and dilatations." Moreover it is also shown that another topology which induces Euclidean topology on every time axis has the same group of homeomorphisms. (Received January 13, 1969.) (Author introduced by Professor Kirti K. Oberai.)

69T-C20. ARNOLD LENT, and ADI BEN-ISRAEL, Technological Institute, Northwestern University, Evanston, Illinois 60201. An asymptotic theorem for linear systems of index zero or one.

Let $A^{\#}$ denote the group inverse of $A \in C^{n \times n}$ (I. Erdelyi, J. Math. Anal. Appl. 17 (1967), 119-132; P. Robert, J. Math. Anal. Appl. 22 (1968), 658-669). Theorem. Let the system (1) $\dot{x} = -Ax + b(t)$, $x(0) = 0$, satisfy (i) rank $A$ = rank $A^2$, (ii) $\sigma(A) \subset \{0\} \cup \{z \in C : \operatorname{Re} z > 0\}$, (iii) $b(t) = b(\infty) + f(t)$, $b(\infty) \in R(A)$, $f(t) \in L^1[0,\infty]$. Then the solution of (1) satisfies $\lim_{t\to\infty} x(t) = A^{\#} b(\infty) + (I - A^{\#}A)\int_0^{\infty} f(x)ds$. Corollary. Let (1), (ii), (iii) be as in the theorem and let (i') $R(A) = R(A^*)$, replace (i). Then $\lim_{t\to\infty} x(t) = A^{+} b(\infty) + P_{N(A)} \int_0^{\infty} f(s)ds$. (Received February 5, 1969.)

# Geometry

69T-D12. FREDERIC CUNNINGHAM, JR., Bryn Mawr College, Bryn Mawr, Pennsylvania 19010. Simply connected Kakeya sets having small area.

A plane set K is a Kakeya set if a segment of unit length can be moved continuously in K so as to return to its original position with its direction reversed. A. S. Besicovitch (Math. Z. 27 (1927), 312-320) showed that there exist Kakeya sets of arbitrarily small area, but his example was not simply connected. Moreover, the diameter of his example increases without bound as the area is decreased. Theorem. Given $\epsilon > 0$, there exists a simply connected polygonal Kakeya set of area less than $\epsilon$ contained in a circle of diameter $2 + \epsilon$. The set referred to is not star-shaped. In fact a star-shaped Kakeya set necessarily has area at least $\pi/128$. (Received January 7, 1969.)

69T-D13. WITHDRAWN.

69T-D14. JAYME M. CARDOSO, Federal University of Parana, Curitiba, Parana, Brazil. On the radical axis of two circles.

It is shown that given two circles in a plane, its radical axis is the axis of the planar homology which transforms each of the two curves in the other. If the circles are concentrics the homology is an homothety and if the circles have the same radius the homology is a symmetry. (Received February 11, 1969.)

# Logic and Foundations

69T-E23. STEVEN K. THOMASON, University of California, Berkeley, California 94720. Sublattices of the degrees of unsolvability.

A set S of (Turing) degrees is said to be a sublattice of the degrees if every two members of S have a g.l.b. in the upper semilattice of degrees, and both the g.l.b. and l.u.b. are again members of S. Note that every initial segment of degrees which is a lattice is a sublattice of the degrees. Theorem. There is a sublattice of the degrees isomorphic to any given finite lattice. (Received December 23, 1968.)

69T-E24. RONALD B. JENSEN, Seminar für Logik, University of Bonn, Lennestrasse 33a, 53 Bonn, West Germany. Automorphism properties of Souslin continua.

A Souslin Continuum (SC) is a complete, dense, linear ordering whose intervals satisfy the countable chain condition, but is not imbeddable in the real continuum. Consider the following properties of continua S: ($A_1$) S has exactly $\omega_2$ automorphisms. ($A_2$) S has exactly $2^{\omega}$ automorphisms. ($A_3$) Any two open intervals of S are isomorphic. ($A_4$) No two distinct open intervals of S are isomorphic. ($A_5$) S is isomorphic to $S^{-1}$. If $V = L[A]$ for an $A \subset \omega_1$, then there are SC satisfying $A_1 \wedge A_3 \wedge A_5$, $A_2 \wedge A_3 \wedge A_5$, $A_2 \wedge A_3 \wedge \neg A_5$, $A_4 \wedge A_5$, $A_4 \wedge \neg A_5$. The existence of SC satisfying all but the first of these combinations follows from the weaker assumption ($\diamondsuit$) There exists $\Gamma \subset \bigcup_{\alpha<\omega_1} \omega_1^{\alpha}$ such that $\Gamma \cap \omega_1^{\alpha}$ is countable for each $\alpha$ and if $f \in \omega_1^{\omega_1}$, then $\bigvee \alpha f \restriction \alpha \in \Gamma$. ($\diamondsuit$) implies the continuum hypothesis, but the reverse implication is open. (Received December 2, 1968.) (Introduced by Professor Gisbert Hasenjaeger.)

69T-E25. DOV GABBAY, Hebrew University, Jerusalem, Israel. Interpretation theorems for intuitionistic logic. Preliminary report.

Let I be the intuitionistic and C the classical predicate calculus respectively. Let $I \subseteq X \subseteq C$ be an intermediate logic. Consider the following propositions: (*) for every $\Phi$, $\vdash_C \Phi$ iff $\vdash_X \sim\sim \Phi$. (**) for every $\Phi$ such that $\sim, \wedge, \forall$ are the only connectives of $\Phi$, $\vdash_C \Phi$ iff $\vdash_X \Phi$, (***) for every $\Phi$, $\vdash_C \Phi$ iff $\vdash_X \sim \Phi \rightarrow \Phi$. It is known that for $X = I$ the above propositions are false. A counter-example is (1) $\sim (\forall x \sim\sim \Phi(x) \wedge \sim \forall x \Phi(x))$. Theorem I. $MH = I + (1)$ is the smallest extension of I such that (*) holds. (Similarly, such that (**) or (***) holds.) Theorem II. MH is complete for the class of all Kripke models whose tree T fulfills the following property:(#) $(\forall x)(\exists y \geq x)(\forall z \geq y)(z = y)$. To prove II, we associate (in a given model $\underline{A}$ of (1)) a dense subset $T_\Phi$ of the tree with every substitution instance of (1). We then use the family of all generic branches to transform $\underline{A}$ into a model fulfilling (#). Conjecture. Monadic MH is decidable. (Received December 9, 1968.) (Author introduced by Professor Azriel Levy.)

69T-E26. DALE MYERS, University of California, Berkeley, California 94720. Separation theorems for difference hierarchies.

Let $\Omega$ (= $\langle B, \cup, \cap, -, 0,1 \rangle$) be a Boolean algebra with symmetric difference operation +. Let R be a subset of B containing 0,1 and closed under $\cup$, $\cap$. Let $D_0(R) = \{0\}$ and $D_{n+1}(R) = \{r \cap - a: r \in R, a \in D_n(R)\}$. A set $S \subseteq B$ has the first separation property iff $(\forall s, t \in S)[s \cap t = 0 \Rightarrow (\exists u \in S \cap S^-)[s \leq u \,\&\, t < - u]]$, where $S^- = \{- s: s \in S\}$ [see Addison, The theory of models, pp. 1-16]. Theorem. If $R^-$ has the first separation property then (i) so does $(D_n(R))^-$ and (ii) $D_{n+1}(R) \cap (D_{n+1}(R))^- = \{r + a: r \in R \cap R^-, a \in D_n(R)\}$. Now suppose $\Omega$ is in addition a closure algebra with closure operation $\bar{\ }$ and that $R = \{x \in B: \bar{x} = x\}$. For $x \in B$ let (x) be the universe of the universe of the subalgebra of $\Omega$ generated by x. A finite set $T \subseteq B$ is independent iff for all elements h of the cartesian product $\times_{t \in T}(t)$: (i) $\bigcap_{t \in T} h(t) \neq 0 \Rightarrow (\exists t \in T)[h(t) \neq 0]$ and (ii) $\overline{\bigcap_{t \in T} h(t)} = \bigcap_{t \in T} \overline{h(t)}$. Theorem. For any $n \in \omega$ $D_n(R^-)$ fails to have the first separation property if n is even and there is an independent set of n + 1 elements of the difference $R^- \sim R$ or if n is odd and there is an independent set of n elements of $R^- \sim R$ one of which satisfies $\bar{x} \notin R^-$. Application. Using a result of Shoenfield one can show for the Boolean set algebra of classes of structures of pure predicate logic that $(D_n(\bigvee_k^0))^-$ has the first separation property while $D_n(\bigvee_k^0)$ does not for $n \in \omega$ and $k \geq 2$. (Received January 90, 1969.) (Author introduced by Professor J. W. Addison.)

69T-E27. MARY POWDERLY, Fairfield University, Fairfield, Connecticut 06430. Further consequences of a lemma of H. Tong.

A previous abstract (these *Notices* 13 (1966), 383) indicated some of the theorems that were immediate consequences of a lemma of H. Tong and the Axiom of Choice. The maximal principles of Teichmuller-Tukey, Kuratowski, Gottschalk, Wallace, Kurepa, Chang, and Rubin also have short (and in some cases, much shorter) and/or immediate proofs using this lemma. Theorems on cardinals and ordinals using one or another of the many statements equivalent to the Axiom of Choice also follow readily from the lemma with aid of the Axiom of Choice. A recent study of many basic theorems in Algebra, Analysis, and Topology (eg., Prime Ideal Theorem, Hahn Banach Theorem, Existence of a Hamel Basis, Tychonoff's Theorem) which use the Axiom of Choice or its equivalents showed that this lemma frequently provided shorter proofs and also provided a unifying and simplifying technique useful in solving a variety of problems. In all cases encountered where applications of the Transfinite Recursion Theorem occured, this technique could also be used. As has been pointed out previously, this lemma does not depend on the Axiom of Choice. (Received January 8, 1969.)

69T-E28. EDISON FARAH, University of Sao Paulo, Sao, Paulo, Brazil. A new form of the axiom of substitution.

Let ZF be the Zermelo-Fraenkel system of Set Theory (the underlying logic of ZF is the functional calculus of first order with equality). Let $\varphi(x,y)$ be a formula of ZF, where the occurrences of the variables x and y are free. Definition 1. $\varphi(x,y)$ is one-many iff $(\forall x, t, y)[(\varphi(x,y) \& \varphi(t,y)) \Rightarrow x = t]$. Definition 2. The set m' is a $\varphi$-correspondent of the set m iff $(\forall y)[y \in m' \Rightarrow (\exists x,u)(x \in u \in m \& \varphi(x,y))]$ and $(\forall x)[((\exists u,y)(x \in u \in m \& \varphi(x,y))) \Rightarrow (\exists ! y)(y \in m' \& \varphi(x,y))]$. (For instance, if $\varphi(x,y)$ is $x = y$, the union of the sets of m is a $\varphi$-correspondent of m.) Now let $A_I$, $A_{II}$, $A_{IV}$ and $A_V$ denote, respectively, the axioms of Extensionality, Power-Set, Infinity and Foundation of ZF, and consider the following axiom-scheme: $A_{III}$. For every set m and every one-many formula $\varphi(x,y)$ there is a set which is $\varphi$-correspondent of m. Then, the system ZF* whose axioms are $A_I$, $A_{II}$, $A_{III}$, $A_{IV}$ and $A_V$ is such that each theorem of ZF is an axiom of a theorem of ZF*. (In particular, the axioms of Union, Selection, Choice and Substitution of ZF are theorems of ZF*.) Further, if ZF is consistent, ZF* is also consistent. (Received December 9, 1968.) (Author introduced by Professor N. C. A. Da Costa.)

69T-E29. BARUCH GERSHUNI, Frans van Mierisstraat 37 huis, Amsterdam, Nederland. Properties of classes and sets.

1. As totalities may appear in our restricted theory only plural classes, singular classes and sets. The sets are the totalities known under this name hitherto. The singular classes are essentially the totalities called hitherto simply classes. But they do not contain the sets as a part. The plural classes are, to the knowledge of the author, new. An individuum is not a totality, but may be grasped as an improper plural class. 2. As elements may appear only the singulars, i.e. the singular classes, the sets and the individua. Plural classes are therefore not elements. 3. The way of writing a singular class may be grasped as the representation of a special plural class. 4. There is a difference between a singular class (a,b,c,...) and the improper plural class written identically: (a,b,c,...). The first has the elements a b c ...; the second has as its unique element itself. 5. Thus there is a difference between the elementship of a plural class and that of a singular class (or a set). We call the first direct and it is denoted by $\epsilon_1$; the second is called indirect and is denoted by $\epsilon_2$. 6. (Axiom of determination of totalities) Two totalities are equal iff they (a) are of the same kind and (b) have the same elements. Thus there follows 4. (Received January 6, 1969.)

69T-30. MIRO BENDA, University of Wisconsin, Madison, Wisconsin 53706. Isomorphisms of reduced direct products.

A filter D is said to be $\varkappa^+$-good whenever for any function $f: S_\omega(\varkappa) \to D$ for which $s,s' \in S_\omega(\varkappa)$ and $s \subseteq s'$ implies $f(s') \subseteq f(s)$ there is a function $g: S_\omega(\varkappa) \to D$ such that $g(s) \subseteq f(s)$ for any $s \in S_\omega(\varkappa)$

and $g(s \cup s') = g(s) \cap g(s')$ for any $s, s' \in S_\omega(\kappa)$. Theorem ($2^\kappa = \kappa^+$). Let $\mathfrak{A}, \mathfrak{B}$ be elementarily equivalent structures of power less than $\kappa^{++}$ and of length less than $\kappa^+$. If D is a filter over $\kappa$ which contains (as a subset) a $\kappa^+$-good, $\omega$-incomplete filter, then $\mathfrak{A}^\kappa/D \cong \mathfrak{B}^\kappa/D$. Corollary (CH). If $\mathfrak{A}, \mathfrak{B}$ are as in the theorem with $\kappa = \omega$ and if D is a countably incomplete filter over $\omega$, then $\mathfrak{A}^\omega/D \cong \mathfrak{B}^\omega/D$. The result in the theorem is connected with the following problem which has been raised by several authors: Does $\mathfrak{A}^\kappa/D \cong \mathfrak{B}^\kappa/D$ hold if D is a $\kappa$-regular ultrafilter over $\kappa$? ($\mathfrak{A}, \mathfrak{B}$ are as in the theorem.) Thus, in view of the theorem, the answer to this important problem is (in some sense) reduced to the purely set-theoretical question: Is it true that any $\kappa$-regular ultrafilter over $\kappa$ contains a $\kappa^+$-good filter? (Received January 24, 1969.)

69T-E31. E. M. KLEINBERG, Rockefeller University, New York, New York 10021. Strong partition properties. Preliminary report.

Let $\lambda$ and $\gamma$ denote limit ordinals less than uncountable cardinal $\kappa$. The partition property $\kappa \to (\kappa)^\lambda$ ($\kappa \to (\kappa)^{<\lambda}_{2^\gamma}$) is "For each partition of the $\lambda$ sequences (the $\alpha$ sequences for each $\alpha < \lambda$) from $\kappa$ into 2 ($2^\gamma$) pieces there exists a size $\kappa$ subset, x, of $\kappa$, such that (such that for each $\alpha < \lambda$) the size $\lambda$ ($\alpha$) sequences from x meet but one member of the partition." Let $AC_\kappa$ be that form of the axiom of choice which states that the product of $\kappa$ nonempty sets is nonempty. Theorem 1. ZF $\vdash$ "$\forall\lambda(\kappa \to (\kappa)^\lambda)$ iff $\forall\lambda\forall\gamma(\kappa \to (\kappa)^{<\lambda}_{2^\gamma})$." Theorem 2. ZF $\vdash$ "$\forall\lambda(\kappa \to (\kappa)^\lambda)$ implies that there are at least as many normal $\kappa$-additive, 0-1 measures on $\kappa$ as there are regular cardinals less than $\kappa$." Theorem 3. ZF + $AC_\kappa \vdash$ "It is not the case that $\kappa \to (\kappa)^{\omega+\omega}$." Theorem 4. ZF + $AC_\omega \vdash$ "If $\forall\lambda(\kappa \to (\kappa)^\lambda)$ then there exists a $\kappa$-additive 0-1 measure on $2^\kappa$ such that any well ordered subset of $2^\kappa$ has measure 0." (Received January 29, 1969.) (Author introduced by Professor Hao Wang.)

69T-E32. ALBERT J. SADE, 364 Cours de la République, Pertuis, Vaucluse, France. Foncteurs monadiques en logique trivalente.

Si l'on cherche a construire une logique strictement trivalente, c'est-à-dire dans laquelle existent seulement trois attitudes noétiques: l'affirmation, la négation et l'indécision la seule solution est, pour les 3 foncteurs propositionnels monadiques: affirmation = x, déni = x + 1, aporie = x + 2. Cette restriction ne peut etre maintenue avec l'apparition des fonctions dyadiques. Si l'on considère les éléments 0,1,2 du corps du 3$^e$ ordre comme exprimant le faux = 0, le vrai = 1, le neutre = 2, il existe 27 fonctions propositionnelles monadiques, qui sont les 27 polynômes $ax^2 + bx + c$ sur le corps du 3$^e$ ordre, englobant toutes les définitions connues de la négation, de la possibilité, de la contingence, du tertium, etc. A chacune d'elles est associée une représentation ensembliste au moyen de régions, qui fournit sa forme normale disjonctive. Toutes ces fonctions, sauf 3 d'entr'elles, sont exprimables au moyen des deux fonctions M'' = x + 1 et N' = $2x^2 + 1$, les 3 autres au moyen de R = 2x, et forment un groupoïde $\Sigma$ dont la table fournit la construction des tautologies. Le groupe d'automorphisme de $\Sigma$ est isomorphe à $\mathfrak{S}_3$ et contient un seul automorphisme non identique respectant les tautologies. (Received February 4, 1969.)

69T-E33. JOAN RAND MOSCHOVAKIS, Occidental College, Los Angeles, California. Consistency of Church's thesis with Vesley's schema in intuitionistic analysis. Preliminary report.

Let I be Kleene's full system of intuitionistic analysis [Kleene and Vesley, The foundations of intuitionistic analysis, Amsterdam (North-Holland), 1965], VS be Vesley's Schema $\forall\alpha\forall x\,\exists\beta(\bar{\beta}(x) = \bar{\alpha}(x)\ \&\ \neg A(\beta))\ \&\ \forall\alpha(\neg A(\alpha) \supset \exists\beta B(\alpha,\beta)) \supset \forall\alpha\exists\beta(\neg A(\alpha) \supset B(\alpha,\beta))$ (where $A(\alpha)$ is a formula of I containing free only $\alpha$), and CT be Church's Thesis in the form of the axiom schema $\exists\alpha A(\alpha) \supset \exists\alpha(GR(\alpha)\ \&\ A(\alpha))$ (where $GR(\alpha)$ expresses "$\alpha$ is general recursive," and $A(\alpha)$ is a formula of I containing free at most $\alpha$). (Vesley has shown that VS is consistent with I by Kleene's ${}_s$realizability, and that Brouwer's "creating subject" refutations, such as $\neg\forall\alpha(\neg\forall x\alpha(x) = 0 \supset \exists x\alpha(x) \neq 0)$, can be obtained in I + VS.) Theorem. There is a function-realizability notion, ${}^G$realizability, such that (i) if $\Gamma$, E are formulas of I, $\Gamma$ are ${}^G$realizable, and $\Gamma \vdash_{I}$ E, then E is ${}^G$realizable; (ii) all instances of VS and $\neg\neg$CT are ${}^G$realizable; (iii) for no formula A of I are both A and $\neg A$ ${}^G$realizable. Corollary. The system obtained from I by adjoining VS and CT as axiom schemata is consistent. Proofs are both classically and intuitionistically valid. The theorem incidentally provides a direct proof of the fact (apparently due to Scarpellini) that CT is consistent with I. (Received February 6, 1969.)

69T-E34. SAHARON SHELAH, Hebrew University, Jerusalem, Israel. On models with orderings.

This notice generalizes theorems in Keisler ["Models with orderings" in Logic, methodology and philosophy of science. III] by using a more elaborate partition theorem. Definition 1. M is a $\langle\varkappa,\mu \mid \lambda\rangle$-model if $P_1^M$ is $\varkappa$-like ordered by $<^M$, $P_2^M$ is $\mu$-like ordered by $<^M$, and $|P_3^M| = \lambda \geq \aleph_0$. Definition 2. $\chi : \langle\varkappa,\mu \mid \lambda\rangle \to \langle\varkappa_1,\mu_1 \mid \lambda_1\rangle$ if every first-order theory T, $|T| \leq \chi$, which has a $\langle\varkappa,\mu \mid \lambda\rangle$-model has also a $\langle\varkappa_1,\mu_1 \mid \lambda_1\rangle$-model. Let $\varkappa$, $\mu$ denote strong limit singular cardinals, and $\varkappa_1, \mu_1$ singular cardinals. Theorem 1. Let $\varkappa > \lambda$, $\varkappa_1 > \lambda_1 \geq \chi$. $\chi : \langle\varkappa,\varkappa \mid \lambda\rangle \to \langle\varkappa_1,\varkappa_1 \mid \lambda_1\rangle$ if cf $\varkappa_1 \leq \lambda_1$ or cf $\varkappa \geq \beth_{\alpha+\omega} > \beth_\alpha \geq \lambda$, or cf $\varkappa > \lambda$, cf $\varkappa_1 = \lambda_1^+$, $\lambda_1 = \Sigma_{\chi<\lambda_1}(\lambda_1)^\chi$. Theorem 2. Let $\varkappa > \mu$, $\varkappa_1 > \mu_1 > \chi$, cf $\varkappa_1 < \mu_1$, $\chi : \langle\varkappa,\mu \mid \mu\rangle \to \langle\varkappa_1,\mu_1 \mid \mu_1\rangle$ if cf $\varkappa \neq$ cf $\mu$ or cf $\varkappa_1 =$ cf $\mu_1$. Theorem 3. If T has a $\langle\varkappa,\varkappa \mid \varkappa\rangle$-model omitting a type p, then it has a $\langle\varkappa_1,\varkappa_1 \mid \varkappa_1\rangle$-model, $\varkappa_1 > |T|$, $\lambda =$ cf $\varkappa_1$, omitting p, if (1) $2^{|T|+\lambda} <$ cf $\varkappa$, or (2) cf $\varkappa > |T| + \lambda = \beth_\delta$, cf $\delta = \omega$, or (3) cf $\varkappa \geq \mu_{|T|}$ or (4) $(2^{\lambda+|T|})^+ \times \omega$ divides $\delta$, where $\varkappa = \beth_\delta$. Corollary. If M is a model of ZF + AC, $\{a_i : i \in I\}$ are different regular cardinals in M, $\{\lambda_i : i \in I\}$ are regular cardinals, then M has an elementary extension in which the outer cofinality of $a_i$ is $\lambda_i$. (See an article of Keisler and Morley in Israel J. Math. 6, 49.) (Received February 11, 1969.) (Author introduced by Professor Michael O. Rabin.)

## Statistics and Probability

69T-F6. THEODORE E. HARRIS, University of Southern California, Los Angeles, California 90007. Changing random point processes. I.

$M_1$ = set of counting measures (CM) $\alpha$ in $R_1$ with $\alpha(\{x\}) \leq 1$, $\alpha((-\infty, x)) = \alpha((x,\infty)) = \infty$, $\alpha(A) < \infty$ for bounded $A \subset R_1$. $M_2 = \{\xi : \xi$ is CM in $R_2$, ${}_1\xi \in M_1$, ${}_2\xi \in M_1\}$, where ${}_1\xi(A) = \xi(A \times R_1)$, ${}_2\xi(A) = \xi(R_1 \times A)$, $A \subset R_1$. For $\xi \in M_2$, $\mathbf{x} \in R_1$, get $\xi + x$ by adding x to each coordinate of each atom of $\xi$. $\mathfrak{M}_1$ and $\mathfrak{M}_2$ are the usual $\sigma$-fields in $M_1$ and $M_2$. $\mathcal{P}_2$ = class of pr. measures P on $\mathfrak{M}_2$ with $P(A + x) = P(A)$, $A \in \mathfrak{M}_2$, $x \in R_1$, and $\int {}_1\xi((0,1))dP = 1$. $M_{0i} = \{\xi : {}_i\xi(\{0\}) = 1\}$, $i = 1,2$; $P_{0i}(A)$ = "conditional" pr. of A (Palm sense) given $\xi \in M_{0i}$; $w(\xi)$, $\xi \in M_{01}$, = unique $y \in R_1$ such that $\xi(\{(0,y)\}) = 1$; suppose $f(\xi)$ bd. meas. mapping of $M_{02} \to R_1$. Theorem. $\int_{M_{01}} f(\xi - w(\xi))dP_{01} = \int_{M_{02}} f(\xi)dP_{02}$. This is related to a result of F. Spitzer about stationarity relative to a moving point in certain moving systems, and also implies a known result about stationarity of spaceiings under 1-dim. Palm measure. (Received January 24, 1969.)

# Topology

69T-G36. PING-FUN LAM, Wesleyan University, Middletown, Connecticut 06457. Normal dynamical systems.

Let $(X,h)$ be a dynamical system, i.e. $X$ is a compact metric space and $h$ is a homeomorphism of $X$. A point $x \in X$ is said to be regularly approximated by periodic points, if for every $\epsilon > 0$, there exists a periodic point $y$ such that $d(h^i(x), h^i(y)) < \epsilon$; $i = 0,1,\ldots,n-1$, where $n$ is the period of $y$ and $d$ is a fixed metric for $X$. Such an $x$ is necessarily a positively recurrent point. Definition. A dynamical system is said to be normal {seminormal} if every positively recurrent point is regularly approximated by periodic points. Theorem 1. Every dynamical system is isomorphic to a closed subsystem of a normal system. Theorem 2. A homomorphic image of a normal {seminormal} dynamical system is normal {seminormal}. Theorem 3. Let $(X,h)$ be a seminormal expansive dynamical system, $(Y,g)$ an expansive dynamical system, and $\varphi$ a homomorphism of $(X,h)$ onto $(Y,g)$. Then for every periodic point $y \in Y$ there exists a periodic point $x \in X$ such that $\varphi(x) = y$. Interesting examples of normal expansive dynamical systems include the symbolic flows and the p-adic solenoids with their shifts. Theorem 3 extends a result of G. A. Hedlund on symbolic flows. (Received August 19, 1969.)

69T-G37. TUDOR GANEA, Universite de Paris, 1 Rue de Messine, Paris 8, France. Fibrations over co-H-spaces.

Let $F \to E \to B$ be a fibration with $\pi_1(B) = 0$ and cat $B \leqq 1$. The well-known construction of a one-sided homotopy inverse for the comultiplication on $B$ yields a homotopy equivalence $\varphi : B \vee B \to B \vee B$ with $q \circ \varphi \simeq d$ and $\varphi \circ i \simeq \varphi$, where $d$ is the folding map, and $i$, $j$ and $q$ are the inclusions and projection with $q \circ i = 1 : B \to B$, $q \circ j = 0$. Hence, the fibre spaces $D$ and $Q$ induces over $B \vee B$ by $d$ and $q$ from $E \to B$ are homotopically equivalent; also, $D = E \cup G$, $F = E \cap G$, and $Q = E \cup H$, $F = E \cap H$, where $G$ and $H$ are induced over $B$ by $d \circ j$ and $q \circ j$ so that $G \simeq E$ and $H \simeq F \times B$. Theorem. $E/F \simeq D/E \simeq Q/E \simeq F \times B/F$. Hence, in the homology sequence of the pair $(E,F)$, we may replace $H_*(E,F)$ by $H_*(F \times B, F)$ and obtain the exact sequence first proved by M. Ginsburg [Proc. Amer. Math. Soc. 15 (1964), 423-431, Theorem 3.1] by means of elaborate algebraic arguments. (Received November 18, 1968.)

69T-G38. CHARLES J. MOZZOCHI, Trinity College, Hartford, Connecticut 06106. On locally compact symmetric generalized topological groups.

Let $(G, \cdot, \mathcal{T})$ be a symmetric generalized topological group with natural left uniformity $\mathcal{U}(G)$. Theorem. If $(G, \cdot, \mathcal{T})$ is locally compact, then $\mathcal{U}(G)$ is complete. Theorem. If $(G, \cdot, \mathcal{T})$ is locally compact, then there exists a regular Borel measure in $G$ such that $\mu(U) > 0$ for every nonempty open Borel set $U$, and $\mu(xE) = \mu(E)$ for every Borel set $E$. Remark. The proof or disproof of the (essential) uniqueness of the measure in the second theorem is an open question. (Received January 3, 1969.)

69T-G39. KENNETH C. MILLETT, Massachusetts Institute of Technology, Cambridge, Massachusetts. 02139. Nonsingular sections to Euclidean bundles.

This paper investigates the relationship between the existence of a nonzero section and a splitting of a Euclidean fiber bundle. For example, if a vector bundle has a nonzero section it splits off an $\epsilon^1$, as a direct summand. W. Browder has shown that there is an $(R^k,0)$-bundle over a finite simplicial complex which has a nonzero section, but which does not split off an $\epsilon^1$. Thus one of the following statements is false. (A) A Euclidean bundle, $(\xi^{k+1},0)$, with nonzero section, is equivalent to $(\xi^k \oplus \epsilon^1, \epsilon^1)$, with $\epsilon^1$ containing the section. (B) Every $(\xi^{k+1}, \epsilon^1)$ is equivalent to a $(\xi^k \oplus \epsilon^1, \epsilon^1)$. We note that B is almost certainly false and give the following theorem towards proving A below the stable range. Theorem. Suppose that $(\xi^{k+1}, 0)$ over a finite complex of dimension m has a nonzero section. If $k \geqq 3$ and $m \leqq 2k - 3$, the bundle is equivalent to a $(\xi^k \oplus \epsilon^1, \epsilon^1)$, with the line bundle containing the section. The proof employs a generalization of the techniques of Černavskiĭ for modifying homeomorphism of Euclidean space. (Received January 8, 1969.)

69T-G40. ROBION C. KIRBY, University of California, Los Angeles, California 90024, and LAWRENCE C. SIEBENMANN, Institute for Advanced Study, Princeton, New Jersey 08540. A straightening theorem and a Hauptvermutung for pairs.

Theorem A. Let $M^m$ and $V^{m+q}$ be PL manifolds without boundary, and let $h: M \to V$ be a proper, locally flat topological embedding. If $q \geqq 3$, there exists a topological isotopy $H_t$, $0 \leqq t \leqq 1$, of $1_V = H_0$ so that $H_1 h$ is a PL locally flat embedding. Suppose further that h is PL locally flat on a neighborhood of C with C closed in M, and let $\epsilon: V \to [0,\infty)$ be continuous with $\epsilon|h(M - C) > 0$; then $H_t$ can be chosen so that $d(H_t, 1_V) < \epsilon$, $0 \leqq t \leqq 1$. If $q \leqq 2$ and $m + q \geqq 6$ ($\geqq 5$ if $q \leqq 1$), then $H_t$ as just described exists iff an obstruction vanishes in $H^3(M, C; \pi_3(TOP/PL))$ (Čech). We know $\pi_3(TOP/PL) \subset Z_2$. Theorem B. With the data of Theorem A, suppose h is a PL locally flat inclusion $M^m \subsetneq V^{m+q}$, and let $G: V \to V'$ be a homeomorphism PL locally flat on M and PL on a neighborhood of C in V. If $q \leqq 2$ and $m + q \geqq 6$ ($\geqq 5$ if $q \leqq 1$), then there exists an isotopy $G_t$ of $G = G_0$, satisfying $d(G_t, G) < \epsilon$ and fixing $G|M$, to a homeomorphism $G_1$ PL near M. If $q \geqq 3$ and $n + q \geqq 5$, then $G_t$ as described exists iff an obstruction vanishes in $H^3(M, C; \pi_3(TOP/PL))$; and when G is isotopic to a PL homeomorphism it is also isotopic fixing M. Theorems A and B are derived by a pairwise application of the authors' arguments in On the triangulation of manifolds and the Hauptvermutung (Bull. Amer. Math. Soc., to appear). The new information above for $q \geqq 3$ can also be deduced from a PL theorem of Richard Miller (Thesis, Univ. of Michigan, Ann Arbor, 1968). (Received January 22, 1969.)

69T-G41. DANIEL R. McMILLAN, JR., University of Wisconsin, Madison, Wisconsin 53706. Compact decompositions of 3-manifolds which yield 3-manifolds.

Throughout, let $M^3$ and $N^3$ denote closed, piecewise-linear 3-manifolds, and let f be a monotone, continuous mapping of $M^3$ **onto** $N^3$. Let $S_f$ consist of all points $x \in M^3$ such that $f^{-1}f(x)$ is nondegenerate (i.e., the components of $S_f$ are exactly the nondegenerate point-inverses of f). Suppose that the closure X of $f(S_f)$ is 0-dimensional, and that for each component C of $S_f$ there is a 3-cell in $M^3$ containing C in its interior. Theorem. $\overline{S_f}$ is definable by cubes-with-handles. Using techniques of R. Bean, we have Corollary. Each component of $S_f$ is cellular in $M^3$, and there is a pseudo-isotopy of $M^3$ onto

$M^3$ which shrinks the collection of components of $\bar{S}_f$ to points. In particular, $M^3$ and $N^3$ are homeomorphic. An important step in the proof of the Theorem is showing that each component C of $S_f$ is strongly acyclic: For each open set $U \subset M^3$ such that $C \subset U$, there is an open set V such that $C \subset V \subset U$ and such that for $i > 0$, the image of $H_i(V;Z)$ in $H_i(U,Z)$ is zero under the inclusion-induced homomorphism. Results of Haken and Waldhausen are then used to derive the Theorem (see also Abstract 663-542, these *Notices* 16 (1969), 244). (Received January 28, 1969.)

69T-G42. RICHARD T. MILLER, University of Chicago, Chicago, Illinois 60637. Close isotopies on piecewise linear manifolds.

Suppose $n \leq q - 3$. Let M be a compact p.l. n-manifold, let $\hat{M}$ be a compact p.l. n-submanifold of M, and let Q be a p.l. q-manifold. Theorem 1. $q \geq 5$ if $g: M \to \text{int}(Q)$ is a locally-flat embedding such that $g|\hat{M}$ is piecewise linear, and if N is a neighborhood of $g(\overline{M - \hat{M}})$ in Q, then for $\epsilon > 0$ there is an embient isotopy of Q that is fixed outside N, that moves points less than $\epsilon$, and that takes g to a piecewise linear embedding of M in Q. Theorem 2. If $g: M \to \text{int}(Q)$ is a piecewise linear embedding and if N is a neighborhood of $g(\overline{M - \hat{M}})$ in Q, then for $\epsilon > 0$, there is a $\delta > 0$ such that if $h: M \to \text{int}(Q)$ is a p.l. embedding with $h|\hat{M} = g|\hat{M}$ and with h(x) within $\delta$ of g(x) for each $x \in M$, then there is a p.l. ambient isotopy of Q that is fixed outside N, that moves points less than $\epsilon$, and that takes h to g. The proof of Theorem 2 is done from the definitions. Theorem 1 is a corollary of Theorem 2 and the fact that all orientation preserving homeomorphisms of Rq onto itself ($q \geq 5$) are stable. (Received January 22, 1969.)

69T-G43. JAMES M. BOYTE, Virginia Polytechnic Institute, Blacksberg, Virginia 24061. A remark on paracompactness.

For the definition of point paracompactness, see J. M. Boyte Abstract 660-2, these *Notices* 15 (1968), 1007. For the following theorem we are assuming no separation axioms on (X,T). By a paracompact space, we mean a space X such that every open cover of X has an open locally finite refinement covering X. Theorem 1. A topological space (X,T) is paracompact if and only if (X,T) is point paracompact and every open cover of X has a $\sigma$-locally finite refinement. The theorem by E. Michael follows as an immediate corollary. Corollary 1. If (X,T) is regular then (X,T) is paracompact if and only if every open cover of X has a $\sigma$-locally finite refinement. Theorem 2. A first countable $T_2$ space (X,T) is regular if and only if given a countable closed set F and $a \notin F$ then there exists disjoint open sets N(a) and N(F) containing a and F respectively. (Received February 3, 1969.) (Author introduced by Dr. Ernest P. Lane.)

69T-G44. CHARLES J. HIMMELBERG, JACK R. PORTER, and FRED S. VAN VLECK, University of Kansas, Lawrence, Kansas 66044. Fixed point theorems for condensing multifunctions.

Let E be a locally convex linear space, and let $\mathcal{B}$ be a basis of neighborhoods of O composed of convex sets. If $\Omega \subset E$, define $Q(\Omega)$ to be the collection of all $B \in \mathcal{B}$ such that $S + B \supset \Omega$ for some totally bounded subset S of E. If $T \subset E$, then a multifunction $F : T \to E$ is condensing iff for some choice of basis $\mathcal{B}$ of convex neighborhoods of O, we have $Q(F(\Omega)) \supsetneq Q(\Omega)$ for every bounded, but not totally bounded, subset $\Omega$ of T. Theorem 1. If T is a nonempty complete convex subset of a separated locally convex space E, and if $F : T \to T$ is a condensing multifunction with convex values, closed graph, and bounded range, then F has a fixed point. Theorem 2. If T is a nonempty complete convex subset of a locally convex space, and if $F : T \to T$ is a l.s.c. multifunction with closed convex values, then F has a fixed point if either of the following conditions is satisfied: (i) T is compact and metrizable or (ii) the subspace uniformity on T is metrizable, and F is condensing and has bounded range. (Received February 3, 1969.)

69T-G45. LUDVIK JANOS, University of Florida, Gainesville, Florida 32601. On Lipschitz-homeomorphisms.

Let $(X, \rho)$ be a metric space and $f : X \to X$ a homeomorphism of X onto f(X) for which there exist $c_1$ and $c_2$, $0 < c_1 \leqq c_2$ such that $c_1 \rho(x,y) \leqq \rho(f(x),f(y)) \leqq c_2\rho(x,y)$ for all $x,y \in X$. In this case f is said to be a $(c_1,c_2)$-map or a Lipschitz-homeomorphism, since both f and $f^{-1}$ have the Lipschitz property. We investigate the question of how such maps can be linearized in topological vector spaces. Theorem 1. Let $(X,\rho)$ be separable and $f : X \to X$ a $(c_1,c_2)$-map where $0 < c_1 \leqq c_2 < 1$ and $f(X) = X$. Then for any $c \in (0,1)$ there exists a topological embedding $\mu : X \to H$ of X into a separable Hilbert space H such that $\mu(f(x)) = c\mu(x)$ for all $x \in X$. Theorem 2. Dropping the condition $f(X) = X$ in the Theorem 1 and requiring $(X,\rho)$ to be totally bounded, we get the same conclusion. (Received February 6, 1969.)

69T-G46. WITHDRAWN.

69T-G47. SURENDRA NATH PATNAIK, Université de Montréal, Montréal 3, Québec, Canada. Two-categories of sheaves.

For each topological space X, a category of the second type or a 2-category (see e.g. J.-M. Maranda, Canadian J. Math. 17 (1965)) denoted by T(X,(Cat)) is constructed. The objects of T(X,(Cat)) are the categories A, B, C,... . For each pair of categories (A,B), there is given the category M(A,B) whose objects are functors F*, G*,... from S(X,A) to S(X,B) and whose morphisms are the natural transformations $F^* \to^{\alpha^*} G^*$,... where F*, G* are the functors from S(X,A) to S(X,B) induced by the functors F, G, from A to B. $\alpha^*$ is induced by the natural transformation $F \to^{\alpha} G$. (Here S(X,A) denotes the category of sheaves over the space X with values in A.) To each category A there is associated the identity functor 1 : S(X,A). There is easily given a functor * from $M(B,C) \times M(A,B)$ to M(A,C) for each triple (A,B,C) satisfying the conditions A1 and A2 of Maranda's paper cited above. The study of 2-categories T(X,(Cat)) for $X \in$ (Top) in a way parallels that of the category of sheaves S(X,A) for $A \in$ (Cat). A sample result: Proposition. A continuous map $f : X \to Y$ induces in a natural manner the 2-functor $f^{**} : T(X,(Cat)) \to T(Y,(Cat))$. (Received February 11, 1969.)

69T-G48. THOMAS B. RUSHING, University of Georgia, Athens, Georgia. Locally flat embeddings of PL manifolds are $\epsilon$-tame in codimension three.

Definition. A topological embedding $f : M^k \to Q^n$ of the PL k-manifold M into the PL n-manifold Q is said to be allowable if $f^{-1}(\partial Q)$ is a PL (k - 1)-submanifold (possibly empty) of $\partial M$. Theorem.

Let $f : M^k \to Q^n$, $n - k \geq 3$, be an allowable embedding of the PL manifold $M^k$ into the PL manifold $Q^n$ such that $f|f^{-1}(\partial Q)$ and $f|(M - f^{-1}(\partial Q))$ are locally flat. Then, f is $\epsilon$-tame. Remark 1. If $M' \subset f^{-1}(\partial Q)$ is a PL (k - 1)-manifold, if $M'' \subset M - f^{-1}(\partial Q)$ is a PL k-manifold, and if $f|M' \cup M''$ is PL, then f can be $\epsilon$-tame by an isotopy $e_t$ such that $e_t|f(M' \cup M'') = 1$ for all $t \in [0,1]$. Remark 2. If $f|f^{-1}(\partial Q)$ is PL, then the taming isotopy is fixed on $\partial Q$. (Received February 11, 1969.)

69T-G49. WITHDRAWN.

69T-G50. DAVID E. KULLMAN, University of Kansas, Lawrence, Kansas 66044. A note on $\sigma$-paracompact spaces. Preliminary report.

The class of $\sigma$-paracompact spaces, as defined by A. V. Arhangel'skiĭ [Russian Math. Surveys. 21 (1966), 115-162], includes all paracompact spaces and all regular developable spaces. The concept of $\overline{G}_\delta$-diagonal was defined by C. J. R. Borges [Canad. J. Math. 20 (1968), 795-804]. A. Okuyama [Sci. Rep. Tokyo Kyoiku Daigaku, Sec. A, 9 (1967), 236-254] has investigated topological spaces with $\sigma$-locally finite networks. Theorem 1. A regular $\sigma$-paracompact space with a $G_\delta$-diagonal has a $\overline{G}_\delta$-diagonal. Theorem 2. Let X be a completely regular space. Then the following are equivalent: (a) X is developable; (b) X is a $\sigma$-paracompact p-space with a $G_\delta$-diagonal; (c) X is a p-space with a $G_\delta$-diagonal and a $\sigma$-locally finite network. (Received February 11, 1969.)

69T-G51. VICTOR BELFI, Rice University, Houston, Texas 77001. Nontangential homotopy equivalences.

This paper provides a procedure for constructing manifolds which are homotopy equivalent but not homeomorphic to certain $\pi$-manifolds. Specifically, the central theorem is this: Suppose N is a closed $C^\infty$ $\pi$-manifold of dimension 4k ($k \geq 2$) and L is a closed, simply-connected $C^\infty$ $\pi$-manifold. (If dim $L \equiv 2 \pmod 4$, we must also assume that there is some framing of L which has zero Kervaire invariant.) Then there exists a countable sequence of closed $C^\infty$ manifolds $\{M_i\}$ having the following properties: (1) each $M_i$ is homotopy equivalent to $N \times L$, (2) $M_i$ is not homeomorphic to $M_j$ if $i \neq j$. (3) no $M_i$ is a $\pi$-manifold. It follows from (3) that no $M_i$ is homeomorphic to $N \times L$. The basic idea of the proof of the theorem is to apply a slight variation of Browder-Novikov surgery in a very special situation. The principal corollary is an application to simply-connected Lie groups. Taking M and L to be Lie groups satisfying the conditions of the theorem, we obtain a countable sequence of topologically distinct manifolds, all of which are homotopy equivalent to the same Lie group. The recently proved result that simply-connected homotopy equivalent Lie groups are isomorphic implies that none of these manifolds is homeomorphic to any Lie group if N is also taken to be simply-connected. (Received February 11, 1969.) (Author introduced by Professor Morton L. Curtis.)

69T-G52. JOEL M. COHEN and HERMAN R. GLUCK, University of Pennsylvania, Philadelphia, Pennsylvania 19104. Homotopy in chain complexes.

We obtain the following results: Theorem A. Let $f, g: C \to D$ be chain maps, where C is a free chain complex whose homology in each dimension is a direct sum of cyclic groups and D is a torsion free chain complex. If $f_* = g_* : H_*(C;G) \to H_*(D;G)$ for every cyclic group G, then f is chain homotopic to g. The main tool is the Stacked Bases Theorem (see Abstract 666-18, this issue of these *Notices*). Theorem B. Let $f: C \to D$ be a chain map between chain complexes which in each dimension are direct sums of cyclic groups (thus including chain complexes which are finitely generated in each dimension). If $f_* : H_*(C;G) \cong H_*(D;G)$ for each cyclic group G, then f is a chain equivalence. (Received February 5, 1969.)

# Miscellaneous Fields

69T-H20. E. M. WRIGHT, University of Aberdeen, Aberdeen, United Kingdom. Graphs on many nodes with many edges. II. Preliminary report.

The graphs considered are on n nodes with p edges, each pair being joined by one undirected edge or not joined. If the nodes are labelled, the number of different graphs is $F_{np} = N!/p!\,(N-p)!$, where $N = n(n-1)/2$. Let $T_{np}$ be the number of different graphs if the nodes are unlabelled. The necessary and sufficient condition that $n!\,T_{np} \sim F_{np}$ for large n is that $\{\min(q, N-q)/n\} - (1/2)\log n \to \infty$ as $n \to \infty$. This improves the result of Abstract 68T-H5, these *Notices* 15 (1968), 814. Under the same necessary and sufficient condition, we have $n!\,T_{np} - F_{np} \sim F_{np} N\beta^{n-2} e^{-\gamma}$, where $\lambda = p/N$, $\beta = \lambda^2 + (1-\lambda)^2$, $\gamma = 4\beta^{-2}\lambda(1-\lambda)(1-2\lambda)$. (Received January 6, 1969.)

69T-H21. PAUL S. SCHNARE, University of Florida, Gainesville, Florida 32601. Multiple complementation in the lattice of partitions.

Given a fixed set X let $\xi$ (resp., $\mathfrak{F}$) denote the lattice of partitions or equivalence relations (resp., complete fields of subsets) of X. Using a characterization of $\mathfrak{F}$ by M. C. Rayburn (Canad. J. Math. (to appear)) and the dual-isomorphism of the lattice of principal topologies on X with the lattice of quasi-orders on X due to A. K. Steiner, the author independently found an immediate proof of the following known result. Theorem A (Ore). $\mathfrak{F}$ is dual-isomorphic to $\xi$. Extending the result of Ore, Duke J. Math. 9 (1942), that $\xi$ is multiply complemented the author establishes: Theorem B. Every proper (i.e. $\neq 0,1$) element of $\xi$ has (1) at least $n-1$ complements if $n = |X| < \aleph_0$; (2) at least $|X|$ and at most $2^{|X|}$ complements if $\aleph_0 \leq |X|$. Moreover, these bounds are best possible. As an immediate corollary, Theorem B holds if $\xi$ is replaced by $\mathfrak{F}$. (Cf., P. S. Schnare, Multiple complementation in the lattice of topologies, Fund. Math. 62 (1968) and Infinite complementation in the lattice of topologies, Fund. Math. 64 (1968).) (Received January 16, 1969.)

69T-H22. MARTIN S. AIGNER, and T. A. DOWLING, University of North Carolina, Chapel Hill, North Carolina 27514. A geometric characterization of the line graph of a symmetric BIB design. Preliminary report.

Let G be a graph, d(x,y) the distance, and $\Delta(x,y)$ the number of vertices adjacent to both x and y, $x,y \in V(G)$. The line graph of a symmetric BIB design is readily seen to satisfy conditions (1)-(6) below. Theorem. Let $v,k,\lambda$ be integers such that $0 < \lambda \leq k \leq v$ and $\lambda(v-1) = k(k-1)$, and let G be a graph with the following properties: (1) $|V(G)| = vk$. (2) $\deg u = 2(k-1)$, $u \in V(G)$. (3) $\Delta(x,y) = k-2$, $(x,y) \in E(G)$. (4) $\Delta(x,y) \leq 2$, $(x,y) \notin E(G)$. (5) If $d(x,y) = 2$ and $\Delta(x,y) = 1$, there exist exactly $k - \lambda$ vertices z such that $d(x,z) = 1$, $d(y,z) = 3$. (6) $d(x,y) \leq 3$, $x,y \in V(G)$. Then G is the line graph of a symmetric balanced incomplete block design $\pi(v,k,\lambda)$ or $v = 7$, $k = 4$, $\lambda = 2$, in which case there is exactly one exception. A characterization in terms of the eigenvalues of the adjacency matrix was given by A. J. Hoffman and D. K. Ray-Chaudhuri (Trans. Amer. Math. Soc. 11 (1965), 238-252). (Received January 17, 1969.)

69T-H23. HARVEY J. CHARLTON, North Carolina State University, Raleigh, North Carolina 27606. Paving 2-complexes with closed arcs of lengths bounded from zero.

By induction using the classification theorem for compact 2-manifolds [Massey, Algebraic Topology: An Introduction, p. 10] one can easily show: Given a compact 2-manifold $\exists\ \epsilon > 0$ and a collection of closed arcs of lengths greater than $\epsilon$ which pave the manifold. With a little more trouble one can show: Given a 2-complex with simplices whose diameters are bounded from zero $\exists\ \epsilon > 0$ and a collection of arcs of lengths greater than $\epsilon$ which pave the complex. (Received January 22, 1969.)

69T-H24. JAMES J. KAPUT, Southeastern Massachusetts Technological Institute, North Dartmouth, Massachusetts 02747. Nonuniversal adjunctions.

Let $F : \underline{A} \to \underline{C}$ be a functor. We say F is locally left adjunctable (l.l.a.) if for any $f : X \to F(A)$ in $\underline{C}$ there exists an object $f(X)$ and morphism $f_1 : f(X) \to A$ in $\underline{A}$ and a morphism $f_0 : X \to F(f(X))$ in $\underline{C}$ such that $f = F(f_1)f_0$. Moreover, if $f = F(h)g$ with say $h : A' \to A$, then there exists a unique $t : f(X) \to A'$ such that $ht = f_1$ and $F(t)f_0 = g$. It is clear that if F has a left adjoint G, then the front adjunction $e_X : X \to FG(X)$ serves as the "$f_0$" for every morphism f with codomain X, etc. Theorem. F is l.l.a. iff F can be fibered by a suitable functor with a left adjoint. Theorem. Suppose F is l.l.a. and $f : X \to F(A)$. There exists a group $G(f(X))$ of automorphisms of $f(X)$ such that if $a,a' : f(X) \to A'$ satisfy $F(a)f_0 = F(a')f_0$, then there exists a unique r in $G(f(X))$ with $a'r = a$ and $F(r)f_0 = f_0$. These groups turn out to be familiar ones when examples in the category of fields and in categories of spaces which have covering spaces of some type are considered. (Received February 6, 1969.)

69T-H25. HENRY S. LIEBERMAN, 48 Gay Street, Newtonville, Massachusetts 02160, and PETER PERKINS, Holy Cross College, Worcester, Massachusetts 01610. Control automata.

A control automaton over a pair of finite sets, $\Sigma_1$ and $\Sigma_2$, each with special symbols, $\delta_1 \in \Sigma_1$ and $\delta_2 \in \Sigma_2$, is a 4-tuple $C = \langle S,M,s_0,F \rangle$ where S is a finite, nonempty set, $s_0 \in S$, $F \subseteq S$ and $M : S \times (\Sigma_1 \times \Sigma_2) \to S$ and $M(s, \delta_1, \delta_2) = s$ for all $s \in S$. M is extended to be defined on $S \times (\Sigma_1^* \times \Sigma_2^*)$ after tapes of unequal length have been equalized using the appropriate special symbol. C is controllable iff for each $x \in \Sigma_1^*$ there is a $y \in \Sigma_2^*$ such that $M(s_0,x,y) \in F$. C is n-sequentially controllable iff there exists $f : \Sigma_1^* \to \Sigma_2^*$ such that for all $x \in \Sigma_1^*$, $M(s_0,\bar{x},f(\bar{x})) \in F$ where $\bar{x} = x\,\delta_1^k$ for some k; and f is defined so as to model the ability of a controller to consider the current state and next n-symbols on the environment tape ($\epsilon\ \Sigma_1^*$) when choosing the next control symbol ($\epsilon\ \Sigma_2$). Both controllability and, for each n, n-sequential controllability are shown to be equivalent to universal acceptance by associated (ordinary) finite automata. Hence, by known results, these properties are effectively decidable. Finally, a probabilistic control automaton is defined reflecting uncertainty in state transitions and in occurrence of environment symbols. An algorithm is given for optimizing the probability that a given control tape, when paired with an environment tape, will move the automaton from $s_0$ to a state in F. (Received February 5, 1969.)

69T-H26. R. B. KIRK, Southern Illinois University, Carbondale, Illinois 62901. Locally-compact, B-compact spaces. Preliminary report.

In a previous paper, the author introduced the concept of a B-compact space as a completely-regular Hausdorff space in which every finite, regular Baire measure is net-additive. (A finite, regular Baire measure m is net-additive if for every downward directed system of zero sets $\{Z_\tau\}$ with $Z_\tau \downarrow \varphi$, then $m(Z_\tau) \downarrow 0$.) In the present paper, locally-compact, B-compact spaces are studied. It is shown that a finite intersection of B-compact spaces is B-compact. Futhermore, it is shown that a countable product of locally-compact, B-compact spaces is again B-compact. This result is best possible (modulo the continuum hypothesis) since it is known that $R^{\mathfrak{c}}$ is not B-compact, where $\mathfrak{c}$ is the power of the continuum. Finally, the following conjecture is offered. Let X be locally-compact. Then X is B-compact if and only if X is realcompact. (Received October 24, 1968.)

69T-H27. H. E. GORMAN, Roosevelt University, Chicago, Illinois 60605. The Brandt condition and invertibility of modules.

Let L be a finite-dimensional, symmetric algebra with 1 over K, the quotient field of a domain R. Kaplansky (Submodules of quaternion algebras, J. London Math. Soc., to appear) has shown that when L is a quaternion algebra and R is a Bezout domain, invertibility of a module is equivalent to its satisfying a relation between itself, its dual and its discriminant. We call this relation the Brandt condition, a module which satisfies it a Brandt module and L a Brandt algebra if a contained module is invertible if and only if it satisfies the Brandt condition. When R is a Prüfer ring, we show that L is a Brandt algebra if (i) L contains only invertible modules; (ii) L is 3-dimensional. We show that if L is generated over K by a noncubic element, then L is not Brandt, and that the only matrix algebra over L which is Brandt is $M_2(K)$. Finally, we generalize a theorem of Fadeev (An introduction to the multiplicative theory of modules of integral representations, Trudy Mat. Inst. Steklov 80 (1965), 145-182; English transl., Trudy Mat. Inst. Steklov 80 (1965), 164-210) by showing that, when R is a valuation ring and L has the property that Brandt modules are invertible, then any module or its dual is invertible. (Received September 20, 1968.)

# Errata - Volume 15

SARAHAN SHELAH. Classes with homogeneous models only, Abstract 68T-E2, Page 803.

Line 11: Replace "$|T| < \lambda_1 < \lambda$ ( $\chi < \lambda_1 < \lambda$)" by "$|T|^+ < \lambda_1 < \lambda$ ($\lambda < \lambda_1 < \lambda$)" and replace "$\lambda < \beth_\omega$" by "$\lambda \leqq \beth_\omega$".

AL SOMAYAJULU. Convolution involving the Möbius function and an analogue of Selberg's formula, Abstract 662-16, Page 1034.

Line 4: Replace $\sum_{d|n} g(d)$ by $\sum_{d|n;\ (d,m)=1} g(d)$.

F. N. SPRINGSTEEL. Turing machines and marking automata languages, Abstract 68T-E12, Page 806.

Line 7: Replace "$w_i \in A^*, SA^*$" with "$w_i \in A^*SA^*$".

# Errata - Volume 16

DENNIS ALLEN, JR. A structure theory for finite regular semigroups, Abstract 663-258, Pages 159-160.

Replace the first five lines of the abstract by the following: "Let S be an arbitrary finite regular semigroup. Then by generalizing the Rees theorem for regular finite zero-simple semigroups we construct a set W, a map $\varphi: W \to S$, and a finite number of associative binary operations $*_1, \ldots, *_t$ on W each making $\varphi$ an epimorphism and W a generalized regular Rees matrix semigroup such that if $W^{(k)}$ denotes W with the product $*_k$ for $1 \leqq k \leqq t$, then the following conditions are satisfied: (i) $\varphi$ separates D-classes and is one-to-one on subgroups of each $W^{(k)}$, and (ii) the set $\{W^{(1)}, \ldots, W^{(t)}\}$ is minimal."

STEPHEN FRIEDBERG. Closed subalgebras of group algebras, Abstract 663-491, Page 229.

The last sentence should read as follows: "This theorem leads to the existence of a closed subalgebra A without factorization, i.e. $A \neq A * A$, and $A = L^1$-closure of ($A \cap L^p(G)$) where $1 < p < 2$."

CAULTON IRWIN. Green's function for singular boundary value problems. Preliminary report, Abstract 663-655, Page 278.

Theorem 1 should read "If $[P_A - I]^{-1}$ exists, then $K_A$ is the...".

H. F. KREIMER. A Galois theory for separable algebras, Abstract 663-97, Pages 113-114.

The sentence beginning on line three which reads "Assume the G is faithfully represented..." should read "Assume that every finite group of automorphisms of $\Lambda$ over $\Gamma$ is faithfully represented...".

SUSAN WILLIAMSON. Ramification theory for extensions of degree p, Abstract 663-575, Page 255.

Line 11: Replace "i = (ap/p - 1) - pg" by "i = (ap/p - 1) - pg - t with $0 \leqq t \leqq p - 1$", and replace "i = (a/p - 1)- g" by "i = (a/p - 1) - g - 1".

It was incorrectly stated in Abstract 69T-A6, Page 310, that the author, Michael Rich, was introduced by Dr. E. I. Lezak. Mr. Lezak has not as yet received his Ph.D.

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