Notices

OF THE

AMERICAN

MATHEMATICAL

SOCIETY

CODEN: AMNOAN
Volume 19, Number 2

Pages 89- 156, A289- A424

February 1972
Issue 136

Notices

OF THE

AMERICAN MATHEMATICAL SOCIETY

Edited by Everett Pitcher and Gordon L. Walker

CONTENTS

Six Hundred Ninety-Second Meeting
Biltmore Hotel
New York, New York
March 27–30, 1972

The six hundred ninety-second meeting of the American Mathematical Society will be held at the Biltmore Hotel, Madison Avenue at 43rd Street, New York, New York, from Monday, March 27, through Thursday, March 30, 1972. The Metropolitan New York Section of the Mathematical Association of America will hold a one-day meeting on Tuesday, March 28, in the Music Room of the Biltmore Hotel. The program for this meeting is given below.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, there will be four one-hour addresses. Professor Alex Rosenberg of Cornell University will present an address entitled "Commutative ring theory and the structure of Witt rings" at 11:00 a.m. on Monday, March 27; Professor David A. Buchsbaum of Brandeis University will speak on "Complexes, ideals, resolutions" at 2:00 p.m. on Monday, March 27; Professor James B. Ax of the State University of New York at Stony Brook will lecture on "Some topics in differential algebraic geometry" at 11:00 a.m. on Tuesday, March 28; and Professor Gian-Carlo Rota of the Massachusetts Institute of Technology will speak on "Combinatorial theory" at 2:00 p.m. on Tuesday, March 28.

There will be sessions for ten-minute contributed papers both mornings and afternoons of Monday and Tuesday. There will be no provision for late papers.

SYMPOSIUM ON STOCHASTIC DIFFERENTIAL EQUATIONS

With the support of the Office of Naval Research, a symposium on Stochastic Differential Equations will be held on Wednesday and Thursday, March 29 and 30. This topic was selected by the AMS-SIAM Committee on Applied Mathematics whose members are Donald G.M. Anderson, Hirsh G. Cohen, Joaquin B. Diaz, Stanislaw M. Ulam, Richard S. Varga (chairman), and Calvin H. Wilcox. The co-chairmen of the Organizing Committee are Joseph B. Keller and Henry P. McKean, Jr., of the Courant Institute of Mathematical Sciences, New York University. The hour speakers at the symposium will be Professor Richard Bucy of the University of Southern California; Professor Wendell Fleming of Brown University; Professor Bruce Knight of Rockefeller University; Professor Melvin Lax of the City College of the City University of New York; Professor Henry P. McKean, Jr., of the Courant Institute of Mathematical Sciences, New York University; Dr. John A. Morrison of the Bell Telephone Laboratories; Professor George Papanicolau of the Courant Institute of Mathematical Sciences, New York University; and Professor P. P. Varaiya of the University of California, Berkeley. The symposium will be held in the Grand Ballroom of the Biltmore Hotel on both days. Each hour address will be followed by an intermission of forty-five minutes which will include a discussion period and coffee break.

MATHEMATICAL ASSOCIATION OF AMERICA METROPOLITAN NEW YORK SECTIONAL MEETING

All sessions of the Association's sectional meeting will be held in the Music Room, located on the first floor, of the Biltmore Hotel. Registration for the meeting will take place in the corridor adjacent to the Music Room from 9:15 a.m. to 10:00 a.m. The morning session of the meeting, chaired by Dr. Morton J. Hellman of Long Island University, will in-

clude a presentation entitled "Functional analysis and valuation theory" at 10:00 a.m. by George Bachman and Edward Beckenstein of the Polytechnic Institute of Brooklyn, and a lecture by Donald J. Newman of Yeshiva University at 11:00 a.m. on "Speeds of polynomial approximation." There will be a break for lunch from 12:00 noon to 2:00 p.m. followed by an afternoon session chaired by Dr. Israel H. Rose, Lehman College. A business meeting will be conducted from 2:00 p.m. to 3:00 p.m. after which a lecture entitled "England was lost on the playing fields of Eton: A parable for mathematics" will be presented by Dr. A. B. Willcox, Executive Director of the Mathematical Association of America. "Physical applications of mathematics" is the title of the talk to be given at 4:00 p.m. by Dr. Paul C. Rosenbloom of the Teachers College, Columbia University.

REGISTRATION

The registration desk will be located in the Key Room of the Biltmore Hotel on the nineteenth floor adjacent to the Grand Ballroom. The desk will be open from 8:30 a.m. to 4:30 p.m. on Monday, March 27, through Wednesday, March 29; and from 8:30 a.m. to 3:30 p.m. on Thursday, March 30.

The registration fees for the meeting are as follows:

Member	$3.00
Student and unemployed	1.00
Nonmember	5.00

ACCOMMODATIONS

Persons intending to stay at the Biltmore Hotel should make their own reservations with the hotel. A reservation form and a listing of room rates will be found on the last page of the January *Notices*. The deadline for receipt of reservations is March 20, 1972.

TRAVEL

The Biltmore Hotel is located on Madison Avenue at 43rd Street on the east side of New York City. Walkways to Grand Central Station are located under the hotel and signs are posted directing persons to the lobby of the hotel.

Those arriving by bus may take the Independent Subway System from the Port Authority Bus Terminal. There is shuttle bus service from LaGuardia and Kennedy Airports directly to Grand Central Station. Starters can direct participants to the correct bus.

Air passengers arriving at Newark Airport can take a shuttle bus to the West Side Terminal and take either a subway or taxi to the hotel.

Those arriving by car will find many parking facilities in the neighborhood in addition to those at the hotel. Parking service can be arranged through the hotel doorman at a cost of $6 for a 24-hour period. There will be an additional charge for extra pickup-delivery service if it is required. The parking fee is subject to New York City taxes.

MAIL ADDRESS

Registrants at the meeting may receive mail addressed in care of the American Mathematical Society, The Biltmore Hotel, Madison Avenue at 43rd Street, New York, New York 10017.

SYMPOSIUM ON STOCHASTIC DIFFERENTIAL EQUATIONS

WEDNESDAY

First Session, Grand Ballroom (19th Floor)

9:00-10:00 a.m. Some stochastic questions in neurophysiology
Professor Bruce Knight, Rockefeller University

10:45-11:45 a.m. Wave propagation and conductivity in random media
Professor Melvin Lax, The City College of The City University of New York

Second Session, Grand Ballroom (19th Floor)

1:30-2:30 p.m. Some problems and methods for the analysis of stochastic equations
Professor George Papanicolau, Department of Mathematics, University Heights and Courant Institute of Mathematical Sciences, New York University

3:15-4:15 p.m. Analysis of some stochastic ordinary differential equations
Dr. John A. Morrison, Bell Telephone Laboratories

THURSDAY

Third Session, Grand Ballroom (19th Floor)

9:00-10:00 a.m. Optimal control of partially observable processes
Professor P. P. Varaiya, University of California, Berkeley

10:45-11:45 a.m. Optimal control of diffusion processes
Professor Wendell H. Fleming, Brown University

Fourth Session, Grand Ballroom (19th Floor)

1:30-2:30 p.m. Building and evaluating nonlinear filters
Professor Richard S. Bucy, University of Southern California

3:15-4:15 p.m. Nonlinear prediction theory
Professor Henry P. McKean, Jr., Courant Institute of Mathematical Sciences, New York University

The time limit for each contributed paper is ten minutes. To maintain this schedule, the time limit will be strictly enforced.

MONDAY, 9:00 A. M.

Session on Analysis I, Music Room (1st floor)

9:00-9:10
(1) Mean Cesàro summability of Laguerre and Hermite series
Dr. Eileen L. Poiani, Saint Peter's College (692-B1)

9:15-9:25
(2) Strongly regular matrices, almost convergence and Banach limits
Mr. Joseph Peter Duran, University of Rochester (692-B3)

9:30-9:40
(3) Summability methods obtained by substitution of power series
Professor Paul Gordon, Drexel University (692-B12)

9:45-9:55
(4) Approximation by a class of generalized Szasz operators. Preliminary report
Professor John J. Swetits*, Old Dominion University, and Professor Bruce Wood, University of Arizona (692-B4)

10:00-10:10
(5) Approximation by rational functions of constant modulus
Professor Stanley J. Poreda, Clark University (692-B20)

10:15-10:25
(6) An optimal bicubic spline on a rectilinear mesh over a rectangle. Preliminary report
Dr. Jonathan D. Young, Lawrence Radiation Laboratory, University of California, Berkeley (692-B18)

10:30-10:40
(7) Length, area, and curvature of Bernstein-type polynomials
Dr. Martin E. Price, Wayne State University (692-B19)

MONDAY, 9:00 A. M.

Session on Functional Analysis I, Vanderbilt Suite (1st floor)

9:00-9:10
(8) Measure-induced seminormed topologies on a Banach algebra
Professor Warren Page, City University of New York, New York City Community College (692-B9)

9:15-9:25
(9) Nonarchimedian Banach algebras
Mr. Richard F. Freund* and Mr. Teddy T. Wong, Courant Institute, New York University (692-B26)

9:30-9:40
(10) On property A.A. in W-* algebras
Professor Paul Willig, Stevens Institute of Technology (692-B10)

9:45-9:55
(11) The trace-class of a full Hilbert algebra
Dr. Michael R. W. Kervin, Virginia Polytechnic Institute and State University (692-B16)

10:00-10:10
(12) The p-classes of a Hilbert module
Professor James F. Smith, Le Moyne College (692-B33)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

MONDAY, 9:00 A. M.

Session on Geometry, French Suite (1st floor)

9:00-9:10
(13) Polar reciprocal convex bodies
Professor Heinrich W. Guggenheimer, Polytechnic Institute of Brooklyn (692-D1)

9:15-9:25
(14) A new characterization of circles. Preliminary report
Dr. Hans-Heinrich W. Herda, Boston State College (692-D5)

9:30-9:40
(15) Elementary geometric applications of a maximum principle for nonlinear elliptic equations
Professor Dimitri Koutroufiotis, University of California, Santa Barbara (692-D2)

9:45-9:55
(16) On the inequality of E. Heinz and E. Hopf
Professor Johannes C. C. Nitsche, University of Minnesota (692-D4)

10:00-10:10
(17) Constant volume envelopes of a convex cone. Preliminary report
Mr. Salvador D. Gigena, University of Pennsylvania (692-D3)

MONDAY, 9:00 A. M.

Session on Ring Theory, Georgian Room (18th floor)

9:00-9:10
(18) Absolutely torsion-free rings. Preliminary report
Dr. Robert A. Rubin, University of Kentucky (692-A2)

9:15-9:25
(19) Rings with a polynomial identity
Professor S. K. Jain, Ohio University (692-A13)

9:30-9:40
(20) Rings which are almost polynomial rings
Professor Paul M. Eakin, University of Kentucky, and Professor James L. Silver, Jr.*, King College (692-A23)

9:45-9:55
(21) The spectrum of a noncommutative ring. Preliminary report
Professor Ancel C. Mewborn* and Mr. G. L. Norwood, University of North Carolina at Chapel Hill (692-A25)

10:00-10:10
(22) Schur subgroups over the 2-adic field
Professor Toshihiko Yamada, Queen's University (692-A14)

10:15-10:25
(23) Groups on modules. Preliminary report
Mr. Pedro Pablo Sanchez, University of Michigan (692-A22)

10:30-10:40
(24) The Grothendieck ring of certain metacyclic groups
Professor John J. SantaPietro, Rutgers University (691-16-22)

MONDAY, 9:00 A. M.

Session on Topology I, Biltmore Suite (1st floor)

9:00-9:10
(25) On E-compact spaces and E-perfect mappings
Dr. Jung H. Tsai, State University of New York, College at Geneseo (692-G1)

9:15-9:25
(26) On realcompactness and Wallman realcompactifications
Professor Anthony J. D'Aristotle, State University of New York, College at Geneseo (692-G4)

9:30-9:40
(27) Ordinal subspaces of topological spaces
Dr. John Warren Baker, Florida State University (692-G18)

9:45-9:55
(28) Ultrafilters and epi-reflective hulls. Preliminary report
Mr. George Reynolds, Wesleyan University (692-G15)

10:00-10:10
(29) Two theorems in uniform spaces. Preliminary report
Mr. Michael D. Rice, Wesleyan University (692-G16)

10:15-10:25
(30) Products of Fréchet spaces. Preliminary report
Professor Peter W. Harley III, University of South Carolina (692-G20)

MONDAY, 11:00 A. M.

Invited Address, Grand Ballroom (19th floor)
Commutative ring theory and the structure of Witt rings
Professor Alex Rosenberg, Cornell University

MONDAY, 2:00 P. M.

Invited Address, Grand Ballroom (19th floor)
Complexes, ideals, resolutions
Professor David A. Buchsbaum, Brandeis University

MONDAY, 3:15 P. M.

Session on Analysis II, Music Room (1st floor)

3:15-3:25
(31) A new type of variational theory sufficiency theorem
Professor Russell D. Rupp, State University of New York at Albany (692-B27)

3:30-3:40
(32) An integral analog of the Leibniz rule. Preliminary report
Professor Thomas J. Osler, Rensselaer Polytechnic Institute (692-B32)

3:45-3:55
(33) Weighted norm inequalities for fractional integrals
Professor Benjamin Muckenhoupt* and Professor Richard Wheeden, Rutgers University (692-B34)

4:00-4:10
(34) On stability for linear Volterra integral equations using associated differential equations in the Banach algebra $\mathcal{M}_\infty$. Preliminary report
Professor John M. Bownds*, Rensselaer Polytechnic Institute, and Professor Jim M. Cushing, IBM, T.J. Watson Research Center, Yorktown Heights, New York (692-B15)

4:15-4:25
(35) A discontinuous Hill's equation. Preliminary report
Professor Norman J. Finizio, University of Rhode Island (692-B35)

4:30-4:40
(36) Nonlinear boundary value problems and several Lyapunov functions
Professor Stephen R. Bernfeld and Professor V. Lakshmikantham, University of Rhode Island, and Professor S. Leela*, State University of New York, College at Geneseo (692-B36)

4:45-4:55
(37) Asymptotic self invariant sets and functional differential equations in Banach spaces. Preliminary report
Miss P. S. Kamala, University of Rhode Island (692-B37)

5:00-5:10
(38) Asymptotic behaviour of perturbed nonlinear differential equations. Preliminary report
Professor Athanassios G. Kartsatos, University of South Florida (692-B38)

MONDAY, 3:15 P. M.

Session on Functional Analysis II, Vanderbilt Suite (1st floor)

3:15-3:25
(39) Remarks on the Radon-Nikodym theorem for vector measures
Dr. Joseph Diestel, Kent State University (692-B6)

3:30-3:40
(40) A very weak topology for the Mikusiński field
Professor Gregers L. Krabbe, Purdue University (692-B8)

3:45-3:55

(41) Abstract distribution theory in De Wilde spaces
Professor Milos A. Dostal and Professor Michael J. Kascic, Jr.*, Stevens Institute of Technology (692-B23)

4:00-4:10

(42) A spectral analysis of the reduced transport operator
Dr. L. C. Baird* and Dr. P. F. Zweifel, Virginia Polytechnic Institute and State University (692-B28)

MONDAY, 3:15 P. M.

Session on Group Theory, Georgian Room (18th floor)

3:15-3:25

(43) On some conditions which do not characterize finite abelian groups. Preliminary report
Mr. John H. Ying, State University of New York at Binghamton (692-A4)

3:30-3:40

(44) On extending characters of a normal subgroup. Preliminary report
Professor Richard L. Roth, University of Colorado (692-A6)

3:45-3:55

(45) Groups of exponent 4. Preliminary report
Professor Seymour Bachmuth* and Professor Horace Y. Mochizuki, University of California, Santa Barbara (692-A9)

4:00-4:10

(46) $\mathfrak{F}$-projectors in finite π-solvable groups
Dr. Ben C. Brewster, State University of New York at Binghamton (692-A15)

4:15-4:25

(47) On 3-Engel groups
Professor Luise-Charlotte Kappe* and Professor Wolfgang P. Kappe, State University of New York at Binghamton (692-A16)

4:30-4:40

(48) Metabelian Levi-formations
Professor Wolfgang P. Kappe* and Professor Luise-Charlotte Kappe, State University of New York at Binghamton (692-A17)

4:45-4:55

(49) Complete exact sequences. Preliminary report
Professor Donald Cook, Hollins College (692-A19)

MONDAY, 3:15 P. M.

Session on Topology II, Biltmore Suite (1st floor)

3:15-3:25

(50) Metrizability and graph-distinguishable correspondences. Preliminary report
Dr. Eric John Braude, Seton Hall University (692-G7)

3:30-3:40

(51) A note on monotone normality. Preliminary report
Mr. Jack W. Pope, University of North Carolina at Chapel Hill (692-G19)

3:45-3:55

(52) An exact sequence of braid groups
Dr. Charles H. Goldberg, Trenton State College (692-G8)

4:00-4:10

(53) Some topological methods for plane geometry
Professor Harold Bell, University of Cincinnati (692-G10)

4:15-4:25

(54) Note on symbolic transformation groups. Preliminary report
Mr. Wayne Lawton, Wesleyan University (692-G13)
(Introduced by Professor Walter H. Gottschalk)

TUESDAY, 9:00 A. M.

Session on Algebra, French Suite (1st floor)

9:00-9:10

(55) Congurence relations on lattices
Professor Melvin F. Janowitz, University of Massachusetts (692-A3)

9:15-9:25
(56) Baer semigroups as an equational class
Professor Donald H. Adams, University of Massachusetts (692-A11)

9:30-9:40
(57) Regularity and reductions in local Noether lattices
Professor Michael E. Detlefsen, Slippery Rock State College (692-A24)

9:45-9:55
(58) Localizability of the embedding problem with symplectic kernel
Professor Jack Sonn, Adelphi University (692-A27)

10:00-10:10
(59) A complete normal surface without effective Cartier divisors
Mr. Thomas A. McIntyre, University of Notre Dame (692-A8)

10:15-10:25
(60) The collineation group of a normal projective abelian variety
Dr. Federico Gaeta, State University of New York at Buffalo (692-A12)

10:30-10:40
(61) Every algebraic set in n-space is the intersection of n hypersurfaces
Dr. David Eisenbud*, Brandeis University, and Dr. E. Graham Evans, Massachusetts Institute of Technology (692-A21)

TUESDAY, 9:00 A. M.

Session on Complex Analysis, Biltmore Suite (1st floor)

9:00-9:10
(62) Projective structures on Riemann surfaces. Preliminary report
Professor Richard Mandelbaum, University of Massachusetts (692-B14)

9:15-9:25
(63) Convex functions of negative order
Professor Robert W. Redding, Clark University and Worcester State College (692-B21)

9:30-9:40
(64) On spiral-like univalent functions. Preliminary report
Professor Frank R. Keogh, University of Kentucky (692-B24)

9:45-9:55
(65) An analytic structure on a set of polar divisors. Preliminary report
Dr. John Kasdan, City University of New York, City College (692-B39)

10:00-10:10
(66) On the maximum modulus theorem for nonanalytic functions in several complex variables
Professor Mario O. Gonzalez, University of Alabama (692-B17)

TUESDAY, 9:00 A. M.

General Session, Vanderbilt Suite (1st floor)

9:00-9:10
(67) A quick version of Godel's theorem
Mr. Donald R. Perlis, Cooper Union (692-E1)
(Introduced by Professor Martin D. Davis)

9:15-9:25
(68) Fixed point theorems for a sum of nonlinear operators
Dr. Sankatha P. Singh, Memorial University of Newfoundland (692-G5)

9:30-9:40
(69) On distributive quasi-groups of prime order $n > 2$. Preliminary report
Mr. Boris Bohun-Chudyniv, Seton Hall University (692-A28)

9:45-9:55
(70) On the number of idempotent linear transformations of a vector space
Professor Jin Bai Kim, West Virginia University (692-A29)

TUESDAY, 9:00 A. M.

Session on Partial Differential Equations, Park Lounge (18th floor)

9:00-9:10
(71) Global solutions of Hamilton-Jacobi boundary value problems by variational methods. Preliminary report
Mr. Stanley H. Benton, Tulane University (692-B2)

9:15-9:25
(72) Shadows are not what they used to be
Professor Jeffrey B. Rauch* and Mr. Michael E. Taylor, University of Michigan (692-B5)

9:30-9:40
(73) Criteria for the solvability of noncoercive semilinear operator equations
Professor Melvyn S. Berger* and Professor Martin Schechter, Belfer Graduate School, Yeshiva University (692-B13)

9:45-9:55
(74) An improved determination for decay at infinity of solutions to a convolution equation. Preliminary report
Professor Kuang-Ho Chen, Louisiana State University in New Orleans (692-B30)

10:00-10:10
(75) Some exact solutions of boundary layer equations in a rotating system
Dr. Lokenath Debnath and Mr. Michael A. Hall*, East Carolina University (692-B7)

10:15-10:25
(76) The initial-boundary value problem for the Navier-Stokes equations with data in L^p
Professor Jeff E. Lewis, University of Minnesota (692-B31)

10:30-10:40
(77) Solution of the generalized heat equation along its generalized characteristics
Dr. Michael Schilder, Passaic County College (692-B40)
(Introduced by Professor Monroe D. Donsker)

TUESDAY, 9:00 A. M.

Session on Probability Theory, Georgian Room (18th floor)

9:00-9:10
(78) Conditional expectations and submartingale sequences of random Schwartz distributions
Professor George Yu-Hua Chi, University of Pittsburgh (692-F1)

9:15-9:25
(79) Hilbert module valued stationary stochastic processes. Preliminary report
Professor Parfeny P. Saworotnow, Catholic University of America (692-F2)

9:30-9:40
(80) Operator-measure theoretic approach to infinitely divisible probability distributions
Professor Pesi R. Masani, Indiana University (692-F3)

9:45-9:55
(81) On a random integrodifferential system. Preliminary report
Professor Dhandapani Kannan, University of Georgia (692-F4)

TUESDAY, 11:00 A. M.

Invited address, Grand Ballroom (19th floor)
Some topics in differential algebraic geometry
Professor James B. Ax, State University of New York at Stony Brook

TUESDAY, 2:00 P. M.

Invited Address, Grand Ballroom (19th floor)
Combinatorial theory
Professor Gian-Carlo Rota, Massachusetts Institute of Technology

TUESDAY, 3:15 P. M.

Session on Algebraic Topology, Park Lounge (18th floor)

3:15-3:25
(82) The group of self-equivalences of a localization. Preliminary report
Mr. Gerald Lieberman and Mr. David L. Smallen*, University of Rochester (692-G2)

3:30-3:40
(83) Comodule spectra and the Adams spectral sequence
Professor Harold M. Hastings, Hofstra University (692-G3)

3:45-3:55
(84) Quasi-commutativity of H-spaces
Professor Francis D. Williams, New Mexico State University (692-G6)

4:00-4:10
(85) Polynomial-dual coalgebra structure in Hopf algebras over the integers. Preliminary report
Mr. P. Brian Shay, City University of New York, Graduate School (692-G9)

4:15-4:25
(86) The Kervaire invariant of manifolds with few nonzero Stiefel-Whitney classes. Preliminary report.
Mr. Stavros G. Papastavridis, Princeton University (692-G14)

TUESDAY, 3:15 P. M.

Session on Applied Mathematics, Georgian Room (18th floor)

3:15-3:25
(87) k-part splittings and operator-parameter-overrelaxation
Professor John de Pillis, University of California, Riverside (692-C1)

3:30-3:40
(88) Convergence theorems for parameter estimation by quasi-linearization
Mr. George M. Groome, Jr., Brown University (692-C2)

3:45-3:55
(89) A recurrence scheme for converting from one orthogonal expansion into another
Dr. Herbert E. Salzer, Brooklyn, New York (692-C3)

4:00-4:10
(90) The exterior Neumann problem for the Helmholtz equation
Dr. John F. Ahner* and Dr. Ralph E. Kleinman, University of Delaware (692-C4)

4:15-4:25
(91) Non-null electromagnetic fields in Einstein-Maxwell theory admitting a Killing vector field. Preliminary report
Professor George C. Debney, Virginia Polytechnic Institute and State University (692-C5)

TUESDAY, 3:15 P. M.

Session on Combinatorics, Biltmore Suite (1st floor)

3:15-3:25
(92) r-ways in graphs
Professor John L. Leonard, University of Arizona (692-A1)

3:30-3:40
(93) The full homeomorphs of K_5. Preliminary report
Dr. Peter V. O'Neil, College of William and Mary (692-A5)

3:45-3:55
(94) Partitions of finite abelian groups into symmetric sum-free sets. Preliminary report
Dr. Earl Glen Whitehead, Jr., Courant Institute, New York University (692-A7)

4:00-4:10
(95) The number of partially ordered sets. Preliminary report
Professor Kim Ki-Hang Butler, Pembroke State University (692-A10)

4:15-4:25
(96) Excluding minors in combinatorial geometry
Professor Larry D. Shatoff, Colgate University (692-A18)

4:30-4:40
(97) Planarity of Cayley diagrams. Preliminary report
Professor Elvira Strasser Rapaport, State University of New York at Stony Brook, and Professor Henry W. Levinson*, Rutgers University (692-A20)

4:45-4:55
(98) Some (0, 1)-matrix equations
Dr. William G. Bridges, University of Wyoming (692-A26)

Session on Global Analysis, Vanderbilt Suite (1st floor)

3:15-3:25
(99) Homoclinic points of analytic symplectic diffeomorphisms. Preliminary report
Dr. Edward Zehnder, Courant Institute, New York University (692-B29)
(Introduced by Professor Jurgen K. Moser)

3:30-3:40
(100) A classification of the singularities of (X, f)
Dr. Okan Gurel, IBM, White Plains, New York (692-G11)

3:45-3:55
(101) Generating forms of Lagrangian submanifolds
Dr. Jedrzej Zie Śniatycki* and Mr. W. M. Tulczyjew, University of Calgary (692-G12)

4:00-4:10
(102) Green's lines on Riemannian manifolds. Preliminary report
Professor Richard Katz, California State College at Los Angeles (692-G17)
(Introduced by Dr. M. Glasner)

Walter Gottschalk
Associate Secretary

Middletown, Connecticut

PRESENTORS OF TEN-MINUTE PAPERS

Following each name is the number corresponding to the speaker's position on the program

Adams, D. H. #56
Ahner, J. F. #90
Bachmuth, S. #45
Baird, L. C. #42
Baker, J. W. #27
Bell, H. #53
Benton, S. H. #71
Berger, M. S. #73
Bohun-Chudyniv, B. #69
Bownds, J. M. #34
Braude, E. J. #50
Brewster, B. C. #46
Bridges, W. G. #98
Butler, K. K.-H. #95
Chen, K.-H. #74
Chi, G. Y.-H. #78
Cook, D. #49
D'Aristotle, A. J. #26
Debney, G. C. #91
Detlefsen, M. E. #57
Diestel, J. #39
Duran, J. P. #2
Eisenbud, D. #61
Finizio, N. J. #35
Freund, R. F. #9
Gaeta, F. #60
Gigena, S. D. #17
Goldberg, C. H. #52
Gonzalez, M. O. #66
Gordon, P. #3
Groome, G. M., Jr. #88
Guggenheimer, H. W. #13
Gurel, O. #100
Hall, M. A. #75
Harley, P. W., III #30
Hastings, H. M. #83
Herda, H.-H. W. #14
Jain, S. K. #19
Janowitz, M. F. #55
Kamala, P. S. #37
Kannan, D. #81
Kappe, L.-C. #47
Kappe, W. P. #48
Kartsatos, A. G. #38
Kascic, M. J., Jr. #41
Kasdan, J. #65
Katz, R. #102
Keogh, F. R. #64
Kervin, M. R. W. #11
Kim, J. B. #70
Koutroufiotis, D. #15
Krabbe, G. L. #40
Lawton, W. #54
Leela, S. #36
Leonard, J. L. #92
Levinson, H. W. #97
Lewis, J. E. #76
Mandelbaum, R. #62
Masani, P. R. #80
McIntyre, T. A. #59
Mewborn, A. C. #21
Muckenhoupt, B. #33
Nitsche, J. C. C. #16
O'Neil, P. V. #93
Osler, T. J. #32
Page, W. #8
Papastavridis, S. G. #86
Perlis, D. R. #67
de Pillis, J. #87
Poiani, E. L. #1
Pope, J. W. #51
Poreda, S. J. #5
Price, M. E. #7
Rauch, J. B. #72
Redding, R. W. #63
Reynolds, G. #28
Rice, M. D. #29
Roth, R. L. #44
Rubin, R. A. #18
Rupp, R. D. #31
Salzer, H. E. #89
Sanchez, P. P. #23
SantaPietro, J. J. #24
Saworotnow, P. P. #79
Schilder, M. #77
Shatoff, L. D. #96
Shay, P. B. #85
Silver, J. L., Jr. #20
Singh, S. P. #68
Smallen, D. L. #82
Śmith, J. F. #12
Śniatycki, J. Z. #101
Sonn, J. #58
Swetits, J. J. #4
Tsai, J. H. #25
Whitehead, E. G., Jr. #94
Williams, F. D. #84
Willig, P. #10
Yamada, T. #22
Ying, J. H. #43
Young, J. D. #6
Zehnder, E. #99

Six Hundred Ninety-Third Meeting
St. Louis University
St. Louis, Missouri
March 27-April 1, 1972

The six hundred ninety-third meeting of the American Mathematical Society will be held at St. Louis University, St. Louis, Missouri, from March 27 to April 1, 1972, in conjunction with the 1972 spring meeting of the Association for Symbolic Logic. Further information on the ASL meeting is given below. St. Louis University is located near the corner of Grand Avenue and Lindell Boulevard about two miles west of downtown St. Louis. The sessions will be held in Ritter Hall, which houses the Department of Mathematics, and in three underground lecture halls nearby, namely Lee Lecture Hall (Lecture Hall I), Baer-Fuller Lecture Hall (Lecture Hall II), and Kelley Auditorium (Lecture Hall III).

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings there will be four one-hour addresses. Professor Wolfgang M. Schmidt of the University of Colorado will address the Society on Friday, March 31, at 11:00 a.m.; his subject will be "Irregularities of distribution in Euclidean space and on the sphere." Professor Allen Devinatz of Northwestern University will speak on Friday, March 31, at 1:45 p.m.; his talk will be entitled "The deficiency index problem for ordinary selfadjoint differential operators." Professor Lawrence Markus of the University of Minnesota will address the Society on Saturday, April 1, at 11:00 a.m.; his topic will be "Lie dynamical systems." Professor Gilbert Baumslag of Rice University will speak on Saturday, April 1, at 1:45 p.m.; his subject will be "Groups with a single defining relation." All the hour addresses will be presented in Kelley Auditorium.

By invitation of the same committee, there will be five special sessions of selected twenty-minute papers, to be held at various times on Friday, March 31, and Saturday, April 1. The Associate Secretary has arranged one such session on the subject of Analytic Number Theory, as an extension of the Symposium described below; this session will meet Friday morning, Friday afternoon, and Saturday morning and will have the following speakers: Paul T. Bateman, Ronald J. Evans, Janos Galambos, J. M. Gandhi, Wlodzimierz Holsztyński, Marvin I. Knopp, Carlos J. Moreno, George B. Purdy, Lawrence J. Risman, Robert S. Spira, Kenneth B. Stolarsky, Henry E. Thomas, Jr., and Jack P. Tull. Professor William M. Boothby of Washington University has organized a simultaneous special session on the subject of Differential Geometry; the speakers will be Richard L. Bishop, James B. Carrell, Lawrence W. Conlon, Robert B. Gardner, Stephen J. Greenfield, Franz W. Kamber, Wu-Hsiung Huang, Richard S. Millman, Howard A. Osborn, William F. Pohl, Nolan R. Wallach, and Frank W. Warner III. Another special session, to be held Friday afternoon and Saturday morning, has been arranged by Professor Saunders Mac Lane of the University of Chicago on the subject of Category Theory; the speakers will be Eduardo J. Dubuc, John W. Duskin, Jr., John W. Gray, John R. Isbell, Paul C. Kainen, Fred E. J. Linton, Ernest G. Manes, William J. Mitchell, and Joan M. Negrepontis. Professor Lee A. Rubel of the University of Illinois has organized a special session for Saturday on the subject of Vector Spaces of Analytic Functions; the speakers will be Patrick R. Ahern, Earl R. Berkson, James G. Caughran, Ronald R. Coifman, Richard M. Crownover, Charles L. Fefferman, Stephen D. Fisher, Joel H. Shapiro, Allen L. Shields, and B. A. Taylor. The fifth special session, to be held Saturday afternoon, has been arranged by Professor Thomas E. Storer of the University of Michigan on the subject of Combinatorial Theory; the speakers will be George W. Dinolf, John R. Doner, Eugene C. Johnsen, David W. Matula, and Richard J. Turyn. There will also be seven sessions on Friday and Saturday for the presentation of contributed ten-minute papers.

COUNCIL MEETING

The Council of the Society will meet on Friday, March 31, at 5:00 p.m. in Banquet Room 3 of the Ramada Inn, 303 South Grand Avenue. There will be an intermission for dinner.

SYMPOSIUM ON ANALYTIC NUMBER THEORY

With the support of the National Science Foundation, there will be a symposium on Analytic Number Theory on Monday through Thursday, March 27-30. This topic was chosen by the Committee to Select Hour Speakers for Western Sectional Meetings, which during 1971 consisted of Paul T. Bateman (chairman), Roger C. Lyndon, and Lawrence Markus. The Organizing Committee of the symposium, responsible for the planning of the program and the choice of speakers, consists of Harold G. Diamond (chairman), Patrick X.Gallagher, Hugh L. Montgomery, Wolfgang M. Schmidt, and Harold M. Stark. The speakers at the Symposium will be Alan Baker, Bruce C. Berndt, Enrico Bombieri, Sarvadaman Chowla, John Coates, Harold G. Diamond, Peter D.T.A. Elliott, Patrick X. Gallagher, Larry J. Goldstein, Emil Grosswald, Christopher Hooley, Martin N. Huxley, Wolfgang B. Jurkat, Donald J. Lewis, Kurt Mahler, Hugh L. Montgomery, Harald G. Niederreiter, Karl K. Norton, Andrew P. Ogg, Walter V. Philipp, Hans-Egon Richert, Klaus F. Roth, Charles A. Ryavec, Daniel Shanks, Harold M. Stark, Paul Turán, A. I. Vinogradov (tentative), Peter J. Weinberger, and possibly one or two others. The detailed program is given below, although it is still subject to minor changes.

MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

Invited addresses and sessions for contributed papers are scheduled for the spring meeting of the Association for Symbolic Logic. Programs, listing titles and times for individual papers, will be available at the registration desk located in the lobby of Ritter Hall.

There will be three sessions for contributed papers: at 1:25 p.m. on Wednesday, March 29; and at 9:00 a.m. and 1:30 p.m. on Thursday, March 30. An invited address entitled "Equivalence of measurable cardinals and determinacy of games" will be presented by Professor D. A. Martin of Rockefeller University at 4:00 p.m. on Wednesday. Professor C.E.M. Yates of the University of Manchester and the University of Colorado will speak at 10:30 a.m. on Thursday, March 30; the title of his lecture will be "Abundance in the degrees of unsolvability." The sessions for contributed papers as well as the lectures presented by Professors Martin and Yates will be given in the Baer-Fuller Lecture Hall (Lecture Hall II).

"The unsolvability of Hilbert's tenth problem" is the title of the lecture to be presented by Professor Martin Davis of the Courant Institute of Mathematical Sciences, New York University, on Thursday, March 30; this talk will be given in Kelley Auditorium (Lecture Hall III) at 4:00 p.m.

The Council of the Association will meet at 8:00 p.m. on Wednesday, March 29, in the Board Room (Room 111) of Ritter Hall.

REGISTRATION

The registration desk will be in the lobby of Ritter Hall, which is located on Grand Avenue about a block south of Lindell Boulevard. The desk will be open on Monday from 1:00 p.m. to 5:00 p.m.; on Tuesday, Wednesday, Thursday, and Friday from 9:00 a.m. to 5:00 p.m.; and on Saturday from 8:00 a.m. to 4:00 p.m.

The registration fees for the meeting are as follows:

Member	$3.00
Student and unemployed	1.00
Nonmember	5.00

ACCOMMODATIONS

Two motels within walking distance of the campus which are holding blocks of rooms, and for which reservations can be made using the form on the last page of the January *Notices*, are

RAMADA INN (150-room block)
(2 blocks south of campus)
303 South Grand Boulevard
St. Louis, Missouri 63103
Phone: (314) 534-8300

Singles $15.00
(one person, one bed)
Doubles $18.00
(two persons, one double bed)
Twin Doubles $21.00
(two persons, two double beds)

Additional persons can be accommodated in rooms for an added charge of $3 per night plus $3 if a rollaway bed is required.

TRAVELODGE (80-room block)
(1 block east of campus)
3420 Lindell Boulevard
St. Louis, Missouri 63103
Phone: (314) 371-6700 or
(800) 255-3050-toll free

Singles $13.00 - $14.00
(one person, one double bed)
Doubles $16.00 - $17.00
(two persons, one double bed)
Twin Doubles $18.00 - $19.00
(two persons, two double beds)

Participants should write or telephone the following nearby hotels if reservations are desired at any of these locations:

HOLIDAY INN, MIDTOWN
(6 blocks west of campus)
4483 Lindell Boulevard
St. Louis, Missouri 63108
Phone: (314) JE5-9600

BEL AIR WEST LUXURY MOTEL
(8 blocks west of campus)
4630 Lindell Boulevard
St. Louis, Missouri 63108
Phone: (314) 367-7500

RODEWAY INN, DOWNTOWN
(8 blocks east of campus)
2600 Market Street
St. Louis, Missouri 63103
Phone: (314) 534-4700

HOLIDAY INN, DOWNTOWN
(10 blocks east of campus)
2211 Market Street
St. Louis, Missouri 63103
Phone: (314) 231-3232

A limited amount of University housing will be available in Lewis Memorial Residence, a former hotel now used as a dormitory. Rates are $5 per night for single rooms and $8 per night for double rooms. Requests for reservations should be addressed to Mr. Bohr, Director of Lewis Memorial Residence, 3701 Lindell Boulevard, St. Louis, Missouri 63108.

FOOD SERVICE

Lunch will be available in the Busch Memorial Center, the student center of St. Louis University. On Monday, Tuesday, and Wednesday regular lunch service will be available. On Thursday, Friday, and Saturday, most offices of the University are closed and so a special fixed-menu lunch at $1.50 has been arranged for the convenience of those attending the meeting. In order to make this special arrangement economically feasible for the host institution, those attending the meeting are urgently requested to eat their noon meals at the Busch Center if at all possible. A list of possible restaurants for the evening meal will be available at the registration desk.

TRAVEL

Amtrak offers direct train service to St. Louis from Chicago, Kansas City, New York, and Washington. Union Station is approximately one mile east of the campus of St. Louis University. Sleeping cars are available from New York and Washington. St. Louis is served by about a dozen different air lines. Limousine service is available from the airport to all the hotels listed. Those coming by car should leave the Daniel Boone Expressway (US Route 40) at the Grand Avenue Exit and go two blocks north on Grand Avenue.

ENTERTAINMENT

A cash bar will be set up in the Knights Room in the Pius XII Memorial Library from 5:00 p.m. to 6:30 p.m. on Monday through Friday evenings. In addition there will be a beer party on Thursday evening at the Busch Brewery for the first 175 who indicate a desire to attend; the beer will be complimentary, but there may be a charge of 75 cents for service. Tourist attractions in St. Louis include Forest Park, the Gateway Arch, and the Missouri Botanical Garden.

PARKING

Since the meeting will be held during the spring vacation of St. Louis University, there should be no serious parking problem. All the hotels listed have ample park-

ing facilities.

ADDRESS FOR MAIL AND TELEGRAMS

The registration desk will be adjacent to the office of the Mathematics Department, so that persons attending the meetings may be addressed at Mathematics Meetings, c/o Department of Mathematics, St. Louis University, St. Louis, Missouri 63121.

SYMPOSIUM ON ANALYTIC NUMBER THEORY

MONDAY, 1:30 P.M.

First Session, Kelley Auditorium

1:30-1:45 p.m. Welcoming ceremonies

1:45-2:10 p.m. Connections between the large sieve and the Turán-Kubilius inequality
Professor Peter D. T. A. Elliott, University of Colorado

2:25-3:15 p.m. An invariance principle for additive function
Professor Walter V. Philipp, University of Illinois

3:45-4:10 p.m. Classification of transcendental numbers
Professor Kurt Mahler, The Ohio State University

4:25-4:50 p.m. Metric theorems on the distribution of sequences
Professor Harald G. Niederreiter, Southern Illinois University and University of Illinois

TUESDAY, 8:00 A.M.

Second Session, Kelley Auditorium

8:00-8:25 a.m. On the difference between consecutive primes
Dr. Martin N. Huxley, University College, Cardiff

8:35-9:25 a.m. The distribution of zeros of the zeta function
Dr. Hugh L. Montgomery, Trinity College, Cambridge

9:35-10:00 a.m. Study of Euler products associated with Beurling's generalized prime number systems
Professor Charles A. Ryavec, University of Colorado

10:30-11:20 a.m. New density results for the zeta function and Dirichlet L functions
Professor Enrico Bombieri, University of Pisa

11:30-11:55 a.m. On the connection between primes and the zeros of the zeta function
Professor Paul Turán, Hungarian Academy of Sciences and University of Budapest

TUESDAY, 1:30 P.M.

Third Session, Kelley Auditorium

1:30-2:20 p.m. Complex quadratic fields with class-number two
Professor Harold M. Stark, Massachusetts Institute of Technology

2:35-3:00 p.m. Conjectures related to the class-number of imaginary quadratic fields with large discriminant
Professor Sarvadaman Chowla, Pennsylvania State University

3:30-4:20 p.m. On Iwasawa's analogue of the Jacobian for a totally real number field
Dr. John Coates, Harvard University

4:35-5:00 p.m. Some comments on arithmetic densities
Professor Larry J. Goldstein, University of Maryland

WEDNESDAY, 8:00 A.M.

Fourth Session, Kelley Auditorium

8:00-8:25 a.m. To be announced

8:40-9:30 a.m. Systematic evidence for the extended Riemann hypothesis
Dr. Daniel Shanks, Naval Ship Research and Development Center

10:00-10:50 a.m. Primes in short intervals of arithmetic progressions modulo p^r
Professor Patrick X. Gallagher, Columbia University

11:05-11:55 a.m. Some arithmetic properties of polynomials
Professor Christopher Hooley, University College, Cardiff

WEDNESDAY, 1:30 P.M.

Fifth Session, Kelley Auditorium

1:30-2:20 p.m. Effective methods in Diophantine problems II
Dr. Alan Baker, Trinity College, Cambridge

2:35-3:00 p.m. Gaps between values of positive definite quadratic forms
Professor Donald J. Lewis, University of Michigan

3:30-4:20 p.m. Rational points on certain elliptic modular curves
Professor Andrew P. Ogg, University of California, Berkeley

4:35-5:00 p.m. Some transformation formulas which generalize those of the Dekekind eta function
Professor Bruce C. Berndt, University of Illinois

THURSDAY, 8:00 A.M.

Sixth Session, Kelley Auditorium

8:00-8:25 a.m. Title to be announced
Dr. A. I. Vinogradov, Leningrad Branch, Steklov Institute, Academy of Sciences of the USSR

8:40-9:05 a.m. Bounds in the least positive integer which is relatively prime to n and not a kth power modulo n
Professor Karl K. Norton, University of Colorado

9:20-9:45 a.m. The distribution of values of Euler's phi function
Professor Harold G. Diamond, University of Illinois

10:15-10:40 a.m. On the number of solutions of the equation $\phi(x) = n$
Professor Emil Grosswald, Temple University

10:55-11:20 a.m. On the Mertens conjecture and related problems
Professor Wolfgang B. Jurkat, Syracuse University

11:30-11:55 a.m. Report on attempts to disprove the Mertens conjecture
Dr. Peter Weinberger, University of Michigan

THURSDAY, 1:30 P.M.

Seventh Session, Kelley Auditorium

1:30-2:20 p.m. Brun's method and the fundamental lemma
Professor Hans-Egon Richert, University of Marburg

2:30-3:20 p.m. Estimation of the area of the smallest triangle obtained by selecting three out of n points in a disc of unit area
Professor Klaus F. Roth, Imperial College, London

3:20-3:30 p.m. Closing ceremonies

Those attending the Symposium may also be interested in the address to be given to the Association for Symbolic Logic by Professor Martin D. Davis of the Courant Institute, New York University. It will be entitled "The unsolvability of Hilbert's tenth problem" and will be presented at 4:00 P.M. on Thursday, March 30, in Kelley Auditorium. In addition, the Special Session on Analytic Number Theory and Invited Address of Professor Wolfgang M. Schmidt, which forms part of the regular meeting of the American Mathematical Society on March 31 and April 1, may be regarded as extensions of the Symposium.

PROGRAM OF THE SESSIONS

The time limit for each contributed paper in the general sessions is ten minutes and in the special sessions is twenty minutes. To maintain this schedule, the time limits will be strictly enforced.

FRIDAY, 8:30 A. M.

Special Session on Analytic Number Theory I, Kelley Auditorium

8:30-8:50
(1) Extrapolation techniques related to transcendence proofs. Preliminary report
Professor Kenneth B. Stolarsky, University of Illinois (693-A26)

9:00-9:20
(2) Modular forms on Hecke's modular group. Preliminary report
Professor Ronald J. Evans, Jackson State College (693-A18)
(Introduced by Professor Paul T. Bateman)

9:30-9:50
(3) A problem of Rademacher in modular forms
Professor Marvin I. Knopp, University of Wisconsin and University of Illinois at Chicago Circle (693-A22)

10:00-10:20
(4) Applications of the large sieve to problems in algebraic geometry
Mr. Carlos Julio Moreno, University of Illinois (693-A23)

10:30-10:50
(5) Distribution of arithmetical functions
Professor Janos Galambos, Temple University (693-A7)

FRIDAY, 9:00 A. M.

Special Session on Differential Geometry I, Lee Lecture Hall

9:00-9:20
(6) Closed minimal curves in Riemannian manifolds with boundary
Dr. Wu-Hsiung Huang, Wayne State University (693-D13)

9:30-9:50
(7) Remarks on global hypoellipticity. Preliminary report
Professor Stephen J. Greenfield* and Professor Nolan R. Wallach, Rutgers University (693-G2)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

10:00-10:20
(8) Holomorphically injective complex torus actions
Professor J. B. Carrell, Louisiana State University (693-D12)

10:30-10:50
(9) Bundle homogeneous holomorphic fiber bundles and holomorphic connections
Professor Richard S. Millman, Southern Illinois University (693-D2)

FRIDAY, 9:00 A. M.

Session on Functional Analysis and Several Complex Variables, 225 Ritter Hall

9:00-9:10
(10) Analytic representations of tempered distributions and partial differential equations in infinite dimensions
Professor Thomas A.W. Dwyer III, Northern Illinois University (693-B31)

9:15-9:25
(11) On the Weyl spectrum of a Hilbert space operator
Dr. John V. Baxley, Wake Forest University (693-B29)

9:30-9:40
(12) Analytic representation of the finite Fourier transform. Preliminary report
Dr. Richard D. Carmichael, Wake Forest University (693-B5)

9:45-9:55
(13) Separating algebras, strongly separating algebras, and peak modulus sets. Preliminary report
Mr. Michael E. Frazier, University of Michigan (693-B19)

10:00-10:10
(14) Linear functionals and multipliers on H^p of the polydisc
Mrs. Arlene Parsekian Frazier, University of Michigan (693-B21)
(Introduced by Professor Peter L. Duren)

10:15-10:25
(15) Representation of linear functionals on H^p spaces, $0 < p < 1$, over bounded symmetric domains
Professor Josephine M. Mitchell*, State University of New York at Buffalo, and Professor Kyong T. Hahn, Pennsylvania State Univers ity (693-B25)

10:30-10:40
(16) The value distribution of most holomorphic mappings. Preliminary report
Professor Paul M. Gauthier* and Mr. Ngo Van Que, Université de Montréal (693-B3)

FRIDAY, 9:30 A. M.

Session on Automata Theory and Logic, Baer-Fuller Lecture Hall

9:30-9:40
(17) The composition of threshold functions. Preliminary report
Professor R. Arthur Knoebel, New Mexico State University (693-C1)

9:45-9:55
(18) The dual space of the nonstandard hull of a normed space
Professor C. Ward Henson* and Professor Lawrence C. Moore, Jr., Duke University (693-E1)

10:00-10:10
(19) On violating GCH below the least measurable cardinal. Preliminary report
Professor Donald H. Pelletier, York University (693-E2)

10:15-10:25
(20) On intersections of the models of a theory. Preliminary report
Professor David W. Kueker, University of Michigan (693-E3)

10:30-10:40
(21) K-descriptions in free variable logics
Professor Robert G. Jeroslow, University of Minnesota (693-E4)

FRIDAY, 11:00 A. M.

Invited Address, Kelley Auditorium
Irregularities of distribution in Euclidean space and on the sphere
Professor Wolfgang M. Schmidt, University of Colorado

FRIDAY, 1:45 P. M.

Invited Address, Kelley Auditorium

The deficiency index problem for ordinary selfadjoint differential operators
Professor Allen Devinatz, Northwestern University

FRIDAY, 3:00 P. M.

Special Session on Category Theory I, Baer-Fuller Lecture Hall

3:00-3:20
(22) Machines in a category. Preliminary report
Professor M. A. Arbib and Professor E. G. Manes*, University of Massachusetts (693-A35)

3:30-3:50
(23) Duality of functors in the category of Banach spaces
Professor Joan M. Negrepontis, Sir George Williams University (693-A39)

4:00-4:20
(24) Some structure of the category of 2-categories. Preliminary report
Professor John W. Gray, University of Illinois (693-A34)

4:30-4:50
(25) Protriples
Professor Michael Barr, McGill University, and Professor John R. Isbell*, State University of New York at Buffalo (693-A13)

FRIDAY, 3:00 P. M.

Special Session on Analytic Number Theory II, Kelley Auditorium

3:00-3:20
(26) Waring's problem for fourth powers. Preliminary report
Mr. Henry Edward Thomas, Jr., University of Michigan (693-A27)

3:30-3:50
(27) Real zeros of the Dedekind zeta function of an imaginary quadratic field. Preliminary report
Dr. George B. Purdy, University of Illinois (693-A25)

4:00-4:20
(28) Computer study of some lattice point problems. Preliminary report
Professor Zaven A. Karian, Denison University, and Professor Jack P. Tull*, Ohio State University (693-A21)

4:30-4:50
(29) Multiplicative arithmetic functions and the representation of integers as sums of squares
Professor Paul T. Bateman, University of Illinois (693-A17)

FRIDAY, 3:00 P. M.

Special Session on Differential Geometry II, Lee Lecture Hall

3:00-3:20
(30) Differential geometry in PL
Professor Howard A. Osborn, University of Illinois (693-D4)

3:30-3:50
(31) Convex functions on spheres. Preliminary report
Professor Stephanie B. Alexander and Professor Richard L. Bishop*, University of Illinois (693-D7)

4:00-4:20
(32) Curvature functions for 2-manifolds
Professor Frank W. Warner* and Professor Jerry L. Kazdan, University of Pennsylvania (693-D3)

4:30-4:50
(33) A difference tensor and characterizations of euclidean spaces
Professor Robert B. Gardner, University of North Carolina at Chapel Hill (693-D9)

FRIDAY, 3:00 P. M.

Session on Algebra, 200 Ritter Hall

3:00-3:10
(34) Large abelian subgroups of infinite solvable and nilpotent groups. Preliminary report
Dr. Vance Faber, University of Colorado, Denver Center (693-A28)

3:15-3:25
(35) Groups with a two variable commutator identity. Preliminary report
Professor J. M. Gandhi and Professor Daryl Kreiling*, Western Illinois University (693-A3)

3:30-3:40
(36) Left perfect rings that are right perfect and a characterization of Steinitz rings
Dr. William H. Rant, Jacksonville State University (693-A16)

3:45-3:55
(37) Dual generalizations of the Artinian and Noetherian conditions. Preliminary report
Dr. Robert C. Shock, Southern Illinois University (693-A12)

4:00-4:10
(38) Generalized heredity in radical classes. Preliminary report
Mr. Richard L. Tangeman, Arkansas State University (693-A5)

4:15-4:25
(39) On saturated formations of solvable Lie algebras. Preliminary report
Mr. Julius W. Overbeck, Arkansas State University (693-A31)

FRIDAY, 3:00 P. M.

Session on Functions of a Complex Variable, 225 Ritter Hall

3:00-3:10
(40) On the nonseparability of L^{∞}/H^{∞}. Preliminary report
Professor Earl R. Berkson and Professor Lee A. Rubel*, University of Illinois (693-B7)

3:15-3:25
(41) Representation of H^{p}-functions
Professor D. J. Patil, University of Wisconsin-Milwaukee (693-B20)

3:30-3:40
(42) On α-convex functions. Preliminary report
Professor Petru Mocanu, Babeş-Bolyai University, Cluj, Romania, and Professor Maxwell O. Reade*, University of Michigan (693-B13)

3:45-3:55
(43) Distortion of hyperbolic area under quasiconformal mappings
Professor John A. Kelingos, Vanderbilt University (693-B22)

4:00-4:10
(44) On Julia directions of functions of finite order. Preliminary report
Mr. David Drasin* and Mr. Allen W. Weitsman, Purdue University (693-B24)

4:15-4:25
(45) On the double points of the image of the unit circle under a polynomial mapping. Preliminary report
Professor John R. Quine, Florida State University (693-B26)

4:30-4:40
(46) Inequalities for orthogonal polynomials and certain generalized hypergeometric series with only real zeros. Preliminary report
Professor George Gasper, Northwestern University (693-B12)

SATURDAY, 8:30 A. M.

Special Session on Category Theory II, Baer-Fuller Lecture Hall

8:30-8:50
(47) The infinitary simplicial category
Professor Eduardo Dubuc, University of Rochester (693-A38)
(Introduced by Professor Saunders Mac Lane)

9:00-9:20
(48) Realizing commutative cubes. Preliminary report
Professor Paul C. Kainen, Case Western Reserve University (693-A9)

9:30-9:50
(49) Categories of Topoi and the theory of sets
Mr. William J. Mitchell, University of Chicago (693-A30)

10:00-10:20
(50) Three categorical novelties
Professor Fred E.J. Linton, Wesleyan University (693-A29)

10:30-10:50
(51) Nonabelian triple cohomology: Extensions and obstructions. Preliminary report
Professor John W. Duskin, Jr., State University of New York at Buffalo (693-A14)

SATURDAY, 8:30 A. M.

Special Session on Vector Spaces of Analytic Functions I, Kelley Auditorium

8:30-8:50
(52) On R. P. measures on the torus
Professor Patrick Ahern, University of Wisconsin (693-B27)
(Introduced by Professor Lee A. Rubel)

9:00-9:20
(53) Totally hyponormal operators and analytic functions. Preliminary report
Professor Earl R. Berkson* and Professor Lee A. Rubel, University of Illinois, and Professor James P. Williams, Indiana University (693-B11)

9:30-9:50
(54) Spectra of composition operators on H^2
Professor James G. Caughran, University of Toronto (693-B15)

10:00-10:20
(55) A theory of H^p spaces associated with compact Lie groups. Preliminary report
Professor Ronald R. Coifman* and Professor Guido L. Weiss, Washington University (693-B16)

10:30-10:50
(56) Commutants of shifts on Banach spaces
Professor Richard M. Crownover, University of Missouri-Columbia (693-B2)

SATURDAY, 8:45 A. M.

Session on Analysis and Differential Equations, 225 Ritter Hall

8:45-8:55
(57) Approximation in norms defined by derivations. Preliminary report
Professor James M. Briggs, University of Wisconsin-Whitewater (693-B18)

9:00-9:10
(58) Weak and strong Φ variation
Professor Casper Goffman*, Purdue University, and Professor John J. Loughlin, Virginia Polytechnic Institute and State University (693-F1)

9:15-9:25
(59) Unique extensions of abstract measures
Mr. Gerald A. Kraus, Southern Illinois University (693-B28)

9:30-9:40
(60) Interpolation sets for convolution measure algebras
Professor Colin C. Graham, Northwestern University (693-B4)

9:45-9:55
(61) Nonzero solutions of boundary value problems of second order ordinary and delay-differential equations. Preliminary report
Professor Grant B. Gustafson* and Professor Klauss Schmitt, University of Utah (693-B1)

10:00-10:10
(62) Oscillation phenomena for linear differential systems in a B*-algebra
Dr. C. Max Williams, University of Oklahoma (693-B30)
(Introduced by Professor William T. Reid)

10:15-10:25
(63) Quasilinear elliptic equations with rapidly increasing coefficients
Dr. Jean-Pierre Gossez, University of Chicago (693-B23)

10:30-10:40
(64) Some a priori estimate for elliptic second order equations with Dirichlet boundary data
Professor Alan R. Elcrat, Wichita State University (693-B6)

SATURDAY, 9:00 A. M.

Special Session on Analytic Number Theory III, 223 Ritter Hall

9:00-9:20
(65) The average order of $L(1, \chi)$
Professor Robert S. Spira, Michigan State University (693-A8)

9:30-9:50
(66) A new proof of the three squares theorem
Mr. Lawrence James Risman, Fitchburg State College (693-A4)

10:00-10:20
(67) Applications of ergodic theory to number theory. Preliminary report
Professor Wlodzimierz Holsztyński, University of Michigan (693-A20)

10:30-10:50
(68) On certain sums of fractional parts
Professor J. M. Gandhi*, Western Illinois University, and Professor Kenneth S. Williams, Carleton University (693-A19)

SATURDAY, 9:00 A. M.

Special Session on Differential Geometry III, Lee Lecture Hall

9:00-9:20
(69) Orbits of codimension two
Professor Lawrence W. Conlon, St. Louis University and Washington University (693-D1)

9:30-9:50
(70) Homogeneous spaces of positive curvature
Professor Nolan R. Wallach, Rutgers University (693-D6)

10:00-10:20
(71) A theorem of géométrie finie
Professor William F. Pohl, University of Minnesota (693-D8)

10:30-10:50
(72) Characteristic classes of modules over a sheaf of Lie algebras. Preliminary report
Professor Franz W. Kamber* and Professor Philippe Tondeur, University of Illinois (693-D5)

SATURDAY, 9:30 A. M.

Session on Ordered Algebraic Structures, 200 Ritter Hall

9:30-9:40
(73) Some properties of the lattice generalizations of G_{32}. Preliminary report
Dr. Gerald C. Schrag, Central Missouri State College (693-A6)

9:45-9:55
(74) A new characterization of orthomodular partially ordered sets. Preliminary report
Dr. L. A. Cammack, Culver-Stockton College (693-A2)

10:00-10:10
(75) On construction of orders in abelian groups
Professor Donald P. Minassian, Butler University (693-A10)

10:15-10:25
(76) Integrally closed principal elements in a Noether lattice. Preliminary report
Mr. Eugene I. Furuyama, Washington State University (693-A15)

10:30-10:40
(77) On semigroups, groups and lattices of tables of subgroups of a group. Preliminary report
Professor Lal M. Chawla* and Professor Leonard E. Fuller, Kansas State University (693-A1)

SATURDAY, 11:00 A. M.

Invited Address, Kelley Auditorium
Lie dynamical systems
Professor Lawrence Markus, University of Minnesota

SATURDAY, 1:45 P. M.

Invited Address, Kelley Auditorium
Groups with a single defining relation
Professor Gilbert Baumslag, Rice University

SATURDAY, 3:00 P. M.

Special Session on Combinatorial Theory, Baer-Fuller Lecture Hall

3:00-3:20
(78) Combinatorial structure in cyclic neofields. Preliminary report
Professor Eugene C. Johnsen, University of California, Santa Barbara (693-A36)

3:30-3:50
(79) CIP neofields and combinatorial designs
Mr. John R. Doner, University of Michigan (693-A33)

4:00-4:20
(80) Hadamard matrices, algebras, and composition theorems
Dr. Richard J. Turyn, Raytheon Company, Sudbury, Massachusetts (693-A32)

4:30-4:50
(81) An extremal problem for nonseparable combinatorial geometries. Preliminary report
Dr. George W. Dinolt, University of Michigan at Dearborn (693-A37)

5:00-5:20
(82) The employee party problem
Professor David W. Matula, Washington University (693-A11)

SATURDAY, 3:00 P. M.

Special Session on Vector Spaces of Analytic Functions II, Kelley Auditorium

3:00-3:20
(83) Some recent results on H^p
Professor Charles L. Fefferman, University of Chicago (693-B14)

3:30-3:50
(84) Invariant subalgebras for the backward shift. Preliminary report
Professor Stephen D. Fisher, Northwestern University (693-B10)

4:00-4:20
(85) Absolutely p-summing composition operators on H^p spaces
Professor Joel H. Shapiro, Michigan State University (693-B9)

4:30-4:50
(86) Singular inner factors of analytic functions
Professor James G. Caughran, University of Toronto, and Professor Allen L. Shields*, University of Michigan (693-B17)

5:00-5:20
(87) Discrete sufficient sets for some spaces of entire functions
Professor B. A. Taylor, University of Michigan (693-B8)

SATURDAY, 3:00 P. M.

Session on Geometry and Topology, Lee Lecture Hall

3:00-3:10
(88) Compact homogeneous spaces possessing invariant contact, symplectic or cosymplectic structures
Professor Philip B. Zwart, Washington University (693-D11)
(Introduced by Professor William M. Boothby)

3:15-3:25
(89) Second order connections. II
Professor Robert H. Bowman, Arkansas State University (693-D10)

3:30-3:40
(90) On the existence of certain linear homeomorphisms of a convex polyhedral disk
Professor Chung-wu Ho, Southern Illinois University at Edwardsville (693-G1)

3:45-3:55
(91) Paracompactness and transfinite convergence
Mr. Jerrel K. Yates, Huntingdon College (693-G3)

Paul T. Bateman
Associate Secretary

Urbana, Illinois

PRESENTORS OF TEN-MINUTE PAPERS

Following each name is the number corresponding to the speaker's position on the program.

Ahern, P. #52
Bateman, P. T. #29
Baxley, J. V. #11
Berkson, E. R. #53
Bishop, R. L. #31
Bowman, R. H. #89
Briggs, J. M. #57
Cammack, L. A. #74
Carmichael, R. D. #12
Carrell, J. B. #8
Caughran, J. G. #54
Chawla, L. M. #77
Coifman, R. R. #55
Conlon, L. W. #69
Crownover, R. M. #56
Dinolt, G. W. #81
Doner, J. R. #79
Drasin, D. #44
Dubuc, E. #47
Duskin, J. W., Jr. #51
Dwyer, T.A.W., III #10
Elcrat, A. R. #64
Evans, R. J. #2
Faber, V. #34
Fefferman, C. L. #83
Fisher, S. D. #84
Frazier, A. P. #14
Frazier, M. E. #13
Furuyama, E. I. #76
Galambos, J. #5
Gandhi, J. M. #68
Gardner, R. B. #33
Gasper, G. #46
Gauthier, P. M. #16
Goffman, C. #58
Gossez, J.-P. #63
Graham, C. C. #60
Gray, J. W. #24
Greenfield, S. J. #7
Gustafson, G. B. #61
Henson, C. W. #18
Ho, C. #90
Holsztyński, W. #67
Huang, W.-H. #6
Isbell, J. R. #25
Johnsen, E. C. #78
Jeroslow, R. G. #21
Kainen, P. C. #48
Kamber, F. W. #72
Kelingos, J. A. #43
Knoebel, R. A. #17
Knopp, M. I. #3
Kraus, G. A. #59
Kreiling, D. #35
Kueker, D. W. #20
Linton, F.E.J. #50
Manes, E. G. #22
Matula, D. W. #82
Millman, R. S. #9
Minassian, D. P. #75
Mitchell, J. M. #15
Mitchell, W. J. #49
Moreno, C. J. #4
Negrepontis, J. M. #23
Osborn, H. A. #30
Overbeck, J. W. #39
Patil, D. J. #41
Pelletier, D. H. #19
Pohl, W. F. #71
Purdy, G. B. #27
Quine, J. R. #45
Rant, W. H. #36
Reade, M. O. #42
Risman, L. J. #66
Rubel, L. A. #40
Schrag, G. C. #73
Shapiro, J. H. #85
Shields, A. L. #86
Shock, R. C. #37
Spira, R. S. #65
Stolarsky, K. B. #1
Tangeman, R. L. #38
Taylor, B. A. #87
Thomas, H. E., Jr. #26
Tull, J. P. #28
Turyn, R. J. #80
Wallach, N. R. #70
Warner, F. W. #32
Williams, C. M. #62
Yates, J. K. #91
Zwart, P. B. #88

NEWS ITEMS AND ANNOUNCEMENTS

ROCKY MOUNTAIN JOURNAL OF MATHEMATICS-RESEARCH PAPERS

The Editors of the Rocky Mountain Journal of Mathematics will not accept research papers until further notice because of the large backlog of papers that have been received and are being processed. It is an editorial policy of the Rocky Mountain Journal of Mathematics to attempt to divide the Journal equally between expository papers and research papers. Consequently, the Editors will continue to accept expository and survey papers. The end of the moratorium on research papers will be announced in these *Notices*.

QUARTERLY OF APPLIED MATHEMATICS, SPECIAL ISSUE

The April issue of the Quarterly of Applied Mathematics will be a special issue with the title "The future of applied mathematics." It will be comprised of contributions from several distinguished applied mathematicians who were invited to lecture at the 25th Anniversary Conference of the Division of Applied Mathematics at Brown University. This issue can be purchased separately for six dollars ($6) from the American Mathematical Society. When ordering, please specify Volume 30, Issue 1.

PRELIMINARY ANNOUNCEMENTS OF MEETINGS

Six Hundred Ninety-Fourth Meeting
University of California, Berkeley
Berkeley, California
April 22, 1972

The six hundred ninety-fourth meeting of the American Mathematical Society will be held at the University of California, Berkeley, California, on Saturday, April 22, 1972.

By invitation of the Committee to Select Hour speakers for Far Western Sectional Meetings, there will be two one-hour addresses. Professor Roger Richardson of the University of Washington will lecture at 11:00 a.m.; the title of his lecture is "On the variation of isotropy subgroups." Professor Joseph L. Taylor of the University of Utah will give the second address at 2:00 p.m.; he will speak on "Functions of several non-commuting variables." Both lectures will be given in Room 10 of Evans Hall.

There will be sessions for contributed papers in the morning and afternoon. Abstracts should be submitted to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904, so as to arrive prior to the deadline of March 9, 1972. Late papers will be accepted for presentation at the meeting, but late papers will not be listed in the printed program of the meeting.

The registration desk will be located in the ground level lobby of Evans Hall. Registration will begin at 8:30 a.m. on Saturday.

There are numerous motels and hotels in Berkeley, some of which are listed below:

BERKELEY MOTEL
2001 Bancroft Way, Zip Code 94704
Phone: (415) 845-9644

Double	$ 8.50 up
Twin	10.00 up
Family room	13.50

BERKELEY TRAVELODGE
1820 University Avenue, Zip Code 94703
Phone: (415) 843-4262

Single	$11.00 up
Double	14.00 up
Twin	17.00 up

FLAMINGO MOTEL
1761 University Avenue, Zip Code 94703
Phone: (415) 841-4242

Single	$12.00 up
Double	14.00 up
Twin	16.00 up

HOTEL DURANT
2600 Durant Avenue, Zip Code 94704
Phone: (415) 895-8981

Single	$12.00 up
Double	15.50 up
Twin	16.50 up

HOTEL SHATTUCK
Shattuck Avenue and Allston Way, Zip Code 94704
Phone: (415) 845-7300

The Hotel Durant and the Hotel Shattuck are within easy walking distance of the campus. Participants should write or telephone hotels or motels if reservations are desired.

Several airlines serve the San Francisco and Oakland airports. Taxi fare from Oakland Airport to downtown Berkeley is approximately $8.00, and limousine service is about $3.00. The Hotel Durant is a terminal for Oakland Airport Limousine Service. Taxi service from San Francisco Airport to Berkeley is approximately $14.50. One can take a limousine from the San Francisco Airport to the downtown San Francisco terminal, then a taxi to the bus terminal, and a bus to

Berkeley. Helicopter service is available from San Francisco Airport to the Berkeley Heliport. The fare, purchased locally, is $9.00. When the San Francisco-Berkeley flight is purchased as a portion of a ticket from point of departure, the fare may be reduced by an amount varying from $4.50 to $8.00, depending on the airline.

Persons driving to the meeting on the freeway from any direction should take the University Avenue turnoff. Drive east on University Avenue (toward the hills) to the campus; turn left off University onto Oxford Street. Turn right off Oxford onto Hearst Avenue and continue up Hearst to LaLoma Avenue. Participants may park in the AB lots of the parking structure H which is located on the northwest corner of Hearst and LaLoma.

Kenneth A. Ross
Associate Secretary

Eugene, Oregon

NEWS ITEMS AND ANNOUNCEMENTS

DOCTOR OF ARTS DEGREE

A study has recently been completed by Dr. Ralph D. Norman on reactions to the new degree of Doctor of Arts. The results have been compiled under the title "A Study of Some Attitudes Towards the Doctor of Arts Degree in the Southwest." The study involved 151 institutions granting associate, bachelor's, master's, and doctor's degrees, and the answers received from these institutions are compared. An attempt was made to ascertain the degree of approval of the D.A.; willingness to hire, pay, and promote D.A. degree holders on an equal basis with those holding the Ph.D.; employment in certain disciplines; appropriate year levels taught; perceived prestige; comparative adequacy of training for undergraduate teaching; and percentage of D.A.'s on the faculties. This study is currently available for $2.50, to cover production and mailing costs, from Dr. Ralph D. Norman, Associate Dean, College of Arts and Sciences, Orgega Hall 203B, University of New Mexico, Albuquerque, New Mexico 87106.

THE ANNALS OF MATHEMATICAL STATISTICS

The Council of the Institute of Mathematical Statistics has recommended to the membership that, as of January 1973, the Annals of Mathematical Statistics be superseded by two journals: the Annals of Statistics and the Annals of Probability under the editorship of Ingram Olkin and Ronald Pyke, respectively. Both journals will become, subject to the vote of the membership, official journals of the IMS and will appear in alternate months. From January 1972, papers submitted for publication in each journal should be sent to the appropriate editor. In cases of doubt, a paper may be sent to either editor. Because some papers might naturally be in either journal, some members of the Editorial Board will be common to both journals. Editorial addresses are Ingram Olkin, Editor, Annals of Statistics, Department of Statistics, Stanford University, Stanford, California 94305 (before June 15, 1972, address Professor Olkin at the Educational Testing Service, Princeton, New Jersey 08540); Ronald Pyke, Editor, Annals of Probability, Department of Mathematics, University of Washington, Seattle, Washington 98195.

THEODORE S. MOTZKIN SCHOLARSHIP FUND

A scholarship fund in memory of Theodore S. Motzkin of the University of California, Los Angeles, is being set up at Hebrew University in Jerusalem; Professor Motzkin was formerly on the staff of Hebrew University. Those interested in contributing, even a small amount, should make checks payable to T. S. Motzkin Scholarship Fund, Hebrew University, and send them to Professor George B. Dantzig, Operations Research Department, Stanford University, Stanford, California 94305.

ACADEMIC EMPLOYMENT PROSPECTS FOR SEPTEMBER 1972

by R. D. Anderson

During December 1971, a one page questionnaire prepared by the author, on behalf of the AMS Committee on Employment and Educational Policy, was sent to chairmen of all departments in the mathematical sciences in the seven states of Indiana, Louisiana, Maryland, Missouri, New Jersey, Oklahoma, and Washington to get information on academic job prospects for Ph.D.'s for 1972–1973. The seven states represent 15 percent of the nation's population. Based on estimates of those conducting the survey, it is believed that well over 90 percent of the total demand in these states was represented by those departments returning questionnaires. Returns from three of the states were essentially complete.

Departmental chairmen were asked to give facts (positive or negative) on departmental-size increments (and on increments of doctorate-holding faculty) from 1970-1971 to 1971-1972 and their expectations for such increments from 1971-1972 to 1972-1973 and from 1972-1973 to 1973-1974. Where an answer was not explicitly given as a single integral value, the most conservative consistent integral numerical value was arbitrarily used. In general, the figures for the known increments are not seriously inconsistent with earlier faculty figures available to the AMS.

The following table gives some of the results from the survey.

Statement of the AMS Committee on
Employment and Educational Policy

Over the next ten or twenty years there will be more than twice as many new mathematical Ph.D.'s seeking academic positions than there will be such positions available to them unless (1) the present rate of production of Ph.D.'s is drastically cut; or (2) business, industry and government greatly and unexpectedly increase their professional employment of mathematical Ph.D.'s; or (3) an unlikely new pattern of really large scale suitable nonprofessional employment of Ph.D.'s emerges.

We urge all graduate departments to consider seriously the implications of such expectations and to act accordingly. The future employment problems will be most acute for those students whose training equips them only for academic jobs.

The committee believes that all departments should advise their prospective graduate students as well as their own undergraduate majors of the academic employment outlook so that all prospective students may be properly apprised of their own employment prospects.

****

The members of the AMS Committee on Employment and Educational Policy are Richard D. Anderson, William L. Duren, Jr. (chairman), John W. Jewett, and Gail S. Young. For background material on the problems of employment of Ph.D. mathematicians and other scientists, readers are referred to articles published in these *Notices* (April 1971, pp. 486-490; August 1971, pp. 718-722; October 1971, pp. 865-866; November 1971, pp. 1021-1026) and in Science (9 April 1971, pp. 132-140; 27 August 1971, pp. 784-793).

TABLE I
Faculty Size Increments for Holders of Doctorates
Seven State Survey
By Classes of Schools

	1970-1971 to 1971-1972	1971-1972 to 1972-1973	1972-1973 to 1973-1974
University Mathematics Departments	21	4	13
Public College Mathematics Departments	45	46	48
Private College Mathematics Departments	14	28	22
Other Mathematical Science Departments	12	14	14
Two-year College Departments	5	10	7
Total	97	102	104

The figures for the university mathematics departments are skewed by one department which had a +7 for the previous increment and a -9 for this next one (because of specifically temporary visiting positions under an NSF Science Development grant). Generally, however, all but a few of the individual increments given were in the -2 to +2 range. The projected increase in private college hiring is encouraging and probably reflects some better economic expectations.

The comparison of expected doctorate level increments for 1972-1973 as compared with the observed increments for 1971-1972 may be the most valid basis of predicting the market. Here the numbers indicate an overall situation essentially the same as that of last year. In a somewhat analogous CBMS survey in the late fall of 1970, however, department chairmen apparently overestimated their prospective faculty growth for this year by over 2 to 1. Hopefully the chairmen are more realistic this year but since many 1972-1973 higher education budgets are not yet determined, the gross projections for Ph.D. hiring for next fall are not fully reliable. Academic hiring should be optimistically estimated at last year's level and pessimistically at a substantially lower level (which would be a real disaster). The author uses the optimistic estimates in Table III below.

Of the seven states surveyed, the comparison of next fall's projections with last fall's hiring showed two states (Indiana and New Jersey) with slight increases, two states (Louisiana and Maryland) with slight decreases, and the other three states with the same levels.

Since states like California and New York have about 10 and 9 percent of the nation's population respectively, and other states like Pennsylvania, Illinois, and Texas each has almost 6 percent of the nation's population, it is clear that the funds for (expansion of) public higher education in these five states for next year will be a major determinant as to whether the market is merely bad or is disastrous. The author has asked about twenty-five mathematicians in various academic positions in the larger states to estimate comparisons of the increments in next summer's Ph.D. hiring in public colleges and universities with last summer's. The consensus is that the situation should be about the same as last summer's. This qualitative judgment tends to justify projections from the Seven State Survey.

The Survey also asked for total faculty increments as distinct from doctorate-holding faculty increments. For the four-year colleges and universities, the total faculty increments were lower than the corresponding doctorate-holding faculty increments: 53, 64, and 80 for the three years as compared to 92, 92, and 97. In the two-year colleges the total faculty increments were relatively much higher: 28, 18, and 32 as compared to 5, 10, and 7. Clearly the growth of two-year colleges at the expense of the growth of four year colleges significantly reduces the demand for Ph.D.'s.

The Prospects for Pure Mathematicians

In the Seven State Survey, chairmen were also asked to give priorities (1,2,3,

4,5) on any possible hiring of Ph.D.'s for 1972-1973 by type of primary mathematics training desired (pure mathematics, applied mathematics, computer related mathematics, statistics, mathematics education). In the table below are given the numbers of respondents who (partially) answered the question on priorities and those who listed pure mathematics training as priority 1 or 2.

TABLE II
Priority Preference for Ph.D.'s in Pure Mathematics

	Total Number of Answers	Pure Math. Priority 1	Pure Math. Priority 2
University Mathematics Departments	20	8	4
Public College Mathematics Departments	52	19	7
Private College Mathematics Departments	55	19	10
Other Mathematical Science Departments	17	0	0
Two-year College Departments	40	7	5

With appropriate increments assigned to the various categories of the approximately 100 additional Ph.D.'s (shown in Table I) to be added to the faculty, 29 should be in pure mathematics if priority 1 is used as the basis and 42 if priority 1 and 2 are both used. (The net additions for those listing priority 1 were actually 21 out of 100 but this was skewed by the school with nine temporary positions being dropped.) Thus, although about 60 percent of the new Ph.D.'s are in pure mathematics, not more than about 40 percent of the academic demand is for pure mathematicians (even using the combined priority 1 and 2 levels). When the nonacademic demand is included, the percentage demand for pure mathematicians goes down to even less of the total demand.

It should be noted further that many or most chairmen are pure mathematicians, and thus the trend away from pure mathematicians is probably a reflection of faculty needs in colleges (e.g. for computer oriented people). From the survey, one even expects less than half of the replacements of retiring faculty to be in pure mathematics. In fact, the long-range academic demand for Ph.D.-level pure mathematicians may well be a very low figure, e.g. 200-400 per year, as contrasted to our current production of over 800 pure mathematics Ph.D.'s. For this next year, the situation for pure mathematicians probably will not be quite as critical as that suggested by the survey since many college (as distinct from university) departments will not be able to get the "nonpure" Ph.D.'s they seek. It seems quite clear, however, that within a very few years the academic demand for many types of mathematicians will be extremely low in terms of the projected supply. With the time lags in Ph.D. programs, it would appear folly at this date to switch programs drastically from pure to applied mathematics. Indeed, Ph.D. physicists and engineers, some of whom are working as applied mathematicians, have even worse current job prospects than do mathematicians.

A BALANCE SHEET FOR ACADEMIC JOBS IN MATHEMATICS

The following table represents an estimate of job status for next fall. See the notes below for explanations of the figures (all rounded to the nearest hundred).

TABLE III

	Academic Job Seekers		Academic Jobs Available	
1.	900	(new Ph.D.'s not already having jobs)	500	(from survey)
2.	200	(currently professionally unemployed)	100	(death and retirement)
3.	500	(nonretainees)	500	(jobs of nonretainees)
	1,600	Total job seekers	1,100	Total jobs
4.	-300	Nonpure mathematicians	-300	
	1,300	Pure mathematicians seeking jobs	800	Jobs for pure mathematicians

5. Prospective professionally unemployed pure mathematicians:

$$500 \pm 200.$$

1. Of 1400 new Ph.D.'s in the U.S. and Canada, about 300 will locate abroad or take jobs in government, business, or industry (last year's figures). Another 200 (or more) new Ph.D.'s already currently hold academic positions. Of the 700 (7×100) jobs apparently available (from the Survey), about 200 are already held by prospective new Ph.D.'s.
2. It is estimated that about 200 Ph.D.'s are professionally unemployed for this academic year—100 new Ph.D.'s and 100 nonretained Ph.D.'s. The usual estimate for the number of deaths and retirements is 200 per year but some were already accounted for in line 1.
3. The number of nonretainees is estimated at 500. Their positions are considered available since the figures of line 1 allowed for those not being replaced.
4. Last year something over 300 new Ph.D.'s not in pure mathematics took academic jobs. The Survey indicates adequate demand for such mathematicians this summer.
5. The error estimate of ± 200 is a guess, concurred in by other members of the AMS Committee on Employment and Educational Policy. Of the 500 prospective unemployed, perhaps 200 will be new Ph.D.'s, 100 presently unemployed, and 200 nonretainees.

The author wishes to thank Donald W. Bushaw, Deborah T. Haimo, John W. Jewett, James E. Lightner, H. O. Pollak, and Billy E. Rhoades for their help in conducting the Survey in their states. The responsibility for the estimates and conclusions are the author's.

MATHEMATICAL SCIENCES EMPLOYMENT REGISTER

The Employment Register is a project sponsored jointly by the American Mathematical Society, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics. There are two distinct components to the Register. One component is a published listing of applicants and available positions, subscriptions to which may be obtained by writing to the Mathematical Sciences Employment Register, Post Office Box 6248, Providence, Rhode Island 02904. The second component is the open Register, which is held at the annual meetings each January and sometimes at the spring meeting of the Society in New York City. The purpose of the open Register is to provide opportunities for applicants and employers to meet for interviews.

The current state of the job market, in which the applicants now greatly outnumber the positions available, has caused a number of questions to be raised about the operation of both components of the Register. At the annual meeting in Atlantic City in January 1971, there were difficulties in scheduling interviews because both the number of applicants and its ratio to the number of employers were much larger than in the past (582 and 4.5, respectively). For the meeting in Las Vegas in January 1972, the computer program which scheduled the interviews was modified to account for these difficulties, and the operation of the Register proved quite successful. At Las Vegas, there were 406 applicants, with a ratio of 9.7 applicants per employer. In view of changes taking place in hiring patterns, the open Register previously scheduled for the spring of 1972 has been postponed to the summer meeting, which will be held at Dartmouth College in August.

At the meeting in Las Vegas, a proposal was made for improving the service which the printed lists were designed to provide. The new proposal is being considered by several groups concerned with the problems faced by mathematicians seeking employment at this time. This proposal to improve the Register arose in discussions of the MAA Committee to Facilitate Employer-Employee Contacts (S. H. Douglas, Joseph Landin, R. D. Mayer, H. J. Osner, B. E. Rhoades (chairman), and R. J. Thompson) and was developed further by the joint AMS-MAA-SIAM Committee on Employment Opportunities (Richard A. Handelsman, Judah I. Rosenblatt, and R. J. Thompson (chairman)), whose charge is to oversee the actual operation of the Register. The proposal calls for the creation of a centralized registry for both applicants and employers which would permit the applicants to have access to information about as many of the available jobs as possible and at the same time allow a preliminary screening of the applicants according to criteria specified by the employers. One of the desired objectives is to make it unnecessary for applicants to expend either the effort or the money needed to send out hundreds of letters, many of which go to prospective employers who have no positions to be filled; another is to relieve some of the work of sorting through the large number of applications received by each employer. Successful operation of the proposed system requires that all (or essentially all) positions and applicants be listed. A number of the leading institutions have already expressed their interest in participating, if only to obtain some relief from the burden of correspondence they now face. The general proposal was received with interest at the meeting of the MAA Board of Governors at Las Vegas, and in the evening panel discussion presented by the AMS Committee on Employment and Educational Policy. Precise details of how the system would work, including estimates of the cost, are currently being developed.

Members of the mathematical community with opinions on the proposal, or suggestions to make regarding it, are urged to communicate them to the Mathematical Sciences Employment Register in care of the undersigned at the address given above. They may be assured that they will receive careful attention by all who are concerned with improving the services provided by the Register.

L. K. Durst

SUPPORT OF THE MATHEMATICAL SCIENCES BY THE NATIONAL SCIENCE FOUNDATION

by

William H. Pell, Head

Mathematical Sciences Section, NSF

On the occasion of the tenth anniversary of the founding of the National Science Foundation, Arthur Grad, then Program Director for the Mathematical Sciences, NSF, gave in these pages* an account of the administrative structure and activities of the Foundation. Grad was impelled to do this by the rapid growth of the Foundation and the concomitant proliferation of its administrative structure. In the decade which has ensued since Grad's paper appeared, this growth has been even more pronounced, so that the Foundation now exhibits what one might call the characteristically complex morphology of a mature Federal agency. The result is that the support of each science is diffused widely within the Foundation—one part handles research grants, another fellowships, still another summer institutes for college teachers, and so on. One might say roughly that on the large scale the NSF structure is functional rather than disciplinary. Within the Research Directorate (that portion having to do with research grants), however, subdivisions are indeed disciplinary.

The particularization of support of the mathematical sciences has caused a certain amount of confusion in the mathematical community, and the writer feels that this opening year of the third decade of our operation is an opportune time to update Grad's discussion and to discuss aspects of the support of our discipline by NSF. It should be understood that these remarks are not intended to be exhaustive, and that mathematical elements may appear in programs which are not mentioned here. Further, because of the presumed interests of most readers of these *Notices*, the description of the functioning of the research granting area (for mathematics) is more detailed than that of other parts of the Foundation.

The Mathematical Sciences Section

This Section is one of four in the Division of Mathematical and Physical Sciences, the others being Astronomy, Chemistry, and Physics. Further, this Division is one of six under the Assistant Director for Research, the others being Biological and Medical Sciences, Engineering, Environmental Sciences, Social Sciences, and Materials Research.

The Mathematical Sciences Section handles proposals for research in pure and applied mathematics, for travel to international conferences, and for research conferences and symposia. The Section is currently made up of five Program Directors and a Section Head (the writer). The Programs and Program Directors are as follows:

Classical Analysis and Geometry—
John V. Ryff
Modern Analysis and Probability—
William G. Rosen
Topology and Foundations—
Ralph M. Krause
Algebra—
Alvin I. Thaler
Applied Mathematics and Statistics—
B. R. Agins
Special Projects—
William H. Pell

Each of the Program Directors pursued graduate study in mathematics at a major university and followed this by some years of academic life. Thus, each possesses some expertise in certain areas of the mathematical sciences, as well as some understanding of the administrative operation of university departments and their attendant problems.

---

*Arthur Grad, Support of Mathematical Research and Training in the National Science Foundation, these *Notices* 7(1960), pp. 442–447.

Since proposals are accepted from the entire spectrum of mathematics and applied mathematics, each program necessarily subsumes numerous subdisciplines. Moreover, since boundaries between fields are perforce illy defined, some arbitrariness exists in the assignment of proposals to programs. This frequently results in the collaboration of Program Directors in the reviewing process for a proposal.

The Special Projects Program deals with conference and international travel grants, as well as proposals of an unorthodox nature that pose policy problems for the Foundation. Only conferences, symposia, and seminars directly related to research can be supported. Examples are the symposia on special topics occasionally held in conjunction with AMS spring sectional meetings, small conferences at universities and colleges–again on research topics–and AMS Summer Research Institutes.

Since 1969 a special feature of this program has been a yearly series of so-called Regional Conferences. The subject of each of these conferences is a topic of intense current research activity, with a series of lectures given by a leader in the field, the contents of which subsequently appear in the CBMS Regional Conference Series in Mathematics published by the AMS or in the CBMS Regional Conference Series in Applied Mathematics published by SIAM. These conferences are designed to stimulate interest and activity in the subject of the conference in the geographic region around the host school. The Regional Conferences do not compete with the ordinary research conferences discussed in the preceding paragraph.

International Travel Grants are for the purpose of facilitating travel outside the United States and its possessions, and Canada and Mexico. Travel may be to international meetings of significance, in connection with sabbaticals, or simply to pursue research abroad (where facilities or collaborative personnel commend this).

Far and away the most important task of the Section is the processing of proposals for research, and the most crucial activity in dealing with a proposal is evaluation of the research proposed. Evaluation is normally made by reviewers external to the Foundation–experts in the area of the proposed research. The number of reviewers is usually three, although for proposals involving several investigators perhaps twice this many, or even more, will be used. The decision to support or not to support a proposal depends upon the evaluation of the merit of the proposed research and the competence of the proposer, aside from the few cases of such unusual nature as to require a policy decision from without the Section. This is not always clear to proposers, who believe that this decision may be influenced by the budget. Usually reviewers disregard the budget, although occasionally remarks are directed toward it and may indeed be solicited. This is most apt to happen when computer time is involved, about which reviewers may in some instances be expected to be more knowledgeable than the NSF staff. The support level is, however, so intimately related to our funding resources that reviewers' remarks on the budget cannot be expected to play any large role in the deliberations.

Program Directors maintain a continuing dialogue with as many members of the academic community as possible, for only in this way can they remain aware of the research interests of the academic community, have a reasonable knowledge of the complexion of dozens of departments in the mathematical sciences, and, hopefully, keep abreast of the continually shifting currents of mathematical activity. By the same token, such contacts enable the academic community to become acquainted with the Program Directors and to convey to them information regarding mathematical research, its practitioners, and the institutions employing them. The Foundation has two formal devices for promoting such personal interchange: the site visit and the Advisory Panel. A site visit is merely a visit by one or more of the NSF staff to a campus at which the Foundation supports or expects to support research. The visit is to some extent, of course, focussed on those departments and persons whose research is supported, but it affords the NSF staff the opportunity to meet many other persons and to obtain background information on those departments (primarily mathematics) with which the Foundation is concerned. In recent

years the heavy workload in the NSF office has restricted severely the number of site visits which could be made. The Advisory Panel is regarded as a very important adjunct to the staff. This consists of nine persons prominent in research in the mathematical sciences who meet with the members of the Mathematical Sciences Section, its Division Director, and occasionally people from other parts of the Foundation for two days twice a year. In choosing Panel members an attempt is made to achieve broad representation over the fields of mathematics, various types of schools (private and state universities, large and small), and geographic areas. This Panel is the official conduit of the mathematical community to the Mathematical Sciences Section and Research Directorate, and conversely. The Panel cannot make Foundation policy; it is, as the name indicates, advisory. In this sense it is relied upon heavily. It is privy to all actions of the Section, and therefore in a certain sense serves the mathematical community as a monitor of operation of the Section. Elsewhere in these pages the Panel has been discussed, and its members listed, by its present chairman (W. J. LeVeque, Letters to the Editor).

The current magnitude of the Section's operations is indicated by the following statistics. In Fiscal Year 1971 1104 proposals were received and 539 grants were awarded, of which 20 were for conferences. There were 360 declinations and 36 withdrawals of proposals. (The fact that the sum of grants, declinations, and withdrawals does not equal proposals received is due to the fact that some proposals are held over from one year to the next.) The Section budget was approximately $13 million.

These grants provided support for about 1300 investigators of professorial rank, 68 postdoctoral research associates, and 330 graduate students. The figures on research associates and students are somewhat misleading, however, since they do not indicate what fraction of full support was given. Full-time support is usually measured in terms of full-time equivalents in each category: in the case of the research associates, it was about 45 and in the case of research assistants about 250 in Fiscal Year 1971.

Some research in the mathematical sciences is supported outside the Research Directorate altogether. Among its other programs, the Office of Computing Activities supports basic research in the theory of computation, numerical analysis and computational mathematics, theory of formal languages, intelligent systems, and other topics concerned with the theoretical foundations of computer science. Obviously there is some overlap between this program and those of the Mathematical Sciences Section, which is accommodated by consultation among the respective Program Directors. Furthermore, during recent years, the Foundation has developed a major program of Research Applied to National Needs (RANN). In some of the projects supported by RANN, mathematics is likely to appear in a supporting role as part of an interdisciplinary effort.

Education Programs

The Foundation supports a large number of programs having to do with the promotion and improvement of education in science, and, in particular, mathematics, at all levels of the educational process. These are administered by three divisions under the Assistant Director for Education: Graduate, Undergraduate, and Pre-college Education in Science. Each division deals with educational matters for all of the sciences at the level indicated by its name. The fact that these operate quite independently of the research-granting area of the Foundation has been, and continues to be, the source of much confusion in the educational community. The basic reason for this lies in the fact that the research and educational processes cannot really be separated. The Foundation has not been unaware of this, and so it is that although the Division of Graduate Education in Science has been primarily responsible for the support of graduate students and postdoctorals, the Mathematical Sciences Section has also supported students in the late phase of their graduate careers, as well as a limited number of postdoctoral research associates.

Programs for the support of graduate and postdoctoral personnel have been cut back or eliminated during the past

two years. At present the following programs are operative:

(1) Graduate Fellowship Program. This gives financial support to worthy students studying for the master's or Ph.D. degree. During the late 1960's about 500 fellowships were awarded each year in the mathematical sciences, but this number has declined sharply over the past two years. The number for 1972–1973 is expected to be about 250.

(2) NATO Fellowship and NSF-NATO Travel Grant Programs. NATO supports with its own funds two fellowship programs which are administered by NSF in cooperation with the Department of State. The NATO Postdoctoral Fellowships are similar in terms and conditions to the now suspended NSF Postdoctoral Program, except that the recipient must hold the fellowship in a NATO country or a country that cooperates with NATO. A few are awarded each year in the mathematical sciences. The NATO Senior Fellowships have shorter terms, normally one to three months. Each summer NATO supports a number of international conferences of one to eight weeks on various topics at an advanced level. NSF awards a limited number of travel grants to enable advanced graduate students and recent postdoctorals to participate.

Other programs of the Education Division which may be of interest to readers of the *Notices* are listed below. These relate to curriculum or course development, improvement of teaching effectiveness, updating of teaching background, providing teaching equipment, etc. Some are aimed at the college level, others at the secondary or elementary level.

(1) Advance Science Education Program
(2) College Teacher Programs
  (a) Summer Institutes
  (b) Short Courses
(3) Secondary School Teacher Programs
  (a) Academic Year Institutes
  (b) Summer Institutes and Short Courses
  (c) In-Service Institutes
(4) Programs for the Improvement of Science Education
  (a) College Science Improvement Programs. These apply not only to individual four-year schools, but to consortia of such, as well as to two-year colleges. The most recent program here is designed to aid the traditionally black, and ethnic minority institutions.
  (b) Undergraduate Science Course Improvement Program
  (c) Pre-college Curriculum and Instruction Development Program
(5) Visiting Scientists Program. This program provides for visits of productive scientists with unusual expository ability to colleges and small universities for two or three days to give lectures, hold seminars, and talk with students and faculty. In short, the program aims at motivation and enlightenment at institutions with limited educational resources and opportunities. (This program will be suspended after this year.)

International Cooperative Activities

This is a gradually developing series of programs, none very large, designed to facilitate research, foster exchange of information between U.S. and foreign scientists, and strengthen U.S. science through such interchange. These programs are administered by the Office of International Programs, under the Director of National and International Programs. Cooperative programs of somewhat varying nature have been established with the following countries: Australia, Republic of China, France, India, Italy, Japan, and Romania. The Mathematical Sciences Section has had most experience with the U.S.-Japan Program. Under this program conferences have been held in both countries, with each country paying expenses of its citizens in going to the other. Generally, three types of activity are included in a program: (1) cooperative research; (2) seminars; and (3) visits of scientists.

Science Information Services

The Office of Science Information Services, under the Assistant Director for National and International Programs, makes awards for the development and improvement of information systems; for the operational support of information systems; for the publication of journals and monographs, translations of journals and research articles, production and

publication of abstracts, indexes, and bibliographic aids; and for research in science information.

The reader should be aware that the list of programs above is not an exhaustive list of those which in some way or at some time impinge upon the enterprise of research and education in the mathematical sciences. The writer does feel, however, that the more important programs have been included. A comparison of the discussion above with that given by Grad will bring home most forcibly the many changes that have taken place in the structure of the Foundation as a result of its growth over the past decade. This expansion has perforce meant that contact between research and educational operations, desirable at all educational levels, but especially so at the undergraduate and graduate level, is somewhat less close than in the early days of the Foundation. Nevertheless, frequent contact still is maintained. Members of the Mathematical Sciences Section are often asked to do reviews of proposals from many of the programs listed above, to evaluate the credentials of scientists who apply for various types of awards, to make site visits to campuses, and to suggest reviewers for proposals and consultants for other portions of the Foundation. The Education Divisions and other parts of the Foundation also maintain contact with the academic community through advisory committees and panels much like the Advisory Panel of the Mathematical Sciences Section, except, of course, that such bodies are multidisciplinary, reflecting the constituencies of the nonresearch arms of the Foundation.

In conclusion, it is hoped that these remarks will make clearer the structure of NSF, the way in which it operates, and, in particular, the ways in which activities having to do with mathematics are handled.

NEWS ITEMS AND ANNOUNCEMENTS

1972 NSF GUIDE TO PROGRAMS

A revised guide to all the programs of the National Science Foundation, reflecting major program changes in the last year, has just been published. The new Guide to Programs replaces the edition of June 1970. The guide describes 64 NSF programs in the areas of scientific research, science education, computing activities, international scientific activities, science information, science policy, and institutional grants for science. The new volume also includes an updated NSF organization chart.

Single copies of the NSF Guide to Programs (NSF 71-22) may be obtained from the Distribution Section, National Science Foundation, Washington, D. C. 20550. Additional copies are available for 75 cents from the Superintendent of Documents, U. S. Government Printing Office, Washington, D. C. 20402.

E. T. KOBAYASHI MEMORIAL FUND

In the name of her two sons, Mrs. Kobayashi wants to thank every contributor to the E. T. Kobayashi Memorial Fund for his kind generosity.

STATISTICAL LABORATORY CATHOLIC UNIVERSITY OF AMERICA

The Statistical Laboratory of the Catholic University of America will transfer its activity to Bowling Green State University, Bowling Green, Ohio, on September 1, 1972. Professors R. G. Laha and Eugene Lukacs have accepted appointments as full professors and Professor V. K. Rohatgi has accepted an appointment as an associate professor in the Department of Mathematics of Bowling Green State University.

NATIONAL SCIENCE FOUNDATION BUDGET FOR 1973

On January 24, 1972, the National Science Foundation released details of its Fiscal Year (FY) 1973 budget request. For purposes of comparison, the release also gave estimates of FY 1972 expenditures as well as figures for the actual expenditures in FY 1971. The tables that appear below were constructed from two of those included in the press release. The first table presents a general breakdown of the whole NSF budget; in the second, the figures for Scientific Research Support are broken down by scientific discipline. (In an accompanying note, it is explained that the $275.3 million budgeted for research includes $12 million for research equipment which was formerly budgeted separately.) For the second year in a row (cf. these *Notices* 18(1971), p. 352) mathematics is not once mentioned in the "Program summary" which, along with the tables, comprised the text of the release.

National Science Foundation Budget Allocations
Fiscal Years 1971–1973 (In Millions of Dollars)

	FY 1971	FY 1972		FY 1973	
	Actual	Estimate	% Increase	Estimate	% Increase
Scientific Research Project Support	$180.4	$246.6	37	$275.3	12
National and Special Research Programs	49.8	85.6	72	109.1	27
National Research Centers	36.9	40.2	8.9	42.3	5.2
National Sea Grant Program	6.1	-0-	-100	-0-	
Computing Activities in Education and Research	15.0	21.0	40	20.5	-2.4
Science Information Activities	10.7	9.8	-8.4	9.5	-3.1
International Cooperative Scientific Activities	2.2	4.0	82	4.7	18
Research Applied to National Needs	34.0	56.0	65	80.0	43
Intergovernmental Science Program	.8	1.0	25	1.0	00
Institutional Improvement for Science	34.4	21.0	-39	12.0	-43
Science Graduate Student Support	30.5	20.0	-34	14.0	-30
Science Education Improvement	68.3	66.1	-3.2	70.0	5.9
Planning and Policy Studies	3.2	2.7	-16	2.5	-7.4
Program Development and Management	21.8	24.1	11	26.8	11
Foreign Currency Program	2.0	3.0	50	7.0	133
Total Available for NSF Programs	496.1	601.1	21	674.7*	12
By Appropriations:					
Salaries and Expenses	511.0	619.0	21	646.0	4.4
Foreign Currency	2.0	3.0	50	7.0	133
TOTAL	$513.0	$622.0	21	$653.0	5.0

*Includes $21.7 carryover from FY 1972

National Science Foundation
Scientific Research Project Support
(Millions of Dollars)

	FY 1971 Actual	FY 1972 Estimate	FY 1972 % Increase	FY 1973 Estimate	FY 1973 % Increase
Atmospheric Sciences	$ 9.4	$ 11.6	23	$ 13.0	12
Earth Sciences	8.1	9.2	14	11.1	21
Oceanography	10.0	12.6	26	14.0	11
Biological Sciences	44.3	53.4	21	59.9	12
Physics	26.5	32.9	24	36.0	9.4
Chemistry	19.6	24.1	23	26.8	11
Astronomy	6.7	8.0	19	8.8	10
Mathematics	12.9	13.6	5.4	14.4	5.9
Social Sciences	17.7	22.5	27	24.6	9.3
Engineering	14.1	25.4	80	29.3	15
Materials Research	11.1	33.3	200	37.4	12
TOTAL	$180.4	$246.6	37	$275.3	12

NEWS ITEMS AND ANNOUNCEMENTS

BANGLADESH REFUGEE MATHEMATICIANS

At the suggestion of the AMS Committee in Aid of Pakistani Refugee Mathematicians, a fund is being established to aid in the resettlement of Bangladesh mathematicians. Contributions may be mailed to the Providence office of the Society. Checks should be made out to the American Mathematical Society and marked "for resettlement of Bangladesh mathematicians." Contributions will be exempt from U. S. income tax.

There are between fifteen and twenty mathematicians among the refugees from East Pakistan. Support for each one at the rate of $125 per month is needed for the six-month period until the beginning of the next academic year. It is hoped that a total of close to $15,000 can be collected between now and April 15, 1972. Arrangements are now being made with the International Rescue Committee for the distribution of the funds received. If it turns out to be impossible to distribute the funds as indicated, contributions will be returned to the donors.

A MATHEMATICAL VISIT TO NORTH VIET NAM
by Chandler Davis

I spent most of August 1971 in Hanoi, giving a series of lectures as a guest of the mathematicians there. "Both North Vietnamese mathematicians," commented one Canadian colleague. Let me assure him there were more than that.

Mathematics was scanty here in 1954, to be sure. There was only one practising mathematician with a doctorate: Lè van Thièm, who got his degree at ETH, Zürich, and who is now president of the Association des Mathématiciens Vietnamiens. To speak of "both North Vietnamese mathematicians" in 1954, you'd have had to include Ta Quang Bu'ú, a versatile and lively mathematician-engineer educated in Paris in the '30's, now Minister of Higher Education.

But in 1971 mathematicians abound. When Premier Pham van Dong opened the first nationwide Mathematics Conference in Hanoi this April, there were over 500 mathematicians there. Mostly teachers at the University, the Polytechnic, or the Pedagogical Institutes, but some from other jobs, mostly in government. Most of them are at our graduate student's level, but many hold doctorates or equivalent.

During my visit, I gave 26 hours of lectures at the research level. Only specialists had patience for that much, so by the end I was really just reporting to the functional analysis seminar, which modestly describes itself as the least established of Hanoi's 14 specialized seminars. In the formal meetings and many informal ones, I was asked dozens of well-informed, pertinent questions (I wish I had been up to answering more of them), and of course heard about my hosts' research, which was of considerable interest.

Obviously there must have been a take-off during the peaceful interlude 1954–64. Well–that's not the way the Vietnamese would put it. They say the real take-off came during the Resistance, around 1950, at a time when they held no large cities and their only post-secondary teaching was in two Pedagogical Institutes. Namely, they created a Vietnamese mathematical vocabulary and began writing their own textbooks. Now, the student could begin serious work in mathematics without the delay of perfecting a foreign language. This is described as the key step.

Nevertheless, as soon as the Geneva Agreement was signed in 1954, North Viet Nam began sending a stream of their best students for long periods of study in the USSR.

The U. S. 1965–68 bombing (and some since) didn't slow down development. Educational institutions scattered, mostly to student- and faculty-made temporary buildings in the forests; but enrollments kept rising at all levels. Since 1968 there has been recongregation in Hanoi. The modern Polytechnic building, completed in 1965, is in full use for the first time. (But when more new universities can be built, they will probably not be in Hanoi, for the long-term plan is to decentralize.)

A Minsk-22 computer is installed and its time is well occupied with scientific and teaching uses. Its young staff is contemplating new applications and additional machines.

Vietnamese mathematics has taken off. Its present state is looking around to choose which way to fly.

The students who return from the USSR quickly find themselves fairly senior members of this rapidly expanding mathematical community: they are prominent among the 30 research fellows at the Institute, among the mainstays of the advanced seminars, among the deans and the advanced teachers at the University and other institutions. They also find that the specialties to which they have been hewn

Reprinted from Notes of the Canadian Mathematical Congress, vol. 4, no. 3, December 1971, with the permission of the Canadian Mathematical Congress.

by Soviet masters are too widely separated one from another, and from any Vietnamese application. ("Quantum field theory does not count as an application for us.") They feel lonely. They also feel ill-at-ease in a country still at war, in a society where scholarship, like other labour, is expected to be socially useful.

The question of fitting research mathematics usefully into society, difficult enough in a rich country, is acute in Viet Nam. Though mathematicians have made improvements and adaptations of standard methods of linear programming and bridge computations for local needs, and though they are all happy to teach such subjects to engineers, they are not all able to build research careers in them. On the other hand, they do want very much to do some kind of serious research. The strong feeling on this point is partly national pride: they remember the pleasant surprise with which they discovered, in the decade 1955–65, that it was possible to do mathematical research in Viet Nam. Many young people, like the logician Pham dình Diệu and the analyst Nguyễn dình Trí, are struggling toward a new understanding of the role of mathematics. The functional analysis seminar is trying to find a single direction of concentration which will enlist its considerable but disparate resources. But nobody is letting research wait until global policy problems are solved.

Tentative solutions were agreed upon at the Mathematics Conference this year. I missed the Conference but can report its conclusions. Five general priorities for North Vietnamese mathematics. In order, they are–

1. Computers, cybernetics, finite mathematics, numerical analysis.
2. Mathematical economics, operations research.
3. Probability and statistics, prediction theory.
4. Analysis (including functional analysis).
5. New directions.

Clearly all five points are so broad that much selection is called for within them. The rationale for point #5 is that some new field may in the future acquire importance like those of ##1-4, and they want advance familiarity so they'll be prepared to catch up. Algebraic topology was mentioned to me as a field which had got some priority as a "new direction"; I suggested René Thom's ideas on structural stability, they agreed that that might be added.

How about fields which don't qualify at all under the five points but in which there is research interest? Number theory and classical geometry are two such which we discussed. They are still supported, through a back door. The main Pedagogical Institute trains teachers of geometry and elementary algebra; it has taken the policy (its Rector being a research geometer, Nguyễn canh Toàn) that research in number theory and geometry enriches teaching. Indeed all the Institute's teachers are expected to be active in research; this doesn't have to be theorem-proving but can be research in education.

Many aspects of the above picture are familiar. The most striking thing about Vietnamese mathematical activity is how much there is of it. Remember that these same people are writing and updating a full set of undergraduate texts, since no course is based on a text in a foreign language.

I note also that research activity is not spurred by hope of appointment to a "better" institution (there are so few) or of promotion (there are no ranks, though there are administrative positions carrying considerable power). Salaries are pretty uniform too, and not much above the rest of the population: the equivalent of our full professors make about 80 dong per month, while salaries of unskilled workers don't go much below 50 dong per month. Even a 3 : 1 ratio of salaries, if it existed, wouldn't be much in effect, since essentials are rationed and are either free of very cheap. Practically, then, no salary differentials in the society.

Will the Vietnamese preserve this after the end of wartime austerity? Related questions: Will the atmosphere of cheerful diligence be maintained? Will the generosity which leads the better students to coach the weaker be supplanted by competitiveness as positions become fewer relative to the student population? I can only report that they are proud of their egalitarianism and seem to regard it as a permanent principle.

The North Vietnamese work hard at keeping in touch with the rest of the world. As to the Soviet bloc, they achieve this by sending graduate students for "kandidatur", and by much importing of books. They hope to re-establish similar relations with China, but aren't sure this can be done quickly.

As to the capitalist countries, they wish to improve the contact. Visits by our mathematicians are quite expensive for both sides and will remain infrequent. Brief visits here by their mathematicians are a possibility, but, again, would be few. I got non-committal response to my enquiry whether they might send graduate students to Canada or France.

The main contact, of course, is books and journals. Their Central Library of Science and Technology, serving about 10,000 readers in the Hanoi area, tries to acquire major journals and important advanced books in mathematics and other sciences. 120 mathematics journals are on subscription. The Vietnamese research journal in mathematics is about to resume publication, and will be offered in exchange. Still the Library will not get everything the mathematicians need, for lack of funds. U. S. journals arrive irregularly. Also, the Library having been set up only in 1960, back files are short. One more problem is transportation: It is not easy for a reader in (say) Vinh to drop into Hanoi to consult a book. Even the Pedagogical Institute 15 km. away would find it very useful to build up a little math library of its own. I brought back lists of specific and not-altogether-specific book needs.

I also brought back dozens of questions and messages for other mathematicians here. The mails to and from North Viet Nam have been slow and uncertain, no one seems to know why, but one high official expressed confidence they'll improve. I hope the mathematicians there will hear from us. It seems to be the most effective way for us to offer the encouragement and help they deserve.

LETTERS TO THE EDITOR

Editor, the *Notices*

Do permit this one long-time dues-paying member to congratulate the management of the AMS and MAA on the selection of Las Vegas as a site for the most recent annual meetings.

For once, the rooms were of adequate size, the lighting and projection facilities worked properly, and the public address systems provided comprehensible sound. There were spacious places for discussions, the hotel rates were not excessive, and the food costs relatively moderate.

Congratulations!

The other aspects of Las Vegas–the garishness of the gambling and the entertainment–doubtless also serve a valuable function by either (a) appealing to those amongst us who have a taste for those things, or (b) for those of a different sensibility, preserving that sense of psychological (or even physical) isolation which is often considered conducive to scholarly and academic thought.

Finally, the taxi fares seemed the most reasonable in the United States.

A. N. Feldzamen

SITUATION WANTED ADVERTISEMENTS

Mathematicians who are unemployed, or who are under notice of involuntary unemployment, are to be allowed free nondisplay situation wanted advertising in the *Notices*. Nonmembers of the Society must be introduced by a member, and the applicant should be, in the judgment of the editors, a mathematician. (For example, free advertisements will not be accepted from professionals in other disciplines such as physics or chemistry.) This service will not normally be available to graduate students seeking their first postdoctoral positions; however, veterans recently released from service will qualify.

These situation wanted advertisements are for regular employment only, not for part-time, free lance, or consulting work. An exception will be made in the case of retired mathematicians who may advertise for part-time positions or consulting work. A nonemployed mathematician may have two free advertisements during the year; retired mathematicians may have one per year.

The applicant will be requested to provide the following information if it is not included in the body of the advertising text: (1) name of the university or institution where he was last employed; (2) date of termination of service; (3) highest degree; (4) field. Application forms may be obtained from the Editorial Department, American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904. Advertising copy offered by nonmembers of the American Mathematical Society must carry the signature of a member. These advertisements should not exceed fifty words (not more than six 65-space typed lines), including the address of the advertiser; excess words will be charged at the rate of $0.15 per word, minimum charge of $1. Anonymous listings will be carried for an additional fee of $5; correspondence for such applicants will be forwarded to them. The deadline for receipt by the Editorial Department of the Society will be the same as that for abstracts; this date is found on the inside front cover of each issue of the *Notices*.

Situation wanted advertisements from employed members of the Society for full- or part-time positions or consulting work will be accepted at the rate of $0.15 per word for each insertion, with a minimum charge of $1. Nonmembers of the Society will be charged $0.50 per word, with a minimum charge of $15.

In printing these advertisements, the American Mathematical Society assumes no obligation as to the qualification of prospective employees or the responsibility of employers. The AMS expects that each user of this section will consider himself morally obligated to acknowledge all replies to his advertisement.

This new service supplements, but does not supplant, the Employment Register. All unemployed mathematicians are urged to avail themselves of the services of the Register. This advice is particularly applicable to graduate students who are not eligible for the new service.

Sample Advertisements*

Regular

MATHEMATICS PROFESSOR, TEACHING AND RESEARCH. Ph.D. 1965. Age 34. Specialty: analysis. Five published articles. Five years experience in teaching and academic research, including one in foreign university. Fluent in German. Midwest U. S. preferred. References and resume available upon request. Available March 1972. Isaac Newton, 211 Front Street, Joplin, Missouri 64801.

Anonymous

RESEARCH MATHEMATICIAN. Ph.D. 1967. Age 35. Specialty: statistics and probability. Three published articles. Seven years total experience in teaching and academic and industrial research in U. S. and abroad. Military obligation completed. References upon request. Western U. S. preferred; will relocate if necessary. Available immediately. SW96

*See current listings in the Advertisement Section of these *Notices*.

Return to Editorial Department, American Mathematical Society
P. O. Box 6248, Providence, Rhode Island 02904

APPLICATION FORM FOR SITUATION WANTED ADVERTISEMENT
Notices OF THE AMERICAN MATHEMATICAL SOCIETY

Name_____ Highest degree earned_____

Address (street, city, state, zip code) Field_____

_____ Unemployed ☐

_____ Employed ☐

_____ Retired ☐

Name of the university or institution where last employed_____

Date of termination of service_____

Member of the American Mathematical Society Yes ☐ No ☐

Signature of AMS member for nonmember applicants_____

Text of Advertisement

Free listings, when applicable, are limited to fifty words (equivalent of six 65-space typed lines). Extra words are $0.15 each. PLEASE TYPE AND DOUBLE SPACE.

All fees must be prepaid with order

Number of words_____

Free advertisement (fifty words), extra words $0.15 each, $1 minimum $_____

Employed mathematician (member), $0.15 per word, $1 minimum $_____

Employed mathematician (nonmember), $0.50 per word, $15 minimum $_____

If anonymous listing is desired, please check here ☐ ; fee $5 $_____

TOTAL $_____

_____
Signature

SPECIAL MEETINGS INFORMATION CENTER

The purpose of this center is to maintain a file on prospective symposia, colloquia, institutes, seminars, special years, meetings of other associations, and to notify the organizers if conflicts in subject matter, dates or geographical area become apparent. A first announcement will be published in the *Notices* if it contains a call for papers, place, date, and subject, where applicable; a second announcement must contain reasonably complete details of the meeting in order for it to be published. Information on the pre-preliminary planning will be stored in the files, and will be available to anyone desiring information on prospective conferences. All communications on special meetings should be sent to the Special Meetings Information Center of the American Mathematical Society.

March 7-11, 1972
SYMPOSIUM ON VARIOUS ASPECTS OF DELAY AND FUNCTIONAL DIFFERENTIAL EQUATIONS
Park City, Utah
Program: Approximately twelve invited lectures; limited number of contributed papers
Sponsor: Department of Mathematics, University of Utah
Information: Professor W. J. Coles, Department of Mathematics, University of Utah, Salt Lake City, Utah 84112

March 15-18, 1972
CONFERENCE ON COMPLEX ANALYSIS (SINGULARITIES OF ANALYTIC SPACES)
Rice University, Houston, Texas
Program: Twelve lecturers, including Ph. A. Griffiths, Reese Harvey, Henry Laufer, David Lieberman, David Mumford, R. Narasimhan, O. Riemanschneider, H. Rossi, Wilfried Schmid
Participation: All interested mathematicians invited to attend
Information: Professor R. O. Wells, Jr., Department of Mathematics, Rice University, Houston, Texas 77001

March 24-25, 1972
SYMPOSIUM ON THE RESPONSIBILITIES OF COMPUTER SCIENCE EDUCATION
Washington University, St. Louis, Missouri
Program: This second technical symposium will include sessions and panels on the following topics: Undergraduate Computer Science Curricula, Computer Science in Small Colleges and Junior Colleges, Employers' Needs in Computer Science, Computer Science and Social Relevance. A special feature will be the presentation of current dissertation research in computer science.
Sponsor: ACM-Special Interest Group on Computer Science Education
Information: Dr. David Matula, Campus Box 1045, Washington University, St. Louis, Missouri 63130

May 22 - July 7, 1972
SUMMER SEMINAR IN DIMENSION THEORY
Franklin and Marshall College, Lancaster, Pennsylvania 17604
Program: An introduction to "classical" dimension theory and recent results. Presented from the standpoint of coverings and mappings
Participation: For college teachers of mathematics who have taken a course in point-set topology
Information: Professor George M. Rosenstein, Jr., Franklin and Marshall College, Lancaster, Pennsylvania 17604
Deadline: May 15, 1972 (application)

May 25 - August 13, 1972
SEMINAR IN ALGEBRA (in conjunction with Summer Research Institute of Canadian Mathematical Congress)
Carleton University, Ottawa, Ontario
Program: Selected topics in current research in group theory and ring theory
Information: V. Dlab, Department of Mathematics, Carleton University,

Colonel By Drive, Ottawa K1S 5B6, Canada
Deadline: May 1, 1972

June 5-23, 1972
SHORT COURSE IN CATEGORY THEORY
Colgate University, Hamilton, New York 13346
Program: Lectures by Peter J. Hilton; participants will study impact of category theory on various branches of mathematics as well as its impact on the teaching of undergraduate mathematics
Support: National Science Foundation
Participants: Thirty college teachers from non-Ph.D. granting institutions. Commitment by participant to conduct a seminar for his colleagues based on lecture notes is a condition for acceptance into the program.
Information and applications: Professor Malcolm W. Pownall, Department of Mathematics, Colgate University, Hamilton, New York 13346 (The deadline for applications is March 1, 1972, but it may be possible to accept some late applications received prior to March 15.)

June 6-8, 1972
CONFERENCE ON DIOPHANTINE APPROXIMATION AND ITS APPLICATIONS
Washington, D. C.
Program: Invited address and contributed papers
Speakers: (preliminary list) A. Baker, J. Coates, W. Schmidt, E. G. Straus, H. Stark, P. Turán, and S. Zaremba
Sponsor: Mathematical Research Center of the United States Naval Research Laboratory
Contributed papers: Limited number. Submit name, address, title, presentation time, and abstract to the chairman of the Organizing Committee by April 15, 1972.
Information: Chairman of the Organizing Committee, Charles F. Osgood, Code 7840, Naval Research Laboratory, Washington, D. C. 20390

ıne 8-10, 1972
CANADIAN MATHEMATICAL CONGRESS
Memorial University of Newfoundland, St. John's, Newfoundland
Program: Main topic of meetings will be applied mathematics. Fifth annual Jeffery-Williams Lecture by Philip J. Davis, Brown University, on "Applying diverse areas of mathematics to the approximate computation of integrals."
Participation: Open
Information: Professor W. J. Blundon, Department of Mathematics, Memorial University of Newfoundland, St. John's, Newfoundland, Canada

June 10-12, 1972
CONFERENCE ON COMBINATORIAL MATHEMATICS
University of Newcastle, New South Wales, Australia
Program: In addition to sessions of research papers, participants are asked whether they would like to see any of the following: short announcements of research results, group discussion seminars, instructional lectures.
Contributed papers: Papers are invited on any part of combinatorial mathematics-pure or applied.
Information and applications: Conference on Combinatorial Mathematics, Faculty of Mathematics, University of Newcastle, New South Wales, Australia 2308

June 12 - August 4, 1972
INSTITUTE IN FLUID MECHANICS OF THE ENVIRONMENT
Colorado State University, Fort Collins, Colorado
Program: Subject matter will be focused on the description, analysis and measurement of atmospheric and oceanic motions affecting air and water pollution, local weather modification, soil and water conservation, wind forces on structures, and urban planning. Courses include geophysical fluid mechanics, turbulent diffusion, data processing, numerical analysis,

and instrumentation and measurements.

Support: National Science Foundation

Participants: Twenty-five university faculty in engineering and science

Information: Dr. J. E. Cermak, Department of Civil Engineering, Fluid Mechanics Program, Colorado State University, Fort Collins, Colorado 80521

June 19-21, 1972
SYMPOSIUM ON POPULATION DYNAMICS
Mathematics Research Center, University of Wisconsin, Madison, Wisconsin

Program: The symposium will embrace all applications of mathematics to population studies. Fifteen invited addresses: Franciso Bayo, Social Security Administration; Chin Long Chiang, University of California, Berkeley; Stuart E. Dreyfus, University of California, Berkeley; Ralph B. Ginsberg, University of Pennsylvania; Jan M. Hoem, Central Bureau of Statistics, Oslo, Norway; Nathan Keyfitz, University of California, Berkeley; David D. McFarland, University of Chicago; Robert McGinnis, Cornell University; Jane Menken, Princeton University; Beresford Parlett, University of California, Berkeley; Robert G. Potter, Brown University; Samuel H. Preston, University of California, Berkeley; Mindel C. Sheps, University of North Carolina

Sponsor: Mathematics Research Center

Organizing Committee: T. N. E. Greville (chairman), N. Keyfitz, L. B. Ball, K. E. Taeuber, H. Winsborough

Participation: Open

Information: Professor T. N. E. Greville, Mathematics Research Center, University of Wisconsin, Madison, Wisconsin 53706

June 19-23, 1972
CONFERENCE ON ERGODIC THEORY IN STATISTICAL PHYSICS AND PROBABILITY THEORY
Texas Christian University, Fort Worth, Texas

Program: Ten lectures by George W. Mackey, Harvard University, entitled "The notion of ergodicity and its significance for probability theory and statistical physics"; informal lectures by participants on current research; discussion groups

Support: National Science Foundation (pending)

Participants: Applications for support should include a brief vita, indication of research interests, whether or not applicant wishes to present a paper; terminal year graduate students should have major professor send a letter of recommendation.

Information and applications: Professor Robert S. Doran, Conference Director, Department of Mathematics, Texas Christian University, Fort Worth, Texas 76129

June 19-24, 1972
THIRD INTERNATIONAL SYMPOSIUM ON MULTIVARIATE ANALYSIS
Wright State University, Dayton, Ohio

Program: Topics to be discussed are characterization problems, classification procedures, distribution theory, econometrics, multivariate analysis of variance, nonparametric methods, reliability, sequential methods, time series and stochastic processes, and applications.

Speakers: R. Barlow and F. Proschan, H. Chernoff, A. K. Chattopadhyay and K. C. S. Pillai, A. P. Dempster, R. Farrell, D. A. S. Fraser, R. Gnanadesikan, A. T. James, G. Kallianpur, J. Kiefer, J. B. Kruskal, E. Lukacs, E. Parzen, C. R. Rao, M. M. Rao, M. Rosenblatt, Yu. A. Rozanov, P. K. Sen and M. Ghosh, J. N. Srivastava, R. A. Wijsman, and H. Wold and E. Lyttkens

Contributed papers: Anyone interested may send abstract (not to exceed 200 words) to the symposium chairman, P. R. Krishnaiah, ARL(LB) Bldg. 450, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio 45433

June 19 - July 29, 1972
ADVANCED INSTITUTE ON STATISTICAL ECOLOGY
Pennsylvania State University, University Park, Pennsylvania

Program: The institute will cover multi-

variate methods, sampling, systems analysis, population dynamics, statistical distributions, statistical measurement.

Participation: Open to those at the doctorate or postdoctoral level who have completed one year each of calculus, biology, statistics, and linear algebra and programming.

Information: Professor G. P. Patil, 330 McAllister Building, University Park, Pennsylvania 16802

Deadline: April 1, 1972 (application for visitors)

June 19 - August 11, 1972
SUMMER INSTITUTE FOR TEACHERS OF MATHEMATICS IN DEVELOPING COLLEGES
University of Montana, Missoula, Montana

Program: Elementary calculus from an advanced standpoint and abstract mathematical systems will be covered.

Participation: For teachers of college mathematics without a Ph.D.

Support: Office of Education (pending)

Information: Professor H. E. Reinhardt, Chairman, Department of Mathematics, University of Montana, Missoula, Montana 59801

Deadline: April 10, 1972 (application)

June 21-24, 1972
INTERNATIONAL CONFERENCE ON PADÉ APPROXIMANTS, CONTINUED FRACTIONS AND RELATED TOPICS
University of Colorado, Boulder, Colorado

Program: Recent mathematical developments of the subject and on applications to approximation of functions, theoretical physics, and various areas of numerical analysis; free exchange of ideas from workers with diverse viewpoints. Invited expository papers as well as contributed papers.

Proceedings: To be published as a special issue of the Rocky Mountain Journal of Mathematics

Support: Air Force Office of Scientific Research (pending)

Participant support: Some funds for travel and subsistence if grant approved.

Contributed papers: Abstract deadline April 15, 1972, to Professor Jones

Information: Professor William B. Jones, Department of Mathematics, University of Colorado, Boulder, Colorado 80302

June 26 - August 4, 1972
SUMMER INSTITUTE ON DETERMINISTIC METHODS IN OPERATIONS RESEARCH
Cornell University, Ithaca, New York

Program: Two main series of lectures on the theory, computation, and applications of mathematical programming and of game theory; general survey talks on other analytical techniques used in operations research

Support: National Science Foundation

Participants: Thirty college teachers of mathematics, operations research, industrial engineering, management science, mathematical economics, or related areas

Information and applications: Professor William F. Lucas, Institute Director, Upson Hall, Cornell University, Ithaca, New York 14850

July 12-22, 1972
ST. ANDREWS MATHEMATICAL COLLOQUIUM
University of St. Andrews, St. Andrews, Scotland

Program: Courses of approximately seven lectures each by P. R. Halmos, "The connection between linear algebra and operator theory"; F. Harary, "New directions in graph theory"; S. Vajda, "Mathematical programming-constrained optimization". Seminar on algebra directed by P. M. Cohn; seminar on global analysis directed by J. Eells

Sponsor: Edinburgh Mathematical Society

Fees: Registration, $2.60 (one pound) before May 1, 1972; double thereafter. Membership fee, $13 (five pounds). Inclusive fee (payable on arrival) for membership, board, lodging in a university residence, $72.80 (twenty-eight pounds)

Information and application forms (enclose self-addressed envelope): Dr. T. S. Blyth, Mathematical Institute, University of St. Andrews, North Haugh, St. Andrews, Fife, Scotland

July 17–26, 1972
ADVANCED STUDY INSTITUTE ON DECOMPOSITION AS A TOOL FOR SOLVING LARGE-SCALE PROBLEMS
Cambridge, England
Support: NATO
Tuition: Approximately $150
Information: Professor D. M. Himmelblau, Department of Chemical Engineering, The University of Texas at Austin, Austin, Texas 78712

August 21–26, 1972
13th INTERNATIONAL CONGRESS OF THEORETICAL AND APPLIED MECHANICS
Moscow State University, Moscow, U. S. S. R.
Program: Will encompass the entire field of the science of analytical, solid and fluid mechanics, including applications. Computational methods as such will not be included. There will be a number of general and sectional lectures given by speakers on the invitation of the International Program Committee, and about 230 contributed papers.
Accommodations: A limited number in the hostels of the university, rest in the hotels in the city
Contributed papers: Limited to approximately 50 papers from residents in the United States, the initial selection to be made by the U. S. National Committee on Theoretical and Applied Mechanics of the National Academy of Sciences–National Research Council and the final selection to be made by the International Program Committee. Each contributor should send five copies of a 500 word summary to Professor G. F. Carrier, Pierce Hall, Harvard University, Cambridge, Massachusetts 02138, before March 15, 1972. An abstract of not more than 100 words to be included in the program booklet should also be sent.
Support: A limited number of grants for support of travel are available from the National Academy of Sciences for those who are unable to obtain support from other sources. Applications for travel grants must be submitted by April 15, 1972. Applications available from Ad Hoc Committee on Travel Grants, Room JH 432, National Academy of Sciences, 2101 Constitution Avenue, Washington, D. C. 20418.
Information: Secretary of the Organizing Committee, Professor G. K. Mikhailov, Leningrad Avenue 7, Moscow A-40, U. S. S. R.

August 28 - September 3, 1972
CONFERENCE ON GROUP ALGEBRAS OF LOCALLY COMPACT GROUPS AND HARMONIC ANALYSIS
Jabłonna (near Warsaw), Poland
Sponsor: Mathematical Institute of the Polish Academy of Sciences
Information: Professor Stanisław Hartman, Mathematical Institute of the Polish Academy of Sciences, Wrocław 9, ul. Kopernika 18, Poland

August 29 - September 4, 1972
VI INTERNATIONAL CONFERENCE ON NONLINEAR OSCILLATIONS
Poznań, Poland
Program: Papers will be presented on the following topics: analytical methods of the theory of nonlinear oscillations; qualitative methods of the theory of nonlinear oscillations and theory of stability; application of the theory of oscillations to mechanics; application of the theory of oscillations to electrical engineering and electronics.
Sponsor: Institute of Fundamental Technical Research of the Polish Academy of Sciences
Information: Polish Academy of Sciences, Institute of Fundamental Technical Research, Organizing Committee of the VI International Conference on Nonlinear Oscillations, Warsaw, Świętorkrzyska 21, room 334, Poland

September 4–15, 1972
NATO ADVANCED STUDY INSTITUTE ON TOPOLOGICAL VECTOR SPACES
Université Libre de Bruxelles, Belgium
Program: Lectures by H. Buchwalter (Lyon), H. Hogbe-Nlend (Bordeaux), J. Horváth (University of Maryland), J. Lindenstrauss (Jerusalem), L. Waelbroeck (Brussels), J. Wloka (Kiel).
Participation: Participating students

will be expected to contribute a sum of $40 towards expenses. Some financial support should be available to most students who cannot obtain support elsewhere.

Information and applications: Professor L. Waelbroeck, Université Libre de Bruxelles, Département de Mathématique, 166 Chaussee de la Hulpe, B-1170 Brussels, Belgium. Deadline for applications May 15, 1972. Application should state whether a grant is necessary; if necessary, state whether funds are needed for all or part of travel and subsistence.

October 16-18, 1972
SIAM 1972 FALL MEETING
Sheraton-Crest Inn, Austin, Texas

Program: 25th Anniversary of Modern Numerical Mathematics, Numerical Analysis and Computation.

Contributed papers: Members and those sponsored by members. Submit abstracts to SIAM office in duplicate by June 30, 1972. Abstracts will be printed and available at meeting or from SIAM office.

Information and abstract forms: H. B. Hair, Society for Industrial and Applied Mathematics (SIAM), 33 South 17th Street, Philadelphia, Pennsylvania 19103

December 11-15, 1972
INTERNATIONAL SYMPOSIUM ON UNCERTAINTIES IN HYDROLOGY AND WATER RESOURCE SYSTEMS
University of Arizona, Tucson, Arizona

Purpose: To bring together ideas and problems from hydrology and other disciplines, particularly those which involve stochastic processes and applied mathematics in decision theory

Program: Invited speakers, sessions for contributed papers

Contributed papers: Submit abstract of two pages by June 30, 1972.

Information: Professor J. L. Denny, Department of Mathematics, or Professor Chester C. Kisiel, Department of Hydrology and Water Resources, both at the University of Arizona, Tucson, Arizona 85721

ASSISTANTSHIPS AND FELLOWSHIPS IN MATHEMATICS IN 1972-1973

The Assistantships and Fellowships listed below are in addition to those listed on pages 1123-1227 of the December 1971 issue of these *Notices*.

TYPE	STIPEND		TUITION	SERVICE REQUIRED	
of financial assistance (with number anticipated in 1972-1973)	amount in dollars	9 or 12 months	if not included in stipend (dollars)	hours per week	type of service

ILLINOIS

University of Illinois at Urbana-Champaign

URBANA, ILLINOIS 61801
J. N. Snyder, Head
Department of Computer Science
Applications must be filed by February 15, 1972

NUMBER OF DEGREES AWARDED IN 1971
Baccalaureate degrees by institution 5203
Master's degrees by department 21

Ph. D. degrees awarded during last three years by field of specialization: computer science and numerical analysis (16). TOTAL 16.

Fellowship (5)	2500	12			
Teaching Assistantship (5)	2900	9		20	Teaching
Research Assistantship (10)	2900	9		20	Research
Tuition Waiver (2)		12			

NORTH CAROLINA

University of North Carolina

CHAPEL HILL, NORTH CAROLINA 27514
Frederick P. Brooks, Jr., Chairman
Department of Computer Science
Applications must be filed by February 1, 1972

NUMBER OF DEGREES AWARDED IN 1971
Baccalaureate degrees by institution 2719
Master's degrees by department 6

Ph. D. degrees awarded during last three years by field of specialization: computer science and numerical analysis (3). TOTAL 3.

Teaching Assistantship (10)	3400	9	*	20	Teaching
Research Assistantship (16)	3400	9	*	20	Research

*North Carolina resident $199.00; nonresident $986.50

OHIO

Bowling Green State University

BOWLING GREEN, OHIO 43403
Louis C. Graue, Chairman
Department of Mathematics
Applications must be filed by March 15, 1972*

NUMBER OF DEGREES AWARDED IN 1971
Baccalaureate degrees by institution 2775
Baccalaureate degrees by department 78
Master's degrees by department 90

Fellowship (2)	2400	12			
Teaching Fellowship (4)	2000-4000	9		10-20	Teaching
Teaching Assistantship (25)	1400-2800	9		10-20	Teaching
In-Service Institute (30)**					
Summer Institute (25)**					

*For first consideration
**Already in progress

TYPE of financial assistance (with number anticipated in 1972-1973)	STIPEND amount in dollars	9 or 12 months	TUITION if not included in stipend (dollars)	SERVICE REQUIRED hours per week	type of service

Case Western Reserve University

CLEVELAND, OHIO 44106
Harold B. Houser, Acting Chairman
Department of Biometry

NUMBER OF DEGREES AWARDED IN 1971
Baccalaureate degrees by institution 1086
Master's degrees by department 1

Teaching Assistantship (2)	4500-5000	12		20	Teaching, Research
Research Assistantship (1)	4000-4500	12		20	Research

OKLAHOMA

University of Tulsa

TULSA, OKLAHOMA 74104
Thomas W. Cairns, Head
Department of Mathematical Sciences
Applications must be filed by March, 1972

NUMBER OF DEGREES AWARDED IN 1971
Baccalaureate degrees by institution 843
Baccalaureate degrees by department 18
Master's degrees by department 3

Fellowship (3)	2100-2700	9		35/hr.	
Teaching Assistantship (4)	2400	9		6	

WEST VIRGINIA

West Virginia University

MORGANTOWN, WEST VIRGINIA 26506
J. C. Eaves, Chairman
Department of Mathematics
Applications should be filed by April, 1972*

NUMBER OF DEGREES AWARDED IN 1971
Baccalaureate degrees by institution 1857
Baccalaureate degrees by department 18
Master's degrees by department 11

Teaching Assistantship (16)	2200-2800	9		6	Teaching

*Late applications will be considered if positions are still available.

CANADA

Carleton University

OTTAWA, ONTARIO K1S 5B6, CANADA
V. Dlab, Chairman
Department of Mathematics
Applications must be filed by March 1, 1972

NUMBER OF DEGREES AWARDED IN 1971
Baccalaureate degrees by institution 782
Baccalaureate degrees by department 51
Master's degrees by department 11

Ph. D. degrees awarded during last three years by field of specialization: analysis and functional analysis (1); probability and statistics (1). TOTAL 2.

Fellowship (3)	3800-4700	12	485	4	Teaching
Teaching Fellowship (3)	3800-4200	12	485	4	Teaching
Teaching Assistantship (15)	800-1800	9	485	4-8	Teaching
Research Assistantship (12)	3180	12	485		
Scholarship (4)	2250-3800	12	485		

University of Saskatchewan

SASKATOON, SASKATCHEWAN, CANADA
G.H.M. Thomas, Head
Department of Mathematics

NUMBER OF DEGREES AWARDED IN 1971
Master's degrees by department 4

Ph. D. degrees awarded during last three years by field of specialization: analysis and functional analysis (1); computer science and numerical analysis (2). TOTAL 3.

Teaching Fellowship (10)	3500	12	450	3	

SUMMER GRADUATE COURSES

The following is a list of graduate courses being offered in the mathematical sciences during the summer of 1972. Another list will appear in the April issue of these *Notices*.

ARKANSAS

UNIVERSITY OF ARKANSAS
Fayetteville, Arkansas 72701
Application deadline: May 31
Information: Department of Mathematics,

May 31 - August 20
Point Set Topology
Probability Theory
Functions of a Complex Variable I
Measure Theory
Topics in Topology

ILLINOIS

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Urbana, Illinois 61801
Information: James W. Armstrong, Graduate Supervisor, Department of Mathematics

June 19 - August 12
Broad range of undergraduate courses plus graduate courses in group theory, logical foundations of mathematics, partial differential equations, and mathematical methods in physics.

KENTUCKY

UNIVERSITY OF LOUISVILLE
Louisville, Kentucky 40208
Application deadline: June 12
Information: R. H. Geeslin, Chairman, Department of Mathematics

June 12 - July 28
Math 471 - Mathematical Logic (M.A.T. only)
Math 505 - Analysis I
Math 541 - Elementary Topology
Math A551 - Geometry I
Math 699 - Seminar in Functional Analysis

July 3 - August 11
Math 595 - Computer in Math Teaching*

*Limited enrollment

MISSOURI

CENTRAL MISSOURI STATE COLLEGE
Warrensburg, Missouri 64093
Application deadline: May 1
Information: H. Keith Stumpff, Mathematics Department

June 1 - August 12
M 4161 - Advanced Calculus I
M 4181 - Vector Analysis
M 4711 - Modern Algebra I
M 5221 - Projective Geometry
M 4510 - Linear Programming
M 5711 - Modern Algebra II
M 5812 - Problems in Teaching Elementary Mathematics
M 5852 - Problems in Teaching Secondary Mathematics

SAINT LOUIS UNIVERSITY
St. Louis, Missouri 63103
Information: Raymond Freese, Chairman Mathematics Department

June 20 - July 28
Mt 140 - Synthetic Projective Geometry
Mt 171 - Vector Analysis
Mt 211a - Modern Algebra
Mt 238 - Foundations of Mathematics
Mt 244 - Geometric Transformations

NEW YORK

STATE UNIVERSITY OF NEW YORK AT BUFFALO
Amherst, New York 14226
Information: Stuart P. Hastings, Department of Mathematics

May 22 - July 7
Course in Algebraic Geometry with a set of lectures on the Riemann-Roch Theorem delivered by Alexandre Grothendieck, Collège de France.

NORTH CAROLINA

EAST CAROLINA UNIVERSITY
Greenville, North Carolina 27834
Application deadline: May 8 First Summer Term, June 14 Second Summer Term
Information: John M. Howell, Dean, Graduate School

June 5 - July 11
Math 365G - Theory of Numbers I
Math 385G - Advanced Calculus I
Math 419 - Properties of Infinite Series
Math 444 - Matrix Algebra I

July 13 - August 18
Math 366G - Theory of Numbers II
Math 386G - Advanced Calculus II
Math 371G - Theory of Equations I
Math 445 - Matrix Algebra II

UNIVERSITY OF NORTH CAROLINA AT CHARLOTTE
Charlotte, North Carolina 28213
Application deadline: May 15
Information: Phillip E. Johnson, Department of Mathematics

June 19 - August 11
Math 600 - Foundations of Mathematics
Math 608 - Topics in Geometry-Topology (Linear Algebra)

OHIO

KENT STATE UNIVERSITY
Kent, Ohio 44242
Application deadline: May 13
Information: B. H. McCandless, Mathematics Department

June 19 - July 21
515 - Foundations of Mathematics I
523 - Introduction to Analysis I
571 - Introduction to Modern Algebra I
661 - Functions of a Real Variable
641 - Introduction to Topology I
- Advanced Topics in Analysis*

July 24 - August 25
516 - Foundations of Mathematics II
524 - Introduction to Analysis II
572 - Introduction to Modern Algebra II
662 - Functions of a Real Variable II
642 - Introduction to Topology II
- Advanced Topics in Analysis*

OHIO STATE UNIVERSITY
Columbus, Ohio 43210
Application deadline: June 1
Information: Marshall C. Yovits, Chairman, Department of Computer and Information Science

June 20 - September 1
- Numerical Analysis
- Computer Systems Programming I
- Digital Computer Organization
- Introduction to Linguistic Analysis
- Computer Systems Programming II
- Theory of Indexing
- Selected Topics in the Mathematics of Information Handling I
- Comparative Operating Systems
- Telecommunications
- Theory of Computability
- Advanced Seminar in Computer and Information Science

PENNSYLVANIA

BUCKNELL UNIVERSITY
Lewisburg, Pennsylvania 17837
Application deadline: June 1
Information: Coordinator of Graduate Studies

June 26 - August 4
- Topics in Algebra
- Topics in Analysis
- Topics in Geometry
- Seminar

TEXAS

UNIVERSITY OF HOUSTON
Houston, Texas 77004
Application deadline: April 10 First Summer Term, June 16 Second Summer Term
Information: Graduate School

June 5 - July 11
Math 477A - Linear Algebra
Math 683 - Infinitesimal Generators
Math 695A - Algebraic Number Theory

June 5 - August 19
Math 431 - Introduction to Analysis
Math 437 - Point Set Topology

July 13 - August 19
Math 439 - Set Theory and Logic
Math 683 - Stieltjes Integral Equations
Math 695B - Analytic Number Theory

*To be determined

VIRGINIA

VIRGINIA COMMONWEALTH UNIVERSITY
Richmond, Virginia 23220
Application deadline: May 15
Information: W. A. Glynn, Chairman, Department of Mathematical Sciences

June 12 - July 21
509 - Introduction to Topology
511 - Applied Linear Algebra
601 - Abstract Algebra I
401 - Algebraic Structures I
405 - Modern Geometry
407 - Advanced Calculus I

July 24 - August 25
402 - Algebraic Structures II
408 - Advanced Calculus II

WISCONSIN

UNIVERSITY OF WISCONSIN-MILWAUKEE
Milwaukee, Wisconsin 53201
Application deadline: April 7
Information: R. L. Hall, Department of Mathematics

June 19 - August 12
- Asymptotic and Perturbation Methods
- Summability Theory
- Topics in Ring Theory

CANADA

CARLETON UNIVERSITY
Ottawa, Ontario K1S 5B6, Canada
Application deadline: April 15
Information: D. Dawson, Department of Mathematics

May 25 - August 13
Math 510 - General Algebra
Math 680 - Seminar in Algebra
Math 580 - Seminar in Probability and Mathematical Statistics

VISITING MATHEMATICIANS

Supplementary List

The following is a continuation of the lists of visiting mathematicians printed in the August, October, November, and January issues of these *Notices*.

American and Canadian Mathematicians Visiting Abroad

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Brauer, Richard (U.S.A.)	Universitet Aarhus, Denmark	Finite Groups	Spring term 1972
Cooke, Kenneth L. (U.S.A.)	University of Florence, Italy	Functional Differential Equations	2/72-6/72
Ferguson, LeBaron O. (U.S.A.)	Faculté des Sciences, Nancy, France	Approximation Theory	
Myhill, John (U.S.A.)	University of Leeds, England	Intuitionistic Analysis	
Redheffer, Raymond M. (U.S.A.)	Universität Karlsruhe, Fed. Rep. of Germany	Function Theory	to 3/72

Foreign Mathematicians Visiting in the United States

Prakash, Nirmala (India)	Massachusetts Institute of Technology	Differential Geometry

BACKLOG OF MATHEMATICS RESEARCH JOURNALS

Information on the backlog of papers for research journals is published in the February and August issues of these *Notices* with the cooperation of the respective editorial boards. Since all columns in the table are not self-explanatory, we include further details on their meaning.

Column 3. This is an estimate of the number of printed pages which have been accepted but are not necessary to maintain copy editing and printing schedules.

Column 5. The first (Q_1) and third (Q_3) quartiles are presented to give a measure of normal dispersion. They do not include misleading extremes, the result of unusual circumstances arising in part from the refereeing system.

The observations are made from the latest issue of each journal received at the Headquarters Offices before the deadline for the appropriate issue of these *Notices*. Waiting times are measured in months from receipt of manuscript in final form to receipt of final publication at the Headquarters Offices. When a paper is revised, the waiting time between an editor's receipt of the final revision and its publication may be much shorter than is the case otherwise, so these figures are low to that extent.

	1	2	3		4	5		
JOURNAL	No. issues per year	Approx. no. pages per year	BACKLOG 12/31/71	BACKLOG 6/30/71	Est. time for paper submitted currently to be published (in months)	Observed waiting time in latest published issue (in months) Q_1	Med.	Q_3
Acta Informatica	4	384	200	0	8-9		###	
American J. of Math.	4	1500	1150	472	8	8	10	11
Annals of Math. Stat.	6	2200	0	0	9	10	12	15
Annals of Math.	6	1200	NR*	NR*	12	11	13	19
Arch. Rational Mech.	22	1790	164	0	5-6	7	7	8
Canad. J. of Math.	6	1200	800	100	12	7	7	10
Comm. Math. Physics	16	1337	250	0	5-6	6	7	8
Duke Math. J.	4	800	0	340	6		**	
Illinois J. of Math.	4	700	1000	1700	24	31	32	33
Indiana Univ. Math. J.	12	1200	600	500	6	9	9	11
Inventiones Math.	14	1220	254	0	7	7	9	10
J. Amer. Stat. Assoc.	4	NR*	NR*	99	NR*		***	
J. Assoc. for Comp. Mach.	4	700	300	NR*	15	6	8	9
J. Diff. Geometry	4	830	800	600	9-12	12	15	17
J. Math. Physics	12	2450	0	400	6	7	8	9
J. Symbolic Logic	4	800	0	0	10	13	15	18
Linear Algebra and Appl.	4	NR*	NR*	0	NR*	12	19	28
Math. Biosciences	6	NR*	NR*	120	NR*		***	
Math. Systems Theory	4	NR*	NR*	0	NR*	12	14	16
Math. of Comp.	4	1000	0	0	8	9	10	12
Math. Annalen	20	1683	255	0	2-9	10	10	12
Math. Zeitschrift	22	2040	168	180	13	9	10	10
Michigan Math. J.	4	400	150	150	14	9	13	20
Numerische Math.	10	900	NR*	94	13-14	11	15	22
Operations Research	6	1500	200	400	14	18	23	26
Pacific J. of Math.	12	3300	NR*	NR*	12	11	13	15
Proceedings of AMS	12	3250	50	300	11	11	12	14
Proc. Nat'l Acad. Sci.	12	5000#	0	0	12	3	3	4
Quarterly of Appl. Math.	4	520	520	650	12	13	15	16
SIAM J. of Appl. Math.	8	1600	0	0	8-10	8	9	11
SIAM. J. on Computing	4	600	0	0	8-10		##	
SIAM. J. on Control	4	700	0	0	8-10	9	10	11
SIAM J. on Math. Anal.	4	700	0	0	8-10	9	10	12
SIAM J. on Numer. Anal.	4	800	0	0	8-10	11	15	17
SIAM Review	4	600	0	0	8-10	11	11	13
Transactions of AMS	12	5500	100	200	12	11	13	17
Z. Wahrscheinlichkeitstheorie	19	1620	NR*	260	15	13	15	17

*NR means that no response was received to a request for information.
**Duke Mathematical Journal has just resumed accepting papers for publication. Latest issue is last of old backlog, hence not relevant.
***Date of receipt of manuscript not given in this journal.
#All subjects.
##New journal–first issue not yet received.
###No recent copy available for checking.

PERSONAL ITEMS

SERGIO A. ALBEVERIO of Princeton University has been appointed a lecturer at the University of Oslo, Norway.

STEPHEN R. ALPERT of Lehigh University has been appointed to an assistant professorship in the Department of Computer Science at Worcester Polytechnic Institute.

WALLACE E. BARNES of Westinghouse Learning has been appointed to the chairmanship of the Natural Sciences Department at Robert Morris College.

JACOB T. B. BEARD, JR., of the University of Tennessee has been appointed to an assistant professorship at the University of Texas at Arlington.

HERSCHEL L. BENTLEY of Bucknell University has been appointed to an associate professorship at the University of Toledo.

STEPHEN R. BERNFELD of the University of Missouri, Columbia, has been appointed to a visiting assistant professorship at the University of Rhode Island for the year September 1971 to September 1972.

JOHN W. BESSMAN, JR., of Mercer University has been appointed a senior research scientist at Scope Electronics, Incorporated, Reston, Virginia.

THEODORE S. BOLIS of the Claremont Graduate School has been appointed to a lecturer and research associate at SUNY at Binghamton.

RAJ C. BOSE of the University of North Carolina, Chapel Hill, has been appointed to a professorship at Colorado State University.

JOHN M. BOWNDS of the University of Arizona has been appointed to a visiting assistant professorship at Rensselaer Polytechnic Institute.

ALFRED T. BRAUER of the University of North Carolina and Wake Forest University has been awarded the Hegel Medal of Humboldt Universität, the former University of Berlin.

JOEL V. BRAWLEY, JR., of Clemson University is on sabbatical leave. He is spending his leave as a visiting associate professor at North Carolina State University.

ROBERT P. BURN of St. John's College, India, has been appointed a senior lecturer at Homerton College, Cambridge, England.

PHILIP CALABRESE of the Naval Postgraduate School, Monterey, California, has been appointed to an assistant professorship at California State College, Bakersfield.

ALBERTO P. CALDERÓN of the University of Chicago has been appointed to a professorship at the Massachusetts Institute of Technology.

WILLIAM G. CHANG of Michigan State University has been appointed to an assistant professorship at Cleveland State University.

CHARLES A. CHENEY of Carnegie-Mellon University has been appointed to an assistant professorship at Indiana State University.

W. ROBERT COLLINS of the University of Massachusetts has been appointed to an assistant professorship at the Christopher Newport College of the College of William and Mary.

S. H. COX, JR., of the University of Texas has been appointed to an acting assistant professorship at the University of California, Los Angeles.

JOHN F. DALY of St. Louis University has been appointed to a visiting professorship at Gonzaga University.

WILLIAM H. DAVENPORT of the University of Alabama has been appointed a mathematician at the U.S. Army Missile Command, Redstone Arsenal, Huntsville, Alabama.

DONALD A. DAWSON of McGill University has been appointed to a professorship at Carleton University.

THEODORUS J. DEKKER of the Mathematical Centre, Amsterdam, has been appointed to a professorship at the University of Amsterdam.

JOHN L. DeVAULT of the GTS Corporation has been appointed area manager with Tetra Tech, Incorporated, Houston, Texas.

RAYMOND F. DICKMAN, JR., of

the University of Miami has been appointed to an associate professorship at Virginia Polytechnic Institute and State University.

JOSEPH DIESTEL of the University of South Florida has been appointed to an associate professorship at Kent State University.

JAMES R. DRAKE of the Frank J. Seiler Research Laboratory has been appointed to an assistant professorship at Drake University.

ERNEST J. ECKERT of California State College at Los Angeles has been appointed to a professorship at Aalborg Teknikum, Aalborg, Denmark.

THOMAS ERBER of the Illinois Institute of Technology has been awarded an honorary professorship of Physics by the University of Graz, Graz, Austria.

GRAEME FAIRWEATHER of Rice University has been appointed to an associate professorship at the University of Kentucky.

ROBERT P. FEINERMAN of Harvard University has been appointed to an assistant professorship at Herbert H. Lehman College (CUNY).

CHARLES D. FEUSTEL of the Institute for Defense Analyses has been appointed to an assistant professorship at Virginia Polytechnic Institute and State University.

JOHN R. FISHER of the University of California, Riverside, has been appointed to an assistant professorship at the California State Polytechnic College, Pomona.

ROBERT B. GARDNER of the Institute for Advanced Study has been appointed to an associate professorship at the University of North Carolina, Chapel Hill.

SAMUEL GRAFF has been appointed to an assistant professorship at John Jay College of Criminal Justice (CUNY).

DOUGLASS L. GRANT of McMaster University has been appointed to an assistant professorship at Xavier College, Sydney, Nova Scotia, Canada.

GREGORY F. GRUSKA of the U.S. Army Signal Corps, STRATCOM, has been appointed a reliability engineer with Chevrolet, Warren, Michigan.

JOHN R. HIGGINS of the University of Minnesota has been appointed a lecturer at the Cambridgeshire College of Arts and Technology, Cambridge, England.

KYONG T. HAHN of Pennsylvania State University has been appointed to a visiting professor at the University of California, Berkeley.

RONALD E. HARRELL of the University of Maryland has been appointed to an assistant professorship at Allegheny College.

JAMES D. HARRIS of NASA, Langley Research Center, has been appointed to an assistant professorship at Tennessee Technological University.

DONALD G. HAZLEWOOD of Syracuse University has been appointed to an assistant professorship at Southwest Texas State University.

RAYMOND A. HEITGER of Ball State University has been appointed a teaching assistant at the University of Toledo.

GEORGE W. JOHNSON has been appointed to a visiting assistant professorship at the University of South Carolina.

KEITH E. JOHNSON of the University of Georgia has been appointed to an assistant professorship at the University of South Alabama.

PAUL M. KAHN of Equitable Life Assurance Society has been appointed assistant vice-president and associate actuary at Beneficial Standard Life Insurance Company, Los Angeles, California.

KEITH M. KENDIG of SUNY at Albany has been appointed to an associate professorship at Cleveland State University.

DAVID A. KLARNER of Reading University has been appointed a research associate at Stanford University.

BENJAMIN G. KLEIN of New York University has been appointed to an assistant professorship at Davidson College.

ANDREW KRAUS has been appointed a senior systems analyst with ITT-Federal Electric Corporation, Vandenberg Air Force Base, California.

DONALD L. KREIDER of Dartmouth College has been appointed to a visiting professorship at the Massachusetts Institute of Technology.

WILLIAM D. KUNKIN of the Massachusetts Institute of Technology has been appointed to an assistant professorship at Cleveland State University.

JONATHAN K. LEE of Indiana University has been appointed to an assistant professorship at the University of Florida.

PHILIP F. LEE of Carleton University has been appointed to an assistant professorship at Mississippi State University.

PHILIP A. LEONARD of Arizona State University is on leave for the academic year 1971-1972. He is spending his leave as a research associate at Carleton University.

H. MELVIN LIEBERSTEIN of Wichita State University has been appointed to the Second Chair of Mathematics at the University of Newcastle, Newcastle, Australia.

S. J. LOMONACO, Jr., of Texas Instruments has been appointed to an associate professorship at SUNY at Albany.

HENDRICUS G. LOOS of the Douglas Advanced Research Laboratories has been appointed to a professorship at Cleveland State University.

RICHARD MANDELBAUM of Newark College of Engineering has been appointed to an assistant professorship at the University of Massachusetts.

JESSE D. MASON of the University of Georgia has been appointed to an assistant professorship at the University of Utah.

RAYMOND J. McGIVNEY has been appointed to an associate professorship at the University of Hartford.

FRANCIS S. McGOWAN of Rockland Community College has been appointed a senior systems analyst at the New York University Medical Center.

ISAAC S. METTS, JR., of the Walter Reed Army Institute of Research has been appointed to an assistant professorship at The Citadel.

PAUL T. MIELKE of the Committee on the Undergraduate Program in Mathematics has been appointed to a professorship and to the chairmanship of the Department of Mathematics at Wabash College.

B. ARTHUR MILLER of Syracuse University has been appointed to an assistant professorship at Mount Allison University.

MYRON W. MILLER of the University of Colorado has been appointed a medical statistician at the Computer Science Corporation, Marshall Space Flight Center, Huntsville, Alabama.

V. NARDI of the University of Padova has been appointed to a research professorship at the Istituto Avogadro di Tecnologia, Rome, Italy.

S. NEGREPONTIS of McGill University has been elected to the First Chair of Mathematical Analysis at the University of Athens, Greece.

KAJ L. NIELSEN of the Battelle Memorial Institute has been appointed to the chairmanship of the Department of Mathematics at Butler University.

GLORIA OLIVE of the University of Wisconsin-Superior has been appointed a senior lecturer at the University of Otago, Dunedin, New Zealand.

EDWARD W. PACKEL of Reed College has been appointed to an assistant professorship at Lake Forest College.

JAMES F. PORTER of Syracuse University has been appointed to an assistant professorship at the University of Arkansas.

LAWRENCE D. PORTER of the Lawrence Radiation Laboratory has been appointed a scientific specialist with EG&G, Incorporated, Albuquerque, New Mexico.

GERARD P. PROTOMASTRO of Clemson University at Sumter has been appointed to an assistant professorship at Loyola University of New Orleans.

CHESTER W. RICHARDS of the University of Mississippi has been appointed a management intern in the Office of the Secretary of Defense, the Pentagon.

RONALD E. RIETZ of the University of Minnesota has been appointed to an assistant professorship at Gustavus Adolphus College.

ROBERT L. ROSENBERG of Drexel University has been appointed to an assistant professorship at Glassboro State College.

LAWRENCE J. ROSENBLUM of Ohio State University has been appointed a mathematician with NAVSTIC, Suitland Federal Center, Maryland.

HARRY L. ROSENZWEIG of Haverford College has been appointed to an assistant professorship at Western Maryland College.

GARY G. SACKETT of the University of New Mexico has been appointed to an

assistant professorship at Idaho State University.

STEPHEN H. SAPERSTONE of Howard University and the Center for Naval Analysis has been appointed to an assistant professorship at the George Mason College of the University of Virginia.

JOEL L. SCHIFF of the University of California, Los Angeles, has been appointed a lecturer at the University of Auckland, Auckland, New Zealand.

ARLO W. SCHURLE of Indiana University has been appointed to an associate professorship at the University of North Carolina at Charlotte.

SHERWOOD D. SILLIMAN of the University of Wisconsin-Madison has been appointed to an assistant professorship at Cleveland State University.

GUSTAVUS J. SIMMONS of Rolamite, Incorporated, has been appointed division supervisor at Sandia Laboratories, Sandia Base, Albuquerque, New Mexico.

DAVID ALLEN SINGER of Princeton University has been appointed to an assistant professorship at Cornell University.

RUTH SILVERMAN of Washington University has been appointed to an assistant professorship at Lehigh University.

JAMES F. SLIFKER of SUNY at Binghamton has been appointed to a visiting associate professorship at Haverford College.

ANN K. STEHNEY of SUNY at Stony Brook has been appointed to an assistant professorship at Wellesley College.

RICHARD M. SUMMERVILLE of Syracuse University has been appointed to an associate professorship and to the chairmanship of the Department of Mathematics at Armstrong State College.

GEORGE R. TALBOT of North American Rockwell Corporation, Autonetics Division, has been appointed to a professorship at Pacific States University.

AUDREY A. TERRAS of the University of Puerto Rico has been appointed to an assistant professorship at Brooklyn College (CUNY).

HAROLD H. TESSEREAU of the University of Missouri, St. Louis, has been appointed to an associate professorship at Macon Junior College.

ERNST-JOCHEN THIELE of the Technische Universität Hannover has been appointed to a professorship at the Freie Universität Berlin.

CHRISTINE TREASH of Mount Allison University has been appointed to an assistant professorship at Sweet Briar College.

JOSEPH A. TROCCOLO of the University of Wisconsin-Madison has been appointed to an assistant professorship at Cleveland State University.

ENRIQUE VALLE-FLORES of the Universidad Nacional Autónoma de México has been appointed to a professorship at the Universidad de Sonora, Hermosillo, Sonora, Mexico.

JAAK VILMS of Purdue University has been appointed to an associate professorship at Colorado State University.

BHUSHAN L. WADHWA of Indiana University, Bloomington, has been appointed to an assistant professorship at Cleveland State University.

MASAMI WAKAE of the University of Manitoba has been appointed to a professorship at Soka University, Tokyo, Japan.

JOHN D. WELLER of the Institute for Defense Analyses has been appointed a member of the senior research staff at the Urban Institute, Washington, D.C.

H. IAN WHITLOCK of the Illinois Institute of Technology has been appointed to an assistant professorship at the Bronx Community College (CUNY).

LOUIS F. WILLIAMS, JR., of the University of Florida has been appointed to an assistant professorship at the University of South Alabama.

JOEL A. WINTHROP of the University of California, Davis, has been appointed to an assistant professorship at the University of Missouri, Columbia.

JAMES C. S. WONG of McMaster University has been appointed to an assistant professorship at the University of Calgary.

WOJBOR A. WOYCZYNSKI of Carnegie-Mellon University has been appointed adjunct and lecturer at the Wroclaw University and the Institute of Mathematics of the Polish Academy of Sciences.

YOSHIKATSU YAMAMURA has been appointed to an assistant professorship at Hosei University, Tokyo, Japan.

PROMOTIONS

To Dean, School of Science. Rensselaer Polytechnic Institute: GEORGE H. HANDELMAN.

To Chairman, Department of Mathematics. Carleton University: VLASTIMIL DLAB; Radford College: JOSEPH C. NICHOLS; Indian Institute of Technology, India: PADAM C. JAIN.

To Chairman, School of Mathematics. University of Minnesota: JOHANNES C. C. NITSCHE.

To Senior Associate. Daniel H. Wagner, Associates: LAWRENCE D. STONE.

To Professor. Creighton University: JOHN N. MORDESON; Herbert H. Lehman College (CUNY): PAUL R. MEYER; Massachusetts Institute of Technology: DANIEL G. QUILLEN; Northeastern University: BOHUMIL CENKL; Purdue University: LOWELL W. BEINEKE; Western Connecticut State College: GLORIA F. BRUNELL; University of Graz, Austria: JOHANN HANS HIJTMANEK.

To Associate Professor. Brandeis University: DAVID I. LIEBERMAN; Brooklyn College (CUNY): CHARLES F. GODINO; Herbert H. Lehman College (CUNY): MELVIN C. FITTING, JOHN C. MINEKA; Kent State University: HELEN I. MEDLEY; Manhattan College: TYN MYINT-U; Saint Joseph College: SISTER JOHN FRANCES GILMAN; Stanislaus State College: LOUIS A. FELDMAN; University of Montana: MICHAEL J. FISHER, MERLE E. MANIS, I. KEITH YALE; University of South Alabama: LEON E. MATTICS; University of Wisconsin-Superior: FRANCIS G. FLOREY; University of Tokyo: DAISUKE FUJIWARA.

To Assistant Professor. Massachusetts Institute of Technology: EUGENE M. KLEINBERG; Purdue University, Calumet Campus: SIGRID E. WAGNER.

To Reader. Imperial College of Science and Technology, University of London: THOMAS KOVARI.

To Visiting Lecturer. University of Illinois: JANET E. FORBES.

INSTRUCTORSHIPS

Lowell Technological Institute: PETER D. ROSENBAUM; Luther College: WALTER E. WILL; Massachusetts Institute of Technology: CHARLES N. FRIEDMAN, ANDREW G. MARKOE, GEORGE METAKIDES, DOUGLAS C. RAVENEL; SUNY at Buffalo: DALLAS E. WEBSTER; SUNY at Stony Brook: PHILLIP C. LYNCH; University of North Carolina at Charlotte: NICK M. STAVRAKAS; University of South Alabama: RONALD C. LINTON.

DEATHS

Professor RICHARD COURANT of NYU-Courant Institute of Mathematical Sciences died on January 27, 1972, at the age of 84. He was a member of the Society for 36 years.

Professor TUDOR GANEA of the University of Washington died on August 10, 1971, at the age of 48. He was a member of the Society for 8 years.

Dr. KURT-RUDIGER KANNENBERG of Bochum, Federal Republic of Germany, died on November 4, 1971, at the age of 43. He was a member of the Society for 6 years.

Professor HANNA NEUMANN of the Australian National University died on November 14, 1971, at the age of 57. She was a member of the Society for 16 years.

Professor EDWARD RAYHER of Hartwick College died on December 24, 1971 at the age of 54. He was a member of the Society for 22 years.

Professor HERBERT J. REBASSOO of Luther College died on December 12, 1971, at the age of 58. He was a member of the Society for 14 years.

LETTERS TO THE EDITOR

Editor, the *Notices*

The resolution passed at the Summer Meeting at The Pennsylvania State University by both the Board of Governors of the MAA and the Council of the Society, concerning lack of adequate liaison between the NSF and the mathematical community, failed to discriminate among the various sections of the NSF. The Mathematical Sciences Section of the Foundation, which is responsible for the awarding of research grants in pure mathematics, statistics and applied mathematics, has a different relationship with the mathematical community from that of the other offices of the Foundation. As Chairman of the Advisory Panel for the Mathematical Sciences Section, I am writing to point out the existence of this Panel and to delineate its role.

New panel members are chosen by the staff of the Mathematical Sciences Section (MSS) from names put forward by current members, attention being given to areas of specialization as well as types and locations of institutions represented. Meetings of the Panel are held twice a year in Washington, and are attended by the staff of the MSS and by the head (presently Dr. William E. Wright) of the Division of Mathematical and Physical Sciences. At present, the members of the Panel are G. J. Lieberman, A. P. Mattuck, A. Nerode, R. S. Palais, H. O. Pollak, H. F. Weinberger, J. H. Wells, and F. B. Wright and myself.

The function of the Panel is not to second-guess the staff on individual decisions concerning orthodox research proposals. Rather it is, first, to give advice intended to influence the evolution of policy, as that is required by changing circumstances. Occasionally this involves a specific proposal (e.g., a request for the support of the colloquium lecture series at a certain institution), but more frequently we are concerned with broader questions such as the weighting of research grants between younger and more established mathematicians, the fraction of the available funds which should be allocated to graduate students as research associates, and the optimal emphasis on applied mathematics. Certainly we are listened to, and on the whole our advice is followed. (An example of an exception would be our recommendation that a limitation be put on the summer NSF earnings of senior mathematicians; this would require a policy change at a much higher level, and has not been acted upon affirmatively.)

The second function of the Panel is to convey information–both ways. At one meeting a year several panel members give brief descriptions of recent progress and current research trends in their fields, for the general edification of staff members who must be au courant with an enormous spectrum of research. In the opposite direction, staff members explain the budgetary situation, both in the mathematical sciences and overall, indicating shifts in emphasis and changes in programs, and review in a broad way the recent history of grants made and proposals declined. (The Panel also receives lists of individual actions taken.)

As is made clear in the article by Dr. Pell in this issue of the *Notices*, the MSS is by no means the only branch of the NSF which has impact on the mathematical community: graduate fellowships, CUPM, information services and summer institutes, for example, are all handled by other offices. These other units are not organized along discipline lines (there is no mathematical sciences section of the Office of Science Information Services, for example), and correspondingly there are no other exclusively mathematical advisory panels than the one under discussion, although of course a group of mathematicians serves as the selection panel for graduate fellowships in mathematics. (See CBMS Newsletter, October 1971, page 2.) The Advisory Panel does hear several presentations a year from representatives of these other units, but up to the present time has had no significant input to, nor impact on, the policies

of other offices. In light of recent developments, we feel that some way should be found to modify this situation.

With the above facts in mind, the Panel unanimously adopted the following resolution at its meeting of October 15, 1971:

> In response to the recent resolution of the Board of Governors of the MAA on the activities of the NSF, the Advisory Panel reaffirms strongly its confidence in the activities of the Mathematical Sciences Section of the NSF, and its liaison with the mathematical community. We view with concern, however, the lack of routine communication to the Section, and to its Advisory Panel, of information about current trends and activities in other parts of the Foundation that affect the mathematical community.

As the representatives of the mathematical sciences community, all of us will welcome suggestions for improving the services the Foundation renders to this community and to the nation. (This does not include protests about individual declinations.)

William J. LeVeque
Chairman, Advisory Panel for
the Mathematical Sciences

Editor, the *Notices*

With respect to M. Solveig Espelie's letter in the November *Notices* questioning whether the University of Witwatersrand in Johannesburg abides with Title VII of the 1964 U. S. Civil Rights Act, as a long time American resident of Switzerland let me assure you that neither Swiss academic institutions nor Swiss corporations abide with the letter or spirit of the Act. Non-compliance is also rampant throughout other European countries.

This may come as a shock, but outside the United States (with a few exceptions) it is difficult to find any countries which abide by U. S. laws, or even share American concepts of how society should be organized.

I suppose it is naive to expect middle class American university professors to be less persuaded of their moral purity than American middle class society as a whole. ...[C]an't we leave the running of other peoples' countries to them, and get on with the business at hand, which is, presumably, mathematics.

Kent Gordis

Editor, the *Notices*

At the last summer meeting of the Mathematical Association of America at Pennsylvania State University, a panel was held on "Women in Mathematics". A report on this panel in the November issue of the American Mathematical Monthly summarized part of the remarks of one of the speakers, Professor Mary Ellen Rudin, as follows: "There is little overt discrimination except on the new Ph.D. who may suffer in job opportunities because of the high dropout rate of her sisters". Any system which treats a person according to presumed characteristics of a class to which the person belongs is by definition discriminatory. So this statement implies that women mathematicians do face very serious discriminatory attitudes. What about the truth of the presumed characteristic "high dropout rate"? This has been quite a common excuse for not taking the aspirations of women seriously. Recently Helen Astin (The Woman Doctorate in America; Origins, Career, and Family, Russell Sage Foundation, New York 1969) found that women who receive Ph.D.'s are likely to use them in a professional capacity. She found that 91 percent of the women who received Ph.D.'s in 1957–58 were employed in 1964, and 79 percent of them had not interrupted their careers during that time. Therefore this study puts to rest the tired cliché of women Ph.D.'s dropping out of professions in general. It certainly casts doubt about the validity of this cliché about women mathematicians. Hence an appropriate subject for an American Mathematical Society study would be the career pattern of women who have received a Ph.D. in mathematics. Certainly until such a study has been made, one should not act upon the basis of this unsubstantiated old saw.

Vera S. Pless
Applied Mathematics
Data Sciences Lab

The resolution apparently prepared and proposed by the Mathematics Action Group and approved by voice vote of those present at the Business Meeting of the Society at Las Vegas raises two fundamental questions: (1) Is the Society an organization primarily for mathematics or primarily for its members? (2) Are potentially controversial resolutions, presented at Business Meetings of the Society without prior notice and voted on only by those present and with only limited discussion and consideration, a proper way of determining attitudes or policies of the Society on important issues? This letter addresses itself to the first of these questions as the answer to the second seems self-evidently negative.

It is my contention that, as shown by its publications, meetings, and other activities, the AMS has been, is, and should be a society for mathematics and specifically for mathematical research and scholarship. It follows almost as a corollary that potential officers and other members of the governing boards of the Society should be selected primarily on the basis of demonstrated involvement in and dedication to mathematical research and scholarship.

The Society is governed by a president, a Council which makes decisions on scientific policy matters, and a Board of Trustees which makes decisions on financial policy matters.

Nominations for offices, including memberships on editorial boards, are offered by a nominating committee to the Council, which formally makes the nominations to the membership on the ballot. A nominating committee is appointed annually by the president. Suggestions for future nominations are explicitly solicited from the membership with the annual ballot. Customarily the only contested positions on the ballot are those for member-at-large of the Council and for vice-president, with twice as many being nominated as are to be elected. The aforementioned resolution of the Las Vegas Business Meeting asked that nomination by petition for the position of member-at-large be provided for (and thus encouraged). Although the resolution itself was not restricted to the position of member-at-large, much of the discussion seemed to center on this position. It is my contention that such a different method of nomination threatens to change the nature of the AMS from a society dedicated to mathematical research and scholarship to one concerned with problems of special interest groups and pressure groups. Indeed, at the present time those groups pushing for a change appear to have a raison d'être quite apart from that of mathematical research. The change could tend to make the AMS more like a union than a professional society.

The present nominating procedure virtually insures that nominees for the position of member-at-large on the Council have been judged first by a nominating committee and by the Council to be deeply and personally involved with mathematical research and scholarship. The electorate, with many Society members having almost no knowledge of the research of more than a few of the nominees, presumably must choose Council members from among the nominees on the basis of their general reputation and visibility in the community.

With there being perhaps 2,000 Society members whose research might well qualify them for at-large Council membership, only about 300 will be nominated in the professional lifetime of an individual member. Because the composition of the nominating committee varies from year to year, there is a kind of partially random process of selection of nominees from among those reasonably qualified. This seems much fairer and sounder than nomination by petition which means nomination by special interest groups perhaps on the basis of nonmathematical qualifications and perhaps with little or no consideration of the individual's involvement with research.

Over the past five years, there have been fifty different people (ten per year) listed on the ballot for the position of member-at-large. They came from thirty-two different institutions with only Berkeley having as many as four nominees of whom only one was elected. All major geographic areas of Society membership were represented among the nominees. The twenty-five elected members came from twenty different institutions. Those

nominated (and those elected) appear to be from all the major and many of the not-so-major fields of mathematical research. Two had been awarded Fields medals at international congresses. Most were in their thirties or forties. Three of those nominated were women and all of these were elected and are presently serving their three-year terms on the Council. The one common characteristic of the nominees seems to be dedication to and competence in mathematical research and scholarship.

I submit that in light of the record of the nominating procedure, there is not only no valid cause for its change but, indeed, the proposed change threatens the generally sound and responsible basis on which the Society has been governed.

R. D. Anderson

Editor, the *Notices*

There was a time, not long ago, when a black mathematician in this country could not get a job commensurate with his competence; to be considered for one, he had to be at least twice as talented and productive as a white competitor. That is wrong. Race, and sex, are independent of mathematical merit and should never be allowed to influence professional decisions.

The time is coming, very soon, when a black mathematician, or a woman, to be considered for a job or a place on the Council of the AMS will need to be mathematically at most half as talented and productive as a white male competitor. That is foolish. Race and sex are independent of mathematical merit and should never be allowed to influence professional decisions.

Whatever the Society may do in the future, most if its time, energy, and money now are spent on the publication of scholarly journals and books. I think that decisions about scholarly matters should be made by the best qualified members of the Society, not the most representative or the most oppressed ones.

It appears that the majority of the membership of the Society has a different opinion and favors "liberalization" of the Council. While I feel a little lonesome down here, I can recognize a sign of the times, and I see no virtue in making waves in the chosen course of my colleagues. Effective immediately, I resign from the Committee on Nominating Procedures and wish it smooth sailing from now on.

P. R. Halmos

Editor, the *Notices*

I have a serious complaint on the 1972 meeting:

In the prospectus you announce a long list of chic hotels all demanding about $14-$16 per day. Accordingly, you reserved me a room at the Thunderbird for $14. But I soon learned the Westward Ho charged $7 a day and I moved after one night.

Why did you list only the high priced hotels? Many colleagues, stuck in the Thunderbird, found Westward Ho filled when they learned with horror that $7 rooms existed. Most of us do not have limitless budgets. I am furious at you. Next time do as the American Statistical Association did in August 1971 at Fort Collins-hold the meeting at a university and house everyone in dorms for about $12 a day which included meals.

Eugène H. Lehman

EDITORIAL COMMENT

It is AMS policy to hold August meetings at universities with housing in dormitories.

Editor, the *Notices*

The words "negative" and "minus" are not synonymous. In fact, I can't think of any instance in which a literate mathematician would regard them as interchangeable. For the nonprofessional, however, the distinction is not merely obscured but often turned around: the terms "minus number" and "negative four" are by now so firmly entrenched in the underground that they are surfacing in all the textbooks, or in any case in all the remedial and semi-remedial textbooks that cater to the student in his own vernac-

ular–precisely those that should most be concerned with setting him straight. This is the same student who is stymied by the absolute value of x that insists on being minus x if x is negative. My own children–grades 8 and 10 of public school–are well acquainted with minus numbers and negative four, and won't have it otherwise; their teachers have all learned their mathematics from the remedial textbooks. Can't something be done? Can't we put the book-publishers on notice? It isn't a question of linguistic permissiveness; it's a question of ground-level communication in a technical subject, and I consider this insensitivity to "good English," if you like, a gross disservice. Far better that my students (and children) learn the language first, and learn it right, before they are bedeviled with the properties of the void set and other such insanities of the Modern Mathematicians.

J. A. Zilber

MEMORANDA TO MEMBERS

COMMITTEE ON WOMEN IN MATHEMATICS

The American Mathematical Society has appointed a Committee on Women in Mathematics to identify and to recommend to the Council those actions which, in the opinion of the committee, the Society should take to alleviate some of the disadvantages that women mathematicians now experience and to document their recommendations and actions by presenting data. The members of the committee are Mary Gray, I. N. Herstein, Cathleen S. Morawetz (chairman), Charles B. Morrey, Jr., and Jane Cronin Scanlon. The committee would like to receive relevant information, comments and recommendations from members of the Society. The committee plans to compile a roster of women mathematicians with Ph.D.'s who are available for employment, and requests such mathematicians to submit a vita, including a list of publications and references, together with information on the type of positions sought. Committee address: Professor Cathleen S. Morawetz, New York University, 251 Mercer Street, New York, New York 10012.

CORPORATE MEMBERS AND INSTITUTIONAL ASSOCIATES

The Society is pleased to announce that, as of December 31, 1971, the following companies and corporations are supporting the Society through Corporate Memberships:

Academic Press, Incorporated
Bell Telephone Laboratories, Incorporated
The Boeing Company
Eastman Kodak Company
Ford Motor Company
General Motors Corporation
International Business Machines Corporation
North American Rockwell Corporation
RCA Corporation
Texaco, Incorporated
TRW, Incorporated

The following corporations and companies are supporting the society as Institutional Associates:

Addison-Wesley Publishing Company
Chelsea Publishing Company
Chemical Rubber Company
Consultants Bureau Enterprises, Incorporated
Dover Publications, Incorporated
Princeton University Press
Prindle, Weber & Schmidt, Incorporated
Shell Development Company
Springer-Verlag, Heidelberg
Union Oil Company of California

NEW MAILING LABELS

Beginning with this issue of these *Notices*, new mailing labels will be used. The advantage of these labels is that they may be detached easily without defacement of the cover. The use of these labels should provide a valuable service both to the Society and to the members. When corresponding with the Society about fiscal matters, changes of address, promotions, or when placing orders for books and journals, the label affixed to the letter or order will enable clerks to process such correspondence with the least possible delay. As all future issues of the *Notices* will use these peel-off labels, members will always have a small supply on hand for their use.

COMBINED MEMBERSHIP LIST, 1972-1973
CHANGE OF ADDRESS OR POSITION

In the past, it has been customary for a return postcard to be sent to each member of the Society with a request for changes in the listing in CML or a confirmation of the current listing. Since we must reduce the cost of the CML operation, this procedure is being discontinued. If there have been any changes in address or position, and the Society has not been notified, members are requested to fill in the form below and return it to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904, no later than May 15, 1972.

Peel off the *Notices* mailing label and place in the space provided below. The mailing label contains the customer code, and with this information, clerks are able to process orders more efficiently. If your mailing label is lost, damaged, or being used for other purposes, please print your complete name and customer code in the space provided.

Name_____ Position _____

Institution or business _____

Location of institution or business (city and state only) _____

If the change in position is temporary, please check here. ☐

Mailing address (if different from above) _____

_____

Date the above changes become effective _____

NEW AMS PUBLICATIONS

SELECTED TRANSLATIONS IN MATHEMATICAL STATISTICS AND PROBABILITY

Volume 10

314 + vi pages; List Price $16.00; Member Price $12.00

The following authors are represented in this volume of SELECTED TRANSLATIONS IN MATHEMATICAL STATISTICS AND PROBABILITY: M. Arató, F. E. Binet, Zoltán Daróczy, Ju. S. Davidovič, A. Dobó, A. Ja. Dorogovcev, Ĭ. Ĭ. Ežov, M. Ĭ. Jadrenko, N. Kalinauskaĭte, Ju. V. Kartošov, V. F. Kibec, V. S. Koroljuk, G. L. Kulīnīč, M. N. Marušin, Pál Medgyessy, O. G. Mikluhin, József Mogyoródi, È. A. Nadaraja, V. N. Nagornīĭ, O. Ĭ. Ponomarenko, G. Ĭ. Prizva, A. Rényi, G. Sâmboan, D. S. Sīl'vestrov, È. S. Štatland, R. L. Stratonovič, S. Szajcz, R. Theodorescu, Van-an', I. N. Volodin.

TRANSLATIONS OF MATHEMATICAL MONOGRAPHS

Volume 29

LINEAR DIFFERENTIAL EQUATIONS IN BANACH SPACE
By S. G. Kreĭn

390 + vi pages; List Price $24.20; Member Price $18.15

This volume deals with the theory of linear differential equations in Banach space with unbounded operator coefficients, and presents a number of fundamental results of the theory of strongly continuous semigroups of operators. The contents reflect the work of the Seminar on Differential Equations at Voronezh University which the author has directed for more than ten years. The results presented in this book are not always formulated in the most general form. The author tried to choose some mean level of generality to which he had to lower some results and pull up others. Compensation for the loss arising in this way will be found in remarks and in the references to the literature appearing at the end of the book.

Volume 31

DIFFERENTIAL EQUATIONS OF THE SECOND ORDER WITH RETARDED ARGUMENT. SOME PROBLEMS OF THE THEORY OF VIBRATIONS OF SYSTEMS WITH RETARDATION
By S. B. Norkin

285 + vi pages; List Price $18.50; Member Price $13.88

The theory of differential equations with deviating arguments is a relatively new and rapidly developing branch of the theory of ordinary differential equations. This volume is concerned with the case of equations with a "concentrated" delay. For ease of reading, almost all the proofs are included, and references are given for all results that have already appeared in a monograph. The reader of this book should have a knowledge of elementary analysis and ordinary differential equations; only in special places does the author use elementary facts of the theory of functions of a real variable and functional analysis. The book will be of interest to mathematicians working in the theory of ordinary differential equations and to physicists and research engineers working on systems with retardation.

ABSTRACTS PRESENTED TO THE SOCIETY

Preprints are available from the author in cases where the abstract number is starred.

The papers printed below were accepted by the American Mathematical Society for presentation by title. The abstracts are grouped according to subjects chosen by the author from categories listed on the abstract form. The miscellaneous group includes all abstracts for which the authors did not indicate a category.

An individual may present only one abstract by title in any one issue of the *Notices* but joint authors are treated as a separate category. Thus, in addition to abstracts from two individual authors, one joint abstract by them may also be accepted for an issue.

Algebra & Theory of Numbers

72T-A26. KIM KI-HANG BUTLER, Pembroke State University, Pembroke, North Carolina 28372. The number of nonisomorphic partially ordered sets. Preliminary report.

An unsolved problem in combinatorial analysis asks for G(n), the number of nonisomorphic partial orderings which may be defined on a finite set containing n elements. In the present paper we give a partial solution to this problem by interpreting a partial order relation as a reduced idempotent Boolean matrix. The main results of this paper are as follows: (i) Enumerating the nonisomorphic partial order relations which may be defined on a finite set containing n elements is equivalent to enumerating E*(n), the set of nonisomorphic idempotent reduced $n \times n$ Boolean matrices. (ii) The set E*(n) and a special subset E*(n,m) are defined. (iii) The number of interest is $|E^*(n)| = \sum_{r=0}^{n(n-1)/2} |E^*(n,r)|$. (iv) Formulas for $|E^*(n,0)|$, $|E^*(n,1)|$, $|E^*(n,2)|$, $|E^*(n,(n(n-1)/2)-2)|$, $|E^*(n,(n(n-1)/2)-1)|$, $|E^*(n,n(n-1)/2)|$ are given. (Received October 13, 1971.)

*72T-A27. S. K. JAIN, Ohio University, Athens, Ohio 45701. A counterexample to Amitsur's conjecture.

S. A. Amitsur in his paper "Rings of quotients and Morita contexts," J. Algebra 17(1971), 273-298, conjectures that in the statement (1) of Theorem 4 the requirement of semiprimeness can be dropped (cf. Remark 10A, p. 282). We provide the following example which shows that semiprimeness cannot be dropped: Let R be the lower triangular 2×2 matrix ring over the rationals Q. Then $Z({}_RR) = 0$ and $U = \begin{pmatrix} Q & 0 \\ Q & 0 \end{pmatrix}$ is a faithful uniform left ideal but R is not prime. If Amitsur's conjecture were true then the ring with the above conditions should be prime (cf. Theorem 4 and Theorem 10B). This example also shows that we cannot even replace the semiprimeness with the condition that there are no nonzero closed nilpotent ideals, i.e., by potent rings introduced by R. E. Johnson (Trans. Amer. Math. Soc. 119(1965), 524-534). (Received November 1, 1971.)

*72T-A28. ANDREAS Z. ZACHARIOU, Johns Hopkins University, Baltimore, Maryland 21218. The Riesz representation theorem and the Gram-Schmidt process. Preliminary report.

The following results complement those announced by the author in Abstract 687-15-4, these *Notices* 18(1971), 760. Theorem 1. Let $(V, \langle\ ,\ \rangle)$ be a real inner-product space and $B = \{x_1,\ldots,x_n,\ldots\}$ a basis of V. Let V_n be the subspace of V spanned by $\{x_1,\ldots,x_n\}$, $n = 1,2,\ldots$. Let $f_n : V_n \to R$ be linear functionals with $f_n(x_k) = \delta_{nk}$. Let y_n be the normalized Riesz point of f_n and $C = \{y_1,\ldots,y_n,\ldots\}$. Then (1) C is the

orthonormal basis of V, obtained from B by the Gram-Schmidt (G-S) process. (2) $y_n = (G_{n-1}G_n)^{-1/2} \det A$, where G_k is the Gramian of $x_1,\ldots,x_k$ and $A = (a_{ij})$ is given by: $a_{ij} = \langle x_i, x_j \rangle$ for $1 \leqq i \leqq n$, $1 \leqq j \leqq n-1$ and $a_{in} = x_i$ for $1 \leqq i \leqq n$. (3) Each y_n depends (up to sign) on $x_1,\ldots,x_{n-1}$ only. In particular: if $V = R^n$ then $y_n = \epsilon\, G_{n-1}^{-1/2}\,(x_1 \wedge \ldots \wedge x_{n-1})$, where $\epsilon = 1$ or -1, according as $\det(x_1,\ldots,x_n)$ is > 0 or < 0.

Theorem 2. Let $B = \{A_1,\ldots,A_n\}$ be a basis of R^n and A the matrix with rows $A_1,\ldots,A_n$. Let Z_n be the nth element of the orthogonal basis of R^n obtained from B by the G-S process. Then the nth column of A^{-1} is $Z_n/\|Z_n\|^2$. Remark. The fact that C in Theorem 1 is orthonormal implies that $G_k > 0$ for all k; in fact, let $(x_i) = A(y_i)$ for some matrix A. Then we have $G(x_i) = AG(y_i)A^t = AA^t$, and so $G_k = \det G(x_i) = (\det A)^2 > 0$. This is a short proof for the nonexistence of negative Gramians. (Received November 2, 1971.)

*72T-A29. WITHDRAWN.

*72T-A30. HUGO SUI-HWAN SUN and H. C. YEUNG, Fresno State College, Fresno, California 93710. Embedding a semigroup into a ring. Preliminary report.

Theorem. Every semigroup may be embedded into a ring of matrices over a field of characteristic 2. (Received November 3, 1971.)

72T-A31. STANLEY N. BURRIS, University of Waterloo, Waterloo, Ontario, Canada and MICHAEL KARL KWATINETZ, University of California, Berkeley, California 94720. A metamathematical observation and its application to the cardinality of certain classes of mathematical structures. Preliminary report.

Let A be denumerable and Λ a first-order language in which (1) all variables range over elements of A, and (2) for any n-ary relation R on A, $n \in \omega$, there is a constant n-placed predicate denoting R. Let $\bar{\Lambda}$ be obtained by adding to Λ: (i) a variable predicate Φ ranging over sets $X \subseteq A$ and (ii) a symbol for denumerable disjunction of formulas. (The use of Λ and $\bar{\Lambda}$ was suggested by A. Tarski.) Let $S = \{X : X \subseteq A\}$ be provided with the ordinary product topology. For the induced topological space it is seen that $F \subseteq S$ is Borelian iff there is a formula ψ in $\bar{\Lambda}$, with Φ as the only free variable, such that $F = \{X \subseteq A : X \text{ satisfies } \psi\}$. (Cf. a related remark of Ryll-Nardzewski, "The theory of models," 1965, p. 339.) By this remark important families of subsets of A, or of finitary relations on A, or of functions from A into B, $\overline{\overline{B}} \leqq \aleph_0$, prove to be Borelian in an appropriate topological space. A known theorem on the cardinality of Borelian sets implies all results of Abstract 691-08-8, these *Notices* 19(1972), A-46. Some further implications are: for every denumerable algebra $\mathfrak{A}$, the following families have cardinality $\leqq \aleph_0$ or $= 2^{\aleph_0}$: those of all simple and all finitely generated subalgebras; those of all homomorphisms, and isomorphisms, from $\mathfrak{A}$ into (or onto) another denumerable algebra $\mathfrak{B}$. (Received November 3, 1971.)

*72T-A32. HENRY E. HEATHERLY, University of Southwestern Louisiana, Lafayette, Louisiana 70501. Regular near-rings.

A near-ring R is regular if its multiplicative semigroup is regular. (A near-ring is a left near-ring satisfying $0x = 0$ for each x.) In this paper structure theory for regular near-rings is established. It is first observed that a regular near-ring with D.C.C. on R-subgroups is the direct sum of a finite number of simple

regular near-rings. Theorem 1. If R is a finite regular d.g. near-ring, then R is the direct sum of ideals each isomorphic to either a full matrix ring over a finite field or to some $T_0(G)$, where G is an invariantly simple nonabelian group. ($T_0(G)$ is the near-ring composed of the set $\{f \in \text{Map}(G,G) : 0f = 0\}$ under the operations of pointwise addition and composition on the (not necessarily abelian) group (G,+).) Theorem 2. If R is a near-ring with no nonzero nilpotent elements and R has D.C.C. on R-subgroups, then R is regular. Furthermore, R is the direct sum of ideals R_i such that each R_i is either (1) a near-field or (2) a simple near-ring with no divisors of zero, which satisfies $xR_i = R_i$ for each nonzero $x \in R_i$, and every idempotent is a left identity. Corollary. If R is finite, then there exists an integer n such that $x^n = x$ for each $x \in R$. Corollary. If R has a nonzero right distributive element, then R is the direct sum of near-fields. Some examples of regular near-rings are discussed. Equivalent conditions are given for a regular near-ring to be a near-field. It is observed that a distributive regular near-ring is a ring. (Received November 4, 1971.)

*72T-A33. R. B. J. T. ALLENBY, University of Leeds, Leeds 2, England and ROBERT J. GREGORAC, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. The free product of π_c groups amalgamated along retracts is π_c.

A group G is π_c if, whenever $g_1, g_2 \in G$ and $g_1 \notin \langle g_2 \rangle$, then there is a finite image $G\theta$ of G such that $g_1\theta \notin \langle g_2\theta \rangle$. Let $\{G_i : i \in I\}$ be a collection of groups which are π_c. Suppose for each $i \in I$ that K_i is a retract of G_i and for each $i, j \in I$, K_i is isomorphic to K_j. Then the generalized free product of the groups G_i amalgamated along the subgroups K_i is again π_c. This extends the corresponding result for residually finite groups (Evans and Boler, Abstract 71T-A186, these *Notices* 18(1971), 807). The methods of proof are different, requiring several cases. Evans and Boler's corollary on knot spaces then immediately extends to this case as well, that is, a knot space has a fundamental group which is π_c if and only if each of its prime components has a fundamental group which is π_c. (Received November 5, 1971.)

*72T-A34. AVIEZRI S. FRAENKEL, Weizmann Institute of Science, Rehovot, Israel. Equality of the largest moduli of an exactly covering system.

If every nonnegative integer occurs in exactly one of the integer sequences $a_i n + b_i$, $n = 0,1,2,\ldots, 0 < a_1 \leqq \ldots \leqq a_m$, $i = 1,\ldots,m$ (= an exactly covering system), then $a_{m-1} = a_m$. The standard proof of this fact uses a generating function and roots of unity. We deduce it from either one of the following identities: $\sum_{i=1}^{m} (b_i^{2n}/a_i - nb_i^{2n-1} + \sum_{j=1}^{n} (-1)^{j-1} C_{2j}^{2n} B_j b_i^{2n-2j} a_i^{2j-1}) = (-1)^{n-1} B_n$; $\sum_{i=1}^{m} (b_i^{2n+1}/a_i - \frac{1}{2}(2n+1) b_i^{2n} + \sum_{j=1}^{n} (-1)^{j-1} C_{2j}^{2n+1} B_j b_i^{2n-2j+1} a_i^{2j-1}) = 0$, valid for every integer $n \geqq 1$, which are proved first where B_k is the kth Bernoulli number. Two connections between this proof and the standard proof are given at the end, as well as an example of the type of congruences deducible from the two identities. (Received November 10, 1971.)

*72T-A35. AVIEZRI S. FRAENKEL, Weizmann Institute of Science, Rehovot, Israel and ITSHAK BOROSH, University of Illinois, Urbana, Illinois 61801. A generalization of Wythoff's game.

Let a, b be positive integers. Three different characterizations of the winning positions of the following game are presented: Two players remove alternately matches from two piles, by taking either a positive multiple of b matches from a single pile, or else by taking $k > 0$, $\ell > 0$ matches from the two piles respectively, subject to $k - \ell \equiv 0 \pmod{b}$, $|k - \ell| < ab$. The case $a = b = 1$ is the classical game known as Wythoff's game, or tsianshidsi [to appear in J. Combinatorial Theory]. (Received November 10, 1971.)

*72T-A36. AVIEZRI S. FRAENKEL, J. LEVITT and M. SHIMSHONI, Weizmann Institute of Science, Rehovot, Israel. Characterization of the set of values $f(n) = [n\alpha]$, $n = 1,2,\ldots$.

Let $\alpha > 1$. Denoting by $[x]$ the integer part of x, we give complete answers to the following two questions: (1) Find $f(n) = [n\alpha]$ as a function of n for all positive integers n. (2) Characterize the set of all f(n). Our answers to both questions depend on a counting system based on the convergents of the simple continued fraction expansion of α. If $1 < \alpha < 2$, α irrational, then $\{f(n)\}$ is precisely the set of all integers ending in an even number of zeros in this counting system, and $\{[n\beta]\}$ is the set of all those ending in an odd number of zeros, where $\beta = \alpha/(\alpha - 1)$. Moreover, for all $k \geq 1$, $[k\beta]$ is obtained from $[k\alpha]$ by adjoining a zero at the latter's end if and only if $\alpha = (2 - a + \sqrt{a^2+4})/2$, a any positive integer. Modified results hold when α is rational. To appear in Discrete Mathematics. (Received November 10, 1971.)

72T-A37. J. THOMAS ALLEN, University of California, Berkeley, California 94720. Automorphism groups of algebras and relational systems. Preliminary report.

A relational system $\mathfrak{A} = \langle A, F_i, R_j \rangle_{i\in I, j\in J}$ is a set A, called the universe of $\mathfrak{A}$, together with an indexed family of operations, F_i, and relations, R_j, on A. Its multiplicity type is the map $\mu: \{0,1\} \times \omega \to$ cardinals such that for each n, $\mu(0,n)$ is the cardinality of $\{i : F_i \text{ is n-placed}\}$, and $\mu(1,n)$ is the cardinality of $\{j : R_j \text{ is n-placed}\}$. If μ and ν are multiplicity types, we write "$\mu \leqq \nu$" iff for every $\mathfrak{A}$ of multiplicity type μ there is a $\mathfrak{B}$ of multiplicity type ν, such that $\mathfrak{A}$ and $\mathfrak{B}$ have the same universe and automorphism group. For κ a cardinal, let $\epsilon_n(\kappa)$ denote the multiplicity type of a system with κ n-place operations and no other operations or relations, and let $\rho_n(\kappa)$ denote the multiplicity type of a system with κ n-place relations and no other relations or operations. Theorem 1. $\rho_n(\kappa) \leqq \rho_{n+1}(1)$. Theorem 2. $\rho_n(\kappa) \leqq \epsilon_{n+1}(1)$. Theorem 3. $\epsilon_n(1) \not\leqq \rho_n(\kappa)$. Theorem 4. $\epsilon_n(\kappa) \leqq \epsilon_{n+1}(1)$. Theorem 5. If $1 + \lambda \leqq 2^{\kappa-1}$, then $\epsilon_n(\lambda) \leqq \rho_{n+1}(\kappa)$. These theorems improve some of the results announced by M. Gould in these *Notices*. They were proved using methods developed by Ralph McKenzie and the author which seem to be quite different from those of Gould. (Received November 17, 1971.) (Author introduced by Professor Jerome Malitz.)

*72T-A38. WALTER D. WALLIS, University of Newcastle, New South Wales 2308, Australia. Integral equivalence of Hadamard matrices.

The integral equivalence class of an integer matrix may be specified by its set of invariants, the entries

in the Smith normal form. For an Hadamard matrix the first invariant is 1 and the next is 2; if the Hadamard matrix has order 4m then the product of the ith invariant with the (4m - i + 1)th is 4m. Theorem. An Hadamard matrix of order 4m has at least [log (4m - 1) + 1] invariants equal to 2 (logarithms are to base 2; square brackets denote integer part). Theorem. If there is a skew-Hadamard matrix of order 8m then there are integrally inequivalent Hadamard matrices of order 16m. Theorem. There are precisely eleven integral equivalence classes of Hadamard matrices of order 32. (The author has constructed examples of all eleven classes.) (Received November 11, 1971.)

72T-A39. W. RUSSELL BELDING, University of Notre Dame, Notre Dame, Indiana 46556. Makai subgroups and pre-ordered sets. Preliminary report.

(X,t) is an SOS topological space iff each $x \in X$ has a smallest open set $t(x)$ about it. SOS spaces (X,t) correspond bijectively to pre-ordered sets $(X,\leqq)$. $A(X,\leqq)$ is the automorphism group of $(X,\leqq)$. A subgroup M of $A(X,\leqq)$ is a Makai subgroup iff (1) for every f, g in M either $f < g$ or $g < f$ or $g \| f$; and (2) $M(x) = X$ for at least one (and hence for all) $x \in X$. Assume (X,t) is an SOS space corresponding to the pre-order $(X,\leqq)$. Lemma. There is a $+$ such that $(X,t,+)$ is a semitopological group iff $A(X,\leqq)$ contains a Makai subgroup. Lemma. There is a $+$ such that $(X,t,+)$ is a topological group iff $A(X,\leqq)$ contains a Makai subgroup and $(X,\leqq)$ is an equivalence relation. Lemma. Let $(X,\cap,\cup)$ be a lattice and $x \leqq y$ iff $x \cap y = x$. There is a $+$ such that $(X,+)$ is a lattice group iff $A(X,\leqq)$ contains a Makai subgroup. (Received November 18, 1971.)

72T-A40. HARRY F. SMITH, University of Iowa, Iowa City, Iowa 52240. On generalized alternative rings I and II. Preliminary report.

In (Abstract 71T-A242, these *Notices* 18(1971), 946-947) are listed the defining identities for generalized alternative rings I and II. In addition to the results cited there, we have the following: Theorem 4. A simple, finite-dimensional, degree one, generalized alternative algebra II, over a field of characteristic $\neq 2$, is itself a field. Theorem 5 (Wedderburn principal theorem). Let A be a finite-dimensional generalized alternative algebra II over a field of characteristic $\neq 2$; and let N be the nil radical of A. If A/N is separable, then $A = S + N$ (vector space direct sum) where S is a subalgebra of A, $S \cong A/N$. (Received November 22, 1971.)

*72T-A41. NORBERT H. SCHLOMIUK, Université de Montréal, Montréal, Québec, Canada. On co-H-objects in the category of augmented algebras.

Let C_R be the category of augmented algebras over a commutative ring R with unit. It is known that the category C_R admits coproducts, hence one can consider co-H-objects in C_R. We prove that if R is a field, a co-H-object in C_R contains a nontrivial free graded algebra and we give an example of an object in C_R with an associative comultiplication not admitting an inverse. (Received November 26, 1971.)

72T-A42. JOEL A. WINTHROP, University of Missouri, Columbia, Missouri 65201. A homological characterization of large subgroups.

A preradical L is defined for each extension of abelian groups $H \rightarrowtail M \twoheadrightarrow Q/Z$ by $L(G) = \mathrm{Im}(\mathrm{Tor}(M,G) \rightarrow \mathrm{Tor}(Q/Z,G))$ for any abelian G. A characterization of the large subgroups L of any p-primary abelian group G is given by appropriately selecting H and M for each L. It is shown that the t-functors of Bode are a special case of the functors L. (Received November 26, 1971.)

*72T-A43. LEO J. ALEX, State University College of New York, Oneonta, New York 13820. Some exponential Diophantine equations which arise in finite groups.

In recent work, R. Brauer and the author have classified finite simple groups according to their orders. A first step in these classifications is the solution of the Diophantine equation $1 + x = y$, where x and y are positive integers and the possible primes dividing x or y are known. In this paper this equation is solved when the set of primes involved in x or y is $\{2,3\}$ or $\{2,3,7\}$ or $\{2,3,5,7\}$. (Received November 29, 1971.)

72T-A44. GEORGE A. GRÄTZER and J. SICHLER, University of Manitoba, Winnipeg, Manitoba, Canada. Free products of hopfian lattices. Preliminary report.

A lattice A is called hopfian if any onto endomorphism is an automorphism. A pair of hopfian lattices A and B is constructed (in fact, for every infinite cardinal $\mathfrak{m}$, we construct $2^{\mathfrak{m}}$ such pairs with $|A| = |B| = \mathfrak{m}$) such that $A * B$ (the free product of A and B) is not hopfian. (Received November 29, 1971.)

*72T-A45. GABRIEL SABBAGH, 116 avenue Saint-Exupéry, 92-Antony, France and University of Wisconsin, Madison, Wisconsin 53706. On the coherence of polynomial rings.

Let R be a commutative coherent ring with 1. It is shown in (Soublin, "Anneaux et modules cohérents," J. Algebra 15(1970)) that the ring of polynomials in one variable over R need not be coherent, although this must be the case if R is assumed to be von Neumann regular. Theorem. If R is a von Neumann regular ring or a Prüfer domain, then the ring of polynomials in an arbitrary number of variables over R is coherent. In view of this result, it is perhaps legitimate to ask whether polynomial rings over commutative semihereditary rings are coherent. We can only show that such rings have the following property: every ideal generated by at most two elements is finitely presented. (Received December 1, 1971.)

*72T-A46. S. H. COX, JR., University of California, Los Angeles, California 90024. Commutative endomorphism rings.

Two problems of W. V. Vasconcelos are partially resolved: (1) the total quotient ring T of a noetherian ring R is quasi-frobenius if and only if End(A) is commutative for each ideal A of R. (2) Let R be a ring whose total quotient ring is absolutely flat and E a finitely presented faithful R-module. If End(E) is semiprime then E is isomorphic to an ideal of R. Only commutative rings with unit and unital modules are considered. (Received December 2, 1971.)

*72T-A47. GEORGE A. GRÄTZER and H. LAKSER, University of Manitoba, Winnipeg, Manitoba, Canada. Two observations on the congruence extension property.

A pair of algebras $\mathfrak{A}$, $\mathfrak{B}$ with $\mathfrak{B}$ a subalgebra of $\mathfrak{A}$ is said to have the (principal) congruence extension property (abbreviated as PCEP and CEP, respectively) if every (principal) congruence relation of $\mathfrak{B}$ can be extended to $\mathfrak{A}$. A pair of algebras $\mathfrak{A}, \mathfrak{B}$ is constructed having PCEP but not CEP, solving a problem of A. Day (A. Day, "A note on the congruence extension property," Algebra Universalis 1 (1971), 234-235). A result of A. Day states that if $\mathfrak{B}$ is a subalgebra of $\mathfrak{A}$ and if for any subalgebra $\mathfrak{C}$ of $\mathfrak{A}$ containing $\mathfrak{B}$, the pair $\mathfrak{A}$, $\mathfrak{C}$ has PCEP, then $\mathfrak{A}$, $\mathfrak{B}$ has CEP. A new proof of this theorem that avoids the use of the axiom of choice is also given. (Received December 2, 1971.)

72T-A48. ANNE P. GRAMS, Florida State University, Tallahassee, Florida 32306. Factorization in certain power series rings. Preliminary report.

Let R be a commutative ring with identity. We say that R satisfies property (*) if $R[[X_1,\dots,X_n]]$ is a unique factorization domain (UFD) for each finite set $\{X_1,\dots,X_n\}$ of indeterminates over R; R satisfies property (**) if for some $m \geq 1$, $R[X_1,\dots,X_n][[Y_1,\dots,Y_m]]$ is a UFD for each finite set $\{X_1,\dots,X_n\}$ of indeterminates over R, disjoint from the set $\{Y_1,\dots,Y_m\}$. Let $\{Y_\lambda\}_{\lambda\in\Lambda}$ and $\{Y_\delta\}_{\delta\in\Delta}$ be disjoint sets of indeterminates over R. We consider the three power series rings $R[[\{X_\lambda\}]]_i$, $i = 1, 2, 3$. (For notation, see R. Gilmer, "Power series rings over a Krull domain," Pacific J. Math. 29(1969), 543-549.) Theorem 1. If Λ is infinite, then the following conditions are equivalent. (a) R satisfies property (*). (b) $R[[\{X_\lambda\}]]_2$ is a UFD. (c) $R[[\{X_\lambda\}]]_1[[Y_1,\dots,Y_m]]$ is a UFD for each $m \geq 0$. Theorem 2. If Λ is infinite, then R satisfies property (**) if and only if $R[\{X_\lambda\}][[Y_1,\dots,Y_m]]$ is a UFD for some $m \geq 1$. Theorem 3. (a) (R. Gilmer and Roger Taylor) $R[[\{X_\lambda\}]]_3[[\{Y_\delta\}]]_3 \cong R[[\{X_\lambda\} \cup \{Y_\delta\}]]_3$. (b) $R[[\{X_\lambda\}]]_2[[Y_1,\dots,Y_m]] \cong R[[\{X_\lambda\} \cup \{Y_1,\dots,Y_m\}]]_2$. (c) $R[[\{X_\lambda\}]]_1[[Y_1,\dots,Y_m]] \cong R[[\{X_\lambda\} \cup \{Y_1,\dots,Y_m\}]]_1$ if and only if Λ is finite. (Received December 9, 1971.)

72T-A49. WITHDRAWN.

*72T-A50. N. S. MENDELSOHN and STEPHEN H. Y. HUNG, University of Manitoba, Winnipeg, Manitoba, Canada. On Steiner handcuffed designs.

A handcuffed design $H(v, k, \lambda, u)$ is a system of k-subsets of a v-set called handcuffed blocks. In each block $\{A_1, A_2, \dots, A_k\}$ each element is assumed to be handcuffed to its neighbours. The elements $A_2, A_3, \dots, A_{k-1}$ are called interior elements. The following conditions must be satisfied for a handcuffed design. (1) Every pair of elements is handcuffed λ times and (2) every element appears u times as an interior element of a block. If $\lambda = 1$, the design is called a Steiner handcuffed design. It is proved that the following are necessary and sufficient for the existence of a Steiner handcuffed design. (a) $v \equiv 1 \bmod 4h$ or $v \equiv 1 \bmod (2h-1)$. (b) If $v \equiv 1 \bmod 4h$ then $k = 2h+1$ and $h = 1, 2, 3, \dots$; if $v \equiv 1 \bmod (2h-1)$ then $k = 2h$ and $h = 2, 3, 4, \dots$. (Received December 10, 1971.)

72T-A51. A. J. B. LOPES PINTO, Universidade de Luanda, Luanda, Angola. Alaoglu theorem. Preliminary report.

Let X and E be two topological linear spaces. $K \subset E$ is a closed cone with vertex at 0 which orders E. Let F(X,E) (C(X,E)) be the space of all functions (continuous functions) defined on all of X with values in E endowed with the simple convergence topology. K(X,E) (L(X,E)) is the family of convex (linear) functions; it is closed in F(X,E). It is well known (Bourbaki) that $\mathfrak{F} \subset F(X,E)$ is relatively compact iff for each $x \in E$ the set $\{f(x) : f \in \mathfrak{F}\}$ is relatively compact in E. On the other hand the interval $[-m,m] = (-m+K) \cap (m-K)$ is convex and closed and, for example, if K is a normal cone of a normed space, then $[-m, m]$ is bounded (Krasnoselskii). Let E be endowed with a topology τ such that $[-m, m]$ is $[\tau]$-compact. Alaoglu theorem. If $\mathfrak{F}$ is a family of convex functions on X into E which is uniformly majorized at x_0 (i.e., there exists an open neighborhood V of $0 \in X$ such that $f(x) - f(x_0) \leqq m$ for each $x \in x_0 + V$ and for $f \in \mathfrak{F}$), then $\mathfrak{F}$ is relatively compact in C(X,E). Particular cases are results in Alaoglu ("Weak convergence of linear spaces"), N. Bourbaki ("Espaces vectoriels topologiques," Livre V) and Horvath ("Topological vector spaces and distributions," vol. I). (Received November 26, 1971.)

*72T-A52. WITHDRAWN.

72T-A53. KENNETH KALMANSON, Montclair State College, Upper Montclair, New Jersey 07043. A combinatorial note on certain Minkowski spaces. Preliminary report.

Let (R^n, d) denote a Minkowski space with metric d whose unit sphere U_d is an n-dimensional polyhedron. We say that $S = \{P_1, \ldots, P_r\} \subset R^n$ is a d-cogeodesic subset of (R^n, d) if $d(P_{i_1}, P_{i_r}) = \sum_{k=1}^{r-1} d(P_{i_k}, P_{i_{k+1}})$ for some permutation of the points of S. Then, given a positive integer $r \geqq 2$, there exists a smallest positive integer $N_d(r)$ such that if the number of points of S is greater than $N_d(r)$, then S contains a d-cogeodesic subset of $r+1$ points. In particular, if d is (isometric to) the metric $d(X,Y) = \sum_{i=1}^{i=n} |x_i - y_i|$ in R^n, then $N_d(r) = r^{2^{n-1}} + 1$, $n \geqq 2$. (This is a generalization of a well-known theorem of Erdos and Szekeres concerning sequences of length $mn + 1$.) Upper bounds on $N_d(r)$ are given for other metrics of this type. (Received December 13, 1971.)

*72T-A54. DON GREENWELL and ROBERT L. HEMMINGER, Vanderbilt University, Nashville, Tennessee 37203. Total graphs and forbidden subgraphs. Preliminary report.

A graph G is said to have property P_n, n a positive integer, if G contains no subgraph homeomorphic to the complete graph K_{n+1} or the complete bipartite graph $K([(n+2)/2], \{(n+2)/2\})$; it is said to have property D_n if $dg(v) \leqq n$ for all $v \in V(G)$ and if v is a cutpoint of G whenever $dg(v) = n$. The following problem has been posed by Chartrand, Geller, and Hedetniemi ["Graphs with forbidden subgraphs," J. Combinatorial Theory 10(1970), 12-41]: Determine the values of n for which the total graph, T(G), has property P_{n+1} if and only if G has property D_n. At the time of the Chartrand, Geller, and Hedetniemi paper this statement was known to be true for $n = 2$ and 3. Since then Greenwell has shown it to be true for $n = 4$ and 5. He also showed that for each $n \geqq 6$ there are graphs G with property D_n for which T(G) does not have property P_{n+1}. In this note we will show that the condition is necessary for $n = 6$ and 7 but that it is not necessary for $n \geqq 8$. The proof relies on a forbidden subgraph characterization of graphs having property D_n. (Received December 13, 1971.)

*72T-A55. STUART A. STEINBERG, University of Toledo, Toledo, Ohio 43606. Quotient rings of f-rings. Preliminary report.

An f-ring R with $r(R) = 0$ is a qf-ring if its Utumi maximal left quotient ring Q can be made into an f-ring extension of R [Anderson, Canad. J. Math. 27(1965)]. The f-ring R is said to be left convex if each of its left ideals is a left 1-ideal. Theorem 1. A left convex f-ring R is a qf-ring. If R has regular elements, then it is a left Ore ring and a classical quotient ring of its subring of bounded elements. Theorem 2. If R is left convex and has the maximum condition on polars (i.e., ${}_R R$ is finite dimensional), then it is unital and is the direct sum of totally ordered left convex f-rings. Theorem 3. There is exactly one totally ordered right injective ring U without an identity element. If R is a right injective f-ring without 1, then R is the direct sum of U and a right injective unital f-ring. A right injective unital f-ring is left convex. (Received December 17, 1971.)

72T-A56. WITHDRAWN.

*72T-A57. N. S. MENDELSOHN and Y. S. HO, University of Manitoba, Winnipeg, Manitoba, Canada. Inequalities for t-designs with repeated blocks.

Inequalities for B.I.B. designs have been obtained by R. A. Fisher, H. B. Mann, R. J. Stanton and D. Sprott and D. Raghavarav, relating block size to the other parameters. All these results are generalized to t-designs yielding much sharper inequalities. The main inequality is $b \geq dv(\lambda_1 - \lambda_t)/(t-1)(\lambda_{t-1} - \lambda_t)$. The parameters are all standard except that d represents the number of times a single block is replicated. (Received December 27, 1971.)

*72T-A58. RONALD ALTER, Department of Computer Science, University of Kentucky, Lexington, Kentucky 40506. The Diophantine equation $3^n - 11 = x^2$.

It is known that the Diophantine equation, $x^2 + 7 = 2^n$, has a solution only in the five cases $x = 1, 3, 5, 11$ and 181. It was recently proven that the equation, $x^2 + D = p^n$ where $p = (D+1)/4$ and $D \equiv 3 \pmod 8$, has no solution in integers for $D \geq 19$. In the present paper the remaining case, $D = 11$, is resolved and it is shown that the only integer solution of the title equation occurs when $x = 4$. Also, a sequence of integers related to the title equation is discussed. (Received December 23, 1971.)

*72T-A59. JOSEPH E. KIST, New Mexico State University, Las Cruces, New Mexico 88001. Two characterizations of commutative Baer rings.

If X is a nonempty set, and if Z is the ring of integers, then Z^X is a Baer ring, i.e., the annihilator of each finite subset of Z^X is the principal ideal generated by an idempotent. D. Scott (Proc. Sympos. Foundations of Mathematics, Pergamon Press, London, 1961) has shown that there is a bijection of the set of minimal prime ideals of Z^X upon the set of ultrafilters on X. The author generalizes this result by showing that a semiprime commutative ring A with identity is a Baer ring if and only if the mapping $Q \to Q \cap E$ is a homeomorphism of the space of minimal prime ideals of A with the Boolean space of the Boolean algebra E of idempotents in A. As a

second characterization, it is shown that a semiprime commutative ring A with identity is a Baer ring if and only if the minimal prime spectrum of A is a retract of the prime spectrum of A. (Received January 3, 1972.)

*72T-A60. J. L. BRENNER (University of Arizona), 10 Phillips Road, Palo Alto, California 94303 and J. LIM, University of Victoria, Victoria, British Columbia, Canada. The matrix equations A = XYZ, B = ZYX and related ones.

First assume A, B both invertible. (1) If the equations A = XYZ, B = ZYX [XZY, YXZ, ZXY, YZX] are solvable in any group, then AB^{-1} is a commutator. The converse is true for A = XYZ, B = ZYX [B = XZY, YXZ], but not for B = ZXY or YZX. (2) Application of results of Shoda, Ree and Thompson yields (i) If SL(n,K) is the multiplicative group of all invertible n × n matrices with determinant 1 over a field K, then the equations A = XYZ, B = ZYX (or XZY or YXZ) are solvable in (a) SL(n,2), n > 2; (b) SL(n,3), n > 2; (c) SL(n,m), m > 3; (d) SL(n,K), K algebraically closed. (ii) In case A, B have the same determinant, the same equations are solvable in invertible n × n matrices over C or in invertible 2 × 2 matrices over F_3. (iii) In a connected semisimple algebraic group over an algebraically closed field, the equations are solvable. (iv) If A, B are quaternions with $\|A\| = \|B\|$, same conclusion. If n ≦ 3 and A, B are n × n real or complex singular matrices, A = XYZ, B = ZYX are solvable. For all n, solvability of these equations implies $|\text{rank } A - \text{rank } B| \leqq 2n/3$. For n ≧ 4, the same equations are solvable if A, B are both diagonal and of equal rank. This article will appear in Canad. J. Math. (Received January 6, 1972.)

72T-A61. DAVID KELLY, Queen's University, Kingston, Ontario, Canada. A combinatorial identity.

Latin letters always represent nonnegative integers unless stated otherwise and [r,s] denotes the set of integers x such that r ≦ x ≦ s. Let σ(n,k,m) be the number of maps $\varphi : [1, n] \to [1,m]$ such that $\varphi([1,n]) \supseteq [1,k]$. Theorem. For k ≦ m, $\sigma(n,k,m) = \sum_{i=0}^{k} (-1)^i \binom{k}{i} (m-i)^n$. The theorem is proved by induction on m, starting with the known result when m = k. The case k = 3 was announced by Kim Ki-Hang Butler [Abstract 71T-A68, these *Notices* 18(1971), 549]. Corollary 1. For n < k ≦ m, $\sum_{i=0}^{k} (-1)^i \binom{k}{i} (m-i)^n = 0$. Corollary 2. If a and b are elements of a commutative ring R and n < k, then the following equation holds in R: $\sum_{i=0}^{k} (-1)^i \binom{k}{i} (a+ib)^n = 0$. (Received January 7, 1972.)

*72T-A62. K. BROOKS REID, Louisiana State University, Baton Rouge, Louisiana 70803. The number of graphs on n vertices with 3 cliques.

A clique in a graph G is a maximal complete subgraph of G. M. Rosenfeld (Amer. Math. Monthly 78 (1971), 49-50) proposed the problem of determining c(n,k), the number of nonisomorphic graphs on n vertices with exactly k cliques. Clearly, c(n,1) = 1, and Rosenfeld showed $c(n,2) = [n^2/4]$. We have determined c(n,3) to be: $c(n,3) = (1/240)(n)(n^2-4)(n^2-6) + p(n,3)$, n even; $c(n,3) = (1/240)(n)(n^2-1)(n^2-9) + p(n,3)$, n odd; where, $p(n,3) = (1/72)(n)(2n^2+3n-6)$, n ≡ 0; $p(n,3) = (1/72)(n-1)(2n^2+5n-1)$, n ≡ 1; $p(n,3) = (1/72)(n-2)(2n^2+7n+8)$, n ≡ 2; $p(n,3) = (1/72)(n-3)(2n^2+9n+21) + 1$, n ≡ 3; $p(n,3) = (1/72)(n-4)(2n^2+11n+38) + 2$, n ≡ 4; $p(n,3) = (1/72)(n-5)(2n^2+13n+59) + 4$, n ≡ 5; and all congruences are modulo 6. (Received January 10, 1972.)

72T-A63. DAVID WHITMAN, San Diego State College, San Diego, California 92115. Chain conditions in commutative semigroup rings.

Let R be a commutative ring with identity and S a commutative semigroup with identity. Let R(S) denote the semigroup ring of R over S. Theorem 1. If R(S) satisfies the descending chain condition on ideals then S is finite. Theorem 2. If R(S) satisfies the ascending chain condition on ideals, then S is finitely generated. (Received January 10, 1972.)

72T-A64. E. M. WRIGHT, University of Aberdeen, Aberdeen, Scotland, United Kingdom. Unlabelled graphs with many nodes and edges. II. Preliminary report.

An (n,q) graph is one with n nodes and q edges, in which any two different nodes are or are not joined by a single edge. $F = F(n,q)$ (resp. T) is the number of (n,q) graphs with labelled (resp. unlabelled) nodes; f (resp. t) is the number of these graphs which are connected; $\alpha = f/F$ (resp. $\beta = t/T$) is the probability that an (n,q) graph is connected. We write $\gamma = 2(q/n) - \log n$. Erdös and Renyi (Publ. Math. Debrecen 6(1959), 290-297) showed that if $\gamma \to c$, then $\alpha \to \exp(-e^{-c})$. For fixed n, the number α increases (nonstrictly) with q. Hence, $\alpha \to 1$ or 0 if $\gamma \to \infty$ or $-\infty$. We prove that if $\gamma \to c > 0$, then $\beta \to 1 - e^{-c}$. Since β can decrease as q increases, we cannot deduce that $\beta \to 1$ if $\gamma \to +\infty$ and that $\beta \to 0$ if $\gamma \to c \leqq 0$ or if $\gamma \to -\infty$, but we can prove these results otherwise. (Received January 10, 1972.)

*72T-A65. IRVING REINER and STEPHEN ULLOM, University of Illinois, Urbana, Illinois 61801. A Mayer-Vietoris sequence for class groups.

Let R be a Dedekind domain in the algebraic number field K, let Λ be an R-order in a semisimple K-algebra A, and let Λ' be a maximal R-order in A containing Λ. Denote by $C(\Lambda)$ the "locally free class group" of Λ, and let $D(\Lambda)$ be the kernel of the epimorphism $C(\Lambda) \to C(\Lambda')$. Suppose that Λ is the fibre product of a pair of R-orders Λ_1, Λ_2, relative to maps $\varphi_i : \Lambda_i \to \bar{\Lambda}$, $i = 1, 2$, where $\bar{\Lambda}$ is an R-torsion R-algebra. Let $u(\Lambda_i)$ denote the group of units of Λ_i, and let $u^*(\Lambda_i)$ be its image in $u(\bar{\Lambda})$. Theorem. If A satisfies the Eichler condition, then the sequence $1 \to u^*(\Lambda_1)u^*(\Lambda_2) \to u(\bar{\Lambda}) \to D(\Lambda) \to D(\Lambda_1) + D(\Lambda_2) \to 0$ is exact. (This is analogous to Milnor's Mayer-Vietoris sequence.) A corresponding theorem holds in the non-Eichler case. Applications include calculation of D(ZG) for various groups G (dihedral of order 2p, dihedral or quaternion of order 8). Theorem. If $\{G_i\}$ is any sequence of abelian groups of composite order such that $|G_i| \to \infty$, then also $|D(ZG_i)| \to \infty$. (Received January 11, 1972.)

72T-A66. VASANTI A. JATEGAONKAR, Cornell University, Ithaca, New York 14850. Global dimension of tiled orders over a DVR. Preliminary report.

Throughout, R denotes a DVR with the quotient field K. An R-order Λ in $M_n(K)$ is tiled if $e_{ii} \Lambda e_{jj} \subseteq \Lambda$, where e_{jj} are the usual matrix units in $M_n(K)$ for $1 \leqq i, j \leqq n$. Thus $\Lambda = (\Lambda_{ij}) \subseteq M_n(K)$ for some fractional ideals Λ_{ij} in R. Theorem 1. Up to conjugation there are only finitely many tiled R-orders in $M_n(K)$ whose global dimension is finite. Theorem 2. There is DCC on tiled R-orders of finite global dimension in

$M_n(K)$. This was conjectured by R. B. Tarsey ["Global dimension of orders," Trans. Amer. Math. Soc. 151 (1970), 335-340]. Theorem 3. Let $\Lambda = (\Lambda_{ij}) \subseteq M_n(R)$ be a tiled R-order of finite global dimension. Let $\Gamma = (\Gamma_{ij}) \subseteq M_n(R)$, where $\Gamma_{ij} = \Lambda_{ij}$ for $i \geq j$ and $\Gamma_{ij} = R$, if $i < j$. If Γ is a ring, then gl. dim. $\Gamma < \infty$. For structure theoretic characterization of such rings we refer to [Abstract 71T-A107, these *Notices* 18(1971), 626]. (Received January 12, 1972.) (Author introduced by Professor Alex Rosenberg.)

72T-A67. MIECZYSLAW ALTMAN, Louisiana State University, Baton Rouge, Louisiana 70803. Generalized factorization theorems in Banach algebras. Preliminary report.

A subset U of a Banach algebra A is called an l.a.i. (left approximate identity) for A if for every arbitrary finite subset $\{a_1,\ldots,a_n\}$ of A and $\epsilon > 0$ there exists u in U such that $\|x_i - ux_i\| < \epsilon$ $(i = 1,\ldots,n)$. If $n = 1$, U is called a weak or simple l.a.i. Lemma. If U is a bounded weak l.a.i., then $U \circ U = [w \in A \mid w = v+u-vu; v, u \in U]$ is a bounded l.a.i. Theorem 1. Let X be a left Banach A-module, A having a bounded weak l.a.i. Then $A \cdot X$ is a closed linear subspace of X. For arbitrary z of $A \cdot X$ and $r > 0$ there exist a in A and x in X such that $z = a \cdot x$ and $\|x - z\| < r$, x being in the closure of $A \cdot z$. Other factorization theorems remain also true when replacing the bounded l.a.i. by the bounded weak l.a.i. Theorem 2. Let U be a bounded subset of A satisfying:(a) for arbitrary u of $U \cup U \cdot U$ and $\epsilon > 0$ there exists v in U such that $\|u - vu\| < \epsilon$; (b) for arbitrary a of the form $a = u - vu$ $(u, v \in U)$ and $\epsilon > 0$ there exists w in U such that $\|a - wa\| < \epsilon$. Then $U \circ U$ is a bounded l.a.i. for the Banach algebra generated by U. If U is commutative, then (b) is needless. (Received January 12, 1972.)

*72T-A68. GUENTER R. KRAUSE, University of Manitoba, Winnipeg, Manitoba, R3T 2N2, Canada. A class of rings with projective left socle. Preliminary report.

A left noetherian ring R with identity is called a left quasi-Matlis-ring if the map $E \to \mathrm{Ass}(E)$ induces a bijective map from the set of isomorphism classes of injective, indecomposable left R-modules onto the set of all two-sided prime ideals of the ring R. Theorem. Let R be a left noetherian left quasi-Matlis-ring with zero left singular ideal. Then: (a) R is either a semisimple artinian ring or every essential left ideal contains a product of finitely many prime ideals not associated with the left R-module ${}_RR$. (b) R is left bounded, i.e. every essential left ideal of R contains a nonzero two-sided ideal. (c) The left socle of R is a projective left and right R-module. (Received January 13, 1972.)

*72T-A69. C. J. EVERETT and P. R. STEIN, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico 87544. The asymptotic number of (0,1)-matrices with zero permanent.

It is shown that $Z(n)/f(n) \to 1$, where $Z(n)$ is the number of n-square matrices $X = [x_{ij}]$, $x_{ij} \in \{0,1,\ldots,M-1\}$, $M \geq 2$, with Per $X = 0$, and $f(n) = 2nM^N$, $N = n^2 - n$. The form of f(n) for the case of combinatorial interest, $M = 2$, was suggested by P. Erdös. (Received January 13, 1972.)

*72T-A70. WALTER F. TAYLOR, University of Colorado, Boulder, Colorado 80302. Fixed points of endomorphisms. Preliminary report.

An algebra $\mathfrak{A}$ has the fixed point property iff the following is true: for every endomorphism $f: \mathfrak{A} \to \mathfrak{A}$, there exists a in the universe of $\mathfrak{A}$ such that $f(a) = a$. Theorem. Let $\mathcal{V}$ be a variety of algebras. Every algebra in $\mathcal{V}$ has the fixed point property if and only if there exists a one-variable term α such that the equation $\alpha(x_0) = \alpha(x_1)$ holds identically in $\mathcal{V}$. (Received January 17, 1972.)

72T-A71. THOMAS E. HAYS, University of Tennessee, Knoxville, Tennessee 37916. $\leqq(\aleph)$-stability. Preliminary report.

Definition. A semigroup S is $\leqq(\aleph)$-stable in a category $\mathcal{P}$ of semigroups and homomorphisms if $S \subseteq T$ with $T \in \mathcal{P}$ implies $\leqq(\aleph_S) = \leqq(\aleph_T) \cap (S \times S)$. Results of the following nature are established. Theorem. (a) A semilattice of groups is $\leqq(\aleph)$-stable in the category of all semigroups and homomorphisms. (b) Compact monothetic semigroups are $\leqq(\aleph)$-stable in the category $\mathcal{S}$ of compact semigroups and homomorphisms. (c) Let S be a Γ-compact monoid and T be a Γ-compact semigroup with zero. If S and T are $\leqq(\aleph)$-stable in $\mathcal{S}$, then so is the contact extension of S by T. (d) Compact totally $\aleph$-ordered semigroups are $\leqq(\aleph)$-stable in $\mathcal{S}$. (e) ML-semigroups are $\leqq(\aleph)$-stable in $\mathcal{S}$. (f) Let $S = G\,\mathrm{Horm}(X, S_x, m_{xy})$ with $S_x \leqq(\aleph)$-stable in $\mathcal{S}$ for each $x \in X$. Then S is $\leqq(\aleph)$ stable in $\mathcal{S}$. (Received January 17, 1972.)

*72T-A72. DONALD A. CHAMBLESS, University of Florida, Gainesville, Florida 32601. Representation of the projectable and strongly projectable hulls of a lattice-ordered group.

It is known that a representable ℓ-group G has unique minimal projectable and strongly projectable essential extensions. In this paper these projectable and strongly projectable "hulls" of G are represented as direct limits of certain directed systems of ℓ-groups and ℓ-isomorphisms in a fashion similar to that in which P. Conrad [Proc. London Math. Soc. 19(1969), 444-480] has represented the orthocompletion of G. It is also shown that if G is an f-ring, or f-ring without nonzero nilpotent elements, then so are the projectable and strongly projectable hulls of G. (Received January 17, 1972.)

*72T-A73. WILLIAM J. HEINZER and JACK E. OHM, Purdue University, Lafayette, Indiana 47907. The finiteness of I when R[X]/I is R-flat. II.

Let R be a commutative ring with identity, let X be an indeterminate, and let I be an ideal in R[X]. Theorem. If R[X]/I is a flat R-module, then I is locally principal at primes of R[X]. Theorem. If R has only a finite number of minimal prime ideals and R[X]/I is R-flat, then I is a finitely generated ideal. Also concerning the question of classifying what rings R have the property that R[X]/I is R-flat implies I is finitely generated, an example is given of a quasi-local ring R and an ideal I in R[X] such that R[X]/I is R-flat, but I is not a finitely generated ideal of R[X]. These results are closely connected with the work of Ohm-Rush announced in Bull. Amer. Math. Soc. 77(1971), 793-796; Abstract 683-A7, these *Notices* 18(1971), 356. (Received January 17, 1972.)

Analysis

*72T-B26. H. M. SRIVASTAVA, University of Victoria, Victoria, British Columbia, Canada and J. P. SINGHAL, University of Jodhpur, Jodhpur, India. A unified presentation of certain classical polynomials.

This paper attempts at presenting a unified treatment of the classical orthogonal polynomials, viz. Jacobi, Laguerre and Hermite polynomials, and their generalizations introduced from time to time. The authors study here the polynomial system $\{T_n^{(\alpha,\beta)}(x,a,b,c,d,p,r) \mid n = 0,1,2,\ldots\}$ defined by (*) $n!\, T_n^{(\alpha,\beta)}(x,a,b,c,d,p,r) = (ax+b)^{-\alpha}(cx+d)^{-\beta} \cdot \exp(px^r) D^n \{(ax+b)^{n+\alpha}(cx+d)^{n+\beta}\exp(-px^r)\}$, where $D = d/dx$, and α, β, a, b, c, d, p, r are arbitrary parameters. The results obtained here include a number of linear, bilinear and bilateral generating functions and operational formulas for the polynomials defined by (*). (Received September 24, 1971.)

*72T-B27. CHARLES F. DUNKL and DONALD E. RAMIREZ, University of Virginia, Charlottesville, Virginia 22903. Multipliers on modules over the Fourier algebra. Preliminary report.

Let G be an infinite compact group and $\hat{G}$ its dual (we use the notation from our book ["Topics in harmonic analysis", Appleton-Century-Crofts, New York, 1971]). For $1 \leq p < \infty$, $\mathcal{L}^p(\hat{G})$ is a module over $\mathcal{L}^1(\hat{G}) \cong A(G)$, the Fourier algebra of G. For $1 \leq p, q < \infty$, let $\mathcal{M}_{p,q} = \mathrm{Hom}_{A(G)}(\mathcal{L}^p(\hat{G}), \mathcal{L}^q(\hat{G}))$. If G is abelian, then $\mathcal{M}_{p,p}$ is the space of $L^p(\hat{G})$-multipliers. For $1 \leq p < 2$ and p' the conjugate index of p, $A(G) \cong \mathcal{M}_{1,1} \subset \mathcal{M}_{p,p} = \mathcal{M}_{p',p'} \subsetneq \mathcal{M}_{2,2} \cong L^\infty(G)\hat{}$. Further, the space $\mathcal{M}_{p,p}$ is the dual of a space called $\mathcal{A}_p$, a subspace of $\mathcal{C}_0(\hat{G})$. Using a method of J. F. Price [Trans. Amer. Math. Soc. 152(1970), 321-330] we show that $\bigcup \{\mathcal{M}_{q,q} : 1 \leq q < p\} \subsetneq \mathcal{M}_{p,p} \subsetneq \bigcap \{\mathcal{M}_{q,q} : p < q \leq 2\}$ (where $1 < p < 2$). Finally $\mathcal{M}_{q,p} = \{0\}$ for $1 \leq p < q < \infty$ (the proof of this statement is due to our colleague John Fournier). (Received October 18, 1971.)

*72T-B28. MARY R. EMBRY, University of North Carolina, Charlotte, North Carolina 28213. Strictly cyclic operator algebras on a Banach space.

A strictly cyclic operator algebra $\mathcal{A}$ on a complex Banach space X (dim $X \geq 2$) is a uniformly closed subalgebra of $\mathcal{L}(X)$ such that $\mathcal{A}x = X$ for some x in X. If $Ax = 0$, $A \in \mathcal{A}$ implies that $A = 0$, then $\mathcal{A}$ is separated. Theorem 1. If $\mathcal{A}$ is strictly cyclic and separated, then $\mathcal{A}$ has a maximal, proper, closed invariant subspace. Theorem 2. If $\mathcal{A}$ is strictly cyclic, then $\mathcal{A}'$ (the commutant of $\mathcal{A}$) has a proper closed invariant subspace. Theorem 3. If $A \in \mathcal{L}(X)$, $A \neq zI$ and $\{A\}'$ is strictly cyclic, then $\{A\}'$ has a proper closed invariant subspace. Theorem 4. A transitive subalgebra of $\mathcal{L}(X)$, containing a strictly cyclic algebra which contains the identity operator I, is strongly dense in $\mathcal{L}(X)$. The proofs of Theorems 1-3 depend upon the open mapping theorem and the proof of Theorem 4 depends upon Arveson's density theorem. (Received October 29, 1971.)

*72T-B29. DAVID BORWEIN and J. H. RIZVI, University of Western Ontario, London 72, Ontario, Canada. Strong Abel-type summability.

Let $\epsilon_n^\lambda = \binom{n+\lambda}{n}$, $s_n = \sum_0^n u_r$, $s_\lambda(y) = (1+y)^{-\lambda-1}\sum_{n=0}^\infty \epsilon_n^\lambda s_n(y/(1+y))^n$, $u_\lambda(y) = (1+y)^{-\lambda-1}\cdot\sum_{n=0}^\infty \epsilon_n^\lambda u_n(y/(1+y))^n$, $U_\lambda(y) = \lambda\int_0^y u_\lambda(t)dt$. The sequence $\{s_n\}$ is said to be strongly A_λ-convergent to ℓ if $\int_0^y |s_{\lambda+1}(t) - \ell|\,dt = o(y)$, as $y \to \infty$, and we write $s_n \to \ell[A_\lambda]$. The sequence $\{s_n\}$ is said to be strongly A'_λ-convergent to ℓ if $\int_0^y |U_{\lambda+1}(t) - \ell|dt = o(y)$, as $y \to \infty$ and we write $s_n \to \ell[A'_\lambda]$. If $\{h_n\}$ is a Hausdorff transform of the sequence $\{s_n\}$ and $h_n \to \ell[A_\lambda]$, then $s_n \to \ell[A_\lambda H_\lambda]$. The main results proved are: Theorem 1. For $\lambda > 0$, $s_n \to \ell[A_\lambda]$ if and only if $s_n \to \ell[A'_\lambda]$ and $n \cdot u_n \to o[A_{\lambda-1}]$. Theorem 2. For $\lambda > 0$, $s_n \to \ell[A'_\lambda]$ if and only if $s_n \to \ell[A_{\lambda-1}]$. Theorem 3. If $\lambda > -1$, H_χ is a regular Hausdorff method and $s_n \to \ell[A_\lambda]$, then $s_n \to \ell[A_\lambda H_\lambda]$. (Received November 1, 1971.)

72T-B30. JOHN C. OXTOBY, Bryn Mawr College, Bryn Mawr, Pennsylvania 19010. Horizontal chord theorems.

For any plane set K, the set $H(K) = \{h \geq 0 : (x+h, y) \in K \text{ for some } (x,y) \in K\}$ is called the chord set of K. H. Hopf [Comment. Math. Helv. 9(1937), 303-319] showed that the chord set of any plane continuum is compact and co-additive, and that any compact, co-additive set $H \subset [0,\infty)$ is the chord set of some continuous function f on $[0,B]$, where $B = \sup H$. It is shown that f can always be taken to be of class C^∞. f cannot be required to be analytic on $(0,B)$, because the boundary of the chord set of f would then be countable, whereas the boundary of H need not be. If g is continuous on R, and if for each x_0 and $\epsilon > 0$ the set $\{x : |g(x) - g(x_0)| < \epsilon\}$ is unbounded above, then $H(g) = [0,\infty)$. This generalizes a theorem of M. C. Tews [Amer. Math. Monthly 77(1970), 729-731] that a continuous almost periodic function has every chord. (Received November 3, 1971.)

72T-B31. K. V. RAJESWARA RAO, Purdue University, Lafayette, Indiana 47907. On the boundedness of automorphic forms.

Let Γ be a Fuchsian group acting on the unit disc U of the complex plane with fundamental region Ω in U. Let $\lambda(z) = (1 - |z|^2)^{-1}$ and $dm = \lambda^2 dx\,dy$. For a fixed real number $q > 1$ and a given system of factors of automorphy of dimension $-2q$ for Γ, let $A_q^p(\Gamma)$ be the space of holomorphic, automorphic forms f of dimension $-2q$ such that $\lambda^{-q} f \in L^p(\Omega, dm)$. Let $\alpha_q(z,\zeta)$ be the Bergman kernel for the Hilbert space $A_q^2(\Gamma)$. Theorem. If, for some finite $p \geq 1$, $A_q^p(\Gamma) \subset A_q^\infty(\Gamma)$, then (*) $\sup_{z\in U} \lambda^{-2q}(z)\,\alpha_q(z,z) < \infty$. If (*) holds, then $A_q^p(\Gamma) \subset A_q^\infty(\Gamma)$ for all $p \geq 1$. Remarks. (i) The theorem and proof remain valid for a Kleinian group Γ with U replaced by an invariant union D of components of Γ and λ by the Poincaré metric for D. (ii) The result is an extension of one jointly due to T. A. Metzger and the author (Proc. Amer. Math. Soc. 28(1971), 562-566). (Received November 4, 1971.)

72T-B32. HOWELL K. WILSON, Southern Illinois University, Edwardsville, Illinois 62025.

Nonautonomous Liapunov functions.

In (S) $x' = f(x,t)$, $t \geq \beta$, let f be continuous. If φ is a solution, τ_φ^+ is its escape time. $V(x,t)$ is a gauge function if it is continuous, locally Lipschitzian, and bounded below in t for bounded x. DV is its derivative with respect to (S). The intervals $[a_r, b_r]$ form an ω-test sequence if $b_r \leq a_{r+1}$. Let V be a gauge function for which there is a W satisfying $DV \leq -W$ and define $E = \{x : \limsup_{r\to\infty} \int_{a_r}^{b_r} W(x,s)ds < \infty$ for at least one ω-test sequence with $\sum_{r=1}^{\infty}(b_r - a_r) = \infty\}$. $V(x,t)$ is a nonautonomous Liapunov function for (S) if: (a) for $p \notin E$, there is a $\lambda > 0$ such that $W(x,t) \geq \lambda W(p,t)$ for all x in a neighborhood of p and all $t \in [\beta, \infty) - A$, where A has finite measure; (b) there is a γ such that $W(x,t) \geq \gamma(t)$ for all x and a.a. $t \geq \beta$ and $\liminf_{t\to\infty} \int_b^t W(x,s)\,ds > -\infty$. Equation (S) locally defines uniformly continuous velocities if there is a uniformly continuous M on $[\beta, +\infty)$ such that $\left|\int_s^t f(x,u)\,du\right| \leq |M(t) - M(s)|$ for all $t, s \geq \beta$ and x in a bounded set.

Theorem. Let (S) have a nonautonomous Liapunov function and locally define uniformly continuous velocities. Then, if φ does not escape as $t \to \tau_\varphi^+$, $\varphi(t) \to E$. (Received November 5, 1971.)

72T-B33. WILLIAM D. L. APPLING, North Texas State University, Denton, Texas 76203. Distribution functions and set function summability.

U, F, p, p_A^+, and the notions of subdivision, refinement and summability are as in previous abstracts of the author. "$E \ll D$" means "E is a refinement of D". Suppose A is in p and m is in p_A^+. If $G \ll \{U\}$ and x is in $\mathbb{R}$, let $G(x) = \{I : I \text{ in } G, A(I) < x\}$. For each s in $\mathbb{R}$, let $h(s) = \inf\{\sup\{\sum_{E(s)} m(I) : E \ll D\} : D \ll \{U\}\}$, and for each t in $\mathbb{R}$, let $g(t) = \inf\{h(s) : t < s\}$. The preceding definitions and following theorem are extended to "many-valued" A, but are stated here in "single-valued" form for simplicity. Theorem. The following two statements are equivalent: (1) A is m-summable; (2) the following three statements hold: (i) if x is in $\mathbb{R}$ and $0 < c$, then there is $r > x$ such that if $x < s < r$, then there is $D \ll \{U\}$ such that if $E \ll D$, then $|g(x) - \sum_{E(s)} m(I)| < c$, (ii) $g(x) - g(-x) \to m(U)$, $x \to \infty$, (iii) $\int_{-\infty}^{+\infty} x\,dg(x)$ exists. (Received November 8, 1971.)

*72T-B34. JOSEPH M. LAMBERT, Pennsylvania State University, York, Pennsylvania 17403.

Simultaneous approximation and interpolation in L_1 and C(T).

The concept of simultaneous approximation and interpolation which preserves the norm (SAIN) was introduced in [J. Approximation Theory 2(1969), 355-373]. One can show that (L_1, M, T) has property SAIN where M is the dense subspace of L_1 consisting of those functions with support of finite measure and T is any finite dimensional subspace of L_∞. In the space $C[0,1]$, a function f, which attains its norm finitely often, can be approximated and interpolated at any finite number of linear functionals by some polynomial whose norm equals the norm of f. Both results follow from Theorem. Let X be a normed linear space, M a dense subspace of X, and $L = \text{span}(g_1, g_2, \ldots, g_n)$ a subspace of X*. Given x in X of norm one, let F be the minimal closed unit ball extremal subset containing x. If $F \cap M$ is dense in F, then given $\epsilon > 0$ there exists m in M of norm one such that $g_i(x) = g_i(m)$, $i = 1, \ldots, n$ with $\|x - m\| < \epsilon$. (Received November 10, 1971.)

72T-B35. MAHMUD HAIFAWI, Middle East Technical University, Ankara, Turkey. Orthogonal complements in nonarchimedean Banach spaces.

Let E be a nonarchimedean Banach space satisfying the condition: "For every closed proper subspace V of E there exists at least one element ($\neq 0$) of $E \setminus V$ orthogonal to V." Theorem. Every closed proper subspace of E has an orthogonal complement. Theorem. The orthogonal projection property is valid in E. Theorem. Every discrete nonarchimedean Banach space satisfies the above condition. Theorem. Every finite dimensional n.a. Banach space satisfies the above condition. Theorem. E possesses an orthogonal basis. Furthermore, one can give an example of a space satisfying the above condition having a nonspherically complete subspace, which shows that spherical completeness is not a necessary condition for the existence of orthogonal complements. (Received September 27, 1971.) (Author introduced by Dr. Tunc Geveci.)

*72T-B36. GORDON G. JOHNSON, University of Houston, Houston, Texas 77004. Mean value iteration in Hilbert space.

Theorem. If E is a convex, compact subset of a Hilbert space and T is a function from E into E satisfying the condition that $\|Tx - Ty\| \leq \|x - y\|$ if each of x and y is in E, then the point sequence $x_1, x_2, \ldots$ converges to a fixed point of T where x_1 is any point in E and for each positive integer n, $x_{n+1} = [1/(n+1)]\,[Tx_n + nX_n]$. (Received November 5, 1971.)

72T-B37. SHAWKY E. SHAMMA and DONALD P. SQUIER, University of West Florida, Pensacola, Florida 32504. Some representations of solutions of elliptic equations.

The Dirichlet problem for the region of the plane inside closed smooth curve C for second-order elliptic equations is considered. It is shown that under certain circumstances the solution u can be written uniquely in the form $u(P) = \int_C F(P,Q)g(Q)\,ds_Q$ where F(P,Q) is the fundamental solution of the elliptic equation and $g \in L^2$ if the boundary value function f is absolutely continuous with square integrable derivative ($f \in W$); and $u(P) = \rho(F(P, \cdot))$ where ρ is a unique bounded linear functional on W if $f \in L^2$. These representations are valid in the exterior of C also. As special cases with slight modifications, the exterior Dirichlet problems for the Helmholtz and Laplace equations are mentioned. It is shown also that if kernel F(P',Q), with P' and Q on C, has a complete set of eigenfunctions $\{\psi_k(P')\}$ then u(P) can be expanded in a series of their extensions $\{\psi_k(P)\}$ where $\psi_k(P) = \lambda_k \int_C F(P,Q)\,\psi_k(Q)\,ds_Q$. (Received November 5, 1971.)

72T-B38. AVRAHAM UNGAR, Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario, Canada. P-algebras. Preliminary report.

The term P-algebras refers to algebras of primitive solutions of basic partial differential equations. To say roughly, a P-algebra is a triple $\{P,A,F\}$ where P is a set of solutions of the partial differential equation $A\varphi = 0$ and F is a family of functions (satisfying conditions that make it wide enough) whose elements are called analytic functions such that if $f \in F$ and $p_1, p_2, \ldots, p_n$ are elements of P then $f(p_1, p_2, \ldots, p_n)$ is a solution of $A\varphi = 0$. A partial differential equation having a primitive solution is said to be basic. A well-known

example of a P- algebra is the z-algebra of the complex numbers where z is the primitive solution $z = x + iy$ of Laplace's equation. The n-dimensional Laplace equation is basic while the heat equation is not. A general procedure for constructing P-algebras has been established. Example. $u = \exp\{-\sinh^{-1}(z/r)+i\theta\}$ is a primitive Laplace function and every function f(u) satisfying $\nabla f(u) = \dot{f}(u)\nabla u$ (the analyticity condition) is also a Laplace function. Some properties of the algebra of complex numbers are shown to remain valid also for P-algebras. As an application to the theory of wave propagation, the Cagniard problem (see Cagniard, McGraw-Hill, New York, 1962) has been solved in a simple way without using Laplace transforms. (Received December 2, 1971.)

*72T-B39. THOMAS H. MacGREGOR, State University of New York, Albany, New York 12203. Applications of extreme point theory to problems on univalent functions. Preliminary report.

Let S denote the family of analytic functions which are univalent for $|z| < 1$ and satisfy $f(0) = 0$ and $f'(0) = 1$, and let St, K and C denote the subfamilies of S consisting of starlike, convex and close-to-convex mappings, respectively. In [Trans. Amer. Math. Soc. 156(1971), 91-107] the extreme points of St, K and C were determined. This information is used to solve extremal problems (not necessarily linear) over St, K and C as well as over families of related functions. This provides a systematic development of a number of classical results as well as the solution to some unsolved problems, as the following. If $f(z) = \sum_{n=1}^{\infty} a_n z^n$ is analytic for $|z| < 1$ and if $|f(z)| \leqq |F(z)|$ for $|z| < 1$, where $F \in C$, then $|a_n| \leqq n$ for $n = 1, 2, \ldots$. If $f \in C$ then $(1/2\pi)\int_0^{2\pi} |f(re^{i\theta})|d\theta \leqq r/(1-r^2)$ for $0 < r < 1$. Similar results hold for L^p means of f as well as for such means of derivatives of f where $f \in C$. (Received November 15, 1971.)

72T-B40. JAMES K. FINCH, University of Illinois, Urbana, Illinois 61801. Sufficient conditions for an operator to fail to have the single valued extension property.

Let T be a closed linear operator with domain and range in a complex Banach space X. Following N. Dunford, we say that T has the single valued extension property if the only vector valued analytic function f satisfying $(\lambda I - T)f(\lambda) = 0$ is $f = 0$. If T satisfies any of the following conditions, it fails to have the single valued extension property. (1) If Y is an invariant subspace, $T(Y) = Y$, but T is not one-one on Y. (2) T is a semi-Fredholm operator and the nullity of T is strictly greater than the deficiency. (3) The ascent of T is infinite and the descent is finite. (4) The intersection of the point spectrum with the semi-Fredholm resolvent has interior. (Received November 16, 1971.)

*72T-B41. MARGARET C. WAID, District of Columbia Teachers College, Washington, D. C. 20009. Strong maximum principles for time degenerate parabolic operators.

We study the degenerate parabolic operator $Lu = \sum_{i,j=1}^{n} a^{ij}u_{x_ix_j} + \sum_{i=1}^{n} b^i u_{x_i} - cu_t + du$ where u, a^{ij}, b^i, c, and d are bounded, real-valued functions defined on a bounded domain $D \subset R^{n+1}$. Assume that L is parabolic. Though classically $c \equiv 1$, we assume only that c is nonnegative. Using proofs substantially different from classical ones, which rely heavily on the minimum of c in $\bar{D}$ being positive, we prove the usual strong maximum principles. This work is contained in the author's dissertation written at Texas Tech University under the direction of Wayne T. Ford. (Received November 18, 1971.)

*72T-B42. DATTATRAYA J. PATIL, University of Wisconsin, Milwaukee, Wisconsin 53201. Recapturing H^2 functions from boundary values on small sets.

If E is a set of positive measure on the unit circle, then it is well known that a function f in H^2 is uniquely determined by its boundary values on E. In the present work a constructive algorithm for recapturing such a function f from its values on E is given. The main tool is the following Theorem. If T is a positive selfadjoint operator on a complex Hilbert space H and T is one-one, then for every $x \in H$, we have $\lambda(I+\lambda T)^{-1} Tx \to x$ as $\lambda \to \infty$, λ real. If H is H^2 of the disc and T is the Toeplitz operator corresponding to the characteristic function of E then this theorem leads to the algorithm. Necessary and sufficient conditions for $f \in L^2(E)$ to be a restriction of a function in H^2 are also obtained. (Received November 19, 1971.)

*72T-B43. SIMON COHEN, Polytechnic Institute of Brooklyn, Brooklyn, New York 11201. A measure induced metric topology for a Banach algebra.

Let X be a semisimple Banach algebra and m a probability measure on the maximal ideal space $\mathcal{M}$ of X. If m is positive on nonempty open sets, then $d(x,y) = m(\hat{x} \neq \hat{y})$, where $\hat{x}, \hat{y}$ are the associated Gelfand functions, defines a metric on X. To begin with, the interplay between the algebraic structure of X and the metric topology is studied. Denoting the resultant metric space by X_d, it is shown, for example, that X_d is a bounded topological ring. Furthermore, we show that the units of X are open in X_d iff d is trivial. An example is given in which each maximal ideal is dense in the metric ring X_d. Secondly, questions relating to completeness and convergence are investigated. We show that if $\mathcal{M}$ is countably infinite, X_d is incomplete. A strengthening of the concept of convergence in measure is introduced in the general measure space. It is then shown that a sequence $(x_n) \to x$ in X_d iff $(\hat{x}_n) \to \hat{x}$ strongly in measure. Lastly, we show that X_d is locally compact iff d is trivial. (Received November 3, 1971.)

*72T-B44. ANNE E. DRINKWATER, McGill University, Montreal 110, Quebec, Canada. Integral representation for multiply superharmonic functions. Preliminary report.

Let Ω_i, $i = 1, 2, \ldots, n$, be connected, locally connected Hausdorff spaces which are locally compact but not compact. For each i, let Ω_i be a harmonic space of Brelot with a potential > 0 satisfying the Axioms 1, 2, 3 and 4. Let M^+ be the set of all positive multiply superharmonic functions on $\Pi_{i=1}^n \Omega_i$, and $M = M^+ - M^+$. It is shown, that with respect to a certain topology τ on M, the cone M^+ has a compact, metrizable base, hence that there is integral representation for the elements of M in terms of signed radon measures on the compact base carried by the set of extreme elements. The topology τ is constructed in the following way. Let B_i be the countable base for Ω_i consisting of completely determining regular open domains. If $\omega_i \in B_i$, $x_i \in \omega_i$, for each i, then for each $m \in M^+$ define $[(\omega_1,x_1)\times\ldots\times(\omega_n,x_n)](m) = \int\ldots\int m(\Sigma_1,\ldots,\Sigma_n) d\rho_{x_1}^{\omega_1}(\Sigma_1)\ldots d\rho_{x_n}^{\omega_n}(\Sigma_n)$. This defines a linear functional on M^+ which can easily be extended to M. The topology τ is the weakest topology on M such that all linear functionals of the above form are continuous. (Received November 22, 1971.) (Author introduced by Professor K. N. Gowrisankaran.)

*72T-B45. JON C. HELTON, Arizona State University, Tempe, Arizona 85281. Existence theorems for sum and product integrals.

If H is a bounded function, then $H \in OL^1$, OL^2, OL^3, and OL^4 on $[a,b]$ if (1) $H(p^-,p^-) = H(p^+,p^+) = 0$, (2) $H(p^-,p^-)$ and $H(p^+,p^+)$ exist, (3) $H(p^-,p) = H(p,p^+) = 0$, and (4) $H(p^-,p)$ and $H(p,p^+)$ exist, respectively, for each $p \in [a,b]$, and $H \in OL^{ij}$ only if $H \in OL^i \cap OL^j$. If $D = \{x_q\}_0^n$ is a subdivision of $[a,b]$, then J is a modified refinement of D if there exist sequences $\{y_q\}_1^n$, $\{z_q\}_1^n$, and $\{L(q)\}_1^n$ such that $x_{q-1} < y_q < z_q < x_q$, L(q) is a subdivision of $[y_q,z_q]$ and $J = \cup_1^n L(q)$. $G \in AZ$ on $[a,b]$ only if G is bounded and if $\epsilon > 0$ then there exists a subdivision D of $[a,b]$ such that if $J = \{L(q)\}_1^n$ is a modified refinement of D then $\Sigma_1^n \Sigma_{L(q)} |G| < \epsilon$. See B. W. Helton [Pacific J. Math. 16(1966), 297-322] for more details. Theorem. If functions defined on $[a,b]$ from $R \times R$ to R are considered and G is a function, then each of the following consists of two equivalent statements: (1) (a) if $H \in OL^1$, then $HG \in OA^\circ$, (b) $G \in OL^3 \cap OB^\circ$; (2)(a) if $H \in OL^2$, then $HG \in OA^\circ$, (b) $G \in OL^3 \cap OB^\circ \cap OA^\circ$; (3)(a) if $H \in OL^3$, then $HG \in OA^\circ$, (b) $G \in AZ$; (4) (a) if $H \in OL^4$, then $HG \in OA^\circ$, (b) $G \in OL^4 \cap AZ$; (5)(a) if $H \in OL^{13}$, then $HG \in OA^\circ$, (b) $G \in OB^\circ$; (6)(a) if $H \in OL^{14}$, then $HG \in OA^\circ$, (b) $G \in OL^4 \cap OB^\circ$; (7)(a) if $H \in OL^{23}$, then $HG \in OA^\circ$, (b) $G \in OB^\circ$; (8)(a) if $H \in OL^{24}$, then $HG \in OA^\circ$, (b) $G \in OB^\circ \cap OA^\circ$. Further, in each case (b) is necessary and sufficient for $HG \in OM^\circ$. (Received November 22, 1971.)

72T-B46. WILLIAM H. FORD, University of Illinois, Urbana, Illinois 61801. Numerical solution of pseudo-parabolic partial differential equations. Preliminary report.

The initial boundary value problem for the pseudo-parabolic equation $(a(x,t)u_{xt})_x + (b(x,t)u_x)_x - q(x,t,u) = r(x,t)u_t$, $0 < x < 1$, $0 < t \leq T$, is considered. Two finite difference schemes are developed and, with appropriate conditions on the coefficients and the solution u, are shown by means of stability analysis to converge in the discrete L_2 norm with truncation error $O((\Delta x)^2)$. These schemes are applied to the nonlinear equation $(a(x,t)u_{xt})_x + (b(x,t)u_x)_x = F(x,t,u,u_x,u_t)$ and are shown to converge in the uniform norm with truncation error $O((\Delta x)^2)$. An a priori error bound is developed for a continuous time Galerkin approximation to the solution of the problem $u_t - \Sigma_{ij}(a_{ij}(x)u_{x_j t})_{x_i} - \Sigma_{ij}(b_{ij}(x,t,u)u_{x_j})_{x_i} = f(x,t)$, $i,j = 1,\ldots,n$, $x \in G$, $t > 0$, $u(x,t) = 0$, $x \in \partial G$, $t \geq 0$, $u(x,0) = u_0(x)$, $x \in G$, where G is a bounded domain in R^n, and the matrix $(a_{ij}(x))$ is a.e. uniformly positive definite. (Received November 22, 1971.)

*72T-B47. HUGO TEUFEL, JR., Wichita State University, Wichita, Kansas 67208. Forced second order nonlinear oscillation.

Sufficient conditions are given for the oscillation of all solutions of $x'' + F(t,x,x') = f(t)$ where $xF \geq a(t)xN(x) > 0$, $x \neq 0$, N nondecreasing. The conditions are: Theorem 1. $a(t) \equiv 1$, $\int_0^t f$ bounded on $[0,\infty)$, and there are δ, $\Delta > 0$ such that in each $[T,\infty)$ there are intervals of length Δ on which $f < -\delta$, and $f > \delta$. Theorem 2. a(t) nonincreasing, $\int^\infty a = \infty$, $\int_0^t f = o(\int_0^t a)$, the δ, Δ, condition holds and the Δ-intervals are regularly distributed. (Received November 23, 1971.)

*72T-B48. ERNEST E. BURNISTON and C. E. SIEWERT, North Carolina State University, Raleigh, North Carolina 27607. The use of Riemann problems in solving a class of transcendental equations.

A method of finding explicit expressions for the roots of a certain class of transcendental equations is discussed. In particular it is shown by determining a canonical solution of an associated Riemann boundary-value problem that expressions for the roots may be derived in closed form. The explicit solutions to two transcendental equations, $\tan \beta = \omega\beta$ and $\beta \tan \beta = \omega$, are discussed in detail, and several additional specific results are given. (Received November 24, 1971.)

*72T-B49. JOSEPH L. BERTORELLI, State University College of New York, Potsdam, New York 13676. Exponential decay of solutions of an acoustic equation.

Consider the equation $u_{tt} - \Delta u + qu = 0$ in $[0,\infty) \times \mathcal{D}$ where $\mathcal{D}$ is a region exterior to a smooth, bounded, star-shaped body in R^3. The solution is assumed to satisfy the Dirichlet boundary condition. Let $q \geqq 0$, $u(0,x)$ have distributional derivatives of the first order, and $u_t(0,x)$ be in L^2. Let $\partial\mathcal{D}$ and the support of q, $u(0,x)$, $u_t(0,x)$ be contained in a ball centered at the origin of finite radius. Theorem. If energy of the solution u on a bounded set $S \subset R^3$ at time t is $E(u,S,t) = \int_S |\nabla u|^2 + u_t^2 + qu^2 |_t dV$ then $E(u,S,t) \leqq C e^{-\alpha t} E(u,\mathcal{D},0)$ for some positive constants C and α. This result can be generalized to the equation $u_{tt} - \Sigma(a^{ij}u_i)_j + qu = 0$ where $a^{ij}(x) = \delta_{ij}$ for $|x| \geqq$ some R_0 and $\ell\Sigma\xi_i^2 \leqq \Sigma a^{ij}\xi_i\xi_j \geqq \ell\Sigma\xi_i^2$ and $|x|\Sigma a^{ij}_{|x|}\xi_i\xi_j \leqq \alpha_0\Sigma a^{ij}\xi_i\xi_j$ for all $x \in \mathcal{D}$ and some $\alpha_0 < 1$. (Received November 26, 1971.)

*72T-B50. JOHN E. COURY, University of British Columbia, Vancouver 8, British Columbia, Canada. Some results on lacunary Walsh series. Preliminary report.

Let $S(x)$ be a lacunary Walsh series with degree of lacunarity greater than or equal to two. The following results are proved: Theorem 1. Suppose that $S(x)$ converges to 0, or is constant, on a set of measure exceeding $\frac{1}{2}$. Then $S(x)$ vanishes identically. Theorem 2. Suppose that $S(x)$ converges to 0, or is constant, on a set E of positive measure. Then $S(x)$ is a finite sum and the measure of E is a dyadic rational. Theorem 3. A necessary and sufficient condition that $S(x)$ be absolutely convergent is that there exist a set B of the second category such that $S(x)$ has bounded partial sums for each x in B. Corollary. If $S(x)$ converges to 0 on the complement of a set of the first category, then the series is identically 0. These theorems are analogues for the Walsh system of certain classical results for lacunary trigonometric series. (Received November 29, 1971.)

*72T-B51. T. V. LAKSHMINARASIMHAN, Madras Christian College, Madras-59, India. On an inequality of S. Bernstein concerning the maximum modulus of an entire function and its derivative. Preliminary report.

Let $f(z)$ be an entire function of finite order ρ and finite type τ. Then Bernstein ("Leçons sur les propriétés extremales et la meilleure approximation des functions analytic de une variable reelle," Paris, 1926) and Kovari ("A note on entire functions," Acta Math. Acad. Sci. Hungar. 8(1957), 87-90) had shown that

$\rho\tau \leqq \limsup_{r\to\infty}[M(r,f')/r^{\rho-1}M(r,f)] \leqq e\rho\tau$. These inequalities are extended by replacing τ by the L-type σ_L (with respect to the order ρ) defined by $\sigma_L = \limsup_{r\to\infty}[\log M(r,f)/r^{\rho}L(r)]$ where $L(r)$ is a "slowly increasing function". Thus we have Theorem. $\rho\sigma_L \leqq \limsup_{r\to\infty}[M(r,f')/r^{\rho-1}L(r)\,M(r,f)] \leqq e\rho\sigma_L$. Further, by specialising $L(r)$, we deduce that if $\rho(r)$ is a positive continuous function and is a proximate order of $f(z)$ defined by (i) $\lim_{r\to\infty}\rho(r) = \rho$, (ii) $\lim_{r\to\infty}\rho'(r)\,r\log r = 0$ and if $\sigma = \limsup_{r\to\infty}[\log M(r,f)/r^{\rho(r)}]$, then Corollary. $\rho\sigma \leqq \limsup_{r\to\infty}[M(r,f')/r^{\rho(r)-1}M(r,f)] \leqq e\rho\sigma$. (The result in the Corollary is contained in a paper presented at the 36th Conference of the Indian Mathematical Society at Gorakhpur, India, 1970.) (Received November 30, 1971.)

*72T-B52. JOHN S. LANCASTER, Indiana University, Bloomington, Indiana 47401 and University of Hawaii, Hilo, Hawaii 96720. Weyl's theorem holds for compact perturbations. Preliminary report.

Let $\mathcal{H}$ be a separable, infinite dimensional Hilbert space and let $\mathcal{B}(\mathcal{H})$ be the algebra of bounded linear operators on $\mathcal{H}$. If T is in $\mathcal{B}(\mathcal{H})$, the Weyl spectrum of T is the set $\omega(T) = \bigcap\{\sigma(T+K)\colon K \text{ is compact on } \mathcal{H}\}$, where σ denotes the usual spectrum of an operator. Weyl's theorem is said to hold for T if $\sigma(T) - \pi_{00}(T) = \omega(T)$, where $\pi_{00}(T)$ is the set of isolated eigenvalues of finite multiplicity for T. Theorem. There exists a compact operator K on $\mathcal{H}$, of arbitrarily small norm, such that Weyl's theorem holds for T + K. In particular, the collection of operators on $\mathcal{H}$ for which Weyl's theorem holds is dense in $\mathcal{B}(\mathcal{H})$. (Received December 2, 1971.)

72T-B53. C. L. RISHISHWAR, Gur Narain Khatri College, Kanpur-1, India. On certain integral formulae for the functions represented by the series $\sum_{n=0}^{\infty} a_n e^{\lambda_n\cdot\psi(z)}$.

Consider the series (*) $\sum_{n=0}^{\infty} a_n e^{\lambda_n\cdot\psi(z)}$ where $\lim_{n\to\infty}\sup n/\lambda_n = D < \infty$, $0 = \lambda_0 < \lambda_1 < \lambda_2 < \lambda_3 < \ldots < \lambda_n \to \infty$ as $n \to \infty$, $\{a_n\}$ $(n = 0, 1, 2, 3, \ldots)$ is a sequence of complex numbers and $\psi(z)$ is an analytic function of complex variable z, analytic in the region $\log|e^{\psi(z)}| \leqq R$, and satisfying the conditions: (a) $\psi(z)$ has an inverse, (b) $\psi(z) = \log|e^{\psi(z)}| + i\,c$, where c is a real function depending on z. Theorem. Let $F(z) = \sum_{n=0}^{\infty} a_n e^{\lambda_n\cdot\psi(z)}$ be a function analytic in the region $\log|e^{\psi(z)}| \leqq R$, then, for every z in this region $a_n = \lim_{C\to(\pi \text{ or } \infty)}(1/2Ci)\int_{\psi^{-1}(R-Ci)}^{\psi^{-1}(R+Ci)} F(z)\cdot e^{-\lambda_n\cdot\psi(z)}\cdot\psi'(z)dz$, where $\psi(z) = \log|e^{\psi(z)}| + i\,c$, and integration being taken along the boundary of the region $\log|e^{\psi(z)}| \leqq R$; also when $\lim c \to \pi$, is considered, λ_n s are taken to be integers. Integral representations for F(a) and F'(a), where a is any point in the region determined by $\log|e^{\psi(z)}| \leqq R$, have been obtained. Taking $\psi(z) = \log z$ and $\lambda_n = n$, or $\psi(s) = s$ corresponding known results, for the Taylor series and Dirichlet series respectively follow from the results obtained in the paper. (Received December 2, 1971.) (Author introduced by Dr. B. L. Bhatia.)

72T-B54. MILOS A. DOSTAL and MICHAEL J. KASCIC, JR., Stevens Institute of Technology, Hoboken, New Jersey 07030. Inductive limits with partition of unity.

Marc De Wilde in his paper "Inductive limits and partitions of unity" (Manuscripta Math. 5(1971), 45-58) introduced the concept of LF-space with partition of unity. We study further properties of these spaces and derive an abstract analogue of Laurent Schwartz's spaces $\mathcal{D}(\Omega)$, $\mathcal{D}'(\Omega)$, $\mathcal{E}(\Omega)$, $\mathcal{E}'(\Omega)$ and their mutual relations. The emphasis is on properties of operators acting on these spaces, in particular openness and analogues of

Hörmander's notion of strong P-convexity. The main tool of this approach is the imposition of a bilinear form on the LF-spaces in question that injects them into their duals. (Received December 6, 1971.)

*72T-B55. BERNARD N. HARVEY, California State College, Long Beach, California 90801. On J-unitary-dissipative and unbounded J-nonnegative operators in a J-space. Preliminary report.

A J-space H is a Hilbert space with inner product $[x,y]$ and another inner product $(x,y) = [Jx,y]$ where $J = J^*$ and $J^2 = I$. We define the J-adjoint T^+ of a linear operator T ($\mathcal{D}(T)$ dense) by $(Tx,y) = (x,T^+y)$ for appropriate x and y. Theorem 1. A linear operator U with $U^*JU = J$ and $\mathrm{Im}(Ux,x) \geq 0$ is a J-orthogonal direct sum $U_1 \oplus U_2$ where $(-1)^j \in \rho(U_j)$. Thus $A_j = (-1)^j i(U_j + (-I)^j)(U_j - (-I)^j)^{-1}$ is bounded ($j = 1,2$) and $(A_j x,x) \geq 0$. Hence $A_j = \int_{-\infty}^{\infty} t dE_j(t) + S_j$ where $S_j^2 = (E_j(b) - E_j(a))S_j = 0$ for all real $a < b$ with $0 \notin [a,b]$ and $(S_j x,x) \geq 0$. The function $E_j(t)$ ($j = 1,2$) is a J-spectral function with critical point zero. (See Ju. Smul'jan, Mat. Issled. 1(1966), 172-210.) Theorem 2. Let A be densely defined, $A = A^+$, $(Ax,x) \geq 0$, and $ir \in \rho(A)$ ($r > 0$). Then A is a J-orthogonal direct sum $A_1 \oplus A_2$, where A_1 is bounded and $(A_1 x,x) \geq 0$, and thus A_1 is given as above, and $A_2 = \int_{-\infty}^{\infty} t^{-1} dE_2(t)$ on $\mathcal{D}(A_2)$. Both integrals are strongly convergent improper integrals on $\mathcal{D}(A_j)$ with singularity 0. Again $\{E_j(t)\}$ ($j = 1,2$) is a J-spectral function with critical point zero. To prove this, $ir \in \rho(A)$ implies that $U = (A + riI)(A - riI)^{-1}$ satisfies the hypotheses of Theorem 1. Then $A = ir(U+I) \cdot (U-I)^{-1}$ yields Theorem 2. (Received December 6, 1971.)

*72T-B56. JAMES K. BROOKS and PAUL W. LEWIS, University of Florida, Gainesville, Florida 32601. Weakly compact operators on function spaces. Preliminary report.

If E and F are Banach spaces and Σ is the Borel σ-algebra of a compact Hausdorff space H, then a finitely additive set function $m: \Sigma \to B(E,F^{**})$ with finite semivariation is called a representing measure provided that $\bar{m}_z$ is a regular Borel measure for each $z \in F$. The function m is called vsr if $\{A_n\} \searrow \varphi \Rightarrow \tilde{m}(A) \to 0$. Theorem 1. If $L: C(H,E) \to F$ is weakly compact and $L \leftrightarrow m$, then m is vsr. Conversely, if E is reflexive and $L \leftrightarrow m$ is vsr, then L is weakly compact. It is shown that this theorem contains, and provides a converse to, recent results in the literature. General necessary and sufficient conditions of a summability type for m to map to B(E,F) are unknown. An application of Theorem 1 provides Theorem 2. If m does not map Σ into B(E,F), then there is a y in F***, a disjoint sequence $\{A_i\} \subset \Sigma$, and $\{x_i\}$ in the closed unit ball of E so that $\sum_{i=1}^{\infty} |y(m(A_i)x_i)| = \infty$. Theorem 3. If $C(H)^*$ is separable and $m \leftrightarrow L: C(H) \to F$ is a continuous linear operator, then the following are equivalent. (1) L is compact; (2) L is weakly compact; (3) $m: \Sigma \to F$; (4) m is countably additive; (5) m is vsr. (Received December 6, 1971.)

*72T-B57. MANFRED STOLL, University of South Carolina, Columbia, South Carolina 29208. Hardy-type spaces of harmonic functions on symmetric spaces of noncompact type.

Let G be a connected semisimple Lie group with finite center, K a maximal compact subgroup, G/K the corresponding symmetric space of noncompact type, and K/M the Furstenberg boundary of G/K. Let $G = KCl(A^+)K$ be the Cartan decomposition of G. For $0 < p < \infty$, $\mathcal{H}^p(G/K)$ denotes the space of harmonic functions f on G/K for which $\sup_{a \in A^+} \int_K |f(kaK)|^p dk$ is finite, and $\tilde{\mathcal{H}}^p(G/K)$ denotes the space of harmonic functions f

for which there exists a harmonic function F such that $|f(gK)|^p \leq F(gK)$ for all gK in G/K. A Poisson integral representation theorem for functions in $\mathfrak{H}^p(G/K)$, $p \geq 1$, is proved, and it is shown that $\mathfrak{H}^p(G/K)$ is isometrically isomorphic to $L^p(K/M)$, $p > 1$, and $\mathcal{C}(K/M)^*$ for $p = 1$. It is also proved that the space $\mathfrak{H}^p(G/K)$ and $\tilde{\mathfrak{H}}^p(G/K)$ are equivalent for all $p \geq 1$. By an example it is shown that $\tilde{\mathfrak{H}}^p(G/K)$ in general is a proper subspace of $\mathfrak{H}^p(G/K)$ for $0 < p < 1$. A metric is defined on $\tilde{\mathfrak{H}}^p(G/K)$, $0 < p < 1$, with respect to which it is a complete linear topological metric space. (Received December 8, 1971.)

72T-B58. JOSEPH K. CROSS, New College, Sarasota, Florida 33578. Sequence spaces λ for which weakly in λ implies strongly in λ. Preliminary report.

A sequence space λ is called an ASI space provided that whenever (x_n) is a sequence in a normed space, and (x_n) is weakly in λ, then (x_n) is strongly in λ. Let a^k, $k = 1,2,\ldots$, be any countable collection of scalar sequences, and let λ be the smallest solid (i.e., normaler) sequence space which contains the a^k. Then λ is an ASI space. Let λ be a perfect sequence space; then λ is an ASI space iff every $\sigma(\lambda, \lambda^\times)$-bounded subset of λ is contained in the solid hull of a point in λ. Let λ be a perfect sequence space; then λ is an ASI space iff there is a solid sequence space μ such that $\lambda = \mu^\times$, and the normal topology on λ induced by μ (i.e., the topology generated by the seminorms $(a_n) \to \sum |a_n b_n|$, $\forall\ (a_n) \in \lambda$, $\forall\ (b_n) \in \mu$) is nuclear. (Received December 9, 1971.)

72T-B59. MARTIN SCHECHTER, Belfer Graduate School of Science, Yeshiva University, New York, New York 10033. Proper boundary points of the spectrum. Preliminary report.

Let A be a bounded operator on a Hilbert space X. A boundary point λ of the spectrum $\sigma(A)$ of A will be called proper if there is a sequence $\{\lambda_n\}$ of points of the resolvent set $\rho(A)$ of A such that $\|(\lambda_n - \lambda)(\lambda_n - A)^{-1}\| \to 1$ as $n \to \infty$. Theorem. If λ is a proper boundary point of $\sigma(A)$, then (a) $N(A - \lambda) = N(A^* - \bar{\lambda})$, (b) for any bounded sequence $\{x_n\} \subset X$ one has $(A - \lambda)x_n \to 0$ iff $(A^* - \bar{\lambda})x_n \to 0$, and (c) if λ is not an isolated point of $\sigma(A)$, then there exists an orthonormal sequence $\{x_n\} \subset X$ such that $(A - \lambda)x_n \to 0$ and $(A^* - \bar{\lambda})x_n \to 0$. This generalizes results of Schreiber, Sz.-Nagy-Foiaş, Saito, Putnam and others. (Received December 10, 1971.)

*72T-B60. DOUGLAS MOREMAN, Auburn University, Auburn, Alabama 36830. Uniform spherical convexity of metric spaces. Preliminary report.

This paper represents a continuation of work mentioned in Abstract 689-G6, these *Notices* 18(1971), 1065. In the context of a complete metric space S: Definition. The statement that S is uniformly spherically convex at the point P means that if $\epsilon > 0$ then there exists $\delta > 0$ such that if $r > 0$ and A and B are points at a distance from P of r or less and at a distance from each other of $\epsilon \cdot r$ or more than the spherical interval $\mathscr{I}(A, B)$ (which is the common part of all spherical domains that contain A and B) contains a point at a distance from P of $r - r\delta$ or less. Theorem. If S is a normed linear space then S is uniformly spherically convex (at each of its points) if and only if S is uniformly convex. Theorems. If S is uniformly spherically convex at one of its points then S is spherically convexly perfectly compact. If S is uniformly spherically convex at a point P, and Q is a point, and M is a closed and spherically convex point set then there is a point of M that is

nearer to P than is any other point of M and there is a point of M that is as near to Q as is any other point of M. If S is uniformly spherically convex then every infinite and bounded point set has a center. (Received December 10, 1971.)

72T-B61. ERWIN O. KREYSZIG, University of Karlsruhe, Karlsruhe, Germany. On pairs of analytic functions.

Let f and g be analytic functions which are defined in a domain D of the complex plane and whose derivatives are related by a Möbius transformation T. Then the two corresponding surfaces of the real part have the same Gaussian curvature at corresponding points if and only if T leaves the chordal distance invariant. This theorem includes a result by R. Jerrard (Colloq. Math. 21(1970), 127-132) as a special case. (Received November 5, 1971.)

*72T-B62. NATHANIEL A. FRIEDMAN, State University of New York, Albany, New York 12203. Bernoulli shifts induce Bernoulli shifts. Preliminary report.

Recently Paul Shields utilized a construction of the author to show that certain Bernoulli shifts with finite entropy induce Bernoulli shifts on a dense class of measurable subsets of the unit interval. By generalizing the construction it can be shown that Theorem. Every Bernoulli shift with finite or infinite entropy induces Bernoulli shifts on a dense class of subsets. (Received November 11, 1971.)

72T-B63. DAVID A. BRANNAN and WILLIAM E. KIRWAN, Queen Elizabeth College, University of London, London, England. On functions of bounded boundary rotation. Preliminary report.

Let V_k ($k \geq 2$) denote the class of functions $f(z) = z + a_2 z^2 + \dots$ that are analytic in $|z| < 1$ and satisfy the two conditions (i) $f'(z) \neq 0$, and (ii) $\int_0^{2\pi} |\mathrm{Re}\,(1+zf''/f')|d\theta \leq k\pi$. The coefficient problem for V_k has attracted the attention of many authors. Using elementary techniques the present authors show that if $k \geq 4$ then $|a_n| \leq A_n$ for all $n \geq 2$, and that if $2 \leq k \leq 4$ then $|a_n| \leq A_n$ for $2 \leq n \leq 5$, where $\{(1+z)^{k/2}(1-z)^{-k/2} - 1\}/k = z + A_2 z^2 + \dots$. It was previously known that $|a_n| \leq A_n$ for $2 \leq n \leq 4$ (see, e.g., Schiffer and Tammi, J. Analyse Math. 17(1966), 109-144 and Ann. Acad. Sci. Fenn. Ser. A I 396(1967)). (Received November 29, 1971.)

72T-B64. KOK-KEONG TAN, Dalhousie University, Halifax, Nova Scotia, Canada. Remark on pointwise periodic isometries.

Definition. Let X be a normed space. Then $f: X \to X$ is said to be (i) isometric iff $\|f(x) - f(y)\| = \|x - y\|$, for all $x, y \in X$; (ii) pointwise periodic iff for each $x \in X$, there is a positive integer n such that $f^n(x) = x$. The following result is a consequence of Corollary 1 of the Abstract 71T-B82, these *Notices* 18 (1971), 555, and a theorem of Mazur and Ulman. Theorem. Let X be a real reflexive Banach space with normal structure (in particular, a real uniformly convex Banach space or a real Hilbert space) and $f: X \to X$ be pointwise periodic and isometric. If f maps the closed unit ball of X into itself, then f must be linear. (Received December 20, 1971.)

*72T-B65. H. M. SRIVASTAVA, University of Victoria, Victoria, British Columbia, Canada. On the reducibility of Appell's function F_4.

In this paper the author gives the following general result. Theorem. Let $\{c_n\}$ be a sequence of arbitrary complex numbers. Also let $I(\nu,\sigma,x,y) = \sum_{m,n=0}^{\infty} \{c_{m+n} x^m y^n\}/\{(\nu)_m (\sigma)_n m!n!\}$. Then, under appropriate convergence conditions, (*) $I(\nu,\sigma,x,x) = \sum_{n=0}^{\infty} \{c_n (\nu+\sigma+n-1)_n x^n\}/\{(\nu)_n (\sigma)_n n!\}$ and (**) $I(\nu,\nu,x,-x) = \sum_{n=0}^{\infty} \{(-1)^n c_{2n} x^{2n}\}/\{(\nu)_n (\nu)_{2n} n!\}$. The series transformation (*) incorporates, as a special case, a familiar reduction formula for Appell's function F_4, which was obtained by J. L. Burchnall [Quart. J. Math. Oxford Ser. 13(1942), 90-106] while investigating solutions of equivalent systems of hypergeometric differential equations. It is also pointed out that an F_4 reduction formula, given recently by R. K. Saxena [Canad. Math. Bull. 9(1966), 215-222] is erroneous, and that its corrected version is a special case of Burchnall's formula. This paper is scheduled to appear in Canad. Math. Bull. (Received December 13, 1971.)

72T-B66. WITHDRAWN.

72T-B67. ROBERT T. GLASSEY, Division of Applied Mathematics, Brown University, Providence, Rhode Island 02912. Asymptotic behavior of a class of nonlinear wave equations. Preliminary report.

Consider the Cauchy problem for the equation (*) $u_{tt} - \Delta u + m^2 u + F(u) = 0$ $(x \in R^n,\ t > 0)$ where $m > 0$. The free equation is (*) with $F(u) \equiv 0$. Let $G(u) = \int_0^u F(s)\,ds$. The function $G(\cdot)$ will be said to have property (p) if whenever $\|v(t)\|_{L^p} \to 0$ as $t \to \infty$ for all p, $2 < p \leq 2n/n-2$ $(n \geq 3)$, then $\int G(v(x,t))\,dx \to 0$ as $t \to \infty$. Make the standing assumptions that: (i) $\frac{1}{2}m^2 s^2 + G(s) \geq 0$ for all s; (ii) $G(u_0) < 0$ for some u_0. The following results are obtained: (1) Suppose that arg F(s) = arg s and that G has property (p). Then (*) has complex-valued solutions u(x,t) such that there does not exist any finite energy solution $u_+(x,t)$ of the free equation which is asymptotic to u in energy norm as $t \to \infty$. (2) Let $G(s) = o(s^2)$ as $s \to 0$. Then there exist real Cauchy data and a positive constant m_0, depending on this data, on n, and on the value of $G(u_0)$, with the property that if $0 < m \leq m_0$, the corresponding real-valued solution u(x,t) of (*) satisfies $\sup_x |u(x,t)| \not\to 0$ as $t \to \infty$. If moreover $G(\cdot)$ has property (p), the same conclusion as in (1) is valid. These results extend a theorem of E. Roffman (see Bull. Amer. Math. Soc. 76(1970), 70-71). (Received December 14, 1971.)

*72T-B68. ANTHONY C. THOMPSON and M. S. VIJAYAKUMAR, Dalhousie University, Halifax, Nova Scotia, Canada. An order-preserving representation theorem for Banach algebras and some examples. Preliminary report.

Let $\mathcal{A}$ be a complex Banach algebra with unit e of norm one; $M = \{f \in \mathcal{A}' : f(e) = 1 = \|f\|$ and $f(J) = (0)$ for some maximal left ideal J of $\mathcal{A}\}$, and $\Omega = w^*\text{-cl } M$. We represent $\mathcal{A}$ on the compact, Hausdorff space Ω, which depends only on the algebraic and metric structures of $\mathcal{A}$. If $\mathcal{A}$ is commutative, then Ω is the spectrum of $\mathcal{A}$, and we have a generalisation of Gelfand's representation theorem. The space Ω induces a wedge K on $\mathcal{A}$ (see Abstract 663-639, these *Notices*) 16(1969), 274; take $K = K_2$). The representation is norm reducing and maps K into the cone $P = \{f \in C(\Omega) : \text{Re } f \geq 0$ and $\text{Im } f \geq 0\}$; it is thus a continuous, linear, order homomorphism which is also an isomorphism (a homeomorphism) iff K is a cone (normal cone). By

considering algebras of n by n complex matrices we show (among other things) that (1) there exist noncommutative Banach algebras for which the set M is unchanged if "right" is substituted for "left" in its definition, (2) in general, M and the maximal left ideals are not in one-to-one correspondence, and (3) in general, M is not w*-closed. (Received November 8, 1971.)

72T-B69. GERASIMOS E. LADAS, V. LAKSHMIKANTHAM and J. S. PAPADAKIS, University of Rhode Island, Kingston, Rhode Island 02881. Oscillations of higher order functional differential equations. Preliminary report.

Consider the functional differential equations (1) $y^{(n)}(t) + (-1)^{n+1}p(t)y(g(t)) = 0$ and (2) $y^{(n)}(t) + (-1)^{n}\cdot p(t)[y(t) - y(g(t))] = 0$ where $n \geq 1$ in (1) and $n \geq 2$ in (2). Assume that $p(t) \geq 0$, $g(t) < t$, $\lim_{t\to\infty} g(t) = \infty$, $g'(t) \geq 0$ and $\lim\sup_{t\to\infty} \int_{g(t)}^{t} [g(t) - g(s)]^{n-1}p(s)\,ds > (n-1)!$. Theorem 1. Every bounded solution of (1) is oscillatory. Theorem 2. Let $y(t)$ be a solution of (2) with $y'(t)$ bounded. Then $y'(t)$ oscillates. The boundedness assumption in these theorems is unnecessary if $n = 1$ in equation (1) or $n = 2$ in equation (2). A classification of the solutions of (1) is also obtained with respect to the oscillatory behavior and the asymptotic behavior as $t \to \infty$. Clearly the above oscillation theorems are not valid if $g(t) \equiv t$. That is, these oscillations are caused by the delay. (Received December 15, 1971.)

*72T-B70. BADRI N. SAHNEY, Department of Mathematics, Statistics and Computer Science, University of Calgary, Calgary-44, Alberta, Canada and V. VENU GOPAL RAO, Institute of Advanced Studies, Australian National University, Canberra, A.C.T. 2600, Australia. Error bounds in the approximation of functions.

Let $f(x) \in \text{Lip}\,\alpha$, $0 < \alpha < 1$, in the range $(-\pi, \pi)$, and periodic with period 2π, outside this range. Also let (*) $f(x) \sim \frac{1}{2}a_0 + \sum_{\nu=1}^{\infty}(a_\nu \cos \nu x + b_\nu \sin \nu x) = \sum_{\nu=0}^{\infty} A_\nu(x)$. We write $\varphi(x,t) = (f(x+t) + f(x-t))/2 - f(x)$. We define the norm as $\|f\|_p = \{\int^{2\pi} |f(x)|^p\,dx\}^{1/p}$, $p \geq 1$, and let the degree of approximation be given by $E_n^*(f) = \min_{T_1,\ldots,T_n} \|f - T_n\|_p$ where $T_n(x)$ is some nth trigonometrical polynomial. We define a generating sequence $\{p_n\}$ such that it is nonnegative, nonincreasing and (**) $P(n) = p_0 + p_1 + \ldots + p_n \to \infty$ as $n \to \infty$. Approximation of functions belonging to Lip α class by means of (c, δ), $0 < \delta \leq 1$, means of its Fourier series is due to Chapman and Riesz [see G. Alexits, "Convergence problems of orthogonal series," Pergamon Press, New York, (1961)]. The following is the main theorem in our paper. Theorem. If $f(x)$ is periodic and belongs to the class $\text{Lip}(\alpha, p)$, $0 < \alpha \leq 1$, and let the sequence $\{p_n\}$ be defined as in (**) and if $(\int_1^n (P(y))^q/(y^{q\alpha+2-q})dy)^{1/q} = O(P(n)/n^{\alpha+1/q-1})$ then $E_n^*(f) = \max_{0\leq x\leq 2\pi} \|f - N_n\|_p = O(1/n^\alpha)$ where $N_n(x)$ is the (N, p_n) means of (*) and $T_n(x) = N_n(x)$ in particular. (Received December 15, 1971.)

*72T-B71. HENDRIK J. KUIPER, Arizona State University, Tempe, Arizona 85281. On positive solutions of nonlinear elliptic eigenvalue problems.

L is a linear second order partial differential operator and D a well behaved bounded region in R^n. Consider $Lu = \lambda F(x, u(x))$ with zero boundary conditions. $F(x,t)$ is a negative function on $D \times R^1$ which may have jump discontinuities with respect to t. Then under certain natural conditions, there exists a continuum of solutions (λ, u), with $u(x) \geq 0$, connecting $(0,0)$ and ∞ in the space $R^1 \times S$, where S is a Banach space of

real valued functions such as $C(\bar{D})$. It is shown that if A is a compact continuous operator in a Banach space X, positive with respect to a cone K, then there exists a continuum of positive solutions of $u = \lambda Au$ which connects (0,0) and ∞ in $R^1 \times X$. The same result is obtained for the discontinuous operator $L^{-1}F$ by considering it to be the limit of a sequence of continuous operators, each of which possesses such a continuum of positive solutions. A crucial preliminary result states that if u is a positive generalized solution of $Lu = f < 0$ then the function u takes sets of positive measure in R^n onto sets of positive measure in R^1. (Received December 20, 1971.)

*72T-B72. DHARMA P. GUPTA, Motilal Nehru Regional Engineering College, Allahabad-4, India. On Nörlund summation of Jacobi series. Preliminary report.

Let f(x) be a function defined in the closed interval $[-1,+1]$ such that $(1-x)^{\alpha}(1+x)^{\beta} f(x) \in L[-1,+1]$, $\alpha > -1$, $\beta > -1$. The Nörlund summation of the Fourier-Jacobi expansion (*) $f(x) \sim \sum_{n=0}^{\infty} a_n P_n^{(\alpha,\beta)}(x)$ is considered in this paper. The main result reads as follows: Theorem. Let $\{p_n\}$ be a nonnegative nonincreasing sequence such that $\sum_a^n (P_k/k^{\alpha+3/2}\log k) = O(P_n/n^{\alpha+1/2})$, a being fixed positive integer, and $\sum_n (n^{\alpha+1/2}/P_n) < \infty$. If $\int_0^t |[f(\cos\varphi) - A](\sin\varphi/2)^{2\alpha+1}(\cos\varphi/2)^{2\beta+1}|d\varphi = O(t^{2\alpha+2}/\log 1/t)$ then the series (*) is summable (N,p_n) at the point $x = 1$ to the sum A, provided $-1/2 \leq \alpha < 1/2$, $\beta > -1/2$, and the antipole condition $\int_{-1}^{b} (1+x)^{\beta/2-3/4}|f(x)|\,dx < \infty$, b fixed, is satisfied. A similar theorem holds for the other end point. (Received December 20, 1971.)

*72T-B73. J. P. SHARMA and DHARMA P. GUPTA, Motilal Nehru Regional Engineering College, Allahabad-4, India. Very strong Nörlund summability of orthogonal series. Preliminary report.

Let $\{p_n\}$ be a nonnegative, monotonic increasing or decreasing sequence of real numbers such that $P_n = p_0 + p_1 + p_2 + \ldots + p_n \to \infty$ as $n \to \infty$, and $np_n = O(P_n)$. It is shown that if the orthogonal series $\sum a_n \varphi_n(x)$ is $(\bar{N},p_n)$ summable almost everywhere in $[a,b]$ to a certain function f(x), then it is also strongly $(\bar{N},p_n)$ summable almost everywhere in $[a,b]$ to the same function. For the convergence almost everywhere of the sequence $\bar{T}_n^{\nu}(x)$, where $\bar{T}_n^{\nu}(x) = (1/P_n)\sum_{k=0}^{n} p_k S_{\nu_k}(x)$, $k = 0, 1, 2, \ldots$, it is necessary and sufficient that the sequence of partial sums $S_{\nu_{n_k}}(x)$ should be convergent almost everywhere in $[a,b]$, where $\{n_k\}$ is a positive increasing sequence of numbers such that $1 < q \leq n_{k+1}/n_k \leq r$, $k = 0, 1, 2, \ldots, q$ and r being constants, and $\{\nu_n\}$ being an index sequence such that $1 \leq \nu_1 < \nu_2 < \nu_3 < \ldots$. For the convergence almost everywhere of the sequence $\{\bar{T}_n^{\nu}(x)\}$, to a certain function f(x), it is necessary and sufficient that the orthogonal series $\sum a_n \varphi_n(x)$ be very strong $(\bar{N},p_n)$ summable almost everywhere to this function f(x). (Received December 20, 1971.)

*72T-B74. MAX JODEIT, JR., University of Chicago, Chicago, Illinois 60637. Rearrangements and differentiation. Preliminary report.

If $x_1, \ldots, x_n$ are real numbers let $x_1^*, \ldots, x_n^*$ denote a rearrangement of them in decreasing order. Let $u_j = |x_j - x_{j+1}|$, $v_j = x_j^* - x_{j+1}^*$, $1 \leq j < n$. Theorem 1. $v_j^* \leq u_j^*$. Suppose F is a continuous nonnegative function of bounded variation on $[0,1]$, with derivative f. Let g denote the derivative of F^*, the decreasing

rearrangement of F. Theorem 2. $|g|^* \leq |f|^*$. If $F(0) = F(1) = 0$ and now F^* denotes the symmetric decreasing rearrangement of F, the same conclusion holds. The proof uses classical real-variable methods, as found in Saks, "Theory of the integral." Applications include Theorem 404 of Hardy, Littlewood and Polya, "Inequalities," and Theorems 2.1 and 4.1 of G. F. D. Duff, Canad. J. Math. 19(1967), 1153-1178, MR 37 #357. Also see Canad. J. Math. 22(1970), 408-430, MR 41 #1955, for another paper of Duff; see especially (4.18), p. 416. (Received December 13, 1971.)

*72T-B75. KARI HAG, University of Michigan, Ann Arbor, Michigan 48104. Quasiconformal boundary correspondences and extremal mappings in higher dimensions. Preliminary report.

Using a boundary correspondence theorem of Gehring, we prove that an n ($\geq$ 3)-dimensional quasiconformal mapping $f: D \to D'$ induces a quasiconformal mapping f* between the boundaries of the domains, provided these are sufficiently "smooth". Moreover, the inner and outer dilatations are not increased, i.e. $K_I(f^*) \leq K_I(f)$, $K_O(f^*) \leq K_O(f)$. This extends results of Gehring and Väisälä. The theorem is applied to determine solutions to various extremal problems for quasiconformal mappings. In particular, we determine the outer dilatation in the class of all quasiconformal mappings between an infinite circular cylinder and the unit ball. This, in turn, enables us to determine the extremal mapping for the outer dilatation between two infinite convex cones. (Received December 13, 1971.) (Author introduced by Professor Frederick W. Gehring.)

*72T-B76. ARTHUR SARD, University of California, San Diego, La Jolla, California 92037. Approximation based on nonscalar observations.

Splines based on nonscalar observations were introduced in the author's "Optimal approximation", J. Functional Analysis 1(1967), 222-244 and ibid. 2(1968), 368-369. The present paper develops the theory under the best possible hypotheses and suggests new applications. In one of these the observation of a function consists of its boundary values and the co-observation is its gradient. The splines then turn out to be generalized harmonic functions, and our theory pertains to the Dirichlet problem. (Received December 23, 1971.)

72T-B77. JOHN DAUNS, Tulane University, New Orleans, Louisiana 70118. Categorical W-tensor product.

If A and B are von Neumann algebras and $A \bar{\otimes} B$ denotes their categorical C*-tensor product with the universal property, then the von Neumann tensor product $A \triangledown B$ of A and B is defined as $A \triangledown B = (A \bar{\otimes} B)^{**}/J$, where $J \subset A \bar{\otimes} B^{**}$ is an appropriate ideal. It has the universal property. (Received December 27, 1971.)

72T-B78. JOHN C. MORGAN II, Department of Statistics, University of California, Berkeley, California 94720. Infinite games and singular sets. Preliminary report.

A generalization of the classical Banach-Mazur game is presented in which the players are permitted to choose sets from any family of sets. The sets for which the players have winning strategies are characterized for some special families of sets; e.g. all perfect subsets of the unit interval, all closed subsets of the unit interval with positive Lebesgue measure, all subsets of the unit interval having order type $1 + \lambda + 1$, etc. For

:his characterization, the concept of a singular set is introduced and leads to a universal method of classifying 'negligible sets" in analysis, e.g. sets of the first category, sets of Lebesgue measure zero, sets always of the first category, etc. (Received December 27, 1971.) (Author introduced by Professor David Blackwell.)

*72T-B79. THOMAS G. KURTZ, University of Wisconsin, Madison, Wisconsin 53706. A limit theorem for perturbed operator semigroups with applications to random evolutions.

Let U(t) and S(t) be strongly continuous contraction semigroups on a Banach space L with infinitesimal operators A and B respectively. Suppose the closure of $A + \alpha B$ generates a semigroup $T_\alpha(t)$. The behavior of $T_\alpha(t)$ as α goes to infinity is examined. In particular, suppose S(t) converges strongly to P. If the closure of PA generates a semigroup T(t) on $\mathcal{R}(P)$, then $T_\alpha(t)$ goes to T(t) on $\mathcal{R}(P)$. If PA = 0 and if BVf = -f for $f \in \mathcal{N}(P)$, conditions are given that imply $T_\alpha(\alpha t)$ converges on $\mathcal{R}(P)$ to a semigroup generated by the closure of PAVA. The results are used to obtain new and known limit theorems for random evolutions, which in turn give approximation theorems for diffusion processes. (Received December 27, 1971.)

*72T-B80. K. P. S. BHASKARA RAO and M. BHASKARA RAO, Indian Statistical Institute, Calcutta 35, India. On topological properties of measure algebras. Preliminary report.

Let $\mathcal{A}$ be a field of sets and μ a charge on $\mathcal{A}$. Let $\mathcal{A}(\mu)$ be the collection of all equivalence classes of $\mathcal{A}$ under the equivalence relation $A \sim B$ if $\mu(A \Delta B) = 0$. $\mathcal{A}(\mu)$ is a metric space. In this paper, some topological properties of $\mathcal{A}(\mu)$ are characterised in terms of $\mathcal{A}$ and μ. Some sample results are: (1) $\mathcal{A}(\mu)$ is compact if and only if $\mu = \sum a_i \mu_i$ with $a_i \geq 0$ and μ_i is 0-1 valued charge on $\mathcal{A}$ and the family $\mu_i : i \geq 1$ is disjoint. (Definition. A sequence $\mu_n : n \geq 1$ of 0-1 valued charges on an algebra $\mathcal{A}$ is said to be disjoint if for any sequence $i_1, i_2, \ldots$ (finite or infinite) of natural numbers, there exists A in $\mathcal{A}$ such that $\mu_{i_k}(A) = 1$ for every $k \geq 1$ and $\mu_p(A) = 0$ for every $p \neq i_k$, $k \geq 1$.) (2) $\mathcal{A}(\mu)$ is locally compact if and only if $\mathcal{A}(\mu)$ is compact. (3) $\mathcal{A}(\mu)$ is a complete metric space if and only if $\mathcal{A}(\mu)$ is a complete Boolean algebra and μ is countably additive on $\mathcal{A}(\mu)$. If, further, $\mathcal{B}$ is a sigma-algebra and λ is a measure on $\mathcal{B}$, then (4) $\mathcal{B}(\lambda)$ is compact if and only if it is locally compact if and only if it is totally disconnected if and only if λ is nonatomic. (5) $\mathcal{B}(\lambda)$ is connected if and only if λ is nonatomic. (Received December 17, 1971.) (Introduced by Dr. M. G. Nadkarni.)

*72T-B81. HASKELL P. ROSENTHAL, University of California, Berkeley, California 94720. On subspaces of L^p.

Given (real or complex) Banach spaces X and Y and $K < \infty$, Y is said to be K-isomorphic to X if there is an invertible linear operator $T : X \to Y$ with $\|T\| \, \|T^{-1}\| \leq K$; Y imbeds in X if Y is linearly homeomorphic to a subspace of X; Y is K-complemented in X if $Y \subset X$ and there is a linear projection P from X onto Y with $\|P\| \leq K$. The theory of p-absolutely summing operators is applied in this paper, to obtain linear topological invariants of subspaces of L^p. The main structural result obtained is the Theorem. Let $1 \leq p < 2$, μ a measure on some measureable space, and X a closed linear subspace of the (real or complex) space $L^p(\mu)$. Then either X imbeds in $L^{p'}(\mu)$ for some $p' > p$, or for all $\epsilon > 0$, there exists a linear subspace

Y of X which is $(1+\epsilon)$-complemented in $L^p(\mu)$ and $(1+\epsilon)$-isomorphic to ℓ^p. Corollary. Every closed linear subspace of $L^1(\mu)$ of infinite dimension, contains a subspace with an unconditional basis. (Received December 3, 1971.)

72T-B82. WITHDRAWN.

72T-B83. JOSEPH PETER DURAN, University of Rochester, Rochester, New York 14627. Matrices containing Banach-Hölder summation. Preliminary report.

A Banach-Hölder (B-H_0) limit is a Banach limit φ such that $\varphi(x) = \varphi(H_0 x)$ for every bounded real sequence $x = (x_0, x_1, \ldots)$, where H_0 is the Hölder-Césaro matrix. A bounded real sequence x is said to be B-H_0 summable to $\alpha \in \mathbb{R}$ if $\varphi(x) = \alpha$ for each B-H_0 limit φ. α is called the B-H_0 sum of x. Theorem. The regular matrix $A = (a_{m,n})$ sums every B-H_0 summable sequence to its B-H_0 sum if and only if $\lim_m \sum_{n=0}^{\infty} |(n/n+1)\, a_{m,n} - \sum_{\ell=n+1}^{\infty} a_{m,\ell}/\ell + 1| = 0$. Corollary. The method of summation by logarithmic means sums every B-H_0 summable sequence. Because of a theorem of Eberlein (Abstract 691-46-21, these *Notices* 19(1972), A-164) which implies trivially that B-H_0 summation coincides with Banach-Hausdorff summation (Eberlein, Proc. Amer. Math. Soc. 1(1950), 662-665), the theorem characterizes a class of matrices which contain the collective Hausdorff method for bounded sequences. This complements a result of Fuchs (Proc. Amer. Math. Soc. 1(1950), 66-70) that there is no matrix which contains the collective Hausdorff method for all sequences. The corollary generalizes the classical result that the logarithmic method contains (C,1) summability. (Received January 6, 1972.)

*72T-B84. ANDRE de KORVIN, Indiana State University, Terre Haute, Indiana 47809 and CHARLES ALEX CHENEY and RICHARD A. ALO, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213. A Vitali-Hahn-Saks theorem. I.

Let M and N be two von Neumann subalgebras of $\mathcal{L}(h)$. If L is a von Neumann algebra L_* will denote its predual. Theorem. Let F be a collection of continuous linear mappings from M into N such that $\rho_0 \varphi \in M_*$ for every $\rho \in N_*$ and $\varphi \in F$. Assume F is pointwise bounded on the projections of M. Then F is uniformly bounded on norm bounded subsets of M. Theorem. Let φ_n be a sequence of continuous linear mappings from M into N and assume $\rho_0 \varphi_n \in M_*$ for all $\rho \in N_*$. Assume $\lim \varphi_n(P) = \varphi(P)$ for all projections P of M. Then φ can be extended to a continuous linear map from M into N. Theorem. Let $\psi_n = \varphi_n|_{P \cap L}$ where $P \cap L$ are the projections common to M and to an abelian subalgebra L then $\{\psi_n\}$ is equicontinuous at 0 for the weak topology. (Received January 6, 1972.)

*72T-B85. ANDRE de KORVIN and LAWRENCE E. KUNES, Indiana State University, Terre Haute, Indiana 47809. A topology associated with $\mathcal{L}^p$ spaces. I.

Let $\mathcal{C}$ denote a ring of subsets of T, μ a positive finite measure on $\mathcal{C}$, m a measure from $\mathcal{C}$ into L(E,F) (bounded linear operators from E into F). Let $\tilde{m}_q(A) = \sup |\sum m(A_i)x_i|$ where the sup is taken over simple functions $\sum X_{A_i} \cdot x_i$ with p-norm less or equal to 1. $p_{m,A}$ is a seminorm on F* defined by $p_{m,A}(y^*)$

$= \bar{m}_{y^*}(A)$ where $\bar{m}_{y^*}$ is the variation of m_{y^*} with $\langle m_{y^*}(A), x\rangle = \langle y^*, m(A)x\rangle$. On F* consider the weak topology defined by $p_{m,A}$ as m ranges over all measures with $\tilde{m}_q(A) < \infty$ and $A \in \mathcal{C}$. Let σ^* be the unit sphere of F* and let (σ^*, δ_q) be the relative topology. $(\sigma^*, \delta_{m,q})$ is the relative topology in σ^* as m is kept fixed with $\tilde{m}_q(A) < \infty$. m is called right continuous if for $A_n \searrow \varphi$ then $m(A_n) \to 0$. If $(\sigma^*, \delta_{m,q})$ is compact, different conditions are obtained for $\tilde{m}_q$ to be right continuous involving the q-variation of m_{y^*}. Also if σ^* is not compact the right continuity of $\tilde{m}_q$ is studied. The conditions are somewhat comparable with the variational semiregularity as defined by P. Lewis. (Received January 6, 1972.)

*72T-B86. RICHARD A. ALO, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213 and ANDRE de KORVIN and LAWRENCE E. KUNES, Indiana State University, Terre Haute, Indiana 47809. A topology associated with $\mathcal{L}^p$ spaces. II.

The notations are as defined previously. Lemma. The following conditions are equivalent: (σ^*, δ_q) is Hausdorff, $\bigcup_m \bigcup_A m(A)\sigma$ (m ranges over all measures such that $\tilde{m}_q(A) < \infty$ and $A \in \mathcal{C}$, σ is the unit sphere of E) is dense in F, (σ^*, δ_q) is stronger than the wk* topology on σ^*. Theorem. If (σ^*, δ_q) is Hausdorff then (σ^*, δ_q) is compact if and only if (σ^*, δ_q) coincides with σ^* with its wk* topology. If (σ^*, δ_q) and (σ^*, δ_r) are Hausdorff then (σ^*, δ_q) is compact if and only if (σ^*, δ_r) is. Theorem. If (σ^*, δ_q) is Hausdorff then the following conditions are equivalent: (1) The topology generated by $p_{m,A}$ (m and A fixed) is Hausdorff. (2) If $\tilde{n}_q(A) < \infty$ then $(\overline{m_{y^*}})_q(A) = 0$ implies $(\overline{n_{y^*}})_q(A) = 0$. (3) The topology defined in (1) is stronger then wk* topology. (Received January 6, 1972.)

*72T-B87. JAMES C. S. WONG, University of Calgary, Calgary, Alberta, Canada. An ergodic property of locally compact amenable semigroups.

Let S be a locally compact topological semigroup and M(S) the Banach algebra of all bounded regular Borel measures on S with convolution as multiplication and variation norm. For $F \in M(S)^*$, $\mu \in M(S)$ define $\mu \odot F$ and $F \odot \mu \in M(S)$ by $(\mu \odot F)(\nu) = F(\mu * \nu)$ and $(F \odot \mu)(\nu) = F(\nu * \mu)$, $\nu \in M(S)$. $M \in M(S)^{**}$ is called a mean iff $M(F) \geq 0$ whenever $F \geq 0$ and $M(1) = 1$ where $1 \in M(S)^*$ is defined by $1(\mu) = \mu(S)$, $\mu \in M(S)$. M is topological left invariant if $M(\mu \odot F) = M(F)$ $\forall$ $F \in M(S)^*$ and $\mu \in M_0(S) = \{\mu \in M_0(S) : \mu \geq 0 \text{ and } \|\mu\| = 1\}$. Theorem. The following statements are equivalent: (1) $M(S)^*$ has a topological left invariant mean. (2) For each $F \in M(S)^*$, there is a net $\mu_\alpha \in M_0(S)$ such that $F \odot \mu_\alpha \to \beta.1$ weak* in $M(S)^*$ for some constant β, and (3) for each $\mu \in M(S)$, $|\mu(S)| = \inf\{\|\mu * \nu\| : \nu \in M_0(S)\}$. Equivalence of (1) and (3) is an analogue of a result of H. Reiter on ergodic property of locally compact amenable groups (see [Greenleaf, "Invariant means on topological groups," Van Nostrand, New York, 1969, p. 77, Theorem 3.7.3]). Equivalence of (1) and (2) is an extension in a slightly different form of a result by this author in ["Topologically stationary locally compact groups and amenability," Trans. Amer. Math. Soc. 144(1969), 351-363]. (Received January 10, 1972.)

*72T-B88. PETER G. CASAZZA, University of Iowa, Iowa City, Iowa 52240. On a geometric condition related to boundedly complete bases and normal structure in Banach spaces.

A basis $\{x_n\}$ of a Banach space X is said to satisfy property A if for every number $C > 0$, there exists

a number $r_c > 0$ such that $\| \sum_{i=1}^{n} \alpha_i x_i \| = 1$ and $\| \sum_{i=n+1}^{\infty} \alpha_i x_i \| \geq C$ imply $\| \sum_{i=1}^{\infty} \alpha_i x_i \| \geq 1 + r_c$. It is known that property A implies (1) $\{x_n\}$ is a boundedly complete basis, (2) X is isomorphic to a locally uniformly convex (l.u.c.) space, and (3) every convex, weakly compact subset of X has normal structure. In this paper, we construct a reflexive Banach space X, with an unconditional basis $\{x_n\}$, such that: (a) X is l.u.c., (b) X has normal structure, and (c) there does not exist an equivalent norm on X with respect to which $\{x_n\}$ satisfies property A; showing, with a single example, that the converse of (1), (2), and (3) are invalid even with the weaker conclusion that $\{x_n\}$ be equivalent to a basis satisfying property A. (Received January 11, 1972.)

72T-B89. LAWRENCE A. ZALCMAN, Stanford University, Stanford, California 94305. Analyticity and the Pompeiu problem. II. Preliminary report.

We announce extensions of the results of our previous notice (Abstract 691-30-3, these *Notices* 19(1972), A-110) to functions defined in discs (with either the hyperbolic or the Euclidean geometry). In particular, new versions of Morera's theorem and the converse to the mean value theorem for harmonic functions are obtained to resolve some problems posed at the end of our earlier paper. Techniques employed include pure real variable methods as well as consideration of a differential equation of generalized Euler-Poisson-Darboux type. Related results are obtained for functions defined in squares and other regions. (Received January 11, 1972.)

72T-B90. ARNO C. SEROLD, University of Colorado, Boulder, Colorado 80302. Discontinuous translation-invariant linear forms exist on $\ell_p(Z)$. Preliminary report.

It is proved that there exist discontinuous, translation-invariant linear forms on the Banach spaces $\ell_p(Z)$, $1 < p \leq \infty$, $c(Z)$ and $c_0(Z)$. The proof employs the general criterion stated by G. H. Meisters in Abstract 72T-B24, these *Notices* 19(1972), A-15, but otherwise requires different methods. (Received January 17, 1972.)

72T-B91. DAVID G. COSTA, Brown University, Providence, Rhode Island 02912. Decay and growth of solutions of higher order hyperbolic equations. Preliminary report.

Let $u(x,t)$, $x \in \mathbb{R}^n$, n odd, $t \geq 0$, be a solution of a strictly hyperbolic homogeneous equation of order $m \geq 2$, m even, subject to smooth initial data with compact support. Theorem. Assume the equation is spherically symmetric and none of the characteristic roots is zero. Then, $\sup_x |u(x,t)| = O(t^{-(n-1/2)})$ (as $t \to +\infty$) if $2 \leq m \leq (n+3/2)$, and $= O(t^{-(n+1)+m})$ if $(n+3/2) < m < +\infty$. Furthermore, these powers are optimal. This result partly generalizes a well-known result for the wave equation. (Received January 17, 1972.)

*72T-B92. GARY H. MEISTERS and WOLFGANG M. SCHMIDT, University of Colorado, Boulder, Colorado 80302. Translation-invariant linear forms on $L^2(G)$ for compact Abelian groups G. Preliminary report.

The first author has proved previously [J. Functional Analysis 8(1971), 173-188] that translation-invariant linear forms on $\mathcal{D}(R^n)$, as well as on several other spaces of C^∞ test functions and distributions, are necessarily continuous. The analogous result is proved here for the Hilbert space $L^2(G)$, where G is a compact Abelian group with a finite number of components, as well as for a one-parameter family of spaces $\mathfrak{F}_p(G)$ of pseudo-measures on G, for $1 \leqq p < \infty$. These results are new even when G is the circle group T. Some results are: Theorem. For each f in $L^2(G)$ with $\int_G f(x)\,dx = 0$ and for each integer $m \geqq 3$, there is a set N(f) consisting of almost all (in the sense of Haar measure) m-tuples $(a_1,\ldots,a_m)$ in G^m such that for each m-tuple $(a_1,\ldots,a_m)$ in N(f) there are vectors $g_1,\ldots,g_m$ in $L^2(G)$ such that $f(x) = \sum_{j=1}^m [g_j(x) - g_j(x - a_j)]$ for almost all x in G. Corollary. If $\varphi : L^2(G) \to C$ is a translation-invariant linear form (not assumed continuous or positive) on the Hilbert space $L^2(G)$, then there is a complex constant c such that $\varphi(f) = c\int_G f(x)\,dx$ for all f in $L^2(G)$. These results are best possible in the sense that (1) $m = 2$ fails for some f in $L^2(G)$ satisfying $\int_G f(x)\,dx = 0$, (2) $\cap\{N(f) : f \in L^2(T)\} = \emptyset$, (3) the corollary fails for groups with infinitely many components. (Received January 17, 1972.)

72T-B93. JOE B. HILL, Auburn University, Montgomery, Alabama 36109. Renorming and the regular functionals. Preliminary report.

(S,g) denotes a linear space S with norm g, ((S,g)*,g*) denotes the first conjugate space of (S,g) with conjugate norm g*. A point f in (S,g)* is called regular if there exists an x in S, $g(x) = 1$ and $f(x) = g^*(f)$. R_g denotes the set of regular functionals in (S,g)*. Suppose H is a closed linear manifold in S and $f \in (S,g)^*$. The statement that f is irregular on H means that for each $y \in H$, such that $g(y) = 1$, $f(y) < \text{lub}\{f(x) \mid x \in H, g(x) = 1\}$. Theorem 1. Suppose (S,g) is a normed linear space and $f \in (S,g)^*$. Then there exists a norm h on S such that (1) h is equivalent to g, (2) $f \in R_h$ and (3) $R_g \subseteq R_h$. Theorem 2. Suppose (S,g) is a normed linear space and H is a closed linear manifold in S. Suppose further that $f \in (S,g)^*$ and f is irregular on H. Then there exists a norm h on S such that (1) h is equivalent to g and (2) $f \notin R_h$. (Received January 19, 1972.)

Applied Mathematics

*72T-C7. PETER M. GOORJIAN, USAAMRDL, Ames Research Center, Moffett Field, California 94035. An invalid equation in the general momentum theory of the actuator disc.

The general momentum theory of the actuator disk model of a propeller, including a helicopter rotor, (Aerodynamic Theory, vol. 4, pp. 182-193) is incorrect. An unverified equation is included in the theory which, when combined with the other equations, leads to a contradiction. Specifically, the equation of axial momentum for the propeller states that the total thrust is expressible in terms of integrals of the wake parameters. Then the differential form of that equation is applied to the separate annular elements of the propeller. It is this application which leads to a contradiction. The proof employs a maximal method to obtain bounds on the solutions to the resulting system of ordinary differential equations. With these bounds no solution exists which satisfies the boundary conditions of the system. (Received November 15, 1971.)

*72T-C8. MICHAEL A. GOLBERG, University of Nevada, Las Vegas, Nevada 89109. A generalized Rayleigh quotient for eigenvalue problems nonlinear in the parameter.

A functional is given which generalizes the Rayleigh quotient to eigenvalue problems for linear operators, where the eigenvalue parameter appears nonlinearly. Particular emphasis is given to the development of perturbation type results for eigenvalues and characteristic values which generalize the classical results. Applications are made to eigenvalue and characteristic value problems for integral and matrix operators and to the critical length problem for integral operators. Both symmetric and nonsymmetric operators are treated. (Received December 6, 1971.)

*72T-C9. KENDALL E. ATKINSON, Indiana University, Bloomington, Indiana 47401. A Schauder-Leray degree theory for collectively compact operator approximations. Preliminary report.

Let X be a Banach space, and let K be a completely continuous operator from the open region $D \subset X$ into X. Let K_n, $n \geq 1$, be a family of approximating operators defined from D into X, and let them satisfy: (i) $K_n(x) \to K(x)$, for all $x \in D$; (ii) $\{K_n\}$ is a collectively compact family, i.e. $\bigcup_n K_n(B)$ has compact closure in X for all bounded sets $B \subset D$; and (iii) $\{K_n\}$ is equicontinuous at all $x \in D$. Define $\Phi(x) \equiv x - K(x)$ and $\Phi_n(x) \equiv x - K_n(x)$, $x \in D$. Let T be any closed, bounded, and connected subset of D with a nonempty interior; let S be its boundary. Suppose that $\Phi(x) \neq 0$ for all $x \in S$. Then for all sufficiently large n, say $n \geq N$, $\Phi_n(x) \neq 0$ for all $x \in S$. More importantly, the Schauder-Leray rotations of Φ and Φ_n on S are the same for all $n \geq N$. These results lead to existence and convergence theorems for the fixed points of the operators K_n vis-a-vis those of the original nonlinear operator K. Collectively compact approximations arise in the replacement of a nonlinear integral operator by nonlinear numerical integral operators. (Received December 6, 1971.)

72T-C10. NORMAN M. STEEN and GEORGE D. BYRNE, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. An algorithm for nonlinear parameter estimation. Preliminary report.

We seek an n-vector θ^* which minimizes $R(\theta) = \sum_{k=1}^{m} w_k [y_k - f_k(\theta)]^2$. Let θ° be an estimate of θ^* and δ_g and δ_t denote the gradient and Taylor increments at θ°. Thus $\delta_g = A\delta_t$, where $A = [a_{ij}] = [2\sum_{k=1}^{m} w_k \{\partial f_k(\theta^\circ)/\partial\theta_i\}\{\partial f_k(\theta^\circ)/\partial\theta_j\}]$. Assume A^{-1} exists, the $\lambda_i(A)$ are distinct, and the $w_k > 0$ have been multiplied by $c > 0$ so that $\lambda(A) > 2$. Let $\tilde{R}(\theta+\delta) = \sum_{k=1}^{m} w_k [y_k - f_k(\theta) - \sum_{j=1}^{n} (\partial f_k(\theta)/\partial\theta_j)\delta_j]^2$ and $\delta_p = \omega(p)[p\Delta + \delta_t]$, where $\Delta = \delta_g - \delta_t$ and $\omega(p) = (1 + p\Delta^T A\Delta/\delta_g^T\Delta)^{-1}$. The salient features of the algorithm are that the norm and direction of δ_p are so chosen that $\tilde{R}(\theta^\circ + \delta_p)$ is expected to be an adequate approximation to $R(\theta^\circ + \delta_p)$ and that $\tilde{R}(\theta^\circ + \delta_p)$ is minimal on a restricted class of vectors. The algorithm admits δ_p such that $\delta_p^T \delta_g < 0$. An estimate of $\|\delta_p\|$ is obtained via some pseudonorm. Theorem. Let $\delta_g \neq 0$ and let A and δ_p be as above. Then there exists $r_0 < 1$ such that for each r, $r_0 \leqq r \leqq 1$, there is a unique $p \in [0,1]$ for which $r\|\delta_t\| = \|\delta_p\|$ and $\tilde{R}(\theta^\circ + \delta_p)$ is minimal on the set $\{\delta, \|\delta\| = \|\delta_p\|, \delta = \mu[q\Delta + \delta_t], \mu > 0\}$. In all tests to date this algorithm has outperformed those of Marquardt [SIAM J. Appl. Math. 2(1963), 431-441] and Jones [Comput. J. 13(1970), 301-308]. (Received December 9, 1971.)

72T-C11. RAYMOND M. REDHEFFER, Mathematisches Institut Universität Karlsruhe, Karlsruhe, Germany. Lower bounds for solutions of nonlinear equations.

Let $x = X(s)$ be a smooth curve, with arc $s > 0$, in a region $B \subset E_n$. Let $u(x) > 0$, $u \in D^{(2)}(B)$, and $\rho(s) = \inf_x |X(s) - x|$ $(x \in \partial B)$, $M(s) = \inf_x u(x)$ $(|X(s) - x| < \rho(s)/2)$. Suppose $a(x,u')u'' \leqq \epsilon_1^2 u + \epsilon_2 |u'|$, $a \geqq 0$, $pa(x,p)p \geqq |p|^2$, $\mathrm{tr}(a) \leqq \epsilon_0$ with $\epsilon_k \geqq$ constant, $\epsilon = (\epsilon_1 + \epsilon_2)/2$. Then $M(s) \geqq M(0)[e^{2h\rho(s)/\rho(0)}]^{-\epsilon_0}[e^{2s}e^{\rho(s)}/e^{\rho(0)}]^{-\epsilon}$, $h = \int_0^s dt/\rho(t)$. Corollary. Let $n = 2$, $B = \{x \mid |x| < 1\}$ and let u satisfy, besides the above, $-\delta_1^2 u - \delta_2|u'| \leqq a(x,u')u''$, $\delta_k \geqq 0$ constant. Then $u(x) \geqq ce^{-5\epsilon_0}e^{-3\epsilon\delta^{(7/3)\epsilon_0}}u(0)$, $|x| < 1/2$, where $c \geqq 3/16$, $c \to 3/8$ for $\delta_1 \to 0$, and $\delta = (1/2\delta_2)\log(1 + \delta_2/2\delta_1)$ or $1/4$, whichever is smaller. (Received November 8, 1971.)

*72T-C12. LOKENATH DEBNATH, East Carolina University, Greenville, North Carolina 27834. On the growth of unsteady hydromagnetic multiple boundary layers.

A study is made of the development of hydromagnetic multiple boundary layers in an electrically conducting rotating fluid bounded by an infinite disk or by two parallel disks in the presence of a uniform magnetic field. The velocity field of the unsteady boundary layer flow generated by the nontorsional oscillations of the disk(s) is calculated for small and large values of time. The simultaneous effects of the external magnetic field and the Coriolis force with a special reference to weak and strong rotation have been explored. Characteristic features of the flow phenomena are examined in some detail. The Laplace transform treatment together with the Heaviside expansion theorem has been employed to carry out the mathematical calculation. (Received December 23, 1971.)

*72T-C13. MITCHELL WAND, Artificial Intelligence Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. The elementary algebraic theory of generalized finite automata. Preliminary report.

A clone of operations is a generalization of a monoid of transformations to functions of finite numbers of variables [Cohn, 1965]. Clones are seen to play the same role in the theory of generalized finite automata on trees as monoids play in the theory of finite automata on strings. The generalizations of several elementary constructions in the algebraic theory of automata are considered, including the input clone of an automaton and the syntactic clone of an event. It is shown that the Nerode and Myhill theorems reflect an ambiguity in the usual constructions of a free monoid. (Received December 23, 1971.)

72T-C14. O. P. BHUTANI, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio 45433 and R. S. SETH, Indian Institute of Technology, New Delhi, India. Differential effects of shock-fronts in dissociating gas flows.

Assuming the shock configuration to be a continuous differentiable function of coordinates along which the velocity components, pressure, density and relaxation behind the shock surface are differentiable functions, we have in this paper studied the following topics: (1) gradients of flow variables behind a shock; (2) components of vorticity. In short, we have carried over the most comprehensive and detailed works on differential effects of shock-fronts in conventional gasdynamics by Truesdell, Thomas, Kanwal and Taub to the case of dissociating gasdynamics. The interesting results that have emerged out of this study are summed up in the form of theorems. (Received November 4, 1971.) (Authors introduced by Dr. Oved Shisha.)

72T-C15. MEERA M. BLATTNER, Los Angeles Harbor College, Wilmington, California 90744. The unsolvability of the equality problem for sentential forms of context-free grammars. Preliminary report.

Let grammar $G = (V, \Sigma, S, P)$ where V is a finite set, $\Sigma \subset V$, $S \in (V - \Sigma)$, and P is a set of production rules. Define $\text{Sent}(G) = \{\alpha | \alpha \in V^* \text{ and } S \overset{*}{\Rightarrow} \alpha\}$. Sent(G) is the set of sentential forms of G. Questions about the structure of sentential forms of grammars arose in applications of languages to theoretical biology (Lindenmayer, 1968) and the development of OL-languages. In this paper it is shown that the question of Sent(G) = Sent(G') is undecidable for two context-free grammars G and G'. The proof makes use of linear grammars and a-transducers. Sent(G) = Sent(G') is also an undecidable question for c-finite and bounded grammars. Finally, since the sentential forms of context-free grammars are special cases of OL-languages, then L(G) = L(G') is undecidable for OL-systems G and G'. (Received January 11, 1972.)

*72T-D4. W. ROY UTZ, University of Missouri, Columbia, Missouri 65201. The motion of a billiard ball on a triangular table.

By a cycle on a triangular billiard table one means a path from the initial position of the ball to an intersection, A, of a cushion followed by intersections with the other two cushions (in either order) and then the return of the ball to A. Thus a cycle, aside from its initial path to A, is a triangle with A as a vertex. C. V. Boys (Math. Gaz. 15(1930), 78-81) has given a construction for cycles when they are possible. Contrary to the impression given in the paper cited, cycles are not possible from all positions in all acute angled triangles, even, if the initial cushion is specified. Limitations on initial positions of the ball are given for acute angled triangles and periodicity is considered. The question of oscillation in the sense of A. M. Leontovič (Soviet Math. Dokl. 3(1962), 1049-1052) for infinite tables is posed for triangular tables with small angles and the possibility of cycles on other polygonal tables is considered. Results are secured by elementary methods involving a covering manifold for the table. (Received November 10, 1971.)

*72T-D5. RICHARD S. MILLMAN, Southern Illinois University, Carbondale, Illinois 62901. The holomorphic connection as a C^∞ phenomenon.

The purpose of this paper is to give a necessary and sufficient condition which answers the following question: To what extent does the existence of a holomorphic connection depend only on the underlying real (C^∞) structure of the bundle? More precisely, if ξ and ξ_0 be holomorphic principal fibre bundles which are equivalent as real bundles (i.e. there is a smooth but not necessarily holomorphic bundle isomorphism between them), the question which we treat in this paper is the following: If ξ_0 has a holomorphic connection, does ξ? The necessary and sufficient condition which we obtain reduces this problem to the problem of global solution of certain differential equations. These equations are solved for bundles with a compact Kahler base and abelian structure group thus proving that the existence of a holomorphic connection is a C^∞ phenomenon in this case. (Received November 11, 1971.)

72T-D6. JOSEPH A. ERBACHER, University of Southern California, Los Angeles, California 90007. A characterization of Riemannian manifolds of constant curvature. Preliminary report.

Let $\bar{M}$ be a C^∞ Riemannian manifold, and let M be a C^∞ compact, relatively orientable submanifold of codimension 1 of $\bar{M}$, possibly with boundary. For sufficiently small s, let M_s denote the set of points lying on geodesic normals to M (and on a fixed side of M) at a distance s from M. Let A(s) denote the volume of M_s. Let $D = d/ds$, and let c be a constant. Let $L_n = (D^2+c)(D^2+9c)\cdots(D^2+(n-1)^2c)$ for n even, $L_n = D(D^2+4c)\cdots(D^2+(n-1)^2c)$ for n odd. The author has proven the following theorem, announced as a conjecture in Abstract 71T-D30, these *Notices* 18(1971), 1106. Theorem. Let $\dim \bar{M} = n$. If $L_n A = 0$ for all compact, relatively orientable hypersurfaces M of $\bar{M}$, then $\bar{M}$ has constant curvature c. (Received November 15, 1971.)

72T-D7. RICHARD H. ESCOBALES, JR., University of Notre Dame, Notre Dame, Indiana 46556. Submersions from spheres. Preliminary report.

Let π be a Riemannian submersion from a connected complete Riemannian manifold M onto a Riemannian manifold B. Assume the fibres $\pi^{-1}(x)$ are totally geodesic submanifolds of M for every $x \in B$. Proposition. Let π, M and B satisfy the above hypotheses. If M is δ pinched, B is $\delta/4$ pinched. Now let $M = S^n$ where S^n is the unit sphere of constant curvature 1. We apply the above proposition together with some known results to obtain the following Theorem. Assume $\pi : S^n \to B$ is a Riemannian submersion with totally geodesic fibres of dimension one or greater. Then B is simply connected and even dimensional. In fact B is homeomorphic to a sphere or isometric to a compact symmetric space of rank one. (Received November 29, 1971.)

72T-D8. ISAAC CHAVEL, City University of New York, City College, New York, New York 10031 and HARRY E. RAUCH, City University of New York, Graduate Center, New York, New York 10036. Holomorphic embedding of complex curves in spaces of constant holomorphic curvature.

Theorem. Let CP_n be an n-(complex) dimensional complex projective space with its Fubini-Study metric, M a compact, connected, orientable 2-(real) dimensional manifold, $f : M \to CP_n$ a minimal Riemannian immersion such that f(M) is not homologous to zero. If the second fundamental form of M in CP_n is formally holomorphic (cf. definition below) with respect to the complex structure on M induced by its Riemannian metric, then f(M) is a complex submanifold of CP_n. Definition. The second fundamental form is formally holomorphic if the projection of its covariant differential on the (0,1)-forms on M vanishes. (Received December 9, 1971.)

Logic and Foundations

*72T-E5. JOHN HAYS, 927 South Douglas, Springfield, Missouri 65802. Matrices for absolute (uncomplemented or positive) lattice logics. Preliminary report.

These are "truth tables" for analogue of intuitionist positive statement calculus (with modal interpretations, and the classical as special case), i.e. distributive lattices L represented by factor lattices on natural numbers $n = \pi^k p_i^{m_i}$: max = n, min = 1. L_n is constructed from k atoms (primes) by meet (gcd), join (lcm), and augment (product: Abstract 71T-A245, these *Notices* 18(1971), 947), with a proper join-irreducible (p^r, $r > 1$) constructed by pure augment. Each matrix has 2^k states, an infinite number of orbits (agreeing with Gödel, 1933), of which j are reserved for valuations v = 1 or 0 if $j = \max m_i$, and the remaining orbits with lowest value, - (blank). (Classically, j = 1. Note. Matrix is not strictly 3-valued, since each first orbit is 2-valued.) Given any independent assignment of values for atoms p_i, value of proper join-irreducible p_i^r, $1 < r \leq j$, repeats orbital values of p_i; $v(a \wedge b) = \min(v(a), v(b))$; $v(a \vee b) = \max(v(a), v(b))$. Definition (subjuncture, analogue of implication). $a \curvearrowright b = \max\{c \mid a \wedge b = a \wedge c\}$. (Suprajuncture, $\curvearrowright$, is its dual, also analogue of implication.) Lemma. Values for atoms and meet implicitly determine values for subjuncture. Matrices for complete analogue of statement calculus can be obtained by complementation and extension to the free lattice. (Received October 29, 1971.)

72T-E6. WILFRID A. HODGES, Bedford College, Regent's Park, London N.W.1, England. Embedding orderings in models of arithmetic. Preliminary report.

Theorem 1. Let T be any countable complete extension of Peano arithmetic with induction for all formulae, and let $\aleph$ be a cardinal which is regular but $\aleph \not\to (\aleph)^2_2$. Then there is no linearly ordered set of cardinality $\aleph$ which is order-embeddable in all models of T of cardinality $\aleph$. Theorem 2. Let $\aleph$ be either a singular strong limit number or a cardinal such that $\aleph \to (\aleph)^2_2$. Then $\aleph$ itself is order-embeddable in all models of Peano arithmetic of cardinality $\aleph$. (Received November 1, 1971.)

72T-E7. JOSEPH GRIFFIN, Pennsylvania State University, University Park, Pennsylvania 16802. On cylindric algebras of structures. Preliminary report.

Let $\mathfrak{A}^*$ denote the cylindric algebra determined by the structure $\mathfrak{A}$ (see Henkin's paper in "Mathematical interpretation of formal systems," North-Holland, 1955). For structures $\mathfrak{A}$, $\mathfrak{B}$ of the same similarity type, elementary equivalence (hence isomorphism) of $\mathfrak{A}$ and $\mathfrak{B}$ is sufficient but not necessary for $\mathfrak{A}^*$ and $\mathfrak{B}^*$ to be isomorphic, but structures of different types can have isomorphic cylindric algebras. Definitions. $\mathfrak{B}$ is a definitional expansion of $\mathfrak{A}$ if $\mathfrak{A}$ is a reduct of $\mathfrak{B}$ and for each predicate letter P_α in the language of $\mathfrak{B}$ there is φ_α in the language of $\mathfrak{A}$ such that $\mathfrak{B} \models P_\alpha v_1 \ldots v_{n_\alpha} \leftrightarrow \varphi_\alpha$; and $\mathfrak{A}$ and $\mathfrak{B}$ are definitionally equivalent if there exist structures $\bar{\mathfrak{A}}$ and $\bar{\mathfrak{B}}$, isomorphic to $\mathfrak{A}$ and $\mathfrak{B}$ respectively, with a common definitional expansion. Theorem. If $\mathfrak{A}$ and $\mathfrak{B}$ are finite structures of finite order then $\mathfrak{A}^*$ and $\mathfrak{B}^*$ are isomorphic iff $\mathfrak{A}$ and $\mathfrak{B}$ are definitionally equivalent. Also, for arbitrary structures $\mathfrak{A}$ and $\mathfrak{B}$, $\mathfrak{A}^*$ is a subalgebra of $\mathfrak{B}^*$ iff $\mathfrak{A}$ is a reduct of some definitional expansion of $\mathfrak{B}$. (Received November 3, 1971.) (Author introduced by Professor Hugo B. Ribeiro.)

72T-E8. GEORGE F. McNULTY, University of California, Berkeley, California 94720. Undecidable properties of finite sets of equations. Preliminary report.

For notation see [Abstract 70T-E47, these *Notices* 17(1970), 675]. T is base-decidable modulo U if the set of finite sets of equations extending U to a base of T is recursive. Example. The theory of semilattices is base-decidable modulo the associative law. T is compatibility-decidable if the inconsistent theory is base-decidable modulo T. The theory of semigroups is compatibility-decidable (Perkins, Ph.D. Thesis, University of California, Berkeley, 1966). Example. The theory of all semilattices, lattices, Boolean algebras, groups, Abelian groups, rings, or rings with unit is compatibility-decidable. Theorem. Fix a similarity type including an operation symbol of rank at least two or two operation symbols of rank one. (1) The family of finite sets of equations from which T is derivable is recursive iff T is comprised entirely of tautologies. (2) The family of consistent finite sets of equations from which T is derivable is recursive iff T has no consistent finitely based extensions. (3) The family of irredundant sets of equations is not recursive. (4) The family of one-based finite sets of equations is not recursive. (5) If there is an operation of rank at least two then the family of consistent singleton sets of equations is not recursive. (1) was found independently by V. L. Murskii [Dokl. Akad. Nauk SSSR 196(1971)]. (Received November 8, 1971.)

72T-E9. JOHN GREGORY, 4517 Jonwall Court, Columbia, South Carolina 29206 and University of Maryland, College Park, Maryland 20742. Beth definability for infinitary finite-quantifier languages. Preliminary report.

(1) Beth definability fails for $L_{\omega\omega}$. This is a consequence of the known fact that there is an infinite rigid dense linear ordering. For infinite cardinal K, consider the statement A(K): there are two nonisomorphic $L_{\omega K}$-elementarily equivalent structures of cardinality K with no more than K relations. A(K) implies that there are two such structures which are also rigid. (2) A(K) implies not Beth($L_{K+\omega}$, $L_{\omega K}$), i.e., there is an implicit $L_{K+\omega}$ definition of relations for which there is no corresponding explicit $L_{\omega}K$ definition. Morley has shown A(K) for regular $K > \omega$. Thus, (3) if uncountable K is regular, then not Beth($L_{K+\omega}$, $L_{\omega K}$). (Received November 10, 1971.)

*72T-E10. ROHIT J. PARIKH, State University of New York at Buffalo, Amherst, New York 14226. D-structures and their semantics. Preliminary report.

Definition 1. A D-structure M of (finite) type μ is a pair F, H where F is a family of finite relational structures of type μ and H is a family of homomorphisms (preserving atomic sentences) between elements (diagrams) of F. All identity maps are in H. M is rigid if all maps in H are inclusions, transitive if M is a category. Definition 2. $D \in F$. A sentence of $L_{\mu * D}$ augmented by $\Box$ and $\Diamond$. We define $M, D \models A$ by induction on the complexity of A. (1) A is atomic. $M, D \models A$ if A holds in D. (2) $M, D \models B \wedge C$ iff $M, D \models B$ and $M, D \models C$. (3) $M, D \models \neg B$ iff $M, D \not\models B$. (4) $M, D \models (\exists x)B(x)$ iff $\exists \alpha \in |D|$ and $M, D \models B(\alpha)$. (5) $M, D \models \Diamond B(\alpha_1 \ldots \alpha_n)$ iff $\exists g \in H, g: D \to D'$ and $M, D' \models B(g(\alpha_1), \ldots, g(\alpha_n))$. $\vee, \forall, \Box$ are dual to $\wedge, \exists, \Diamond$. The constants from $|D|$ are displayed in (5). Classical structures, intuitionistic structures (cf. Fitting) and the *-structures of Ehrenfeucht can be regarded as appropriate kinds of D-structures. E.g., classical structures are rigid, directed, D-structures with classical $\exists, \forall$ translated as $\Diamond\exists$ and $\Box\forall$. The same translation holds for Ehrenfeucht structures but H consists of monomorphisms $D \to D'$ with $\overline{\overline{D'}} - \overline{\overline{D}} \leq 1$. For intuitionistic structures assume M transitive ($D \in F$ may be infinite) and use $\Box$ before every operator. (Received December 13, 1971.)

*72T-E11. EUGENE M. KLEINBERG, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. The equiconsistency of two large cardinal axioms. Preliminary report.

The following two theories are equiconsistent: ZFC + "there exists a Rowbottom cardinal", ZFC + "there exists a Jonsson cardinal". (Recall that a cardinal κ is said to be Jonsson if every structure of power κ has a proper elementary substructure of power κ, and that a cardinal κ is said to be Rowbottom if for every cardinal λ, $\omega < \lambda < \kappa$, every (two-cardinal) structure of type $\langle \kappa, \lambda \rangle$ has an elementary substructure of type $\langle \kappa, \omega \rangle$.) (Received November 16, 1971.)

*72T-E12. EDISON FARAH, Instituto de Matemática e Estatística, Universidade de São Paulo, São Paulo, Brazil. Two classical definitions of finite sets and a particular form of the Axiom of Choice.

In this paper the underlying axiomatic set theory is the Zermelo-Fraenkel system without the Axiom of Choice (AC). A set is countable iff it is equipollent to the set ω of all natural numbers $0,1,2,\ldots$ ($0=\emptyset$, $1=\{0\}$, $2=\{0,1\},\ldots$). A set is ω-finite (finite in the ordinary arithmetical sense) iff it is equipollent to some natural number. A set s is D-finite (finite according to Dedekind's first definition) iff s is not equipollent to any proper subset of s. Every ω-finite set is D-finite; further, denoting by (P) the proposition "every D-finite set is ω-finite" and by AC' the proposition "every countable set of nonempty sets admits a choice-function", it is also known that AC' implies (P). In this paper the author shows that (P) implies the following particular form of AC: "for every countable set s of nonempty D-finite sets there is a countable subset of s which admits a choice-function". The author also remarks that, in AC' $\Rightarrow$ (P), AC' can be replaced by the following weaker proposition: "for every countable set s of nonempty sets there is a countable subset of s which admits a choice-function". (Received November 19, 1971.)

*72T-E13. DALE MYERS, University of Hawaii, Honolulu, Hawaii 96822. The failure of the $L_{\omega_1\omega}$ analogue of $\underset{\sim}{\Pi}^1_1$ uniformization.

A class is in Π_1 ($\underset{\sim}{\Pi}_1$) iff it is the class of all countable models of a sentence of the form $\forall \underline{R}\sigma$ where $\underline{R}$ is a sequence of predicate variables and σ is a formula of $L_{\omega\omega}$ ($L_{\omega_1\omega}$). Many results concerning Π^1_1 have analogues which hold for $\underset{\sim}{\Pi}_1$, but not, apparently, the $\underset{\sim}{\Pi}^1_1$ uniformization theorem. Definition. $\underset{\sim}{\Pi}_1$ uniformizes Π_1 iff every Π_1 class X of structures of the form $(\mathfrak{A},R)$, where R is a relation on the universe of $\mathfrak{A}$, includes a $Y \in \underset{\sim}{\Pi}_1$ such that $\{\mathfrak{A}:$ for some R, $(\mathfrak{A},R) \in X\} = \{\mathfrak{A}:$ for some R, $(\mathfrak{A},R) \in Y\}$ and $(\mathfrak{A},R) \cong (\mathfrak{A},S)$ whenever $(\mathfrak{A},R)$, $(\mathfrak{A},S) \in Y$. Lemma. If $\underset{\sim}{\Pi}_1$ uniformizes Π_1, then there is a $\underset{\sim}{\Pi}^1_1$ set of well-orderings of ω containing exactly one representative of each countable well-order type. By results of Solovay the existence of such a set implies the negation of $(\forall X \subseteq \omega)(\omega^{L[X]} \neq \omega_1 \neq 2^{\omega})$ and hence Theorem. (1) Con(ZFC + $\exists$ an inaccessible cardinal) $\Rightarrow$ Con(ZFC + $\underset{\sim}{\Pi}_1$ does not uniformize Π_1), and (2) the existence of a measurable cardinal and $2^{\omega} \neq \omega_1$ implies $\underset{\sim}{\Pi}_1$ does not uniformize Π_1. Open problem. Does ZFC $\vdash$ $\underset{\sim}{\Pi}_1$ does not uniformize Π_1? (Received November 15, 1971.)

*72T-E14. JOSEPH BARBACK, State University College of New York, Buffalo, New York 14222 and WILLIAM D. JACKSON, Detroit Institute of Technology, Detroit, Michigan 48201. Recursively enumerable sets and regressive isols.

Let a recursive function be called complete if it has the property that each subset of its range can be obtained as the image of the function upon a set where it is increasing. Each recursively enumerable set that is nonempty can be shown to be the range of a recursive function that is complete. Let α be a nonempty recursively enumerable set and let α_R denote the collection of all regressive isols that belong to the extension of the set α to the isols. Then the following characterization of α_R can be given when f is chosen to be a complete recursive function whose range is α: $\alpha_R = \{Y \in \Lambda_R \mid \exists A \in \Lambda_R : Y = D_f(A)\}$, where Λ_R denotes the collection of all regressive isols and D_f denotes the canonical extension to the isols of the function f. (Received October 29, 1971.)

72T-E15. NICHOLAS J. DeLILLO, Manhattan College, New York, New York 10471. A formal characterization of ordinal numbers. Preliminary report.

This paper presents the axioms for a first-order finitely axiomatized theory ORD which characterizes certain elementary properties of ordinal numbers, in that (i) models $\mathcal{A}$ of ORD can be chosen whose domain may be any ordinal number, and (ii) the predicate symbols in the alphabet of ORD may be interpreted in $\mathcal{A}$ in the standard manner. Using certain well-known properties of ordinal numbers, it is proven that ORD is an example of a theory in which the proof-theoretic notions of explicit and implicit definability may be illustrated. (Received November 26, 1971.)

72T-E16. MELVIN C. FITTING, Bedford Park Boulevard West, Bronx, New York 10468 and Herbert H. Lehman College, City University of New York, Bronx, New York 10468. A modal logic analog of Smullyan's fundamental theorem.

We show a constructive version of the fundamental theorem of quantification theory (a relative of Herbrand's theorem) for the first order modal logic S4. To do this, we give an alternative formulation of first order S4 (which we call λS4) in which, intuitively, constant symbols can name different constants in different possible worlds of a Kripke model. We prove constructively the equivalence of this version of first order S4 and more ordinary formulations. Then we prove the following Theorem. There is a constructive way of associating with each formula X of λS4 an infinite sequence, $R_1, R_2, R_3, \ldots$ of λS4 formulas so that (1) X is a theorem of λS4 iff for some i, $R_i \supset X$ is a theorem of the propositional part of λS4; (2) from a first order proof of X one can determine an R_i so that $R_i \supset X$ is a theorem of the propositional part of λS4 and produce a more propositional proof of it; (3) from a propositional proof of $R_i \supset X$ one can produce a first order proof of X. (Received December 3, 1971.)

72T-E17. JEROME I. MALITZ, University of Colorado, Boulder, Colorado 80302. The numbers of countable rigid nonisomorphic models of complete theories.

A structure is rigid if its only endomorphism is the identity function. The theories mentioned below are sets of sentences of the first order predicate calculus. Theorem. For every $n \leq \omega$ there is a complete theory T_n of finite type such that T_n has exactly n nonisomorphic rigid countable models. This answers a question of A. Ehrenfeucht. (Received December 6, 1971.)

72T-E18. TELIS K. MENAS, Group in Logic and the Methodology of Science, University of California, Berkeley, California 94720. A partition theorem for $P_\kappa(\lambda)$. Preliminary report.

For cardinals κ, λ such that $\lambda \geq \kappa$, let $P_\kappa(\lambda)$ denote the set of all subsets of λ of cardinality strictly less than κ and $X = \{\langle x, y\rangle \in P_\kappa(\lambda) \times P_\kappa(\lambda) : x \subseteq y\}$. If $X = A_0 \cup A_1$ is a partition of X into two disjoint sets A_0 and A_1, then $A \subseteq P_\kappa(\lambda)$ is said to be homogeneous with respect to this partition if $(\exists i < 2)\ (\forall x, y \in A)$ $(x \subsetneq y \rightarrow \langle x, y\rangle \in A_i)$. Theorem. Let κ be supercompact. Then for every cardinal $\lambda \geq \kappa$ there exists a normal measure on $P_\kappa(\lambda)$ such that for every partition of X there exists a homogeneous set of measure one. The

theorem was conjectured by T. Jech and solved by R. Solovay for certain small λ and from the assumption that GCH holds. (Received December 3, 1971.) (Author introduced by Professor Jack H. Silver.)

*72T-E19. M. MAKKAI, University of Manitoba, Winnipeg, Manitoba, Canada. Vaught sentences and regular relations.

Following recent work of Robert Vaught, we consider infinitary sentences, called Vaught sentences, of the form $\Phi = \langle Q_n x_n \vee (k_n < \omega) : n < \omega \rangle \bigwedge_{n<\omega} N^{k_0 \ldots k_{n-1}}(v_0, \ldots, v_{n-1})$ where $Q_n = \forall$ or $\exists$, and the $N^{\underline{k}}$ are finitary first-order formulas. The ω_1, ω-approximations of Φ are defined by Vaught as the sentences $\varphi_\nu^\emptyset$ ($\emptyset$ is the empty sequence) where $\varphi_0^{\underline{k}} = n^{\underline{k}}$, $\varphi_{\nu+1}^{\underline{k}} = Q_n x_n \vee \{\varphi_\nu^{\underline{k}, k} : k \in \omega\}$ and $\varphi_\lambda^{\underline{k}} = \bigwedge\{\varphi_\nu^{\underline{k}} : \nu < \lambda\}$; here $\underline{k} = k_0, \ldots, k_{n-1}$, $n < \omega$, $\nu, \lambda < \omega_1$ and λ is limit. With every regular relation R (cf. P. Lindström, Theoria 32(1966), 172-185) we associate a class V_R of Vaught sentences such that Theorem. (i) Any ω_1, ω-approximation of any element of V_R is preserved under R, (ii) K is the class of all countable (including finite) models of some element of V_R iff K is the class of all countable models of some $\Sigma^1_{\sim 1}$ formula $\exists S_1 \ldots S_n \ldots \varphi$, $\varphi \in L_{\omega_1, \omega}$, and for any countable $\mathfrak{A}$ and $\mathfrak{B}$, $\mathfrak{A}$ R $\mathfrak{B}$ and $\mathfrak{A} \in K$ imply $\mathfrak{B} \in K$. This generalizes our results in Abstract 69T-E79, these *Notices* 16(1969), 981. Using methods of Vaught, we can deduce as a corollary a general interpolation and preservation theorem for regular R and for L_A, A any admissible $\subseteq$ HC, of the form of the corollary in our above abstract. (Received December 13, 1971.) (Author introduced by Professor George A. Grätzer.)

*72T-E20. GIORGIO M. GERMANO and A. MAGGIOLO-SCHETTINI, Laboratorio di Cibernetica del C.N.R., 80872 Arco Felice, Napoli, Italy. A characterization of partial recursive functions via sequence functions.

We consider partial functions mapping any sequence of natural numbers $r \in \{\Lambda\} \cup N \cup N^2 \cup \ldots$ on a sequence of natural numbers $s \in N \cup N^2 \cup \ldots$. For any such functions f_1 and f_2 we consider the following three operations. (1) The juxtaposition $(f_1 \wedge f_2)$, such that $(f_1 \wedge f_2)r = s$ if there exist s_0, s_1, s_2 with: $f_i r$ defined (for $1 \leq i \leq 2$), $s_0 = r$, $s_i = f_i s_0$ (for $1 \leq i \leq 2$), $s = s_1 s_2$; $(f_1 \wedge f_2)r$ undefined otherwise. (2) The composition $(f_1 \circ f_2)$, such that $(f_1 \circ f_2)r = s$ if there exist s_0, s_1, s_2 with: $f_i s_{i-1}$ defined (for $1 \leq i \leq 2$), $s_0 = r$, $s_i = f_i s_{i-1}$ (for $1 \leq i \leq 2$), $s = s_2$; $(f_1 \circ f_2)r$ undefined otherwise. (3) The repetition $(f_1 | f_2)$, such that $(f_1 | f_2)r = s$ if there exist $s_0, \ldots, s_k$ ($0 \leq k$) with: $f_i s_j$ defined (for $1 \leq i \leq 2$, $0 \leq j \leq k$), $s_0 = r$, $s_j = f_2 s_{j-1}$ (for $1 \leq j \leq k$), $s = s_k$, $f_1 s_j \neq 0$ (for $0 \leq j < k$), $f_1 s_k = 0$; $(f_1 | f_2)r$ undefined otherwise. Let F be the closure of the set containing the zero-function, the successor-function, the precedessor-function and the projection-functions with respect to juxtaposition, composition and repetition. Let F_1 be the set of functions of F whose domain is contained in N. Let R be the set of partial recursive functions. Theorem 1. $R \subseteq F_1$. Theorem 2. F is computable. Corollary. $F_1 \subseteq R$. Theorem 3. $R = F_1$. (Received December 13, 1971.)

72T-E21. BARUCH GERSHUNI, Ibn Gvirol street 43, Tel Aviv, Israel. Some achievements of the simplified Elementary Theory of Totalities (ETT). I.

The state of the elementary theory of classes and sets in 1966 is (partly) characterized by the circumstances that (I) there is only one sort of totalities apart, viz. classes obeying some axioma; (II) sets are special

classes obeying special axioms; and (III) only sets and not classes which are not sets are allowed to be elements of classes. The axioms to which the classes and sets were obeying were developed by von Neumann, Bernays and Gödel. This state of affairs was totally changed by the emergence of the ETT, a simplified version of which is here outlined. There was emphasized the difference between the (synthetic) singular and the plural totalities and stress was laid upon the circumstance whether a totality is allsidedly packed up or not packed up at all. (The not simplified ETT knows also partly packed up totalities.) The former (packed up) totalities are the so called sets and the latter the classes. The sets are now as before written with braces, the classes -- without any brackets. The braces symbolize the fact that a set being allsidedly packed up is a unity. Thus the sets are not classes, but totalities apart. The classes may be singulars and plurals, while the sets are always singulars. Any class may be transformed to a set by enclosing it in braces (by letting the braces operator operate on it). (Received November 22, 1971.)

72T-E22. WITHDRAWN.

*72T-E23. JOHANN ANDREAS MAKOWSKY, Swiss Federal Institute of Technology, 8006 Zürich, Switzerland. A note on almost strongly minimal theories. Preliminary report.

Among $\aleph_1$-categorical theories almost strongly minimal theories have a rather simple structure. (See Baldwin-Lachlan, "On strongly minimal sets", J. Symbolic Logic 36(1971), 79-96 and Baldwin, Abstract 70T-E59, these *Notices* 17(1970), 834.) A purely algebraic characterization of these theories shall be given. A theory T has weak intersection property (w.i.p.) if for all models $\mathfrak{A}$ of T and all submodels $\mathfrak{A}_1$, $\mathfrak{A}_2$ of $\mathfrak{A}$, $\mathfrak{A}_1 \cap \mathfrak{A}_2$ is either finite or a model of T. Theorem. Let T be $\aleph_1$-categorical and model-complete. Then the following are equivalent: (i) T is almost strongly minimal, (ii) There is a principal extension T' of T with w.i.p. Corollary. Let T be $\aleph_1$-categorical and complete. T is almost strongly minimal iff some principal extension $(T^*)'$ of T^* (full expansion of T) has w.i.p. (Received December 20, 1971.) (Author introduced by Professor Ernst P. Specker.)

*72T-E24. EGON BÖRGER, Universität Münster, 44 Münster, Roxelerstrasse 64, West Germany. Some reduction classes of Horn formulae with short disjunctions.

Let $K_0, \ldots, K_r$ be fixed binary predicate symbols for an appropriately chosen r. We construct for every first order formula c closed Horn formulae $a_1, b_1, \ldots, a_3, b_3$ in prenex conjunctive normal form with binary disjunctions, where b_i (resp. a_i) has a prefix $\forall\exists\ldots\exists\forall$ ($\forall\exists\exists\forall$) and a matrix built up from individual variables and $K_0, \ldots, K_r$ (resp. additional monadic predicate symbols) so that c is satisfiable (sf) iff b_1 (the same for a_1), b_2 (a_2), b_3 (a_3) are respectively sf by antisymmetric and antitransitive, disjunct, reflexive relations. (This strengthens the reduction types in: S. Aanderaa, "On the dec. pr. for form. in which all disj. are bin.," Proc. Second Scand. Log. Sympos., 1971,pp. 1-18.) Let d_n be $\forall z_0 \ldots \forall z_{n+1} (K_0 z_0 z_1 \wedge \bigwedge_{0<j<n} K_1 z_j z_{j+1} \wedge K_2 z_n z_{n+1} \rightarrow K_3 z_n z_{n+1})$ and e_n be $\exists z_0 \ldots \exists z_n (K_0 z_0 z_0 \wedge \bigwedge_{j<n} K_1 z_j z_{j+1} \wedge K_2 z_n z_n)$. We exhibit $a_1, \ldots, a_4$, $b_1, \ldots, b_4$ as above except that both a_i and b_i have the prefixes $\exists\forall$, $\forall\exists\forall$, $\exists\forall\exists\forall$, $\forall\exists\exists\forall$ resp. and that b_i for $2 \leq i \leq 4$ contains, besides bin. disj., exactly one ternary disj. so that an arbitrary c is sf iff $a_1 \wedge a_2 \wedge d_{\bar{c}}$ (resp. $a_i \wedge d_{\bar{c}}$, $b_1 \wedge b_2 \wedge e_{\bar{c}}$, $b_i \wedge e_{\bar{c}}$, $i = 3, 4$) is sf, where $\bar{c}$ is a Gödel-number of c. (Received December 20, 1971.)

72T-E25. G. P. MONRO, "Laputa", Seaview Avenue, Ferny Creek, Victoria 3786, Australia. Decomposable cardinals.

An infinite cardinal m is decomposable if there exist cardinals $p, q < m$ such that $p + q = m$. Well-orderable cardinals are not decomposable, so in the presence of the axiom of choice there are no decomposable cardinals. Let N be the Halpern-Lévy model for the independence of the axiom of choice from the Boolean prime ideal theorem. We show that in N every non-well-orderable cardinal is decomposable; in particular, $N \models 2^{\omega}$ is decomposable. (Received December 21, 1971.) (Author introduced by Professor John C. Shepherdson.)

*72T-E26. GERALD E. SACKS, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. Proximity of k-sections. Preliminary report.

U and V are objects of finite type. $S_k U$ (the k-section of U) is the set of all objects of type k recursive in U. ${}^{n}E$ is the equality predicate restricted to objects of type less than n. Theorem. If U is of type n, ${}^{n}E$ is recursive in U, and k is less than n, then there exists a V of type $k + 1$ such that ${}^{k+1}E$ is recursive in V and $S_k U = S_k V$. The proof opens with a downward Skolem-Löwenheim construction that defines the class of forcing conditions needed to erect a certain generic class (rather then set) that serves as the "kernel" of V. (Received December 27, 1971.)

*72T-E27. C. J. ASH, Monash University, Clayton, Victoria 3168, Australia. The theory of n-valued Post algebras.

The finitary Boolean extension, $\mathfrak{A} \circ B$, of an arbitrary structure $\mathfrak{A}$ by a Boolean algebra B was introduced by Foster. Theorem 1. The theory of $\mathfrak{A} \circ B$ can be effectively obtained from the theories of $\mathfrak{A}$ and of B. Theorem 2. $\mathfrak{A} \circ 2^I/F < \mathfrak{A}^I/F$. From Theorem 1 and Foster's theorem we utilise Tarski's results on Boolean algebras to obtain Theorem 3. For each n, the theory of n-valued Post algebras is decidable, and each Post algebra has a decidable theory. We also adapt Rabin's method to obtain Theorem 4. The theory of n-valued Post algebras with quantification over ideals is decidable. Lastly, Theorem 5. If $\mathfrak{A}_n$ is the prime n-valued Post algebra and B the countable atomless Boolean algebra, then $\mathfrak{A}_n \circ B$ is the free Post algebra on $\aleph_0$ generators, which is also $\aleph_0$-universal-homogeneous. (Received December 27, 1971.) (Author introduced by Professor J. N. Crossley.)

*72T-E28. NEIL JONES, Department of Computer Science, Pennsylvania State University, University Park, Pennsylvania 16802 and ALAN L. SELMAN, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213. Automata theoretic solutions to a problem of H. Scholz.

The spectrum of a first order formula φ is the set of all natural numbers n for which φ has a model of cardinality n. Automata theoretic characterizations are given for the class of spectra. Theorem 1. A set of natural numbers is a spectrum if and only if it is accepted by a nondeterministic Turing machine within time 2^{cx}, where x is the length of the input, and $c > 0$. A scene is a k-dimensional cubical array of symbols from a fixed alphabet. A spectrum automaton is a multihead nonwriting deterministic finite state automaton which scans the vertices of a scene. A scene is accepted by a spectrum automaton if the machine goes into an accepting state,

if started in its initial state at the origin of the scene. An integer n, $n > 0$, is accepted if and only if there is a scene with sides of length n which is accepted. Theorem 2. A set is a spectrum if and only if it is accepted by a spectrum automaton. Theorem 3. If the set of tautologies is recognizable within polynomial time by a deterministic Turing machine, then the complement of every spectrum is a spectrum. (Received January 4, 1972.)

*72T-E29. STEVEN K. THOMASON, Simon Fraser University, Burnaby 2, British Columbia, Canada. An incompleteness theorem in modal logic.

We construct a finitely axiomatized normal extension L of the modal propositional logic T which is not complete for the usual relational semantics; i. e. there is a formula which is not a thesis of L but which is valid in every frame (W, R) which validates all theses of L. (Received January 7, 1972.)

*72T-E30. WILLIAM BOOS, State University of New York, Buffalo, New York 14226. A boolean extension which effaces the mahlo property.

Recall that a cardinal κ is '0-mahlo' iff regular, 1-mahlo iff mahlo, i.e. $\{\lambda < \kappa | \lambda \text{ regular}\}$ is stationary in κ, α-mahlo for $0 < \alpha \leq \kappa$ iff, $\forall\, \beta < \alpha, \{\lambda < \kappa | \lambda\ \beta\text{-mahlo}\}$ is stationary in κ. Theorem. Assume κ is κ-mahlo and the GCH. Then for each $\alpha < \kappa$ there are complete boolean algebras $\mathbf{B}_\alpha$ and $\mathbf{C}_\alpha$ such that $[\![\check{\kappa}$ is α-mahlo but not $(\alpha+1)^{\check{}}$-mahlo$]\!]^{\mathbf{B}_\alpha}$ and $[\![\check{\kappa}$ is the least $(\alpha+1)^{\check{}}$-mahlo$]\!]^{\mathbf{C}_\alpha} = 1$. The proof extends an unpublished observation of Jensen on forcing with thin sets. Using methods of Kunen-Paris and Mitchell-Silver one then has the following applications: Theorem. (i) Con(ZFC + κ measurable) $\Rightarrow$ Con(ZFC + κ has a nontrivial κ^+-saturated ideal and κ is the first strong mahlo cardinal). (ii) Con(ZFC + κ weakly compact) $\Rightarrow$ Con(ZFC + κ is the first weak mahlo cardinal and has the tree property). (Received January 10, 1972.)

72T-E31. DANIEL GOGOL, Staten Island Community College, Staten Island, New York 10301. Solution to a problem of Henkin. Preliminary report.

Our theorem concerns the infinitary language with a denumerable set of variables, atomic formulas of the types P(S) (where S is a sequence of variables of type w) and x eq y, a negation symbol, finite conjunctions and finite strings of quantifiers. Leon Henkin has proven ("Some remarks on infinitely long formulas", Infinitistic Methods, 1959, pp. 167-183) that with a certain standard-type set of logical axioms and rules of inference a set of formulas in this language has a model if it satisfies the following three conditions: (i) it is consistent; (ii) any two infinite sequences of variables appearing in any formulas in the set are "comparable" (i.e., the set of variables which do not appear in exactly the same places in the two sequences is finite); (iii) any infinite sequence of variables in any formula in the set has an infinite set of variables missing from it. We have shown that if we add the additional condition that the set of formulas is finite, then it must have a denumerable model. Our proof uses Henkin's result and requires a much different method. (Received January 19, 1972.) (Author introduced by Professor Martin D. Davis.)

*72T-E32. STANLEY N. BURRIS, Department of Pure Mathematics, University of Waterloo, Waterloo, Ontario, Canada. Scott sentences for mono-unary algebras. Preliminary report.

Theorem 1. There is a countable ordinal α such that for any countable mono-unary algebra $\langle A,\{f\}\rangle$ there is a Scott sentence φ for $\langle A,\{f\}\rangle$ in $\mathcal{L}_{\omega_1\omega}$ with quantifier rank less than α. Theorem 2. Let τ be a sentence in $\mathcal{L}_{\omega_1\omega}$. The number of isomorphism types of countable mono-unary algebras satisfying τ is either countable or 2^{ω}. (Received January 20, 1972.)

Statistics and Probability

*72T-F3. STEVEN I. ROSENCRANS, University of New Mexico, Albuquerque, New Mexico 87106. Mapping second-order ACP's into first-order ACP's. Preliminary report.

Let u be a solution of the abstract Cauchy problem $\frac{1}{2}e^2(t)u_{tt} + f(t)u_t = Au$ (the linear operator A independent of t), $u(0) = \varphi$, $u_t(0) = 0$. Then if $\{x_t, t \geq 0\}$ is the diffusion process $dx = f\,dt + e\,db$, $x(0) = 0$, the function $t \to Eu(x_t) \equiv v(t)$ satisfies $v_t = Av$, $v(0) = \varphi$. (Some mild growth condition must be satisfied by u.) In the case $e \equiv$ const, $f(t) = \text{const}^2/t$ this representation, which is an integral over a Bessel process, is similar to a representation introduced (nonprobabilistically) by J. A. Donaldson ("New solution integral representations of Cauchy's problem for abstract parabolic equations," to appear). In this same special case, Kac's formula for the solution to the telegraph equation (as extended by Stanley Kaplan) is combined with the above-mentioned transformations to give a probabilistic derivation of the well-known transformation that maps solutions of the wave equation to solutions of the Darboux equation. Finally, some applications are given to the study of problems that are not well-posed. (Received October 18, 1971.)

*72T-F4. S. CAMBANIS and B. S. RAJPUT, University of North Carolina, Chapel Hill, North Carolina 27514. Some zero-one laws for Gaussian processes.

Let $\{\xi_t, t \in T\}$, T any interval of the real line, be a real separable Gaussian process with zero mean. Then it is shown that with probability zero or one the paths of ξ_t are: (1) bounded, (2) of bounded variation (if T compact), (3) continuous, (4) uniformly continuous, (5) absolutely continuous, (6) differentiable, (7) in the class $\text{Lip}_\alpha(T)$, $\alpha > 0$, (8) free of oscillatory discontinuities. These results are obtained by using a zero-one law for Gaussian processes which slightly extends a result due to Kallianpur (Trans. Amer. Math. Soc. 149(1970), 199-211) and Jain (Proc. Amer. Math. Soc. 29(1971), 585-587) by relaxing the hypotheses on the covariance of the process and its parameter set. The zero-one law in (1) was recently proven by Landau and Shepp (Sankhyā Ser. A 32(1970), 369-378). (Received January 14, 1972.)

*72T-G16. T. THRIVIKRAMAN, Mar Athanasius College, Kothamangalam, Kerala, India. On Hausdorff quotients of spaces and Magill's theorem.

All spaces are T_2. F(X) (respectively P(X), E(X)) denotes the set of T_2 quotients of X with finite number of (respectively $\not> 1$, any number of) multiple points. The order is that of "refines" in the corresponding partitions. Main theorems. (1) Two normal spaces X and Y are homeomorphic if and only if the lattices F(X) and F(Y) are isomorphic. (2) If X and Y are normal spaces, then any lattice isomorphism of P(X) to P(Y) (respectively F(X) to F(Y)) induces and is induced by a lattice isomorphism of F(X) to F(Y) (respectively E(X) to E(Y)). (3) A is the category of normal spaces, morphisms being closed continuous functions on closed subsets; B is the category of complete lattices, morphisms being isomorphisms from "primary" dual ideals to intervals. Let E associate to each $X \in A$, the corresponding $E(X) \in B$. E is a contravariant functor with proper definition of E on morphisms. (4) Any nontrivial meet-complete lattice homomorphism from E(X) to E(Y) is a bijection on the dual atoms. These interpreted to compactifications give Magill's theorem, its generalisations and limits to further generalisations. (Received October 6, 1971.) (Author introduced by Professor M. Rajagopalan.)

*72T-G17. JAMES R. BOONE, Texas A & M University, College Station, Texas 77843. Examples relating to mesocompact and sequentially mesocompact spaces.

A Tychonoff sequentially mesocompact space, which is not mesocompact, is presented. Also, an example is given which establishes that sequential mesocompactness is not invariant under perfect mappings. (Received October 21, 1971.)

*72T-G18. THOMAS W. RISHEL, Dalhousie University, Halifax, Nova Scotia, Canada. Sequential, Fréchet and k-spaces.

The following results give internal characterizations of sequential, Fréchet and k-spaces. Theorem 1. A T_1-space Y is sequential iff given $U \subseteq Y$, $y \in \text{Int } U$ iff there exists a network $\{U_1, U_2, \ldots\}$ at y such that $y \in U_n \cap U$ for every $n \in N$ (the natural numbers). Theorem 2. A T_1-space Y is Fréchet iff given $U \subseteq Y$, $y \in \text{Int } U$ iff there exists a network $\{U_1, U_2, \ldots\}$ at y such that $y \in \text{Cl}(U_n \cap U)$ for every $n \in N$. Definition. A k-sequence at y is a (countable) decreasing family of sets $\{U_1, U_2, \ldots\}$ such that $y \in \bigcap \{U_n : n \in N\} = C$, and C is compact. Theorem 3. A regular space Y is k iff given $U \subseteq Y$, $y \in \text{Int } U$ iff there exists a k-sequence at y such that $y \in U_n \cap U$ for every $n \in N$. (Received November 1, 1971.)

72T-G19. LUDVIK JANOS, Istituto Matematico "Ulisse Dini", 50134 Firenze, Italy. On weak contractions.

Let X be a completely regular space and $f: X \to X$ a continuous selfmapping. If there exists a family of pseudometrics $\{\rho_i \mid i \in \mathfrak{A}\}$, generating the topology of X and such that $\rho_i(f(x), f(y)) \leqq c\rho_i(x,y)$ for all $i \in \mathfrak{A}$ and

some $c \in (0,1)$ we say that f is a weak contraction. If X is metrizable then a natural question arises, whether contraction in this weak sense implies contraction. In case X is compact this is true, but in general these two concepts are not equivalent. Theorem. Let L be a linear topological space of all continuous functions on the half-open interval $[0,\frac{1}{2})$, with the topology of uniform convergence on compacta, and $P: L \to L$ the operator of integration: $(Pf)(x) = \int_0^x f(t)dt$. The space L is metrizable and P is a weak contraction, but not a contraction. (There is no metric on L with respect to which P is a contraction.) (Received November 1, 1971.)

*72T-G20. JOHN L. BRYANT, Florida State University, Tallahassee, Florida 32306. An example of a wild $(n-1)$-sphere in S^n in which each 2-complex is tame.

By modifying Bing's example [Duke Math. J. 28(1961), 1-15] Gillman obtains a 2-sphere Σ in S^3 with the property that every closed, nowhere dense subset of Σ is tame in S^3 [Duke Math. J. 31(1964), 247-254]. Let S be the $(k+2)$-sphere in S^{k+3} obtained by suspending Gillman's example k-times $(k \geq 2)$. Theorem 1. Every 2-complex topologically embedded in S is tame in S^{k+3}. Theorem 2. Every m-complex topologically embedded in S $(m \leq k)$ can be homeomorphically approximated in S by complexes that are tame in S^{k+3}. Theorem 2 is a consequence of the following theorem, which is an extension of a result of Seebeck [Proc. Amer. Math. Soc. 29(1971), 197-201]. Theorem 3. Suppose that M is an $(n-1)$-manifold in an n-manifold N $(n \geq 6)$ and that every 2-complex in M can be homeomorphically approximated in M by complexes that are tame in N. Then the same holds true for m-complexes in M whenever $m \leq n-3$. A natural conjecture is that every m-complex in S is tame in S^{k+3} whenever $m \leq k$. (Received November 5, 1971.)

72T-G21. WITHDRAWN.

72T-G22. ROBERT W. HEATH, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. A postscript to a note on quasi-metric spaces. Preliminary report.

In "A note on quasi-metric spaces" (see Abstract 687-54-11, these *Notices* 18(1971), 786), the author inaccurately stated that the Niemytski plane is a Moore space that is not quasi-metrizable. One does in fact obtain a non-quasi-metrizable Moore space by modifying the Niemytski plane in the following way: the points of the modified space include only the irrationals on the X-axis and the points with both coordinates rational off the X-axis; also basis elements are either (i) open spherical discs centered on points off the X-axis (as in the standard Niemytski plane), or (ii) for each irrational number x and each positive number h_0, any set consisting of the point $(x,0)$ together with all points $(x+k,h)$ such that $|k| < |h| < h_0$ (i.e., the point $(x,0)$ on the X-axis together with the union of interiors of two congruent triangles having common vertex at $(x,0)$, vertical axis of symmetry, vertical angle 90° and altitude h_0). This space is a non-quasi-metrizable Moore space. The Niemytski plane itself is shown to be a complete quasi-metrizable space. (Received November 10, 1971.)

72T-G23. LOWELL W. BEINEKE, Purdue University, Fort Wayne, Indiana 46805 and RAYMOND E. PIPPERT, Western Michigan University, Kalamazoo, Michigan 49001 and Purdue University, Fort Wayne, Indiana 46805. Enumerating dissectible polyhedra by their automorphism groups. Preliminary report.

Dissectible polyhedra are those which can be built by adding tetrahedra along triangular faces. Labeled

and rooted polyhedra have been counted by the authors [Math. Ann. 191(1971), 87-98]. Formulas are obtained for the number of those which are unrooted and unlabeled. This is done by finding the number with each of the seventeen possible automorphism groups. The approach parallels that of Guy [Bull. Malayan Math. Soc. 5(1958), 57-60] in the 2-dimensional case. (Received November 10, 1971.)

*72T-G24. SIDNEY A. MORRIS, University of New South Wales, Kensington, New South Wales 2033, Australia. Varieties of topological groups generated by solvable and nilpotent groups.

As an extension of the work of Su-shing Chen and the author, the following is proved: Theorem. If Ω is a class of solvable locally compact groups and G is a connected locally compact group in the variety $\underline{\underline{V}}(\Omega)$ of topological groups generated by Ω, then G is solvable; that is any connected locally compact group G which can be "manufactured" from Ω by (repeated) operations of taking subgroups, quotient groups and arbitrary cartesian products is solvable. It is shown that the connectedness restriction on G cannot be removed in general, but can be if G is assumed to be a Lie group. Similar results are also obtained for nilpotent groups. (Received November 4, 1971.)

*72T-G25. STEPHEN KENTON, Eastern Connecticut State College, Willimantic, Connecticut 06226. Cauchy completions of neometric spaces.

The standard metric space completion theory is generalized to neometric spaces (a neometric is a continuous semimetric). Necessary and sufficient conditions are given for a neometric space (X,d) to have a Cauchy completion (a Cauchy complete neometric space in which (X,d) is dense and isometrically embedded). The completion is necessarily unique up to an isometry. Examples are given of a noncompletable neometric space and a noncomplete neometric space which is completable. (Received November 15, 1971.) (Author introduced by Professor O. I. Litoff.)

*72T-G26. GEORGE MICHAEL REED, Ohio University, Athens, Ohio 45701. On dense subspaces of certain topological spaces.

Consider the following well-known relationships between certain abstract spaces. (1) Each M_1-space is a stratifiable space (M_3-space). (2) Each stratifiable space is a paracompact σ-space. (3) Each σ-space is semistratifiable. The converses of statements (2) and (3) are known to be false and the validity of the converse of statement (1) is an open question. However, it follows from the author's results (Abstract 679-G1, these *Notices* 17(1970), 1033) that in each of statements (1), (2), and (3), a first countable space of the second type contains a dense subspace which is of the first type. It is the purpose of this paper to investigate this relationship for nonfirst countable spaces. The following results are obtained: (i) Each stratifiable space in which each point has a σ-closure preserving local base has a dense subspace which is an M_1-space. (ii) There exists a paracompact σ-space (due to R. W. Heath, Proc. Emory University Topology Conference, 1970, pp. 119-122) which has no dense stratifiable subspace. (iii) Each semistratifiable space has a dense subspace which is a σ-space. (Received November 18, 1971.)

72T-G27. ALLAN C. COCHRAN and ROBERT B. TRAIL, University of Arkansas, Fayetteville, Arkansas 72701. Complete regularity for convergence spaces. Preliminary report.

We consider regularity and complete regularity for convergence spaces and their relation to Stone-Cech compactifications. Slightly modifying a definition in "On convergence groups and convergence uniformities," D. C. Kent, Fund. Math. 60(1967), 213-222, we define a convergence space (E,τ) to be K-regular if when x does not belong to the adherence of a set A, there exists a complete neighborhood cover of A and a neighborhood of x which are disjoint and K-completely regular if there exists a continuous function which separates x and A. Lemma. For principal convergence structures, K-regularity is equivalent to (*) $\mathfrak{F} \in \tau x$ iff $\overline{\mathfrak{F}} \in \tau x$. Theorem. A regular convergence space which is compact and principal is topological. (See D. C. Kent and G. D. Richardson, "Regular compactifications of convergence spaces," Abstract 689-G13, these *Notices* 18(1971), 1067, for compatible results.) Corollary. If the Stone-Cech compactification of a principal convergence space is regular, then it is topological. Corollary. A convergence space which is almost pretopological has a K-regular Stone-Cech compactification iff it is K-completely regular. With an analogous definition of normality, we find the convergence space separation axioms comparable to the topological separation axioms. (Received November 19, 1971.)

72T-G28. CHARLES E. AULL, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. $\delta\theta$-bases. Preliminary report.

A base $\mathcal{U}$ for a topological space X is a $\delta\theta$-base if $\mathcal{U}$ can be written as $\mathcal{U} = \cup\mathcal{U}_n$ in such a way given an open set $T \subset X$ and a point $x \in T$, there is $n \in N$ such that $\mathcal{U}_n$ has countable order at X and for some member U_n, $x \in U_n \subset T$, a weak σ-space (see author's Abstract 689-G5, these *Notices* 18(1971), 1065), with a $\delta\theta$-base is quasidevelopable. Consequently a T_3 σ-space is metrizable iff it has a $\delta\theta$-base and is collectionwise normal. A hereditary separable T_3 space with a $\delta\theta$-base is metrizable. A T_3 stratifiable space is metrizable iff it has a $\delta\theta$-base. (Received November 17, 1971.)

*72T-G29. PETER FLETCHER, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061 and WILLIAM F. LINDGREN, Slippery Rock State College, Slippery Rock, Pennsylvania 16057. Topological properties related to completeness. Preliminary report.

A quasi-uniform space $(X,\mathcal{U})$ is almost complete provided that every open $\mathcal{U}$-Cauchy filter has a cluster point. A filter has the countable subcollection intersection property (C.C.I.P.) if every countable subcollection has a cluster point. A topological space is almost realcompact if every open ultrafilter with the C.C.I.P. has a cluster point. Theorem. A topological space is almost realcompact if and only if there is a compatible almost complete countably precompact transitive quasi-uniformity. Corollary. A topological space is almost realcompact if and only if the upper semicontinuous quasi-uniformity is almost complete. It is known that every orthocompact space admits a compatible complete quasi-uniformity. It follows from a result of R. W. Heath, "A postscript to a note on quasi-metric spaces", Abstract 72T-G22, these *Notices* 19(1972), that his modification of the "Niemytski plane" is not orthocompact. It also follows that it is consistent with Zermelo-Frankel set theory that there exist a nonorthocompact separable, perfectly normal topological space. (Received November 26, 1971.)

72T-G30. MARTA B. P. HERRERO, University of Chicago, Chicago, Illinois 60637. Homology operations on $BU \times Z$ and $BO \times Z$. Preliminary report.

The Whitney sum and the tensor product of complex (real) vector bundles induce H-space structures on $BU \times Z$ ($BO \times Z$) which will be denoted by $\oplus$ and #, and the products induced in homology will be denoted by * and •, respectively. Although $(BU \times Z, \#)$ and $(BO \times Z, \#)$ are not infinite loop spaces, their mod p homology admits operations $\tilde{Q}^i$ which satisfy most of the properties of Dyer-Lashof operations. These operations are defined and evaluated by applying the techniques used by J. P. May in "Homology operations on infinite loop spaces", Proc. Sympos. Pure Math., vol 22, Amer. Math. Soc., Providence, R. I., 1971, to the space of Fredholm operators on a Hilbert space, which is homotopy equivalent to $BU \times Z$ ($BO \times Z$). The operations are first evaluated on $*$-indecomposable elements and a "mixed Cartan formula" evaluates $\tilde{Q}^i(x*y)$ in terms of both products and the Dyer-Lashof operations on $(BU \times Z, \oplus)$ and $(BO \times Z, \oplus)$, which were determined by S. Kochman in his thesis (University of Chicago, 1970). (Received December 1, 1971.)

72T-G31. KEITH R. ALLEN, University of North Carolina, Charlotte, North Carolina 28216. Dendritic compactification. Preliminary report.

A dendritic space is a connected space in which any two distinct points can be separated by a third point. Methods of J. de Groot and J. M. Aarts (Canad. J. Math. 21(1969), 96-105) show that a dendritic space X has a dendritic compactification if and only if X is rim compact. This compactification is unique, and is the same as Freudenthal's compactification. A result of R. F. Dickman, Jr. may be used to characterize the dendritic compactification X^* of a rim compact, dendritic, metric space X in terms of the set M of all continuous, monotone mappings of X into the closed unit interval: X^* is the unique T_2 compactification of X such that both, (a) each $f \in M$ has a unique, continuous, monotone extension f^* to X^*, and, (b) given distinct points p and q of $cl_{X^*}(X^* - X)$, there is an $f \in M$ such that $f^*(p) \neq f^*(q)$. Moreover, X^* is the largest T_2 compactification of X which satisfies (b), and the smallest which satisfies (a). (Received December 2, 1971.)

72T-G32. P. L. SHARMA, Indian Institute of Technology, Kanpur-16, U. P., India and S. A. NAIMPALLY, Lakehead University, Thunder Bay, Ontario, Canada. Pseudometric approach to proximities.

In this paper we present a new method of studying proximities via pseudometrics. Our approach is direct using the concepts of base and subbase introduced by Sharma (Pacific J. Math. 37(1971), 515-526) in contrast to that of Leader (Proc. Amer. Math. Soc. 16(1965), 493-495) who uses generalized uniformities. We then use our method to solve two of the open problems posed by Hunsaker in his doctoral thesis ("Proximities and uniform structures induced by a family of real functions," Washington State University, 1967). (Received December 6, 1971.)

*72T-G33. HSU-TUNG KU and MEI-CHIN KU, University of Massachusetts, Amherst, Massachusetts 01002. Differentiable actions of S^1 on homotopy complex and quaternion projective spaces.

The notation is as in Abstract 690-G5, these *Notices* 18(1971), 1090. Theorem 1. Let S^1 act freely and differentiably on a homotopy $(2n+1)$-sphere Σ^{2n+1} ($n \geq 5$) such that the orbit space Σ^{2n+1}/S^1 admits a

differentiable action of S^1 with a component of the fixed point set of codimension 2. Then $I_{4k+2}(S^1, \Sigma^{2n+1}) = 0$, for $k \geq 0$ if n is odd; $I_{4k}(S^1, \Sigma^{2n+1}) = 0$, for $k \geq 1$ if n is even. Corollary 1. For each n, there exist at most finitely many homotopy complex projective n-spaces which admit differentiable S^1-actions with a component of the fixed point set of codimension 2. Corollary 2. If M^6 is a homotopy complex projective 3-space which admits a differentiable S^1-action with a component of the fixed point set of codimension 2, then M is diffeomorphic to CP^3. Theorem 2. Let S^3 act freely and differentiably on a homotopy $(4n+3)$-sphere Σ^{4n+3} $(n \geq 3)$ such that the orbit space Σ^{4n+3}/S^3 admits a differentiable action of S^1 with a component of the fixed point set of codimension 4. Then $I_{4k}(S^3, \Sigma^{4n+3}) = 0$, for $k \geq 1$. Corollary 3. Let M^{4n} be a homotopy quaternion projective n-space which admits a differentiable S^1-action with a component of the fixed point set of codimension 4. Then M^{4n} is PL homeomorphic to QP^n. (Received December 8, 1971.)

72T-G34. CHARLES L. HAGOPIAN, Arizona State University, Tempe, Arizona 85281. Semi-aposyndetic dendroids. Preliminary report.

If a metric space is both compact and connected it is called a continuum. A continuum M is said to be semi-aposyndetic if for each pair of distinct points x and y in M there exists a subcontinuum F of M such that the sets M - F and the interior of F relative to M each contain a point of $\{x,y\}$. Here it is proved that every hereditarily unicoherent semi-aposyndetic continuum is arcwise connected. (Received December 8, 1971.)

72T-G35. HWANG-WEN PU, Texas A & M University, College Station, Texas 77843. Another Ascoli theorem for multi-valued functions. Preliminary report.

Various Ascoli theorems for multi-valued functions have been proved by Lin and Rose ("Ascoli's theorem for spaces of multifunctions," Pacific J. Math. 34(1970), 741-747), Mancuso ("An Ascoli theorem for multi-valued functions," J. Austral. Math. Soc., to appear), and Smithson ("Uniform convergence for multifunctions," Pacific J. Math., to appear). However, the fact that a multi-valued function on a space X to a space Y can be considered as a single-valued function on X to the space of nonempty subsets of Y is not used in any one of the above-mentioned papers. In the present paper, this fact is used and another Ascoli theorem for multi-valued functions is obtained. It is shown that this theorem includes Smithson's. (Received December 10, 1971.)

72T-G36. JUN-ITI NAGATA, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. Characterizations of σ - and Σ-spaces.

Theorem 1. The following conditions are equivalent for a regular space Y: (i) Y is a σ-space (A. Okuyama, "A survey of the theory of σ-spaces," General Topology and its Applications, 1971, pp. 57-63), (ii) there is a half-metric space (X,X') (J. Nagata, "Problems on generalized metric spaces. II," Proc. Houston Topology Conference, 1971) and a perfect mapping f from X onto Y satisfying f(X') = Y, (iii) there is a half-metric space (X,X') and a closed continuous mapping f from X onto Y such that f(X') = Y. (The equivalence of (i) and (ii) was announced at the Houston conference.) Definition. A pair (X,X') of a topological space X and its subspace X' is called a half-M-space if X has a sequence $\mathcal{U}_1, \mathcal{U}_2, \ldots$ of open covers such that (i) $\mathcal{U}_1 > \mathcal{U}_2^* > \mathcal{U}_2 > \ldots$, (ii) if $x \in X'$ and $x_n \in S(x, \mathcal{U}_n)$, $n = 1, 2, \ldots$, then the point sequence $\{x_n\}$

clusters in X. Theorem 2. A T_1-space Y is a Σ-space (K. Nagami, "Σ-spaces," Fund. Math. 65(1969), 169-192) if and only if there is a half-M-space (X,X') and a perfect mapping f from X onto Y such that f(X') = Y. (Received December 13, 1971.)

*72T-G37. GEORGE R. GORDH, JR., University of Kentucky, Lexington, Kentucky 40506. Indecomposable Hausdorff continua and mappings of connected linearly ordered spaces.

It is shown that if each proper subcontinuum of an indecomposable Hausdorff continuum M is a generalized arc, then M contains uncountably many composants. The result is obtained as a corollary to several theorems concerning mappings from linearly ordered topological spaces into compact Hausdorff continua. (Received December 15, 1971.)

72T-G38. WOLFGANG J. KRIEGER, Ohio State University, Columbus, Ohio 43210. Subsystems of subshifts of finite type. Preliminary report.

Let Ω be a finite alphabet. A closed set $C \subset \Omega^{\mathbb{Z}}$ that is invariant under the shift S, $(Sx)_i = x_{i+1}$, $x \in \Omega^{\mathbb{Z}}$, is said to be of finite type if for some $I \in \mathbb{N}$ and some $\mathcal{A} \subset \Omega^I$, $C = \bigcap_{j=-\infty}^{\infty} \{x \in \Omega^{\mathbb{Z}} : (x_{j+i})_{i=1}^{I} \in \mathcal{A}\}$. Theorem. Let C be a topologically transitive subshift of finite type, and let T be an ergodic measure preserving transformation of the Lebesgue measure space (X,B,μ) whose entropy is smaller than the topological entropy of C. Then there exists a shift invariant probability Borel measure ν on C such that the systems (X,μ,T) and (C,ν,S) are measure theoretically isomorphic. Theorem. Let C be a topologically transitive subshift of finite type, and let T be a minimal expansive homeomorphism of the Cantor discontinuum D whose topological entropy is smaller than the topological entropy of C. Then there exists a homeomorphism U of D onto a closed shift invariant subset of C such that UT = SU. The existence of (topologically transitive) subshifts of finite type with n fixed points whose topological entropy is arbitrarily small shows that the word "minimal" cannot be deleted from the statement of this theorem. (Received December 20, 1971.)

*72T-G39. ROSS GEOGHEGAN and R. RICHARD SUMMERHILL. School of Mathematics, Institute for Advanced Study, Princeton, New Jersey 08540. Concerning the shapes of finite-dimensional compacta.

A closed subset X of euclidean space E^n is a Z_k-set ($k \geq 0$) if U nonempty, k-connected and open in E^n implies U - X nonempty and k-connected; X is a strong Z_k-set if whenever $\epsilon > 0$ and P is a compact subpolyhedron of E^n having dimension $\leq k+1$, there is an ϵ-push h of (E^n,X) with $h(X) \cap P = \emptyset$. 1-ULC means uniformly locally simply connected. Sh(X) denotes the (Borsuk) shape of X, and $\hat{X}$ denotes the quotient point of E^n/X. Theorem. Let X and Y be nonempty compacta in E^n satisfying one of the following conditions: (a) X and Y strong Z_{n-k-2} sets ($k \geq 0$, $n \geq 2k+2$); or (b) X and Y Z_{n-k-2} sets ($k \geq 2$, $n \geq 2k+2$); or (c) $E^n - X$ and $E^n - Y$ 1-ULC ($n \neq 4$, $n \geq \max(2 \dim X+2, 2 \dim Y+2)$). Then the following are equivalent: (1) Sh(X) = Sh(Y); (2) $(E^n/X,\hat{X})$ and $(E^n/Y,\hat{Y})$ are homeomorphic pairs; (3) $(E^n/X, E^n/X-\hat{X}, \hat{X})$ and $(E^n/Y, E^n/Y-\hat{Y}, \hat{Y})$ are homotopy equivalent triples; (4) $E^n - X$ and $E^n - Y$ are homeomorphic. (Received December 20, 1971.)

*72T-G40. ARLO W. SCHURLE, University of North Carolina, Charlotte, North Carolina 28213. Strongly cellular subsets of E^3.

A set Z in E^n is said to be strongly cellular if there is an n-cell C with boundary S and a homotopy $H: C \times I \to C$ such that $H_0 = Id$, $H_t|Z = Id$ for all t, $H_t|S$ is a homeomorphism and Z misses $H_t(S)$ for $t < 1$, $H_t(S)$ misses $H_u(S)$ if $t \neq u$, and $H_1(C) = Z$. Theorem. A one-dimensional subset of E^3 is strongly cellular if and only if it is a tame dendrite. (Received December 23, 1971.)

72T-G41. LOUIS M. FRIEDLER, University of Alberta, Edmonton, Alberta, Canada. Quotients of proximity spaces. Preliminary report.

Let (X, δ) be a proximity space, $f: (X, \delta) \to Y$ any function onto a set Y. Define $A \not\delta' B$ in Y iff there is a function $g: Y \to I$ such that $g(A) = 0$, $g(B) = 1$ and $g \circ f$ is a p-map. Theorem 1. δ' is the quotient proximity on Y. Theorem 2. If X is normal then every proximity quotient of (X, δ) generates the quotient topology iff δ is the elementary proximity and every (topological) quotient of X is completely regular. Theorem 3. If (X, δ) is separated, every p-map on X with separated range is a proximity quotient map iff the induced topology on X is compact. Proximity open maps are also considered and other mapping characterizations of domains and ranges are obtained. (Received December 27, 1971.)

72T-G42. DAVID J. LUTZER, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. Mappings of M_1-spaces. Preliminary report.

In [Pacific J. Math. 11(1961), 105-125], J. Ceder introduced M_1, M_2, and M_3 spaces. Borges [Pacific J. Math. 17(1966), 1-16] proved that closed mappings preserve M_3-spaces. The author has been able to prove: Theorem 1. Closed mappings preserve M_2-spaces. Theorem 2. Perfect irreducible mappings preserve M_1-spaces. (Recall that a perfect mapping is a closed, continuous, onto mapping with compact fibres and that a mapping f from X onto Y is irreducible if there does not exist a closed proper subspace $S \subset X$ such that $f[S] = Y$.) Therefore one has Corollary 3. If every closed subspace of an M_1-space X is also an M_1-space, then so is every perfect image of X. The author would appreciate any comments on the novelty of these results. (Received January 3, 1972.)

72T-G43. YAICHI SHINOHARA, University of Georgia, Athens, Georgia 30601. On the signature of a link with two bridges. Preliminary report.

Let L be a link with two bridges of type (α, β) [H. Schubert, Math. Z. 65(1956), 133-170] and $\sigma(L)$ the signature of L [K. Murasugi, Trans. Amer. Math. Soc. 117(1965), 387-422]. Then $\sigma(L) = {}^2N_{2\alpha,\beta} - (\alpha - 1)$, where $N_{p,q}$ is the number of qx $(1 \leqq x \leqq [p - 1/2])$ whose remainder mod p of smallest absolute value is negative. Geometrically, in the Schubert normal form of L, $\sigma(L)$ is the number of times that L crosses under one of two bridges from left to right minus the number of times that L crosses under the bridge from right to left. (Received January 10, 1972.)

72T-G44. CHARLES RODGER HAMMONS, Morehead State University, Morehead, Kentucky 40351. On paracompact p-space completions of some uniform spaces. Preliminary report.

Let X be a completely regular T_1 space and δX be the completion of the uniform space $[X;\mathcal{U}]$ where $\mathcal{U}$ is the largest admissible uniform structure on X. X is a WM-space if there exists a normal sequence of open covers $\{\mathcal{U}_n\}$ of X such that if $\{x_n\}$ is a C-embedded sequence, $x \in X$ and $x_n \in st(x,\mathcal{U}_n)$ for all n, then $\{x_n\}$ has a cluster point. X is a WM^1-space if there exists a normal sequence of open covers $\{\mathcal{U}_n\}$ of X such that if $f \in C(X)$, $x \in X$ and f is bounded on $X \setminus st(x,\mathcal{U}_n)$ for all n, then $f \in C^*(X)$. Among the results are the following. Theorem. Conditions (1), (2), (3) and (4) are equivalent. If X is countably paracompact or δ-normally separated, then (1) is equivalent to (5): (1) δX is a paracompact p-space. (2) There is a WZ relatively pseudocompact map $f:X \to Y$ onto a metric space Y. (3) X is a WM-space. (4) X is a WM^1-space. (5) X is an M-space. Theorem. If $f:X \to S$ is a WZ relatively pseudocompact map onto a metric space S and $g:Y \to T$ is a perfect map onto a metric space T, then the product map $f \times g:X \times Y \to S \times T$ is a WZ relatively pseudocompact map. Corollary. The product of a WM-space and a paracompact p-space is a WM-space. It is noted that Isiwata also studied the class of WM-spaces in [Proc. Japan Acad. 45(1969), 359-367] using a different definition. (Received January 10, 1972.)

*72T-G45. J. VAN DALEN and E. WATTEL, Vrije Universiteit, Amsterdam, The Netherlands. A topological characterization of ordered spaces. Preliminary report.

Theorem 1. A T_1 space is a subspace of a totally ordered space if and only if there exists an open subbase of the space which is the union of two nests of open sets. Theorem 2. A T_1 space can be supplied with an order such that the order topology and the original topology coincide if and only if there exists a subbase $\mathcal{S}$ of the space which satisfies the following properties: (i) The subbase $\mathcal{S}$ is the union of two nests. (ii) If $S_0 \in \mathcal{S}$ and $S_0 = \bigcap \{S \mid S_0 \subset S \in \mathcal{S} \setminus \{S_0\}\}$ then $S_0 = \bigcup \{S \mid S \subset S_0 \text{ and } S \in \mathcal{S} \setminus \{S_0\}\}$. Theorem 2A. A connected T_1 space is orderable if and only if there is a subbase for the space satisfying (i). This generalizes results of J. de Groot and P. S. Schnare, "A characterization of products of compact totally ordered spaces," (to be published). (Received December 27, 1971.) (Authors introduced by Dr. M. A. Maurice.)

72T-G46. J. VAN DALEN, Vrije Universiteit, Amsterdam, The Netherlands. A topological characterization of products of totally ordered spaces.

Theorem. A T_1-space X is homeomorphic to the topological product of totally ordered spaces, if and only if X has an open subbase $\mathcal{S}$, satisfying the following conditions: (i) if $S_0 \cup S_1 = X = S_0 \cup S_2$ with $S_i \in \mathcal{S}$ for $i = 0, 1, 2$, then either $S_1 \subset S_2$ or $S_2 \subset S_1$; (ii) for each $S \in \mathcal{S}$ there exists $S' \in \mathcal{S}$ such that $S' \cup S = X$; (iii) if $S_0 \in \mathcal{S}$ and $S_0 = \bigcap \{S \mid S_0 \subset S \in \mathcal{S} \setminus \{S_0\}\}$, then $S_0 = \bigcup \{S \mid S \subset S_0 \text{ and } S \in \mathcal{S} \setminus \{S_0\}\}$; (iv) every covering $\mathcal{C}$ of X by members of $\mathcal{S}$ has a subcovering $\mathcal{C}'$ such that if $S_1 \in \mathcal{C}'$ and $S_2 \in \mathcal{C}'$, then either $S_1 \subset S_2$ or $S_2 \subset S_1$ or $S_1 \cap S_2 = \emptyset$ or $S_1 \cup S_2 = X$. (Received January 7, 1972.) (Author introduced by Dr. M. A. Maurice.)

*72T-G47. HAROLD R. BENNETT, Texas Tech University, Lubbock, Texas 79406 and ERNEST S. BERNEY, Idaho State University, Pocatello, Idaho 83201. Quasi-developable spaces and spaces with bases of countable order. Preliminary report.

Theorem. In a T_3 quasi-developable space compact sets are of countable character. Theorem. A T_3 space with a G_δ-diagonal has a base of countable order if it is either a p-space or a $W-\Delta$ space. Theorem. A hereditarily weak θ-refinable T_3 space which has a base of countable order is a quasi-developable space. (Received January 14, 1972.)

72T-G48. KAI WANG, University of Chicago, Chicago, Illinois 60637. Free S^1-actions and the group of diffeomorphisms.

Let $S^{2p-1} \times_G S^{2q-1}$, $S^{2p-1} \times_G D^{2q}$ and $D^{2p} \times_G S^{2q-1}$ be the orbit spaces of the diagonal actions of S^1 on $S^{2p-1} \times S^{2q-1}$, $S^{2p-1} \times D^{2q}$ and $D^{2p} \times S^{2q-1}$ respectively. Let $D_0(S^{2p-1} \times_G S^{2q-1})$, or D_0 be the group of pseudo-isotopy classes of diffeomorphisms of $S^{2p-1} \times_G S^{2q-1}$ which are homotopic to identity. For $u \in D_0$, define $P(u) = S^{2p-1} \times_G D^{2q} \cup_f D^{2p} \times_G S^{2q-1}$ where $f \in u$, and it is clear $P(u)$ depends only on u and is homotopy equivalent to CP^{p+q-1} by a homotopy equivalence h_u. Let T_M be the stable tangent bundle of M. Let $CP = CP^{p+q-1}$. Theorem 1. For u, v in D_0, $h_{uv}^* T_{P(uv)} = h_u^* T_{P(u)} + h_v^* T_{P(v)} + T_{CP}^{-1}$. Theorem 2. For $u \in D_0$, $P(u)$ is tangential equivalent to CP iff $u^r = 1$ for some r. Theorem 3. The set of $u \in D_0$ which are of finite order is a finite normal subgroup of D_0. Theorem 4. For $k \geq 3$, and (a) $n \geq 6$ if n is even, (b) $n \geq 5$ if n is odd and k is odd, and (c) $n \geq 7$ if n is odd and k is even, then there are infinitely many topologically distinct decomposable smooth free S^1 actions on homotopy $(2n+1)$-spheres with characteristic homotopy $(2k+1)$-spheres. (Received January 17, 1972.)

72T-G49. WILLIAM M. FLEISCHMAN and S. WILLIAMS, State University of New York, Buffalo, New York 14226. The G_δ topology on a compact space. Preliminary report.

Several years ago Arhangelski proved that a compact space satisfying the first axiom of countability consists of at most $2^{\aleph_0}$ points. Solution to this long unsolved problem brought to light again some related questions, one of which was recently communicated to the first author by Professor Juhasz and stated as follows: Let $(X, <)$ be compact linearly ordered topological space when given the interval topology τ. Furthermore, let τ_δ be the topology generated by τ - G_δ-sets. Is (X, τ_δ) $(\omega, 2^{\aleph_0})$-compact? We have answered this question in the affirmative even when (X, τ) is assumed to be Lindelöf or (X, τ) is the finite product of Dedekind complete linearly ordered sets and τ is the product of the interval topologies. Cardinal generalizations (on the δ and $\aleph_0$) go through as well. Moreover, if $<$ is a well-ordering, then (X, τ_δ) is Lindelöf. (Received January 13, 1972.)

72T-G50. HUGH R. COOMES, William Paterson College, Wayne, New Jersey 07470. Colocally acyclic spaces. Preliminary report.

If X is a topological space and P is a topological property then X is colocally P if for each point x of X and each neighborhood U of x there is a neighborhood V of x contained in U such that X - V has property P. Let X be a compact Hausdorff n-dimensional space which is colocally acyclic in dimensions less than n. The coefficient group L is a field or the integers. Theorem 1. X is connected, $H^p(X;L) = 0$, $1 \leq p \leq n-1$, $H^n(X;L) \neq 0$, and X is cohomologically locally connected (clc_L) if and only if rank $H^n(X;L)$ is finite. Theorem 2. For each point x in X, the local Betti numbers $p^i(x,L)$ are zero for $i \leq n-1$, and $0 < p^n(x,L) \leq$ rank $H^n(X;L)$. Theorem 3. If X is colocally acyclic in dimension n then X is an orientable Wilder n-manifold. Corollary. An n-dimensional H-spheroidal space of Borsuk ("Über Späroidale und H-Späroidal Räume," Mat. Sb. 1(1936), 643-660) is an orientable Wilder n-manifold over Z with the cohomology groups of an n-sphere. (Received January 18, 1972.)

The March Meeting in New York, New York
March 27-30, 1972

Algebra & Theory of Numbers

*692-A1. JOHN L. LEONARD, University of Arizona, Tucson, Arizona 85721. r-ways in graphs.

Two points of a graph are joined by an r-way, or more explicitly, a line r-way, if there are r line-disjoint paths between the points. The number $\ell_r(n)$ is the smallest integer such that any graph with n points and $\ell_r(n)$ or more lines must contain an r-way. We show that $\ell_5(n) = \lceil (5n-3)/2 \rceil$ and $\ell_6(n) = 3n-2$, and exhibit graphs illustrating the sharpness of these results. (Received November 22, 1971.)

*692-A2. ROBERT A. RUBIN, University of Kentucky, Lexington, Kentucky 40506. Absolutely torsion-free rings. Preliminary report.

Call a ring Λ absolutely torsion-free (ATF) if for every finite kernel functor σ (i.e. a topologizing filter of left ideals not containing 0) $\sigma(\Lambda) = 0$. Since a commutative ring is ATF iff it is an integral domain, ATF rings may be viewed as noncommutative analogs of domains. Now an ATF ring is a prime ring, but there are even primitive rings that are not ATF. However, if Λ is either Goldie or finite as a module over its center, then Λ is ATF iff it is prime. The class of ATF rings is closed under the formation of polynomial rings, overrings in the maximal quotient ring, and under Morita equivalence, but not under subrings. The maximal ring of quotients of an ATF ring is simple self-injective, and regular, and satisfies the d.c.c. iff the ATF ring is finite-dimensional. An interesting class of ATF rings are the hereditary noetherian prime rings (HNP). Techniques used in deriving properties of ATF rings show that if Λ is HNP then every ring between Λ and its maximal quotient ring is itself a ring of quotients of Λ with respect to an idempotent kernel functor, and hence is HNP itself. (Received December 30, 1971.)

*692-A3. MELVIN F. JANOWITZ, University of Massachusetts, Amherst, Massachusetts 01002. Congruence relations on lattices.

Consider the following axioms on a bounded lattice L: (A) $a/0 \to c/d$ with $c > d$ implies $c/d \to a_1/a_2$ for suitable a_1, a_2 such that $a \geq a_1 > a_2$. (B) $a > b$ implies $\exists t \equiv 1\ (\Theta_{a/b})$ such that $t \not\geq a$. (C) $a > b$ implies $\exists t \equiv 1\ (\Theta_{a/b})$ such that $t \geq b$, $t \not\geq a$. Theorem. Axiom A is equivalent to the assertion that for every congruence relation Θ on L, $a \equiv 0\ (\Theta^*) \Leftrightarrow$ the interval $[0,a]$ contains only trivial congruence classes modulo Θ; Axioms A and B are both valid iff for every congruence relation Θ on L, the kernel of Θ^* is the set of lower bounds of $\{t \in L; t \equiv 1\ (\Theta)\}$. Theorem. If L is complete and if both L and its dual satisfy Axioms A and B, then the lattice of congruence relations of L is a Stone lattice. (Note. If L is lower continuous, the conditions on the dual are not needed.) Theorem. L is weakly modular and satisfies the dual of (C) iff for every congruence relation Θ on L, Θ^* is given by the rule $a \equiv b\ (\Theta^*) \Leftrightarrow [0,a] \cap \ker\Theta = [0,b] \cap \ker\Theta$. Theorem. Let L be weakly modular, and suppose both L and its dual satisfy (C). Let Θ be a congruence relation on L, and $J = \ker\Theta^*$. Then J is a central element of $\overline{L}$, the completion by cuts of L, and $a \equiv b\ (\Theta^*) \Leftrightarrow [0,a] \vee J = [0,b] \vee J$ in $\overline{L}$. (Received January 4, 1972.)

*692-A4. JOHN H. YING, State University of New York at Binghamton, Binghamton, New York 13901. On some conditions which do not characterize finite abelian groups. Preliminary report.

A finite abelian group G satisfies the following conditions: (A) each subgroup of G is isomorphic to a quotient group of G, and (B) each quotient group of G is isomorphic to a subgroup of G. Though finite groups satisfying (A) are easily seen to be nilpotent, they need not be abelian. Theorem. A finite nonabelian p-group G satisfies condition (A) if and only if G is isomorphic to $L_p^n \times A$ for some integer $n \geq 2$ or G is isomorphic to $S_p \times B$. Here L_p^n is the group generated by a,b with defining relations $a^{p^n} = b^{p^n} = 1$ and $[a,b] = a^{p^{n-1}}$, S_p is the nonabelian group of order p^3 and exponent $p > 2$, A is a finite abelian p-group with $\exp(A) < p^n$ and B is a finite elementary abelian p-group. Corollary. A finite nonabelian p-group G satisfies conditions (A) and (B) if and only if G is isomorphic to $S_p \times B$. The corollary extends results by Spencer ("Self dual finite groups," Abstract 677-20-4, these *Notices* 17(1970), 767). (Received January 14, 1972.)

692-A5. PETER V. O'NEIL, College of William and Mary, Williamsburg, Virginia 23185. The full homeomorphs of K_5. Preliminary report.

Let K_5 denote the complete graph on five vertices. A graph G is a full homeomorph of K_5 if (1) G has a subgraph homeomorphic to K_5, and (2) for each vertex v of G, G - v is planar. This paper gives an explicit description of all full homeomorphs of K_5, up to isomorphism. In particular, eight graphs $g_1, \ldots, g_8$ and two graph operations P and Q are determined such that each full homeomorph of K_5 is isomorphic to a graph obtained by applying P and/or Q to some g_i. (Received January 17, 1972.)

692-A6. RICHARD L. ROTH, University of Colorado, Boulder, Colorado 80302. On extending characters of a normal subgroup. Preliminary report.

Let K be a normal subgroup of the finite group G, G/K abelian, χ an irreducible character of G and σ an irreducible component of $\chi|_K$. Using theory developed in a paper of the author's ("A dual view of the Clifford theory of characters of finite groups," Canad. J. Math. 23(1971), 857-865), the following theorem is proved: Theorem. There exists a subgroup M such that σ extends to a character ρ of M and $\rho^G = \chi$. For given M, ρ may be chosen in precisely $e_K(\chi)$ ways where $e_K(\chi)$ is the usual Clifford index. If I is the inertial group of σ and J the dual inertial group of χ (see paper cited), then $J \subseteq M \subseteq I$ and $I/M \cong M/J$. (Received January 17, 1972.)

*692-A7. EARL GLEN WHITEHEAD, JR., Courant Institute, New York University, New York, New York 10012. Partitions of finite abelian groups into symmetric sum-free sets. Preliminary report.

A set, S, contained in an additive group, G, is said to be symmetric sum-free if and only if $(S+S) \cap S = \emptyset$ and $S^{-1} \subset S$. Let M(G) be the minimal number of symmetric sum-free sets needed to partition the nonzero elements of a finite abelian group G. In this paper we establish lower and upper bounds for M(G) for certain classes of groups. (Received January 17, 1972.)

*692-A8. THOMAS A. McINTYRE, University of Notre Dame, Notre Dame, Indiana 46556. A complete normal surface without effective Cartier divisors.

This paper gives an example of a complete, normal surface whose only divisors are those which are linearly equivalent to zero. This surface was originally constructed by M. Nagata (Illinois J. Math. 2(1958), 490-498) as an example of a complete, normal, nonprojective surface. This result removes the only possible counterexample in the literature to the conjecture that a complete, normal, divisorial surface must be projective. Also, a criterion for a counterexample to this conjecture is given. (Received January 17, 1972.)

692-A9. SEYMOUR BACHMUTH and HORACE Y. MOCHIZUKI, University of California, Santa Barbara, California 93106. Groups of exponent 4. Preliminary report.

There exists a group of exponent 4 which has solvability class at least four. Until recently there was hope that groups of exponent 4 are solvable of class three. (Cf. A. L. Tritter, "A module-theoretic computation related to the Burnside problem", Proc. Conference (Oxford, 1967), Pergamon, Oxford, 1970, pp. 189-198.) (Received January 17, 1972.)

692-A10. KIM KI-HANG BUTLER, Pembroke State University, Pembroke, North Carolina 28372. The number of partially ordered sets. Preliminary report.

Extensions of these results (Abstract 72T-A8, these *Notices* 19(1972), A-3) are given in this paper. Let H(q) (q = 1, 2, ..., m) be the reduced regular $\mathcal{D}$-inequivalent subgroups of the semigroup B(n) of all binary relations on a set of n elements. Let o(H(q)) denote the order of H(q). If G*(n) denotes the number of different

partial orderings of n elements, then $G^*(n) = \sum_{q=1}^{m} n!/o(H(q))$ where $1 \leqq o(H(q)) \leqq n!$, $o(H(q))|n!$, and m is equivalent to the number of nonisomorphic reduced regular $\mathcal{D}$-classes in B(n). Moreover, (1) H(q) (q=1, 2, ..., m) are nonisomorphic reduced regular $\mathcal{H}$-classes in B(n). (2) Let $\Gamma(H(q))$ be the Schützenberger groups of H(q) (q = 1, 2, ..., m). Then $\Gamma(H(q)) \cong H(q)$. Hence $\Gamma(H(q))$ are simply transitive groups of permutations of H(q). (3) G*(n) is equivalent to the number of reduced idempotents in B(n). (4) G*(n) is equivalent to the number of partial order graphs with n vertices. (Received January 18, 1972.)

*692-A11. DONALD H. ADAMS, University of Massachusetts, Amherst, Massachusetts 01002. Baer semigroups as an equational class.

A Baer semigroup (M. F. Janowitz, "A semigroup approach to lattice theory", Canad. J. Math. 18 (1966), 1212-1223) is a semigroup S with zero in which the left [right] annihilator of each element is a principal left [right] ideal generated by an idempotent. We introduce, using the Axiom of Choice, two unary operations, $x \to x^{\ell}$ and $x \to x^{r}$ where x^{ℓ} and x^{r} are idempotents generating the left and right annihilators respectively of x. The resulting algebra of type $\langle 2, 1, 1, 0\rangle$ coordinatises bounded lattices in the same way as Baer semigroups do. They are called coordinatising algebras, and are characterised by: Theorem. An algebra $(B; \cdot, {}^{\ell}, {}^{r}, 0)$ of type $\langle 2, 1, 1, 0\rangle$ is a coordinatising algebra if and only if it satisfies these identities: (i) $x(yz) = (xy)z$; (ii) $0x = x0 = 0$; (iii) $0^{\ell}x = 0^{r}x = x = x0^{\ell} = x0^{r}$; (iv) $0^{rr} = 0^{\ell\ell} = 0$; (v) $(xy)^{\ell}xy^{\ell} = (xy)^{\ell}x$; (vi) $x^{r}y(xy)^{r} = y(xy)^{r}$. Coordinatising algebras therefore form an equational class. Theorem. If $\mathcal{L}$ is an equational class of bounded lattices, then the class of all coordinatising algebras coordinatising members of $\mathcal{L}$ is equational. This is proved by showing that coordinatising algebra homomorphisms induce lattice homomorphisms. Identities are found that characterise the class of coordinatising algebras whose lattices of left annihilators are distributive. (Received January 19, 1972.)

692-A12. FEDERICO GAETA, State University of New York at Buffalo, Amherst, New York 14226. The collineation group of a normal projective abelian variety.

The subgroup S of the torus group $\mathbb{C}^n/G$ leaving invariant a complete linear system of divisors $|D|$ on $\mathbb{C}^n/G$ can be decomposed in $\nu(<\infty)$ ways as a direct sum $S = S_1 \oplus S_2$, where $S_1 \approx S_2$ are of type $T = (t_1 t_2 \ldots t_n)$ (t_j elementary divisors of $A \,|\, G \times G$, $A = \mathrm{Im}\, H$ (H Hermitean > 0). $\exists\, t = t_1 t_2 \ldots t_n$ linearly independent divisors $D_j \in |D|$ invariant by any $S \in S_1$. S_2 permutes $\{D_1 D_2 \ldots D_t\}$. ν is the index of the congruence subgroup $\Delta(T)$ of the modular group M(T). For T = 3, n = 1 we obtain the "configuration of flexes", for $T = 2 \cdot 1_n$ the properties of "θ-constants" appearing in Rauch-Farkas recent work on Schottky relations are geometrically interpreted. (Received January 21, 1972.)

*692-A13. S. K. JAIN, Ohio University, Athens, Ohio 45701. Rings with a polynomial identity.

Theorem 1. Let R be semiprime, left rationally complete and satisfy a polynomial identity. Then $R = S \times T$ where S is a product of finite dimensional central simple algebras (the dimensions bounded) and T is a von Neumann regular ring with zero socle and satisfies a polynomial identity. Theorem 2. Let (R, V, W, S,

f, g) be a pre-equivalence data (also called Morita context), where R, S have 1 and f, g are not zero. Let R satisfy a polynomial identity. Then S contains a nonzero right (left) ideal with a polynomial identity. The Theorem 2 has a number of interesting corollaries. In case the ring S itself satisfies a polynomial identity and has the same multilinear identities as those of a one-sided ideal in it then the one-sided ideal contains a nonzero two-sided ideal. An application of this is given. (Received January 21, 1972.)

*692-A14. TOSHIHIKO YAMADA, Queen's University, Kingston, Ontario, Canada. Schur subgroups over the 2-adic field.

Let p be a prime number and Q_p the p-adic rational number field. Let k be a subfield of a cyclotomic extension of Q_p. Denote by $S(k|Q_p)$ the subgroup of the Brauer group Br(k) consisting of those algebra classes $[A]$ appearing in some $Q_p[G]$. Theorem. Let $p = 2$. If k contains $\sqrt{-1}$ or $\sqrt{-2}$, then $S(k|Q_2) = 1$. If k is unramified over Q_2, or if $k = Q_2(\zeta_s + \zeta_s^{-1})$ where $s = 2^c$ $(c \geq 2)$, then $S(k|Q_2)$ is the subgroup of order 2 of $B_r(k)$. Remark. For any odd prime p and k/Q_p, the Schur subgroup $S(k|Q_p)$ was completely determined by the author (cf. J. Math. Soc. Japan 23(1971), 295-310). (Received January 24, 1972.)

*692-A15. BEN C. BREWSTER, State University of New York, Binghamton, New York 13901. $\mathfrak{F}$-projectors in finite π-solvable groups.

Let G be a finite group and let π be a set of primes. Then $O_{\pi'}(G)$ is the largest normal π'-subgroup of G. A saturated formation $\mathfrak{F}$ is called π-saturated if $G/O_{\pi'}(G) \in \mathfrak{F}$ implies $G \in \mathfrak{F}$. There are numerous π-saturated formations; for instance, if $\mathfrak{F}$ is any saturated formation, then the class of all extensions of π'-groups by $\mathfrak{F}$-groups is a π-saturated formation. Theorem. If G is π-solvable and $\mathfrak{F}$ is a π-saturated formation, then G has $\mathfrak{F}$-projectors and any two $\mathfrak{F}$-projectors of G are conjugate in G. This result extends the work of Gaschütz [Math. Z. 80(1963), 300-305] and Lausch [J. Austral. Math. Soc. 10(1969), 241-250]. (Received January 24, 1972.)

*692-A16. LUISE-CHARLOTTE KAPPE and WOLFGANG P. KAPPE, State University of New York, Binghamton, New York 13901. On 3-Engel groups.

Let $[x,_1y] = x^{-1}y^{-1}xy$ and $[x,_ny] = [[x,_1y],_{n-1}y]$. A group G is called an n-Engel group if $[x,_ny] = 1$ for all $x, y \in G$. The following conditions for a group are investigated: (i) maximal class n subgroups are normal, (ii) normal closures of elements have nilpotency class n at most, (iii) all normal closures are n-Engel groups, (iv) G is an (n+1)-Engel group. Each of these four conditions is a consequence of the preceding condition. It has been shown previously by the second author that these conditions are equivalent for $n = 1$. Here the question is settled for $n = 2$ as follows: conditions (ii), (iii) and (iv) are equivalent. Counterexamples show that (i) does not follow from the other conditions. Moreover, the class of groups defined by (i) is not closed under homomorphisms and no obvious restrictions on commutator or power structure are imposed. (Received January 24, 1972.)

*692-A17. WOLFGANG P. KAPPE and LUISE-CHARLOTTE KAPPE, State University of New York, Binghamton, New York 13901. Metabelian Levi-formations.

Let E be a subgroup closed formation. A Levi-formation, L(E), is the class of all finite groups G such that the normal closures $\langle x^G \rangle$ are in E for all $x \in G$. A complete classification of all metabelian Levi-formations is given. The classification is more complex for groups of order divisible by 3. Disregarding this case the main result is as follows. Theorem. L(E)-groups are metabelian for a subgroup closed formation of groups of order prime to 3 if and only if E-groups are extensions of nilpotent groups by elementary abelian 2-groups and nilpotent E-groups are abelian. (Received January 24, 1972.)

692-A18. LARRY D. SHATOFF, Colgate University, Hamilton, New York 13346. Excluding minors in combinatorial geometry.

We determine conditions that a single point extension of a combinatorial geometry (simple matroid) exclude a minor which is excluded by the original geometry. Denote by G(S) a geometry on a set S. Let M be a modular cut of a combinatorial geometry G(S) and suppose P is a flat of G(S). The set determined by P and M is the collection of all flats L such that either $P \subseteq L$ and $L \in M$, or $P \subseteq L$ and L is not covered by any element of M. When ordered by inclusion, the set determines combinatorial geometry. Denote by G-1 any subgeometry of G with one less point than G. Theorem. Let G(S) be a combinatorial geometry and M a nonempty modular cut of G determining the combinatorial geometry $G'(S \cup e)$. Let F be a combinatorial geometry that does not appear as a minor of G. F does not appear as a minor of G' if and only if: (i) If for some (F-1) we have $r(F) = r(F-1)$, if $f:(F-1) \to G_{[A,B]}$ is an isomorphism of (F-1) onto an upper minor of G, and if the image under f of the sets M*, the modular cut of (F-1) determining F, is the collection $\{D : D \subseteq B-A, D \cup A \in M\}$ then $\bar{A}$ is covered by an element of M. (ii) Let C be any flat of G not covered by an element of M. Then the combinatorial geometry determined by C and M does not contain a subgeometry isomorphic to F. (Received January 24, 1972.)

*692-A19. DONALD COOK, Hollins College, Hollins College, Virginia 24020. Complete exact sequences. Preliminary report.

All groups are Abelian and have the p-adic topology. If G is a group, let $p^\omega G = \bigcap_{n<\omega} p^\eta G$ and G* be the completion of $G/p^\omega G$. Then * is a functor from Abelian groups to metric groups. A subgroup H of G is a subspace of G if the p-adic topology on H is the relative on H. This paper is a study of the sequences $0 \to A \to B \to C \to 0$ for which the induced sequence $0 \to A^* \to B^* \to C^* \to 0$ remains exact. If A is a subspace of B and $p^\omega B = 0$ then $0 \to A^* \to B^* \to C^* \to 0$ is exact if and only if A is closed in B. Moreover, A is a closed p-pure subgroup of B if and only if $B^* = A^* \oplus (B/A)^*$. Thus there is a 1-1 correspondence between p-pure closed subgroups of B and summands of B*. In general the sequence $0 \to A \to B \to C \to 0$ is exact if and only if the induced sequence $0 \to p^\omega A \to p^\omega B \to p^\omega C \to 0$ is exact. Hence the sequences $0 \to A \to B \to C \to 0$ for which the induced sequences $0 \to p^\eta A \to p^\eta B \to p^\eta C \to 0$ remain exact for $\eta \leqq \omega$ are exactly the sequences for which the sequence $0 \to A^* \to B^* \to C^* \to 0$ is split exact. (Received January 24, 1972.)

692-A20. ELVIRA STRASSER RAPAPORT, State University of New York, Stony Brook, New York 11790 and HENRY W. LEVINSON, Rutgers University, New Brunswick, New Jersey 08903. Planarity of Cayley diagrams. Preliminary report.

Theorem 1. Let $G = \langle x; R \rangle$ be a finite presentation of a group with x and R closed under cyclic permutation and inversion. Let Γ^* be the graph of G, and L* the subgraph of Γ^* consisting of a vertex, v, together with all circuits of Γ^* corresponding to elements of R and incident to v, together with all edges of Γ^* incident to these circuits. Let L be an embedding of L* in an orientable 2-manifold M. If every circuit of L bounds on M and the clockwise succession of edges of each vertex of L is the same, then Γ^* is planar. The proof employs Edmonds' embedding technique. Theorem 2 gives certain conditions on G by which L* may be constructed without knowledge of how to construct Γ^*. Theorem 3 delimits further the assumption of the solution of the word problem for G to construct Γ^*. If the maximum genus of Γ^* is 0, then the solution of the word problem in G is corollary to Theorem 3. (Received January 25, 1972.)

*692-A21. DAVID EISENBUD, Brandeis University, Waltham, Massachusetts 02154 and E. GRAHAM EVANS, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. Every algebraic set in n-space is the intersection of n hypersurfaces.

In 1882 Kronecker proved that every radical ideal in $K[x_1, \ldots, x_n]$ is the radical of an ideal generated by $n + 1$ elements. We can show that every radical ideal is the radical of an ideal generated by n elements. More generally, Theorem 1. Let R be a noetherian ring of Krull dimension n, and suppose that R is of the form $S[x]$ for some ring S. Then any radical ideal in R is the radical of an ideal generated by n elements. Corollary 1. Every algebraic set in affine n-space is the intersection of n hypersurfaces. To extend the corollary to algebraic sets in projective n-space, one needs a graded version of Theorem 1. Theorem 2. Let R be a noetherian graded ring of dimension $n + 1$, and suppose that R is a graded polynomial ring of the form $S[x]$, where S is a graded ring, $S = \Sigma_{i \geq 0} S_i$, and that S_0 is artinian. Let $S_+ = \Sigma_{i>0} S_i$. Then every homogeneous radical ideal of R, which is contained in S_+R, is the radical of an ideal generated by n homogeneous elements. Corollary 2. Every algebraic set in projective n-space is the intersection of n hypersurfaces. The proofs of these theorems are closely related to the proof given by Martin Kneser in ["Über die Darstellung algebraischer Raumkurven als Durchschnitte von Flächen," Arch. Math. 11(1960), 157-158], where the case $n = 3$ of Corollary 2 is proved. (Received January 25, 1972.)

692-A22. PEDRO PABLO SANCHEZ, University of Michigan, Ann Arbor, Michigan 48104. Groups on modules. Preliminary report.

Let R be a ring with identity, let M be a left R-module and let G be a group of R-automorphisms of M. In this paper, sufficient conditions are found on M and R under which theorems about linear groups are extended to G. For example, if R has finite characteristic, M is a finite dimensional module (in the sense of Goldie) and G is locally finite, then G has a normal unipotent subgroup U such that G/U is a finite direct sum of linear groups over locally finite fields. The behavior of U and G/U is further discussed under stronger restrictions on R and M. (Received January 20, 1972.)

*692-A23. PAUL M. EAKIN, University of Kentucky, Lexington, Kentucky 40506 and JAMES L. SILVER, JR., King College, Bristol, Tennessee 37620. Rings which are almost polynomial rings.

If A is a commutative ring with identity and B is a unitary A-algebra, B is locally polynomial over A provided that, for every prime p of A, $B_p = B \otimes_A A_p$ is a polynomial ring over A_p. For example, the ring $Z[\{X/p_i\}_{i=1}^{\infty}]$, where $\{p_i\}_{i=1}^{\infty}$ is the set of primes of Z and X is an indeterminate, is locally polynomial over Z, but is not a polynomial ring over Z. If B is locally polynomial over A, the following results are obtained. B is faithfully flat over A. If A is an integral domain, so is B. If p is any prime of A, then pB is a prime of B. If B is a Krull ring, so is A and the class group of B is isomorphic to the class group of A. If A is a Krull ring and B is contained in an affine domain over A, then B is a Krull ring. If A is a noetherian normal domain and B is contained in an affine ring over A, then B is a normal affine ring over A. If M is a module over a ring A, the content of an element x of M over A is defined to be the smallest ideal A_x of A such that x is in $A_x M$. A module is said to be a content module over A if A_x exists for every x in M. M is a content module over A if and only if arbitrary intersections of ideals of A extend to M. Projective modules are content modules. If B is a locally polynomial ring over a Dedekind domain A, then B is a content module over A if and only if B is Dedekind. (Received January 26, 1972.)

*692-A24. MICHAEL E. DETLEFSEN, Slippery Rock State College, Slippery Rock, Pennsylvania 16057. Regularity and reductions in local Noether lattices.

A d-sequence of degree n in a local Noether lattice (LNL) is a sequence of words of degree n in the underlying semigroup of the lattice satisfying certain conditions on the factors of the words. If (L,M) is a LNL of altitude k satisfying the union condition, there exists an infinite set of principal elements, $\{A_i\}$, in L such that the join of any k of them is M. Also, there exists a d-sequence of degree n of length $\binom{n+d-1}{d-1}$ for all $d \geq 2$ and for all $n \geq 1$ in the underlying subsemigroup generated by the A_i. Since any d-sequence of degree n in this semigroup can be extended to a minimal base for M^n, we show that a LNL satisfying the union condition is regular if and only if its maximal element is a minimal reduction. (Received January 27, 1972.)

692-A25. ANCEL C. MEWBORN and G. L. NORWOOD, University of North Carolina, Chapel Hill, North Carolina 27514. The spectrum of a noncommutative ring. Preliminary report.

Let R be a ring. If ${}_RV$ is an indecomposable injective R-module and $S = \mathrm{Hom}_R(V,V)$, then $h(V) = \{x \in V \mid xs = 0, \text{ all } s \in \mathrm{Rad}S\}$. Define Spec R = $\{V \mid V$ is indecomposable and injective with $h(V) \neq (0)\}$. A topology for Spec R is proposed which generalizes the hull-kernel topology for the space of prime ideals of a commutative ring. Properties of the topological space Spec R are explored. If R is the incidence algebra over a field F of a finite partially ordered set T, then Spec R is homeomorphic to T with its order topology. (Received January 27, 1972.)

692-A26. WILLIAM G. BRIDGES, University of Wyoming, Laramie, Wyoming 82070. Some (0,1)-matrix equations.

Various matrix equations in the $n \times n$ (0,1)-matrix Z impose tight restrictions on the number of row sums (valences in digraph interpretations, replications in the design interpretations) possible in the matrix. Such theorems are discussed. In particular the equations (a) $f(Z) = D + \lambda J$ where f is a polynomial, D is diagonal, $\lambda \neq 0$ and J the matrix of ones, and (b) $ZZ^t = D + R$ where Z^t is the transpose of Z, D is diagonal and R is of rank one are studied. (Received January 27, 1972.)

692-A27. JACK SONN, Adelphi University, Garden City, New York 11530. Localizability of the embedding problem with symplectic kernel.

Let k be a number field, K/k a finite Galois extension with Galois group $\bar{G}$, $\gamma : \bar{G} \to G$ an isomorphism, $\Sigma : 1 \to N \xrightarrow{\iota} E \xrightarrow{\epsilon} G \to 1$ an exact sequence of finite groups. The embedding problem $P = P(K/k, \Sigma, \gamma)$ is to construct a (solution) field $L \supset K$, Galois over k such that there is an isomorphism $\beta : \bar{E} = G(L/k) \to E$ for which $\epsilon\beta = \gamma \mathrm{Res}_{L/K}$. Let $\mathfrak{p}$ be a prime of k. An embedding problem P induces a "local" embedding problem which has an "improper" solution if P has. P is called localizable iff there exists a finite set S of primes of k, and an improper local solution to P at each prime of S, such that every extension L/K, Galois over k, which "localizes" to these solutions at the primes of S, is a solution field to P. Let N have trivial center. P is called irreducible iff $E \overset{\cong}{\subset} \mathrm{Aut}\, N$. Theorem. Let $P = P(K/k, \Sigma, \gamma)$, k arbitrary, P irreducible, and $N = PSp(2m, q)$, the projective symplectic group of degree 2m over GF(q), $m > 1$. Then there is a finite set S of primes $\mathfrak{p}$ of k and subgroups $G_{\mathfrak{p}}$ of G for each $\mathfrak{p} \in S$ such that each $G_{\mathfrak{p}}$ is isomorphic to a Galois group over $k_{\mathfrak{p}}$, such that if $G_{\mathfrak{p}}$ is the decomposition group of a prime of K above $\mathfrak{p}$, for each $\mathfrak{p} \in S$, then P is localizable. (See Abstract 673-81, these *Notices* 17(1970), 415.) (Received January 27, 1972.)

692-A28. BORIS BOHUN-CHUDYNIV, Seton Hall University, South Orange, New Jersey 07079. On distributive quasi-groups of prime order $n > 2$. Preliminary report.

In a previous paper (B. Bohun-Chudyniv, "On distributive idempotent groupoids of nonprime order $2n + 1 \geq 9$" Abstract 691-17-5, these *Notices* 19(1972), A-82) distributive groupoids of nonprime order were constructed using algorithms introduced by V. Bohun-Chudyniv (Abstract 622-61, these *Notices* 12(1965), 344) for the construction of double loops and double quasi-groups. No distributive groupoids of prime order can be constructed by these algorithms. The aims of the present paper are: (1) to show that the above algorithms can be used to construct distributive quasi-groups of every prime order; (2) to show that all distributive quasi-groups of prime order are principal isotopes of cyclic groups of the same order. (Received January 27, 1972.)

*692-A29. JIN BAI KIM, West Virginia University, Morgantown, West Virginia 26505. On the number of idempotent linear transformations of a vector space.

Let M and N be two subspaces of a finite dimensional vector space V over a finite field F with q elements. We count the number of all idempotent linear transformations T of the vector space V such that

$R(T) \subset M$ and $N \subset N(T)$, where $R(T)$ and $N(T)$ denote the range and the null space of T, respectively. (Received January 27, 1972.)

Analysis

692-B1. EILEEN L. POIANA, Saint Peter's College, Jersey City, New Jersey 07306. Mean Cesàro summability of Laguerre and Hermite series.

Inequalities of the type $\|\sigma_n(f,x)W(x)\|_p \leqq C\|f(x)W(x)\|_p$ are proved, where $\sigma_n(f,x)$ is the nth $(C,1)$ mean of the Laguerre or Hermite series of f, $W(x)$ is a suitable weight function of particular form, C is a constant independent of $f(x)$ and n, and the norm is taken over $(0,\infty)$ in the Laguerre case and $(-\infty,\infty)$ in the Hermite case for $1 \leqq p \leqq \infty$. Both necessary and sufficient conditions for these inequalities to remain valid are determined. For $p < \infty$ and $f(x)W(x) \in L^p$, mean summability results are derived by use of the appropriate density theorems. Choice of the weight function is also justified. Detailed proofs are presented for the Laguerre expansions, and the analogous results for Hermite series follow as corollaries. (Received October 26, 1972.)

*692-B2. STANLEY H. BENTON, Tulane University, New Orleans, Louisiana 70118. Global solutions of Hamilton-Jacobi boundary value problems by variational methods. Preliminary report.

Letting B be a closed set (boundary) in $R \times R^n$, $f \in C(B)$, consider the functional $J(\alpha) = f(s,y) + \int_s^t L(z,\alpha(z), \alpha'(z))dz$ defined for absolutely continuous curves joining $(s,y) \in B$ and (t,x) with $s < t$. Let $H(t,x,q) = \sup\{\langle p,q\rangle - L(t,x,p) : p \in R^n\}$ where $\langle \cdot, \cdot \rangle$ denotes the Euclidean inner product. The author shows that, under quite general conditions, $u(t,x) = \min\{J(\alpha) : \alpha(t) = x\}$ exists, is locally Lipschitzian, satisfies the Hamilton-Jacobi equation: $u_t(t,x) + H(t,x,u_x(t,x)) = 0$ almost everywhere off B, and approaches $f(s,y)$ as $(t,x) \to (s,y) \in B$ from the complement of B. The conditions needed include convexity of $L(t,x,p)$ in p, a minimal amount of smoothness, compatibility conditions between L and f, a growth condition on $L(t,x,p)$ in p, and a priori boundedness of α' for at least one extremal α joining any two given points. This result is achieved by combining the techniques of Conway, Hopf, Aizawa, Kikuchi, and Fleming. The case where B is a hyperplane $t =$ constant, called the Cauchy problem, has been done previously by W. H. Fleming. The general case considered by the author includes the Cauchy problem, boundary value problem, mixed problem, and the exterior problem. (Received November 15, 1971.)

*693-B3. JOSEPH PETER DURAN, University of Rochester, Rochester, New York 14627. Strongly regular matrices, almost convergence and Banach limits.

Let $\mathscr{S}$ be the set of strongly regular positive matrices. Let C_A denote the bounded convergence field of a member A of $\mathscr{S}$. Lorentz (Acta Math. 80(1948), 167-190) has shown that F, the space of almost convergent sequences is not the countable intersection of bounded convergence fields of regular matrices. Theorem 1. $F = \bigcap\{C_A | A \in \mathscr{S}\}$. Let M be the set of Banach limits and let $p \in \beta\mathbb{N} - \mathbb{N}$ ($\beta\mathbb{N}$ = the Stone-Čech compactification of $\mathbb{N}$). Denote by p' the linear functional on ℓ^∞ which is given by evaluation at p. Denote the adjoint of a member A of $\mathscr{S}$ by A'. Theorem 2. M = weak-*-closure of $\{A'p' | A \in \mathscr{S}\}$. (Received December 3, 1971.)

692-B4. JOHN J. SWETITS, Old Dominion University, Norfolk, Virginia 23508 and BRUCE WOOD, University of Arizona, Tucson, Arizona 85721. Approximation by a class of generalized Szasz operators. Preliminary report.

Let $E(u) = \exp(u)$ and let $G(z) = \sum_{n=0}^{\infty} g_n z^n$ be analytic in the disk $|z| < R$, $R > 1$. Assume $g_n \geq 0$, $n = 0, 1, 2, \ldots$. Let $M(x) = x + \varphi(x)$ and $Q(x)$ be real valued functions defined on $[0, +\infty)$ such that $\varphi(x) \geq 0$ and $0 < R_1 \leq Q(x) \leq R_2 < R$. Using the formula $E(tm(x))\, G(t^{Q(x)}) = \sum_{n=0}^{\infty} R_n(x) t^n$ introduced by D. Zeitlin [Amer. Math. Monthly 74(1967), 1056-1062], we associate with each real valued function, f, defined on $[0, \infty)$ the positive linear operator $P(f, u, x) = [E(M(ux))G(Q(ux))]^{-1} \sum_{n=0}^{\infty} R_n(ux) f(n/u)$. The special case $p = 1$, $\varphi(x) = 0$, $G(z) = 1$, yields the well-known Szasz operator. In this paper we investigate some of the convergence properties of the family of operators $\{P(f, u, x)\}_{u>0}$. Theorem. Let $f(x)$ be continuous at x_0 and suppose $|f(x)| \leq e^{Ax}$ for some $A > 0$. If $\varphi(ux) \to 0$ $(u \to \infty)$ uniformly at x_0 and $Q(ux)$ converges uniformly at x_0 $(u \to \infty)$, then $P(f, u, x) \to f(x_0)$ $(u \to \infty)$ uniformly at x_0. (Received December 20, 1971.)

*692-B5. JEFFREY B. RAUCH and MICHAEL E. TAYLOR, University of Michigan, Ann Arbor, Michigan 48104. Shadows are not what they used to be.

We study mixed initial boundary value problems for second order wave equations with time independent coefficients. The underlying region is the exterior (assumed connected) of a compact obstacle, K, in $\mathbf{R}^n$. Suitable boundary conditions (for example, Dirichlet or Neumann conditions) are imposed at ∂K. Viewing the solution as a signal or disturbance propagating in time, we prove: (1) Every open set exterior to the initial disturbance is reached by the signal at some positive time (nonexistence of perfect shadows). (2) Every open set is hit by the signal at some time t, $-\infty < t < +\infty$ (a unique continuation principle). (3) For a suitable generalization of the Klein Gordon equation, every open set is reached by the signal at some time $t \geq 0$ (generalization of Morawetz theorem). (Received December 20, 1971.)

*692-B6. JOSEPH DIESTEL, Kent State University, Kent, Ohio 44242. Remarks on the Radon-Nikodym theorem for vector measures.

Let (Ω, Σ) be a measurable space. Let X be a Banach space. Suppose $\mu : \Sigma \to X$. Consider $u = \int d\mu$ the associated linear continuous operator on $B(\Omega, \Sigma)$ (bnd, real-valued, Σ-measurable maps on Ω) to X. Theorem 1. u is weakly compact if and only if μ is strongly bounded. Theorem 2. u is absolutely summing if and only if μ has finite variation. Theorem 3. u is nuclear if and only if μ has an approximate Radon-Nikodym derivative (in the sense of Bochner) with respect to its variation $|\mu|$. Corollary 4. A Banach space X has the Radon-Nikodym property if and only if every integral operator into X is nuclear. Corollary 5. A vector measure $\mu : \Sigma \to X$ has a Radon-Nikodym derivative in class of Bochner $|\mu|$-integrable functions if and only if μ can be factored through ℓ_1 via a set function of finite variation. This last corollary, in a sense, reduces the Radon-Nikodym question to the classical results of Dunford and Pettis. (Received December 20, 1971.)

*692-B7. LOKENATH DEBNATH and MICHAEL A. HALL, East Carolina University, Greenville, North Carolina 27823. Some exact solutions of boundary layer equations in a rotating system.

An initial value investigation is made of the unsteady hydrodynamic boundary layer flows generated in a homogeneous, incompressible, viscous rotating fluid between two infinite parallel disks by elliptic harmonic oscillations of the disks. An exact solution of the steady and the transient velocity field is obtained explicitly by using the Heaviside operational calculus and the theory of residues. The transient effects are shown to die out exponentially in the limit $t \to \infty$ and the final steady flow is set up. The structure of the associated boundary layers is determined for the resonant and nonresonant frequencies. It is found that the ultimate steady flow consists of a diffusive Rayleigh layer and modified Stokes layers on the disks. Several limiting cases of interest have been recovered with their physical significances. The present study is a generalization of a problem recently considered by Thornley (Quart. J. Mech. Appl. Math. 21(1968), 451-461). (Received December 23, 1971.)

*692-B8. GREGERS L. KRABBE, Division of Mathematical Sciences, Purdue University, Lafayette, Indiana 47907. A very weak topology for the Mikusiński field.

The Mikusiński field Q can be equipped with a linear Hausdorff topology $\mathfrak{T}$; the result is a topological ring with continuous division. If a sequence converges in the sense of Mikusiński, then it converges in the topology $\mathfrak{T}$. Although the series $1 - \lambda s + \lambda^2 s^2/2! + \ldots + (-\lambda)^k s^k/k! + \ldots$ does not converge in the sense of Mikusiński, it does converge in the topology $\mathfrak{T}$ to the translation operator $\exp(-\lambda s)$. If q^{-1} denotes the reciprocal of an element q of the Mikusiński field, then the endomorphism $q \mapsto q^{-1}$ is continuous in the topology $\mathfrak{T}$. The topology $\mathfrak{T}$ is an extension of the one I defined in [Math. Ann. 162(1966), 237-245]; the basic idea can be viewed in the following way: a sequence converges in the topology $\mathfrak{T}$ iff its generalized Laplace transformation (in the sense of V. A. Ditkin and L. Berg) converges (uniform convergence on compacts with respect to the chordal metric). (Received December 27, 1971.)

692-B9. WARREN PAGE, City University of New York, New York City Community College, Brooklyn, New York 11201. Measure-induced seminormed topologies on a Banach algebra.

Several seminorm generated topologies P_h, P_{reg}, P, P_1 and P_+ are defined on a complex commutative Banach algebra $(X, \|\ \|)$ with identity. These seminorms are defined using certain subclasses of Borel measures $Y'M(\mathcal{M}, S; Y)$, where $M(\mathcal{M}, S; Y)$ denotes the vector space (VS) of measures from X'^S Gelfand-Borel measurable space $(\mathcal{M}, S)$ into a locally convex T_2 topological space (LCT_2VS) Y with continuous dual Y'. These {P}-topologies exhibit a strong bond amongst themselves and their properties are intimately related to many properties of $(X, \|\ \|)$. Theorem. X is semisimple iff each {P}-topology on X defined relative $Y'M(\mathcal{M}, S; Y)$ is T_2 for every LCT_2 VS Y. Relating these {P}-topologies to some additional topologies on X explored by W. Page [Abstract 691-46-8, these *Notices* 19(1972), A-160], one obtains Theorem. Let A be a linear injection from X into a T_2 VS N. Then X is semisimple and every {P}-topology is T_2 if A is P or P_+-continuous. Other basic properties of these {P}-topologies are also investigated. These include bornological, barreled, Montel, normable, metrizible, completeness and full completeness considerations. Applications to A^*, B^* and regular

algebras lead to some especially interesting results. Theorem. Every $B^*(A^*)$-involution on X reduces to a unique $B^*(A^*)$-involution on X. Several unusual characterizations of dim X for B^*-algebras and for semisimple algebras are also presented. (Received December 28, 1971.)

692-B10. PAUL WILLIG, Stevens Institute of Technology, Hoboken, New Jersey 07030. On property A.A. in W-* algebras.

A W-* algebra R has property A.A. if there is a sequence of *-automorphisms (Φ_n) of R such that for every $A \in R$, $(\Phi_n(A))$ is a central sequence in R. Theorem 1. If R is a type I W-* algebra on separable Hilbert space, R does not have property A.A. Theorem 2. If R and S are factors of type II_1 on separable Hilbert space, it is sufficient but not necessary for $R \otimes S$ to have property A.A. that both R and S have the property. Corollary. If R is a type II_1 factor on separable Hilbert space, R has property A.A. if and only if there are type II_1 factors S_1 and S_2 having property A.A. such that R is isomorphic to $S_1 \otimes S_2$. (Received January 10, 1972.)

692-B11. WITHDRAWN.

*692-B12. PAUL GORDON, Drexel University, Philadelphia, Pennsylvania 19104. Summability methods obtained by substitution of power series.

The Euler summability method is obtained by substituting into the series $\sum a_n x^n$ the expression $x = f(y) = y/(1-qy)$, thereby obtaining a series $\sum b_n y^n$. In this paper we consider more general expressions for f(y), $f(y) = \sum f_i y_i$ with $f(1) = 1$. A class of sequences naturally associated with such methods is given by σ^*: $\{s_n\}$ is in σ^* if $\overline{\lim}_n(|a_n|)^{1/n} < 1$ where $a_n = s_n - s_{n-1}$. We obtain both matrix conditions (for an arbitrary summability method) and functional conditions (for the series substitution method) for regularity with respect to σ^*. Also, we show that if $f_i \geq 0$ the method is regular in the usual sense. Examples, including several from the theory of stability of numerical processes, are given of summability methods regular with respect to σ^*. (Received January 13, 1972.)

692-B13. MELVYN S. BERGER and MARTIN SCHECHTER, Belfer Graduate School, Yeshiva University, New York, New York 10033. Criteria for the solvability of noncoercive semilinear operator equations.

Let H be a real Hilbert space and L a linear selfadjoint operator mapping H into itself with closed range and such that $\dim \operatorname{Ker} L < \infty$. Then we obtain necessary and sufficient conditions for the solvability of nonlinear operator equations of the form (*) $Lu = \mathcal{N}'(u) + f$ where $\mathcal{N}'(u)$ is a C^1 gradient map of H into itself. A typical result is Theorem. If $\mathcal{N}(u)$ is a C^2 convex weakly continuous real-valued functional such that (i) $\mathcal{N}''(u)$ is injective on Ker L, (ii) $\mathcal{N}(u) \leq (\frac{1}{2}\lambda_1 - \epsilon)\|u\|^2$ as $\|u\| \to \infty$ where λ_1 is the first strictly positive eigenvalue of L, and (iii) $(\mathcal{N}'(w+z), z) - \mathcal{N}(w+z) \to \infty$ as $\|z\| \to \infty$ $(z \in \operatorname{Ker} L)$ uniformly over bounded sets $\|w\|_H \leq M$ in H. Then a necessary and sufficient condition for the solvability of (*) is the existence of a $u \in H$ such that $[\mathcal{N}'(u)+f]$ is orthogonal to Ker L. An analogue of the theorem holds if we weaken (ii) to $\overline{\lim}\, \mathcal{N}(u)\|u\|^{-2} < \infty$ as $\|u\| \to \infty$. Applications are given to semilinear elliptic boundary value problems. (Received January 17, 1972.)

*692-B14. RICHARD MANDELBAUM, University of Massachusetts, Amherst, Massachusetts 01002. Projective structures on Riemann surfaces. Preliminary report.

Let $\mathcal{P}(M)$ be the space of branched projective structures on the Riemann surface M of genus g. Let $j: \mathcal{P}(M) \to H^1(M, PL(1,\mathbb{C}))$ be the canonical embedding map into the space of flat projective bundles on M. Suppose for any nonnegative integer B, $\mathcal{P}_B(M)$ is the subspace of $\mathcal{P}(M)$ of branched structures of total order B and $j_B = j|\mathcal{P}_B(M)$. Furthermore, let $S^B(M)$ be the B-fold symmetric product of M, represented by a fundamental region in M^B and $T^B(M) \subset S^B(M)$ the subset of $S^B(M)$ consisting of points $(p_1,\ldots,p_B) \in S^B(M)$ such that all the p_i's $(i = 1,\ldots,B)$ are distinct. Then we have: Theorem A. For any positive integer B and any partition $\sigma = (\sigma_1,\ldots,\sigma_n)$ of B with $1 \leq n \leq B$ (where partitions are also distinguished by their order), there exists a complex analytic vector bundle $H_B(\sigma)$ with fiber C^{3g-3+n} and base space $T^n(M)$ such that if $\mathcal{P}_{(B,\sigma)}(M)$ is the subspace of $\mathcal{P}_B(M)$ of branched structures of total order B and type σ (where a branched structure is of type σ iff its branching divisor δ has the expression $\delta = \sum_{i=1}^{i=n} \sigma_i p_i$ for distinct points $p_1 \ldots p_n$), then $\mathcal{P}_{(B,\sigma)}(M)$ can be realized as a complex analytic subvariety of $H_B(\sigma)$. Furthermore, if $B \leq 3g - 2$ then $\dim \mathcal{P}_{(B,\sigma)}(M) = 3g - 3 + n$. A consequence of this theorem is Theorem B. $\mathcal{P}_B(M)$ can be realized as a finite union of complex analytic varieties such that j_B restricted to each such variety is a complex analytic map. (Received January 19, 1972.)

*692-B15. JOHN M. BOWNDS, Rensselaer Polytechnic Institute, Troy, New York 12181 and JIM M. CUSHING, IBM, T. J. Watson Research Center, Yorktown Heights, New York 10598. On stability for linear Volterra integral equations using associated differential equations in the Banach algebra $\mathcal{M}_\infty$. Preliminary report.

Using several representation formulas previously established by the authors, stability theorems for finite systems of linear Volterra integral equations are proved by referring to a so-called associated differential equation (ADE). If the kernel K(x,t) is of Pincerle-Goursat type then the ADE consists of a finite system of linear differential equations. However, if the expansion for K(x,t) cannot be so terminated, then the ADE is an equation with coefficient lying in the algebra of infinite matrices $\mathcal{M}_\infty$. The stability theorems which are proved require specific but natural conditions on the otherwise general kernel; the kernel need not be of convolution type. (Received January 20, 1972.)

*692-B16. MICHAEL R. W. KERVIN, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. The trace-class of a full Hilbert algebra.

Let A denote an arbitrary full Hilbert algebra, and H its Hilbert space completion. A left (right) centralizer on A is a continuous linear operator T on H with the property $T(xy) = T(x)y$ (resp. $T(xy) = xT(y)$) for all x and y belonging to A. The trace-class of A is the set $\tau(A) = \{xy | x \text{ and } y \text{ belong to } A\}$. One shows that this set is in fact a *-ideal of A, and is invariant under the action of left or right centralizers. $\tau(A)$ also possesses another norm τ. In contrast to the situation for H*-algebras (see the paper of Saworotnow and Friedell [Proc. Amer. Math. Soc. 26(1970), 95-100]), τ is not an algebra norm and is incomplete, unless A were to begin with an H*-algebra in a trivially equivalent norm. The topological dual of $\tau(A)$ in the norm τ can

be identified (as a Banach space) with the set of left centralizers on A. Considered as an algebra, $\tau(A)$ possesses two (generally inequivalent) norms; nevertheless the one-sided ideals of $\tau(A)$ which are closed in either topology are the same. (Received January 21, 1972.)

692-B17. MARIO O. GONZALEZ, University of Alabama, University, Alabama 35486. On the maximum modulus theorem for nonanalytic functions in several complex variables.

Let $w = f(z_1,\dots,z_n) = u(x_1,\dots,y_n) + iv(x_1,\dots,y_n)$ be a complex function of the n complex variables $z_1,\dots,z_n$. The following strong form of the maximum modulus theorem holds. Theorem. Let $A \subset \mathbb{C}^n$ be open, and suppose: (1) f is not constant in A. (2) The functions u and v are differentiable in A (in the real sense). (3) $|f_{z_j}| - |f_{\bar{z}_j}| \neq 0$ at each point of A for some $j \in \{1, 2,\dots,n\}$ (not necessarily the same j at different points). Then $|f(z_1,\dots,z_n)|$ does not attain a maximum value anywhere in A. A weak form of the maximum modulus theorem is also valid, as well as the corresponding minimum modulus theorems, provided that, in the last case, f does not vanish in A. (Received January 24, 1972.)

*692-B18. JONATHAN D. YOUNG, Lawrence Radiation Laboratory, University of California, Berkeley, California 94720. An optimal bicubic spline on a rectilinear mesh over a rectangle. Preliminary report.

For a function of two variables, we construct an optimal bicubic spline which interpolates to specified function values at the grid-points of a rectilinear mesh over a rectangle. Additional conditions in the form of normal derivatives may be specified. By optimality, we mean that third derivative discontinuities are minimized in the least square sense. (Received January 24, 1972.)

*692-B19. MARTIN E. PRICE, Wayne State University, Detroit, Michigan 48202. Length, area, and curvature of Bernstein-type polynomials.

The Bernstein polynomials $B_n f$ of a real function f on $[0,1]$ are known to diminish total variation, i.e. $VB_n f \leq Vf$ for all n and f. Correspondingly it is shown that the Kantorovitch polynomials $K_n f$ of a summable f diminish a type of generalized variation. For parametric curves $F = (f_1 f_2 f_3)$, the polynomial curves $B_n F$ and $K_n F$ are respectively length diminishing and generalized length diminishing. $B_n F$ are also shown to diminish total curvature for C^2 curves F as well as Iseki's generalization, "bend" for arbitrary curves. If $z = f(x,y)$ is defined on the unit square, the Bernstein polynomials $B_{n,m} f$ are not in general area-diminishing; however, sufficient conditions are given for convergence in area. A similar theorem is proved for the two variable Kantorovitch polynomials when area is replaced by the generalized area of Cesari and Goffman. (Received January 24, 1972.)

692-B20. STANLEY J. POREDA, Clark University, Worcester, Massachusetts 01610. Approximation by rational functions of constant modulus.

Theorem. If a function f is continuous and has constant modulus on the unit circle U, then given any $\epsilon > 0$ there exists a rational function R(z) of the form, $R(z) = Kz^k \prod_{i=1}^{m} [(1 - \bar{a}_i z)/(z - a_i)]$, where K is a constant, $k \in Z^+$ and such that $\max_{|z|=1} |f(z) - R(z)| < \epsilon$. (Received January 24, 1972.)

*692-B21. ROBERT W. REDDING, Clark University and Worcester State College, Worcester, Massachusetts 01610. Convex functions of negative order.

Definition 1. For α real, let $C(\alpha)$ denote the class of functions convex of order α, i.e. $f(z) \in C(\alpha)$ iff $f(0) = 0$, $f'(0) = 1$, $\mathrm{Re}\{1+zf''(z)/f'(z)\} > \alpha$, $|z| < 1$. For $0 \leqq \alpha < 1$, the class $C(\alpha)$ has been closely scrutinized, while the case where $\alpha < 0$ has been largely overlooked. Theorem 1. $C(\alpha) \subset S$ for $-\frac{1}{2} \leqq \alpha < 1$. For $\alpha < -\frac{1}{2}$, $f(z) = \int_0^z (1-w)^{2(\alpha-1)}\,dw \in C(\alpha)$ and $f(z) \notin S$. Theorem 2. Let $f(z) \in C(\alpha)$, $-\frac{1}{2} \leqq \alpha \leqq 0$. Then $\mathrm{Re}\{1+zf''(z)/f'(z)\} > 0$ provided $|z| < 1/(1-2\alpha)$. Theorem 2 is sharp. An integral relationship between the classes $C(\alpha)$ and $C(0)$ is obtained and distortion theorems follow. Definition 2. For α real, let $S(\alpha)$ denote the class of functions starlike of order α, i.e. $f(z) \in S(\alpha)$ iff $f(0) = 0$, $f'(0) = 1$, $\mathrm{Re}\{zf'(z)/f(z)\} > \alpha$, $|z| < 1$. Theorem 3. For $\alpha < 0$, $S(\alpha) \nsubseteq S$. (Received January 24, 1972.)

692-B22. WITHDRAWN.

692-B23. MILOS A. DOSTAL and MICHAEL J. KASCIC, JR., Stevens Institute of Technology, Hoboken, New Jersey 07030. Abstract distribution theory in De Wilde spaces.

Dieudonné and Schwartz introduced the concept of LF-space in order to provide a suitable functional analysis setting for the theory of distributions. Since then the problem has arisen of similarly reformulating the theory of operators on distribution spaces. All the previous work in this area indicates that the concept of LF-space is too general to allow a satisfactory treatment. However, results of Pták, Słowikowski, Kascic and others show that inroads can be made by suitably restricting the class of LF-spaces. We introduce a new class of LF-spaces that combines the notion of an abstract partition of unity (due to M. De Wilde) and the projective decomposition (due to W. Słowikowski). This approach is not only germane to the problem of generalizing the notion of P- and strong P-convexity, but at the same time is interesting from the categorical point of view. Actually, it leads in a very natural way to an abstract analog of the Schwartz distribution theory. In particular, we obtain simple abstract counterparts of support and singular support, etc. (Received January 24, 1972.)

692-B24. FRANK R. KEOGH, University of Kentucky, Lexington, Kentucky 40506. On spiral-like univalent functions. Preliminary report.

For a real α, with $|\alpha| < \frac{1}{2}\pi$, a function $f(z) = \sum_1^\infty a_n z^n$ which is analytic and univalent for $|z| < 1$ is said to be spiral-like of order α in the sense of Špaček ("Contributions à la theorie des fonctions univalentes", Časopis Pěst. Mat. 62(1933), 12-19) if $\mathrm{Re}[e^{i\alpha} zf'(z)/f(z)] > 0$ for $|z| < 1$. Using a technique of Clunie, it is proved that if $\sum_1^\infty n|a_n|^2 \leqq 1$, then $|a_n| \leqq 2\cos\alpha(n^2 + 2n\cos 2\alpha + 1)^{-1/2}$. An example is constructed to show that the $2\cos\alpha$ in the numerator cannot be reduced. A function $f(z) = z^{-1} + \sum_1^\infty a_n z^n$ which is meromorphic and univalent for $|z| < 1$ is said to be spiral-like of order α if $\mathrm{Re}[e^{i\alpha} zf'(z)/f(z)] < 0$ for $|z| < 1$. Using the same technique, the sharp inequality $|a_n| \leqq 2\cos\alpha/(n+1)$ is proved. (Received January 24, 1972.)

692-B25. WITHDRAWN.

*692-B26. RICHARD F. FREUND and TEDDY T. WONG, Courant Institute, New York University, New York, New York 10012. Nonarchimedian Banach algebras.

Let X be any Banach algebra with identity over a complete, nonarchimedian valued field, F, such that for all $x, y \in X$, $\|x+y\| \leq \max(\|x\|, \|y\|)$. Furthermore, for all $x \in X$, the spectrum of x is nonempty. Finally, it is assumed, (*) that for all invertible x, $\|x^{-1}\| \leq \|x\|^{-1}$. Theorem. For all nonzero x, x^{-1} exists. Three immediate consequences are: Corollary 1. X is isomorphic to F. Corollary 2. For all nonzero $x \in X$, the spectrum of x consists of just one point. Corollary 3. If (*) is replaced by the assumption that for all $x, y \in X$, $\|xy\| = \|x\| \|y\|$, then X is isomorphic to F. (Received January 25, 1972.)

692-B27. RUSSELL D. RUPP, State University of New York, Albany, New York 12203. A new type of variational theory sufficiency theorem.

A sufficiency theorem appropriate to applications of Hestenes' method of multipliers is established. It is a generalization of the lemma of Hahn, which has been used by previous authors to prove variable end point sufficiency theorems. The classical and control formulations of the problems of Mayer, Lagrange, and Bolza in both parametric and nonparametric form are included. The method of proof is indirect. (Received January 25, 1972.)

*692-B28. L. C. BAIRD and P. F. ZWEIFEL, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. A spectral analysis of the reduced transport operator.

Nuclear reactor design requires an analysis of the manner in which neutrons will travel through the fuel. The relevant equations contain an entity called the "reduced transport operator". It is an integral operator on the function space, $L^2(-1, 1)$. "Functions" outside this space are commonly treated as eigenfunctions when engineers and physicists discuss the operator. We show that this questionable procedure gives rigorous results if one constructs certain approximate eigenfunctions which we call "function packets". (Received January 25, 1972.)

692-B29. EDWARD ZEHNDER, Courant Institute, New York University, New York, New York 10012. Homoclinic points of analytic symplectic diffeomorphisms. Preliminary report.

It is well known that the behaviour of the iterates of a diffeomorphism near an elliptic fixed point is rather intricate. It is often taken for granted, that any neighbourhood of such a fixed point contains homoclinic points. This is indeed the typical case compatible with the analytic structure: In the space Σ of analytic area preserving diffeomorphisms in R^2 near the origin we introduce the topology generated by the seminorms $P_k(W) = |w_k|$, where W are the generating functions and (w_k) the coefficients of the Taylor expansion of W. Let $\Sigma_e \subset \Sigma$ be the open subset with an elliptic fixed point of general type. Then the subset $\Sigma_h \subset \Sigma_e$ having homoclinic points in every neighbourhood of the fixed point is residual in Σ_e. (Received January 26, 1972.)

692-B30. KUANG-HO CHEN, Louisiana State University, New Orleans, Louisiana 70122. An improved determination for decay at infinity of solutions to a convolution equation. Preliminary report.

Let T be a finite distribution with its Fourier transform F, an entire function of finite exponential type, satisfying the conditions: (i) the gradient $\nabla f_j(s)$ of each of its irreducible factors does not vanish at each point $s \in N(f_j) = \{s \in R^n : f_j(s) = 0\}$; (ii) with f as product of all distinct irreducible factors of F and I(s) as the set of all indices of f_j such that $s \in N(f_j)$, the matrix $|\nabla f_j(s)|$, $j \in I(s)$, is of rank $|I(s)|$, the number of elements in I(s). With $k_j(s)$ as the number of nonzero principal curvatures of $N(f_j)$ at the point s, let $\ell = \inf\{1 + k_j(s) - |I(s)| \geq 0\}$, $j \in I(s)$, $s \in N(f)$, then there exists at most one solution u of the convolution equation $T * u = \phi$, for a distribution ϕ, satisfying at infinity $u(x) = o(|x|^{-d})$ for any $d \geq n - 1 - \ell/2$. Further, if for each $s \in R^n$ there is $\eta \in R^n$ such that $|s - \eta| \leq a \log(1 + |s|)$ and $|F(\eta)| \geq (a + |\eta|)^{-a}$, then the solution, with the behavior at infinity, is a smooth function with compact support provided $\phi \in C_0^\infty(R^n)$. (Received January 26, 1972.)

*692-B31. JEFF E. LEWIS, University of Minnesota, Minneapolis, Minnesota 55455. The initial-boundary value problem for the Navier-Stokes equations with data in L^p.

Let $S_T = (0,T) \times R_+^n$, and consider the initial-boundary value problem for the Navier-Stokes equations: find a vector function u(t,x,y) and a scalar pressure function p(t,x,y) such that $u_t - \Delta u + u \cdot \nabla u + \nabla p = 0$ and $\nabla_{xy} \cdot u = 0$ in S_T, $u(0,x,y) = g(x,y)$ in R_+^n, and $u(t,x,0) = a(t,x)$ in $(0,T) \times R^{n-1} = \partial S_T$. Define the $L^{p,q}$ norm of a function f(t,x) by $\|f\|_{pq} = (\int_0^T (\int |f(t,x)|^p dx)^{q/p} dt)^{1/q}$. A function $u(t,x,y) \in L^{P,Q}(S_T)$ is a weak solution to the above problem if $\iiint_{S_T} u \cdot (\Phi_t + \Delta\Phi + (u \cdot v)\Phi)\, dx\, dy\, dt + \iint_{R_+^n} g \cdot \Phi(0,x,y)\, dx\, dy + \iint_{\partial S_T} a \cdot (\partial\Phi/\partial y)(t,x,0) + a^n q(t,x,0)\, dx\, dt = 0$, for all good divergence free vector functions Φ which vanish at $t = T$ and $y = 0$, and all $q(t,x,y) \in C_0^\infty(R^{n+1})$. Theorem. Suppose that $g(x) \in L^r(R_+^n)$ is weakly divergence free, $a(t,x) \in L^{p,q} \cap L^{P(n-1)/n,Q}(2S_T)$, where (1) $(n-1)/p + 2/q = n/P + 2/Q = 1$, (2) $n - 1 < p < \infty$, $n < P < \infty$, (3) $n < r < P(n-1)/n$, (4) $P/2 < p < P$, $Q/2 < q < Q$. Then there is a weak solution $u \in L^{P,Q}(S_T)$, where T depends on the norm of the data. The techniques for handling the nonlinear term are similar to those of Fabes and Riviere (Abstract 671-32, these *Notices* 16(1969), 1070). (Received January 26, 1972.)

*692-B32. THOMAS J. OSLER, Rensselaer Polytechnic Institute, Troy, New York 12181. An integral analog of the Leibniz rule. Preliminary report.

The fractional derivative of the function f(z) with respect to g(z) is a generalization of the familiar derivative $d^\alpha f(z)/(dg(z))^\alpha$ to arbitrary (rational, irrational or complex) values of α. It is defined as $D^\alpha_{g(z)} f(z) = \Gamma(\alpha+1)/2\pi i \int_C f(t)(g(t) - g(z))^{-\alpha-1} g'(t)\, dt$ where C starts at $t = 0$, encloses $t = z$ once in the positive sense and returns to $t = z$. In this paper we prove the "integral analog of the Leibniz rule": $D^\alpha_{g(z)} u(z) v(z) = \int_{-\infty}^{\infty} \binom{\alpha}{w} D^{\alpha-w}_{g(z)} u(z)\, D^w_{g(z)} v(z)\, dw$. The region in the complex z-plane in which this integral converges is described. A relationship between this integral and the integral form of Parseval's theorem familiar from Fourier analysis is revealed. By selecting specific functions for u(z) and v(z), a number of interesting definite integrals relating the special functions of mathematical physics are evaluated using the integral analog of the Leibniz rule. Other generalizations of this integral are also examined. (Received January 26, 1972.)

*692-B33. JAMES F. SMITH, Le Moyne College, Syracuse, New York 13214. The p-classes of a Hilbert module.

This paper generalizes results of Giellis [Proc. Amer. Math. Soc. 29(1971), 63-68] on a right Hilbert A-module H (where A is a proper H*-algebra), as defined by Saworotnow [Duke Math. J. 35(1968), 191-198]. In terms of previous results of the author on the p-classes of an H*-algebra (to appear in Pacific J. Math.; see Abstract 688-B3, these *Notices* 18(1971), 927), for $0 < p \leq \infty$ a nonnegative extended-real value $\|f\|_p$ is associated with each $f \in H$, and the p-class H_p is defined as $\{f \in H: \|f\|_p < \infty\}$. Theorem 1. For $0 < p < p' \leq \infty$ and any $f \in H$, $\|f\|_{p'} \leq \|f\|_p$; hence $H_p \subset H_{p'}$, and $H_p = H$ if $p \geq 2$. For $1 \leq p \leq \infty$, $(H_p, \|\cdot\|_p)$ is a right normed A-module. $(H_2, \|\cdot\|_2)$ is H with its original Hilbert space norm, and $(H_1, \|\cdot\|_1)$ is the trace class of H with its norm π (as defined by Giellis). Theorem 2. For $1 \leq p \leq 2$, let q be such that $(1/p) + (1/q) = 1$; then there exists a conjugate-linear isometry of $(H_p, \|\cdot\|_p)$ onto the dual of $(H_q, \|\cdot\|_q)$; hence H_p is complete in its norm. (Received January 26, 1972.)

*692-B34. BENJAMIN MUCKENHOUPT and RICHARD WHEEDEN, Rutgers University, New Brunswick, New Jersey 08903. Weighted norm inequalities for fractional integrals.

The principal problem considered is the determination of all nonnegative functions, W(x), such that $\|T_\gamma[f(x)]\, W(x)\|_q \leq C\|f(x)\, W(x)\|_p$ where $0 < \gamma < 1$, $1 < p < 1/\gamma$, $1/q = 1/p - \gamma$, C is a constant independent of f, and $T_\gamma[f(x)] = \int_{-\infty}^{\infty} f(x-y)|y|^{\gamma-1}dy$. The main result is that W(x) is such a function if and only if $([1/|I|]\int_I [W(x)]^q dx)^{1/q} ([1/|I|]\int_I [W(x)]^{-p'} dx)^{1/p'} \leq K$ where I is any interval, $|I|$ denotes the length of I, $p' = p/(p-1)$ and K is a constant independent of I. A similar result is also obtained for n dimensional fractional integrals. (Received January 26, 1972.)

692-B35. NORMAN J. FINIZIO, University of Rhode Island, Kingston, Rhode Island 02881. A discontinuous Hill's equation. Preliminary report.

Fundamental solutions and a stability diagram (via Floquet theory) are presented for a Hill's equation whose periodic term is of the form $\sum_{-\infty}^{\infty} (-1)^n \delta(t - n\pi)$ where δ denotes the Dirac delta function. A comparison of the properties of the derived stability diagram with that of the general Hill's equation is made. Stability diagram(s) for the corresponding damped equation are also presented. (Received January 27, 1972.)

*692-B36. STEPHEN R. BERNFELD and V. LAKSHMIKANTHAM, University of Rhode Island, Kingston, Rhode Island 02881 and S. LEELA, State University College of New York, Geneseo, New York 14454. Nonlinear boundary value problems and several Lyapunov functions.

We employ several Lyapunov functions and the theory of differential inequalities to obtain the existence of solutions of the boundary value problem (S) $x'' = f(t,x,x')$, $x(a) = \gamma$, $x(b) = \delta$, where $f: [a,b] \times R^2 \to R$ is continuous. In our first result we show that the solutions of second order differential equations lie between two given prescribed functions. We next obtain bounds on the derivatives of the solutions and are thus able to deduce the existence of solutions of the boundary value problem. Suitable Lyapunov functions are obtained to give

conditions obtained by K. Schrader ("Solutions of second order ordinary differential equations," J. Differential Equations 4(1968), 510-518). (Received January 27, 1972.)

*692-B37. P. S. KAMALA, University of Rhode Island, Kingston, Rhode Island 02881. Asymptotic self invariant sets and functional differential equations in Banach spaces. Preliminary report.

This paper is devoted to the study of stability criteria of a given functional differential equation in a Banach space relative to an asymptotic self invariant set. This has been achieved by treating the functional differential equation as a perturbation of another differential equation in Banach space whose stability properties in variation are assumed to be known. Our method of approach depends on the use of nonlinear variation of constants formula for differential equations in Banach spaces, construction of smooth Lyapunov functions and the theory of differential inequalities. (Received January 27, 1972.)

*692-B38. ATHANASSIOS G. KARTSATOS, University of South Florida, Tampa, Florida 33620. Asymptotic behaviour of perturbed nonlinear differential equations. Preliminary report.

Some results are given concerning the asymptotic behaviour of nonoscillatory solutions of the differential equation: (*) $x^{(n)} + P(t)\,G(x) = Q(t,x)$, n = even, where P, G, Q are continuous, $xG(x) > 0$ for $x \neq 0$ and G(x) is increasing in $(-\infty, +\infty)$. One of the results proved is the following: Assume that (i) $P(t) \geqq M/t^{n-2}$ for all large t, where M is a positive constant, (ii) $G(u) \geqq u$ for all large positive u and (iii) $\int^{\infty} t^{n-2} Q_0(t)\,dt < +\infty$, where Q_0 satisfies $|Q(t,u)| \leqq Q_0(t)$ for all large t and all positive u. Then if x(t) is a solution of (*) with $x(t) > 0$ eventually, it must satisfy $\lim_{t\to\infty} x(t) = 0$. This theorem extends a recent result of M. Hammett ["Nonoscillation properties of a nonlinear differential equation," Proc. Amer. Math. Soc. 30(1971), 92-96]. The whole work is a step forward towards the following Conjecture. All solutions of (*) are oscillatory if $Q(t) \equiv 0$ and either (i) $\int^{\infty} P(t)\,dt = +\infty$, or (ii) $\int^{\infty} t^{n-1} P(t)\,dt = +\infty$, $\int_{\epsilon}^{\infty} [G(u)]^{-1}\,du < +\infty$ and $\int_{-\epsilon}^{-\infty} [G(u)]^{-1}\,du < +\infty$, where ϵ is some positive constant. (Received December 30, 1971.)

*692-B39. JOHN KASDAN, City University of New York, City College, New York, New York 10031. An analytic structure on a set of polar divisors. Preliminary report.

The set S of polar divisors of the form $2P + Q$ is studied on the curve $x^4 + y^4 + z^4 = 0$. It is shown that S may, in a natural manner, be given the structure of a Riemann surface of genus 27. The proof proceeds by considering S as a branched cover of the curve and considering its behavior at the ramification points. The construction appears capable of being generalized to other curves. (Received January 27, 1972.)

*692-B40. MICHAEL SCHILDER, Passaic County College, Paterson, New Jersey 07505. Solution of the generalized heat equation along its generalized characteristics.

It will be shown how the heat (diffusion) equation, $p_t + (fp)_x = \lambda^2/2 p_{xx} + Lp/\lambda^2$, may be expanded in an asymptotic expansion valid for small λ when the two ordinary equations, $v''(t) = f_t(t,v(t)) + f(t,v(t))\,f_x(t,v(t)) - L_x(t,v(t))$ and $w''(t) = (f_{xt}(t,v(t)) + f_{xx} f + f_x^2 - L_{xx}(t,v(t)))\,w(t)$ have been solved. Special attention will be given to numerical computation of p in regions where the equation for v has multiple solutions and the case when p

has boundary conditions. This type of heat equation arises in current medical and pollution problems. An application is given to analyzing oxygen flow from a red blood cell into the tissues. (Received January 27, 1972.)

Applied Mathematics

*692-C1. JOHN de PILLIS, University of California, Riverside, California 92502. k-part splittings and operator-parameter-overrelaxation.

Given $\mathcal{B}(\mathcal{H})$, all bounded linear operators on Hilbert space $\mathcal{H}$, and invertible $A \in \mathcal{B}(\mathcal{H})$, where $A = A_1 + \ldots + A_k$, where A_1^{-1} exists. Given $k-1$ initial vectors $\{x_0, x_1, \ldots, x_{k-2}\} \subset \mathcal{H}$, and fixed $y_0 \in \mathcal{H}$, define $\{x_n\}$ inductively by $A_1 x_{n+k-1} + A_2 x_{n+k-2} + \ldots + A_k x_n = y_0$, for all $n = 0, 1, 2, \ldots$. Under what constraints on the coefficient operators $A_1, A_2, \ldots, A_k$ are we assured that for all initial $x_0, x_1, \ldots, x_{k-2}$, we always have $x_n \to x$, the solution vector x of the linear system $Ax = y_0$? A special case of our results is the following theorem where $k = 3$. Theorem. Invertible $A = A^* \in \mathcal{B}(\mathcal{H})$ has operator-parameter-overrelaxation splitting $A = [W^* DW + S_1] + [D - W^* DW + S_2] + [S_3] = A_1[W] + A_2[W] + A_3$, where invertible $A_1[W] \in \mathcal{B}(\mathcal{H})$, $W \in \mathcal{B}(\mathcal{H})$, $0 < D \in \mathcal{B}(\mathcal{H})$. Suppose $X = X^* \in \mathcal{B}(\mathcal{H})$, $XD = DX$, $XW = WX$, and $XD + S_1^* - S_2 - S_3 > 0$. If there exists $0 < P \in \mathcal{B}(\mathcal{H})$ such that $\begin{pmatrix} a & b \\ c & d \end{pmatrix} > 0$ on $\mathcal{H} \oplus \mathcal{H}$, where $a = P$, $b = S_3$, $c = S_3^*$, $d = XD + S_1^* - S_2 - S_3 - P$, then for all overrelaxation splittings where $\|W^{-1}\|^2 \|I+X\| < 2$, the sequence $\{x_n\}$ defined by $A_1(W)x_{n+2} + A_2(W)x_{n+1} + A_3 x_n = y_0$, converges to x, $Ax = y_0$, if and only if $A > 0$. (Received December 28, 1971.)

*692-C2. GEORGE M. GROOME, JR., Division of Applied Mathematics, Brown University, Providence, Rhode Island 02912. Convergence theorems for parameter estimation by quasi-linearization.

The convergence properties of a quasi-linearization algorithm for least squares estimation of parameters in nonlinear systems governed by differential equations are examined. The study was motivated by quadratic convergence claims made for the algorithm by several authors. It is first shown that the algorithm is naturally viewed as an iterative process in the space of continuous functions. Local convergence theorems are then obtained by applying Ostrowski's theorem. In the absence of system modelling and observation errors, quadratic convergence is established. However, in other cases exactly linear convergence of the algorithm is proved by adapting an idea of Ortega and Rockoff [SIAM J. Numer. Anal. 3(1966), 497-513]. An analogy is drawn between the convergence properties of this algorithm and those of the Gauss-Newton method. (Received January 5, 1972.)

692-C3. HERBERT E. SALZER, 941 Washington Avenue, Brooklyn, New York 11225. A recurrence scheme for converting from one orthogonal expansion into another.

A generalization of a scheme of Hamming and Feigenbaum for converting any polynomial $P_n(x)$ into a Chebyshev series is combined with a recurrence scheme of Clenshaw for summing any finite series whose terms satisfy a 3-term recurrence formula. In the special cases involving orthogonal polynomials, this combination provides a 5-term recurrence scheme for obtaining directly the coefficients in the expansion of $P_n(x)$ into any series of orthogonal polynomials when given the coefficients in the expansion of $P_n(x)$ into any other series of orthogonal polynomials. This present scheme requires neither the explicit expression for any (beyond the first)

orthogonal polynomial involved, nor the expression of powers (beyond the first) in terms of orthogonal polynomials, thus having computational advantages in a simplified time-saving program, with reduced storage and fewer operations. Extensions to sums involving nonpolynomial functions, not necessarily orthogonal, are indicated just briefly. (Received January 14, 1972.)

*692-C4. JOHN F. AHNER and RALPH E. KLEINMAN, University of Delaware, Newark, Delaware 19711. The exterior Neumann problem for the Helmholtz equation.

Neumann's method of solving the interior potential problem for a surface on which is specified the normal derivative of the potential (e.g., E. Goursat, "A course in mathematical analysis," Vol. III, Part One), is generalized to the exterior problem for the Helmholtz equation. Let S be a piecewise smooth, closed, bounded, simply connected, regular surface in E^3. An integral representation of the field induced by an exterior source (regular solution of the Helmholtz equation interior to S) is derived which differs from the standard result using Green's identities in that the representation is continuous as the field point approaches S from the exterior, is continuous for all points on the surface including points where the normal is not uniquely defined, and the integrand is bounded even at the singularity of the free space Green's function. Furthermore, if S is strictly convex, it is proven that for sufficiently small but nonzero k, the integral equation for the field on S may be found by a standard Neumann series. (Received January 26, 1972.)

*692-C5. GEORGE C. DEBNEY, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. Non-null electromagnetic fields in Einstein-Maxwell theory admitting a Killing vector field. Preliminary report.

Using the complex null tetrad as basis for the tangent space a Killing vector is introduced into the system of Einstein's equations with Maxwell's equations, where the electromagnetic field is restricted to be of the non-null variety. Since any electromagnetic field has a bivector $F_{\mu\nu}$ associated with it, as does any Killing vector field ($K_{\mu;\nu}$, the Killing bivector or KBV), we subsequently restrict ourselves to the cases where both $F_{\mu\nu}$ and $K_{\mu;\nu}$ have a principal null direction in common, call it k^μ. The following lemma leads to a version of an "Electromagnetic Goldberg-Sachs Theorem". Lemma. If k^μ is a degenerate principal null direction for the Weyl conformal tensor, then k^μ is geodesic and shearfree. (The converse was proved some time ago by Sachs.) In studying static versions of these space-times it was surprising to discover the highly restrictive nature of the geodesic and shearfree condition: the space must be Petrov Type D whenever the KBV is itself not null. (Received January 26, 1972.)

Geometry

*692-D1. HEINRICH W. GUGGENHEIMER, Polytechnic Institute of Brooklyn, Brooklyn, New York 11201. Polar reciprocal convex bodies.

Let K_n be the set of convex bodies in n-space, S_n that of symmetric convex bodies, V the volume, and K_P^* the polar reciprocal of the convex body K for the point P. Then $\min_{K\in K_n} \min_{P\in \operatorname{int} K} V(K)V(K_P^*) = (n+1)^{n+1}/(n!)^2$, $\min_{K\in S_n} \min_{P\in \operatorname{int} K} V(K)V(K_P^*) = 4^n/n!$. (Received November 17, 1971.)

*692-D2. DIMITRI KOUTROUFIOTIS, University of California, Santa Barbara, California 93106. Elementary geometric applications of a maximum principle for nonlinear elliptic equations.

Let S_0 be a two-dimensional, compact, connected manifold of class C^2 and $S_0 \xrightarrow{f} E^3$ a C^2-immersion; let K and H be the Gaussian and mean curvatures of the immersed surface $S = f(S_0)$. We have the following estimates on the size of S: (1) If S is simple and $0 < \alpha \leqq |H|$, then the largest sphere which fits inside S has radius less than α^{-1}, unless S is a sphere; compare this with a result by E. Heinz, Math. Ann. 129(1955); (2) if $|H| \leqq \alpha$, $\alpha > 0$, then the smallest sphere enclosing S has radius larger than α^{-1}, unless S is a sphere; (3) if $0 < \alpha \leqq K$, then the largest sphere which fits inside S has radius smaller than $(\sqrt{\alpha})^{-1}$, unless S is a sphere; this result goes back to Blaschke, Kreis and Kugel; (4) if $K \leqq \alpha$, $\alpha > 0$, then the smallest sphere enclosing S has radius larger than $(\sqrt{\alpha})^{-1}$, unless S is a sphere. All four results are obtained by applying a weak form of the maximum principle for nonlinear elliptic equations due to A. D. Aleksandrov. (Received January 17, 1972.)

692-D3. SALVADOR D. GIGENA, 3834 Marshall Road, Drexel Hill, Pennsylvania 19026 and University of Pennsylvania, Philadelphia, Pennsylvania 19104. Constant volume envelopes of a convex cone. Preliminary report.

Let E be a finite dimensional real vector space, E* a dual space of E and K, K* dual convex cones contained respectively in E and E*. Assume that both Int(K) and Int(K*) are nonempty and consider the envelope hypersurface S of the set of hyperplanes in E such that the volume of the corresponding truncated cones is constant. Theorem. S is analytic and convex. Each point in S is the barycenter of the intersection of the corresponding tangent hyperplane with K. A canonical Riemannian structure is defined on S and a comparison with two other, naturally defined, volume-elements is established. Theorem. The Riemannian volume-element is the geometric mean between the prismatic volume-elements on S and its dual hypersurface. The comparison is expressed in terms of the mean square fractional volume of simplexes contained in a convex body, vertices varying uniformly on the interior, called the Fractional Volume and being a positive functional on affine equivalence classes of bounded convex sets. Theorem. Given a hyperplane P in E, the Fractional Volume is minimum for a quadric cone and maximum for a cone subtended by a simplex. (Received January 20, 1972.)

692-D4. JOHANNES C. C. NITSCHE, Institute of Technology, University of Minnesota, Minneapolis, Minnesota 55455. On the inequality of E. Heinz and E. Hopf.

Let $z = z(x,y)$ be a minimal surface defined over the disk $x^2 + y^2 < R^2$. The value $K_0 = K(0,0)$ of its Gaussian curvature at the origin satisfies the inequality $|K_0| \leqq c W_0^{-2} R^{-2}$. Here $W_0 = W(0,0)$ and $W(x,y) = [1+z_x^2(x,y)+z_y^2(x,y)]^{1/2}$. This inequality proved by E. Hopf sharpens an earlier inequality by E. Heinz. The constant c is independent of the particular solution of the minimal surface equation under consideration. Still undetermined is the best value for c, i.e., the value of the universal constant $c^{(0)} = \sup |R^2 W_0^2 K_0|$ where the supremum is taken over all minimal surfaces $z = z(x,y)$ in the disk $x^2 + y^2 < R^2$. So far the bounds $\pi^2/2 = 4.93\ldots \leqq c^{(0)} \leqq 6.24\ldots$ are known. R. Finn and R. Osserman (J. Analyse Math. 12(1964), 351-364) showed that $\sup |R^2 W_0^2 K_0| = \pi^2/2$ for solutions $z(x,y)$ satisfying the additional conditions $z_x(0,0) = z_y(0,0) = 0$. In the present paper the following theorem is proved: "Let $z = z(x,y)$ be a minimal surface over the disk $x^2 +$

$y^2 < R^2$ satisfying the symmetry relations $z(x,y) = z(x,-y)$, or $z(x,y) = -z(x,-y)$. Then $|K_0| \leq (\pi^2/2) W_0^{-2} R^{-2}$." (Received January 25, 1972.)

692-D5. HANS-HEINRICH W. HERDA, Boston State College, Boston, Massachusetts 02115. A new characterization of circles. Preliminary report.

Let C be a closed, rectifiable curve in E^n with positive perimeter p. Choose any point $x \in C$ and associate with it the unique point $x' \in C$ whose distance (measured on C) from x is p/2. Denote the line segment joining x and x' (and also its length) by s_x and call s_x the pseudo-diameter of C at x. The function $f: C \to R^+$ defined by setting $f(x) = s_x$ is continuous with respect to distance on C. Because C is compact, $s = \min_{x \in C} s_x$ exists. Call s the inner diameter of C. Proofs of the following will be sketched: Theorem 1. $p/s \geq \pi$ always, and $p/s = \pi$ implies that C is a circle. Theorem 2. If C is a plane curve which bounds a convex region, and if $s_x \equiv s$, then C is continuously differentiable. The generalizations of Theorem 1 to normed linear spaces and to surfaces in higher dimensions will be discussed. Various conjectures will be mentioned. (Received January 25, 1972.)

Logic and Foundations

692-E1. DONALD R. PERLIS, Cooper Union, Cooper Square, New York, New York 10003. A quick version of Godel's theorem.

Let N be any formal theory in the language of arithmetic, such that the set of Godel numbers of the axioms of N is effectively decidable, and such that N represents at least all computable functions. It then readily follows that it is impossible to determine effectively when a formula represents a function in N. But this is seen to necessitate the existence of a formula F(y) such that $E!\, yF(y)$ is a theorem of N but for no number k is $F(\bar{k})$ a theorem of N. If then we assume N is ω-consistent, we have that for some k_0 also the negation of $F(\bar{k}_0)$ is not a theorem of N. Thus N is incomplete. Church's thesis translates the expressions 'effectively decidable' and 'computable' above into 'recursive', and the result is the familiar form of Godel's theorem. (Received January 24, 1972.)

Statistics and Probability

*692-F1. GEORGE YU-HUA CHI, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. Conditional expectations and submartingale sequences of random Schwartz distributions.

Let (Ω, Σ, P) be a fixed probability space, $\mathcal{D}$ the real Schwartz space, and $\mathcal{D}'$ its strong dual. $\mathcal{D}$ and $\mathcal{D}'$ are partially ordered by $\mathcal{C}$ and $\mathcal{C}'$ respectively, where $\mathcal{C}$ is the positive cone of nonnegative functions in $\mathcal{D}$ and $\mathcal{C}'$ its dual in $\mathcal{D}'$. $\mathcal{C}$ is a strict-$\mathcal{B}$ cone and $\mathcal{C}'$ is normal, where $\mathcal{B}$ is the family of all bounded subsets of $\mathcal{D}$. Let X, Y be two random Schwartz distributions. $X \leq Y$ if and only if $Y(\omega) - X(\omega) \in \mathcal{C}'$ for almost all $\omega \in \Omega(P)$. Integrability of random Schwartz distributions is discussed and properties of integrals of random Schwartz distributions proved. The monotone convergence theorem, the dominated convergence theorem and Fatou's lemma are proved. Generalized conditional expectation is introduced. Every integrable random Schwartz

distribution has, relative to a given sub σ-field of Σ, an unique (up to equivalence) generalized conditional expectation. Properties of generalized conditional expectations and the conditional form of nonnegative monotone convergence theorem are proved. Sub (super)-martingale sequences are defined via the partial order relations and a convergence theorem proved. Generalized potential is introduced and the Riesz decomposition theorem proved. (Received January 4, 1972.)

692-F2. PARFENY P. SAWOROTNOW, Catholic University of America, Washington, D. C. 20017. Hilbert module valued stationary stochastic processes. Preliminary report.

Let A be a proper H*-algebra, let H be a Hilbert module over A [Duke Math. J. 35(1968), 191-197] and let G be a locally compact Abelian group. An abstract stationary process is a mapping $t \to \xi(t)$ of G into H such that $(\xi(t+r), \xi(s+r)) = (\xi(t), \xi(s))$ for all $t, r, s \in G$ (here $(\ ,\ )$ denotes the generalized scalar product on H). The function $B(t) = (\xi(t), \xi(0))$ is called the correlation function of $\xi(t)$. It is shown that each abstract stationary process is of the form $\xi(t) = (\int_{\hat{G}} (\alpha, t)\, dP_\alpha) f$, where $P: E \to P_E$ is a generalized spectral measure [Duke Math. J. 37(1970), 145-150] on $\hat{G}$ and $f \in H$. Consider $\tau(A) = \{xy \mid x, y \in A\}$. A $\tau(A)$-valued function B(t) is a correlation function for some abstract stationary process if and only if B(t) is positive definite in the sense that $\sum_{i,j} a_i^* B(t_j - t_i) a_j$ is positive for all $\{a_1, \ldots, a_n\} \subset A$, $\{t_1, t_2, \ldots, t_n\} \subset G$ [theory of $\tau(A)$ can be found in Proc. Amer. Math. Soc. 26(1970), 95-104]. (Received January 17, 1972.)

*692-F3. PESI R. MASANI, Indiana University, Bloomington, Indiana 47401. Operator-measure theoretic approach to infinitely divisible probability distributions.

Let $\mathcal{H}$ be an infinite dimensional Hilbert space. With every strongly continuous group $(U_t, t \in \mathbb{R})$ of unitary operators on $\mathcal{H}$ onto $\mathcal{H}$ associate: $T_U(a,b] = \{U_b - U_a - \int_a^b U_t dt\}/\sqrt{2}$ and $R_U(t) = U_t - I - \int_0^t (t-s) U_s ds$. Then $2T_U(0,a]^* T_U(0,b] = R_U(b-a) - R_U(b) - R_U(-a)$ [Bull. Amer. Math. Soc. 76(1970), 516]. Theorem. (a) Φ is the characteristic function of an infinitely divisible probability distribution over $\mathbb{R}$, iff. $\exists$ a s.continuous group $(U_t, t \in \mathbb{R})$ of unitary operators on $\mathcal{H}$ onto $\mathcal{H}$, $\exists \alpha \in \mathcal{H}$ & $\exists a \in \mathbb{R}$ such that $\forall t \in \mathbb{R}$, $\Phi(t) = \exp\{iat + (R_U(t)\alpha, \alpha)\}$. (b) This representation is unique in that for any two, $a_1 = a_2$ & $(R_{U_1}(\cdot)\alpha_1, \alpha_1) = (R_{U_2}(\cdot)\alpha_2, \alpha_2)$. The Lévy-Khinchine formula for Φ follows at once on taking the spectral resolution: $U_t = \int_{\mathbb{R}} e^{it\lambda} E(d\lambda)$, the bounded canonical measure being $|E(\cdot)\alpha|^2$. The proof rests on three results: (1) A continuous $x(\cdot)$ on $\mathbb{R}$ to $\mathcal{H}$ is a helix, iff. $x(b) - x(a) = T_U(a,b](\alpha)$ [these *Notices* 17(1970), 541]; (2) $K(\cdot,\cdot)$ is a continuous, positive definite kernel on $\mathbb{R} \times \mathbb{R}$ to $\mathbb{C}$, iff. $\exists$ a continuous $x(\cdot)$ on $\mathbb{R}$ to $\mathcal{H}$ such that $K(s,t) = (x(s), x(t))$ (Kolmogorov); (3) a complex version of Gangolli's Lemma 2.5 [Ann. Inst. H. Poincaré Sect. B (N. S.) 3(1967), 126]. The paper makes explicit connections noted earlier by I. J. Schoenberg and M. G. Kreĭn. (Received January 17, 1972.)

692-F4. DHANDAPANI KANNAN, University of Georgia, Athens, Georgia 30601. On a random integrodifferential system. Preliminary report.

We study a stochastic system $x'(t,\omega) = \xi(t,\omega) + k(t)x(t,\omega) + \int_0^t K(t,s)x(s,\omega)\, ds$, $x(0,\omega) = x_0(\omega)$ $(0 \leq t < \infty)$ where the derivative and the integral are suitably defined (sample-pathwise or in mean-square sense),

in terms of variation of constants formula (in suitable sense). A perturbation theorem is obtained. The delay-system ($K(t,s) = K(t-s)$) arises, for instance, in physics (when a particle moves such that the time scale of the molecular motion is no longer shorter than that of the particle under motion) and we obtain several stochastic stability theorems for the delay system. We also give several stochastic properties of the solution of the system. (Received January 27, 1972.)

Topology

*692-G1. JUNG H. TSAI, State University College of New York, Geneseo, New York 14454. On E-compact spaces and E-perfect mappings.

Notations and terminology are that of S. Mrówka ("Further results on E-compact spaces. I," Acta Math. 120(1968), 161-185). In this paper we generalize the concept of perfect mapping. Let X, Y be E-completely regular spaces. Then a mapping $\varphi: X \to Y$ is called E-perfect if $\Phi_E[\beta_E X - X] \subseteq \beta_E Y - Y$, where Φ_E is the extension of φ to $\beta_E X$. It follows directly from the definition that if X is E-completely regular, Y is E-compact and if there exists an E-perfect mapping from X onto Y, then X is E-compact. A criterion that a mapping is E-perfect will be presented. As interesting applications of this, preservation of inverse images of realcompact spaces will be considered. (Received December 20, 1971.)

692-G2. GERALD LIEBERMAN and DAVID L. SMALLEN, University of Rochester, Rochester, New York 14627. The group of self-equivalences of a localization. Preliminary report.

Let X be a topological space and P a collection of prime numbers. Then Sullivan (Geometric Topology. I: "Localization, periodicity, and Galois symmetry", M.I.T., 1970) has defined the notion of the "Localization of X at the collection P", called X_P. In this paper we study the group $Eq(X_P)$ of self-equivalences (homotopy classes of base point preserving homotopy equivalences) of X_P for certain spaces X. We prove in particular that: Theorem. $Eq(S_1^n \vee \ldots \vee S_k^n)_P \approx GL(k; Z_p)$ $(k \geq 1)$, where S_i^n $(i = 1, \ldots, k)$ is an n-sphere, P any collection of primes, $Z_P = \{r/s \mid r, s \in Z, (r,s) = 1, p \in P \Rightarrow p \not| s\}$ the usual algebraic localization of the integers "at P", and $GL(k; Z_P)$ the collection of all $k \times k$ matrices with coefficients in Z_P and unit determinant. As a corollary we recover known results about self-equivalences of spheres: Let P be the collection of all primes. Then $Eq(S_1^n \vee \ldots \vee S_k^n) = Eq(S_1^n \vee \ldots \vee S_k^n)_P \approx GL(k; Z)$. (Received January 4, 1972.)

*692-G3. HAROLD M. HASTINGS, Hofstra University, Hempstead, New York 11550. Comodule spectra and the Adams spectral sequence.

Adams' geometric formulation of the stable Adams spectral sequence ("Lectures on generalized cohomology," Lecture Notes in Math., vol. 99, Springer, New York, 1969, pp. 1-138) requires finiteness assumptions since cohomology is used. Here these finiteness restrictions are removed by working entirely in homology. Following Adams, let E be a strict commutative ring spectrum, and E_* the associated homology theory. Assume that $E_*(E)$ is a flat $E_*(S^0)$-module. Let X and Y be spectra, and consider $E_*(X)$ ($E_*(Y)$) as a right (left) comodule over the Hopf algebra $E_*(E)$. The cotensor product $E_*(X) \square_{E_*(E)} E_*(Y)$ can then be defined geometrically by introducing comodule spectra in the category of E-module spectra. Theorem. If $E_*(X)$ or

$E_*(Y)$ is a flat $E_*(S^0)$-module, there is a spectral sequence with $E^2 = \text{Cotor}^{E_*(E)}(E_*(X), E_*(Y))$ which converges weakly to the homotopy groups of a spectrum with the same E_*-homology as $X \wedge Y$. Cosimplicial methods are used. The proof of convergence is dual to that in (J. Milnor, "On axiomatic homology theory," Pacific J. Math. 12(1962), 337-341); see also (A. K. Bousfield and D. M. Kan, "Homotopy with respect to a ring," Proc. Sympos. Pure Math., vol. 22, Amer. Math. Soc., Providence, R. I., 1971). (Received January 5, 1972.)

*692-G4. ANTHONY J. D'ARISTOTLE, State University College of New York, Geneseo, New York 14454. On realcompactness and Wallman realcompactifications.

Recently Alo, Shapiro, and Weir (Abstract 685-G3, these *Notices* 18(1971), 546) proved the following Theorem. Let X and Y be Tychonoff spaces with respective strong delta normal bases $\mathcal{B}_X$ and $\mathcal{B}_Y$. If f is a continuous mapping from X into Y such that $f^{-1}(A) \in \mathcal{B}_X$ whenever $A \in \mathcal{B}_Y$, then there exists a continuous extension f* of f from $\rho(\mathcal{B}_X)$ into $\rho(\mathcal{B}_Y)$. ($\rho(\mathcal{B}_X)$, $\rho(\mathcal{B}_Y)$ are respectively the Wallman realcompactifications of X and Y.) In this paper we first observe that this theorem may be obtained as a corollary to a result of Frink in his original paper on Wallman-type compactifications (Amer. J. Math. 86(1964), 602-607). We then proceed to sharpen Frink's result and to provide a number of equivalent conditions for extendibility from X to $\rho(\mathcal{B}_X)$. In the case of real-valued functions, we show that these conditions are equivalent to countable $\mathcal{B}$-uniform continuity, a notion which the present author introduced in an earlier paper (Abstract 682-54-3, these *Notices* 18(1971), 205). (Received January 10, 1972.)

692-G5. SANKATHA P. SINGH, Memorial University of Newfoundland, St. John's, Newfoundland, Canada. Fixed point theorems for a sum of nonlinear operators.

The following theorems, which generalize some earlier known results, have been given. Theorem 1. Let X be a reflexive Banach space, C a nonempty closed, bounded and convex subset of X. Let A and B be two mappings of C into X such that A is a 1-set contraction with I - A demiclosed, and B is strongly continuous. Then there exists some x in C such that $Ax + Bx = x$. Theorem 2. Let C be a nonempty weakly compact convex subspace of a Banach space X. If $T: C \to C$ is a nonlinear operator such that $T = A + B$, where $A: C \to C$ is a nonexpansive mapping and $B: C \to C$ is completely continuous and I - T is convex. Then T has a fixed point provided that $\inf_{x \in C} \|x - Tx\| = 0$. (Received January 12, 1972.)

*692-G6. FRANCIS D. WILLIAMS, New Mexico State University, Las Cruces, New Mexico 88001. Quasi-commutativity of H-spaces.

Two H-spaces (X,m) and (Y,n) are called H-equivalent if there is a homotopy equivalence $f: X \to Y$ which is an H-map. Given an H-space, (X,m), let $m': X \times X \to X$ be given by $m'(x,y) = m(y,x)$. Problem 34 of the 1970 Neuchatel Conference on H-Spaces (Lecture Notes in Math., vol. 196, 1971) asks whether a given H-space (X,m) is always H-equivalent to (X,m') when X is a finite CW-complex. Theorem. If $X = SO(3)$, there is an H-space multiplication m for X such that (X,m) is not H-equivalent to (X,m'). The proof involves analysis of the Postnikov invariants of X. (Received January 17, 1972.)

*692-G7. ERIC JOHN BRAUDE, Seton Hall University, South Orange, New Jersey 07079. Metrizability and graph-distinguishable correspondences. Preliminary report.

Definitions. A family $\mathcal{A}$ of subsets of a Hausdorff space T is distinguishable if a separable metric space M and a continuous map $q: T \to M$ exist such that $q(A) \cap q(T \sim A) = \emptyset$ for all A in $\mathcal{A}$; a subset $S \subseteq T$ is distinguishable if $\{S\}$ is; and a correspondence $F: X \to Y$ is graph-distinguishable if F is distinguishable in $X \times Y$. Example. Continuous graph-distinguishable maps do not generally preserve distinguishable sets but (Theorem 1) they do preserve compact distinguishable sets (compact zerosets). Theorem 2. If $\{F(x): x \in X\}$ and $\{F^{-1}(y): y \in Y\}$ are distinguishable, then the correspondence $F: X \to Y$ is graph-distinguishable. The converse holds if each of X and Y is either separable or compact. Corollary 2.1. The composition of graph-distinguishable correspondences between separable or compact spaces is graph-distinguishable. Corollary 2.2. Graph-distinguishable upper semicontinuous compact valued correspondences with separable or compact domains preserve $\mathcal{L}$-analytic sets [Proc. London Math. Soc. (3) 23(1971), 409-427]. Corollary 2.3. A compact space Y is metrizable if, and only if there exists a compact space X and a distinguishable correspondence $F: X \to Y$ such that $F^{-1}(y) \neq F^{-1}(y')$ for $y \neq y'$. Corollary 2.3.1 (Sneider, 1945). A compact space is metrizable if and only if it has a G_δ diagonal. (Received January 19, 1972.)

*692-G8. CHARLES H. GOLDBERG, Trenton State College, Trenton, New Jersey 08625. An exact sequence of braid groups.

Let M denote a closed surface $\neq S^2$ or P^2; F_nM the set of all imbeddings of a fixed n-point set P_n in M (with C-O topology); and x^0 a fixed basepoint of F_nM. Let M^n denote the n-fold cartesian product of M with itself or equivalently the set of all maps of P_n into M. Let i denote the natural inclusion $i: F_nM \to M^n$ and i_* the induced homomorphism $i_*: \pi_1(F_nM,x^0) \to \pi_1(M^n,x^0)$. Let D^2 denote an open disk in M such that D^2 contains the n points $x_1^0, x_2^0, \ldots, x_n^0$ which constitute image(x^0). Let F_nD^2 denote the set of all imbeddings of P_n in D^2; j the natural inclusion $j: F_nD^2 \to F_nM$; and j_* the induced homomorphism $j_*: \pi_1(F_nD^2,x^0) \to \pi_1(F_nM,x^0)$. Then $\pi_1(F_nM,x^0)$ is isomorphic to the Fox-Artin group of n-strand unpermuted braids on the surface M, and $\pi_1(F_nD,x^0)$ is isomorphic to the classical Artin group of unpermuted braids (on the plane). Replacing $\pi_1(M^n,x^0)$ by the isomorphic group $\Pi_{i=1}^n \pi_1(M,x_i^0)$, we state the main result: Theorem. The following sequence of (not necessarily abelian) groups is "exact" in the sense that the kernel of each homomorphism is equal to the normal closure of the image of the preceding homomorphism: $1 \to \pi_1(F_nD^2,x^0) \to \pi_1(F_nM,x^0) \to \Pi_{i=1}^n \pi_1(M,x_i^0) \to 1$. All proofs are geometrical, including simplified new proofs of previously known lemmas. (Received January 19, 1972.)

692-G9. P. BRIAN SHAY, City University of New York, Graduate School, New York, New York 10036. Polynomial-dual coalgebra structure in Hopf algebras over the integers. Preliminary report.

A sequence of obstructions inhibiting polynomial dual coalgebra structure in torsion-free cocommutative, graded, even, connected Hopf algebras over integral domains of characteristic 0 is described. To each element P_1 of the primitives P(H) of H, a set $O^n(P_1)$ in $H/P(H) \otimes Z/nZ$ may be assigned if $O^j(P_1)$ contains 0 for all

$j < n$. If 0 is in $O^n(P_1)$, an element P_n is chosen, with $P_n \overset{\Delta}{\to} \Sigma_{i+j=n} P_i \otimes P_j$. The plot is to use Newton's formula to define iteratively indecomposables in terms of primitives. If all the obstructions are cleared for a basis of P(H), the 'nice' coalgebra structure is shown to occur. A start is made on computing the action of the mod p Steenrod algebra on H*(BU) by choosing a basis natural from the coalgebraic point of view. $P^i(c_{i+1})$ is given in closed form. The Hopf algebra automorphisms of H*(BU) are shown to be topological and few. Hopf algebras which are λ-rings and the universal cocommutative Hopf algebra for algebra maps into a given algebra are studied, looking to applications in complex K theory of homotopy commutative H spaces with even dimensional cells. (Received January 21, 1972.)

*692-G10. HAROLD BELL, University of Cincinnati, Cincinnati, Ohio 45221. Some topological methods for plane geometry.

For a nonempty closed subset of the plane A let E(A) be the set of points in the plane z for which there are two distinct points in the set A whose distance to z is the (minimal) distance from z to A. As is well known E(A) is empty iff A is convex. In fact the "less convex" A the more extensive E(A). In this paper the set E(A) and several related sets are studied. Some of the results are: (1) A proof that the set of points in the plane equidistant from two disjoint closed connected subsets of the plane is a one-manifold. (2) A direct elementary constructive proof of the topological part of the Shoenflies theorem. (3) A notion tangency and normal lines for arbitrary plane continua. (4) A procedure for approximating plane continua "closely" with continua whose boundary is the union of a finite number of simple closed curves and arcs coupled with an automatic procedure for extending continuous functions to these approximating continua. (Received January 24, 1972.)

*692-G11. OKAN GUREL, IBM, White Plains, New York 10604. A classification of the singularities of (X,f).

Singularities of the pair (X,f) describing a dynamic system, where $f: X \to X^d$, X, the phase space, X^d, the velocity space, can be classified by referring to the characteristic orientations [see Abstract 658-185, these *Notices* 15(1968), 774]. These are pairs of characteristic orientations $w^n_{m_i}$ where $n = 0, 1, 2, \ldots$, and $i = 1, 2$, for $n \geq 2$, $i = 1, 2, 3, 4$, for $n = 3, \ldots$. If the orbital structure in the neighborhood of a singularity can be characterized by characteristic singularities of the mth level, then the pair (X,f) where $X \subset R^n$, and $m \leq n$, is said to have the stability of the mth level. For $n = 0$, trivial orientations, for $n = 1$, the two translations, $w^1_{m_1}$, $w^1_{m_2}$, and for $n = 2$, the two rotations, $w^2_{m_1}$, $w^2_{m_2}$, are obtained. As a general rule, characteristic orientations of all the dimensions lower than n are also suborientations of the set of orientations in n. Indicating k as the number of a pair of 1st level characteristic orientations, $w^1_{m_1}$, and $w^1_{m_2}$, for $k = 0, 1, \ldots, \infty$, classical singularities, node, saddlepoint, focus of the two dimensional phase plane, $n = 2$, are classified in terms of $w^1_{m_1}$, $w^1_{m_2}$, $w^2_{m_1}$, $w^2_{m_2}$, and in addition, new singularities, such as pseudo-node, pseudo-saddlepoint, degenerate node, degenerate saddlepoint are introduced. Various examples can be given to illustrate the concept of levels of stability both for isolated singular points and for limit cycles. (Received January 24, 1972.)

*692-G12. JĘDRZEJ ZIEŚNIATYCKI and W. M. TULCZYJEW, Department of Mathematics, Statistics and Computing Science, University of Calgary, Calgary 44, Alberta, Canada. Generating forms of Lagrangian submanifolds.

In this paper the existence of generating forms for a class of Lagrangian submanifolds of special sympletic manifolds is established and the composition properties of generating forms of sympletic relations are derived. Lagrangian submanifolds of symplectic Banad manifolds have been introduced and studied extensively by A. Weinstein (Advances in Math. 6(1971), 329-346). A Lagrangian submanifold of a symplectic manifold is a submanifold for which the restriction of the symplectic form vanishes, and which in addition satisfies certain maximality conditions. Special symplectic manifolds are manifolds isomorphic to cotangent bundles. Symplectic relations are Lagrangian submanifolds of products of symplectic manifolds. Their composition is understood in the set-theoretical sense. This work is motivated by physical applications. The formulation of a physical theory in terms of symplectic geometry is considered to be an essential step towards its quantization. The study of generating forms could be useful in formulating a Schrödinger type quantum theory of a physical system. (Received January 24, 1972.)

*692-G13. WAYNE LAWTON, Wesleyan University, Middletown, Connecticut 06457. Note on symbolic transformation groups. Preliminary report.

Let $n \geq 2$ be an integer, let $P = \{0, 1, \ldots, n-1\}$ be given the discrete topology, let T be an infinite discrete group and let P^T be the set of maps from T to P with the product topology. We define the left symbolic transformation group over T to P to be the pair (P^T, T) where T acts by left translation on maps from T to P. An endomorphism of (P^T, T) is defined to be a continuous map from P^T to P^T which is equivariant with respect to the action of T. We show for a large class of groups T that any one-to-one endomorphism of (P^T, T) is onto. This class of groups, denoted by $S \cup R$, includes the following: (1) Profinite groups, (2) Locally finite groups, (3) Abelian groups, (4) Any subgroup of a group in this class. The proof that the above classes of groups are $S \cup R$-groups is combinatoric in nature and a counterexample has not been found. Profinite groups and Abelian groups admit totally bounded zero dimensional topological group structures, a fact which may indicate a counterexample or a different proof. Other relationships between dynamical properties of (P^T, T) and group properties of T are suggested. (Received January 25, 1972.)

692-G14. STAVROS G. PAPASTAVRIDIS, Princeton University, Princeton, New Jersey 08540. The Kervaire invariant of manifolds with few nonzero Stiefel-Whitney classes. Preliminary report.

Consider the cobordism theory determined by a complex X and a bundle ξ over it. Suppose that we have a map from (X, ξ) to the v_{q+1}-cobordism, so it make sense to talk about the Kervaire invariant of manifolds in (X, ξ). The main theorem is that if $|X| < 2q$, and the ξ has nonzero Stiefel-Whitney classes only in dimensions which are powers of two, and those Stiefel-Whitney classes have zero cup products with each other, and if $q + I \neq 2^i - 2$ then every manifold in (X, ξ) has Kervaire invariant zero. If $q + I = 2^i - 2$ then the existence of manifolds in (X, ξ) of Kervaire invariant one is detected by a specified secondary cohomology operation. Also

in the case where dim $X \leqq q$ we reduce the problem of finding manifolds of Kervaire invariant to specific homotopy problems. (Received January 19, 1972.)

692-G15. GEORGE REYNOLDS, Wesleyan University, Middletown, Connecticut 06457. Ultrafilters and epi-reflective hulls. Preliminary report.

Theorem 1. Let $\mathcal{S}$ be a subcategory of Top, $\mathcal{B}$ the epi-reflective hull in Top and $X \in |\mathrm{Top}|$. Then these are equivalent: (a) $X \in |\mathcal{B}|$; (b) If $\mathfrak{F}$ is a free ultrafilter of closed sets on X, then there exists $f: X \to E$, $E \in |\mathcal{S}|$ with $f(\mathfrak{F})$ having no cluster point; (c) If $\mathfrak{F}$ is a free ultrafilter of closed sets on X, then there exist $f: X \to E$ and an open cover $\mathcal{U}$ of E with $f^{-1}(\mathcal{U}) < X - \mathfrak{F}$. The equivalence of (a) and (c) was proved by P. Zenor. Taking $\mathcal{S}$ to be a single space E, this theorem gives a characterization of the simple epi-reflective subcategories or E-compact spaces. Hušak has shown that the epi-reflective subcategory of α-realcompact (every $\mathcal{Z}$-ultrafilter with the α-intersection property is fixed) spaces are simple. Making use of this notion it is then shown that the epi-reflective subcategory of m-bounded spaces (every subset of cardinality $\leqq$ m has compact closure) is not simple for any $m \geqq \aleph_0$. Some attention is then given to characterizing various known topological properties in terms related to properties (b) and (c) of Theorem 1. For example Theorem 2. A space X is realcompact iff for each free $\mathcal{Z}$-ultrafilter of zero sets, there exists a σ-discrete cozero refinement of $X - \mathfrak{F}$. (Received January 27, 1972.)

692-G16. MICHAEL D. RICE, Wesleyan University, Middletown, Connecticut 06457. Two theorems in uniform spaces. Preliminary report.

Two theorems are presented which explore the relationship between uniform spaces and the theory of ultrafilters. These give an alternate description of the epi-reflective hull of separable metric spaces in terms of the $\mathcal{M}$-fine functor, whose study was initiated by A. W. Hager. As a corollary we show that the epi-reflective hull of (0,1) with the metric uniformity does not contain the precompact uniform spaces. Sample Result. If μX is a separable uniform space, these are equivalent: (1) $m\mu X$ is complete, where m denotes the $\mathcal{M}$-fine functor, (2) each ultrafilter of zero sets of the form Z(f) (where f is a uniformly continuous real valued function) with the countable intersection property is fixed, (3) to each filter $\mathfrak{F}$ generated by a free ultrafilter of closed sets, there exists a uniformly continuous function to a separable metric space such that the image of $\mathfrak{F}$ has no cluster point, (4) μX is uniformly isomorphic to a closed subspace of a product of separable metric spaces. (Received January 27, 1972.)

692-G17. RICHARD KATZ, California State College, Los Angeles, California 90032. Green's lines on Riemannian manifolds. Preliminary report.

The notion of Green's lines on a Riemann surface or a locally (flat) Euclidean space has proved useful in the investigation of various classes of harmonic functions and "radial" limits. It is shown that a generalization of this notion can be extended to open Riemannian manifolds so that a number of the analogous properties hold. (Received January 27, 1972.)

*692-G18. JOHN WARREN BAKER, Florida State University, Tallahassee, Florida 32306. Ordinal subspaces of topological spaces.

Let $\Gamma(\alpha)$ denote the set of ordinals not exceeding α with the interval topology. If X is a topological space and α is an ordinal, $X^{(\alpha)}$ denotes the αth derived set of X. All spaces are assumed to be Hausdorff. Theorem 1. Suppose X is regular and countably compact, λ is a denumerable ordinal, and n is a positive integer. If $\text{Card}(X^{(\lambda)}/X^{(\lambda+1)}) \geq n$, then X contains a subset homeomorphic to $\Gamma(\omega^{\lambda}n)$. Theorem 2. Suppose X is 0-dimensional and first countable. If λ is a denumerable ordinal and n is a positive integer with $\text{Card}\, X^{(\lambda)} \geq n$, then there is a subset Z of X homeomorphic to $\Gamma(\omega^{\lambda}n)$. Theorem 3. A dispersed (i.e., scattered), regular, countably compact set is sequentially compact. Also a counterexample is given to a conjecture due to Z. Semadeni ("Sur les ensembles clairseme," Rozprawy Mat. 39(1959)). (Received January 27, 1972.)

692-G19. JACK W. POPE, University of North Carolina, Chapel Hill, North Carolina 27514. A note on monotone normality. Preliminary report.

The concept of an elastic space was defined by H. Tamano and J. E. Vaughan (Abstract 672-620, these *Notices* 17(1970), 262). C. Borges (Abstract 71T-G158, these *Notices* 18(1971), 840) has announced that every elastic space is monotonically normal. Dropping from the definition of elastic space the requirement that the framing involved be transitive, we prove that the resulting spaces satisfy the conclusion of Lemma 2.1 of Borges (Pacific J. Math. 17(1966), 3) and hence are monotonically normal. (Received January 27, 1972.)

692-G20. PETER W. HARLEY III, University of South Carolina, Columbia, South Carolina 29208. Products of Fréchet spaces. Preliminary report.

The following theorem is proved, which illustrates the depth of the nonproductive nature of Fréchet spaces. Theorem. Let X be normal, Hausdorff, and first countable. Let A be a closed subset of X satisfying the following property: there is a sequence $U_1, U_2, \ldots, U_i, \ldots$ of open sets in X with $U_i \subset U_i + 1$ such that (1) A is covered by $U_1, U_2, \ldots$, and (2) $A \cup U_i$ is not open. Then if Y is a Hausdorff Fréchet space, $X/A \times Y$ (X/A is clearly Fréchet) is Fréchet if and only if Y is discrete. (Received January 27, 1972.)

The March Meeting in St. Louis, Missouri
March 27-April 1, 1972

Algebra & Theory of Numbers

*693-A1. LAL M. CHAWLA and LEONARD E. FULLER, Kansas State University, Manhattan, Kansas 66502. On semigroups, groups and lattices of tables of subgroups of a group. Preliminary report.

For any subgroup A of a group G, the inner table P(A) is defined as the set of all ordered pairs (x,y) where x,y and xy $\in$ A. The outer table Q(A) consists of all ordered pairs (x,y) where x,y $\in$ A' = G - A and xy $\in$ A. The complete table R(A) is the set of all ordered pairs (x,y) where x,y $\in$ G and xy $\in$ A. These tables are also partitioned into subsets $P_a(A)$, $Q_a(A)$, $R_a(A)$ by requiring xy = a where a is any given element in A. Necessary and sufficient conditions for a subset of G $\times$ G to be an inner, outer, or complete table of a subgroup A are determined. In each of the sets of tables P(A), Q(A), R(A) two possible multiplications are defined so that each of the resulting semigroups with carriers P(A), Q(A), R(A) can be mapped homomorphically onto the underlying semigroup of A whereas each of the quotient sets $\{P_a(A) \mid a \in A\}$, $\{Q_a(A) \mid a \in A\}$, $\{R_a(A) \mid a \in A\}$, with the natural multiplication in it is a group isomorphic to the subgroup A. We also prove that the lattice of all subgroups of G can be isomorphically embedded in the lattice of all right ideals of the semigroup of complete tables of G. It is also shown that a certain lattice of tables is isomorphic to the lattice of all subgroups of a group. (Received January 3, 1972.)

693-A2. L. A. CAMMACK, Culver-Stockton College, Canton, Missouri 63435. A new characterization of orthomodular partially ordered sets. Preliminary report.

J. C. Abbott has shown that boolean algebras can be characterized as a particular equational class of implication algebras. Modifying the axiom scheme slightly yields a generalization including the class of orthomodular lattices. Synthesizing these results and methods introduced by S. Gudder and R. Schelp applied to a certain class of partial algebras we show that a similar characterization can be realized for orthomodular partially ordered sets. Theorem. Every QTT-partial implication algebra $(I, R, *, 1)$ determines a partially ordered set $(I, \leqq)$ in which every principal order filter is an orthomodular partially ordered set. Conversely, if I is an orthomodular partially ordered set, then there exists a relation $R \subseteq I \times I$ and a map $*$ from R into I such that $(I, R, *, 0, 1)$ is an OM-partial implication algebra. Corollary. Every implication algebra $(I, .)$ determines a union semilattice $(L, \vee, 1)$ in which every principal order filter is a boolean algebra. (Received January 13, 1972.)

*693-A3. J. M. GANDHI and DARYL KREILING, Western Illinois University, Macomb, Illinois 61455. Groups with a two variable commutator identity. Preliminary report.

Let G be a group and let $\lambda(x)$ and $\rho(x)$ denote the mappings which take all elements y of G onto $[x, y] = x^{-1}y^{-1}xy$ and $[y, x] = y^{-1}x^{-1}yx$ respectively. Denote by L(m, n) (R(m, n)) the family of all groups satisfying the identity $\lambda^m(x) = \lambda^{m+n}(x)$ ($\rho^m(x) = \rho^{m+n}(x)$). Gupta and Heineken, "Groups with a two-variable commutator

identity," Math. Z. 95(1967), 276-287, proved that $R(2,1) = R(2,2)$ but were unable to prove $R(m,1) = R(m,2)$. The purpose of this note is to prove the Theorem. $R(m,1) = R(m,2)$. (Received January 18, 1972.)

*693-A4. LAWRENCE JAMES RISMAN, Fitchburg State College, Fitchburg, Massachusetts 01420. A new proof of the three squares theorem.

A theorem of Fein, Gordon, and Smith on the representation of -1 as a sum of two squares is shown to yield a new proof of the three squares theorem. A positive integer k can be represented as a sum of three squares if and only if $k \neq 4^a n$ with $n \equiv 7 \pmod 8$ and $a \geq 0$. This proof depends on the Brauer group and class field theory, not on ternary quadratic forms. (Received January 20, 1972.)

693-A5. RICHARD L. TANGEMAN, Arkansas State University, State University, Arkansas 72467. Generalized heredity in radical classes. Preliminary report.

Let W be a homomorphically closed subring closed class of not necessarily associative rings. Let S be a subclass of $W \times W$ satisfying (1) If $(I,R) \in S$ then I is a subring of R; (2) If $(J,R) \in S$ and I is an ideal of R then $(I \cap J, J) \in S$; and (3) If $(I,R) \in S$ and φ is a homomorphism of R then $(I\varphi, R\varphi) \in S$. Say that a class $M \subseteq W$ is hereditary relative to S if $R \in M$ and $(I,R) \in S$ imply $I \in M$. Theorem. If M is homomorphically closed and hereditary relative to S then LM, the lower radical determined by M in W, is hereditary relative to S. Corollary. If M is homomorphically closed and hereditary to subrings (left ideals, right ideals, maximal ideals), so is LM. (Received January 20, 1972.)

693-A6. GERALD C. SCHRAG, Central Missouri State College, Warrensburg, Missouri 64093. Some properties of the lattice generalizations of G_{32}. Preliminary report.

Let $\mathcal{L}(X,\mathcal{S})$ denote the quasilogic of the space $(X,\mathcal{S})$ and let $G^*(n,k)$ be the dual of the generalized Petersen graph $G(n,k)$. We have seen [Abstract 674-65, these *Notices* 17(1970), 539] that $\mathcal{L}(G^*(n,k))$ is a generalization of Greechie's lattice G_{32} if $n/(n,k) \neq 3$. In this paper we consider and denote by $H(n,k)$ only those generalized Petersen graphs $G(n,k)$ such that $k \neq 1$ and $n/(n,k) \neq 3$ or 4. (Note that this implies that $n \geq 5$ since $1 < 2k < n$.) This is equivalent to considering only those generalizations of G_{32} which are orthomodular lattices [ibid.]. Lemma. Let L be an orthomodular lattice such that (1) L satisfies the convention of Greechie's atomistic loop lemma, (2) every atom is contained in a loop, and (3) every atom is contained in exactly two blocks. Then L is nonmodular and any two nontrivial elements have a common complement. (A counterexample will be given for orthomodular posets.) Theorem. Each $\mathcal{L}(H^*(n,k))$ is a nonmodular, irreducible, hyper-irreducible, orthomodular lattice which has the relative center property and any two nontrivial elements have a common complement. Furthermore, each $\mathcal{L}(H^*(n,k))$ is an example of a Loomis dimension lattice which is not an orthomodular geometry. (Received January 20, 1972.)

*693-A7. JANOS GALAMBOS, Temple University, Philadelphia, Pennsylvania 19122. Distribution of arithmetical functions.

While extensive studies are available on problems of the asymptotic distribution of additive and multiplicative functions (see the author's papers, Ann. Inst. H. Poincaré Sect. B 6(1970), 281-305, and Bull. London Math. Soc. 3(1971), to appear), very few results are known for arithmetical functions not belonging to one of the two classes above. Independently, Novoselov (Izv. Akad. Nauk SSSR Ser. Mat. 28(1964), 307-364) and the present author (Ann. Univ. Sci. Budapest Eötvös Sect. Math. 7(1964), 25-31) gave a general method which is applicable to a wide class of arithmetical functions. By making use of a more general result on Dirichlet series than the one applied in the author's paper of 1964, referred to above, some general results on asymptotic distributions will be given. From probability theory, the continuity theorem of characteristic functions is applied, while to evaluate (asymptotically) the characteristic function (Fourier transform) of an arithmetical function, properties of Dirichlet series are used. Several examples are included to illustrate the results. (Received January 20, 1972.)

*693-A8. ROBERT S. SPIRA, Michigan State University, East Lansing, Michigan 48823. The average order of $L(1, \chi)$.

Let χ be a nonprincipal character mod k, and $L(s, \chi)$ the Dirichlet L-series corresponding to χ. It is shown that the average order over all such χ's of $|L(1, \chi)|^2$ is $(\pi^2/6)\,\Pi_{p|k}(1-p^{-2})$. (Received December 27, 1971.)

*693-A9. PAUL C. KAINEN, Case Western Reserve University, Cleveland, Ohio 44106. Realizing commutative cubes. Preliminary report.

We use the notation and definitions of the author's paper ("Weak adjoint functors," Math Z. 122(1971), 1-9). Let $T: \mathcal{A} \to \mathcal{B}$ be a covariant functor and let $S: \mathcal{B} \to \mathcal{A}$ be a weak right adjoint to T with adjunction η. Suppose further that $TS = 1$ and that $\eta_{AB}: \mathcal{A}(A, SB) \to \mathcal{B}(TA, B)$ is given by applying T. Also assume that $\mathcal{A}$ has weak colimits and that T preserves weak colimits. A diagram $\delta: \mathcal{D} \to \mathcal{B}$ is realizable (with respect to T) if there is another diagram $\delta': \mathcal{D} \to \mathcal{A}$ such that $T\delta' = \delta$. Theorem. Let $\mathcal{D}$ be any category which is generated by some directed d-dimensional cube Q(d). Then any diagram $\delta: \mathcal{D} \to \mathcal{B}$ can be realized. Here Q(d) denotes the directed graph whose vertices are d-tuples of binary numbers and whose arcs join a vertex each of its immediate successors under the natural (lexicographic) ordering. Corollary. Let $\delta: \mathcal{D} \to \mathcal{B}$ be any diagram which can be extended to a diagram on some cube Q(d). Then δ can be realized. For example, any diagram obtained by pasting together two commutative squares along a common arc can be realized. We hope to apply this result to the algebraic topological problem of realizing diagrams with respect to some homology functor. (Received January 24, 1972.)

*693-A10. DONALD P. MINASSIAN, Butler University, Indianapolis, Indiana 46208. On construction of orders in abelian groups.

A claim (see H. H. Teh, Proc. Cambridge Philos. Soc. 57(1961), 476-482, "only if" of Theorem 2, p. 480) on Archimedean full orders fails. The statement (p. 478, line 5) that the direct product of n copies of the additive rational numbers admits no full order of Archimedean rank more than n is proved without Hahn's embedding theorem. The direct product of I copies of the additive rational numbers, where the cardinality i of I does not exceed that of the real numbers, admits a full order of Archimedean rank m where m is any cardinal number less than or equal to i. (All direct products are restricted direct products, in case of an infinite number of summands.) (Received January 25, 1972.)

*693-A11. DAVID W. MATULA, Washington University, St. Louis, Missouri 63130. The employee party problem.

A set of n employees have intermingled such that each pair have met with (independent) probability p. The boss wishes to host a party for a maximum number of the n employees such that every pair of the employees invited have met, and wonders how big a party to plan. Let D_{np} be the random variable giving the size of the largest subset of employees every pair of which have met. We show that the density of D_{np} is quite spiked. For example, with $n = 1000$, $p = .5$, we exhibit bounds showing that $\text{Prob}\{D_{1000,.5} = 15\} > .8$. Our main result is the following. Theorem. For any $0 < p = 1/b < 1$, $\epsilon > 0$, let $d = 2\log_b n - 2\log_b \log_b n + 2\log_b(e/2) + 1$. Then $\lim_{n\to\infty} \text{Prob}\{\lfloor d - \epsilon \rfloor \leq D_{np} \leq \lfloor d + \epsilon \rfloor\} = 1$. (Received January 25, 1972.)

693-A12. ROBERT C. SHOCK, Southern Illinois University, Carbondale, Illinois 62901. Dual generalizations of the Artinian and Noetherian conditions. Preliminary report.

For module M the dual to (I) every nonzero factor module contains a nonzero minimal submodule is (III) every nonzero submodule of M contains a maximal submodule. Modules with (I) are called Soc modules, those with (III) Max modules. Elementary properties are discussed. Clearly, M is Artinian if and only if M has (I) and (II) the socle of every factor module is finitely generated. We prove M is Noetherian if and only if (II) and (III) hold. Also the T. C. condition of R. P. Kurshan (J. Algebra 15(1970), 376-386) implies (III) but not conversely. Take successive intersections of the radical, M, rad M, rad(rad M),... to obtain R-radical of M. An Artinian module is Noetherian if and only if the R-radical is zero. A module M is completely reducible if and only if the socle of M is essential and the radical of M is zero and cyclic submodules have finitely generated socles. (Received January 25, 1972.)

693-A13. MICHAEL BARR, McGill University, Montreal, Quebec, Canada and JOHN R. ISBELL, State University of New York at Buffalo, Amherst, New York 14226. Protriples.

Over a left extremal-complete category, the categories of models of clones are (up to equivalence) precisely the locally tripleable categories. They are also characterized by Beck's tripleableness criterion minus the existence of an adjoint; and they may be described as the categories of models of certain special inverse limits (formal limits, or limits in clones) of triples, called protriples. Over a suitable base category the

category of protriples is left and right complete, and there are tensor products $P \otimes \theta$ of a protriple P and an equational theory θ. (Received January 26, 1972.)

*693-A14. JOHN W. DUSKIN, JR., State University of New York at Buffalo, Amherst, New York 14226. Nonabelian triple cohomology: Extensions and obstructions. Preliminary report.

Extensions. Recent work of the author has shown that the coefficient groups used in triple cohomology may be generalized to groupoid (or even category) coefficients with $H^1(X;\mathcal{G})$ defined as homotopy classes of simplicial maps of the cotriple resolution of X into the nerve of the groupoid $\mathcal{G}$. If the "underlying object" functor U is tripleable, then $H^1(X;\mathcal{G})$ classifies isomorphism classes of U-split $\mathcal{G}$-torsors above X. These in turn may be identified with extensions of X by some (not necessarily abelian) kernel, and, for suitable choices of coefficient groupoids, all extensions of X may be classified in this manner. Examples range from Dedecker cohomology for groups to crossed groups in simplicial sets, and include arbitrary algebra extensions. Obstructions. If the coefficient groupoid is commutative, its endomorphism subgroupoid is supplied with a canonical (Giraud) discrete functor into a groupoid $\mathcal{G}^{\#}$ which has the connected component object $[0]$ of $\mathcal{G}$ as its own "object object". Classical obstruction theory may be seen as an exploitation of the properties of this discrete functor D in terms of cocycles. In each of the classical cases it is possible to realize all elements of $H^2_\theta = (X, A)$ in this fashion by constructing a suitable coefficient groupoid. These in turn are easily seen to be equivalent to certain "2-term extensions of X". (Received January 26, 1972.)

*693-A15. EUGENE I. FURUYAMA, Washington State University, Pullman, Washington 99163. Integrally closed principal elements in a Noether lattice. Preliminary report.

For q, p primary, the n-symbolic primary power, $q^{(n)}$, is defined as the unique p primary component of the normal primary decomposition of q^n. A lattice element m is a-dependent on lattice element b if there exists a positive integer k such that $m^{k+1} \leqq b(m \cup b)^k$. The integral closure of b is defined to be the join of all elements a-dependent on b and an element is said to be integrally closed iff it equals its integral closure (definition due to John P. Lediaev). The main results are: Theorem. Let m be a proper integrally closed principal element of a Noether lattice which is not a zero divisor. Let p be a prime associated with m. Then p is of rank one and there exists a principal element c such that $c_p = p$. Theorem. Let m be as above, $p_1, \ldots, p_r$ be the primes which belong to m. Then $p_1, \ldots, p_r$ are all minimal primes of m. The element m has only one normal decomposition, and this is of the form $m = p_1^{(n_1)} \cap \ldots \cap p_r^{(n_r)}$. For each prime p of rank one not belonging to 0, one can associate with each integrally closed principal element m a positive integer n such that $p^{(n)}$ is the p primary component of the normal decomposition of m. Since in a Noether lattice each element is a join of principal elements this defines a set of valuations for a Noether domain lattice which has integrally closed principal elements. (Received January 24, 1972.)

693-A16. WILLIAM H. RANT, 705 North Church Street, Jacksonville, Alabama 36265 and Jacksonville State University, Jacksonville, Alabama 36265. Left perfect rings that are right perfect and a characterization of Steinitz rings.

A proof is given to show all flat left modules of a ring are free if and only if the ring is a local ring with a

left T-nilpotent maximal ideal. We characterize a left perfect ring whose radical R has the property that $IR^n = \{0\}$ for some positive integer n if I is a finitely generated right ideal contained in R. We cite an example of a left perfect ring which does not have this property. It is shown that if the set of irreducible elements of a left perfect ring is right T-nilpotent then the ring is right perfect. (Received January 26, 1972.)

693-A17. PAUL T. BATEMAN, University of Illinois, Urbana, Illinois 61801. Multiplicative arithmetic functions and the representation of integers as sums of squares. Preliminary report.

For fixed s suppose we define the arithmetic function r_s by the power series expansion $\{\sum_{n=-\infty}^{\infty} x^{n^2}\}^s = \{1+2\sum_{n=1}^{\infty} x^{n^2}\}^s = 1 + \sum_{n=1}^{\infty} r_s(n)x^n$. Put $f_s(n) = (2s)^{-1}r_s(n)$. Familiar formulas for $r_s(n)$ show that f_s is multiplicative when $s = 1, 2, 4, 8$. On the other hand it is easy to show that f_s is not multiplicative for any other value of s [Amer. Math. Monthly 76(1969), 190-191]. In correspondence with the author, Jean Lagrange has raised a similar question for the arithmetic function φ_s defined by $\varphi_s(n) = (2s)^{-1}r_s(n^2)$. As Lagrange points out, familiar formulas of Hurwitz and others for $r_s(n^2)$ show that φ_s is multiplicative for $s = 1, 2, 3, 4, 5, 6, 7, 8$. In this paper we show that φ_s is not multiplicative for other values of s. (Received January 27, 1972.)

693-A18. RONALD J. EVANS, Jackson State College, Jackson, Mississippi 39217. Modular forms on Hecke's modular group. Preliminary report.

Let $H = \{z : \text{Im } z > 0\}$. Let $M(\lambda, k, \gamma)$ denote the space of functions f for which $f(\tau) = \sum_{n=0}^{\infty} a_n e^{2\pi i n\tau/\lambda}$ where $\lambda > 0$, $\tau \in H$, and $f(-1/\tau) = \gamma(\tau/i)^k f(\tau)$ where $k > 0$, $\gamma = \pm 1$. Let $M_0(\lambda, k, \gamma)$ denote those $f \in M(\lambda, k, \gamma)$ for which $a_n = O(n^c)$ as $n \to \infty$, for some real c. It was proved by H. Petersson in 1949 that if $\lambda = 2\cos(\pi/q)$ for an integer $q \geq 3$, then dim $M(\lambda, k, \gamma)$ is equal to dim $M_0(\lambda, k, \gamma)$. A more elementary proof which uses no non-Euclidean geometry is given here. The key to the proof is the determination of a positive lower bound for $|ic + (cx+d)/y|$ as $\tau = x + iy \in H$ approaches the x-axis, where (c, d) is the bottom row of any matrix for a transformation in Hecke's modular group. (Received January 27, 1972.)

693-A19. J. M. GANDHI, Western Illinois University, Macomb, Illinois 61455 and KENNETH S. WILLIAMS, Carleton University, Ottawa K15 5B6, Ontario, Canada. On certain sums of fractional parts.

Let a and b be integers and m an integer > 1. We let $C_m(a, b) = \sum_{x=0}^{m-1} \{ax+b/m\}$, $R_m(a, b) = \sum_{x=0;(x,m)=1}^{m-1} \{ax+b/m\}$, where $\{y\}$ denotes the fractional part of y. Certain special cases of these sums are known. We evaluate $C_m(a, b)$ and $R_m(a, b)$ in general. (Received January 27, 1972.)

693-A20. WLODZIMIERZ HOLSZTYŃSKI, University of Michigan, Ann Arbor, Michigan 48104. Applications of ergodic theory to number theory. Preliminary report.

The author discusses some infinite ergodic systems on the unit interval and their applications in number theory. (Received January 27, 1972.)

693-A21. ZAVEN A. KARIAN, Denison University, Granville, Ohio 43023 and JACK P. TULL, Ohio State University, Columbus, Ohio 43210. Computer study of some lattice point problems. Preliminary report.

The authors discuss a computer study of a class of lattice point problems which can be treated by the convolution method. The corresponding generating functions are of the form $\zeta(a_1 s)\zeta(a_2 s)\ldots\zeta(a_r s)$. (Received January 27, 1972.)

693-A22. MARVIN I. KNOPP, University of Wisconsin, Madison, Wisconsin 53706 and University of Illinois at Chicago Circle, Chicago, Illinois 60680. A problem of Rademacher in modular forms.

In [Amer. J. Math. 61(1939), 127-156] Rademacher utilized the known Fourier coefficients of $J(\tau)$, the modular invariant, to derive the functional equation $J(-1/\tau) = J(\tau)$. He later raised the question of deriving the functional equation of $1/\eta(\tau)$, the well-known modular form of dimension 1/2, from its known Fourier coefficients, by suitably generalizing the method applied to $J(\tau)$. We here point out that Rademacher's problem is in a certain sense solved by the results in the Ph.D. thesis of D. Niebur [University of Wisconsin, 1968], and that this solution is connected with the deep question of identically vanishing Poincaré series. (Received January 27, 1972.)

*693-A23. CARLOS JULIO MORENO, Center for Advanced Study, University of Illinois, Urbana, Illinois 61801. Applications of the large sieve to problems in algebraic geometry.

A recent conjecture of A. Weil states that the global zeta-function of an elliptic curve defined over the rationals is associated with a cusp form of level -2 corresponding to a certain arithmetic subgroup of the modular group. This conjecture has been verified by Shimura and Eichler for the so-called Fricke curves. Using analytic techniques the author verifies that most of the classical results in prime number theory hold for these Fricke curves. These results are then used to show that the analogue of the Hoheisel phenomenon in the analytic theory of primes holds for the traces of the Frobenius endomorphisms defined on the Fricke curves. (Received January 27, 1972.)

693-A24. WITHDRAWN.

*693-A25. GEORGE B. PURDY, Center for Advanced Computation, University of Illinois, Urbana, Illinois 61801. Real zeros of the Dedekind zeta function of an imaginary quadratic field. Preliminary report.

The author extends the computer work of Low [Acta Arith. 14(1968), 117-140] on the real zeros of the Dedekind zeta function of an imaginary quadratic field. This is equivalent to considering the zeros of the functions defined by $L(s,x) = \sum_{n=1}^{\infty} \chi(n)n^{-s}$ (Re $s > 0$), where χ is a real residue character with $\chi(-1) = -1$. No counterexamples to the extended Riemann hypothesis were found, but three "close shaves" are discussed in detail, including one overlooked by Low. An error analysis is given to show that the results are decisive. (Received January 27, 1972.)

*693-A26. KENNETH B. STOLARSKY, University of Illinois, Urbana, Illinois 61801. Extrapolation techniques related to transcendence proofs. Preliminary report.

An extrapolation method used by A. Baker to study linear forms in the logarithms of algebraic numbers is further refined and used to study a general extrapolation problem involving a class F of functions holomorphic in a large disc. Roughly, F consists of functions such that the moduli of their first I derivatives (i) are bounded by a single function of exponential growth, (ii) assume small values at the first J integers, and (iii) do not take values in some subinterval of (0,1) for integer arguments. Roughly, the conclusion is that an f in F is small at any integer less than J exp I. Previous methods would only give J exp $(\log^2 I)$. (Received January 27, 1972.)

693-A27. HENRY EDWARD THOMAS, JR., University of Michigan, Ann Arbor, Michigan 48104. Waring's problem for fourth powers. Preliminary report.

The only unsolved case of Waring's problem is to demonstrate that each natural number is the sum of nineteen fourth powers of nonnegative integers. To date, the best results were that every natural number is the sum of thirty fourth powers and that every natural number larger than $10^{10^{88.39}}$ is the sum of nineteen fourth powers. Pillai claimed without proof that every natural number is the sum of twenty-seven biquadrates. Using the method of Hardy and Littlewood as presented by Davenport and Jing-run Chen, we show that every natural number larger than $10^{1403.4}$ is the sum of nineteen biquadrates. We develop an algorithm with which a computer can rapidly find those numbers in a specified interval that are not the sum of, say, seven fourth powers. Using this algorithm, an IBM System/360 computer was able to show that nearly every number between $4 \cdot 10^6$ and $40 \cdot 10^6$ and congruent to 1, 2, 3, 4, 5, or 6 modulo 16 is the sum of seven biquadrates. This is sufficient to show that every natural number is the sum of twenty-three fourth powers. (Received January 27, 1972.)

693-A28. VANCE FABER, University of Colorado, Denver, Colorado 80202. Large abelian subgroups of infinite solvable and nilpotent groups. Preliminary report.

If m is a cardinal number, let $\exp m = 2^m$. The author has shown ("Large abelian subgroups of some infinite groups," Rocky Mt. J. Math. 1(1971), 677-685) that every infinite SI*-group G has an abelian subgroup A such that $\exp(\exp|A|) \geq |G|$. Theorem 1. Every infinite solvable group G has an abelian subgroup A such that $\exp|A| \geq |G|$. A. Ehrenfeucht and the author have shown ("Do infinite nilpotent groups always have equipotent

abelian subgroups?", Indag. Math. (to appear)) that assuming G.C.H. there exists an infinite nilpotent group G having the property that every abelian subgroup A of G has $\exp|A| \leqq |G|$. Theorem 2. If A is a maximal abelian subgroup of an infinite nilpotent group G, then $\exp|A| \geqq |G|$. (Received January 27, 1972.)

*693-A29. FRED E. J. LINTON, Wesleyan University, Middletown, Connecticut 06457. Three categorical novelties.

(1) Dixmier's topological characterization of dual Banach spaces flows from the easy tripleableness of the functor "passage to the dual space" on Banach spaces. (2) Isbell's conjecture that the dual of a category cotripleable over sets is tripleable over sets can be verified via a tiny modification of a known composite tripleableness lemma. (3) Supernaturality is ultimately reducible to an utter triviality. (Received January 27, 1972.)

*693-A30. WILLIAM J. MITCHELL, University of Chicago, Chicago, Illinois 60637. Categories of Topoi and the theory of sets.

Let BT be the theory of a boolean topos as defined by Lawvere and Tierney, and let Z_0 be a set of axioms for set theory with comprehension for bounded formulas only. Given a model $\underline{C}$ of BT in which the epi parts of maps $x \to\!\!\!\!\to z \mapsto 1$ split, a model $M(\underline{C})$ of Z_0 is defined. If maps between models of Z_0 are properly defined, then M is a functor left adjoint to the forgetful functor C which takes a model M of Z_0 to its category of sets. This construction is then modified to define the class of constructible sets $L(\underline{C})$ in an arbitrary model $\underline{C}$ of BT. Then $L(\underline{C})$ is a model of Z_0 with the axiom of choice, so the axiom of choice is consistent relative to BT. (Received January 27, 1972.)

693-A31. JULIUS W. OVERBECK, Arkansas State University, State University, Arkansas 72467. On saturated formations of solvable Lie algebras. Preliminary report.

A nonempty homomorphically closed class Γ of finite dimensional solvable Lie algebras is called a formation if every solvable Lie algebra L contains a unique minimal ideal L_Γ such that $L/L_\Gamma \in \Gamma$. A formation is called saturated if every solvable Lie algebra L contains a subalgebra H which is maximal with respect to being in Γ, and remains so under every homomorphism of L; it is locally defined if $L \in \Gamma$ if and only if the induced derivation algebra of every chief-factor of L lies in some fixed formation Γ_1. If L is a Lie algebra, then F(L) is the Fitting subalgebra of L. Barnes and Gastineau-Hills [Math. Z. 106(1968), 343-354] have shown that every locally defined formation is saturated, and that in general there are saturated formations that are not locally definable. In this paper we prove Theorem. Let Γ be a saturated formation of solvable Lie algebras over an arbitrary field. Then Γ is locally definable if and only if for each L in Γ and each irreducible L/F(L)-module V, the semidirect sum of L/F(L) with V is in Γ. Corollary. The saturated formation of supersolvable Lie algebras is locally definable if and only if the ground field is algebraically closed. Some analogues to theorems from finite group theory are also noted (Received January 27, 1972.)

693-A32. RICHARD J. TURYN, Equipment Development Laboratories, Raytheon Company, Sudbury, Massachusetts 01776. Hadamard matrices, algebras, and composition theorems.

An Hadamard matrix H has entries ± 1 and satisfies $HH' = nI$. It has been conjectured that at least one exists of order 4t for every integer t. It is sufficient to show this for t odd. We consider various composition theorems for Hadamard matrices which are suggested by various algebraic forms. Complex (fourth roots of 1, $HH^* = nI$) Hadamard matrices suggest several composition theorems for (real) Hadamard matrices; the orders of the resulting matrices in general contain a smaller power of 2. A composition theorem for a special class of complex matrices leads to the following Theorem. There exist Williamson (i.e. of quaternion type) matrices of order $4 \cdot 9^t$ (symmetric, with constant diagonal) and abelian difference sets with $v = 4 \cdot 9^t$, $n = 9^t$. Other composition theorems, related to subalgebras of abelian group algebras, are discussed. Some theorems related to quarternion algebras concern the construction of Baumert-Hall units. If a Williamson matrix of order 4w exists (e.g. w odd = q + 1/2, q a prime power, $w = 9^t$, $w \leqq 31$) and a set of Baumert-Hall units of order 4t exist (constructed by Baumert and Hall for $t = 3$) then an Hadamard matrix of order 4tw exists. Baumert-Hall units of order 4t and 20t can be constructed for $t = 1 + 2^a 10^b 26^c$, all odd $t \leqq 23$ and 31. (Received January 27, 1972.)

693-A33. JOHN R. DONER, University of Michigan, Ann Arbor, Michigan 48104. CIP neofields and combinatorial designs.

A finite CIP neofield is a neofield of finite order with cyclic multiplication group, and in which addition has the inverse property and is (hence) commutative. (A simpler characterization is that a CIP neofield is a set with an addition and multiplication which satisfies all of the field axioms except for associativity of addition.) The question of the existence of these structures for any finite order v is completely resolved via consideration of an equivalent question concerning a partition of $\mathbf{Z}_{v-1}$ into subsets of a particular structure. The additive structure of a finite CIP neofield of even order is a completely symmetric loop, which provides a construction for a cyclic Steiner triple system the existence of which is in fact equivalent to the existence of the original CIP neofield. For a CIP neofield of odd order the additive loop is not completely symmetric, but constructions of Steiner triple systems somewhat analogous to the above construction can be carried out. (Received January 27, 1972.)

693-A34. JOHN W. GRAY, University of Illinois, Urbana, Illinois 61801. Some structure of the category of 2-categories. Preliminary report.

The category of 2-categories, besides being cartesian closed, carries another nonsymmetric monoidal closed structure; i.e., a tensor product with (different) right adjoints to the product with one variable fixed. Each adjunction is self-enriched and the two internal hom-functors commute with each other. These properties follow from a suitable notion of multilinearity involving squares of 1-cells with given 2-cells between the compositions in the square. Associativity of the tensor product involves commutative cubes with such squares as faces and the coherence of this associativity follows from the result that a hypercube of such cubes is automatically commutative. The calculations have a curious connection with the standard presentation of braid groups. This structure is the basis for the study of cartesian quasi-limits and quasi-adjoint functors. (Received January 27, 1972.)

693-A35. M. A. ARBIB and E. G. MANES, University of Massachusetts, Amherst, Massachusetts 01002. Machines in a category. Preliminary report.

More conventional approaches to a categorical automata theory have been hindered by an undue emphasis on the free monoid generated by a set of inputs. By replacing the input set with a functor and expressing the "run" of a machine by an existential axiom, it is shown how dynamics induces all other machine-theoretic "processing" requirements. An endofunctor X of a category K is an input process if the underlying K-object functor from Dyn(X) to K has a left adjoint, where Dyn(X) is the category of pairs $(Q, \partial : QX \to Q)$; adding an initial state $t : I \to Q$ and an output map $b : Q \to Y$ defines a machine, M. The unique dynamorphic extension of t, $IX^{@} \to Q$, is the reachability map of M, the output of which is the behavior. M is state-behavior if X has a right adjoint (subsuming the conventional approach and J. A. Goguen, "Minimal realization of machines in a closed category," Bull. Amer. Math. Soc., to appear). M is decomposable if X has form $- + X_0$; a minimal realization algorithm is obtained which includes the usual one for linear systems. Other examples include algebra automata and stochastic automata. (Received January 27, 1972.)

693-A36. EUGENE C. JOHNSEN, University of California, Santa Barbara, California 93106. Combinatorial structure in cyclic neofields. Preliminary report.

Thomas Storer and the author have recently been investigating the existence and construction of certain combinatorial structures in finite cyclic neofields. These structures include loop difference sets and Steiner triple systems. Loop difference sets consisting of the eth powers or the eth powers plus zero have been obtained in certain cyclic neofields, right inverse property cyclic neofields, and commutative inverse property cyclic neofields for certain low even values of e. To serve this purpose, the elementary parts of the theory of cyclotomy for finite fields have been extended to finite cyclic neofields. A Steiner triple system of order n with a cyclic automorphism group which is sharply transitive on the elements has been shown by the author to be equivalent to a commutative inverse property cyclic neofield of order $n + 1$. (A corresponding result for a general sharply transitive automorphism group has also been obtained.) These together with other occurrences of neofields in the literature seem to indicate a general usefulness of these algebraic structures in combinatorial constructions. (Received January 27, 1972.)

*693-A37. GEORGE W. DINOLT, University of Michigan, Dearborn, Michigan 48128. An extremal problem for nonseparable combinatorial geometries. Preliminary report.

Let E be a finite set and $\underline{M}$ a matroid on E. Let $\underline{B}$ be the set of bases (maximal independent sets) of $\underline{M}$. The common cardinality of the members of $\underline{B}$, r, is called the rank of $\underline{M}$. A matroid is nonseparable if and only if each pair of elements of E is contained in some circuit (minimal dependent set) of $\underline{M}$. If every circuit of $\underline{M}$ has 3 or more elements, $\underline{M}$ is called a combinatorial geometry. Theorem. If $\underline{M}$ is a nonseparable combinatorial geometry of rank r on a set E, $|E| = n$, then if $r = n = 1$, $|\underline{B}| = 1$; if $r = 4$ and $n = 7$, then $|\underline{B}| \geq 20$; in all other cases $|\underline{B}| \geq \frac{1}{2}(r-1)(n-r)(n-r-1) + (n-r)$. The proof of the theorem is by an induction argument on $n + r$ using the facts that the restriction or contraction of a nonseparable matroid to the complement of a single

point of E must be nonseparable and that the number of bases in a nonseparable matroid of rank r on n elements must be at least $r(n-r)+1$. (Cf. this author, "An extremal problem for nonseparable matroids," Théorie des Matroids, C. P. Bruter, ed., Lecture Notes in Math., vol. 211, Springer-Verlag, New York, 1971, pp. 31-49.) (Received January 27, 1972.)

693-A38. EDUARDO DUBUC, University of Rochester, Rochester, New York 14627. The infinitary simplicial category.

We present here a simplicial category $\mathbb{M}$ whose objects are all ordinal numbers and (-1). The usual simplicial category Δ is the $\aleph_0$-truncation of $\mathbb{M}$. In a sense, it can be said this introduces the notion of simplices for higher (than finite) dimensions. $\mathbb{M}$ is a monoidal category (as Δ) and there is a canonical monoid in $\mathbb{M}$, $(-1) \to 0 \leftarrow 0 \otimes 0 = 1$. $\mathbb{M}$ is divided into its "face" part $\mathbb{M}_m$ and its "degeneracy" part $\mathbb{M}_e$. We have the following universal property: Theorem. If $\mathbf{P}$ is monoidal cocomplete, and $I \to P$ is a "pointed object" in $\mathbf{P}$ ($I \to P \leftarrow P \otimes P$ is a monoid in $\mathbf{P}$), there exists a functor $\mathbb{M}_m \xrightarrow{P} \mathbf{P}$ ($\mathbb{M} \xrightarrow{P} \mathbf{P}$), a morphism of monoidal categories, which sends $(-1) \to 0$ into $I \to P$ ($(-1) \to 0 \leftarrow 0 \otimes 0$ into $I \to P \leftarrow P \otimes P$). An additional important property is the following Theorem. Given a regular ordinal φ, if $P \in \mathbf{P}$ (as above) is φ-copresentable (i.e., the functors $-- \otimes P$ and $P \otimes --$ preserve φ-filtered colimits), then the functor $\mathbb{M}_m \xrightarrow{P} \mathbf{P}$ becomes stationary after φ (actually, $P(\varphi+\omega) \cong P\varphi$), and furthermore, there is a map $P\varphi \otimes P\varphi \to P\varphi$ which makes $I \to P\varphi \leftarrow P\varphi \otimes P\varphi$ the free monoid over $I \to P$. In the category $\mathbb{M}$, 0 is not φ-copresentable for any regular ordinal φ. (Received January 27, 1972.)

693-A39. JOAN M. NEGREPONTIS, Sir George Williams University, Montreal, Quebec, Canada. Duality of functors in the category of Banach spaces.

This paper will present a further development of the duality notions initiated by Svarc, Linton, and others with applications. (Received January 27, 1972.)

Analysis

*693-B1. GRANT B. GUSTAFSON and KLAUSS SCHMITT, University of Utah, Salt Lake City, Utah 84112. Nonzero solutions of boundary value problems for second order ordinary and delay-differential equations. Preliminary report.

A fixed-point theorem for completely continuous operators on a cone k in a Banach space is established, in order to study the question of existence of nonzero solutions to the boundary value problem $x''(t) + f(t,x(t)) = 0$, $ax(0) - bx'(0) = cx(T) + dx'(T) = 0$. The functional equation $x''(t) + f(t,x_t) = 0$ with similar boundary conditions is also studied. (Received November 8, 1971.)

*693-B2. RICHARD M. CROWNOVER, University of Missouri, Columbia, Missouri 65201. Commutants of shifts on Banach spaces.

We consider a class of operators on an arbitrary complex Banach space which may be expressed as

certain unilateral shift operators on a Banach space of sequences. An analytic structure is associated in such a way that each element of the Banach space may be expressed as an analytic function on a neighborhood in the spectrum of the operator. This identification enables us to view the commutant of the operator as an algebra of multiplications by bounded analytic functions on this neighborhood. We thus are led to a commutant theory similar to that for the unilateral shift on the sequence space ℓ^2. An application of the present theory is made to a commutant problem of Shields and Wallen. For the special case in which the shift operator is an isometry, sharper estimates on the size of the convergence set for the commutant are available. It is shown that the commutant in this case is a subalgebra of the space H^∞ on the unit disk. Other applications of the analytic structure and commutant theory are made to factorization of power series with coefficients in ℓ^p, the spectrum of a shift, and the nonexistence of roots. An approximation theory for the commutant is discussed for special cases of considerable interest. (Received December 20, 1971.)

693-B3. PAUL M. GAUTHIER and NGO VAN QUE, Université de Montréal, Montréal, Québec, Canada. The value distribution of most holomorphic mappings. Preliminary report.

Leon Brown and the first author have shown that nearly all holomorphic (meromorphic) functions have a Picard behaviour (equidistribution of values). The purpose of this preliminary report is to point out that similar theorems hold for holomorphic mappings. For example, if X is a Stein space of dimension at least n, then nearly all (in the topological sense) holomorphic mappings from X into n-dimensional complex space are surjective. The same is true for mappings into complex projective n-space. The methods are standard category arguments. (Received January 3, 1972.)

*693-B4. COLIN C. GRAHAM, Northwestern University, Evanston, Illinois 60201. Interpolation sets for convolution measure algebras.

Examples of regular, symmetric commutative convolution measure algebras A are given for which: (1) the union of two interpolation sets for A is not (necessarily) an interpolation set; (2) a compact interpolation set is not a set of spectral synthesis. It is also shown that for G a nondiscrete LCA group, there exist two interpolation sets for M(G) whose union is not an interpolation set. These generalize results of Körner (Proc. Cambridge Philos. Soc. (67), 559-568). (Received January 7, 1972.)

693-B5. RICHARD D. CARMICHAEL, Wake Forest University, Winston-Salem, North Carolina 27109. Analytic representation of the finite Fourier transform. Preliminary report.

Let $A = (a_1, \ldots, a_n)$ be a fixed vector of positive real numbers. $K(A)$ is the set of complex valued infinitely differentiable functions whose support is in $\{t: |t_j| \leq a_j,\ j = 1, \ldots, n\}$. The finite Fourier transform $F_A(\varphi)$, $\varphi \in K(A)$, is a one-one mapping of $K(A)$ onto the space $Z(\pi)$ of all complex valued infinitely differentiable functions which can be extended to be entire functions $\psi(z)$ satisfying $|z^\alpha \psi(z)| \leq C_\alpha \exp(\pi(|y_1| + \ldots + |y_n|))$ for every n-tuple α of nonnegative integers. The distributional finite Fourier transform of $U \in K'(A)$ is an element $V \in Z'(\pi)$ defined by $\langle V, \psi \rangle = (2^n a_1 \ldots a_n)^{-1} \langle U, \check{\varphi} \rangle$, where $\varphi \in K(A)$, $\psi = F_A(\varphi) \in Z(\pi)$, and $\check{\varphi}(t) = \varphi(-t)$; and it is a one-one mapping of $K'(A)$ onto $Z'(\pi)$. We give several representation theorems of this

distributional finite Fourier transform which extend some results of Warmbrod [SIAM J. Appl. Math. 17(1969), 930-956]. Let C be an open convex cone in $\mathbb{R}^n$, and put $T^C = \mathbb{R}^n + iC$. We further show that this finite Fourier transform can be represented as the distributional boundary value of functions which are analytic in T^C and which satisfy a specified growth condition. (Received January 7, 1972.)

*693-B6. ALAN R. ELCRAT, Wichita State University, Wichita, Kansas 67208. Some a priori estimate for elliptic second order equations with Dirichlet boundary data.

An explicit a priori estimate for an elliptic second order partial differential equation in n-variables with bounded measurable coefficients is given. This estimate is obtained under the following assumptions: (1) The boundary of the region in question has nonnegative mean curvature; (2) The coefficients of the principal part have bounded first order (distribution) derivatives; (3) The differential operator is "sufficiently close" to being selfadjoint. The estimate is of the form $\|U\|_2 \leq C\|LU\|_0$ where $\| \ \|_2$, $\| \ \|_0$ are the norms in $W^2_{2,0}$, L_2, respectively. Suppose the operator is $AD^2U + B \cdot \nabla U - aU$, the matrix A being positive definite. In case this operator is selfadjoint, a bound $C = C(a_0)$ can be given for $a_0 \in (-\lambda\nu^2, \infty)$ where $a_0 = \inf a$, $\xi A\xi \geq \nu^2|\xi|^2$ for real n-vectors ξ, and λ is the lowest eigenvalue of Δ with Dirichlet boundary condition. The explicit nature of this bound is of value in giving constructive existence theorems for nonlinear elliptic equations, and it is this purpose which motivates the above work. (Received January 10, 1972.)

*693-B7. EARL R. BERKSON and LEE A. RUBEL, University of Illinois, Urbana, Illinois 61801. On the nonseparability of L^∞/H^∞. Preliminary report.

Theorem. L^∞/H^∞ is not separable. Two proofs of this result are given. The first proof uses a theorem of Iversen in the theory of cluster sets. A corollary of the authors' theorem is that H^1 is not reflexive, a result that was proved by A. E. Taylor in 1950. The authors actually prove that H^1 is not even homeomorphic as a topological space to its second dual. The second proof uses the known nonreflexivity of H^1, and derives the theorem from it by using some results in functional analysis. (Received January 11, 1972.)

*693-B8. B. A. TAYLOR, University of Michigan, Ann Arbor, Michigan 48104. Discrete sufficient sets for some spaces of entire functions.

Let E denote the space of all entire functions f of exponential type (i.e. $|f(z)| = O(\exp(B|z|))$ for some $B > 0$). Let $\mathcal{K}$ denote the space of all positive continuous functions k on the complex plane $\mathbb{C}$ with $\exp(B|z|) = O(k(z))$ for each $B > 0$. For $k \in \mathcal{K}$ and $S \subset \mathbb{C}$, let $\|f\|_{k,S} = \sup\{|f(z)|/k(z) : z \in S\}$. We prove that the two families of seminorms $\{\| \ \|_{k,\mathbb{C}}\}_{k\in\mathcal{K}}$ and $\{\| \ \|_{k,S}\}_{k\in\mathcal{K}}$, where $S = \{n+im : -\infty < n, m < +\infty\}$, determine the same topology on E. (Received January 11, 1972.)

*693-B9. JOEL H. SHAPIRO, Michigan State University, East Lansing, Michigan 48823. Absolutely p-summing composition operators on H^p spaces.

An analytic function φ taking the unit disc into itself defines a composition operator C_φ on the Hardy

space H^p by the equation $C_\varphi f = f \circ \varphi$. J. Ryff [Duke Math. J. 33(1966), 347-354] proved that C_φ is bounded on H^p, and H. J. Schwartz [Thesis, University of Toledo, 1969] showed that C_φ is compact on H^p whenever $\int_0^{2\pi} [1 - |\varphi(e^{it})|]^{-1} dt < \infty$. We show that this condition on φ actually characterizes the absolutely p-summing composition operators on H^p for $2 \leq p < \infty$; and is sufficient for C_φ to be absolutely p-summing on H^p for $1 < p < 2$, and nuclear on H^1. In particular, C_φ is absolutely p-summing on H^p $(1 < p < \infty)$ and nuclear on H^1 whenever φ maps the unit disc into a polygon inscribed in the unit circle. In contrast, if $\|\varphi\|_\infty = 1$ and $\varphi' \in H^\infty$, then C_φ is not even compact on H^p $(p \geq 1)$. (Received January 11, 1972.)

*693-B10. STEPHEN D. FISHER, Northwestern University, Evanston, Illinois 60201. Invariant subalgebras for the backward shift. Preliminary report.

Let D be a domain in the complex plane which contains the origin. The backward shift operator T is defined by $Tf(z) = z^{-1}(f(z) - f(0))$ for f holomorphic in D. Let Γ be a simple closed curve; Γ is said to have zero continuous analytic capacity if the only functions which are continuous on the Riemann sphere and holomorphic off Γ are the constants. Let $A(\Gamma)$ be the algebra of functions which are continuous on Γ and have a holomorphic extension to the region bounded by Γ. Theorem. If Γ has zero continuous analytic capacity and if A is a nontrivial closed subalgebra of $A(\Gamma)$ which contains the constants and is invariant under T, then $A = A(\Gamma)$. If Γ has positive continuous analytic capacity, then the theorem does not necessarily hold. Various extensions and applications of the theorem are possible; for example, if D is bounded by a finite number of disjoint simple closed curves each of which has zero c.a. capacity, then the subalgebras of A(D) which are T-invariant and contain 1 can be described. Another application is to describe the T-invariant subalgebras of the space of all holomorphic functions on D which are closed in the topology of uniform convergence on compact subsets and contain 1. (Received January 13, 1972.)

693-B11. EARL R. BERKSON and LEE A. RUBEL, University of Illinois, Urbana, Illinois 61801 and JAMES P. WILLIAMS, Indiana University, Bloomington, Indiana 47401. Totally hyponormal operators and analytic functions. Preliminary report.

Let K be a Hilbert space, [K] its ring of bounded operators. Call $A \in [K]$ totally hyponormal if its commutant $\{A\}'$ consists of hyponormal operators. In this case $\{A\}'$ must be commutative, and one gets: Theorem. An analytic Toeplitz operator T on H^2 is totally hyponormal if and only if $\{T\}' = \{U\}'$, U the unilateral shift. Corollary. The totally hyponormal isometries of H^2 into H^2 which commute with U are precisely the analytic Toeplitz operators induced by functions of the form $\varphi(z) = \lambda(z - z_0)(1 - \bar{z}_0 z)^{-1}$ for λ, z_0 constants satisfying $|z_0| < 1 = |\lambda|$. It is shown that if ψ is a weak-star generator of H^∞ (such functions have been characterized in terms of their mapping properties by D. Sarason), then the polynomials in ψ are dense in H^2 in the latter's norm topology, and this in turn implies that the Toeplitz operator induced by ψ is totally hyponormal. Related matters concerning the bilateral shift are treated, and a characterization of the general totally hyponormal isometry is presented. (Received January 13, 1972.)

693-B12. GEORGE GASPER, Northwestern University, Evanston, Illinois 60201. Inequalities for orthogonal polynomials and certain generalized hypergeometric series with only real zeros. Preliminary report.

The main tool the author used in ["An inequality of Turán type for Jacobi polynomials," Proc. Amer. Math. Soc., to appear] was the fact that if $p_n = p_n(z)$ is a polynomial of degree n with zeros $z_1, z_2, \ldots, z_n$, then $p_n'/p_n = \sum_1^n (z - z_k)^{-1}$ and $-(p_n'/p_n)' = \sum_1^n (z - z_k)^{-2}$, where a prime denotes differentiation with respect to z. It is found that these identities can also be used to obtain inequalities of Turán type for orthogonal polynomials of a discrete variable (Charlier, Krawtchouk, and Meixner polynomials) provided that the differentiations are performed with respect to appropriate parameters. In studying the Hahn polynomials we are led to determine when certain generalized hypergeometric series have only real zeros. (Received January 13, 1972.)

*693-B13. PETRU MOCANU, Babès-Bolyai University, Cluj, Romania and MAXWELL O. READE, University of Michigan, Ann Arbor, Michigan 48104. On α-convex functions. Preliminary report.

The authors continue their investigations of the class of functions $f(z) = z + \ldots$, analytic in the unit disc Δ, satisfying $[f(z)f'(z)/z] \neq 0$ and (*) $\mathrm{Re}[(1-\alpha)zf'(z)/f(z) + \alpha(1 + z(f''(z)/f'(z)))] \geq 0$ there, with $\alpha \geq 0$. The authors can now show that for certain negative values of α, functions f(z) satisfying (*) are univalent in Δ (and even starlike). The authors also show that the same f(z) may satisfy (*) for two different values of α. (Received January 12, 1972.)

*693-B14. CHARLES L. FEFFERMAN, University of Chicago, Chicago, Illinois 60637. Some recent results on H^p.

E. M. Stein and the author have recently found simpler and more precise methods than were known before to study Fourier analysis on the H^p spaces. In the setting of the upper half-space $R_+^{n+1} = \{(x_1, \ldots, x_n, t) \mid t > 0\}$ our results include the following. (1) The dual of H^1 may be naturally identified with $BMO(R^n)$, a Banach space defined by F. John and L. Nirenberg. (2) H^p is isomorphic to a space of distributions on R^n whose "maximal functions" are in L^p, $0 < p < \infty$. (This is closely related to work of Burkholder-Gundy-Silverstein.) (3) Suitable singular and strongly singular integrals are bounded on H^p. (Received January 14, 1972.)

693-B15. JAMES G. CAUGHRAN, University of Toronto, Toronto 181, Ontario, Canada. Spectra of composition operators on H^2. Preliminary report.

Let φ be a nonconstant analytic function taking the unit disk into itself. Then the composition operator C_φ defined by $\{C_\varphi f\}(z) = f\{\varphi(z)\}$ is a bounded linear operator on the Hilbert space H^2. We investigate the spectrum of these operators and determine the spectra of operators in certain classes of composition operators. (Received January 17, 1972.)

693-B16. RONALD R. COIFMAN and GUIDO L. WEISS, Washington University, St. Louis, Missouri 63130. A theory of H^p spaces associated with compact Lie groups. Preliminary report.

Let G be an n-dimensional Lie group and $\mathfrak{L}$ its Lie algebra (the left invariant derivations) which we

assume endowed with an inner product (,) satisfying $([X,Y],Z) = -(Y,[X,Z])$ (this implies $\mathfrak{g}$ is the Lie algebra of a compact group). A function u on $\mathbf{R}^+ \times G = \{(x_0,x) : x \in G,\ x_0 > 0\}$ is harmonic provided $\Delta u = (\Delta + \partial^2/\partial x_0^2)u = 0$, where $\Delta = \sum_{j=1}^n X_j^2$ with $\{X_1, X_2, \ldots, X_n\}$ an orthonormal basis of $\mathfrak{g}$. Writing $X_0 = \partial/\partial x_0$ and extending the inner product to $\mathbf{R} \oplus \mathfrak{g}$ by requiring $\{X_0, X_1, \ldots, X_n\}$ to be orthonormal, we consider the following generalization of the Cauchy-Riemann equations (when $n \geq 2$): for $F = (f_1, f_2, \ldots, f_n) : \mathbf{R}^+ \times G \to \mathbf{R}^{n+1}$ satisfies (a) $X_i f_j - X_j f_i = ([X_i, X_j], F)$ and (b) $\sum_{j=0}^n X_j f_j = 0$. For such an F it can be shown that $|F|^p$ is "subharmonic" for $p \geq (n-1)/n$. This permits us to develop an H^p-space theory for $(n-1)/n \leq p < \infty$, which, when n = 2, reduces to the classical one associated with the disc (and G is the circle group). (Received January 17, 1972.)

*693-B17. JAMES G. CAUGHRAN, University of Toronto, Toronto 181, Ontario, Canada and ALLEN L. SHIELDS, University of Michigan, Ann Arbor, Michigan 48104. Singular inner factors of analytic functions.

Let S be a singular inner function. We give a review of recent work and unsolved problems concerning the function class to which S' belongs. We also comment on the set of omitted values of S. Finally, if f is in H^p for some $p > 0$, we discuss the set of c such that f - c has a nontrivial singular inner factor. (Received January 18, 1972.)

*693-B18. JAMES M. BRIGGS, University of Wisconsin, Whitewater, Wisconsin 53190. Approximation in norms defined by derivations. Preliminary report.

A linear mapping D from the polynomials $P([0,1])$ into $C([0,1])$, the continuous functions on $[0,1]$, is a derivation provided that $D(fg) = fD(g) + gD(f)$ for all polynomials f and g. Each such derivation is of the form D_w where $w \in C([0,1])$ and $D_w(f) = wf'$, where f' denotes the derivative of f. If $\|\cdot\|$ denotes the supremum norm on $C([0,1])$, then to each derivation D_w corresponds the norm $\|\cdot\|_w$ on $P([0,1])$ defined by $\|f\|_w = \|f\| + \|D_w(f)\|$. Let $C^1(w)$ denote the Banach algebra completion of $P([0,1])$ under the norm $\|\cdot\|_w$. The algebras $C^1(w)$ are determined for a large class of functions w. (Received January 17, 1972.)

693-B19. MICHAEL E. FRAZIER, University of Michigan, Ann Arbor, Michigan 48104. Separating algebras, strongly separating algebras, and peak modulus sets. Preliminary report.

In answer to the question of Csordas and Reiter (Abstracts 682-46-9 and 682-46-10, these *Notices* 18(1971), 178), there are nonseparating essential function algebras (different construction by L. Eifler). Examples illustrate a hierarchy among defining spaces on which algebras are separating. Much of Csordas-Reiter's paper reduces to the observation that approximately regular algebras are separating. Counterexamples, a theorem, and unsolved questions inquire whether tensor products of separating algebras are separating. Call A strongly separating if for all disjoint, closed $K, L \subseteq X$ there exists $f \in A$ such that $f(K) \cap f(L)$ is empty. Algebras separating but not strongly are supplied from several complex variables and otherwise by glueing. Various conditions to imply strong separation are investigated. Define peak modulus set (PMS) naturally. B. A. Taylor showed that (the product of a semicircle and a circle) with one point added within the unit torus fails

to be a PMS for the 2-disc algebra. Question. Are the red squares in a four-square checkerboard flattened torus a PMS for the 2-disc algebra? (Received January 17, 1972.)

*693-B20. D. J. PATIL, University of Wisconsin, Milwaukee, Wisconsin 53201. Representation of H^p-functions.

Let D be the open unit disc in the plane and C its boundary. Let H^p $(1 \leq p \leq \infty)$ be the Hardy spaces on D. Theorem I. Let $E \subset C$ with $m(E) > 0$, $1 \leq p \leq \infty$, $f \in H^p$ and let g be the restriction of f to E. For each $\lambda > 0$ define h_λ, g_λ on D by $h_\lambda(z) = \exp\{-(1/4\pi)\log(1+\lambda)\int_E (e^{i\theta}+z)/(e^{i\theta}-z)d\theta\}$, $g_\lambda(z) = \lambda h_\lambda(z)(1/2\pi i)$ $\cdot\int_E \{\overline{h}_\lambda(w)g(w)/(w-z)\}dw$. Then as $\lambda \to \infty$, $g_\lambda \to f$ uniformly on compact subsets of D. Moreover, for $1 < p < \infty$, $\|g_\lambda - f\|_p \to 0$ as $\lambda \to \infty$. Theorem II. Let $E \subset C$, $0 < m(E) < m(C)$. A function $g \in L^p(E)$, $1 < p < \infty$, is the restriction to E of some $f \in H^p$ if and only if $\sup_{\lambda>0}\|g_\lambda\|_p < \infty$. (Received January 20, 1972.)

693-B21. ARLENE PARSEKIAN FRAZIER, University of Michigan, Ann Arbor, Michigan 48104. Linear functionals and multipliers on H^p of the polydisc.

Let Δ^n denote the n-dimensional polydisc and $H^p(\Delta^n)$ $(0 < p \leq \infty)$ the usual Hardy space of analytic functions. The Hardy-Littlewood inequality on the growth of H^p functions (see Duren, "Theory of H^p spaces," p. 87) is extended to the polydisc. As an application of this generalization, the continuous linear functionals on $H^p(\Delta^n)$ $(0 < p < 1)$ are completely represented by functions satisfying a growth condition on the partial derivative of order $(m, \ldots, m)$, where m is the integer such that $m \leq 1/p < m + 1$. Let $B^p(\Delta^n)$ $(0 < p < 1)$ be the natural generalization of the space B^p (see Duren, Romberg, and Shields, "Linear functionals on H^p spaces with $0 < p < 1$," J. Reine Angew. Math. 238(1969), 32-60). Characterizations are also given for the coefficient multipliers of $H^p(\Delta^n)$ $(0 < p < 1)$ into the spaces $B^q(\Delta^n)$ $(0 < q < 1)$, $H^q(\Delta^n)$ $(q \geq 1)$, and ℓ^q $(q \geq 2)$. (Received January 20, 1972.)

693-B22. JOHN A. KELINGOS, Vanderbilt University, Nashville, Tennessee 37203. Distortion of hyperbolic area under quasiconformal mappings.

Suppose D is a simply connected domain in the complex plane. For each measurable subset E denote by $m_h(E)$ and $R_h(E)$ the hyperbolic area and the hyperbolic radius of E, respectively. The author proves the following distortion Theorem. For each K, $1 \leq K < \infty$, and each R, $0 < R < \infty$, there exists an increasing continuous function $\psi_{K,R}$ with $\psi(0) = 0$, $\psi(\infty) = \infty$, such that, if f is a K-quasiconformal mapping of D, then $m_h(f(E))/\pi = \psi_{K,R}(m_h(E)/\pi)$ for each measurable set $E \subset D$ with $R_h(E) \leq R$. A specific formula for ψ is obtained together with asymptotic behavior. The proof uses a parametric representation for quasiconformal mappings. (Received January 10, 1972.)

*693-B23. JEAN-PIERRE GOSSEZ, University of Chicago, Chicago, Illinois 60637. Quasilinear elliptic equations with rapidly increasing coefficients.

Let Ω be an open bounded subset of R^n with smooth boundary and consider the Dirichlet problem for the

differential operator $A(u) = \sum_{|\alpha|\leq m} (-1)^{|\alpha|} D^{\alpha}(p(D^{\alpha}u)) + \sum_{|\alpha|=m} (-1)^{|\alpha|} D^{\alpha} A_{\alpha}(x, u, \ldots, D^{m-1}u) + \sum_{|\alpha|<m} (-1)^{|\alpha|} D^{\alpha} A_{\alpha}(x, u, \ldots, D^{m}u)$, where $p: R \to R$ is any strictly increasing odd continuous function with $p(+\infty) = +\infty$ and where the A_{α}'s satisfy some growth condition involving p together with the sign condition $\sum_{|\alpha|\leq m} A_{\alpha}(x, \xi)\, \xi_{\alpha} \geq 0$, $x \in \Omega$, $\xi \in R^{s_m}$. Define $M(t) = \int_0^t p(s)\, ds$ and let $\overline{M}$ be the N-function conjugate to M. Theorem. Given $f \in W^{-m}E_{\overline{M}}(\Omega)$, there exists $u \in W_0^m L_M(\Omega)$ satisfying $p(D^{\alpha}u)$ and $A_{\alpha}(\xi(u)) \in L_{\overline{M}}(\Omega)$ for all α, $|\alpha| \leq m$, and $a(u, v) = f(v)$ for all $v \in W_0^m E_M(\Omega)$, where $a(u, v)$ is the Dirichlet form associated with A. The particularity of the above problem is that no global a priori bound holds in general, i.e. f bounded in $W^{-m}E_M(\Omega)$ does not force the possible solutions u to remain bounded in $W_0^m L_M(\Omega)$. This theorem is a particular case of a general existence theorem whose proof is based upon the consideration of a new class of mappings of monotone type between Banach spaces. (Received January 24, 1972.)

*693-B24. DAVID DRASIN and ALLEN W. WEITSMAN, Purdue University, Lafayette, Indiana 47907. On Julia directions of functions of finite order. Preliminary report.

Let f(z) be an entire transcendental function of order $\rho \leq \infty$, and let $E = \{\theta; 0 \leq \theta \leq 2\pi$ and $\arg z = \theta$ is a direction of Julia for $f(z)\}$. Then E is a nonempty, closed subset of $[0, 2\pi]$. Polya (Math. Z. (1929)) and Anderson-Clunie (Math. Z. (1969)) showed, respectively, that when $\rho = \infty$ or 0, E need satisfy no other conditions. When $\frac{1}{2} < \rho < \infty$, the classical indicator theory leads to further restrictions on E: see Theorem 67 of Cartwright, "Integral functions." We note that these conditions are not sufficient when $\rho > 1$, and discuss the characterization of E for functions of finite order. (Received January 26, 1972.)

693-B25. JOSEPHINE M. MITCHELL, State University of New York at Buffalo, Amherst, New York 14226 and KYONG T. HAHN, Pennsylvania State University, University Park, Pennsylvania 16802. Representation of linear functionals on H^p spaces, $0 < p < 1$, over bounded symmetric domains.

Let D be a bounded symmetric domain in C^N $(N > 1)$ with Bergman-Šilov boundary b. Let f be holomorphic on D. $f \in H^p$ $(p > 0)$ if $\|f\|_p = \sup_{0 \leq r < 1} (V^{-1} \int_b |f(rt)|^p\, ds_t)^{1/p} < \infty$ (ds_t euclidean volume element on b). In this paper we generalize results of Duren, Romberg and Shields [J. Reine Angew. Math. 238(1969)] for the disc to bounded symmetric domains. Theorem 1. Let $T \in (H^p)^*$, $p > 0$. There is a unique holomorphic function G on D such that (1) $T(f) = \lim_{r \to 1} V^{-1} \int_b f(\rho t)\overline{G}(r\rho^{-1}t)\, ds_t$ $(\equiv (f_{\rho}, G_{r\rho^{-1}}))$, $0 < r < \rho < 1$, for all $f \in H^p$. Conversely if G is holomorphic on D and (1) holds for all $f \in H^p$, then $T \in (H^p)^*$. Corollary. Theorem 1 holds for bounded star-shaped circular domains in C^N with a Bergman-Šilov boundary. The following lemma gives deeper results for the classical Cartan domains R_j. Lemma. There exists p_0, $0 < p_0 < 1$, such that the Szegö kernel of R_j satisfies the inequality $\|S_{t_0}\|_p \leq C(p, N)$ for $0 < p < p_0$, $t_0 \in b$, where $C(p, N)$ is a constant depending only on p and N. p_0 is sharp for the ball R_I with $m = n = 2$ and R_{II} and R_{III} with $n = 2$. Results following from the lemma are: Theorem 2. Let $Np_0/(N+q) < p < Np_0/(N+q-1)$ $(q = 0, 1, 2, \ldots)$ and $j \neq IV$. Then the derivative $G^{(q-1)} \in \Lambda_{\alpha}$ where $\alpha = N(p_0/p - 1) - q + 1$. Conversely if $G^{(q-1)} \in \Lambda_{\alpha}$, then $\lim_{r \to 1} (f_r, G)$ exists for all $f \in H^p$ and defines a functional in $(H^p)^*$. For R_{IV} replace N by $\frac{1}{2}N$. Also the Banach spaces $(H^p)^*$ and Λ_{α}^{q-1} are equivalent. (Received January 26, 1972.)

693-B26. JOHN R. QUINE, Florida State University, Tallahassee, Florida 32306. On the double points of the image of the unit circle under a polynomial mapping. Preliminary report.

Let $p(z)$ be a polynomial of degree n. We say w is a double point of the curve $p(e^{i\varphi})$, $0 \leqq \varphi \leqq 2\pi$, if there exist z_1 and z_2 distinct such that $|z_1| = |z_2| = 1$ and $p(z_1) = p(z_2) = w$. Theorem. If $p(z)$ is not of the form $g(z^m)$ where g is a polynomial and m is an integer, $m > 1$, then the curve $p(e^{i\varphi})$ has at most $(n-1)^2$ double points. This bound is sharp. The theorem is based on a conjecture of C. J. Titus (Acta. Math. 106(1961), p. 60). The method of proof relates to the problem of computing the polynomials univalent in $|z| < 1$. (Received January 26, 1972.)

693-B27. PATRICK AHERN, University of Wisconsin, Madison, Wisconsin 53706. On R.P. measures on the torus.

A measure on the torus is called an R.P. measure if its Poisson integral is the real part of a holomorphic function in the polydisc. Some results on the structure of R.P. measures are given with some applications to factoring holomorphic functions. (Received January 26, 1972.)

693-B28. GERALD A. KRAUS, Southern Illinois University, Carbondale, Illinois 62901. Unique extensions of abstract measures.

Let μ be a measure on a semiring $\mathscr{S}$ of subsets of a set X, as defined by N. G. de Bruijn and A. C. Zaanen [Indag. Math. 16(1954), 456-466]. Let μ^* be the outer measure on 2^X induced by μ. Let $\mathscr{M}$ be the σ-algebra of all (Carathéodory) μ^*-measurable sets. Let $\mathscr{E}$ be any semiring of subsets of X which contains $\mathscr{S}$. If μ^* acts as a measure on $\mathscr{E}$ then $\mathscr{E}$ is contained in $\mathscr{M}$. Theorem 1. If λ is a measure on $\mathscr{E}$ which extends μ then each μ^*-measurable set A with $\mu^*A < \infty$ is λ^*-measurable and $\lambda^*A = \mu^*A$. Theorem 2. If $\mathscr{E} \subset \mathscr{M}$ then there is a unique measure λ on $\mathscr{E}$ which extends μ and has the following property: Each $E \in \mathscr{E}$ with $\lambda E < \infty$ is μ-σ-finite. Theorem 3. If $\mathscr{E}$ is a σ-ring and $\mathscr{E} \subset \mathscr{M}$ then there is a unique measure on $\mathscr{E}$ which extends μ iff the following holds: For each locally μ-null set $L \in \mathscr{E}$ with $\mu^*L = \infty$ there is a set $S \in \mathscr{S}$ with $\mu S = \infty$ and $\mu^*(S \sim L) < \infty$. The extension theorems of P. H. Maserick [Pacific J. Math. 17(1966), 137-148] and W. Rinow [Math. Nachr. 42(1969), 289-294] are immediate consequences of the above theorems. (Received January 27, 1972.)

*693-B29. JOHN V. BAXLEY, Wake Forest University, Winston-Salem, North Carolina 27109. On the Weyl spectrum of a Hilbert space operator.

Let T be a closed (possibly unbounded) linear operator in a Hilbert space H. Question. To what extent can the Weyl spectrum $\omega(T)$ differ from the set A of points in the spectrum of T which are not isolated eigenvalues of finite algebraic multiplicity? In addition to providing an answer to this question, conditions on T are given which guarantee that $\omega(T) = A$. The techniques are an extension of those used by the author (Rev. Roumaine Math. Pures Appl. 16(1972), 1163-1166) and the results complement those of R. Rouldin (Trans. Amer. Math. Soc., to appear). (Received January 27, 1972.)

693-B30. C. MAX WILLIAMS, University of Oklahoma, Norman, Oklahoma 73069. Oscillation phenomena for linear differential systems in a B*-algebra.

This paper extends results for a linear selfadjoint matrix differential system to the corresponding differential system $u' = a(t)u + b(t)v$, $v' = c(t)u - a^*(t)v$, where the coefficient functions and solutions assume their values in a B*-algebra. The concepts of disconjugacy and nonoscillation for the linear differential system on a compact interval are considered. A hermitian form, which is a generalization of the Dirichlet functional, is introduced, and necessary and sufficient conditions for the linear system to be nonoscillatory on a compact interval are expressed in terms of spectral properties of this hermitian form. With the aid of these conditions, several Sturmian type comparison theorems are established. (Received January 27, 1972.)

*693-B31. THOMAS A. W. DWYER III, Northern Illinois University, DeKalb, Illinois 60115. Analytic representations of tempered distributions and partial differential equations in infinite dimensions.

We extend the Fischer-Fock space $F_1(E)$ of entire functions on a Hilbert space E defined in [Bull. Amer. Math. Soc. 77(1971), 725-730] to countably Hilbert spaces. If $E = \text{inv lim}_\ell E_\ell$ with the $E_{\ell+1} \to E_\ell$ injective and with Hilbert-Schmidt norms $\leqq 1$ then $E' = \text{dir lim}_\ell E'_\ell$. Let $P(^nE') = \text{inv lim}_\ell P_H(^nE'_\ell)$ and $P(^nE) = \text{dir lim}_\ell P_H(^nE_\ell)$, where $P_H(^nE'_\ell)$ and $P_H(^nE_\ell)$ are as in [Bull. Amer. Math. Soc., loc. cit.]. The duality between $P_H(^nE'_\ell)$ and $P_H(^nE_\ell)$ extends to $P(^nE')$ and $P(^nE)$. Let $F(E') = \text{inv lim}_\ell F_1(E'_\ell)$ and $F(E) = \text{dir lim}_\ell F_1(E_\ell)$. The Borel duality $\langle\ ,\ \rangle$ between $F_1(E'_\ell)$ and $F_1(E_\ell)$ extends to $F(E')$ and $F(E)$. Given P' in $\sum_n P(^nE)$, the differential operator $P'(d)$ and $P' \cdot$ are defined as in [Bull. Amer. Math. Soc., loc. cit.], as well as $P(d)$ and $P \cdot$ for P in $\sum_n P(^nE')$. If the $E_{\ell+1} \to E_\ell$ have Hilbert-Schmidt norms $\leqq 2^{-1/2}$ then $P \cdot F(E') \subset F(E')$ and $P'(d)F(E') \subset F(E')$, analogously on E. The Borel duality gives $\langle P'(d)f, g'\rangle = \langle f, P' \cdot g'\rangle$. Using [Lemma 3, Bull. Amer. Math. Soc., loc. cit.] on component spaces E_ℓ and extending products of functions from E to E_ℓ we get $P'(d)F(E') \supset F(E')$, analogously on E. If $E = S =$ rapidly decreasing functions on R then $F(S)$ and $F(S')$ represent spaces of tempered distributions in infinite dimension of Kristensen-Mejlbo-Poulsen [Comm. Math. Phys. 1(1965), 175-214]. (Received January 27, 1972.)

Applied Mathematics

*693-C1. R. ARTHUR KNOEBEL, New Mexico State University, Las Cruces, New Mexico 88001. The composition of threshold functions. Preliminary report.

Assume all switching functions are positive, nonconstant, and depend on all arguments. Let T be the set of all threshold functions, J (M) the set of joins (meets) of arguments, and $\{\iota\}$ the set with only the identity function. A set is OK if it is one of these four. Let $f: 2^m \to 2$ and $g: 2^n \to 2$. Denote the composition $f(x_1, \ldots, x_{m-1}, g(x_m, \ldots, x_{m+n-1}))$ by $f \circ g$, and set $f^* = \{g \in T | f \circ g \in T\}$. Easy result. If $f \circ g \in T$, then $f \in T$ and $g \in T$. We give partial converses. We assume $f \in T$ has weights $w_1, \ldots, w_m$. Theorem. If the weights are integral and $w_m = 1$, then $f^* = T$. Corollary. If f is symmetric, then $f^* = T$. Theorem. If x_m is not a minimal argument of f, then $f^* \subseteq J \cup M$. Proposition. If $m \leqq 4$, then f^* is OK. Counterexample. There is $f \in T$ with $m = 11$ such that f^* is not OK. Let $\sim$ be an equivalence on $m - 1$ with equivalence

classes of size $m_1,\ldots,m_k$, and L be the finite array of lattice points $(n_1,\ldots,n_k)$ such that $0 \leqq n_h \leqq m_h + 1$ $(1 \leqq h \leqq k)$. Theorem. That f* is OK for all $f \in T$ such that $i \sim j$ implies $w_i = w_j$ is equivalent to the following: If three parallel planes π_1, π_2, π_3 divide L into four regions $R_{\infty 1}$, R_{12}, R_{23}, $R_{3\infty}$, respectively such that $R_{\infty 1} \neq \emptyset$ and $R_{12} = \emptyset$ and $d(\pi_1, \pi_2) > d(\pi_2, \pi_3)$, then the points of R_{23} are coplanar. Proposition. This always holds for two-dimensional lattices. (Received January 27, 1972.)

Geometry

693-D1. LAWRENCE W. CONLON, St. Louis University, St. Louis, Missouri 63103 and Washington University, St. Louis, Missouri 63130. Orbits of codimension two.

Let K be a compact connected Lie group acting smoothly on a connected manifold M. Suppose the principal orbit has codimension 2 in M. Let $N \subset M$ be any orbit. Theorem. $\exists$ K-invariant neighborhood U of N in M such that $U^* = U - N$ contains at most two strictly singular orbit types and at most one singular maximal orbit type. The principal orbit is a sphere bundle over each singular orbit in U^*. If $M \cong R^n$, there is no singular maximal orbit and we may take $N = \{0\}$ and $U = M$. The general result easily reduces to the linear case which itself is proven by Morse theoretic methods due principally to Bott and Samelson. The crucial observation here is that a compact linear Lie group with principal orbit of codimension two in R^n automatically has a 2-dimensional K-transversal domain in the sense of [L. Conlon, "Variational completeness and K-transversal domains," J. Differential Geometry 5(1971), 135-147]. (Received December 23, 1971.)

*693-D2. RICHARD S. MILLMAN, Southern Illinois University, Carbondale, Illinois 62901. Bundle homogeneous holomorphic fiber bundles and holomorphic connections.

Let M be a compact complex manifold and $\xi : G \to P \to M$ be a holomorphic principal fiber bundle (h.p.f.b.). Let A(M) be the identity component of the group of holomorphic diffeomorphisms of M, $F(\xi)$ be the identity component of the group of holomorphic bundle automorphisms of ξ. Let $\pi : F(\xi) \to A(M)$ assign to a bundle automorphism the map it induces on M. ξ is called bundle homogeneous if π is surjective. Theorem 1. If ξ has a holomorphic connection then ξ is bundle homogeneous. The converse is false; however, Theorem 2. If M is compact, complex parallelisable then an h.p.f.b. with base space M admits a holomorphic connection if and only if it is bundle homogeneous. If M is a complex torus, this theorem is due to Y. Matsushima ("Fibrés holomorphes sur un tore complexe," Nagoya Math. J. 14(1959), 1-24). Applying these theorems gives an example of a real product bundle (R. Millman, "Complex structures on real product bundles with applications to differential geometry," Trans. Amer. Math. Soc. (to appear)) over the Iwasawa manifold which does not admit a holomorphic connection. Corollary. The Atiyah obstruction is not a topological invariant. (Received December 27, 1971.)

*693-D3. FRANK W. WARNER and JERRY L. KAZDAN, University of Pennsylvania, Philadelphia, Pennsylvania 19104. Curvature functions for 2-manifolds.

If M is a compact 2-manifold, the Gauss-Bonnet Theorem gives obvious sign conditions for a given

function K to be the curvature of some Riemannian metric. One asks if these conditions are sufficient. Theorem. Yes for $\chi(M) \leqq 0$. (J. Moser has shown Yes for P^2. Others have partial results for S^2.) For a large class of open 2-manifolds every C^∞ function is shown to be a curvature. We seek the given function as the curvature of a metric that is pointwise conformal to some prescribed metric. This leads one to the equation $\Delta u = k - Ke^{2u}$, where k is the curvature of the given metric. In the case $\chi(M) \leqq 0$, this equation may not be solvable for K but is for $K \circ \varphi$ if φ is a suitably chosen diffeomorphism of M. The results follow from a detailed investigation of existence and nonexistence theorems for this equation. (Received January 4, 1972.)

*693-D4. HOWARD A. OSBORN, University of Illinois, Urbana, Illinois 61801. Differential geometry in PL.

Any PL manifold possesses a natural structure sheaf and a derivation into a sheaf $\mathcal{E}$ of differential forms; by taking exterior products and global sections one obtains a de Rham complex and a de Rham theorem as in [Bull. Amer. Math. Soc. 77(1971), 386-391]. The sheaves $\mathcal{E}$ are part of a fibered category of sheaves $\mathfrak{F}$ of modules over PL structure sheaves; the category also contains the category of (sheaves of local sections of) vector bundles on PL manifolds as a full subcategory. There is a Chern-Weil construction which assigns real Chern classes to complex sheaves $\mathfrak{F}$ and real Euler classes to real oriented sheaves $\mathfrak{F}$ in such a way that the usual axioms are satisfied; in particular, these classes are precisely the usual real Chern and Euler classes on the full subcategory. (Received January 4, 1972.)

693-D5. FRANZ W. KAMBER and PHILIPPE TONDEUR, University of Illinois, Urbana, Illinois 61801. Characteristic classes of modules over a sheaf of Lie algebras. Preliminary report.

An involutive subbundle L of the tangent bundle T_X of a (smooth, complex analytic or algebraic) manifold $(X, \underline{O})$ defines a filtration of the de Rham complex $\Omega^\cdot_X$, $F^p\Omega^\cdot_X = \Lambda^p \underline{Q}^* \Omega^\cdot_X$, where $\underline{Q}$ is the sheaf of sections of the bundle $Q = T/L$. The corresponding spectral sequence in hypercohomology starts with $E_1^{pq} = \mathrm{Ext}^q_{\underline{U}}(X; \underline{O}, \Lambda^p \underline{Q}^*)$ and converges to the de Rham cohomology $H^\cdot_{DR}(X) = \mathbb{H}^\cdot(X, \Omega^\cdot_X)$. Here $\underline{U}$ denotes the twisted universal envelope of the sheaf of Lie algebras $\underline{L}$ over $\underline{O}$. For a $\underline{U}$-module $\underline{E}$ (locally free of rank r over $\underline{O}$) and an invariant polynomial $\varphi \in I^p(\underline{g})$, $\underline{g} = \underline{gl}(r)$, there exist characteristic classes $\varphi_A(\underline{E}) \in Z_\infty E_1^{pp}$ (of Atiyah-type) and $\varphi_{DR}(\underline{E}) \in F^p H^{2p}_{DR}(X)$ (of de Rham-type) which correspond to each other via E_∞^{pp}. As $F^p H_{DR}(X) = 0$ for $p > k = \mathrm{rk}_{\underline{O}} \underline{Q}$, this generalizes the vanishing theorems of Bott. A combinatorial construction of these classes is giving using Čech-cohomology and the truncated Weil algebra $W(\underline{g})/J$, where $J = S^{k+1}\underline{g}^* \cdot W(g)$. (References. R. Bott, Proc. Sympos. Pure Math., vol. 16, Amer. Math. Soc., Providence, R. I., 1970, pp. 127-131; F. Kamber and Ph. Tondeur, Mem. Amer. Math. Soc. No. 113(1971).) (Received January 7, 1972.)

693-D6. NOLAN R. WALLACH, Rutgers University, New Brunswick, New Jersey 08903. Homogeneous spaces of positive curvature.

The algebraic condition on a compact simply connected even homogeneous space, M, that is necessary and sufficient for the existence of a homogeneous Riemannian structure of strictly positive curvature on M is

given. The examples of spaces satisfying this condition are given (including three new spaces having positive curvature. Odd dimensional analogues are discussed. (Received January 7, 1972.)

*693-D7. STEPHANIE B. ALEXANDER and RICHARD L. BISHOP, University of Illinois, Urbana, Illinois 61801. Convex functions on spheres. Preliminary report.

A convex function on a riemannian manifold is a real-valued function whose restriction to every geodesic is convex. The sublevel sets of a convex function are totally convex, that is, contain every geodesic segment having its ends in the set. A theorem is formulated which gives necessary and sufficient conditions for a smooth filtration by totally convex sets to be the sublevel filtration given by a smooth convex or strictly convex function. An application shows the nonexistence, in general, of maximal open subsets which support a strictly convex function. Specifically, the complement of a closed geodesic segment of any length greater than π in a unit 2-sphere supports a strictly convex function, but is not contained in any maximal such domain. The example is generalized to higher-dimensional spheres. (Received January 7, 1972.)

*693-D8. WILLIAM F. POHL, University of Minnesota, Minneapolis, Minnesota 55455. A theorem of géométrie finie.

In the Kodira Festschrift ("Sur les variétés d'ordre fini", Global analysis, Princeton Univ. Press, 1969, pp. 397-401) R. Thom gave a proof of the following. Let $f: M \to P_2(C)$ be an immersion of a closed (real) surface M in the complex projective plane. Suppose that there is a dense subset $A \subset P_2^*(C)$ of the Grassmann manifold of all (complex) lines in $P_2(C)$ such that each line of A meets f(M) in exactly m points, for m a fixed integer. Then f(M) is a (complex) algebraic curve. Thom remarks that his proof is incomplete. A more complete proof is given here. (Received January 10, 1972.)

*693-D9. ROBERT B. GARDNER, University of North Carolina, Chapel Hill, North Carolina 27514. A difference tensor and characterizations of euclidean spaces.

Let $X: M_m \to R^{m+1}$ be an imbedding of a piece of hypersurface in euclidean space with induced metric I and with a negative difinite second fundamental form II. As such one may introduce a tensor defined by subtracting the Levi-Civita connections of the Riemannian metrics of I and -II, this tensor will be called the difference tensor of the imbedding. The local behavior of this difference tensor depends essentially on the umbilic structure of the imbedding and results in various a priori inequalities. By coupling these locally defined inequalities with a global integral formula of the author which is valid for arbitrary pairs of metrics, new characterizations of the sphere by differential and integral inequalities are derived. (Received January 11, 1972.)

*693-D10. ROBERT H. BOWMAN, Arkansas State University, State University, Arkansas 72467. Second order connections. II.

The purpose of this paper is the development of certain implications of the second order connection, introduced previously by the present writer. If M is an n-dimensional C^∞ manifold, we show that a linear second order connection on M determines a "covariant derivative" ∇' on TM which satisfies the usual conditions

over the ring $\mathfrak{F}'(TM)$, the vertical lift of the ring $\mathfrak{F}(M)$ of C^∞ functions on M. Using the properties of ∇', we obtain equations analogous to those of Gauss and Weingarten, and an analog of the second fundamental form. If A, B, $C \in \mathfrak{X}'(TM)$, the module of C^∞ vector fields on TM over the ring $\mathfrak{F}'(TM)$, we obtain the maps Tor(A,B) and R(A,B)C which are $\mathfrak{F}'(TM)$ multilinear analogs of the torsion and curvature tensors. From the components of R we obtain equations analogous to those of Gauss and Codazzi, as well as an additional equation which defines a "vertical curvature tensor" on M. Finally, we obtain an invariant which we call the second order curvature of M. This yields as a special case the usual (first order) curvature of M. (Received January 24, 1972.)

693-D11. PHILIP B. ZWART, Washington University, St. Louis, Missouri 63130. Compact homogeneous spaces possessing invariant contact, symplectic or cosymplectic structures.

The main object of this paper is the study of homogeneous compact, contact, symplectic, and cosymplectic spaces. These are compact manifolds which possess a contact, symplectic, or cosymplectic structure which is left invariant by the action of a Lie group acting transitively and effectively on the manifold. In the contact case, the Lie group is reductive with center of dimension ≤ 1. (This result was already obtained in a different manner by A. Lichnerowicz.) In the symplectic case, the Lie group G is a direct product $S * R$ of a compact semi-simple Lie group S and a solvable Lie group R. The manifold $M = G/K$ is a product of the two compact homogeneous symplectic spaces $M_S = S/K \cap S$ and $M_R = R/K \cap R$, the symplectic structure on M being the "sum" of that on M_S and that on M_R. Also, although $R/K \cap R$ is not a group in general, a group multiplication can be defined on M_R so that M_R is a compact abelian Lie group and its symplectic form is an invariant skew form of maximum rank. In the cosymplectic case, the results are the same as in the symplectic case except for the fact that M_R is a compact homogeneous cosymplectic space. (Received January 26, 1972.)

693-D12. J. B. CARRELL, Louisiana State University, Baton Rouge, Louisiana 70803. Holomorphically injective complex torus actions.

A complex torus acting on a compact complex manifold M determines a complex torus action if the natural map $T \times M \to M$ is holomorphic. We wish to classify all holomorphically injective complex torus actions; i.e. those for which the orbit map $f_x(t) = tx$ induces a surjection $f_x^*: \underline{h}^{1,0}(M) \to \underline{h}^{1,0}(T)$. For example, if M is compact Kaehler, then (T,M) is holomorphically injective. Theorem. (T,M) is holomorphically injective iff there exists a properly discontinuous holomorphic action (W,N) on a simply connected complex manifold W and an $m \in \mathrm{Hom}(N,T)$, whose restriction to each isotropy group of (W,N) is an injection, such that $M = (T \times W)/N$ with the natural T action where the action of N on $T \times W$ is $(t,w)\alpha = (tm(\alpha), w\alpha)$. Moreover, Theorem. If (T,M) is a complex torus action and M is Hodge, then M is a holomorphic fibre bundle over T/Δ with connected fibre and structure group Δ, where Δ is finite. The latter theorem is not in general true if M is compact Kaehler. (Received January 6, 1972.)

693-D13. WU-HSIUNG HUANG, Wayne State University, Detroit, Michigan 48202. Closed minimal curves in riemannian manifolds with boundary.

Let $\Lambda(M)$ denote the space of H^1-mappings from the unit circle to a compact riemannian manifold M with boundary. By a minimal curve in M, we mean an element in $\Lambda(M)$ minimizing the length with respect to the variations within M. Theorem. If M contains a nonzero homotopy class (of arbitrary dimension), then M has at least a closed geodesic in the interior of M or a closed minimal curve. (Received January 27, 1972.)

Logic and Foundations

*693-E1. C. WARD HENSON and LAWRENCE C. MOORE, JR., Duke University, Durham, North Carolina 27706. The dual space of the nonstandard hull of a normed space.

Given a normed space (E, ρ), let $^*M$ be an $\aleph_1$-saturated enlargement of a structure for type theory which contains (E, ρ) and its dual space (E', ρ'). Let $(\hat{E}, \hat{\rho})$ be the nonstandard hull of (E, ρ), relative to $^*M$, as defined by Luxemburg ("A general theory of monads", Applications of Model Theory, Holt, Rinehart and Winston, New York, 1969). Similarly $(\hat{E}', \hat{\rho}')$ is the nonstandard hull of (E', ρ'). Then $(\hat{E}, \hat{\rho})$ is a Banach space and $(\hat{E}', \hat{\rho}')$ is a subspace of the dual space of $(\hat{E}, \hat{\rho})$. Theorem. The following conditions are equivalent: (i) The dual space of $(\hat{E}, \hat{\rho})$ is $(\hat{E}', \hat{\rho}')$; (ii) $(\hat{E}, \hat{\rho})$ is reflexive; (iii) For some $0 < r < 1$ and some $n \geq 1$ there do not exist elements $x_1, \ldots, x_n$ of E, $y_1, \ldots, y_n$ of E', all of norm 1, such that if $j \leq i$, then $r < \langle x_i, y_j \rangle$ and if $i < j$, then $0 = \langle x_i, y_j \rangle$. Among the normed spaces which satisfy condition (iii) are the spaces ℓ_p and $L_p([0,1])$ for $1 < p < \infty$. (Received January 19, 1972.)

693-E2. DONALD H. PELLETIER, York University, Downsview, Ontario, Canada. On violating GCH below the least measurable cardinal. Preliminary report.

Let V be a model of ZF+GCH+"there exists a measurable cardinal" and let $\varkappa_0$ denote the least measurable cardinal in V. Let $F: \varkappa_0 \to \varkappa_0$ be a function which is "absolute with respect to the extension" (as explained in A. Levy and R. Solovay, "Measurable cardinals and the continuum hypothesis", Israel J. Math. 5(1967)) and which satisfies (i) $\alpha < \beta \to F'\alpha \leq F'\beta$, (ii) if $\aleph_\alpha$ is regular then $\mathrm{cf}(\aleph_{F'\alpha}) > \aleph_\alpha$, and (iii) if $\aleph_\lambda$ is singular then $F'\lambda = \sum_{\alpha<\lambda} F'\alpha$. Then there exists a Boolean-valued model, $V^{(\Gamma)}$, of ZF satisfying (a) the class of cardinals of $V^{(\Gamma)}$ is precisely $\{\check{\varkappa} | \varkappa$ is a cardinal in V$\}$, (b) $\check{\varkappa}_0$ is measurable, (c) if $\alpha < \varkappa_0$ and $\aleph_\alpha$ is regular then $2^{\check{\aleph}_\alpha} = \check{\aleph}_{F'\alpha}$, (d) if $\lambda < \varkappa_0$ and $\aleph_\lambda$ is singular then $2^{\check{\aleph}_\lambda}$ is the least cardinal $\geq \check{\aleph}_{F'\alpha}$ for every $\alpha < \lambda$ and which is not cofinal with $\check{\aleph}_\lambda$, and (e) the axiom of choice holds for sets of cardinality $< \check{\varkappa}_0$. (Received January 26, 1972.)

693-E3. DAVID W. KUEKER, University of Michigan, Ann Arbor, Michigan 48104. On intersections of the models of a theory. Preliminary report.

Let T be a finitary first-order theory. Two models of T are compatible if there is some model of T containing both as submodels. If $\mathfrak{A}$ is a model of T then the core of T in $\mathfrak{A}$, $C_T(\mathfrak{A})$, is the set of all elements of $\mathfrak{A}$ which belong to every model of T compatible with $\mathfrak{A}$. $D_T^{\Sigma}(\mathfrak{A})$ $[D_T^{\Pi}(\mathfrak{A})]$ is the set of all a of $\mathfrak{A}$ such that $\mathfrak{A} \models \varphi[a]$ for some existential [universal] formula $\varphi(x)$ with $T \vdash \exists^{\leq k} x \varphi(x)$ for some $k \in \omega$. $D_T^{\Sigma!}(\mathfrak{A})$ is like

$D_T^{\Sigma}(\mathfrak{A})$ except we require $T \vdash \exists^{=k} x \varphi(x)$. Theorem. For every model $\mathfrak{A}$ of T, $D_T^{\Sigma!}(\mathfrak{A}) \subseteq C_T(\mathfrak{A}) \subseteq D_T^{\Sigma}(\mathfrak{A}) \cap D_T^{\Pi}(\mathfrak{A})$. Examples show that in general no inclusion can be changed into equality. However, Corollary 1. If T is complete for existential sentences, then both inclusions are equalities. Corollary 2. If any two models of T can be embedded in some model of T, and further, if any isomorphism of $C_T(\mathfrak{A})$ into $\mathfrak{A}$ (a model of T) maps $C_T(\mathfrak{A})$ into itself, then $C_T(\mathfrak{A}) = D_T^{\Sigma!}(\mathfrak{A})$ for every model $\mathfrak{A}$ of T. These results generalize results of A. Robinson (for convex T) and Kreisel (on hard cores). (Received January 27, 1972.)

*693-E4. ROBERT G. JEROSLOW, University of Minnesota, Minneapolis, Minnesota 55455.

K-descriptions in free variable logics.

We give a uniform method, with certain universality properties, for obtaining "encodings" of syntactical notions. The encoding of a Post Canonical System (P.C.S.) is called its K-description, since it is essentially the canonical representation of Kreisel (see "Mathematical logic," vol. 3, p. 154). A logic is K-complete if it contains K-descriptions of every P.C.S. With minimal hypotheses, a logic is K-complete if it has a K-description of a certain P.C.S. (which can be explicitly given) iff it relatively interprets Peano arithmetic in a strong sense. With every P.C.S. F there is associated a P.C.S. F' which essentially consists of the proofs of the theorems of F. Free variable logics extending a certain theory of concatenation C (which can be specified) contain K-descriptions of every proof P.C.S. F'. We prove a version of the third derivability condition of Hilbert-Bernays for extensions of C in terms of K-descriptions and thereby prove Godel's second underivability theorem for these extensions. C is far weaker than the system EA of elementary arithmetic studied by Poszgay (Math. Z. (1968)). Thus, our result subsumes known results on the second theorem for free variable systems. We also discuss aspects of K-descriptions in logics with quantifiers. (Received January 27, 1972.)

Statistics and Probability

*693-F1. CASPER GOFFMAN, Purdue University, Lafayette, Indiana 47907 and JOHN J. LOUGHLIN, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. Weak and strong Φ variation.

Let Φ be a convex function on the nonnegative reals with $\Phi(0) = 0$. A continuous f on $[a,b]$ is of bounded strong Φ variation if $\sup \sum \Phi(|f(x_i) - f(x_{i-1})|) < \infty$, for the partitions of $[a, b]$. Since $\liminf \sum \Phi(|f(x_i) - f(x_{i-1})|) = 0$, as the norm of the partition goes to zero, if $\lim_{x \to 0} x^{-1}\Phi(x) = 0$, the weak Φ variation is defined as $\liminf \sum \Phi[w_f([x_{i-1}, x_i])]$, where $w_f(I)$ is the oscillation of f on I. Of special interest is the case $\Phi(x) = x^p$, $p \geq 1$, in terms of which weak and strong variation dimensions are defined. For Brownian motion, both dimensions are shown to be 2 with probability 1. This idea was just overlooked by P. Lévy in his early writings so that this merely fills a small gap in the work of that illustrious geometer. It is to be expected that the idea should be of use to probabilists; indeed, it has already been put to great use by knowledgeable people who were orally advised of its existence. (Received January 24, 1972.)

Topology

693-G1. CHUNG-WU HO, Southern Illinois University, Edwardsville, Illinois 62025. On the existence of certain linear homeomorphisms of a convex polyhedral disk.

Let D be a convex polyhedral disk in R^2. For each simplicial subdivision K of D, L(K) denotes the set of all linear homeomorphisms from D onto D which are fixed on Bd(D) and are linear on each simplex of K. For each $f \in L(K)$, let f(K) be the induced triangulation on D. The space L(K) is equipped with the compact open topology. Certain general properties of the space are established. In particular, we have: Theorem 1. Let K be an indecomposable subdivision of D. For each inner vertex v of K, there is an $f \in L(K)$ such that St(f(v), f(K)) is strictly convex. Theorem 2. If K is an indecomposable subdivision of D, for each nonbinding pair of vertices v_1, v_2 of K, there is an $f \in L(K)$ such that both $St(f(v_1), f(K))$ and $St(f(v_2), f(K))$ are strictly convex. We shall show how these theorems can be applied to attack certain homotopy problems of the space L(K). (Received January 3, 1972.)

*693-G2. STEPHEN J. GREENFIELD and NOLAN R. WALLACH, Rutgers University, New Brunswick, New Jersey 08903. Remarks on global hypoellipticity. Preliminary report.

We study differential operators D which commute with a fixed normal elliptic operator E on a compact manifold M. We use eigenfunction expansions relative to E to obtain simple conditions giving global hypoellipticity (GH) or global analytic hypoellipticity (GAH) for D. These conditions are equivalent to D having parametrices in certain spaces of functions or distributions. (GH) and (GAH) operators are Fredholm of index 0 in these spaces. An example is given by M = compact homogeneous space and E = Casimir operator. When D is a pure first order operator and left-invariant, we obtain an invariant vector field on M. If M is a compact Lie group, and D is a (GH) or (GAH) vector field, then M is abelian. On each torus we can create such vector fields. With an appropriate definition, we can also prove that the only compact surface admitting (GH) vector fields is the torus. Also, such vector fields can be completely described. (Received January 12, 1972.)

*693-G3. JERREL K. YATES, Huntingdon College, Montgomery, Alabama 36106. Paracompactness and transfinite convergence.

Let P denote the set of ordinal numbers preceding the regular initial ordinal ξ. The space S satisfies Axiom ξ provided there is a collection $\{G_x | x \in P\}$ which satisfies all three parts of Axiom Ω [Abstract 683-G16, these *Notices* 18(1971), 386]. Let S denote a space satisfying Axiom ξ and Axiom 0 of R. L. Moore. The space S has property C' if for each region R and point p of R, there is an x in P such that if g is a region in G_x containing p then $St(g, G_x)$ is a subset of R. The space T is ξ-paracompact if for each open cover G of T, there is a cardinal number $\alpha < |P|$ such that G has a locally type α refinement. Theorem 1. S is paracompact iff S has property C'. Theorem 2. S is paracompact iff S is ξ-paracompact. Theorem 3. If T is a regular Hausdorff space such that (1) if G is a collection of closed subsets of T and $|G| < |P|$ then G* is closed and (2) T has a basis equally numerous with P, then T is ξ-paracompact. Theorem 4. If the generalized continuum hypothesis is true and S is a normal space which contains a dense subset equally numerous with P, then S is paracompact. (Received January 25, 1972.)

ERRATA
Volume 18

ROBERT E. ATALLA. P-sets and F-spaces, Abstract 71T-G156, Page 839.

Professor A. I. Veksler has pointed out that Theorems 4 and 5 have already appeared in his paper "P-sets in topological spaces", Soviet Math. Dokl. 11(1970), 953-956 (Dokl. Akad. Nauk. SSR 193 (1970), no. 3). His paper is on P-sets in basically disconnected spaces.

GORDON L. BAILES, JR. Right inverse semigroups, Abstract 689-A20, Page 1043.

In line 5 "regular" should be replaced by "orthodox". In lines 8 and 9, delete "$\mathcal{R}$ is a congruence" and "S is a band of groups". In line 10 after "$\mathcal{D} = \mathcal{L} = \mathcal{J}$.", insert "S is a band of groups if and only if $\mathcal{R}$ is a congruence.", and, in the same line, change "unions" to "bands".

K. DEMYS. Proof of a basic property of positive definite binary n-adic forms, Abstract 690-A1, Page 1074.

Amend paragraph headed S_5 to read: S_5: If $0 < \cos\theta < 1$ is assumed, then by the expansion of the above trinomial (see S_3) we have $x^n + y^n + \Delta_1 + \Delta_2 = z^n$, where Δ_1 is the sum of the positive terms and Δ_2, of the negative terms of the expansion. But since $x, y > 0$ (by A), $|\Delta_1| > |\Delta_2|$ for all $n > 1$. But by hypothesis $x^n + y^n = z^n$, whence $\Delta_1 + \Delta_2 = 0$. Hence the assumption that $\cos\theta$ was a positive fraction is untenable.

JUDITH GREEN. Beth's Theorem and Craig's Theorem for finite quantifier infinitary languages. Preliminary report, Abstract 71T-E90, Page 968.

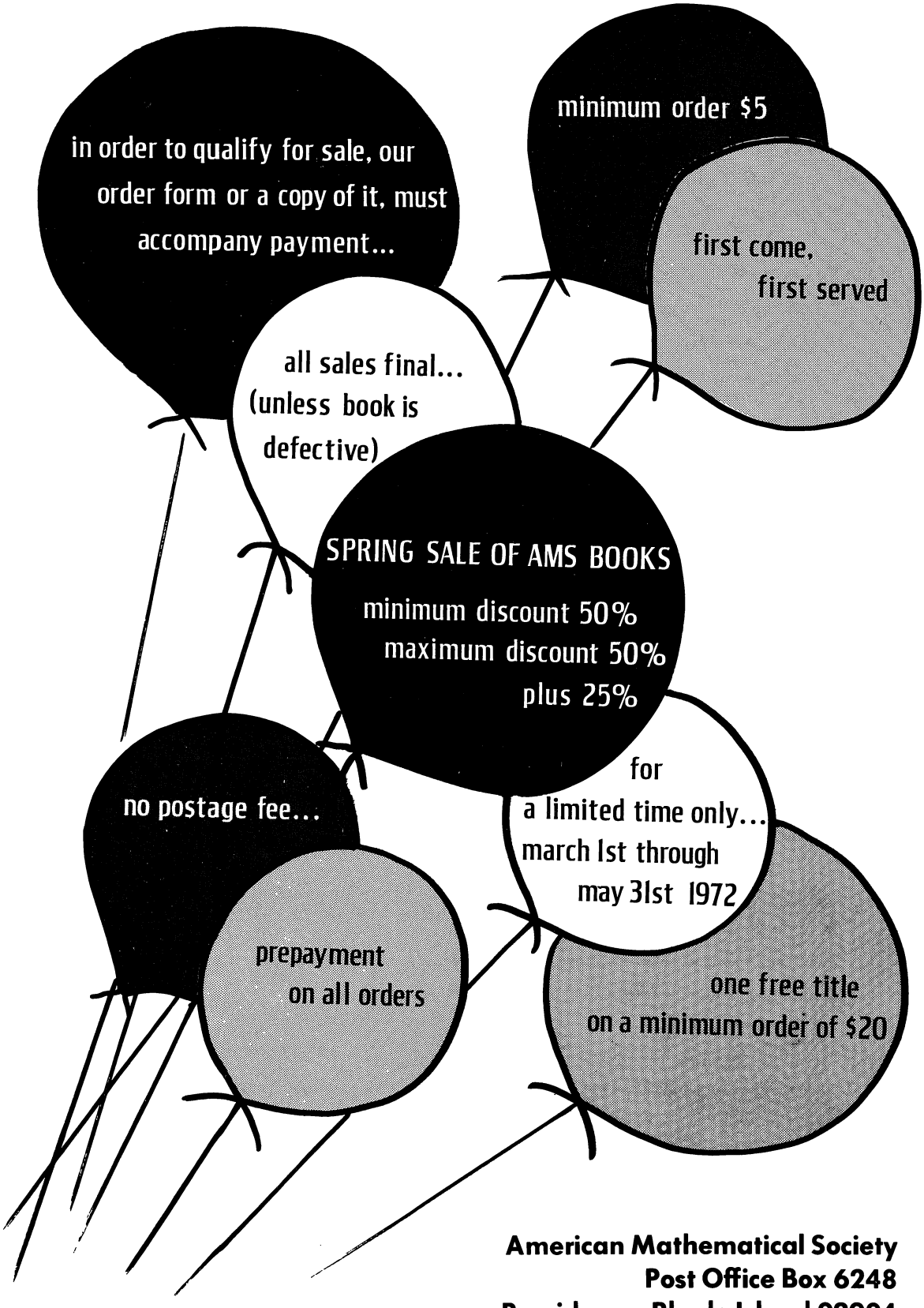
The author found a gap in the proof and withdraws the statements made about the prenex form of the defining and interpolating sentences. The example in Malitz's thesis confirms that no interpolating sentence of this form need exist. However, the generalization of Keisler's consistency property will yield new proofs of the interpolation theorems proved by Malitz and Chang.

MICHAEL A. MACHTEY. The recursive primitive recursive degrees are not a lattice. Preliminary report, Abstract 690-E4, Page 1086.

The Theorem stated in the abstract is false; the honest p.r.-degrees are a distributive lattice. The result announced in the title is true, following from a different theorem.

HARVEY ROSEN. Shrinking wild cellular subsets of 2-spheres in S^3, Abstract 689-G27, Page 1071.

Line 5: At the end of the theorem, add "and the local enveloping genus of an arc on f(S) at its endpoint f(A) is finite."



American Mathematical Society
Post Office Box 6248
Providence, Rhode Island 02904

Volume		Code	List	Member
	COLLOQUIUM PUBLICATIONS			
20.	Interpolation and approximation by rational functions in the complex domain, J. L. Walsh	C20	$9.80	$7.35
	MEMOIRS OF THE AMERICAN MATHEMATICAL SOCIETY			
54.	Generalizations of a theorem of Carathéodory, J. R. Reay	M54	0.82	0.62
55.	Foundations of relative homological algebra, S. Eilenberg and J. C. Moore	M55	0.82	0.62
58.	A Hankel convolution complex inversion theory, F. M. Cholewinski	M58	0.77	0.58
59.	Special trigonometric series in k-dimensions, S. Wainger	M59	0.87	0.65
60.	Torsion in SU-bordism, P. E. Connor and E. E. Floyd	M60	1.22	0.92
63.	Translation-invariant subspaces of $L^p(G)$, H. P. Rosenthal	M63	0.92	0.69
64.	Deformation group theory of pseudogroup structures, V. W. Guillemin and S. Sternberg	M64	0.87	0.65
65.	On the enumeration of non-planar maps, W. G. Brown	M65	0.72	0.54
66.	The undecidability of the domino problem, R. Berger	M66	1.02	0.77
67.	The modified algorithm of Jacobi-Perron, L. Bernstein	M67	0.77	0.58
69.	Analytic continuation of the irreducible unitary representations of the universal covering group of SL(2,R), P. J. Sally, Jr.	M69	0.97	0.73
71.	Algebraic groups and modular Lie algebras, J. E. Humphreys	M71	0.87	0.65
72.	The metastable homotopy of S^n, M. Mahowald	M72	0.87	0.65
73.	Two papers on similarity of certain Volterra integral operators, S. J. Osher	M73	0.87	0.65
75.	Locally compact transformation groups and C*-algebras, E. G. Effros and F. Hahn	M75	0.92	0.69
76.	A class of functional equations of neutral type, J. Hale and K. R. Meyer	M76	0.87	0.65
77.	Monotone processes of convex and concave type, R. T. Rockafellar	M77	0.87	0.65
78.	Measurable, continuous and smooth vectors for semigroups and group representations, R. T. Moore	M78	0.87	0.65
79.	Unitary representation for solvable Lie groups, J. Brezin	M79	0.97	0.73
80.	Endomorphisms of linear algebraic groups, R. Steinberg	M80	0.92	0.69
82.	Cohomology of coalgebras, D. Jonah	M82	0.87	0.65
	PROCEEDINGS OF SYMPOSIA IN APPLIED MATHEMATICS			
2.	Electromagnetic theory	PA02	2.05	1.54
4.	Fluid dynamics	PA04	3.75	2.81
5.	Wave motion and vibration theory	PA05	3.75	2.81
7.	Applied probability	PA07	2.75	2.06
9.	Orbit theory	PA09	3.85	2.89
11.	Nuclear reactor theory	PA11	4.60	3.45
13.	Hydrodynamic instability	PA13	4.20	3.15
14.	Mathematical problems in the biological sciences	PA14	3.50	2.63
15.	Experimental arithmetic, high speed computing, and mathematics	PA15	4.80	3.60
17.	Applications of nonlinear partial differential equations in mathematical physics	PA17	4.25	3.19
18.	Magneto-fluid and plasma dynamics	PA18	3.65	2.74
	PROCEEDINGS OF SYMPOSIA IN PURE MATHEMATICS			
2.	Lattice theory	PP02	4.45	3.34
6.	1960 Institute on finite groups	PP06	3.45	2.59
9.	Algebraic groups and discontinuous subgroups	PP09	7.00	5.25
10.	Singular integrals	PP10	7.75	5.81
11.	Entire functions and related parts of analysis	PP11	8.15	6.11
12.	Number theory	PP12	4.00	3.00
	SELECTED TRANSLATIONS IN MATHEMATICAL STATISTICS AND PROBABILITY			
1.	Twenty-five papers	SP01	7.80	5.85
3.	Eighteen papers	SP03	5.55	4.16
4.	Thirty-five papers	SP04	6.15	4.61
5.	Nineteen papers	SP05	6.05	4.54
7.	Twenty-four papers	SP07	7.95	5.96
8.	Eleven papers	SP08	6.25	4.69
	TRANSLATIONS OF MATHEMATICAL MONOGRAPHS			
13.	Additive theory of prime numbers	TM13	5.75	4.31
15.	Intrinsic geometry of surfaces	TM15	6.15	4.61
17.	Expansions in eigenfunctions of selfadjoint operators	TM17	16.10	12.08
19.	The operator of translation along the trajectories of differential equations	TM19	7.95	5.96
20.	Statistical problems with nuisance parameters	TM20	6.25	4.69
	MISCELLANEOUS			
	Norbert Wiener Memorial	NW	2.05	1.54
	MATHEMATICAL SURVEYS			
3.	Geometry of polynomials	S03	6.60	4.95
9.	Linear approximation	S09	8.65	6.49

Volume	Code	List	Member

AMS TRANSLATIONS—Series 1

Volume	Code	List	Member
1. Algebra	T101	$3.00	$2.25
3. Series and approximation	T103	2.80	2.10
4. Differential equations	T104	3.00	2.25
7. Algebraic topology	T107	3.00	2.25
8. Topology and topological algebra	T108	$3.00	$2.25
11. Probability and physical problems	T111	2.80	2.10

AMS TRANSLATIONS—Series 2

Volume	Code	List	Member
1. Topology, function theory, and differential equations	T201	4.90	3.68
3. Logic, algebra, and number theory	T203	4.05	3.04
6. Algebra and group theory	T206	12.20	9.15
8. Function theory, probability, and differential equations	T208	9.00	6.75
10. Analysis	T210	6.45	4.84
11. Topology	T211	6.10	4.58
12. Analysis, probability, and topology	T212	5.45	4.09
13. Number theory and theory of operators	T213	5.50	4.13
14. Analysis	T214	5.30	3.98
19. Number theory and function theory	T219	5.15	3.86
22. Analysis	T222	5.90	4.43
23. Logic and quantum electrodynamics	T223	5.35	4.01
24. Differential equations	T224	4.95	3.71
25. Analysis, applied mathematics, and algebraic topology	T225	5.35	4.01
26. Analysis	T226	5.60	4.20
27. Algebra	T227	4.90	3.68
28. Analysis	T228	5.50	4.13
29. Logic and differential equations	T229	6.05	4.54
30. Topology	T230	5.70	4.28
32. Functions of complex variables	T232	6.25	4.69
33. Differential equations and information theory	T233	6.90	5.18
34. Analysis	T234	6.50	4.88
35. Analysis and applied mathematics	T235	5.80	4.35
36. Groups and semigroups	T236	6.25	4.69
37. Algebra, number theory, and differential geometry	T237	6.80	5.10
38. Topology and game theory	T238	5.45	4.09
39. Topology and logic	T239	4.80	3.60
40. Functional analysis and numerical analysis	T240	4.70	3.53
41. Partial differential equations	T241	5.10	3.83
42. Differential equations	T242	4.70	3.53
43. Series and functions of complex variables	T243	5.15	3.86
44. Approximations and integrals	T244	4.35	3.26
50. Algebra	T250	8.25	6.19
51. Differential equations, functional analysis, and measure theory	T251	8.65	6.49
52. Topology, algebra, and number theory	T252	7.25	5.44
55. Topology and algebra	T255	7.95	5.96
56. Differential equations and functional analysis	T256	7.75	5.81
57. Foundations, measure theory, and analysis	T257	7.95	5.96
58. Algebra, number theory, and topology	T258	6.85	5.14
59. Logic and algebra	T259	7.45	5.59
60. Analysis	T260	7.45	5.59
61. Functional analysis and differential equations	T261	8.15	6.11
62. Functional analysis	T262	7.45	5.59
63. Algebra, algebraic geometry, and algebraic topology	T263	7.35	5.51
64. Logic and group theory	T264	6.75	5.06
65. Partial differential equations and functional analysis	T265	7.75	5.81
66. Group theory, algebraic geometry, and algebraic topology	T266	7.15	5.36
67. Functional analysis and partial differential equations	T267	7.55	5.66
68. Differential equations and functional analysis	T268	6.95	5.21
69. Algebra	T269	6.75	5.06
70. Thirty-one invited addresses at the International Congress of Mathematicians in Moscow, 1966	T270	7.05	5.29
71. Algebra, topology, complex variables, and linear programming	T271	6.25	4.69
72. Analysis	T272	7.25	5.44
73. Algebra, topology, algebraic and differential geometry	T273	6.85	5.14
74. Analysis	T274	6.85	5.14
75. Equations related to mechanics and heat	T275	6.75	5.06
76. Algebra and analysis	T276	6.95	5.21
77. Series and approximations	T277	7.05	5.29
78. Topology	T278	6.65	4.99
79. Functional analysis and differential equations	T279	7.15	5.36
80. Functions of real and complex variables	T280	7.35	5.51
81. Functions of real variables	T281	7.35	5.51
83. Logic and theory of functions	T283	6.65	4.99
84. Algebra, algebraic geometry, and topology	T284	7.25	5.44
85. Functional analysis and geometry	T285	6.80	5.10
86. Real and complex functions, series, differential and integral equations	T286	7.20	5.40
87. Differential and difference equations, functional analysis, games, and control	T287	7.70	5.78
88. Real and complex function theory	T288	8.25	6.19
89. Differential equations	T289	7.60	5.70

TRANSACTIONS OF THE MOSCOW MATHEMATICAL SOCIETY

12. (1963)	W12	5.55	4.16	15. (1966)	W15	9.90	7.43
13. (1965)	W13	6.05	4.54	18. (1968)	W18	10.15	7.61
14. (1965)	W14	5.85	4.39	19. (1968)	W19	8.65	6.49

(more)

Volume		Code	List	Member
	PROCEEDINGS OF THE STEKLOV INSTITUTE OF MATHEMATICS			
74.	Difference methods of solution of problems of mathematical physics	WS74	$5.15	$3.86
75.	Algebraic surfaces	WS75	7.55	5.66
76.	Two-dimensional manifolds of bounded curvature	WS76	4.95	3.71
77.	Theory and applications of differentiable functions of several variables	WS77	5.65	4.24
78.	Extremal properties of polynomials	WS78	2.80	2.10
79.	Mathematical statistics and probability	WS79	6.85	5.14
80.	Algebraic number theory and representations	WS80	6.15	4.61
81.	Topological imbeddings in Euclidean space	WS81	7.35	5.51
82.	Ergodic problems in the theory of congruences and of diophantine approximations	WS82	3.55	2.66
83.	Boundary value problems of mathematical physics, Part III	WS83	4.95	3.71
84.	Numerical methods and inequalities in functional spaces	WS84	5.25	3.94
85.	Defining relations and algorithmic problems for groups and semigroups	WS85	4.15	3.11
86.	Sequences of convergence for series	WS86	2.70	2.03
87.	Unsteady motions of compressible media with blast waves	WS87	4.05	3.04
88.	Approximation of functions in the mean	WS88	5.25	3.94
89.	Theory and applications of differentiable functions of several variables. II	WS89	9.55	7.16
90.	Geodesic flows on closed Riemann manifolds with negative curvature	WS90	7.85	5.89
91.	Boundary problems for differential equations	WS91	6.45	4.84
92.	Boundary value problems of mathematical physics, Part IV	WS92	10.10	7.58
94.	Extremal problems of the geometric theory of functions	WS94	6.15	4.61

(For more detailed information on these volumes, consult the Catalog of Publications, Fall 1971. In order to qualify for discounts, the following order form (or copy) must be used. Prepayment required. All sales final.)

ORDER FORM

(Please insert quantity in space provided below.)

_____C20
_____M54
_____M55
_____M58
_____M59
_____M60
_____M63
_____M64
_____M65
_____M66
_____M67
_____M69
_____M71
_____M72
_____M73
_____M75
_____M76
_____M77
_____M78
_____M79

_____M80
_____M82
_____PA02
_____PA04
_____PA05
_____PA07
_____PA09
_____PA11
_____PA13
_____PA14
_____PA15
_____PA17
_____PA18
_____PP02
_____PP06
_____PP09
_____PP10
_____PP11
_____PP12

_____SP01
_____SP03
_____SP04
_____SP05
_____SP07
_____SP08
_____TM13
_____TM15
_____TM17
_____TM19
_____TM20
_____NW
_____S03
_____S09

_____T101
_____T103
_____T104
_____T107
_____T108
_____T111
_____T201
_____T203
_____T206
_____T208
_____T210
_____T211
_____T212
_____T213
_____T214
_____T219
_____T222
_____T223
_____T224
_____T225

_____T226
_____T227
_____T228
_____T229
_____T230
_____T232
_____T233
_____T234
_____T235
_____T236
_____T237
_____T238
_____T239
_____T240
_____T241
_____T242
_____T243
_____T244
_____T250
_____T251
_____T252

_____T255
_____T256
_____T257
_____T258
_____T259
_____T260
_____T261
_____T262
_____T263
_____T264
_____T265
_____T266
_____T267
_____T268
_____T269
_____T270
_____T271
_____T272
_____T273
_____T274
_____T275

_____T276
_____T277
_____T278
_____T279
_____T280
_____T281
_____T283
_____T284
_____T285
_____T286
_____T287
_____T288
_____T289
_____W12
_____W13
_____W14
_____W15
_____W18
_____W19

_____WS74
_____WS75
_____WS76
_____WS77
_____WS78
_____WS79
_____WS80
_____WS81
_____WS82
_____WS83
_____WS84
_____WS85
_____WS86
_____WS87
_____WS88
_____WS89
_____WS90
_____WS91
_____WS92
_____WS94

Send orders to Department MC, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02904.

Date_____ Amount enclosed_____

Peel off the NOTICES mailing label and place in the space provided to the right. The mailing label contains the customer code, and with this information, clerks are able to process orders more efficiently. If your mailing label is lost or damaged, please print your complete mailing address in the space provided.

NOTE: If your order totals $20 or more, you are entitled to one free book. To receive this book, please indicate in the spaces provided to the right the book codes for three choices in order of preference. This covers the possibility of stock depletion.

1._____
2._____
3._____

Payment may be made by using international money orders or UNESCO coupons. The name and address of the bank of the American Mathematical Society is the Rhode Island Hospital Trust National Bank, Providence, R. I. 02906.

MULTIVARIABLE CALCULUS WITH LINEAR ALGEBRA AND SERIES

WILLIAM F. TRENCH and BERNARD KOLMAN, both at Drexel University

This expansion of the authors' ELEMENTARY MULTIVARIABLE CALCULUS presents a modern but not extreme treatment of linear algebra, the calculus of several variables, and series at a level appropriate for the sophomore science, engineering or mathematics major who has completed a standard first year calculus course.

By way of introduction to the various concepts of the multivariable calculus, the Authors usually recall the corresponding notion from the one dimensional calculus. They have not hesitated to omit proofs of difficult theorems, preferring instead to illustrate their meanings with numerous examples. In particular, there are few "epsilon-delta" arguments in this book. Simple notions are presented before their generalizations (for example, real valued functions are introduced before vector valued functions, rather than as special cases of the latter) and examples illustrate each theorem and definition. Each section contains, in addition to an ample number of routine exercises, a set of theoretical exercises designed to fill in the gaps in proofs and extend results obtained in the text. A separate answer booklet is available to instructors.

CONTENTS: *PART 1:* Chapter 1: Linear Equations and Matrices. Linear Systems and Matrices. Solutions of Equations. The Inverse of a Matrix. Determinants. Chapter 2: Vector Spaces and Linear Transformations. Vector Spaces. Linear Independence and Bases. Linear Transformations. Rank of a Matrix. More About R^n. Eigenvalues and Eigenvectors. *PART II:* Chapter 3: Vectors and Analytic Geometry. Lines and Planes. Vector in R^3. Motion in R^3. Parametrically Defined Curves. Coordinate Systems in R^3. Surfaces in R^3. Chapter 4: Differential Calculus of Real Valued Functions. Functions, Limits and Continuity. Directional and Partial Derivatives. Differentiable Functions. The Mean Value Theorem. Graphs and Tangent Planes. Implicit Functions. The Gradient. Taylor's Theorem. Maxima and Minima. Lagrange Multipliers. Chapter 5: Differential Calculus of Vector Valued Functions. Functions, Limits and Continuity. Differentiable Functions. The Chain Rule. Vector and Scalar Fields. Implicit Functions. Inverse Functions and Coordinate Transformations. Chapter 6: Integration. Multiple Integrals. Iterated Integrals. Change of Variables. Physical Applications. Line Integral. Surface Integrals of Scalar Fields. Surface Integrals of Vector Fields. The Divergence Theorem; Green's and Stokes' Theorems. Improper Integrals. *PART III:* Chapter 7: Series. Infinite Sequences. Infinite Series. Power Series.

March 1972, about 700 pp., approx. $12.50

ELEMENTARY MULTIVARIABLE CALCULUS

by BERNARD KOLMAN and WILLIAM F. TRENCH

The level of this modern treatment of the calculus of several variables is appropriate for the sophomore science, engineering, or mathematics major who has completed a standard first year calculus course and an introduction to linear algebra. An answer booklet is available to instructors.

CONTENTS: VECTORS AND ANALYTIC GEOMETRY: The Vector Space R^n. Lines and Planes. Vectors in R^3. Motion in R^3. Parametrically Defined Curves. Coordinate Systems in R^3. Surfaces in R^3.

DIFFERENTIAL CALCULUS OF REAL-VALUED FUNCTIONS: Functions, Limits and Continuity. Directional and Partial Derivatives. Differentiable Functions. The Mean Value Theorem. Graphs and Tangent Planes. Implicit Functions. The Gradient. Taylor's Theorem. Maxima and Minima. The Method of Lagrange Multipliers.

DIFFERENTIAL CALCULUS OF VECTOR VALUED FUNCTIONS: Functions, Limits and Continuity. Differentiable Functions. The Chain Rule. Vector and Scalar Fields. Implicit Functions. Inverse Functions and Coordinate Transformations.

INTEGRATION: Multiple Integrals. Iterated Integrals. Change of Variables. Physical Applications. Line Integrals. Surface Integrals of Scalar Fields. Surface Integrals of Vector Fields. The Divergence Theorem; Green's and Stokes' Theorem. Answers to Selected Problems. Index.

1971, 522 pp., $12.50

ACADEMIC PRESS NEW YORK AND LONDON
111 FIFTH AVENUE, NEW YORK, N.Y. 10003

In 1972 Math is still our favorite subject...

CALCULUS AND ANALYTIC GEOMETRY, ALTERNATE EDITION
by George B. Thomas, Jr., *Massachusetts Institute of Technology*

Many teachers of calculus would still like to teach their courses in the style and approach of the Third Edition of Thomas' *Calculus and Analytic Geometry.* This new version of Thomas is essentially the full Third Edition with one content change: the section on limits has been rewritten and substantially expanded. Sixty-three new illustrations have also been included, along with minor corrections. Unlike the Fourth Edition, which was a major departure, this Alternate Edition does not treat linear algebra and vector analysis.

March 1972

CALCULUS: A LINEAR APPROACH, Volumes I and II
By John B. Fraleigh, *University of Rhode Island*

This text presents the analytic geometry needed for calculus; a linear approach to basic ideas and applications of differential and integral calculus for functions of one variable, infinite series, and an introduction to differential calculus of functions of several variables.
Volume II presents conic sections, polar coordinates, and the elementary differential geometry of plane curves, followed by an introduction to determinants. The text also treats multiple integrals and applications and concludes with a chapter introducing the notation of differential forms and presenting Stokes' theorem.

Volume I: 586 pp, 230 illus (1971) $11.95
Volume II: March 1972

ELEMENTARY ALGEBRA FOR COLLEGE STUDENTS

Third Edition

By Irving Drooyan and William Wooton, both of Los Angeles Pierce College.

This is a beginning college text for the student without high school algebra. The third edition contains new problems; reviews at the end of each chapter; a second color to simplify fractions, and evaluate expressions; and transform equations. Student Supplement available.

1972 298 pages $8.95

PRE-ALGEBRA

By Harry H. Jonas, American River College.

This text gives the student the opportunity to learn the mathematical concepts that span the gap between routine arithmetic drill and the abstract principles of algebra. The book introduces integers, the real number, and radicals and exponents using rational numbers.

1972 299 pages $8.50

INTERMEDIATE ALGEBRA:

A College Approach

By Milton D. Eulenberg, Theodore S. Sunko, and Howard A. James, all of Wright College, Chicago.

This introductory text develops the basic algebraic skills and mathematical logic that college students need for further study of mathematics. There is strong computational emphasis in the text and in the exercises. Instructor's Manual available.

1972 Approx. 320 pages In Press

TRIGONOMETRY

By Louis L. Wilson and Bill D. New, both of Cerritos College.

This text uses the circular or mapping function approach with a unique order of presentation. It first develops the sine and cosine functions, then the tangent function. Cosenant, secants, and cotangent are then introduced in terms of reciprocal ratios.

1972 In Press

MATHEMATICS:

The Alphabet of Science, Second Edition

By Margaret F. Willerding, San Diego State College; and Ruth A. Hayward, General Dynamics Corporation.

This revision of a successful math text presents a broad view of number theory, geometry, algebra, computer science, probability, statistics, and logic. It includes a new section on compound statements and deMorgan's Laws.

1972 In Press

AN INTRODUCTION TO THE THEORY OF NUMBERS

Third Edition

By Ivan Niven, University of Oregon; and the late Herbert S. Zuckerman.

Treats the basic topics of elementary number theory and its connections with other parts of mathematics. The text is comprehensive, yet lets the professor choose the material he wants to present.

1972 In Press

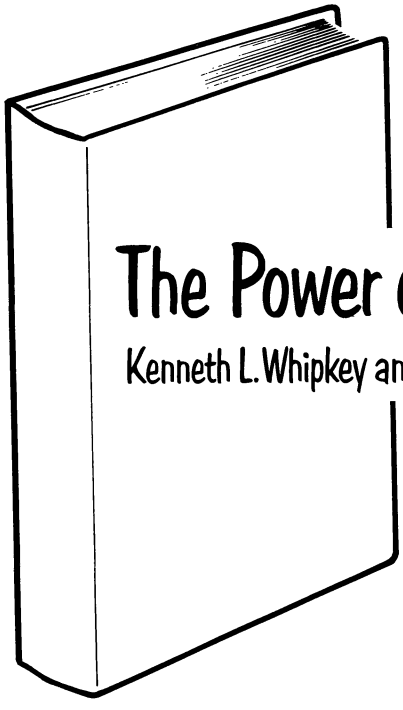
For more information, please contact your Wiley Representative, or write Ben Bean, Dept. 611-B, N.Y. office. Please include title of course, enrollment, and present text.

Prices subject to change without notice.

wiley

JOHN WILEY & SONS, Inc.

605 Third Avenue, New York, N.Y. 10016
In Canada: 22 Worcester Road, Rexdale, Ontario



Now a student doesn't have to be a math genius to understand calculus

Because the Whipkeys don't push the student against calculus but ease him in.

They take up one topic at a time. Explain concepts again and again. Give examples, simple and complex, and spell out the main idea of each example beforehand to let the student know where he's heading.

The Whipkeys also use the derivative and the integral to solve meaningful problems in biology...business...economics...or population control. (Biology and economics terms included).

And what's more, there's a syllabus of 50 class sessions that suggests ideas on how to use the text.

Contents:
Review and Preparation for the Study of Calculus. Functions, Inequalities, and Absolute Value. Limits, Derivatives and Continuity. Differentiation Techniques. Applications of the Derivative. Integration. Logarithmic and Exponential Functions; Review of the Great Ideas of the Calculus. Partial Derivatives and Their Applications. Appendix. Answers to Selected Problems.

THE POWER OF CALCULUS
By KENNETH L. WHIPKEY, Westminster College; and MARY NELL WHIPKEY. 1972 In Press

For more information, please contact your Wiley Representative, or write Ben Bean, Dept. 913-B, New York office. Please include title of course, enrollment, and present text.

JOHN WILEY & SONS, INC. wiley
605 Third Avenue, New York, N.Y. 10016, In Canada: 22 Worcester Road, Rexdale, Ontario

SITUATION WANTED

Correspondence to applicants listed anonymously should be directed to the Editorial Department, American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02904. The code which appears at the end of each anonymous listing should appear on the envelope in order that correspondence can be forwarded by the Editorial Department.

MATHEMATICS PROFESSOR, TEACHING OR TEACHING AND RESEARCH. Ph.D. 1971. Age 30. Specialty: commutative rings. One published article. One year full-time teaching experience in temporary position at Stephens College. U. S. preferred. References and resume available upon request. Available September 1972. David E. Brown, Box 2404, Stephens College, Columbia, Missouri 65201.

MATHEMATICS PROFESSOR, TEACHING AND RESEARCH. Ph.D. 1970. Age 34. Specialty: statistics and probability. Two articles under abstracts and seven others submitted for publication. Thirteen years experience in teaching and research. References and resume available on request. Available June 1972. Ramesh Gupta, Black Lantern Apt. 16, Oswego, New York 13126.

MATHEMATICS PROFESSOR, TEACHING AND RESEARCH. Ph.D. 1970. Age 32. Specialty: algebra (ring theory). One article submitted for publication. Eight years experience in teaching and research. References and resume available on request. Available immediately. Pushpa Gupta, Black Lantern Apt. 16, Oswego, New York 13126.

MATHEMATICS PROFESSOR, Ph.D. 1965. Age 35. Seven years college teaching. Specialty: logic and foundations; also interested algebra. One paper, another submitted. Strongly interested in curricular changes meeting changed circumstances in mathematics. Available now. Resume, references on request. W. M. Lambert, Jr., 17130 Birchcrest, No. 3, Detroit, Michigan 48221.

MATHEMATICS PROFESSOR, ADMINISTRATOR. Ph.D. 1964. Age 37. Specialties: complex analysis, summability. Author of several published articles. Eleven years experience in teaching and research. Desires position involving administration and teaching. Vita and references available on request. Available, summer '72. D. L. Prullage, 624 Center Street, Bethlehem, Pennsylvania 18018.

TEACHING POSITION. Ph.D. 1965. Age 35. Married. Eight years teaching experience at graduate and undergraduate levels. Three years administrative experience as N.S.F. Summer Institute Director. Interests: applications to social sciences, operations research, computer science. Dedicated in excellence in teaching. Resume and references available upon request. Box 105, Bethlehem, Pennsylvania 18016.

ANONYMOUS

MATHEMATICIAN. Ph.D. 1965, University of Illinois. Age 33. Ten years teaching experience. Have taught algebra, analysis, geometry, and topology courses at the graduate level. Research specialty is differential topology. Year of postdoctoral study at the Institute for Advanced Study. Eight publications, including topology textbook submitted for publication. Presently teaching at a liberal arts college. Seek an associate professorship or chairmanship at a university or liberal arts college. Available June 1972. SW1

MATHEMATICS PROFESSOR, TEACHING AND RESEARCH. Ph.D. 1965. Age 32. Specialty: general mathematical systems. Eight published articles. Eight years experience in teaching and research, including some in foreign university. Fluent in two foreign languages. References and resume available upon request. Available June 1972. SW2

ADVERTISERS'

INDEX