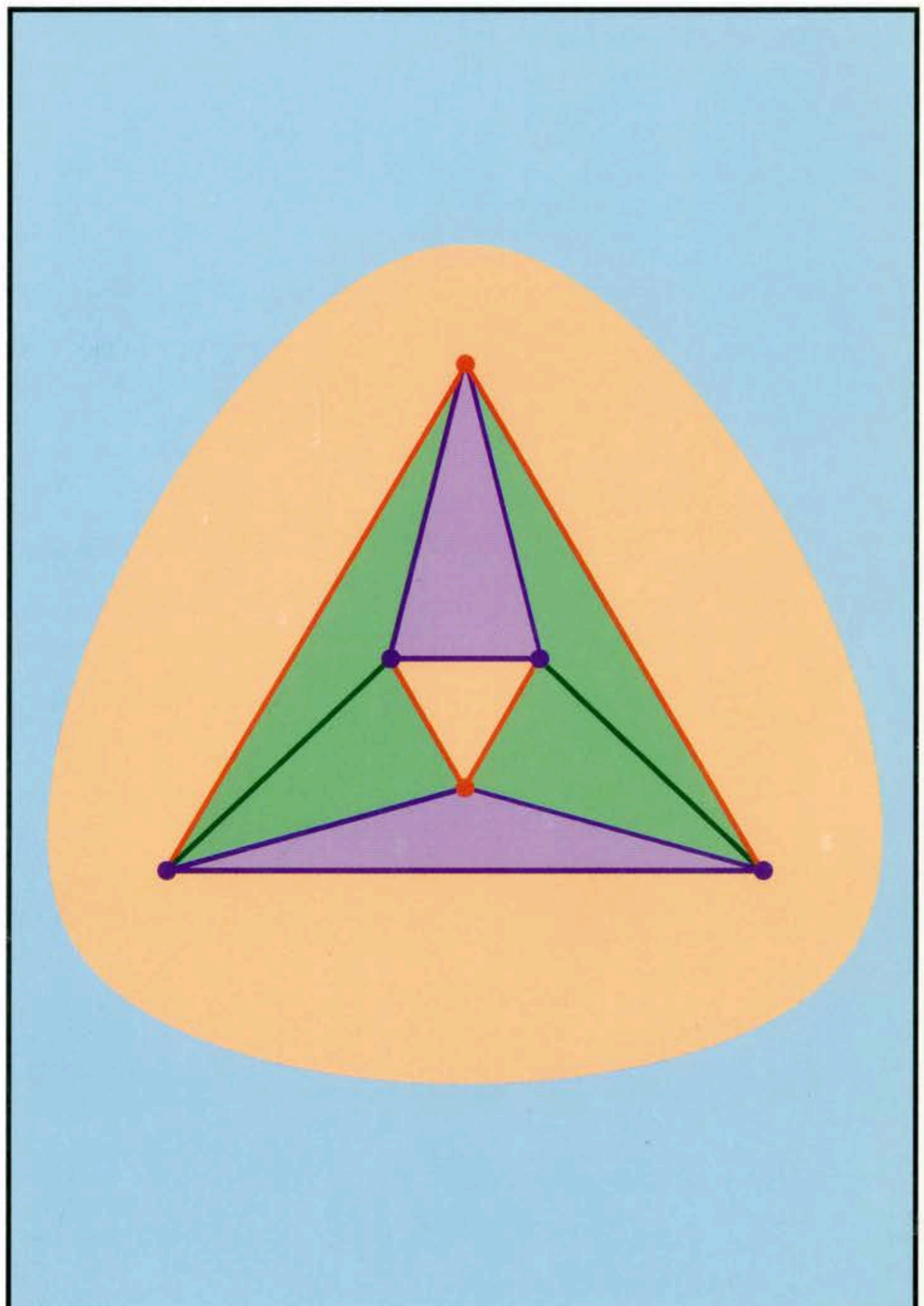
ISSN 0002-9920

Notices

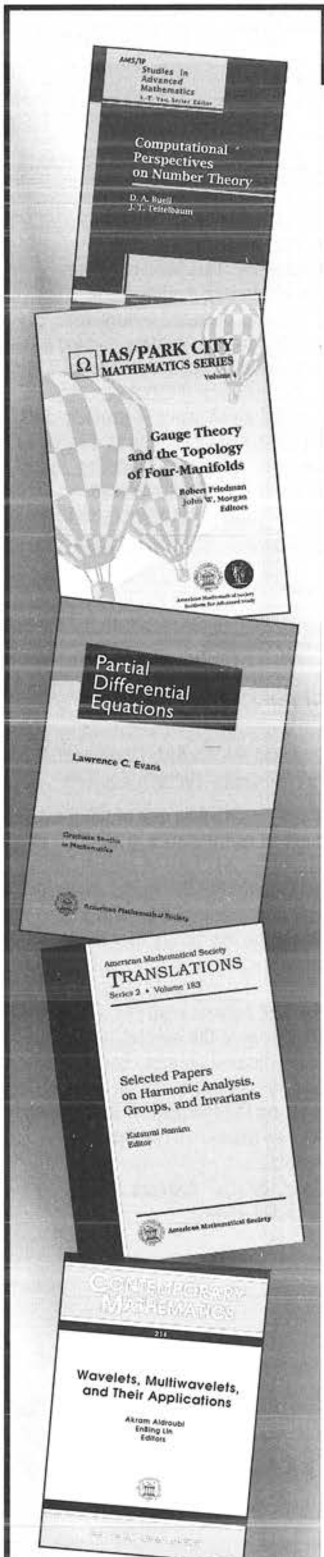
of the American Mathematical Society

February 1998

Volume 45, Number 2



Convex Partition of a Grassmannian (*See page 232*)



American Mathematical Society

New Titles from the AMS

Computational Perspectives on Number Theory

Proceedings of a Conference in Honor of A. O. L. Atkin

D. A. Buell, *Center for Computing Sciences, Bowie, MD*, and **J. T. Teitelbaum**, *University of Illinois at Chicago*, Editors

This volume contains papers presented at the conference "Computational Perspectives on Number Theory" held at the University of Illinois at Chicago in honor of the retirement of A. O. L. Atkin. In keeping with Atkin's interests and work, the papers cover a range of topics, including algebraic number theory, p-adic modular forms and modular curves. Many of the papers reflect Atkin's particular interest in computational and algorithmic questions.

AMS/IP Studies in Advanced Mathematics, Volume 7; 1998; 232 pages; Softcover; ISBN 0-8218-0880-X; List $59; All AMS members $47; Order code AMSIP/7NT82

Supplementary Reading

Gauge Theory and the Topology of Four-Manifolds

Robert Friedman and **John W. Morgan**, *Columbia University, New York*, Editors

The lectures in this volume provide a perspective on how 4-manifold theory was studied before the discovery of modern-day Seiberg-Witten theory. One reason the progress using the Seiberg-Witten invariants was so spectacular was that those studying $SU(2)$-gauge theory had more than ten years' experience with the subject. The tools had been honed, the correct questions formulated, and the basic strategies well understood. The knowledge immediately bore fruit in the technically simpler environment of the Seiberg-Witten theory.

Gauge theory long predates Donaldson's applications of the subject to 4-manifold topology, where the central concern was the geometry of the moduli space. One reason for the interest in this study is the connection between the gauge theory moduli spaces of a Kähler manifold and the algebro-geometric moduli space of stable holomorphic bundles over the manifold. The extra geometric richness of the $SU(2)$-moduli spaces may one day be important for purposes beyond the algebraic invariants that have been studied to date. It is for this reason that the results presented in this volume will be essential.

Members of the Mathematical Association of America (MAA) and the National Council of Teachers of Mathematics (NCTM) receive a 20% discount from list price.

IAS/Park City Mathematics Series, Volume 4; 1998; 221 pages; Hardcover; ISBN 0-8218-0591-6; List $39; All AMS members $31; Order code PCMS/4NT82

Local Properties of Distributions of Stochastic Functionals

Yu. A. Davydov, *University of Lille I, Villeneuve d'Ascq, France*, **M. A. Lifshits**, *MANCOMTECH Training Center, St. Petersburg, Russia*, and **N. V. Smorodina**, *Radiation Hygiene Institute, St. Petersburg, Russia*

This book investigates the distributions of functionals defined on the sample paths of stochastic processes. It contains systematic exposition and applications of three general research methods developed by the authors.

(i) The method of stratifications is used to study the problem of absolute continuity of distribution for different classes of functionals under very mild smoothness assumptions. It can be used also for evaluation of the distribution density of the functional.

(ii) The method of differential operators is based on the abstract formalism of differential calculus and proves to be a powerful tool for the investigation of the smoothness properties of the distributions.

(iii) The superstructure method, which is a later modification of the method of stratifications, is used to derive strong limit theorems (in the variation metric) for the distributions of stochastic functionals under weak convergence of the processes.

The research methods and basic results in this book are presented here in monograph form for the first time. The text would be suitable for a graduate course in the theory of stochastic processes and related topics.

Translations of Mathematical Monographs, Volume 173; 1998; 184 pages; Hardcover; ISBN 0-8218-0584-3; List $75; Individual member $45; Order code MMONO/173NT82

Recommended Text

Partial Differential Equations

Lawrence C. Evans, *University of California, Berkeley*

This text gives a comprehensive survey of modern techniques in the theoretical study of partial differential equations (PDEs) with particular emphasis on nonlinear equations. The exposition is divided into three parts: 1) representation formulas for solutions, 2) theory for linear partial differential equations, and 3) theory for nonlinear partial differential equations.

Included are complete treatments of the method of characteristics; energy methods within Sobolev spaces; regularity for second-order elliptic, parabolic and hyperbolic equations; maximum principles; the multidimensional calculus of variations; viscosity solutions of Hamilton-Jacobi equations; shock waves and entropy criteria for conservation laws; and much more.

The author summarizes the relevant mathematics required to understand current research in PDEs, especially nonlinear PDEs. While he has reworked and simplified much of the classical theory (particularly the method of characteristics), he primarily emphasizes the modern interplay between functional analytic insights and calculus-type estimates within the context of Sobolev spaces. The book's wide scope and clear exposition make it a suitable text for a graduate course in PDEs.

Graduate Studies in Mathematics; 1998; approximately 712 pages; Hardcover; ISBN 0-8218-0772-2; List $75; All AMS members $60; Order code GSM-EVANSNT82

Selected Papers on Harmonic Analysis, Groups, and Invariants

Katsumi Nomizu, *Brown University, Providence, RI*, Editor

This volume contains papers that originally appeared in Japanese in the journal *Sūgaku*. Ordinarily the papers would appear in the AMS translation of that journal, but to expedite publication the Society has chosen to publish them as a volume of selected papers. The papers range over a variety of topics, including representation theory, differential geometry, invariant theory, and complex analysis.

American Mathematical Society Translations—Series 2, Volume 183; 1998; 143 pages; Hardcover; ISBN 0-8218-0840-0; List $59; Individual member $35; Order code TRANS2/183NT82

Wavelets, Multiwavelets, and Their Applications

Akram Aldroubi, *Vanderbilt University, Nashville, TN*, and **EnBing Lin**, *University of Toledo, OH*, Editors

This volume contains refereed research articles on the active area of wavelets and multiwavelets. The book draws upon work presented by experts in the field during the special session on "Wavelets, Multiwavelets and Their Applications" at the Joint Mathematics Meetings in San Diego (January 1997).

Wavelets were implicit in mathematics, physics, signal or image processing, and numerical analysis long before they were given the status of a unified scientific field in the late 1980s. They continue to be one of the few subjects that have attracted considerable interest from the mathematical community and from other diverse disciplines where they have had promising applications. The topic is in full evolution, with active research efforts emerging from the fruitful interaction of various mathematical subjects and other scientific disciplines.

Contemporary Mathematics, Volume 216; 1998; 175 pages; Softcover; ISBN 0-8218-0793-5; List $49; Individual member $29; Order code CONM/216NT82

All prices subject to change. Charges for delivery are $3.00 per order. For optional air delivery outside of the continental U. S., please include $6.50 per item. *Prepayment required.* Order from: **American Mathematical Society**, P. O. Box 5904, Boston, MA 02206-5904, USA. For credit card orders, fax (401) 455-4046 or call toll free 800-321-4AMS (4267) in the U. S. and Canada, (401) 455-4000 worldwide. Or place your order through the AMS bookstore at http://www.ams.org/bookstore/. Residents of Canada, please include 7% GST.

MATHEMATICS from Birkhäuser

Coming February 1998 —

Harmonic Analysis on the Heisenberg Group

S. Thangavelu, *Indian Statistical Institue, Bangalore*

The Heisenberg group has been implicitly present in much of mathematics and physics for a long time, mostly in the area of analysis. This book presents a more restricted area of harmonic analysis (Fourier transforms, convolution algebra, and related ideas). It provides a useful survey of the field, bringing together a number of recent results in a unique coherent framework, and discusses representation theory of the group and its relationships to the theory of classical special functions.

Contents: *1. The Group Fourier Transform* • The Heisenberg group • The Schrödinger representations • The Fourier and Weyl transforms • Hermite and special Hermite functions • Paley-Wiener theorems for the Fourier transform • An uncertainty principle on the Heisenberg group • Notes and references • *2. Analysis of the Sublaplacian* • Spectral theory of the sublaplacian • Spectral decomposition for L^p functions • Restriction theorems for the spectral projections • Paley-Wiener theorem for the spectral projections • Bochner-Riesz means for the sublaplacian • A multiplier theorem for the Fourier transform • Notes and references • *3. Group Algebras and Applications* • The Heisenberg motion group • Gelfand pairs, spherical functions and group algebras • An algebra of radial measures • Analogues of Wiener Tauberian theorem • Spherical means on the Heisenberg group • A maximal theorem for spherical means • Notes and references • *4. The Reduced Heisenberg Group* • The reduced Heisenberg group • A Wiener Tauberian theorem for L^p functions • A maximal theorem for spherical means • Mean periodic functions on phase space • Notes and references • Index

1998 Approx. 212 pp. Hardcover ISBN 0-8176-4050-9 **$54.50 (tent.)**
Progress in Mathematics, Volume 159

Lise Meitner and the Dawn of the Nuclear Age

P. Rife, *University of Hawaii–Manoa, Maui*

In this fascinating biography, Patricia Rife interprets both the life and times of Lise Meitner (1878–1968), providing a rich background of the scientific discoveries and social milieu which affected the research, events, personalities, and politics of 20th century quantum physics. She asks the central question of why, after a dramatic escape from Nazi Germany and priority evidence of Meitner's role in the interpretation of nuclear fission, was she not awarded the Nobel Prize?

February 1998 Approx. 400 pp, 24 illus. Hardcover
ISBN 0-8176-3732-X **$44.50 (tent.)**

Available —

Geometry of Foliations

P. Tondeur, *University of Illinois, Urbana*

This volume describes research on the differential geometry of foliations, in particular Riemannian foliations. There are chapters on the Hodge theory for the transversal Laplacian, applications of the heat equation method to Riemannian foliations, spectral theory for Riemannian foliations, Connes' point of view of foliations as examples of noncommutative spaces, and infinite-dimensional examples of Riemannian foliations.

1997 312 pp. Hardcover ISBN 3-7643-5741-X **$98.00**
Monographs in Mathematics, Volume 90

Compactifications of Symmetric Spaces

Y. Guivarc'h, *Université de Rennes-I, France;* **L. Ji,** *University of Michigan, Ann Arbor* **& J.C. Taylor,** *McGill University, Montreal, Quebec*

Symmetric spaces are of central importance in many branches of mathematics. Compactifications of these spaces have been studied from the points of view of representation theory, geometry, and random walks. This book is devoted to the study of the inter-relationships between these various compactifications.

March 1998 Approx. 240 pp. Hardcover ISBN 0-8176-3899-7
$69.50 (tent.)
Progress in Mathematics, Volume 156

Complex Analysis: Fundamentals of the Classical Theory of Functions

J. Stalker, *Princeton University, NJ*

The classical theory of one complex variable is one of the most beautiful and useful subjects in mathematics. This clear, concise introduction is based on the premise that "anything worth doing is worth doing with interesting examples." Content is driven by techniques and examples rather than by definitions and theorems. The examples here, many of which are treated at a level of detail unmatched in similar introductory texts, are chosen from the analytic theory of numbers and classical special functions, but while they are mostly geared towards number theory, and mathematical physics, the techniques are generally applicable.

February 1998 Approx. 160 pp. Hardcover ISBN 0-8176-4038-X
$39.50 (tent.)

VISIT - www.birkhauser.com

New Textbook —

Laws of Chaos

Invariant Measures and Dynamical Systems in One Dimension

A. Boyarsky & P. Góra, *both, Concordia University, Montreal, Quebec, Canada (Eds.)*

Excellent pedagogical resource for studying probabilistic concepts in the analysis of one dimensional chaotic systems. This book combines three important areas of modern mathematics, including dynamical systems, measure theory, and ergodic theory, making it a truly up-to-date text for a graduate course for students of applied sciences.

Rich in examples and graphical illustrations, this book includes over one hundred pages of problem sets and their solutions. Practical applications as diverse as random number generation, computer modeling, rotary drills, the pendulum, population dynamics, and projective geometry are discussed.

1997 416 pp., 50 Illus. Hardcover ISBN 0-8176-4003-7
$59.95
Probability and Its Applications

Bernhard Riemann — 1826-1866

Turning Points in the Conception of Mathematics

D. Laugwitz, *Technische Hochschule Darmstadt* **& A. Shenitzer,** *York University, Ontario, Canada*

Bernhard Riemann is regarded as one of the most important philosophers of mathematics of all time, as well as a man who created a vast field of mathematics known as Riemannian Geometry. This book, originally written in German and now presented here in an English language translation, describes Riemann's development of a conceptual approach to mathematics at a time when conventional algorithmic thinking dictated that formulas and figures, rigid constructs, and transformations of terms were the only legitimate means of studying mathematical objects. Laugwitz has described in deep and compelling terms the work of Riemann and its impact on the thoughts of these giants of mathematics as well as the entire community of mathematics and physics.

March 1998 Approx. 350 pp., 40 illus. Hardcover
ISBN 0-8176-4040-1 **$75.00 (tent.)**

Birkhäuser

Boston • Basel • Berlin

TO ORDER:

CALL: 1 800 777-4643

FAX: 201 348-4505

E-MAIL: orders@birkhauser.com

Order Via our On-Line Order Form at:

www.birkhauser.com

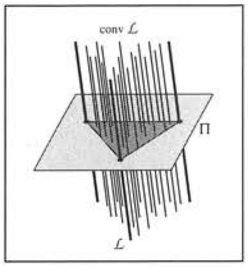
Please mention reference #Y004 when ordering.

Prices are valid in North America only and are subject to change without notice. For price and ordering information outside North America, please contact Birkhäuser Verlag, AG, P.O. Box 133, Klosterberg 23, CH-4010, Basel, Switzerland. Phone ++41 61 205 07 07; Fax ++41 61 205 07 92 or E-mail: sales@birkhauser.ch

Table of Contents

February 1998

Feature Articles



Memorial Article

Communications

Departments

From the AMS

Notices

of the American Mathematical Society

SUBSCRIPTION INFORMATION: Subscription prices for Volume 45 (1998) are $300 list; $240 institutional member; $180 individual member. (The subscription price for members is included in the annual dues.) A late charge of 10% of the subscription price will be imposed upon orders received from nonmembers after January 1 of the subscription year. Add for postage: Surface delivery outside the United States and India—$15; in India—$36; expedited delivery to destinations in North America—$35; elsewhere—$70. Subscriptions and orders for AMS publications should be addressed to the American Mathematical Society, P.O. Box 5904, Boston, MA 02206-5904. All orders must be prepaid.

ADVERTISING: *Notices* publishes situations wanted and classified advertising, and display advertising for publishers and academic or scientific organizations.

SUBMISSIONS: Letters to the Editor, Forum pieces, articles, and reviews may be sent to the editor by e-mail at notices@math.sunysb.edu, by fax at 516-751-5730, or by postal mail at P. O. Box 333, East Setauket, NY 11733. Electronic submissions are preferred. Correspondence with the managing editor may be sent to notices@ams.org.

NOTICES ON e-MATH: Most of this publication is available electronically through e-MATH, the Society's resource for delivering electronic products and services. To access the *Notices* on e-MATH, use the URL:

http://www.ams.org/notices/

Those with VT100-type terminals or without WWW browsing software can connect to e-MATH via Telnet (telnet e-math.ams.org; login and password are "e-math") and use the Lynx option from the main menu.

[*Notices of the American Mathematical Society* is published monthly except bimonthly in June/July by the American Mathematical Society at 201 Charles Street, Providence, RI 02904-2213. Periodicals postage paid at Providence, RI, and additional mailing offices. POSTMASTER: Send address change notices to *Notices of the American Mathematical Society*, P.O. Box 6248, Providence, RI 02940-6248.] Publication here of the Society's street address and the other information in brackets above is a technical requirement of the U.S. Postal Service. All correspondence should be mailed to the post office box, not the street address. Tel: 401-455-4000; e-mail: notices@ams.org.

Editorial

A Look Ahead

The *Notices* reports on mathematics, education, the profession, and opinion, among other things, in addition to recording information about the AMS and its activities. It serves a diverse readership of 30,000, and each article strives to inform a large number of readers about some topic or event.

The number and scope of expository mathematics articles in the *Notices* have increased dramatically in the past three years. But there is still a need: Some measure of success will have been achieved when potential authors are as eager to accept an invitation to write an article for the *Notices* as they are to accept an invitation to give an AMS hour address. This state should not be hard to achieve, as an article in the *Notices* has impact on more people and more mathematics than an invited address does.

In keeping with the idea that each article in the *Notices* hopes to interest a large number of readers, the Editorial Board often urges an invited author of a mathematics article to think in terms of giving a kind of colloquium in written form and to keep in mind some particular "sizable target audience" while writing. Rarely can such an audience consist entirely of experts, and consequently successful articles in the *Notices* are bounded above in level—at approximately the level of the more widely appreciated of the AMS invited hour addresses. Ideally these articles include some history and provide a context for appreciating the mathematics. They lead to the frontier and illuminate interconnections between different areas of mathematics—with that sizable target audience still in mind. The *Notices* continues to seek expository articles that fit this description.

Concerning education articles, the *Notices* recognizes the importance of the viewpoints mathematicians bring to education from pre-school through graduate school. It wants to promote discussion among mathematicians and mathematics educators, and it seeks to inform its readers about mathematics education. In this spirit, the Editorial Board welcomes expository articles related to mathematics education on any level. Articles on education should be well written and well informed. They should be enlightening to a sizable number of readers. In order to prevent acrimonious debate, which inhibits serious discussion, negative comments should not be directed toward personalities, specific programs, or whole fields of inquiry.

The *Notices* reports about many things concerning the mathematics profession. Among these are pieces about the job market, faculty salaries, grant support for research, conditions for graduate students, notable achievements by individuals and institutions, backlogs in journals, and other items. In a future issue the *Notices* will begin an in-depth study of the cost of publishing from a number of points of view.

Opinion pieces are a valuable tradition in the *Notices*. Some readers have wondered in the past whether the views expressed in the editorial column are the views of the AMS. In order to resolve this confusion, the *Notices* henceforth will clearly delineate personal opinions from official positions. Editorials will be statements about the *Notices* or will be elaborations of policies of the AMS written by the Editorial Board or certain officials. Pieces that largely represent just an individual opinion will appear elsewhere: in an "In My Opinion" column, as on the facing page, or in the "Letters to the Editor", or in special pieces clearly marked as opinion items. In addition, the *Notices* continues its tradition that essentially everything other than advertising, certain lists, and information about meetings is signed by the author.

—*Anthony W. Knapp*

Not to Miss

ABET Changes

Over a period of several years, the accrediting process for engineering schools will be undergoing a substantial revision that may affect the mathematics profession in serious ways. Allyn Jackson reports in "Engineering Accreditation Board Issues New Criteria" on the nature of these changes, Mary Beth Ruskai comments on the opposite page on possible ways of dealing with these changes, and a "Communication" by Solomon A. Garfunkel and Gail S. Young considers the related problem of declining enrollments in mathematics courses.

—*A.W.K.*

Commentary

In My Opinion

What Do Engineers Really Want?

The revision in ABET standards, dramatic decline in calculus enrollments, and proposals to shift some mathematics instruction to engineering faculty have focused attention on the design of mathematics courses for students in engineering. Many mathematicians believe that this is a call for cookbook courses that emphasize formulas and computation. Some may even feel that they would rather abandon the courses than the mathematics they hold dear. But I claim that a better understanding of what engineers and physical scientists really need to know can make applied mathematics *more*, not less, satisfying to teach. Let me illustrate with a few examples from the traditional calculus curriculum.

Example 1: Some years back a well-known mathematician publicly stated that asking students to differentiate something like $x^{\sin x}$ was silly because no one in his physics department would know how to work such a problem or would care. But he was only half right. *Every* physicist I asked found the problem unfamiliar but immediately rewrote the function as $e^{(\sin x)(\ln x)}$ and realized that differentiation was then straightforward. Although engineers and physicists may not use functions like $x^{\sin x}$, or even $5^{\sin x}$, they *do* need to know that $a^b = e^{b \ln a}$. Thus, the real issue is not "Will an engineer encounter this problem?" but "Is asking students to differentiate $x^{\sin x}$ an effective way to teach the meaning and use of exponential functions?" Is there a more effective way? Is memorizing a formula for differentiating a^x counterproductive?

Example 2: Too often traditional topics are taught year after year as if they were ends in themselves, with little memory of the motives for introducing them into the curriculum. I can see two important reasons for teaching the standard "solid of revolution" volume problems. One is to provide students with experience relating a practical quantity that is easily approximated by a sum to a definite integral. The other is that converting a problem such as "compute the volume remaining when a hole of radius r is drilled through the center of a sphere of radius R" to an integral involving specific functions may help the student develop the skill needed to analyze more realistic applied problems. However, if we confine ourselves to asking them to use a standard formula to find the volume when $f(x) = \ldots$ is rotated around the x-axis between $x = 1$ and $x = 5$, then *neither* goal will be met.

Example 3: A chemist once complained that he taught the chain rule in physical chemistry because the students did not learn the multivariable form in calculus. How can this be true? Chemists, physicists, and engineers will never need to compute anything like $\frac{\partial}{\partial x} y \sin(x^3 y)$, much less subsequently find $\frac{\partial}{\partial u}$ under some bizarre change of variables. Even transformations to standard, e.g., spherical, coordinates rarely require the explicit use of the chain rule, because the formulas for divergence, Laplacian, etc., are readily available. But the thermodynamic relationship

$$\left(\frac{\partial u}{\partial T}\right)_P = c_V + \left(\frac{\partial u}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P$$

is nothing more than a rewriting of the chain rule and the definition $c_V = \left(\frac{\partial u}{\partial T}\right)_V$. Chemical engineers and physical scientists use the chain rule to study the relationship between various quantities that arise in thermodynamics. Much mathematical meat is buried in the (possibly unfamiliar) notation $\left(\frac{\partial V}{\partial T}\right)_P$. First, it is assumed that P, V, T satisfy an "equation of state" of the form $f(P, V, T) = 1$ and that this implicitly defines three functions $P = p(V, T)$, etc. Then the expression above means $\frac{\partial V}{\partial T}$ in the usual sense, with the subscript P to indicate that V is regarded as a function of P and T. Developing relationships such as

$$\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V = -1$$

requires a basic understanding of the meaning of the implicit function theorem as well as the chain rule. Why should we leave this instruction to physical chemists and engineers? True, neither they nor their students want rigorous proofs. But not all theory is irrelevant. The implicit function theorem will fail at precisely those points where a phase transition occurs. We counter the attitude that counterexamples are mathematical pathologies by explaining how the failure of certain hypotheses is related to real physical phenomena.

Some may object that we are beginning to cross the line and teach things best left to engineers. But the boundary is fuzzy precisely because mathematical concepts, not just formulas, are important in applied fields. We should not strive for zero overlap, but for better communications between mathematicians and those in other fields.

—*Mary Beth Ruskai*
Associate Editor

When Is a Set of Lines in Space Convex?

Jacob E. Goodman

The Definition of a Convex Set

In $\mathbb{R}^d$, a set S of points is *convex* if the line segment joining any two points of S lies completely within S (Figure 1). The purpose of this article is to describe a recent extension of this concept of convexity to the Grassmannian and to discuss its connection with some other ideas in geometry. More specifically, the extension is to the so-called "affine Grassmannian" $G'_{k,d}$, the open manifold that parametrizes all the k-dimensional *flats* (translates of linear subspaces) in $\mathbb{R}^d$. In other words, rather than convex sets of points, we will be talking about convex sets of lines, for example, or of planes. Much of the material that this article deals with is based on joint work of the author's with Richard Pollack [7, 8], as well as with Rephael Wenger and others [3, 10].

Which properties of convex point sets would we expect to hold also for convex sets of k-flats?

The basic setup for point sets is this. To any set S of points is associated a set conv S containing S, called its convex hull (Figure 2), that satisfies:

1. monotonicity: $S \subset T \implies \operatorname{conv} S \subset \operatorname{conv} T$.

2. idempotence: $\operatorname{conv}(\operatorname{conv} S) = \operatorname{conv} S$.

3. antiexchange: $\operatorname{conv} S = S,\ x, y \notin S,\ x \neq y,\ y \in \operatorname{conv}(S \cup \{x\}) \implies x \notin \operatorname{conv}(S \cup \{y\})$ (Figure 3).

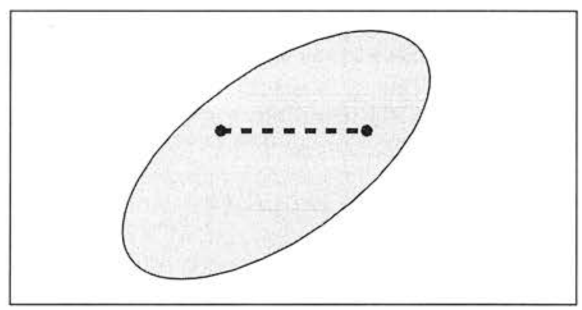


Figure 1. A convex point set.

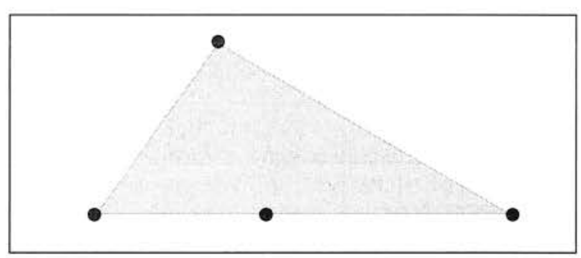


Figure 2. The convex hull of a set of four points.

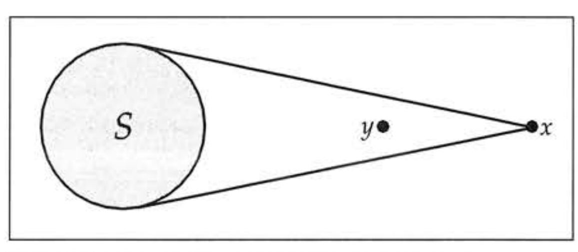


Figure 3. The antiexchange property.

Jacob E. Goodman is professor of mathematics at the City College of the City University of New York. His e-mail address is jegcc@cunyvm.cuny.edu.

Research partially supported by NSF grant DMS-9322475, NSA grant MDA904-95-H-1012, and PSC-CUNY grant 666366. This article is based on an address presented at the Montreal meeting of the American Mathematical Society in September 1997 under the title "Convex geometry on affine Grassmannians and geometric transversals". I am grateful to Hugo Rossi for inviting me to write an article for the Notices *on my recent work and to Richard Pollack and Rephael Wenger for their helpful comments.*

—J. E. Goodman

(Property 3 says essentially that conv induces a partial ordering on the complement of any convex set, which can be thought of intuitively as representing how "far away" points are from the set.)

4. nonsingular affine invariance: conv commutes with the action of the affine group. (If $\mathrm{A}(d,\mathbb{R})$ is the group of nonsingular affine transformations—nonsingular linear transformations plus translations—

$$\mathrm{A}(d,\mathbb{R}) = \left\{ \begin{bmatrix} L & \alpha \\ 0 & 1 \end{bmatrix} \;\middle|\; L \in \mathrm{GL}(d,\mathbb{R}),\ \alpha \in \mathbb{R}^d \right\},$$

and if $\sigma \in \mathrm{A}(d,\mathbb{R})$, then $\operatorname{conv}(\sigma S) = \sigma(\operatorname{conv} S)$ for $S \subset \mathbb{R}^d$; see Figure 4.) Then once we know what the convex hull of a set is, a set S is said to be *convex* if $\operatorname{conv} S = S$.

The first three of these are usually taken to be the defining properties of what is known as an abstract "convex hull" operation, i.e., of a convexity structure, and the last condition means that this convexity structure is a natural one in the affine space $\mathbb{R}^d$.

But if we want to define a similar convexity structure on $G'_{k,d}$, for example on the set of lines in 3-space, we immediately run up against the following apparent obstacle. One of the characteristic features of a convex set of points is that it is connected. Yet we can show:

Theorem 1 [8]. There is no notion of convexity for lines or higher-dimensional flats that is nonsingular-affine-invariant, that satisfies the antiexchange property, and in which all convex sets are connected.

This is not difficult to prove. It means that we have to give up something. It turns out that if one is willing to give up the connectedness itself, to drop one's insistence that a convex set of flats should always be connected, then a rich theory emerges, one that extends many of the properties of convexity for point sets and that links up to a number of problems that have recently been studied under the heading of what is sometimes called "geometric transversal theory".

There are two ways to describe what the convex hull of a set of k-flats is, both of which extend characterizations of the convex hull of a point set. First let us go back to points for a moment.

Definition 1. Let us say that a point x in a flat F is *surrounded by* a set of points S in F if any *hyperplane* (flat of codimension 1) H in F passing through x lies strictly between two parallel hyperplanes H_1 and H_2 in F, each containing a point of S; in other words, any such hyperplane is "trapped" by points of S if it tries to escape by continuous translation to infinity (Figure 5).

Then it is easy to see that

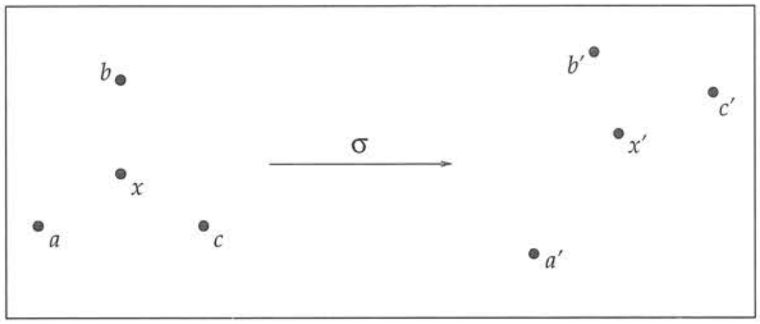


Figure 4. $x \in \operatorname{conv}\{a,b,c\} \Longrightarrow x' \in \operatorname{conv}\{a',b',c'\}$.

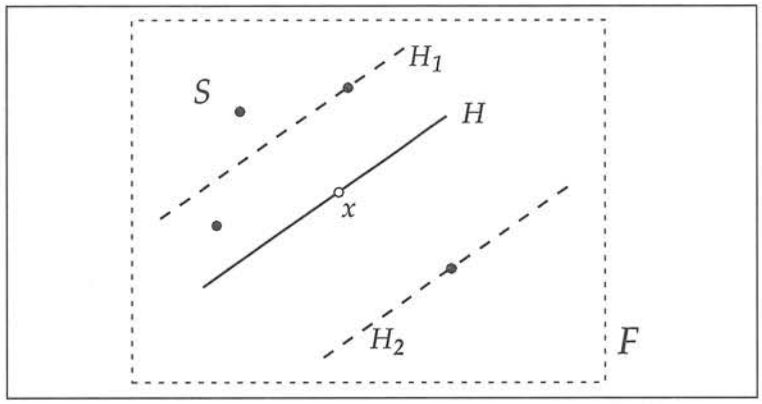


Figure 5. Point x is surrounded by set S.

A) $x \in \operatorname{conv} S$ iff there is a flat F containing x within which x is surrounded by the points of S lying in F. (One cannot simply say "iff x is surrounded by the points of S", since S may be a lower-dimensional set.)

And of course we also have

B) $x \in \operatorname{conv} S$ iff every convex point set meeting every point of S also meets x. (This is trivially true: it just amounts to saying that $\operatorname{conv} S$ is the intersection of all the convex point sets containing S.)

It turns out that "surrounded by" still makes perfectly good sense when the basic objects are flats of some fixed dimension k from 1 to $d-1$ rather than simply points, and that (A) and (B) are still equivalent in that setting. And it also turns out that they imply the four basic properties of the convex hull operator in $\mathbb{R}^d$: monotonicity, idempotence, antiexchange, and invariance with respect to the affine group, and, happily, many other properties as well.

So here is our definition of the convex hull of a set of k-flats in d-space:

Definition 2. A k-flat l' belongs to the *convex hull* $\operatorname{conv} \mathcal{L}$ of a set $\mathcal{L}$ of k-flats in $\mathbb{R}^d$ if it satisfies either of the following two conditions:

A) There is a flat F containing l' within which l' is surrounded by the flats of $\mathcal{L}$ lying in F;

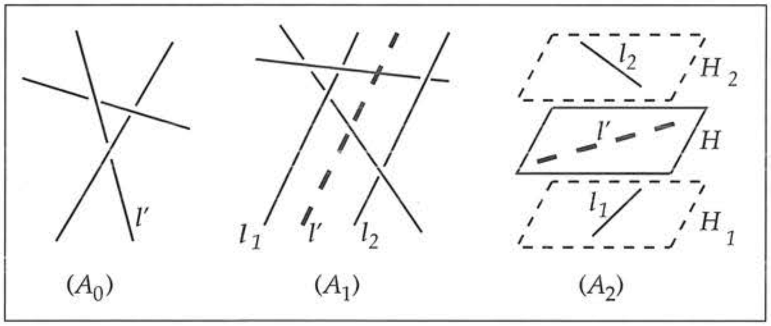


Figure 6. Line l' is surrounded by other lines.

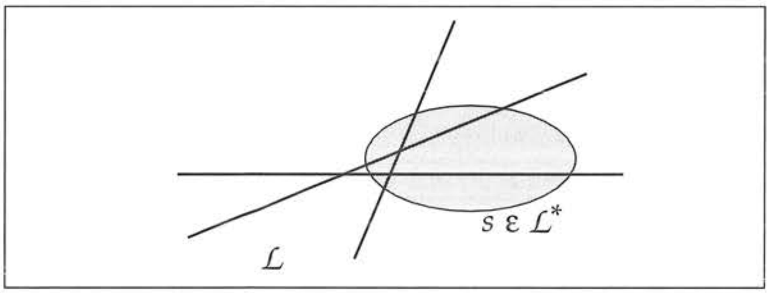


Figure 7. The dual of a set of lines.

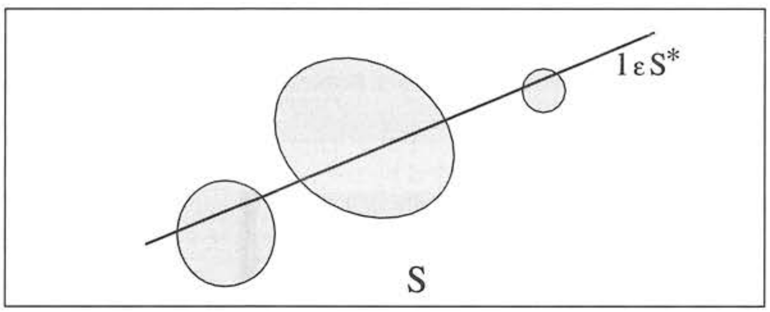


Figure 8. The dual of a set of convex point sets.

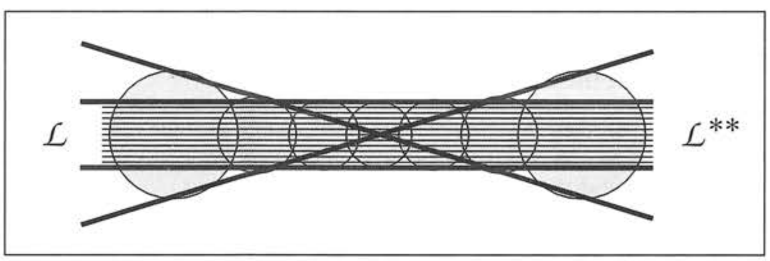


Figure 9. The convex hull of a set of lines.

B) Every convex point set meeting all the members of $\mathcal{L}$ also meets l'.

In (A) "surrounded by" has the same meaning as before: l' is *surrounded by* a set $\mathcal{L}$ of k-flats in a flat F if any hyperplane $H \subset F$ containing l' is sandwiched strictly between two other hyperplanes H_1 and H_2, each containing members of $\mathcal{L}$; in other words, every hyperplane $H \subset F$ containing l' is trapped by members of $\mathcal{L}$.

So, for example, just looking at the special case of lines in $\mathbb{R}^3$, we have:

If $\mathcal{L} \subset G'_{1,3}$, a line l' is in conv $\mathcal{L}$ if and only if (Figure 6)

A$_0$) $l' \in \mathcal{L}$, or

A$_1$) l' lies between two parallel lines $l_1, l_2 \in \mathcal{L}$, or

A$_2$) any plane Π passing through l' can be moved parallel to itself a positive distance in either direction until it contains lines of $\mathcal{L}$.

(A$_2$) describes the "generic" way in which a line is surrounded by other lines, while (A$_0$) and (A$_1$) describe the degenerate ways.

And then we have

Theorem 2 [8]. (A) and (B) in Definition 2 are equivalent.

Theorem 3. The operator conv on $G'_{k,d}$ satisfies

1. monotonicity: $\mathcal{L}_1 \subset \mathcal{L}_2 \implies \text{conv}\,\mathcal{L}_1 \subset \text{conv}\,\mathcal{L}_2$.

2. idempotence: conv (conv $\mathcal{L}$) = conv $\mathcal{L}$.

3. antiexchange: conv $\mathcal{L} = \mathcal{L}$, $l_1, l_2 \notin \mathcal{L}$, $l_1 \neq l_2$, $l_2 \in \text{conv}\,(\mathcal{L} \cup \{l_1\}) \implies l_1 \notin \text{conv}\,(\mathcal{L} \cup \{l_2\})$.

4. nonsingular affine invariance: conv commutes with the action of the affine group. (If $\sigma \in \mathrm{A}(d, \mathbb{R})$ induces $\sigma_k : G'_{k,d} \to G'_{k,d}$, then $\text{conv}\,(\sigma_k \mathcal{L}) = \sigma_k(\text{conv}\,\mathcal{L})$ for $\mathcal{L} \subset G'_{k,d}$.)

There is a nice way to look at condition (B) in the definition. Given a set $\mathcal{L}$ of k-flats in $\mathbb{R}^d$, let us define its *dual*, $\mathcal{L}^*$, to be the set of all convex point sets S, each of which meets all the flats of $\mathcal{L}$ (Figure 7). And given a family S of convex point sets, define its *k-dual* S^* (actually S^{*_k}, but simply S^* if we fix k) to be the set of all k-flats l meeting all the members of S, the so-called *k-transversals* of the family S (Figure 8). Then condition (B) just amounts to saying that the convex hull of a set $\mathcal{L}$ of flats is its double dual $\mathcal{L}^{**}$. So if $k = 1$, say, start with a set $\mathcal{L}$ of four lines as in Figure 9, look at all the convex point sets meeting them ($\mathcal{L}^*$), and then look at *all* the lines meeting those convex sets ($\mathcal{L}^{**}$). They constitute the convex hull.

It is trivial to see that this concept of duality satisfies the usual conditions:

1. $\mathcal{L}_1 \subset \mathcal{L}_2 \implies \mathcal{L}_1^* \supset \mathcal{L}_2^*$; $S_1 \subset S_2 \implies S_1^* \supset S_2^*$;

2. $\mathcal{L}_1 \subset \mathcal{L}_1^{**}$; $S_1 \subset S_1^{**}$.

These imply immediately that $\mathcal{L}_1^{***} = \mathcal{L}_1^*$ and $S_1^{***} = S_1^*$, and it follows that a set $\mathcal{L}$ of flats is convex if and only if it is self-dual ($\mathcal{L}^{**} = \mathcal{L}$). Equivalently,

A set $\mathcal{L}$ of k-flats is convex iff $\mathcal{L}$ is the set of all common k-flat transversals of some family S of convex point sets.

If S can be taken to consist of a finite family, we say that $\mathcal{L}$ is *finitely presented*; if S consists of a *single* convex point set, we call $\mathcal{L}$ *principal*.

This, incidentally, is one of the main reasons why we are interested in convex sets of flats: because of their connection to what is known as "geometric transversal theory". Before discussing this connection, though, let us look at some examples of convex sets, say for lines in $\mathbb{R}^3$.

Some Examples of Convex Sets of Lines

Recall that there are two equivalent ways to think of a set of lines being convex: one is that every line surrounded by the set must belong to the set; the other is that it is the set of all common line transversals to some family of convex point sets.

1. First, an example of a convex set of lines that we find useful in guiding our intuition: Consider the 1-sheeted hyperboloid $x^2 + y^2 - z^2 = 1$ (Figure 10). On it there is a set of rulings: the line $\{x = 1, z = y\}$ and all of the lines obtained by rotating it about the z-axis, as well as a second such set generated by $\{x = 1, z = -y\}$. Consider the first of these sets; call it $\mathcal{L}$. What is conv $\mathcal{L}$?

It turns out that it consists precisely of all the lines "inside" the hyperboloid, together with the members of $\mathcal{L}$ themselves.

It is not hard to see this, using the "surrounding" criterion. For example, the z-axis belongs to conv $\mathcal{L}$ because any plane containing the z-axis, when translated (in either direction perpendicular to itself) a unit distance, passes through one of the rulings in our set. The same thing holds (with modification) for *any* line inside the hyperboloid. On the other hand, if l' is not inside the hyperboloid, it is not hard to find a plane through l' that can be translated *away* from the origin so as never to pass through one of our rulings.

2. Here is a second example. Let $\mathcal{L}$ be a (necessarily discontinuous) section of the tangent bundle to a unit sphere in $\mathbb{R}^3$; i.e., choose a tangent line at each point (Figure 11).

Then it is not hard to see that $l' \in$ conv $\mathcal{L}$ if and only if either l' meets the interior of the sphere or $l' \in \mathcal{L}$.

3. A third example consists of a set $\mathcal{L}$ of parallel lines, such as the three extreme lines in Figure 12. Take any plane Π cutting them, and take the convex hull $\overline{S}$ of their trace S in that plane. Then the convex hull of $\mathcal{L}$ consists precisely of the lines through $\overline{S}$ parallel to the original set $\mathcal{L}$.

4. What happens, though, if we take, say, a *finite* set of lines in $\mathbb{R}^3$, no two parallel, as in Figure 13? It is easy to see, from the "surrounding" criterion, that such a set is already convex!— there is no line outside the set that is surrounded by the set. So this is an example, perhaps an extreme one, of a convex set of lines that is disconnected, as predicted by Theorem 1.

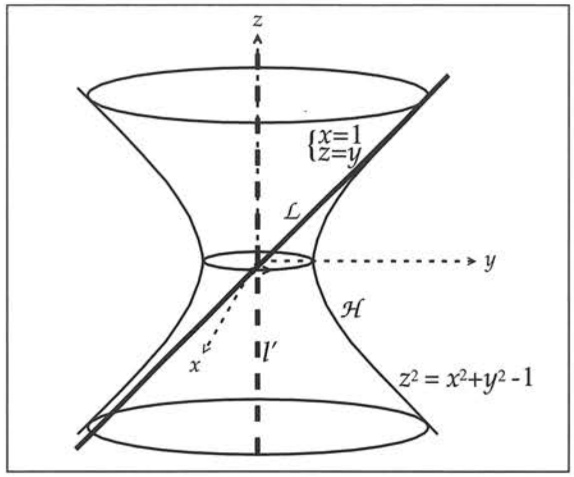


Figure 10. Rulings on a hyperboloid.

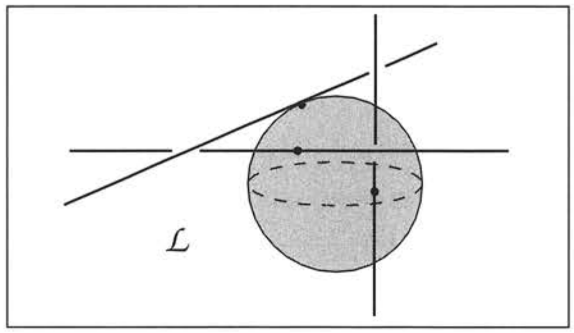


Figure 11. Lines tangent to a sphere.

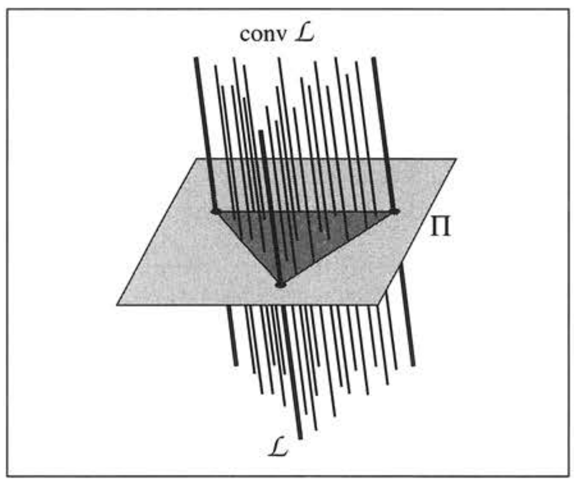


Figure 12. A set of parallel lines.

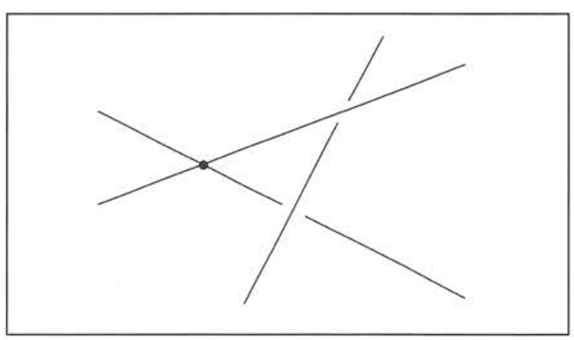


Figure 13. A finite set of lines without parallels.

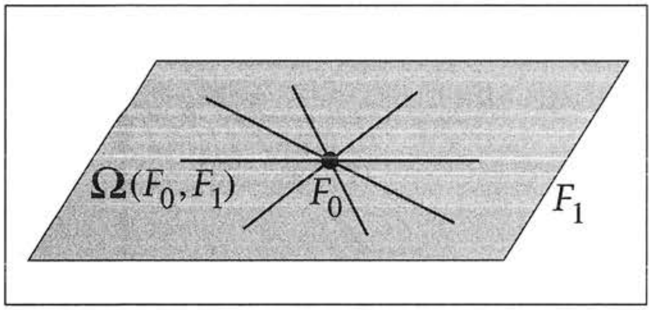


Figure 14. A partial flag of flats.

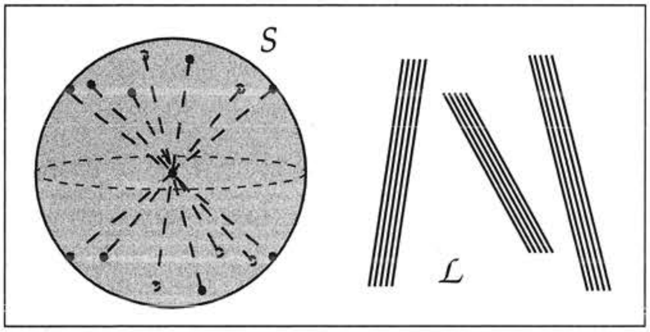


Figure 15. A parallel-closed set of lines.

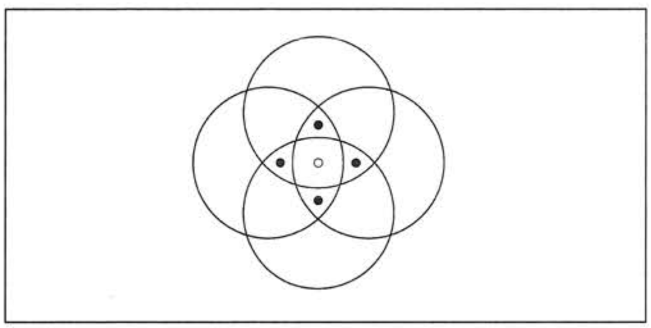


Figure 16. Helly's theorem in the plane.

Nothing like this happens, of course, with point sets, but for a very simple reason: any two points are "parallel" in the sense that each is a translate of the other. And we have seen that for a set of parallel lines the convex hull *is* connected. So this is a major difference between convexity for point sets and for sets of flats—the possibility that for flats a convex set may turn out to be disconnected or even discrete.

5. Certain Schubert varieties in an affine Grassmannian are convex. These are defined as follows: Let $\Phi : F_0 \subset F_1 \subset \cdots \subset F_k$, where the inclusions are strict, be a (partial) flag of flats in $\mathbb{R}^d$; an example is shown in Figure 14 for $d = 3$ and $k = 1$. The *Schubert variety* $\Omega(F_0, \ldots, F_k)$ determined by Φ, which consists of all the k-flats $l \in G'_{k,d}$ with $\dim(l \cap F_i) \geq i$ for all $i = 0, \ldots, k$, and the algebraic set $\Omega^0(F_0, \ldots, F_k)$ determined by the *strict* Schubert conditions coming from Φ, which consists of all the k-flats $l \in G'_{k,d}$ with $\dim(l \cap F_i) = i$ for all $i = 0, \ldots, k$, are convex in certain cases, for example if $d = 3$ and $k \leq 3$. We do not yet know all of the values of k and d for which this holds; in fact, this seems to be an interesting problem.

6. For a final example consider a centrally symmetric point set S on a 2-sphere in $\mathbb{R}^3$, and let $\mathcal{L}$ be the set of all lines parallel to the lines joining the center of the sphere to the points of S, a "parallel-closed set" (Figure 15). Then it turns out that $\mathcal{L}$ is convex if and only if the set S is the complement of a union of great circles.

These parallel-closed families, in fact, turn out to be rather interesting. They can be thought of as duals of families of "convex point sets at infinity", in an appropriate sense. We will say more about them below.

Geometric Transversal Theory

Let us look at convexity on the affine Grassmannian from the point of view of what is called "geometric transversal theory". Helly's theorem [12], one of the cornerstones of combinatorial geometry, says that for a family of at least $d + 1$ compact convex sets in $\mathbb{R}^d$, if every $d + 1$ are pierced by a point, then all the sets are pierced by a single point (Figure 16).

In 1935 Vincensini asked whether a similar theorem could be proved if "pierced by a point" was replaced by "met by a line", or by a plane, or a flat of dimension k in d-space. This question led to a series of papers by, among others, Santaló, Klee, Hadwiger, Debrunner, Grünbaum, Danzer, Valentine, and Eckhoff, giving various generalizations of Helly's theorem to line transversals and higher-dimensional transversals and to families of convex bodies of various kinds (parallelopipeds, translates of a fixed body, and so on). An excellent survey of work in geometric transversal theory up to 1962 is given in [4]. In the past two decades there has been a sort of explosion of interest, with work by Katchalski, Lewis, Liu, Zaks, Tverberg, Dol'nikov, Alon, Kalai, Kleitman, Montejano, and many other people. In recent years the thread has also been picked up by computer scientists, for example Agarwal, Aronov, Avis, Edelsbrunner, Pellegrini, Robert, Sharir, Shor, and Wenger, among others, who are interested in the combinatorial complexity of the space of transversals of a family of convex sets, both for lines and for higher-dimensional "stabbers", in connection with the complexity of algorithms in computational geometry. Up-to-date surveys include [5, 9, 17, 18].

The author's own interest in the subject dates to a paper with Pollack [7], in which we generalized a theorem of Hadwiger's on line transversals:

Theorem 4 (Hadwiger's Transversal Theorem) [11]. A family S of pairwise disjoint compact convex sets in the plane has a line transversal if and only if there is a linear ordering of S such that every three convex sets are met by a directed line consistently with that ordering.

Thus if there is some numbering of the sets such that any three line up in increasing order, as in Figure 17, then the family has a line transversal. Without the assumption about the sets having a consistent order the theorem would be false; in fact, there is no "pure" Helly-type theorem that says that if enough of the sets have a common line transversal, then all of them do: the example shown in Figure 18, consisting of a family of four line segments and a centrally located point, generalizes.

But if we are interested in the existence of, say, a *planar* transversal for a family of compact convex sets in 3-space, how can we generalize this notion of "consistent linear order"? The answer is provided by the concept of the *order type* of a set of points in the plane or in general in $\mathbb{R}^d$. This is a very simple idea that has proven useful in a great many situations where one wants to talk about higher-dimensional "order" properties of a set of points.

Definition 3. If S is a set of labeled points, $P_1 = (x_1^1, \ldots, x_1^d), \ldots, P_n = (x_n^1, \ldots, x_n^d) \in \mathbb{R}^d$, the *order type* of S is the family of orientations of the $(d+1)$-tuples of points, i.e., the mapping that associates to each $(d+1)$-tuple $i_0 < \ldots < i_d$ the sign of the determinant

$$\begin{vmatrix} 1 & x_{i_0}^1 & \cdots & x_{i_0}^d \\ \vdots & & \ddots & \\ 1 & x_{i_d}^1 & \cdots & x_{i_d}^d \end{vmatrix}.$$

So, for example, the sets S and T in Figure 19 have the same order type, labeled as they are, since each triple on the left is counterclockwise if and only if the corresponding triple on the right is, but the sets S and U do not, since P_1, P_2, P_3 and R_1, R_2, R_3 have opposite orientations. In fact, it is easy to see that there is *no* labeling in which their order types agree.

The order type of a set of points is also known as a "realizable acyclic oriented matroid". Roughly speaking, an oriented matroid is an axiomatically defined structure that generalizes many of the incidence and order properties of a configuration of points. This notion will occur in Theorem 6 below. A more precise definition, which we do not need, may be found in [2].

Now the Hadwiger theorem for line transversals requires that the sets be met consistently by directed lines. What do we mean by the order in which a family of convex sets meets a directed line l? Simply choose any point from the intersection of each set with l, and look at their order along l (Figure 20). The pairwise disjointness hypothesis guaran-

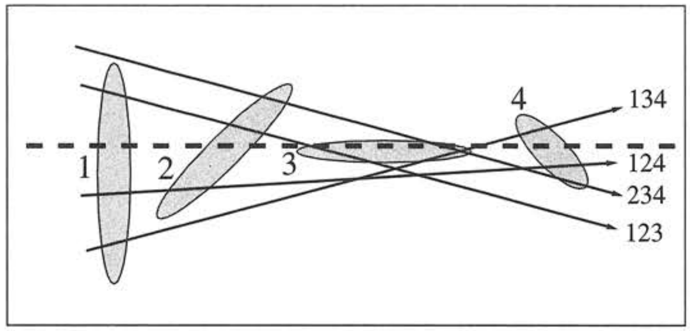


Figure 17. Hadwiger's transversal theorem.

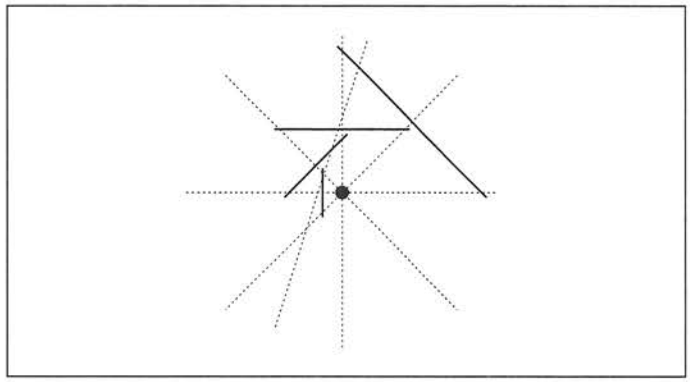


Figure 18. Any 4 sets have a common transversal, but all 5 do not.

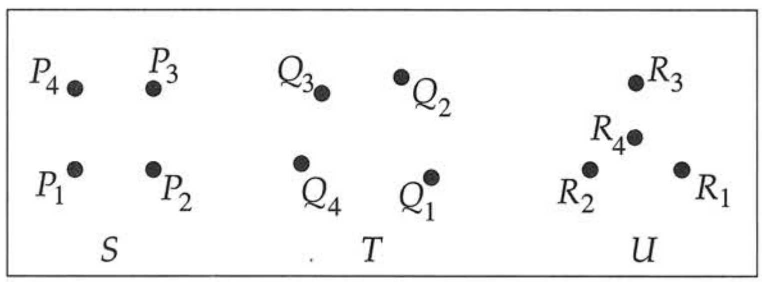


Figure 19. U has a different order type from S and T.

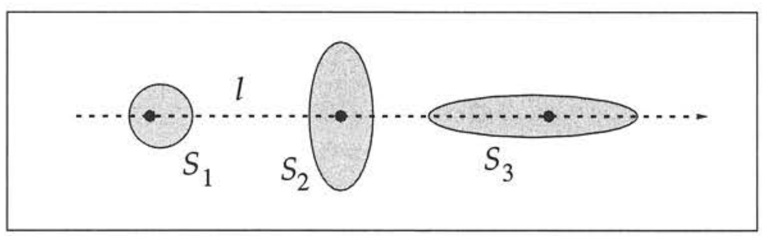


Figure 20. The order of the 3 sets is well-defined.

tees that for a given line transversal meeting any number of the sets, the order in which it meets them will not depend on which point we choose from each set.

How can we guarantee the same thing for, say, an oriented plane transversal? A moment's reflection shows that we should assume that no three of the sets have a line transversal (if they did, as in Figure 21, the choice of points x, y_i, z could turn out clockwise, counterclockwise, or degener-

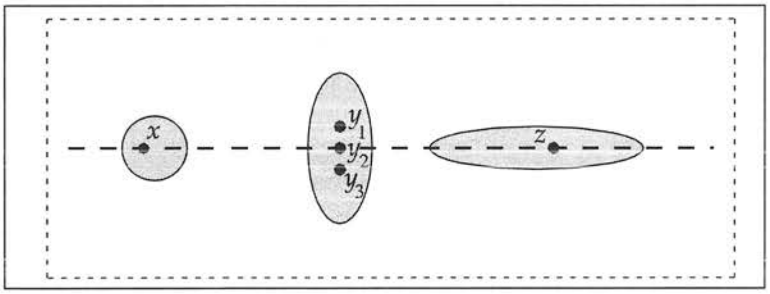


Figure 21. The orientation of the 3 sets is ambiguous.

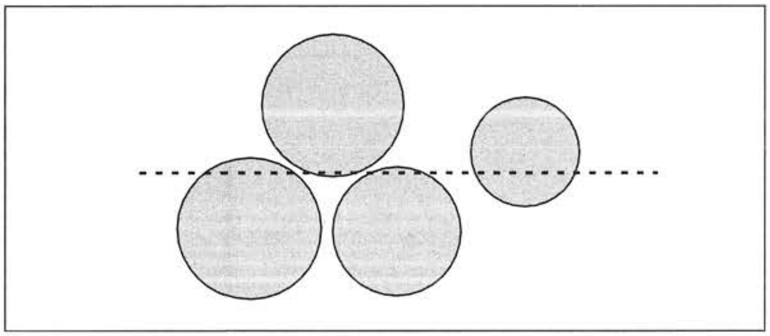


Figure 22. 0-separated sets in the plane.

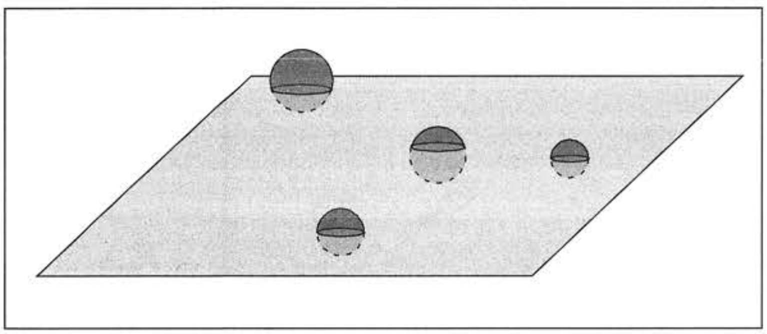


Figure 23. 1-separated sets in 3-space.

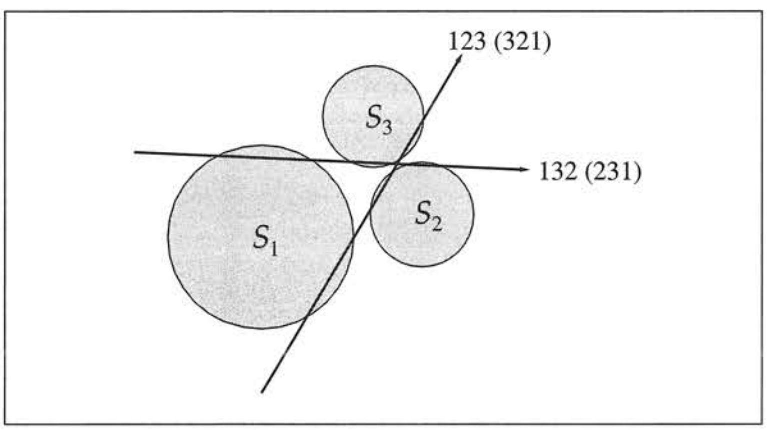


Figure 24. Two geometric permutations.

ate, depending on which side of the line we chose them). So we call a family of point sets k*-separated* if no $k+2$ of them have a common k-transversal; thus 0-separated means that no two have a point in common, which simply means that the sets are pairwise disjoint (Figure 22), as in the original Hadwiger theorem, and 1-separated means that no three line up (Figure 23).

Then the generalized Hadwiger theorem turns out to be:

Theorem 5 [7]. A $(d-2)$-separated family S of compact convex sets in $\mathbb{R}^d$ has a hyperplane transversal if and only if there is a set S of points in $\mathbb{R}^{d-1}$ such that every $d+1$ convex sets are met by an oriented hyperplane consistently with the order type of S.

This result has since been generalized further by Pollack and Wenger [15] and more recently still further by Anderson and Wenger to the following, which eliminates entirely the separatedness condition in Hadwiger's original theorem, drops the assumption that the order type of the sets is realizable by points, and also subsumes an earlier theorem of Katchalski:

Theorem 6 [1]. Let S be a finite family of connected sets in $\mathbb{R}^d$. Then S has a hyperplane transversal if and only if, for some k with $0 \le k < d$, there is a rank $k+1$ acyclic oriented matroid on S such that every $k+2$ members of S are met by an oriented k-flat consistently with that oriented matroid.

The outstanding problem in this connection is to find a similar result, if one exists, for transversals of codimension greater than 1. Here something beyond the order type will be needed, just as the original Hadwiger theorem needed something beyond a Helly-type condition: For each n there is an example, due to Aronov et al., of a linearly ordered family of convex sets in 3-space such that every n are met by a line transversal consistently with the order, but no line transversal exists for the entire family (see [9], Theorem 2.9).

Geometric Permutations

A second thread in geometric transversal theory, also involving hyperplane transversals, has a more combinatorial flavor. It involves the idea of what is called a "geometric permutation". A family of convex sets in the plane may be met by directed lines in several different orders, even if the sets are disjoint, as in Figure 24. Each such order (together with its reverse) is called a *geometric permutation.* If there are n mutually disjoint sets, how many geometric permutations can they have?

It was shown by Katchalski, Lewis, and Zaks [14] that there can be as many as $2n-2$ (see Figure 25, where half of the permutations for $n=5$ are indicated), and by Edelsbrunner and Sharir [6] that this is, in fact, the maximum number. What about the same problem for hyperplane transversals in $\mathbb{R}^d$?

Again, the notion of *order type* tells us what a "geometric permutation" should mean in this case: a pair consisting of a $(d-1)$-dimensional order type determined by intersecting a $(d-2)$-separated family of convex sets with a hyperplane transversal, together with its reverse, as shown in Figure 26. Then we have the following result, which generalizes, at least asymptotically, the upper bound in the plane due to Edelsbrunner and Sharir:

Theorem 7 [3]. Let S be a $(d-2)$-separated family of n compact convex sets in $\mathbb{R}^d$. Then S has $O(n^{d-1})$ geometric permutations induced by hyperplane transversals.

The key step in the proof turns out to involve looking at the space of oriented hyperplanes that are *common tangents* to a $(k-2)$-separated family of $k \leq d$ strictly convex sets in $\mathbb{R}^d$ and showing that it is homeomorphic to 2^k copies of the sphere S^{d-k}. For example, Figure 27 indicates one of the four circles of oriented planes that are common tangents to the two spheres shown.

Now it is clear that any two k-flat transversals inducing distinct geometric permutations on a $(k-1)$-separated family of compact convex sets must belong to distinct connected components of the entire space of transversals; for example, in the plane, if two lines meet a family of pairwise disjoint sets in different orders, then one line cannot be moved continuously to the other, keeping it transversal to all the sets, since that would violate the separatedness. The converse was shown by Wenger to hold in the case of hyperplane transversals:

Theorem 8 [16]. Let S be a $(d-2)$-separated family of compact convex sets in $\mathbb{R}^d$. Two hyperplane transversals induce the same geometric permutation on S if and only if they lie in the same connected component of the space of transversals to S.

So in terms of the convexity structure on the space $G'_{d-1,d}$ of hyperplanes, this last result gives a description of the various connected components of a convex set of hyperplane transversals if the presentation is given by a separated family: each connected component corresponds to a different geometric permutation; see Figure 28. (This shows again why convex sets of higher-dimensional flats cannot be expected to be connected, in general, as indicated in Example 4 above: because of the possibility that they may correspond to distinct geometric permutations.)

Summing up, one might say that geometric transversal theory involves the study of the space of k-flat transversals to a given family of convex point sets with regard to its topological properties, its combinatorial properties, and so on. The theory of convexity on the affine Grassmannian explores the *inverse* problem: given a set $\mathcal{L}$ of k-flats, under what circumstances is $\mathcal{L}$ the set of all k-transversals to some family of convex point sets? The answer, provided by Theorem 2 above, is: if

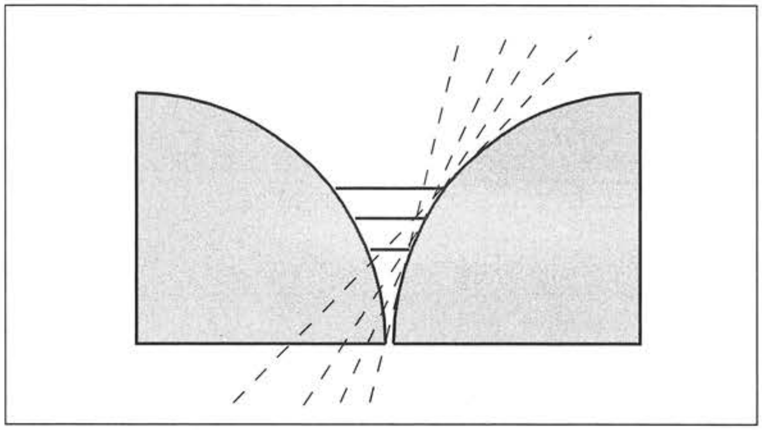


Figure 25. 5 sets with 8 geometric permutations.

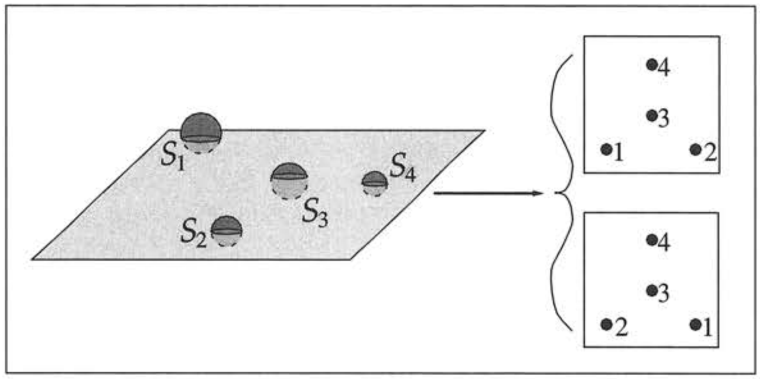


Figure 26. A planar geometric permutation.

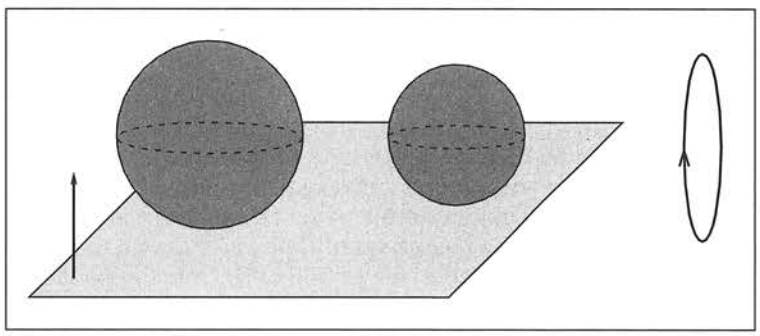


Figure 27. A circle of oriented tangent planes.

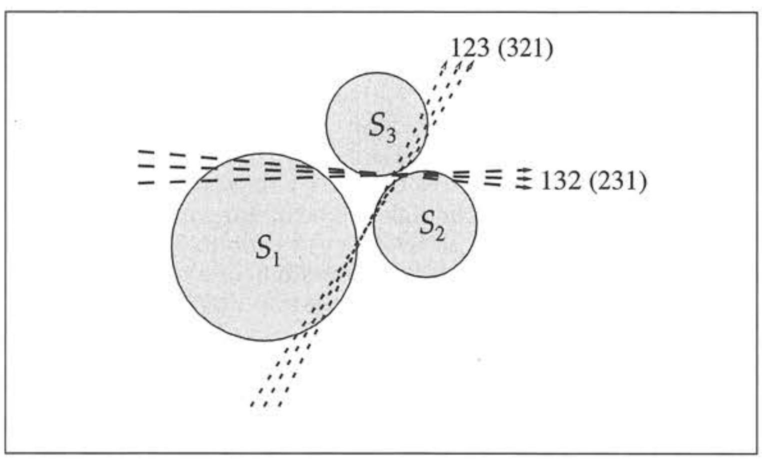


Figure 28. The two connected components of a convex set.

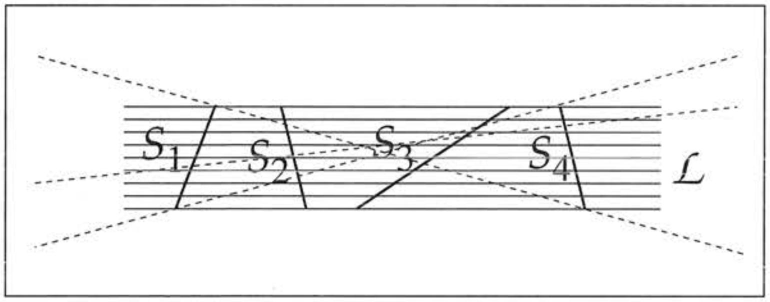


Figure 29. $\mathcal{L}$ **is not finitely presented.**

and only if every k-flat surrounded by $\mathcal{L}$ already belongs to $\mathcal{L}$.

Properties of Convex Sets: Some Results and Questions

Of the various properties of convex point sets that extend to convex sets on the affine Grassmannian, here are just a few.

We have already mentioned the fact that convex sets of k-flats are preserved under nonsingular affine transformations. Just as is true for convex point sets, convex sets of k-flats are also preserved under restriction to subspaces and under restriction to direction:

Theorem 9 [8]. Let $\mathcal{L} \subset G'_{k,d}$ be convex.

i) If F is an m-flat in $\mathbb{R}^d$ with $m \geq k$, the restriction $\mathcal{L}|_F$ of $\mathcal{L}$ to F, consisting of all the members of $\mathcal{L}$ that lie in F, is a convex subset of $G'_{k,m}$.

ii) If F is an m-flat in $\mathbb{R}^d$ with $m \leq k$, then the set $\mathcal{L}\|_F$ of all flats in $\mathcal{L}$ parallel to F is convex.

Another property that extends nicely from convex point sets to convex sets of flats is the Krein-Milman property. For point sets this says that every compact convex set of points is the convex hull of its extreme points. If we define an *extreme flat* of a convex set of flats in $\mathbb{R}^d$ as one that does not lie in the convex hull of the remaining flats of the set, then we have

Theorem 10. Every compact convex set of flats in $\mathbb{R}^d$ is the convex hull of its set of extreme flats.

So far the questions we have been dealing with have been largely ones that make sense for convex point sets; we have seen that many of the standard properties of convex point sets extend to convex sets of flats.

Now let us consider a different kind of question that does not come up for convex point sets, one that arises from the "double dual" characterization of the convex hull of a set of flats. This can be thought of as the question of *canonical representation.*

If a convex set $\mathcal{L}$ of k-flats is the set of transversals to a *finite* set $S = \{S_1, \dots, S_n\}$ of convex point sets, recall that $\mathcal{L}$ is called *finitely presented.* It is easy to see that not all convex sets of k-flats are finitely presented, for example, the set of all lines in the plane lying between two parallel lines (Figure 29). Here one needs infinitely many convex point sets S_i to limit their common transversals to just the lines of $\mathcal{L}$.

But suppose $\mathcal{L}$ *is* finitely presented. Then we may be able to throw away convex point sets S_i that are not needed in the presentation to get an *irredundant* presentation, and we may be able to shrink each set S_i as much as possible to get a *minimal* presentation. More precisely,

Definition. If $\mathcal{L}$ is a convex set of k-flats in $\mathbb{R}^d$ and S a family of convex point sets, the presentation $\mathcal{L} = S^*$ is *irredundant* if $\mathcal{L} \subsetneq S_0^*$ for every proper subset S_0 of S. On the other hand, if S^* becomes strictly smaller whenever any $S \in S$ is replaced by a proper subset, the presentation $\mathcal{L} = S^*$ is called *minimal.*

Not every convex set of k-flats has a minimal, irredundant presentation, for example the set of lines between two parallels, but it is easy to prove

Theorem 11 [8]. If $\mathcal{L} \subset G'_{k,d}$ is finitely presented by a set S of compact convex point sets, then S can be refined to a family S_0 of compact convex point sets that gives a minimal, irredundant presentation of $\mathcal{L}$.

In the special case where $\mathcal{L}$ is a finite set of hyperplanes in general position in $\mathbb{R}^d$ (such a set is easily seen to be convex by the surrounding criterion), we can prove more:

Theorem 12 [8]. If $\mathcal{L}$ is a finite subset of $G'_{d-1,d}$ consisting of n hyperplanes in general position, then $\mathcal{L}$ has a minimal, irredundant presentation by $2(d-1)(n-d)+2^d$ compact convex point sets.

Figure 30 shows such a presentation for the case of n lines in the plane ($d = 2$ in this case, so that $2(d-1)(n-d)+2^d = 2n$).

We have no such result for the case of k-flats with $1 \leq k \leq d-2$; it would be nice to know what happens there.

Another question that does not arise for convex point sets but does for convex sets of flats of positive dimension is the following: If a convex set of k-flats can have more than one component, is it true, at least, that each component by itself is convex? With no restriction at all on the convex set, the answer turns out to be no—there is a rather complicated example [10] of a set of lines in 3-space whose convex hull has two connected components, one of which is nonconvex. But if we restrict our convex sets in fairly natural ways, the answer seems to be yes, at least in certain dimensions.

For example, we have:

Theorem 13 [10]. Let S be a finite family of pairwise disjoint compact convex point sets in $\mathbb{R}^3$, and let S^* be the convex set of lines dual to S. If $\mathcal{L}$ is a connected component of S^* (see Figure 31), then $\mathcal{L}$ is convex. Moreover, $\mathcal{L}$ is itself the space of line transversals of some finite family of pairwise disjoint compact convex point sets.

We conjecture that a similar theorem holds in any dimension.

Finally, here is a combinatorial question that also has no counterpart for point sets; this has to do with partitioning the Grassmannian into proper convex subsets.

For point sets, $\mathbb{R}^d$ can be partitioned into two proper convex subsets, for example a closed half-space and its complement. For k-flats in $\mathbb{R}^d$, though, it turns out that this is impossible in general, even though there *is* always a partition into a finite number. We can prove, for example,

Theorem 14 [8]. The lines in $\mathbb{R}^3$ cannot be partitioned into two nonempty convex sets.

On the other hand, there *is* a partition of $G'_{1,3}$ into *three* nonempty convex sets. For example, here is one way to get it:

Take an octahedron centered at the origin in $\mathbb{R}^3$, and 3-color its boundary complex so that

i) opposite faces get the same color;

ii) the color of each vertex agrees with that of some incident edge;

iii) the color of each edge agrees with that of some incident face; and

iv) whenever a vertex and an incident face have the same color, then every edge incident to both has the same color as well.

The image on the cover of this issue of the *Notices* shows an example of such a coloring.

Then take for $\mathcal{L}'_1, \mathcal{L}'_2, \mathcal{L}'_3$ the sets of lines joining the origin to the points in each color class and for $\mathcal{L}_i$ the set of all lines in $\mathbb{R}^3$ parallel to the lines of $\mathcal{L}'_i$. Then it turns out that $\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3$ form a convex partition of the set of all lines in $\mathbb{R}^3$.

This is, in fact, an example of a partition into *parallel-closed* convex sets. For partitions of this kind we can prove:

Theorem 15 [8]. Let $G_0 \subset G_1 \subset \cdots \subset G_{d-1}$ be a flag of subspaces of $\mathbf{P}^{d-1}$, with $\dim G_i = i$, and set $G_{-1} = \varnothing$. Let S be the set of all subsets $\{i_0, \ldots, i_{k-1}\}$ with $i_0 < \ldots < i_{k-1}$ of the integers $0, \ldots, d-1$, and let S_{d-1} consist of all k-sets $i_0 < \ldots < i_{k-1}$ in S with $i_{k-1} = d-1$. For each element $\sigma = \{i_0, \ldots, i_{k-1}\}$ of S, put $i_{-1} = 1$ and $i_k = d$, and let Φ_σ be the Schubert cell defined by

$$\Phi_\sigma = \{\phi \in G_{k-1,d-1} \mid \dim \phi \cap G_i = j \text{ for } i_j \le i < i_{j+1},\ -1 \le j \le k-1\}.$$

Let $\mathcal{L}_\sigma = \delta^{-1}(\Phi_\sigma)$, where $\delta : \mathbb{R}^d \setminus \{O\} \to \mathbf{P}^{d-1}$ is the canonical map. Then each $\mathcal{L}_\sigma$ is a parallel-closed convex set of k-flats, as is the union

$$\mathcal{L}_{d-1} = \bigcup \mathcal{L}_{i_0, \ldots, i_{k-2}, d-1},$$

and

$$G'_{k,d} = \left(\bigcup_{\sigma \in S \setminus S_{d-1}} \mathcal{L}_\sigma \right) \cup \mathcal{L}_{d-1}$$

is a partition of $G'_{k,d}$ into $\binom{d-1}{k} + 1$ nonempty parallel-closed convex sets.

In particular, this gives an upper bound on the smallest number (> 1) of convex sets into which $G'_{k,d}$ can always be partitioned.

The only lower bound we have is for the case of parallel-closed convex sets:

Theorem 16 [8]. If $G'_{k,d} = \bigcup_{i=1}^n \mathcal{L}_i$ is a partition of $G'_{k,d}$ into $n > 1$ nonempty parallel-closed convex sets, then $n \ge d - k + 1$.

(For example, for lines in 3-space, at least three parallel-closed sets are needed.)

We suspect that the lower bound of $d - k + 1$ holds for *all* convex sets, not just parallel-closed sets, but at the moment we have no proof.

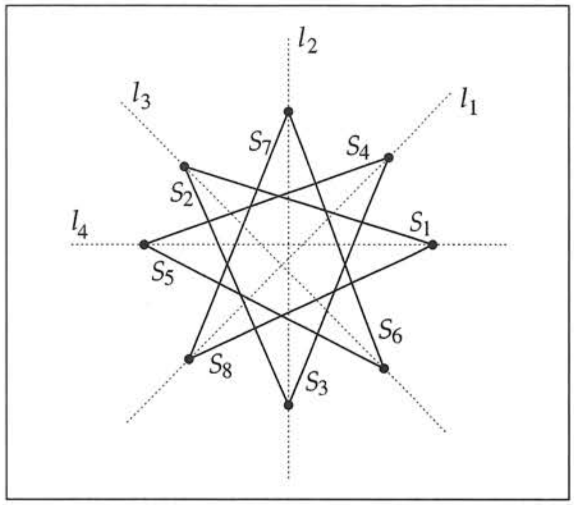


Figure 30. A minimal, irredundant presentation.

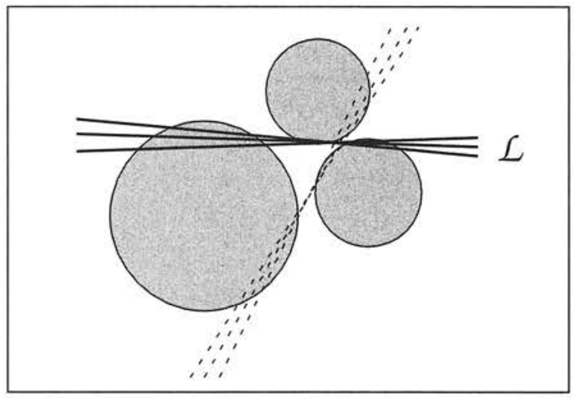


Figure 31. A connected component of a convex set of lines.

References

[1] L. Anderson and R. Wenger, *Oriented matroids and hyperplane transversals*, Adv. Math. **119** (1996), 117–125.

[2] A. Björner, M. Las Vergnas, B. Sturmfels, N. White, and G. Ziegler, *Oriented Matroids*, Cambridge Univ. Press, Cambridge, 1993.

[3] S. Cappell, J. E. Goodman, J. Pach, R. Pollack, M. Sharir, and R. Wenger, *Common tangents and common transversals*, Adv. Math. **106** (1994), 198–215.

[4] L. Danzer, B. Grünbaum, and V. Klee, *Helly's theorem and its relatives*, Convexity, vol. 7, Proc. Sympos. Pure Math., Amer. Math. Soc., Providence, RI, 1963, pp. 101–180.

[5] J. Eckhoff, *Helly, Radon and Carathéodory type theorems*, Handbook of Convex Geometry (P. Gruber and J.M. Wills, eds.), North-Holland, Amsterdam, 1993, pp. 389–448.

[6] H. Edelsbrunner and M. Sharir, *The maximum number of ways to stab n convex non-intersecting sets in the plane is 2n-2*, Discrete Comput. Geom. **5** (1990), 35–42.

[7] J. E. Goodman and R. Pollack, *Hadwiger's transversal theorem in higher dimensions*, J. Amer. Math. Soc. **1** (1988), 301–309.

[8] ——, *Foundations of a theory of convexity on affine Grassmann manifolds*, Mathematika **42** (1995), 305–328.

[9] J. E. Goodman, R. Pollack, and R. Wenger, *Geometric transversal theory*, New Trends in Discrete and Computational Geometry (J. Pach, ed.), Springer-Verlag, Berlin, 1993, pp. 163–198.

[10] ——, *On the connected components of the space of line transversals to a family of convex sets*, L. Fejes Tóth Festschrift (I. Bárány and J. Pach, eds.), Discrete Comput. Geom. **13** (1995), pp. 469–476.

[11] H. Hadwiger, *Über Eibereiche mit gemeinsamer Treffgeraden*, Portugal. Math. **16** (1957), 23–29.

[12] E. Helly, *Über Mengen konvexer Körper mit gemeinschaftlichen Punkten*, Jber. Deutsch. Math. Verein **32** (1923), 175–176.

[13] M. Katchalski, T. Lewis, and A. Liu, *The different ways of stabbing disjoint convex sets*, Discrete Comput. Geom. **7** (1992), 197–206.

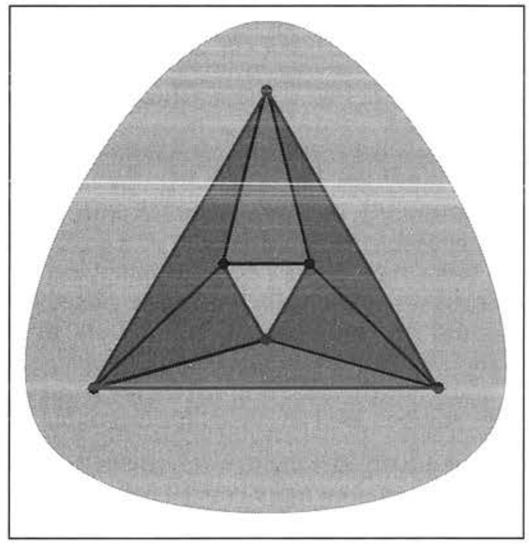
[14] M. Katchalski, T. Lewis, and J. Zaks, *Geometric permutations for convex sets*, Discrete Math. **54** (1985), 271–284.

[15] R. Pollack and R. Wenger, *Necessary and sufficient conditions for hyperplane transversals*, Combinatorica **10** (1990), 307–311.

[16] R. Wenger, *Geometric permutations and connected components*, Tech. Rep. no. TR-90-50, DIMACS, 1990.

[17] ——, *Helly-type theorems and geometric transversals*, Handbook of Discrete and Computational Geometry (J. E. Goodman and J. O'Rourke, eds.), CRC Press, Boca Raton, FL, 1997, pp. 63–82.

[18] ——, *Progress in geometric transversal theory*, Discrete and Computational Geometry: Ten Years Later (Proc. Mount Holyoke Conf.), (B. Chazelle and J. E. Goodman, and R. Pollack, eds.), Amer. Math. Soc., Providence, RI (to appear).



About the Cover

Convex Partition of a Grassmanian: The cover figure is a *Schlegel diagram* (a planar projection from a point close to an interior point of some facet) of the regular octahedron in $\mathbb{R}^3$. Its coloring partitions the Grassmannian $G'_{1,3}$ of lines in 3-dimensional space into three sets, as follows: Color each line through the center of the octahedron by the color of its points of intersection with the octahedron; this makes sense, since the face coloring is centrally symmetric. Then color all parallel translates of a given line by the same color. The set of lines in each color class turns out to be convex.

The Mathematics of Lars Valerian Ahlfors

Frederick Gehring; Irwin Kra; Steven G. Krantz, Editor; and Robert Osserman

Lars Valerian Ahlfors died October 11, 1996, and a memorial article appears elsewhere in this issue. This feature article contains three essays about different aspects of his mathematics. Ahlfors's contributions were so substantial and so diverse that it would not be possible to do all of them justice in an article of this scope (the reference [36] serves as a detailed map of Ahlfors's contributions to the subject). These essays give the flavor of some of the ideas that Ahlfors studied.

—Steven G. Krantz, Editor

Conformal Geometry

Robert Osserman

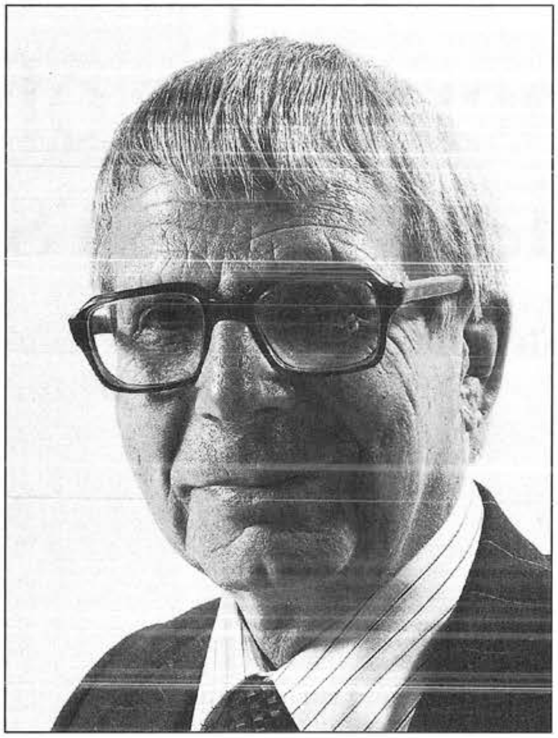
There are two directions in which one can pursue the relations between Riemann surfaces and Riemannian manifolds. First, if a two-dimensional Riemannian manifold is given, then not only lengths but also angles are well defined, so that it inherits a conformal structure. Furthermore, there always exist local isothermal coordinates, which are local conformal maps from the plane into the surface. (That was proved by Gauss for real analytic surfaces and early in the twentieth century for the general case by Korn and Lichtenstein.) The set of all such local maps forms a complex structure for the manifold, which can then be thought of as a Riemann surface. One then has all of complex function theory to bring to bear in studying the geometry of the surface. The most notable successes of this approach have been in the study of minimal surfaces, as exemplified in the contributions to that subject made by some of the leading function theorists of the nineteenth century: Riemann, Weierstrass, and Schwarz.

In the other direction, given a Riemann surface, one can consider those metrics on the surface that induce the given conformal structure. By the Koebe uniformization theorem, such metrics always exist. In fact, for "classical Riemann surfaces" of the sort originally considered by Riemann, which are branched covering surfaces of the plane, there is the natural euclidean metric obtained by pulling back the standard metric on the plane under the projection map. One can also consider the Riemann surface to lie over the Riemann sphere model of the extended complex plane and to lift the spherical metric to the surface. Both of those metrics prove very useful for obtaining information about the complex structure of the surface.

For simply connected Riemann surfaces the Koebe uniformization theorem tells us that they are all conformally equivalent to the sphere, the plane, or the unit disk. Since the first case is distinguished from the other two by the topological property of compactness, the interesting question concerning complex structure is deciding in the noncompact case whether a given surface is conformally the plane or the disk, which became known as the parabolic and hyperbolic cases, respectively. In 1932 Andreas Speiser formulated the "problem of type", which was to find criteria that could be applied to various classes of Riemann surfaces to decide whether a given one was parabolic or hyperbolic. That problem and variants of it became a central focus of Ahlfors's work for several decades. He started by obtaining conditions

Steven G. Krantz is professor of mathematics at Washington University in St. Louis. His e-mail address is `sk@math.wustl.edu`.

Robert Osserman is professor of mathematics emeritus at Stanford University and special projects director at MSRI.



Lars Valerian Ahlfors

for a branched surface to be of parabolic type in terms of the number of branch points within a given distance of a fixed point on the surface, first using the euclidean metric and later realizing that a much better result could be obtained from the spherical metric. But perhaps his main insight was that one could give a necessary and sufficient condition by looking at the totality of all conformal metrics on the surface.

The problem of type may be viewed as a special case of the general problem of finding conformal invariants. There one has some class of topologically defined objects, such as a simply or double-connected domain or a simply connected domain with boundary and four distinguished points on the boundary, and one seeks to define quantities that determine when two topologically equivalent configurations are conformally equivalent. One example is the "extremal length" of a family of curves in a domain, which is defined by a minimax expression in terms of all conformal metrics on the domain and is thereby automatically a conformal invariant. Ahlfors and Beurling, first independently and then jointly, developed the idea into a very useful tool that has since found many further applications.

From the first, Ahlfors viewed classical results like Picard's theorem and Bloch's theorem as special cases of the problem of type, in which conditions such as the projection of a Riemann surface omitting a certain number of points would imply that the surface was hyperbolic and hence could not be the image of a function defined in the whole plane. He felt that Nevanlinna theory should also be fit into that framework. Finally, in 1935 he produced one of his most important papers, in which he used the idea of specially constructed conformal metrics (or "mass distributions" in the terminology of that paper) to give his own geometric version of Nevanlinna theory. When he received his Fields Medal the following year, Carathéodory remarked that it was hard to say which was more surprising: that Nevanlinna could develop his entire theory without the geometric picture to go with it or that Ahlfors could condense the whole theory into fourteen pages.

Not satisfied that he had yet got to the heart of Nevanlinna theory from a geometric point of view, Ahlfors went on to present two further versions of the theory. The first, also from 1935, was one of his masterpieces: the theory of covering surfaces. The guiding intuition of the paper is this: If a meromorphic function is given, then the fundamental quantities studied in Nevanlinna theory, such as the counting function, determined by the number of points inside a disk of given radius where the function assumes a given value, and the Nevanlinna characteristic function, measuring the growth of the function, can be reinterpreted as properties of the Riemann surface of the image of the function, viewed as a covering surface of the Riemann sphere. The counting function, for example, just tells how many points in the part of the image surface given by the image of the disk lie over the given point on the sphere. The exhaustion of the plane by disks of increasing radii is replaced by an exhaustion of the image surface. Ahlfors succeeds in showing that by using a combination of metric and topological arguments (the metric being that of the sphere and its lift to the covering surface), one can not only recover basically all of standard Nevanlinna theory but that—quite astonishingly—the essential parts of the theory all extend to a far wider class of functions than the very rigid special case of meromorphic functions, namely, to functions that Ahlfors calls "quasiconformal"; in this theory the smoothness requirements may be almost entirely dropped, and, asymptotically, images of small circles—rather than having to be circles—can be arbitrary ellipses as long as the ratio of the radii remains uniformly bounded.

Of his three geometric versions of Nevanlinna theory, Ahlfors has described the one on covering surfaces as a "much more radical departure from Nevanlinna's own methods" and as "the most original of the three papers," which is certainly the case. (According to Carathéodory that paper was singled out in the decision of the selection committee to award the Fields Medal to Ahlfors.) Nevertheless, the last of the three, published two years later in 1937, was destined to be probably at least as influential. Here the goal was to apply the methods of differential geometry to the study of covering surfaces. The paper is basically a symphony on the theme of Gauss-Bonnet. The explicit relation between topology and total curvature of a surface, now called the "Gauss-Bonnet theorem", had not been around all that long at the time, perhaps first appearing in Blaschke's 1921 *Vorlesungen über Differentialgeometrie* [15].

It occurred to Ahlfors that if one hoped to develop a higher-dimensional version of Nevanlinna theory, it might be useful to have a higher-dimensional Gauss-Bonnet formula, a fact that he mentioned to André Weil in 1939, as Weil recounts in his collected works. (A letter from Weil says, "I learnt from Ahlfors, in 1939, all the little I ever knew about Gauss-Bonnet (in dimension 2).") When Weil spent the year 1941–42 at Haverford, where Allendoerfer was teaching, he heard of Allendoerfer's proof of the higher-dimensional Gauss-Bonnet theorem; and remembering Ahlfors's suggestion, he worked with Allendoerfer on their joint paper, proving the generalized Gauss-Bonnet theorem for a general class of manifolds that need not be embedded. That in turn led to Chern's famous intrinsic proof of the general Gauss-Bonnet theorem. As for Ahlfors's idea of adapting the method to obtain a higher-dimensional Nevanlinna theory, that had to wait until the paper by Bott and Chern in 1965.

The year following his Gauss-Bonnet Nevanlinna theory paper, there appeared a deceptively short and unassuming paper called "An extension of Schwarz's lemma" [8, v. 1, p. 350]. The main theorem and its proof take up less than a page. That is followed by two brief statements of more general versions of the theorem and then four pages of applications. Initially, it was the applications that received the most attention and that Ahlfors was most pleased with, since they were anything but a straightforward consequence of the main theorem. That is particularly true of the second application, which gives a new proof of Bloch's theorem in a remarkably precise form. Bloch's theorem states that there is a uniform constant B such that every function analytic in the unit disk, and normalized so that its derivative at the origin has modulus 1, must map some subdomain of the unit disk one-to-one conformally onto a disk of radius B. Said differently, the image Riemann surface contains an unbranched disk of radius B. The largest such B is known as Bloch's constant. In 1937 Ahlfors and Grunsky [8, v. 1, p. 279] published a paper giving an upper bound for B that they conjectured to be the exact value. The conjectured extremal function maps the unit disk onto a Riemann surface with simple branch points in every sheet over the lattice formed by the vertices obtained by repeated reflection over the sides of an equilateral triangle, where the center of the unit disk maps onto the center of one of the triangles. One obtains the map by taking three circles orthogonal to the boundary of the unit disk that form an equilateral triangle centered at the origin with 30° angles and mapping the interior of that triangle onto the interior of a euclidean equilateral triangle. Under repeated reflections one gets a map of the entire unit disk onto the surface described. Ahlfors and Grunsky write down an explicit expression for the function of that description with the right normalization at the origin and thereby get the size of the largest circular disk in the image, which is just the circumscribed circle of one of the equilateral triangles whose vertices are the branch points of the image surface. The size of that circle turns out to be a bit under 1/2, or approximately .472, which is therefore an upper bound for Bloch's constant B.

Photograph of Lars Ahlfors by Bill Graustein, the nephew of Harvard mathematician William Caspar Graustein. Ahlfors was at the time the William Caspar Graustein Professor of Mathematics.

As one application of his generalized Schwarz lemma, Ahlfors proves Bloch's theorem with a lower bound for B of $\sqrt{3}/4 \approx .433$. The method is worth describing, since it so typifies Ahlfors's approach to many problems in function theory.

First we give the statement of his generalized Schwarz lemma. The original Schwarz lemma said in particular that for an analytic function mapping the unit disk into the disk, with $f(0) = 0$, one has $|f'(0)| < 1$, unless f is a rotation, in which case $|f'(0)| = 1$. Pick observed in 1916 that since every one-to-one conformal map of the unit disk onto itself is an isometry of the hyperbolic metric of the disk, one could drop the assumption that the origin maps into the origin and conclude that for any analytic map of the unit disk into itself, the hyperbolic length of the image of any curve is at most equal to the hyperbolic length of the original curve and, in fact, those lengths are strictly decreased unless the map is one-to-one onto or an isometry preserving all hyperbolic lengths. Ahlfors's great insight came from viewing the Schwarz-Pick lemma as a statement about two conformal metrics on the unit disk: the original hyperbolic metric and the pullback of the hyperbolic metric on the image. That led him to a truly far-reaching generalization, in which he replaces the second conformal metric by any one whose curvature is bounded above by the (constant) negative curvature of the original hyperbolic metric. The conclusion is that, again, the lengths of curves in the second metric are at most equal to the original hyperbolic lengths. That is the main result of his paper and is the content of the Ahlfors-Schwarz lemma. The proof requires the metrics to be smooth, but in his generalized versions he shows that the conclusion continues to hold in particular for various piecewise smooth metrics. In order to apply the result, Ahlfors constructs specific conformal metrics adapted to specific problems. As already mentioned, that is anything but straightforward and may take considerable ingenuity in each case. For Bloch's theorem he constructs a conformal metric based on the conjectured optimal surface described above and uses it to obtain the lower bound of $\sqrt{3}/4$ for Bloch's constant.[1]

When his collected papers [8] were published in 1982, Ahlfors commented that this particular paper "has more substance than I was aware of," but he also said: "Without applications my lemma would have been too lightweight for publication." It is a lucky thing for posterity that he found applications that he considered up to his standard, since it would have been a major loss for us not to have the published version of the Ahlfors-Schwarz lemma. As elegant and important as his applications were, I believe that they have long ago been dwarfed by the impact of the lemma itself, which has proved its value in countless other applications and has served as the underlying insight and model for vast parts of modern complex manifold theory, including Kobayashi's introduction of the metric that now bears his name and Griffiths's geometric approach to higher-dimensional Nevanlinna theory. It demonstrates perhaps more strikingly than anywhere else the power that Ahlfors was able to derive from his unique skill in melding the complex analysis of Riemann surfaces with the metric approach of Riemannian geometry.

Kleinian Groups

Irwin Kra

Of the many significant contributions of Lars Ahlfors to the modern theory of Kleinian groups, I will discuss only two closely related contributions: the finiteness theorem (AFT) and the use of Eichler cohomology as a tool for proving this and related results. Both originated in the seminal paper [2].

For the purposes of this note a *Kleinian group* G will always be finitely generated, nonelementary, and of the second kind. Thus it consists of Möbius transformations (a subgroup of $\mathrm{PSL}(2,\mathbb{C})$) and acts discontinuously on a nonempty maximal open set $\Omega \subset \mathbb{C} \cup \{\infty\}$, the *region of discontinuity* of G, whose complement Λ in $\mathbb{C} \cup \{\infty\}$, the *limit set* of G, is an uncountable perfect nowhere dense subset of the Riemann sphere.

In the early sixties not much was known about Kleinian groups. Around the beginning of this century Poincaré suggested a program for studying discrete subgroups of $\mathrm{PSL}(2,\mathbb{C})$; Poincaré's program was based on the fact that $\mathrm{PSL}(2,\mathbb{R})$ acts on the upper half plane H^2, a model for hyperbolic 2-space. The quotient of H^2 by a discrete subgroup (a *Fuchsian* group) of $\mathrm{PSL}(2,\mathbb{R})$ is a 2-dimensional orbifold (a Riemann surface with some "marked" points). By analogy $\mathrm{PSL}(2,\mathbb{C})$ acts on H^3, hyperbolic 3-space, and the quotient of H^3 by a torsion free discrete subgroup of $\mathrm{PSL}(2,\mathbb{C})$ is a 3-dimensional hyperbolic manifold. The study of subgroups of $\mathrm{PSL}(2,\mathbb{R})$ was successful because of its connection to classical function theory and to 2-dimensional topology and geometry, about which a lot was known, including the uniformization theorem classifying all simply connected Riemann surfaces. Poincaré's program was to take advantage of the connection of $\mathrm{PSL}(2,\mathbb{C})$ to 3-dimensional topology and geometry to study groups of Möbius transformations. However, in 1965 very little was known about hyperbolic 3-manifolds. Research in the field seemed to be stuck and going nowhere. Ahlfors completely ignored Poincaré's program and took a different route to prove the finiteness theorem. He used complex analytic methods, and his result described the Riemann surfaces that can be represented by a Kleinian group. About fifteen years later in the mid-seventies, as a result of the fundamental contributions of W. Thurston [34], 3-dimensional topology came to the forefront in the study of Kleinian groups. However, in the interest of keeping this presentation to reasonable length, I ignore this subject.

The history of Ahlfors's work on Kleinian groups is also part of the history of a remarkable collaboration between Lars Ahlfors and Lipman Bers.[2] Ahlfors's finiteness theorem says that the ordinary set Ω of a (finitely generated) Kleinian group G factored by the action of the group is an orbifold Ω/G of finite type (finitely many "marked" points *and* compactifiable as an orbifold by adding a finite number of points).

Bers [11], in approximately 1965, reproved an equivalent known result in the Fuchsian case, a much simpler case to handle. The finiteness the-

[1] *It may be noted that Ahlfors's lower bound for the Bloch constant stood for many years. The bound was shown to be strict by M. Heins [21] in 1962; C. Pommerenke [30] contributed some refinements in 1970; the lower estimate was improved by* 10^{-14} *by M. Bonk [16] in 1990; the related Landau constant was improved by* 10^{-335} *by H. Yanagihara [35] in 1995; and finally the lower bound for the Bloch constant was improved by* 10^{-4} *by H. Chen and P. Gauthier [17] in 1996.*

Irwin Kra is Distinguished Service Professor at the State University of New York, Stony Brook. His e-mail address is irwin@math.sunysb.edu.

[2] *Although they co-authored only one paper [9], their work and the work of many of their students was intertwined. See [27].*

orem for PSL(2, ℝ) had been known for a long time, and Bers reproved it using modern methods, Eichler cohomology. Bers constructs Eichler cohomology classes from analytic potentials (by integrating cusp forms sufficiently many times, using methods developed by Eichler [18] for number theory). To be specific, let G be a Fuchsian group (finitely generated) operating on the upper half plane H^2. Fix an integer $q \geq 2$. Let φ be a holomorphic q-form for G on H^2. Choose a holomorphic function F on H^2 whose $(2q-1)^{\text{st}}$ derivative is φ. Then, for every element $g \in G$,

$$\chi_g = (F \circ g)(g')^{1-q} - F \tag{1}$$

is the restriction to H^2 of a polynomial of degree at most $2q-2$; we denote the vector space of such polynomials by Π_{2q-2}. The polynomials constructed satisfy the *cocycle condition*

$$\chi_{g_1 \circ g_2} = (\chi_{g_1} \circ g_2)(g_2{}')^{1-q} + \chi_{g_2}, \quad \text{all } g_1 \text{ and } g_2 \in G. \tag{2}$$

One obtains in this way a holomorphic potential F for the automorphic form φ and a cohomology class $[\chi] = \beta(\varphi)$ in $H^1(G, \Pi_{2q-2})$. If G is finitely generated, then $H^1(G, \Pi_{2q-2})$ is a finite-dimensional vector space. Now if G is of the first kind and φ is a cusp form, then, as a consequence of the Riemann-Roch theorem, $\beta(\varphi) = 0$ if and only if $\varphi = 0$. The space of cusp forms for G on H^2 is finite dimensional if and only if H^2/G is a finitely punctured compact surface. From the injectivity of the linear map β, Bers concludes the finiteness result in this case. Bers's argument for groups of the second kind is more complicated but proceeds along similar lines.

Ahlfors generalized Bers's methods to a much wider class of subgroups of PSL(2, ℂ). This generalization was completely nontrivial. It required the passage from holomorphic potentials to smooth potentials. This involved a conceptual jump forward—a construction of Eichler cohomology classes via an integral operator, producing a conjugate linear map α that assigns an Eichler cohomology class $\alpha(\varphi)$ to a bounded holomorphic q-form φ for the group G. In addition, there appeared a very difficult technical obstacle that Ahlfors had to surmount to prove the injectivity of α. To surmount this obstacle, Ahlfors introduces a "mollifier", a function used to construct an approximate identity. Ahlfors works only with the case $q = 2$. Using a modified Cauchy kernel, he constructs a potential for $\lambda^{2-2q}\bar{\varphi}$, a continuous function F on ℂ whose $\bar{z}$-derivative is $\lambda^{2-2q}\bar{\varphi}$. Here λ is a weight function whose exact form need not concern us. To be more specific, without loss of generality, we can assume that $\infty \in \Lambda$. Choose $2q-2$ distinct finite points $a_1, \ldots, a_{2q-2} \in \Lambda$. Form the potential $F_{\lambda^{2-2q}\bar{\varphi}}$ for $\lambda^{2-2q}\bar{\varphi}$,

$$F(z) = F_{\lambda^{2-2q}\bar{\varphi}}(z) = \frac{(z-a_1)\ldots(z-a_{2q-2})}{2\pi i} \times \int\int_\Omega \frac{\lambda^{2-2q}(\zeta)\overline{\varphi}(\zeta)\, d\zeta \wedge \overline{d\zeta}}{(\zeta - z)(\zeta - a_1)\ldots(\zeta - a_{2q-2})}.$$

Then as before, (1) defines an element of Π_{2q-2}, and these polynomials satisfy (2). There is no analogue of the Riemann-Roch theorem. Let $\mathcal{R}$ be the span of the rational functions R^b with $b \in \Lambda \setminus \{a_1, a_2, \ldots, a_{2q-2}, \infty\}$, where $R^b(\zeta) = \frac{b-a_1)\cdots(b-a_{2q-2})}{(\zeta-b)(\zeta-a_1)\cdots(\zeta-a_{2q-2})}$. Ahlfors needs to establish the density of $\mathcal{R}$ in the Banach space $\mathbb{A}$ of integrable holomorphic functions on Ω. He needs to use Stokes's theorem. However, the functions confronting him are not smooth at the boundary, *and* the boundary is not even rectifiable. The outline of Ahlfors's argument follows. Assume that $q = 2$. It is easily seen that $\alpha(\varphi) = 0$ if and only if $F_{\lambda^{2-2q}\bar{\varphi}}$ vanishes on Λ. A bounded measurable function[3] μ induces a bounded linear functional l_μ on $\mathbb{A}$ by the formula

$$l_\mu(\psi) = \int\int_\Omega \psi(z)\mu(z) dz \wedge \overline{dz}, \ \psi \in \mathbb{A}.$$

Hence the injectivity of α is equivalent to the density of $\mathcal{R}$ in $\mathbb{A}$. It suffices by Hahn-Banach and the Riesz representation theorem to prove that for every bounded measurable function[3] μ on Ω the condition

$$\int\int_\Omega \mu(z) r(z) dz \wedge \overline{dz} = 0, \quad \text{all } r \in \mathcal{R},$$

is enough to guarantee that $\mu = 0$, a.e.[4] The hypothesis on μ tells us that F_μ vanishes on $\Lambda = \partial\Omega$. A fake "proof" of the density, using divergent integrals, is provided by

$$\begin{aligned} l_\mu(\psi) &= \int\int_\Omega \mu\psi dz \wedge \overline{dz} \\ &= \int\int_\Omega \left(\frac{\partial F_\mu}{\partial \bar{z}}\right)\psi dz \wedge \overline{dz} \\ &= \int\int_\Omega \frac{\partial}{\partial \bar{z}}\left(F_\mu \psi\right) dz \wedge \overline{dz} \\ &= -\int\int_\Omega \bar{\partial}\left(F_\mu \psi\, dz\right) \\ &= -\int_{\partial\Omega} F_\mu \psi\, dz = 0, \quad \text{all } \psi \in \mathbb{A}. \end{aligned}$$

To convert the above fake proof into a real one, choose a smooth function j on ℝ with values in the closed interval $[0, 1]$ with the properties that

---

[3] *In particular, $\lambda^{-2}\bar{\varphi}$ with φ a bounded holomorphic 2-form on Ω for G.*

[4] *Bers's paper [11] shows that every bounded linear functional on $\mathbb{A}$ is of this form with $\mu = \lambda^{-q}\bar{\varphi}$ and φ a holomorphic bounded 2-form. But this observation does* not *simplify the argument.*

it vanishes on $(-\infty, 1]$ and takes on the value 1 on $[2, \infty)$. Then for $n \in \mathbb{Z}^+$, define

$$\omega_n(z) = j\left(\frac{n}{\log\log\frac{1}{\delta(z)}}\right), \quad z \in \Omega,$$

where $\delta(z)$ is the distance from z to $\partial\Omega$. Let $R > 0$. We now let Ω_R and Λ_R be the intersection of Ω with the disc and circle centered at the origin and radius R, respectively. Because ω_n vanishes in a neighborhood of Λ, we can use integration by parts to conclude that

$$\int\int_{\Omega_R} \omega_n \psi \mu \, dz \wedge \overline{dz}$$
$$= -\int_{\Lambda_R} \omega_n \psi F_\mu - \int\int_{\Omega_R} \psi F_\mu \frac{\partial \omega_n}{\partial \bar{z}} \, dz \wedge \overline{dz},$$

and from here conclude that

$$\left| \int\int_{\Omega_R} \psi \mu \, dz \wedge \overline{dz} \right|$$
$$\leq \left| \int_{\Lambda_R} \psi F_\mu \, dz \right| \leq \text{ const. } R \log R \int_{\Lambda_R} |\psi \, dz|.$$

Standard potential-theoretic estimates for the growth of $|F_\mu|$ are used to obtain the last inequality. The finiteness of $\int\int_\Omega |\psi \, dz \wedge \overline{dz}|$ now implies that the last integral in the series of inequalities tends to zero as R becomes large. This completes the proof of the injectivity of α for $q = 2$. It is important to observe that the argument did *not* require G to be finitely generated. Ahlfors's "mollifier" is so delicate that it has not found any other uses even though it has been around for more than thirty-three years. In particular, it is an open problem to determine necessary and sufficient conditions on G for α to be injective for a fixed $q > 2$. For finitely generated G, once one has the injectivity for $q = 2$, it is easy to obtain the same conclusion for bigger q. Alternative approaches to the finiteness theorem are found in [33, 14, 28].[5]

In his proof of the finiteness theorem [2], Ahlfors made a small mistake: he left out the possibility of infinitely many thrice-punctured spheres appearing in Ω/G. (Such surfaces admit no moduli (deformations) and alternatively carry no nontrivial integrable quadratic differentials.) That deficiency was remedied in subsequent papers by work of Bers [13], Greenberg [20], and Ahlfors himself [3]. Ahlfors initially limited his work to quadratic differentials (which include squares of "classical" abelian differentials), in part because this case and the abelian case are the only ones with geometric significance. Perhaps more significantly, it was Ahlfors's style to make the pioneering contributions to a field and leave plenty of room for others to continue in the same area. In this particular case much remained to be done.

Bers [12] saw that if he studied the more general case of q-differentials, he would be able to improve on the results of Ahlfors and get quantitative versions of the finiteness theorem that have become known as the Bers area theorems. The first of these theorems [12] states: *If G is generated by N-motions, then* $\text{Area}(\Omega/G) \leq 4\pi(N-1)$. This paper by Bers led, in turn, to investigations of the structure of (the Eichler cohomology groups of) Kleinian groups by Ahlfors [4], based on his earlier paper [3], and this author [23, 24]. Whereas Ahlfors used meromorphic Eichler integrals (in a sense going back to Bers's studies [11]) to describe the structure of the cohomology groups, I relied on smooth potentials (in a sense combining methods of Ahlfors [2] and Bers [12]). Ahlfors's generosity, as evidenced by the footnote in [4] regarding the relation between his and my approach to this problem, was very much appreciated; his remarks were most encouraging to a young mathematician.

Since the minimal area of a hyperbolic orbifold is $\pi/21$, Bers's area theorem gives an upper bound on the number of (connected) Riemann surfaces represented by a nonelementary Kleinian group as $84(N-1)$. Ahlfors [3] lowered that bound to $18(N-1)$. Even after some important work of Abikoff [1], there is still no satisfactory bound on the number of surfaces that a Kleinian group represents, especially if one insists on using only 2-dimensional methods. Bers's paper [12] also showed that the thrice-punctured spheres issue can be resolved "without new ideas." It, together with Ahlfors's discoveries on Kleinian groups, led fifteen years later to work on the vanishing[6] of Poincaré and relative Poincaré series [25, 26].

The so-called measure zero problem first surfaced during the 1965 Tulane conference.[7] In his 1964 paper Ahlfors remarked that perhaps of greater interest than the theorems (AFT) that he has been able to prove were the ones he was *not* able to prove. First of these was the assertion that the limit set of a finitely generated Kleinian group has two-dimensional Lebesgue measure zero.[8] This

[5] *This paper deals with the topological aspects of AFT.*

[6] *A different, more classical, approach to the problem of deciding when a relative Poincaré series vanishes identically is found in the earlier work of Hejhal [22].*

[7] *The first of the periodic meetings, roughly every four years, of researchers in fields related to the mathematical interests of Lars Ahlfors and Lipman Bers. The tradition continues; the next meeting of the Ahlfors-Bers Colloquium will take place in November 1998.*

[8] *Although quasiconformal mappings do not appear in the proof of AFT, Ahlfors's motivation and ideas came from*

has become known as the Ahlfors measure zero conjecture. It is still unsolved, although important work on it has been done by Ahlfors, Maskit, Thurston, Sullivan, and Bonahan. In some sense the problem has been solved for analysts by Sullivan [31], who showed that a nontrivial deformation of a finitely generated Kleinian group cannot be supported entirely on its limit set; topologists are still interested in the measure zero problem. Formulae in Ahlfors's nonsuccessful attempt [5] to prove the measure zero conjecture led Sullivan [32] to a finiteness theorem on the number of maximal conjugacy classes of purely parabolic subgroups of a Kleinian group. The measure zero problem not only opened up a new industry in the Kleinian groups "industrial park", it also revived the connection with 3-dimensional topology following the fundamental work of Marden [29] and Thurston [34]. It showed that Poincaré was not at all wrong when he thought that we could study Kleinian groups by 3-dimensional methods. The second "theorem" that Ahlfors "wanted" to establish for his 1964 paper can be rephrased in today's language to say that *a finitely generated Kleinian group is geometrically finite*; a counterexample was produced shortly thereafter [19].

Quasiconformal Mappings

Frederick Gehring

In 1982 Birkhäuser published two fine volumes of Lars Ahlfors's collected papers [8] and his fascinating commentaries on them. Volume 2 contains forty-three articles: twenty-one of these are directly concerned with quasiconformal mappings and Teichmüller spaces, twelve with Kleinian groups, and ten with topics in geometric function theory. This distribution shows clearly the dominant role that quasiconformal mappings played in this part of Ahlfors's work. Moreover, quasiconformal mappings play a key role in several other papers, for example, the important finiteness theorem for Kleinian groups. For this reason I have chosen quasiconformal mappings as the subject of this survey. In particular, I will consider the four papers [6, 7, 9, 10], which have had great impact on contemporary analysis.

On Quasiconformal Mappings [6]

In his commentary to this paper Ahlfors wrote that "It had become increasingly evident that Teichmüller's ideas would profoundly influence analysis and especially the theory of functions of one complex variable....The foundations of the theory were not commensurate with the loftiness of Teichmüller's vision, and I thought it was time to re-examine the basic concepts."

The quasiconformal mappings considered by Grötzsch and Teichmüller were assumed to be continuously differentiable except for isolated points or small exceptional sets. Teichmüller's theorem concerned the nature of the quasiconformal mappings between two Riemann surfaces S and S' that have minimum maximal dilatation. This and the fact that any useful theory that generalizes conformal mappings should have compactness and reflection properties led Ahlfors to formulate a geometric definition that was free of all a priori smoothness hypotheses.

A *quadrilateral* Q is a Jordan domain Q with four distinguished boundary points. The *conformal modulus* of Q, denoted mod(Q), is defined as the side ratio of any conformally equivalent rectangle R. Grötzsch showed that if $f : D \to D'$ is K-quasiconformal in the classical sense, then

$$(1) \qquad \frac{1}{K}\mathrm{mod}(Q) \le \mathrm{mod}(f(Q)) \le K\mathrm{mod}(Q)$$

for each quadrilateral $Q \subset D$. Ahlfors used this inequality to define his new class of quasiconformal mappings: a homeomorphism $f : D \to D'$ is K-quasiconformal if (1) holds for each quadrilateral $Q \subset D$. Ahlfors then established all of the basic properties of conformal mappings for this general class of homeomorphisms, including a uniform Hölder estimate, a reflection principle, a compactness principle, and an analogue of the Hurwitz theorem. He did all of this in nine pages.

The major part of this article was, of course, concerned with a statement, several interpretations, and the first complete proof of Teichmüller's theorem.

In his commentary on this paper Ahlfors modestly wrote that "My paper has serious shortcomings, but it has nevertheless been very influential and has led to a resurgence of interest in quasiconformal mappings and Teichmüller theory."

This is an understatement! Ahlfors's exposition made Teichmüller's ideas accessible to the mathematical public and resulted in a flurry of activity and research in the area by scientists from many different fields, including analysis, topology, algebraic geometry, and even physics.

Next, Ahlfors's geometric approach to quasiconformal mappings stimulated analysts to study

his work on quasiconformality. Ahlfors knew that a finitely generated Kleinian group had a finite-dimensional deformation space and for a family of such groups a deformation should be induced by a quasiconformal map whose Beltrami coefficient is supported on the ordinary set. These assertions would follow if the limit set had measure zero. They follow from Sullivan's theorem [31].

Frederick Gehring is Distinguished University Professor Emeritus, University of Michigan, Ann Arbor. His e-mail address is fgehring@math.lsa.umich.edu.

Ahlfors in the classroom.

this class of mappings in the plane, in higher-dimensional euclidean spaces, and now in arbitrary metric spaces. His inspired idea to drop all analytic hypotheses eventually led to striking applications of these mappings in other parts of complex analysis, such as discontinuous groups, classical function theory, complex iteration, and in other fields of mathematics, including harmonic analysis, partial differential equations, differential geometry, and topology.

The Boundary Correspondence under Quasiconformal Mappings [10]

In the previous paper Ahlfors proved that a quasiconformal mapping $f : D \to D'$ between Jordan domains has a homeomorphic extension to their closures. A classical theorem due to F. and M. Riesz implied that the induced boundary correspondence ϕ is absolutely continuous with respect to linear measure whenever ∂D and $\partial D'$ are rectifiable and f is conformal. Mathematicians asked whether this conclusion holds when f is K-quasiconformal.

By composing f with a pair of conformal mappings, one can reduce the problem to the case where $D = D' = H$, where H is the upper half plane and $\phi(\infty) = \infty$. Next, if x and t are real with $t > 0$ and if Q is the quadrilateral with vertices at $x - t$, x, $x + t$, ∞, then $\mathrm{mod}(Q) = 1$ and inequality (1) implies that

$$\frac{1}{\lambda} \le \frac{\phi(x+t) - \phi(x)}{\phi(x) - \phi(x-t)} \le \lambda, \tag{2}$$

where $\lambda = \lambda(K)$. Inequality (2) is a quasisymmetry condition that it was thought would imply that ϕ is absolutely continuous.

In 1956 Ahlfors and Beurling published a paper in which they exhibited for each $K > 1$ a K-quasiconformal mapping $f : H \to H$ for which the boundary correspondence $\phi : \partial H \to \partial H$ is completely singular. The importance of this example was, however, overshadowed by the authors' main theorem, which stated that inequality (2) characterizes the boundary correspondences induced by quasiconformal self-mappings of H. The sufficiency part consisted in showing that the remarkable formula

$$\begin{aligned} f(z) = {} & \frac{1}{2y} \int_0^y \left[\phi(x+t) + \phi(x-t) \right] dt \\ & + \frac{i}{2y} \int_0^y \left[\phi(x+t) - \phi(x-t) \right] dt \end{aligned} \tag{3}$$

yields a K-quasiconformal self-mapping of H with $K = K(\lambda)$ whenever ϕ satisfies (2). Moreover, f is a *hyperbolic quasi-isometry* of H, a fact that turns out to have many important consequences.

In 1962 it was observed that a self-quasi-conformal mapping of the n-dimensional upper half space H^n induces an $(n-1)$-dimensional self-mapping ϕ of the $(n-1)$-dimensional boundary plane ∂H^n; this fact, for the case $n = 3$, was an important step in the original proof of Mostow's rigidity theorem. It was then natural to ask if every quasiconformal self-mapping ϕ of ∂H^n admits a quasiconformal extension to H^n. This question was eventually answered in the affirmative by Ahlfors in 1963 for $n = 3$, by Carleson in 1972 for $n = 4$, and by Tukia-Väisälä in 1982 for all $n \ge 3$.

Riemann's Mapping Theorem for Variable Metrics [9]

If $f : D \to D'$ is K-quasiconformal according to (1), then f is differentiable with $f_z \neq 0$ a.e. in D, and

$$\mu_f = \frac{f_{\bar z}}{f_z} \tag{4}$$

is measurable with

$$|\mu_f| \le k = \frac{K-1}{K+1} \tag{5}$$

a.e. in D. The *complex dilatation* μ_f determines f uniquely up to postcomposition with a conformal mapping.

The main result of this article states that for any μ that is measurable with $|\mu| \le k$ a.e. in D, there exists a K-quasiconformal mapping f that has μ as its complex dilatation. Moreover, if f is suitably normalized, then f depends holomorphically on μ.

The above result, known by many as the "measurable Riemann mapping theorem", has proved to be an enormously influential and effective tool in analysis. It is a cornerstone for the study of Teichmüller space; it was the key for settling outstanding questions of classical function theory, including Sullivan's solution of the Fatou-Julia problem on wandering domains; and it currently plays a major role in the study of iteration of rational functions. Indeed, application of this theorem has become a verb in this area. In a lecture at the 1986 International Congress of Mathematicians a distinguished French mathematician was heard to explain that "before mating two polynomials, one must first Ahlfors-Bers the structure."

Quasiconformal Reflections [7]

A Jordan curve C is said to be a *quasicircle* if it is the image of a circle or line under a quasiconformal self-mapping of the extended plane; a domain D is a *quasidisk* if ∂D is a quasicircle. Quasicircles can be very wild curves. Indeed, for $0 < a < 2$ there exists a quasicircle C with Hausdorff dimension at least a.

Nevertheless, the first theorem of this elegant paper contains the following remarkable characterization for this class of curves. A Jordan curve C is a quasicircle if and only if there exists a constant b such that

$$(6) \qquad |z_1 - z_2| \leq b|z_1 - z_3|$$

for each ordered triple of points $z_1, z_2, z_3 \in C$. The proof for the sufficiency of (6) depends on the fact that the function in (3) is a hyperbolic quasi-isometry.

The fact that quasicircles admit such a simple geometric description is the reason why these curves play such an important role in many different areas of analysis. Inequality (6) is universally known as the "Ahlfors condition", and many regard it as the best way to define the notion of a quasicircle. This is an example of a beautiful theorem that is in danger of becoming a definition!

The second main result of this paper asserts that the set S of Schwarzian derivatives

$$(7) \qquad S_f = \left(\frac{f''}{f'}\right)' - \frac{1}{2}\left(\frac{f''}{f'}\right)^2$$

of conformal mappings f that map the upper half plane H onto a quasidisk D is an open subset of the Banach space of holomorphic functions ϕ with norm

$$(8) \qquad \|\phi\|_H = \sup_H |\phi(z)|^2 y^2.$$

This fact is the key step in proving that the Bers universal Teichmüller space $T(1)$ is the interior of S with the topology induced by the norm (8). It also led to another surprising connection between quasiconformal mappings and classical function theory, namely, that a simply connected domain D is a quasidisk if and only if each function f, analytic with small Schwarzian S_f in D, is injective. Here the size of S_f is measured by

$$(9) \qquad \|S_f\| = \sup_D |S_f(z)|^2 \rho_D(z)^{-2},$$

where ρ_D is the hyperbolic metric in D.

Lars Ahlfors lectured on this beautiful paper at the Oberwolfach Tagung in 1963. He, Olli Lehto, and I were scheduled to speak the same morning. After several hours of socializing and wine the evening before, Olli and I tried to excuse ourselves so that we could get some sleep before our talks. We were told by Lars that that was "a very silly idea indeed" and that it would be far better to relax and drink with the pleasant company. The next day Lars's talk went extremely well, and he was subsequently asked if he believed that staying up late always improved his lectures. "I am not sure," he replied, "but at least they always sound better to me!"

Conclusion

Quasiconformal mappings first appeared under this name in Ahlfors's paper *Zur theorie der Überlagerungsflächen* in 1935. In his commentary for this article he wrote, "Little did I know at the time what an important role quasiconformal mappings would come to play in my own work." This class of mappings offers a stripped-down picture of the geometric essentials of complex function theory and, as such, admits applications of these ideas to many other parts of analysis and geometry. They constitute just one illustration of the profound and lasting effect that the deep, central, and seminal character of Lars Ahlfors's research has had on the face of modern mathematics.

References

[1] W. Abikoff, *The Euler characteristic and inequalities for Kleinian groups*, Proc. Amer. Math. Soc. **97** (1986), 593–601.

[2] L. V. Ahlfors, *Finitely generated Kleinian groups*, Amer. J. Math. **86** (1964), 413–429, and **87** (1965), 759.

[3] ____, *Eichler integrals and Bers's area theorem*, Mich. Math. J. **15** (1968), 257–263.

[4] ____, *The structure of a finitely generated Kleinian group*, Acta Math. **122** (1969), 1–17.

[5] ____, *Some remarks on Kleinian groups*, Lars Valerian Ahlfors: Collected Papers, Volume 2, 1954–1979, Birkhäuser, Basel, 1982, pp. 316–319.

[6] ____, *On quasiconformal mappings*, J. Analyse Math. **3** (1954), 1–58; correction loc. cit., 207–208.

[7] ____, *Quasiconformal reflections*, Acta Math. **109** (1963), 291–301.

[8] ____, Lars Valerian Ahlfors: Collected Papers (Rae Michael Shortt, ed.), Birkhäuser, Boston, 1982.

[9] L. V. Ahlfors and L. Bers, *Riemann's mapping theorem for variable metrics*, Ann. of Math. **72** (1960), 345–404.

[10] L. V. Ahlfors and A. Beurling, *The boundary correspondence under quasiconformal mappings*, Acta Math. (1956).

[11] L. Bers, *Automorphic forms and Poincaré series for infinitely generated Fuchsian groups*, Amer. J. Math. **87** (1965), 196–214.

[12] ____, *Inequalities for finitely generated Kleinian groups*, J. Analyse Math. **18** (1967), 23–41.

[13] ____, *On Ahlfors's finiteness theorem*, Amer. J. Math. **89** (1967), 1078–1082.

[14] ____, *On Sullivan's proof of the finiteness theorem and the eventual periodicity theorem*, Amer. J. Math. **109** (1987), 833–852.

[15] W. BLASCHKE, *Vorlesungen über Differentialgeometrie und Geometrische Grundlagen von Einstein's Relativitätstheorie*, Springer-Verlag, Berlin, 1921–1929.

[16] M. BONK, *On Bloch's constant*, Proc. Amer. Math. Soc. **110** (1990), 889–894.

[17] H. CHEN and P. M. GAUTHIER, *On Bloch's constant*, J. Analyse Math. **69** (1996), 275–291.

[18] M. EICHLER, *Eine Verallgemeinerung der Abelschen Integrale*, Math. Z. **67** (1957), 267–298.

[19] L. GREENBERG, *Fundamental polyhedra for Kleinian groups*, Ann. of Math. **84** (1966), 433–441.

[20] ____, *On a theorem of Ahlfors and conjugate subgroups of Kleinian groups*, Amer. J. Math. **89** (1967), 56–68.

[21] M. HEINS, *On a class of conformal metrics*, Nagoya Math. J. **21** (1962), 1–60.

[22] D. A. HEJHAL, *On monodromy groups and Poincaré series*, Bull. Amer. Math. Soc. **84** (1978), 339–376.

[23] I. KRA, *On cohomology of Kleinian groups*, Ann. of Math. **89** (1969), 533–556.

[24] ____, *On cohomology of Kleinian groups: II*, Ann. of Math. **90** (1969), 575–589.

[25] ____, *On the vanishing of and spanning sets for Poincaré series for cusp forms*, Acta Math. **153** (1984), 47–116.

[26] ____, *Cusp forms associated to loxodromic elements of Kleinian groups*, Duke Math. J. **52** (1985), 587–625.

[27] ____, *Creating an American mathematical tradition: The extended Ahlfors-Bers family*, A Century of Mathematical Meetings, Amer. Math. Soc., Providence, RI, 1996, pp. 265–280.

[28] R. S. KULKARNI, *Some topological aspects of Kleinian groups*, Amer. J. Math. **100** (1978), 897–911.

[29] A. MARDEN, *The geometry of finitely generated Kleinian groups*, Ann. of Math. **99** (1974), 383–462.

[30] C. POMMERENKE, *On Bloch functions*, J. London Math. Soc. **2** (1970), 689–695.

[31] D. SULLIVAN, *On the ergodic theory at infinity of an arbitrary discrete group of hyperbolic motions*, Riemann Surfaces and Related Topics, Ann. of Math. Stud., vol 97, Princeton Univ. Press, Princeton, NJ, 1980, pp. 465–496.

[32] ____, *A finiteness theorem for cusps*, Acta Math. **147** (1981), 289–299.

[33] ____, *Quasiconformal homeomorphisms and dynamics I, Solution of the Fatou-Julia problem on wandering domains*, Ann. of Math. **122** (1985), 401–418.

[34] W. P. THURSTON, *Three dimensional manifolds, Kleinian groups and hyperbolic geometry*, Bull. Amer. Math. Soc. (N.S.) **6** (1982), 357–381.

[35] H. YANAGIHARA, *On the locally univalent Bolch constant*, J. Analyse Math. **65** (1995), 1–17.

[36] *Commentationes in honorem Lars V. Ahlfors*, LXXX Annos NATO, Ann. Acad. Sci. Fenn. Ser. A I Math. Dissertationes **13** (1988).

The AMS and Mathematics Education: The Revision of the "NCTM Standards"

Roger Howe

Readers of the *Notices* will be aware, through recent articles by Allyn Jackson [1] and maybe otherwise also, that there is currently great ferment in mathematics education, from kindergarten through graduate school. Today's mathematics education reform (=: mathedreform) movement advocates sweeping changes in the philosophy, pedagogy, and content of mathematics education in the U.S. If implemented thoroughly, it will have widespread effects on American society; in particular it would affect future mathematical research. Thus, it behooves the AMS to work to ensure that the changes are beneficial to American children and to American mathematics.

The NCTM Standards

The central documents in the current debates on K–12 mathematics education are the "NCTM Standards" [2], a collection of three volumes containing ideas about curriculum, pedagogy, and assessment published by the National Council of Teachers of Mathematics (NCTM). These have been enormously influential, especially among bodies which set broad educational policy. For example, many state "frameworks" for the mathematics curriculum have borrowed heavily from the "Standards", sometimes adopting them almost wholesale. They have also inspired the creation of numerous novel curricula. In particular, the Division of Education and Human Resources of the National Science Foundation (NSF) has sponsored several projects for the development of "Standards"-based curricula [3]. The enormous public relations success of the "Standards" has placed NCTM firmly in the driver's seat of the school mathedreform bus.

NCTM is now planning to issue a revised version of its "Standards", tentatively in the year 2000, and it has assembled a writing team for the purpose. There have been calls for revision from within the mathedreform community for several years [4], and some have always considered a revision to be part of the plan. Revision will allow NCTM to take into account extensive comments from some university mathematicians and to eliminate ambiguities which have allowed, most visibly in California, what some regard as excesses in the direction of changes suggested in the "Standards" [1, 5].

To further the revision process, NCTM has solicited input from most of the other mathematics professional organizations (AMS, MAA, AWM, ASA, ASL, SIAM,...). These have responded by creating committees, known collectively as ARGs (Association Resource Groups) to provide comment as solicited by the NCTM writing group. The AMSARG was appointed as a subcommittee of the Committee on Education (CoE) by Hyman Bass, chair of CoE. AMSARG consists of Richard Askey, Wayne Bishop, Roger Howe (chair), Alfred Manaster, David Moore, Judy Roitman, Mark Saul, and William Thurston. (Manaster and Roitman are also on the NCTM revision writing team.)

So far NCTM has solicited two reports from AMSARG. NCTM's questions and our reports are reproduced elsewhere in this issue of the *Notices*. To provide some background for the reports, this article summarizes some of the most basic issues in the mathedreform debate, then gives a brief sum-

Roger Howe is professor of mathematics at Yale University.

mary of the reports. This should be sufficient background for understanding most of the remarks in the reports, although some of the comments will not be thoroughly intelligible to someone not familiar with the "Standards".

Though AMSARG is not large, we have found that we are far from being of one mind on many issues, and the debates we conducted to frame our reports were both intense and extensive. Nevertheless, after the discussions there were significant points of agreement, as formulated in the reports. Very probably there are aspects of the issues that were not reflected in our thinking, and AMS members with ideas they think would illuminate NCTM's questions in useful ways are invited to communicate with us by writing to: Ms. Monica Foulkes, American Mathematical Society, 1527 18th St., NW, Washington, DC 20036-1358, e-mail mxf@ams.org, with a cover note indicating a message for AMSARG.

Issues

The debate on mathematics education is complicated. It involves many overlapping, sometimes contradictory, concerns. Here are some of the issues that figure in discussions of K–12 education.

1) Relative performance.

International comparisons of mathematics competence have regularly put the U.S. far behind the leaders in aggregate performance. The latest and largest such comparison is TIMSS (Third International Mathematics and Science Study). In TIMSS, U.S. eighth-graders come out below the median among the forty participating nations [6], and fourth-graders somewhat above, but still in the middle of the pack [7]. In studies with fewer participating countries, the U.S. has tended to place near the bottom. Asian countries tend to do well. Harold Stevenson and colleagues have presented a detailed picture of contrasts between U.S. and Asian mathematics classrooms (see, e.g., [8]) in practice and achievement.

2) Equity.

It is no secret that in the U.S. achievement varies widely from state to state and within any region tends to increase with community wealth. Some states put in performances which bear international comparison; others would come in near the bottom of any heap [9]. Similarly, suburbs outscore inner cities. This geographical variation tends to get correlated with ethnic data. Though not related to geography, gender variation gets attention too as an equity issue.

3) Technology.

Technology affects mathematics education in at least three distinct ways:

(i) The increased role of information processing in society creates new job opportunities for mathematically apt people, and it creates demand for higher levels of mathematical training for the less apt.

(ii) Computer technology can be used in the classroom. Calculators and computer graphics software offer alternatives to chalk for presenting mathematics and invite consideration of how they can best be used.

(iii) On the most fundamental level, technology requires rethinking not only of the "how" but of the "what" of teaching mathematics. It is pretty clear that in the future no one is going to get a job based on the ability to add long columns of numbers accurately. Recently we have seen the appearance of calculators and computer software that can perform much of the repertoire of undergraduate mathematics and beyond. Even if everything had been fine with U.S. math education, we would have to pay attention now to how the availability of sophisticated calculational tools changes what is important to teach. The automation of computation challenges the notion that mastery of computational technique should be the main criterion of mathematical success. The relation between computational expertise and conceptual understanding, and how each supports the other, is complex and requires careful study and thought.

4) Demography.

More students are taking more advanced degrees, and vastly more mathematics, than was the case when today's senior academics were getting their degrees. Over 750,000 students a year take calculus [10, 11], which has become a prerequisite for pursuit of a large variety of desirable careers. A larger percentage of a given age cohort now does graduate study of some kind than matriculated in colleges before World War II. A useful mantra here is "College is high school; graduate school is college", but even that does not account for the extent of the change. At the same time that mathematics course enrollments were swelling, the average SAT scores of college matriculants were decreasing. (Both these trends seem to have reversed to some extent in the 1990s.) The remarks above refer to the demography of academia and of mathematics courses. There is also the society-wide trend toward a more diverse population.

5) Subject matter.

Much of the school curriculum has been in place for a long time. One can, and probably should, ask if there are parts which are no longer germane. Also, research has created new fields of inquiry and application. Statistics figures prominently here. Some pieces of operations research and discrete mathematics also vie for a place in the curriculum. Although it is not much discussed, it should be relevant to AMS members that research in pure mathematics has shed new light on topics of school mathematics, and this insight has only very partially and imperfectly trickled down to the K–12 classroom.

6) Pedagogy.

a) Classroom procedure. The traditional lecture is questioned as the sole means of conveying subject matter. Lectures are admitted to be an efficient way to present information, but absorption rates are questioned. More varied practice is advocated. Doctrines of "constructivism", which emphasizes the role of the internal mental processes and installed database of the individual student in his or her learning, enjoy wide currency among educators. Small group work is often put forward, sometimes as a corollary of constructivism, as a way to ensure "active, hands-on" learning. Problem solving is put forth as a major method and goal. Contextualism, the view that school mathematics should be rooted in the problems of the real world, asks more specifically for "real-world problems" with "real data". There are calls for deemphasis of memorization and of repetitive practice.

b) Rigor and technique. AMS members should find it easy to approve of calls for "mathematical power", "increased understanding", and "mathematical reasoning". However, implementation of teaching consistent with these slogans may involve tradeoffs that some will find troublesome. In this area, there are two issues especially relevant to the AMS. One is how far mathematical reasoning should be developed, in particular, whether it includes an appreciation of and ability to deal with proof. A second concern is the extent to which technical skills are necessary for conceptual understanding, or, put more starkly, how far technical skills can be neglected before understanding and mathematical power also flag. The technique debate has been of concern to the general public also. Deemphasis of "basic skills" has been one of the red flags that have aroused parental ire in some communities.

7) Teacher preparation and certification.

There is substantial evidence that teachers in other countries, especially high-scoring countries like

National Council of Teachers of Mathematics
1906 Association Drive, Reston, VA 20191-1593
(703) 620-9840; fax: (703) 476-2970

19 November 1996

Roger Howe
Department of Mathematics
Yale University
P.O. Box 208283
New Haven, CT 06520-8283

Dear Roger,

This follows up on an earlier letter in which Gail Burrill asked that you set up a Review Group within the American Mathematical Society to help us as we update the NCTM Standards for the coming century. We are very pleased that you have appointed such a group, and we look forward to fruitful collaboration as we proceed with this project.

Our first request is to get your reactions to the statements of the content standards as they are now phrased. To that end I have enclosed copies of the "barebones" curriculum standards from the 1989 *Curriculum and Evaluation Standards for School Mathematics*, without any of the explanatory or illustrative material that followed each of them. (This set of standards is on green paper.) With regard just to these curriculum standards, here are three questions to which we would like your Review Group to respond:

1. Your view of Mathematics. Consider the nature of Mathematics—its content, processes, and procedures—that you feel is important for students from pre-Kindergarten through grade 12. Do the current statements of the Standards adequately communicate your view of the discipline?
2. Consistency and growth. Do the statements of the current curriculum Standards convey a sense of consistency and growth in content themes as the student moves across the grade levels? (Content themes would include, for example, ideas of measurement, number sense, and algebraic and geometric thinking.)
3. Expected understanding of content. Do the statements of the content Standards adequately reflect the mathematical understanding expected of a student graduating in the 21st century? Do they reflect the needs of students who are planning post-secondary study in a Mathematics-related discipline?

As you know, one of the goals of the updated document is to merge the classroom-related aspects of curriculum, teaching, and assessment so that the interrelationships among them are clear. (We have also enclosed standards from the *Professional Standards for Teaching Mathematics* and from the *Assessment Standards for School Mathematics*, on blue and gray paper, respectively.) Exactly how this is to be done is the subject of the fourth question:

4. Blending the three sets of Standards. The NCTM project to reissue a Standards document in the year 2000 intends to meld together the dimensions of content, teaching, and assessment. What suggestions could you make as to the most effective ways of blending these ideas?

The Commission on the Future of the Standards and the Writing Groups will be reconvening again on February 1, 1997, and we would very much like to have your input by mid-January so that we can compile responses from several associations in time for that meeting. Of course there will be other opportunities for you to provide advice later. We also welcome your group's suggestions of other ways they can be involved. Some groups are thinking about holding forums or discussions, although we are not specifically requesting this now.

We appreciate your willingness to help NCTM with this important project. Please let me know if there is anything I can do to facilitate your response.

Sincerely yours,

Mary M. Lindquist
Chair

Enclosures

Japan, know more mathematics than do U.S. teachers [8, 12, 13]. Assuming that it would be desirable for U.S. teachers to have better mastery of their subject matter, it is not obvious that the simple step of requiring more mathematics courses for future teachers will achieve this goal. Standard undergraduate courses do not dwell heavily on the relevance of their subject matter to the high school curriculum, and teacher candidates may not be able to make the connections on their own.

8) Assessment.

Along with calls for changes in subject matter and pedagogy, there is a strong chorus urging changes in the way mathematics achievement is evaluated. Multiple-choice tests are decried as inadequate. There are calls for tests which require writing and human evaluation. There are calls for evaluation by means of "portfolios" of accumulated work and for complex profiles of progress to replace simple letter grades. Debates about measurement complicate all other aspects of mathedreform. They cloud the criteria for success, and they make agreement on what constitutes progress harder to reach.

9) High performers.

This issue is relatively neglected in K–12 discussions but is one that should concern AMS members. There is plenty of evidence of problems here. In the TIMSS only 5 percent of U.S. students performed in the top 10 percent internationally [6]. In graduate programs a declining and now minority portion of students are U.S. born [14].

10) New curricula.

There has been extensive development of new mathematics programs which attempt to implement the reform ideas. NSF has sponsored several programs at the high school and junior high school level, and there are also commercially developed new programs. These programs have been a rich mine of controversy. It is remarkably difficult to get a clear picture of their effects [1]. There is heartfelt anecdotal testimony both positive and negative. When statistical evidence, favorable or unfavorable, becomes available, its significance can be disputed on the basis of the issues sketched above, as well as on technical statistical grounds.

The list of ten issues above attempts to present the main ingredients of mathedreform as distinct issues, although some interactions are noted. In the give-and-take of debate, the issues of course interact in complex ways, and sometimes the interactions are comparable or even greater in importance than the separate ingredients.

The ARG Reports

Here is a brief summary of the reports of AMSARG. The full reports appear on pages 270–276 in this issue of the *Notices*.

First Report: This was in response to general questions about the "Standards". The main point of the AMSARG report is that the revised "Standards" need to be more specific and less ambiguous. Particular areas in which we sought greater emphasis and clarity are: (i) in the role of the teacher as a leader of the class and as guarantor of mathematical closure, (ii) in the time development of the curriculum—specification of what should be learned when, (iii) in clarifying the notion of algorithm as a key feature in the automatization of mathematics, (iv) in developing the notion that proof and logical argument are the appropriate continuation of successful nurturing of mathematical reasoning ability, and (v) in articulating the connections between algebra and geometry. We also made a suggestion about the organization of the "Standards" revision.

Second Report: The second round of questions focussed on the ideas of proof and algorithm. Our report articulated some of the important features of algorithms and of the concept of algorithm. It also addressed the role of the algorithms of arithmetic. In this connection it noted that decimal notation is itself a highly sophisticated algorithm, whose structure strongly constrains the form of algorithms for performing arithmetic operations. Concerning proof, we stated our sense that mathematics should be taught as a subject which makes sense in itself and which helps to make sense of the world. We also argued that proof is the ultimate formulation of this sense-making practice and that ability to understand and construct proofs should be the end result of the successful development of reasoning skills. We did not advocate strict axiomatic developments of whole areas, but rather a practice of "semilocal" proof, in which substantial numbers of results are shown to follow from a much smaller set of accepted principles. We stated a preference for developing specific examples of abstract structures rather than a formal treatment of them in high school.

References

[1] A. Jackson, *The Math Wars, Parts I and II*, Notices Amer. Math. Soc. **44** (June/July and August, 1997), 619–702; 817–827.

[2] *Curriculum and Evaluation Standards for School Mathematics* (1989), *Professional Standards for Teaching Mathematics* (1991), *Assessment Standards for School Mathematics* (1995), National Council of Teachers of Mathematics, Reston, VA.

[3] *NSF-funded Mathematics Instructional Materials*, prepared by Mary Bahns, the McKenzie Group, Inc., Washington, DC, for the NSF, July 1995.

[4] Z. Usiskin, *Towards the Second Edition*, UCSMP Newsletter no. 12 (Winter 1993), University of Chicago School Mathematics Project, Chicago, IL.

[5] L. Cheney, *Once Again, Basic Skills Fall Prey to a Fad*, New York Times, Op-Ed page (August 11, 1997).

[6] *Pursuing Excellence, A Study of U. S. Eighth Grade Mathematics and Science Teaching, Learning, Curriculum, and Achievement in International Context*, (NCES 97-198), U.S. Department of Education, Na-

tional Center for Education Statistics, Washington, D.C., U.S. Government Printing Office, 1996.

[7] *Pursuing Excellence, A Study of U.S. Fourth-Grade Mathematics and Science Achievement in International Context*, National Center for Education Statistics, U.S. Department of Education, U.S. Government Printing Office, Washington, DC, 1997. [Editor's Note: Much of the TIMSS material may be found at the Web site `http://nces.ed.gov/TIMSS/`.]

[8] H. Stevenson and J. Stigler, *The Learning Gap*, Summit Books, New York, 1992.

[9] *Indicators of Science and Mathematics Education 1995* (NSF 96-52) (L. Slater, ed.), Division of Research, Evaluation, and Communication, Directorate for Education and Human Resources, National Science Foundation, Arlington, VA, 1996.

[10] D. Rung, *A Survey of Four-Year and University Mathematics in Fall 1995: A Hiatus in Both Enrollment and Faculty Increases*, Notices Amer. Math. Soc. **44** (September 1997), 923–931.

[11] *1997 Advanced Placement Program, Candidate Grade Distributions*, The College Board, New York.

[12] *Japanese University Entrance Examination Problems in Mathematics* (Ling-Erl Eileen T. Wu, ed.), Math. Assoc. Amer., 1993. (Also included as an insert in the December 1992 FOCUS.)

[13] K. Kodaira, ed., *Japanese High School Mathematics; Mathematics I, Mathematics 2, Algebra and Geometry, Basic Analysis* (translated by H. Nagata), Math. World, vols. 8, 9, 10, 11, Amer. Math. Soc., Providence, RI, 1996.

[14] *International Benchmarking of U.S. Mathematics Research*, National Academy Press, Washington, DC, 1997.

Lars Valerian Ahlfors (1907–1996)

Raoul Bott; Clifford Earle; Dennis Hejhal; James Jenkins; Troels Jorgensen; Steven G. Krantz, Editor; Albert Marden; Robert Osserman

Lars Valerian Ahlfors was born in Helsingfors, Finland, on April 18, 1907. He studied mathematics with Ernst Lindelöf at Helsingfors University and earned the doctorate in 1928. Ahlfors accepted a position at Harvard University and moved to Cambridge in 1935. He spent most of his professional life at Harvard, with a period in Europe (primarily at the University of Helsinki) from 1938 to 1946.

Lars Ahlfors retired from his position at Harvard in 1977. He continued to study mathematics until the end of his life on October 11, 1996.

Lars Valerian Ahlfors was arguably the preeminent complex function theorist of the twentieth century. With a career spanning more than sixty years, Ahlfors made decisive contributions to areas ranging from meromorphic curves to value distribution theory, Riemann surfaces, conformal geometry, extremal length, quasiconformal mappings, and Kleinian groups ([7] serves as a map of Ahlfors's contributions to the subject). Ahlfors was both role model and mentor to his graduate students and to the many mathematicians around the world who learned from his example. He is remembered warmly, both as a mathematician and as a man.

Ahlfors's life contained many firsts. In 1936 he received, along with Jesse Douglas, one of the first two Fields Medals. Ahlfors's remarks concerning that occasion, taken from his *Collected Papers* [4], typify his honesty and humility:

> I was in for the surprise of my life when in 1936, at the International Congress in Oslo, I was told only hours before the ceremony that I was to receive one of the first two Fields Medals ever awarded. The prestige was perhaps not yet the same as it is now, but in any case I felt singled out and greatly honored. The citation by Carathéodory mentions explicitly my paper "Zur Theorie der Überlagerungsflächen," which threw some new light on Nevanlinna's theory of meromorphic functions. The award contributed in great measure to the confidence I felt in my work.

During World War II Ahlfors was forced to pawn his Fields Medal in order to obtain the money to secure safe passage from Finland to Zurich at the end of the war. Ahlfors's remarks on the incident were:

> I can give one very definite benefit [of winning a Fields Medal]. When I was able to leave Finland to go to Sweden, I was not allowed to take more than 10 crowns with me, and I wanted to take a train to where my wife was waiting for me. So what did I do? I smuggled out my Fields Medal, and I pawned it in the pawn shop and got enough money. I had no other way, no other way at all. And I'm sure that it's the only Fields Medal that has been in a pawn shop... As soon as I got a little money, some people in Switzerland helped me retrieve it.

Another remarkable achievement of Lars Ahlfors is that he was invited to be a plenary speaker at the International Congress of Mathematicians on three different occasions. No other mathematician in the past seventy years has achieved such a distinction. His plenary addresses were about three completely distinct areas in which he had made seminal contributions.

Steven G. Krantz is professor of mathematics at Washington University in St. Louis. His e-mail address is `sk@math.wustl.edu`.

Ahlfors's text *Complex Analysis* [1], which was first published in 1953 and has gone through three editions, has been the definitive text in complex function theory for the past forty-five years; there are few other texts in modern mathematics that have played such a dominant role.

At a conference held in Storrs, Connecticut, to commemorate Ahlfors's seventy-fifth birthday, Ahlfors remarked that "Retirement is wonderful. I can't perish anymore, so I don't have to publish." The upshot of his remarks was that he could devote himself full time to understanding the new developments in his field without the pressure of writing them up. He seemed to think of this as a mathematician's paradise. The NSF program officer in complex analysis told me about fifteen years ago that Lars Ahlfors's entire NSF proposal consisted of a single sentence: "I will continue to study the work of Thurston." Ahlfors was seventy-five years old at the time, and he was devoting himself to learning the newest ideas in the subject!

In what follows, seven mathematicians relate their personal reminiscences of Lars Ahlfors.

—Steven G. Krantz, Editor

Early photograph of Lars Ahlfors, who is reputed to have worn a beret until the day he died.

James Jenkins

Lars Ahlfors returned to Harvard in the fall of 1946, the same time that I arrived to begin my graduate work there. David Widder was at that time the chairman of the mathematics department, and he was recruiting people to work with Ahlfors. Since I had some background in function theory from my work at the University of Toronto, he approached me in this context (not that I was reluctant).

In those days there was no mathematics building at Harvard; indeed, the faculty members did not have regular offices. Some of the senior people had faculty studies in Widener Library, and Ahlfors shared one with Julian Coolidge, professor emeritus and a geometer of the old school. I had my first few meetings with Ahlfors there. On one occasion he had the duty of testing my proficiency in foreign languages with Coolidge present. In a passage in French making a mildly pejorative statement about "géomètres", I translated the word as "geometers", and Ahlfors was quick to point out that it was used in the more general sense of "mathematicians".

Every faculty member at Harvard had an affiliation with one of the residential houses (which had a certain analogy with colleges at the English universities). Ahlfors's affiliation was with Dunster House, and he had a sort of office there. We met there a few times, but afterwards always at his home. Ahlfors was very conscientious in meeting with his graduate students, but his role was largely reactive rather than presenting suggestions or ideas (at least in my case).

In the spring of 1947 Ahlfors gave a course on the method of the extremal metric, which was then in its infancy. In the year 1947–48 he gave a course on the calculus of variations. In the year 1948–49 he gave a course on Riemann surfaces, directed largely to a generalization of his work on Schwarz's lemma to finite-bordered Riemann surfaces. Ahlfors did not conduct a regular seminar at any time during my stay. One year he gave a series of talks on Teichmüller's papers, probably with a view to preparing himself for his work on quasiconformal mappings.

Ahlfors had a remarkable command of English for one for whom it was not the native language. However, English is tricky. One semester, when he was teaching a course in elementary calculus, he asked students on the examination to find the three first derivatives of an explicit function of one real variable (creating considerable havoc). Also, under stress one tends to revert: at a tense moment in a lecture "*j*" came out as "*yot*".

There were occasionally people who came to work with Ahlfors, for example, a Swiss by the name of Haefeli. In the year 1948–49 Arne Beurling came to Harvard as visiting professor. He gave two rather well-known one-semester courses on his brands of functional analysis and harmonic analysis. He and Ahlfors were working on a book on extremal length and in fact had a rough draft of some of the material. No book ever appeared, but some of the contents showed up in Ahlfors's book *Conformal Invariants* [2].

James Jenkins is professor of mathematics emeritus at Washington University in St. Louis.

Robert Osserman

When I reached the point in my graduate studies where it was time to select a Ph.D. advisor, I naturally consulted fellow students who were more advanced. One of those was a young student from Finland named Vidar Wolontis. He may not have been totally objective in his recommendation that I study with his countryman Ahlfors, but he did mention one reason that I still remember. He said, "Look at Ahlfors's bibliography. It is really short. Only a relatively small number of papers, but every one significant." I later learned of Gauss's famed motto: "Pauca sed matura" or "Few, but ripe." Perhaps Ahlfors's unstated motto was "Pauca sed egregia"—"Few, but outstanding." That was an idea that appealed to me greatly, and I did become Ahlfors's student, a decision I have always felt lucky to have made.

Ahlfors's style as mentor suited me perfectly. He was always available but never intrusive. He provided me with several opportunities that proved important to my work. One was a 1951 summer program in Stillwater, Oklahoma, where he was invited to give a series of lectures. He asked if I would like to be the official note taker. It was actually a paid position, or at least what passed for pay in those days—$150, if I remember right. (Perhaps I should add that when I started graduate school at Harvard, the tuition was $600 per year, so that my summer salary was one quarter of a year's tuition. Also, I paid $10 per month for a room that came with a piano in it.) I worked closely with Ahlfors on reworking his lectures and learned an enormous amount in the process. Murray Gerstenhaber was also there for the program, and he too was a great help to me in working through the lectures. When the program ended, Murray and Ahlfors and I drove in Murray's newly acquired car from Stillwater to Chicago. A highlight of the trip was dinner the first night in a fancy restaurant in Missouri on what Murray reminded me recently was Bastille Day. I remember Ahlfors's great delight in celebrating the end of a month of living under the restrictions imposed by the dry state of Oklahoma.

Later in 1951 Ahlfors suggested that I accompany him to Princeton for a special conference for the one-hundredth anniversary of the birth of the Riemann surface in Riemann's doctoral dissertation, presented in public in 1851. The keynote lecture was given by Ahlfors, who presented a masterful overview of the evolution of Riemann's ideas during the preceding century.

This conference was particularly significant for me; right after Ahlfors's talk, Lipman Bers stood up and said he had two problems concerning Riemann surfaces that he thought the group in attendance would be ideally suited to work on. After we returned to Harvard Ahlfors suggested that I work on one of the two problems, and he had some ideas about how to proceed. It turned out that the particular approach he suggested did not work, but I was able to solve the problem by a different method, and it became part of my Ph.D. thesis.

A high point of the Princeton conference was a cameo appearance by Albert Einstein, who expressed his own indebtedness to Riemann and the importance to him in particular of Riemann's introduction of the idea of a Riemann surface. What Einstein was referring to was of course the idea of a Riemannian manifold, and it was a legitimate confusion to think that a Riemann surface might be just a two-(real-like)-dimensional Riemannian manifold. As the terminology evolved, by the middle of this century the term "Riemann surface" meant a one-dimensional complex manifold or, equivalently, a two-(real)-dimensional surface with a conformal structure. A Riemannian manifold could have any dimension and was endowed with a Riemannian metric, so that lengths, for example, make sense on a Riemannian manifold but not on a Riemann surface. The two subjects had diverged so much that there were many practitioners of each who had nothing to do with the other. On the other hand, there was, and is, a considerable area of overlap; it was precisely in exploiting that area that Ahlfors was most ingenious and adept, especially in the use of conformal metrics to study analytic mappings and Riemann surfaces. My own later work also made significant use of the connection, but chiefly in the opposite direction: applying the theory of Riemann surfaces to problems in differential geometry.

In the years following my doctorate Ahlfors continued in his supportive role. He made a number of suggested changes in a paper I sent him for submission to the *Proceedings of the National Academy of Sciences*, including the deletion of the word "unfortunately" that I used at one point, which he described as "too passionate."

In later years I had less contact with Ahlfors as I drifted away from the areas in which he worked and more toward differential geometry. However, the mathematical style and professional ideals that I had imprinted during my graduate work persisted throughout my career. On the mathematical side I was always drawn to areas that involved analysis with a geometric flavor. And professionally I have tried to live up to Ahlfors's high standards. I might add that one of his characteristics that I have always greatly admired was his scrupulous honesty and generosity in giving credit to others. Ahlfors's influence and legacy will permeate mathematical life for a long time to come, and I am very grateful to have been able to be a part of it.

Robert Osserman is professor of mathematics emeritus at Stanford University and special projects director at MSRI.

Clifford Earle

Toward the end of the "Author's Preface" to his *Collected Papers* [4] Ahlfors wrote, "I have enjoyed the many excellent students Harvard has had to offer, many of whom I have watched become leaders in my own or some other field." Many of them, both undergraduates and graduate students, attended Math 213, the two-semester graduate course in complex function theory that Ahlfors taught many times at Harvard. His well-known book *Complex Analysis* [1] grew in part from his experience in Math 213 and has been a standard textbook in such courses for many years.

The final paragraph of the preface of the first edition of *Complex Analysis* [1] begins as follows:

> One more point: the author makes abundant and unblushing use of the words "clearly", "obviously", "evidently", etc. They are not used to blur the picture. On the contrary, they test the reader's understanding, for if he does not agree that the omitted reasoning is clear, obvious, and evident, he had better turn back a few pages and make a fresh start.

Ahlfors's classroom manner belied these blunt statements, which disappeared in the later editions of the book. When he used a word like "obviously" in his Math 213 lectures, the "obvious" statement was always followed by a friendly glance around the classroom and then, more often than not, by the polysyllable "Well" and an extended explanation or example.

Higher-level graduate courses on topics in geometric function theory and Riemann surface theory played an important role in Ahlfors's teaching of graduate students and postgraduate visitors to Harvard. These courses were often given before books on their subjects had been published. Ahlfors made his detailed handwritten lecture notes available for reading in the library after class. Such notebooks were the basis of his books *Lectures on Quasiconformal Mappings* [3] and *Conformal Invariants* [2].

Besides his classroom notes, Ahlfors, of course, shared his latest research papers with his graduate students and encouraged the reading of classic texts. Carathéodory's two volumes on function theory [6] and the books of Rolf Nevanlinna were high on his reading list.

But from 1959 through the 1960s an Ahlfors graduate student's most memorable experience was participation in the Ahlfors seminar. In this weekly seminar graduate students gave expository talks about papers in the literature and Ahlfors lectured about his recent research, but most of the talks were given by short- or long-term visitors who lectured about topics of their choice, usually related to their latest work. Regular attendance at the seminar exposed the participants to a broad range of topics in complex analysis. Among the visitors who gave a lecture or series of lectures in the seminar during those years are Accola, Beardon, Belinskii, Gehring, Hayman, Kodaira, Osserman, Pommerenke, Rickman, Rodin, Sario, Väisälä, Weill, and Wermer. This stream of visitors was made possible by the dramatic increase in federal support of mathematics after the Sputnik flight in 1957 and represents a highly effective use of such funding to benefit both mathematics research and the education of young mathematicians (if these can be separated) simultaneously.

Clifford Earle is professor of mathematics at Cornell University. His e-mail address is `cliff@math.cornell.edu`.

Dennis Hejhal

During the fall of 1964 I was a fifteen-year-old high school student at Lane Technical High School in Chicago. I was reading some analytic function theory on my own at the time and discussing it with one of my teachers. She informed me that Ahlfors's book was a classic, that she had used it during her grad school days, and that maybe I should try to work through her copy. I did so and encountered a variety of questions in connection with the exercises. As I recall today, there was an error in one that prompted me to write a letter to Ahlfors to ask about the matter.

As a lowly high school student I was pleasantly shocked when I promptly got a reply from the author, and even more so when it was addressed to someone called "Dr. D. A. Hejhal". I immediately wrote back with a fuller letter stating that I was still in high school and was just beginning my study of function theory.

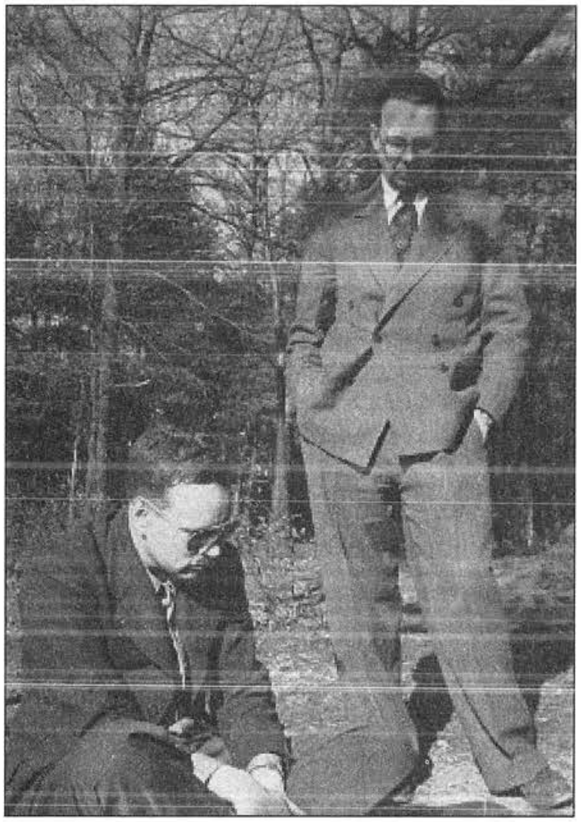
Thus began my correspondence and long-distance "tutelage" with Lars Ahlfors.

I was profoundly influenced by the guidance he offered me, as I sent him my various reports telling him what I was up to. Very early on I developed a sense of the quality Ahlfors represented. I also felt encouraged that a Harvard mathematician of his caliber would have the time and interest to correspond with a high school student.

Our early exchanges covered a variety of topics: what type of books I had access to, whether I knew any university faculty, what I wanted to study next, my project for the National Science Fair, etc.

He did not really try to influence my choice of study topics, but rather encouraged me to go where

Dennis Hejhal currently holds a joint professorship in mathematics at Uppsala University and the University of Minnesota. His e-mail address is `hejhal@math.umn.edu`.



Ahlfors (standing in top photograph and in foreground of bottom photo) at Harvard math picnics , circa 1950–1952.

my interests took me. About the only thing he did strongly suggest, after two or three letters, was that I take a copy of one of his letters and "report to" the chairman's office at the University of Chicago mathematics department with it, basically seeking someone to help "look after me."

Herstein was the acting chairman, and the first thing he did was give me a note saying that it was okay for Hejhal to check out library books in his name. The Eckhart Hall Library is a first-rate mathematics library, and this was like opening the door to infinity for me. This high school student had never seen anything like it!

In this way, through Ahlfors, I came to appreciate that mathematics includes components of scholarship and history concomitant to research. In a 1985 paper Ahlfors credits me with providing him with some decisive historical references thanks to my "gargantuan appetite for reading." I can only respond by saying that it was he who both whetted and precipitated that appetite—and that he exaggerated.

During my undergraduate days at the University of Chicago, I continued to keep Ahlfors posted with progress reports and copies of my papers. Paul Sally turned out to be the ideal local person to keep me headed in the right direction. One paper that I wrote while in my second year, on kernel functions (answering a question posed to me by R. R. Coifman), was thought possibly to be suitable for a Ph.D. thesis. I was not sure what course was best and received a mixture of views. Building on the sense of trust that had evolved earlier, I decided to listen to Ahlfors; he wrote that rather than "racing," he felt it better that someone in my shoes substantially deepen their knowledge first and work on more things before taking a Ph.D.

This ultimately led to my going to Stanford in 1970 for my graduate work (a place that has rightly been described as a "hotbed" of complex analysis in those days, particularly for someone enthusiastic about kernel functions and conformal mappings). Since Ahlfors was planning to be away from Harvard during part of that time, he concurred that Stanford was the natural place for my graduate work.

Though new influences abounded there for me (Pólya, Bergman, Schiffer, Royden, Cohen, Zalcman), it has to be said that it was Ahlfors who, in effect, started me in serious motion down the wonderful path of complex analysis.

It was only in the spring of 1972—just before I applied for a job—that Lars Ahlfors and I first met in person. He invited me to come to Harvard for a month.

There is an interesting story behind this invitation. Whenever I play "tour guide" for my visitors in Sweden, we always go to the Mittag-Leffler Institute near Stockholm. The library there contains many old books, spread out over the third and fourth floors. On the third floor in the corner, looking out over "his books", is an enormous bronze statue of Mittag-Leffler (commissioned by same!) holding his reading glasses. In 1972 my work with Max Schiffer on the Szegö kernel function led me to discover the wondrous gold mine of a book by H. F. Baker [5]. Ahlfors was visiting the Mittag-Leffler Institute when I wrote him describing my results, citing the reference to Baker's book. I offered to come to the Mittag-Leffler Institute if he wanted to hear more. I soon received a reply. Ahlfors said it would be preferable to wait until he returned to Harvard; let's set something up, etc. Then, with obvious delight, he went on to say that he found Baker's book, "but that in order to get the book, I first had to get a ladder, prop it up against the statue, and quite literally stand *on top of* Mittag-Leffler to grab it from the (nearly inaccessible) shelf above his head."

That scene with the sixty-five-year-old Ahlfors would have been priceless to catch in a photo. Isaac Newton's famous quotation[1] could have served as its title.

One presumes the same story was told to Lennart Carleson, then director of the Mittag-Leffler Institute, because shortly afterward Baker migrated to a more easily accessible spot on the fourth-floor balcony level.

With Ahlfors safely back at Harvard, my one-month visit took place as planned. During my stay there I was offered a Benjamin Peirce Assistant Professorship, thus leading to the next stage in our relationship, viz., working as a junior colleague with my former long-distance mentor.

I remember my Harvard days with great warmth. The atmosphere that Ahlfors created there—e.g., in our weekly animated discussion sessions and in the people to whom he introduced me (Weil, Selberg, Bers, Carleson, etc.)—is one that I am deeply grateful for and will always remember.

During my days as an impressionable student/apprentice I would have to say that, in his old-fashioned way, Ahlfors taught me much about being a mathematician and about constantly seeking to achieve something of quality.

One hears today various things about "the dumbing down of society". In my dealings with Lars Ahlfors I always got the impression that here was somebody who truly believed in excellence, someone who not only would *not* lower the bar but who would instead *raise it* to facilitate bringing out talent that he intuited was there.

I feel myself exceedingly fortunate to have encountered someone like this in my formative days as a mathematician. One could not have asked for a better start.

Troels Jorgensen

It was my good fortune to know Lars Ahlfors for a quarter of a century. He was an inspiring mathematician and a very special person. His intuition for and joy in mathematics, combined with a great capacity for work, made him a harmonious craftsman through a career that spanned over sixty years. He savored the recognition and awards he received yet was very modest about his accomplishments. At a banquet celebrating his seventieth birthday Ahlfors said that "He liked to go fishing where the fish are, rather than trying exclusively for the big one."

Lars was grateful for the opportunity to serve at Harvard and took much pride in his colleagues, enjoying their professional successes as well as their good company. A strong constitution allowed him to stay fit without spending much time on exercise and to share many a happy evening with his family and friends from around the world. Yet he was always ready for work the next morning. As with his mathematical interests, his interest in other human affairs was often strong and addressed forcefully: "That is just it! That is just it!" would resound when something met with his approval. One could have a wonderful time discussing almost any matter with Lars.

Lars often recalled his delight when, as a young man, he found out that one could become a mathematician. The ultimate blessing, however, was his lifelong happy marriage to Erna. The example set by Lars and Erna of affection for each other and

Ahlfors's Students

Lars Ahlfors had a large number of Ph.D. students, many of whom are leading mathematicians today. The most complete list that could be assembled includes these names:

- Robert D. M. Accola, Brown University
- S. Thomas Bagby, Indiana University
- Harvey Cohn, City University, New York
- Clifford J. Earle Jr., Cornell University
- John Fay, St. Joseph's College
- Paul R. Garabedian, Courant Institute
- Dale Husemoller, Haverford College
- James A. Jenkins, Washington University
- Roger B. Kirchner, Carleton College
- Saul Kravetz (deceased)
- Henry Landau, AT&T Bell Labs
- Albert Marden, University of Minnesota
- Peter Evans Martin, Dickinson College
- Audrey Wishad McMillan (retired)
- Robert Osserman, Stanford University, now at MSRI
- Henry O. Pollak, AT&T Bell Labs
- J. Ian Richards, University of Minnesota (deceased)
- Halsey Royden, Stanford University (deceased)
- Ernest C. Schlesinger, Connecticut College
- George Sethares, Bridgewater State College
- George Springer, Indiana University
- F. Ulrich (deceased)
- Eoin Whitney (deceased)
- Vidar Wolontis

There were so many others who found inspiration in his work. A partial list includes Lipman Bers, Arne Beurling, Christopher Bishop, Frederick Gardiner, Frederick Gehring, David Hamilton, Dennis Hejhal, Peter Jones, Troels Jorgenson, Linda Keen, Irwin Kra, Bernard Maskit, Howard Masur, and Seppo Rickman.

[1] *"If I have seen further than others, it is by standing on the shoulders of giants."*

Troels Jorgensen is at Michigan State University. His e-mail address is jorgensen@psl.msu.edu.

Ahlfors with wife Erna around 1987.

for the whole world should be remembered, along with his mathematics, with deep admiration.

Albert Marden

Lars Ahlfors has been a role model of course for his taste and style in mathematical research, but also for his conduct of professional life.

As an undergraduate I took 2.5 courses from Lars: Math 105 (Advanced Calculus) as a sophomore in 1953, Math 213 (Complex Analysis) as a junior, and Math 240 (Conformal Invariants) as a senior. It was characteristic of his sensitivity that he gave me an *A* in the latter course. As I look back over my meticulously taken course notes, it is painful to recognize how immature I was, too immature to understand the material. Instead, what penetrated was the clarity, elegance, and utter authority with which Ahlfors wove together analysis and geometry to resolve a seemingly intractable problem. I remember following him into the Sever Hall faculty room as he lit up a cigarette and considered how to answer my questions. After getting medical school out of my system, I began thesis work under Ahlfors about 1960. I generalized some of Accola's results connecting extremal length of homology classes to various canonical harmonic differentials and finished in 1962.

In 1981 I arranged for Lars to be invited for a quarter as the first Ordway Professor at Minnesota. That was at the beginning of his search for higher-dimensional analogs of the theory of Kleinian groups, in which he was the pioneer. Later, Burt Rodin brought Lars and Erna to La Jolla for a quarter, and, thanks to Burt, I had the good fortune to be there as well. We watched the sun set from Lars and Erna's apartment on the shore as we enjoyed their incomparable hospitality.

By the early 1980s Thurston's work had transformed the subject of Kleinian groups beyond all recognition. Lars had great respect for Thurston but railed against the lack of detailed written proofs, as this was a great frustration to his effort to understand the new theory. Lars was a genius at computing and finding the right perspective and formula for a situation, but he did not have the insight and turn of mind of a three-dimensional topologist.

In our time perhaps only Nevanlinna himself had anything approaching the universal esteem with which Lars was regarded in complex analysis. One reason was that Lars made germinal contributions to most of the active areas in the field. One could argue that Ahlfors defined the field by the scope of his work. Another reason was the objectivity and evenhandedness with which Ahlfors regarded and supported other mathematicians. One did not gain his preference simply by living in a particular political territory. Many appreciated his formal, refined, and reserved personal style; at the same time he was tolerant and understanding.

Albert Marden is professor of mathematics at the University of Minnesota. His e-mail address is `am@math.umn.edu`.

Raoul Bott

Ahlfors was a concept for me long before we met when my family and I moved here (to Cambridge) in 1959. Nevanlinna had visited our former home in Michigan, and in any case Lars's name came up often there in a department much concerned with the theory of complex variables. Much later, in the 1970s and in a collaboration with Chern, I was finally introduced to Lars's work firsthand and even had the foolish hope of trying to improve on it. This was his virtuoso achievement on equidistribution, to which Herman Weyl alluded as "a vineyard planted by that grand gardener from the North."

And so now—miracle of miracles—in 1959 it appeared that we would be colleagues henceforth! At that time the department fell into several age groups. Walsh and Widder were the most senior, in their sixties. Then came the trio of "foreigners": Ahlfors, Brauer, and Zariski. These were followed by Garrett Birkhoff, Lynn Loomis, George Mackey, and Andy Gleason. John Tate and I were

Raoul Bott is Casper Graustein Professor of Mathematics at Harvard University. His e-mail address is `bott@math.harvard.edu`. *His remarks were delivered at a memorial service for Ahlfors.*

the new and youngest additions to the professorial stable at ages thirty-five and thirty-six. Actually, in 1959 Barry Mazur, Shlomo Sternberg, and David Mumford were also already on board as Junior Fellows and assistant professors.

I was of course scared to death to join a department that included such names as Ahlfors, Brauer, Zariski, and I tried hard to be on my best behavior. The department was extremely hospitable; we were wined and dined, and soon learned that nobody would bite us.

Still, it was only after a delightful evening at the Ahlforses' that Phyl and I truly felt at ease here. Somehow, in their house, matters of protocol or hierarchy—as well as being on one's good behavior—were completely missing. We were there to have a good time and to bare our souls to each other, helped along with the obligatory Skol in vodka, often followed by champagne.

In the department Lars was a quiet, reserved, and benevolent presence. He was always perfectly attired, attended the meetings punctually, and only now and then entered the discussions, often decisively. For me it was always a pleasure to run into him in the corridors. It reassured me: this was indeed a good place to be.

At home Lars was the warmest of hosts, a delightful drinking companion, a devotee of beauty and class in every form—from literature, music, and painting to the admiration of the opposite sex. Yet there was no impropriety in Lars's gallantry. All his life he was enthralled with his wonderful wife, Erna, who had such a mercurial way of dealing with his natural Finnish reserve and his shyness.

I now remember with special affection a summer month we spent together with the Ahlfors family on the coast of Rhode Island, with the many minor misadventures that invariably occur in a joint rental venture. For instance, each family had a dog, both male, who unfortunately would not tolerate each other. So I remember playing bridge with Erna and Lars and Phyl after our young brood had been put to bed; Lars and I would each hold a growling dog at our feet, with some sort of makeshift barrier between them. Amazingly, this did the job; "out of sight, out of mind" really works for dogs.

In other ways we were not so evenly matched: our four youngsters, two of them under five years old, against one spirited teenager of the Ahlforses'. Actually, on second thought and on observing my grandchildren, maybe four kids under five years of age is an even match for one child over fourteen. In any case, it was a memorable month, with Erna and Phyl outdoing each other with their consummate culinary skills.

Lars was a great Harvard patriot. On many occasions he delivered himself of the opinion that Harvard was the best of all universities and the Boston/Cambridge area the most desirable place to live. Of course, he came here at a time when Harvard had a rather grander way about it. At dinner parties Lars would reminisce: it was understood that men would turn up in black tie and the ladies in evening gowns. Although he relished these memories, he was ever on the side of us youngsters and did not seem to mind our much more slovenly ways.

Lars was also a man of action and had an instinct for achieving his ends in the simplest way, be it in the exposition of his mathematics or in real life. When, already in retirement, Lars found himself accosted by a knife-wielding assailant on the doorstep of his Beacon Street apartment in downtown Boston, he did not hesitate. It happened that he was returning from a shopping expedition with a bottle of whiskey (first class, of course) under his arm. So he instinctively hit the chap over the head with the bottle and managed to open his door to safety before the assailant recovered his wits. Thereafter, it was mainly the loss of that fine whiskey that Lars lamented.

Let me end by giving thanks that we have had Lars among us for so many years. We must spend our time not in mourning but in celebration of a wonderful life—a life of brilliant achievement, great professional success, lived amidst much human warmth and mirth. Yes, there was great tragedy also. But who, in a life span of nearly ninety years, can avoid that, especially in this century of ours?

And so we must be consoled by the thought that mortal man cannot wish for much more than to be remembered, not only as a pioneer in the annals of the culture of his day, but also with love, affection, and gratitude in the hearts of his family, friends, and colleagues.

References

[1] L. V. Ahlfors, *Complex analysis, an introduction to the theory of analytic functions of one complex variable*, McGraw-Hill, New York, 1953, 1966, 1979.

[2] ____ , *Conformal invariants: Topics in geometric function theory*, McGraw-Hill, New York, 1973.

[3] ____ , *Lectures on quasiconformal mappings*, manuscript prepared with the assistance of Clifford J. Earle Jr., Van Nostrand, Princeton, NJ, 1966.

[4] ____ , *Lars Valerian Ahlfors: Collected papers*, in two volumes (Rae Michael Shortt, ed.), Birkhäuser, Boston, 1982.

[5] H. F. Baker, *Abel's theorem and the allied theory, including the theory of the theta functions*, Cambridge Univ. Press, Cambridge, 1897.

[6] C. Carathéodory, *Theory of functions of a complex variable*, in two volumes (translated by F. Steinhardt), Chelsea, New York, 1954.

[7] *Commentationes in honorem Lars V. Ahlfors*, LXXX Annos NATO, Ann. Acad. Sci. Fenn. Ser. A I Math. Dissertationes **13** (1988).

The Sky Is Falling

Solomon A. Garfunkel and Gail S. Young

The reason for writing this piece is our belief that our profession is in desperate trouble—immediate and present danger. The absolute numbers and the trends are clear. If something is not done soon, we will see mathematics department faculties decimated and an already dismal job market completely collapse. Simply put, we are losing our students.

In 1989, concerned by dropping undergraduate enrollment in mathematics courses during a period when the use of mathematics in other fields had grown dramatically, we conducted a new type of study. This study, supported by the Exxon Education Foundation, looked at the enrollment in mathematics courses above the level of calculus being offered in departments other than mathematics. To avoid possible confusion, we chose to exclude those courses taught in departments of statistics.

We published a pamphlet, "Math Outside of Math" [1], with our findings and sent a copy to every mathematics department chair. The *Notices* [2] published an article containing the major results of the survey. Briefly, we found that enrollment in advanced mathematics courses offered in non-math departments in the academic year 1988–89 was 173,200 (plus or minus 28,500). The figure for enrollment in mathematics department advanced courses for 1990, as given in the 1990 Conference Board of Mathematical Sciences (CBMS) Survey of Mathematics [3], was 119,000. This figure, however, covers just the fall semester.

Solomon A. Garfunkel is executive director of COMAP Inc., Lexington, MA. His e-mail address is `sol@mail.comap.com`.

Gail S. Young is a former president of the Mathematical Association of America. He lives in Cortland Manor, NY. His e-mail address is `youngg@mindspring.com`.

In addition to the raw data in the quantitative part of our survey, we asked the responding departments why they in fact offered what were essentially advanced mathematics courses. A summary of their responses was included in the pamphlet and in the *Notices* article. We note here that three of the major reasons given were that some of the material they wished to cover was not offered in their math department; that some of the material they wanted was indeed offered in the math department, but in too many courses to fit their majors' schedules; and that our courses had no useful applications in their field.

The penultimate paragraph of our report is worth repeating: "We are anxious to get the reactions of the broader mathematical community. Towards that end we have written to chairs and put an article in the *Notices* summarizing our findings in order to begin a public debate...." The *Chronicle of Higher Education* wrote an article about our study before its publication. In two weeks we received some 250 letters—none from mathematicians. After the publication of the *Notices* article, we received nine letters from mathematicians, only two of whom expressed alarm (the other seven basically said, so what?). We began to feel like Cassandra or, more to the point, Chicken Little.

But now with the publication of the 1995 CBMS Survey [4], it is clear that the sky is actually falling. The survey divides undergraduate mathematics classes into four categories: Remedial, Pre-Calculus, Calculus, and Advanced. Here are the figures for 1985, 1990, and 1995 in thousands for four-year colleges and universities. For an excellent summary of the data, see [5].

	1985	1990	1995	% Change 1990–1995	% Change 1985–1995
Remedial	251	261	222	-15	-12
Pre-Calculus	593	592	613	4	4
Calculus	637	647	539	-17	-15
Advanced	138	119	96	-19	-30

While we do not claim to know the reasons for each of these declines and it is likely that they are quite different for the different course groupings, the net result has strong implications for the academic job market. Even if we leave out the Remedial courses, totaling the remaining three groups, we have a drop of 17 percent in the past five years—130,000 students. Even with an overestimate of average class size of 40 students, that means a loss of 3,250 sections. With a teaching load of four courses, we just lost over 800 jobs. See [6] for actual trends in faculty composition data.

It is important to emphasize what we know and what we need to learn more about. Our original study of advanced enrollment was to our knowledge a base-line study. We documented what was (to us) a surprisingly large undergraduate enrollment in advanced mathematics courses in non-math departments. We now note a further decline in advanced mathematics enrollment, as documented by CBMS. It is conceivable that this falling enrollment is due to some other factors such as a drop in total enrollment in advanced quantitative work. However, given our fears and the draconian nature of the recent CBMS numbers, we are committed to undertaking a Math Outside of Math II study to see if in fact the decline in advanced enrollment within mathematics departments is matched by an increase in enrollment outside of math departments. Based on our prior experience and anecdotal data, we firmly believe this to be the case. But whatever the reasons, one fact stands out—in the ten-year interval from 1985–95 we have lost 30 percent of our enrollment in advanced mathematics—30 percent!

We have more numbers, more charts. But we want to stop here to reiterate our main points. The 1995 CBMS report shows a marked decrease in enrollment in mathematics courses at four-year colleges and universities. It clearly shows that calculus and especially advanced enrollment are in sharp decline. We believe that this decline is an extension of a trend we pointed out in 1990, namely, that students are increasingly taking their advanced mathematical training in non-mathematics departments. These trends taken together are costing jobs and, we fear, eventually whole departments.

We would like to be wrong. There is very little pleasure (well, maybe a wee bit) in saying we told you so. What is important is the future of undergraduate mathematics instruction—for our students and ourselves. We have to take a long look at what is actually happening. Where and how is mathematics being taught at our nations' colleges? We are especially concerned, as this is yet another example of a phenomenon which may handicap new faculty, whom we are training for a world which may no longer exist. We have to stop eating our young!

We would like to close with one more thought. This is not an issue of left or right, of reform or status quo. This is an issue of survival. We are losing students; we are losing faculty. We need to understand why and, if possible, find ways to reverse these trends. If we decide not to or we are not capable of doing so, then we must plan for the new reality and train our graduates accordingly. To quote from the last line of Geoff Davis's recent *Notices* article [7]: "The future of the profession and the next generation of mathematicians depend upon it."

References

[1] *Math Outside of Math: A Study of Mathematics Enrollments in Non-Mathematics Departments*, COMAP, 1990.

[2] Solomon A. Garfunkel and Gail S. Young, *Mathematics Outside of Mathematics Departments*, Notices Amer. Math. Soc. **37** (April 1990).

[3] *Statistical Abstract of Undergraduate Programs in the Mathematical Sciences in the United States*, Fall 1990 CBMS Survey, Math. Assoc. Amer., 1991.

[4] *Statistical Abstract of Undergraduate Programs in the Mathematical Sciences in the United States*, Fall 1995 CBMS Survey, Math. Assoc. Amer., 1997.

[5] Donald C. Rung, *A Survey of Four-Year and University Mathematics in Fall 1995: A Hiatus in Both Enrollment and Faculty Increases*, Notices Amer. Math. Soc. **44** (September 1997).

[6] James W. Maxwell, *Changes in Mathematics Faculty Composition, Fall 1990 to Fall 1996*, Notices Amer. Math. Soc. **44** (November 1997).

[7] Geoff Davis, *Mathematicians and the Market*, Notices Amer. Math. Soc. **44** (November 1997).

Engineering Accreditation Board Issues New Criteria

The Accreditation Board for Engineering Technology (ABET) has issued a new set of criteria, Engineering Criteria 2000. The new criteria are being published in January 1998 and phased in over three years, starting in the 1998–99 accreditation cycle. This change has raised concerns about the effect the new criteria could have on mathematics departments.

Established in 1932, ABET is overseen by twenty-two member organizations, including most of the major professional engineering societies in the country. The purpose of ABET accreditation is to identify engineering, engineering technology, and engineering-related degree programs that meet a certain standard of quality. The ABET accreditation process is quite extensive, comprising site visits, interviews with faculty and administrators, and reviews of documentation about the programs. Many mathematicians are familiar with ABET, having been asked by their administrations to supply documentation—some of it very detailed—about the mathematics courses engineering students are required to take. Accreditation is generally granted for six years.

There are 1,430 ABET-accredited degree programs in the U.S. While participation in ABET accreditation is voluntary, most schools consider it a necessity for their engineering programs. As a result, ABET wields considerable influence. University and college administrations take seriously the possibility of withdrawal of ABET accreditation and are often prepared to supply resources to insure that the ABET criteria are fulfilled.

In 1996 ABET undertook a revision of its criteria, which had been in place, with some revisions, since the founding of the organization. The most important change in the new criteria is a shift away from prescribing what engineering programs should be like and a shift toward examining the outcomes of engineering education. The old criteria set standards for things like faculty size, curricular requirements, and the quality of laboratories and other facilities. The new criteria focus far less on such operational details and look instead for the educational objectives of engineering programs and evaluation procedures to track whether objectives are met. For example, one of the new criteria says that engineering programs must demonstrate that their graduates have "an ability to apply knowledge of mathematics, science, and engineering." Evaluation procedures might include asking program graduates and their employers about whether this aim has been achieved.

When it comes to the study of mathematics, neither the old nor the new ABET criteria is very specific. The old criteria had a section devoted to standards for curricular content, which specified that engineering students had to study differential and integral calculus and differential equations. "Encouraged" but not required were probability and statistics, linear algebra, numerical analysis, and advanced calculus. The new criteria are even less specific and do not mention particular mathematics topics, saying only that engineering programs should have "one year of a combination of college level mathematics and basic sciences." ("One year" means a full year's course load, which in a semester system would be 32 semester hours in a 128-semester-hour curriculum.)

Some have been under the impression that at one time ABET required that mathematics courses be taught in mathematics departments and that the new ABET criteria eliminated this requirement. However, according to David Kaufman, the director of the ABET Engineering Accreditation Commission, the ABET criteria never contained any such requirement.

Many mathematics departments have watched with growing concern as engineering schools attempt to shift the teaching of some mathematics courses to engineering departments. A combination of factors underlies this trend, including decreasing engineering enrollment, which leaves engineering departments with more faculty than they need to cover their courses, and dissatisfaction with instruction in mathematics departments. Many mathematicians worry that the new ABET criteria, being so much more vague and flexible than the old ones, could stimulate this trend. Whether such worries are justified is difficult to evaluate. There was nothing in the old criteria to prevent engineering departments from teaching their own mathematics courses, and there is nothing in the new criteria to encourage them in this direction. Kaufman believes that the new criteria will have little influence one way or the other on this dynamic. He predicts that engineering programs will continue to need mathematics departments to teach mathematics courses. Weak ties between a mathematics and engineering department would not be an asset for an engineering department undergoing ABET accreditation review, he noted. In addition, Kaufman points out that one aspect implicit in the new ABET criteria is an interdisciplinary emphasis, which would encourage interactions between mathematics and engineering.

The new ABET criteria have aroused concern not only among mathematicians but among engineers as well. In the past ABET's provision of minimum curricular requirements for an engineering program served as a guide, and some feel betrayed by the withdrawal of this guidance. Kaufman says that ABET would like to encourage more diversity, leaving it up to individual programs to set their own goals. One of ABET's main functions will then be to see whether programs have mechanisms in place to evaluate whether they are achieving their goals. Kaufman does not anticipate many changes in the kind of information that mathematics departments will be asked to provide for the ABET accreditation process under the new criteria.

The AMS Committee on Education has reviewed the new ABET criteria and will continue to monitor possible impacts on mathematics departments. The new and old criteria are available on the ABET Web site, `http://www.abet.ba.md.us/`.

—Allyn Jackson

Book Review

The End of Science: Facing the Limits of Knowledge in the Twilight of the Scientific Age

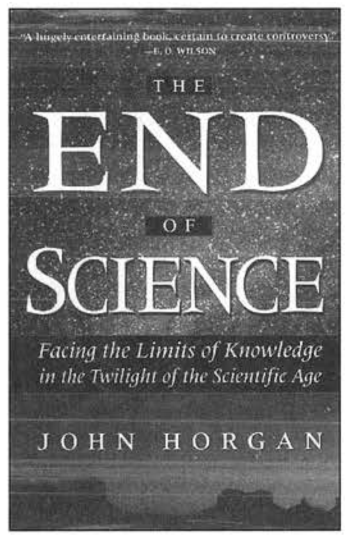
Reviewed by David Hoffman

The End of Science: Facing the Limits of Knowledge in the Twilight of the Scientific Age
John Horgan
Helix Books
Addison Wesley, 1996

Four years ago John Horgan, then a senior staff writer for *Scientific American,* published in that journal "The Death of Proof", a piece that claimed that "video proofs" were transforming mathematics. Among other things, "The Death of Proof" suggested that Wiles's announced resolution of Fermat's Last Theorem might be a "splendid anachronism", one of the last great formal proofs. As it turns out, Horgan's ten-page mathematics-as-we-knew-it-is-over article was merely a finger exercise for a full-scale requiem for science—the book under review here.

The bulk of *The End of Science* consists of a series of interviews, some originally done for *Scientific American,* linked together by the thesis that science as we know it is coming to an end because it is close to achieving its goal, explaining nature. Horgan credits his agent for turning an amorphous idea into a marketable proposal. The agent is pretty good. The book has been widely publicized, and Horgan has appeared on the high-toned TV and radio talk shows. Short pieces based on *The End of Science* have appeared in magazines and newspapers, including the *New York Times* and the *International Herald Tribune.*

Horgan writes in a sly and entertaining manner, often taking the imagined reader's side against scientists whom he portrays as blinded by their own success and ambition. There is little serious explanation of scientific ideas in the book. Nonetheless, it must be taken seriously. Writers like Horgan and books like *The End of Science* influence the public image of science.



I should say here that I was interviewed for Horgan's "Death of Proof" article and felt misled by him. Because of this I was not favorably disposed toward this book before I read it. I liked it less afterward. But more disturbing to me is the fact that a former colleague who read an article by Horgan—extracted from *The End of Science* and published in *Technology Review*—thought it profound. I think it is profoundly wrong. My main focus in this review will be on the aspects of *The End of Science* that I found most troubling: a clear antipathy toward mathematical thinking and a fundamental misunderstanding of the uses of mathematics in science. Books like this cloud the

David Hoffman is associate director for external collaboration at the Mathematical Sciences Research Institute and senior scientist in the Visualization Group of the National Energy Research Supercomputing Center of the Lawrence Berkeley National Laboratory. His e-mail address is `david@msri.org`.

public understanding of what we do, to our detriment.

Is Science PM?

To support his thesis that science is reaching its limit, Horgan concocts the notion of "ironic science", which he has explained in the following way.

> Some of the most prominent scientists in the world traffic in hypotheses that are remarkably postmodern in character. I like to call this type of theorizing ironic science. The concept of irony is central to that wellspring of postmodernism, literary criticism....The job of a literary critic is....not to pin down the true meaning of the text—an impossible task—but to invent new meanings, ones that challenge received wisdom and provoke further dialogue. Similarly, ironic science advances hypotheses that, while often profound, should not be considered literally true. My favorite example of ironic science is superstring theory....[1]

So-called ironic science is blooming, according to Horgan, because the job of science is largely done. Its theories (quantum mechanics and general relativity, molecular biology and evolution) give a framework that will last for thousands of years. They explain the explainable, Horgan tells us. To him they are true. We are approaching the limits of knowledge; further theorizing will only lead to speculation that cannot be verified or falsified by experiment. While sometimes beautiful, such intellectual activity has more in common with *The Inferno* of Dante than with the *De Revolutionibus* of Copernicus. Strong scientists, those not content with working out the mere consequences of the now-and-future canon, must become practitioners of ironic science. Or do something else.

I have a problem with the word *invent* as it is used here. Horgan not only misrepresents the scientific enterprise, he also does a disservice to serious literary theorists by confusing fabrication with invention. Surely serious critics of any school do not make up meanings; they seek them in the text and in the way the text is used in social interaction. That is, they *invent* them in the classical sense of the word; they do not fabricate them as Horgan implies.

Horgan is confused not only about what he believes scientists are doing but, as a self-styled postmodern critic of science, about what he himself is doing. Can it be that a critic of any sort would draw conclusions without reading the text, no matter what else is considered important? Horgan draws sweeping conclusions about scientific theory that he does not pretend to understand by merely interviewing scientists. When he does not like their answers, he refutes them with derisive personal comments, the gist of which is that scientists, like authors, are not to be trusted when explaining their own work.

Horgan's main argument to support the contention that current scientific theories are the final theories goes something like: Because they are true. Horgan presents other lines of reasoning that contradict one another. On the one hand, the lack of fundamental advances in some scientific areas is taken as evidence that these fields are close to their intellectual conclusion. On the other, the proliferation of theoretical advances in some areas of science in the last two decades is not an indication that these sciences are thriving. Rather, Horgan interprets this as evidence that these disciplines are rapidly completing their tasks in a flurry of activity that marks the end of their life cycle; they are approaching the "limits of knowledge in the twilight of the scientific age." Science cannot continue past this limit. He wants to have it both ways.

These dubious arguments are topped by what Horgan calls "the sun principle". For example, generalizing from the statement that "No one really knows what causes sunspots," Horgan concocts this extrapolation: "Our ability to describe the universe with simple, elegant models stems in large part from our lack of data, our ignorance. The more clearly we can see the universe in all its glorious detail, the more difficult it will be for us to explain with a simple theory how it came to be that way." I find this simplistic and wrong. For me, the recent visual constructions made from Hubble-telescope data evoke a sense of wonder and the faith that the eventual understanding of what these images mean will fundamentally alter our view of the history of the cosmos. Horgan ignores the complex interaction among theory, experiment, and instrument building which characterizes modern science, in particular the regularity with which acquisition of fundamentally new data challenges the very theory that initiated the search for it.

Body Blows

But Horgan has other, indirect ways to build up his case; he substitutes personalities and gossip for ideas and analysis. The thesis of the book requires the assumption that "Science" is a well-defined, generally understood entity. To explain in any detail even one scientific theory, let alone argue that all theories are part of a single endeavor, is beyond the scope of Horgan's book. Horgan solves this problem by replacing "Science" by "scientists". Many of the scientists in the book, especially ones that Horgan does not like, are presented as weird

[1] *Quoted from* Postmodern irony may be profound, but it's strange science, *International Herald Tribune, July 17, 1996.*

people, put out that their privileged place in the world is being challenged. Their understandable annoyance with Horgan is reported as defensiveness and interpreted as arrogance. Horgan loves to play the role of the exposer, the clear-seeing child in his private version of *The Emperor's New Clothes.* But he is no innocent. The book is filled with mean-spirited remarks about clothing, grooming, skin texture, thick accents, and other personal characteristics. Condescending and occasionally crossing the border into downright nastiness, these asides set the tone of the book.

Here are a few examples.

- About the evolutionary biologist Richard Dawkins:

> He is an icily handsome man, with predatory eyes, a knife-thin nose, and incongruously rosy cheeks. He wore what appeared to be an expensive, custom-made suit. When he held out his finely veined hands to make a point, they quivered slightly. It was the tremor not of a nervous man, but of a finely tuned high-performance competitor in the war of ideas: Darwin's greyhound.

This is mild; Horgan actually admires Dawkins.

- About the mathematician Mitchell Feigenbaum:

> When amused, Feigenbaum did not smile so much as grimace....his already protuberant eyes bulged still farther from their sockets, and his lips peeled back to expose twin rows of brown, piglike teeth stained by countless cigarettes and espressos (both of which he consumed during our meeting). His vocal cords, cured by decades of exposure to these toxins, yielded a voice as rich and resonant as a basso profundo's and a deep, villainous snicker.

- About the physicist Andrei Linde:

> For someone so publicly playful and inventive, Linde can be surprisingly dour....When I arrived at the gray, cubist house they were renting, Linde gave me a perfunctory tour. In the backyard, we encountered Kallosh, who was rooting happily in a flower bed. 'Look, Andrei,' she cried, pointing to a nest filled with cheeping birds on a tree branch above her. Linde, his pallor and squint betraying his unfamiliarity with sunlight, merely nodded. When I asked if he found California relaxing, he muttered, 'Maybe too relaxing.' As Linde recounted his life story, it became apparent that anxiety, even depression, had played a significant role in motivating him.

- About the physicist David Bohm:

> His skin was alarmingly pale, especially in contrast to his purplish lips and dark wiry hair. His frame, sinking into a large armchair, seemed limp and languorous, but at the same time suffused with nervous energy. He cupped one hand over the top of his head; the other rested on the armrest. His fingers, long and blue veined, with tapered yellow nails, were splayed. He was recovering, he told me, from a recent heart attack.

The tone of these remarks serves a thematic purpose, namely, to convey the sense that the heroic age of science is over, its great lights dead or dying. What few new heroes there are do not belong in the same class as their predecessors. George Andrews has called Horgan the self-styled "Jack Kevorkian of Science". To me, this refers to more than the Horganian theme of the twilight of the scientific age. I do not mean to challenge the view of Kevorkian as a humanitarian who, by facilitating the suicide of those with painful and terminal illness, eases their transition to finality. Rather, I want to call attention to the recurring motif of morbidity in Horgan's interviews. The bodies of scientists are offered up in place of a body of scientific knowledge. Age and decay become evidence for the theme that we are entering the twilight of the scientific age.

Given the people Horgan interviews, it should be expected that some of them would express regret, even sadness, at coming to the end of a career without having understood, to their satisfaction, important aspects of the scientific problem that motivated them in their youth. Disappointment expressed by scientists at their own human limitations—that the problem was more difficult than expected or that they will not live to see a complete resolution—is interpreted as a comment on the limits of science itself. Missing an opportunity to explore and present in a sympathetic manner human stories of how personality, theory, and career interact in the course of the lives of some very interesting people, Horgan misrepresents these personal remarks as statements about the state of science itself. Worse, in some of the interviews, scientists are engaged in conversations that evidently have nothing to do with the notion of the "end of science". Horgan writes it up in a way that makes his case; the scientists have no opportunity to respond to the picture Horgan is painting.

The Death of Proof

This questionable journalistic technique was already in use in the preparation of the "Death of

Proof" article. When I was interviewed for that one, Horgan did not mention that the article was to be about the demise of traditional proofs and the rise of "video proofs". Rather, I was led to believe that it was a survey of the various uses of computers in mathematics. About six weeks before the article was published, Horgan phoned me, ostensibly to clarify some technical matter in his article. He soon changed the topic of conversation, asking me what I thought about the notion of a "video proof". At first I had no idea what he was talking about. He said he was referring to the notion that animations were replacing traditional proofs in avant-garde mathematics. I asked him where he had gotten such an idea. He would not say. I asked if this had anything to do with the article, and he changed the subject. Only when the "The Death of Proof" appeared in print did I have a clear idea of what the article was about.

Some of the mathematicians discussed in the article claimed that they were quoted out of context, and many letters of protest were written. The article angered me, and in October 1993 I wrote a letter to the editor of *Scientific American*, which began

> "The Death of Proof" by John Horgan defies logic and accuracy in favor of controversy and sensationalism. The juxtaposition of Wiles' announced proof of Fermat's Last Theorem (wrongly referred to as an "anachronism") with the video *Not Knot*, made to explain and illustrate ideas in geometry (incomprehensibly called "a video proof") is silly. However, when the two are interpreted as landmarks in the downfall of rigorous mathematics, it is idiotic. I am very much in favor of the use of computer graphics in mathematical research and communication, but is there one respectable mathematician willing to explain and defend the notion of video proofs as a replacement for traditional mathematical methods?

Horgan was stung by the torrent of howls and complaints he received from mathematicians. We were the group of scientists, he said, who whined most about not getting enough press coverage. But when we got it, we complained that it was wrong or inaccurate. He expressed particular disappointment with me personally. I had let him down. He thought I was "with it". When I asked "With what?" he did not answer. I guess now I know what he meant.

What Really Gets Horgan's Goat

Horgan's favorite example of ironic science is superstring theory, which in his mind

> ... for the last 15 years has represented the cutting edge of physics. Sometimes called a "theory of everything," it posits that all the matter and energy in the universe, and even space and time, stem from infinitesimal loops of ur-stuff writhing in a hyperspace of 10 (or more) dimensions.

For Horgan the absurdly high-dimensional space in which superstrings live is only slightly less ridiculous than their small size: "The tiny domain that superstrings supposedly inhabit is even less accessible than the quasars haunting the edge of the universe. A superstring is to a proton in size as a proton is to the solar system."[2]

Edward Witten is Horgan's "leading practitioner" of string theory, "the most spectacular practitioner of naive ironic science" that Horgan has ever encountered. According to Horgan, Witten "believes in his speculations, even though they have not been empirically verified," and he is like other naive scientists who "believe they do not invent their theories so much as they discover them...." Witten"—like a Texan who thinks that everyone but Texans has an accent—does not acknowledge that he or she [a scientist] has taken any philosophical stance at all." Horgan thinks that Witten is the sort of scientist who believes that he is "just a conduit through which truths pass from Platonic realm to the world."

What does this have to do with the content of string theory? Nothing. A clue to why Horgan may think it is relevant can be found in the article, based on *The End of Science*, which appeared in the *International Herald Tribune*.[3] In it Horgan mentions the paper by the physicist Alan Sokal which was published in the journal *Social Text*. Sokal held up superstring theory as a breakthrough, one that would free science from its dependence on the fictional notion of objectivity. After its publication Sokal announced that his paper was in fact a parody of current so-called postmodern writing about science. But according to Horgan: "superstring theory is exactly the kind of science that subverts conventional notions of truth...it is highly unlikely that we will know whether superstring theory is true. That is what makes it ironic."[4]

In fact, in Horgan's view of things it is mathematics that "subverts conventional notions of truth." When Witten, apparently none too happy about having agreed to be interviewed by Horgan, suggests that he should write profiles (presumably for *Scientific American*) of five mathematicians, Horgan's response is not to Witten but to us: "Wit-

[2] *International Herald Tribune*, Postmodern irony, *July 17, 1996.*

[3] *Ibid.*

[4] *Ibid.*

ten did not realize that he was providing fodder for those who claimed he was less a physicist than a mathematician." When one of Witten's colleagues described him to Horgan as "possessing the greatest mathematical mind since Newton," it is miscast by Horgan to mean that Witten is a mathematician and therefore not a real physicist. Continuing with this theme, Horgan writes: "In the late 1980s Witten created a technique—which borrowed from both topology and quantum field theory—that allows mathematicians to uncover deep symmetries.... As a result of his finding, Witten won the 1990 Fields Medal, the most prestigious prize in *mathematics*" (emphasis in the original). The italics emphasize that Witten's work is mathematics, i.e., not really physics, not really science at all.

Witten tells Horgan that the development of the sort of mathematics that will allow increased understanding of string theory will also lead to the construction of experiments to test and refine it. Horgan does not appear to understand what Witten is trying to say, and Witten's exasperation with Horgan is evident: "I don't think I've succeeded in conveying to you its (string theory's) wonder, its incredible consistency, remarkable elegance, and beauty." This is interpreted by Horgan as a claim that "superstring theory is too beautiful to be wrong." "I asked Witten how he responded to claims of his critics that superstring theory is not really physics at all," Horgan continues. "Witten replied that it predicted gravity." Since, to Horgan, gravity needs no prediction—it has been discovered and verified, so it is true—this response is taken by Horgan as further indication of the occult nature of string theory. Horgan certainly knows better, but he refuses to relent on this point.

The high dimension and the small scale of superstring theory make it a priori ironic to Horgan. In his mind it has no connection with reality—it is unable to serve any real purpose in a true, nonironic scientific endeavor. It is not testable and does not further our understanding of what the world and the universe are really like. Horgan misses the point that the theory is not one of an infinite number of theoretical explanations, that there is nothing arbitrary about its formulation. Is it less problematic to assume that every physical system is associated to an infinite-dimensional, normed vector space whose unit-length vectors represent its states? Of course, the difference is that quantum mechanics provides a theory that is testable on a small scale and can give numerical predictions in agreement with the most delicate experiments. While no significant experiments have been done to test string theory, this does not mean that no such experiment is possible, as Horgan assumes. In fact, there are plans to attempt to detect supersymmetry at the Large Hadron Collider at CERN, a facility that is expected to be operational in a decade. Supersymmetry, a symmetry between bosons and fermions, is one of the main predictions of string theory. I do not know if the plans for this experiment were known to Horgan at the time of his interview of Witten or at the time of publication of *The End of Science*.

Horgan furthers his case that superstring theory has little connection with reality by trying to portray Witten in the same vein. First he sets up Witten as "the smartest physicist of them all," the Albert Einstein of the end of the twentieth century, and he makes it quite clear that he is interviewing Witten at the Institute for Advanced Study. But instead of a shaggy-haired icon with a charming accent and a violin, rivaling Charlie Chaplin in popularity, we have presented to us by Horgan a mean-spirited caricature of a man: one who talks in a "highly abstract, impersonal mode of speech"; who "paused frequently—for 51 seconds at one point—casting his eyes down and squeezing his lips together like a bashful teenager"; and who "now and then—for no reason I could discern—broke into convulsive, hiccupping laughter as some private joke flitted through his consciousness." This portrayal is meant to convey, by association, a fundamental difference between general relativity and superstring theory. On one part of Long Island, where I grew up in the fifties, it was believed that only four people in the world understood relativity theory—one for each dimension of space-time, I suppose—but this did not make it suspect, rather the opposite. Science extended reality and instilled wonder, not ridicule. Horgan's caricature of superstring theory casts it as a fabrication devoid of meaning. That it seems to predict gravity, as Witten tries to emphasize, makes no sense at all to Horgan—gravity is already explained. Witten has already slipped up and admitted that he is a mathematician, that he is not dealing with reality at all, just speculation and austere abstraction whose meaning has nothing to do with physics and can never be verified or disproved.

Revolution

> Meeting a friend in a corridor, Wittgenstein said: "Tell me, why do people always say that it was *natural* for men to assume that the sun went around the earth rather than that the earth was rotating?" His friend said, "Well, obviously, because it just *looks* as if the sun is going around the earth." To which the philosopher replied, "Well, what would it have looked like if it had looked as if the earth was rotating?"

—from Jumpers[5], *a play by Tom Stoppard.*

[5]*Grove Press, Evergreen Edition, New York, 1974, p. 75.*

Horgan's objections to string theory and to Witten are based on a fundamental misunderstanding of the relationship between mathematics and physical theory. "Let's give superstring believers the benefit of the doubt, if only for a moment," he says.

> Let's assume that some future Witten, or even Witten himself, finds an infinitely pliable geometry that accurately describes the behavior of all known forces and particles. In what sense will such a theory explain the world? I have talked to many physicists about superstrings, and none has been able to help me understand what, exactly, a superstring *is*: it is some kind of mathematical ur-stuff that generates matter and energy and space and time but does not itself correspond to anything in our world....The true meaning of superstring theory, of course, is embedded in the theory's austere mathematics. I once heard a professor of literature liken James Joyce's gobbledygookian tome *Finnegan's Wake* to the gargoyles atop the Cathedral of Notre Dame, built solely for God's amusement. I suspect that if Witten ever finds the theory he so desires, only he—and God—will appreciate its beauty.

Let's leave aside the fact that Horgan has to drag in James Joyce, the Cathedral of Notre Dame, and some unnamed—probably misquoted—teacher of his in order to give a scholarly patina to his snide equating of string theory with gobblydegook. Let's ignore his short count of people able to appreciate string theory: one—or two, if you count God—down from the fabled four who understood general relativity according to fifties' lore on Long Island. The real problem is not this nonsense. It is something more fundamental. The Nobel Prize-winning chemist Hans Krebs once wrote that "Those ignorant of the historical development of science are not likely ever to understand fully the nature of science and scientific research."[6] Horgan has no comprehension of the way in which a new and radical theory—at first defying common sense, and often mathematical—can become accepted as physical reality, usually after a period of rejection and incredulity. There are many examples of this phenomenon in the history of science, and I will briefly discuss two of them.

The most famous example is Newton's postulation of gravity without any attempt to describe a means for its transmission. This was considered by many of Newton's contemporaries, Leibniz included, as unacceptable. The attribution of "occult qualities" to matter was considered a serious mistake that threatened to destroy faith in not only natural philosophy, which seeks reasons, but also in divine wisdom, which provides them.[7]

The second example, and in my opinion the clearest one, was the slow acceptance of the motion of the earth. Published in 1543, the *De Revolutionibus* of Copernicus was accepted as the greatest astronomical work since Ptolemy. But according to Thomas Kuhn, "the success of the *De Revolutionibus* does not imply the success of its central thesis, namely that the earth was rotating and moving around the sun. The faith of most astronomers in the earth's stability was at first unshaken. Authors who applauded Copernicus' erudition, borrowed his diagrams, or quoted his determination of the distance from the earth to the moon, usually either ignored the earth's motion or dismissed it as absurd."[8] This was the case even though there were some astronomical computations that were improvements over values derived from Ptolemaic principles.[9]

The motion of the earth was not accepted until well into the seventeenth century. Apart from the religious and scriptural objections, "the debate about the earth's motion became bitter and intense. The earth's motion, it was said, violated the first dictate of common sense; it conflicts with long-established laws of motion."[10] It was generally felt that the desire to make the motion of the stars seem simpler was insignificant reason for positing the rotation of the earth and its revolution around the sun. It was considered a convenient geometric device, with no substantive basis in

[6]*Hans A. Krebs,* The history of the tricarboxylic acid cycle, *Perspectives in Biology and Medicine vol. 14, 1970, pp. 154–170.*

[7]*For a fascinating discussion of conflicting views on the meaning of gravity among Newton's contemporaries, see* Newtonian Studies, *Alexandre Koyré, Harvard University Press, Cambridge MA, 1965: in particular, Appendix B to Chapter III.*

[8]*Thomas Kuhn,* The Copernican Revolution: Planetary Astronomy in the Development of Western Thought, *Random House, New York, 1957. Kuhn is skewered by Horgan in his discussion of his more famous book,* The Structure of Scientific Revolutions. *Horgan tells us that "according to literary theory, Kuhn himself cannot be trusted to provide a definitive account of his own work." Horgan, of course, can. Shortly before his death, Kuhn remarked that he had more in common with those who strongly disagreed with* The Structure of Scientific Revolutions *than with those who believed with Horgan that it was a "seminal postmodern text".*

[9]*But things were not all that simple. There was not an across-the-board improvement, and some of the predictions derived using the Copernican "fiction" that the earth was in motion around a fixed sun were somewhat less accurate than those derived from Ptolemaic theory. The computations, however, were easier.*

[10]*Kuhn, op. cit.*

physical reality. After a long time—and after Kepler and Galileo—it finally became accepted that the solid earth could move around the sun in reality, not just in mathematics. And it is worth noting that the acceptance of the motion of the earth required really serious mathematical work, both pure and applied.

A great triumph of Newton's *Mathematical Principles of Natural Philosophy* (the *Principia*), published in 1686, was the derivation of Kepler's Laws, by means of the newly developed calculus, from the assumption of the inverse-square law for gravitational attraction. So the motion of the earth and planets comes to be explained by a force that can also be used to explain why ripe apples fall to the ground. But what explains or predicts gravity? How can masses act on each other at a distance (through what medium?), and how can they influence each other in a manner that appears to be instantaneous? These are good questions that could not be answered in the eighteenth century. Newton's "system of the world" was accepted without answers to these questions. But the questions did not go away; they are fundamental questions.

The End

Near the beginning of *The End* Horgan anticipated the fact that he would be branded as pathetically shortsighted. In order to deflect this criticism he retells the well-known story of a nineteenth-century commissioner of patents who suggested that the patent office should be closed because there would soon be nothing left to invent. This story is apocryphal, Horgan claims, and has very little basis in historical reality. Fair enough. But just wait a hundred years or so. People who write about, think about, and do science and mathematics at the end of the twenty-first century will not have to dredge up half-true stories of patent commissioners to point out the shortsightedness of their post-postmodern critics. They will have Horgan's *End* to kick around.

A Research Experience for Undergraduates

Shelly Harvey and Andrea Ritter

Every undergraduate interested in graduate school should consider attending a Research Experience for Undergraduates (REU) program. The National Science Foundation yearly sponsors approximately twenty REU programs in mathematics that are held at various universities around the country during the summer. The programs range from eight to ten weeks in length, admit anywhere from six to twelve students, and include topics ranging from algebraic geometry and computational group theory to population dynamics and topology. These programs allow an undergraduate to closely work with a faculty member on some component of the faculty's research. These situations allow students to experience graduate study firsthand very early in their careers.

The authors attended Cal Poly San Luis Obispo, a predominately teaching university. As a result of our REU experiences, we both applied to graduate programs for the fall of 1997 and each of us is now in her first year of graduate studies. Andrea is working on her Ph.D. at the University of North Carolina at Chapel Hill, Shelly is at Rice University in Houston, Texas. Ironically, we both discovered our enjoyment of mathematics during our sophomore proof-oriented mathematics course. Moreover, we were both interested in furthering our education in mathematics. With encouragement from various faculty members, we decided to explore the REU programs. What follows are our personal accounts of life in an REU program.

Andrea:

I started at Cal Poly as an architectural engineering major with a minor in mathematics, but somehow I wound up pursuing mathematics full-time. To help pay for college, I worked as a research analyst for a major trade association. My immediate supervisor was the company statistician, and thus my curiosity in statistics was aroused. After two years I had a strong interest in continuing with statistics, but was undecided as to whether I wished to pursue a graduate degree. A good friend on the Cal Poly mathematics faculty suggested attending an REU and handed me a list of programs. I chose to apply to several programs in statistics and finally wound up at the University of South Alabama in Mobile. My family and friends were against it. They all said the same thing: "Alabama is so hot and sticky. The bugs are humongous there, you know. Don't do it. It's not worth it." Everyone told me I was crazy to leave the relative comfort of the central coast of California for the heat of the Gulf Coast, but it was well worth it.

When I arrived in Mobile, I was immediately assigned a project. My adviser was a sociology professor whose interest was in the social upheaval following a technological disaster, particularly the *Exxon Valdez* oil spill. Hence my project was to design a mathematical model explaining the phenomenon sociologists refer to as The Disaster Syndrome. The work was difficult but interesting. I usually found myself working late into the night as one idea often led to another. I even enjoyed reading the literature, a rare sensation for me.

There were several other students at South—all math or statistics majors working in fields as diverse as economics, psychology, epidemiology, and theoretical statistics. Once a week we got together and discussed our progress, problems we were having, surprising results, and the like, but mostly we were on our own. Weekends were reserved for play; we often got together for a barbecue or a movie. We also had several opportunities to travel around the state. I even made it to the Olympic Games in Atlanta!

Before I arrived I expected to gain only two things: a topic for my senior project, a requirement to graduate from Cal Poly, and an idea of life as a graduate student. The program far exceeded my expectations. I left South with a senior thesis topic, new friends, several recommendations, a possible publication, a clear idea of what research at the graduate level includes, and even a little pocket money. My summer in Alabama allowed me the time to focus and to experience research without

Shelly Harvey is currently working toward a Ph.D. in mathematics at Rice University in Houston, Texas, while Andrea Ritter is working toward a Ph.D. in statistics at the University of North Carolina at Chapel Hill.

extraneous details like classes, tests, and tuition. It convinced me to further my education and helped me find the right path. I would recommend an REU program to anyone with a strong background and interest in any of the mathematical fields, especially ones whose future educational plans are uncertain.

Shelly:

My sophomore year was the first year that I began to understand what mathematics really was about. It was the first time that I realized mathematics wasn't about plugging and chugging, but entailed understanding the foundations of a subject. I decided that I wanted to know more about mathematics. I thought about going on to graduate school. Then one of my mathematics professors told me about a summer program called REU and suggested I try to find out more about it. This sounded intriguing, so I decided to check out the programs.

So there I was, a young female who was on a mission to find out what mathematics was about. At this point I had only taken a few proof-oriented classes and had no clue what the future held. I took a shot at what was probably the best thing I could have ever done and applied to a couple of programs based on the fields of research offered. Within a couple of months I was thrilled to be accepted to the program at Louisiana State University (LSU) in Baton Rouge. Of course, I happily accepted the offer.

Before I arrived in Louisiana I wasn't sure what the experience would be like. I assumed some of the other students and I would read through some material in knot theory and then do some research. After I arrived I found that most of the students were in the same situation as I: we were interested in mathematics and in particular going to graduate school, whatever that meant. I soon found my assumption to be somewhat limited; research was more difficult than I had thought.

The first few weeks were set aside to become familiar with previous work done in the field. Then all of the students picked a question on which they wanted to work. The remaining time we met with our individual mentors and tried to get as much accomplished as was possible in the remaining five weeks. All the while, we gave talks to the rest of the group on our work. At the end of the program everyone was required to write up his or her results in a short paper.

I came away from the program with nothing but positive thoughts. Although the work was difficult, the REU gave me a more realistic look at graduate school. I realized that graduate school was not something that anyone should jump into lightly—it is more than just five extra years of college; it is a major life decision. Moreover, I got to interact with students like me, learn about other fields in mathematics, practice giving talks, and visit new regions of the United States.

When I got back from LSU, I was excited about mathematics and going to graduate school. I dove into mathematics, trying to take as many courses as I could handle. I had a fire in my belly; I wanted to be able to really understand all those articles I had struggled with during the summer. As the year progressed I chose to again apply to REU programs for the following summer. I also advertised the REUs to every student who showed a knack for mathematics.

I spent the next summer at Cornell University in Ithaca, New York. Having gone to the REU program the previous summer, I had an idea of what the upcoming summer would entail. Indeed, the programs were similar, but some aspects were different. At Cornell we had guest speakers give hour-long talks on their fields of interest on Mondays and Wednesdays. Although not every student was interested in every talk, each talk did give us a different perspective of mathematics. I felt that all of the students found an interest in at least one of the areas. We also participated in a weekly jam session. This was a time for the students to practice giving talks in front of an audience and to share the research we had accomplished that week. I found this to be especially beneficial in two respects. First, it allowed me to practice giving talks, not something you get to do every day. Second, it helped me understand that you cannot learn mathematics by reading alone; mathematics is a language that needs to be spoken.

I was particularly lucky that summer. I spent part of the summer investigating different areas of convex geometry. In particular, I read through an abundance of material on sphere packing and geometric crystallography with two other students. However, these readings came to a halt when Jade Vinson (another REU student) and I found a new result. We began writing our result into a paper to be published—"The Voronoi Vectors of a Lattice". Now I was really doing mathematics, not reading it. The fun was just beginning. This was a great experience, a feeling I could not have understood without actually writing the article myself. We worked for about three weeks straight trying to make the paper perfect. The end result: a chance to publish a paper and give a talk at the Mathfest in Seattle, Washington.

I would recommend an REU program to any undergraduate who plans to pursue graduate studies in mathematics. In both of my experiences there were stimulating problems to work on and a cooperative atmosphere with little or no outside distractions from research. This kind of environment is not something that comes up often, even for tenured professors.

Partial List of REUs for Summer 1998

The following is a list of some of the Research Experiences for Undergraduates programs in mathematics that will run in the summer of 1998. This is only a partial list, containing those REU programs funded on multiyear grants; at press time decisions about proposals for REU grants to start in 1998 had not yet been made. For the full list of programs, please consult the Web site of the NSF's Division of Mathematical Sciences, http://www.nsf.gov/mps/dms/reulist.htm, or contact Lloyd Douglas, telephone 703-306-1874, e-mail ldouglas@nsf.gov.

College of William and Mary
Matrix Analysis and Its Applications
David J. Lutzer
djlutz@mail.wm.edu
http://www.math.wm.edu/~lutzer/anncment.html

Cornell University
Analysis on Fractals, Complex Dynamics, Combinatorics
Robert S. Strichartz
reu@math.cornell.edu
http://math.cornell.edu/~michelle/reu.html

Hope College
Algebra, Dynamical Systems, Probability and Number Theory
Tim Pennings
pennings@math.hope.edu
http://www.math.hope.edu/reu.html

University of Illinois at Urbana-Champaign
General Physics with Emphasis on Nonlinear Physics
David Campbell
reu@physics.uiuc.edu
http://www.physics.uiuc.edu/undergrad/REU/REU98/

Indiana University
Algebra, Topology, Analysis, Probability, and Applied Mathematics
Daniel Maki
reu@indiana.edu
http://www.math.indiana.edu/reu/home.html

Michigan Technological University
Probability, Combinatorics, Number Theory, Statistics, Algorithms and Geometry
Anant P. Godbole
anant@mtu.edu
http://math.mtu.edu/

University of Minnesota-Duluth
Discrete Mathematics, Combinatorics and Graph Theory
Joseph A. Gallian
reu@mtholyoke.edu
http://www.d.umn.edu/~jgallian/

University of Missouri-Rolla
Parallel Numerical Computing
Daniel I. Okunbor
okunbor@cs.umr.edu
http://www.cs.umr.edu/nsf/

Oregon State University
Analysis of Algorithms, Geometry, Population Dynamics, and Topology
Dennis J. Garity
reu@math.orst.edu
http://www.orst.edu/~garityd/REU/reuhome.htm

Pennsylvania State University-Erie
Mathematical Biology
J. Carl Panetta
panetta@wagner.bd.psu.edu
http://euler.bd.psu.edu/science/math/REU/index.html

University of Tennessee-Knoxville
Selected Topics in Pure and Applied Math
Suzanne Lenhart
lenhart@math.utk.edu
http://www.math.utk.edu/Docs/reuflyer.html

Trinity University
Dynamical Systems, Algebra and Statistics
Saber Elaydi
selaydi@trinity.edu

Utah State University
Nonlinear Dynamics
Emily Stone
stone@sunfs.math.usu.edu
http://www.physics.usu.edu/reu.html

University of Washington
Inverse Problems
James A. Morrow
morrow@math.washington.edu
http://www.math.washington.edu/~morrow/reu98/reu.html

Williams College
Geometry
Colin Adams
colin.adams@williams.edu
http://www.williams.edu/Mathematics/SMALL.html

Reports of AMS Association Resource Group

The AMSARG is a subcommittee of the AMS Committee on Education, chaired by Roger Howe and charged with representing the AMS to the National Council of Teachers of Mathematics (NCTM) in its revision of the NCTM Standards. Starting in 1989, the NCTM issued three sets of Standards for K–12 mathematics: one set on mathematics curricula, one on assessment, and one on the mathematics teaching profession. The revision will involve updating and refining the existing Standards and blending the three sets into a single document. More background about the work of the AMSARG may be found in the feature article by Roger Howe in this issue of the *Notices*. The questions that the AMSARG was to answer are listed in a letter from Mary M. Lindquist to Roger Howe that is reproduced in a box with his article. Briefly, these questions are: (1) Do the current statements of the Standards adequately communicate your view of the discipline? (2) Do the statements of the current curriculum Standards convey a sense of consistency and growth in content themes as the student moves across the grade levels? (3) Do the statements of the content Standards adequately reflect the mathematical understanding expected of a student graduating in the twenty-first century? (4) What suggestions could you make as to the most effective ways of blending the ideas of content, teaching, and assessment?

—*Roger Howe*

American Mathematical Society NCTM2000 Association Resource Group First Report January 1997

Introduction

We are very pleased to have been invited to participate in this consultative role in the revision of the NCTM Standards. We recognize that there are a variety of communities that have important stakes and interests in this process, and we share your conviction that the inclusion of as many of these communities as possible in each step of the process will result in a stronger document. We discussed a little among ourselves the difficulty of communicating with these various groups, since they have not only such different views of their own roles but also very fundamental questions such as the purposes of education and even the nature of mathematics. We recognize the challenges that face you and hope that we can be helpful.

The interest of the AMSARG in education is evidenced by the intense discussion that has been going on (via e-mail) for the past two months (one transcript runs to 140 printed pages). While a lot of this discussion has been simply the attempts of people who have not worked together and don't know each other personally to come closer, a number of points of agreement, and also of disagreement, have come up consistently. The following is an attempt to summarize these points.

In our discussions many specific observations about individual points in the curriculum Standards were made. We hope to organize these and transmit them as time allows, but in this initial report we discuss mainly what appeared to be the larger issues.

A. The Level of Ambiguity of the Standards

There was considerable agreement that the Standards need to be more explicit and less ambiguous. We considered some examples of that ambiguity that appeared from varying interpretations of the Standards by practitioners and others that arose from our own variations. We agreed that while vignettes may be helpful to many readers, it is important to make the Standards as clear and specific as reasonable and not to rely on the vignettes or other stories to *implicitly* develop standards. That said, vignettes are helpful to many and essential to some readers, especially in clarifying the meaning of terminology. Given this importance, vignettes should be written with great care and should carry serious mathematical content.

A.1. "Decreased Attention" and "Increased Attention" Tables

Several examples of ambiguity appear to have grown out of the "decreased attention" and "increased attention" tables in the Standards. The topics listed in the former have often been misinterpreted to mean almost no (or even absolutely no) attention. This often conflicts with not only the common-sense meaning of "decreased" but also the importance of related topics. An important example is the role of paper and pencil calculation. Another example concerns the apparent conflict between emphasizing the connections among various branches of mathematics—especially algebra and geometry, in our view—and the call for decreased emphasis on conic sections—which provide a beautiful illustration of a deep connection between these two topics. More of our discussion on these two topics is summarized below. There was considerable sentiment, though not consensus, for removing the "Increased-Decreased" tables from the Standards. If this is not done, effort should be made to avoid tendentiousness in them, and they should be supplemented with cautionary language emphasizing the need for judgment and balance in implementing proposed changes.

A.2. The Role of the Teacher

Other examples of ambiguity deal with pedagogy and teacher preparation. The Standards have led some teachers and leaders of teacher in-service development activities to call for a dramatic reduction to the point of almost total elimination of direct instruction. Most deep constructivists, in contrast to naive ones, appear to recognize that it is compatible with the view that individuals construct their own understanding to give the teacher a key role in the classroom, including directing the classroom dialogue, setting goals for student understanding, judging when that understanding has been achieved, and facilitating mathematical closure. We cannot tell whether the first editions of the Standards contain a deliberate overemphasis on reducing direct instruction in order to encourage more active student participation in class activities, but experience makes clear the need to be quite careful in documents like these to say what kinds of balance are needed and not to allow rhetorical excesses. Another example is the confusion—at least in our reading of the Standards—between calling for more mathematics in the preparation of teachers (which all of us enthusiastically support) and calling for changes in the nature of the mathematics taught to teachers (which some of us support to some extent, but about which others have deep concerns).

A.3. But How Specific?

While there was consensus that the Standards should be made more specific, there was enormous disagreement as to how thoroughly they should delineate the curriculum. This was the subject of the most heated discussion in the group.

On the one hand, a number of members argued persuasively for the advantages of a national curriculum. The examples of the Virginia state curriculum; the Japanese, Russian; and Dutch systems, and E. D. Hirsch's Core Knowledge program were adduced. The idea was brought forward that a national curriculum gives a basis for national discussion, so that the entire teaching profession can receive the benefit of research by small groups of teachers into specific classroom techniques. (The Japanese system was seen as the best example of this.) Also, a specific year-by-year curriculum would prevent misjudgment on the part of individual teachers or school districts about the importance of particular topics.

On the other hand, other members argued against a national curriculum. Examples were given where specific curriculum items led to a mechanical or rote mastery of the topics and encouraged assessment procedures which searched only for surface-level understanding of what mathematics looks like. The argument was made that a rigid national curriculum, dictated from above, would erode teacher and local autonomy, would not be accepted politically by school districts, and would place the classroom teacher in the position of a low-level "deliverer" of curriculum rather than a responsible member of an active profession.

We did agree that a call in the Standards for the development of detailed curricula at appropriate levels—whether school, district, state, or national—

would be very helpful, since such curricula could allow teachers to work together more effectively on common issues and could indicate clearly to all the expectations of the schools for student learning. One role of the new Standards could be to guide the development of such detailed curricula.

B. Algorithms

Two unrelated discussions took place with regard to this word. The first was about how arithmetic computation should be taught and particularly the role of the calculator in elementary school. The second was about the nature of algorithms in their more general mathematical context.

With respect to arithmetic computation, there was consensus that the use of calculators should support, but not supplant, other methods of computation, including paper-and-pencil algorithms. Other methods we contrasted with calculators included doing arithmetic mentally and using manipulatives to represent computations.

There was no consensus about how this might translate into classroom practice. Some members voiced concern about using the calculator at all in the early grades. Others pointed out that perhaps this decision should be left to teachers to work out.

The second discussion, about algorithms in general, was a bit more diffuse. The following summary seemed to fit everyone's ideas:

- Kids need to learn certain algorithms.
- They need to do this for three reasons:
  a. efficiency,
  b. mathematical understanding,
  c. the notion of algorithm itself.
- In an age of calculators and spreadsheets the notion of algorithm becomes even more important.

Conventional algorithms for basic arithmetic—addition, subtraction, multiplication, and division—were felt to be worth teaching for reasons (a), (b), (c) above and in particular for their preparatory value for the algebra of polynomials. Beyond that there was little consensus. There will be further comments on algorithms in the point-by-point comments to be submitted later.

There was a clear consensus that the use of algorithms, on whatever level, must be accompanied by understanding of how the algorithm works, not just what it accomplishes, and by discussion (wherever appropriate) of how algorithms can provide additional insight, not just specific answers.

These comments lead to the recommendation that the notion of algorithm in the new Standards be clarified and separated from the notion of arithmetic computation. Further, the status of algorithms and computation, especially in the "Increased Attention, Decreased Attention" lists, should be rethought.

C. Proof

The discussion on this topic was not nearly so thorough as the previous one, probably because of the limited time available. However, there seemed to be a consensus that the sudden appearance of "mathematical structure" in grades 9–12 should be rethought, expressed by one of us as follows: "...[I]t bothers me that there seems to be some very heavy line drawn at proof. I would hope, first of all, that if we really do a good job at developing mathematical reasoning skills in K–8, then proof would seem a natural next step in 9–12. In fact, I would put it almost as a litmus test of success in this area."

While the notion of logical deduction is not completely lacking in the description of K–8 education given in the Standards, the ARG discussion suggests that this strand could be made more prominent and more coherent. In particular, there is a need, once filled by the standard geometry class, for students to learn basic syllogistic logic, including notions such as converse, inverse, and contrapositive.

D. Connection between Algebra and Geometry

There was a consensus that the connections between geometry and algebra, the two salient aspects of precollege mathematics, should be made frequently and early. Existing statements about this contained in the Standards should be strengthened, and more ways to make this connection should be ferreted out. Examples are: the number line, modelling addition with lengths and multiplication with areas, statistical graphics, conic sections, matrices.

While "conic sections" appears under algebra in the column "Decreased Attention" for grades 9–12, the importance of conic sections as a way of showing the connection between algebra and geometry should be reexamined.

E. The Organization of the Standards Documents

The ARG did not discuss directly the idea of "blending" the three Standards documents; discussion centered on the curriculum and evaluation Standards. In regard to these there was a consensus that the first four of these Standards are qualitatively different from the others. This may have been clear also to the original authors, since they are repeated on each of the three instructional levels. There was agreement that perhaps a tighter organization of the document might be achieved if these Standards were separated from the Standards concerned with description of content and discussed under a different and more descriptive title with less differentiation into grade levels.

American Mathematical Society NCTM2000 Association Resource Group Second Report June 1997

Responses to questions posed in the letter of 1 April 1997 from Joan Ferrini-Mundy and Mary Lindquist.

Question 1.

(a) What is meant by "algorithmic thinking"? **(b)** How should the Standards address the nature of algorithms in their more general mathematical context? **(c)** How should the Standards address the matter of invented and standard algorithms for arithmetic computations? (d) What is it about the nature of algorithms that might be important for children to learn?

Response to Question 1.

(a) We do not know a useful reply to this question in the context of K–12 mathematics. "Algorithmic thinking" conjures up no ready images or category of ideas for us. We feel that in some sense the question is not productive. An important feature of algorithms is that they are automatic and so do not require thought once mastered. Thus learning algorithms frees up the brain to struggle with higher-level tasks. On the other hand, algorithms frequently embody significant ideas, and understanding of these ideas is a source of mathematical power. We feel it should be a goal that children should understand why and how the algorithms they use work. Our predilection is that this understanding be achieved as soon as possible—ideally, at the time of introduction of the algorithm. However, we recognize that in some cases operational mastery of an algorithm can support the conceptual understanding, which might be more difficult without such mastery. Thus sometimes it can be sound pedagogy to teach an automatic procedure first and discuss the reasons for its success later. However, we strongly support the principle that such conceptual understanding be a firm goal.

(b) We believe that the notion of an algorithm, as a guaranteed method to solve a problem, can be presented in the elementary grades. This would involve at least the following four aspects:

(1) Presentation of the idea of an algorithm as a procedure *guaranteed* to solve a type of problem, accomplish a class of computation, or some other desired goal. (Examples would not even have to be limited to mathematics; thus, in language, verb conjugation, case formation, plural formation, etc., are (sometimes strictly, sometimes less so) algorithmic.)

(2) Experience with some specific algorithms. We believe that these should include standard algorithms for the four basic operations of arithmetic. (By "standard" we do not mean to imply that there is a unique "standard" algorithm for each arithmetic operation; however, the possibilities for "standard" algorithms for arithmetical operations will necessarily be highly constrained.)

(3) The standard algorithms of arithmetic should be seen as examples in a much broader class of things called algorithms. The fact that computer programs, even the computer games the kids play, are embodiments of algorithms could be mentioned to illustrate what a many-splendored class algorithms form. It would probably be well to cover in detail other algorithms beyond those for the basic arithmetic operations to underscore the fact that "algorithm" does not simply mean "rule for doing arithmetic". The Euclidean algorithm for finding the GCD of two integers is directly relevant to ideas of elementary arithmetic, undergirds some important theoretical facts, and is well suited to calculator implementation.

(4) An algorithm is not the same as what it does. Thus the addition algorithm is to be distinguished from the idea of addition.

Algebra presents a natural context for considering algorithms at a higher level. Some essential ideas are:

(1) The fact that mathematical procedures can be algorithmized is a key to the usefulness of mathematics and the reason that it can be automated: algorithms are the source of the power of computers.

(2) That the guarantee of validity of algorithms is accomplished by *proof* and that this is a fundamental feature of mathematics. (However, the role of proof extends far beyond guaranteeing the correctness of algorithms.)

(3) There can be different algorithms to accomplish the same task, and one algorithm might be better in one context and worse in another. A good example of this could be the comparison of Cramer's Rule with elimination for the solution of linear systems of equations. Cramer's Rule gives an explicit formula for the solution of a system of linear equations in terms of determinants. Elimination does not provide a formula, but instead describes a procedure that will lead to the answer. One's initial predilection might be to prefer the formula; but, in fact, the formula for determinants involves so much computation and is so vulnerable to round-off error that Cramer's Rule is impractical for large systems, and determinants, when needed, are usually computed using elimination. Nevertheless, Cramer's Rule is important in some cases and for conceptual purposes. Such compar-

isons should be encouraged throughout the curriculum.

(4) There is a strong connection between algebraic formulas and algorithms. In particular, an algebraic expression is a sort of "loose algorithm": it is a recipe for producing some quantity from others by means of algebraic operations. Thus "$y = 3x + 2$" can be translated: "to get y from x, multiply x by three and then add two". However, an algebraic formula is not quite an algorithm, because algebraic notation has built into it ambiguities that are known not to matter to the final outcome. Thus, in computing a sum of terms an algorithm would specify which pair of terms to add first, which further term to add to the result, and so on; but an algebraic expression does not specify an order of addition, because the associative law for addition tells us that the order of addition does not affect the final outcome. This algorithmic viewpoint can be usefully applied to the understanding of identities, which are seen as a statement that two (rough) algorithms are equivalent, in the sense that they yield equal results. For example, the standard identity $a^2 - b^2 = (a + b)(a - b)$ says that the two procedures are:

(i) Take two numbers, square the first, square the second, subtract.

(ii) Take two numbers, add them, subtract the second from the first, multiply the two results. Both yield the same final result.

(5) Algorithms have a recursive structure, and this recursive structure is a source of power: once one problem has been solved, the solution can be applied to further problems. The quadratic formula provides an example here: taking the algorithmic point of view toward formulas, one sees that the quadratic formula gives a procedure for finding the roots of an arbitrary quadratic equation in one unknown. It does so by expressing the solution in terms of the standard arithmetic operations and the operation of taking a square root. Thus it presupposes the ability to perform these operations.

(6) The recursive nature of algorithms is analogous to the recursive nature of mathematics itself. This recursive structure is a prime feature of logical deduction and of axiomatic systems, in which you can use either the basic postulates, or previous theorems, in proving a new result.

Some Further Comments on Question 1 (b)

It probably would be valuable to revisit the algorithms of arithmetic from the higher perspective of algebra.

Geometric constructions are effectively algorithms. (We do not mean that devising a construction is algorithmic, but that a completed construction can be read as a set of instructions to do various basic operations to produce the desired geometric object.)

It is natural to discuss algorithms in relation to computers. Getting a computer reliably to do a multistep calculation, perhaps via a spreadsheet program, rather than formal programming would provide excellent hands-on training in algorithms. A simple programming environment like Logo offers experience in the recursive aspect of algorithms.

Related to machine computation, it is an interesting issue what algorithms calculators actually implement to compute the functions they offer. We do not suggest that this should be in the school curriculum, but it would be desirable for secondary teachers to know this so they could discuss it with their more advanced and interested students. The CORDIC algorithm uses very strongly the structure of elementary functions, especially the addition laws for trig functions, so it illustrates the power of such structural facts.

(c) We are aware of some suggestive studies (for example, by C. Kamii), as well as the practice of some foreign countries (e.g., Switzerland, Japan) which do well on TIMMS, that support the idea that extensive practice with mental computation helps develop strong number sense. Since the standard algorithms tend to be optimized for pencil-and-paper computation and not for mental computation, practice in mental arithmetic will probably lead to alternative algorithms. In particular, in practical problems involving addition or multiplication, estimation usually is a consideration, and for purposes of estimation the natural way to add is to combine like digits from left to right rather than from right to left, as in the standard pencil-and-paper algorithm, which is concerned instead with minimizing the amount of rewriting. We can believe that investigating and comparing the methods that arise may well help understanding of arithmetic. More generally, we find plausible the idea that devising personal ways to deal with arithmetic problems can promote number sense. On the other hand, we suspect it is impractical to ask all children personally to devise an accurate, efficient, and general method for dealing with addition of any numbers—even more so with the other operations. Therefore, we hope that experimental periods during which private algorithms may be developed would be brought to closure with the presentation of and practice with standard algorithms. Also, we hope care would be taken to ensure that time spent developing and testing private algorithms will not significantly slow overall progress. We believe that neither pure rote mastery of algorithms nor purely privately invented algorithms can optimize learning of arithmetic. Finding a good balance between the two is a delicate business and a matter for much practice and study. Guidance here (and elsewhere) might be found by examination of curricular materials from high-ranking TIMMS countries.

We note that to use invented algorithms in teaching, as opposed to their private use by students, will require teachers to be quite expert about the alternative algorithms which are possible. We suspect that the range of algorithms that will arise and that survive a test of reasonable generality will not be huge, and it could be a beneficial research activity to investigate and classify these and incorporate the results into teachers' manuals so that teachers could be prepared to discuss invented algorithms profitably as they arise. We understand that Japanese teachers' manuals frequently discuss the ramifications of a given topic and survey possible student responses. Such manuals would be most desirable in the U.S. We hope that children who invent algorithms could usually be brought to understand the relation between their method and the standard algorithm.

Regarding the algorithms for arithmetic, an important point to be made is that our way of writing numbers, e.g., decimal notation, is an algorithm, a very sophisticated and powerful algorithm. It produces very high information density and is marvelously adapted to computation. Furthermore, it is the result of a lengthy process of development and was not essentially complete until late in the first millennium (and not generally adopted in Europe until the sixteenth century). It incorporates in its very structure all the basic operations of rational algebra—counting, addition, multiplication, and exponentiation. Finally, it conditions the other algorithms we use—for example, an addition algorithm is not something that tells us how to add—which is a primitive intuition; an addition algorithm is something that tells us how to express the sum of two numbers, each expressed in standard decimal form, in standard decimal form also. It is probably not appropriate to tell all of this to children, but some propaganda to help them appreciate what a marvelous machine they are operating (whether or not they are using a calculator!) might be useful. Ethnomathematics might help supplement and reinforce the comparisons available through the traditional study of roman numerals, which were also a decimal system, but less systematic than our Hindu-Arabic one. Teachers should be deeply aware of the algorithmic qualities of our decimal notation and of the reasons for its power. In particular, they should keenly appreciate that our decimal notation is a highly unnatural creation, one which took about four of our five millennia of civilization to produce, and that its efficiency and apparent simplicity are the result of the sophistication of its construction. From a practical point of view, we suspect that a sufficiently deep appreciation of the beauty, power, and sophistication of the decimal system could help teachers bridge the gap between standard arithmetic algorithms and the ones invented by their students.

Standard algorithms may be viewed analogously to spelling: to some degree they constitute a convention, and it is not essential that students operate with them from day one or even in their private thinking; but eventually, as a matter of mutual communication and understanding, it is highly desirable that everyone (that is, nearly everyone—we recognize that there are always exceptional cases) learn a standard way of doing the four basic arithmetic operations. (The standard algorithms need not be absolutely unique, just as there are variant spellings between, say, the U.S. and England, but too much variation leads to difficulties.) We do not think it is wise for students to be left with untested private algorithms for arithmetic operations—such algorithms may only be valid for some subclass of problems. The virtue of standard algorithms—that they are *guaranteed* to work for *all* problems of the type they deal with—deserves emphasis.

We would like to emphasize that the standard algorithms of arithmetic are more than just "ways to get the answer"—that is, they have theoretical as well as practical significance. For one thing, all the algorithms of arithmetic are preparatory for algebra, since there are (again, not by accident, but by virtue of the construction of the decimal system) strong analogies between arithmetic of ordinary numbers and arithmetic of polynomials. The division algorithm is also significant for later understanding of real numbers. For all its virtues, decimal notation suffers a significant drawback over, say, standard notation for fractions: decimal numbers (meaning decimal fractions with finitely many terms) do not allow division. This can be remedied at the cost of using infinite decimal expansions, but this is a big leap, and the general infinite decimal is not rational. To understand that rational numbers correspond to repeating decimals essentially means understanding the structure of division of decimals as embodied in the division algorithm. We do not see that naive use of calculators can be of much help here: the length of repeat of a decimal will typically be comparable to the size of the denominator, so that $7/23$ or $5/29$ will not reveal any repeating behavior on standard calculators.

(d) The most important thing is that they exist; there are uniform ways of solving an entire class of problems.

It is important to understand that in learning an algorithm you are confronting the essence of the phenomenon with which the algorithm deals, since it is guaranteed to accomplish its aim. Also, that, once learned, the algorithm gives you automatic mastery over the topic.

Thirdly, there is the demystification of machines: your calculator and your computer perform algorithms—in fact, that is all they do. Limitations on the algorithms limit the processes they can handle.

Again, it is important to distinguish between an algorithm and what it accomplishes.

Question 2.

(a) What mathematical reasoning skills should be emphasized across the grades? **(b)** How should the Standards address mathematical proof? Why? **(c)** How should the Standards address topics within mathematical structure?

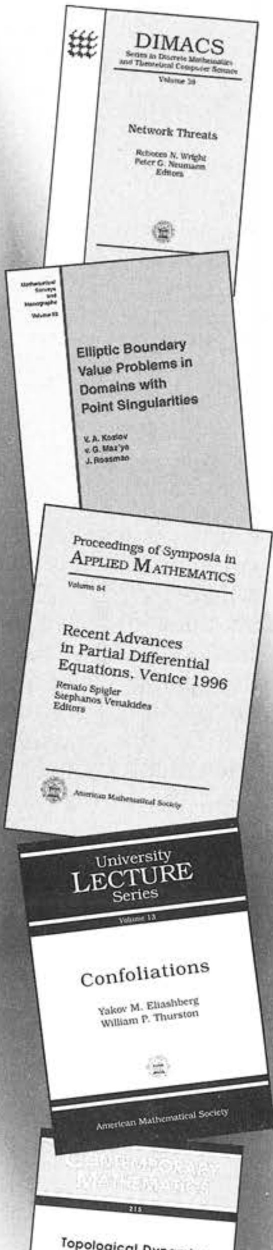
(a) The most important thing to emphasize about mathematical reasoning is that it exists—more, that it is the heart of the subject, that mathematics is a coherent subject, and that mathematical reasoning is what makes it so. This need not be taught in so many words, in fact, probably should not be. Mathematics should simply be taught as a subject where things make sense and where you can figure out why they are the way they are. There are significant exceptions, of course: there are axioms, such as the field axioms in algebra. They should be introduced as summaries of what we know from our experience—intuitively acceptable general rules—and sold as firm principles we can rely on when we are in less familiar territory. Not everything need be justified ab initio—sometimes an important property or fact may take some getting used to and is best introduced first and justified later. In such cases it should be pointed out that the item can be justified and will be later, preferably at some identifiable time. If the idea that you can and should figure (most) things out (maybe with help from classmates or teachers or from the text) can be inculcated in elementary school, then illustrations of how you use axioms, like the field axioms, to extend your range—to go from arithmetic of numbers to arithmetic of polynomials—can be introduced in junior high. Also, in algebra derivations of important formulas should be given. Derivation as proof—as the justification for formulas and statements of algebra—should be pointed out. Also, there should be practice with proof in the sense of "local proof": statements should be given, to be justified on the basis of logic, together with simpler facts which are agreed to be taken for granted. Throughout, mathematical reasoning in the form of translating from concrete problem contexts to mathematics, then back again after mathematical processing, should also be developed. The complexity of the mathematical processing and of the translation step should be gradually increased with student age.

(b) We feel that if mathematical reasoning is handled well in elementary and junior high school, then students should be ready to see (fairly) formal proof in high school. Issues of formal logic—syllogisms, negation and/or statements, converses, contrapositives, inverses, and the beginnings of quantification—should receive serious attention. Also, the necessity of using language carefully should be discussed—the need to specify hypotheses and conclusions, to be clear on the difference between "if" and "if and only if" statements, the need for careful definitions rather than simply "intuition". Also, the principle that reasoning has to start somewhere and that the starting point is defined by the axioms should be explained clearly. The value of stating obvious facts as a basis from which to arrive at nonobvious ones should be demonstrated in toy examples as well as in mathematically significant ones. (This fits well with the notion of "semilocal" proof, mentioned below.) The history of non-Euclidean geometry might be discussed as an illustration of how rigor can sharpen and reform intuition. (The 2,000 year debate which led to non-Euclidean geometry could never have started if Euclid had not given his axiomatic foundations of geometry.) The traditional (meaning for students of our generation) place to learn formal proof was in geometry. With greater appreciation both of the difficulties and cumbersomeness of a full axiomatization of Euclidean geometry as well as of the importance of other geometries, we are hesitant to recommend a full axiomatic treatment of Euclidean plane geometry. Nevertheless, we feel that the opportunities for reasoning in this subject are very rich and that it has great intuitive appeal for many mathematically talented students. Thus we feel that reasoning should play a large role in geometry courses, perhaps in a kind of "semilocal" proof, where a few assumptions are used to justify fair-sized collections of theorems, of which some of the more elementary and "intuitively obvious" of the statements are accepted without proof (while other, similar statements are proved to illustrate the issues involved), and the less obvious results are proved or assigned as exercises. Neither would all proofs have to be synthetic; the derivation of the equation of an ellipse or hyperbola from their metric definitions is a nice combination of geometry and algebra.

(c) We are not enthusiastic about formal treatment of algebraic systems in K–12. It would be enough, and very beneficial, for some groundwork for understanding formalization to be laid by a deeper investigation of more concrete topics. This would include a good working understanding of the field axioms and their usefulness in algebra. It could include an understanding of a concrete but nonstandard algebraic system, such as modular arithmetic, and a deeper understanding of the complex numbers, and polynomials. Similarly, transformational geometry, linked to both synthetic geometry and to matrix algebra, could provide a rich intuitive background for the abstractions of algebra. Integrating these ideas into the curriculum presents challenges enough.

American Mathematical Society

Recently Published Titles from the AMS



Confoliations

Yakov M. Eliashberg, *Stanford University, CA*, and **William P. Thurston**, *University of California, Davis*

This book presents the first steps of a theory of confoliations designed to link geometry and topology of three-dimensional contact structures with the geometry and topology of codimension-one foliations on three-dimensional manifolds. Developing almost independently, these theories at first glance belonged to two different worlds: The theory of foliations is part of topology and dynamical systems, while contact geometry is the odd-dimensional "brother" of symplectic geometry.

However, both theories have developed striking similarities. Confoliations—which interpolate between contact structures and codimension-one foliations—should help us to understand better links between the two theories. These links provide tools for transporting results from one field to the other.

University Lecture Series, Volume 13; 1997; 66 pages; Softcover; ISBN 0-8218-0776-5; List $16; All AMS members $13; Order code ULECT/13RT82

Elliptic Boundary Value Problems in Domains with Point Singularities

V. A. Kozlov and **V. G. Maz′ya**, *Linköping University, Sweden*, and **J. Rossmann**, *Rostock University, Germany*

This monograph systematically treats a theory of elliptic boundary value problems in domains without singularities and in domains with conical or cuspidal points. This exposition is self-contained and a priori requires only basic knowledge of functional analysis. Restricting to boundary value problems formed by differential operators and avoiding the use of pseudo-differential operators makes the book accessible for a wider readership.

The authors concentrate on fundamental results of the theory: estimates for solutions in different function spaces, the Fredholm property of the operator of the boundary value problem, regularity assertions and asymptotic formulas for the solutions near singular points. A special feature of the book is that the solutions of the boundary value problems are considered in Sobolev spaces of both positive and negative orders. Results of the general theory are illustrated by concrete examples. The book may be used for courses in partial differential equations.

Mathematical Surveys and Monographs, Volume 52; 1997; 414 pages; Hardcover; ISBN 0-8218-0754-4; List $99; Individual member $59; Order code SURV/52RT82

Network Threats

Rebecca N. Wright, *AT&T Labs Research, Florham Park, NJ*, and **Peter G. Neumann**, *SRI International, Menlo Park, CA*, Editors

The volume offers a timely assessment of avoiding or minimizing network threats. Presented here is an interdisciplinary, system-oriented approach that encompasses security requirements, specifications, protocols, and algorithms. The text includes implementation and development strategies using real-world applications that are reliable, fault-tolerant, and performance oriented. The book would be suitable for a graduate seminar on computer security.

DIMACS: Series in Discrete Mathematics and Theoretical Computer Science, Volume 38; 1997; 110 pages; Hardcover; ISBN 0-8218-0832-X; List $29; All AMS members $23; Order code DIMACS/38RT82

Perspectives on Quantization

Lewis A. Coburn, *State University of New York at Buffalo*, and **Marc A. Rieffel**, *University of California, Berkeley*, Editors

This book presents the proceedings of a 1996 Joint Summer Research Conference sponsored by AMS-IMS-SIAM on "Quantization" held at Mount Holyoke College (Northampton, MA). The purpose of the conference was to bring together researchers on various mathematical aspects of quantization.

In the early work of Weyl and von Neumann at the beginning of the quantum era, the setting for this enterprise was operators on Hilbert space. This setting has been expanded, especially over the past decade, to involve C^*-algebras—noncommutative differential geometry and noncommutative harmonic analysis—as well as more general algebras and infinite-dimensional manifolds. The applications now include quantum field theory, notable conformal and topological field theories related to quantization of moduli spaces, and constructive quantum field theory of supersymmetric models and condensed matter physics (the fractional quantum Hall effect in particular).

The spectrum of research interests which significantly intersects the topic of quantization is unusually broad, including, for example, pseudodifferential analysis, the representation theory of Lie groups and algebras (including infinite-dimensional ones), operator algebras and algebraic deformation theory. The papers in this collection originated with talks by the authors at the conference and represent a strong cross-section of the interests described above.

Contemporary Mathematics, Volume 214; 1998; 195 pages; Softcover; ISBN 0-8218-0684-X; List $39; Individual member $23; Order code CONM/214RT82

Recent Advances in Partial Differential Equations, Venice 1996

Renato Spigler, *University of Padova, Italy*, and **Stephanos Venakides**, *Duke University, Durham, NC*, Editors

The work of Lax and Nirenberg on partial differential equations (PDEs) over the last half-century has dramatically advanced the subject and has profoundly influenced the course of mathematics.

A large number of mathematicians honored these two exceptional scientists in a week-long conference in Venice (June, 1996) on the occasion of their 70th birthdays.

This volume contains the proceedings of the conference, which focused on the modern theory of nonlinear PDEs and their applications. Among the topics treated are turbulence, kinetic models of a rarefied gas, vortex filaments, dispersive waves, singular limits and blow-up of solutions, conservation laws, Hamiltonian systems and others. The conference served as a forum for the dissemination of new scientific ideas and discoveries and enhanced scientific communication by bringing together such a large number of scientists working in related fields. The event allowed the international mathematics community to honor two of its outstanding members.

Proceedings of Symposia in Applied Mathematics, Volume 54; 1997; 393 pages; Hardcover; ISBN 0-8218-0657-2; List $59; Individual member $35; Order code PSAPM/54RT82

Topological Dynamics and Applications

M. G. Nerurkar, *Rutgers University, Camden, NJ*, **D. P. Dokken**, *St. Paul, MN*, and **D. B. Ellis**, *Beloit College, WI*, Editors

This book is a very readable exposition of the modern theory of topological dynamics and presents diverse applications to such areas as ergodic theory, combinatorial number theory and differential equations. There are three parts: 1) The abstract theory of topological dynamics is discussed, including a comprehensive survey by Furstenberg and Glasner on the work and influence of R. Ellis. Presented in book form for the first time are new topics in the theory of dynamical systems, such as weak almost-periodicity, hidden eigenvalues, a natural family of factors and topological analogues of ergodic decomposition. 2) The power of abstract techniques is demonstrated by giving a wide range of applications to areas of ergodic theory, combinatorial number theory, random walks on groups and others. 3) Applications to non-autonomous linear differential equations are shown. Exposition on recent results about Floquet theory, bifurcation theory and Lyapanov exponents is given.

Contemporary Mathematics, Volume 215; 1998; 334 pages; Softcover; ISBN 0-8218-0608-4; List $69; Individual member $41; Order code CONM/215RT82

All prices subject to change. Charges for delivery are $3.00 per order. For optional air delivery outside of the continental U. S., please include $6.50 per item. *Prepayment required.* Order from: **American Mathematical Society**, P. O. Box 5904, Boston, MA 02206-5904, USA. For credit card orders, fax (401) 455-4046 or call toll free 800-321-4AMS (4267) in the U. S. and Canada, (401) 455-4000 worldwide. Or place your order through the AMS bookstore at http://www.ams.org/bookstore/. Residents of Canada, please include 7% GST.

Mathematics People

Presidential Early Career Awardees Named

Sixty young researchers have been chosen to receive the second annual Presidential Early Career Awards for Scientists and Engineers. The presidential honor is the highest bestowed by the U.S. Government on outstanding young scientists, mathematicians, and engineers who are in the early stages of their independent research careers. The awards, which include a five-year grant of up to $500,000, are made by nine governmental agencies. Twenty of the awards are made through the National Science Foundation.

The awardees include two in the mathematical sciences. LORI ANN FREITAG DIACHIN is in the mathematics and computer science division at Argonne National Laboratory. She works in the area of visualization of data sets and real-time simulation and analyses. KATHERINE OKIKIOLU is on the mathematics faculty at the University of California, San Diego. Her area of research is geometric analysis, particularly the determinant of the Laplacian under smooth perturbations.

—*Allyn Jackson*

AAAS Fellows Elected

In September 1997 the Council of the American Association for the Advancement of Science elected 270 members as Fellows of AAAS. Elected to the mathematics section of AAAS are: WILLIAM H. JACO, Oklahoma State University; CORA SADOSKY, Howard University; DAVID A. SMITH, Duke University; and WILHELM STOLL, University of Notre Dame.

—*from AAAS Announcement*

Dantzig Prize Recipients Announced

ROGER FLETCHER of the University of Dundee and STEVE ROBINSON of the University of Wisconsin, Madison, received the 1997 George B. Dantzig Prize. The prize is given every three years by the Mathematical Programming Society and the Society for Industrial and Applied Mathematics. Fletcher was cited for his fundamental contributions to algorithms in almost all areas of nonlinear optimization and especially to the development of nonlinear conjugate gradient and variable-metric methods for unconstrained optimization. Robinson was cited for his work on the solution, stability, and sensitivity of nonlinear optimization problems and for introducing generalized equations and the strong-regularity condition into optimizaton.

—*from* SIAM News

Deaths

BERNARD ALTSHULER, of NYU Medical Center, died on October 4, 1997. Born on June 22, 1919, he was a member of the Society for 48 years.

RAYMOND F. BELL, of Glen Ferris, WV, died in October 1993. Born on June 9, 1911, he was a member of the Society for 22 years.

Editor's Note: Due to a clerical error, the December 1997 issue of *Notices* erroneously reported the August 11, 1997, death of Masami Okada. The name of the deceased should have been Akihito Uchiyama (see the January issue, page 95). The *Notices* regrets this error.

Mathematics Opportunities

News from The Fields Institute

The Fields Institute for Research in Mathematical Sciences at the University of Toronto is holding eight workshops as part of its 1998-99 program, Probability and Its Applications. Information about the first three of the workshops is given below.

Mathematical Physics of Polymers and Percolation, August 24-29, 1998. Organizing Committee: J. Cardy (University of Oxford), F. den Hollander (University of Nijmegen), G. Slade (McMaster University), and S. G. Whittington (University of Toronto).

The following individuals have been invited to speak: M. Aizenman (Princeton University), M. Batchelor (Australian National University), M. Bousquet-Melou (Université Bordeaux I), J. Cardy (Oxford University), J. T. Chayes (Microsoft Research and UCLA), A. J. Guttmann (University of Melbourne), F. den Hollander (University of Nijmegen), E. J. Janse van Rensburg (York University), G. Lawler (Duke University), A. Owczarek (University of Melbourne), A. Pisztora (Carnegie Mellon University), Y. Saint-Aubin (CRM, Université de Montréal), H. Saleur (University of Southern California), G. Slade (McMaster University), T. Spencer (Institute for Advanced Study), and S. G. Whittington (University of Toronto).

Each speaker will give two lectures. There are limited funds available to assist attendance of graduate students and postdocs; please contact `slade@mcmaster.ca` for further information.

Hydrodynamic Limits, October 5-10, 1998. Organizing Committee: S. Feng (McMaster University), A. Lawniczak (University of Guelph), and S. R. S. Varadhan (Courant Institute).

On October 5-6, S. R. S. Varadhan will present a minicourse on Hydrodynamic Limits. The following individuals have been invited to speak during October 7-10: B. Boghosian (Boston University), A. De Masi (Universita di L'Aquila), T. Funaki (University of Tokyo), J. Lebowitz (Rutgers University), D. Levermore (University of Arizona), J. Quastel (University of Toronto), F. Rezakhanlou (University of California, Berkeley), T. Seppalainen (Iowa State University), H. Spohn (Ludwig Maximilians Universität), and H. T. Yau (Courant Institute). Poster and discussion sessions are planned.

Monte Carlo Methods, October 25-29, 1998. Organizing Committee: N. Madras (York University), R. Neal (University of Toronto), and J. Rosenthal (University of Toronto).

The following individuals have been invited to speak: Bernd Berg (Florida State University), David M. Ceperley (University of Illinois), James Allen Fill (Johns Hopkins University), Daan Frenkel (FOM Institute, The Netherlands), Peter Green (University of Bristol), Karl Jansen (CERN), Anthony D. Kennedy (Florida State University), Xiao-Lo Meng (University of Chicago), Jesper Moller (Aalborg University), Duncan Murdoch (Queen's University), Gareth O. Roberts (Cambridge University), Alistair Sinclair (University of California, Berkeley), Alan D. Sokal (New York University), Stuart G. Whittington (University of Toronto), and David B. Wilson (Institute for Advanced Study).

The focus of the workshop will be on Monte Carlo methods (mostly Markov chain methods) that have been used effectively in statistical physics and/or statistics and that are based on ideas that seem to be widely applicable. The speakers have backgrounds in physics, statistics, and probability, and the program is intended to promote discussion across these disciplines. The emphasis will be on new methods, practical applications, and analysis of methods.

For further information about these workshops, consult the Web site `http://www.math.yorku.ca/Probability/Fields.html` or contact: The Fields Institute for

Research in Mathematical Sciences, 222 College Street, Second Floor, Toronto, Ontario, M5T 3J1, Canada; telephone: 416-348-9710; fax: 416-348-9385; e-mail: probability@fields.utoronto.ca.

—from Fields Institute Announcement

1998 NSF-CBMS Regional Conferences

Contingent upon funding from the National Science Foundation (NSF), the Conference Board of the Mathematical Sciences (CBMS) will hold six NSF-CBMS Regional Conferences in the summer of 1998. These conferences are intended to stimulate interest and activity in mathematical research.

Each five-day conference features a distinguished lecturer who speaks on a topic of current research. Support for about thirty participants is provided for each conference. The conference organizers invite both established researchers and interested newcomers, including postdoctoral researchers and graduate students.

The title of each conference appears below, followed by the name of the principal speaker, the date, the location, and the names of the organizers, who can be contacted for more information.

Wavelet Analysis as a Tool for Computational and Harmonic Analysis, Ronald Coifman, May 4–8, 1998, University of Central Florida, Orlando. Contact Lokenath Debnath, telephone: 407-823-2478 or 407-823-2754, e-mail: ldebnath@pegasus.cc.ucf.edu or ijmms@pegasus.cc.ucf.edu.

Blocks of Finite Reductive Groups, Deligne-Lusztig Varieties, and Complex Reflections Groups, Michel Broué, May 25–29, 1998, University of North Texas, Denton. Contact J. Matthew Douglass, telephone: 817-565-2155, e-mail: douglass@unt.edu, World Wide Web: http://hilbert.math.unt.edu/cbms.html.

Lectures on Division Algebras, David J. Saltman, June 14–18, 1998, Colorado State University, Fort Collins. Contact Frank DeMeyer, telephone 970-491-6327, e-mail: demeyer@math.colostate.edu, World Wide Web: http://www.math.colostate.edu/.

Ergodic Theory, Groups, and Geometry, Robert J. Zimmer, June 22-26, 1998, University of Minnesota. Contact Scot Adams or Dave Witte, telephone: 612-625-5507; e-mail: adams@math.umn.edu, dwitte@math.okstate.edu; World Wide Web: http://www.math.umn.edu/~adams/CBMS/main.html.

For information on submitting a proposal to organize an NSF-CBMS Regional Conference, see the announcement about the request for proposals in this section of the *Notices*.

—from CBMS Announcement

Request for Proposals for 1999 NSF-CBMS Regional Conferences

The National Science Foundation (NSF), with the sponsorship of the Conference Board of the Mathematical Sciences (CBMS), intends to support up to five NSF-CBMS Regional Research Conferences in 1999.

These five-day conferences feature a distinguished lecturer who delivers ten lectures on a sharply focused topic of current research in the mathematical sciences. The lecturer subsequently prepares a monograph which, depending on the topic, is published by the AMS, or the Society for Industrial and Applied Mathematics, or jointly by the American Statistical Association and the Institute of Mathematical Statistics. Support is provided for about thirty conference participants, including postdoctoral researchers and graduate students.

Colleges and universities with at least some research competence in the field of the proposal are eligible to apply. Since a major goal of these conferences is to attract new researchers into the field of the conference and to stimulate new research activity, institutions that are interested in upgrading or improving their research efforts are especially encouraged to apply. Proposals should reach the NSF by **April 6, 1998**.

For further information on the NSF-CBMS Regional Conferences and guidelines for preparing proposals, contact: Conference Board of the Mathematical Sciences, 1529 18th Street, NW, Washington, DC 20036; telephone 202-293-1170; fax 202-265-2384; World Wide Web http://www.maa.org/cbms/cbms.html. See also the announcement elsewhere on this page about the upcoming NSF-CBMS Regional Conferences, to be held in the summer of 1998.

—from CBMS Announcement

ICM-98 Call for Presentations of Mathematical Software

The 1998 International Congress of Mathematicians (ICM-98) will take place in Berlin, Germany, August 18–27, 1998. In addition to the scientific program, a "Section of Special Activities" is planned. One of these activities will be a session on mathematical software, to be held on two afternoons during the congress. The focus of this session will be the presentation of a broad spectrum of mathematical software systems ranging from general purpose to specialized systems, e.g., systems from numerical analysis, computer algebra, optimization, mathematical visualization, or mathematical education. The session is intended to attract a broad audience: ICM attendees, students, teachers, etc., with a special interest in mathematical software.

Submissions for the session on mathematical software are encouraged from all fields of mathematics where software systems are used. Systems which are available free of charge (e.g., public domain) are especially invited and will be preferred when the presentations are chosen. Talks in which various commercial packages are compared from an independent viewpoint, pointing out particular strengths and weaknesses of the systems, are also sought.

There will also be in connection with ICM-98 a software exhibition and book fair, which may be more suitable for the demands of vendors of commercial software systems. Please contact the chairman of the local arrangements committee, Rolf H. Moehring (e-mail: `moehring@math.tu-berlin.de`) for details about the exhibition.

The program committee, a group of internationally renowned mathematicians and experts on mathematical software, will evaluate the entries and select a number of contributions according to quality and thematic balance. The committee will be chaired by Johannes Grabmeier of IBM Germany.

Submissions should be sent, preferably by electronic mail, to: ICM-98—Session on Mathematical Software, c/o W. Neun, Konrad-Zuse-Zentrum (ZIB), Takustr. 7, D-14195 Berlin, Germany; e-mail: `neun@zib.de`. Submissions must be received by **March 1, 1998**. Notifications of acceptance will be made on April 1, 1998. For further information, consult the ICM-98 Web site, `http://elib.zib.de/ICM98/`.

—from ICM-98 Announcement

Summer Mathematics Program for Women

Bryn Mawr and Spelman Colleges extend a special invitation to women who will be entering graduate programs in mathematics in the fall of 1998 to participate in a post-baccalaureate summer enrichment program. The Enhancing Diversity in Graduate Education (EDGE) Program will consist of a four-week summer session and an accompanying graduate school mentoring and support network component. The summer program consists of two core courses in analysis and algebra/linear algebra. A set of minicourses in vital areas of mathematical research, short-term visitors from academia and industry, guest lectures, graduate student mentors, and problem sessions will round out the summer experiences.

Applicants to the program should be women who are (i) graduating seniors who have applied to graduate programs in the mathematical sciences, (ii) recent recipients of undergraduate degrees who are now entering graduate programs, or (iii) first-year graduate students. *Women from minority groups who fit one of the above categories are especially encouraged to apply.* Final acceptance to the program is contingent upon acceptance to a graduate program in the mathematical sciences.

The 1998 summer session will be conducted on the campus of Bryn Mawr College outside Philadelphia, Pennsylvania, June 15–July 10 and will be co-directed by Rhonda Hughes (Bryn Mawr College) and Sylvia Bozeman (Spelman College). A stipend of $1,800 plus room and board will be awarded to participants.

Applications should consist of the following: (i) a statement describing the expected value of this program to the applicant's academic goals; (ii) two letters of recommendation from mathematical sciences faculty familiar with the applicant's work; (iii) a transcript and vita; and (iv) a list of graduate programs to which the applicant has applied, together with a ranked list of her two or three top choices.

The deadline is **March 1, 1998**. Applications should be sent to: EDGE Program, Department of Mathematics, Bryn Mawr College, Bryn Mawr, PA 19010.

Major funding for the program is provided by the National Science Foundation. Visit the program's Web site at `http://www.brynmawr.edu/Acads/Math/`.

Editor's Note: This is a corrected version of the announcement that appeared in the January 1998 issue of the *Notices*.

—EDGE Program Announcement

Grants for Computing Equipment

The Division of Mathematical Sciences (DMS) of the National Science Foundation plans a limited number of grants for the purchase and support of computing equipment through its Scientific Computing Research Environments for the Mathematical Sciences (SCREMS) program.

Some awards may be as high as $200,000, provided a case is made for substantial impact and cost effectiveness. The DMS expects to provide about $1,000,000 for this activity in fiscal year 1998, pending availability of funds. In fiscal year 1997, twenty-one awards were made.

The solicitation is available on the World Wide Web at `http://www.nsf.gov/mps/dms/screms.htm`. The announcement appears on the NSF home pages and search engines as publication number NSF 98-31. A text version is also available. The deadline for proposals is **February 20, 1998.** (This is a change from previous years' deadlines.) The mailing address is: Division of Mathematical Sciences, National Science Foundation, 4201 Wilson Boulevard, Room 1025, Arlington, VA 22230; the telephone number is 703-306-1870. For specific questions or information not on the announcement, please send e-mail to `screms@nsf.gov`.

—from DMS Announcement

For Your Information

"Streaming Video" Project at MSRI

In October 1997 the Mathematical Sciences Research Institute (MSRI) made public its streaming-video archive, which is accessible from MSRI's home page. Videos of lectures are joined with lecture notes, making it possible to follow the lectures in complete detail. Lectures can be paused, and one can skip backwards and forwards, for example, to repeat a difficult passage. In addition to lectures from MSRI, lectures from Pacific Northwest Geometry Seminars, talks given at the Ahlfors symposium (held at Stanford University in September 1997), colloquia from UC Berkeley, and some of the main presentations at the 45th Annual Meeting of SIAM (Stanford, July 1997) are available. The selection includes lectures by Simon Donaldson, Don Knuth, Boris Rozovskii, Ron Stern, Margaret Wright; minicourses by Cameron Gordon (Combinatorial Methods in Low-Dimensional Topology) and Tatiana Toro (Geometric Measure Theory, Harmonic Analysis and Potential Theory); and a visit from hyperbolic realtor Mel Slugbate. MSRI has made video recordings of a large number of lectures during the past year, and we have built up a valuable video library. Each week new streaming-video lectures will be made available from this library and from current events at the Institute.

The streaming-video archive is part of MSRI's Multicast Backbone Project, supported by the National Science Foundation. Our goals are to implement practical methods of Internet communication in order to make the mathematical activities of MSRI nationally and internationally available and to help mathematics departments and centers profit by our experience to do the same. Some of the advantages of the streaming-video archive are: (1) accessibility: the lectures are available, in principle, any time and anywhere on the Internet; (2) simplicity of use: all that is required is a Web browser and a plug-in streaming-video player; and (3) wide support in the computer industry: there are many vendors and a strong movement toward interoperability.

Comments about the streaming-video archive are welcome. Here is a list of relevant Web addresses: MSRI's home page: `http://www.msri.org/`; MSRI's streaming video introduction page: `http://nextra.msri.org/computing/av/`; list of available lectures on streaming video: `http://nextra.msri.org/computing/av/lectures.html`; Realnetworks' home page (where the plug-in is available): `http://www.real.com/`; Industrial-Strength Streaming Video (a survey article on commercially available streaming video servers, compressors, and viewers as of September 1997): `http://www.newmedia.com/NewMedia/97/12/feature/Streaming_Video.html`.

—*David Hoffman, MSRI*

Cornell Digital Library of Mathematics Books

Cornell University has a digital library that contains a substantial collection of mathematics books, primarily from the nineteenth century. They are available for viewing on the World Wide Web. In addition, through funding from the Andrew W. Mellon Foundation and the Charles E. Culpeper Foundation, Cornell is collaborating with the University of Michigan on a digital library collection called Making of America (MOA). MOA focuses on books and journals published between 1850 and 1900 and contains some holdings in science, mathematics, and technology. Both the Cornell mathematics collection and the MOA collection are available at the Web site `http://library3.cit.cornell.edu/MOA/moa-main_page.html`.

—*Allyn Jackson*

Editor's Note: Due to printer error, an advertisement for Wolfram Research featuring outdated conference information appeared in the December issue of the *Notices*. Wolfram Research intended to announce their Worldwide Mathematica Conference to be held June 18–21, 1998, in Chicago, Illinois. The *Notices* regrets this error.

In the "About the Cover" box on page 1429 of the December 1997 issue, the name of Linda Buttel was misspelled and Simon Levin was referred to as a mathematician biologist. He is a mathematical biologist. The *Notices* regrets these errors.

From the AMS

1997 Election Results

In 1997, the Society elected a president-elect, a vice-president, a trustee, five members-at-large of the Council, two members of the Editorial Boards Committee and three members of the Nominating Committee. Terms for these positions are three years, beginning on February 1, 1998, and ending on January 31, 2001, except for the president-elect, whose term is for one year, and the trustee, whose term is for five years ending on January 31, 2003. Members elected to the Nominating Committee begin serving immediately. Their terms end on December 31, 2000.

President-Elect

Elected as the new president-elect is **Felix Browder** from Rutgers University. Browder will serve as president-elect for one year. On February 1, 1999, Browder will become president of the Society.

Vice-President

Elected as the new vice-president is **Jennifer Tour Chayes** from Microsoft, Redmond, WA.

Trustee

Elected as the new trustee is **Roy L. Adler** from IBM, Yorktown Heights, NY.

Member-at-large of the Council

Elected as new members-at-large of the Council of the Society are

Robert L. Bryant from Duke University

Jane M. Hawkins from the University of North Carolina

Karen V. H. Parshall from the University of Virginia

M. Beth Ruskai from the University of Massachusetts at Lowell

Michael Starbird from the University of Texas at Austin

Editorial Boards Committee

Elected as new members of the Editorial Boards Committee are

David Jerison from Massachusetts Institute of Technology

Abel Klein from the University of California, Irvine

Nominating Committee

Elected as new members of the Nominating Committee are

Paul H. Rabinowitz from the University of Wisconsin

Elias M. Stein from Princeton University

Sylvia M. Wiegand from the University of Nebraska

Suggestions for elections to be held in the fall of 1998 are solicited by the 1998 Nominating Committee. Send electronic mail to `nomcomm@sol.math.uiuc.edu`. The Nominating Committee will be meeting during the Annual Meeting in Baltimore in January.

Positions to be filled in the 1998 election are:

Vice-president

Trustee

Five Members-at-large of the Council

Suggestions for nominations for election for two positions on the Editorial Boards Committee and three positions on the 1999 Nominating Committee are also welcome and can be sent to the Secretary.

—*Robert M. Fossum*
Secretary

1998 Frank and Brennie Morgan AMS-MAA-SIAM Prize for Outstanding Research in Mathematics by an Undergraduate Student

The prize is awarded each year to an undergraduate student (or students having submitted joint work) for outstanding research in mathematics. Any student who is an undergraduate in a college or university in the United States or its possessions, or Canada or Mexico, is eligible to be considered for this prize.

The prize recipient's research need not be confined to a single paper; it may be contained in several papers. However, the paper (or papers) to be considered for the prize must be submitted while the student is an undergraduate; they cannot be submitted after the student's graduation. The research paper (or papers) may be submitted for consideration by the student or a nominator. All submissions for the prize must include at least one letter of support from a person, usually a faculty member, familiar with the student's research. Publication of research is not required.

The recipients of the prize are to be selected by a standing joint committee of the AMS, MAA, and SIAM. The decisions of this committee are final. The 1998 prize will be awarded for papers submitted for consideration no later than **March 31, 1998,** by (or on behalf of) students who were undergraduates in December 1997.

Nominations and submissions should be sent to:

Morgan Prize Committee
c/o Robert M. Fossum, Secretary
American Mathematical Society
University of Illinois
Department of Mathematics
1409 West Green Street
Urbana, IL 61801-2975

Questions may be directed to the chairperson of the Morgan Prize Committee:

Martha J. Siegel
Department of Mathematics
Towson State University
Towson, MD 21204-7097
telephone 410-830-2980
e-mail: siegel-m@toe.towson.edu

AMERICAN MATHEMATICAL SOCIETY

CALL FOR SUGGESTIONS

There will be a number of contested seats in the 1998 AMS Elections. Your suggestions are wanted by:

The Nominating Committee
for vice-president, trustee,
and five members-at-large of the council

and by

The President
for three Nominating Committee members
and two Editorial Boards Committee members.

In addition

The Editorial Boards Committee
requests suggestions for appointments to
various editorial boards of Society publications.

Send your suggestions for any of the above to:

Robert M. Fossum
American Mathematical Society
Department of Mathematics
University of Illinois
1409 West Green Street
Urbana, IL 61801
e-mail: r-fossum@uiuc.edu

1998 AMS Election

Nominations by Petition

Vice-President or Member-at-Large

One position of vice-president and member of the Council *ex officio* for a term of three years is to be filled in the election of 1998. The Council intends to nominate at least two candidates, among whom may be candidates nominated by petition as described in the rules and procedures.

Five positions of member-at-large of the Council for a term of three years are to be filled in the same election. The Council intends to nominate at least ten candidates, among whom may be candidates nominated by petition in the manner described in the rules and procedures.

Petitions are presented to the Council, which, according to Section 2 of Article VII of the bylaws, makes the nominations. The Council of 23 January 1979 stated the intent of the Council of nominating all persons on whose behalf there were valid petitions.

Prior to presentation to the Council, petitions in support of a candidate for the position of vice-president or of member-at-large of the Council must have at least fifty valid signatures and must conform to several rules and operational considerations, which are described below.

Editorial Boards Committee

Two places on the Editorial Boards Committee will be filled by election. There will be four continuing members of the Editorial Boards Committee.

The President will name at least four candidates for these two places, among whom may be candidates nominated by petition in the manner described in the rules and procedures.

The candidate's assent and petitions bearing at least 100 valid signatures are required for a name to be placed on the ballot. In addition, several other rules and operational considerations, described below, should be followed.

Nominating Committee

Three places on the Nominating Committee will be filled by election. There will be six continuing members of the Nominating Committee.

The President will name at least six candidates for these three places, among whom may be candidates nominated by petition in the manner described in the rules and procedures.

The candidate's assent and petitions bearing at least 100 valid signatures are required for a name to be placed on the ballot. In addition, several other rules and operational considerations, described below, should be followed.

Rules and Procedures

Use separate copies of the form for each candidate for vice-president, member-at-large, or member of the Nominating and Editorial Boards Committees.

1. To be considered, petitions must be addressed to Robert M. Fossum, Secretary, P.O. Box 6248, Providence, Rhode Island 02940, and must arrive by 28 February 1998.
2. The name of the candidate must be given as it appears in the *Combined Membership List* (*CML*). If the name does not appear in the list, as in the case of a new member or by error, it must be as it appears in the mailing lists, for example on the mailing label of the *Notices*. If the name does not identify the candidate uniquely, append the member code, which may be obtained from the candidate's mailing label or the Providence office.
3. The petition for a single candidate may consist of several sheets each bearing the statement of the petition, including the name of the position, and signatures. The name of the candidate must be exactly the same on all sheets.
4. On the next page is a sample form for petitions. Copies may be obtained from the secretary; however, petitioners may make and use photocopies or reasonable facsimiles.
5. A signature is valid when it is clearly that of the member whose name and address is given in the left-hand column.
6. The signature may be in the style chosen by the signer. However, the printed name and address will be checked against the *Combined Membership List* and the mailing lists. No attempt will be made to match variants of names with the form of name in the *CML*. A name neither in the *CML* nor on the mailing lists is not that of a member. (Example: The name Robert M. Fossum is that of a member. The name R. Fossum appears not to be.)
7. When a petition meeting these various requirements appears, the secretary will ask the candidate to indicate willingness to be included on the ballot. Petitioners can facilitate the procedure by accompanying the petitions with a signed statement from the candidate giving consent.

Nomination Petition for 1998 Election

The undersigned members of the American Mathematical Society propose the name of

____

as a candidate for the position of (check one):

- ☐ **Vice President**
- ☐ **Member-at-Large of the Council**
- ☐ **Member of the Nominating Committee**
- ☐ **Member of the Editorial Boards Committee**

of the American Mathematical Society for a term beginning 1 February, 1999.

Name and address (printed or typed)	
	Signature
	Signature
	Signature
	Signature
	Signature
	Signature

Add this Cover Sheet to all of your Academic Job Applications

How to use this form

1. Using the facing page or a photocopy, (or a TEX version which can be downloaded from the e-math "Employment Information" menu, `http://www.ams.org/profession/employ.html`), fill in the answers which apply to *all* of your academic applications. Make photocopies.

2. As you mail each application, fill in the remaining questions neatly on one cover sheet and include it *on top of* your application materials.

The Joint Committee on Employment Opportunities has adopted the cover sheet on the facing page as an aid to job applicants and prospective employers. The form is now available on e-math in a TeX format which can be downloaded and edited. The purpose of the cover form is to aid department staff in tracking and responding to each application.

Mathematics Departments in Bachelor's, Master's and Doctorate granting institutions have been contacted and are expecting to receive the form from each applicant, along with any other application materials they require. Obviously, not all departments will utilize the cover form information in the same manner. Please direct all general questions and comments about the form to: `emp-info@ams.org` or call the Professional Programs and Services Department, AMS, at 800-321-4267 extension 4105.

JCEO Recommendations for Professional Standards in Hiring Practices

The JCEO believes that every applicant is entitled to the courtesy of a prompt and accurate response that provides timely information about his/her status. Specifically, the JCEO urges all institutions to do the following after receiving an application:

(1) Acknowledge receipt of the application—immediately; and
(2) Provide information as to the current status of the application, as soon as possible.

The JCEO recommends a triage-based response, informing the applicant that he/she
(a) is not being considered further;
(b) is not among the top candidates; or
(c) is a strong match for the position.

Academic Employment in Mathematics

AMS STANDARD COVER SHEET

Last Name ______________________

First Name ______________________

Middle Names ______________________

Address *through June 1998* ______________________

Home Phone ______________________

e-mail Address ______________________

Current Institutional Affiliation ______________________

Work Phone ______________________

Highest Degree and Source ______________________

Year of Ph.D. (optional) ______________________

Ph.D. Advisor ______________________

If the Ph.D. is not presently held, date on which you expect to receive __________

Indicate *the mathematical subject area(s) in which you have done research using, if applicable, the 1991 Mathematics Subject Classification printed on the back of this form. If listing more than one number, list first the one number which best describes your current primary interest.*

Primary Interest ______________________

Secondary Interests *optional* ______________________

Give a brief synopsis *of your current research interests (e.g. finite group actions on four-manifolds). Avoid special mathematical symbols and please do not write outside of the boxed area.*

____

____

Most recent, if any, position held *post Ph.D.*

University or Company ______________________

Position Title ______________________ Dates __________

Indicate the position for which you are applying and position posting code, if applicable

____

If unsuccessful for this position, would you like to be considered for a temporary position?

☐ Yes ☐ No If yes, please check the appropriate boxes.

☐ Postdoctoral Position ☐ 2+ Year Position ☐ 1 Year Position

List the names, affiliations, and e-mail addresses *of up to four individuals who will provide letters of recommendation if asked. Mark the box provided for each individual whom you have already asked to send a letter.*

☐ ______________________

☐ ______________________

☐ ______________________

☐ ______________________

This form is provided courtesy of the American Mathematical Society.

This cover sheet is provided as an aid to departments in processing job applications. It should be included with your application material.

Please print or type. Do not send this form to the AMS.

1991 Mathematics Subject Classification

00 General
01 History and biography
03 Logic and foundations
04 Set theory
05 Combinatorics
06 Order, lattices, ordered algebraic structures
08 General mathematical systems
11 Number theory
12 Field theory and polynomials
13 Commutative rings and algebras
14 Algebraic geometry
15 Linear and multilinear algebra, matrix theory
16 Associative rings and algebras
17 Nonassociative rings and algebras
18 Category theory, homological algebra
19 K-theory
20 Group theory and generalizations
22 Topological groups, Lie groups
26 Real functions
28 Measure and integration
30 Functions of a complex variable
31 Potential theory
32 Several complex variables and analytic spaces
33 Special functions
34 Ordinary differential equations
35 Partial differential equations
39 Finite differences and functional equations
40 Sequences, series, summability
41 Approximations and expansions
42 Fourier analysis
43 Abstract harmonic analysis
44 Integral transforms, operational calculus
45 Integral equations
46 Functional analysis
47 Operator theory
49 Calculus of variations, optimal control
51 Geometry
52 Convex and discrete geometry
53 Differential geometry
54 General topology
55 Algebraic topology
57 Manifolds and cell complexes
58 Global analysis, analysis on manifolds
60 Probability theory and stochastic processes
62 Statistics
65 Numerical analysis
68 Computer science
70 Mechanics of particles and systems
73 Mechanics of solids
76 Fluid mechanics
78 Optics, electromagnetic theory
80 Classical thermodynamics, heat transfer
81 Quantum theory
82 Statistical mechanics, structure of matter
83 Relativity and gravitational theory
85 Astronomy and astrophysics
86 Geophysics
90 Economics, operations research, programming, games
92 Biology and other natural sciences, behavioral sciences
93 Systems theory, control
94 Information and communication, circuits

AMERICAN MATHEMATICAL SOCIETY

Leroy P. Steele Prizes

Call for Nominations

The selection committee for this prize requests nominations for consideration for the 1999 award. Further information about this prize can be found in the November 1997 Notices, pp. 1350-1353 (also available at http://www.ams.org/general/prizes.html).

Three Leroy P. Steele Prizes are awarded each year in the following categories: (1) the Steele Prize for Lifetime Achievement: for the cumulative influence of the total mathematical work of the recipient, high level of research over a period of time, particular influence on the development of a field, and influence on mathematics through Ph.D. students; (2) the Steele Prize for Mathematical Exposition: for a book or substantial survey or expository-research paper; and (3) the Steele Prize for Seminal Contributions to Research: for a paper, whether recent or not, that has proved to be of fundamental or lasting importance in its field, or a model of important research.

Nominations with supporting information should be submitted to the Secretary, Robert M. Fossum, Department of Mathematics, University of Illinois, 1409 West Green Street, Urbana, IL 61801-2975. Include a short description on the work that is the basis of the nomination, including complete biographic citations. A curriculum vitae should be included. The nominations will be forwarded by the Secretary to the prize selection committee, which will, as in the past, make final decisions on the awarding of prizes.

Deadline for nominations is March 31, 1998.

AMS
AMERICAN MATHEMATICAL SOCIETY

Mathematics Calendar

The most comprehensive and up-to-date Mathematics Calendar information is available on e-MATH at http://www.ams.org/mathcal/.

February 1998

* 19–21 **Workshop on Current Trends in Research on Mathematics Education**, Centre de Recerca Matemàtica, Bellaterra, Spain.
Coordinators: A. J. Bishop and N. Gorgoriu.
Information: e-mail: crm@crm.es, Web: http://www.crm.es/.

* 20–21 **DIMACS Workshop on Ethical and Social Dimensions of Information Technology**, Princeton University, Princeton, New Jersey.
Sponsor: DIMACS Center.
Organizers: B. Chazelle, J. Feigenbaum, and H. Nissenbaum.
Contacts: H. Nissenbaum, e-mail: helen@phoenix.princeton.edu.
Local Arrangements: S. Barbu, Princeton Univ.; e-mail: barbu@cs.princeton.edu; tel: 609-609-1771.
WWW Information: http://dimacs.rutgers.edu/Workshops/index.html.

March 1998

* 30–April 3 **Third Workshop on Randomized Parallel Computing (Call for Papers)**, Orlando, Florida.
Sponsor: IEEE Computer Society, Technical Committee on Parallel Processing.
Forum: Randomization has played a vital role in the domains of both sequential and parallel computing in the past two decades. This workshop is a forum for bringing together both theoreticians and practitioners who employ randomized techniques in parallel computing.
Topics: Topics include, but are not limited to: network algorithms, PRAM algorithms, architectures, I/O systems, scheduling, network fault tolerance, reconfigurable networks, optical networks, various applications, programming models and languages, implementation experience.
Information: Check the Web using the URLs: http://www.ippsxx.org; http://www.cise.ufl.edu/~raj/WPRC98.html.

April 1998

* 4–5 **Rivière-Fabes Symposium on Analysis and PDE**, University of Minnesota, Minneapolis, Minnesota.
Speakers: N. Garofalo (Purdue Univ.), C. Kenig (Univ. of Chicago), Z. Zhao (Univ. of Missouri-Columbia).
Support: A limited amount of financial support will be available to out-of-town graduate students who participate in the symposium.
Information: M. Jodeit (jodeit@math.umn.edu); http://www.math.umn.edu/arb/RFPoster.html. Mailing address: School of Mathematics, Univ. of Minnesota, 206 Church St., S.E., 127 Vincent Hall, Minneapolis, MN 55455.

* 14–July 17 **Topology Semester**, Centre de Recerca Matemàtica, Bellaterra, Spain.
Organizers: J. Aguadõ, C. Broto, and C. Casacuberta.
Information: e-mail: crm@crm.es; Web: http://www.crm.es/.

May 1998

* 2–4 **Quadratic Forms and Orthogonal Groups: Conference in Honor of the 70th Birthday of O. Timothy O'Meara**, University of Notre Dame, Notre Dame, Indiana.
Principal Speakers: M. Aschbacher, J. Conway, M. Knebusch, T. Y. Lam, G. Margulis, G. Prasad, C. Riehm.
Contributed Papers: A limited number of short talks on topics close to Professor O'Meara's work on quadratic forms and classical groups will be scheduled.
Organizers: M. Dyer, A. Hahn, B. Pollak, and W. Wong.
Information: Write to any one of the organizers at the University of Notre Dame, Notre Dame, IN 46556-5683, or e-mail: wong.1@nd.edu.

* 20–23 **Continued Fractions: From Analytic Number Theory to Constructive Approx-**

This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.

An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.

In general, announcements of meetings and conferences held in North America carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. Meetings held outside the North American area may carry more detailed information. In any case, if there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences in the mathematical sciences should be sent to the Editor of the *Notices* in care of the American Mathematical Society in Providence or electronically to notices@ams.org or mathcal@ams.org.

In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the *Notices* prior to the meeting in question. To achieve this, listings should be received in Providence **six months** prior to the scheduled date of the meeting.

The complete listing of the Mathematics Calendar will be published only in the September issue of the *Notices*. The March, June, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.

The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through e-MATH on the World Wide Web. To access e-MATH, use the URL: http://e-math.ams.org/ (or http://www.ams.org/). (For those with VT100-type terminals or for those without WWW browsing software, connect to e-MATH via Telnet (telnet e-math.ams.org; login and password e-math) and use the Lynx option from the main menu.)

imation, University of Missouri, Columbia, Missouri.
Plenary Speakers: R. A. Askey, B. C. Berndt, D. C. Bowman, A. Bultheel, H. H. Chan, M. H. Ismail, W. B. Jones, L. J. Lange, L. Lorentzen, D. R. Masson, P. Nevai, O. Njastad, W. van Assche.
Program: In addition to the plenary talks, the conference program will include sessions for 25-minute contributed talks.
Abstracts: Abstracts of talks (TEX files by e-mail or hard copies) should be sent by May 8, 1998, to one of the addresses below.
Registration: There will be a conference fee of $25 payable on arrival (graduate students exempt). Registration forms can be obtained upon request by e-mail or from our conference Web site.
Organizing Committee: M. Ashbaugh, B. Berndt, F. Gesztesy, N. Kalton, J. Lange, and I. Verbitsky.
Information: e-mail: cf@math.missouri.edu or by regular mail to F. Gesztesy, Dept. of Mathematics, Univ. of Missouri, Columbia, MO 65211. Conference home page: http://www.math.missouri.edu/~cf/. We have applied for funds to defray travel and lodging costs. Preference will be given to graduate students and recent Ph.D.s. Those interested in applying for such funds please contact F. Gesztesy by April 30, 1998.

*25-27 **The Delft Meeting on Functional Analysis and Nonlinear Partial Differential Equations**, Delft University of Technology, Delft, The Netherlands.
Invited Lecturers: H. Amann (Univ. Zürich), H. Brezis (Univ. Paris VI and Rutgers Univ.), G. Da Prato (Scuola Normale Superiore di Pisa), D. G. de Figueiredo (Unicamp Campinas), J. P. Gossez (Univ. Libre de Bruxelles), H.-Ch. Grunau (Univ. Bayreuth), J. Hulshof (Rijksuniversiteit Leiden), E. Mitidieri (Univ. degli Studi di Trieste), M. Plum (Univ. Karlsruhe), S. I. Pohozaev (Steklov Mathematical Institute, Moscow), P. H. Rabinowitz (Univ. of Wisconsin, Madison), J. Serrin (Univ. of Minnesota, Minneapolis).
Organizers: Ph. Climent and G. Sweers.
Information: http://aw.twi.tudelft.nl/~sweers/meeting/meeting.html. Registration: http://aw.twi.tudelft.nl/~sweers/meeting/register.html. Contact: tini@twi.tudelft.nl.

*27-June 2 **Advanced Course on Classifying Spaces and Cohomology of Groups**, Centre de Recerca Matemàtica, Bellaterra, Spain.
Speakers: W. G. Dwyer (Univ. of Notre Dame, Indianapolis), H.-W. Henn (Univ. d'Strasbourg and Max-Planck Institute).
Organizers: C. Broto and C. Casacuberta (Univ. Autonoma de Barcelona).
Information: e-mail: crm@crm.es; Web: http://www.crm.es/.

June 1998

*4-10 **1998 Barcelona Conference on Algebraic Topology**, Centre de Recerca Matemàtica, Bellaterra, Spain.
Organizers: J. Aguadõ, C. Broto, and C. Casacuberta.
Speakers: A. K. Bousfield (Univ. of Illinois at Chicago), F. R. Cohen (Univ. of Rochester), W. G. Dwyer (Univ. of Notre Dame), Y. Felix (Univ. Catholique de Louvain), J. P. Greenlees (Univ. of Sheffield), J. Lannes (École Polytechnique, Paris), W. Luck (Univ. of Munster), R. Milgram (Stanford Univ.), G. Mislin (ETH, Zurich), D. Ravenel (Univ. of Rochester).
Information: e-mail: crm@crm.es; Web:: http://www.crm.es/.

*7-19 **NATO ASI—1998 CRM Summer School**, Banff, Alberta, Canada.
Organizing Committee: B. Gordon (Oklahoma), J. D. Lewis (Alberta), S. Müller-Stach (Essen), S. Saito (Tokyo), N. Yui (Queen's).
Program: This institute will offer an in-depth account of the arithmetic and geometry of algebraic cycles from several points of view, including arithmetic methods, transcendental methods, topological methods, and motives and K-theory methods.
Speakers: A. Beilinson (IAS), S. Bloch (Chicago), J.-L. Colliot-Thélène (Paris-Sud), H. Esnault (Essen), E. Friedlander (Northwestern) [tentative], P. Gajer (Johns Hopkins), B. van Geemen (Torino), H. Gillet (Illinois), B. Gordon (Oklahoma), M. Green (UCLA), U. Jannsen (Köln), A. Langer (Münster), B. Lawson (Stony Brook), J. Lewis (Alberta), S. Müller-Stach (Essen), K. Murty (Toronto), J. Nekovář (Cambridge), D. Ramakrishnan (Cal Tech), W. Raskind (Southern Cal), M. Saito (Kobe), S. Saito (Tokyo) T. Saito (Tokyo), C. Schoen (Duke), A. Scholl (Durham, UK), C. Soulé (IHES), V. Voevodsky (Northwestern), N. Yui (Queen's), D. Zagier (MPIM Bonn), Y. Zarhin (Penn State).
Information: And application form: L. Pelletier, CRM, Université de Montreal, C.P. 6128, Succ. Centre-ville, Montréal (Quebec), Canada H3C 3J7; e-mail: Banff98@CRM.UMontreal.ca; Web: http://www.CRM.UMontreal.ca/Banff98.html.

*15-17 **Conference on Advances in Applied and Computational Mathematics**, Mathematical Sciences Research Institute, Berkeley, California.
Theme: A conference will be held to celebrate the birthday of Alexandre Chorin. The theme of the conference will be applied mathematics and scientific computing and will cover topics in fluid mechanics, combustion, turbulence, materials sciences, statistical mechanics, and related issues. The mathematical community is warmly invited to attend.
Invited Speakers: Include Chorin's former students and S. Abarbanel (Tel Aviv), G. I. Barenblatt (Berkeley), M. Benartzi (Jerusalem), A. Ghoniem (MIT), J. Glimm (Stonybrook), O. Hald (Berkeley), P. Lax (Courant), J.-L. Lions (Paris), A. Majda (Courant), C. Morawetz (Courant), A. K. Oppenheim (Berkeley), R. Piva (Rome), R. Somerville (Scripps), O. Widlund (Courant).
Information: Send e-mail to berkeley-98@math.berkeley.edu or regular mail to: Conference on Advances in Applied and Computational Mathematics, MailStop 50A-2152, Lawrence Berkeley National Laboratory, Berkeley, CA 94720. Hotel and transportation information will be available online in the coming months at http://math.berkeley.edu/berkeley-98/chorin.html.
Financial Support: A limited amount of funding is available for partial support of people wishing to attend. Students, recent Ph.D.s, women, and minorities are particularly encouraged to apply. To apply for funding, please send a letter explaining your interest in the conference together with a vita or bibliography and budget for travel/living expenses. If you are a student, also solicit a letter from a faculty advisor. All information should be received by February 15, 1998. MSRI is committed to the principles of Equal Opportunity and Affirmative Action.

*24-26 **Centennial Congress on M. C. Escher (1898-1972)**, University of Rome "La Sapienza", Rome, Italy.
Contributed Papers: Submit a 1-page abstract on a topic directly relating to Escher's life or work to USA address below by February 15, 1998. Only a small number can be accepted for presentation; notification of acceptance by March 15, 1998.
Special Session: Small special session following on June 27-28, 1998, in Ravello (Amalfi coast), Italy. Only those registered for the Rome congress may participate in the special session in Ravello. Reservations no later than February 1998.
Registration: Registration fee US$120. Deadline is April 1, 1998. Registration form available from: USA: e-mail: escher98@moravian.edu; fax: 610-861-1462; post: D. Schattschneider, Moravian College, 1200 Main St., Bethlehem, PA 18018-6650; Italy: e-mail: escher98@mat.uniroma1.it; fax: 39-6-44701007; post: M. Emmer, Dipartimento di Matematcia, Université de Roma "La Sapienza", Piazzale A. Moro, 00185, Rome, Italy.
Information: Contact: Centro Universitario Europeo per i Beni Culturali, Villa Rufolo, 84010 Ravello (SA), Italy; tel: 39-89-858101, 857669; fax: 39-89-857711; e-mail: cuebc@amalficoast.it; Web: http://www.mat.uniroma1.it/escher98/.

*29-July 1 **Parallel Computing and Algorithms in Economics and Finance (Call for Papers)**, University of Cambridge, Cambridge, United Kingdom.
Organizers: E. J. Kontoghiorghes (Institut d'informatique, Neuchatel), H.-H. Naegeli (Institut d'informatique, Neuchatel).
Theme: This session will deal with papers addressing methods for solving economic and finance problems on parallel computers. Contributions on parallel computing and algorithms that potentially can be used in economics and finance are welcome.

Sessions themes include, but are not limited to: parallel computing applications to economics and finance, software developments, tools and parallel algorithms useful for large-scale estimation problems.

Abstracts: Authors wishing to present a paper are invited to submit an abstract (maximum two pages) by January 10, 1998. Contributions by post (hard copy), e-mail, or fax should be submitted to: E. J. Kontoghiorghes, Institut d'informatique, Universite de Neuchatel, Rue Emile-Argand 11, CH-2007 Neuchatel, Switzerland; e-mail: erricos.kontoghiorghes@info.unine.ch; fax: +41-0-32-718-27-01.

July 1998

* 9–10 **Workshop on New Methods in Applied and Computational Mathematics (NEMACOM'98)**, Hervey Bay, Queensland, Australia.

Information: Because of clashes with some other conferences (SIAM annual meeting, EMAC'98 in Adelaide), it has been decided to change the date of NEMACOM'98 to the 9th and 10th of July 1998. For further information and updates about the conference, see the Web page: http://www.cs.tamu.edu/faculty/oliveira/nemacom98/, or e-mail: nemacom98@cs.tamu.edu.

* 20–25 **Exactly Solvable Models in Mathematical Physics**, Chelyabinsk University of Technology, Chelyabinsk, Russia

Workshop Topics: Multidimensional integrability, tetrahedron equation, functional tetrahedron equation and quantization of its solutions, connection with quantum strings; low-dimensional integrability.

Information: Contact I. G. Korepanov, e-mail: igor@prima.tu-chel.ac.ru.

* 26–August 1 **XV Escola de Algebra**, Canela, Rio Grande do Sul, Brazil.

Organizers: Instituto de Matematica, Univ. Federal do Rio Grande do Sul.

Program: This is a broad meeting including all areas of research in algebra, such as commutative and non-commutative algebra, ring and group theory, algebraic geometry, number theory, and representation theory. The planned activities include several minicourses, conference cycles, talks of 50 minutes and research communications of 20–25 minutes.

Information: Anyone interested in giving a communication at the conference should contact the coordinator of the Organizing Committee, M. Ferrero; e-mail: algebra@mat.ufrgs.br.

* 29–August 7 **Frontiers of Combinatorics**, Los Alamos National Laboratory, Los Alamos, New Mexico.

Description: The objectives of the workshop are (1) to evaluate new combinatorial problems arising from the achievements and potentialities of molecular biology and (2) to discern profitable new developments of combinatorial theories. Topics for discussion will include, for example, combinatorial designs, phylogenetic trees, categorical sequences and pertinent enumerations, classifications, and construction techniques.

Application: A preliminary version of a manuscript is due April 31, 1998.

Organizer: D. C. Torney, 505-667-9452, dct@lanl.gov.

August 1998

* 7–10 **European Summer School: Markov Chain Monte Carlo Methods**, Rebild, Denmark.

Topics: An advanced course on MCMC: Probabilistic aspects of MCMC, Bayesian statistics and MCMC, complex models and MCMC, BUGS/graphical models, new advances/special topics, perfect simulation.

Information: http://www.maths.nott.ac.uk/hsss/Workshops/summerschool.html; or L. G. Nielsen, Aalborg University, Denmark; fax: +45-98-15-81-29; e-mail: grubbe@math.auc.dk.

* 10–17 **The 4th International Conference on Theory of Groups (GROUP-KOREA 1998)**, Pusan National University, Pusan, Korea.

Speakers: B. Amberg (Mainz), Y. G. Baik (Pusan), L. Bokut (Novosibirsk), M. Bridson (Oxford), C. Campbell (St. Andrews), M. Dunwoody (Southampton), S. Gersten (Uta), F. Grunerwald (Duesseldorf), K. Gupta (Manitoba), N. Gupta (Manitoba), M. Herzog (Tel Aviv), J. Howie (Edinburgh), N. Ito (Meijo) G. Kim (Kyungsan), C. Maclachlan (Aberdeen), J. Mennicke (Bielefeld), A. Yu. O'lshanski (Moscow), C. Praeger (Nedlands), S. Pride (Glasgow), A. Rhemtull (Edmonton), D. Robinson (Urbana-Champaign), G. Rosenberger (Dortmund), J. Saxl (Cambridge), H. Sim (Pusan), J. Stallings (Berkeley), F. Tang (Waterloo), J. Wilson (Birmingham), M. Xu (Beijing), E. Zermanov (Yale), H. Zieschang (Bochum), B. Zimmermann (Trieste), A. Zvi (Ramat-Ganm).

Contacts: A. Kim, Organizing Committee, Groups-Korea 1998, Dept. of Mathematics, Pusan National Univ., Pusan 609-735, Korea; tel: 051-510-2200; fax: 051-581-1458; e-mail: ackim@arirang.pusan.ac.kr; S. Pride, e-mail: sjp@maths.gla.ac.uk; D. Johnson, e-mail: dlj@maths.nott.ac.uk.

Information: For more information about the program, registration, submission of abstracts, hotels, etc., visit the conference home page at http://arirang.math.pusan.ac.kr/.

* 15–23 **International Workshop on Nonlinear and Improperly Posed Problems**, Kocaeli, Turkey.

Organizing Institutions: Russian Academy of Science, Institute of Mathematical Modelling, Russia; Moscow State University, Russia; University of Nebraska, Lincoln, Nebraska; Colorado State University, Fort Collins, Colorado; University of Kocaeli, Izmit, Turkey; Isik University, Maslak-Istanbul, Turkey.

Focus: The aim of the workshop is to discuss new classes of nonlinear and ill-posed problems arising from engineering and technology. These involve new nonlinear equations of mathematical physics, nonclassical boundary conditions, and new inverse and improperly posed problems. Both analysis and computational methods for these problems will be considered.

Abstracts: A 600-word abstract should be sent to one of the following: N. Ardelyan, Dept. of Computational Mathematics and Cybernetics, Moscow St. Univ., Leninskie, Gori, Moscow, Russia, ardel@cs.msu.su; S. Cohn, Dept. of Math. and Stat., Univ. of Nebraska, 834 Oldfather Hall, Lincoln, NE 68588-0323, scohn@math.unl.edu; P. DuChateau, Dept. of Math., Colorado State Univ., Fort Collins, CO 80523-0001, pauld@lagrange.math.colostate.edu; A. Hasanov, Applied Mathematical Sciences Research Center, Kocaeli Univ., Ataturk Bulvari, 41300, Izmit, Turkey, kcluniv2@turnet.net.tr; M. Idemen, Isik Univ., Buyukdere caddesi, 80670 Maslak-Istanbul, Turkey, idemen@ISIKUN.edu.tr; W. Rundell, Dept. of Mathematics, Texas A&M Univ., College Station, TX 77843-3368, rundell@math.tamu.edu; A. A. Samarskii, Chair of Organizing Committee, Russian Academy of Science, Institute of Mathematical Modelling, Russia; T. Shores, Dept. of Math. and Stat., Univ. of Nebraska, 834 Oldfather Hall, Lincoln, NE 68588-0323, tshores@math.unl.edu.

* 24–29 **Fields Institute Workshop on Mathematical Physics of Polymers and Percolation**, The Fields Institute for Research in Mathematical Sciences, Toronto, Ontario, Canada.

Scientific and Organizing Committee: J. Cardy (Univ. of Oxford), F. den Hollander (Univ. of Nijmegen), G. Slade (McMaster Univ.), S. Whittington (Univ. of Toronto).

Invited Speakers: M. Aizenman (Princeton Univ.), M. Batchelor (Australian National Univ.), M. Bousquet-Melou (Univ. Bordeaux 1), J. Cardy (Oxford Univ.), J. T. Chayes (Microsoft Research and UCLA), A. J. Guttmann (Univ. of Melbourne), F. den Hollander (Univ. of Nijmegen), E. J. Janse van Rensburg (York Univ.), G. Lawler (Duke Univ.), A. Owczarek (Univ. of Melbourne), A. Pisztora (Carnegie Mellon Univ.), Y. Saint-Aubin (CRM, Univ. de Montreal), H. Saleur (Univ. of Southern California), G. Slade (McMaster Univ.), T. Spencer (Institute for Advanced Study), S. G. Whittington (Univ. of Toronto). Each speaker will give two lectures. Details regarding registration and accommodations will appear in a future announcement.

Funding: There are limited funds available to assist attendance of graduate students and postdocs. Please contact slade@mcmaster.ca for further information.

Organizers: The workshop is being organized in connection with The Fields Institute program in Probability and Its Applications, August 1998 – June 1999, and has been endorsed by the International Association of Mathematical Physicists.

Information: See the program Web site http://www.math.yorku.ca/Probability/Fields.html or write to: The Fields Institute for Research in Mathematical Sciences, 222 College Street, Second Floor, Toronto, Ontario, M5T 3J1, Canada; tel: 416-348-9710; fax: 416-348-9385; e-mail: probability@fields.utoronto.ca.

September 1998

* 2–5 **1998 Conference on Computational Physics (CCP 1998)**, Granada, Spain.

Program: The CCP 1998, to be held at the Exhibition and Conference Centre in Granada, initiates a new series that continues the tradition of both the APS-EPS Physics Computing conferences (Boston 1989, Amsterdam 1990, San José 1991, Prague 1992, Albuquerque 1993, Lugano 1994, Pittsburgh 1995, Krakow 1996, Santa Cruz 1997), and the Asian ICCP conferences (Beijing 1988 and 1993, Taiwan 1995, and Singapore 1997), some of which have also been supported by the IUPAP. The CCP 1998 is planned to cover ALL fields of computational physics and, in particular, modelling collective phenomena in complex systems, including biology, chemistry, economy, environmental sciences, geology and sociology. A principal aim is to favor contacts and the exchange of ideas and methods between these different fields of science. Therefore, the final program will consist of invited lectures and other contributions on computer-aided simulation and modelling and their applications. In addition to invited lectures and oral and poster contributions, we are planning industrial and commercial exhibits and technical presentations, e.g., educational and scientific software, computers and workstations, and emerging technologies concerning computing and networking. Science is expected to meet computation at the CCP 1998! A social program and postconference tours will be offered.

Sponsor: The European Physical Society, the International Union of Pure and Applied Physics, and the American Physical Society.

Organizers: EPS Computational Physics Board and the Institute Carlos I for Theoretical and Computational Physics of the University of Granada.

Call for Papers: A camera-ready one-page abstract, including title, author name(s), affiliation(s) and address(es), should be submitted no later than May 1, 1998, either by airmail or preferably by e-mail using LaTeX format. Further instructions are available at http://dalila.ugr.es/~ccp1998/ or by mail or e-mail (see addresses below) upon request. Contributions accepted will be allocated either short oral or poster presentations. All accepted abstracts are planned to appear in a pre-conference book. It is likely that conference proceedings including selected contributions will be published.

Information: Conference on Computational Physics–CCP 1998, c/o J. Marro, Instituto Carlos I, Facultad de Ciencias, Universidad de Granada, E-18071 Granada, Spain; e-mail: ccp1998@goliat.ugr.es; telefax: +34-58-242-862 and 246-387; tel: -242-860; URL: http://dalila.ugr.es/~ccp1998/ and http://www.ugr.es/ccp1998/. Updated information may also be obtained by sending an e-mail message with any subject and no content (nobody will read it) to the address: info_ccp@landau.ugr.es.

* 17–20 **The Third Annual Conference on Research in Undergraduate Mathematics Education**, Century Center, South Bend, Indiana.

Focus: This conference is a forum for researchers in collegiate mathematics education and includes the following themes: results of current research, contemporary theoretical perspectives and research paradigms, application of learning theory to teaching practice, technology in mathematics learning, and general issues in the psychology of mathematics education as it pertains to the study of undergraduate mathematics. The program will include plenary addresses, invited speakers, panel discussions, and contributed paper sessions.

Call for Proposals: One-page proposals for papers should be submitted via e-mail by May 1 to: G. Tolias, tolias@calumet.purdue.edu.

Conference Chairpersons: G. Tolias (Purdue Univ., Calumet); A. Brown (Indiana Univ., South Bend).

Co-Organizers: J. Clark, E. Dubinsky, J. Kleiman, D. Mathews, M. McDonald, and D. Vidakovic.

Sponsor: Exxon Education Foundation.

Hosts: Indiana University, South Bend; Southwestern Michigan College.

Conference WWW Page: http://galois.oxy.edu/mickey/rume98.html.

October 1998

* 2–4 **Midwest Conference on the History of Mathematics (with a special session on History of Logic)**, Iowa State University, Ames, Iowa.

Organizing Committee: I. Anellis, D. Cameron, D. Kullman, J. Murdock, and J. D. Wine.

Purpose: The purpose of this conference is to encourage study of the history of mathematics at all levels and for all purposes. You are welcome to submit a paper whether you are a historian, a mathematician with an interest in the history of your field, or a teacher interested in the use of history of mathematics in the classroom. Possible topics include mathematical biography, history of a topic in mathematics or logic, and the use of history in teaching mathematics. Contributed papers may be 25 minutes or 50 minutes. Abstracts may be submitted, by January if possible, through the Web page (below), by e-mail to jmurdock@iastate.edu, or to J. Murdock, Mathematics Department, Iowa State University, Ames, IA 50011.

Information: See http://www.math.iastate.edu/jmurdock/conference.html, or write or send e-mail to the addresses given above.

* 5–10 **Fields Institute Workshop on Hydrodynamic Limits**, The Fields Institute for Research in Mathematical Sciences, Toronto, Ontario, Canada.

Organizing Committee: S. Feng (McMaster Univ.), A. Lawniczak (Univ. of Guelph), S. R. S. Varadhan (Courant Institute).

Invited Speakers: B. Boghosian (Boston Univ.), A. De Masi (Univ. di L'Aquila), T. Funaki (Univ. of Tokyo), J. Lebowitz (Rutgers Univ.), D. Levermore (Univ. of Arizona), J. Quastel (Univ. of Toronto), F. Rezakhanlou (Univ. of California, Berkeley), T. Seppalainen (Iowa State Univ.), H. Spohn (Ludwig Maximilians Univ.), H. T. Yau (Courant Institute). Poster and discussion sessions are planned. Details regarding these sessions, registration, and accommodations will appear in a future announcement.

Information: See the program Web site http://www.math.yorku.ca/Probability/Fields.html or write to: The Fields Institute for Research in Mathematical Sciences, 222 College Street, Second Floor, Toronto, Ontario, M5T 3J1, Canada; tel: 416-348-9710; fax: 416-348-9385; e-mail: probability@fields.utoronto.ca.

* 25–29 **Fields Institute Workshop on Monte Carlo Methods**, The Fields Institute for Research in Mathematical Sciences, Toronto, Ontario, Canada.

Organizing Committee: N. Madras (York Univ.), R. Neal (Univ. of Toronto), J. Rosenthal (Univ. of Toronto).

Invited Speakers: B. Berg (Florida State Univ.), D. M. Ceperley (Univ. of Illinois), J. A. Fill (Johns Hopkins Univ.), D. Frenkel (FOM Institute, The Netherlands), P. Green (Univ. of Bristol), K. Jansen (CERN), A. D. Kennedy (Florida State Univ.), X.-L. Meng (Univ. of Chicago), J. Moller (Aalborg Univ.), D. Murdoch (Queen's Univ.), G. O. Roberts (Cambridge Univ.), A. Sinclair (Univ. of California, Berkeley), A. D. Sokal (New York Univ.), S. G. Whittington (Univ. of Toronto), D. B. Wilson (Institute for Advanced Study).

Focus: The focus of the workshop will be on Monte Carlo methods (mostly Markov chain methods) that have been used effectively in statistical physics and/or statistics and which are based on ideas that seem to be widely applicable. Speakers have backgrounds in physics, statistics, and probability, and the program is intended to promote discussion across these disciplines. The emphasis will be on new methods, practical applications, and analysis of methods.

Information: Details regarding registration and accommodations will appear in a future announcement. For further information about the workshop and the program, please see the program Web site `http://www.math.yorku.ca/Probability/Fields.html` or write to: The Fields Institute for Research in Mathematical Sciences, 222 College Street, Second Floor, Toronto, Ontario, M5T 3J1, Canada; tel: 416-348-9710; fax: 416-348-9385; e-mail: `probability@fields.utoronto.ca`.

December 1998

* 14-18 **First International Conference on Semigroups of Operators, Theory and Applications**, Marriott Hotel, Newport Beach, California.

Purpose: As indicated in the title, the conference will deal with both the abstract theory of semigroups of operators as well as applications; the latter can include relevant branches of science and engineering: for example, control engineering (structures, aeroelasticity) and mathematical physics, in addition to areas within mathematics, such as PDE and stochastic processes.

International Program Committee: W. Arendt (Germany), A. V. Balakrishnan (USA), C. J. K. Batty (UK), R. Datko (USA), H. Komatsu (Japan), C. S. Kubrusly (Brazil), I. Lasiecka (USA), G. Lumer (Belgium), Yu. Lyubich (Israel), J. Mazon (Spain), A. McIntosh (Australia), R. Nagel (Germany), S. Y. Shaw (Taiwan), B. Simon (USA), Q. P. Vu (USA, chairman).

Abstracts: Please submit title and a 200-word abstract to: Q. P. Vu, Dept. of Mathematics, 321 Morton Hall, Ohio University, Athens, OH 45701-2979; e-mail: `qvu@bing.math.ohiou.edu`, before June 14, 1998.

* 15-18 **International Conference on Nonlinear Programming and Variational Inequalities**, Hong Kong

Objective: The conference aims to review and discuss recent advances and promising research trends in some areas of nonlinear programming and variational inequalities.

Topics: Nonlinear complementarity problems, variational inequality problems, nonsmooth optimization problems, minimax problems, multilevel optimization problems, structured optimization problems, quadratic and nonquadratic methods.

Information: e-mail: `maopt@cityu.edu.hk`; `http://www.cityu.edu.hk/ma/`.

The following new announcements will not be repeated until the criteria in the next to the last paragraph at the bottom of the first page of this section are met.

July 2000

* 19-26 **The Third World Congress of Nonlinear Analysts (WCNA-2000)**, Catania, Italy.

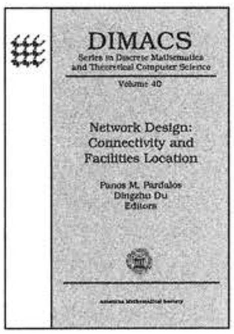
Focus: The scientific program will consist of several invited lectures, organized sessions, and symposia covering recent trends in nonlinear problems (by academic, industrial, and government experts) arising in such diverse disciplines as: aerospace sciences, atmospheric sciences, biological sciences, chemical sciences, cosmological sciences, economics, engineering & technological sciences, environmental sciences, geophysical sciences, medical & health sciences, numerical & computational sciences, oceanographic sciences, physical sciences, social sciences, and mathematical sciences.

There will be opportunities to present short communications, organize informal seminars, and propose special sessions. All accepted articles will be published in the proceedings of the WCNA-2000 should room permit. More details concerning travel facilities, social events, preregistration, accommodations, submission of abstracts, scientific program, and invited lectures will be provided in the second announcement, which will be sent to all interested parties **after** December 1998.

Information: Write to: WCNA-2000, Florida Institute of Technology, Applied Mathematics Program, Melbourne, FL 32901; e-mail: `dkermani@winnie.fit.edu`; or fax: 407-674-7412.

New Publications Offered by the AMS

Applications



Network Design: Connectivity and Facilities Location

Panos M. Pardalos, *University of Florida, Gainesville,* and **Dingzhu Du**, *University of Minnesota, Minneapolis,* Editors

Connectivity and facilities location are two important topics in network design, with applications in data communication, transportation, production planning, and VLSI designs. There are two issues concerning these topics: design and optimization. They involve combinatorial design and combinatorial optimization. This volume features talks presented at an interdisciplinary research workshop held at DIMACS in April 1997. The workshop was attended by leading theorists, algorithmists, and practitioners working on network design problems.

Finding the solution of design problems and the optimal or approximate solution of the related optimization problem are challenging tasks because no polynomial time algorithms are known. Such problems include some variations of Steiner tree problems (such as multiple-connected Steiner network, independent flow problem, and subset-interconnection designs), topology network design, nonlinear assignment problems (such as quadratic assignment problems), problems in facilities location and allocation, and network problems appearing in VLSI design.

The focus of this book is on combinatorial, algorithmic, and applicational aspects of these problems. The volume would be suitable as a textbook for advanced courses in computer science, mathematics, engineering, and operations research.

This text will also be of interest to those working in discrete mathematics and combinatorics.

Contents: S. Arora, Nearly linear time approximation schemes for Euclidean TSP and other geometric problems; **R. Battiti** and **A. Bertossi**, Differential greedy for the 0-1 equicut problem; **M. Brazil**, **D. A. Thomas**, and **J. F. Weng**, Gradient-constrained minimal Steiner trees; **S.-W. Cheng**, The Steiner tree problem for terminals on the boundary of a rectilinear polygon; **D. Cieslik**, Using Hadwiger numbers in network design; **C. Duin**, Reducing the graphical Steiner problem with a sensitivity test; **A. Eisenblätter**, A frequency assignment problem in cellular phone networks; **T. Erlebach**, **K. Jansen**, **C. Kaklamanis**, and **P. Persiano**, An optimal greedy algorithm for wavelength allocation in directed tree networks; **K. Holmqvist**, **A. Migdalas**, and **P. M. Pardalos**, A GRASP algorithm for the single source uncapacitated minimum concave-cost network flow problem; **K. Jansen**, Approximation results for the optimum cost chromatic partition problem; **M. Karpinski** and **A. Zelikovsky**, Approximating dense cases of covering problems; **S. Guha** and **S. Khuller**, Connected facility location problems; **N. Deo** and **N. Kumar**, Constrained spanning tree problems: Approximate methods and parallel computation; **W.-J. Li** and **J. M. Smith**, Star, grid, ring topologies in facility location & network design; **S. O. Krumke**, **M. V. Marathe**, **H. Noltemeier**, **R. Ravi**, and **S. S. Ravi**, Network improvement problems; **M. V. Marathe**, **R. Ravi**, and **R. Sundaram**, Improved results on service-constrained network design problems; **R. A. Murphey**, **P. M. Pardalos**, and **L. Pitsoulis**, A greedy randomized adaptive search procedure for the multitarget multisensor tracking problem; **W. B. Powell** and **Z.-L. Chen**, A generalized threshold algorithm for the shortest path problem with time windows; **J. D. P. Rolim** and **L. Trevisan**, A case study of de-randomization methods for combinatorial approximation algorithms; **S. Voß** and **K. Gutenschwager**, A chunking based genetic algorithm for the Steiner tree problem in graphs; **D. M. Warme**, A new exact algorithm for rectilinear Steiner trees; **P.-J. Wan** and **A. Pavan**, A scalable TWDM lightwave network based on generalized de Bruijn digraph; **J. F. Weng**, A new model of generalized Steiner trees and 3-coordinate systems; **R. Wessäly**, A model for network design; **C. S. Adjiman**, **C. A. Schweiger**, and **C. A. Floudas**, Nonlinear and mixed-integer optimization in chemical process network systems; **M. Brazil**, **J. H. Rubinstein**, **D. A. Thomas**, **J. F. Weng**, and **N. C. Wormald**, Shortest networks on spheres.

DIMACS: Series in Discrete Mathematics and Theoretical Computer Science, Volume 40

March 1998, 461 pages, Hardcover, ISBN 0-8218-0834-6, LC 97-45788, 1991 *Mathematics Subject Classification*: 03B05, 90A05, 68T15, 68Q42, 90C27, 90C30, 90B40, 68T01, 68Q15; 68Q22, 68Q25, 68P10, **Individual member $47**, List $79, Institutional member $63, Order code DIMACS/40N

Topics in Semidefinite and Interior-Point Methods

Panos M. Pardalos, *University of Florida, Gainesville*, and **Henry Wolkowicz**, *University of Waterloo, ON, Canada*, Editors

This volume contains refereed papers presented at the workshop on "Semidefinite Programming and Interior-Point Approaches for Combinatorial Optimization Problems" held at The Fields Institute in May 1996. Semidefinite programming (SDP) is a generalization of linear programming (LP) in that the nonnegativity constraints on the variables is replaced by a positive semidefinite constraint on matrix variables. Many of the elegant theoretical properties and powerful solution techniques follow through from LP to SDP. In particular, the primal-dual interior-point methods, which are currently so successful for LP, can be used to efficiently solve SDP problems.

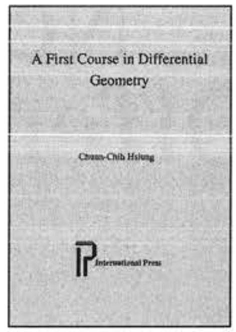
In addition to the interesting theoretical and algorithmic questions, SDP has found many important applications in combinatorial optimization, control theory and other areas of mathematical programming. SDP is currently a very hot area of research. The papers in this volume cover a wide spectrum of recent developments in SDP. The volume would be suitable as a textbook for advanced courses in optimization.

Contents: *Theory:* **A, Shapiro**, Optimality conditions and sensitivity analysis of cone-constrained and semi-definite programs; **L. Porkolab** and **L. Khachiyan**, Testing the feasibility of semidefinite programs; **M. V. Ramana**, Polyhedra, spectrahedra, and semidefinite programming; **L. Faybusovich**, Infinite-dimensional semidefinite programming: Regularized determinants and self-concordant barriers; *Applications:* **M. Laurent**, A tour d'horizon on positive semidefinite and Euclidean distance matrix completion problems; **S. E. Karisch** and **F. Rendl**, Semidefinite programming and graph equipartition; **C. R. Johnson**, **B. K. Kroschel**, and **M. Lundquist**, The totally nonnegative completion problem; **J. Gu**, The multi-SAT algorithm; **M. R. Emamy-K.**, How efficiently can we maximize threshold pseudo-Boolean functions?; **G. Xue**, **D.-Z. Du**, and **F. K. Hwang**, Faster algorithm for shortest network under given topology; **A. Mockus**, **J. Mockus**, and **L. Mockus**, Bayesian heuristic aproach (BHA) and applications to discrete optimization; **B. Mirkin**, Approximation clustering: A mine of semidefinite programming problems; *Algorithms:* **K. M. Anstreicher** and **M. Fampa**, A long-step path following algorithm for semidefinite programming problems; **C. Helmberg** and **R. Weismantel**, Cutting plane algorithms for semidefinite relaxations; **E. de Klerk**, **C. Roos**, and **T. Terlaky**, Infeasible-start semidefinite programming algorithms via self-dual embeddings; **S. Lucidi** and **L. Palagi**, Solution of the trust region problem via a smooth unconstrained reformulation.

Fields Institute Communications, Volume 18

February 1998, 250 pages, Hardcover, ISBN 0-8218-0825-7, LC 97-43573, 1991 *Mathematics Subject Classification*: 68Q10, 90C06, 90C27, 68Q25, 90C05, 90C25, 90C30, 90C10, **Individual member $41,** List $69, Institutional member $55, Order code FIC/18N

Geometry and Topology



A First Course in Differential Geometry

Chuan C. Hsiung, *Lehigh University, Bethlehem, PA*

This book is designed to introduce differential geometry to beginning graduate students and advanced undergraduates. The text covers the traditional topics: curves and surfaces in a three-dimensional Euclidean space. Unlike most classical books on the subject, however, the author pays more attention to the relationships between local and global properties rather than to local properties only.

Most global theorems for curves and surfaces in the book can be extended to either higher-dimensional spaces or more general curves and surfaces or both. Geometric interpretations are given along with analytic expressions. This enables students to make use of geometric intuition—a precious tool for studying geometry and related problems.

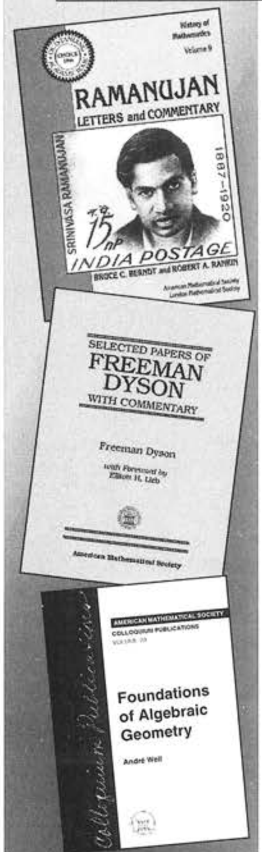
International Press publications are distributed worldwide, except in Japan, by the American Mathematical Society.

Contents: Euclidean spaces; Curves; Local theory of surfaces; Global theory of surfaces; Appendix 1. Proof of existence theorem 1.5.1, chapter 2; Appendix 2. Proof of the first part of theorem 7.3, chapter 3; Bibliography; Answers and hints to exercises; Index.

International Press

November 1997, 343 pages, Hardcover, ISBN 1-57146-046-2, LC 97-073201, 1991 *Mathematics Subject Classification*: 53-01, **All AMS members $36,** List $45, Order code INPR/24N

PUBLICATIONS *of* CONTINUING INTEREST



World-Class Mathematicians and Their Work

The world's great mathematicians publish with the AMS. We are pleased to present to you a selection of such titles. The Society is dedicated to continuing to bring the work of outstanding scientists to the mathematical community.

The Calculus of Variations in the Large

M. Morse

Colloquium Publications, Volume 18; 1934; ISBN 0-8218-1018-9; 368 pages; Softcover; **Individual member $33,** List $55, Institutional member $44, Order Code COLL/18CI82

Dynamical Systems

George D. Birkhoff

His research in dynamics constitute the middle period of Birkhoff's scientific career, that of maturity and greatest power.

—***Yearbook of the American Philosophical Society***

Colloquium Publications, Volume 9; 1927; ISBN 0-8218-1009-X; 305 pages; Softcover; **All AMS members $23,** List $29, Order Code COLL/9CI82

Foundations of Algebraic Geometry

André Weil, *Institute for Advanced Study, Princeton, NJ*

Colloquium Publications, Volume 29; 1946; ISBN 0-8218-1029-4; 363 pages; Softcover; **All AMS members $24,** List $29, Order Code COLL/29CI82

Fourier Transforms in the Complex Domain

Raymond E. A. C. Paley and **Norbert Wiener**

Colloquium Publications, Volume 19; 1934; ISBN 0-8218-1019-7; 183 pages; Softcover; **Individual member $27,** List $45, Institutional member $36, Order Code COLL/19CI82

The Kernel Function and Conformal Mapping

S. Bergman

Mathematical Surveys and Monographs, Volume 5; 1950; ISBN 0-8218-1505-9; 257 pages; Softcover; **Individual member $27,** List $45, Institutional member $36, Order Code SURV/5CI82

Ramanujan: Letters and Commentary

Bruce C. Berndt, *University of Illinois, Urbana*, and **Robert A. Rankin**, *University of Glasgow, Scotland*

This commendable collection ... is a unique contribution to the history of mathematics ... It has brought together precious documents scattered in many places and provides the reader with a wealth of interesting matters related to one of the luminaries in the world of mathematics.

—***CHOICE***

Delightful reading ... a useful reference on English analysts and number theorists of Hardy's time ... has many pictures, some of them quite marvelous ... What Berndt and Rankin have done is to make a great deal of primary material available to ... scholars.

—***Zentralblatt für Mathematik***

Berndt, with the experience he has gained editing Ramanujan's notebooks, and Rankin, one of the veterans in this field, who knew Hardy, Littlewood, Watson, and other British contemporaries of Ramanujan, have combined perfectly to produce this book.

—***American Mathematical Monthly***

This book helps us realize who helped and how in more detail than was possible before reading some of the letters.

—***Mathematical Reviews***

The book is very readable, contains much material not available elsewhere, and can be read at a variety of levels ... highly recommended to anyone with an interest in Ramanujan.

—***Bulletin of the London Mathematical Society***

Co-published with the London Mathematical Society. Members of the LMS may order directly from the AMS at the AMS member price. The LMS is registered with the Charity Commissioners.

Customers in India, please contact *Affiliated East-West Press Private Ltd.*, 62-A Ornes Road, Kilpauk, Madras, 600 010, INDIA; Fax 044-825-7258.

History of Mathematics, Volume 9; 1995; ISBN 0-8218-0287-9; 347 pages; Hardcover; **All AMS members $47,** List $59, Order Code HMATH/9CI82

Selected Papers of Freeman Dyson with Commentary

Freeman Dyson, *Institute for Advanced Study, Princeton, NJ*

All papers are carefully reproduced. The value of the book is greatly enhanced by author's commentary on each selected paper.

—***Zentralblatt für Mathematik***

Wide in subject matter, contains many deep, often classic, results, and is written in an impeccable style ... has a wealth of topics ... Dyson's versatility, mathematical strength and depth are well known and his comments and sometimes provocative opinions are thought-inspiring and a pleasure to read. I cannot do better than recommending this volume by repeating the old saying: Study the masters!

—***Aernout C. D. van Enter, Mathematical Reviews***

This book is jointly published by the AMS and the International Press.

Collected Works, Volume 5; 1996; ISBN 0-8218-0561-4; 601 pages; Hardcover; **All AMS members $47,** List $59, Order Code CWORKS/5CI82

All prices subject to change. Charges for delivery are $3.00 per order. For optional air delivery outside of the continental U. S., please include $6.50 per item. *Prepayment required.* Order from: **American Mathematical Society,** P. O. Box 5904, Boston, MA 02206-5904, USA. For credit card orders, fax (401) 455-4046 or call toll free 800-321-4AMS (4267) in the U. S. and Canada, (401) 455-4000 worldwide. Or place your order through the AMS bookstore at http://www.ams.org/bookstore/. Residents of Canada, please include 7% GST.

For more publications in your subject area visit the AMS Bookstore: www.ams.org/bookstore/

Classified Advertisements

Positions available, items for sale, services available, and more

ALABAMA

UNIVERSITY OF ALABAMA IN HUNTSVILLE
Mathematical Sciences Department

The Mathematical Sciences Department at the University of Alabama invites applications for a tenure-track faculty position with the rank of assistant professor beginning in August 1998. Applicants must possess a Ph.D. in mathematics or applied mathematics and demonstrate evidence of excellent teaching ability and outstanding research potential in an area of interest to current departmental faculty and students. These areas include applied probability/statistics, stochastic analysis, nonlinear functional analysis, differential equations/dynamical systems, numerical analysis, and discrete mathematics. Send a letter of application, curriculum vitae with AMS Standard Cover Sheet, transcripts, and three letters of reference to the Chairman, Department of Mathematical Sciences, University of Alabama in Huntsville, Huntsville, AL 35899. The review of applications will begin February 1, 1998. Women and minorities are particularly encouraged to apply. The University of Alabama in Huntsville is an Affirmative Action/Equal Opportunity Institution.

CALIFORNIA

CALIFORNIA STATE UNIVERSITY MONTEREY BAY
Vacancy in Mathematical Sciences and Applications (#MBIMSA-9808)
Associate Professor or Professor, Tenure-Track (Fall 1998)

CSUMB and the Institute for Mathematical Sciences and Applications seek a senior faculty member to: teach mathematics courses and classes which integrate mathematics into a variety of disciplines, including teacher education; design and implement outcomes-based curriculum in a collaborative setting; play a leading role in the development of the Institute and its "culture of innovation"; and assist in the integration of modern teaching technologies with classroom instruction. Minimum qualifications: earned doctorate in applied mathematics, evidence of effective teaching, record of achievement in scholarship and funded research. Preferred qualifications include specialization in one or more of the following: numerical solutions of partial differential equations; applied mathematics (digital signal processing, wavelets, digital image processing); computational dynamical systems and mathematical physics. **Priority filing date**: Monday, February 16, 1998, at 5:00 p.m. For complete announcement and application information, see CSUMB Web page (`http://www.monterey.edu`) or contact: Faculty Recruitment Office, CSU Monterey Bay, 100 Campus Center, Seaside, CA 93955-8001; 408-582-3569. (e-mail: `faculty_recruitment@monterey.edu`). AA/EEO/ADA Employer.

CONNECTICUT

UNIVERSITY OF CONNECTICUT
Hartford Campus
Assistant Professor
Mathematics

The Department of Mathematics anticipates an opening for a tenure-track position at the assistant professor level at the Hartford campus starting fall 1998. This position requires teaching two courses per semester and service at the Hartford campus, located in West Hartford, 30 miles from the main campus. In addition, an office will be provided at the Storrs campus, and active participation

---

Suggested uses for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services.

The 1997 rate is \$100 per inch or fraction thereof on a single column (one-inch minimum), calculated from top of headline. Any fractional text of $^1/_2$ inch or more will be charged at the next inch rate. No discounts for multiple ads or the same ad in consecutive issues. For an additional \$10 charge, announcements can be placed anonymously. Correspondence will be forwarded.

Advertisements in the "Positions Available" classified section will be set with a minimum one-line headline, consisting of the institution name above body copy, unless additional headline copy is specified by the advertiser. Headlines will be centered in boldface at no extra charge. Ads will appear in the language in which they are submitted.

There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified advertising.

Upcoming deadlines for classified advertising are as follows: March issue–December 19, 1997; April issue–January 21, 1998; May issue–February 23, 1998; June/July issue–April 24, 1998; August issue–May 15, 1998; September issue–June 15, 1998.

U.S. laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the U.S. cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to U.S. laws. Details and specific wording may be found on page 1373 (vol. 44).

Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the U.S. and Canada, or 401-455-4084 worldwide, for further information.

Submission: Promotions Department, AMS, P. O. Box 6248, Providence, Rhode Island 02940, or via fax, 401-331-3842, or send e-mail to `classads@ams.org`. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 02904. Advertisers will be billed upon publication.

in the research activities and interaction with the faculty at the Storrs campus is expected. Candidates must have a Ph.D. in mathematics and demonstrate evidence of excellent teaching ability and outstanding research potential. Preference will be given to candidates whose research and teaching interests strengthen programs within the department, especially applied harmonic analysis, linear algebra, partial differential equations and probability. Salary commensurate with experience.

The review of applications will begin January 15, 1998, and will continue until the position is filled. Send résumé and at least three letters of recommendation to: Head, Department of Mathematics, U-9, University of Connecticut, Storrs, CT 06269-3009. We encourage applications from underrepresented groups, including women, minorities, and people with disabilities. (Search #98A201).

FLORIDA

UNIVERSITY OF MIAMI
Department of Mathematics & Computer Science
Faculty Position - Assistant Professor

The Department announces a junior level tenure-track position in algebraic combinatorics or related areas, beginning fall semester 1998. Applicants must have a Ph.D. in mathematics, excellent research potential, and a strong commitment to teaching.

To apply, send curriculum vitae and three letters of recommendation to:

A. M. Zame, Chairman
Department of Mathematics
& Computer Science
University of Miami
P.O. Box 249085
Coral Gables, FL 33124-4250
e-mail: zame@math.miami.edu

Screening of applications will begin immediately and will continue until the position is filled.

The University of Miami is an Equal Opportunity/Affirmative Action Employer.

GEORGIA

GEORGIA INSTITUTE OF TECHNOLOGY
The Southeast Applied Analysis Center

The Southeast Applied Analysis Center, in the School of Mathematics, invites applications for postdoctoral/visiting positions in applied mathematics. Fields of interest include combinatorics and algorithms, scientific computing, stochastic analysis, differential equations and modelling. Applicants should arrange for a curriculum vitae, at least three letters of recommendation, and a summary of research plans to be sent to Professor Leonid A. Bunimovich, Director, SAAC, School of Mathematics, Georgia Institute of Technology, Atlanta, GA 30332-0160. Georgia Tech, a member of the University System of Georgia, is an Equal Opportunity/Affirmative Action Employer.

GEORGIA SOUTHERN UNIVERSITY
Allen E. Paulson College of Science and Technology
Chair
Department of Mathematics and Computer Science

Nominations and applications are invited for the position of chair of the Department of Mathematics and Computer Science in the College of Science and Technology at Georgia Southern University for an appointment to begin July 1, 1998.

A doctorate in one of the mathematical sciences or in computer science is required, as are knowledge and expertise in both mathematics and computer science. Candidate must have distinguished record of teaching, scholarship, and service and qualify for appointment as an associate or full professor in the department. A strong commitment to excellence in undergraduate and graduate instruction, faculty scholarly activity, and faculty development is essential. Candidates must possess excellent communication and interpersonal skills. Evidence of interest, administrative skills, and ability to lead a multifaceted department is required.

Duties include the support, leadership, and administration of a diverse department which has programs leading to bachelor's degrees in mathematics and computer sciences (CSAB accredited) and a master's degree in mathematics with concentrations in applied mathematics, computer science, and statistics. The department offers core curriculum courses required of all undergraduates, as well as content courses for mathematics education degree programs through the Ed.D. level. There are 38 faculty members at the Ph.D. and M.S. level in mathematics, computer science, mathematics education, and statistics, and another 21 faculty with mathematics/learning support joint appointments. Department faculty have been leaders in integrating technology into the teaching of mathematics, while the vigorous research programs conducted by faculty reflect the breadth of the department. In-house computing facilities strongly support these efforts.

Georgia Southern University, a unit of the University System of Georgia, was founded in 1906 and became a comprehensive regional university in 1990. The 634-acre campus is located in Statesboro, a community of approximately 30,000 residents 50 miles northwest of historic Savannah and 3 hours southeast of Atlanta. Fall quarter 1997 enrollment of approximately 14,000 reflects a decade of expansion in size and scope, resulting in a faculty of well over 600. More information can be found at http://www.gasou.edu/.

Send letter of application describing qualifications and reasons for seeking the position, curriculum vitae, evidence of commitment to excellence in teaching and scholarship, and three letters of recommendation to: Dr. Bill Ponder, Chair, Mathematics/Computer Science Search Committee, P.O. Box 8044, Georgia Southern University, Statesboro, GA 30460-8044. Postmark deadline is February 20, 1998. Additional supporting documents may be requested of leading candidates. Nominations should be received in time for nominees to comply with application requirements by the postmark date.

The names of applicants and nominees, résumés, and other general nonevaluative information are subject to public inspection under the Georgia Open Records Act. Georgia Southern is an Equal Opportunity/Affirmative Action Institution. Persons who need accommodation(s) in the application process under the Americans with Disabilities Act should notify the search chair.

GORDON COLLEGE
Division of Natural Science and Nursing
Assistant/Associate Professor Mathematics

Two tenure-track positions beginning August 1998. Successful candidates will have a doctorate in mathematics **or** a master's degree in mathematics and a related doctorate. Mathematics teaching assignment will include mathematical modeling, precalculus, and calculus. Qualifications to teach another area such as computer science a plus. College-level teaching experience preferred. A strong commitment to the principles of two-year college education is essential. Rank and salary are dependent on credentials and experience. Application deadline is February 16, 1998. Send résumé, teaching philosophy, unofficial transcripts, and names and telephone numbers of three references to: Judith M. Malachowski, Ph.D., RN; Chair, Division of Natural Sciences and Nursing; Gordon College, 419 College Drive; Barnesville, GA 30204. Gordon College, the University System of Georgia, is a two-year institution located within convenient driving distance of Atlanta and Macon. AA/EOE/ADA.

INDIANA

INDIANA UNIVERSITY SOUTH BEND
Department of Mathematics and Computer Science
Visiting Assistant Professor of Mathematics

The Department of Mathematics and Computer Science invites applications for a two-year visiting position in mathematics at the assistant professor level starting August 1998. Applicants must have completed all requirements for a doctoral degree in mathematics, mathematics education, or a closely related field by August 1998. The responsibilities of this position include teaching three courses per semester and service to the department. Preference will be given to candidates who can provide leadership in our developmental mathematics program. Salaries and benefits are competitive. The department currently has 15 full-time faculty and 40 associate faculty.

IUSB is an Equal Opportunity/Affirmative Action Employer; women and minority candidates are encouraged to apply. Send a curriculum vitae, a statement on teaching, and three letters of recommendation, at least two of which should address teaching, to Hiring Committee, Department of Mathematics and Computer Science, Indiana University South Bend, South Bend, IN 46634. Completed applications received by March 15, 1998, will be given full consideration.

PURDUE UNIVERSITY CALUMET

The Department of Mathematics, Computer Science and Statistics is seeking applicants for one tenure-track assistant professorship to begin fall 1998. Responsibilities include teaching service courses as well as courses for mathematics majors and secondary teaching mathematics majors. Additional responsibilities include maintaining an active program of research or comparable scholarship and service to the department and the University in the form of committee assignments, curriculum development, etc. The candidate must have a Ph.D. in any area of mathematics or applied mathematics by August 1998. Successful candidates must be able to document strong teaching abilities and research or scholarship records. Salary is commensurate with experience and qualifications.

Submit a letter of application, curriculum vitae, and three letters of reference (at least one of which addresses teaching ability and one of which addresses scholarly activities) to:

Weihua Ruan, Chair
Mathematics Search Committee
Dept. of Mathematics,
Computer Science and Statistics
Purdue University Calumet
2200-169th St.
Hammond, IN 46323-2094

Use of AMS Application Cover Sheet is recommended. Review of applications will begin February 17, 1998, and will continue until the position is filled.

Located in northwestern Indiana close to Chicago, Purdue University Calumet enrolls more than 9,000 students in more than 80 associate's, bachelor's, and master's degree programs in 16 academic departments. The 12-building commuter campus is situated on 180 wooded acres, less than one hour by car or train from Chicago.

Purdue University Calumet is an Equal Opportunity/Affirmative Action Employer, and applications from women and minorities are especially encouraged.

UNIVERSITY OF NOTRE DAME
Department of Mathematics
Notre Dame, IN 46556
McAndrews Visiting Assistant Professorship

The Mathematics Department invites applications for a postdoctoral position in mathematical logic. The position is for the academic year 1998–99, with renewal for a second year assured provided that teaching performance is satisfactory. The teaching load is three courses per year. Preference will be given to candidates with research interests in computability (recursion theory), but strong candidates in other areas of logic will receive serious consideration. Preference will be given to applicants who received their degrees after May 1995. The materials for a complete application consist of a cover letter, C.V., a short thesis summary, and three letters of recommendation. At least one of the letters should address the candidate's ability to communicate effectively in the classroom. These materials should be sent by January 1, 1998, to Alexander J. Hahn, Chair, Department of Mathematics, University of Notre Dame, Notre Dame, IN 46556. Evaluation of candidates will begin in January.

MASSACHUSETTS

NORTHEASTERN UNIVERSITY
College of Arts and Sciences
Trustee Professorship in Mathematics

Underscoring its long-standing commitment to distinction in practice-oriented education, Northeastern University is launching a major new program of endowed Trustee Professorships.

Northeastern is seeking nominations and applications for a Trustee Professorship in Mathematics, with a focus on applied and industrial mathematics.

Criteria demonstrated by successful candidates will include:

- International distinction as a scholar.
- Research outcomes of practical significance.
- Ability to establish and direct a center for applied and industrial mathematics.

Founded in 1898, Northeastern is a national research university that is student centered, practice-oriented, and urban. A private institution of higher education with approximately 27,000 full- and part-time students, Northeastern has long been recognized as a world leader in the integration of work and learning through its cooperative education program and other practical learning opportunities for students. Northeastern University is located on an attractive campus in the heart of Boston's cultural district.

Review of applications will begin immediately. Applicants must submit a personal statement of interest and qualifications and a current vita. Nominations, applications, and supporting materials should be sent to Trustee Professorship Search Committee, c/o Donna Marlowe, Department of Mathematics, Northeastern University, 567 Lake Hall, Boston, MA 02115.

Northeastern is an Equal Opportunity/ Affirmative Action Title IX Employer.

UNIVERSITY OF MASSACHUSETTS AMHERST
Department of Mathematics & Statistics

The Department of Mathematics & Statistics (www.math.umass.edu/) invites applications for several tenure-track positions at the assistant professor level. In addition, several two-year non-tenure-track positions will be available. The search will focus within the following areas: algebraic geometry, applied analysis, geometric analysis, Lie theory, number theory, probability, scientific computation, and statistics. Exceptional promise in research and in teaching (at all levels of the curriculum) is required. Although this search focuses on junior-level appointments, candidates for more senior-level appointments will be considered. Applicants should send a curriculum vitae and at least three letters of recommendation to: Search Committee, Department of Mathematics & Statistics, University of Massachusetts, Amherst, MA 01003-4515. Review of applications will begin immediately. Applications will continue to be accepted until all positions are filled. Please include the AMS Application Cover Sheet. Equal Opportunity/Affirmative Action Employer.

UNIVERSITY OF MASSACHUSETTS, AMHERST
Department of Mathematics & Statistics

The Department of Mathematics & Statistics (www.math.umass.edu/) invites applications for tenure-track positions at the assistant professor level. In addition, several non-tenure-track positions will be available. Applicants should have a background in theoretical statistics and have an interest in applications, interdisciplinary work, and computation. Exceptional promise in research and in teaching (at all levels of the curriculum) is required. Although this search focuses on junior-level appointments, candidates for more senior-level appointments will be considered. Applicants should send a curriculum vitae and at least three letters of recommendation to: Search Committee, Department of Mathematics & Statistics, University of Massachusetts, Amherst, MA 01003-4515. Review of applications will begin immediately. Applications will continue to be accepted until all positions are filled. Please include the AMS Application Cover Sheet. Equal Opportunity/Affirmative Action Employer.

MISSOURI

UNIVERSITY OF MISSOURI-ST. LOUIS

The Department of Mathematics and Computer Science at the University of Missouri-St. Louis seeks to make an appointment at the rank of associate or full professor to take a leadership role in the implementation of a new Ph.D. program in applied mathematics. (The department also has an open position in computer science at the assistant or associate level.) The successful candidate should have a strong research record in applied mathematics, computational science and/or industrial mathematics, a commitment to quality teaching, and a history of funding from major granting agencies. The salary is competitive. Applications, including a curriculum vitae, a letter of application, and a completed AMS Standard Cover Sheet should be sent to

Ray Balbes, Chair
University of Missouri-St. Louis
Department of Mathematics
and Computer Science
8001 Natural Bridge Road
St. Louis, MO 63121-4499

Applicants should also arrange for at least three letters of recommendation to be sent. The University of Missouri is an Affirmative Action/Equal Opportunity Employer committed to excellence through diversity. For more information please visit our Web site, `http://www.math.umsl.edu/`.

NEBRASKA

UNIVERSITY OF NEBRASKA-LINCOLN
Department of Mathematics and Statistics

Applications are invited for a tenure-track position at the assistant/associate professor level starting in fall 1998. Candidates must have a Ph.D. in mathematics by August of 1998. Candidates must demonstrate evidence of excellent teaching ability and outstanding research potential in an area that can contribute to the department's involvement in the Arts and Sciences Discrete and Experimental Mathematics area of strength. Strong preference will be given to candidates with interests in geometric or combinatorial group theory, semigroup theory, or a closely related area, although outstanding candidates in other areas may also be considered. Ability to contribute to the department's mathematics education activities is a plus. For more details on this position, see our Web site at `http://www.math.unl.edu/`. Send vita and three letters of recommendation to the DEM Search Committee, Department of Mathematics and Statistics, University of Nebraska-Lincoln, Lincoln, NE 68588-0323. The review of applications will begin February 1, 1998, and will continue until suitable candidates are selected. Women and minority candidates are particularly encouraged to apply. The University of Nebraska is committed to a pluralistic campus community through Affirmative Action and Equal Opportunity and is responsive to the needs of dual-career couples. We assure reasonable accommodation under the Americans with Disabilities Act. Please contact Mavis Hettenbaugh at 402-472-4395 for assistance.

NEVADA

UNIVERSITY OF NEVADA, LAS VEGAS
Department of Mathematical Sciences

The Department of Mathematical Sciences at the University of Nevada, Las Vegas, is seeking a tenure-track, assistant professor-level math education coordinator. The position will start in fall 1998 pending budgetary approval. Candidates must hold a Ph.D. in mathematical sciences, with a minimum of two years experience in math education. Duties include coordination of mathematical content courses for future elementary and secondary teachers and collaboration with the Department of Instructional and Curricular Studies in the UNLV College of Education on curricular matters dealing with math education. Other responsiblities include undergraduate and graduate teaching, research publications in refereed journals, and university and professional service. Salary is commensurate with qualifications and experience.

Applicants should send a completed AMS cover sheet, a curriculum vitae, and three letters of reference to:

Professor George Miel, Chairman
Department of Mathematical Sciences
University of Nevada, Las Vegas
4505 Maryland Parkway
Las Vegas, NV 89154-4020

Evaluation of applicants will begin immediately and will continue until a suitable applicant is selected. University of Nevada, Las Vegas, is an Affirmative Action/Equal Opportunity Employer. Minorities, women, veterans, and the disabled are encouraged to apply. For more information, see the World Wide Web site at `http://www.nscee.edu/unlv/math/`.

NEW JERSEY

WILLIAM PATERSON UNIVERSITY
Department of Mathematics

The faculty of William Paterson University seek to create a welcoming and nurturing campus climate for a diverse faculty, staff, and student body. In this spirit, the Department of Mathematics invites applications for two (possibly three) tenure-track positions starting September 1, 1998. The requirements are a Ph.D. in mathematics, strong evidence of commitment to quality teaching, and an ongoing research program.

The department is committed to improving retention and recruitment of students via advisement, precollege programs, and curriculum development. The department is also involved in teacher training programs. For one position (at the assistant/associate professor level) preference will be given to candidates with demonstrated interest or experience in these activities (position AA).

For the other position(s) (at the assistant professor level) preference will be given to candidates whose field of interest is either in areas of applied mathematics such as mathematical physics, applied analysis, and PDE, or in areas of discrete mathematics such as combinatorics, optimization, graph theory, and computational mathematics (position BB).

Send a letter of interest clearly indicating the position, current vita, graduate transcript (unofficial), and three letters of reference (at least one letter or other evidence should specifically address the applicant's ability to teach effectively) to Dr. S. Maheshwari, Chairperson, Math. Dept., WPUNJ, Wayne, NJ 07470. Review of applications will begin immediately and will continue till the positions are filled. Salary is commensurate with experience. Evidence of U.S. citizenship or permanent residency is required. WPUNJ is an Equal Opportunity/AA Employer and actively seeks applications from women, minorities, and underrepresented groups.

NEW YORK

THE CITY COLLEGE OF NEW YORK

The Department of Mathematics invites applications for a tenure-track position as assistant or associate professor beginning fall 1998. A Ph.D. in applied or computational mathematics with a demonstrated interest in the field of mathematical finance is required. Appropriate work or consulting experience in the finance industry is highly desirable. Excellent credentials in undergraduate teaching and a record of scholarly publication are essential. Level of appointment will be based on experience and demonstrated expertise. The appointee will be expected to initiate and direct, in collaboration with the Economics Department, a specialization in the mathematics of finance for undergraduate mathematics majors and master's degree candidates. In addition, the successful candidate will be expected to teach undergraduate and graduate courses in the general curriculum of the department and maintain an active research program. Applications should include (1) covering letter indicating candidate's interest and qualifications, (2) AMS Standard Cover Sheet, and (3) curriculum vitae. In addition, candidates should arrange to have three letters of reference, including one which addresses teaching qualifications, sent directly to:

Faculty Search Committee
Department of Mathematics
The City College of New York
138th St. & Convent Avenue, NAC 8133
New York, NY 10031

Complete applications received by February 1, 1998, are assured full consideration. The City College of New York is an Equal Employment/Affirmative Action Employer and encourages applications from women and underrepresented minorities.

UNIVERSITY OF ROCHESTER

Assistant Professor. This is a non-tenure-track appointment for one to three years for new or recent Ph.D.s beginning in fall 1998. Qualifications include a Ph.D. in mathematics and outstanding promise in research and teaching. Applicants in all areas of mathematics will be considered, but preference will be given to applicants whose research interests are compatible with those in the department. Strong evidence of interest in and experience in the teaching of undergraduates is essential.

Consideration of applicants will begin on 1/15/98. Send a letter of application, current curriculum vitae, minimum of three letters of recommendation (at least one of which addresses teaching), and description of research to:

Douglas Ravenel, Chair
Department of Mathematics
University of Rochester
Rochester, NY 14627

The University of Rochester is an Equal Opportunity/Affirmative Action Employer.

LEHMAN COLLEGE (CUNY)
Department of Mathematics and Computer Science

Tenure-track positions are available starting September 1, 1998, for an assistant professor in mathematics and an assistant/associate professor in computer science. Both positions require an earned doctorate, outstanding research record or potential, and commitment to excellence in teaching. Appointment rank and salary commensurate with qualifications and experience. Application Procedure: Send curriculum vitae with a cover letter (indicate position you are applying for) and at least three letters of recommendation to Professor Robert Feinerman, Chair, Department of Mathematics and Computer Science, Lehman College, 250 Bedford Park Blvd. W., Bronx, NY 10468. Application deadline is March 2, 1998. Use of the AMS Cover Sheet for Academic Employment is encouraged. Additional information at http://www.lehman.cuny.edu/. AA/EEO/ADA Employer.

CITY UNIVERSITY OF NEW YORK, QUEENS COLLEGE

Applications are invited for the Daniel Gorenstein Visiting Professorship at Queens College, City University of New York, for the 1998-99 academic year. Applications may be made for either one semester or two. Applicants should offer a strong research record as well as demonstrated excellence in undergraduate teaching. Applicants should have at least ten years' experience beyond the Ph.D. in university or industrial positions. Appointment is to visiting associate/full professor. Applications from faculty who will be on sabbatical are welcome.

Queens College is a liberal arts institution with undergraduate and master's level programs in mathematics and mathematics education. Advanced courses are often taught in conjunction with our Mathematica laboratory. Queens College is located in Flushing, New York, and is easily accessible from Manhattan by public transportation.

Applicants should send a letter of intent indicating semester(s) of interest, a current vita, a brief description of research interests, and three letters of reference to Gorenstein Chair Search Committee, Department of Mathematics, Queens College, Flushing, NY 11367. All material must be received by March 16, 1998.

Queens College is an Equal Opportunity/Affirmative Action Employer.

OHIO

CASE WESTERN RESERVE UNIVERSITY

The Department of Mathematics anticipates making one tenure-track appointment and possibly a visiting appointment, beginning August 1998.

Required: Ph.D. in mathematics; exceptional promise, with accomplishments commensurate with experience, in research and teaching. All fields of pure and applied mathematics will be considered, with particular interest in fields that fit in well with our current specializations in algebra, analysis, differential equations/dynamical systems, geometry, imaging, numerical algebra/analysis, and probability.

A complete application should contain AMS Cover Sheet, letter of application (including e-mail address and fax number), curriculum vitae, and relevant (p)reprints. Candidates should also have three letters of recommendation sent.

Mail all materials to: Appointments Committee, Department of Mathematics, Case Western Reserve University, Cleveland, OH 44106-7058. No e-mail or fax applications will be accepted. Review of applications will begin after February 1. CWRU is an Affirmative Action/Equal Opportunity Employer.

OKLAHOMA

THE UNIVERSITY OF OKLAHOMA
Department of Mathematics

Applications are invited for two full-time, tenure-track positions beginning August 16, 1998. The positions are initially budgeted at the assistant professor level, but an appointment at the associate professor level may be possible for an exceptional candidate with qualifications and experience appropriate to that rank. Normal duties consist of teaching two courses per semester, conducting research, and rendering service to the department, university, and profession at a level appropriate to the faculty member's experience. Both positions require an earned doctorate and research interests that are compatible with those of the existing faculty; preference will be given to applicants with potential or demonstrated excellence in research and prior successful undergraduate teaching experience. For one of the positions, additional preference will be given to applicants with research interests in applied or computational mathematics. Salary and benefits are competitive. For full consideration, applicants should send a completed AMS cover sheet, curriculum vitae, a description of current and planned research, and three letters of recommendation (at least one of which must address the applicant's teaching experience and proficiency) to:

Search Committee
Department of Mathematics
University of Oklahoma
Norman, OK 73019-0315
Phone: 405-325-6711
Fax: 405-325-7484
e-mail: search@math.ou.edu

Screening of applications will begin on February 15, 1998, and will continue until the position is filled.

The University of Oklahoma is an Equal Opportunity/Affirmative Action Employer. Women and minorities are encouraged to apply. UO has a policy of being responsive to the needs of dual-career couples.

TENNESSEE

THE UNIVERSITY OF MEMPHIS
Department of Mathematical Sciences

Applications are invited for three anticipated tenure-track positions, two in mathematics and one in applied statistics/biostatistics, for the 1998-99 academic year. Desired research areas in mathematics include graph theory and combinatorics, and partial differential equations. Consideration will also be given to persons with expertise in approximation theory, functional analysis, and operator theory. Desired areas of expertise for the position in applied statistics/biostatistics include survival analysis, epidemiological modeling of AIDS and cancer, applied stochastic models of biomedical systems, AIDS or cancer or toxicological risk assessment, longitudinal data analysis, and analysis of discrete correlated data/contingency tables. Applications for possible visiting positions are encouraged also. The department offers degrees at all levels, including the Ph.D., and provides a very favorable research environment in terms of library and computing facilities, teaching load, and travel opportunities.

Applicants must have a Ph.D. by September 1, 1998, and a strong potential for excellence in teaching and research. Selection will begin December 2, 1997. Applications may continue to be accepted until all positions are filled. Women and minorities are strongly urged to apply. Successful candidates must meet all applicable Immigration Reform Act Laws. Electronic applications will not be accepted.

Applicants should submit a résumé and direct three letters of reference to:

Prof. C. Rousseau,
Chair-Search Committee
Department of Mathematical Sciences
The University of Memphis
Memphis, TN 38152-6429

An Equal Opportunity/Affirmative Action Employer.

TEXAS

ST. EDWARD'S UNIVERSITY
Assistant Professor of Mathematics

Full-time, tenure-track position. Appointment beginning mid-August 1998. Ph.D. in mathematics strongly preferred. Research interest in algebra or topology desirable. Duties include course instruction and supervision of undergraduate research. Application letter, vita, and three reference letters to: Mathematics Search Committee, Box 1043, St. Edward's University, 3001 South Congress Ave., Austin, Texas 78704-6489. Applications reviewed from mid-January until position is filled. (`http://www.cs.stedwards.edu/` for Web site or `mathsrch@server.cs.stedwards.edu` for e-mail.) St. Edward's University embraces excellence through diversity and especially encourages applications from underrepresented groups.

UNIVERSITY OF TEXAS AT ARLINGTON
Department of Mathematics

The department invites applications for possibly three anticipated tenure-track positions beginning with the fall semester 1998. One of the positions will be an assistant professor in numerical analysis. We also seek candidates in other areas of mathematics which are complementary to those of the current faculty and would enhance and support the goals of the department. Candidates must show strong potential for excellence in teaching and research. Application deadline is January 31, 1998, or until positions are filled. Salary and rank are commensurate with qualifications, which must include the Ph.D. degree (an earned doctorate by August 1998). Please send a résumé and three letters of recommendation to:

Chairperson
Faculty Recruiting Committee
University of Texas at Arlington
Department of Mathematics
Box 19408
Arlington, TX 76019-0408

The University of Texas at Arlington is an Affirmative Action/Equal Opportunity Employer.

THE UNIVERSITY OF TEXAS AT AUSTIN
Austin, Texas 78712
Department of Mathematics

Openings for fall 1998 include a number of instructorships, some of which have R. H. Bing Faculty Fellowships attached to them, and two or more positions at the tenure-track/tenure level.

Instructorships at The University of Texas at Austin are postdoctoral appointments, renewable for two additional years. It is assumed that applicants for instructorships will have completed all Ph.D. requirements by August 31, 1998. Other factors being equal, preference will be given to those whose doctorates were conferred in 1997 or 1998. Candidates should show superior research ability and have a strong commitment to teaching. Consideration will be given only to persons whose research interests have some overlap with those of the permanent faculty. Duties consist of teaching undergraduate or graduate courses and conducting independent research. The projected salary is $33,500 for the nine-month academic year.

Each R. H. Bing Fellow holds an instructorship in the Mathematics Department, with a teaching load of two courses in one semester and one course in the other. The combined instructorship-fellowship stipend for nine months is $36,500, which is supplemented by a travel allowance of $1,000. Pending satisfactory performance of teaching duties, the fellowship can be renewed for two additional years. Applicants must show outstanding promise in research. Bing Fellowship applicants will automatically be considered for other departmental openings at the postdoctoral level, so a separate application for such a position is unnecessary.

An applicant for a tenure-track or tenured position must present a record of exceptional achievement in her or his research area and must demonstrate a proficiency at teaching. In addition to the duties indicated above for instructors, such an appointment will typically entail the supervision of M.A. or Ph.D. students. The salary will be commensurate with the level at which the position is filled and the qualifications of the person who fills it.

Those wishing to apply for any of the aforementioned positions are asked to send a vita and a brief research summary to the above address, c/o Recruiting Committee. Transmission of the preceding items via e-mail (address: `recruit@math.utexas.edu`) is encouraged. Applications must be supported by three or more letters of recommendation, at least one of which speaks to the applicant's teaching credentials. The screening of applications will begin on December 1, 1997.

The University of Texas at Austin is an Equal Opportunity Employer.

HONG KONG

THE HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY

The Department of Mathematics invites applications for up to two positions at the rank of professor, associate professor, or assistant professor in the area of statistics. Applicants who are specialized in areas other than the above and have demonstrated outstanding performance and contributions to their own fields of specialization will also be considered.

Exceptionally strong research and teaching experience are required. Applicants must demonstrate excellence in teaching and proven ability to teach effectively in English.

Starting rank and salary will depend on qualifications and experience. Generous fringe benefits, including medical and dental benefits, annual leave, and children's education allowance are provided; housing benefit will also be provided where applicable. Initial appointment will normally be on a three-year contract; immediate tenure may be considered for appointment at professorial level. A gratuity will be payable upon successful completion of contract. Reappointment will be subject to mutual agreement.

Applicants should fill in the application form in MATH home page (`http://www.math.usthk/recruit.dir/`), send a curriculum vitae, and ask at least three referees to send letters of recommendation, preferably before February 28, 1998, directly to:

The Director of Personnel
The Hong Kong University of
Science and Technology
Clear Water Bay
Kowloon, Hong Kong
fax: 852-2358-0700

PORTUGAL

CENTER FOR MATHEMATICAL ANALYSIS, GEOMETRY AND DYNAMICAL SYSTEMS
Departamento de Matemática
Instituto Superior Técnico
Av. Rovisco Pais
1096 Lisboa Codex, Portugal
Postdoctoral Positions

The Center for Mathematical Analysis, Geometry and Dynamical Systems of the Department of Mathematics of Instituto Superior Técnico, Lisbon, Portugal, invites applications for two postdoctoral positions for research in mathematics. Positions are for one year, with the possibility of extension for a second year upon mutual agreement. Selected candidates will be able to take up their position between September 1, 1998, and January 1, 1999.

Applicants should have a Ph.D. in mathematics obtained after December 31, 1995. They must show very strong research promise in one of the areas in which the mathematics faculty of the Center is currently active. There are no teaching duties associated with these positions.

Applicants should send a curriculum vitae; reprints, preprints and/or dissertation abstract; description of research project (of no more than 1,000 words); and three letters of reference directly to the director at the above address.

To insure full consideration, complete application packages should be received by **March 15, 1998**. Additional information about the Center and the positions is available at `http://www.math.ist.utl.pt/cam/`.

The Center for Mathematical Analysis, Geometry and Dynamical Systems of the Department of Mathematics of Instituto Superior Técnico, Lisbon, Portugal, does not discriminate in employment on the basis of color, age, sex, race, religion, or national origins.

SULTANATE OF OMAN

SULTAN QABOOS UNIVERSITY
College of Science

The Department of Mathematics & Statistics offers bachelor's degrees in mathematics & statistics. It is responsible for teaching all mathematics service courses to engineering, education, economics, and agriculture. All mathematics faculty are expected to teach calculus or algebra service courses in addition to advanced mathematics courses. Good experience in the teaching of calculus is always taken positively into account when appointments are made. All teaching is in English.

The Department of Mathematics & Statistics has the following vacancies for September 1998:

Professor/Associate Professor in Statistics. Applicants must have wide experience in research and curriculum development both at undergraduate and graduate levels. The appointee is expected to be in overall charge of the undergraduate and master's program, which is expected to start in 1999.

Associate/Assistant Professor/Lecturer in Algebra. The applicants for associate professor are expected to possess good research and organizational experience in order to develop the algebra curriculum. The applicants for assistant professor must possess 4 years of teaching experience after they obtained their doctorate. Applicants who do not satisfy this condition will not be considered.

Assistant Professor in Computational Mathematics. Appointees are expected to teach numerical techniques and also symbolic manipulation software. They are also expected to join a strong research group of fluid dynamics and mathematical modelling.

Assistant Professor/Lecturer in Analysis.

Lecturer in the general area of mathematics.

Apart from a very attractive tax-free base salary, the University offers free furnished accommodations, two years' renewable employment contract with end of service gratuity, annual leave with return air tickets, free medical treatment in government hospitals.

Enquiries with a full curriculum vitae, and names and fax numbers of three referees should be addressed, quoting our Ref:ADV/SCI/03/97, to:

The Director, Personnel Affairs
Sultan Qaboos University
P.O. Box 50, Al-Khod - 123
Sultanate of Oman
(The position will be open until filled.)

UNITED KINGDOM

UNIVERSITY OF CAMBRIDGE
Faculty of Mathematics
University Lectureship in Mathematics

Applications are invited for this lectureship, established as a joint post in the Department of Pure Mathematics and Mathematical Statistics and the Department of Applied Mathematics and Theoretical Physics.

Applicants should work in one of the areas of strong interaction between the two departments: these include geometry and the geometrical aspects of theoretical physics, dynamical systems, partial differential equations.

Salary will be age related on the scale for University lecturers (£=A319,371-=A329,875 p.a.). Appointment will be for three years in the first instance, from April 1, 1998, or as soon as convenient thereafter.

Further particulars may be obtained from Dr. V. Chamberlain, Faculty Office, DAMTP, Silver Street, Cambridge CB3 9EW, to whom applications (including curriculum vitae, publications list and names, and addresses and e-mail addresses of not more than 3 referees) should be sent so as to arrive no later than February 23, 1998.

The University follows an Equal Opportunities Policy.

AMERICAN MATHEMATICAL SOCIETY

Electronic Research Announcements of the American Mathematical Society

Volume 3, 1997

CONTENTS (year to date)

The American Mathematical Society's electronic-only journal, *Electronic Research Announcements of the AMS* (ERA-AMS), is available on the World Wide Web at the URL: http://www.ams.org/era/

ERA-AMS publishes high-quality research announcements of significant advances in all branches of mathematics. Authors may submit manuscripts to any editor. All papers are reviewed, and the entire Editorial Board must approve the acceptance of any paper. Beginning with Volume 3, papers will be posted as soon as they are accepted and processed by the AMS.

ERA-AMS offers you ...

- decreased turn-around time from submission to print
- fast access to your specific area of interest
- up-to-the-minute research information

To obtain submission information and the template, send email to: **era-info@ams.org** with the word "help" in the subject line.

For more information, contact: **cust-serv@ams.org**
800-321-4267, 401-455-4000, fax 401-455-4046

http://www.ams.org/era/

The Ph.D. Program in Mathematics with Option in Education
Offered by Department of Mathematics at the University of Arizona

The Department of Mathematics at the University of Arizona is committed to quality education as well as quality research. To that end, the department includes among its Ph.D. programs a program to prepare outstanding mathematics educators who are competent in both mathematics and pedagogy.

Students are required to pass the regular Department Qualifying Examination (covering graduate level algebra, real analysis, and topology and undergraduate material in linear algebra, rigorous advanced calculus and complex analysis) and at least 36 graduate credits in mathematics. They must also take 12 credits in a supporting minor (ordinarily education) and have two or more years of pre-college teaching experience (which may be completed through practicums). The dissertation is ordinarily a research study in mathematics education, an expository or an historical paper in mathematics.

Teaching assistantships and other forms of financial support are available. Faculty in the Department who have a special interest in this program include Marta Civil, David Gay, Frederick Stevenson, Elias Toubassi and Stephen Willoughby, but many other members of the Department also participate.

For more information write to:
Mathematics Graduate Program
Department of Mathematics
University of Arizona
PO Box 210089, 617 N Santa Rita
Tucson, AZ 85721-0089
(520) 621-2068
e-mail: graduate@math.arizona.edu
http://www.math.arizona.edu/

The University of Arizona is an Equal Opportunity/Affirmative Action/ADA Employer

Ω *IAS/Park City*
Mathematics Institute

Solicitation of Future Research Topics

The IAS/Park City Mathematics Institute is an unusual mathematics program which actively seeks to bring about interaction among the various members of the mathematics community. The PCMI Summer Session brings together high school teachers, undergraduate and graduate students, college and university faculty, research mathematicians, and researchers in mathematics education to participate in separate but overlapping programs of research and education.

Proposals for future research topics as well as organizers for the three-week Summer Session are now being solicited. Any area of mathematics will be of interest. The proposal (at most two pages) should contain a topic and a list of possible courses and lecturers for the Graduate Summer School and Research Program. Past topics have included *Geometry and Topology of Manifolds and Quantum Field Theory, Nonlinear Partial Differential Equations in Differential Geometry, Higher Dimensional Complex Geometry, Gauge Theory and the Topology of Four-Manifolds, Nonlinear Wave Phenomena, Probability, Symplectic Geometry,* and *Representation Theory of Lie Groups.*

For information about the IAS/PCMI contact Anne Humes (*humes@ias.edu*) or access *http://www.ias.edu/park.htm.*

Institute for Advanced Study/Park City Mathematics Institute, Olden Lane, Princeton, NJ 08540; 1-800-726-4427.

AMERICAN MATHEMATICAL SOCIETY

Application for Membership 1998

(January–December)

Date . . . . 19 . . . .

Please read the "Membership Categories" section of this form to determine the membership category for which you are eligible. Then fill out this application and return it as soon as possible.

. . . .

Family Name ***First*** ***Middle***

Place of Birth . . . .

City ***State*** ***Country***

Date of Birth . . . .

Day ***Month*** ***Year***

If formerly a member of AMS, please indicate dates . . . .

Check here if you are now a member of either MAA ☐ or SIAM ☐

Degrees, with institutions and dates . . . .

. . . .

. . . .

Present position . . . .

Firm or institution . . . .

. . . .

City ***State*** ***Zip/Country***

Primary Fields of Interest (choose five from the list at right)

. . . .

Secondary Fields of Interest (choose from the list at right)

. . . .

. . . .

Address for all mail

. . . .

. . . .

. . . .

Telephone number(s) . . . .

Electronic address . . . .

. . . .

Signature

Prepayment Methods and Mailing Addresses

All prices quoted in U.S. dollars.
Payment by check must be drawn on U.S. bank if paid in U.S. dollars.

Send checks, money orders, UNESCO coupons to American Mathematical Society, P.O. Box 5904, Boston, MA 02206-5904

To use credit cards, fill in information requested and mail to American Mathematical Society, P.O. Box 6248, Providence, RI 02940-6248 or call (401) 455-4000 or 1-800-321-4AMS.

For Foreign Bank Transfers: American Mathematical Society, State Street Bank and Trust Company, 225 Franklin St., ABA #011000028, Account #0128-262-3, Boston, MA 02110.

American Express ☐ *Discover* ☐ *VISA* ☐ *MasterCard* ☐

. . . .

Account number ***Expiration date***

M8NO

Fields of Interest

If you wish to be on the mailing lists to receive information about publications in fields of mathematics in which you have an interest, please consult the list of major headings below. These categories will be added to your computer record so that you will be informed of new publications or special sales in the fields you have indicated.

EME	Education/Mathematics Education
00	General
01	History and biography
03	Mathematical logic and foundations
04	Set theory
05	Combinatorics
06	Order, lattices, ordered algebraic structures
08	General algebraic systems
11	Number theory
12	Field theory and polynomials
13	Commutative rings and algebras
14	Algebraic geometry
15	Linear and multilinear algebra; matrix theory
16	Associative rings and algebras
17	Nonassociative rings and algebras
18	Category theory, homological algebra
19	*K*-theory
20	Group theory and generalizations
22	Topological groups, Lie groups
26	Real functions
28	Measure and integration
30	Functions of a complex variable
31	Potential theory
32	Several complex variables and analytic spaces
33	Special functions
34	Ordinary differential equations
35	Partial differential equations
39	Finite differences and functional equations
40	Sequences, series, summability
41	Approximations and expansions
42	Fourier analysis
43	Abstract harmonic analysis
44	Integral transforms, operational calculus
45	Integral equations
46	Functional analysis
47	Operator theory
49	Calculus of variations and optimal control; optimization
51	Geometry
52	Convex and discrete geometry
53	Differential geometry
54	General topology
55	Algebraic topology
57	Manifolds and cell complexes
58	Global analysis, analysis on manifolds
60	Probability theory and stochastic processes
62	Statistics
65	Numerical analysis
68	Computer science
70	Mechanics of particles and systems
73	Mechanics of solids
76	Fluid mechanics
78	Optics, electromagnetic theory
80	Classical thermodynamics, heat transfer
81	Quantum theory
82	Statistical mechanics, structure of matter
83	Relativity and gravitational theory
85	Astronomy and astrophysics
86	Geophysics
90	Economics, operations research, programming, games
92	Biology and other natural sciences, behavioral sciences
93	Systems theory; control
94	Information and communication, circuits

Membership Categories

Please read the following to determine what membership category you are eligible for, and then indicate below the category for which you are applying.

*For **ordinary members** whose annual professional income is below $45,000, the dues are $96; for those whose annual professional income is $45,000 or more, the dues are $128.*

*The **CMS cooperative rate** applies to ordinary members of the AMS who are also members of the Canadian Mathematical Society and reside outside of the U.S. For members whose annual professional income is $45,000 or less, the dues are $82; for those whose annual professional income is above $45,000, the dues are $109.*

*For a **joint family membership,** one member pays ordinary dues, based on his or her income; the other pays ordinary dues based on his or her income, less $20. (Only the member paying full dues will receive the* Notices *and the* Bulletin *as a privilege of membership, but both members will be accorded all other privileges of membership.)*

*Minimum dues for **contributing members** are $192. The amount paid which exceeds the higher ordinary dues level and is purely voluntary may be treated as a charitable contribution.*

*For either **students** or **unemployed individuals,** dues are $32, and annual verification is required.*

*The annual dues for **reciprocity members** who reside outside the U.S. and Canada are $64. To be eligible for this classification, members must belong to one of those foreign societies with which the AMS has established a reciprocity agreement, and annual verification is required. Reciprocity members who reside in the U.S. or Canada must pay ordinary member dues ($96 or $128).*

*The annual dues for **category-S members,** those who reside in developing countries, are $16. Members can chose only one privilege journal. Please indicate your choice below.*

*Members can purchase a **multi-year membership** by prepaying their current dues rate for either two, three, four or five years. This option is not available to category-S, unemployed, or student members.*

1998 Dues Schedule (January through December)

Ordinary member .... ☐ $96 ☐ $128

CMS cooperative rate .... ☐ $82 ☐ $109

Joint family member (full rate) .... ☐ $96 ☐ $128

Joint family member (reduced rate) .... ☐ $76 ☐ $108

Contributing member (minimum $192) .... ☐

Student member (please verify)[1] .... ☐ $32

Unemployed member (please verify)[2] .... ☐ $32

Reciprocity member (please verify)[3] .... ☐ $64 ☐ $96 ☐ $128

Category-S member[4] .... ☐ $16

Multi-year membership .... $....... for .... years

[1] ***Student Verification*** (sign below)

I am a full-time student at ....

.... *currently working toward a degree.*

[2] ***Unemployed Verification*** (sign below) *I am currently unemployed and actively seeking employment.*

[3] ***Reciprocity Membership Verification*** (sign below) *I am currently a member of the society indicated on the right and am therefore eligible for reciprocity membership.*

....

Signature

[4] ☐ *send* **NOTICES** ☐ *send* **BULLETIN**

Reciprocating Societies

- ☐ Allahabad Mathematical Society
- ☐ Australian Mathematical Society
- ☐ Azerbaijan Mathematical Society
- ☐ Berliner Mathematische Gessellschaft e.V.
- ☐ Calcutta Mathematical Society
- ☐ Croatian Mathematical Society
- ☐ Cyprus Mathematical Society
- ☐ Dansk Matematisk Forening
- ☐ Deutsche Mathematiker-Vereinigung e.V.
- ☐ Edinburgh Mathematical Society
- ☐ Egyptian Mathematical Society
- ☐ Gesellschaft für Angewandte Mathematik und Mechanik
- ☐ Glasgow Mathematical Association
- ☐ Hellenic Mathematical Society
- ☐ Icelandic Mathematical Society
- ☐ Indian Mathematical Society
- ☐ Iranian Mathematical Society
- ☐ Irish Mathematical Society
- ☐ Israel Mathematical Union
- ☐ János Bolyai Mathematical Society
- ☐ The Korean Mathematical Society
- ☐ London Mathematical Society
- ☐ Malaysian Mathematical Society
- ☐ Mathematical Society of Japan
- ☐ Mathematical Society of the Philippines
- ☐ Mathematical Society of the Republic of China
- ☐ Mongolian Mathematical Society
- ☐ Nepal Mathematical Society
- ☐ New Zealand Mathematical Society
- ☐ Nigerian Mathematical Society
- ☐ Norsk Matematisk Forening
- ☐ Österreichische Mathematische Gesellschaft
- ☐ Palestine Society for Mathematical Sciences
- ☐ Polskie Towarzystwo Matematyczne
- ☐ Punjab Mathematical Society
- ☐ Ramanujan Mathematical Society
- ☐ Real Sociedad Matemática Española
- ☐ Saudi Association for Mathematical Sciences
- ☐ Sociedad Colombiana de Matemáticas
- ☐ Sociedad Española de Matemática Aplicada
- ☐ Sociedad de Matemática de Chile
- ☐ Sociedad Matemática de la Republica Dominicana
- ☐ Sociedad Matemática Mexicana
- ☐ Sociedad Uruguaya de Matemática y Estadística
- ☐ Sociedade Brasileira Matemática
- ☐ Sociedade Brasileira de Matemática Aplicada e Computacional
- ☐ Sociedade Paranaense de Matemática
- ☐ Sociedade Portuguesa de Matemática
- ☐ Societat Catalana de Matemàtiques
- ☐ Societatea de Ştiinţe Matematice din România
- ☐ Societatea Matematicienilor din Romania
- ☐ Société de Mathématiques Appliquées et Industrielles
- ☐ Société Mathématique de Belgique
- ☐ Société Mathématique de France
- ☐ Société Mathématique Suisse
- ☐ Society of Associations of Mathematicians & Computer Science of Macedonia
- ☐ Society of Mathematicians, Physicists, and Astronomers of Slovenia
- ☐ South African Mathematical Society
- ☐ Southeast Asian Mathematical Society
- ☐ Suomen Matemaattinen Yhdistys
- ☐ Svenska Matematikersamfundet
- ☐ Union Mathemática Argentina
- ☐ Union of Bulgarian Mathematicians
- ☐ Union of Czech Mathematicians and Physicists
- ☐ Union of Slovak Mathematicians and Physicists
- ☐ Unione Matematica Italiana
- ☐ Vijnana Parishad of India
- ☐ Wiskundig Genootschap

Order Form

ORDERED BY

Name ______________________

Address ______________________

____

City ______________________

State ______________________

Zip ______________________

Country ______________________

Code ______________________

e-mail ______________________

MAIL TO (IF DIFFERENT)

Name ______________________

Address ______________________

____

City ______________________

State ______________________

Zip ______________________

Country ______________________

Code ______________________

e-mail ______________________

Qty.	Code	Title	Price	Total
Shipping and Handling ($3.00 per order).				$3.00
Or for optional delivery by air outside the continental U.S, pay $6.50 per item.				
Residents of Canada, please include 7% GST.				
			Total	

For orders with remittances:
(Payment must be made in U.S. currency drawn on a U.S. bank)
American Mathematical Society
P.O. Box 5904
Boston, MA 02206-5904

For credit card orders:
American Mathematical Society
P.O. Box 6248
Providence, RI 02940-6248
800-321-4AMS (4267)
(401) 455-4000, worldwide
cust-serv@ams.org

PAYMENT METHOD

☐ Check or Money Order ☐ American Express ☐ Discover
☐ MasterCard ☐ Visa

Card Number ______________________ Expiration Date __________

Signature ______________________

Please send me information about AMS membership

☐ individual
☐ institutional
☐ corporate
☐ institutional associate

CHARGE BY PHONE IN US AND CANADA 800.321.4AMS

Publications, videotapes, and miscellaneous items are sent via UPS to U.S. addresses and as printed matter elsewhere unless another delivery method is requested. Charges for delivery are $3.00 per order, or for air delivery outside the continental U.S., please include an additional $6.50 per item. **Journal back numbers**, *Mathematical Reviews* indexes, and review volumes are sent via surface mail to any destination unless air delivery is requested. Postage for surface mail is paid by the AMS. Air delivery rates, which will be quoted upon request, must be paid by the purchaser.

Customers in these areas should request price information and order directly from the indicated distributors: **AUSTRALIA, NEW ZEALAND, PAPUA NEW GUINEA:** D. A. Information Services Pty. Ltd., 648 Whitehorse Road, Mitcham, Vic 3132, Australia. Tel. (+613 or 03) 9210 7777, Fax (+613 or 03) 9210 7788; exclusive distributor of AMS books. **EUROPE, MIDDLE EAST, AFRICA:** Oxford University Press, Walton Street, Oxford OX2 6DP England. Tel: (0) 1865-556767, Telefax (0) 1865-267782; exclusive distributor of AMS books. Individual members can order either through OUP or direct from the AMS to receive member discount. **INDIA:** International Book Agency, Flat No. 2, Nirala Market, Nirala Nagar, Lucknow, 226-020, India. Tel: 91-522-370506, Fax Nos.: 91-522-222062, 210376; exclusive distributor of AMS books. **INDIA, SRI LANKA, NEPAL, BANGLADESH, PAKISTAN:** Globe Publication Pvt. Ltd., B-13, 3rd Floor, A Block, Shopping Complex, Naraina Vihar, Ring Road, New Delhi, India. Tel.:5460211, 5460212, Fax: 5752535; exclusive distributor of AMS paper journals only. **KOREA:** Sejong Books, Inc., 81-4 Neung-Dong, Kwangjin-Ku, Seoul 143-180, Korea. Tel: 011-822-498-0300, Fax Nos. 011-822-3409-0321 or 0302, e-mail: sjbk@inet.co.kr; recommended distributor of AMS books. **JAPAN:** Maruzen Co. Ltd., P. O. Box 5050, Tokyo International 100-31, Japan. Tel. Tokyo: 03-3272-7211, Telex J26516; exclusive distributor of AMS books and journals.

VISIT THE AMS BOOKSTORE AT http://www.ams.org/bookstore/

Fold here

NO POSTAGE
NECESSARY
IF MAILED
IN THE
UNITED STATES

BUSINESS REPLY MAIL

FIRST-CLASS MAIL PERMIT NO. 554B PROVIDENCE, RI

POSTAGE WILL BE PAID BY ADDRESSEE

Membership and Customer Services
AMERICAN MATHEMATICAL SOCIETY
P. O. Box 6248
Providence, RI 02940-9943

Fold here

Staple here

CMS Summer 1998
M E E T I N G

University of New Brunswick, Saint John, New Brunswick, June 13–15, 1998

The Canadian Mathematical Society and the University of New Brunswick cordially invite researchers, educators, and students to the 1998 Summer Meeting of the Canadian Mathematical Society. The scientific programme will take place at Oland Hall of the University of New Brunswick (Saint John), Saint John, New Brunswick, from Saturday, June 13, to Monday, June 15, 1998.

PLENARY SPEAKERS: Kenneth Davidson (Waterloo), **Detlef Gromoll** (SUNY Stony Brook), **Erwin Lutwak** (Polytechnic University, Brooklyn), **Stephen Schanuel** (SUNY Buffalo).

PRIZE AND PUBLIC LECTURES: The **Jeffery-Williams Lecture** will be given by **George Elliott**, University of Toronto. The **Krieger-Nelson Lecture** will be given by **Catherine Sulem**, University of Toronto. **F. William Lawvere**, SUNY Buffalo, will deliver a public lecture on Monday, June 15.

SYMPOSIA:

Category Theory (Org: **Richard Wood**, Dalhousie University); M. Barr (McGill), M. Bunge (McGill), P. Freyd (Pennsylvania), A. Joyal (UQAM), F. W. Lawvere (SUNY Buffalo), M. Makkai (McGill), S. Niefield (Union), R. Pare (Dalhousie), J. W. Pelletier (York), S. Schanuel (SUNY Buffalo), M. Tierney (Rutgers), W. Tholen (York), R.F.C. Walters (Sydney).

Convex Geometry (Org: **A.C. Thompson**, Dalhousie University); L. Batten (Manitoba), T. Bisztriczky, (Calgary), J. Bracho (Nat. Univ. Mexico), R. Dawson (SMU), B. Dekster (Mt. Allison), R. Erdahl (Queens), C. Fisher (Regina), R. Gardner (W. Washington), P. Goodey, (U. of Oklahoma), E. Grinberg (Temple and Polytechnic University), P. Gruber (Vienna), D. Klain (Georgia Tech), A. Koldobsky (Univ. Texas, San Antonio), J. D. Lewis (Edmonton), B. Monson (UNB), K. Rybnikov (Queens), R. Schneider (Freiburg, Germany), R. Vitale (U. Conn), A. Weiss (York), E. Werner (Case Western Reserve), G. Zhang (Polytechnic University).

Operator Theory (Org: **Heydar Radjavi**, Dalhousie University); H. Bercovici (Indiana), M.D. Choi (Toronto), R. Curto (Iowa), K. Davidson (Waterloo), D. Farenick (Regina), D. Hadwin (New Hampshire), M. Lamoureux (Calgary), L. Livshits (Colby), V. Lomonosov (Kent State), G. MacDonald (U.P.E.I.), L. Marcoux (Alberta), B. Mathes (Colby), E. Nordgren (New Hampshire), V. Paulsen (Houston), S. Power (Lancaster, U.K.), R. Rosenthal (Toronto), P. Semrl (Maribor, Slovenia), A. Sourour (Victoria).

Relativity and Geometry (Org: **Jacques Hurtubise and Niky Kamran**, McGill University); R. Bielawski (Max-Planck-Institute, Bonn), C. Boyer (New Mexico), A. Coley (Dalhousie University), A. Dancer (McMaster), P. Ehrlich (Florida), T. Ilmanen (Max-Planck-Institute, Leipzig), M. Kossowski (South Carolina) H. Kunzle (Alberta), R. McLenaghan (Waterloo), M. Min-Oo (McMaster), B. Tupper (New Brunswick), J. Wainwright (Waterloo), M. Wang (McMaster), G. Weinstein (Alabama).

Education - Mathematicians Teaching Statistics (Org: **Maureen Tingley and Barry Monson**, University of New Brunswick - Fredericton); Robert Dawson (SMU), David Hamilton (Dalhousie).

Combinatorics (Self-supporting session) (Org: **Katherine Heinrich and Brian Alspach,** Simon Fraser University, and **Abraham Punnen** University of New Brunswick - Fredericton); Speakers to be announced.

Graduate Student Seminar (Org: **Jennifer Mills**, University of New Brunswick (Saint John); A special session is being organized for graduate students. Anyone interested in participating in the organization of this programme should contact the Meeting Director at the following address: **md-s98@cms.math.ca.**

CONTRIBUTED PAPERS: Contributed papers of 15 minutes duration are invited and **graduate students are particularly urged to participate**. For an abstract to be eligible, the abstract must be received **before March 15, 1998**. The abstract must be accompanied by its contributor's registration form and appropriate fees.

SUBMISSION OF ABSTRACTS: The CMS publishes abstracts for all scheduled talks. Abstracts for Plenary Speakers, Prize Lecturers and Invited Special Session Speakers for the scientific and education programme will appear in the **April** issue of the *CMS Notes*. Abstracts for Contributed Papers will appear in the **May/June** issue of the *CMS Notes*. All abstracts will also be available on the Canadian Mathematical Electronics Services (Camel).

PLENARY SPEAKERS, PRIZE LECTURERS AND INVITED SPECIAL SESSION SPEAKERS FOR THE SCIENTIFIC AND EDUCATION PROGRAMME: These speakers are asked to submit their abstracts to the CMS as instructed by their organizers. Abstracts may be sent electronically, following instructions given below. Abstracts may also be prepared on the standard CMS form available from the session organizer or the CMS office in Ottawa. Abstracts should be sent to the Abstracts Coordinator, CMS Executive Office, 577 King Edward, P.O. Box 450, Station A, Ottawa, Ontario CANADA K1N 6N5, **so as to arrive by the invited speaker deadline of February 15, 1998.**

CONTRIBUTED PAPERS: Those submitting contributed papers may submit their abstracts electronically, following instructions given below, or by using the standard CMS form available from the CMS office in Ottawa or in the **January/February** issue of the *CMS Notes*. Abstracts should be sent to the Abstracts Coordinator, CMS Executive Office, 577 King Edward, P.O. Box 450, Station A, Ottawa, Ontario CANADA K1N 6N5, **so as to arrive by the contributed papers deadline of March 15, 1998**.

ELECTRONIC SUBMISSION OF ABSTRACTS: This service is available only to those who use the TeX typesetting system. Files should include the speaker's name, affiliation, complete address, title of talk and the abstract itself. Files may be sent by e-mail to the Abstracts Coordinator at: **abstracts@cms.math.ca.** *Please note the appropriate deadline given above for the submission of your abstract. Please note that we cannot accept abstracts sent by FAX.*

SOCIAL EVENTS: Social events include a cash-bar reception on Friday evening during evening registration at the Saint John Hilton, a delegates' luncheon on Saturday, June 13 (the cost of this luncheon is included in most registration categories), and a seafood banquet on Sunday, June 14 (tickets for the banquet will be available).

REGISTRATION: Forms are available from the CMS Executive Office, 577 King Edward, Suite 109, PO Box 450, Station A, Ottawa, Ontario, CANADA K1N 6N5 Tel: 613-562-5702, FAX: 613-565-1539, e-mail: **meetings@cms.math.ca.** Payment may be made by cheque, or by VISA or MasterCard. Although *registration fees are given in Canadian dollars*, delegates may send cheques in U.S. dollars by contacting their financial institution for the current exchange rate. Speakers should contact their organizers for special speaker rates. **Electronic pre-registration** is available on our Camel site at **http://camel.math.ca/Events/summer98/**. This site also has the latest information on the meetings.

	Before May 15	After May 15
CMS/AMS/MAA members with grants	$200	$285
CMS/AMS/MAA members without grants	110	140
Non-members with grants	325	425
Non-members without grants	165	210
One-day fee	110	140
Teachers/students/postdocs/retired/unemployed	80	105
Sunday night Banquet	45	45

ACCOMMODATION: It is recommended that those attending the conference book early to avoid disappointment. Blocks of rooms have been reserved at three different facilities and will be held until the dates given below. Reservations not in by that date will be on a request only, space available basis. Attendees should make their own reservations. Please mention that you are participating in the CMS Summer Meeting. The conference rate is extended up to two days pre and post convention.

Saint John Hilton
One Market Square
Saint John, New Brunswick, CANADA E2L 4Z6
Check-in: 3:00 pm, Check-out: 12:00 noon
Reservation Deadline: **April 30, 1998**
Rates:
$95.00 single/double Deluxe
$95.00 triple/quadruple Deluxe
$125.00 single/double Plaza Room
$125.00 Mini Suite
Applicable taxes: 15% Harmonized Sales Tax (HST)
Phone: (506) 693-8484
Reservations Fax: (506) 658-0783
Toll-free reservations: 1-800-561-8282

Rooms are held until 4:00 p.m. unless a first night deposit is received or guaranteed for late arrival to a credit card. Deposit will be returned if the reservation is cancelled three days prior to the requested date.

Howard Johnsons Hotel
400 Main Street at Chesley Drive
Saint John, New Brunswick, CANADA E2K 4N5
Check-in: 3:00 pm, Check-out: 1:00 pm
Reservation Deadline: **May 11, 1998**
Rates:
$75.00 single/double/triple/quadruple
Applicable taxes: 15% Harmonized Sales Tax (HST)
Phone: (506) 642-2622; Fax: (506) 658-1529
Toll-free reservations: 1-800-4SJ-HOJO (475-4656)

Only single rooms with pull-out couches available.

Sir James Dunn Residence
University of New Brunswick (Saint John)
P.O. Box 5050
Saint John, New Brunswick, CANADA E2L 4L5
Check-in: 8:00 am–4:00 pm (Monday to Friday)
after hours - security will check you in
Check-out: 12:00 noon
Reservation Deadline: **May 11, 1998**
Rates: $28.40 single room and $41.50 double room
Call, fax, or e-mail for detailed information on rooms.
Phone: (506) 648-5755; Fax: (506) 648-5662
E-mail: **flewwt@unbsj.ca** (Theresa Flewwelling).
Net: **http://www.unbsj.ca.**

Payments are required upon arrival. Cash, VISA, MasterCard only. No personal cheques please.

Child care: Both the Saint John Hilton and the Howard Johnson will provide complimentary cribs. Both hotels can arrange for child care, given 24 hours notice. The YMCA-YWCA Saint John also has child care facilities. It is located near both hotels.

TRAVEL: Participants are warned to check their airline tickets to make sure they are destined for Saint John, New Brunswick and NOT St. John's, Newfoundland. Upon arrival, taxis are available for the 20 minute ride to the hotels. The Inter City Bus (SMT) arrives at downtown Saint John, the bus station being located approximately 2 kilometers from the Hilton and approximately 2.5 kilimeters from the Howard Johnson. The nearest railway station is in Moncton, New Brunswick. There will be bus service from Moncton to Saint John. By car, from highway #1, take the Millidgeville exit.

ACKNOWLEDGEMENTS: The Meeting Committee wishes to extend its thanks to the members of the Mathematics Department at the University of New Brunswick, for their support.

MEETING COMMITTEE: *Meeting Director*: Robert Rosebrugh (Mount Allison), *Local Arrangements Committee Chair*: Abraham Punnen (UNBSJ), *Category Theory*: Richard Wood (Dalhousie), *Convex Geometry*: A.C. Thompson (Dalhousie), *Combinatorics* Katherine Heinrich and Brian Alspach (SFU) and Abraham Punnen (UNBSJ), *Education - Mathematicians Teaching Statistics*: Maureen Tingley and Barry Monson (UNBF), *Graduate Seminar*: Jennifer Mills (UNBSJ), *Operator Theory*: Heydar Radjavi (Dalhousie), *Relativity and Geometry*: Jacques Hurtubise and Niky Kamran (McGill), *Other members*: Monique Bouchard (CMS) – Ex-officio, Mohammad Hamdan (UNBSJ), Jon Thompson (UNBF), G.P. Wright (CMS) – Ex-officio.

International Congress of Mathematicians Berlin, Germany August 18-27, 1998

Second Announcement

The Organizing Committee is pleased to invite you to attend the International Congress of Mathematicians in Berlin, August 18–27, 1998.
Bundespräsident Professor Roman Herzog has personally expressed a most cordial welcome to all participants.
The Congress will be held under the auspices of the International Mathematical Union and under the sponsorship of the German Mathematical Society (DMV), the Federal Ministry of Education, Science, Research and Technology, and the Senate of Berlin.
This announcement describes the Congress and gives related information. It explains how to register and how to submit a short communication or a poster representation. It also contains the necessary forms for securing accommodation during the Congress.

Contents

A. Location of the Congress

A.1 Venue

The mathematical tradition in Germany is linked to the International Congresses. Felix Klein at the opening of the Congress on Mathematics and Astronomy in Chicago 1893 stressed that the mathematicians must form international unions. At the second congress in Paris 1900 David Hilbert presented his list of problems, the third congress was held in Heidelberg 1904, and now the German Mathematical Society is happy to welcome you at the end of the century to Berlin.

Berlin has a long mathematical tradition. Euler, one of the greatest mathematicians of all time, worked and lived here for 25 years. For the 19th century, names from Dirichlet to Jacobi, from Kummer to Weierstraß stand out. The exodus of mathematicians from Berlin because of the Nazi terror was dramatic and tragic. After the second world war the city was divided, and mathematics and all other sciences had to overcome many difficulties. Now the city is united again and promises to become a great mathematical center through its three universities and two mathematical research institutes.

Berlin is known throughout the world as a political center, as a symbol of separation and then of unification. But Berlin is also a wonderful city with lakes and woods, historical sites and sights, museums and galleries, theaters and concert halls, and not the least, its famous nightlife. There will be enough to do and to see for everybody.

A.2 The Congress

The activities of the Congress are divided between two locations. The program of the first day, August 18, 1998, including the opening ceremony, will take place at the International Congress Center (ICC). On all other days, the lectures, seminars and short communications will be held at the Technical University (TU) which is situated downtown.

The address of the Congress is:

ICM'98
c/o Prof. Dr. J. Winkler
TU Berlin, MA 8–2
Str. des 17. Juni 135
D-10623 Berlin, Germany

Phone: +49 30 314-24105
Fax: +49 30 314-21604
E-mail: icm98@zib.de
WWW: http://elib.zib.de/ICM98

A.3 Deadlines

May 1, 1998	Early Registration at reduced rate
May 1, 1998	Submission of abstracts for • Plenary Lectures • Invited Lectures • Short Communications • Poster Sessions
May 15, 1998	Hotel accommodation
June 15, 1998	Cancellation (with partial refund) of • tickets for the opera • tickets for the tourist program
July 1, 1998	Submission of manuscripts of • Invited Lectures for the Proceedings • Plenary Lectures for the Proceedings
July 15, 1998	Cancellation (with partial refund) of • private accommodation • registration

Further Deadlines

March 1, 1998	Submission for the Session on Mathematical Software
April 3, 1998	Submission of videos for the VideoMath Festival

B. Scientific Program

The tentative program is summarized on page 330. Some changes may become necessary depending on the number of speakers. The final program will be available electronically by June 1, 1998.

B.1 Opening and Closing Ceremonies

The opening ceremony will be held in the International Congress Center (ICC) at 10:00 on Tuesday, August 18, 1998. As part of the program, the Fields medals and the Nevanlinna prize will be awarded. The ICC seats up to 5000 people and allows a good view from all seats.

After the opening session, there will be a lunch reception at the ICC followed by addresses on the works of the Fields medalists and the Nevanlinna prize winner and the first Plenary Lecture.

The closing ceremony, including the last Plenary Lecture, is scheduled for Thursday, August 27, at 14:00 in the main lecture hall (Auditorium Maximum) of TU Berlin.

B.2 Plenary Lectures

At the recommendation of the Program Committee, appointed by the International Mathematical Union (IMU), the Organizing Committee has invited 21 mathematicians to give one-hour Plenary Lectures. The names of the speakers, together with their affiliations and fields of research, are listed below. The Plenary Lectures will inform participants of major developments, problems, and trends in mathematics.

The IMU General Assembly, the Program Committee, and the Organizing Committee have emphasized that these lectures should be comprehensible to a wide spectrum of mathematicians. All plenary speakers have agreed to prepare addresses for a general mathematical audience.

Jean-Michel Bismut (Université Paris-Sud, Orsay, France): Differential Geometry and Global Analysis

Christopher Deninger (Universität Münster, Germany): Arithmetic Algebraic Geometry, L-Functions of Motives

Persi Diaconis (Mathematics and ORIE, Cornell University, Ithaca, USA): Statistics, Probability, Algebraic Combinatorics

Giovanni Gallavotti (Universitá La Sapienza, Roma, Italy): Dynamical Systems, Statistical Mechanics, Probability

Wolfgang Hackbusch (Universität Kiel, Germany): Numerical Analysis, Scientific Computing

Helmut H. W. Hofer (Courant Institute, New York University, USA): Global Analysis, Dynamical Systems

Ehud Hrushovski (Hebrew University of Jerusalem, Israel): Logic

I. G. Macdonald (Queen Mary and Westfield College, University of London, England): Lie Groups, Algebraic Combinatorics

Stéphane Mallat (École Polytechnique, CMAP, Palaiseau, France): Applied Mathematics, Signal Processing

Dusa McDuff (SUNY Stony Brook, USA): Symplectic Topology

Tetsuji Miwa (RIMS, Kyoto University, Japan): Integrable Systems, Infinite Dimensional Algebras

Jürgen Moser (ETH Zürich, Switzerland): Dynamical Systems, Partial Differential Equations

George C. Papanicolaou (Stanford University, USA): Applied Mathematics, Probability

Gilles Pisier (Université Paris VI, France and Texas A&M University, College Station, USA): Functional Analysis

Peter Sarnak (Princeton University, USA): Number Theory

Peter W. Shor (AT&T Labs, Florham Park, USA): Computer Science

Karl Sigmund (University of Vienna, Austria): Mathematical Ecology, Evolutionary Game Theory

Michel Talagrand (C.N.R.S., Université Paris VI, France): Probability, Statistical Mechanics, Functional Analysis, Measure Theory

Cumrun Vafa (Harvard University, Cambridge, USA and Tehran, Iran): String Theory, Quantum Field Theory and Quantum Gravity

Marcelo Viana (IMPA, Rio de Janeiro, Brazil): Dynamical Systems, Ergodic Theory

Vladimir Voevodsky (Northwestern University, Evanston, USA): Algebraic Cycles and Motives

The first Plenary Lecture will be given by J. Moser and will be delivered in the afternoon of the opening day at the International Congress Center. The other addresses will be given in the Auditorium Maximum (lecture hall H 105) of TU Berlin. The Auditorium Maximum has a capacity of 1200 seats. The talks will also be shown in an adjoining lecture hall via closed-circuit television, so that an audience of more than 2000 people can be reached.

B.3 Invited Lectures

Also at the recommendation of the IMU Program Committee, more than 160 mathematicians have been invited to give 45-minute lectures in specified sections. These lectures are intended to be surveys of significant topics in the respective area of research. The speakers have been asked to make their lectures comprehensible for the general mathematical community, and they agreed to do so.

These Invited Lectures will take place in the afternoons of August 19–22 and 24–26, beginning at 14:00. The Invited Lectures will be given in several large lecture rooms at TU Berlin. Usually there will be about six or seven lectures simultaneously. The list of sections is as follows, where the number in parentheses indicates the expected number of presentations in that section.

1. Logic (5)
2. Algebra (8)
3. Number Theory and Arithmetic Algebraic Geometry (9)
4. Algebraic Geometry (7)
5. Differential Geometry and Global Analysis (13)
6. Topology (8)
7. Lie Groups and Lie Algebras (10)
8. Analysis (14)
9. Ordinary Differential Equations and Dynamical Systems (10)
10. Partial Differential Equations (10)
11. Mathematical Physics (12)
12. Probability and Statistics (13)
13. Combinatorics (8)
14. Mathematical Aspects of Computer Science (6)
15. Numerical Analysis and Scientific Computing (6)
16. Applications (12)
17. Control Theory and Optimization (7)
18. Teaching and Popularization of Mathematics (6)
19. History of Mathematics (3)

B.4 Short Communications and Posters

All Ordinary Members (see I.1) of the Congress will have the opportunity to present their mathematical work in the form of a Short Communication or a Poster — provided that

- they have registered by May 1 and marked in the registration form, that they want to present their work,
- they have submitted an abstract by that date,
- their contribution has been accepted by the Local Scientific Committee.

Only one Short Communication or Poster (and thus only one abstract) is allowed for each member. Each Short Communication lasts 15 minutes including discussion. Short Communications are grouped into time slots of 45 minutes for three presentations. The rooms for Short Communications are equipped with a blackboard and an overhead projector. Each Poster session lasts 105 minutes; during that period the authors should stand by their posters and be available for questions and discussion. Authors presenting a Poster are advised to bring the material of the Poster with them when they come to the Congress since no facilities for preparing posters are available on site. The size of the individual poster panels is as follows: width 180 cm, height 120 cm.

The abstract for a Short Communication or a Poster must include the appropriate section number (see B.3) and 1991 MS classification number (see N.) so that the Communications and Posters can be grouped in a coherent way for presentation. Abstracts may be submitted in English, French, Russian or German. Abstracts of accepted Posters and Short Communications which are properly prepared and received by the deadline will be reproduced and distributed to all Ordinary Members when they pick up their registration package. The Local Scientific Committee will notify authors of acceptance/rejection of their contribution.

Instructions on how to prepare an abstract are in Section D.3. Abstracts which do not conform to the stipulated rules will be returned to the author for resubmission. Late papers will not be accepted. However, it is possible to present them in ad-hoc sessions that will be organized and announced during the conference.

B.5 Organized Sessions

We invite Ordinary Members to organize sessions of Short Communications on their own initiative. Such sessions and their organizers will be included in the final program after a review by the Local Scientific Committee. Ordinary Members who want to organize such a session should contact the Local Arrangements Chair (`moehring@math.tu-berlin.de`) with their program and their request for time slots within the schedule of Short Communications.

All participants of these organized sessions have to submit an abstract for their Short Communication as indicated in Section D.3. The organizer of such a session should inform the Local Arrangements Chair about the submissions that belong to this session.

B.6 Informal Seminars

During the Congress it will also be possible to organize informal mathematical seminars on site.

Ordinary Members who wish to organize such spontaneous seminars are asked to make all arrangements among themselves, and to request a room either in advance (`moehring@math.tu-berlin.de`), or during ICM'98 from the congress office or from the Local Arrangements Chair. Such seminars may take place during the afternoons. If the congress office is notified before 15:00 the previous day, an announcement of the seminar can be included in the daily newsletter to be distributed to all participants on the following day.

B.7 Special Activities

Various suggestions for sessions of general mathematical interest have been brought to the attention of the Organizing Committee. In accordance with IMU and the Program Committee, the Organizing Committee has opened a new *Section of Special Activities* to cover the most important topics. This section is not part of the official scientific program as specified by the Program Committee. The section is still taking shape. It will probably include

- Sessions on **Mathematical Software.**
- A workshop entitled **Berlin as a Center of Mathematical Activity** organized by the International Commission on the History of Mathematics (ICHM).
- Talks, discussions, and roundtables related to **Electronic Publishing**.
- The **Noether Lecture,** a roundtable and a luncheon organized by a group representing *Women in Mathematics.*
- Further roundtables, e.g., on educational topics, and other events such as the presentation of O. Lehto's new book on the IMU.

The Deutsche Mathematiker-Vereinigung DMV will present an exhibition on the suppression and expulsion of mathematicians from the Berlin universities by the Nazis. There will also be lectures concerning the impact of the Nazi regime on mathematics.

C. Other Events of Mathematical Interest

C.1 Events for the General Public

The International Congress provides a unique opportunity to inform the general public about some of the recent developments and future challenges of mathematics. Planned are:

- A special lecture by Andrew Wiles (Princeton), on the development of number theory in the last twenty years.
- Several lectures for a general, in particular, nonmathematical audience.
- The VideoMath Festival, where films of mathematical interest are shown.
- The exhibition *Hands-on Mathematics,* aiming in particular at teachers and high school students.
- Small exhibitions related to mathematics, e.g. mathematical posters, mathematics and ceramics.

The lecture by Wiles will be in the Auditorium Maximum at TU Berlin, August 19, 19:30. All other presentations will take place in the evening, August 21–22 and 24–28 at the Urania, a lecture institute of long tradition situated not far from TU Berlin. (There will be a modest admission fee for some events at the Urania.) The exhibitions will be situated in the Urania and TU. You will find a detailed program on the WWW-server by June 1, 1998, and in the final program.

C.2 Book Exhibitions etc.

Book, educational media, and computer software exhibits are located near the main lecture hall and in the large atrium in the main building of the University. They will be open on August 19–22 and 24–26 from 10:00 to 18:00, and on August 27 from 10:00 to 15:00. All participants are encouraged to visit the exhibits during the Congress.

C.3 Other Mathematical Conferences

Traditionally there are several smaller conferences scheduled at various places immediately before or after the Congress. Starting on page 334, you find a list of such satellite conferences (as of December 1, 1997), together with the addresses where further information may be obtained. Please do not direct inquiries about these conferences to ICM'98.

D. Publications

The material described in the items D.1–D.4 will be included in the registration package that each Ordinary Member will receive at the registration counter.

D.1 Program

All Ordinary Members will receive copies of the official ICM'98 program. The program will show in detail the dates, times and locations of all Plenary and Invited Lectures. It will also contain a complete listing of all Short Communications and Poster presentations. See the ICM'98 WWW server, after June 1, 1998.

D.2 List of Participants

A list of all participants who have registered by July 15, 1998 will be distributed to all Ordinary Members. A list of all ICM'98 members, including their mailing addresses, will be kept at the ICM'98 registration counter throughout the Congress. Ordinary Members are asked to check their own listing for accuracy while they are in Berlin, as this list will be used to prepare the official list of participants for inclusion in the Proceedings and to prepare mailing labels for shipment of the Proceedings.

D.3 Abstracts

Abstracts of Plenary and Invited Lectures, Short Communications and Posters will be reproduced and distributed in printed form to all Ordinary Members at the beginning of the Congress. These abstracts will also be available on the internet under the following address:

`http://www.mathematik.uni-bielefeld.de/ICM98`

Abstracts of Short Communications and Posters should be written in English, French, Russian or German and should have the following form (compare also the enclosed example):

- Section Number (see B.3)
- 1991 Mathematics Subject Classification number (see N.)
- Name and affiliation of author(s)
- Title
- Abstract text (no more than 120 words)

Example Abstract

Section: 2

1991 MS Classification: 17, 18, 55

Loday, Jean-Louis, Université de Strasbourg, France: **Leibniz algebras and their (co)homology.**
A *Leibniz algebra* is a vector space equipped with a product satisfying a variation of the Jacobi identity: $[x,[y,z]] = [[x,y],z] - [[x,z],y]$. There is a dual notion in the sense of Koszul duality for operads. For any Leibniz algebra $\mathbf{g}$ there is a (co)homology theory $HL(\mathbf{g})$, which satisfies various properties including the following: $HL^*(\mathbf{g})$ is a dual Leibniz algebra. Applications to non-commutative rational homotopy theory will be presented. Part of these results is joint work with T. Pirashvili.

Reference: J.-L. Loday, and T. Pirashvili, Universal enveloping algebras of Leibniz algebras and (co)homology, *Math. Ann.* **296** (1993), 139–158.

Abstracts should be submitted electronically via the forms provided by the above WWW address or by e-mail using the subject "`ICM'98 abstract`" to

`rehmann@mathematik.uni-bielefeld.de`

TeX and html forms supporting electronic submission will be available under the above WWW address.

Submission is also possible by fax or by ordinary mail to

ICM'98 Abstracts
c/o Prof. Dr. Ulf Rehmann
Fakultät für Mathematik
Universität Bielefeld
Postfach 100131
D-33501 Bielefeld, Germany
Fax: +49 521 106-4743

However, electronic submission is strongly encouraged.

The Deadline for submission of abstracts is May 1, 1998.

D.4 The Berlin Intelligencer

The *Mitteilungen* of the German Mathematical Society (DMV) and Springer-Verlag Berlin/Heidelberg will together prepare, publish, and present to all participants a magazine called the "Berlin Intelligencer", which is meant to welcome you to Berlin and to the ICM'98 and to guide your visit to Berlin and to some of its manifold aspects and attractions.

D.5 Proceedings

All Plenary and Invited Lectures as well as the congress report will be published in the Proceedings of the International Congress of Mathematicians, Berlin, 1998. These Proceedings will appear as a special volume of DOCUMENTA MATHEMATICA (Journal der Deutschen Mathematiker-Vereinigung), whose electronic version is available at

`http://www.mathematik.uni-bielefeld.de/documenta`
or `http://www.math.uiuc.edu/documenta`

A printed version will be distributed to all Ordinary Members of the Congress (except for the student members). There will also be a free internet version under the above WWW addresses.

D.6 Daily News

A newsletter containing program changes, announcements of informal seminars and information of general interest to ICM'98 participants will be available each day at the TU conference office (H 2036). Participants with announcements for the newsletter should be sure to submit them to the conference office no later than 15:00 the day before.

E. Social Program

E.1 Opening Lunch Buffet

On August 18, from 12:30 to 14:30 (between the Opening Ceremony and the Afternoon Session) a complimentary lunch buffet will be offered to all participants. It will be served in the International Congress Center (ICC).

E.2 ICM'98 Party

The Organizing Committee plans to stage a complimentary party for all participants. It will probably take place in the evening of August 26, 1998 on the campus of TU Berlin.

E.3 Opera

On the evening of Sunday, August 23, at 19:30, a special performance of the opera ***The Magic Flute*** by Wolfgang Amadeus Mozart will be staged at the Deutsche Oper Berlin. For this performance, a large contingent of tickets has been reserved for participants of ICM'98. The tickets are available in five price categories, with prices ranging from 52 DM to 88 DM. Participants who wish to attend this special performance must purchase tickets in advance — using the registration form.

E.4 Tourist Program

Several excursions (bus tours) will be offered by DER-CONGRESS during August 19–26, 1998. For a description of these tours see Section K.2

Reservations can be made on the registration form; if seats are left, it will also be possible to make bookings during the Congress (please contact the Tourist Program counter at TU Berlin). All tours will start in front of the TU Main Building, Str. des 17. Juni 135.

E.5 Footloose Tours

To convey some of the many facets of Berlin to the ICM'98 participants, and in particular to accompanying persons, several Berlin mathematicians, their friends and spouses have volunteered to offer informal tours of Berlin including visits to special museums, sights, and shopping streets. The tours are free of charge, except for possible entrance fees. They will be announced shortly before the Congress by means of Circular Letters from the ICM'98 e-mail service. They will also be posted on the WWW-server of ICM'98 or during the Congress on an information board. It will be possible to register by e-mail. To maintain the informal character of the tours, group sizes are tightly restricted. Places will be assigned on a first–come–first–serve basis.

E.6 Activities for Accompanying Persons

Accompanying persons do not have to register, see I.2. They are invited to participate in all activities of the social program. The Footloose Tours are especially planned for this group of visitors to guide them to nonstandard places of interest, to meet other people, and to explore Berlin in a casual way. Moreover, the events for the general public in the Urania, see C.1, are also intended for accompanying persons with some interest in mathematics.

F. Travel

F.1 Passports and Visas

For citizens of the countries of the European Community an identity card is sufficient. Citizens of other countries must be in possession of a valid passport, and for certain countries visas are required as well. Please consult the German Embassy or Consulate in your country for details. We have contacted the German Foreign Office on visa issues and do not expect difficulties. If you need a personal invitation to attend the Congress, you can request an official invitation letter (see Section H.2).

We advise you to apply for a visa at least three months before the date on which you plan to leave for Germany. If you have not received a visa one month before the beginning of ICM'98, contact us by fax or e-mail (SUBJECT: visa). In order to help you, we need the following information: name, address and date of birth, passport number, date and place of visa application.

F.2 Congress Agents

DER-CONGRESS has been appointed by the Organizing Committee to handle registration for the Congress, hotel reservations, etc. for all individual participants of the Congress.

Please send all correspondence related to the Congress (inquiries, requests, etc.) to the official congress address, as indicated in A.2. DER-CONGRESS can be contacted directly at:

DER-CONGRESS
Congress Organisation
Bundesallee 56
D-10715 Berlin, Germany

Phone:	+49 30 857 903-0
Fax:	+49 30 857 903-26
E-Mail:	`icm@der-congress.de`

Participants from Japan are requested to contact the official ICM'98 agent for Japan who will forward the registration form to DER-CONGRESS:

Nikkei Culture Inc.
Mr. Sumiyoshi
9-5, 1 Chome, Otemachi
Chiyoda-ku
Tokyo 100, Japan

Phone: (03) 5259 2666
Fax: (03) 5259 2664

F.3 Arriving in Berlin

Berlin is easy to reach by all means of transportation. It has three airports, **Berlin Tegel** (aviation code TXL on your airline ticket), **Berlin Tempelhof** (THF) and **Berlin Schönefeld** (SXF). If you come to Berlin by train, you will arrive at one of the three main stations, which are **Zoologischer Garten** (in the western city center, walking distance to TU Berlin), **Lichtenberg** and **Hauptbahnhof** (both of them are in the east of Berlin and further away from ICM'98 activities). For more geographical details, see `http://www.berlin.de`

All airports and train stations are well connected to all main locations in Berlin by public transport — Berlin is well served by a far reaching network of bus, U-Bahn (subway) and S-Bahn (inner city railway) lines. The standard fare for a single ride AB-ticket that is valid on all busses, subways etc. will be DM 3.90. You will need this only on your day of arrival, since with the congress registration materials you will receive your Local Transportation Ticket (see F.4).

If you come by car, we suggest you leave it at the hotel and use the ICM'98 Local Transportation Ticket. There is ample parking space at the ICC, but parking is very limited at TU Berlin.

Many hotels are in the Western city center, close to the *Zoologischer Garten* ("Zoo Station") Train Station, which is not only a train station, but also a U-Bahn (subway) and S-Bahn (inner city railway) station with good connections in all directions. (To get to TU Berlin, for example, you would take the U2 subway line, direction *Ruhleben*, and leave it at the next stop, *Ernst-Reuter-Platz*).

In this section, we therefore explain how to reach *Zoologischer Garten* from the airports and from the other train stations. To locate your personal accommodation or hotel you might try to consult the Berlin city map which is online at

`http://www.kulturbox.de/perl/berlininfo`

Detailed city maps (including a street index) are also posted at all bus and subway stations in Berlin, which could help you to locate your destination after arrival in Berlin. The main connections to reach the congress locations are explained in Section F.5.

From **Tegel** Airport:

The two bus lines with the numbers X9 and 109 operate the route to the city center. The X9 is a so-called express bus, which is fast, but makes only few stops. (But it does stop at TU's U-Bahn station, *Ernst-Reuter-Platz.*) The 109 bus is much slower, but it takes a route down *Kurfürstendamm* which is nice and convenient for the many hotels on and near this (main shopping) street. Both lines pass or end at *Zoologischer Garten.*

Duration: approx. 20 minutes with the X9 and approx. 30 minutes with the 109 bus.

From **Tempelhof** Airport:

Take the U6 subway line, direction *Alt-Tegel*, up to the *Stadtmitte* station. There switch to the U2 subway line, direction *Ruhleben*, which will take you to the *Zoologischer Garten* station.

Duration: approx. 30 minutes.

From **Schönefeld** Airport:

This airport is just outside the city limits, southeast of

Berlin. To get to the center of Berlin, walk from the terminal building to the S-Bahn station. From there the S9 line will take you to *Zoologischer Garten.*

Duration: approx. 50 minutes.

From **Lichtenberg** train station:

This station is in the east of Berlin. From there two S-Bahn lines, S5 direction *Charlottenburg* and S7 direction *Potsdam Stadt*, will take you directly to the *Zoologischer Garten.*

Duration: approx. 30 minutes.

From **Hauptbahnhof** train station:

This station is in the east of Berlin. Take one of the four S-Bahn lines operating from *Hauptbahnhof* in the western direction, whose destination may be *Potsdam Stadt*, *Westkreuz* or *Charlottenburg.* Any of these will take you to *Zoologischer Garten.*

Duration: approx. 20 minutes

F.4 Your Local Transportation Ticket

With your congress materials, distributed at registration in Berlin, you will receive a special ticket that is valid for all types of public transportation (*Bus, U-Bahn, S-Bahn*) in Berlin and in Potsdam, for the complete duration of the Congress (Aug. 17-27, 1998). The ticket is included in the registration fee. Additional such tickets, e.g. for accompanying persons, may be bought at the registration counter for DM 47,–.

F.5 Reaching the Conference Locations

TU Berlin is located at *Ernst-Reuter-Platz*, which is a stop for the U2 subway line (one stop away from *Zoologischer Garten*), for the X9 express bus (from Berlin Tegel airport), and for the 145 and 245 bus lines. Leaving the *Ernst-Reuter-Platz* subway station take the exit marked *Technische Universität.* Above ground you will see the three conference buildings: the Architecture Building (A) with orange sun blinds right on Ernst-Reuter-Platz, the red-and-blue Math Building (MA), and the (huge white) Main Building (H), facing each other on the street called *Str. des 17. Juni*. TU Berlin is also located within walking distance from the S-Bahn station *Tiergarten.*

To reach ICC (e.g. from *Zoologischer Garten*, see F.3) take the subway line U2 until the stop *Kaiserdamm* and follow the *ICC* signs from there (a good 5 minutes walk). ICC is also served by the bus lines 104 (directly from Tempelhof airport) and by the X21, X49, 149, and 204 bus lines which all stop directly at ICC, by the S-Bahn line S45/46 (station *Witzleben*), and by the S-Bahn lines S3, S7, S75, and S9 (station *Westkreuz*, 5 minutes walk).

F.6 Taxis (Cabs)

There is, of course, also the possibility to take a taxi. To get an idea about the costs, here are some typical prices:
Zoologischer Garten — ICC approx. 18 DM
✈ Tegel — ICC or TU Berlin approx. 25 DM
✈ Tempelhof — ICC or TU Berlin approx. 25 DM
✈ Schönefeld — ICC or TU Berlin approx. 70 DM
Lichtenberg — ICC or TU Berlin approx. 35 DM
Hauptbahnhof — ICC or TU Berlin approx. 30 DM

Please note that all the rates quoted here are average rates, based on normal traffic conditions.

G. Mail and Messages

G.1 Mail

All mail, telegrams, and faxes for persons attending the Congress should be addressed to:

Name of the Participant
c/o ICM'98, Prof. Dr. J. Winkler
TU Berlin, MA 8–2
Str. des 17. Juni 135
D-10623 Berlin, Germany
Fax: +49 30 314-21604

Incoming items will be posted at the conference office in the main building.

G.2 Telephone Messages

For urgent messages to a participant, call

+49 30 314-24105
or +49 30 314-25224.

To avoid mistakes, only simple messages will be taken and will be put in the respective mailbox. Unless it is a matter of life and death, it will not be possible for our secretarial staff either to check the presence of a specific person or to check whether the message actually reaches the intended person.

At the registration desk area at TU Berlin, there will be public phones accepting telephone-cards. Telephone-cards will be sold at the conference office.

G.3 Personal Messages

Participants wishing to exchange personal messages during ICM'98 should use the mailboxes. We regret that messages left in the box after the Congress cannot be forwarded to participants.

G.4 Public E-Mail Service

During the conference days public e-mail service will be available to Ordinary Members of ICM'98. For this, a pool of UNIX–workstations is offered, located in the TU Math-building (room MA 241). Opening hours during the Congress are from 9:00 to 18:00.

H. Miscellaneous Information

H.1 Official Languages

English, French, Russian, and German are the official languages of the Congress. Announcements, correspondence, and all other business matters will be carried out in English.

H.2 Invitation Letter

An Official Invitation Letter will be sent by the Organizing Committee upon request (see the second page of the registration form). This personal invitation is intended only to facilitate participants' travel and visa arrangements, but we regret that we cannot provide any financial or other support.

H.3 Libraries

Several (mathematics) libraries will be available for Ordinary Members of ICM'98. The reading room of the main library (main building H 3503) features a rich collection of reference works in all fields of science, especially of technology. The mathematics library is located in the building of the Department of Mathematics (MA 163). Please note that books cannot be checked out for any reason.

Libraries at TU Berlin are open Monday through Friday from 9:00 to 18:00.

Furthermore, there are also mathematics libraries at Freie Universität Berlin and at Humboldt–Universität Berlin. Please contact the conference office for additional information concerning addresses and opening hours.

H.4 Climate and Clothing

The Congress takes place during late summer where the temperature is around 23°C (73°F) during the day and 13°C (55°F) at night. It may be useful to bring a sweater and raincoat.

H.5 Electric Current

The electric current is 220 V (50 Hz). Please note that adapters may be necessary.

H.6 Bank Services

The two closest banks, both within walking distance, are:

Hypo Bank	Deutsche Bank
Ernst-Reuter-Platz 9–10	Otto-Suhr-Allee 6
10585 Berlin	10585 Berlin

Opening hours are:

Monday and Wednesday 9:00–15:30
Tuesday and Thursday 9:00–18:00
Friday 9:00–12:30
Saturday and Sunday closed.

H.7 Credit Cards

Participants are reminded that smaller shops and restaurants may not be prepared for credit card payment.

H.8 Shopping Hours

Shopping hours are from 9:00–20:00 Monday to Friday and from 9:00 to 16:00 on Saturdays. All shops are closed on Sundays.

H.9 First Aid and Health Insurance

The congress fee does not include insurance for the participants against accidents, sickness, or loss of personal property. All participants are strongly advised to make necessary arrangements for short–term health and accident insurance in advance. In any case, the organizers refuse all liability to cover health or accident expenses of participants unless expenses are due to an act of negligence by ICM'98.

During the conference, First Aid will be available in the main building. In case of emergency, please contact the registration counter, the congress office, or one of the lecture room attendees.

H.10 Child Care

Child care can be arranged upon request.
Please contact the Local Arrangements Chair (moehring@math.tu-berlin.de).

I. Registration

I.1 Membership

It has been a long tradition to call any person who has registered for ICM'98 an *Ordinary Member* of the Congress. Registration is required in order to be admitted to the scientific program of the Congress. Ordinary members will receive a registration package including a congress badge, the program, an abstract book, one complimentary public transport ticket valid for the duration of the Congress, and other material at the registration counter as well as a complimentary copy of the Proceedings when published. Furthermore the registration fee includes the Opening Ceremonies with the Lunch Buffet, free coffee during coffee breaks, and the ICM'98 Party.

Please do wear your congress badge at all congress activities or whenever you want to be recognized as a congress member; in any case, be prepared to show the badge when asked to do so.

Students not having completed their PhD may choose to register at the reduced student rate by supplying an official student certificate of their university. Student registration does not include the Proceedings volume.

I.2 Accompanying Persons

To minimize bureaucracy, accompanying persons will not be registered for ICM'98. However, accompanying persons are welcome to all social and public events of the Congress. For these occasions, blank name badges for accompanying persons will be available at the registration counter. Participants are encouraged to order additional public transport tickets (at reduced Congress rates of DM 47,–) for accompanying persons on the registration form.

I.3 Registration

There are two possible ways for ICM'98 registration. You may register electronically using the WWW registration form on the ICM'98–Server with the following URL:

`http://elib.zib.de/ICM98/RegistrationForm.html`

Alternatively, you can complete the attached registration forms (page 338) and return them to DER-CONGRESS, *as soon as possible.* **For participants who need hotel accommodation and ticket reservations for the tourist program we need the registration form by May 1, 1998.** Please note that all registrations have to be submitted on official registration forms. Please use a separate form for each participant. For additional copies of the registration form, please write to ICM'98 or copy the attached registration form (page 338). The registration will be considered as binding when it is received by DER-CONGRESS, and it will be confirmed as soon as the required fees have been received.

I.4 Registration Fees

Registration	until May 1, 1998	after May 1, 1998
Full registration fee	DM 450,–	DM 600,–
Students*	DM 200,–	DM 350,–

*Registration as a student requires an official certificate of the university to be attached.

The registration fee includes all the conference materials and the Proceedings of the Congress (exept for students), a free public transportation ticket for the duration of the Congress (August 17–27, 1998), the Lunch Buffet at the Opening Ceremony, and the ICM'98-Party.

I.5 Methods of Payment

The payment should be made in advance by one of the following methods:

a) bank cheque to be paid in German Currency (DM) to DER-CONGRESS

b) bank transfer in German Currency (DM) to DER-CONGRESS, Commerzbank AG Frankfurt/Main, Account no. 589 32 35, Sort Code 500 400 00. A copy of the remittance order must be attached to the registration form. *ICM'98* as well as the name of the participant (or, if payment is made for more than one person, all names) must be clearly marked on the remittance.

c) credit card: VISA / Diners Club / EUROCARD / MasterCard.

All fees for bank services which might occur have to be paid by the participants themselves and will, if not paid, be charged upon registration at the registration counter in the TU Berlin.

For details see the last page of the registration form.

I.6 Cancellations and Changes

Changes of Registration and Name Changes

A handling fee of DM 25,– will be charged for changes in registration except for name changes. These will be treated as cancellations and new registrations. The handling fee will not be charged if the complete registration is cancelled.

Cancellation of Registration

The following fees will remain due if the cancellation for the Congress is received by DER–CONGRESS in written form by July 15, 1998:

Participant	DM 150,–
Student	DM 100,–

No refund will be made for cancellations received after July 15, 1998. The full fee is also payable if the participant does not show up at the Congress. Kindly note that only written cancellations can be accepted.

I.7 Registration Counter

The registration counter is located in the main building of TU Berlin close to the Auditorium Maximum (H 105) where all Plenary Lectures will be given. The conference office (H 2036) is also located in the main building close to the large atrium of TU Berlin. The registration counter and the conference office will be operated during the following hours:

Monday,	August 17, 1998	14:00–20:00
Tuesday,	August 18, 1998	8:00–20:00
Wednesday,	August 19, 1998	8:00–20:00
Thursday,	August 20, 1998	8:30–18:00
Friday,	August 21, 1998	8:30–18:00
Saturday,	August 22, 1998	8:30–18:00
Sunday,	August 23, 1998	closed
Monday,	August 24, 1998	8:30–18:00
Tuesday,	August 25, 1998	8:30–18:00
Wednesday,	August 26, 1998	8:30–18:00
Thursday,	August 27, 1998	8:30–16:00

At this location, registered participants will receive their badges, congress documents, and their vouchers for all events that have been confirmed. The documents will not be mailed before the Congress. If fees have been forwarded late and therefore not yet been credited to the account of DER-CONGRESS on the day of arrival, a copy of the remittance order should be presented. The following credit cards will be accepted for on-site registration: VISA, EUROCARD, MasterCard, Diners Club.

On the opening day, there will be an additional information counter situated in the International Congress Center (ICC Berlin), which is open as follows:

Tuesday, August 18, 1998	9:00–18:00

Kindly note that this counter is operated for information purposes only. Therefore you cannot register on-site and you will not receive your personal congress documents at this location.

Everyone is invited to attend the ICM'98 Opening Day at ICC: Registration prior to this event is not necessary (but recommended).

J. Accommodation

J.1 Hotels

Many (large and small) hotels in Berlin are located in the Western City Center, in convenient (walking) distance to Zoo station and TU Berlin. General information about hotels may be obtained from the ICM'98 WWW-site.

If you want to request accommodation for the Congress via DER-CONGRESS, please notify that using the WWW or the attached registration form. Please note that telephone requests cannot be accepted. Your requests for hotel reservation will be processed as soon as the pre-payment of DM 420,– per room, payable to DER-CONGRESS has been received. (This includes a hotel deposit of DM 400,– which will be deducted from your final hotel bill, and a handling fee of DM 20,– for DER-CONGRESS).

Please note that when all the hotel rooms of a particular category have been booked, DER-CONGRESS will no longer be able to guarantee further rooms in a desired hotel category, and thus reserves the right to confirm reservations in another category.

DER-CONGRESS will hand over the reservation lists to the hotels two weeks before the Congress. If changes should be necessary after that date, you are kindly requested to contact the hotel directly by fax, and to submit a copy to DER-CONGRESS.

If you plan to arrive at your hotel after 18:00 on the scheduled day of arrival, please mark this on your registration form, as reserved rooms will be kept until that time. If it is necessary to cancel your reservation, you are kindly requested to cancel until May 15, 1998. Later the hotels may charge non-occupancy fees to the customer. Please understand that in case of cancellation an administrative fee of DM 50,– per person will be charged.

You will receive the corresponding hotel voucher together with your congress documents at the registration counter: this voucher has to be presented to your hotel at some point during your stay, but not necessarily at your arrival.

Group reservations are possible. Please contact DER-CONGRESS for separate contracts.

J.2 Private/Low Budget Accommodation

We are pleased to also offer the possibility to reserve accommodation in private households, until May 1, 1998. After that date DER-CONGRESS will not be able to guarantee that such rooms will be available. The prices per day are in the following range:

Single rooms from DM 62.– to 86.– incl. breakfast.
Double rooms from DM 86.– to 123.– incl. breakfast.
Apartments (double use) from DM 100.– to 150.– without breakfast.

Apartments and houses for more than two persons can be arranged on request.

The handling fee for the reservation in a private household is DM 30,– per person and will be charged by DER-CONGRESS.

Many of these rooms and apartments are equipped with TV and radio, and some also have a phone. However, most of the single and double rooms do not have a private bathroom. In this case the guests are usually sharing the bathrooms with their hosts. Apartments for two persons normally consist of one living/sleeping room, a kitchen, and a bathroom. At some apartments, you might have to pay a single fee of DM 40,– or 50,– for cleaning up after departure. In addition, some hosts would like to have a deposit in advance. Please be aware of the fact that some of the hosts may only be able to communicate in German.

A reservation form will be sent out by DER-CONGRESS upon receipt of the handling fee. Please fill in and return this form by fax or regular mail to DER-CONGRESS. The confirmation of reservation will be sent out directly afterwards, on which the address of the host is provided. To clarify all further details, such as time of arrival, payment conditions, etc. you should contact your host immediately after receipt. If you have not contacted your host four weeks prior to arrival at the latest, the reservation will be released without further notice.

Please understand that your host cannot process cheque or credit card payment, so kindly have cash ready on the day of arrival to cover the costs for the complete stay. Of course, pre-payment will be deducted.

Cancellations of private accommodation will be accepted free of charge until July 15, 1998. After that date the full amount for the stay is payable to the host if required. The handling fee cannot be reimbursed in any case.

J.3 Student Dormitories

If you prefer a reservation of a bed in a student dormitory (prices range between DM 26,– and DM 41,– per bed, incl. breakfast) you can contact Youth Hostels in Berlin directly under the following address:

`http://www.jugendherberge.de/berlin.html`
Phone: +49 30 262-3024
Fax: +49 30 262-9529

Reservation can then be made on your own using fax or e-mail.

J.4 Camping

`http://www.ecamp.de/cgi-bin/db.cgi`

J.5 Liability

In all cases DER-CONGRESS shall only act as an agent and shall not be liable for any losses, accidents, personal injury, or damage to property of any kind and origin whatsoever. The liability of the persons or companies instructed to provide the services shall not be affected. Amendments shall not be valid unless mutually agreed in writing and signed by the two contracting parties. The place of jurisdiction for fully qualified merchants, persons who do not have a general place of jurisdiction in Germany, and persons who have relocated their place of residence to a foreign country after the conclusion of the agreement shall be Frankfurt am Main.

Outline of the Program

Day	9–10	10–11	11–12	12–13	13–14	14–16	16–18	19–22
Tue., Aug. 18		Opening Ceremony and Prize Winners		Lunch Buffet		Laudationes and Plenary Lecture		
Wed., Aug. 19	Plenary Lecture		Plenary Lecture			Invited Lectures Communications Posters	Invited Lectures Communications Posters	Special Lecture A. Wiles
Thu., Aug. 20	Plenary Lecture		Plenary Lecture			Invited Lectures Communications Posters	Invited Lectures Communications Posters	Urania Activities Popular Lectures Exhibitions
Fri., Aug. 21	Plenary Lecture		Plenary Lecture			Invited Lectures Communications Posters	Invited Lectures Communications Posters	Urania Activities Popular Lectures Exhibitions
Sat., Aug. 22	Plenary Lecture		Noether Lecture			Invited Lectures Communications Posters	Invited Lectures Communications Posters	Urania Activities Popular Lectures Exhibitions
Sun., Aug. 23	Optional Excursions							Opera
Mon., Aug. 24	Plenary Lecture	Plenary Lecture	Plenary Lecture			Invited Lectures Communications Posters	Invited Lectures Communications Posters	VideoMath-Festival
Tue., Aug. 25	Plenary Lecture	Plenary Lecture	Plenary Lecture			Invited Lectures Communications Posters	Invited Lectures Communications Posters	VideoMath-Festival
Wed., Aug. 26	Plenary Lecture	Plenary Lecture	Plenary Lecture			Invited Lectures Communications Posters	Invited Lectures Communications Posters	ICM'98 Party
Thu., Aug. 27	Plenary Lecture	Plenary Lecture	Plenary Lecture			Plenary Lecture Closing Ceremony		Urania

K. Social and Tourist Program

Overview

	Time	Price in DM
Tuesday August 18, 1998		
Opening Ceremony in the ICC followed by a Lunch Buffet	10:00–14:30	free of charge
Wednesday August 19, 1998		
City Sights	9:00–12:00	40,–
From Prussian Kings to Egyptian Pharaohs	13:00–17:00	45,–
Thursday August 20, 1998		
Potsdam's Parks and Palaces	8:00–18:00	150,–
City Sights with Visit to the Pergamon Museum	10:00–14:00	50,–
Visit to the Hamburger Bahnhof	14:00–17:30	40,–
Friday August 21, 1998		
Dresden	8:00–19:00	150,–
City Sights	9:00–12:00	40,–
Boat trip on the River Havel	13:30–17:15	55,–
Saturday August 22, 1998		
Berlin's Famous Waterways	9:45–12:15	50,–
City Sights with Visit to the Pergamon Museum	10:00–14:00	50,–
Potsdam/Sanssouci	13:00–17:00	65,–
Sunday August 23, 1998		
Dresden	8:00–19:00	150,–
Potsdam's Parks and Palaces	8:00–18:00	150,–
Neuruppin and Rheinsberg Palace	9:00–17:00	110,–
Impressions of the Spreewald	9:00–17:00	125,–
From Prussian Kings to Egyptian Pharaohs	10:00–14:00	45,–
City Sights with Visit to the Pergamon Museum	13:00–17:00	50,–
City Sights	14:00–17:00	40,–
Visit to the Hamburger Bahnhof	14:00–17:30	40,–
Deutsche Oper ***The Magic Flute***	19:30 h	52,– to 88,–
Monday August 24, 1998		
City Sights	9:00–12:00	40,–
Boat trip on the River Havel	13:30–17:15	55,–
Tuesday August 25, 1998		
Berlin's Famous Waterways	9:45–12:15	50,–
City Sights with Visit to the Pergamon Museum	10:00–14:00	50,–
Potsdam/Sanssouci	13:00–17:00	65,–
Wednesday August 26, 1998		
City Sights	9:00–12:00	40,–
ICM'98 Party	18:00–open end	free of charge

All bus tours start in front of the TU Berlin, Main Building, Str. des 17. Juni 135.

Please note that the return times of the bus tours are approximate and depend on traffic conditions. Luncheons do not include beverages. The program is subject to change. All tours are provided with English speaking guides.

K.1 Social Events

Opening Lunch Buffet

Date:	August 18, 1998
Time:	12:30–14:30 (after Opening Ceremony)
Place:	International Congress Center Berlin ICC Berlin Messedamm 22/Entrance Neue Kantstraße 14055 Berlin

Public Transportation: U-Bahn line 2, station *Kaiserdamm*, bus routes 104, 105, 149, 219, 700 and S-Bahn lines 45/46, station *Witzleben*, lines 3/7/9/75, station *Westkreuz.*

ICM'98 Party

Probable Date: August 26, 1998
Time: 18:00 after the lectures
Place: Campus of the TU Berlin
Public Transportation: U-Bahn line 2, station *Ernst-Reuter-Platz*

Opera: *The Magic Flute*

Date:	August 23, 1998
Time:	19:30 h
Place:	Deutsche Oper Bismarckstr. 35 10627 Berlin

Public Transportation:
U-Bahn line U2, station *Deutsche Oper*

Categories*:	1. Upper circle	DM 52,–
	2. Stalls (row 21-26)	DM 55,–
	3. Dress circle (row 3-7)	DM 75,–
	4. Stalls (row 14-20)	DM 77,–
	5. Stalls (row 10-13)	DM 88,–

*If tickets in the desired category are sold out, tickets in another category will be reserved and confirmed by DER-CONGRESS.

K.2 DER-CONGRESS Tourist Program

City Sights with Visit to the Pergamon Museum

This tour takes you to the historical and modern centers of Berlin and gives impressions of its culture, history, and open-minded inhabitants. You will see places of interest around the famous *Kurfürstendamm* and the historic part of *Berlin-Mitte*, for example the Kaiser Wilhelm Memorial Church, New National Gallery, the French and German Cathedrals, Brandenburg Gate and the Reichstag, the avenue *Unter den Linden* and much more. The Pergamon Museum houses a collection of antiquities as well as the famous altar of Pergamon, the Ishtar Gate and the Marketgate of Milet.

Thursday,	August 20, 1998	10:00–14:00
Saturday,	August 22, 1998	10:00–14:00
Sunday,	August 23, 1998	13:00–17:00
Tuesday,	August 25, 1998	10:00–14:00

DM 50,– per person

Dresden

On the way to Dresden you will pass by the lake *Teupitzer See*, some distant parts of the Spreewald and the coal mining area of Niederlausitz. Dresden is one of the most beautiful baroque towns in Europe. After a sightseeing tour through the city and lunch you will visit the picture gallery *Old Masters* with the famous painting *Sixtinische Madonna* or the dazzling jewellery exhibition in the *Green Vault,* or you can discover the city on your own. On the return journey to Berlin you will have a look at the famous china factory in Meissen where you can witness the production of the *white gold*, as the delicate porcelain is called.

Friday,	August 21, 1998	8:00–19:00
Sunday*,	August 23, 1998	8:00–19:00

DM 150,– per person

*Please note: Should you decide to attend the Dresden tour on Sunday the time remaining will not be sufficient to join ***The Magic Flute*** in the Opera as well.

City Sights

This tour takes you to the historical and modern centers of Berlin and gives you a feeling for the culture, history, and inhabitants. You will see places of interest around the famous *Kurfürstendamm* and *Berlin-Mitte*, for example the Kaiser Wilhelm Memorial Church, the New National Gallery, the French and German Cathedral, the Reichstag with the Brandenburg Gate as well as the avenue *Unter den Linden.*

Wednesday,	August 19,1998	9:00–12:00
Friday,	August 21,1998	9:00–12:00
Sunday,	August 23,1998	14:00–17:00
Monday,	August 24,1998	9:00–12:00
Wednesday,	August 26,1998	9:00–12:00

DM 40,– per person

Boat Trip on the River Havel

A substantial part of Berlin consists of lakes, rivers, and forests. Enjoy the unique forest, the beautiful sandy bays, and the tree-shaded shores of Lake Wannsee, Lake Stölpchensee, Lake Griebnitzsee, see the Peacock Island from the waterside. Coffee and cake included.

Friday,	August 21, 1998	13:30–17:15
Monday,	August 24, 1998	13:30–17:15

DM 55,– per person

Berlin's Famous Waterways

This river cruise on the Spree presents you the historic center of the city with its most important sights from a completely different angle. Coffee and cake included.

Saturday,	August 22, 1998	9:45–12:15
Tuesday,	August 25, 1998	9:45–12:15

DM 50,– per person

Neuruppin & Rheinsberg Palace

This will be a coach trip to the little town of Neuruppin which is situated on the lake *Ruppiner See*. During a sightseeing tour you will get to know the home town of Theodor Fontane and Karl Friedrich Schinkel. After lunch the trip continues to Rheinsberg, a *picture book for lovers* as Tucholsky called it. The town, which is situated on the east bank of the *Griebericksee*, is famous in particular for its palace, Schloß Rheinsberg. It was built in 1566 as a moated castle, and was reconstructed in 1737/40 by Knobelsdorff into a major architectural work during Friedrich the Great's reign. The Rheinsberg Palace houses a museum which is open to the public. The tour includes a walk through the palace gardens, which were originally designed in Baroque-style and changed into English-style gardens at the end of the 18th century.

Sunday, August 23, 1998 9:00–17:00

DM 110,– per person

From Prussian Kings to Egyptian Pharaohs

Charlottenburg Palace used to be the summer residence of the Prussian kings. The Palace was renovated with much care and houses a collection of paintings by Watteau, Caspar David Friedrich, and Spitzweg. The Egyptian Museum in Charlottenburg is the home of a collection of more than 1500 exhibits, which originated from all dynasties of Ancient Egypt including the famous bust of Queen Nefertiti and the Pharaoh Echnaton.

Wednesday,	August 19, 1998	13:00–17:00
Sunday,	August 23, 1998	10:00–14:00

DM 45,– per person

Impressions of the Spreewald

This day-tour to the Spreewald offers an unparalleled experience of nature. The area, which was originally settled by the Sorbs, is traversed by numerous arms of the River Spree, these often being the only links between houses and fields. The tour will take you to Burg and Leipe, two small idyllic towns in a romantic setting. Spreewald specialities will be served at lunch. Enjoy a trip along the network of waterways on one of the traditional Spreewald punts.

Sunday, August 23, 1998 9:00–17:00

DM 125,– per person

Potsdam's Parks and Palaces

Potsdam is famous for its splendid palaces and gardens. The palace of Sanssouci in Potsdam was a favourite refuge of the great Prussian King Friedrich II. Here, the *philosopher of Sanssouci*, as he was called, entertained some of the most educated men of his time, among them the French philosopher Voltaire. A visit of the palaces and gardens of Sanssouci will be followed by a guided tour to the Cecilienhof, the place where the Potsdam Agreement was signed in 1945. This excursion will also include a city tour of the small garrison town with its Dutch Quarters and Russian Colony as well as a lunch in a typical German restaurant.

Thursday,	August 20, 1998	8:00–18:00
Sunday,	August 23, 1998	8:00–18:00

DM 150,– per person

Potsdam/Sanssouci

This tour shows old and modern Potsdam. You will visit the well-preserved, worldfamous Rococo Palace Sanssouci with its art treasures and beautiful park. A city tour through Potsdam and its surroundings will follow.

Saturday,	August 22, 1998	13:00–17:00
Tuesday,	August 25, 1998	13:00–17:00

DM 65,– per person

Visit to the Hamburger Bahnhof

In November 1996, the newly renovated Hamburger Bahnhof was opened as the Museum for Contemporary

Art, Berlin. The Hamburger Bahnhof has approximately 10,000 qm of exhibition space for the Collection of Erich Marx. The Marx Collection is composed primarily of works by major artists from the past thirty years such as Joseph Beuys, Andy Warhol, Robert Rauschenberg, Cy Twombly, Roy Lichtenstein, and Anselm Kiefer. Above and beyond these artists, the collection shows also works by painters of the Italian Transavanguardia, of Minimal Art, and the *Junge Wilden* (*Wild Youth*) painters from Berlin. The renovation and partial rebuilding of the former terminal train station is the realization of plans by Josef Paul Kleihues.

Thursday,	August 20, 1998	14:00–17:30
Sunday,	August 23, 1998	14:00–17:30

DM 40,– per person

K.3 Cancellation

In case of cancellation of tours or tickets for the Deutsche Oper after June 15, 1998, the full fee must be paid. There will be no refunds. The tours will only take place if there is a minimum number of participants. If this condition is not fulfilled by the registration deadline, the tour will be cancelled and the amount paid refunded. No further claims can be accepted. If the participants choose some other means of transportation, no refunds will be made. Participants who do not show up for the tour are not eligible for refunds.

L. Satellite Conferences

• **Algebraic and Arithmetic Geometry.** Ruhrgebiet, Germany; August, 10–15. Organizing Committee: H. Esnault (Essen), G. Frey (Essen), E. Viehweg (Essen), G. Faltings (MPI Bonn).
Contact address: `icm.sat@uni-essen.de`

• **Algebraic Number Theory and Diophantine Analysis.** Graz, Austria; August, 31–September, 4. Organizing Committee: F. Halter-Koch, W. Tichy (Graz). Contact address: `tichy@ftug.th-graz.ac.at`

• **Analysis and Control of Differential Systems.** Constanta, Romania, August 10-15. Organizing Committee: V. Arnatu (Iasi), D. Tiba (Bucharest). Contact address: `dtiba@imar.ro`

• **XII-th Conference on Analytic Functions.** Lublin, Poland, August 30 to September 5. Organizing Committee: J. G. Krzyz (Lublin), J. Lawrynowicz (Lodz and IM PAN), J. Siciak (Krakow), J. Szynal (Lublin), E. Zlotkiewicz (Lublin), M. Nowak – secretary.
Contact address: `nowakm@golem.umcs.lublin.pl`

• **Commutative Algebra in Honour of David Rees's 80th Year.** Exeter, England; August, 13-16. Organizing Committee: R. Y. Sharp (Sheffield) and P. Vámos (Exeter).
Contact address: `car-meet@maths.ex.ac.uk`

• **Computer Algebra ISSAC 98.** Rostock, Germany; August, 13–15. Organizing Committee: (General Chair) V. Weispfenning, B. Trager, R. M. Corless, A. Widiger, K. Hantzschmann, J. Apel, O. Gloor. Contact address: `issac98@alice.fmi.uni-passau.de`

• **Conformal geometry and geometric function theory.** Berlin; August, 11–15. Organizing Committee: M. Bonk, Chr. Pommerenke, S. Rohde (TU Berlin).
Contact address: `rohde@math.tu-berlin.de`

• **The Fourth International Conference on Difference Equations and Applications.** Poznan, Poland, August 27-31. Organizing Committee: A. Gleska, K. Janglajew, A. Marlewski (secretary), J. Morchalo (vice-chairman), Z. Pawlak, J. Rakowski (chairman), E. Schmeidel.
Contact address: `icdea98@math.put.poznan.pl`

• **Differential Geometry and Applications.** Brno, Czech Republic; August, 10–14. Program Committee: O. Kowalski (Prague) and P. W. Michor (Vienna), co-chairmen; D. Alekseevski (Moscow), T. Friedrich (Berlin), I. Kolar (Brno), D. Krupka (Opava).
Contact address: `kowalski@karlin.mff.cuni.cz`

• **1st Conference of the European Society for Research in Mathematics Education.** Osnabrück, Germany, August 27-31. Program Committee E. Cohors-Fresenborg (Osnabrück, Germany) M. Hejny (Prague, Czech Republic) B. Jaworski (Oxford, United Kingdom) J. da Ponte (Lisbon, Portugal) A. Rouchier (Orleans, France). Contact address: `erme@mathematik.uni-osnabrueck.de`

• **Empirical Processes in Non- and Semiparametric Statistics.** Berlin; August 31 – September 3. Local organizers: E. Mammen (Heidelberg), M. Nussbaum (WIAS Berlin).
Contact address: `satellite@wias-berlin.de`

• **Function Spaces V.** Poznan (Poland), August 28 to September 3. Organizing Committee: Z. Ciesielski, H. Hudzik, J. Musielak, Prof. M. Nowak and L. Skrzypczak. Contact address: `funsp5@math.amu.edu.pl`

• **Conference on Functional Analysis, Partial Differential Equations, and Applications.** Rostock, Germany, August 31-September 4. Organizing Committee: G. Wildenhain, S. Prößdorf, P. Takac, J. Roßmann. Contact address:
`juergen.rossmann@mathematik.uni-rostock.de`

• **Geometric Combinatorics.** Montenegrin Adriatic coast, Yugoslavia, August 28 – September 4. Organizing Committee: I. Barany (Budapest), S. Vrecica (Belgrade), R. Zivaljevic (Belgrade).
Contact address: `geomcom@matf.bg.ac.yu`

• **Geometric Methods in Fourier and Functional Analysis.** Kiel, Germany; August, 10–14. Organizing Committee: H. König (Kiel), D. Müller (Kiel) J. Lindenstrauss (Jerusalem), E. M. Stein (Princeton).
Contact address: `conf98@math.uni-kiel.de`

• **Geometry and Topology.** Aarhus, Denmark; August, 10–16. Organizing Committee: K. Grove, I. Madsen, E. K. Pedersen. Contact address: `tag98@mi.aau.dk`

• **Graph Theory.** Hindsgavl, Denmark; August, 9–15. Organizing Committee: B. Toft (Odense).
Contact address: `btoft@imada.ou.dk`

• **International Conference on the History of Computing.** Paderborn, Germany; August, 14-16. Organizing Committee: R. Rojas (Berlin), U. Hashagen (Paderborn), G. Widiger (Berlin).
Contact address: `rojas@inf.fu-berlin.de`

• **History of Mathematics (ISHIM '98).** Göttingen, Germany, August, 10-14. Organizing Committee: H. J. Becker (Göttingen), D. Rowe (Mainz), P. Ullrich (Münster).
Contact address: `ISHIM98@www.sub.uni-goettingen.de`

• **Junior Mathematical Congress.** Potsdam, Germany, August, 17-22. Local Organizing Committee: A. Brückner, M. Fritzsche, R. Gutschmidt, T. Jahnke, Chr. Saalfrank, H.-J. Sprengel, M. Weese.
Contact address: `junmc98@rz.uni-potsdam.de`

• **Lattice Theory and Universal Algebra.** Szeged, Hungary; August, 3-7. Organizers: G. Czédli, L. Zádori and T. Géza.
Contact address: `algebra@math.u-szeged.hu`

• **Linear Algebraic Groups and Related Structures.** Bielefeld; August, 10–14. Organizing Committee: I. Kersten (Bielefeld), A. Merkurjev (St. Petersburg), U. Rehmann (Bielefeld). Contact address: `rehmann@mathematik.uni-bielefeld.de`

• **Logic.** FU Berlin, Germany; August, 28–30. Organizing Committee: A. Baudisch (HU Berlin), R. B. Jensen (HU Berlin), S. Koppelberg (FU Berlin).
Contact address: `sabina@math.fu-berlin.de`

• **Logic Colloquium 1998.** Praha, Czech Republic, August 9–15. Local Organizing Committee chair: P. Hajek (Institute of Computer Science, Prague, Czech Republic). Contact address: `lc98@math.cas.cz`

• **International Conference on Operations Research 1998 (OR'98).** Zürich, Switzerland; August 31 – September 3. President of the Organizing Committee: H.-J. Lüthi.
Contact address: `luethi@ifor.math.ethz.ch`

• **Partial Differential Equations (Theory and Numerical Solutions).** Prague, Czech Republic; August 10-16. Chairman: J. Necas (Prague), W. Jäger (Heidelberg). Contact address: `pde98@karlin.mff.cuni.cz`

• **Poisson Geometry.** Warsaw, Poland; August, 3–15. Organizing Committee: J.-P. Dufour (Montpellier), J. Grabowski (Warsaw), A. Weinstein (Berkeley), S. Zakrzewski (Warsaw), M. Zhitomirskii (Haifa).
Contact address: `szakrz@fuw.edu.pl`

• **International Conference dedicated to the 90th Anniversary of L. S. Pontryagin.** Steklov Institute of Mathematics Moscow, Russia, August, 31 to September, 6. Organizing Committee: Co-Chairmen: Yu. S. Osipov, V. A. Sadovnichii Vice-Chairmen: R. I. Grigorchuk, A. B. Kurzhanskii, A. S. Mishchenko.
Contact address: `pont@genesis.mi.ras.ru`

• **Representation Theory of Algebras.** Bielefeld, Germany, August 31 to September 5. Organizing Committee: D. Happel (Chemnitz), H. Lenzing (Paderborn), C. M. Ringel (Bielefeld), K. W. Roggenkamp (Stuttgart). Contact address: C. M. Ringel, Fakultät für Mathematik, Universität Bielefeld, POBox 100 131, D-33501 Bielefeld, Germany.

• **Representations of finite groups and combinatorics.** Magdeburg, Germany; August, 10–14. Organizing Committee: C. Bessenrodt (Magdeburg), A. Morris, J. B. Olsson (Kopenhagen).
Contact address: `icmsat98@uni-magdeburg.de`

• **Colloquium on Topology.** Gyula, Hungary, August, 9-15. Organizing Committee: A. Csaszar (chairman). Contact address: `gyula@math-inst.hu`

M. ICM'98 Sponsors

To make ICM'98 attractive for as many mathematicians as possible the local Organizing Committee has set a registration fee that is quite modest compared to the length and the size of the Congress and compared to the service that will be offered and included in the fee.

Nevertheless, there will still be many mathematicians for whom participation is economically unaffordable. To help at least some of them attend ICM'98, large support programs have been set up by the ICM'98 Organizing Committee in cooperation with the International Mathematical Union. These programs offer financial support for active young as well as mature mathematicians from developing countries and for mathematicians from Eastern Europe. Details can be found in the ICM'98 World Wide Web server.

The costs of organizing an event such as ICM'98 in a big city like Berlin are enormous. To keep them as low as possible, the ICM'98 organization depends – to a very large extent – on the dedication, time, and energy of many volunteers from the Berlin universities and mathematical research institutes as well as on the help of many colleagues throughout Germany. In addition to working free of charge they often cover arising costs from their own or other sources. The Organizing Committee would like to thank all persons and institutions involved.

Even with all this help, it would still be impossible to finance a large and costly event such as ICM'98 by registration fees only. The organizers are grateful to the

- Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie
- Senat von Berlin
- Deutsche Forschungsgemeinschaft
- Land Sachsen

for their substantial financial contributions to the organization of ICM'98. Generous support has also been obtained from foundations and companies. Sponsoring from industry is hard to obtain in economically hard times, and we are particularly grateful to companies who recognize the importance of mathematics. The final list of sponsors will be announced at the Congress and published in the proceedings volume. Up to now financial contributions have been obtained from

- Allianz Lebensversicherung
- Siemens AG
- Stemmler-Stiftung
- Silicon Graphics
- Deutsche Telekom
- Herlitz AG
- Springer Verlag
- Nikkei Culture
- Minolta
- Sender Freies Berlin
- SUN Microsystems

We would also like to express our thanks for contributions from the International Mathematical Union, the Deutsche Mathematiker-Vereinigung, and the Deutsche Gesellschaft für Versicherungsmathematik. Donations have been given by many mathematicians from all over the world who particularly contributed to the funds for the support programs for mathematicians from developing countries and Eastern Europe.

Finally, we would like to thank the state of Saxony, one of the sixteen German states, who helped to make the IMU General Assembly possible.

The IMU General Assembly will take place in Dresden, Saxony, from August 15 to 16, directly before ICM'98.

N. 1991 Mathematics Subject Classification

0	General
1	History and biography
3	Mathematical logic and foundations
4	Set theory
5	Combinatorics
6	Order, lattices, ordered algebraic structures
8	General algebraic systems
11	Number theory
12	Field theory and polynomials
13	Commutative rings and algebras
14	Algebraic geometry
15	Linear and multilinear algebra; matrix theory
16	Associative rings and algebras
17	Nonassociative rings and algebras
18	Category theory, homological algebra
19	K-theory
20	Group theory and generalizations
22	Topological groups, Lie groups
26	Real functions
28	Measure and integration
30	Functions of a complex variable
31	Potential theory
32	Several complex variables and analytic spaces
33	Special functions
34	Ordinary differential equations
35	Partial differential equations
39	Finite differences and functional equations
40	Sequences, series, summability
41	Approximations and expansions
42	Fourier analysis
43	Abstract harmonic analysis
44	Integral transforms, operational calculus
45	Integral equations
46	Functional analysis
47	Operator theory
49	Calculus of variations and optimal control; optimization
51	Geometry
52	Convex and discrete geometry
53	Differential geometry
54	General topology
55	Algebraic topology
57	Manifolds and cell complexes
58	Global analysis, analysis on manifolds
60	Probability theory and stochastic processes
62	Statistics
65	Numerical analysis
68	Computer science
70	Mechanics of particles and systems
73	Mechanics of solids
76	Fluid mechanics
78	Optics, electromagnetic theory
80	Classical thermodynamics, heat transfer
81	Quantum Theory
82	Statistical mechanics, structure of matter
83	Relativity and gravitational theory
85	Astronomy and astrophysics
86	Geophysics
90	Economics, operations research, programming, games
92	Biology and other natural sciences, behavioral sciences
93	Systems theory; control
94	Information and communication, circuits

O. Organizing Committee

The Organizing Committee was set up in 1994 and has been growing since. At the moment of this writing the following people are involved:

M. Grötschel	TU Berlin and ZIB (President)
F. Hirzebruch	MPI Bonn (Honorary President)
M. Aigner	FU Berlin (Vice-President)
J. Sprekels	HU Berlin and WIAS (Finances)
J. Winkler	TU Berlin (Secretary)
R. Möhring	TU Berlin (Local arrangements)

E. Behrends	FU Berlin
G. Berendt	FU Berlin
A. Beutelspacher	U Giessen
J. Brüning	HU Berlin
W. Dalitz	ZIB
G. Fischer	U Düsseldorf (Proceedings)
G. Frey	U Essen
U. Fuchs	FU Berlin
S. Hartmann	TU Berlin
C. Helmberg	ZIB
K.H. Hoffmann	TU München
H. Kurke	HU Berlin
E. Letzner	FU Berlin
M. Pohst	TU Berlin (Chair Local Scientific Committee)
K. Radbruch	U Kaiserslautern
U. Rehmann	U Bielefeld (Abstracts, Proceedings)
W. Römisch	HU Berlin
R. H. Schulz	FU Berlin
M. Teuchert	WIAS Berlin
G. M. Ziegler	TU Berlin

International Congress of Mathematicians, Berlin, August 18-27, 1998

Please note, to grant the early registration fee and to ensure that all other services offered will be available (e.g. hotel reservation and tourist program) this form should be completed and **returned at the latest by May 1st, 1998 to**

DER-CONGRESS
Congress Organisation
Bundesallee 56
D-10715 Berlin, Germany

Tel: +49 30 857903-0
Fax: +49 30 857903-26
E-Mail: der@der-congress.de

REGISTRATION FORM FOR ONE PARTICIPANT

Please print or type! (If there are more participants, please ask for additional forms or copy.)

Please mark: Prof. O Dr. O Mr. O Mrs. O Ms. O

Name* ____________ First Name* and Initial ____________

Phone ____________ Fax ____________

e-Mail ____________

Contact address

Affiliation ____________

Street or P.O. Box* ____________

Zip-Code ____________ City* ____________

Country* ____________

The asterisk * indicates a mandatory field

Address to appear in the list of participants, if different from above

Affiliation ____________

City ____________ Country ____________

Please state here how your name / address should be printed on the name badge:

First Name/Nickname ____________ Name ____________

Affiliation and/or city ____________

Country ____________

I hereby accept that my personal data (name, address, e-Mail, phone and fax number) will be passed on to the organizer of the ICM '98, to be used for information purposes in the Internet with respect to this congress.	Please mark yes O no O

I herewith accept the conditions as laid down in the Second Announcement.

Signature ____________ Date ____________

10 Sections

Please mark the sections you will probably attend (up to three choices).

☐ 1. Logic
☐ 2. Algebra
☐ 3. Number Theory and Arithmetic Algebraic Geometry
☐ 4. Algebraic Geometry
☐ 5. Differential Geometry and Global Analysis
☐ 6. Topology
☐ 7. Lie Groups and Lie Algebras
☐ 8. Analysis
☐ 9. Ordinary Differential Equations and Dynamical Systems
☐ 10. Partial Differential Equations
☐ 11. Mathematical Physics
☐ 12. Probability and Statistics
☐ 13. Combinatorics
☐ 14. Mathematical Aspects of Computer Science
☐ 15. Numerical Analysis Scientific Computing
☐ 16. Applications
☐ 17. Control Theory and Optimization
☐ 18. Teaching and Popularization of Mathematics
☐ 19. History of Mathematics

☐ I plan to give a talk in section _________

☐ I plan to present a poster in section _________

☐ I need an invitation letter

20 Registration Fees

DM

	Early Registration **until May 1st, 1998**		Registration **after May 1st, 1998**		
Registration fee	01	DM 450,--	02	DM 600,--	______
Student Registration fee[1]	03	DM 200,--	04	DM 350,--	______

[1] Registration as a student requires an official certificate of the University to be attached to this form.

Hotel Accommodation

DM

☐ I do not need hotel accommodation booked via DER-CONGRESS

☐ Please arrange my hotel reservation as indicated below:

Date of Arrival: ____________________ Date of Departure: _______________

Possible arrival after 18:00 ☐

		☐ Single room	☐ Double room
Category I	☐	DM 200 - 290	DM 256 - 360
Category II	☐	DM 151 - 199	DM 201 - 255
Category III	☐	DM 111 - 150	DM 156 - 200
Category IV	☐	DM 75 - 110	DM 110 - 155

Alternatively, I would prefer category ________________

The rates are per night and include breakfast, service charge and VAT.

Pre-payment is DM 420,--[2] per room x ________ room(s) = DM ______

[2] DM 400,-- is hotel deposit and will be deducted from the final hotel invoice, DM 20,-- is handling charge, payable to DER-CONGRESS.

Private Accommodation

DM

There have been made reservations in private households. (For information see section J.2. in the Second Announcement).

☐ Please arrange my accommodation in a private household.

Handling fee per person is DM 30,-- x ________ person(s) = DM ______

Reservations will only be processed after receipt of payment.

Sum carried forward DM _________

Sum carried forward DM ________

30	Social Events				DM
	Tuesday • August 18, 1998	**Time/hrs**	**Price DM**	**Person(s)**	
01	Opening Ceremony in the ICC followed by a Lunch Buffet	10.00 - 14.30	free of charge	________	
	Sunday • August 23, 1998				
	Deutsche Oper "The Magic Flute"	19:30			
02	Category 1. Upper circle		52,--	________	________
03	Category 2 . Stalls (row 21-26)		55,--	________	________
04	Category 3 . Dress circle (row 3-7)		75,--	________	________
05	Category 4 . Stalls (row 14-20)		77,--	________	________
06	Category 5 . Stalls (row 10-13)		88,--	________	________
	Alternatively, I would prefer category ________				

70	Tourist Program				
	Day / Tour	**Time/hrs**	**Price DM**	**Person(s)**	
	Wednesday • August 19, 1998				
01	City Sights	09.00 - 12.00	40,--	________	________
02	From Prussian Kings to Egyptian Pharaohs	13.00 - 17.00	45,--	________	________
	Thursday • August 20, 1998				
03	Potsdam's Parks and Palaces	08.00 - 18.00	150,--	________	________
04	City Sights with Visit to the Pergamon Museum	10.00 - 14.00	50,--	________	________
05	Visit of the Hamburger Bahnhof	14.00 - 17.30	40,--	________	________
	Friday • August 21, 1998				
06	Dresden	08.00 - 19.00	150,--	________	________
07	City Sights	09.00 - 12.00	40,--	________	________
08	Boat trip on the River Havel	13.30 - 17.15	55,--	________	________
	Saturday • August 22, 1998				
09	Berlin's Famous Waterways	09.45 - 12.15	50,--	________	________
10	City Sights with Visit to the Pergamon Museum	10.00 - 14.00	50,--	________	________
11	Potsdam/Sanssouci	13.00 - 17.00	65,--	________	________
	Sunday • August 23, 1998				
12	Dresden	08.00 - 19.00	150,--	________	________
13	Potsdam's Parks and Palaces	08.00 - 18.00	150,--	________	________
14	Neuruppin and Rheinsberg Palace	09.00 - 17.00	110,--	________	________
15	Impressions of the Spreewald	09.00 - 17.00	125,--	________	________
16	From Prussian Kings to Egyptian Pharaohs	10.00 - 14.00	45,--	________	________
17	City Sights with Visit to the Pergamon Museum	13.00 - 17.00	50,--	________	________
18	City Sights	14.00 - 17.00	40,--	________	________
19	Visit of the Hamburger Bahnhof	14.00 - 17.30	40,--	________	________
	Monday • August 24, 1998				
20	City Sights	09.00 - 12.00	40,--	________	________
21	Boat Trip on the River Havel	13.30 - 17.15	55,--	________	________
	Tuesday • August 25, 1998				
22	Berlin's Famous Waterways	09.45 - 12.15	50,--	________	________
23	City Sights with Visit to the Pergamon Museum	10.00 - 14.00	50,--	________	________
24	Potsdam/Sanssouci	13.00 - 17.00	65,--	________	________
	Wednesday • August 26, 1998				
25	City Sights	09.00 - 12.00	40,--	________	________

75	Public Transport Ticket (valid August 17 - 27, 1998)	DM
	One Public Transport Ticket is included in the registration fee!	
	I need **additional** public transport tickets for accompanying persons DM 47,-- x ________person(s) = DM	________

Total amount DM ________

Name, First name:
Address:
Total amount: DM

PAYMENT - please tick method of payment

I wish to pay by:

1. ☐ enclosing a **bank cheque** in German Currency (DM) made payable to DER-CONGRESS, Congress Organisation

2. ☐ **bank transfer** made through (name of your bank) ____________________
(as per enclosed copy)
in German Currency to DER-CONGRESS, Congress Organisation
Commerzbank AG, Frankfurt/Main, Account no. 589 32 35, Sort Code 500 400 00

All charges due on bank transfers have to be paid by the participants.

Please always state your name and the Congress ICM'98.

3. ☐ **Credit Card:**

Please note: In case of credit card payment via e-Mail your data could get lost or can be abused. We therefore recommend to use mail or fax service for the transmission.

I hereby authorise DER-CONGRESS to debit the below mentioned credit card account with the total value of the items booked by me on this form. I also consent to DER-CONGRESS debiting or crediting my credit card account with the amount of any subsequent change(s) to the items booked.

☐ VISA ☐ Diners Club ☐ EUROCARD ☐ MasterCard

Cardnumber

|_|_|_|_|_|_|_|_|_|_|_|_|_|_|_|_|

Expiry Date

|_|_|_|_|

Name of Cardholder

|_|_|_|_|_|_|_|_|_|_|_|_|_|_|_|_|_|_|_|_|_|_|_|_|_|

Address for which card issued if different from address on this registration form:

____

____

Cardholder's Signature: ____________________

Meetings & Conferences of the AMS

PROGRAM ALERT: In order that AMS meeting programs include the most timely information for each speaker, abstract deadlines have been moved to dates much closer to the meeting. What this means is that most meeting programs will appear in the *Notices* *after* the meeting takes place. However, complete meeting programs will be available on e-MATH about two to three weeks after the abstract deadline. *Remember*, e-MATH is your most comprehensive source for up-to-date meeting information. See `http://www.ams.org/meetings/`.

Louisville, Kentucky

University of Louisville

March 20–21, 1998

Meeting #931

Southeastern Section
Associate secretary: Robert J. Daverman
Announcement issue of *Notices*: January 1998
Program issue of *Notices*: May 1998
Issue of *Abstracts*: Volume 19, Issue 2

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: January 28, 1998

Invited Addresses

Anders Bjorner, Royal Institute of Technology, Stockholm, Sweden, *The combinatorial topology of graph properties.*

Andrew Bruckner, University of California at Santa Barbara, *The X, Y, Z's in the characterization problem for derivatives.*

Philippe DiFrancesco, University of North Carolina at Chapel Hill, *Folding and coloring problems in mathematics and physics.*

Abigail Thompson, University of California at Davis, *Recognizing the 3-sphere.*

Special Sessions

Algebraic Combinatorics (Code: AMS SS N1), **Anders Bjorner**, Royal Institute of Technology, and **Michelle L. Wachs**, University of Miami.

Applied Probability and Actuarial Science (Code: AMS SS P1), **Grzegorz Rempala**, **Krzysztof Ostraszewski**, **Ewa M. Kubicka**, and **Bogdan Gapinski**, University of Louisville.

Banach Space Theory (Code: AMS SS F1), **Patrick N. Dowling** and **Beata Randrianantoanina**, Miami University, Ohio.

Boundary Value Problems for Differential Equations (Code: AMS SS J1), **Paul W. Eloe**, University of Dayton.

Combinatorics and Enumerative Geometry (Code: AMS SS A1), **Kequan Ding**, University of Illinois, Urbana-Champaign, and **Chi Wang**, University of Louisville.

Combinatorics and Graph Theory (Code: AMS SS B1), **Andre E. Kezdy**, **Grzegorz Kubicki**, and **Jenoe Lehel**, University of Louisville.

Discrete Mathematics, Classification Theory and Consensus (Code: AMS SS C1), **Robert C. Powers**, University of Louisville.

Fractal Geometry and Related Topics (Code: AMS SS D1), **Ka-Sing Lau**, University of Pittsburgh, and **Weibin Zeng**, University of Louisville.

Functional Equations and Inequalities (Code: AMS SS E1), **Thomas Riedel** and **Prasanna Sahoo**, University of Louisville.

Geometry of Affine Space (Code: AMS SS M1), **Gene Freudenburg**, University of Southern Indiana, and **David Wright**, Washington University.

Low-Dimensional Topology (Code: AMS SS R1), **Abigail A. Thompson**, University of California, Davis, and **Martin Scharlemann**, University of Califronia, Santa Barbara.

Modern Function Theory (Code: AMS SS Q1), **David Minda** and **David A. Herron**, University of Cincinnati.

Real Analysis (Code: AMS SS G1), **Udayan B. Darji** and **Lee Larson**, University of Louisville.

Semigroups, Algorithms, and Universal Algebra (Code: AMS SS H1), **Ralph N. McKenzie**, Vanderbilt University, and **Steven Seif**, University of Louisville.

Spectral Geometry (Code: AMS SS K1), **Ruth Gornet**, Texas Tech University, and **Peter Anton Perry**, University of Kentucky.

Spectral Theory, Mathematical Physics and Disordered Media (Code: AMS SS L1), **Peter David Hislop**, University of Kentucky, and **Gunter H. Stolz**, University of Alabama at Birmingham.

The Use of the History of Mathematics and Science in the University and School Classroom (Code: AMS SS I1), **Richard M. Davitt**, University of Louisville.

Manhattan, Kansas

Kansas State University

March 27–28, 1998

Meeting #932

Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: January 1998
Program issue of *Notices*: June 1998
Issue of *Abstracts*: Volume 19, Issue 2

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: February 2, 1998

Invited Addresses

Gopal Prasad, University of Michigan-Ann Arbor, *Title to be announced.*

Mikhail Vishik, University of Texas at Austin, *Title to be announced.*

Clarence Eugene Wayne, Pennsylvania State University, University Park, *Title to be announced.*

Zihong Jeff Xia, Northwestern University, *Title to be announced.*

Special Sessions

Abstract Harmonic Analysis (Code: AMS SS H1), **Sadahiro Saeki**, Kansas State University.

Cohomology of Finite Groups (Code: AMS SS F1), **John S. Maginnis**, Kansas State University, and **Stephen F. Siegel**, University of Massachusetts.

Groups and Geometry (Code: AMS SS I1), **Ernest E. Shult**, Kansas State University.

Integrable Systems and Their Applications. (Code: AMS SS M1), **Kirill L. Vaninsky**, Kansas State University.

Lie Groups, Algebraic Groups, Their Arithmetic and Representation Theory (Code: AMS SS O1), **Gopal Prasad**, University of Michigan-Ann Arbor.

Linear Operators and Holomorphic Function Spaces (Code: AMS SS G1), **V. V. Peller**, Kansas State University.

Mathematics Education and the Internet (Code: AMS SS C1), **Andrew G. Bennett**, Kansas State University.

Nonlinear Problems (Code: AMS SS D1), **Lev Kapitanski**, Kansas State University, and **Clarence Euge ne Wayne**, Pennsylvania State University.

Numerical Analysis and Computational Mathematics (Code: AMS SS L1), **Qisu Zou** and **Huanan Yang**, Kansas State University.

Partial Differential Equations and Inverse Problems (Code: AMS SS A1), **Alexander G. Ramm**, Kansas State University.

Pictorial Methods in Low Dimensional Topology (Code: AMS SS B1), **David R. Auckly**, University of California Berkeley.

Quantum Groups and Applications (Code: AMS SS J1), **Ya S. Soibelman** and **Volodymyr V. Lyubashenko**, Kansas State University.

Quantum Topology (Code: AMS SS K1), **David N. Yetter** and **Louis Crane**, Kansas State University.

Representation Theory of Lie Algebras, Algebraic Groups and Quantum Groups (Code: AMS SS E1), **Zongzhu Lin**, Kansas State University, and **Daniel Ken Nakano**, Utah State University.

Philadelphia, Pennsylvania

Temple University

April 4–6, 1998

Meeting #933

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: January 1998
Program issue of *Notices*: June 1998
Issue of *Abstracts*: Volume 19, Issue 2

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: February 11, 1998

Invited Addresses

Tobias H. Colding, Courant Institute-New York University, *Title to be announced.*

Martin Davis, University of California, Berkeley, *Title to be announced.*

Ezra Getzler, Max-Planck-Institute and Northwestern University, *Title to be announced.*

Yanyan Li, Rutgers University, *Title to be announced.*

Elias M. Stein, Princeton University, *Title to be announced.*

Special Sessions

Differential Geometric Methods in Hydrodynamics (Code: AMS SS J1), **Gerard K. Misiolek**, University of Notre Dame and California Institute of Technology.

Harmonic Analysis and Its Applications to PDEs (Code: AMS SS G1), **Cristian E. Gutierrez**, Temple University, and **Guozhen Lu**, Wright State University.

Heat Kernel Analysis on Lie Groups (Code: AMS SS H1), **Leonard Gross**, Cornell University, and **Omar Hijab**, Temple University.

Mathematical Pedagogy (Code: AMS SS I1), **Orin N. Chein**, Temple University.

Modular Identities and Q-Series in Number Theory (Code: AMS SS A1), **Marvin I. Knopp** and **Boris Datskovsky**, Temple University.

Nonlinear Partial Differential Equations (Code: AMS SS K1), **Yanyan Li**, Rutgers University.

PDEs in Several Complex Variables (Code: AMS SS B1), **Shiferaw Berhanu** and **Gerardo Mendoza**, Temple University.

Radon Transforms and Tomography (Code: AMS SS C1), **Eric L. Grinberg**, Temple University, and **Eric Todd Quinto**, Tufts University.

Rings and Representations (Code: AMS SS E1), **Maria E. Lorenz**, Ursinus College, and **Martin Lorenz**, Temple University.

Sparse Elimination Methods in Polynomial System Solving (Code: AMS SS L1), **Ioannis Z. Emiris**, INRIA, Sophia-Antipolis, France, and **J. Maurice Rojas**, Massachusetts Institute of Technology.

Sparse Matrix Computations (Code: AMS SS M1), **Jesse Barlow**, Pennsylvania State University, and **Daniel B. Szyld**, Temple University.

The History of American Mathematics (Code: AMS SS D1), **David E. Zitarelli**, Temple University, and **Karen H. Parshall**, University of Virginia.

Topology of Manifolds and Varieties (Code: AMS SS F1), **Georgia Triantafillou**, Temple University, and **Sylvain E. Cappell**, New York University-Courant Institute.

Davis, California

University of California

April 25-26, 1998

Meeting #934

Western Section
Associate secretary: William A. Harris Jr.
Announcement issue of *Notices*: February 1998
Program issue of *Notices*: June 1998
Issue of *Abstracts*: Volume 19, Issue 2

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: March 4, 1998

Invited Addressess

Edward Frenkel, University of California, Berkeley, *Recent progress in geometric Langlands correspondence.*

Ian Putnam, University of Victoria, *Interactions between C^*-algebras and dynamics.*

Boris Rozovsky, University of Southern California, *Wiener chaos and stochastic PDEs.*

William Thurston, University of California, Davis, *Three-manifolds, foliations and circles.*

Special Sessions

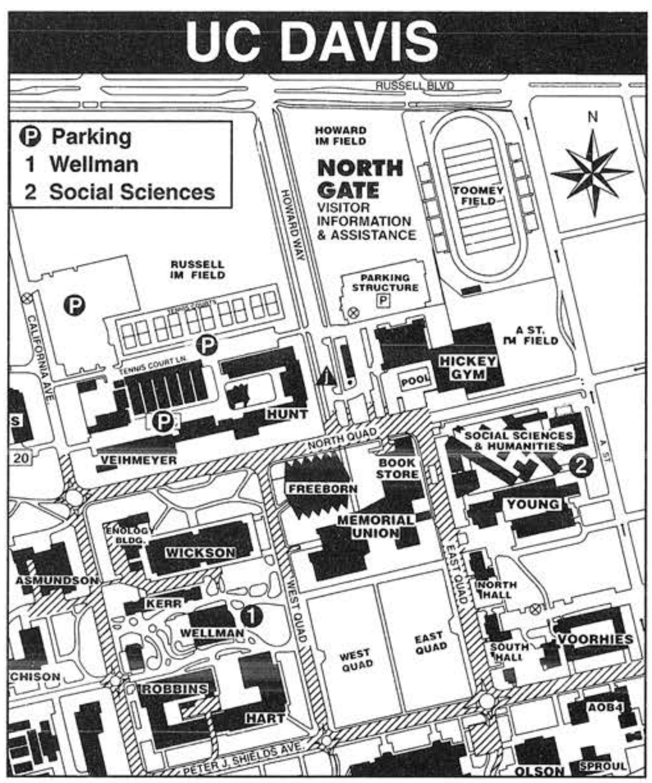
C^*-algebras and Dynamics (Code: AMS SS A1), **Jerry Kaminker**, Indiana Univ.-Purdue Univ. at Indianapolis; **Ian Fraser Putnam**, University of Victoria; and **Jack Spielberg**, Arizona State University.

Differential Equations with Applications (Code: AMS SS B1), **Sally Sailai Shao**, Cleveland State University and **Tatsuhiko J. Tabara**, Golden Gate University.

Dualities in Mathematics and Physics (Code: AMS SS C1), **Edward Frenkel** and **Nocolai Reshetikhin**, University of California, Berkeley.

Dynamical Systems and Mathematical Physics (Code: AMS SS D1), **Motohico Mulase** and **Bruno L. Nachtergaele**, University of California, Davis.

Finite Groups and Representations (Code: AMS SS E1), **Kenechukwu Kenneth Nwabueze**, University of Brunei Darussalam.



Geometric Analysis (Code: AMS SS F1), **Chikako Mese**, University of Southern California, and **Richard M. Schoen**, Stanford University.

Graph Theory (Code: AMS SS H1), **David Barnette**, University of California, Davis.

Mathematical Biology (Code: AMS SS I1), **Alexander Isaak Mogilner**, Univeristy of California, Davis.

Mathematical Physics and Topology (Code: AMS SS J1), **Gregory J. Kuperberg** and **Albert Schwarz**, University of California, Davis.

Nonlinear Analysis (Code: AMS SS K1), **John K. Hunter** and **Blake Temple**, University of California, Davis.

Random Fields and Stochastic Partial Differential Equations (Code: AMS SS L1), **Arthur J. Krener**, University of California, Davis, and **Boris Rozovsky**, University of Southern California.

The Geometry and Topolgy of 3-manifolds (Code: AMS SS G1), **Dmitry Fuchs**, **Joel Hass**, **Ramin Naimi**, and **William Thurston**, University of California, Davis.

Accommodations

Participants should make their own arrangements directly with the hotel of their choice and state that they will be attending the AMS Western Section Meeting. **All rooms will be on a space available basis after the deadline given.** The AMS is not responsible for rate changes or for the quality of the accommodations.

Aggie Inn, 245 First Street, Davis, CA; 916-756-0352; $72/single or double; one-half mile to campus.

Best Western, 123 B Street, Davis, CA; 916-756-7890; $54/single and $59/double; one-half mile to campus.

Motel 6, 4835 Chiles Road, Davis, CA; 916-753-3777; $35.99/single and $41.99/double; three miles to campus.

Ramada Inn, 110 F Street, Davis, CA; 916-753-3600; $60/single and $66/double; one-quarter mile to campus. **Deadline for reservations is April 10.**

Food Service

There are a number of restaurants located off-campus as well as the ASUCD Coffee House in the Memorial Union on campus. A list of restaurants will be available at the registration desk.

Local Information

Please visit the website maintained by the UC Davis Department of Mathematics at `http://math.ucdavis.edu/`.

Other Activities

AMS Book Sale: Examine the newest titles from AMS! Most books will be available at a special 50% discount offered only at meetings. AMS representatives will be on hand to demonstrate and discuss the newest electronic jounals, the preprint server and other products and member services available on e-MATH. Complimentary coffee will be served, courtesy of AMS Membership Services.

Smoller Conference

A conference on Nonlinear Analysis and Mathematical Physics will be held in Davis on April 24, 1998, cosponsored by the California Coordinating Committee on Non-Linear Science (CCCNLS), Mathematical Sciences Research Institute (MSRI), Berkeley, UC Davis, and the University of Michigan. This conference is dedicated to the 60th anniversary of Joel Smoller. Joel Smoller is a pioneering mathematician at the University of Michigan whose contributions have influenced modern analysis during the last several decades. Speakers for the conference include Avner Friedman, James Glimm, Peter Lax, Tai Ping Liu, and Shing-Tung Yau. For more information on this conference, please contact the UC Davis Mathematics Department at `boire@math.ucdavis.edu/`.

Parking

Parking is located in Lots #14 and #15. Alternate lots are #25 and #35. Parking is free on the weekend, unless there is a scheduled activity on the campus where vendors appear. In this case, the parking would be $3.00. Parking machines on campus accept credit cards, dollar bills, nickels, dimes, and quarters. Please note that you must activate one of the silver buttons before inserting money. Parking is permitted in visitor/student/staff lots on campus (not including special permit areas) and is free of charge on Saturday and Sunday, April 25–26.

Participants attending the Smoller Conference on Friday, April 24, should park in Lot #2 located near the Alumni Center (AGR Room). Alternate lots are #1 and #2B. The fee for parking on Friday is $3.00.

Registration and Meeting Information

Registration will take place in the lounge in Wellman Hall from 7:30 a.m. to 5:00 p.m., Saturday, April 25, and 8:00 a.m. to 11:00 on Sunday, April 26. Lectures will take place in the Social Sciences Building and Wellman Hall.

Registration Fees: (payable on-site only) $30/AMS members; $45/nonmembers; $10/emeritus members, students, or unemployed mathematicians. Fees are payable by cash, check, VISA, MasterCard, Discover, or American Express.

Travel

By Air: The Sacramento Metropolitan Airport is the closest commercial airport to the campus. It is ten miles from downtown Sacramento and twenty miles from UC Davis.

The following specially negotiated rates are available only for the period April 22–29: 5% discount off first class and any published USAirways promotional round-trip fare, or 10% discount off unrestricted coach fares with seven-day advance reservations and ticketing required. These discounts are valid providing all rules and restrictions are met and are applicable for travel from the continental U.S., Bahamas, Canada, and San Juan, P.R. Discounts are not combinable with other discounts or promotions. Additional restrictions may apply on international travel. For reservations call (or have your travel agent call) 800-334-8644 between 8:00 a.m. and 9:00 p.m. Eastern Daylight Time. **Refer to Gold file Number 73670341.**

Driving: Driving east on I-80 from the San Francisco Bay Area or driving west on I-80 from Sacramento, take the UC Davis exit onto Old Davis Road and past the South Gate information kiosk on your way to the campus. From the north on Highway 113, take either the Russell Boulevard or UC Davis (Hutchinson Drive) exits and go east until you reach the campus.

By Train or Bus: Both Amtrak and Greyhound have stations in downtown Davis which are approximately one-half mile from the eastern border of the campus. Taxi service is available.

Weather

The daytime temperatures typically range from 70–80 degrees Fahrenheit, and in the 50 degree range at night. Some light rain is possible.

Chicago, Illinois

DePaul University-Chicago

September 12–13, 1998

Meeting #935

Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: June 1998
Program issue of *Notices*: November 1998
Issue of *Abstracts*: Volume 19, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: May 26, 1998
For abstracts: July 21, 1998

Invited Addresses

Vitaly Bergelson, Ohio State University, *Title to be announced.*

Sheldon Katz, Oklahoma State University, *Title to be announced.*

Ralf Spatzier, University of Michigan, *Title to be announced.*

Vladimir Voevodsky, Northwestern University, *Title to be announced.*

Special Sessions

Algebraic Coding (Code: AMS SS C1), **William C. Huffman**, Loyola University of Chicago, and **Vera S. Pless**, University of Illinois at Chicago.

Algebraic Combinatorics: Association Schemes and Related Topics (Code: AMS SS L1), **Sung Yell Song**, Iowa State University.

Commutative Algebra (Code: AMS SS J1), **Irena V. Peeva**, Massachusetts Institute of Technology, and **Michael Stillman**, Cornell University.

Complex Dynamics (Code: AMS SS H1), **Shmuel Friedland**, University of Illinois at Chicago.

Complexity of Geometric Structures on Manifolds (Code: AMS SS F1), **Melvin G. Rothenberg** and **Shmuel A. Weinberger**, University of Chicago.

Ergodic Theory and Topological Dynamics (Code: AMS SS G1), **Roger L. Jones**, DePaul University, and **Randall McCutcheon**, Wesleyan College.

Fourier Analysis (Code: AMS SS E1), **Marshall Ash**, DePaul University, and **Mark A. Pinsky**, Northwestern University.

K-Theory and Motivic Cohomology (Code: AMS SS D1), **Kevin Knudson**, Northwestern University, and **Mark Walker**, University of Nebraska-Lincoln.

Number Theory (Code: AMS SS I1), **Jeremy T. Teitelbaum** and **Yuri Tschinkel**, University of Illinois at Chicago.

Orthogonal Polynomial Series, Summability and Conjugates (Code: AMS SS M1), **Calixto P. Calderon**, University of Illinois at Chicago, and **Luis A. Caffarelli**, University of Texas at Austin.

Rigidity in Geometry and Dynamics (Code: AMS SS K1), **Steven E Hurder**, University of Illinois at Chicago, and **Ralf J. Spatzier**, University of Michigan.

Stochastic Analysis (Code: AMS SS A1), **Richard B. Sowers**, University of Illinois-Urbana, and **Elton P. Hsu**, Northwestern University.

Topics in Mathematics and Curriculum Reform (Code: AMS SS B1), **Richard J. Maher**, Loyola University Chicago.

Winston-Salem, North Carolina

Wake Forest University

October 9–10, 1998

Meeting #936

Southeastern Section
Associate secretary: Robert J. Daverman
Announcement issue of *Notices*: August 1998
Program issue of *Notices*: December 1998
Issue of *Abstracts*: Volume 19, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: June 23, 1998
For abstracts: August 18, 1998

Invited Addresses

David F. Anderson, University of Tennessee, *Title to be announced.*

Idris Assani, University of North Carolina, Chapel Hill, *Title to be announced.*

Marcy Barge, Montanta State University, *Title to be announced.*

Roger Temam, University of Paris XI and Indiana University, *Title to be announced.*

Special Sessions

Abelian Groups and Modules (Code: AMS SS B1), **Ulrich Albrecht**, Auburn University.

Combinatorics and Graph Theory (Code: AMS SS A1), **Bruce Landman**, University of North Carolina.

Noncommutatuve Algebra (Code: AMS SS C1), **Ellen Kirkman** and **James Kuzmanovich**, Wake Forest University.

Recent Results on the Topology of Three-Manifolds (Code: AMS SS D1), **Hugh Nelson Howards**, Wake Forest University.

State College, Pennsylvania

Pennsylvania State University

October 24–25, 1998

Meeting #937

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: August 1998
Program issue of *Notices*: January 1999
Issue of *Abstracts*: Volume 19, Issue 4

Deadlines

For organizers: January 22, 1998
For consideration of contributed papers in Special Sessions: July 7, 1998
For abstracts: September 1, 1998

Invited Addresses

Jeffrey Adams, University of Maryland, College Park, *Title to be announced.*

Nigel D. Higson, Pennsylvania State University, *Title to be announced.*

Tasso J. Kaper, Boston University, *Title to be announced.*

Kate Okikiolu, University of California, San Diego, and MIT, *Title to be announced.*

Special Sessions

C-Algebraic Methods in Geometry and Topology* (Code: AMS SS B1), **Nigel D. Higson**, Pennsylvania State University, and **Erik Guentner** and **John D. Trout Jr.**, Dartmouth College.

Mathematical Modeling of Inhomogeneous Materials: Homogenizaton and Related Topics (Code: AMS SS D1), **Leonid Berlyand**, Pennsylvania State University, and **Karl Voss**, Yale University.

Modeling of Phase Transitions of Partially Ordered Physical Systems (Code: AMS SS C1), **Maria-Carme T. Calderer**.

Partitions and q-Series (Code: AMS SS A1), **George E. Andrews** and **Ken Ono**, Pennsylvania State University.

Tucson, Arizona

University of Arizona-Tucson

November 14–15, 1998

Meeting #938

Western Section
Associate secretary: William A. Harris Jr.
Announcement issue of *Notices*: September 1998
Program issue of *Notices*: To be announced
Issue of *Abstracts*: Volume 19, Issue 4

Deadlines

For organizers: February 12, 1998
For consideration of contributed papers in Special Sessions: July 29, 1998
For abstracts: September 23, 1998

San Antonio, Texas

San Antonio Convention Center

January 13–16, 1999

Joint Mathematics Meetings, including the 105th Annual Meeting of the AMS, 82nd Meeting of the Mathematical Association of America (MAA), and annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM).
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: October 1998
Program issue of *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 14, 1998
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
For summaries of papers to MAA organizers: To be announced

Gainesville, Florida

University of Florida

March 12–13, 1999

Southeastern Section
Associate secretary: Robert J. Daverman
Announcement issue of *Notices*: To be announced
Program issue of *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: June 11, 1998
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Urbana, Illinois

University of Illinois, Urbana-Champaign

March 18-21, 1999

Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: To be announced
Program issue of *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: June 18, 1998
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Special Sessions

Galois Representations (Code: AMS SS C1), **Nigel Boston**, University of Illinois-Urbana, and **Michael Larsen**, University of Missouri.

Nonstandard Analysis (Code: AMS SS B1), **C. Ward Henson** and **Peter Loeb**, University of Illinois-Urbana.

Recent Progress in Elementary Geometry (Code: AMS SS A1), **John E. Wetzel**, University of Illinois-Urbana, and **Clark Kimberling**, University of Evansville.

Las Vegas, Nevada

University of Nevada-Las Vegas

April 10-11, 1999

Western Section
Associate secretary: William A. Harris Jr.
Announcement issue of *Notices*: To be announced
Program issue of *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: July 10, 1998
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Buffalo, New York

State University of New York at Buffalo

April 24-25, 1999

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: To be announced
Program issue of *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: July 24, 1998
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Invited Addresses

Michele M. Audin, University Louis Pasteur, Strasbourg, *Title to be announced.*

Jeff Smith, Purdue University, *Title to be announced.*

Alexander A. Voronov, Massachusetts Institute of Technology, *Title to be announced.*

Gregg J. Zuckerman, Yale University, *Title to be announced.*

Providence, Rhode Island

Providence College

October 2-3, 1999

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: To be announced
Program issue of *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: January 6, 1999
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Austin, Texas

University of Texas-Austin

October 8-10, 1999

Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: To be announced
Program issue of *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: January 6, 1999
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Washington, District of Columbia

Sheraton Washington Hotel and Omni Shoreham Hotel

January 19-22, 2000

Joint Mathematics Meetings, including the 106th Annual Meeting of the AMS, 83rd Meeting of the Mathematical Association of America (MAA), with minisymposia and other special events contributed by the Society for Industrial and Applied Mathematics (SIAM), and the annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM).
Associate secretary: William A. Harris Jr.
Announcement issue of *Notices*: To be announced
Program issue of *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 20, 1999
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
For summaries of papers to MAA organizers: To be announced

Lowell, Massachusetts

University of Massachusetts, Lowell

April 1-2, 2000

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: To be announced
Program issue of *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: July 1, 1999
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Notre Dame, Indiana

University of Notre Dame

April 7-9, 2000

Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: To be announced
Program issue of *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: July 7, 1999
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

New Orleans, Louisiana

New Orleans Marriott and ITT Sheraton New Orleans Hotel

January 10-13, 2001

Joint Mathematics Meetings, including the 107th Annual Meeting of the AMS, 84th Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM).
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: To be announced
Program issue of *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 11, 2000
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
For summaries of papers to MAA organizers: To be announced

Columbia, South Carolina

University of South Carolina

March 16-18, 2001

Southeastern Section
Associate secretary: Robert J. Daverman
Announcement issue of *Notices*: To be announced
Program issue of *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: June 15, 2000
For consideration of contributed papers in Special Sessions:
To be announced
For abstracts: To be announced

Williamstown, Massachusetts

Williams College

October 13–14, 2001

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: To be announced
Program issue of *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: January 11, 2001
For consideration of contributed papers in Special Sessions:
To be announced
For abstracts: To be announced

Meetings and Conferences of the AMS

Associate Secretaries of the AMS

Western Section: William A. Harris Jr., Department of Mathematics, University of Southern California, Los Angeles, CA 90089-1113; e-mail: wharris@math.usc.edu; telephone: 213-740-3794.

Central Section: Susan J. Friedlander, Department of Mathematics, University of Illinois at Chicago, 851 S. Morgan (M/C 249), Chicago, IL 60607-7045; e-mail: susan@math.nwu.edu; telephone: 312-996-3041.

Eastern Section: Lesley M. Sibner, Department of Mathematics, Polytechnic University, Brooklyn, NY 11201-2990; e-mail: lsibner@magnus.poly.edu; telephone: 718-260-3505.

Southeastern Section: Robert J. Daverman, Department of Mathematics, University of Tennessee, Knoxville, TN 37996-1300; e-mail: daverman@novell.math.utk.edu; telephone: 423-974-6577.

The Meetings and Conferences section of the *Notices* gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. Up-to-date meeting and conference information is available on the World Wide Web at www.ams.org/meetings/.

Meetings:

Important Information Regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 183 in the January issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts

Several options are available for speakers submitting abstracts, including an easy-to-use interactive Web form. No knowledge of TeX is necessary to submit an electronic form, although those who use plain TeX, AMS-TeX, LaTeX, or AMS-LaTeX may submit abstracts with TeX coding. To see descriptions of the forms available, visit http://www.ams.org/abstracts/instructions.html or send mail to abs-submit@ams.org, typing help as the subject line, and descriptions and instructions on how to get the template of your choice will be e-mailed to you.

Completed abstracts should be sent to abs-submit@ams.org, typing submission as the subject line. Questions about abstracts may be sent to abs-info@ams.org.

Paper abstract forms may be sent to Meetings & Conferences Department, AMS, P.O. Box 6887, Providence, RI 02940. Note that all abstract deadlines are strictly enforced. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

Conferences: (See http://www.ams.org/meetings/ for the most up-to-date information on these conferences.)

1998:

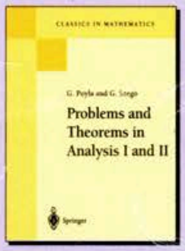
June 21–July 23: Joint Summer Research Conferences in the Mathematical Sciences, South Hadley, MA. See pp. 1412–1416 (November 1997) and pp. 146–148 (January 1998) for details.

Cosponsored Conference:

February 12–18: American Association for the Advancement of Science, Philadelphia, PA. See p. 1550 (December 1997) for details.

SPRINGER FOR MATHEMATICS

Announcing the Latest Releases — Springer's Classics in Mathematics

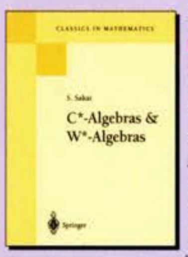


GEORGE POLYA and **GABOR SZEGŐ**

PROBLEMS AND THEOREMS IN ANALYSIS I AND II

"The work is one of the real classics of this century; it has had much influence on teaching, on research in several branches of hard analysis, particularly complex function theory, and it has been an essential indispensable source book for those seriously interested in mathematical problems. Written in the early twenties by two young mathematicians of outstanding talent, taste, breadth, perception, perseverance, and pedagogical skill, this work broke new ground in the teaching of mathematics and how to do mathematical research." –BULLETIN OF THE AMERICAN MATHEMATICAL SOCIETY

1998/APP. 780 PP./SOFTCOVER/$39.95
ISBN 3-540-63640-4

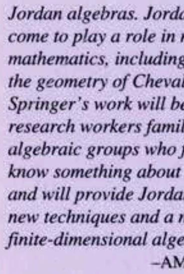


SHOICHIRO SAKAI

C*-ALGEBRAS & W*-ALGEBRAS

"This book is an excellent and comprehensive survey of the theory of von Neumann algebras. It includes all the fundamental results of the subject, and is a valuable reference for both the beginner and the expert." –MATHEMATICAL REVIEWS

1998/APP. 256 PP./SOFTCOVER/$39.95
ISBN 3-540-63633-1



TONNY A. SPRINGER, University of Utrecht, The Netherlands

JORDAN ALGEBRAS AND ALGEBRAIC GROUPS

"This book presents an important and novel approach to Jordan algebras. Jordan algebras have come to play a role in many areas of mathematics, including Lie algebras and the geometry of Chevalley groups. Springer's work will be of service to research workers familiar with linear algebraic groups who find they need to know something about Jordan algebras and will provide Jordan algebraists with new techniques and a new approach to finite-dimensional algebras over fields." –AMERICAN SCIENTIST

1998/APP. 170 PP./SOFTCOVER/$39.95
ISBN 3-540-63632-3

PAULO NEY DE SOUZA, University of California, Berkeley, and **JORGE-NUNO SILVA,** Fac De Ciencias De Lisbon, Portugal

BERKELEY PROBLEMS IN MATHEMATICS

This book is a compilation of approximately nine hundred problems which have appeared on the preliminary exams at Berkeley. It is an invaluable source of problems and solutions for every mathematics student who plans to enter a PhD program. Students who work through this book will develop problem solving skills in areas such as real analysis, multi-variable calculus, differential equations, metric spaces, complex analysis, algebra, and linear algebra. The problems are organized by subject and ordered in an increasing level of difficulty. Tags with the exact exam year provide the opportunity to rehearse complete examinations.

1998/APP. 464 PP., 42 ILLUS./SOFTCOVER/$34.95
ISBN 0-387-94933-X
ALSO IN HARDCOVER $59.95/ISBN 0-387-94934-8
PROBLEM BOOKS IN MATHEMATICS

H. KOCH, Humboldt-Universität zu Berlin, Germany; **A.N. PARSHIN, I.R. SHAFAREVICH,** both of Steklov Mathematical Institute, Moscow, Russia

ALGEBRAIC NUMBER THEORY

Contents: Preface • Basic Number Theory • Class Field Theory • Galois Groups • Abelian Fields • Artin L-Functions and Galois Module Structure • Appendix 1. Fields, Domains and Complexes • Appendix 2. Quadratic Residues • Appendix 3. Locally Compact Groups • Appendix 4. Bernoulli Numbers • Tables • References • Author Index • Subject Index.

1998/269 PP./SOFTCOVER/$49.95/ISBN 3-540-63003-1

R. OSSERMAN, Mathematical Sciences Research Institute, Berkeley, CA (Ed.)

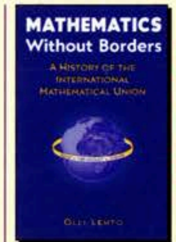
GEOMETRY V
Minimal Surfaces

The theory of minimal surfaces has expanded in many directions over the past decade or two. This volume gathers in one place an over-view of some of the most exciting developments, presented by five of the leading contributors to those developments. H. Fujimoto, who obtained the definitive results on the Gauss map of minimal surfaces, reports on Minimal Surfaces and Nevanlinna Theory. S. Hildebrandt provides an up-to-date account of the Plateau problem and related boundary-value problems. D. Hoffman and H. Karcher describe the wealth of results on embedded minimal surfaces from the past decade, starting with Costa's surface and the subsequent Hoffman-Meeks examples. Finally, L. Simon covers the PDE aspect of minimal surfaces, with a survey of known results both in the classical case of surfaces and the higher dimensional case.

1997/272 PP./HARDCOVER/$99.95/ISBN 3-540-60523-1
ENCYCLOPAEDIA OF MATHEMATICAL SCIENCES, VOLUME 90

Springer

http://www.springer-ny.com



OLLI E. LEHTO, University of Helsinki, Finland

MATHEMATICS WITHOUT BORDERS
A History of the International Mathematical Union

This volume covers events of the IMU through the present day. It is a story of how ideas of the global cultivation of mathematics, across national borders, gradually began to take shape a century ago and how these ideas developed, amidst political difficulties and serious setbacks, into a fruitful worldwide cooperative effort under the aegis of the IMU.

1998/APP. 384 PP., 53 ILLUS./SOFTCOVER/$35.00
ISBN 0-387-98358-9

ARTHUR ENGEL, Johann Wolfgang Goethe Universität, Germany

PROBLEM-SOLVING STRATEGIES

This is a unique collection of competition problems from over twenty major national and international mathematical competitions for high school students, including extensive discussions of problem solving strategies. It is written for trainers and participants of contests of all levels up to the highest level: IMO, Tournament of the Towns, and the non-calculus parts of the Putnam Competition.

1998/APP. 403 PP., 223 ILLUS./$39.95/SOFTCOVER
ISBN 0-387-98219-1
PROBLEM BOOKS IN MATHEMATICS

Third Edition

HERBERT ALEXANDER, University of Illinois, Chicago and **JOHN WERMER,** Brown University, Providence, RI

SEVERAL COMPLEX VARIABLES AND BANACH ALGEBRAS

Many connections have been found between the theory of analytic functions of one or more complex variables and the study of commutative Banach algebras. This book is concerned with developing some of the principal applications of function theory in several complex variables to Banach algebras. The authors do not presuppose any knowledge of several complex variables on the part of the reader and all relevant material is developed within the text. Furthermore, the book deals with problems of uniform approximation on compact subsets of the space of n complex variables.

1998/256 PP./HARDCOVER/$49.00/ISBN 0-387-98253-1
GRADUATE TEXTS IN MATHEMATICS, VOLUME 35

ORDER TODAY!

CALL: 1-800-SPRINGER or Fax: (201)-348-4505

WRITE: Springer-Verlag New York, Inc., Dept. S224, PO Box 2485, Secaucus, NJ 07096-2485

VISIT: Your local technical bookstore

E-MAIL: orders@springer-ny.com

INSTRUCTORS: Call or write for info on textbook exam copies

Your 30-Day return privilege is always guaranteed!

2/98 *Reference: S226*

0002-9920(199802)45:2;1-R