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of the American Mathematical Society

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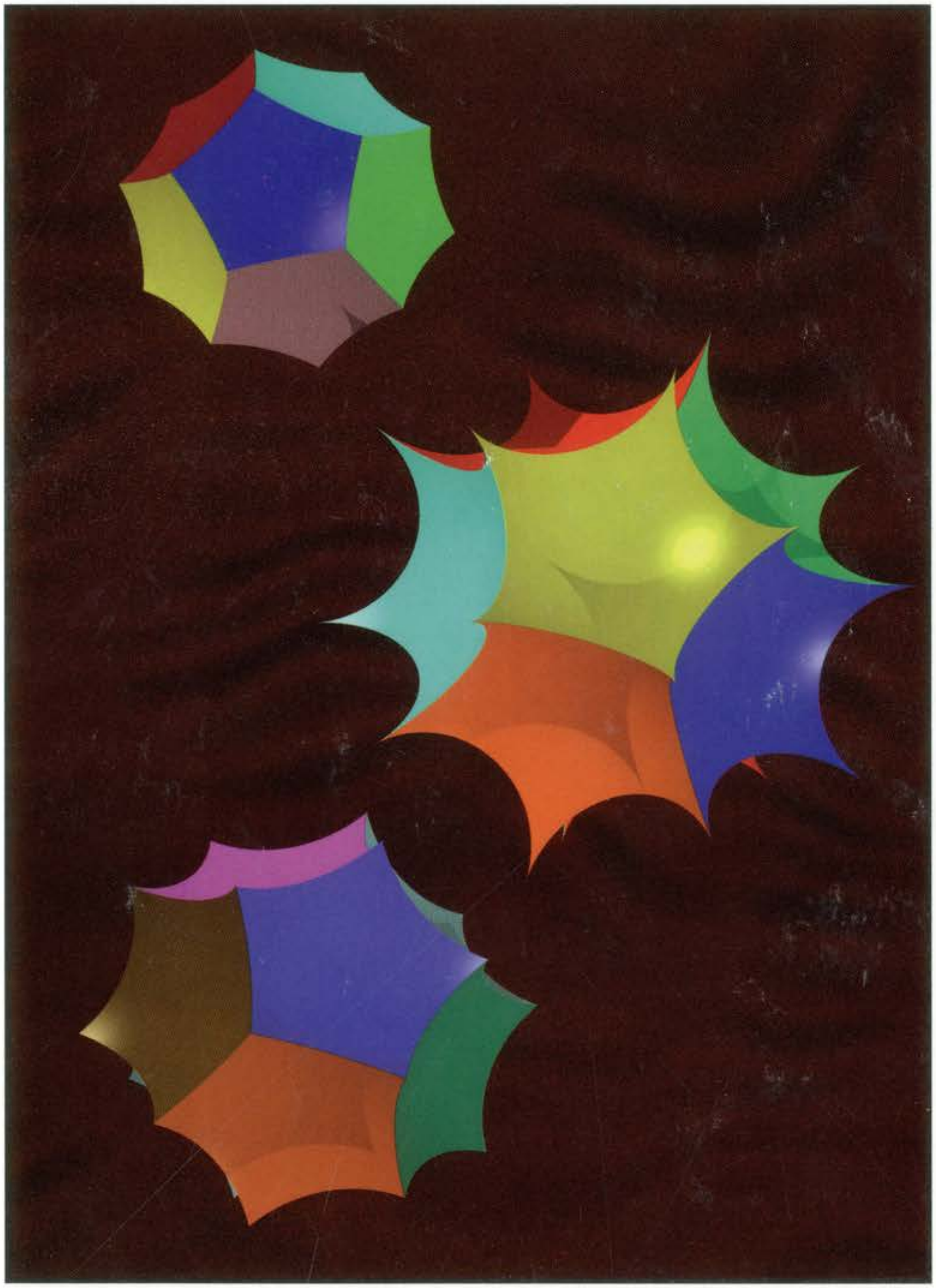

The Tessellating Hyperbolic Dodecahedra (see page 340)

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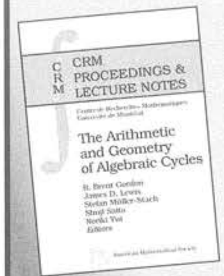
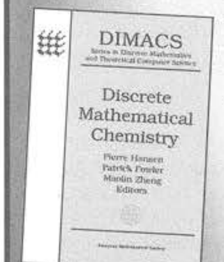
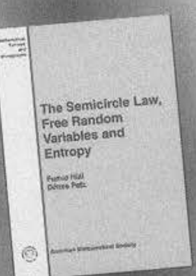
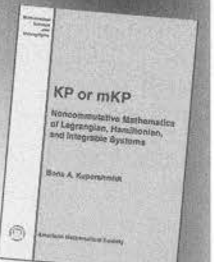
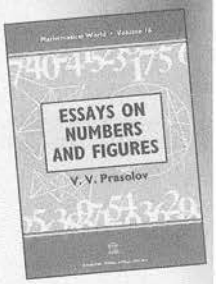
New Titles from the AMS











The Arithmetic and Geometry of Algebraic Cycles

B. Brent Gordon, *University of Oklahoma, Norman*, **James D. Lewis**, *University of Alberta, Edmonton, AB, Canada*, **Stefan Müller-Stach**, *Universität Essen, Germany*, **Shuji Saito**, *Tokyo Institute of Technology, Oh-Okayama, Meguro-ku, Japan*, and **Noriki Yui**, *Queen's University, Kingston, ON, Canada*, Editors

The NATO ASI/CRM Summer School at Banff offered a unique, full, and in-depth account of the topic, ranging from introductory courses by leading experts to discussions of the latest developments by all participants. The papers have been organized into three categories: cohomological methods; Chow groups and motives; and arithmetic methods.

As a subfield of algebraic geometry, the theory of algebraic cycles has gone through various interactions with algebraic K-theory, Hodge theory, arithmetic algebraic geometry, number theory, and topology. These interactions have led to developments such as a description of Chow groups in terms of algebraic K-theory, the application of the Merkurjev-Suslin theorem to the arithmetic Abel-Jacobi mapping, progress on the celebrated conjectures of Hodge, and of Tate, which compute cycles class groups respectively in terms of Hodge theory or as the invariants of a Galois group action on étale cohomology, the conjectures of Bloch and Beilinson, which explain the zero or pole of the L-function of a variety and interpret the leading non-zero coefficient of its Taylor expansion at a critical point, in terms of arithmetic and geometric invariant of the variety and its cycle class groups.

The immense recent progress in the theory of algebraic cycles is based on its many interactions with several other areas of mathematics. This conference was the first to focus on both arithmetic and geometric aspects of algebraic cycles. It brought together leading experts to speak from their various points of view. A unique opportunity was created to explore and view the depth and the breadth of the subject. This volume presents the intriguing results.

CRM Proceedings & Lecture Notes, Volume 24; 2000; 432 pages; Softcover; ISBN 0-8218-1954-2; List $110; Individual member $66; Order code CRMP/24NT003

Discrete Mathematical Chemistry

Pierre Hansen, *GERARD, Montreal, PQ, Canada*, **Patrick Fowler**, *University of Exeter, England*, and **Maolin Zheng**, *Lexis-Nexis, Mianmisburg, OH*, Editors

This volume contains the proceedings from the first DIMACS meeting on discrete mathematical chemistry held at Rutgers University (New Brunswick, NJ). The contributions reflect the presentations and spotlight the breadth of current research on the topic—from the Benzenoid Clar problem to the Wulff-shape of sphere packings. Much of the volume reflects the combined mathematical and physical interest in the new molecules, fullerenes.

This DIMACS conference highlighted the range of opportunities for fruitful and informed collaboration across the mathematics-chemistry boundaries. The interdisciplinary nature of the contributions pays testament to the fact that "real" chemistry and "real" mathematics do indeed interact.

DIMACS: Series in Discrete Mathematics and Theoretical Computer Science; 2000; 392 pages; Hardcover; ISBN 0-8218-0987-3; List $99; Individual member $59; Order code DIMACS-HANSEN2NT003

Independent Study

The Semicircle Law, Free Random Variables and Entropy

Fumio Hiai, *Tohoku University, Sendai, Japan*, and **Dénes Petz**, *Technical University of Budapest, Hungary*

The book treats free probability theory, which has been extensively developed since the early eighties. The emphasis is put on entropy and the random matrix model approach. It is a unique presentation demonstrating the extensive interrelation between the topics. Wigner's theorem and its broad generalizations, such as asymptotic freeness of independent matrices, are explained in detail. Consistent throughout is the parallelism between the normal and semicircle laws. The authors present Voiculescu's multivariate free entropy theory with full proofs and extend the results to unitary operators. Some applications to operator algebras are also given.

The book is the first essentially full-scale presentation on free probability theory and includes improvements of results and proofs in current literature. The combinatorial aspects of the specialized topics are emphasized; many examples are given. The book would be a suitable text for graduate courses in free probability theory.

Mathematical Surveys and Monographs; 2000; approximately 386 pages; Hardcover; ISBN 0-8218-2081-8; List $89; Individual member $53; Order code SURV-PETZNT003

KP or mKP
Noncommutative Mathematics of Lagrangian, Hamiltonian, and Integrable Systems

Boris A. Kupershmidt, *University of Tennessee Space Institute, Tullahoma*

This book develops a theory that can be viewed as a noncommutative counterpart of the following topics: dynamical systems in general and integrable systems in particular; Hamiltonian formalism; variational calculus, both in continuous space and discrete. The text is self-contained and includes a large number of exercises. Many different specific models are analyzed extensively and motivations for the new notions are provided.

Mathematical Surveys and Monographs; 2000; approximately 632 pages; Hardcover; ISBN 0-8218-1400-1; List $109; Individual member $65; Order code SURV-KUPERSHMNT003

Independent Study

Essays on Numbers and Figures

V. V. Prasolov, *Independent University of Moscow, Russia*

This is the English translation of the book originally published in Russian. It contains 20 essays, each dealing with a separate mathematical topic. The topics range from brilliant mathematical statements with interesting proofs, to simple and effective methods of problem-solving, to interesting properties of polynomials, to exceptional points of the triangle. Many of the topics have a long and interesting history. The author has lectured on them to students worldwide.

The essays are independent of one another for the most part, and each presents a vivid mathematical result that led to current research in number theory, geometry, polynomial algebra, or topology.

Mathematical World, Volume 16; 2000; 75 pages; Softcover; ISBN 0-8218-1944-5; List $15; All AMS members $12; Order code MAWRLD/16NT003

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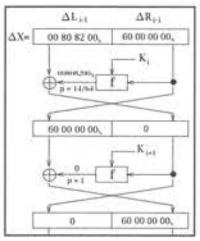

Memorial Article

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of the American Mathematical Society

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From the President of the AMS

Mathematical Challenges of the 21st Century

On August 6–12, 2000, the American Mathematical Society will hold an extraordinary meeting on the UCLA campus under the title "Mathematical Challenges of the 21st Century". As the principal organizer of this meeting, I feel compelled to tell the members of the AMS about the purposes of this meeting and the principles on which it has been organized. (For a detailed listing of the 31 plenary speakers and their topics, see the "Meetings" section of the present issue of the *Notices* or the Web site http://www.ams.org/meetings/; for a discursive description of the meeting, see Allyn Jackson's article "Stellar Lineup for UCLA Meeting" in the February 2000 *Notices*.)

The purposes of the meeting are twofold:

1. To exhibit the vitality of mathematical research and to indicate some of its potential major growing points: these include some of the major classical problems (the Riemann Hypothesis, the Poincaré Conjecture, the regularity of three-dimensional fluid flows) as well as some of the recently developed major research programs like those associated with the names of Langlands and Thurston.

2. To point up the growing connections between the frontiers of research in the mathematical sciences and cutting-edge developments in such areas as physics, biology, computational science, computer science, and finance.

The meeting will aim to raise consciousness within the mathematical community itself, the scientific community more generally, and beyond these (one hopes) policy and opinion makers and society at large.

We hope to raise awareness of the amazing growth in the past several decades of interactions between sophisticated mathematical research and major problems arising in science and society. As I write these lines, I have just been looking at a note in the *Proceedings of the National Academy of Sciences* that describes the application of contemporary knot theory to the function and structure of DNA. The other day I wrote down a list of such topics, many of which might be covered in our meeting. I found the following eighteen (the reader is invited to add to this list):

1. Wavelet theory and harmonic analysis in data compression and statistical inference
2. Knot theory in quantum physics and molecular biology
3. Stochastic analysis and mathematics of finance
4. Dynamical systems, chaos theory, and fractals
5. Mathematical models of pattern perception
6. Complex systems in biology
7. Quantum computing and quantum information theory
8. Mathematical probes of reliability in computational science
9. Algebraic methods in combinatorial problems
10. Quantum field theory and string theory in relation to geometry
11. Noncommutative geometry and models of space and time
12. Mathematical analysis of algorithms
13. Computational molecular biology
14. Mathematical models of turbulence
15. Nonlinear partial differential equations of general relativity theory
16. Calculus of variations and nonlinear PDEs in materials science
17. Prime number theory and cryptology
18. Algebraic number theory in coding theory

The program for "Mathematical Challenges" reflects the way in which mathematics is reaching out to other disciplines, solving problems, and opening new pathways for research, while at the same time drawing in ideas that bring new vitality and richness to the field. The speakers have been asked to give a broad picture of the prospects and challenges in the areas they cover and to do so in terms that are comprehensible to a general mathematical audience. This will be an important event, and I urge and welcome you to attend.

—*Felix E. Browder*

Commentary

In My Opinion

Internet Time

One of the pleasures of mathematics is its timelessness: Euclid's theorems are as useful and correct as they were two millennia ago. But if mathematics is timeless, the world is not; it is moving to Internet time. It is hard to imagine the little food shops along the streets of Rome being replaced by `myfood.com`, but the fact is that globalization and the Internet are having a profound impact on our daily lives. Five years ago few had heard of the Web; now `Amazon.com` is stealing market share from the corner bookstore and Blackwells at Oxford. Toysrus is online, as are Charles Schwab, United Airlines, and even the Louvre. The CEO who hasn't thought about the Internet is the CEO who is on her way out the door.

And at the academy? Universities and colleges have put up Web sites listing academics, athletics, faculty, and research. Some faculty post their research papers. (A *very* informal survey shows that this appears to happen more frequently in physics and computer science, less frequently in mathematics.) Some professors post course-related materials: syllabi, homework solutions, occasional course notes. But by and large, academic teaching has been barely affected by the Web.

The revolution is happening, whether we want it or not. The world is moving to the Internet. If video enables distance learning, the Web does so even more easily. This new technology may completely revamp the academy. Lest we be complacent because universities survived the threat of video courses, note that the Web is a very different beast. The Internet provides instant access to massive amounts of information to anyone with a PC and a modem.

What will these changes mean for mathematics departments? We like to argue that teaching is interactive and cannot be duplicated from afar. This is a weak argument when undergraduate classes run to two hundred and more. If a Web-based course can present the material at a hundredth of the cost of the traditional model, the fact that the Web-based education is not personal will not matter. The Web offers certain advantages. Web courses can be offered when and where a student wants them. Web courses are easily customized, an important consideration when an increasing percentage of students are members of the work force returning for advanced training.

Universities must face the Internet revolution. It behooves university and college leaders to structure education in a Web world.

There is another important issue for mathematicians. How will mathematics research function in the Internet world? The question seems foolish at first. After all, a theorem is a theorem is a theorem. A proof will remain true no matter how many networked machines search for a counterexample. But those arguments may miss the point. The real issue is, will mathematics matter in an Internet world?

Over the centuries mathematical questions have arisen from a variety of sources. For example, two millennia ago research problems arose from the mathematics of astronomy, geography, optics, and war; four centuries ago mathematics was driven by the calculations of the planets and the heavens, by physics, and by art. In the 1700s military needs added a large new source of mathematical problems. In the nineteenth century much mathematical research derived from questions of physics and electromagnetism.

In the last several weeks three fruitful areas of mathematical research have caught my attention. One was cryptography, without which e-commerce could not exist. Another was model checking; using logic to check the correctness of programs, model checking has been remarkably effective in the design of integrated circuits. The third was load balancing: how to shift loads on the Web so that when everyone is checking CNN for the latest report on the hurricane in Honduras, access time remains fast. Mathematics provides the solutions for all of these. In all three cases, the research was done by computer scientists. Even worse—from the point of view of mathematics departments and mathematicians—is that these topics are rarely taught by mathematics departments. Cryptography is mathematics, model checking is mathematics, load balancing is mathematics, but all three are taught and researched in computer science departments.

A knowledge of the classics was once central to the notion of what it meant to be an educated person; now the field is optional. Mathematics is central to science and engineering, yet mathematicians have frequently disconnected from the study of mathematics in those fields. The information revolution will have as profound an effect on the world as the industrial revolution did. The organization and access of massive amounts of information is a deep and fundamentally mathematical problem. Mathematicians largely sat out the computer science revolution; will mathematicians now opt to venture from their ethereal worlds in order to explore and develop the Internet world? If not, I fear that the study and practice of mathematics will be increasingly marginalized.

—Susan Landau
Associate Editor

Encounter with a Geometer, Part II

Marcel Berger

Editor's Note. *Part I of this article appeared in the February 1999 Notices. The article discusses Mikhael Gromov's extraordinary mathematics and its impact from the point of view of the author, Marcel Berger. It is partially based on three interviews of Gromov by Berger, and it first appeared in French in the* Gazette des Mathématiciens *in 1998, issues 76 and 77. It was translated into English by Ilan Vardi and adapted by the author. The resulting article is reproduced here with the permission of the* Gazette *and the author.*

Mikhael Gromov is professor of mathematics at the Institut des Hautes Études Scientifiques and, in addition, is the Jay Gould Professor of Mathematics at the Courant Institute of Mathematical Sciences, spending three months a year at Courant.

As the author said at the beginning of Part I, "The aim of this article is to communicate the work of Mikhael Gromov (MG) and its influence in almost all branches of contemporary mathematics and, with a leap of faith, of future mathematics. It is not meant to be a technical report, and, in order to make it accessible to a wide audience, I have made some difficult choices by highlighting only a few of the many subjects studied by MG. In this way, I can be more leisurely in my exposition and give full definitions, results, and even occasional hints of proofs."

The author's warning in Part I about bibliographical matters applies equally to Part II: "In order to shorten the text, I have omitted essential intermediate results of varying importance, and have therefore neglected to include numerous names and references. Although this practice might lead to some controversy, I hope to be forgiven for the choices."

Riemannian Geometry

Starting in the late 1970s, MG completely revolutionized Riemannian geometry. I mention in this section some results that reflect my taste. This article contains only a small bibliography; further references may be found in my 1998 survey article in *Jahresbericht der Deutschen Mathematiker-Vereinigung.*[1] Except for obvious cases, every Riemannian manifold will be compact; in any case it will always be assumed complete. In (M, g) the letter M stands for the manifold and the letter g its Riemannian metric. This by definition means that at every point m of M there is an inner-product structure $g(\cdot,\cdot)$ on the tangent space T_mM at this point. We begin by describing the various notions of "curvature".

Marcel Berger is emeritus director of research at the Centre National de la Recherche Scientifique (CNRS) and was director of the Institut des Hautes Études Scientifiques (IHÉS) from 1985 to 1994. His e-mail address is berger@ihes.fr.

The author expresses his immense debt to Ilan Vardi and Anthony Knapp—to Vardi for translating the article into English and for improving the clarity of the mathematical exposition, and to Knapp for editing the article into its current form.

[1]*Riemannian geometry during the second half of the twentieth century,* Jahresbericht ***100*** *(1998), 45–208; reprinted with the same title as volume 17 of the University Lecture Series, Amer. Math. Soc., Providence, RI, 2000, ISBN 0-8218-2052-4.*

The curvature tensor is the basic invariant of a Riemannian manifold. Some of its power comes from the fact it has three equivalent definitions. Two of these are in terms of the associated "Levi-Civita connection".

Informally a "connection" on a smooth manifold is a way of computing directional derivatives of vector fields. These directional derivatives, which do not exist in general, will be called "covariant". More precisely, a *connection* is an operator D that assigns to each pair of vector fields x and y on M a vector field D_xy on M, the *covariant derivative* of y with respect to x, in a fashion that is $\mathbb{R}$ linear in y, is $C^\infty(M)$ linear in x, and satisfies $D_x(fy) = x(f)Y + fD_xy$ for all $f \in C^\infty(M)$. The vector $(D_xy)_m$ at a point $m \in M$ depends only on x_m and the values of y on any curve whose velocity vector at m equals x_m. Consequently it is meaningful to speak of a vector field on a curve that is "parallel" along the curve: If σ is the curve and u is its tangent, then a vector field y on σ is *parallel* along σ if $D_uy = 0$ on σ. If σ has domain $[a, b]$, one knows that for each $y \in M_{\sigma(a)}$ there is a unique vector field $Y(t)$ on σ such that $y(a) = y$ and the field $y(t)$ is parallel along σ. The passage from $M_{\sigma(a)}$ to $M_{\sigma(t)}$ in this way is called *parallel transport.* Thus a connection yields a notion of parallel transport along curves. It yields also a notion of absolute (intrinsic) derivatives of all orders for all tensors on the manifold, in particular for functions.

Such intrinsic derivatives, apart from those of first order, do not exist on differentiable manifolds without additional structures.

A Riemannian manifold (M, g) has a unique connection D such that $D_x y - D_y x = [x, y]$ and

$$z(g(x, y)) = g(D_z x, y) + g(x, D_z y)$$

for all vector fields x, y, and z. This is called the *Levi-Civita connection* and will be understood throughout. At every $m \in M$ the *curvature tensor*, for every pair x, y of tangent vectors, is denoted by $R(x, y)$ and is an endomorphism of $T_m M$. There are three equivalent definitions of curvature; the first two are given in terms of the Levi-Civita connection D:

- The curvature can be computed explicitly using the two first derivatives of the metric g, namely,

$$R(x, y)z = (D_y D_x z - D_x D_y z - D_{[y,x]} z).$$

- Geometrically, the value of $R(x, y)$ is the defect from the identity of the parallel transport around an infinitesimal parallelogram with sides generated by x and y. To this tensor of type $(3, 1)$, it is useful to associate the 4-linear differential form $R(x, y, z, t) = g(R(x, y)z, t)$. For numerical functions f, the absolute second derivatives are still symmetric; a special case is the commutativity of partial derivatives in classical differential calculus. The third derivatives are no longer symmetric 3-forms, and the defect is represented exactly by the curvature tensor:

$$D^3 f(x, y, z) - D^3 f(x, z, y) = R(y, z, x, \operatorname{grad} f).$$

- One looks at the defect of (M, g) from being locally Euclidean. This can be achieved, for example, by computing the length of an arc of a small circle $\Gamma(\varepsilon)$ as in Figure 6. This arc, say of angle α, is obtained from ε and from a pair (x, y) of unit vectors in $T_m M$ by going a length ε along all geodesics whose initial tangent vector is contained in the angular sector of angle α determined by x and y. The truncated expansion of this length is given by the formula

$$\operatorname{length}(\Gamma(\varepsilon)) = \alpha\varepsilon\left(1 - \frac{R(x, y, x, y)}{3\sin^2\alpha}\,\varepsilon^2 + o(\varepsilon^2)\right).$$

The symmetries of R show that the second term depends only on the tangent plane P in $T_m M$ that is determined by x and y; its name is the *sectional curvature* of P and is denoted by $K(P)$. Knowledge of $K(P)$ on the complete Grassmannian manifold of tangent planes is equivalent to knowing the curvature tensor.

The real power of the curvature tensor R and the sectional curvature K is that they measure how (M, g) fails to be locally Euclidean. That is, (M, g) is locally Euclidean (i.e., locally isometric to Euclidean space of equal dimension; one says *flat*) if and only if R (or K) vanishes identically.

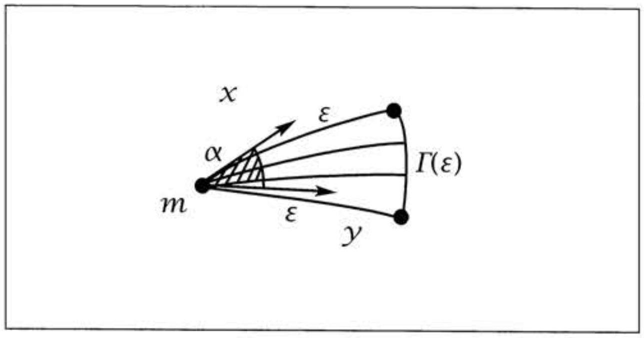

Figure 6. Curvature measures the defect of the manifold from being locally Euclidean. Sectional curvature operates at the two-dimensional level, appearing in the second term of the formula for the length of an arc of a small circle $\Gamma(\varepsilon)$.

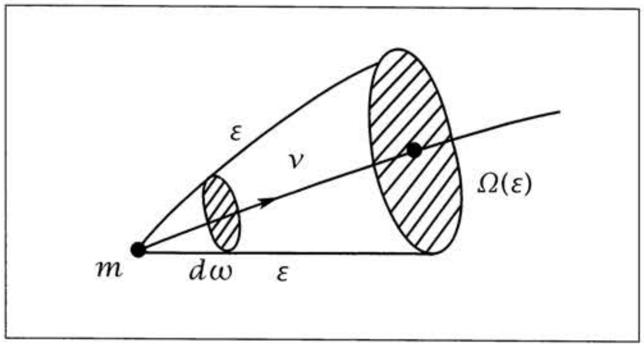

Figure 7. Ricci curvature operates directionally at the d-dimensional level in measuring the defect of the manifold from being locally Euclidean in various tangent directions. Specifically, it appears in the second term of the formula for the $(d-1)$-volume $\Omega(\varepsilon)$ generated within a solid angle.

Moreover, if K is constant everywhere and equal to k, then (M, g) is locally isometric to the standard simply connected space of constant sectional curvature k, namely, a sphere (of radius $1/\sqrt{k}$) if $k > 0$ and a hyperbolic space if $k < 0$ (the canonical hyperbolic space has curvature -1).

Something that is not emphasized in the Riemannian geometry literature is that despite its power, the curvature tensor does not in general determine the metric up to local isomorphism. There is room for strange examples, the reason being that, because of its symmetries, R depends only on $d^2(d^2-1)/12$ parameters, where d is the dimension of M. At present, knowledge of g requires knowing all its second derivatives, but these depend on more parameters, namely, $d^2(d+1)^2/4$ parameters.

However, since g depends only on $d(d+1)/2$ parameters, one could expect strong results with an invariant weaker than R. The natural one is the "Ricci curvature" Ricci, which is a quadratic form that assigns a real number $\operatorname{Ricci}(v)$ to every unit tangent vector v. This time it measures the defect from Euclidian at the level of a solid angle $d\omega$ in the direction of v, as in Figure 7. For this one looks

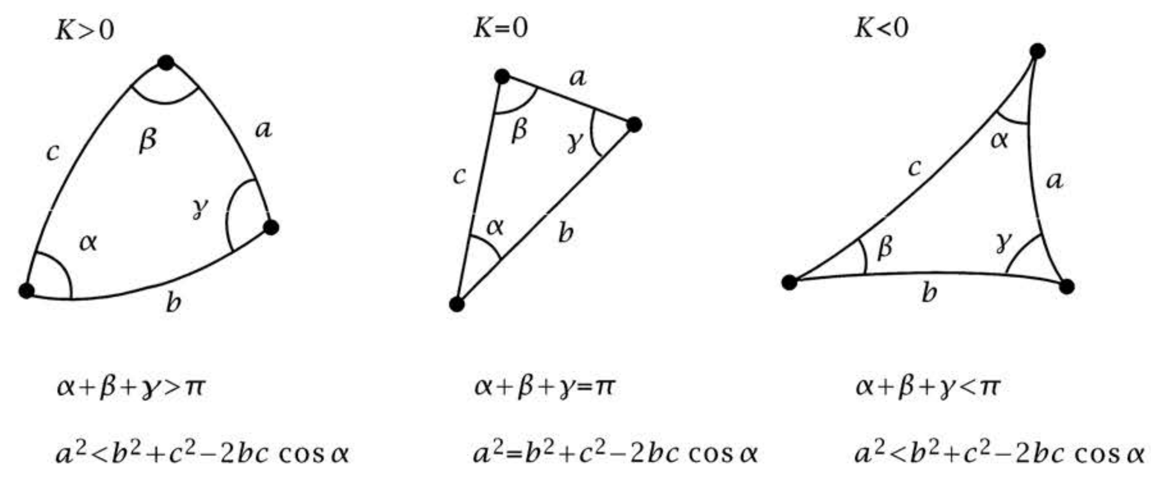

Figure 8. The manifolds with identically zero sectional curvature K are the locally Euclidean ones, and the ones with positive sectional curvature are those for which the sum of the three angles of any triangle is always larger than π, as in spherical geometry. In the negative case the sum of the angles of every sufficiently small triangle is less than π, as in hyperbolic geometry.

at the $(d-1)$-volume $\Omega(\varepsilon)$ generated by the geodesics of length ε starting in $d\omega$. The formula for the volume is

$$\mathrm{Vol}(\Omega(\varepsilon)) = d\omega \cdot \varepsilon^{d-1}\Big(1 - \frac{\mathrm{Ricci}(v)}{3}\,\varepsilon^2 + o(\varepsilon^2)\Big).$$

Algebraically, as a function of R or K, Ricci(v) is nothing but the trace of the sectional curvatures of all planes containing v. This expresses the fact that volumes are determinants and that derivatives of determinants are traces. One also has

$$\mathrm{Ricci}(v) = \sum_{i=2}^{d} K(v, x_i) = \sum_{i=2}^{d} R(v, x_i, v, x_i)$$

for any collection of x_i completing v to an orthonormal basis.

Finally, the *scalar curvature* scal(m) is the mean of the numbers Ricci(v) as v runs through the unit tangent vectors at m. To interpret it geometrically, we look at the limiting expansion of the volumes of small balls of radius ε centered at m. Since we have only to integrate over the unit ball of the v's, the expansion will clearly start with the volume of the Euclidean ball of radius ε; the next order term will be the scalar curvature, multiplied by a suitable coefficient.

When the metric g is multiplied by a scaling factor k, the curvatures are multiplied by k^{-1}. Scaling can thus make the curvature as small as desired. To block this effect of scaling, we can bound the diameter, which is scaled by $\sqrt{k}$.

Sectional Curvatures of Constant Sign

From the work of Élie Cartan, a Riemannian manifold has negative sectional curvature everywhere if and only if the sum of the angles of every sufficiently small triangle is smaller than π. This is true for any triangle when the manifold is simply connected. Similarly, it is not hard to see, via Cartan's argument, that there is positive curvature everywhere if and only if the sum of the angles of every sufficiently small triangle is larger than π. The surprising and basic point in many Riemannian geometry results is that this result about the angles in the positive case holds for any triangle without any extra assumption on the manifold. This was discovered by Alexandrov for surfaces and extended by Toponogov to all abstract Riemannian manifolds. The inequality between the sum of the angles and π translates into an inequality for the sides, as in Figure 8.

We say that a manifold is *negatively curved* if the sectional curvature of every tangent plane is negative, *positively curved* if the sectional curvature of every tangent plane is positive. It is therefore natural to ask the global question: Which compact manifolds enjoy such a property?

For positive sign, the striking fact is that the only examples known today are spheres; projective spaces $\mathbb{K}P^n$ over the real numbers $\mathbb{R}$, the complex numbers $\mathbb{C}$, the quaternions $\mathbb{H}$, and the Cayley numbers $\mathbb{O}$ (only $\mathbb{O}P^1$ and $\mathbb{O}P^2$ exist); and some sporadic examples in dimensions 6, 7, and 13. These last examples are also homogeneous spaces, or almost.

Apart from the very weak topological restriction coming from the positivity of scalar curvature, which is just the nullity of a single topological invariant, there was not a single restriction known for positively curved manifolds before MG's paper

[6], "Curvature, diameter and Betti numbers". In [6] MG showed that positive curvature forces the sum of the Betti numbers over any field to be universally bounded for all dimensions. In fact the proof works also for nonnegative sectional curvature. The proof is a marvelous juxtaposition of subtle algebraic topology and a type of Morse theory for the distance function. Grove and Shiohama in 1977 succeeded in extending the notion of a *critical point* to the distance function even though it is not smooth. MG showed for positively curved manifolds that if a sequence of critical points for the distance to a given point has its distances in geometric progression, then the sequence has to be finite and its length depends only on the dimension. This follows from the Toponogov comparison theorem for triangles. The result about the Betti numbers follows by using various initial points and using Mayer-Vietoris sequences. As MG remarks in [6], the proof works without much change when the condition $K \geq 0$ is replaced by

$$\text{Diameter}(g)^2 \cdot \inf K \geq k,$$

for any $k < 0$. Consequently a lower bound for the sectional curvature and an upper bound for the diameter are enough to control the Betti numbers of the manifold under consideration.

The case of negatively curved manifolds was also completely mysterious before the publication by MG in 1978 of [3], "Manifolds of negative curvature". The content of this paper and more are in book form in [1], *Manifolds of Nonpositive Curvature*, which is a detailed account of a series of lectures given by MG at the Collège de France.

From the work of Hadamard (1898) and Cartan (1926), one knew that if (M^d, g) is negatively curved, then its (simply connected) universal cover has to be diffeomorphic to $\mathbb{R}^d$. This cover can be constructed by considering the geodesics originating at any given point (by means of the exponential map). For a naive person, such as I, one considers the classification as finished, since "everything is in $\pi_1(M^d)$." So the question is reduced to an algebraic problem. Before [1] one did not know much concerning the algebraic structure of such a $\pi_1(M^d)$, seen as a group. This takes us back to the first section of Part I and the notion of hyperbolic group invented by MG to solve this negative curvature problem. Although [1] is a basic advance, one still does not know if the fundamental group of a manifold of negative curvature is any more special than an arbitrary hyperbolic group.

Before quoting results of [1], let me explain why it is so important to study manifolds of negative curvature. I quote MG on a philosophical point: "Almost all geometries are of negative curvature." Negatively curved geometries (either Riemannian or more general) are the kind that one is most likely to encounter in nature. For the moment this affirmation is only heuristic. Except in dimension two! In fact, the most natural way to construct geometries in dimension two is to glue Euclidean regular polygons (the simplest geometric objects we know) along their sides. Then the result is not a smooth surface but a locally Euclidean object with distributional curvatures at the vertices equal to 2π minus the sum of the angles of the polygons that meet there, hence negative as soon as most polygons have more than six sides. By contrast, in larger dimensions, when we glue polyhedra along their faces, we run into difficulty in knowing how to recognize at the vertices whether the distribution curvature can truly be said to be negative. Despite a number of papers on this subject, this question remains mysterious.

In [3] appears the important result of topological finiteness when the total volume of the negatively curved manifold (normalized by the condition $K \leq -1$) is bounded. There is also a finiteness result in the real analytic case. The latter result is obviously false in general: just take larger and larger connected sums. Results in Riemannian geometry using real analyticity are extremely scarce.

In [1], besides the negative case, the nonpositive situation is treated at length. For those manifolds MG introduces on the sphere at infinity the notion of *Tits metric*. This is a basic tool for studying the fine structure of negativity versus nonpositivity. To visualize the situation, the reader should think of *space forms* as basic examples. These were defined in Part I as any compact quotient of a Riemannian globally symmetric space of noncompact type. One has negativity if the rank is 1, but only nonpositivity if the rank is 2 or larger. We will return later to MG's work in the negative-curvature realm and why he thinks this study is so important. For the moment one can reduce things to what he calls a vague conjecture: "*In high dimensions every hyperbolic manifold is arithmetic.*"

We now consider the positive case, but this time for scalar curvature. At present the only classification that geometers have been able to solve in Riemannian geometry is that of manifolds with positive *scalar* curvature. Apart from a few remaining questions when the manifold is not simply connected, one has a complete classification of compact manifolds that can admit a metric with positive scalar curvature. As was mentioned at the end of Part I, a basic result in the field is a 1980 joint paper of MG with Lawson, "The classification of simply connected manifolds of positive curvature". Its basic tool is a geometrically controlled surgery.

MG wanted to find the right conceptual tools to explain existing results concerning positive scalar curvature. He managed to find one and called it "K-area". Its definition is bafflingly simple. Take all nontrivial vector bundles over M, and look for the minimum of the inverse of their largest curvature. More precisely, consider all complex vector bundles over M, put on them Riemannian bundle metrics, and endow them with a connection preserving this metric. They then have a sectional curvature. As soon as such a bundle has a nonzero Chern class, its curvatures cannot all vanish. If $\|R(X)\|$ is the supremum of this curvature over all tangent planes, then the K-area of M is the minimum of $\|R(X)\|^{-1}$, the minimum taken over all nontrivial bundles and all possible Riemannian bundle metrics on them.

In MG's 1996 paper [12], "Positive curvature, macroscopic dimension, spectral gaps and higher signatures", the central statement is the following relation between K-area and the scalar curvature Scal of a Riemannian manifold: there is a universal constant $c(d)$ such that, for every complete spin manifold M^d, $\mathrm{Scal}(M^d) \geq \epsilon^{-2}$ implies $\text{K-area}_{\mathrm{st}}(M^d) \leq c(d)\epsilon^2$. The index "st" means one works on vector bundles stabilized by products with trivial ones. Its importance should not be underestimated; it captures the essence of MG's joint work with Lawson. It has a beautiful corollary: on a torus a Riemannian metric with nonnegative scalar curvature must be flat. This result is very strong because scalar curvature is a very weak invariant, just a numerical function on the manifold. Moreover, this sheds geometric light on scalar curvature; such a light was missing before. In fact, we have seen that one can completely classify the compact manifolds admitting a metric with positive scalar curvature, but the proof is completely nongeometric. In particular, there has not yet been given a single local interpretation of the positivity of scalar curvature.

Finally, we are still far from a complete classification of manifolds with positive sectional curvature, negative sectional curvature, positive Ricci curvature, and similarly for nonnegative and nonpositive curvature.

The Space of All Riemannian Structures: $d_{G\text{-}H}$ and Collapsing

In his ICM address [4], MG launched a whole program of *synthetic Riemannian geometry.* His aim was nothing less than the study of the space of all Riemannian structures in order to give some structure to this space and to study completeness, possible convergences, compact subspaces, compactification, etc. MG says he was inspired by the history text of Klein, *Development of Mathematics in the 19th Century.* The questions as they stand are too general, so it was necessary to look at the subsets consisting of manifolds satisfying various conditions on their curvature, diameter, volume, injectivity radius, etc. The need was to understand in depth the 1969 paper of Cheeger, "Finiteness theorems for Riemannian manifolds", in which it is proved that, apart from $d = 4$, there are only finitely many diffeomorphism types for d-manifolds satisfying the following three conditions: the sectional curvature stays in $[-1, 1]$, the diameter is bounded above, and the volume is bounded below and has a positive lower bound.

MG's starting point is completely elementary: He defines a metric on the set of all complete separable metric spaces. More precisely, there are two possible definitions: one for the set $\mathcal{Z}$ of all compact metric spaces and a variant for the set of all complete separable metric spaces with base point specified. The latter is what was needed for Gromov's proof of Milnor's conjecture about groups of finite type, which was discussed in Part I. In Part II we stick to the metric on the set $\mathcal{Z}$ of all compact metric spaces, since interest will be in only that. The definition begins from the "Hausdorff distance" between compact subsets X and Y of a metric space Z, namely,

$$d_Z(X, Y) =$$
$$\max\left\{\sup_{x\in X}\inf_{y\in Y} d(x,y),\ \sup_{y\in Y}\inf_{x\in X} d(x,y)\right\}.$$

If X and Y are now no longer subsets of the same space, consider all possible pairs $\{f, g\}$ of isometric embeddings $f : X \to Z$, $g : Y \to Z$ of them into a third metric space Z. Compactness ensures that $f(X)$ and $g(Y)$ are closed. Then define $d_{G\text{-}H}(X, Y)$ as the infimum of the $d_Z(f(X), g(Y))$ over all possible Z, f, and g. In effect this distance measures the best possible simultaneous approximation of X by Y and of Y by X. This $(\mathcal{Z}, d_{G\text{-}H})$ is a complete metric space. In this space, compact Riemannian manifolds can be approximated as well as desired by finite (metric) subsets. This approximation property will be basic in the next subsection, on Ricci curvature. The metric $d_{G\text{-}H}$ is called the *Gromov-Hausdorff metric.*

The main question now is to look for compact, or precompact, or even finite, subsets of $\mathcal{Z}$. In this direction Cheeger's finiteness result looks promising. Page 74 of [9], "Volume and bounded cohomology", already written in 1978, implicitly uses a compactness result in order to prove an existence result for extremal metrics under the simplicial volume, as discussed in Part I. This existence result, as well as page 63 of the Filling paper [8], were never to my knowledge written up in detail. About the compactness result, MG says:

> I always took it for granted, but since people asked for more, I wrote it in the 1981 paper *Structures Métriques pour les Variétés Riemanniennes.* [This is [7] in the present article.]

This compactness result has its germ in Cheeger's work, but stating it explicitly had to wait until

1978. The convergence can be only $C^{1,\alpha}$. To see that one cannot do better, just consider a cylinder closed by two hemispheres. The compactness result holds under Cheeger's conditions stated above (K bounded above and below, diameter bounded above, and volume bounded below), so the natural question is whether one can suppress some of these hypotheses. The least natural is the positive lower bound for the volume, but many examples show that, if K is kept inside [-1,1] and the diameter is bounded, a manifold can "collapse" on manifolds (or more general objects) of smaller dimension. The simplest case to visualize is that of flat tori, which can collapse on tori of any dimension, and also to a point. This matter is related to the vanishing of minimal volume, which was defined in Part I.

What we have here is a whole program, sketched out in the ICM 1978 address "Synthetic Riemannian geometry": consider the limit of collapsing and also study the collapsing itself by looking at the inverse images of the collapsed points. This program was essentially completed in 1992 with Cheeger and Fukaya in "Nilpotent structures and invariant metrics on collapsed manifolds", the precise details being too involved to quote here. This work has the following interesting corollary: any Riemannian manifold admits a canonical decomposition into two sets, one where large balls remain diffeomorphic to $\mathbb{R}^d$ and the other where balls admit generalized circle fibrations, i.e., nilpotent structures.

I owe the reader a technical but extremely important notion, that of the *injectivity radius*. It is the largest number r such that all balls of radius r are diffeomorphic to $\mathbb{R}^d$ under the exponential map, namely, the spray made up of geodesics issuing from the center of this ball and of length r. Roughly speaking, collapsing can happen only when the local injectivity radius is small. A key lemma in Cheeger's dissertation was how to prevent collapsing by giving a lower bound for the injectivity radius as a function of the volume, the diameter, and the supremum and infimum of the sectional curvature.

Universality of Ricci Curvature Bounded Below

The most spectacular result of [7], *Structures Métriques pour les Variétés Riemanniennes*, is one that asserts that the two hypothesis Ricci $\geq k$ and diameter bounded above imply precompactness in $\mathcal{Z}$. Roughly put, in this class of Riemannian manifolds there are only finitely many "metric types". The proof is not too hard once one has the right framework, because, from the work of Bishop in 1963, one knows that in dimension d the condition Ricci $\geq (d-1)k$ gives complete control over the volume of balls of a given radius: for any point x the function $\frac{\mathrm{Vol}(B(x,r))}{\mathrm{Vol}(B_k(r))}$ is nonincreasing in the radius r, where $\mathrm{Vol}(B_k(r))$ denotes the volume of a ball of radius r in the simply connnected reference space of constant sectional curvature equal to k.[2] One has only to use a counting argument and a classical metric trick: when disjoint balls of a given radius r are packed as tightly as possible, then the set of balls with the same centers and with radius $2r$ is a covering. In the metric on $\mathcal{Z}$, the finite subset of the manifold made up of the centers of these balls is a good approximation to the manifold as the common radius r goes to zero.

With such a result in hand what can we now hope for? Not everything, as examples show that Ricci ≥ 0 permits an infinite number of homology types. However, in this program one now has many strong results, typically from work of Cheeger and Colding; an informative text is Gallot's lecture at the Séminaire Bourbaki in November 1997. Let us mention one result: in every dimension d there exists an $\eta(d) > 0$ such that if a manifold satisfies

$$\mathrm{Diameter}^2(g) \cdot \mathrm{Inf\ Ricci}(g) > -\eta(d),$$

then its fundamental group is nilpotent up to a subgroup of finite index. Besides many new ideas of "Ricci-synthetic geometry", essential use is made of the basic technique introduced in [5], "Groups of polynomial growth and expanding maps": extract a suitable limit from the sequence $(M^d, \varepsilon^{-1} \cdot g)$ as ε goes to zero. One step consists in showing that this limit, which captures the structure at infinity of (M, g), is pleasant—in fact, a cone. By contrast, for general Riemannian manifolds this limit can be awful. An essential idea of [7] for studying $\pi_1(M)$ is to use geometrically chosen generators with ad hoc loops. MG used this technique in [3] and also in a 1978 paper for his main theorem on almost-flat manifolds; a detailed exposition appears in the 1981 book *Gromov's Almost Flat Manifolds* by Buser and Karcher. This result answered the long-standing question: Which manifolds have almost zero curvature? The first fact is that such manifolds need not be tori, because nilpotent manifolds can be collapsed to a point, or equivalently are the manifolds obtained by successive circle fibrations starting from a point. But there are essentially no more such manifolds, and this is MG's result: manifolds with almost zero curvature have to be almost-nilpotent, i.e., the quotient, up to a subgroup of finite index, of a nilpotent Lie group. The appropriate hypothesis is

$$\mathrm{Diameter}^2(g) \cdot \sup(|K|) \leq \varepsilon(d)$$

for a universal positive $\varepsilon(d)$.

MG's work, "Paul Lévy's isoperimetric inequality", published in 1980, also deals with a lower bound for the Ricci curvature and is just as spectacular; this text, only an IHÉS preprint, appears now as one of the appendices to [13]. It shows that a lower bound on Ricci curvature is enough to completely control the "isoperimetric profile" of

[2] *The volume of the balls in the reference space does not depend on the point, as the space is a homogeneous space.*

a manifold. In (M^d, g) the *isoperimetric profile* is the function of τ given by the lower bound of the $(d-1)$-volume of the boundary ∂D over all domains D whose volume is equal to τ. The proof uses in an essential way geometric measure theory, which has been available to mathematicians since the end of the 1960s. This theory provides absolute minimal objects—here a domain of given volume whose boundary has smallest possible volume—having reasonable singularities; the set of singularities is of codimension at most 8. This control is needed in order to study PDEs on a Riemannian manifold from a geometric point of view.

The Spectrum of Riemannian Manifolds

A Riemannian manifold has a Laplacian, which is the canonical elliptic linear second-order operator given by $\Delta f = -\mathrm{Trace}_g D^2 f$. For compact manifolds this operator has a discrete spectral decomposition: the equation $\Delta\phi = \lambda\phi$ has nonzero solutions for a discrete unbounded set $\{\lambda_i\}$ of eigenvalues in $\mathbb{R}^+$, including 0. This set is called the *spectrum* of the manifold. Moreover, the λ_i always have finite multiplicity, and every eigenfunction is C^∞. Let us list each λ_i as often as its multiplicity and denote by $\{\phi_i\}$ a corresponding orthonormal set of eigenfunctions. Then every L^2 function can be written uniquely as the sum of an L^2-convergent series $\sum_i a_i\phi_i$, where $a_i = \int_M f\phi_i$. For smooth functions f, the series converges in the C^∞ topology. Consequently, in terms of this expansion one can solve various differential equations on the manifold, such as the heat equation, the wave equation, the Schrödinger equation, etc.

The main problem is to try to analyze the spectrum as a subset of $\mathbb{R}^+$. One introduces the *counting function* $N(\lambda) = \#\{\lambda_i \le \lambda\}$. Its asymptotic behavior was found in 1949 by Minakshisundaram and Pleijel:

$$N(\lambda) \underset{\lambda\to\infty}{\sim} \frac{\beta(d)}{(2\pi)^d}\mathrm{Vol}(g)\lambda^{d/2}.$$

In this formula $\beta(d)$ is the volume of the unit ball in $\mathbb{R}^d$, and thus $\mathrm{Vol}(g)$ and the dimension d are the only Riemannian invariants that play a role. This is the Riemannian generalization of Weyl's famous asymptotic formula for bounded domains of the plane. One wants to control the gaps, and this amounts to finding the next term in the asymptotic expansion of $N(\lambda)$. In 1968 Hörmander showed that

$$N(\lambda) = \frac{\beta(d)}{(2\pi)^d}\mathrm{Vol}(g)\lambda^{d/2} + O(\lambda^{(d-1)/2})$$

as $\lambda \to \infty$. The exponent $(d-1)/2$ is optimal, as is shown by the standard sphere, where the gaps are huge, expressing the fact that the eigenfunctions, which are the spherical harmonics, have large multiplicity.

However, this estimate is somewhat unsatisfactory for a Riemannian geometer, since the constant in the O term is not explicit. One would prefer an error term controlled by Riemannian invariants such as curvature, diameter, injectivity radius, etc. Moreover, one would like to control the gaps from the beginning and not only asymptotically. These two questions were settled in [12], "Positive curvature, macroscopic dimension, spectral gaps and higher signatures".

The control is given by the supremum and infimum of the sectional curvature and by the injectivity radius, which was defined above. The proof is very involved and has the striking feature that it is opposite to standard spectral arguments. Normally, when working with elliptic operators, one uses control of the spectrum for numerical functions on the manifold to control the spectrum for sections of various fiber bundles over the manifold by means of the "Kac-Kato-Feynman inequality". Here MG controls the low eigenvalues by controlling the spectrum of suitable bundles via the topology, the Atiyah-Singer index theorem, and techniques of Vafa-Witten and Bochner-Lichnerowicz. Then he applies the Kac-Kato-Feynmann inequality in reverse.

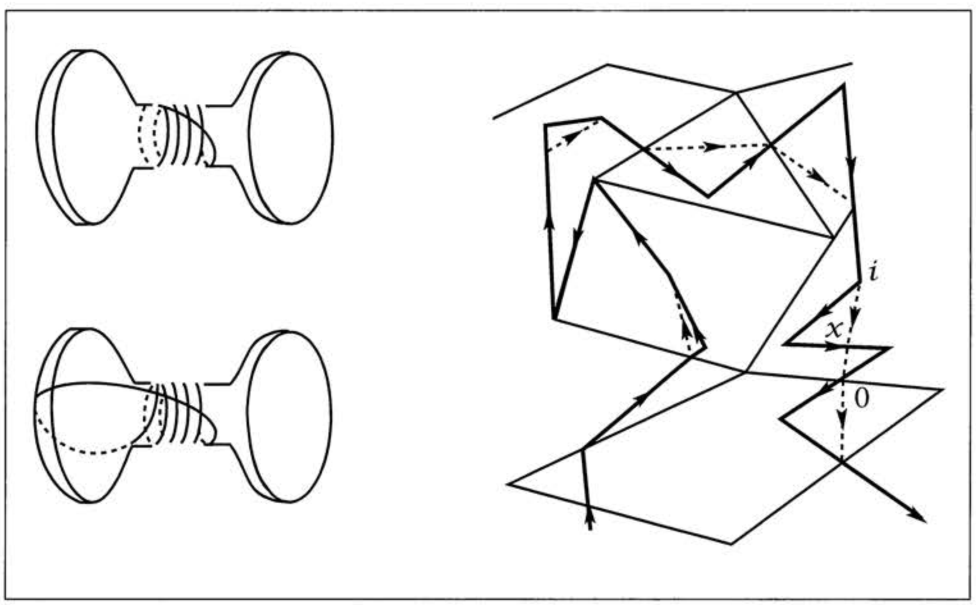

Figure 9. The diagrams on the left show how to unwrap carefully a many-times-twisted rope around a dumbbell without increasing its length too much. The diagram on the right indicates how to deduce from a triangulation of the manifold an efficient triangulation of its loop space.

The second major contribution to the spectrum came much earlier, in the above-mentioned 1980 preprint "Paul Lévy's isoperimetric inequality"; it was made more precise by Bérard, Besson, and

Gallot in 1985. In the spirit of the preceding subsection, a control of the isoperimetric profile enabled MG to completely squeeze the ith eigenvalue within universal bounds. For every i one has:

$$\text{univ}(\text{Inf Ricci}, d, \text{Diam}(g))\, i^{2/d} \le \lambda_i \le \text{univ}(\text{Inf Ricci}, d, \text{Volume}(g))\, i^{2/d},$$

where "univ" refers to some function that depends only on the specified arguments. This estimate agrees with the exponent in the Minakshisundaram-Pleijel asymptotic.

Periodic Geodesics

Having obtained a nice distribution for the spectral counting function $N(\lambda)$, one can hope that there is also a nice distribution for the length of periodic geodesics. We have in mind the following analogy: The Laplacian controls the quantum mechanics of the manifold via Schrödinger's equation, whereas the geodesic flow controls its classical Hamiltonian mechanics. One draws a parallel between ϕ_i as eigenfunction—in terms of stationary modes and pure vibrations—and the periodic geodesics γ, which are stationary motions. One is therefore tempted to draw a parallel between the eigenvalues λ_i, giving the frequencies of the vibrations, and the lengths $L(\gamma)$ of the periodic geodesics γ. In that case the geodesic flow will be completely described by the set of periodic geodesics and their lengths. In particular, it is reasonable to hope for an infinite number of such periodic geodesics, their lengths making up a discrete subset of $\mathbb{R}^+$, and to have an asymptotic expansion for the *counting function* $CF(L)$ defined by

$$CF(L) = \left\{ \begin{matrix} \text{number of periodic geodesics} \\ \text{of length smaller than } L \end{matrix} \right\}.$$

Under this analogy the growth corresponding to $\lambda^{d/2}$ for the spectrum would be exponential in L.

One can take a suitable surface of revolution to see that the set of lengths need not be discrete. These examples show also that there are continuous bands of periodic geodesics. On the other hand, one still does not know whether, for any compact manifold and any Riemannian metric on it, there exist infinitely many periodic geodesics. One does know the existence of infinitely many periodic geodesics in dimension two; the first unsettled case is the three-dimensional sphere. In the counting process, running two or more times along a given periodic geodesic is not to be considered as different from going once; this phenomenon is the major difficulty in getting an infinite number of truly geometrically different periodic geodesics. Let us now examine MG's contributions to the subject.

We begin by briefly describing the extreme case of negatively curved manifolds. In this case there always exists at least one periodic geodesic in any free homotopy class of curves: just take the minimum length curve in the given class. Thus huge fundamental groups yield many periodic geodesics, and one gets almost trivially an exponential growth for the counting function CF. Optimal results are obtained if the manifold is the underlying one of a space form as defined earlier; in this case the factor in the exponential is no less than the one in the constant curvature case, with equality only for isometry with the space-form structure. This was proved by Katok in 1982 for surfaces and for all dimensions by Besson, Courtois, and Gallot in 1996; we quote this result because MG's notion of simplicial volume enters in a fundamental way in the proof.

Let us consider the opposite situation—simply connected manifolds. Before [2], "Homotopical effects of dilatation", there was an almost complete paralysis. Why was this? Morse theory started with Birkhoff's 1913 result yielding at least one periodic geodesic on any convex surface. The basic idea applies to any compact manifold: One considers the set $\Omega(M)$ of all closed curves on the manifold that are homotopic to zero, together with the function on this space given by length. The critical point will be exactly the periodic geodesics. So Morse theory apparently yields as many periodic geodesics as the Betti numbers of this space of curves. There are three difficulties: The first is that this space is infinite dimensional, a difficulty that is overcome by taking a finite-dimensional approximation (Birkhoff already knew how to do this by using the injectivity radius) and by replacing every curve by an approximating one made up of geodesic pieces, a so-called *broken geodesic*. The second difficulty is the computation of the Betti numbers of $\Omega(M)$. The fact that we take curves without a fixed base point makes things quite difficult, but this algebraic-topology difficulty was overcome quite successfully by various topologists. There remains the third difficulty: Morse theory gives only the existence of (many) critical points, but does not say anything about the value of the function—here the length of periodic geodesics. A typical example: If we have an infinite number of critical points (as when the Betti numbers are nonzero for an infinite sequence of dimensions of $\Omega(M)$) and if all the lengths are multiples of a given one, then the geodesics so obtained could be only the covering of a single one.

So the aim is to quantize Morse theory at the level of the values at critical points. Trivial examples show that one cannot expect any results for general functions. But in the Riemannian case and the space $\Omega(M)$ of [2], MG managed to quantify things as in Figure 9. Two ideas were used. The first pertains to the left-hand diagrams in Figure 9, where one has a curve homotopic to zero turning many times around a "thin" part; one contracts it very carefully to a point by contracting each "turn" along the big part of the manifold. Doing this for every turn yields a nice control over the length. If all turns pass through the big part at once, the length will become too large. The subtle part of [2] consists in deducing from a triangulation of M a triangulation of $\Omega(M)$; it is indicated in the right-hand diagram of Figure 9. The dimension of this triangulation equals the sum of the dimensions of the simplices of the triangulation of M that are crossed by a given broken geodesic. The final trick is to use the simple connectivity in order to contract the whole 1-skeleton to a point so that the pieces of broken geodesics running through edges no longer count.

The conclusion is this: For any compact (M, g), there exist two positive constants a and b such that for every length L one has

$$CF(L) \geq a \sum_{k \leq bL} \beta_k(\Omega(M)),$$

where β_k denotes the Betti numbers. The proof consists in contracting the curves of $\Omega(M)$ to a point while permanently controlling, with an eye on the length, the topology in degree i of $\Omega(M)$. This result applies only in the case where all periodic geodesics are nondegenerate; nondegeneracy is essential in order to apply Morse theory.

The above inequality solves the problem of periodic geodesics for "doubly generic" Riemannian manifolds: their counting function $CF(L)$ grows exponentially with L. To explain this statement, we need some algebraic topology. Compact manifolds fall into two classes: those called *rationally elliptic*, and those called *rationally hyperbolic*, most manifolds being rationally hyperbolic. The elliptic ones are those having all their homotopy groups $\pi_k(M^d)$ finite for every $k > 2d - 1$. On the other hand, from work of Felix and Halperin in 1982 one knows that rationally hyperbolic manifolds have Betti numbers $\beta_i(\Omega(M))$ that grow exponentially with i. There remain two difficulties. The first was noted above: how to get periodic geodesics geometrically, i.e., how to eliminate the ones that are multiples of others. But the iterates of a periodic geodesic have their lengths in arithmetic progression, so this difficulty goes away under the exponential. The second is that Morse theory applies only when all the critical points are nondegenerate. Here we will have the notion of nondegenerate periodic geodesic. In full generality a Riemannian manifold might have degenerate periodic geodesics, but not for a "generic metric". One knows by results of Klingenberg, Takens, Anosov, and Rademacher that "bumpy" metrics—those for which all periodic geodesics are nondegenerate—are generic; this genericity is made precise in the Baire category sense. MG's result is that for generic metrics on most manifolds one has exponential growth for the counting function of periodic geodesics. This result addresses the problem stated at the beginning of this subsection.

Which Spaces for Geometry? Gromov's Program

We now summarize Chapter $3\frac{1}{2}$ of [13], *Metric Structures for Riemannian and Non-Riemannian Spaces.* This chapter ends with: "We humbly hope that the general ambiance of X can provide a friendly environment for treating asymptotics of many interesting spaces of configurations and maps." Present models of geometry, even if quite numerous, are not able to answer various essential questions. For example: among all possible configurations of a living organism, describe its trajectory (life) in time; give as a function of time the mean diameter of planar self-avoiding Brownian motion; improve results of statistical mechanics; create a geometric theory of probability, say by quantifying geometrically the law of large numbers. Some of what we come to now is more or less known in probability theory and in statistical mechanics either formally or heuristically. The aim here is to lay a foundation, an axiomatic theory powerful enough to handle the above problems, solved or unsolved.

We now sketch the answer given in Chapter $3\frac{1}{2}$, referring the reader to the book [13] for details and many more results. One thing to realize is that in geometry the notion of measure is ultimately more important than that of metric. Measure arises first in probability theory, since it is needed to make any statistical assertion. In the Riemannian case the measure comes automatically from the metric. This order of events does not preclude the Riemannian measure from being basic. In Riemannian geometry the innovation of Riemann's was to dissociate the metric from the vector-space structure in Euclidean geometry and to replace the vector space by a differentiable manifold. Here MG dissociates the metric and the measure in a Riemannian manifold by introducing the notion of *mm-space*. This is a triple (X, d, μ) in which (X, d) is any complete separable metric space and μ is initially a finite measure on the σ-algebra of Borel sets, i.e., the smallest σ-algebra containing the open sets. Then the measure space is completed by adjoining to the σ-algebra all subsets of Borel sets of measure 0. It is assumed that every one-point set has measure 0. Let $m = \mu(M) > 0$. It is known[3] that any mm-space with $m = \mu(M)$ always admits a measure-preserving parametrization $\phi : [0, m) \to X$, i.e., a one-one onto function ϕ from the complement of a set of measure 0 to the complement of a set of measure 0 such that ϕ preserves measurable sets and the measure.

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[3] *As a result of (§43, IX) in Hausdorff's* Set Theory, *Theorem 2 of the 1942* Annals *paper by Halmos and von Neumann, and an easy supplementary argument. For an exposition, see §2 of V. A. Rohlin, On the fundamental ideas of measure theory,* Translations Amer. Math. Soc. *(1)* ***10*** *(1962), 1–54.*

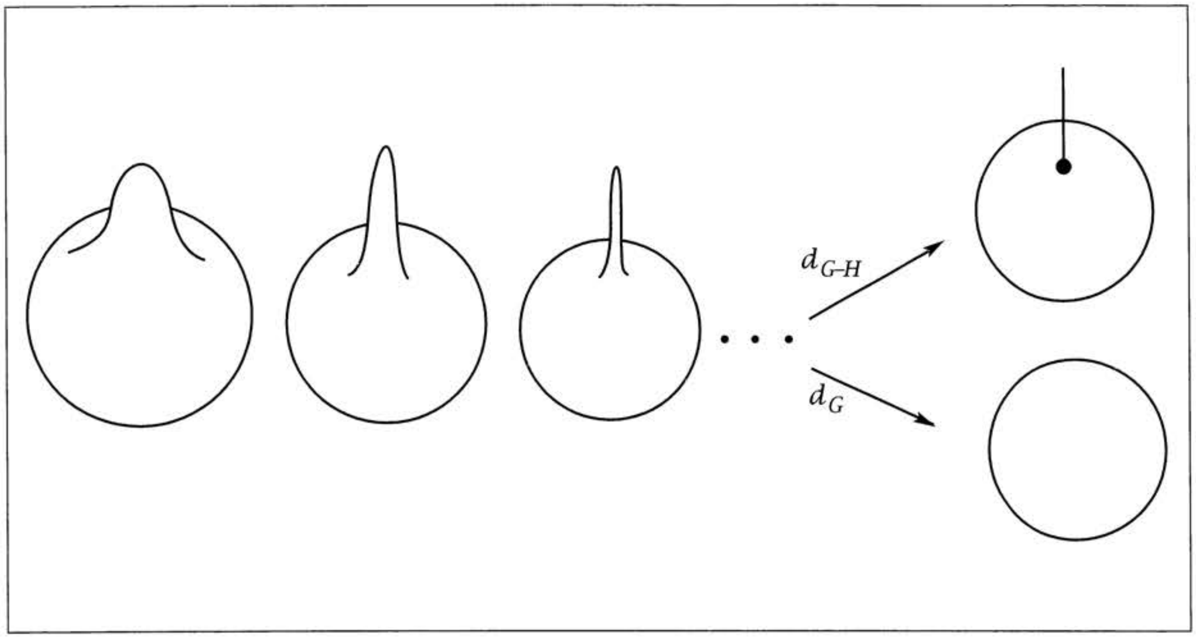


Figure 10. The limiting space of a sequence of spheres equipped with caps that are converging to a "hair" is a sphere with this hair in the metric $d_{G\text{-}H}$ but is only the sphere (with no hair) in d_G. The distinction arises because a hair has zero measure.

The first thing MG does is to consider the set $\mathcal{X}$ of all mm-spaces and to endow it with various structures. It seems that no reasonable measure can exist on $\mathcal{X}$, but MG defines a metric on $\mathcal{X}$, denoted here by d_G. For the sake of simplicity, we define $d_G(X, X')$ only when $\mu(X) = \mu(X') = m$. Unlike the Gromov-Hausdorff metric $d_{G\text{-}H}$ on $\mathcal{Z}$ defined in the section on Riemannian geometry, the metric d_G is not very geometric and is very hard to visualize since already a measure-preserving bijection between the interval $[0, 1]$ and the square $[0, 1]^2$ is hard to visualize. We first consider measure-preserving parametrizations of (X, d, μ) and (X', d', μ'), say $\phi : [0, m) \to X$ and $\phi' : [0, m) \to X'$. We pull back d and d' to real-valued functions on the square $[0, m)^2$, namely, $d \circ (\phi \times \phi)$ and $d' \circ (\phi' \times \phi')$. We introduce the ε *almost-distance* $\varepsilon(\phi, \phi')$ between d and d' as the smallest ε such that the set of $t \in [0, m)^2$ with

$$|d((\phi \times \phi)(t)) - d'((\phi' \times \phi')(t))| \geq \varepsilon$$

is of measure smaller than ε. Then $d_G(X, X')$ is defined as the infimum of $\varepsilon(\phi, \phi')$ over all possible parametrizations ϕ and ϕ' of X and X' of the above type. It is easy to check that this d_G is ≥ 0 and is symmetric and transitive, but it is hard to show that $d_G(X, X') = 0$ implies that X and X' are suitably isomorphic; we return to this point in a moment. The metric space $(\mathcal{X}, d_G)$ so constructed is complete. Heuristically speaking, the Gromov distance d_G is similar to the Gromov-Hausdorff distance $d_{G\text{-}H}$ in the purely metric case, but this time we are asking questions whose answers apply only almost everywhere.

One can compare the nature of $d_{G\text{-}H}$ and d_G somewhat by using a picture (Figure 10). We take a sequence of spheres with hats, the hats converging toward a segment (a hair). For $d_{G\text{-}H}$ the limit will be a sphere with a hair, but for d_G it will be only a sphere. This is satisfactory, since hairs have measure zero and thus can be neglected.

We return to comment on the proof that $d_G(X, X') = 0$ implies that X and X' are suitably isomorphic. There are two things to say. One is that the proof uses MG's notion generalizing sectional curvature to mm-spaces. The other is that the proof gives a typical example of how control of the volume (measure) of metric balls can have strong metric applications. This technique captures some of the essence of MG's precompactness in the section above. The notion of curvature used here for a given (X, d, μ) is a collection $\mu^{X;k}$ of measures, k being any natural number. The measure $\mu^{X;k}$ is defined on the space M_k of all symmetric $k \times k$ real matrices: this is the measure pushed forward from $\mu \times \cdots \times \mu$ on $X \times \cdots \times X$ by the natural map $X \times \cdots \times X \to M_k$ that assigns to k-tuples of points in X the set of their mutual distances. For example,

$$\int_{M_2} f \, d\mu^{X;2} = \int_{X \times X} f \begin{pmatrix} d(x,x) & d(x,y) \\ d(x,y) & d(y,y) \end{pmatrix} d\mu(x) \, d\mu(y).$$

It turns out that knowledge of all these measures, as k varies, allows one to reconstruct the metric d up to isomorphism of metric spaces.

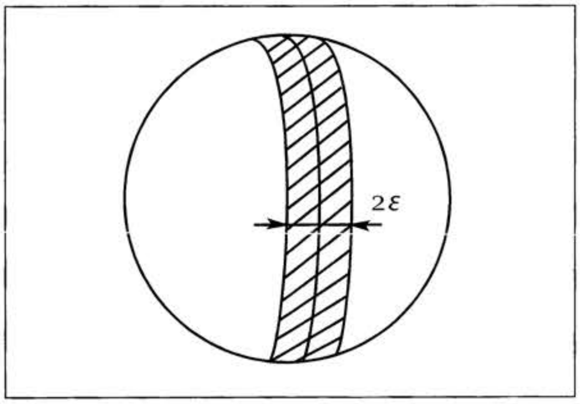


Figure 11. The complement of the ε-neighborhood of an equator of an n-dimensional unit-volume sphere has a volume that, for ε fixed, goes exponentially to zero as the dimension goes to infinity. More precisely, this volume is smaller than $2\exp(-n\varepsilon^2/2)$). Applying the isoperimetric inequality to domains of the sphere, one deduces from this that, as the dimension goes to infinity, any function tends to its central value on a set of measure arbitrarily close to 1.

Next in Chapter $3\frac{1}{2}$, the author introduces the notion of *observable diameter*—more precisely, the notion of κ-observable diameter with magnification λ. We normalize the magnification λ to be 1, since the asymptotic behavior of observable diameter as the dimension goes to infinity does not depend much on λ. The idea is to introduce notions corresponding to physical reality and physical experiments. Physical reality is taken to be a metric space (X, d). An object can be observed only by signals we perceive from it. The signals are Lipschitz functions, and we restrict ourselves to Lipschitz functions with Lipschitz constant 1, i.e., those satisfying $|f(x) - f(y)| \le d(x, y)$ for all x and y. What we perceive, due to the lack of accuracy of our instruments, holds up only to a small error, and observable diameter is intended to capture this variability. The notion of observable diameter can be defined for any geometric concept such as the central radius (the minimal radius of a ball covering the whole metric space), the center of mass, etc. A metric and a measure are enough to define such notions.

The κ-observable diameter of (X, d, μ), denoted by ObsDiam(X, κ), is the smallest real number δ such that, for every Lipschitz numerical-valued function f on X with Lipschitz constant 1, there exists an $A \subset \mathbb{R}$ of $\mathbb{R}$-diameter smaller than δ such that

$$\mu(f^{-1}(A)) \ge \mu(X) - \kappa.$$

From the observer's point of view, this says that if $\mu_* = \mu \circ f^{-1}$ is the pushed-forward measure by f, then

$$\mu_*(A) \ge \mu_*(f(X)) - \kappa.$$

In what follows one will see that ObsDiam(X, κ) is not very sensitive to the parameter κ, so MG suggests taking $\kappa = 10^{-10}$ once and for all; then we can write simply ObsDiam(X). The geometric law of large numbers consists in studying in various contexts the asymptotic behavior of ObsDiam as the dimension goes to infinity. Here are some results about ObsDiam illustrating the subtlety of this notion. In particular, the topology of the space is not the important feature; its metric and its measure are the determining factors.

Historically, the first estimation of observable diameter was for standard spheres. As early as 1919 Paul Lévy studied the so-called *concentration* phenomenon of spheres S^n: most of the measure of the sphere is concentrated around an equator, and this effect becomes more pronounced as the dimension gets large. This is because $\int_0^{\pi/2} \sin^n t\, dt$ is concentrated at $\pi/2$ as $n \to \infty$. In other words, consider the tubular neigborhood $U_\varepsilon(D)$ of a hemisphere D, namely, the set of points whose distance to D is $\le \varepsilon$; then Paul Lévy proved that

$$\mathrm{Vol}(S^n \setminus U_\varepsilon(D)) < 2\exp(-n\varepsilon^2/2).$$

The isoperimetric inequality for spheres can be seen at the level of tubular neighborhoods, and we will use it shortly for domains whose measure is, as with the hemisphere, half of the total volume. For now, consider any function f on the sphere, and look at it close to its central value c. If the isoperimetric inequality is applied to the domain where f takes values smaller than its central value, then the above two facts yield: the set of values of f that are outside the interval $[c - \varepsilon, c + \varepsilon]$ has measure $< 2\exp(-n\varepsilon^2/2)$. In the language of observable diameter,

$$\mathrm{ObsDiam}(S^n) = O(1/\sqrt{n}).$$

Of course, the exact diameter of S^n equals π for every dimension.

In the preceding section we discussed the control that MG obtains over the isoperimetric inequality for manifolds with positive Ricci curvature. MG's result can be immediately translated to the estimate ObsDiam $= O(1/\sqrt{n})$ for any such manifold. But this condition is necessary, as examples show that even nonnegative Ricci curvature will lead to manifolds with ObsDiam $= O(1)$ but not $= o(1)$. Topology is not the factor producing these larger estimates; other examples lead to metrics on S^n close to the standard one and whose observable diameter can be as large as desired. The hardest result on observable diameter given in Chapter $3\frac{1}{2}$ is the following: for complex algebraic submanifolds $X \subset \mathbb{C}P^n$ of degree d and codimension k, one has

$$\mathrm{ObsDiam}(X) = O\left(\frac{\log n}{n}\right)^{1/2d} \text{ as } n \to \infty \text{ with } k \text{ and } d$$

fixed. One can say that algebraicity takes the place of positive Ricci curvature (strictly speaking, there is no such relation). The metric we are considering on X is the one induced in the Riemannian sense, not just the distance on $\mathbb{C}P^n$ restricted to X in the trivial sense. This means the distance between two points is the infimum of the length in $\mathbb{C}P^n$ of all curves joining them. Metric length structures on algebraic manifolds are an extremely difficult subject which is very rarely tackled. MG likes to call this topic "the muddy waters of metric algebraic geometry." This is reflected by the fact that the behavior of the (intrinsic) diameter is still not understood. For a given degree (in a fixed $\mathbb{C}P^n$, of course) the diameter is bounded using general machinery, but does the diameter become infinite as the degree gets large? This is known only for curves in $\mathbb{C}P^2$ and only since Bogomolov's work in 1994. But the question remains open starting with surfaces in $\mathbb{C}P^3$. The proof of the above estimate for the observable diameter of algebraic manifolds needs more than twenty pages and is very involved.

A geometric law of large numbers consists in studying observable geometric quantities on the products $X^n = X \times \cdots \times X$ as $n \to \infty$. First, we have to specify which mm-structure we are considering on X^n, given a fixed mm-space (X, d, μ). For the measure one always takes the product measure, but for the metric there is a choice according to the situation. The extension of the Riemannian case consists in taking the l_2-product metric. This is a special case of the l_p-product metric, given by $(\sum_i \text{dist}_i^p)^{1/p}$. For general mm-spaces one cannot do better than $\text{ObsDiam}(X^n) = O(n^{1/2p})$, but this is better than the real diameter, namely, $O(n^{1/p})$. One more example due to MG is the discrete cube $\{0, 1\}^n$, for which $\text{ObsDiam} \approx n^{1/4}$, whereas the full diameter of the cube is $O(1)$. For the case of the regular simplex, the observable diameter is $O(1/n)$.

Now we turn to the spectrum. MG succeeds in defining a spectrum $\{\lambda_i\}$ for any mm-space. We will work with only the first eigenvalue λ_1, defining it below. Of course, in general we do not have a differential operator like the Laplacian. In the special case of Riemannian manifolds, $\lambda_1(X)$ is characterized as the minimum of

$$\frac{\int_X ||\text{grad} f||^2 \, d\mu}{\int_X f^2 \, d\mu}$$

over all functions with $\int_X f \, d\mu = 0$. On an mm-space all of the above ingredients are defined except for the gradient. But one has only to define $||(\text{grad} f)(p)||$ for a function f on a metric space (X, d) as $\limsup_{\varepsilon \to 0} \frac{|f(x)-f(y)|}{d(x,y)}$ for x and y in the ball of center p and radius ε. MG proves an inequality valid for any mm-space connecting λ_1 and the observable diameter:

$$\text{ObsDiam}(X, \kappa) \leq \log \kappa^{-1} / 2\sqrt{\lambda_1(X)}.$$

Glances at Other Important Results

We present here, even more briefly than above, a series of results whose omission would not do justice to our geometer.

Space Forms

There is nothing more natural for a geometer than to look for geometries that generalize Euclidean geometry. Some names associated to this quest are Clifford, Klein, and Killing. One starts with *space forms* in the strict sense, namely, the geometries that enjoy the basic property of Euclidean space: two triangles with corresponding sides equal are *congruent*, which means that there is an isometry of the space (a local one in general) sending one onto the other. One can also speak of 3-*point transitivity*. If one imposes this condition as well as simple connectivity, only three geometries are possible: Euclidean, spherical, and hyperbolic. This class coincides with simply connected Riemannian manifolds of constant sectional curvature.

We shall take the spaces in question to be compact manifolds that are quotients of the three standard ones. We stick to the compact case and to manifolds for simplicity. In the Euclidean and spherical cases, examples are easy to construct; moreover, one had a complete classification by the end of the 1960s. The hyperbolic case is a completely different story. In dimension two, examples are easy, but a classification is much harder and in fact is the basic content of Teichmüller theory for Riemann surfaces. But starting in dimension three, one had to wait until 1931 to have some examples, and in higher dimensions until Armand Borel in 1963. Borel's construction is based in an essential way on number theory, the corresponding space forms being called *arithmetic*; his construction is valid for all symmetric spaces of arbitrary rank. In a joint 1988 paper with Piatetski-Shapiro, "Nonarithmetic groups in Lobachevsky spaces", MG managed to construct some nonarithmetic examples in all dimensions (even if number theory was always present at the start).

It remains open whether arithmetic examples are more numerous or less numerous than nonarithmetic ones. MG has a program to try to solve this. His idea is to mix suitably the notion of hyperbolic polyhedron with number theory. Part of the difficulty is that the known construction of Riemann surfaces by taking triangles or other polygons in the hyperbolic plane and reflecting them about their sides—this kind of construction was shown by Vinberg in 1984 to be impossible in large dimensions (around 40). The construction works in dimension three, but which dimensions permit such constructions by reflections is an open problem.

One next looks at *space forms of rank one*, the simply connected ones being the symmetric spaces of rank one. The compact simply connected ones are the generalized projective spaces $\mathbb{K}P^n$ met in the Riemannian geometry section. For $\mathbb{K} \neq \mathbb{R}$, there is no classification problem for manifolds that are compact quotients of these because even dimension and positive curvature force simple connectivity up to a two-element group by a theorem of Synge.

The analogous simply connected negative curvature spaces, denoted here by $\mathrm{Hyp}^n(\mathbb{K})$, offer more of a challenge. (For $\mathbb{K} = \mathbb{R}$ one has the standard hyperbolic geometry.) The geometric characterization of these simply connected spaces is that they are 2-*point transitive* in the sense that pairs of points with the same mutual distance can be carried to each other by a global isometry. We again look for compact quotient manifolds of these spaces; they will be the locally 2-point transitive geometries. Such quotients exist, as shown by Borel, but their classification is not finished: For $\mathbb{K} = \mathbb{C}$ one knows only the existence of some nonarithmetic examples. For $\mathbb{H}$ and $\mathbb{O}$, it is shown in a 1992 joint work of MG and Schoen, "Harmonic maps into singular spaces and p-adic superrigidity for lattices in groups of rank one", that all such quotients must be arithmetic. Of this result, MG says that the most important thing about the paper is not this corollary, but the introduction and use of "harmonic maps" with values in singular spaces (Tits buildings in this case). This technique is now widely used.

In this area the work of [11], "Foliated Plateau problem", has not received much attention. However, MG believes that this pair of papers is important. He says:

> One of the essential ideas of this text is that, in treating the solutions of elliptic equations, the right framework is that of foliations. But, if one excepts the trivial case where the tangent bundle is enough, in general one has to go to infinite dimensions to get the space of solutions. There are some holes in this text, but it does several things: it furnishes this general framework and therefore serves to make the problems well posed, and afterward it contains also some things in the spirit of Nevanlinna theory.

Kählerian Manifolds

In much of his work MG examines Kähler manifolds with a vengeance. He absolutely wants to find *robust* results. For example, integrability of an almost-complex structure—the condition that the structure come from a complex structure—is fragile. On the other hand, invariants such as the fundamental group and the spectrum are robust. In the 1989 paper by MG, "Sur le groupe fondamental d'une variété kählérienne", one finds the first known strong restriction for the algebraic structure of possible fundamental groups of Kähler manifolds. The final theorem has not yet been obtained. Classical methods are useless for this result. MG used transcendental methods, namely, L^2-cohomology and the index theorem. He used L^2-cohomology also in other instances, such as in his 1991 paper "Kähler hyperbolicity and L^2-Hodge theory", in which he showed that the two conditions "negative sectional curvature" and "Kähler" determine the expected sign of the Euler-Poincaré characteristic. This is a conjecture of H. Hopf from the 1930s, stated for the general Riemannian case and sectional curvature of constant sign. The case of dimension two follows immediately from the Gauss-Bonnet theorem. The case of dimension four is proved. The conjecture is open starting in dimension six, and examples indicate that the proof cannot follow directly from the higher-dimensional generalization by Allendoerfer and Weil of the Gauss-Bonnet theorem.

Building Examples in Riemannian Geometry

The construction of subtle examples is an important aspect of mathematics. MG has produced a number of these.

In the section on Riemannian geometry, we saw results going beyond the condition "sectional curvature is positive, or nonnegative". It is natural to ask the same kind of question for negative sectional curvature. This cannot be done, as MG constructs for every $\varepsilon > 0$ on the sphere S^3 a Riemannian metric whose diameter is equal to 1 while the curvature satisfies $K < \varepsilon$.

In Part I we met the notion of systolic softness. The basic example furnished by MG is incredibly simple: consider $S^1 \times S^3$, as obtained from $[0,1] \times S^3$ by gluing the two copies $\{0\} \times S^3$ and $\{1\} \times S^3$ with a Clifford translation (i.e., along Hopf fibers) of greater and greater length.

According to MG, his most subtle and important constructions are in the realm of negative sectional curvature. Recall from the beginning of Part II that it is difficult to construct compact manifolds of negative curvature. Borel's examples of space forms have sectional curvature in $[-1, -\frac{1}{4}]$. Of course, one can just deform (not too much) those examples, and sectional curvature will remain negative. However, this approach leaves untouched the question of finding a classification of the set of negatively curved manifolds. The class of these manifolds is very interesting, as it supplies us with objects worthy of study for themselves but also very subtle to deal with, since products automatically yield many vanishing curvatures. Even the case of polyhedra is not simple (except as we saw in dimension two). Finally, these manifolds are linked with the hyperbolic groups seen at the beginning of Part I.

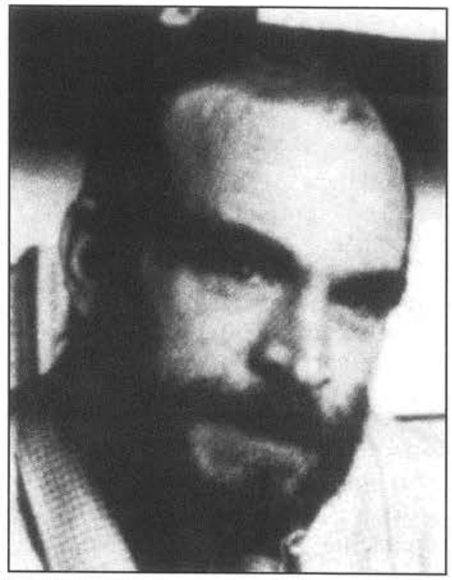
In a 1987 joint paper with Thurston, "Pinching constants for hyperbolic manifolds", one finds two essential constructions for negative curvature that work in every dimension ≥ 4. In both constructions one starts with a compact space form M of hyperbolic type, i.e., the sectional curvature is constant and is equal to -1. Consider in M a totally geodesic submanifold of codimension 2 (i.e., a submanifold N in which geodesics starting in M and tangent to N remain in N). Look now at cyclic coverings of M ramified along N. It is not too hard to endow such a covering with negative curvature, and one can even control the *pinching*, the ratio $\sup K / \inf K$. MG studies the volumes of these objects. A major result of the book [1], *Manifolds of Nonpositive Curvature*, furnishes bounds for the volume as a function of the pinching. This construction yields manifolds whose topology can differ strongly from that of a space form. In the first type of example one can show that the pinching can be as close as desired to 1. Hence the conclusion: for any ε there exist manifolds with curvature in $[-1-\varepsilon, -1+\varepsilon]$, of bounded diameter, that do not admit a metric of constant negative curvature.

A second construction enables MG to obtain examples of a complementary type: for every ε with $0 < \varepsilon < 1$, there exist manifolds of negative curvature that do not admit a metric with curvature in the range $[-1, -1+\varepsilon]$. This result is hard to prove but essential to the understanding of negative curvature. The proof uses the technique of diffusion of cycles discussed in Part I.

Conclusion

If MG has a muse, it is not the axiomatic one of Euclid. MG is instead guided by concepts such as softness versus rigidity, computability, physical reality of objects, etc. In particular, when talking about results, he is concerned with the robustness of the invariants used. His other principle is to avoid empty generalization: "Many theorems are not interesting if one cannot produce examples where the result is not already there". From this point of view the Filling paper [8] is exemplary. In case some of his results do not meet the above criterion, he adds, "Then put them in what is now called foundations."

We have seen time and again that MG's papers are like icebergs: most of the results lie under the surface and are accessible only to exceptional mathematicians who are willing to devote their time to them. So why does MG not write his results in detail? We think that the best way to answer this and other questions is to let MG speak for himself: "Checking in full detail the proof in my head was already so painful that I was left with no energy for more." Let us also quote what he says in an expository paper of 1992, "Stability and pinching":



Mikhael Gromov

> The results we present are, for the most part, not new and we do not provide detailed proofs (these can be found in the papers cited in our list of references). What may be new and interesting for non-experts is an exposition of the stability/pinching philosophy which lies behind the basic results and methods in the field and which is rarely (if ever) presented in print (this common and unfortunate fact of the lack of an adequate presentation of basic ideas and motivations of almost any mathematical theory is, probably, due to the binary nature of mathematical perception: either you have no inkling of an idea or, once you have understood it, this very idea appears so embarrassingly obvious that you feel reluctant to say it aloud; moreover, once your mind switches from the state of darkness to the light, all memory of the dark state is erased and it becomes impossible to conceive the existence of another mind for which the idea appears nonobvious).

Finally, for those who want to know more about MG's process of discovery, we end with the following quotation of his response to his being awarded the AMS Steele Prize in 1997 (*Notices*, March 1997). The response analyzes the results of [10], "Pseudo-holomorphic curves in symplectic manifolds", a paper that was discussed in Part I:

> I saw the light when struggling with Pogorelov's proof of rigidity of convex surfaces where he appeals to the Bers-Vekua theory of quasi-analytic functions. There was nothing seemingly complex-analytic in the linearized system written down by Pogorelov, and

then it struck me that *every* first order elliptic equation or quasilinear system of two equations in two variables has the same principal symbol as Cauchy-Riemann and then the solutions appear as (pseudo) holomorphic curves for the almost complex structure defined by the field of the principal symbols. Now the surface rigidity trivially followed from positivity of the intersections of holomorphic curves. What fascinated me even more was the familiar web of algebraic curves in a surface emerging in its full beauty in the softish environment of general (nonintegrable!) almost complex structures. (Integrability had always made me feel claustrophobic.) And my mind was ready for the miracle; Donaldson's ideas were in the air. So I tried to replay Yang-Mills on my holomorphic curves (strings?) and reluctantly abandoned the idea, being convinced by Pierre Deligne that the area of curves cannot be controlled without a symplectic structure. Everything went smoothly with the symplectic structure, and I even came to understand the definition of quasianalytic functions and of the nonlinear Riemann-mapping theorem of Schapiro-Lavrentiev (albeit I am still unable to read a single line of this style of analysis).

I was happy to see my friends using holomorphic curves immediately after birth: Eliashberg, Floer, McDuff. Eliashberg came across them independently in the contact framework but was unable to publish (staying in the USSR). Floer has morsified them by breaking the symmetry, and I still cannot forgive him for this. (Alas, prejudice does not pay in science.) McDuff started the systematic hunt for them which goes on till present day. And what goes on today goes beyond these lines and the pen behind them.

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About the Cover

While in Euclidean 3-space there is only one regular dodecahedron and its dihedral angle is approximately 116°, in hyperbolic 3-space there exists a continuous family with varying dihedral angle.

In the limiting case for small distances, hyperbolic 3-space looks like Euclidean 3-space, and small hyperbolic dodecahedra therefore have dihedral angles close to 116°. On the other hand, for the largest possible dodecahedron with the vertices on the sphere at infinity, the dihedral angle is precisely 60°.

Of special interest are the three dodecahedra with the intermediate dihedral angles 60°, 72°, and 90°, because they tessellate hyperbolic 3-space. These dodecahedra are shown on the front cover, each at the center of its own Poincaré ball model of hyperbolic space.

Examples of tessellations of hyperbolic 3-space by a bounded region were unknown before 1931.

—*Matthias Weber*

Standing the Test of Time: The Data Encryption Standard

Susan Landau

Fast and hard, that is all that cryptographers have ever wanted: a system that encrypts quickly but is essentially impossible to break. With their reliance on elementary number theory, public-key systems have captured mathematicians' imagination. Public-key algorithms are too slow to be used for most data transmissions, and instead public-key algorithms are used for establishing a key. Then a private-key system does the encryption. Private-key algorithms are typically faster than public-key ones.

The workhorse private-key algorithm is the Data Encryption Standard (DES), which relies on cryptographic design principles that predate public key. With the exception of RC4 in Web browsers and relatively insecure cable-TV signal encryption, DES is the most widely used public cryptosystem in the world. DES is the cryptographic algorithm used by banks for electronic funds transfer, DES is used for the protection of civilian satellite communications, and a variant of DES is used for UNIX password protection.

Proposed in 1975 and approved in 1977 as a Federal Information Processing Standard,[1] DES was immediately attacked by those who felt that its 56-bit key length was insecure. In spite of such claims, DES remained a strong encryption algorithm until the middle of the 1990s—several times longer than the government had reason to expect. Now, however, DES is past the end of its useful lifetime.

In the summer of 1998 DES's insecurity was definitively demonstrated when a $250,000 computer built by the Electronic Frontier Foundation (EFF) decrypted a DES-encoded message in 56 hours. In January 1999 this was improved to 22 hours through a combination of 100,000 networked PCs and the EFF machine. But until a substitute is found, DES remains a *de facto* standard. The National Institute of Standards and Technology (NIST)—whose predecessor, the National Bureau of Standards, certified DES—is currently seeking a successor to the algorithm. The Advanced Encryption Standard (AES) will work in three key lengths: 128, 192, and 256 bits. Fifteen candidates were submitted in June 1998 (there were actually twenty-one submissions, but six candidates had not fulfilled NIST's requirements). In August 1999 NIST eliminated ten of the fifteen. The agency is scheduled to pick DES's successor in the summer of 2000. The winning algorithm will be one whose security should stand well into the new century.

The publication of DES heralded a new era in cryptography. Academic and industrial researchers had an algorithm available for study that the National Security Agency had certified as secure. This helped develop a community of public cryptographers.

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[1] *This means the system is approved for sale to the federal government, an important issue for industry.*

When it came time to replace DES, there was a skilled community to take on the task.

In this article I outline DES, the cryptographic principles that underly its design, the algorithm's twenty-year history, and some of the strongest attacks against the algorithm. In a subsequent article I will present the cryptomathematics that evolved over these two decades and the AES effort. My intent is to illuminate the mathematics and politics behind block-structured cryptosystems.

What Is Wanted in a Cryptosystem?

Assume that the unencrypted message, the *plaintext*, is a string of bits. It is to be transformed into an encrypted string, or *ciphertext*, by means of a *cryptographic algorithm* and *key*. So that the recipient can read the message, encryption must be invertible.

Conventional wisdom holds that in order to defy easy decryption, a cryptographic algorithm should produce seeming chaos; that is, ciphertext should look and test random. In theory an eavesdropper should not be able to determine any significant information from an intercepted ciphertext.

One-time pads, whose keys are strings of random bits at least as long as the message itself, achieve this seeming impossibilty. Encryption is simple: if p_i is the i^{th} bit of the plaintext, k_i is the i^{th} bit of the key, and c_i is the i^{th} bit of the ciphertext, then $c_i = p_i \oplus k_i$, where $\oplus$ is exclusive or, often written XOR, and is simply addition modulo 2. Sender and recipient have a copy of the key. One-time pads must be used exactly once; if a key is ever reused, the system becomes highly vulnerable. In the early 1940s the Soviets made just such a mistake. Western intelligence discovered this and exploited it. Study of the messages encoded with the reused keys proved quite fruitful.[2] The constant need to refresh keying material eliminates much of the advantage of one-time pads. If we could efficiently and securely exchange keys, we could almost as easily securely transmit the plaintext, and we would have little need for a cryptosystem.

Broadly speaking, attacks on a cryptosystem fall into two categories: *passive attacks*, in which the adversary monitors the communication channel, and *active* ones, in which the adversary may transmit messages to obtain information (e.g., ciphertext of chosen plaintext). Passive attacks are easier to mount, but yield less. Attackers hope to determine the plaintext from the ciphertext they capture; an even more successful attack will determine the key and thus compromise a whole set of messages. An assumption first codified by Kerckhoffs in the nineteenth century is that the algorithm is known and that the security of the algorithm rests entirely in the secrecy of the key.

Cryptographers design their algorithms to resist the following list of increasingly aggressive attacks:

- *ciphertext-only:* The adversary has access to the encrypted communications;
- *known-plaintext:* the adversary has some plaintext and its corresponding ciphertext;
- *chosen-text:* the adversary chooses the plaintext to be encrypted, or the adversary picks the ciphertext to be decrypted (chosen ciphertext), or the adversary chooses the plaintext to be encrypted depending on ciphertext received from previous requests (adaptive chosen plaintext).

Chosen-text attacks are largely used to simplify analysis of cryptosystems, but because of such devices as "smart cards" (credit card-sized objects equipped with a small processor), such attacks can occur in practice.

If an algorithm uses a k-bit key, the measure of security is how close the algorithm is to being 2^k-secure, that is, whether there are methods for breaking the system that are significantly better than a brute-force search of the entire key space. Sometimes an algorithm's weakness is readily apparent; such was the case for "Magenta", German Telecom's submission to the AES competition. The "key scheduling" (the order in which key bits are fed to the algorithm) was poorly designed, and this insecurity was discovered by rival cryptographers during the first public meeting to discuss the AES candidates.

Frequently, weaknesses may take years to discover. With DES, one strong form of attack, "differential cryptanalysis", had apparently been known to the algorithm's designers, but "linear cryptanalysis", discovered by Mitsuru Matsui [5] eighteen years after DES was proposed as a Federal Information Processing Standard, seems to be new. DES was indeed at least theoretically vulnerable to this type of attack. Designing secure cryptosystems is a mixture of a few well-known principles, some theorems, and, at least at present, some magic.

Block Cipher Designs

The simplest techniques for encrypting a block of symbols are substitution and permutation. *Substitution* replaces a symbol by another; *permutation* moves the symbols of a block around. Neither simple substitution nor simple permutation work very well by themselves. Frequency analysis, using the relative commonness of letters, pairs, triples, etc., is a strong tool against both.[3] Any message of reasonable length that is encrypted via a substitution or permutation function can be quickly deciphered using this technique; a trained

[2] *Details may be found at* `http://www.nsa.gov:8080/docs/venona/`.

[3] *In English, for example, the letter "e" appears 13% of the time in text, with "t,r,n,i,o,a,s" being the next most frequent letters. Similarly, there are data on the frequency of various letters appearing at the beginning and end of words, etc. Blanks (spaces) can be ignored.*

cryptanalyst can break a simple substitution cipher given only 25 characters of ciphertext.

Nonetheless, substitution and permutation form the backbone of modern cryptosystems. Fifty years ago Claude Shannon observed that the fundamental techniques for encryption are confusion—obscuring the relationship between the plaintext and the ciphertext—and diffusion—spreading the change throughout the ciphertext. Substitution is the simplest type of confusion, and permutation is the the simplest method of diffusion.

Cryptanalysis can be viewed as approximation theory; given ciphertext, determine the plaintext by an approximation process. Seen this way, linear functions of the input and key are poor design choices; such functions can be easily solved. Thus nonlinear functions form the basis of cryptographic design. But cryptographic functions must be invertible and fast-to-compute, and they should have small key size and memory requirements; consequently linear functions nonetheless play an essential role. A proper combination of simple operations such as $\oplus$, substitution, and permutation, produces a cryptosystem whose strength is greater than the sum of its parts.

The Data Encryption Standard (DES)

The three operations—XOR, substitution, and permutation—are all that is behind DES, which is an *iterated block cipher*, a cryptosystem on a block of symbols that sequentially repeats an internal function, called a *round*. It is customary at present to encrypt data using a primitive that operates on a block of symbols of moderate size. Although there are noniterative block ciphers (e.g., the public-key algorithm RSA), iteration is a natural way to proceed because that yields an algorithm with a small set of instructions, an important issue for hardware.

Some kind of self-invertibility is also valuable. This enables one object (a chip, a piece of software) to both encrypt and decrypt. *Feistel ciphers*, in which the $2t$-bit input is split into t-bit halves L_0, R_0 and mapped after r rounds to L_r, R_r, succinctly accomplish this. In the i^{th} round, the right half of the previous round becomes the new left half,

$$L_i := R_{i-1},$$

while the new right half, R_i, is the XOR of the previous left half and a function of a round subkey, K_i, and the previous right half:

$$R_i := L_{i-1} \oplus f(R_{i-1}, K_i).$$

An easy computation allows us to invert, obtaining L_{i-1} and R_{i-1} from L_i and R_i:

$$\begin{aligned} R_{i-1} &= L_i \\ L_{i-1} &= L_{i-1} \oplus f(R_{i-1}, K_i) \oplus f(R_{i-1}, K_i) \\ &= R_i \oplus f(R_{i-1}, K_i), \end{aligned}$$

regardless of the round function f used. Decryption is the algorithm run in reverse, with subkeys used in the opposite order. In order to make decryption a genuine inverse of encryption, the final round of a Feistel cipher switches the ciphertext to (R_r, L_r). Put another way, in decryption the swap is done at the beginning of each round. DES is a 16-round Feistel cipher.

In 1965, when computers were clunky mainframes and the networked world was more science fiction than scientific fact, Congress charged the National Bureau of Standards (NBS) with developing computer standards for civilian use. In the early 1970s, the National Security Agency (NSA) and NBS realized that civilians needed to be able to secure their "sensitive but unclassified" data. Though NSA would have been the usual agency to build such a cryptosystem, the agency was reluctant to create an algorithm for public use. There was concern that work in cracking an NSA-designed algorithm might in turn enable attacks on other NSA-designed systems.

So NBS issued a public solicitation for a cryptosystem. IBM responded. Originally IBM's proposal was to have been a 16-round Feistel cipher[4] with a 64-bit key, but the company modified the submission to work with a 56-bit key. There have been persistent rumors that NSA had pressed for the shorter key length. But IBM claimed the truth was more mundane: IBM engineers had insisted on parity bits for register-to-register transfer of key data, thus decreasing the key length from 64 to 56 bits, with 8 bits for parity. The new algorithm became the Data Encryption Standard (DES).

In encrypting ordinary text, DES begins by grouping the text into 64-bit blocks. DES then performs a number of operations on each block. Throughout the message, a single key of 56 bits determines how the transformation of the blocks is to be carried out. DES iterates sixteen identical rounds of mixing; each round of DES uses a 48-bit subkey.

DES begins with an intitial permutation $\mathbb{P}$ and ends with its inverse, $\mathbb{P}^{-1}$. These permutations are of minor crytographic significance but form part of the official algorithm. Without these permutations the algorithm is not DES.

The selection of subkeys, or "key schedule", begins by splitting the 56-bit key into two 28-bit halves and rotating each half one or two bits (one bit in rounds 1, 2, 9, and 16; two bits otherwise). The two halves are put back together, and then 48 particular bits are chosen and put in order as follows:

[4] *Horst Feistel, whose career had included building cryptographic "Identification-friend-or-foe" systems for the Air Force, was part of the IBM design team.*

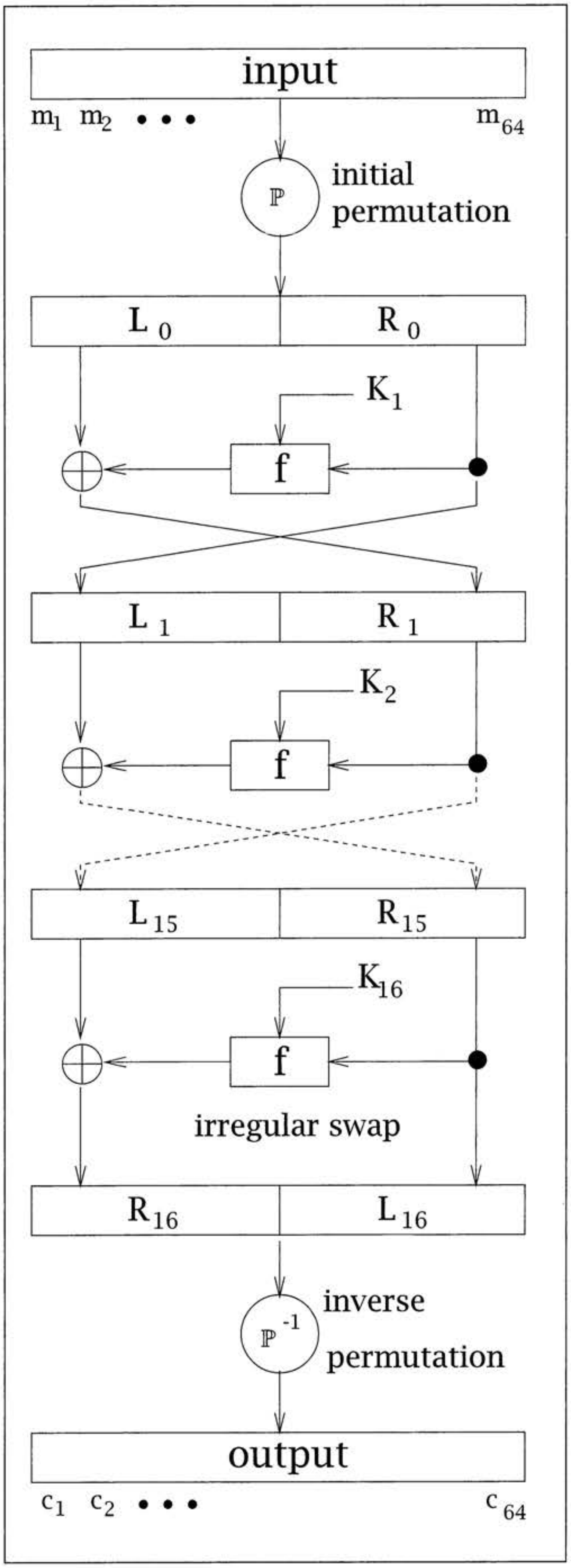

Figure 1. The Data Encryption Standard.

14,	17,	11,	24,	1,	5,
3,	28,	15,	6,	21,	10,
23,	19,	12,	4,	26,	8,
16,	7,	27,	20,	13,	2,
41,	52,	31,	37,	47,	55,
30,	40,	51,	45,	33,	48,
44,	49,	39,	56,	34,	53,
46,	42,	50,	36,	29,	32.

The rotation ensures that a different subset of key bits is used for each of the sixteen rounds of DES.

DES is a standard Feistel construction:

$$L_i := R_{i-1},$$
$$R_i := L_{i-1} \oplus f(R_{i-1}, K_i),$$

where

$$f(R_{i-1}, K_i) = P(S(E(R_{i-1}) \oplus K_i)),$$

with the operations E (expansion), S (S-box lookup), and P (permutation) defined below.

In order to have one bit of input affect more than one bit of output, the right half of the data is expanded to 48 bits. This is done by the expansion operation E, which, beginning with bit 32 and cycling back to the beginning, uses all the bits in order, repeating every fourth and fifth bits. Figure 2 shows this.

Because of E, every bit of the output of a DES encryption depends on every bit of the plaintext and every bit of the key. Indeed, this is true after five rounds of the 16-round cipher.

Now the round subkey and the expanded right "half" of the data are XORed together. The result is passed through the "S-boxes". Each S-box takes input of six bits and outputs four bits. The outputs are concatenated, and as there are eight S-boxes there are 32 bits of output.

The S-boxes are the source of DES's complexity. One way to view the S-boxes is as having "inputs" b_2, b_3, b_4, b_5 and "instructions" b_1 and b_6. There are two possible values for each of b_1, b_6, and two possible values for each of b_2, b_3, b_4, b_5. The S-box may be written as a 4×16 table; for example, here is the "table" for S_5:

$$\begin{pmatrix} 2 & 12 & 4 & 1 & 7 & 10 & 11 & 6 & 8 & 5 & 3 & 15 & 13 & 0 & 14 & 9 \\ 14 & 11 & 2 & 12 & 4 & 7 & 13 & 1 & 5 & 0 & 15 & 10 & 3 & 9 & 8 & 6 \\ 4 & 2 & 1 & 11 & 10 & 13 & 7 & 8 & 15 & 9 & 12 & 5 & 6 & 3 & 0 & 14 \\ 11 & 8 & 12 & 7 & 1 & 14 & 2 & 13 & 6 & 15 & 0 & 9 & 10 & 4 & 5 & 3 \end{pmatrix}$$

The bits b_1, b_6 determine the row, while the bits b_2, b_3, b_4, b_5 determine the column; the output is the entry in the intersection. Note that each possible four-bit entry $0, \ldots, 15$ appears in each row of the S-box output. This is true for all rows of DES S-boxes. I will return to the S-boxes later.

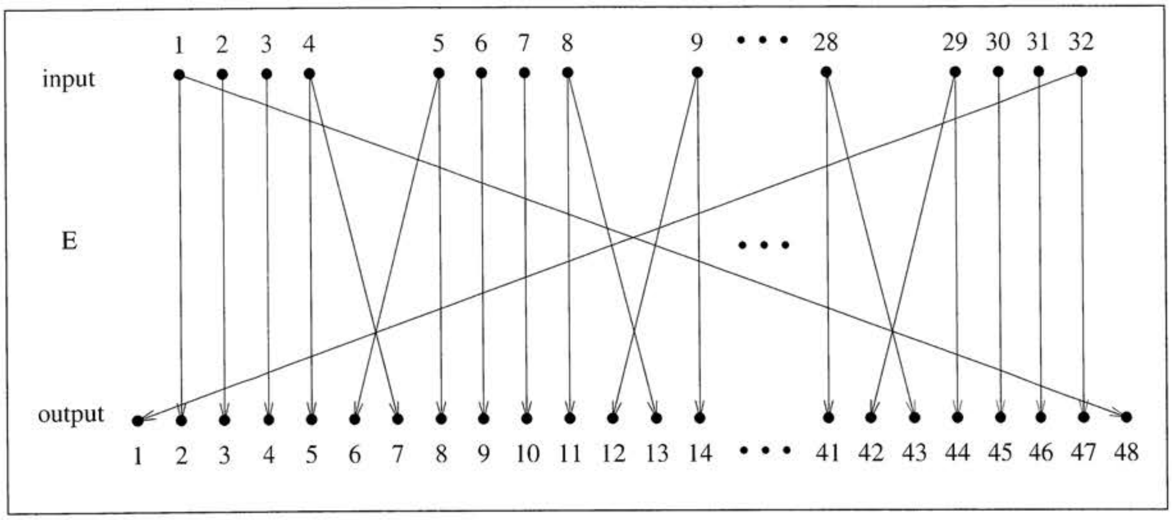

Figure 2. The Expansion Operation ($\mathcal{E}$).

P, a specific 32-bit permutation on the output of the S-boxes, completes the round function. It carries $1, \ldots, 32$ into the following list:

16,	7,	20,	21,	29,	12,	28,	17,
1,	15,	23,	26,	5,	18,	31,	10,
2,	8,	24,	14,	32,	27,	3,	9,
19,	13,	30,	6,	22,	11,	4,	25.

P ensures that the output from one round of DES affects the input to multiple S-boxes in the next round. After completing the 16 rounds, DES ends with a final exchange of the left and right halves (and then $\mathbb{P}^{-1}$).

Some Observations about DES

The simplicity of DES gives rise to some not entirely desirable properties. One is complementation. Let $\overline{X}$ denote the bitwise complement of X. If C is the DES encryption of plaintext $\mathcal{P}$ with key $\mathcal{K}$, then $\overline{\mathcal{P}}$ is the DES encryption of $\overline{\mathcal{P}}$ with key $\overline{\mathcal{K}}$. Although in some cases complementation can simplify DES cryptanalysis by essentially cutting the search space in half, in general this property does not cause a serious weakness in the algorithm.

DES permutations do not form a group. Of course, the set generated by DES permutations does form a group. This group has at least 10^{2499} elements [6]. Lack of group structure is a DES strength; if DES were a group, multiple encryption by different keys, $(\mathcal{E}_{\mathcal{K}_k} \ldots (\mathcal{E}_{\mathcal{K}_1}(\mathcal{P}) \ldots))$, would not be any stronger than single encryption. Instead, it appears to be.

Surprisingly, double encryption, encryption twice by two different keys, $\mathcal{E}_{\mathcal{K}_2}(\mathcal{E}_{\mathcal{K}_1}(\mathcal{P}))$, is actually no stronger than single encryption. This is because of "meet-in-the-middle" attacks. Given a plaintext-ciphertext pair, an adversary computes all 2^{56} possible encipherings of the plaintext, $\mathcal{E}_{\mathcal{K}_i}(\mathcal{P})$, and indexes these. The adversary then computes all possible decipherings of the ciphertext, $\mathcal{E}_{\mathcal{K}_j}^{-1}(C)$, and compares these against the list of encrypted plaintexts. If there is a match, $\mathcal{E}_{\mathcal{K}_i}(\mathcal{P}) = \mathcal{E}_{\mathcal{K}_j}(C)$, then $\mathcal{K}_i, \mathcal{K}_j$ is a possible encryption pair. This pair of keys is checked against another plaintext-ciphertext pair to see whether the key pair is correct. The process continues until the correct encryption pair is found. The time to perform this computation is not much more than the time to break a single DES encryption.

Triple-DES encryption does not fall to meet-in-the-middle attacks. Triple-DES can also be implemented using just two keys, but this DES variation has been shown to be about 2^{56}-bit secure, rather than the 2^{108}-bit security one might expect. In recent years, triple-DES has become popular.

If all DES subkeys are equal, then $\mathcal{E}_{\mathcal{K}} = \mathcal{E}_{\mathcal{K}}^{-1}$. Any key that satisfies this condition is a "weak key", and there are four of them. "Semi-weak keys" are those pairs of keys $\mathcal{K}_i, \mathcal{K}_j$ such that $\mathcal{E}_{\mathcal{K}_i}(\mathcal{E}_{\mathcal{K}_j}(\mathcal{P})) = \mathcal{P}$. DES has six pairs of semi-weak keys. Finally, there are the "possibly weak keys", which generate only four subkeys, used four times each in DES. There are 48 possibly weak keys. As all DES weak, semi-weak, and possibly weak keys are known, they can be avoided and so present no problem to the security of the algorithm.

Attacks on DES

DES's selection was quickly followed by protests. Some researchers objected to the algorithm's small key space. The inventors of public-key cryptography, Whitfield Diffie and Martin Hellman, claimed that a $20 million machine with a million specially designed VLSI chips, each capable of searching one key per microsecond and working in parallel, could break a DES-encoded message in about a day. David Chaum and Jan-Hendrik Evertse effectively used a meet-in-the-middle attack to break a four-round version of DES 2^{19} times faster than exhaustive search. Their technique did not extend past seven rounds.

None of these attacks posed serious threats to DES. Then two Israeli and one Japanese researcher poked harder into the innards of DES and discovered anomolies that led to the first attacks that were theoretically substantially better than exhaustive search. In 1990, looking at the XOR of plaintexts and ciphertexts, Eli Biham and Adi Shamir discovered a "differential-cryptanalysis" attack on DES that required examining only 2^{47} texts—a large number to be sure, but fewer than the 2^{56} that would be required by exhaustive search. Several years later Mitsuru Matsui, examining sums of plaintext and ciphertext bits, discovered relations that, in the aggregate, revealed information about sums of key bits. Matsui's "linear-cryptanalysis" attack on DES required studying 2^{43} encrypted texts—again, a large number, but again fewer than 2^{56}.

Ironically, neither differential nor linear cryptanalysis broke DES. Faster, cheaper chips did. Yet the Biham-Shamir and Matsui attacks are extremely important. These attacks work against *any* block-structured system, and so all block-structured cryptosystems must be designed to be secure against differential and linear cryptanalyis. Indeed, if one looks at the AES finalists, the choice of operations and the number of rounds of each of the candidates were frequently determined by differential and linear cryptanalysis. In what follows I describe the three serious attacks against DES—differential cryptanalysis, linear cryptanalysis, and the EFF DES Cracker—in the order in which they occurred.

Differential Cryptanalysis

As researchers studied DES, oddities about the S-boxes began to surface. Although the S-boxes have balanced output (each possible output appears four times, once in each "row"), there were subtle imbalances. In particular, the output of differences of inputs has an uneven distribution. Biham and Shamir exploited this in their 1990 differential cryptanalysis attack on DES.

Consider S-box 5 on page 344, and denote it by S_5. I use data from the difference distribution table in [1] for S_5; the interested reader can calculate this material directly from the description of S_5 given earlier. Notation is in hexadecimal, using characters $0_x, 1_x, \ldots, 9_x, A_x, B_x, \ldots, F_x$.

For each S-box I can create a "difference distribution table", a table of the distribution of all input XOR (there are 64 of these) and output XOR (there are 16 of these) pairs. The entries in the table are the number of possible pairs of a particular input difference and a particular output difference. Although the entries in the difference distribution table average 4, there is wide variance. This variance is exploited by differential cryptanalysis.

Suppose that the input difference of X and X^* to S_5, namely, $\Delta X = X \oplus X^*$, is 27_x. In this case the output difference has no chance of being $2_x, 4_x$, or F_x; $\frac{2}{64}$ chance of being $1_x, 3_x, 6_x, 7_x, D_x$, or E_x; $\frac{4}{64}$ chance for each of $8_x, 9_x$, or B_x; $\frac{8}{64}$ chance for each of A_x or C_x; and $\frac{12}{64}$ chance of being 0_x or 5_x. If I measure ΔX before and after processing by S_5, I would have a probability distribution on the key bits input to S_5 (or in other words, a probability distribution on the bits of K_5).

More formally, consider a pair of inputs to DES, X, X^*. The round function is the composition of E, $\oplus K$, the S-box transformation, and P. Observe that:

$$E(X) \oplus E(X^*) = E(X \oplus X^*)$$
$$(X \oplus K) \oplus (X^* \oplus K) = X \oplus X^*$$
$$P(X) \oplus P(X^*) = P(X \oplus X^*).$$

Furthermore, the output XOR of f is linear in the function that connects the rounds. If (Y, Y^*) is a pair of 32-bit strings, then

$$(X \oplus Y) \oplus (X^* \oplus Y^*) = (X \oplus X^*) \oplus (Y \oplus Y^*).$$

But the S-boxes are nonlinear, and $\mathrm{DES}(X \oplus X^*) \neq \mathrm{DES}(X) \oplus \mathrm{DES}(X^*)$.

Biham and Shamir discovered *characteristics* to help them push the knowledge gained from the XORs through the rounds. Informally, characteristics are differences in plaintext pairs that have a high probability of causing certain differences in ciphertext pairs. A trivial characteristic is input ΔX=0= output ΔX (that is, begin and end with the same string). This occurs, of course, with probability 1. A more interesting one-round characteristic has 0 as the input difference to seven S-boxes, while the input to the remaining S-box is nonzero and is chosen to maximize the probability the input ΔX may cause in the output. (Since several of the input bits to this remaining box also affect two neighboring S-boxes, these must be zero.) One high-probability way to do this is:

$$S_1 : C_x \to E_x \text{ with probability } \frac{14}{64}$$
$$S_2, \ldots, S_8 : 00_x \to 0_x \text{ with probability } 1.$$

One can now concatenate the above two one-round characteristics to get a two-round characteristic that has probability $\frac{14}{64}$. Indeed, one can put these together to have a 3-round characteristic with probability $(\frac{14}{64})^2 \simeq .05$ ([1], p. 26); see Figure 3.

An *iterative characteristic* is one that can be concatenated with itself. Biham and Shamir developed what they believe is an optimal set. Their differential cryptanalysis attack on DES is:

1. Pick an appropriate difference ΔX.
2. Create an appropriate number of plaintext pairs with this ΔX, encrypt with DES, and store the ciphertext pairs.

3. For each pair, from the plaintext ΔX and the ciphertext pair, determine the expected output difference of as many S-boxes in the last round as possible.
4. For each possible key value, count the number of pairs that result with the expected output change using the value in the last DES round.
5. The right key value is the one suggested by all the key pairs.

Biham and Shamir found a 13-round characteristic that requires encryption of *only* 2^{47} chosen plaintexts. This finds 48 bits of the key used in round 16 and then determines the 8 other bits by exhaustive search (a relatively fast process in this case). The total time is essentially bounded by the time taken to do the 2^{47} encryptions.

Biham and Shamir experimented with various modified versions of DES. Without the permutation P, the algorithm lacks sufficiently quick diffusion from the S-boxes. Reordering the S-boxes leads to higher iterative characteristics that could be exploited by a differential-cryptanalysis attack. Almost every variation on DES that Biham and Shamir tried resulted in a weaker algorithm.

IBM said this was no accident. After the Biham-Shamir attack, Don Coppersmith, an IBM researcher who worked on the DES design, revealed the criteria used in the S-box design two decades earlier.

1. Each S-box should have six bits of input and four bits of output. (In 1974 this was the largest size S-box that could be accommodated if DES were to fit on a single chip.)
2. No output bit of an S-box should be too close to a linear function of the input bits. (The S-boxes are the only nonlinear part of DES. Their nonlinearity is the algorithm's strength.)
3. Each "row" of an S-box should contain all possible outputs. (This randomizes the output.)
4. If two inputs to an S-box differ in exactly one bit, their outputs should differ in at least two bits.
5. If two inputs to an S-box differ exactly in the middle two bits, their outputs must differ by at least two bits. (Criteria (4) and (5) provide some diffusion.)
6. If two inputs to an S-box differ in their first two bits and agree on their last two, the two outputs must differ.
7. For any nonzero 6-bit difference between inputs, no more than 8 of the 32 pairs of inputs exhibiting that difference may result in the same output difference.

Call an S-box "active" if not all input differences to the box are zero. The S-boxes were designed to increase the number of active boxes. This maxim, along with a simplifying assumption that S-box events are statistically independent, ensures that with n active S-boxes, the probability of a particular pattern holding through n boxes is $1/4^n$.

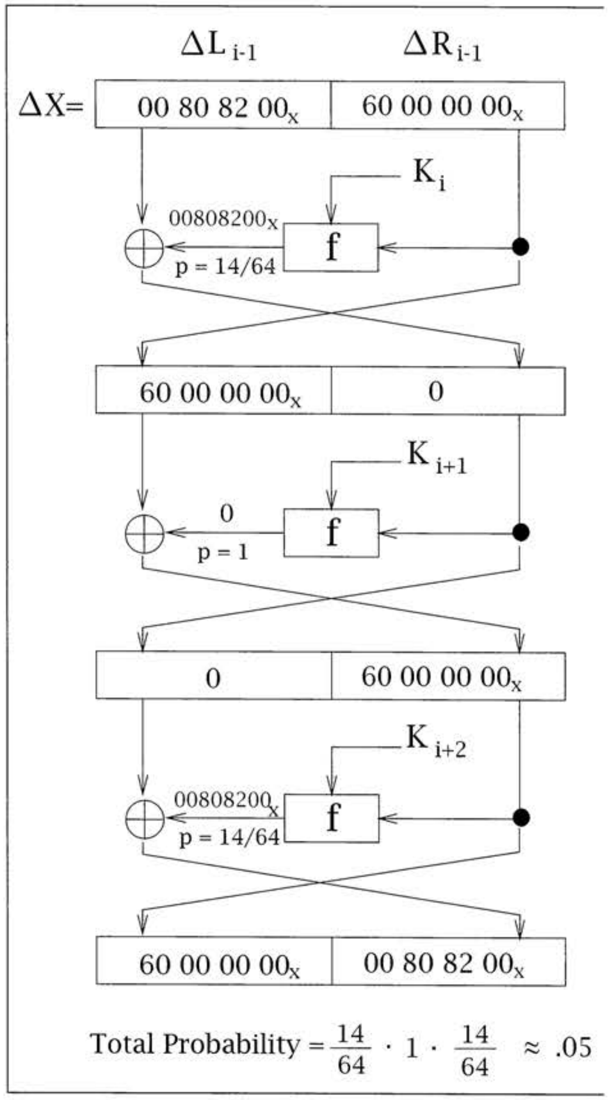

Figure 3. A Three-Round Characteristic.

Coppersmith commented that a better criterion than (2) would have been:

2′. No linear combination of output bits of an S-box should be too close to a linear function of the input bits.

While neither (2) nor (2′) could be perfectly achieved, (2′) would have increased DES's ability to resist differential cryptanalysis. So would larger S-boxes, but these were not possible in the technology of the time. There were also criteria to promote further randomization by the permutation P.

Linear Cryptanalysis

If DES was designed with differential cryptanalysis in mind, it seems clear the algorithm's developers had not anticipated Mitsuru Matsui's linear-cryptanalysis attack. Like many good insights, Matsui's idea was startingly simple. Cryptanalysts cannot expect that the ciphertext will be a linear function of plaintext and key bits (or equivalently, that some key bits are a linear function of the plaintext and ciphertext bits), but some of the bits were not too far off from some linear function.

Assume that input to DES will be random; then the mixing effect of $\mathbb{P}$ and $\mathbb{P}^{-1}$ can be ignored in performing this analysis. Let $B[i]$ denote the i^{th} bit of an array B of any length, and define

$$B[i_1, i_2, \ldots, i_k] = B[i_1] \oplus B[i_2] \oplus \cdots \oplus B[i_k].$$

Let $\mathcal{P}$ be the 64-bit DES data after the initial permutation, C be the 64-bit DES data just before the final permutation, and $\mathcal{K}$ the key. Then find a set of bit positions $i_1, i_2 \ldots, i_a, j_1, j_2, \ldots, j_b, k_1, k_2, k_c$ such that for random plaintext and ciphertext the equation

$$\mathcal{P}[i_1, i_2, \ldots, i_a] \oplus C[j_1, j_2, \ldots, j_b] = \mathcal{K}[k_1, k_2, \ldots, k_c]$$

holds with probability $p \neq 1/2$. (The farther p is from $1/2$, the better.)

One computes the left-hand side for many plaintext-ciphertext pairs, then guesses the value for the right-hand side that occurs most often. This gives one bit of information about the key. If one does this computation for more than $|p - 1/2|^{-2}$ pairs, the chance of a wrong guess is small. Chaining single-round expressions together, one obtains an effective linear expression for DES.

The issue is to determine on which set of key bits to work. One looks at the correlations between the input and output bits in the various S-boxes, and different S-boxes work better than others. Matsui observed, for example, that the parity of the first, second, third, fourth, and sixth input bits of the third S-box agrees with the parity of all of the output bits 38 of 64 times. The second input bit of S_5 agrees with the XOR of all four output bits of S_5 with probability $\frac{12}{64} = 0.19$ (an observation first made by Shamir). Specifically, from the E expansion and the P permutation of the round function, one finds for a fixed key and random input X that the following holds with probability .19:

$$X[17] \oplus f[X, K][3, 8, 16, 25] = K[26],$$

and the following holds with probability $1 - 0.19 = .81$:

$$X[15] \oplus f[X, K][3, 8, 16, 25] = K[26] \oplus 1.$$

Here $f(X, K)$ is the DES round function.

For three rounds of DES, Matsui discovered that:

$$L_0[3, 8, 14, 25] \oplus R_0[17] \oplus R_3[3, 8, 14, 25] \oplus L_3[17] = K_1[26] \oplus K_3[26]$$

with probability .695. Linear cryptanalysis works by chaining together such relations. How many plaintexts must be examined? Matsui showed:

Theorem. *Let N be the number of given random plaintexts, and let p be, as above, the probability that*

$$\mathcal{P}[i_1, i_2, \ldots, i_a] \oplus C[j_1, j_2, \ldots, j_b] = \mathcal{K}[k_1, k_2, \ldots, k_c].$$

If $|p - \frac{1}{2}|$ is sufficiently small, the success rate for the algorithm above is

$$\int_{-2\sqrt{N}|\frac{p-1}{2}|}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx.$$

Corollary. *With the same assumptions as above, the success rate of the algorithm is dependent only on $\sqrt{N}\,|\frac{p-1}{2}|$.*

Matsui calculated the following table:

N	$\frac{1}{4}\lvert\frac{p-1}{2}\rvert^{-2}$	$\frac{1}{2}\lvert\frac{p-1}{2}\rvert^{-2}$	$\lvert\frac{p-1}{2}\rvert^{-2}$
Success Rate	84.1%	92.1%	97.7%

As with differential cryptanalysis, the issue is how to join one-round characteristics into a longer chain.

Theorem. *Let $X_i, 1 \le i \le n$, be independent random variables whose values are 0 with probability p_i or 1 with probability $1 - p_i$. Then the probability that $X_1 \oplus X_2 \oplus \ldots \oplus X_n$ equals 0 is*

$$\frac{1}{2} + 2^{n-1} \prod_{i=1}^{n} \left(p_i - \frac{1}{2}\right).$$

Matsui determined the best linear approximate expressions for DES going up to 20 rounds (recall that DES is 16 rounds). Then he used a combination of reduced-rounds linear approximation and exhaustive search to find the key. His plaintext-ciphertext attack requires, on average, 2^{43} known plaintexts. With a network of twelve workstations in 1994, linear cryptanalysis broke a DES-encoded message in fifty days.

Breaking DES

Despite these successes, differential and linear cryptanalysis attacks are largely theoretical. The attack model is a problem. Consider Table 1, taken from [6], p. 259. In practice it is considerably easier to do an exhaustive search with one known plaintext-ciphertext pair and 2^{55} DES operations than it is to perform linear cryptanalysis requiring 2^{43} known plaintext-ciphertext pairs. Similarly, exhaustive search beats differential cryptanalysis on 16-round DES. Differential and linear cryptanalysis do pose threats to block-structured algorithms, and an attack that is successful even .01% of the time is potentially devastating.

In 1993 Michael Wiener updated the exhaustive-search machine to then-current technology. The

Attack Method	Data Complexity		Storage Complexity	Processing Complexity
	Known	Chosen		
Exhaustive Precomputation	—	1	2^{56}	1 (table lookup)
Exhaustive Search	1	—	negligible	2^{55}
Linear Cryptanalysis	2^{43}	—	for texts	2^{43}
Differential Cryptanalysis	—	2^{47}	for texts	2^{47}
Differential Cryptanalysis	2^{55}	—	for texts	2^{55}

Table 1.

result was a one-million-dollar machine using 57,000 DES chips and a "pipelined" architecture—one that admits sufficient parallelism so as to constantly feed data and perform computations through all components simultaneously. Wiener estimated that the machine could break a DES-encrypted message in three and a half hours [9]. Wiener's proposal raised interest in many circles.

In July 1998, using custom-designed chips and a personal computer, the Electronic Frontier Foundation built "DES Cracker". Costing less than $250,000 and taking less than a year to build, DES Cracker broke a DES-encoded message in fifty-six hours. There was some luck in the process; the key was found after only a quarter of the key space was searched rather than the expected half. DES Cracker was built using 1,536 chips; these searched 88 billion keys per second. There was nothing terribly novel about the decryption machine except that it was built. The machine scales: if EFF spends another $250,000 and links the resulting machines together, it would have a DES "Double-Cracker" that could decode DES-encrypted messages in half the time.

An Advanced Encryption Standard

Now even the U.S. Government had to agree that DES was insecure.[5] The National Institute for Standards and Technology had opened a competition for a DES replacement, the Advanced Encryption Standard (AES), a block-structured algorithm with variable-length keys of 128-, 192-, and 256-bits. Candidates were submitted in June 1998. Though the winner would be a U.S. Federal Information Processing Standard, the AES competition was open to foreign submissions. There were public evaluation meetings. (It was assumed that NSA was also doing some private vetting.) Fifteen candidates passed the initial criteria; after one year of evaluation, there are now five finalists. I will discuss these in a subsequent article.

[5] *Though a 1996 National Research Council report had urged that export controls on cryptography be immediately lifted to include the export of 56-bit DES, this change occurred only after DES Cracker had definitively established that DES was easily breakable. Cryptography controls are controversial (see [3] for a discussion).*

Acknowledgments

I am greatly indebted to Whitfield Diffie, Eli Biham, Germano Caronni, Neil Immerman, and Andrew Odlyzko, whose comments on an earlier version of this paper substantially improved it.

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Jean Leray (1906–1998)

Armand Borel, Gennadi M. Henkin, and Peter D. Lax

***Editor's Note:** Jean Leray, called the "first modern analyst" in an article in* Nature, *died November 10, 1998, in La Baule, France. He is best known for his stunning work in partial differential equations, including the first mathematical description of turbulence in fluid flow and an early application of the idea of a function space to solving differential equations. But the work that he did in algebraic topology and several complex variables has also had a huge impact. He was the one who introduced sheaves and spectral sequences into topology, and he was a pioneer in establishing a general theory of residues in several complex variables.*

Leray was born November 7, 1906, in Nantes, France; went to the École Normale Supérieure; and became a professor first in Nancy, then in Paris, and ultimately, starting in 1947, at the Collège de France. He was a member of the Académie des Sciences de Paris, the National Academy of Sciences of the USA, the Royal Society of London, and at least half a dozen other national academies. He received the Malaxa Prize (Romania, 1938) with J. Schauder, the Grand Prix in mathematical sciences (Académie des Sciences de Paris, 1940), the Feltrinelli Prize (Lincei, 1971), and the Wolf Prize (Israel, 1979) with A. Weil.

His Selected Papers *[L97], edited by Paul Malliavin, are in three volumes: on algebraic topology, partial differential equations, and several complex variables respectively. Each has a detailed introduction that includes thorough references; the respective introductions are by Armand Borel, Peter Lax, and Gennadi Henkin. These authors have kindly prepared from their introductions the abridged versions below, which are intended for a broad* Notices *audience.*

Armand Borel

Jean Leray was first and foremost an analyst. His involvement with algebraic topology was initially incidental and became later, at first for nonmathematical reasons, a topic of major interest for him for about ten years. His papers are written with his own notation and conventions, which, for the most part, have not been adopted and are consequently little read today. But it is there that sheaves and spectral sequences originated and were first used, so this work has exerted an immense influence on the further course of algebraic topology and of the emerging homological algebra. It divides naturally into three parts.

Leray-Schauder

Leray's first contacts with topology came through his collaboration with Juliusz Schauder [LS34], following the pattern of earlier work by Schauder: he had considered continuous transformations of a Banach space B of the form

$$\Phi(x) = x - F(x) \tag{1}$$

where F is a completely continuous operator, defined on B or only on some bounded set, and he had deduced results on elliptic or hyperbolic equations from an extension to that situation of the invariance of domain and of Brouwer's fixed point theorem.

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The paper [LS34] introduces, again in analogy with Brouwer, a *topological degree* $d(\Phi, \omega, b)$, where ω is a bounded open set in B and $b \in B$ does not belong to the boundary of ω. The degree has in particular the property that it can be $\neq 0$ only if $b \in \Phi(\omega)$. The paper also defines an *index* $i(\Phi, a)$ at a point a that is isolated in its fiber $\Phi^{-1}\left(\Phi(a)\right)$. It is an integer, which under some technical assumptions is equal to ± 1. If $\Phi^{-1}(b)$ consists of finitely many points, then $d(\Phi, \omega, b)$ is the sum of the $i(\Phi, a)$ for $a \in \Phi^{-1}(b)$. These notions are applied to a family of transformations

$$\Phi(x, k) = x - F(x, k) \tag{2}$$

depending on a parameter k varying on a closed interval K of the real line. For each $k \in K$, the transformation $F(x, k)$ is as above, defined on $\omega(k)$, and the union of the $\omega(k) \times k$ is bounded in $B \times K$. The goal is to investigate the fixed points of $F(x, k)$, i.e., the zeroes of $\Phi(x, k)$. This is done via a study of $d\left(\Phi(x, k), \omega(k), 0\right)$. It is assumed that for some fixed value k_0 of k the transformation $F(x, k_0)$ has finitely many zeroes. The goal is then to prove, under some conditions, the existence of fixed points for other values of k, some of which depend continuously on k. The results are applied to a variety of functional or differential equations.

Leray's first paper in algebraic topology [L35] is a sequel to [LS34]. Leray gives a formula for the degree of the composition of two transformations of the type (1) and deduces from it:

a. invariance of the domain under assumptions somewhat more general than those of Schauder,
b. a theorem about the invariance of the number of bounded components in the complement of a closed bounded set C: it is the same for C and C' if there exists a homeomorphism f of C onto C' such that the differences $f(x) - x$ for $x \in C$ form a bounded set.

Alexandroff and Hopf would have liked to include (b) for finite-dimensional spaces in their book (*Topologie*, Springer, 1935), but it was too late when they heard about it. They did acknowledge it in a footnote on page 312, though.

The War Years

Apart from [L35], topology in the work of Schauder and Leray-Schauder is definitely a servant, a tool to prove theorems in analysis. It might well have remained so for Leray had it not been for the Second World War. Leray campaigned as an officer, was captured in 1940, and sent to an officers' camp in Austria, where he stayed until the end of the war. There he and some colleagues created a university, of which he became the director ("recteur"). He feared that if his competence in fluid dynamics and mechanics were known to the Germans, he might be required to work for them, so he turned his minor interest in topology into the major one and presented himself as a topologist. Indeed, during those five years he carried out research only in topology.

His first goal was to set up a theory of transformations and equations which would include Leray-Schauder and would be directly applicable to more general spaces without a reduction to finite-dimensional spaces, in contrast with Schauder's work and [LS34]. He also wanted to avoid simplicial approximations, triangulations, subdivisions of complexes, and quasilinearity of the ambient space. So he had to create a new homology theory.

Until about 1935 the main tools in algebraic topology were the simplicial homology groups, with some new ideas of Čech and Vietoris to define homology for more general spaces. Around 1935 a new type of homology group was introduced independently by several people and soon christened cohomology groups by H. Whitney. They were dual to homology groups but had the great advantage of having a product, adding the degrees, soon called the cup-product. For differentiable manifolds an example was well known: the complex of exterior differential forms with its product and exterior differential, the cohomology of which was related to homology by the de Rham theorems. In fact, one of the proponents of a cohomology theory, J. W. Alexander, had indeed been inspired by the exterior differential calculus (*Annals of Math.* (2) **37** (1936), 698–708). Leray viewed it in this way and remarked in [L50a], section 5, that Alexander was the "first to apply this formalism to the topology of abstract spaces." Following Alexander, he wanted to develop directly a theory akin to cohomology and warned the reader later in many papers that he would call his groups homology groups, since he had little use for the traditional homology groups (I shall use cohomology). While developing his ideas, he was pretty much cut off from current research,[1] so that he started essentially from scratch in his own framework.

The outcome was a three-part "Course in algebraic topology taught in captivity", published in 1945 [L45], previously announced in part in some *Comptes Rendus* notes. In the context of Leray's oeuvre, it has to be viewed as a first step. The concepts appearing there for the first time have either been strongly modified or not survived, so that there is little point in supplying many details here. I shall mainly try to give some basic definitions, in particular that of "form on a space", which Leray viewed as the analogue of differential forms in his theory. In the introduction to the second part of [L45], he states that his forms on the space obey most of the rules of the calculus of Pfaffian forms and that the main interest of the paper seems to him to be its treatment of a problem in topology, alien to any assumptions of differentiability, by computations of that nature.

The starting points are the notions of a complex on a space and of a "couverture". Fix a ground ring L (usually $\mathbb{Z}, \mathbb{Z}/m\mathbb{Z}$, or $\mathbb{Q}$). An *abstract complex* is a free finitely generated graded (by degrees in $\mathbb{N}$) L-module, endowed with a differential d, of square zero, which increases the degree by one. A *complex* K on a space E is an abstract complex, to each element C of which is assigned a subset of E, its *support* $|C|$, with some natural properties. An important example is the cochain complex of the nerve of a finite cover of E, the support of a simplex being the intersection of the subsets labeled by the vertices. Given a closed subspace F of E, let $F.K$ be the quotient of K by the submodule of elements not meeting F, the support of $F.C$ being $F \cap |C|$. The complex K is a *couverture* if xK is acyclic for all x, plus a coherent condition for the generator of $H^0(xK; L)$. This is a condition similar to the validity of the Poincaré lemma. The *forms on* E are the elements of a couverture. Since those are finitely generated by definition, infinitely many

[1] *His main information came from some reprints Heinz Hopf had managed to procure for him, mainly by him and some people around him, in particular his paper on the homology of grouplike manifolds* (Annals of Math. ***42*** *(1941), 28–52) and Gysin's Thesis* (Comm. Math. Helv. ***14*** *(1942), 61–122).*

will be needed to define the cohomology of a space in general. Besides, there is not yet a product. Leray defines the latter geometrically, via the notion of intersection $K \circ K'$ of two complexes K and K'. It is the quotient of $K \otimes K'$ by the intersection of the kernels of the natural maps $K \otimes K' \to xK \otimes xK'$ $(x \in E)$. The union of the couvertures (over L) is a differential graded algebra. By definition, its cohomology is the cohomology ring $H^*(E;L)$ of E with coefficients in L.

To compute it, it is not necessary to use all couvertures: a family stable under intersection, which contains couvertures with arbitrary small supports, suffices. This allows one to see that for a finite polyhedron we get back the usual cohomology or that for a compact space it is equivalent to Čech cohomology. The cohomology is mostly used for locally compact spaces. As was noticed later, it is in that case equivalent to the Alexander-Spanier cohomology with compact supports. The paper gives many properties of these cohomology groups, for which I refer to §§6 to 9 in [B]. Let me just mention: a long exact sequence in cohomology (with compact supports) for a space and a closed subspace; generalizations of Hopf's theorem to compact groups, of the Lefschetz fixed point theorem, and of the Leray-Schauder theory; for manifolds: Poincaré and Alexander duality, the Jordan-Brouwer theorem. Among the new results: on a compact space a cohomology class of strictly positive degree is nilpotent. If E has a closed finite cover such that all nonempty intersections are acyclic, the cohomology of E is isomorphic to that of the nerve of the cover.

The Topology of a Continuous Map

Leray had developed a very special theory, but—granted that his cohomology was essentially Čech cohomology, say for compact spaces—the concrete results did not seem to go drastically beyond those of mainstream algebraic topology (even though a closer examination would have revealed a novel approach and more general assumptions for a number of familiar results), so [L45] did not create such a big impression. However, Leray had other goals. For him, algebraic topology should not only study the *topology of a space*, i.e., algebraic objects attached to a space, invariant under homeomorphisms, but also the *topology of a representation* (continuous map), i.e., topological invariants of a similar nature for continuous maps.

Of course, if one is given a continuous map $f: E \to E^*$, there is always an induced homomorphism in homology or cohomology, but Leray had something much deeper in mind, and the implementation of that idea led him to break entirely new ground. That he had conceived of that development while still in captivity is clear from the footnote in the first page of the third part of [L45]. Also, in a conversation with A. Weil in summer 1945 (see A. Weil, *Collected Papers*, II, p. 526), he had spoken of a homology "with variable coefficients" and it is likely that, as an example, the cohomology groups of the fibers of a continuous map were very much on his mind.

The first publications by Leray in that new direction are [L46a] and [L46b], which introduce first versions of sheaves, cohomology with respect to sheaves, and the spectral sequence of a continuous map.

In [L46a] a sheaf $\mathcal{B}$ on the space E associates to each *closed* subset F of E a module (or algebra) over the given ground ring L and to each inclusion $F \supset F'$ a homomorphism $\mathcal{B}(F) \to \mathcal{B}(F')$ with a natural transitivity property. The sheaf $\mathcal{B}$ is *normal* if $\mathcal{B}(F)$ is the inductive limit of the $\mathcal{B}(F')$ for $F' \supset F$. A basic example is the q-th cohomology sheaf $\mathcal{B}^q_E$ of E, which assigns $H^q(F;L)$ to F. (It is normal, since the cohomology has compact supports.) Normality is always assumed. As a further example, the sheaf $\mathcal{B}$ of germs of continuous functions is obtained in this setup by letting $\mathcal{B}(F)$ be the set of equivalence classes of continuous functions defined in open neighborhoods of F, two such functions being equivalent if they coincide on some neighborhood of F.

A form on E with coefficients in the sheaf $\mathcal{B}$ is a finite linear combination $\sum b_i X_i$, where the X_i belong to the basis of a couverture and $b_i \in \mathcal{B}(|X_i|)$. It is asserted that the constructions and results of [L45] extend to that case, whence the definition of the cohomology group (or ring if $\mathcal{B}$ is a sheaf of rings) $H^*(E;\mathcal{B})$ of E with respect to $\mathcal{B}$. Now let $\pi: E \to E^*$ be a continuous map. By definition, the transform $\pi(\mathcal{B})$ of a sheaf on B by π is the sheaf $F^* \mapsto \mathcal{B}(\pi^{-1}(F^*))$, the direct image in this setup. The q-th cohomology sheaf of π is, by definition, $\pi(\mathcal{B}^q_E)$, which assigns $H^q\left(\pi^{-1}(F^*);L\right)$ to F^* (the q-th right-derived functor of the direct image functor, in today's parlance).

The (p,q)-cohomology group of π is $H^p\left(E;\pi(\mathcal{B}^q_E)\right)$. The cohomology ring of π, which I shall denote $H^*(\pi)$, is the direct sum of the (p,q)-cohomology groups, with the product inherited from those on the cohomology of E^* and of the closed subsets of E.

The next *Comptes Rendus* note [L46b] is devoted to the structure of $H^*(\pi)$. By this is meant a procedure allowing one to relate it to the cohomology of E. I shall not try to describe it (see §11 of [B] for some details). One recognizes in it a number of constructions soon to be codified in the notion of a spectral sequence. There is a filtration of $H^*(E;L)$, and the successive quotients are arrived at by a sequence of approximations, starting from subquotients of $H^*(\pi)$ and using the action of differentials on representative forms. Applications to fibre bundles are given in this and the two following *Comptes Rendus* notes (*C. R. Acad. Sci. Paris* **223** (1946), 395–397, 412–415). The last one in particular describes the real cohomology ring of the quotient G/T of a compact simple group G by a maximal torus T when G is *classical*.

In 1947 various improvements were contributed by H. Cartan, J.-L. Koszul, and Leray himself. The analysis of [L46b] led Koszul to what we now call the spectral sequence of a filtered differential graded ring, a notion soon adopted by Leray (under some more general assumptions) and called later by him "spectral ring". Cartan suggested allowing complexes to be differential graded *algebras*, not necessarily free, finitely generated. Cartan and Leray independently introduced the notion of a fine complex (Leray's terminology), i.e., stable under partitions of unity associated to finite covers. Then $H^*(E;L)$ could be defined as the cohomology of just one fine couverture, a considerable conceptual simplification. In [L50a] Leray gives his final exposition of the theory (always cohomology with compact supports of locally compact spaces). He also introduces the cohomology of E with respect to a differential graded sheaf (now called hypercohomology) and shows it to be the abutment of a spectral sequence in which an early term (E_2 nowadays) is the cohomology of E with respect to the derived sheaf $\mathcal{H}\mathcal{B}$ of $\mathcal{B}$, the "fundamental theorem of sheaf theory".

During that period Leray had pursued his work on fibre bundles, in particular, homogeneous spaces of compact connected Lie groups. In his last paper on this topic [L50c], among other results Leray determines the real cohomology ring of G/T, where G is now any compact semisimple group and T a maximal torus, and establishes the Hirsch formula giving the Poincaré polynomial of G/H when H has the same rank as G.

After 1950, as before the war, algebraic topology played only a subservient role in Leray's work and appeared mainly in his theory of residues and in one paper on fixed point theorems [L59c].

Leray's framework is sheaf cohomology and spectral sequences for cohomology with compact supports of locally compact spaces, and his theory has proved to be a very powerful instrument for those spaces. But Leray's ideas penetrated other parts of topology and of mathematics as well. For this, various generalizations of his theory were needed, and we list them briefly.

H. Cartan produced three versions of sheaf theory between 1947 and 1950, of increasing generality. The last one [C] is valid over any regular space. The definition of sheaf is modified in a point of capital importance: a sheaf on the space X now assigns to each *open* subset a module, or ring (in Leray, and in the first two versions of Cartan, closed subsets were used). Injective resolutions are introduced; the fundamental theorem of sheaf theory is proved in full generality (and became a fundamental tool in the construction of derived categories). In 1950 a spectral sequence in singular homology or cohomology, also for general spaces, was introduced by J.-P. Serre, was applied to a very broad (and new) type of fibration, and was used in particular to study homotopy groups of spheres.

The passage to open subsets in the definition of sheaves opened the way to the introduction of sheaves in several complex variables (Cartan, Serre), in algebraic geometry over $\mathbb{C}$ (Kodaira, Spencer, Serre), and over any algebraically closed groundfield (Serre). These generalizations and applications go far beyond Leray's own contributions. Still, the sources of those groundbreaking ideas are the notes [L46a], [L46b], and they are so original that no earlier work by someone else can be viewed as a precursor.

Peter D. Lax

Jean Leray was one of the leading mathematicians of the twentieth century. A large part of his interests center on partial differential equations, especially those arising in mathematical physics. His investigations, some of them going back more than sixty years, still set the agenda of research in the fields in which he worked. The methods he introduced have found their uses in far-flung areas of mathematics.

Leray's papers are well organized; each distinct result has a chapter of its own, and the chapters are divided into short sections devoted to particular technical aspects of the argument. Since a priori estimates lie at the heart of most of his arguments, many of Leray's papers contain symphonies of inequalities; sometimes the orchestration is heavy, but the melody is always clearly audible.

Within the subject of partial differential equations, Leray studied both stationary problems, mostly governed by elliptic equations, and time-dependent problems, governed by parabolic and hyperbolic equations. His 1933 dissertation [L33], in the *Journal de Mathématiques Pures et Appliqués*, deals with stationary problems, using an abstract and extended version of Erhardt Schmidt's method of deformation and bifurcation. A wealth of applications is presented, including the existence of

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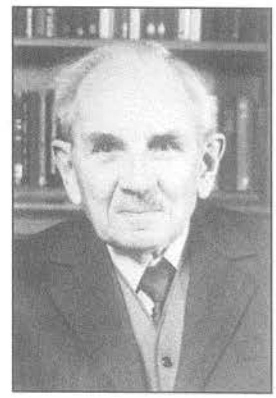

Photographs of Jean Leray courtesy of the Académie des Sciences Archives and Jean Leray relatives.

steady rotating fluids in three dimensions that satisfy the Navier-Stokes equation.

In 1934 Leray and Schauder [LS34] devised the epoch-making method bearing their name, using deformations to prove the existence of solutions for various classes of equations. This method extends Brouwer's notion of the degree to mappings of infinite-dimensional spaces of form **I** plus compact map. Like its finite-dimensional counterpart, the degree remains invariant under continuous deformations at every point that is not the image of a boundary point. To apply this principle in a concrete situation, two sets of a priori estimates have to be made: one showing the compactness of the one-parameter family of mappings employed, the other showing that all points on a sphere of radius R are mapped into points outside of a sphere. In addition, one has to verify for a particular value of the parameter that the degree of the mapping is nonzero. Leray and Schauder gave a number of applications of their method to solve the Dirichlet problem for various classes of quasilinear second-order elliptic equations; the norm they employ is the Hölder norm.

Ever since its appearance the Leray-Schauder degree has been one of the most powerful methods for dealing with nonlinear problems. A quick search of *Mathematical Reviews* disclosed 591 references to papers that make use of it.

Leray returned to elliptic problems again and again. In a 1935 paper in *Commentarii Matematici Helvetici* Leray used degree theory to construct steady ideal fluid flow in the plane around an obstacle and its wake. In a technically formidable paper in 1939 he showed how to use degree theory to construct solutions of boundary value problems for second-order fully nonlinear elliptic equations in two variables, including the Monge-Ampère equation. In the 1960s, in collaboration with J.-L. Lions, he examined results of Vishik and of Minty and Browder from the point of view of degree theory in finite-dimensional space. In the 1970s he and Y. Choquet-Bruhat used a fixed point theorem to solve the Dirichlet problem for second-order elliptic equations in divergence form.

We turn now to Leray's studies of time-dependent problems. In a paper [L34] that appeared in *Acta Mathematica* in 1934 Leray investigates the existence, uniqueness, and smoothness of solutions of the initial value problem for the Navier-Stokes equation in three-dimensional space. Physicists sometimes deride such existential pursuits by mathematicians, saying that they stop just when things are getting interesting, but what Leray found about existence, smoothness, and uniqueness of solutions was far more interesting for the physics of fluids than anything thought of before. He showed that in three space dimensions smooth initial data give rise to solutions that are smooth for a finite time; these solutions may be continued beyond this time only as generalized (weak) solutions of the Navier-Stokes equations. Leray calls these *turbulent* solutions. He shows that if two solutions, one regular and the other turbulent, have the same initial values, then they are equal; but it is still not known if turbulent solutions are uniquely determined by their initial data. Leray's results suggest a scenario for the occurrence of turbulence in fluid flow as the breakdown of smooth solutions as well as the possibility of the branching of weak solutions into different time histories.

Leray shows that in order for a solution to become turbulent at time T the maximum velocity $V(t)$ must blow up like $const/\sqrt{T-t}$ as t approaches T. No such solutions have been found so far. Leray suggested that there may be singular similarity solutions of the form

$$u_i(x,t) = (T-t)^{-1/2}U_i((T-t)^{-1/2}x),$$

u_i denoting the components of velocity. Clearly a solution of this form becomes singular as t approaches T. However, recently Necas, Ruzicka, and Sverak (1996) have shown that the equations that must be satisfied by the functions U_i have no solution of class L^3 in the whole three-dimensional space. Even more recently, Tai-peng Tsai [Tsa] has shown that no similarity solution, unless identically zero, has locally finite energy and locally finite rate of energy dissipation.

In the course of constructing his possibly turbulent solutions Leray used a host of concepts and methods of functional analysis that have since become an indispensable part of the arsenal of analysts: the weak compactness of bounded sequences in L^2, and a weakly convergent sequence is strongly convergent if and only if the limit of the norms is the norm of the limit. Leray defined the weak derivative of an L^2 function in the modern sense, as well as the concept of an L^2 vector field that is divergence free in the weak sense. He used mollifiers to show that a weak derivative is a strong derivative.

Despite much effort, remarkably little has been learned in the last sixty years about the smoothness of the weak solutions constructed by Leray. Scheffer (1976) was the first to study the size of the singular set in space-time; subsequently Caffarelli, Kohn, and Nirenberg (1982) have shown that the one-dimensional Hausdorff measure of the singular set is zero. In particular, the singularities cannot lie along a smooth curve. More recently, simplified derivations of the CKN result have been given by Fang-Hua Lin and Chun Liu (1996), as well as by Gang Tian and Zhouping Xin [TiX].

There has been some advance in existence theory. In 1951 Eberhardt Hopf showed that the Navier-Stokes equations have weak solutions with prescribed initial values in smoothly bounded domains in three-dimensional space, with zero velocity at the boundary. Hopf's proof makes use of the same functional analytic machinery as Leray's, but it is simpler in some details; in particular, instead of mollification he uses a Galerkin procedure to construct approximate solutions. A different approach to existence theory was taken by Fujita and Kato (1964); they used fractional powers of operators and the theory of semigroups.

Our knowledge of smooth solutions has advanced. Leray had shown that if the initial data are sufficiently smooth and tend to zero sufficiently fast near infinity, then a unique smooth solution exists in a time interval [0,T]; the size of this interval may depend on the viscosity γ. Ebin and Marsden (1970), Swann (1971), and Kato (1972) have shown that in domains without boundaries T may be chosen to be independent of the size of viscosity and that as γ tends to zero these solutions with fixed initial data tend to the solution of the inviscid incompressible Euler equations. No comparable result is known for flows in a domain with boundaries.

Leray showed that in the absence of a driving force in the interior or on the boundary, solutions of the Navier-Stokes equation tend to zero as t tends to ∞ and that they regain regularity after a finite time. Much work has been done since on the behavior of driven viscous flows as $t \to \infty$, such as the finiteness of the Hausdorff dimension of the so-called attractor set; see, e.g., Babin and Vishik [BV], Constantin and Foias [CF], Témam [Té], Ladyzhenskaya [Lad], and the literature quoted there.

Major effort has been devoted to devising and implementing effective computational schemes for calculating Navier-Stokes flows, steady and time dependent. Curiously, although for many classes of partial differential equations computations have, in von Neumann's prophetic words, "provided us with those heuristic hints which are needed in all parts of mathematics for genuine progress," computations have so far failed to shed much light on whether there are regular solutions that become turbulent.

After World War II Leray turned his attention to time-dependent hyperbolic partial differential equations. As pointed out long ago by Friedrichs and Lewy, the key to the initial value problem is furnished by energy inequalities. Leray [L53] derived these by multiplying the n^{th} order equation $a(x,D)u = 0$ by mu, where $m(x,D)$ is an $(n-1)^{st}$-order hyperbolic differential operator whose characteristics separate those of a; a natural choice is $m = a_\tau$, $\tau = \partial/\partial t$. The product $(mu)au$ is integrated over a domain in (x,t) space bounded by initial and final surfaces. Integration by parts produces integrals over the bounding spacelike surfaces whose integrands are quadratic forms in the $(n-1)^{st}$ derivatives of u. A criterion of Gårding shows that these energy integrals are positive definite.

In 1958 Calderon showed how energy estimates can be derived by employing singular integral (pseudodifferential) operators as symmetrizers of hyperbolic operators.

In the 1960s Leray became interested in hyperbolic equations with multiple characteristics. A typical example is

$$u_{tt} + u_x = 0\,;$$

this equation has solutions of the form $u = e^{-inx+\sqrt{in}t}$, which shows that solutions do not depend boundedly in the C^N norm on their initial data at $t = 0$, no matter how large N is. It follows that the initial value problem cannot be solved for all C^N initial data. The same conclusion holds for all hyperbolic operators $a(x,D)$ with multiple characteristics unless restrictions, called the Levi-Lax condition and given in [Lax], are placed on the allowable lower-order terms; see Mizohata [Mi]. In the 1960s Ohya had discovered that if the coefficients of $a(x,D)$ and the prescribed initial data are not only C^∞ but in an appropriate Gevrey class, then the initial value problem has a solution that belongs to a Gevrey class.

The importance of Gevrey classes in this context is that they are *not* quasianalytic, i.e., that they contain functions with arbitrarily prescribed compact support. Therefore it is possible to define domains of dependence and domains of influence for Gevrey class solutions. Leray [LO67], in collaboration with Ohya, generalized Ohya's result considerably, including even quasilinear equations and systems of n^{th}-order equations.

Leray's formulation of analytical problems in geometric terms is very much in the spirit of Poincaré, although for Poincaré function spaces were a promised land he saw but did not enter. Like Poincaré, Leray chose to work mostly on problems that came from physics. In marked contrast, the founding members of the Bourbaki movement, most of them Leray's contemporaries, sought inspiration not in nature but in mathematics itself. That Leray remained faithful to nature had a profound effect on postwar French mathematics. For it was his achievements, prestige, and influence that assured a rightful place for his outlook; he was the intellectual guide of the present distinguished French school of applied mathematics. More than that, he provided that balance between the concrete and the abstract that is so essential for the health of mathematics.

Gennadi M. Henkin

The works of Jean Leray in the 1950s and 1960s twice radically changed the direction of the development of contemporary complex analysis.

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Doctoral Students of Jean Leray

René Deheuvels (1953)
István Fáry (ca. 1953)
Philippe-A. Dionne (1962)
Jean Vaillant (1964)
Pham The Lai (1966)
Solange Delache (1968)
Claude Wagschal (1973)
Dominique Schiltz (1987)

The *Notices* is grateful to Claude Wagschal and Daniel Barsky for preparing this list.

Indeed, before the 1950s the theory of functions of several complex variables was based, in general, on traditional constructive methods.

One can mention here a series of works of K. Oka and H. Cartan, who in the period 1936–50, using the Cauchy-Weil formula (1935), solved the "fundamental problems" (the problems of P. Cousin, K. Weierstrass, H. Poincaré, E. Levi, and A. Weil). At the same time in the 1940s Leray, in connection with the study of the topology of continuous mappings and fiber spaces, developed so-called "sheaf theory" (1946, 1950).

Developing the ideas of Leray and Oka, H. Cartan (1950) introduced coherent analytic sheaves. After this it was found (by H. Cartan and J.-P. Serre and later H. Grauert and R. Remmert) that the methods of sheaf theory allowed one not only to reduce constructive methods (integral formulas of the Cauchy-Weil type) to a minimum in the Oka-Cartan theory but also to give a far-reaching generalization of this theory.

Thus, in the 1950s the constructive analytical methods of integral representations were practically driven out of multidimensional complex analysis and were replaced by algebraic methods of sheaf theory. The weakness of sheaf theory is that it does not provide quantitative estimates for solutions of the "fundamental problems".

At the same time in the 1950s Leray systematically brought to the Cauchy problem sharply advanced, necessary analytical methods, in particular residue theory on complex manifolds. In connection with this theory he introduced into consideration the highly general Cauchy-Leray integral formula. This formula led to progress not only for the Cauchy problem but also for a series of other important problems of complex analysis and differential equations that apparently could not be solved if one had to rely on the nonconstructive methods of sheaf theory.

Thus, thanks to Leray, the constructive methods of residue theory and of integral representations occupied again a first-rank place in complex analysis of several variables.

Holomorphic Cauchy Problem

The connection between multidimensional complex analysis and the Cauchy problem has been more apparent in formulas for elementary solutions of elliptic and hyperbolic equations with constant coefficients found in increasing generality in works of G. Herglotz (1926, 1928), L. Fantappie (1943), I. Petrowski (1945), and Leray (1953). Namely, these formulas express elementary solutions $u(x)$ of a homogeneous hyperbolic operator $P\left(-i\frac{d}{dx}\right)$ of an arbitrary order in terms of abelian integrals on the surface

$$\{\xi \in \mathbb{C}P^n: \ P(\xi) = 0, \ x \cdot \xi = 0\}.$$

Starting from the Herglotz-Petrowski-Leray formula (1953), Leray began in [L56] the study of the Cauchy problem for equations with variable coefficients. He stated his program of investigations in the following way in the introduction to [L57]:[2]

> Nous proposons d'étudier globalement le problème linéaire de Cauchy dans le cas complexe, puis dans le cas réel et hyperbolique, en supposant les données analytiques. Notre principal but est la proposition suivante: les singularities de la solution appartiennent aux caractéristiques issues des singularités des données ou tangentes à la variété qui porte les données de Cauchy. C'est l'extension aux équations aux dérivées partielles de la propriété fondamentale des solutions des équations différentielles ordinaires, linéaires et analytiques: leurs singularités sont des singularités des données.

However, the global Cauchy problem (both in the complex and the real domain) turned out to be a theme so large, difficult, and interesting that in spite of the efforts of Leray himself and his successors (Y. Hamada, C. Wagschal, J. Vaillant, D. Schiltz, D. Agnolo, P. Schapira, E. Leichtnam, B. Sternin, V. Shatalov, ...) the formulated problem is not yet solved completely. One of the most brilliant and unfinished ideas of Leray is contained in the work [L56].

Several deep steps in the realization of this program were done in the fundamental series of Leray's papers entitled "Problème de Cauchy I, II,

[2] *"We propose to study globally the linear Cauchy problem in the complex case, then in the real hyperbolic case, assuming the given data to be analytic. Our main goal is the following proposition: the singularities of the solution belong to the characteristics stemming from singularities of the data or tangents to the variety carrying the Cauchy data. This is the extension to partial differential equations of the fundamental property of solutions of ordinary differential equations that are linear and analytic: their singularities are singularities of the data."*

III, IV, VI". The Leray paper entitled "Problème de Cauchy V" has not been published, but in [L56] and in Leray papers in 1962, 1963, and 1964 there are some indications of the ideas of this work.

In the introduction to the article "Problème de Cauchy I" [L57] Leray describes his idea of uniformization of the solution of the Cauchy problem in the following brief and expressive way:[3]

> Ce premier article étudie la solution $u(x)$ du problème de Cauchy près de la variété S qui porte les données de Cauchy. Si S n'est caractéristique en aucun de ses points, alors, $u(x)$ est holomorphe près de S, vu le théorème de Cauchy-Kowalewski, et nos théorèmes n'énoncent rien de neuf. Mais nous admettons que S soit caractéristique en certains de ses points: il s'agit d'un cas sans analogue en théorie des équations différentielles ordinaires, en théorie des équations aux dérivées partielles ce cas joue un rôle fondamental, parce qu'il est celui où $u(x)$ présente les singularités les plus simples: $u(x)$ peut être uniformisé et, sauf des cas exceptionnels, est algébroide.

In the work of Gårding-Kotake-Leray (1964) developing [L57], the authors obtained an asymptotic expansion of the solution of the Cauchy problem in the neighborhood of characteristic points. The Leray uniformization method was applied with success to nonlinear systems in work of Y. Choquet-Bruhat (1966).

A fundamental concept in the Leray program (1957, 1963) is the so-called unitary solution of the Cauchy problem. Denote by ξ^* the hyperplane in $\mathbb{C}^n$ or the point in $(\mathbb{C}P^n)^*$ defined by the equation

$$\xi^* : \ \xi \cdot x = \xi_0 + \xi_1 \cdot x_1 + \cdots + \xi_n \cdot x_n = 0.$$

Let $a(x, \xi)$ be a polynomial of degree m with respect to ξ, independent of ξ_0, with coefficients that are holomorphic with respect to $x \in \Omega$. Let $g(x, \xi)$ be the *principal part* of $a(x, \xi)$, i.e., the term homogeneous in ξ of degree m such that $a(x, \xi) - g(x, \xi)$ is a polynomial in ξ of degree $< m$. A *unitary solution* for the operator $a\left(x, \frac{\partial}{\partial x}\right)$ is, by definition, a solution $U(\xi, y)$ of the Cauchy problem

$$a\left(y, \frac{\partial}{\partial y}\right) U(\xi, y) = 1,$$

where the function $U(\xi, y)$ has a zero of order m on the surface $\xi \cdot y = 0$. Due to zero homogeneity with respect to ξ, the function $U(\xi, y)$ is a function of $y \in \Omega$ and $\xi^* \in (\mathbb{C}P^n)^*$. Let $a^*\left(x, \frac{\partial}{\partial x}\right)$ be the adjoint operator for $a\left(x, \frac{\partial}{\partial x}\right)$, and let $U^*(\xi, y)$ be a unitary solution corresponding to $a^*\left(x, \frac{\partial}{\partial x}\right)$.

The Leray uniformization result [L57] can be applied to describing, in general, the singularities of the multivalued function $U(\xi, y)$ in the neighborhood of *characteristic points* (y, ξ), those with $\xi \cdot y = 0$ and $g(y, \xi) = 0$. The uniformization of unitary solutions of the Cauchy problem is used in an essential way in the fundamental work of Leray [L62] for defining the singular part of the "elementary solution" for a hyperbolic operator. For such an operator $a\left(x, \frac{\partial}{\partial x}\right)$, of degree m, a theorem of J. Hadamard (1923) and I. Petrowski (1937) states the global existence and uniqueness of the *elementary solution* $E(x, y)$ of the equation $a\left(x, \frac{\partial}{\partial x}\right) E(x, y) = \delta(x - y)$ with condition $\operatorname{supp} E \subset \mathcal{E}(y)$, where $\mathcal{E}(y)$ is the union of all timelike paths originating from y. The formulated existence and uniqueness result gives no precise information about the singularities of $E(x, y)$. Such information can be obtained from the following formula for $E(x, y)$ given in [L62]:

$$E(x, y) = \mathcal{L}(U^*(\xi, y)),$$

where $U^*(\xi, y)$ is a unitary solution of the operator a^* adjoint to a and $\mathcal{L}$ denotes a generalized Laplace transform defined in [L62]. The Leray formula, applied to a homogeneous operator with constant coefficients $a(\partial/\partial x)$, turns into the Herglotz-Petrowski-Leray formula. For this case,

$$U^*(\xi, y) = \frac{1}{m!}(\xi \cdot y)^m / a(\xi).$$

From the Leray formula it follows that $E(x, y)$ as a function of x is holomorphic outside of the characteristic conoid $K(y)$, the union of all bicharacteristics originating at y. In addition, the principal part of the singularity of $E(x, y)$ can be computed on the conoid $K(y)$.

The work of Leray [L62] was generalized for the case of nonstrictly hyperbolic equations in works of Atiyah-Bott-Gårding (1970, 1973) and was used by them for the development of the Petrowski lacunas theory (1945) for hyperbolic differential operators. For further results on the holomorphic Cauchy problem and applications,

[3] *"This first article studies the solution $u(x)$ of the Cauchy problem close to the variety S carrying the Cauchy data. If S is characteristic at none of its points, then $u(x)$ is holomorphic near S, by the Cauchy-Kovalevsky theorem, and our theorems say nothing new. But we allow that S is characteristic at certain of its points. This is a case without an analog in the theory of ordinary differential equations; in the theory of partial differential equations this case plays a fundamental role because this is the one for which $u(x)$ presents the simplest singularities: $u(x)$ can be uniformized and, save for some exceptional cases, is algebroidal." (An algebroidal function in a domain D is a function on D that satisfies a monic polynomial equation with coefficients that are holomorphic on D.)*

see [L97], [DS], [Lei], [Shap], [StSh], [V], and references therein.

Theory of Residues on Complex Manifolds

The results of Leray on the Cauchy problem turned out to be closely connected to multidimensional residue theory. Multidimensional residue theory started actually with H. Poincaré's (1887) work, in which Poincaré introduced the 1-form-residue of any rational 2-form in $\mathbb{C}^2$.

Leray [L59a] developed a general residue theory on complex manifolds and applied it to the investigation of concrete integrals depending on parameters arising from solving the Cauchy problem. F. Pham (1967) developed Leray's investigation in a more general context: namely, one can consider the integral $I(t) = \int_{x \in \gamma} \omega(x,t)$ of a rational (algebraic) differential p-form $\omega(x,t)$ depending algebraically on a parameter $t \in T$, with respect to a p-cycle γ on an algebraic manifold X, where γ does not intersect the singularity $S(t)$ of the p-form $\omega(x,t)$.

It was proved that the integral $I(t)$ is a (multivalued) analytic function of the parameter t outside of an analytic manifold $L \subset T$, called the "Landau manifold". For the case considered by Leray, the singularities of $\omega(z,t)$ have the form of poles on the hypersurface $S(t)$ depending linearly on t. To the Landau manifold corresponds a manifold L of such values t when $S(t)$ has a singular (double quadratic) point. For this case Leray (1959), applying the Picard-Lefchetz formula and a residue formula, proved the following:

Let $p = n = \dim_{\mathbb{C}} X$. Then going around the manifold L along a simple loop, beginning and ending in the point $t_0 \in T \setminus L$, the integral $I(t_0)$ turns into

$$I(t_0) + (-1)^{\frac{(n-1)(n-2)}{2}} (2\pi i) N \int_e \operatorname{Res} \omega(x,t),$$

where e is the so-called $(n-1)$-dimensional "vanishing cycle" on $S(t_0)$ and N is a linking index of e with γ. Hence, Leray [L59a] obtained explicit formulas for the singular part $I(t)$ in the neighborhood of L. The only singularities of this integral that can appear are poles, algebraic singularities of the second order, and logarithmic singularities.

Further, Leray (1967), generalizing the work of N. Nilsson (1964), applied the residue theory to the investigation of singularities of integrals of the large class of multivalued analytic forms whose singularities form algebraic submanifolds.

Leray (1956, 1959), developing on the one hand the Herglotz-Petrowski-Leray (1953) formula and on the other hand the theory of the analytic Fantappie functionals (1943), found a formula called by him the Cauchy-Fantappie formula, which led to fundamental progress in analysis.

We formulate here only two direct applications of the Leray formulas to the theory of analytic functionals.

Let D be a linearly concave domain in $\mathbb{C}P^n$ in the sense that for every $z \in D$ there exists a projective hyperplane $\mathbb{C}P^{n-1}_{\xi(z)} = \{w \in \mathbb{C}P^n : \xi(z) \cdot w = 0\}$ depending continuously on z, passing through the point z, and contained in D. Suppose $\{w_0 = 0\}$ is contained in D. The set of projective hyperplanes contained in D forms in the dual space $(\mathbb{C}P^n)^*$ the open set D^*. Let

$$M = \{z \in \mathbb{C}P^n : \widetilde{P}_1(z) = \cdots = \widetilde{P}_r(z) = 0\}$$

be an algebraic subset of $\mathbb{C}P^n$ of dimension k, where the homogeneous polynomials $\widetilde{P}_1, \widetilde{P}_2, \ldots, \widetilde{P}_r$ are such that rank$[\operatorname{grad} \widetilde{P}_1, \ldots, \operatorname{grad} \widetilde{P}_r] = n - k$ almost everywhere on M. Let $\mathcal{H}^*(K)$ denote the space of linear functionals on the space $\mathcal{H}(K)$ of holomorphic functions on $K = \mathbb{C}P^n \setminus D$. For the functional $\mu \in \mathcal{H}^*(K)$ we define the *Cauchy-Fantappie indicatrix* as the function

$$f(\xi) = \mathcal{F}\mu(\xi) = \left\langle \mu, \frac{z_0}{\xi \cdot z} \right\rangle, \quad \xi \in D^*.$$

We have $f \in \mathcal{H}(D^*, \mathcal{O}(-1))$, where $\mathcal{O}(l)$ denotes the line bundle over $(\mathbb{C}P^n)^*$ whose sections are homogeneous functions of $(\xi_0, \xi_1, \ldots, \xi_n)$ of degree l. The main result of the theory of analytic functionals of L. Fantappie (1943), A. Martineau (1962, 1967), and L. Aizenberg (1966) can be formulated as follows:

> The mapping $\mu \mapsto \mathcal{F}\mu$ realizes an isomorphism of the space $\mathcal{H}^*(K)$ and the space $\mathcal{H}(D^*, \mathcal{O}(-1))$.

The main application of analytic functionals according to L. Fantappie (1943, 1956) consists of different methods of integration of partial differential equations with constant coefficients, including an explicit solution of the Cauchy problem. This application can be deduced from the following:

> The functional $\mu \in H^*(K)$ has support on $K \cap M$ if and only if its Cauchy-Fantappie indicatrix $f = \mathcal{F}\mu$ satisfies the system of differential equations
>
> $$\widetilde{P}_j\left(\frac{d}{d\xi}\right) f(\xi) = 0, \quad j = 1, 2, \ldots, r.$$

This last statement (Henkin (1995)) can be interpreted as a variant of the Ehrenpreis (1960, 1970) and Palamodov (1961, 1967) "fundamental principle" for systems with constant coefficients. For further results on residue theory on complex manifolds and applications, see [L97], [A], [BGVY], [BP], [D], [H], [Tsi], and the references therein.

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Reifying the Research: Mathematics Education in Taiwan

Mark Saul

A bronze Confucius sits on a bronze pillow, surrounded by his bronze disciples. The bas-relief decorates the entrance to the Chien-Kuo Senior High School, a prestigious public high school for boys in Taipei.

I am visiting the school in Taipei as a guest of the Nine Nine Cultural and Educational Foundation, to help them set up a mathematics contest modeled after that of the American Regions Mathematics League, an American competition with which I am associated. As a high school classroom teacher, I have asked my hosts to show me what goes on in Taiwan's secondary schools.

We have heard much recently about how students in Pacific Rim countries excel in their study of mathematics. But what is actually happening in these places? How do the numbers translate into practices from which we can learn? My visits to Taiwan presented a picture that reifies and extends what we learn from the research.

The Chien-Kuo School has recently celebrated its one-hundredth birthday, having been founded shortly after the Japanese occupied the island in 1895. After the customary glass of tea with the school's principal, I am ushered into a large room where one hundred young men have gathered to hear my talk about the American Regions Mathematics League competition. One of the examples I give uses Ptolemy's theorem about the diagonals and sides of a cyclic quadrilateral. I speak through an interpreter. My host, Yang-Ming Ho, has worked extensively with these students but had not been told in advance about the content of my talk. When I mention Ptolemy's theorem at one point, he interrupts to give a quick explanation in Chinese. The students do not know this theorem but are intrigued. He challenges them: "Can you prove it?" Then he writes some preliminary formulas on the chalkboard. "Now can you prove it?" His eyes burn with intensity, and the students are mesmerized. I am confident that many will go home and investigate the theorem.

I also wonder how many American high school teachers would have a working knowledge of Ptolemy's theorem at the ready.

The girls in Taipei Wesley High School are dressed in identical uniforms, but each has altered small details of her dress in some personal way. The mathematics class I am visiting consists of forty-five girls, a bit smaller than most high school classes in Taiwan. They are reviewing word problems that lead to quadratic equations. About three-quarters of the way through the lesson, the teacher reminds them that they know three ways of solving quadratic equations: factoring, completing the square, and applying the formula. She presents a problem, then invites the students to use whatever method of solution they find convenient. One student, chosen at random, is called to the board. Here she completes the square, although this method had not been reviewed in the day's lesson.

These students are in the eighth grade. American students usually learn to solve quadratics by factoring in the ninth grade and learn the

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quadratic formula in grade ten or eleven. Many of them will not find the method of completing the square either in their courses or in their textbooks.

In Tung-Shan High School, a private school pressed against the mountains that surround Taipei, sixty students are crowded into a large classroom. These are high school juniors studying precalculus. The teacher stands in front of the room with a microphone, while the students work at tiny desks bolted to the floor in a rectangular array. Their workspace is about half what an American classroom would offer. When I squeeze into the back of the room with my retinue of translator, assistant principal, and four other dignitaries, some of the students offer us their desks and double up in the tiny spaces of others.

The room is quiet. Faint noises float in from outside. (Taiwan's subtropical climate allows for open doors and windows in January and requires them by April or May.) Virtually all the students are on task. I find this particularly remarkable, since their task is merely to listen to the teacher. Only twice does he stop, once for a question ("Shall I do the derivation over for the second case?"), which he answers himself ("Well, I like it, so I will do it again"). The second pause is for a student's question. The student rises to ask it, the teacher replies, and the lecture continues.

This lesson is standard fare in traditional texts worldwide, a derivation of the equation of a hyperbola from its definition as a locus. It is essentially the same as the derivation in the text, which is used by all students in Taiwan. While I cannot read Chinese, I can read mathematics and found an interesting point in the text that I had not known about before. If we choose any point on a hyperbola and drop perpendiculars to the two asymptotes, the product of these perpendiculars is constant (this is easy to prove algebraically). The text uses this fact to prove that the distance from a point on the hyperbola to the nearest asymptote vanishes as we go up the hyperbola. The hour-long lesson I observed did not get this far.

The students in the class do not fidget, do not pass notes, do not gossip. They attend to the teacher or look down at a notebook or text. But the real surprise comes at the end of the period, marked by a gentle chime. The teacher finishes his thought, while the students remain attentive. When the teacher has finished, one student stands and issues a command to the others. They all rise and bow to the teacher, chanting *hsieh-hsieh, lao-tse* (thank you, teacher). Then they file out.

Lunch is no different from recess in an American high school. Kids rush down the corridors (which are open to the subtropical weather) and up the stairs, weaving around the staid visitors, teasing and calling to each other in organized chaos.

So Taiwanese students are not all that different from American students. Outside the classroom they seem to have the same energy and intensity as my own. It is the nature of the social contract that differs. Taiwanese teachers can prepare lessons assuming that the students in front of them will be interested and work hard. American teachers face a greater responsibility: they must create in the classroom an atmosphere in which hard work and intellectual curiosity are standard. I think to myself that if I could master the language (no mean feat!), I could teach in Taiwan. But a Taiwanese colleague would have to learn much more about American life than the English language to succeed in my classroom.

The food in Taiwan is delicious but rarely familiar. It is only after tasting it that one recognizes the fish or beef that has been presented in an exotic way. Over an array of such dishes I speak with Chi-Lin Yen, a professor at Taiwan Normal University. He tells me about the education of teachers. The university entrance examination, taken by most students, is quite rigorous, and the Normal University gets some of the most successful candidates.

He outlines the encyclopedic course of study in mathematics for prospective high school teachers, amounting to some 80 credits in subject matter alone (no wonder Yang-Ming Ho knew so much about Ptolemy's theorem!). Since the Normal University was free for a long time, many people who did not intend to be teachers got their degrees there. Some ended up in teaching. Others ended up as research mathematicians and had the background to pursue this career.

The evening air in downtown Taiwan is full of the aroma of frying, baking, and broiling. People on their way home stop to enjoy a quick snack from the food vendors on every corner. Many of these people are wearing high school uniforms. High school students come downtown one or more nights a week to attend evening classes. These are enrichment classes that virtually all students enroll in. The prices are low, and the parents eager to pay. Yang-Ming Ho runs an after-school center and is its most popular teacher. I visit his class of four hundred (not a misprint!) students. They are assembled in a large room with microphone and TV monitors. It is not like a lecture at a large state university, but more like a television show with a live audience. Ho works the audience, telling jokes and directing remarks at those students he knows well.

Here is a problem he gave, one of a series on division of polynomials with remainder: What is the remainder when the polynomial

$$x^{33} + x^{22} + x^{11} + x + 2$$

is divided by the polynomial $x^2 + x + 1$? Ho explained that we can write

$$P(x) = Q(x)(x^2 + x + 1) + (ax + b)$$

and that we can find the constants a and b by plugging in the roots of the equation $x^2 + x + 1 = 0$. Luckily these roots are the complex cube roots of unity, and luckily most of the 33rd degree polynomial drops out after substitution. The students were all on task. They were all in the tenth grade. Could they all do the problem? We do not really know, since there are no examinations in the enrichment program: students come to get ahead in their regular studies. But I could see that at least 90 percent of the students were interested and were following the discourse closely.

On a return visit eight months later, I see the results of this work. The first competition of the Taiwan Regions Mathematics League attracts 1,200 students and their teachers to a two-day celebration of mathematics. It is held during the students' summer vacation. Its American prototype, now twenty-four years old, serves 1,800 students out of a much larger population and is held (for most of this population) during the school year. It has taken years for this event to become popular. In Taiwan its success was much quicker.

The National Palace Museum of Taipei holds the world's most comprehensive collection of Chinese art. From neolithic ceramics to contemporary scroll paintings, its exhibits display the bottomless cultural wealth of Chinese civilization. In one of the galleries I find a rendering of the god Kuei-Hsing, whose special charge was the success of candidates in the official examinations of the Mandarin system. The students I talk with all know about Kuei-Hsing, and incense burns before his image in local temples. While there are no more Mandarin examinations, Kuei-Hsing has now taken responsibility for success on various local tests. Success in school here seems to be more than a point of personal pride or of self-improvement.

In the entrance to another building there is another copy of the bas-relief of Confucius. This copy is larger and a bit finer, for the building is the National Ministry of Education. I am talking here with Chao-Hsien Lin, the deputy minister of education, trying to get insights into the remarkable classrooms I have visited. His story is interesting but does not always answer the questions I have. Taiwan was a relatively backward part of China when the Japanese annexed it in 1895. By and large, education under the Japanese involved leaving the island for Japan at a certain point in the student's life. When the Nationalist government came to Taiwan, this changed. Particularly in the last few years, an enlightened leadership has emphasized mathematics and the sciences as the keys to economic prosperity. And the key has turned. Taiwan is indeed prosperous.

What changes are coming to Taiwanese education? Lin mentions that the ministry is exploring a variety of alternatives for college admission. The current system relies on a single examination, and the Taiwanese are looking to the American system to develop some alternatives. What if, asks Lin, a good student is ill on the day of the examination?

Tuan Tuan Lee, my hostess as the vice president of the Nine Nine Foundation, nods. This was exactly what happened to her daughter, who had to go to America for her undergraduate work. This proved a silver lining, she explained, as her daughter likes America and has learned a lot, both about business administration and about other cultures.

Lin tells me that many Taiwanese leave the island for graduate education, and those that study mathematics go mostly to the United States. Here is an echo of one of the paradoxes of American education. While our graduate mathematics programs are the envy of the world, our precollege education suffers in comparison to that of other developed countries.

Do the graduate students return to Taiwan? Sometimes, says Lin, and they are returning more and more often. The economic conditions at home are attractive, and they would rather live and work in their own country. While I do not tell him this, Lin has touched on another problem of American education: our best minds have recently been immigrants. Currently, fewer than half our graduate students in mathematics are native-born Americans. From the point of view of other countries, this brain drain is destabilizing. From the American point of view, it ties the success of our educational efforts to the success of our economy. If we slip, we will have very far to fall. In the case of Taiwanese students, this has happened already. If the trend that Lin points out continues, we may get fewer Taiwanese immigrants, and, in particular, fewer mathematicians, in years to come.

One Taiwanese mathematician who has stayed in the U.S. is my guide and host, Peter Shiue of the University of Nevada at Las Vegas. Shiue grew up on a poor fishing island off the coast of Taiwan. His talent was spotted early by an attentive junior high school teacher, and Shiue was sent to school on Taiwan, then in the U.S. He still keeps in touch with his junior high school teacher, who has herself moved to America. What would have become of his talent if she had not been able to spot it? How many American teachers of middle school mathematics would be able to recognize, in a classroom context, a real mathematical talent and not just a diligent and obedient student?

On another visit I was privileged to give a talk at the high school of the Pescadore Islands, where

Peter Shiue had spent his childhood. Thirty students and teachers came to the school, which was open for the talk despite the summer vacation. The students wore their uniforms. All engaged readily in the problems I posed for them.

Later, K. C. Shiue, principal of the school, told me that 80 percent of his students go on to college in Taiwan. Most do not return. The local economy, built on fishing and tourism, provides only a limited number of jobs. But even those who stay work on significant mathematics in high school. And those who leave are well prepared, despite their having attended school in this remote part of the country. How would this compare with a rural American school?

Most of the schools I visited in Taiwan were among the most successful on the island. As private or church schools, they serve students who can afford to pay tuition or who earn scholarships. But this observation does not lessen the achievement. It would be difficult to find an American middle school where eighth-grade students can complete the square, and I know of no after-school enrichment program in mathematics that draws anywhere near the number of students who attend those in Taipei. And few American enrichment programs are as advanced. Even if we compare the best American students and institutions to those in Taiwan, there is still much to learn.

What indeed is to be learned from this extraordinary system? There are those who look longingly at the educational results of Asian countries and urge us to copy their methods. This argument underestimates the enormous influence of culture. It is difficult to envision American students sitting at tiny desks. They have no special god to watch over their success in examinations. They will not flock to after-school programs, nor sit in a room with four hundred others to learn how to do complicated mathematics problems.

Likewise, American teachers cannot be expected to use a microphone to address classes of sixty. Even in large universities, where this form of instruction has been institutionalized, there is a move away from it. And few American school districts will vote for budgets based on their teachers working with only two classes each day, as their Taiwanese colleagues do.

We cannot plug Taiwanese practices directly into American schools. Taiwanese teaching is fashioned for Taiwanese students. But neither can we ignore what their experiences tell us about variations in teaching and learning. Like the cooking, the teaching in Taiwan takes some effort to appreciate. But in both cases the effort is quickly repaid.

Declining Student Numbers Worry German Mathematics Departments

In the United States, shrinking student numbers in undergraduate mathematics programs have become commonplace. Figures from the AMS-IMS-MAA Annual Survey, showing that the number of juniors and seniors majoring in mathematics declined by about 20 percent between 1992 and 1998, will elicit little surprise. What is less well known in the U.S. is that similar declines are occurring elsewhere. One example is Germany, where the number of students choosing mathematics as a university subject has been dropping steadily for several years. Why these declines have occurred in Germany is not an easy question, but it is one that German mathematics departments have had to face as they try to come up with ways to reverse the trend.

The German *Diplom*

The organization of universities in Germany and the degree programs are quite different from what one finds at U.S. universities. Most universities in Germany are state universities which are open to all students and which charge no tuition. Students must choose a subject to study; the subject can be changed later on, but it is not possible, as it is in the U.S., to enroll as an "undeclared major". While some universities have recently established bachelor's and master's degrees, the typical first university degree in the sciences is the *Diplom*, which takes a minimum of four years and can take as many as six.

The first two years of study for *Diplom* students in mathematics consist of a fairly standard set of courses and end with an examination, called the *Vordiplom*, on the course material. After that, students have a great deal of freedom in what they study and how they arrange their coursework, though there are some requirements designed to ensure breadth in the mathematical topics they study. They also must choose a second subject of study; traditionally this was physics, but today it is often computer science or economics. In addition to taking lecture courses, mathematics students attend seminars, in which they lecture to each other about topics they are studying. To receive the *Diplom*, they must write a thesis, which is called the *Diplomarbeit* and is similar to a master's thesis in the U.S., and they must pass a set of oral exams called the *Diplomprüfung*.

In addition to the regular mathematics *Diplom*, many German mathematics departments offer a *Diplom* in such subjects as *Finanzmathematik* (financial mathematics), *Versicherungsmathematik* (actuarial mathematics), and *Wirtschaftsmathematik* (applications of mathematics to economics and/or managerial science), and various combinations of these topics; there are also programs in *Technomathematik* (engineering mathematics). Mathematics departments also educate *Lehramt* students, those intending to teach mathematics in secondary schools.

Data about the numbers of students in a particular subject in Germany usually focus on the numbers of beginning students choosing that subject. This method of counting can be imprecise, because many students later drop out or switch to another subject. Because there are no restrictions on the number of students enrolled to study mathematics, student turnover can be quite high. Nevertheless, trends in the numbers of beginners year to year do give an indication of student interest in a subject.

Figures from the Statistisches Bundesamt, the central clearing house in Germany for national statistics, show a drop of around 20 percent in the number of beginning students in mathematics between 1992 and 1999; for students choosing mathematics in their first semester of university studies, the drop is about 35 percent. Interviews with faculty in mathematics departments around Germany reveal that in many places the numbers have declined further. For example, at Universität Münster, one of Germany's largest universities, the number of beginning *Diplom* students in mathematics dropped about 35 percent, from nearly 300 in 1990 to around 185 in 1998. Smaller departments have not fared much better: Universität Konstanz had 35 beginning students ten years ago and now has just 10; Universität Regensburg saw its beginners decline by about three-quarters in the same period. At Göttingen, which has perhaps the most illustrious history of any mathematics department in Germany, student numbers have fallen from around 80 ten years ago to a little more than

40 today. And in the mathematics department at Bonn, generally considered to be the country's leader in terms of research, beginning student numbers dropped by more than half in just the past four years. For the first time the Bonn department is having trouble finding enough participants for seminars for students in the first year after the *Vordiplom*.

The declines in student numbers have created great pressures on mathematics departments. The state ministries overseeing the universities, needing to cut costs, have begun to look closely at departments where student numbers are declining and to ask hard questions about whether so many professors are needed if the students are not coming. As a result, many mathematics departments are finding it harder to get their administrations to agree to refill positions that become vacant through retirements or resignations. Sometimes the positions are transferred to another department where student numbers are higher, and sometimes the administration insists that the hiring be done in a certain area, often one with ties to an area of application such as computer science.

Mathematics departments in the U.S. also encounter these kinds of problems, but in Germany there is a special twist. Most German universities have a method of assessing the teaching capacity of the faculty; one such is the *Kurrikularnormwert*, which was developed after university enrollments swelled in the 1960s. The universities needed a way to calculate, given the number of faculty, how many students they could accept and when to impose admissions restrictions in certain subjects. Today the *Kurrikularnormwert* is being used in the opposite way, to estimate how many faculty positions are needed, given the number of students enrolled. Under this assessment method, many departments, including mathematics departments, are found to be overstaffed.

What is more, many in German mathematics departments believe that these measures do not provide a fair assessment of their service teaching load. The amount of teaching credit points a department receives for a given student usually depends on that student's subject: For example, a computer science student typically provides more teaching credit points than does a mathematics student. Compounding this problem is a phenomenon that is common in the U.S. as well: mathematics service courses being taught by faculty outside the mathematics department.

Why Are the Numbers Declining?

Asked why the declines in numbers of *Diplom* students have occurred, German mathematicians do not seem to have any easy or obvious answers. One could imagine a demographic explanation: Germany is now experiencing a local minimum in the number of college-age people. However, the proportion going to university has risen. As a result, according to figures from the Statistisches Bundesamt, the number of students attending German universities has declined only about 2 percent since 1992; in fact, in some smaller German states in the eastern part of the country the numbers attending university are up dramatically.

What about job prospects for those receiving the *Diplom* in mathematics? The job market for those in technical subjects did worsen after the reunification of Germany in 1989, though even then mathematics *Diplom* students tended to do fairly well compared with students in other subjects. Today mathematics faculty across the country seem generally to concur with Friedrich Götze of Universität Bielefeld, who calls the opportunities for mathematics students "splendid". "Companies like mathematicians because they are flexible, they are bright people—and they don't give up!" he remarked. Such students are quickly snapped up by banks, insurance companies, and software houses and often receive offers even before completing their degrees. One problem may be that secondary school students considering what subject to choose in university simply do not know that mathematics *Diplom* students have such bright prospects.

In trying to understand the decline in student interest in mathematics in Germany, it is important to note that there have also been comparable declines across the hard sciences, particularly in physics and chemistry. The subjects in which student numbers have been rising include law, business administration, and economics. Biology is also popular: There are admission restrictions for this field, and there are always more students applying than can be admitted. Enrollments in computer science have soared; the number of beginning students rose more than 50 percent between 1992 and 1999. Computer science may be absorbing many students who have interest in and aptitude for studying mathematics. "Mathematics has to try hard to attract these students back," said Karl-Heinz Hoffmann, immediate past president of the German Mathematical Society and head of caesar (center of advanced european studies and research), a research institute founded in 1995 in Bonn.

Views on the decline in student interest in mathematics seem to converge on two explanations. First, mathematics—and indeed the hard sciences generally—is difficult, and today's students are not seeking deep intellectual challenges. "I don't want to say that German students are lazy," said Hoffmann. "But they are looking for an easier way to get a degree" than studying mathematics. Hermann Karcher of Universität Bonn echoes this view: "Mathematics has the reputation of being a tough field to study, and we don't get the message across that it's a lot of fun." Computer science is not an especially easy subject either, but there the image of the hacker turned

Tough Times in Baden-Württemberg

The mathematics departments in the German state of Baden-Württemberg have seen tough times in recent years. The state went through a cost-cutting exercise that mandated deep reductions in mathematics faculty across the state. Two mathematics departments were especially hard hit and barely escaped being closed down altogether.

How did this come about? The state of Baden-Württemberg, home to some of the most important universities in Germany, has nine altogether: Freiburg, Heidelberg, Hohenheim, Karlsruhe, Konstanz, Mannheim, Stuttgart, Tübingen, and Ulm. Pressed by financial difficulties, the state ministry overseeing the universities appointed a "structure commission" in 1996 to provide recommendations for where to cut. The commission consisted mostly of people from industry, university administration, state government, and philanthropic foundations; there was representation in biology and economics, but not in mathematics.

The commission relied on information and data provided by the ministry and did not visit or consult directly with the universities involved, much less individual departments. As a result, the commission labored under some misperceptions. For example, one of the reasons it gave for cutting mathematics departments was that *Diplom* students in mathematics faced poor job prospects. This was "really ridiculous," said Rainer Weissauer of the mathematics department at the University of Mannheim. Weissauer explained that the commission relied on information from the five years after the 1989 reunification in Germany, when there was a general saturation of the job market as workers from the eastern states moved westward. Even then, mathematics *Diplom* students fared better than those in other subjects, but today, as any German mathematics department can attest, there have been for several years now excellent job opportunities for those receiving the *Diplom* in mathematics.

Among the commission's recommendations, presented to the ministry in 1998, was that the number of mathematics faculty across the state be cut by 25 percent. The commission also said that two mathematics departments, in Konstanz and Mannheim, should be closed down; this would mean that the faculty would be reduced by attrition to a small corps for service teaching. In some sense these two departments were natural targets for cuts, because they are the smallest mathematics departments in Baden-Württemberg. In addition, the University of Mannheim is primarily a business school and has no natural sciences apart from mathematics. In the commission's recommendations, mathematics was not singled out for cuts; indeed, reductions were called for in nearly every subject. Only a few areas, such as computer science and business administration, were de facto spared.

The universities, feeling that the recommendations had been imposed from above without adequate consultation, responded with their own recommendations which would amount to a reduction in faculty overall of about 10 per cent. The ministry took into account both sets of recommendations in its final decisions. The recommendation to cut mathematics faculty across the state by 25 percent was retained; the cuts will come through attrition. The ministry decided not to shut down completely the mathematics departments in Mannheim and Konstanz, but their futures are rather uncertain. At the urging of the ministry, the University of Mannheim eliminated the *Diplom* in mathematics and instituted a new *Diplom* in mathematical computer science. The mathematics faculty at Mannheim will decline by about 35 percent, to about six full professors (and no associate professors). All of this has greatly strained the mathematics faculty and has encouraged some with offers elsewhere to leave.

The mathematics department in Konstanz retains its *Diplom* in mathematics but faces the deepest cuts of any department in the university. And it may end up in the same situation as Mannheim. According to Reinhard Racke, who has been dean of mathematics in Konstanz for the past two years, it is possible that the number of full and associate professors would decline more than 50 percent from its original number, to a total of six. The reason the decline could be so steep is a combination of the cuts mandated by the ministry and earlier negotiations about which positions would be refilled. "It's not clear to me that we can continue to survive in the next ten years," said Racke, because the department may not have the personnel to offer a sufficient number of courses for the *Diplom*. On the other hand, student numbers in the department's newly instituted program in mathematical finance are promising: forty-five students enrolled in 1999, compared with just ten in the regular mathematics program.

Similar structure commissions are now operating in two other German states, Nordrhein-Westfalen and Niedersachsen, and the state of Bavaria has also begun a less systematic but nevertheless serious examination of how to reduce spending on its universities. Karl-Heinz Hoffmann, immediate past president of the German Mathematical Society, is on the structure commission for Nordrhein-Westfalen. Hoffmann said that the commission is visiting all the universities in that state.

The entire episode in Baden-Württemberg left the mathematicians, especially those in Mannheim and Konstanz, rather shaken. "We had a terrible three years," said Racke. "It was shocking for all of us—less the fact that we had to give up positions, but more how mathematics was regarded outside and inside the university. This was really shocking."

—A. J.

billionaire is powerful, the allure of the Internet seductive. And herein lies the second explanation: the public image of mathematics. In the public eye, mathematics appears less lucrative, less modern, and less dynamic than other areas. "I can imagine that in these times mathematics is not so interesting," remarked Peter Schneider of Universität Münster. "Devoting one's self to a very abstract, basic, research-oriented subject is not fashionable. The students want to make money."

Reversing the Trend

What can German mathematics departments do to reverse the decline in the numbers of *Diplom* students? According to Hoffmann, departments should offer a greater range of courses that connect mathematics to other areas, such as physics, chemistry, biology, and computer science. "We have to show that mathematics is an essential part of the natural sciences as a whole," he remarked. "We still teach mathematics as we did twenty years ago. But now there is good, strong mathematics all over science, and we have to take this into account."

In his time at the Technische Universität München (from which he is on a five-year leave), Hoffmann worked hard to invigorate the mathematics department there. The department instituted new degrees in *Technomathematik* and *Finanz- und Wirtschaftsmathematik*, which attracted many students, as well as accolades from the university administration. (The numbers in the regular mathematics *Diplom* have not, however, recovered from steep declines in the early 1990s). Among the keys in the turnaround in the department were a new, dynamic university president who supports mathematics and a wave of retirements that allowed the department to bring in a cadre of young, energetic people. Unlike in some mathematics departments, where the threat of faculty reductions has produced a siege mentality in relations between the department and the administration, the feeling at the TU München is upbeat. Said the current mathematics dean, Peter Gritzmann, "There is an atmosphere where, if you work hard and if you have ideas, there are open doors." The success at the TU München has inspired its next-door neighbor, Universität München, to follow suit and establish a new program that combines *Wirtschaftsmathematik* and actuarial science. A wave of retirements in mathematics now under way at Universität München means the mathematics department there has prospects for a renewal.

Topics like financial mathematics are a clear draw for students: The mathematics department at the Universität Konstanz, despite having just emerged from a battle over its very existence (see sidebar), instituted a program (in cooperation with the economics department) in mathematical finance and immediately drew 45 students, more than four times the current number of beginners in its regular mathematics *Diplom* program. But some worry that an overemphasis on specialization produces students who are too narrowly educated. "We should not create curricula that restrict what people can do instead of opening up new possibilities," said Wolfgang Soergel, dean of mathematics at Universität Freiburg. Rüdiger Verfürth, dean of mathematics at Ruhr-Universität Bochum, explained that his department has not considered starting programs in financial or engineering mathematics "since our experience is that the excellent job market of our students is due to their *broad* mathematical education."

Another way in which German mathematics departments are addressing the challenge of low student numbers is by strengthening their connections to local secondary schools. (The local angle is important: In Germany, secondary school students' perceptions of quality differences among universities are not pronounced, as they are in, say, the United Kingdom, where Oxford and Cambridge are perceived to be the top institutions. As a result, German students often enroll in whichever university is closest to their hometown.) To contact potential students, the Bielefeld mathematics faculty, for instance, uses a two-pronged approach. Once a year students from nearby secondary schools are brought to the mathematics department for a set of activities, including talks by former students in the department who have gone on to interesting careers. And Bielefeld mathematics faculty also travel to area schools to talk with students and teachers. "We try to give them insights into what one can do in mathematics," explained Götze. "We want to show the students something different from what they see in their usual studies of mathematics." The importance and value of such efforts are being recognized nationally: For example, the Volkswagen Foundation recently initiated a program to support such school-university linkages in mathematics.

Another approach, but one that will work only over the long term, is improving mathematics teaching in German secondary schools. As in the U.S., many in Germany believe that those intending to become mathematics teachers simply do not learn enough mathematics. There is another aspect peculiar to Germany. Prospective secondary school teachers must study two subjects and write a final paper in one of the subjects. This paper is similar to but less demanding than the *Diplomarbeit*. Those who choose mathematics as one of their subjects often write this paper in the other, presumably easier, subject. Their mathematical backgrounds are therefore not as strong as they could be, and the requirement that they study two subjects reduces the intensity of the study of both subjects. One consequence is that many students coming out of German secondary schools today lack sufficient mathematical preparation to study any scientific and technical subject, let alone mathematics itself. "The mathematical preparation of students in schools leaves a lot to be desired," noted Samuel J. Patterson of Universität Göttingen. "We need that the faculty in universities thinks hard about how education in the schools works."

The decline in numbers of mathematics students has come at a time of new pressures on German universities, which are increasingly viewed as overly bureaucratic, inefficient, and unresponsive to change. For example, *Diplom* programs are seen as taking too long and providing an education that is too abstract and academic, leaving students to make a big adjustment when they enter the work force. Some universities are trying to make studies more flexible by initiating bachelor's degree programs, which would require only three years of study, and master's degree programs, which would require a further two years. One hope is that these new degree programs will make it easier for students from other countries where the bachelor's degree is the norm to study in Germany. Hoffmann chairs a national committee to provide accreditation guidelines for these new degree programs.

Such structural changes are hard to make. What is even harder is tackling the problem of the public image of mathematics, an important factor in the decline in numbers of mathematics students in Germany—and elsewhere. Mathematicians everywhere share this problem. It is an international phenomenon.

—Allyn Jackson

The Code Book: The Evolution of Secrecy from Mary, Queen of Scots to Quantum Cryptography

Reviewed by Jim Reeds

The Code Book: The Evolution of Secrecy from Mary, Queen of Scots to Quantum Cryptography
Simon Singh
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402 pages, $24.95

It is hard to write a good book about the history of cryptography. The subject is technical enough to be a turnoff for many readers. The evidence a historian of cryptography works from is often suspect. Because much of the practice and research in the field was carried out in secret, any particular document or interview must be viewed with suspicion: did the author or interviewee know the full truth? Healthy suspicion about the competency of sources is of course appropriate in all branches of historical research, but in the history of cryptography the proportion of misinformed or deceptive sources is probably greater than generally found in the history of science or of technology. The historian's standard technique of precise and thorough citation of documentary evidence is therefore especially important in the history of cryptography. Unfortunately, for popular works this technique can mean death to readability.

In cryptography technical developments often came in reaction to events and activities which were at the time secret or, conversely, had ceased to be secret. If we do not understand the "who knew what when" details correctly, our reconstructed timetables for technical progress seem to show puzzling fits and starts, apparently unconnected with contemporary events. This makes coherent exposition difficult. Almost every war, however, has notable instances where some cipher message solution foils a plot or wins a battle. Here it is easy to connect the cryptographic or cryptanalytic technicalities with particular historical events, but a book that relies too much on such instances becomes in effect no more than an adventure story anthology.

So it is no surprise that there are few general surveys of the history of cryptography and fewer good ones. The rule of thumb seems to be one new book every thirty years.

In 1902 and 1906 Alois Meister published his immensely scholarly *Die Anfänge der Modernen Diplomatischen Geheimschrift* and *Die Geheimschrift im Dienste der Päpstlichen Kurie*, reproducing and summarizing texts relevant to cryptography in the late medieval and early modern periods. The readership cannot have been large.

At the opposite extreme of readability was the 1939 *Secret and Urgent: The Story of Codes and Ciphers* by the journalist and naval affairs commentator Fletcher Pratt. The book presented a breezy series of thrilling anecdotal historical episodes involving ciphers and code-breaking exploits. Each episode came complete with Sunday supplement-style character sketches and just the

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right amount of technical information about the cipher or cryptanalysis in question. The technical discussions were not always tightly bound to the factual historical setting: although they were always illustrative of the type of ciphers involved in this or that historical episode, they were not necessarily verbatim transcripts of documents in archives. Pratt thus managed to make the technicalities—always clearly explained—seem important and managed to teach a bit of history in the nonrigorous way a historical movie or novel might teach a bit of history. Like many others, I was inspired by this book when I read it in my early teens. It was only much later that I came to realize that its lack of bibliography and detailed footnotes made it useless as a serious history of cryptography.

In 1967, about thirty years after Pratt's book, a much more serious book appeared, *The Codebreakers*, by David Kahn, also a journalist, but one with a far sterner approach to standards of documentation. Where Pratt had two pages of notes and no literature references, Kahn gave 163 pages. Kahn's method (which he pursued over many years with great energy) seems to have been simply this: to read everything about cryptography in all the world's libraries and archives, to interview all cryptographers, and to write it all down as sensibly, as accurately, and with as much detail as possible; his book has 1,180 pages. This is the book I read when I was in college. By then I had grown up enough to appreciate Kahn's comment in his preface that although his love for cryptography had also been sparked by Pratt's book, he was disappointed in the book. Kahn bemoaned Pratt's "errors and omissions, his false generalizations based on no evidence, and his unfortunate predilection for inventing facts."

Unfortunately, Kahn's book was published a short time before the facts about the Polish and British success in breaking the German Enigma cipher of World War II became publicly known and also a short while before the amazing invention of the new number-theoretical "public key cryptography" techniques now pervasive in computers and the Internet. As a result, these interesting and important topics received no treatment.

Now, thirty years after Kahn's book, a new history of cryptography has appeared, again by a journalist: Simon Singh's *The Code Book: The Evolution of Secrecy from Mary, Queen of Scots to Quantum Cryptography*, a bestseller in England in its first months of publication. Singh states in his preface that "In writing *The Code Book*, I have had two main objectives. The first is to chart the evolution of codes...the book's second objective is to demonstrate how the subject is more relevant today than ever before."

Singh's first five chapters cover the history of cryptography up through the end of the Second World War, summarizing material found in earlier books and journal articles, presented by the episodic snapshot method. His remaining three chapters are based mostly on personal interviews with leading participants. Chapter 6 describes the invention and early development of public key cryptography by W. Diffie, M. Hellman, R. Merkle, R. Rivest, A. Shamir, and L. Adleman in the U.S., and independently, but in secret, by J. Ellis, C. Cocks, and M. Williamson in the U.K. Chapter 7 describes the current controversy about the proper role of cryptography in a free society: personal freedom versus the interests of the state, privacy versus wiretapping, key escrow, export of strong cryptography, and so on. The final chapter describes quantum cryptography, the new system of communications made untappable by exploiting the fact that the polarization of a photon is altered when it is measured.

The good news is that Singh's book has all the good qualities of Pratt's. Unfortunately, Kahn's criticism of Pratt's book also applies to Singh's book. Almost every page has small errors of fact. In many places it is clear that Singh does not really understand the material he copies from his sources. Many of these errors are of little consequence when taken individually, but their cumulative effect is to destroy a knowledgeable reader's confidence in the author's standards of accuracy.

Here are just a few examples:

- On page 128 Singh describes the wired code wheels of the Enigma cipher machine (the "rotors", which he oddly calls "scramblers"): "The scrambler, a thick rubber disc riddled with wires..." But the Enigma's rotors were *not* made of rubber but of aluminum, brass, and Bakelite. Singh may have misunderstood a sentence on page 411 of Kahn's book: "The body of a rotor consists of a thick disk of insulating material, such as Bakelite or hard rubber...", accurately describing the rotors, not of an Enigma machine, but of a different cipher machine.
- On page 168 Singh states that A. M. Turing (in his 1937 paper "On computable numbers, with an application to the Entscheidungsproblem") called "this hypothetical device a *universal Turing machine* [Singh's italics]." But of course the terms "Turing machine" and "universal Turing machine" were *not* used by Turing himself; a glance at his paper shows he used "computing machines" and "universal machines".
- On pages 187–8, Singh states that the British WWII code-breaking organization, the "Government Code and Cypher School", was disbanded after the war and then replaced by another, the "Government Communications Headquarters", or GCHQ. In fact, the change occurred in 1942 and was one in name only.
- On page 191 Singh claims the American breaking of the Japanese "Purple" cipher enabled the naval victory at Midway and the assassination

of Admiral Yamamoto. In fact, these were due to the breaking of the "JN-25" code. "Purple" was a machine cipher, roughly equivalent to the German Enigma, whereas "JN-25" was a hand system relying on code books and random number tables.

Singh's unfamiliarity with the technical vocabulary used by his sources seems to have led him into a more serious mistake in the first two chapters. To explain this, I must first summarize material in Kahn's chapters 3 to 6.

From before 1400 until about 1750 only one kind of encryption method was widely used (although others were discussed in books). This method used what were called at the time "ciphers" or "keys". A cipher was a collection of arbitrary symbols or numbers that substituted for letters, syllables (or other letter sequences), names, and common words and phrases found in plain text. By 1700 ciphers with as many as 1,000 or 2,000 substitutions were common, and even larger ones with as many as 10,000 or 50,000 substitutions were in use later on. Although the general trend was towards greater size and complexity, throughout this period ciphers of widely varying size and complexity were used. Modern scholars have used a variety of terms—more or less interchangeably—for this cryptographic genre, including "homophonic cipher", "nomenclator", "code", "code chart", and so on, as the original terms "cipher" and "key" are no longer precise enough to distinguish these methods from more modern ones.

At the same time a theory for another kind of cryptography was being developed, discussed, and elaborated in successive printed cryptography books all through the 1500s and into the 1600s. The set-piece example of this new kind of cryptography, the "Vigenère" cipher, also known as *chiffre indéchiffrable*, was more algebraic in nature, based on Latin squares and what we now know as modular arithmetic. This kind of cryptography was slow to gain acceptance: although available for use in 1575, it was not actually used until the mid-1600s, and then only sparingly. Even at the end of the 1700s Thomas Jefferson's adoption of the Vigenère cipher by the U.S. State Department was an innovation, and when he left office, the department reverted to the older nomenclator technology. Only in the nineteenth century did the Vigenère cipher come into common use and serve as a basis for further technical developments.

Singh, however, seeing one author use the term "nomenclator" to describe a cipher in use in 1586 and another author using the term "homophonic cipher" to describe one in use in 1700, supposes the two ciphers to be different kinds of things. And he invents a theory explaining why the latter kind was devised: he says on page 52 that the "homophonic cipher" was invented in Louis XIV's reign to serve as a more practical alternative to the *chiffre indéchiffrable*. But Kahn (whose book appears in Singh's list of references), on page 107, shows an example of a homophonic cipher, labelled as such, from 1401, about three centuries before Singh's invented invention.

A different kind of misunderstanding occurs in the discussion of the attack on the German Enigma machine in the early 1930s. The mathematical basis for the initial Polish success was the well-known fact that the cycle type of a permutation is invariant under conjugation: when one writes the permutations τ and $\sigma\tau\sigma^{-1}$ as the products of disjoint cycles, the same lengths appear with the same multiplicities. On pages 148–54 Singh explains very clearly how Marian Rejewski applied this fact to the problem of recovering German Enigma keys. If ever there was a real-world story problem handed to mathematics teachers on a silver platter, this would be it.

The sample permutation Singh uses to illustrate the Enigma application decomposes into cycles of length 3, 9, 7, and 7. (Here, of course, the permutation is a permutation of the 26-letter alphabet: $3+9+7+7=26$.) But here is the kicker. The permutations τ which actually occur in the Enigma application are of the form $\tau=\alpha\beta$, where α and β are each the products of 13 disjoint 2-cycles. This forces τ to have even cycle length multiplicities, which Singh's example does not have. That is, Singh presents an imitation example, not an example of an actual Enigma τ permutation he has worked out. This is perfectly adequate for illustrating the mathematical fact of the invariance of cycle type under conjugation, but will not do for illustrating the historical facts of Rejewski's solution of the Enigma cipher.

This is as if a historian of trigonometry, describing some early work, wrote: "In a right triangle with sides of lengths 2, 3, and 4, the angle opposite the side of length 2 was found by taking the inverse sine of the ratio of the opposite side to the hypotenuse, in this case $\arcsin(2/4)=30°$." The formula is correctly stated and worked out, but applied in an impossible context. Which is the worse fault: Singh not bothering to use an actual historical—or even realistic—example, or not knowing that his example is unrealistic?

Singh does better in the remaining chapters, where the story line and technical explanations derive from interviews. His interviewees' personalities are clearly visible, and the technical explanations are usually comprehensible.[1]

Chapter 6, about the invention of public key cryptography, repeats the stories which have been told in public lectures by Diffie, Hellman, and Shamir about their discovery of the basic ideas of

[1] *Whitfield Diffie, however, has complained in a book review (*Times Higher Education Supplement, *10 September 1999) that not everything he told Singh was accurately reported.*

one-way functions and public key cryptography, as well as their discovery of the number-theoretic examples based on modular exponentiation and the difficulty of factoring. More interesting is Singh's description of the secret and somewhat earlier independent discovery of these ideas by Ellis, Cocks, and Williamson at the GCHQ, the secret British government cryptography organization. GCHQ has recently "gone public" in this matter, making Cocks a media celebrity by GCHQ standards. (The chronology of this matter is somewhat hard to assess because not all the relevant GCHQ files have been made available. One result, the Diffie-Hellman exponential key exchange, seems *not* to have been first discovered by GCHQ.)

In this chapter Singh spends many pages discussing the matter of priority of scientific discovery, exulting in the recent declassification of the earlier GCHQ work as if an injustice had been righted. This vision of the abstract reward of "credit", based on strict chronological priority, distracts Singh from looking at the historically more interesting questions of influence of ideas. These include: how were the initial GCHQ discoveries understood by the discoverers' colleagues at the time, how were these ideas developed, and how were they used? The available evidence is scanty, but it seems likely that they were regarded within GCHQ as impractical curiosities and ignored until the rediscoveries on the outside alerted GCHQ to their importance.

The historiographic issue is neatly illustrated in an example at the end of the chapter, referring back to an episode in Chapter 2, which Singh takes as a parallel foreshadowing. One of the techniques for breaking the Vigenère *chiffre indéchiffrable* was first published in 1863 by F. Kasiski, but apparently sometime in the 1850s Charles Babbage had worked out the same method in private. Singh claims (on no evidence whatsoever) that Babbage did not publish his results because of the interests of military secrecy during the Crimean War of 1854. But now Babbage's injustice is also righted: he gets the credit in the end. Regardless of the reasons for Babbage's failure to publish, the following seems clear: Babbage's discovery, since it was unpublished, had no influence on the further development of cryptography. That he made this discovery tells us something about Babbage's mental capabilities; that it was independently rediscovered tells us something (but not much) about its intrinsic level of difficulty. Babbage might have been first, but (in this matter) he was historically unimportant. Society uses credit and priority as a reward to encourage the dissemination of new ideas, and it is not at all clear that a researcher who fails to publish a new idea—whether out of diffidence, patriotism, or employment at a secret research laboratory—is done an injustice when not awarded credit. Righting such imagined wrongs is not what history is about.

Chapter 7, based on interviews with the PGP (Pretty Good Privacy) programmer Philip R. Zimmermann, concentrates on the currently unsettled matter of the proper role of cryptography in a free society. Zimmermann represents the libertarian side: the people should use—must use, if they do not trust their government—the strongest kind of cryptography they can. Governments, however, remembering the invaluable results of cryptanalysis during the Second World War (and presumably since then) would wish to somehow keep the strongest forms of cryptography out of the hands of potential enemies. As the target of a grand jury investigation, Zimmermann suffered from the American government's embarrassingly inept way of trying to make up its mind on this public policy issue.

The final chapter returns to the purely technological, with a discussion of quantum cryptography. Here again, interviewees (D. Deutsch and C. Bennett) carry the story along. The description of the basics of quantum mechanics is painfully incoherent, that of quantum computing is superficial and vague, but the explanation of how polarized photons can carry untappable information is fairly clear.

In the preface—justifying his rejection of a pedantically more accurate title for his book—Singh states "I have, however, forsaken accuracy for snappiness." With hindsight this is ominous. His carelessness with facts will not harm those readers who pick up the book, skim it, and find the subject not to their taste. Nor will it harm the enthusiasts (like myself), who will seek out other, more reliable books.[2] But most, I suspect, will fall in the middle ground: interested readers who will rely on this book alone for their information about cryptography. This group, which will mine Singh's book for years, if not decades, for term-paper and lecture material, and possibly material for other books, will be disserved by the author's lax standards of accuracy.

[2] *My favorites: instead of Singh's Chapters 1-3, people should read D. Kahn,* The Codebreakers: The Story of Secret Writing *(Macmillan, 1967) and F. Bauer,* Decrypted Secrets : Methods and Maxims of Cryptology *(Springer, 1997). Instead of Singh's Chapter 4, read F. H. Hinsley and A. Stripp,* Codebreakers: The Inside Story of Bletchley Park *(Oxford, 1993) and G. Welchman,* The Hut Six Story *(McGraw-Hill, 1982). Instead of Chapter 7, read W. Diffie and S. Landau,* Privacy on the Line: The Politics of Wiretapping and Encryption *(MIT, 1998). All but one of these books are in Singh's "Further Reading" list, pages 388-393.*

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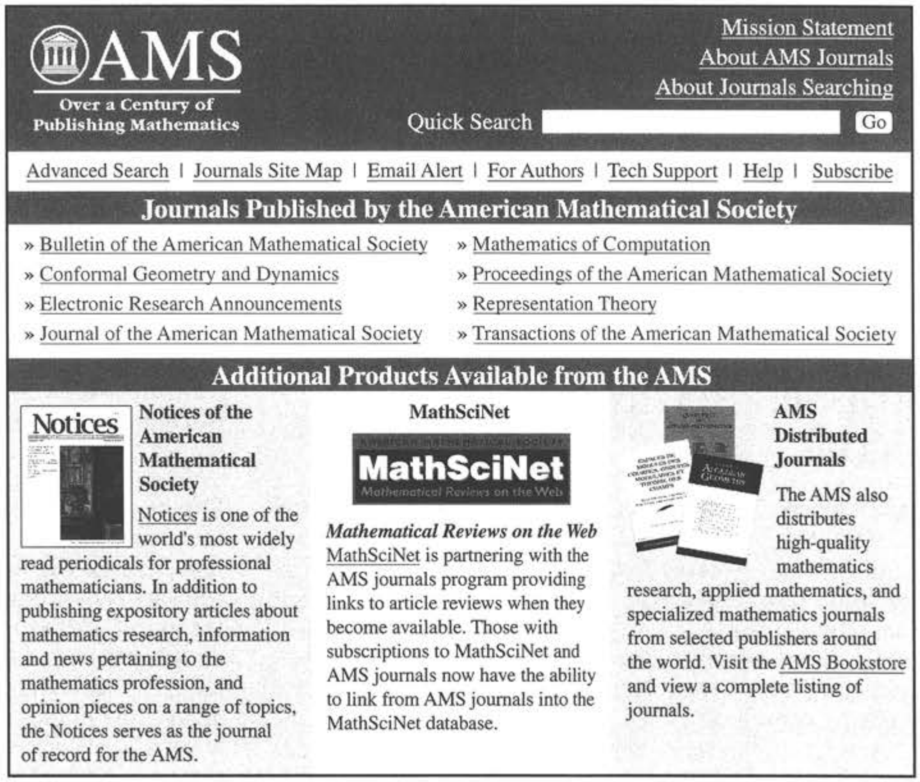

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Mathematics People

Strassen Receives Cantor Medal

The Deutsche Mathematiker Vereinigung (DMV, German Mathematical Society) has awarded the 1999 Georg Cantor Medal to VOLKER STRASSEN of Universität Konstanz. The citation for the medal states: "The Society honors a great scientist who through his multifaceted contributions has given mathematical research decisive impulses and has opened up new areas. His work in probability theory, algebraic complexity theory, and theoretical computer science has been groundbreaking. Those results will always be linked to his name."

Previous recipients of the Cantor Medal are Karl Stein (1990), Jürgen Moser (1992), Erhard Heinz (1994), and Jacques Tits (1996).

—From a DMV announcement

NSF Postdoctoral Research Fellows

The Mathematical Sciences Postdoctoral Research Fellowship program of the Division of Mathematical Sciences (DMS) of the National Science Foundation (NSF) has announced the recipients of its fellowships for 1999. The fellowships are awarded each year for research in pure mathematics, applied mathematics and operations research, and statistics.

The names of the fellows, their Ph.D. institutions (in parentheses), and the institutions at which they will use their fellowships are: MATTHEW BAKER (University of California, Berkeley), Harvard University; DAVID BEN-ZVI (Harvard University), University of Chicago; NATHANIAL BROWN (Purdue University, West Lafayette), University of California, Berkeley; JASON CANTARELLA (University of Pennsylvania), University of Massachusetts, Amherst; CHRISTOPHER CONNELL (University of Michigan, Ann Arbor), University of Illinois, Chicago; BRUNO DE OLIVEIRA (Columbia University), University of Pennsylvania; DARRIN DOUD (University of Illinois, Urbana-Champaign), Harvard University; DAVID FISHER (University of Chicago), Yale University; DONALD FREEMAN (University of Michigan, Ann Arbor), University of Colorado, Boulder; REBECCA GOLDIN (Massachusetts Instistute of Technology), University of Maryland, College Park; THOMAS GRABER (University of California, Los Angeles), Harvard University; MARK HUBER (Cornell University), Stanford University; LEONID KORALOV (State University of New York, Stony Brook), Institute for Advanced Study, Princeton; ANDREW KRESCH (University of Chicago), University of Pennsylvania; MARIA MARTINEZ (University of California, San Diego), Pennsylvania State University; JOSEPH MASTERS (University of Texas, Austin), Rice University; JONATHAN MATTINGLY (Princeton University), Stanford University; MARTIN MOHLENKAMP (Yale University), University of Colorado, Boulder; PETER MUCHA (Princeton University), Massachusetts Institute of Technology; ERIC OLSON (Indiana University, Bloomington), University of California, Irvine; ANNE SHEPLER (University of California, San Diego), University of Wisconsin, Madison; CHADWICK SPROUSE (University of California, Los Angeles), New York University; KRISTIN SWANSON (University of Washington), University of California, San Francisco; JEREMY TYSON (University of Michigan, Ann Arbor), State University of New York, Stony Brook; SALIL VADHAN (Massachusetts Institute of Technology), Massachusetts Institute of Technology; MONICA VAZIRANI (University of California, Berkeley), University of California, San Diego; JEFF VIACLOVSKY (Princeton University), University of Texas, Austin; and KEVIN WHYTE (University of Chicago), University of Utah.

—From an NSF announcement

Correction to the Balzan Prize Announcement

The announcement about Mikhael Gromov receiving the Balzan Prize, which appeared in the February 2000 issue of the *Notices*, gave an incomplete affiliation for Gromov. He is a permanent professor at the Institut des Hautes Études Scientifiques and is also the Jay Gould Professor of Mathematics at the Courant Institute of Mathematical Sciences of New York University, where he spends three months per year.

—*Allyn Jackson*

Deaths

TIMOTHY SWAN HARRIS, of London, England, died on July 22, 1999. Born on December 29, 1942, he was a member of the Society for 9 years.

FREDERICK HOWES, program director, Office of Computational and Technology Research, Germantown, Maryland, died on December 4, 1999. Born on November 21, 1948, he was a member of the Society for 26 years.

NATHAN JACOBSON, Henry Ford II Professor of Mathematics, Emeritus, at Yale University, died on December 5, 1999. Born on September 8, 1910, he was a member of the Society for 67 years. He was president of the AMS in 1971–1972, and he received the AMS Steele Prize for Lifetime Achievement in 1998.

JOHN L. KELLEY, professor emeritus at the University of California at Berkeley, died on November 26, 1999. Born in December 1916, he was a member of the Society for 62 years.

COLINE M. MAKEPEACE, of the Federal Emergency Management Agency, Gaithersburg, Maryland, died on November 8, 1999. Born on May 17, 1929, she was a member of the Society for 36 years.

JÜRGEN MOSER, professor emeritus at the Eidgenössische Technische Hochschule (ETH), Zürich, Switzerland, died on December 17, 1999. Born on July 4, 1928, he was a member of the Society for 43 years. He received the AMS George David Birkhoff Prize in 1968.

SHINGO MURAKAMI, professor emeritus, Osaka University, Japan, died on December 3, 1999. Born on October 21, 1927, he was a member of the Society for 34 years.

RICHARD E. PHILLIPS, of Michigan State University, died on November 9, 1999. Born on December 3, 1936, he was a member of the Society for 37 years.

J. C. SHEPHERD, retired from the University of Maryland, College Park, died on October 23, 1999. Born on August 17, 1922, he was a member of the Society for 51 years.

Mathematics Opportunities

IAS/Park City Mathematics Institute

The Institute for Advanced Study (IAS)/Park City Mathematics Institute (PCMI) will hold its 2000 summer session from July 16–August 5, 2000. The topic is computational complexity theory. The organizers are Avi Wigderson (Institute for Advanced Study and Hebrew University) and Steven Rudich (Carnegie Mellon University).

The IAS/PCMI began in 1991 at the University of Utah as a National Science Foundation Regional Geometry Institute. In 1993 the Institute for Advanced Study assumed sponsorship of the program. Each summer the Institute offers an integrated set of programs for researchers, postdoctorates, graduate and undergraduate students, and teachers.

Further information on the summer program and other IAS/PCMI activities, as well as on application procedures, is available at the Web site `http://www.admin.ias.edu/ma/default.htm`.

Applicants to all programs may apply for financial support. The deadline to apply is **February 15, 2000.**

—From an IAS/PCMI announcement

NSF Biocomplexity Competition

The National Science Foundation (NSF) has announced a special competition that provides an opportunity for mathematical scientists in all fields to engage in multidisciplinary scientific research. The competition, called "Biocomplexity: Integrated Research to Understand and Model Complexity among Biological, Physical, and Social Systems", will support integrated research to achieve better understanding of and to model complexity that arises from the interaction of biological, physical, and social systems. All proposing groups are required to include a quantitative expert, mathematician, or statistician.

The program will also support "incubation" activities that enable groups of researchers who have not historically collaborated on biocomplexity research to develop projects through focused workshops, virtual meetings, and other development and planning activities.

The program will award a total of $50 million in grants to support both research projects and incubation activities. Research projects will be awarded up to $600,000 per year for five years; incubation activities will be awarded up to a total of $100,000 for up to two years without renewal. Full proposals for both research and incubation activities must be received by **March 1, 2000.**

The full program announcement is available on the NSF Web site at `http://www.nsf.gov/cgi-bin/getpub? nsf0022/`. For more information on activities in the areas of mathematical and physical sciences, contact James L. Rosenberger, Statistics Program Director, National Science Foundation, 4201 Wilson Blvd., Room 1025, Arlington, VA 22230; telephone 703-306-1883; fax: 703-306-0555; e-mail: `jrosenbe@nsf.gov`.

—From an NSF announcement

Research Experiences for Undergraduates Sites for 2000

The Research Experiences for Undergraduates (REU) program of the National Science Foundation (NSF) provides opportunities for undergraduates to join research projects each summer to learn how basic research is conducted and to contribute to it. REU "sites" are established in all fields of science, mathematics, and engineering. Each site consists of a group of about ten undergraduates who work in research programs of the host institution. Each student is assigned to a specific research project and works closely with faculty, postdocs, and graduate students.

Undergraduate students are encouraged to apply to the REU sites. What follows is a tentative list of REU sites in the mathematical sciences for the summer of 2000, together with the names of the site directors, who can be contacted for further information.

Auburn University: Discrete Mathematics, Computer Algebra; Overtoun Jenda, jendaov@mail.auburn.edu.

College of William and Mary: Matrix Analysis and Its Applications; David J. Lutzer, djlutz@mail.wm.edu, http://www.math.wm.edu/~lutzer/anncment.html.

Colorado School of Mines: Computer Science, Mathematics; Erik Van Vleck, byoung@mines.edu, http://www.mines.edu/Academic/macs/reu_index.html.

Cornell University: Analysis on Fractals, Complex Dynamics, Combinatorics; Robert S. Strichartz, reu@math.cornell.edu, http://math.cornell.edu/~math/Educate/REU/99REU.html.

Grand Valley State University: Chaotic Dynamical Systems, Fractal Geometry, Differential Equations, Linear Algebra; Steven Schlicker, schlicks@gvsu.edu, http://www.gvsu.edu/mathstat/reu/.

Hope College: Algebra, Dynamical Systems, Probability and Number Theory; Tim Pennings, pennings@math.hope.edu, http://www.math.hope.edu/reu/reu.html.

Indiana University: Algebra, Topology, Analysis, Probability, and Applied Mathematics; Daniel Maki, reu@indiana.edu, http://www.math.indiana.edu/reu/home.html.

Iowa State University: Numerical Analysis, Scientific Computing; Janet Peterson, jspeters@iastate.edu, http://www.math.iastate.edu/reu.html.

Michigan Technological University: Probability, Combinatorics, Number Theory, Statistics, Algorithms and Geometry; Anant P. Godbole, anant@mtu.edu, http://www.math.mtu.edu/~anant/reu/.

Mount Holyoke College: Number Theory, Algebraic Geometry and Applied Analysis; Alan H. Durfee, reu@mtholyoke.edu, http://www.mtholyoke.edu/acad/math/reu/.

Northern Arizona University: Combinatorics, Applied Math, Statistics; Catherine A. Roberts, Catherine.Roberts@nau.edu, http://odin.math.nau.edu/REU/.

Oregon State University: Analysis of Algorithms, Geometry, Population Dynamics, and Topology; Dennis J. Garity, reu@math.orst.edu, http://ucs.orst.edu/~garityd/REU/.

Pennsylvania State University Erie, The Behrend College: Mathematical Biology; J. Carl Panetta, panetta@wagner.bd.psu.edu, http://www.pserie.psu.edu/science/math/REU/index.html.

Rose-Hulman Institute of Technology: Computational Group Theory, Hyperbolic Geometry; S. Allen Broughton, allen.broughton@rose-hulman.edu, http://www.rose-hulman.edu/Class/ma/HTML/REU/NSF-REU.html.

State University of New York, Potsdam: Group Theory, Graph Theory, Topology; Kazem Mahdavi, mahdavk@potsdam.edu, http://www.clarkson.edu/~mcs/reu.html.

Trinity University: Dynamical Systems, Algebra and Statistics; Scott Chapman, schapman@trinity.edu, http://www.math.trinity.edu/mathematics/reu99.htm.

Tulane University: Geometry and Topology; Morris Kalka, reu@math.tulane.edu, http://math.tulane.edu/reu.html.

University of Houston: Geometry, Analysis, Number Theory and Numerical Analysis; Barbara Keyfitz, blk@math.uh.edu, http://www.math.uh.edu/~dean/REU/index.html.

University of Idaho: Discrete Mathematics; Dan Schaal, schaald@ur.sdstate.edu, http://www.sdstate.edu/ma24/http/idahoreu.html.

University of Maryland Eastern Shore: Parallel Numerical Computing; Daniel I. Okunbor, dokunbor@mcs.umes.umd.edu, http://hawk.umes.edu/dokunbor/reu/.

University of Minnesota, Duluth: Discrete Mathematics, Combinatorics and Graph Theory; Joseph A. Gallian, jgallian@d.umn.edu, http://www.d.umn.edu/~jgallian/.

University of Tennessee: Selected Topics in Pure and Applied Mathematics; Suzanne Lenhart, lenhart@math.utk.edu, http://www.math.utk.edu/Docs/reuflyer.html.

University of Washington: Inverse Problems; James A. Morrow, morrow@math.washington.edu, http://www.math.washington.edu/~morrow/reu99/reu.html.

Washington State University: Applied Mathematics, Environmental Science; Valipuram S. Manoranjan, ziya@wsu.edu, http://www.sci.wsu.edu/math/faculty/mano/VSManoranjan.html.

Williams College: Geometry; Colin Adams, colin.adams@williams.edu, http://www.williams.edu/Mathematics/SMALL.html.

Worcester Polytechnic Institute: Applied/Industrial Mathematics; Bogdan Vernescu, vernescu@wpi.edu, http://www.WPI.EDU/~cims/reu/index.htm.

Updated information is available on the Web site of the NSF's Division of Mathematical Sciences, http://www.nsf.gov/mps/bdms/reulist.htm. General information on the REU program, as well as instructions for submitting proposals, is available on the NSF Web site, http://www.nsf.gov/home/crssprgm/reu/start.htm.

—From an NSF announcement

2000 Summer Program for Women in Mathematics

The George Washington University has announced the 2000 Summer Program for Women in Mathematics (SPWM 2000) to be held July 1–August 5, 2000.

SPWM 2000 is an intensive five-week program for mathematically talented undergraduate women who are completing their junior years and may be contemplating graduate study in the mathematical sciences. The goals of this program are to communicate an enthusiasm for mathematics, to develop research skills, to cultivate mathematical self-confidence and independence, and to promote success in graduate school.

Sixteen women will be selected. Each will receive a travel allowance, campus room and board, and a stipend of $1,250. The application deadline is **March 1, 2000.**

For further information see the university's Web site, `http://www.gwu.edu/~math/spwm.html`, or contact the codirectors, Murli M. Gupta (`mmg@gwu.edu`) or E. Arthur Robinson Jr. (`robinson@gwu.edu`), Department of Mathematics, George Washington University, Washington, DC 20052; telephone 202-994-4857; fax 202-994-6760.

—Murli Gupta, George Washington University

Maria Mitchell Women in Science Award

The Maria Mitchell Association offers an annual award to recognize an individual, program, or organization that encourages the advancement of girls and women in studies and careers in science and technology. Maria Mitchell (1818–1889) was the first woman astronomer and first woman astronomy professor in the United States.

The award may be given in the natural and physical sciences, mathematics, engineering, computer science, or technology. The winner will be chosen by a national jury of distinguished educators and scientists and will receive a cash award of $5,000. Funding for the award is provided through the year 2000 by the William R. Kenan Jr. Fund for Engineering, Technology, and Science.

Guidelines and nomination forms are available from the Association's Web site at `http://www.mmo.org/`, or by contacting the Maria Mitchell Women in Science Award Committee at the Maria Mitchell Association, 2 Vestal Street, Nantucket, MA 02554; telephone 508-228-9198. The deadline for nominations is **April 28, 2000.**

—From a Maria Mitchell Association announcement

Project NExT: New Experiences in Teaching

Project NExT (New Experiences in Teaching) is a program for new or recent Ph.D.'s in the mathematical sciences that addresses a broad range of professional issues, focusing on the teaching and learning of undergraduate mathematics. Faculty who are just beginning or just completing their first year of full-time teaching at the college/university level are invited to apply to become Project NExT fellows.

The application deadline is **April 14, 2000.** For more information, consult the Project NExT home page (`http://archives.math.utk.edu/projnext/`) and see the article in the February issue of *Notices*, page 217.

—Elaine Kehoe

For Your Information

AMS Establishes Ky and Yu-Fen Fan Endowment

In the fall of 1999, Ky Fan and his wife, Yu-Fen Fan, made a gift of approximately $1 million to the AMS. The funds will be used to establish the Ky and Yu-Fen Fan Endowment. Income from the endowment will support mathematics in China and mathematically talented high school students in the U.S.

"The gift from Ky and Yu-Fen Fan reflects their commitment to supporting mathematics, particularly in their home country of China, where there is much talent but few resources," said AMS president Felix E. Browder. "Their generosity is remarkable."

Funds from the Ky and Yu-Fen Fan Endowment will primarily be devoted to a program for fostering collaborations between Chinese mathematicians and mathematicians in other parts of the world, especially North America. The program will provide grants to Chinese mathematics departments to bring in visitors from the rest of the world as well as grants to North American departments to bring in visitors from China. There will also be support for occasional conferences in China and for improving mathematics library holdings in Chinese institutions. In addition, the endowment will provide small grants to assist programs in the U.S. that nurture mathematically talented high school students. Half of the Fans' gift will go into the endowment, and the other half into a gift annuity (whereby the donor receives an annuity and the unused portion becomes a donation).

Ky Fan is an emeritus professor of mathematics at the University of California, Santa Barbara. Born on September 19, 1914, in Hangchow, China, he received his B.S. degree from Peking University (1936) and his D.Sc. degree from the University of Paris (1941). He was a member of the Institute for Advanced Study in Princeton from 1945 to 1947 and held positions at the University of Notre Dame, Wayne State University, and Northwestern University before going to U.C., Santa Barbara, in 1965. Elected a member of the Academia Sinica in 1964, Fan served as the director of the Institute of Mathematics there from 1978 to 1984.

Fan was a student and collaborator of M. Fréchet and was also influenced by John von Neumann and Hermann Weyl. The author of approximately 130 papers, Fan made fundamental contributions to operator and matrix theory, convex analysis and inequalities, linear and nonlinear programming, topology and fixed point theory, and topological groups. His work in fixed point theory, in addition to influencing nonlinear functional analysis, has found wide application in mathematical economics and game theory, potential theory, calculus of variations, and differential equations.

Ky and Yu-Fen Fan

"The Ky and Yu-Fen Fan Endowment addresses groups that have deep reservoirs of talent in need of cultivation and resources," Browder said. "The AMS is profoundly grateful to Ky and Yu-Fen Fan. The impact of their generosity will be felt for years to come."

—*Allyn Jackson*

Straley Named MAA Executive Director

The Mathematical Association of America (MAA) has named Tina Straley as its new executive director. Straley, associate vice president for scholarship and graduate studies at Kennesaw State University in Georgia, has served in the Southeastern Section of the MAA, has been an editor of the *MAA Notes*, and has served as a program officer at the National Science Foundation. Straley succeeds Marcia P. Sward, who took a position at the National Environmental Education and Training Foundation.

—*Elaine Kehoe*

Reference and Book List

*The **Reference** section of the* Notices *is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.*

Contacting the *Notices*

The preferred method for contacting the *Notices* is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include Feature Articles, Memorial Articles, book reviews and other Communications, columns for "Another Opinion", and "Forum" pieces. The editor is also the person to whom to send news of unusual interest about mathematics research.

The managing editor is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar". Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are `notices@math.sunysb.edu` in the case of the editor and `notices@ams.org` in the case of the managing editor. The fax numbers are 631-751-5730 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Upcoming Deadlines

February 15, 2000: Applications for IAS/Park City 2000 Summer Program. For further information, see "Mathematics Opportunities" in this issue or `http://wwwadmin.ias.edu/ma/default.htm`.

February 15, 2000: Nominations for the Richard C. DiPrima Prize. Contact Ronald A. DeVore, Chair, DiPrima Prize Selection Committee, c/o A. G. Bogardo, Society for Industrial and Applied Mathematics, 3600 University City Science Center, Philadelphia, PA 19104-2688; telephone 215-382-9800; fax 215-386-7999, e-mail: `bogardo@siam.org`.

March 1, 2000: Proposals for the biocomplexity research and "incubation" activities competition of the National Science Foundation. For more information see "Mathematics Opportunities" in this issue.

March 1, 2000: Applications for the 2000 Summer Program for Women in Mathematics at George Washington University. For more information see "Mathematics Opportunities" in this issue.

March 31, 2000: Nominations for the 2000 Prize for Achievement in Information-Based Complexity. Contact Joseph Traub, `jtraub@cs.columbia.edu`.

April 14, 2000: Applications for Project NExT for 2000-01. See Project NExT Web site `http://archives.math.utk.edu/projnext/`.

April 15, 2000: Applications for the IMA Workshop on "Mathematical Modeling in Industry". For details, see `http://www.ima.umn.edu/modeling/` or contact `ima-staff@ima.umn.edu`.

April 15, and August 15, 2000: Second and third competitions for NRC Research Associateships. For details, see `http://www.nationalacademies.org/rap/`, or contact the National Research Council, Associateship Programs (TJ 2114/D3), 2101 Constitution Avenue, NW, Washington, DC 20418; telephone 202-334-2760; fax 202-334-2759; e-mail: `rap@nas.edu`.

April 28, 2000: Nominations for the Maria Mitchell Women in Science Award. For more information see "Mathematics Opportunities" in this issue.

May 1 and October 1, 2000: Applications for NSF/AWM Travel Grants

Where to Find It

A brief index to information that appears in this and previous issues of the *Notices.*

for Women. For further information see `http://www.awm-math.org/travelgrants.html`, telephone 301-405-7892, e-mail: `awm@math.umd.edu`.

July 31, 2000: Nominations for the Monroe Martin Prize. Contact J. A. Yorke, Director, Institute for Physical Sciences and Technology, University of Maryland, College Park, MD 20742.

MPS Advisory Committee

Following are the names and affiliations of the members of the Advisory Committee for Mathematical and Physical Sciences (MPS) of the National Science Foundation. The date of the expiration of each member's term is given after his or her name. The Web site for the MPS directorate may be found at `www.nsf/gov/MPS/`. The postal address is Directorate for Mathematical and Physical Sciences, National Science Foundation, 4201 Wilson Boulevard, Arlington, VA 22230.

Ronald Brisbois (10/02)
Department of Chemistry
Hamline University

Arturo Bronson (ex-officio)
Materials Center for Synthesis and Processing
University of Texas, El Paso

Tony Chan (10/02)
Department of Mathematics
University of California, Los Angeles

Praveen Chaudhari (chair) (10/00)
IBM T. J. Watson Research Center

Alexandre J. Chorin (10/01)
Department of Mathematics
University of California, Berkeley

Billy Joe Evans (10/02)
Department of Chemistry
University of Michigan, Ann Arbor

Lila M. Gierasch (10/01)
Department of Chemistry
University of Massachusetts, Amherst

Norman Hackerman (10/00)
Robert A. Welch Foundation

Jacqueline N. Hewitt (10/01)
Department of Physics
Massachusetts Institute of Technology

Jiri Jonas (10/00)
Beckman Institute

Bernard V. Khoury (10/01)
American Association of Physics Teachers

Thomas B. W. Kirk (10/01)
Brookhaven National Laboratory

Michael Knotek (10/00)
Universities Research Association
Lockheed Martin

Richard McCray (10/00)
Department of Astrophysical and Planetary Sciences and JILA
University of Colorado, Boulder

Gerard Mourou (10/00)
Center for Ultrafast Optical Science
University of Michigan, Ann Arbor

J. Anthony Tyson (10/01)
Lucent Technologies

Carol S. Wood (10/00)
Department of Mathematics
Wesleyan University

Book List

*The **Book List** highlights books that have mathematical themes and hold appeal for a wide audience, including mathematicians, students, and a significant portion of the general public. When a book has been reviewed in the Notices, a reference is given to the review. Generally the list will contain only books published within the last two years, though exceptions may be made in cases where current events (e.g., the death of a prominent mathematician, coverage of a certain piece of mathematics in the news) warrant drawing readers' attention to older books. Suggestions for books to include on the list may be sent to the managing editor, e-mail:* `notices@ams.org`.

The Applicability of Mathematics as a Philosophical Problem, by Mark Steiner. Harvard University Press, November 1998. ISBN 0-674-04097-X.

The Arithmetic of Life, by George Shaffner. Ballantine Books, August 1999. ISBN 0-345-42631-2.

The Code Book: The Evolution of Secrecy from Mary, Queen of Scots, to Quantum Cryptography, by Simon Singh. Doubleday, October 1999. ISBN 0-385-49531-5. (Reviewed in this issue.)

Complexity and Information, by J. F. Traub and Arthur G. Werschulz. Cambridge University Press, December 1998. ISBN 0-52148-005-1 (hardcover), 0-521-48506-1 (paperback).

Cryptonomicon, by Neal Stephenson. Avon Books, May 1999. ISBN 0-380-97346-4. (Reviewed December 1999.)

Drawbridge Up: Mathematics—A Cultural Anathema (Zugbrücke ausser Betrieb: Die Mathematik im Jenseits der Kultur), by Hans Magnus Enzensberger. A K Peters, December 1999. ISBN 1-56881-099-7.

The Eightfold Way: The Beauty of Klein's Quartic Curve, edited by Silvio Levy. Cambridge University Press, March 1999. ISBN 0-521-66066-1.

The Elegant Universe: Superstrings, Hidden Dimensions, and the Quest for the Ultimate Theory, by Brian Greene. W. W. Norton & Company, February 1999. ISBN 0-393-04688-5.

Emergence: From Chaos to Order, by John Holland. Perseus Press, April 1999. ISBN 0-738-20142-1.

Euclid: The Creation of Mathematics, by Benno Artmann. Springer-Verlag, June 1999. ISBN 0-387-98423-2.

Fermat's Last Theorem for Amateurs, by Paulo Ribenboim. Springer-Verlag, February 1999. ISBN 0-387-98508-5.

**Five More Golden Rules: Knots, Codes, Chaos and Other Great Theories of 20th Century Mathematics*, by John L. Casti. John Wiley & Sons, February 2000. ISBN 0-471-32233-4.

Fragile Dominion: Complexity and the Commons, by Simon Levin. Perseus Books, June 1999. ISBN 0-738-20111-1.

A History of the Circle: Mathematical Reasoning and the Physical Universe, by Ernest Zebrowski Jr. Rutgers University Press, August 1999. ISBN 0-813-52677-9.

The Importance of Being Fuzzy and Other Insights from the Border Between Math and Computers, by Arturo Sangalli. Princeton University Press, December 1998. ISBN 0-691-00144-8.

Imaginary Numbers: An Anthology of Marvelous Mathematical Stories, Diversions, Poems, and Musings, edited by William Frucht. John Wiley & Sons, October 1999. ISBN 0-471-33244-5.

An Imaginary Tale: The Story of $\sqrt{-1}$, by Paul J. Nahin. Princeton University Press, November 1998. ISBN 0-691-02795-1. (Reviewed November 1999.)

In the Light of Logic, by Solomon Feferman. Oxford University Press, September 1998. ISBN 0-195-08030-0.

The Invention of Infinity: Mathematics and Art in the Renaissance, by J. V. Field. Oxford University Press, May 1997. ISBN 0-198-52394-7. (Reviewed January 2000.)

Jacques Hadamard, A Universal Mathematician, by Vladimir Mazýa and Tatyana Shaposhnikova. AMS/London Mathematical Society, January 1998. ISBN 0-821-80841-9.

James Joseph Sylvester: Life and Work in Letters, by Karen Hunger Parshall. Oxford University Press, October 1998. ISBN 0-198-50391-1.

John von Neumann: The Scientific Genius Who Pioneered the Modern Computer, Game Theory, Nuclear Deterrence, and Much More, by Norman Macrae. AMS, October 1999. ISBN 0-821-82064-8.

The Magical Maze: Seeing the World through Mathematical Eyes, by Ian Stewart. John Wiley & Sons, April 1998. ISBN 0-471-19297-X.

The Mathemagician and the Pied Puzzler: A Collection in Tribute to Martin Gardner, edited by Elwyn Berlekamp and Tom Rodgers. A K Peters, March 1999. ISBN 1-568-81075-X.

A Mathematical Mystery Tour: Discovering the Truth and Beauty of the Cosmos, by A. K. Dewdney. John Wiley & Sons, March 1999. ISBN 0-471-23847-3. (Reviewed February 2000.)

**Mathematical Sorcery: Revealing the Secrets of Numbers*, by Calvin C. Clawson. Plenum Press, May 1999. ISBN 0-306-46003-3.

Mathematics and Mathematicians: Mathematics in Sweden before 1950, by Lars Gårding. AMS/London Mathematical Society, 1998. ISBN 0-821-80612-2.

The Mathematics of Ciphers: Number Theory and RSA Cryptography, by S. C. Coutinho. A K Peters, November 1998. ISBN 1-568-81082-2.

**Mathematics: The New Golden Age*, by Keith Devlin. Columbia University Press, 1999.

Mathematics Without Borders: A History of the International Mathematical Union, by Olli Lehto. Springer-Verlag, February 1998. ISBN 0-387-98358-9. (Reviewed November 1999.)

The Moment of Proof: Mathematical Epiphanies, by Donald C. Benson. Oxford University Press, March 1999. ISBN 0-195-11721-2.

Mystic, Geometer, and Intuitionist: The Life of L. E. J. Brouwer, by Dirk Van Dalen. Oxford University Press, April 1999. ISBN 0-198-50297-4.

The Nature of Mathematical Modeling, by Neil Gershenfeld. Cambridge University Press, February 1999. ISBN 0-521-57095-6.

New Directions in the Philosophy of Mathematics: An Anthology, Thomas Tymoczko, Editor. Princeton University Press, revised edition, January 1998. ISBN 0-691-03498-2.

Noeuds: Genèse d'une théorie mathématique (Knots: Genesis of a Mathematical Theory), by Alexei Sossinsky (in French). Seuil, 1999. ISBN 2-02-032089-4.

**The Nothing That Is: A Natural History of Zero*, by Robert Kaplan. Oxford University Press, October 1999. ISBN 0-195-12842-7.

The Number Devil, by Hans Magnus Enzensberger. Metropolitan Books, October 1998. ISBN 0-805-05770-6. (Reviewed January 2000.)

The Number Sense: How the Mind Creates Mathematics, by Stanislas Dehaene. Oxford University Press, October 1997. ISBN 0-195-11004-8.

Philosophy of Mathematics: An Introduction to a World of Proofs and Pictures, by James Robert Brown. Routledge, August 1999. ISBN 0-415-12274-0.

Pioneers of Representation Theory: Frobenius, Burnside, Schur, and Brauer, by Charles W. Curtis. AMS/London Mathematical Society, October 1999. ISBN 0-821-89002-6.

The Queen of Mathematics: A Historically Motivated Guide to Number Theory, by Jay R. Goldman. A K Peters, November 1997. ISBN 1-568-81006-7.

Shadows of the Circle: Conic Sections, Optimal Figures and Non-Euclidean Geometry, by Vagn Lundsgaard Hansen. World Scientific Publishing Company, November 1998. ISBN 9-810-23418-X.

Slicing Pizzas, Racing Turtles, and Further Adventures in Applied Mathematics, by Robert B. Banks. Princeton University Press, September 1999. ISBN 0-691-05947-0.

Small Worlds: The Dynamics of Networks Between Order and Randomness, by Duncan J. Watts. Princeton University Press, November 1999. ISBN 0-691-00541-9.

Statistics on the Table: The History of Statistical Concepts and Methods, by Stephen M. Stigler. Harvard University Press, November 1999. ISBN 0-674-83601-4.

Stephen Smale: The Mathematician Who Broke the Dimension Barrier, by Steve Batterson. AMS, February 2000. ISBN 0-821-82045-1.

Turing and the Computer (The Big Idea), by Paul Strathern. Anchor Books, April 1999. ISBN 0-385-49243-X.

**The Universal History of Numbers: From Prehistory to the Invention of the Computer*, by Georges Ifrah (translated by David Bellos, Sophie Wood, and Ian Monk). John Wiley & Sons, December 1999. ISBN 0-471-37568-3.

What Counts: How Every Brain is Hardwired for Math, by Brian Butterworth. Free Press, August 1999. ISBN 0-684-85417-1.

What is Mathematics, Really?, by Reuben Hersh, Oxford University Press, August 1997. ISBN 0-19-511368-3. (Reviewed October 1999.)

What is Random?: Discovering Chance and Order in Mathematics and the World, by Edward J. Beltrami. Springer-Verlag, August 1999. ISBN 0-387-98737-1

What's Happening in the Mathematical Sciences, 1998–1999, by Barry Cipra. AMS, December 1998. ISBN 0-821-80766-8.

Why Do Buses Come In Threes?, by Rob Eastaway and Jeremy Wyndham. John Wiley & Sons, May 1999. ISBN 0-471-34756-6.

The World According to Wavelets, by Barbara Burke Hubbard. A K Peters, second edition, April 1998. ISBN 1-568-81072-5. (Reviewed October 1999.)

**Added to the "Book List" since the List's last appearance.*

Leroy P. Steele Prizes

Call for Nominations

The selection committee for this prize requests nominations for consideration for the 2001 award. Further information about this prize can be found in the November 1999 *Notices*, pp. 1258-1269 (also available at http://www.ams.org/ams/prizes.html).

Three Leroy P. Steele Prizes are awarded each year in the following categories: (1) the Steele Prize for Lifetime Achievement: for the cumulative influence of the total mathematical work of the recipient, high level of research over a period of time, particular influence on the development of a field, and influence on mathematics through Ph.D. students; (2) the Steele Prize for Mathematical Exposition: for a book or substantial survey or expository-research paper; and (3) the Steele Prize for Seminal Contribution to Research: for a paper, whether recent or not, that has proved to be of fundamental or lasting importance in its field, or a model of important research. In 2001 the prize for Seminal Contribution to Research will be awarded for a paper in Applied Mathematics.

Nominations with supporting information should be submitted to the Secretary, Robert J. Daverman, American Mathematical Society, 312D Ayres Hall, University of Tennessee, Knoxville, TN 37996-1330. Include a short description on the work that is the basis of the nomination, including complete biographic citations. A curriculum vitae should be included. The nominations will be forwarded by the Secretary to the prize selection committee, which will, as in the past, make final decisions on the awarding of prizes.

Deadline for nominations is March 31, 2000.

AMS

AMERICAN MATHEMATICAL SOCIETY

2000 Frank and Brennie Morgan AMS-MAA-SIAM Prize for Outstanding Research in Mathematics by an Undergraduate Student

The prize is awarded each year to an undergraduate student (or students having submitted joint work) for outstanding research in mathematics. Any student who is an undergraduate in a college or university in the United States or its possessions, or Canada or Mexico, is eligible to be considered for this prize.

The prize recipient's research need not be confined to a single paper; it may be contained in several papers. However, the paper (or papers) to be considered for the prize must be submitted while the student is an undergraduate; they cannot be submitted after the student's graduation. The research paper (or papers) may be submitted for consideration by the student or a nominator. All submissions for the prize must include at least one letter of support from a person, usually a faculty member, familiar with the student's research. Publication of research is not required.

The recipients of the prize are to be selected by a standing joint committee of the AMS, MAA, and SIAM. The decisions of this committee are final. The 2000 prize will be awarded for papers submitted for consideration no later than **June 30, 2000,** by (or on behalf of) students who were undergraduates in December 1999.

Nominations and submissions should be sent to:

Morgan Prize Committee
c/o Robert J. Daverman, Secretary
American Mathematical Society
Department of Mathematics
University of Tennessee
Knoxville, TN 37996-1330

Questions may be directed to the chairperson of the Morgan Prize Committee:

Robby Robson
Department of Mathematics
Oregon State University
Corvallis, OR 97331-4605
telephone: 541-737-5171
e-mail: robby@math.orst.edu

AMERICAN MATHEMATICAL SOCIETY

Mathematical Reviews
Associate Editor

Applications and recommendations are invited for a full-time position as an Associate Editor of Mathematical Reviews (MR), to commence as soon as possible after July 1, 2000, and no later than January 1, 2001.

The Mathematical Reviews division of the American Mathematical Society (AMS) is located in Ann Arbor, Michigan, not far from the campus of the University of Michigan. The editors are employees of the AMS; they also enjoy many privileges at the University. At present, MR employs fourteen mathematical editors, about six consultants, and a further sixty nonmathematicians. MR's mission is to develop and maintain the AMS databases of secondary sources covering the published mathematical literature. The chief responsibility is the development and maintenance of the MR Database, from which all MR-related products are produced: the journals *Mathematical Reviews* and *Current Mathematical Publications,* MathSciNet and MathSciDisc, and various other derived products. The responsibilities of an Associate Editor fall primarily in the day-to-day operations of selecting articles and books suitable for coverage in the MR database, classifying these items, determining the type of coverage, assigning those selected for review to reviewers, editing the reviews when they are returned, and correcting the galley proofs. An individual with considerable breadth in pure and applied mathematics is sought; preference will be given to those applicants with expertise in one or more of the following areas: partial differential equations (MSC section 35), numerical analysis (65), social and biological sciences (91, 92). The ability to write good English is essential and the ability to read mathematics in major foreign languages is important. It is desirable that the applicant have several years' relevant academic (or equivalent) experience beyond the Ph.D.

The twelve-month salary will be commensurate with the experience the applicant brings to the position. Interested applicants are encouraged to write (or telephone) for further information. Persons interested in taking extended leave from an academic appointment to accept the position are encouraged to apply.

Applications, including curriculum vitae; bibliography; name, address, and phone number of at least three references; and recommendations should be sent to:

Dr. Jane E. Kister
Executive Editor
Mathematical Reviews
P.O. Box 8604
Ann Arbor, MI 48107-8604

Telephone: 734-996-5257
Fax: 734-996-2916
e-mail: `jek@ams.org`

The closing date for applications is May 1, 2000.

AMS
AMERICAN MATHEMATICAL SOCIETY

The American Mathematical Society is an Equal Opportunity Employer.

Add this Cover Sheet to all of your Academic Job Applications

How to use this form

1. Using the facing page or a photocopy, (or a TEX version which can be downloaded from the e-math "Employment Information" menu, `http://www.ams.org/employment/`), fill in the answers which apply to *all* of your academic applications. Make photocopies.

2. As you mail each application, fill in the remaining questions neatly on one cover sheet and include it *on top of* your application materials.

The Joint Committee on Employment Opportunities has adopted the cover sheet on the facing page as an aid to job applicants and prospective employers. The form is now available on e-math in a TEX format which can be downloaded and edited. The purpose of the cover form is to aid department staff in tracking and responding to each application.

Mathematics Departments in Bachelor's, Master's and Doctorate granting institutions have been contacted and are expecting to receive the form from each applicant, along with any other application materials they require. Obviously, not all departments will utilize the cover form information in the same manner. Please direct all general questions and comments about the form to:
`emp-info@ams.org`
or call the Professional Programs and Services Department, AMS, at 800-321-4267 extension 4105.

JCEO Recommendations for Professional Standards in Hiring Practices

The JCEO believes that every applicant is entitled to the courtesy of a prompt and accurate response that provides timely information about his/her status. Specifically, the JCEO urges all institutions to do the following after receiving an application:

(1) Acknowledge receipt of the application—immediately; and
(2) Provide information as to the current status of the application, as soon as possible.

The JCEO recommends a triage-based response, informing the applicant that he/she
(a) is not being considered further;
(b) is not among the top candidates; or
(c) is a strong match for the position.

This form is provided courtesy of the American Mathematical Society.

This cover sheet is provided as an aid to departments in processing job applications. It should be included with your application material.

Please print or type. Do not send this form to the AMS.

Academic Employment in Mathematics

AMS Standard Cover Sheet

Last Name ______________________________

First Name ______________________________

Middle Names ______________________________

Address *through next June* ______________________________ **Home Phone** ______________

____

______________________________ **e-mail Address** ______________

____

Current Institutional Affiliation ______________________________ **Work Phone** ______________

Highest Degree and Source ______________________________

Year of Ph.D. (optional) ______________________________

Ph.D. Advisor ______________________________

If the Ph.D. is not presently held, date on which you expect to receive ______________

Indicate *the mathematical subject area(s) in which you have done research using, if applicable, the Mathematics Subject Classification printed on the back of this form or on e-MATH. If listing more than one number, list first the one number which best describes your current primary interest.*

Primary Interest ______________________________

Secondary Interests *optional* ______________________________

Give a brief synopsis *of your current research interests (e.g. finite group actions on four-manifolds). Avoid special mathematical symbols and please do not write outside of the boxed area.*

____

____

Most recent, if any, position held *post Ph.D.*

University or Company ______________________________

Position Title ______________________________

Indicate the position for which you are applying and position posting code, if applicable

____

If unsuccessful for this position, would you like to be considered for a temporary position?

☐ Yes ☐ No If yes, please check the appropriate boxes.

☐ Postdoctoral Position ☐ 2+ Year Position ☐ 1 Year Position

List the names, affiliations, and e-mail addresses *of up to four individuals who will provide letters of recommendation if asked. Mark the box provided for each individual whom you have already asked to send a letter.*

☐ ______________________________

☐ ______________________________

☐ ______________________________

☐

2000 Mathematics Subject Classification

00 General
01 History and biography
03 Mathematical logic and foundations
05 Combinatorics
06 Order, lattices, ordered algebraic structures
08 General algebraic systems
11 Number theory
12 Field theory and polynomials
13 Commutative rings and algebras
14 Algebraic geometry
15 Linear and multilinear algebra, matrix theory
16 Associative rings and algebras
17 Nonassociative rings and algebras
18 Category theory, homological algebra
19 *K*-theory
20 Group theory and generalizations
22 Topological groups, Lie groups
26 Real functions
28 Measure and integration
30 Functions of a complex variable
31 Potential theory
32 Several complex variables and analytic spaces
33 Special functions
34 Ordinary differential equations
35 Partial differential equations
37 Dynamical systems and ergodic theory
39 Difference and functional equations
40 Sequences, series, summability
41 Approximations and expansions
42 Fourier analysis
43 Abstract harmonic analysis
44 Integral transforms, operational calculus
45 Integral equations
46 Functional analysis
47 Operator theory
49 Calculus of variations and optimal control, optimization
51 Geometry
52 Convex and discrete geometry
53 Differential geometry
54 General topology
55 Algebraic topology
57 Manifolds and cell complexes
58 Global analysis, analysis on manifolds
60 Probability theory and stochastic processes
62 Statistics
65 Numerical analysis
68 Computer science
70 Mechanics of particles and systems
74 Mechanics of deformable solids
76 Fluid mechanics
78 Optics, electromagnetic theory
80 Classical thermodynamics, heat transfer
81 Quantum theory
82 Statistical mechanics, structure of matter
83 Relativity and gravitational theory
85 Astronomy and astrophysics
86 Geophysics
90 Operations research, mathematical programming
91 Game theory, economics, social and behavioral sciences
92 Biology and other natural sciences
93 Systems theory, control
94 Information and communication, circuits
97 Mathematics education

Mathematics Calendar

The most comprehensive and up-to-date Mathematics Calendar information is available on e-MATH at `http://www.ams.org/mathcal/`.

February 2000

1-5 **Harmonic Maps and Minimal Immersions**, Caparide, (Lisbon), Portugal. (Jan. 2000, p. 80)

7-11 **MSRI Workshop on The Mathematics of Quantum Computation**, MSRI, Berkeley, California. (June/July 1999, p. 713)

8-12 **ANZIAM 2000**, Copthorne Resort, Waitangi, New Zealand. (Sept. 1999, p. 980)

9-13 **IMA Workshop: Resource Recovery**, IMA, University of Minnesota, Minneapolis, Minnesota. (May 1999, p. 596)

12-13 **Workshop on Nonlinear Dispersive Equations**, Stanford University, Palo Alto, California. (Jan. 2000, p. 80)

*14-19 **Spring School: Operator Algebras and Index Theory on Manifolds with Singularities**, University of Potsdam, Potsdam, Germany.

Topics: Boundary value problems, heat equation methods, edge and corner pseudodifferential operators, index theory on singular and non-compact spaces, asymptotics of solutions, anistropic operators, hyperbolic singular problems, nonlinear problems.

Organizers: E. Schrohe, (`schrohe@math.uni-potsdam.de`) and B.-W. Schulze, (`schulze@math.uni-potsdam.de`).

Sponsors: The conference is supported by the Max-Planck-Gesellschaft zur Foerderung der Wissenschaften e.V. and by the Deutsche Forschungsgemeinschaft.

Participation: Graduate students and young researchers are particularly welcome to attend.

Information: For information about the conference and possible support please contact the organizers.

14-25 **MSRI Workshop on the Interactions between Algebraic Geometry and Noncommutative Algebra**, MSRI, Berkeley, California. (June/July 1999, p. 713)

15-18 **Workshop on Combinatorial and Computational Mathematics: Present and Future**, Pohang Univ. of Science and Technology (POSTECH), Pohang, South Korea. (Dec. 1999, p. 1433)

*19-23 **New Trends in Potential Theory and Applications**, University of Bielefeld, Bielefeld, Germany.

Program: This conference is meant to be a continuation of a series of international conferences on Potential Theory and related fields (as, e.g., the ones in Prague '87, Amersfoort '91, Kouty '94, Hammamet '98). The beginning of the new millennium seems appropriate to reflect on the current developments and to specify new promising directions of research in this classical area of mathematics. Emphasis will be given to various applications, in particular, in mathematical physics. We would also like to celebrate the 60th birthday of our colleague and friend, Wolfhard Hansen, on the afternoon of February 22.

Topics: The conference will include the topics: differential geometry, Dirichlet forms, fractals, linear and nonlinear PDE, Schrödinger operators, Markov processes, stochastic analysis.

Information: Anyone interested in taking part in the conference should contact the organizers via e-mail. Further information will be provided soon. V. Metz, Fakultät für Mathematik, Universität Bielefeld, Postfach 100131, D-33501 Bielefeld, Germany; e-mail: `metz@mathematik.uni-bielefeld.de`.

20-22 **International Conference on Stochastic Optimization: Algorithms and Applications**, University of Florida, Center for Applied Optimization, Gainesville, Florida. (Jan. 1999, p. 72)

*20-22 **Workshop on Computing Approximate Solutions to NP-Hard Problems**, Princeton University, Princeton, New Jersey.

Sponsors: DIMACS Center; Rutgers Univer-

This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.

An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.

In general, announcements of meetings and conferences held in North America carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. Meetings held outside the North American area may carry more detailed information. In any case, if there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences in the mathematical sciences should be sent to the Editor of the *Notices* in care of the American Mathematical Society in Providence or electronically to `notices@ams.org` or `mathcal@ams.org`.

In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the *Notices* prior to the meeting in question. To achieve this, listings should be received in Providence **six months** prior to the scheduled date of the meeting.

The complete listing of the Mathematics Calendar will be published only in the September issue of the *Notices*. The March, June, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.

The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through e-MATH on the World Wide Web. To access e-MATH, use the URL: `http://e-math.ams.org/` (or `http://www.ams.org/`). (For those with VT100-type terminals or for those without WWW browsing software, connect to e-MATH via Telnet (`telnet e-math.ams.org`; login and password `e-math`) and use the Lynx option from the main menu.)

sity; and Princeton University, Department of Computer Science.

Organizers: S. Arora (Princeton Univ.), arora@cs.princeton.edu; E. Tardos (Cornell Univ.), eva@cs.cornell.edu; L. Trevisan (Columbia Univ.), luca@cs.columbia.edu.

Local Arrangements: Sandy Barbu, barbu@cs.princeton.edu, 609-258-4562.

Short Description: This workshop focuses on the design and analysis approximation algorithms, heuristics that compute solutions whose value is probably within some fixed factor of the optimal. The workshop will start with a set of plenary talks, geared to a broad audience, surveying the current state of the art in this growing area. These will be followed by talks on recent results in approximation algorithms and lower bounds. We plan to leave enough free time during the workshop that the participants can discuss open problems and make progress during the workshop.

Information: http://dimacs.rutgers.edu/Workshop/Approx/.

*23-24 **Workshop on Faster Exact Solutions for NP-Hard Problems**, Princeton University, Princeton, New Jersey.

Sponsors: DIMACS Center, Rutgers University.

Organizers: M. Paturi (Univ. of California, San Diego), paturi@cs.ucsd.edu; R. Beigel (Univ. of Illinois at Chicago), beigel@uic.edu.

Contact: Mohan Paturi, Univ. of California, San Diego, paturi@cs.ucsd.edu.

Local Arrangements: Sandy Barbu, barbu@cs.princeton.edu, 609-258-4562.

Short Description: We seek out faster exact solutions for NP-hard problems, not only for their own sake, but also in order to gain insights into complexity. This workshop is organized to specifically examine the following topics for NP-hard problems: (1) algorithms with improved exponential-time complexity, (2) relationships among the complexities, (3) obstacles to subexponential algorithms, and (4) heuristics for faster exponential-time algorithms and empirical studies. The workshop will consist mostly of short presentations (30 minutes) and one or two full-length presentations (60 minutes). Anyone who wishes to give a talk should contact the organizers by December 31, 1999.

Information: http://dimacs.rutgers.edu/Workshops/Faster/.

24-26 **Third New Mexico Analysis Seminar**, New Mexico State University, Las Cruces, New Mexico. (Feb. 2000, p. 282)

25-26 **XV SIDIM (Inter-university Mathematical Research Seminar)**, University of Puerto Rico, Mayaguez Campus, Mayaguez, Puerto Rico. (Sept. 1999, p. 980)

26-March 3 **Sven Hagströmer & Mats Qviberg's Foundation Symposium on Geometry and Regularity of Free Boundaries**, KTH, Stockholm, Sweden. (Feb. 2000, p. 282)

28-March 3 **Eighth International Conference on Hyperbolic Problems**, Otto-von-Guericke University, Magdeburg, Germany. (Sept. 1999, p. 980)

March 2000

2-4 **Recent Progress in The Study of Harmonic Measure from a Geometric and an Analytic Point of View**, University of Arkansas Continuing Education Center, Fayetteville, Arkansas. (Dec. 1999, p. 1433)

5-8 **Eighth International Conference on Numerical Combustion**, Amelia Inn and Beach Club, Amelia Plantation, Amelia Island, Florida. (Feb. 2000, p. 282)

6-7 **Data Processing on the Web: A Look into the Future**, DIMACS Center, Rutgers University, Piscataway, New Jersey. (Dec. 1999, p. 1433)

6-10 **4th International Conference on Operations Research Optimization, Probability and Statistics, Mathematical Economics and Algorithms**, Havana, Cuba. (Oct. 1999, p. 1107)

6-10 **International Conference on Differential Geometry and Quantum Physics**, TU Berlin, Germany. (Sept. 1999, p. 980)

6-17 **Homogenization and Effective Media Theories**, Mathematical Sciences Research Institute, Berkeley, California. (Sept. 1998, p. 1053)

11-15 **Arizona Winter School 2000: Topics in the Arithmetic of Function Fields**, The University of Arizona, Tucson, Arizona. (Feb. 2000, p.282)

13-16 **Geometry and Applications**, Sobolev Institute of Mathematics, Novosibirsk, Russia. (Jan. 2000, p. 80)

13-17 **International Conference on Fundamental Sciences: Mathematics and Theoretical Physics**, Singapore, China. (Nov. 1999, p. 1286)

*15-18 **Memphis Lectures on Mathematics**, University of Memphis, Memphis, TN.

Topic: Partial differential equations.

Funding: Limited funding is available for graduate students and young researchers to attend.

Information: Contact either goldstej@msci.memphis.edu or gizhang@memphis.edu.

15-19 **IMA Workshop: Atmospheric Modeling**, IMA, University of Minnesota, Minneapolis, Minnesota. (May 1999, p. 596)

16 **Marvin Rosenblum Day**, University of Virginia, Charlottesville, Virginia. (Jan. 2000, p. 81)

*16 **Yamabe Memorial Lecture**, School of Mathematics, University of Minnesota, Minneapolis, Minnesota.

Speaker: Professor Jeff Cheeger of Courant Institute will speak on analysis on metric measure spaces.

Sponsors: This series has been established jointly by Northwestern University and the University of Minnesota in memory of Hidehiko Yamabe (1923-1960), whose significant work on topological groups and geometry were outstanding contributions to modern mathematics.

Events: Tea will be served at 3:00 p.m. in Vincent Hall 120. A dinner is planned for that evening. Please let Kathy Swedell (612-626-7422) know if you are planning to attend the dinner.

16-18 **Seminar on Stochastic Processes, 2000**, The University of Utah, Salt Lake City, Utah. (Feb. 1999, p. 278)

17-18 **Southeastern Analysis Meeting**, University of Virginia, Charlottesville, Virginia. (Jan. 2000, p. 81)

*17-26 **JAMI 2000: Recent Advances in Homotopy Theory**, Johns Hopkins University, Baltimore, Maryland.

Program: Invited talks and contributed talks. A banquet on March 25 will honor the 60th birthdays of P. Landweber and S. Priddy.

Organizers: J. M. Boardman, D. Davis, J.-P. Meyer, J. Morava, W. S. Wilson.

Speakers: G. Arone (Aberdeen), D. Christensen (IAS), W. Dwyer (Notre Dame), P. Goerss (Northwestern), P. Hu (Chicago), K. Iriye (Osaka), N. Iwase (Kyushu), Y. Kawamoto (Hiroshima), I. Kriz (Michigan), M. Mahowald (Northwestern), J. P. May (Chicago), R. McCarthy (Illinois), J. McClure (Purdue), H. Miller (MIT), N. Minami (Nagoya), L. Nave (MIT), G. Nishida (Kyoto), S. Priddy (Northwestern), D. Ravenel (Rochester), J. Rognes (Oslo), H. Sadofsky (Oregon), K. Shimomura (Kochi), B. Shipley (Purdue), J. Smith (Purdue), P. Turner (Heriot-Watt), V. Voevodsky (IAS), N. Yagita (Ibaragi).

Information: See http://mathnt.mat.jhu.edu/jami9900, or contact W. S. Wilson at wsw@math.jhu.edu.

20-22 **DIMACS Workshop on Cryptography and Intractability**, DIMACS Center, Rutgers University, Piscataway, New Jersey. (Nov. 1999, p. 1286)

20-31 **Superconvergence in Finite Element Methods**, Mathematical Sciences Research Institute, Berkeley, California. (Sept. 1998, p. 1053)

*25-26 **Midwest Partial Differential Equations Seminar**, Purdue University, West Lafayette, Indiana.

Speakers: L. Caffarelli (Univ. of Texas), G. Q. Chen (Northwestern Univ.), T. Christiansen (Univ. of Missouri), M. Feldman (Univ. of Wisconsin), C. Kenig (Univ. of Chicago), D. Kinderlehrer (Carnegie Mellon Univ.), M. Loss (Georgia Institute of Technology), N. Nadirashvili (Univ. of Chicago), T. Riviere (Ecole Normale Suprieure de Cachan).

Information: See the Web site http://www.math.purdue.edu/research/events/mwpde.html.

27-31 **Quantum Groups**, Morelia, Mexico. (Oct. 1999, p. 1108)

31-April 2 **CombinaTexas: The South-Central Regional Combinatorics Conference**, Texas A&M University, College Station, Texas. (Jan. 2000, p. 81)

April 2000

1-2 **AMS Eastern Section Meeting**, University of Massachusetts, Lowell, Massachusetts. (Nov. 1998, p. 1378)
Information: Information will be posted to the meetings pages in e-MATH.

7-9 **AMS Central Sectional Meeting**, University of Notre Dame, Notre Dame, Indiana. (Sept. 1997, p. 1031)
Information: W. Drady, e-mail: wsd@ams.org.

9-16 **The Klee-Grunbaum Festival of Geometry**, Ein Gev, Israel. (June/July 1999, p. 713)

11-14 **Harmonic Maps and Curvature Properties of Submanifolds, 2**, University of Leeds, England. (Sept. 1999, p. 981)

14-16 **AMS Southeastern Section Meeting**, University of Southwestern Louisiana, Lafayette, Louisiana. (Mar. 1999, p. 380)
Information: See the AMS Meetings & Conferences pages on e-MATH, or contact Donna Salter, dls@ams.org.

15 **50th Algebra Day**, Carleton University, Ottawa, Canada. (Jan. 2000, p. 81)

16-19 **FRACTAL 2000: "Complexity and Fractals in the Sciences", 6th International Multidisciplinary Conference**, Singapore. (May 1999, p. 596)

17-18 **Management of Digital Intellectual Property**, DIMACS Center, Rutgers University, Piscataway, New Jersey. (Dec. 1999, p. 1434)

17-28 **Elastic Shells: Modeling, Analysis and Numerics**, Mathematical Sciences Research Institute, Berkeley, California. (Sept. 1998, p. 1054)

23-29 **Spring School on Functional Analysis**, Paseky nad Jizerou, Czech Republic. (Jan. 2000, p. 81)

25-May 6 **NATO Advanced Study Institute, Nonlinear Dynamics in Life and Social Sciences**, Moscow, Russia. (Nov. 1999, p. 1287)

28-30 **Riviere-Fabes Symposium on Analysis and PDE**, University of Minnesota, Minneapolis, Minnesota. (Jan. 2000, p. 81)

May 2000

1-5 **Dispersive Corrections to Transport Equations**, IMA, University of Minnesota, Minneapolis, Minnesota. (Feb. 2000, p. 283)

1-5 **IMA Workshop: Dispersive Corrections to Transport Equations**, IMA, University of Minnesota, Minneapolis, Minnesota. (May 1999, p. 596)

8-12 **Geometric and Topological Aspects of Group Theory**, MSRI, Berkeley, California. (Oct. 1999, p. 1108)

10-12 **ICNPAA-2000, Third International Conference on Nonlinear Problems in Aviation and Aerospace (Methods and Software)**, Daytona Beach, Florida. (June/July 1999, p. 714)

*11-15 **International Conference Dedicated to 150th Birthday of Sofia Kovalevskaya: Theory of Partial Differential Equations and Special Topics of Theory of Ordinary Differential Equations**, Euler International Mathematical Institute, St. Petersburg, Russia.
Organizers: Steklov Institute of Mathematics at St. Petersburg, Euler International Mathematical Institute, St. Petersburg Mathematical Society, St. Petersburg State University.
Program Committee: L. D. Faddeev, chair (St. Petersburg, Russia), O. A. Ladyzhenskaya (St. Petersburg, Russia), V. M. Babich (St. Petersburg, Russia), E. F. Mischenko (Moscow, Russia), M. A. Semenov-Tyan-Shansky (Russia-France), N. N. Ural'tseva (St. Petersburg, Russia).
Topics: Preferable topics connect with famous papers of S. V. Kovalevskaya: Cauchy-Kovalevskaya theorem and Kovalevskaya case of the rotation of a solid body about a fixed point.
Information: http://www.pdmi.ras.ru/EIMI/2000/sofia/.

15-18 **Representation Theory and Computational Algebra**, University of Georgia, Athens, Georgia. (Nov. 1999, p. 1287)

*15-19 **International Conference on Mathematical Physics, Simulation and Approximate Methods**, Obninsk, Russia. (Dec. 1999, p. 1434)
Dedication: This conference is dedicated to the memory of famous Russian mathematician Andrey Tikhonov (1906-1993). He influenced the development of topology, mathematical physics, differential equations theory with small parameter, ill-posed problems, numerical methods, and mathematical simulations in physics.
Topics: The following sections are supposed: (1) conservation laws, (2) mathematical modelling, (3) numerical methods, (4) small parameter methods, (5) ill-posed problems.
Organizer: Chairman of the Program Committee is academician A. Samarskii.
Datelines: Deadline for abstracts: 15 April 2000.
Information: Prof. V. A. Galkin, Dept. Applied Mathematics, IATE, Obninsk, 249020, Russia; e-mail: tikhonov@iate.obninsk.ru, galkin@iate.obninsk.ru.

15-19 **International Workshop and Conference in Mathematical Analysis and Applications**, Department of Mathematics, Chiangmai University, Thailand.

17-20 **Trends in Approximation Theory, an International Symposium Celebrating the 60th Birthday of Larry L. Schumaker, held in conjunction with the 15th Annual Shanks Lecture**, Vanderbilt University, Nashville, Tennessee. (May 1999, p. 596)

18-19 **IMA Tutorial: Simulation of Transport in Transition Regimes**, IMA, University of Minnesota, Minneapolis, Minnesota. (May 1999, p. 597)

18-21 **Year 2000 International Conference on Dynamical Systems and Differential Equations**, Kennesaw State University, Kennesaw, Georgia. (June/July 1999, p. 714)

19-21 **Middle School Mathematics Teacher Preparation**, Branson, Missouri. (Feb. 2000, p. 283)

20-22 **Schloessmann Seminar on Mathematical Models in Biology, Chemistry, and Physics**, Bad Lausick, Germany. (Dec. 1999, p. 1434)

20-25 **Summer School on Stereology and Geometric Tomography**, Sandbjerg Manor, Denmark. (Sept. 1999, p. 981)

*21-24 **Rencontre 2000 des Mathématiciens Algériens (Meeting 2000 of Algerian Mathematicians)**, Algiers, Algeria.
Information: The Web site of the meeting is http://www.ama.ass.dz.

21-26 **Millennial Conference on Number Theory**, University of Illinois, Urbana, Illinois. (May 1999, p. 597)

22-26 **IMA Workshop: Simulation of Transport in Transition Regimes**, IMA, University of Minnesota, Minneapolis, Minnesota. (May 1999, p. 597)

23-27 **Great Plains Operator Theory Symposium (GPOTS)**, The Caribe Hilton Hotel, San Juan, Puerto Rico. (Feb. 2000, p. 283)

23-27 **Summer Symposium in Real Analysis**, University of North Texas, Denton, Texas. (Sept. 1999, p. 981)

24-26 **Advances in Fluid Mechanics**, Montreal, Canada. (Aug. 1999, p. 815)

28-June 2 **Nonlinear Analysis 2000→**, Courant Institute, New York University, New York. (June/July 1999, p. 714)

28-June 3 **Combinatorics 2000**, Hotel Serapo, Gaeta, Italy. (Feb. 2000, p. 283)

28-June 3 **Spring School on Analysis**, Paseky nad Jizerou, Czech Republic. (Jan. 2000, p. 81)

29-June 2 **Second International Encounter on Integer-Valued Polynomials**, CIRM, Luminy, France. (Feb. 2000, p. 283)

29-June 9 **Foliation: Geometry and Dynamics Revisited**, Banach Center, Warsaw, Poland. (June/July 1999, p. 714; Nov. 1999, p. 1287)

29-June 9 **NATO Advanced Study Institute "Special Functions 2000"**, Arizona State University, Tempe, Arizona. (Feb. 2000, p. 283)

June 2000

* 1-3 **7th Chico Topology Conference**, California State University, Chico, Chico, CA.
Focus: The conference is devoted to the topology of continuum theory. There will be four or five 1-hour talks plus sessions for contributed talks.
Organizer: E. J. Vought, Department of Mathematics and Statistics, California State University, Chico, Chico, CA 95929.
Abstracts: Abstracts should be sent in hard copy to E. J. Vought at the above address. The deadline for submission is April 20, 2000.
Information: Contact E. J. Vought at the above address or at either of these e-mail addresses: `eeevought@worldnet.att.net` or `evought@csuchico.edu`.

3-7 **1999-2000 ASL Annual Meeting**, University of Illinois at Urbana-Champaign, Illinois. (Jan. 1999, p. 72)

* 3-8 **The Sixth International Conference on Mathematical Population Dynamics**, Marrakech, Morocco.
Program: The Sixth International Conference on Mathematical Population Dynamics is an interdisciplinary meeting of biologists and mathematicians concerned with populations of bio molecules, genes, cells, and living organisms, as well as other topics of mathematical population biology and epidemiology. The meeting will be focused on (both deterministic and stochastic) mathematical theory, model analysis, and modelling of quantitative data pertaining to cell and molecular biology, epidemiology, cancer research, population genetics, population biology, and other related areas. Proposals for topics and subjects may be submitted to the conference secretaries: M. Alexandersson, U. Olofsson, Z. Taib.
Application: Please send the application by mail, fax, e-mail; or send the online form that can be found at the conference home page, preferably before September 1, 1999, to one of the scientific secretaries and mark the envelop "MPD6".
Information: Department of Mathematics, Chalmers University of Technology, and the University of Göteborg, S-412 96 Göteborg, Sweden; phone: +46 31 772 35 30; fax: +46 31 772 35 08; e-mail: `mpd6@math.chalmers.se`.

4-9 **Ninth Quadrennial International Conference on Graph Theory, Combinatorics, Algorithms, and Applications**, Western Michigan University, Kalamazoo, Michigan. (Oct. 1999, p. 1108)

5-9 **Advances in Convex Analysis and Global Optimization Honoring the Memory of C. Caratheodory (1873-1950)**, Pythagorion, Samos, Greece. (Dec. 1999, p. 1435)

5-9 **IMA Workshop: Multiscale Models for Surface Evolution and Reacting Flows**, IMA, University of Minnesota, Minneapolis, Minnesota. (May 1999, p. 597)

* 5-10 **The Fifth International Petrozavodsk Conference "Probabilistic Methods in Discrete Mathematics"**, Institute of Applied Mathematical Research, Russian Academy of Institute of Applied Mathematical Reasearch, Petrozavodsk, Russia.
Conference Sections: (1) Probabilistic problems in combinatorial analysis, (2) statistical problems in discrete mathematics, (3) mathematical methods of information security, (4) dynamic games.
Conference Languages: Official languages of the conference are Russian and English.
Information: `http://www.krc.karelia.ru/structure/math/conf/probabl/index.shtml/`. Deadline for submission of registration forms and abstracts is March 15, 2000.

7-11 **Ph.D. Euroconference on Complex Analysis and Holomorphic Dynamics**, Platja d'Aro (Costa Brava), Catalonia, Spain. (Jan. 2000, p. 81)

* 7-15 **International Conference on Geometry, Integrability and Quantization**, St. Constantine Resort (near Varna), Bulgaria.
Goal: This second edition of the conference aims, like the previous one, to bring together experts in differential geometry, complex analysis, mathematical physics, and related fields in order to assess recent developments in these areas and to stimulate research in intermediate topics.
Organizers: I. M. Mladenov (Sofia), G. L. Naber (Chico).
Information: For more information, please contact I. M. Mladenov: `mladenov@bgcict.acad.br` or G. L. Naber: `gnaber@csuchico.edu`.

12-15 **Integral Methods in Science and Engineering (IMSE2000)**, Banff Conference Centre, Banff, Alberta, Canada. (June/July 1999, p. 714)

12-15 **Tenth SIAM Conference on Discrete Mathematics**, Radisson Hotel Metrodome, Minneapolis, Minnesota. (Feb. 2000, p. 283)

12-17 **Third International Conference on Differential Equations and Applications (DIFFEQ'2000)**, St. Petersburg State Technical University, St. Petersburg, Russia. (Oct. 1999, p. 1108)

13-16 (NEW DATE) **First AMS-Scandinavian International Mathematics Meeting**, University of Odense, Odense, Denmark. (Mar. 1999, p. 381)

14-17 **International Workshop in Operator Theory and Its Applications (IWOTA 2000)**, Bordeaux Univ., Bordeaux, France. (Aug. 1999, p. 815)

15-17 **2nd Croatian Mathematical Congress**, University of Zagreb, Croatia. (Feb. 2000, p. 283)

16-17 **The Fourth Biennial Symposium on Mathematical Modeling in the Undergraduate Curriculum**, University of Wisconsin-La Crosse, Wisconsin. (Dec. 1999, p. 1435)

* 17-22 **Mathematical Analysis: EuroConference on Partial Differential Equations and Their Applications to Geometry and Physics**, Castelvecchio Pascoli, Italy.
Information: Scientific program and list of speakers available at: `http://www.esf.org/euresco/00/pc00094a/`.
Deadline for Applications: March 24, 2000.
Contact Person: R. Heywood, `rheywood@esf.org`.

18-21 **MCS 2000 International Conference on Monte Carlo Simulation**, Monte Carlo, Monaco. (June/July 1999, p. 714)

19-26 **Dynamical Systems**, Cetraro Cosenza, Italy. (Feb. 2000, p. 283)

19-30 **Probabilistic Combinatorics**, University of Wyoming, Laramie, Wyoming. (Jan. 2000, p. 81)

* 20-24 **Ninth Summer St. Petersburg Meeting in Mathematical Analysis**, Euler International Mathematical Institute, St. Petersburg, Russia.
Organizers: Steklov Institute of Mathematics at St. Petersburg and the Euler International Mathematical Institute.
Organizing Committee: V. I. Vasyunin, S. V. Kisliakov, N. N. Nikolsky.
Information: `http://www.pdmi.ras.ru/EIMI/2000/analysis9/`.

22-24 **CUR 2000: Research in Undergraduate Education**, The College of Wooster, Wooster, Ohio. (Nov. 1999, p. 1287)

* 24-30 **Numerical Methods for Evolution Partial Differential Equations**, Anogia, Crete, Greece.
Background: The Foundation for Research and Technology-Hellas (Institute of Applied and Computational Mathematics) in collaboration with the University of Crete (Department of Mathematics) will continue in 2000 the series Euroconferences in Mathematics on Crete.
Organizers: G. Akrivis (Ioannina, Greece), M. Crouzeix (Rennes, France).
Sponsor: The Training and Mobility of Researchers Programme of the Commission of the European Union.
Main Speakers: T. Gallouet (Marseille, France), R. Nochetto (Maryland), J. Rappaz (Lausanne, Switzerland), V. Thomee (Goeteborg, Sweden), L. Wahlbin (Cornell University).
Support: The Training and Mobility of Researchers Programme financially supports young researchers from the countries of the European Economic Area and Israel, as well as researchers from certain countries in Central and Eastern Europe, to enable them to attend the conferences. There will also be some limited funds from other sources available to support participants not belonging to the above groups. Support can cover (all or certain) travel, living, and registration expenses.
Information: S. Papadopoulou, Dept. of Mathematics, Univ. of Crete, Heraklion, Crete, Greece; fax: 81-393881; e-mail:

souzana@math.uch.gr, or G. Akrivis, Dept. Of Computer Science, Univ. of Ioannina, Ioannina 45110, Greece; e-mail: akrivis@cs.uoi.gr.

* 25-28 **ACA'2000: IMACS Conference on Applications of Computer Algebra**, Steklov Institute of Mathematics at St. Petersburg, St. Petersburg, Russia.

Organizers: Steklov Institute of Mathematics at St. Petersburg, Euler International Mathematical Institute, St. Petersburg Mathematical Society, St. Petersburg State University.

Committee: General chair: N. Vassiliev (St. Petersburg, Russia), Program chairs: V. Edneral (Moscow, Russia), R. Liska (Prague, Czech Republic), M. Wester (Albuquerque, USA).

Focus: The meeting will focus on actual or possible applications of nontrivial computer algebra techniques to other fields and substantial interactions of computer algebra with other fields. The meeting will be run in the standard IMACS format, where individuals are invited to organize a special session. Individuals can propose a special session by contacting the program chairs. All paper submissions must be directed to an organizer of an appropriate special session.

Information: http://www.pdmi.ras.ru/EIMI/2000/imacs/.

26-28 **Heat Transfer 2000, Advanced Computational Methods in Heat Transfer**, Madrid, Spain. (Nov. 1999, p. 1287)

26-28 **The 5th Workshop on Numerical Ranges and Numerical Radii**, Nafplio, Greece. (Dec. 1999, p. 1435)

26-29 **Fifteenth Annual IEEE Symposium on Logic in Computer Science**, Santa Barbara, California. (Feb. 2000, p. 283)

* 26-30 **Conference on Differential Equations and Dynamical Systems in Honor of Waldyr Oliva**, Instituto Superior Tecnico, Lisbon, Portugal.

Confirmed Invited Speakers: P. Collet (Ecole Polytechnique), P. Cordaro (Univ. S. Paulo), E. Faria (Univ. S. Paulo), D. de Figueiredo (UNICAMP), G. Fusco (Univ. L'Aquila), G. Gallavotti (Univ. Roma I), A. Galves (Univ. S. Paulo), J. K. Hale (Georgia Tech), R. Langevin (Univ. Bourgogne), O. Lopes (UNICAMP), W. M. Oliva (Inst. Sup. Técnico), M. Peixoto (IMPA), H. Rodrigues (Univ. S. Paulo), R. Roussarie (Univ. Bourgogne), C. Simó (Univ. Barcelona), J. Sotomayor (Univ. S. Paulo), M. Teixeira (UNICAMP), M. Viana (IMPA), J. Xia (Northwestern Univ.).

Organizers: F. P. da Costa, R. L. Fernandes, P. G. Henriques, and C. Rocha (Center for Mathematical Analysis, Geometry, and Dynamical Systems, IST, Lisbon).

Registration: Participants should send an application to the organizers not later than April 30, 2000. The registration fee is 5000 Portuguese Escudos (free for students).

Information: For additional information please contact the organizers at the address: Dep. Matemática, Instituto Superior Técnico, Avenida Rovisco Pais 1, P-1049-001 Lisboa, Portugal; fax: 351-213523014; e-mail: wolivacf@math.ist.utl.pt; URL address: http://www.math.ist.utl.pt/cam/encontros_oliva.html.

26-30 **Formal Power Series and Algebraic Combinatorics (FPSAC'00)**, Moscow State University, Moscow, Russia. (Sept. 1999, p. 981)

* 26-30 **Nonlinear Modeling and Control, An International Seminar**, Nayanova University, Samara, Russia.

Purpose: The seminar's aim is the exchange of information about recent trends in mathematical modeling and control theory and their applications to various problems in physics, chemistry, biology, medicine, the economy, and industrial concerns.

Call for Papers: Original papers related to the aim of the seminar are solicited. Potential speakers should submit an abstract before April 30. The cover page should contain title, affiliation, and e-mail address of each author. Electronic submissions in LaTex are encouraged.

Information and Submission: V. Sobolev (organizer, e-mail: sable@ssu.samara.ru), or H. Gorelova (seminar coordinator), e-mail: gor@rs34.ssau.ru or gorhel@ssu.samara.ru; Nayanova University, Molodogvardeiskaya 196, Samara, 443001, Russia.

* 26-30 **Using Scientific Notebook and Teach and Learn Mathematics**, Colorado State University, Fort Collins, Colorado.

Information: For more information see http://hardy.math.colostate.edu/workshop/.

27-July 1 **The 18th International Conference on Operator Theory**, University of the West, Timisoara, Romania. (Jan. 2000, p. 81)

28-July 6 **Diophantine Approximation**, Cetraro Cosenza, Italy. (Feb. 2000, p. 284)

30-July 2 **2000 Centennial Vranceanu**, Romanian Academy, Bucharest University, Romania. (Jan. 2000, p. 81)

July 2000

2-15 **NATO Advanced Study Institute 20th Century Harmonic Analysis—a Celebration**, Il Ciocco Resort Hotel, Tuscany, Italy. (Nov. 1999, p. 1287)

3-7 **ALHAMBRA 2000**, Granada, Spain. (Oct. 1999, p. 1108)

3-7 **ANTS IV (Algorithmic Number Theory Symposium IV)**, Korteweg de Vries Institute for Mathematics, University of Amsterdam, The Netherlands. (Oct. 1998, p. 1230)

* 3-7 **Com2MaC Conference on Association Schemes, Codes and Designs**, Pohang University of Science and Technology, Pohang, Korea.

Program: Plenary lectures (45-60 minutes), invited lectures, and contributed presentations of 30 minutes.

Plenary Speakers: E. Bannai, A. Brouwer, C. Godsil, T. Ito, A. A. Ivanov, M. Ozeki, V. Pless, C. E. Praeger, D. K. Ray-Chaudhuri, N. J. A. Sloane, P. Terwilliger, M. Tsfasman, J. H. van Lint, Z.-X. Wan.

Deadlines: Authors are invited to submit an extended abstract before February 29, 2000, by e-mail to sysong@iastakte.edu or at the address: Sung-Yell Song, Department of Mathematics, Iowa State University, Ames, IA 50011, USA. The submitted abstract should include a short summary with a maximum of 200 words. Notification of acceptance will be given by March 15, 2000. Limited funds are available for partial support of particular graduate students and junior researchers, whose abstracts are accepted. All requests should be submitted by March 31, 2000. The authors whose papers are accepted will be required to register for the conference by May 15, 2000.

Information: Com2MaC@postech.ac.kr; sysong@iastate.edu; http://Com2MaC.postech.ac.kr/.

3-7 **Functional Analysis Valencia 2000**, Technical University of Valencia, Valencia, Spain. (Jan. 1999, p. 72)

3-7 **Sixth International Conference on p-Adic Analysis**, Ioannina, Greece. (June/July 1999, p. 715)

* 3-7 **Symposium on Geometry of Submanifolds and Related Problems**, Granada, Spain.

Topics: In this symposium, we will consider the following topics: General theory of submanifolds with emphasis in minimal submanifolds, constant mean curvature hypersurfaces, Willmore immersions, submanifolds of Kaehler manifolds and submanifolds of semi-Riemannian manifolds. We will also be interested in the study of eigenvalues of the Laplace and Dirac operators, mainly in those aspects related with submanifolds, such as lower and upper estimates of eigenvalues on submanifolds and characterization of particular submanifolds by means of these spectral invariants.

Organizers: The conference is organized by S. Montiel (Univ. Granada), O. Hijazi (Univ. Henri Poincaré - Nancy I - France).

Information: More information is available from the Web page http://ugr.es/local/alhambra2000/.

* 3-7 **Symposium on Orthogonal Polynomials**, Granada, Spain.

Contents and Scope: Over the last years the theory of orthogonal polynomials has attracted considerable attention. The main reason for this interest lies in their applicability in areas such as approximation theory, numerical analysis, scattering theory, nuclear physics, solid state physics, digital signal processing, electrical engineering and so forth. The aim of this symposium is to bring together specialists from several areas interested in different aspects of the theory and applications of orthogonal polynomials, as well as to provide a meeting point for the presentation and discussion of new developments in the field.

Organizers: The conference is organized by F. M. Espanol (Univ. Granada, Spain) and M. P. Gonzalez (Univ. Granada, Spain).

Information: More information is available from the Web page `http://ugr.es/local/alhambra2000/`.

* 3-7 **Symposium on Representation Theory of Algebras**, Granada, Spain.

Contents and Scope: This symposium aims to show the state of the art of the broad field representation theory of algebras. This theory includes the study of the structure and representation of ring and algebras; also groups, Lie algebras and superalgebras, quivers, posets, etc.; as well as rings of differential operators, C*-algebras, quantum groups and Hopf algebras.

Organizers: The symposium is organized by A. Verschoren (Univ. Antwerp - Belgium) (`aver@wins.uia.ac.be`) and P. J. Martnez (Univ. Granada - Spain) (`pjara@ugr.es`).

Information: More information is available from the Web page `http://ugr.es/local/alhambra2000/`.

* 3-7 **Symposium on Symmetry**, Granada, Spain.

Contents and Scope: Symmetry has been widely known as a central concept in science since ancient times. Symmetry also plays an important methodological role in modern art and science. Inspired by various cultural traditions, from Europe to Africa and from the Far East to America, symmetry can bridge different branches of science and arts, as well as different human cultures, and thus avoid overspecialization and some related problems. In this symposium, we are interested especially about the modern viewpoints of symmetry: 1. Regularity of periodical and nonperiodical designs, the crystallographic groups and their relationship with the symmetry of ornamentation, and NEC groups. 2. Theory of color. 3. Applications of symmetry theory to other sciences.

Organizers: The symposium is organized by Denes Nagy (U. Tsukuba - Japan) `nagy@bk.tsukuba.ac.jp`.

Information: More information is available from the Web page `http://ugr.es/local/alhambra2000/`.

3-9 **Mathematical Aspects of Evolving Interfaces**, Funchal, Portugal. (Feb. 2000, p. 284)

3-14 **SMS-NATO ASI: Approximation, Complex Analysis, and Potential Theory**, Université de Montréal, Canada. (Dec. 1999, p. 1435)

4-6 **Catop2000**, Department of Mathematics, University of Fribourg, Fribourg, Switzerland. (Oct. 1999, p. 1109)

5-7 **Scandinavian Workshop on Algorithm Theory**, Bergen, Norway. (Nov. 1999, p. 1287)

* 5-8 **The 6th Barcelona Logic Meeting**, Barcelona, Spain

Topics: All areas of mathematical logic, with an emphasis in algebraic logic, model theory and set theory. The scientific program will consist of several one-hour invited lectures and a number of twenty-minute contributed talks.

Invited Speakers: J. L. Balcázar (Univ. Politècnica de Catalunya), J. T. Baldwin (Univ. of Illinois at Chicago), P. Dellunde (Univ. Autònoma de Barcelona), Sy. D. Friedman (Univ. Wien), P. Koepke (Univ. Bonn), J. D. Monk (Univ. of Colorado at Boulder), Y. Peterzil (Univ. of Haifa), A. Pillay (Univ. of Illinois at Urbana-Champaign), Y. Venema (Univ. of Amsterdam), M. Zakharyashev (Keldysh Inst. for Applied Mathematics, Moscow).

Information: Any further information, forms to register and to apply for financial assistance, hotel information, etc., will be available on the Web site of the Congress `http://crm.es/6blm`, or from the e-mail address `6blm@crm.es` or by writing to 6th BLM, Centre de Recerca Matem'atica (CRM), Apartat 50, 08193 Bellaterra (Barcelona), Spain.

5-8 **International Conference on Ordinal and Symbolic Data Analysis - OSDA 2000**, Université Libre de Bruxelles (ULB), Brussels, Belgium. (Feb. 2000, p. 284)

6-8 **The 6th Barcelona Logic Meeting**, Barcelona, Spain. (Nov. 1999, p. 1287)

9-15 **AGRAM Conference on Abelian Groups, Rings and Modules**, The University of Western Australia, Perth, Austalia. (June/July 1999, p. 715)

9-15 **Mathematical Methods for Protein Structure Analysis and Design**, Martina Franca, Taranto, Italy. (Feb. 2000, p. 284)

9-28 **Workshop On Mathematical Models of Individual and Public Choice**, University of California, Irvine, California. (Oct. 1999, p. 1109)

10-14 **2000 SIAM Annual Meeting**, Westin Rio Mar Beach Resort and Country Club, Rio Grande, Puerto Rico. (Feb. 2000, p. 284)

* 10-14 **Third European Congress of Mathematics (3ecm)**, Barcelona, Spain. (Feb. 2000, p. 284)

Organizers: Societat Catalana de Matematiques, under the auspices of the European Mathematical Society.

Plenary Lectures: R. Dijkgraaf, H. Föllmer, H. W. Lenstra Jr., Yu. I. Manin, Y. Meyer, C. Simó, M.-F. Vignéras, O. Viro, A. J. Wiles. There will also be parallel lectures and minisymposia.

Other Activities: The scientific program of the congress also includes parallel lectures, mini-symposia, lectures by EMS prize winners, roundtable discussions, poster sessions, presentations of mathematical software, and exhibitions of video and multimedia with mathematical content.

Deadlines: For registration at a reduced fee: April 1, 2000. For grant requests: January 31. For submission of poster abstracts: March 1. For submissions of mathematical software, video, and multimedia: February 1. For proposals of satellite activities: February 1.

Information: Further information can be found at the Web sites `http://www.iec.es/3ecm/` and `http://www.si.upc.es/3ecm/`, which should also be used for registration. Mail can be sent to Societat Catalana de Matematiques, Institut d'Estudis Catalans, Carrer del Carme 47, E-08001 Barcelona, Spain; e-mail: `3ecm@iec.es`.

13-17 **International Conference on Foundations of Computational Mathematics in honor of Professor Steve Smale's 70th Birthday**, City University of Hong Kong, Kowloon, Hong Kong. (Jan. 2000, p. 81)

16-August 5 **IAS/Park City Mathematics Institute**, Institute for Advanced Study, Princeton, New Jersey. (Dec. 1999, p. 1435)

17-21 **Colloquium on Semigroups**, József Attila University, Bolyai Institute, Szeged, Hungary. (Feb. 2000, p. 284)

17-21 **Ninth International Conference on Fibonacci Numbers and their Applications**, Luxembourg-City, Luxembourg. (Sept. 1999, p. 982)

17-22 **International Congress on Mathematical Physics**, Imperial College, London, United Kingdom. (Nov. 1998, p. 1378)

17-22 **I Colloquium on Lie Theory and Applications**, E.T.S.I. Telecomunicaciones, Vigo, Spain. (June/July 1999, p. 715)

19-26 **Conference on Algebra and Algebraic Geometry with Applications**, Purdue University, West Lafayette, Indiana. (Jan. 2000, p. 82)

19-26 **The Third World Congress of Nonlinear Analysts (WCNA-2000)**, Catania, Italy. (Feb. 1998, p. 296)

* 20-24 **International Workshop on Number Theory in Honor of Professor Chao Ko's 90th Birthday**, Sichuan University, Chengdu, P.R. China.

Topics: This conference will focus on arithmetic algebraic geometry, analytic number theory, arithmetic of function fields, L-functions, diophantine equations, and applications of number theory in cryptography.

Scientific Committee: Y. Wang (Chair), K. Feng (Co-Chair), Q. Sun (Co-Chair), D. Pei, T. Zhan, Z. Jia, S. Zhang (USA), D. Wan (USA), M. Liao (Hong Kong), J. Yu (Taiwan).

Organizing Committee: T. Lu (Chair), Y. Liu (Co-Chair), A. Li (Co-Chair), D. Xu (Secretary-General), S. Yan, Z. Han, Q. Zhang, G. Peng.

Format: In addition to some invited lectures, we anticipate a small number of contributed lectures. Abstracts of contributed papers should be received by April 30, 2000. Abstracts should be typed in LaTeX,

not to exceed one page, and sent by e-mail to G. Peng, (ghpeng@mail.sc.cninfo.net) or Q. Zhang, (sszibbh@mail.sc.cninfo.net).
Information: Please contact G. Peng or Q. Zheng at the above e-mail addresses or visit http://math.uci.edu/~dwan/ko.html/.

21–31 **2000 ASL European Summer Meeting (Logic Colloquium 2000)**, Paris, France. (Oct. 1998, p. 1230)

* 22–28 **New Mathematical Methods in Continuum Mechanics**, Anogia, Crete, Greece.
Background: The Foundation for Research and Technology-Hellas (Institute of Applied and Computational Mathematics) in collaboration with the University of Crete (Department of Mathematics) will continue in 2000 the series Euroconferences in Mathematics on Crete.
Organizers: J. Ball (Oxford, United Kingdom), S. Mueller (Leipzig, Germany).
Sponsor: The Training and Mobility of Researchers Programme of the Commission of the European Union.
Main Speakers: A. Bressan (SISSA Trieste, Italy), G. Francfort (Paris-Nord, France), G. Friesecke (Oxford, United Kingdom), R. James (Minnesota), V. Sverak (Minnesota).
Support: The Training and Mobility of Researchers Programme financially supports young researchers from the countries of the European Economic Area and Israel, as well as researchers from certain countries in Central and Eastern Europe, to enable them to attend the conferences. There will also be some limited funds from other sources available to support participants not belonging to the above groups. Support can cover (all or certain) travel, living, and registration expenses.
Information: S. Papadopoulou, Dept. of Mathematics, Univ. of Crete, Heraklion, Crete, Greece; fax: 81-393881; e-mail: souzana@math.uch.gr or J. Ball, Mathematical Institute, Oxford University, 24-29 St. Giles, Oxford OX1 3LB, United Kingdom; e-mail: ball@maths.ox.ac.uk.

* 24–29 **SIAG OP-SF Summer School 2000**, Laredo, Spain.
Program: The SIAM Activity group (SIAG) on Orthogonal Polynomials and Special Functions (OP-SF) intends to organize a series of summer schools starting this year (2000). The first of such meetings will take place in Laredo, Spain, and its main goal is to give five introductory courses in advanced research topics on Orthogonal Polynomials and Special Functions. Courses will be presented by A. J. Duran (Univ. de Sevilla, Spain), H. T. Koelink (Technische Univ. Delft, The Netherlands), K. T-R. McLaughlin (Univ. of Arizona), J. Prestin (Inst. of Biomathematics and Biometry, Neuherberg, Germany), J. Stokman (Centre de Mathématiques de Jussieu, Univ. Pierre et Marie Curie, France). Some free discussions as well as some informal seminars will also be available.
Participants: The expected audience are graduate and recent postgraduate students (around 25 people who will receive grants for their living expenses and accommodation) and active researchers (around 35 people).
Organizing Committee: The organizing committee is: R. Álvarez-Nodarse (Univ. de Sevilla, Spain), F. Marcellán (Univ. Carlos III, Spain), W. Van Assche (Univ. Katholieke Univ. Leuven, Belgium), and R. Yáñez (Univ. de Granada, Spain).
Information: For further information, please contact R. Álvarez-Nodarse, (ran@cica.es) or F. Marcellán, (pacomarc@ing.uc3m.es).

28–30 **The Third Annual Conference of Bridges: Mathematical Connections in Art, Music, and Science**, Southwestern College, Winfield, Kansas. (Dec. 1999, p. 1436)

* 29–August 4 **Curves and Abelian Varieties over Finite Fields and Their Applications**, Anogia, Crete, Greece.
Background: The Foundation for Research and Technology-Hellas (Institute of Applied and Computational Mathematics) in collaboration with the University of Crete (Department of Mathematics) will continue in 2000 the series Euroconferences in Mathematics on Crete.
Organizers: G. van der Geer (Amsterdam, Holland), R. Schoof (Rome, Italy).
Sponsor: The Training and Mobility of Researchers Programme of the Commission of the European Union.
Main Speakers: N. Elkies (Harvard Univ.), G. van der Geer (Amsterdam, Holland), R. Pellikaan (Eindhoven, Holland), R. Schoof (Rome, Italy), M. Tsfasman (Marseille, France).
Support: The Training and Mobility of Researchers Programme financially supports young researchers from the countries of the European Economic Area and Israel, as well as researchers from certain countries in Central and Eastern Europe, to enable them to attend the conferences. There will also be some limited funds from other sources available to support participants not belonging to the above groups. Support can cover (all or certain) travel, living, and registration expenses.
Information: S. Papadopoulou, Dept. of Mathematics, Univ. of Crete, Heraklion, Crete, Greece; fax: 81-393881; e-mail: souzana@math.uch.gr, or G. van der Geer, Faculty of Mathematics, Univ. van Amsterdam, 1018 WB Amsterdam, The Netherlands; e-mail: geer@wins.uva.nl.

30–August 5 **ICOR 2000**, Innsbruck, Austria. (Feb. 2000, p. 284)

31–August 3 **Third Conference of Balkan Society of Geometers**, Univ. Politehnica of Bucharest, Bucharest, Romania. (June/July 1999, p. 715)

31–August 4 **Numerical Modelling in Continuum Mechanics**, Prague, Czech Republic. (Jan. 2000, p. 82)

31–August 5 **KNOTS 2000, KAIST**, Yongpyong Resort, Kangwon-do, Korea (South of). (Nov. 1999, p. 1288)

* 31–August 18 **Mathematical Geophysics Summer School**, Stanford University, Stanford, California.
Program: The Mathematical Geophysics Summer School (MGSS) is an NSF-funded program being held at Stanford University during the month of August from 1998–2002. Its overall purpose is to attract the attention and interest of theoreticians (applied mathematicians in particular) to the many interesting and important problems in geophysics, as well as to define mathematically, address and solve some of these problems. The topic for MGSS 2000 is Waves and Inhomogeneous Media.
Information: For more information and an application, see the Web page at http://cartan.stanford.edu/mgss/ or e-mail: mgss@math.stanford.edu.

August 2000

August–December **MSRI Program in Algorithmic Number Theory**, Mathematical Sciences Research Institute, Berkeley, California. (Sept. 1998, p. 1054)

August–May **MSRI Program in Operator Algebras**, Mathematical Sciences Research Institute, Berkeley, California. (Sept. 1998, p. 1054)

* 1–6 **Clifford Analysis, Its Applications to Mathematical Physics and Related Topics: In Honor of the Occasion of L. K. Hua's 90th Birthday**, Beijing, China
Scientific Program Committee: X. Ji (Beijing), T. Qian (Australia), M. Mitrea (USA), Q. Lu (Beijing), J. Ryan (USA), W. Lin (Guangzhou), W. Sproessing (Germany), S. Gong (Beijing), A. McIntosh (Australia), Z. Wu (Guangzhou), M. Shapiro (Mexico), R. Delanghe (Belgium), D. Struppa (USA), Z. Wu (USA), E. Meister (Germany), K. Guerlebeck (Germany), Th. Rassias (Greece).
Contact Persons: X. Ji, (xhji@math03.math.ac.cn); J. Ryan, (jryan@comp.uark.edu); and W. Sproessig, (sproessig@math.tu-freiberg.de). Web Page: http://mathe.tu-freiberg.de/beijing2000/. Deadline for abstracts: June 1, 2000.

2–18 **Rings, Modules and Representations - Constanta 2000**, Ovidius University, Constanta, Romania. (Dec. 1999, p. 1436)

* 3–5 **Mathematical Association of America's Mathfest 2000**, UCLA, Los Angeles, CA.

* 6–9 **ISSAC 2000 — International Symposium on Symbolic and Algebraic Computation**, St. Andrews University, Scotland
Information: For more information, please refer to the conference Web page, http://gap.dcs.st-and.ac.uk/issac2000/. E-mail inquiries may be sent to issac2000@dcs.st-and.ac.uk or to one of the conference committee members listed below: General Chair: T. Recio, recio@matesco.

unican.es; Local Arrangements Chair: S. Linton, sal@dcs.st-and.ac.uk; Program Committee Chair: C. Bajaj, bajaj@cs.utexas.edu; Tutorial Chair: J. Schicho, josef.schicho@risc.uni-linz.ac.at; Exhibitor Chair: M. Maza, Numerical Algorithms Group, marc@nag.co.uk; Poster Session Chair: A. Cohen, amc@win.tue.nl; Editor: C. Traverso, traverso@posso.dm.unipi.it; Treasurer: C. Campbell, cmc@st-andrews.ac.uk; Publicity Chair: P. Chin, pchin@wlu.ca.

7-12 **Mathematical Challenges of the 21st Century**, UCLA, Los Angeles, California. (Mar. 1999, p. 381)

7-12 **Nevanlinna Colloquium**, University of Helsinki, Helsinki, Finland. (May 1998, p. 642)

9-12 **Third International Palestinian Conference on Mathematics and Mathematical Education**, Bethlehem University, Palestine. (Feb. 2000, p. 284)

*19-25 **Discrete and Algorithmic Geometry**, Anogia, Crete, Greece.
Background: The Foundation for Research and Technology-Hellas (Institute of Applied and Computational Mathematics) in collaboration with the University of Crete (Department of Mathematics) will continue in 2000 the series Euroconferences in Mathematics on Crete.
Organizers: G. M. Ziegler (Berlin, Germany), E. Welzl (Zurich, Switzerland).
Sponsor: The Training and Mobility of Researchers Programme of the Commission of the European Union.
Main Speakers: G. Kalai (Jerusalem, Israel), R. Seidel (Saarbruecken, Germany), J. Snoeyink (Vancouver, Canada), E. Welzl (Zurich, Switzerland), G. M. Ziegler (Berlin, Germany).
Support: The Training and Mobility of Researchers Programme financially supports young researchers from the countries of the European Economic Area and Israel, as well as researchers from certain countries in Central and Eastern Europe, to enable them to attend the conferences. There will also be some limited funds from other sources available to support participants not belonging to the above groups. Support can cover (all or certain) travel, living, and registration expenses.
Information: S. Papadopoulou, Dept. of Mathematics, Univ. of Crete, Heraklion, Crete, Greece; fax: 81-393881; e-mail: souzana@math.uch.gr, or G. M. Ziegler, Fachbereich Mathematik, Technische Univ., Berlin, Strasse des 17 Juni 135, 10623 Berlin, Germany; e-mail: ziegler@math.tu-berlin.de.

21-24 **International Conference on Geometry, Analysis, and Applications (In Honor of Late Professor V. K. Patodi)**, Banaras Hindu University, Varanasi, India. (Jan. 2000, p. 82)

21-25 **16th IMACS World Congress (IMACS Congress 2000)**, Lausanne, Switzerland. (Oct. 1999, p. 1109)

22-24 **The Fifth Iranian Statistics Conference**, Isfahan University of Technology, Isfahan, Iran. (June/July 1999, p. 715)

September 2000

September 2000-June 2001 **Mathematical Logic**, Mittag-Leffler Institute, Djursholm, Sweden. (Feb. 2000, p. 285)

3-10 **Noncommutative Geometry**, Martina Franca, Taranto, Italy. (Feb. 2000, p. 285)

*4-6 **BEM 22 — 22nd International Conference on the Boundary Element Method**, New Hall, Cambridge University, UK
Organizers: Wessex Institute of Technology (WIT), Ashurst Lodge, Southampton, SO40 7AA, UK.
Information: For further details contact Karen Savage, BEM 22/1485; e-mail: ksavage@wessex.ac.uk; Web site: http://wessex.ac.uk/conferences/2000/; tel: 44 (0) 238 029 3223; fax: 44 (0) 238 029 2853.

*4-15 **Spatial Structures in Biology and Ecology: Models and Methods**, Taranto, Italy.
Description: The European Society for Mathematical and Theoretical Biology (ESMTB) will sponsor a Biomathematics Summer School to be held in Martina Franca (Taranto, Italy).
Deadlines: Scholarship applications: April 30, 2000. Communication of scholarship attributions: May 30, 2000. Reduced-fee registrations: June 30, 2000.
Information: Updated information about the school as well as a direct Web contact form is to be found on the Web page: http://mat.unimi.it/~miriam/ismtb/martina-ss/summer_school.html/.

5-16 **Advanced Course on Algebraic Quantum Groups**, Centre de Recerca Matematica, Campus of the Universitat Autónoma de Barcelona, Bellaterra, Spain. (Jan. 2000, p. 82)

*8-10 **IMA Career Workshop: Connecting Women in Mathematical Sciences to Industry**, IMA, University of Minnesota, Minneapolis, Minnesota.
Organizers: R. E. Chang, S. Lenhart (Univ. of Tennessee), M. H. Wright (Bell Labs).
Information: Institute for Mathematics and its Applications, University of Minnesota, 207 Church St. SE, 400 Lind Hall, Minneapolis, MN 55455. Phone: 612-624-6066, e-mail: staff@ima.umn.edu or Web page: http://ima.umn.edu/women_in_industry.html/.

*11-15 **IMA Short Course: Markov Processes and Markov Random Fields, Information Theory, Statistical Estimation**, IMA, University of Minnesota, Minneapolis, Minnesota.
Organizers: S. Geman (Brown Univ.).
Information: Institute for Mathematics and its Applications, University of Minnesota, 207 Church St. SE, 400 Lind Hall, Minneapolis, MN 55455. Phone: 612-624-6066, e-mail: staff@ima.umn.edu or Web page: http://ima.umn.edu/multimedia/fall/t1.html/.

11-15 **IWOTA-Portugal 2000**, Faro, Portugal. (Sept. 1999, p. 982)

12-15 **International Workshop on Operator Theory and Applications (IWOTA)**, Faro, Portugal. (Oct. 1999, p. 1109)

*15-17 **Homogenization and Materials Science**, University of Akron, Akron, Ohio.
Description: An international conference in honor of the late Professor Ulrich Hornung of Bundeswehr University, Munich, Germany.
Topics: Nonlinear homogenization, effective computational schemes for composites and strongly heterogeneous materials, porous media, polymeric composites, saves in heterogeneous materials, mesoscale models derived from the microscale, evolution of microstructure, and homogenization applications in biology.
Organizing Committee: USA - L. Beryland, A. Friedman, S. I. Hariharan, W. Mattice, and G. Young. Europe - M. Bendsoe, D. Cioranescu, A. Damlamian, and W. Jager.
Honorary Chair of the Scientific Committee: J. L. Lions.
Information: For more information visit http://uakron.edu/ARTSCI/index.html/ and select Conferences, Seminars & Lectures.

*18-22 **IMA Workshop 1: Mathematical Foundations of Speech Processing and Recognition**, IMA, University of Minnesota, Minneapolis, Minnesota.
Organizers: S. Khudanpur (John Hopkins Univ), M. Ostendorf (Boston Univ., Univ. of Washington), R. Rosenfeld (Carnegie Mellon Univ.).
Information: Institute for Mathematics and its Applications, University of Minnesota, 207 Church St. SE, 400 Lind Hall, Minneapolis, MN 55455. Phone: 612-624-6066, e-mail: staff@ima.umn.edu or Web page: http://ima.umn.edu/multimedia/fall/m1.html/.

18-22 **International Data Analysis Conference**, Innsbruck, Austria. (Mar. 99, p. 381)

18-23 **International Congress on Differential Geometry, in Memory of Alfred Gray (1939-1998)**, Bilbao, Spain. (Sept. 1999, p. 982)

19-22 **SCAN 2000: 9th GAMM - IMACS International Symposium on Scientific Computing, Computer Arithmetic, and Validated Numerics**, University of Karlsruhe, Karlsruhe, Germany. (Nov. 1999, p. 1288)

22-24 **AMS Central Section Meeting**, University of Toronto, Toronto, Ontario, Canada. (Nov. 1998, p. 1378)
Information: Information will be posted on the meetings pages of e-MATH.

25–27 **The Third International Workshop on Automated Deduction in Geometry**, ETH, Zurich, Switzerland. (Feb. 2000, p. 285)

October 2000

7–10 **International Conference on the Mathematical Modeling and Computational Experiments (ICMMCE)**, Dushanbe, Tajikistan. (Jan. 2000, p. 82)

* 11–14 **IMA Mini-Symposium: Brain Imaging**, IMA, University of Minnesota, Minneapolis, Minnesota.
Organizers: G. Sapiro, D. Kersten, G. Legge, S. He, X. Hu, K. Ugurbil (Univ. of Minnesota).
Information: Institute for Mathematics and its Applications, University of Minnesota, 207 Church St. SE, 400 Lind Hall, Minneapolis, MN 55455. Phone: 612-624-6066, e-mail: staff@ima.umn.edu or Web page: http://ima.umn.edu/multimedia/fall/ms1.html/.

* 16–20 **IMA Workshop 2: Image Processing and Low Level Vision**, IMA, University of Minnesota, Minneapolis, Minnesota.
Organizers: A. Tannenbaum (Univ. of Minnesota), P. Olver (Univ. of Minnesota), D. McClure (Brown Univ.), P. Perone (Caltech).
Information: Institute for Mathematics and its Applications, University of Minnesota, 207 Church St. SE, 400 Lind Hall, Minneapolis, MN 55455. Phone: 612-624-6066, e-mail: staff@ima.umn.edu or Web page: http://ima.umn.edu/multimedia/fall/m2.html/.

* 19–21 **Midwest Differential Equation Conference on Differential and Difference Equations**, Concordia College, Moodhead, MN
Plenary Speakers: M. Bohner (Missouri-Rolla), L. Erbe (Nebraska-Lincoln), J. Henderson (Auburn), J. Muldowney (Alberta), G. Sell (Minnesota).
Deadlines: Deadlines for contributed paper abstracts and other conference information may be found on the conference Website http://cord.edu/kfaculty/andersod/midwestde.htm/.

23–25 **Third International Conference on Applied Mathematics and Engineering Sciences**, Ecole Hassania des Travaux Publics, Casablanca, Morocco. (Dec. 1999, p. 1436)

23–27 **Third Asian Mathematical Conference (AMC2000)**, Manila, Philippines. (Dec. 1999, p. 1436)

* 30–November 3 **IMA Workshop 3: Mathematical Foundations of Natural Language Modeling**, IMA, University of Minnesota, Minneapolis, Minnesota.
Organizers: R. Rosenfeld (Carnegie Mellon Univ.), S. Khudanpur (John Hopkins Univ.), M. Johnson (Brown Univ.), F. Jelinek (John Hopkins Univ).
Information: Institute for Mathematics and its Applications, University of Minnesota, 207 Church St. SE, 400 Lind Hall, Minneapolis, MN 55455. Phone: 612-624-6066, e-mail: staff@ima.umn.edu or Web page: http://ima.umn.edu/multimedia/fall/m3.html/.

November 2000

3–5 **AMS Northeastern Section Meeting**, Columbia University, New York, New York. (Nov. 1998, p. 1378)
Information: Information will be posted on the meetings pages on e-MATH.

* 13–17 **IMA Workshop 4: Image Analysis and High Level Vision Modeling**, IMA, University of Minnesota, Minneapolis, Minnesota.
Organizers: P. Olver (Univ. of Minnesota), A. Tannenbaum (Univ. of Minnesota), D. Geman, S. Zucker (Yale), Y. Amit (Univ. of Chicago).
Information: Institute for Mathematics and its Applications, University of Minnesota, 207 Church St. SE, 400 Lind Hall, Minneapolis, MN 55455. Phone: 612-624-6066, e-mail: staff@ima.umn.edu or Web page: http://ima.umn.edu/multimedia/fall/m4.html/.

January 2001

January–May **MSRI Program in Spectral Invariants—Analytic and Geometric Aspects**, Mathematical Sciences Research Institute, Berkeley, California. (Sept. 1998, p. 1054)

4–8 **Second Pacific Rim Conference on Mathematics**, Institute of Mathematics, Academia Sinica, Taipei, R.O.C. (Taiwan). (Dec. 1999, p. 1436)

10–13 **Joint Mathematics Meeting**, New Orleans Marriott & ITT Sheraton New Orleans Hotel, New Orleans, Louisiana. (Sept. 1997, p. 1031)

* 17–19 **IMA Mini-Symposium: Fractals in Multimedia**, IMA, University of Minnesota, Minneapolis, Minnesota
Organizers: M. Barnsley (Georgia Tech).
Information: Institute for Mathematics and its Applications, University of Minnesota, 207 Church St. SE, 400 Lind Hall, Minneapolis, MN 55455. Phone: 612-624-6066, e-mail: staff@ima.umn.edu or Web page: http://ima.umn.edu/multimedia/winter/ms.html/.

* 25–26 **IMA Tutorial: Digital Libraries**, IMA, University of Minnesota, Minneapolis, Minnesota.
Organizers: A. Tewfik (Univ. of Minnesota).
Information: Institute for Mathematics and its Applications, University of Minnesota, 207 Church St. SE, 400 Lind Hall, Minneapolis, MN 55455. Phone: 612-624-6066, e-mail: staff@ima.umn.edu or Web page: http://ima.umn.edu/multimedia/winter/tut.html/.

* 29–February 2 **IMA Workshop 5: Digital Libraries: Data Modeling and Representation**, IMA, University of Minnesota, Minneapolis, Minnesota.
Organizers: R. Gray (Stanford Univ.), J. Johnston (AT&T), M. Orchard (Princeton Univ.), S. Shakhar (Univ. of Minnesota).
Information: Institute for Mathematics and its Applications, University of Minnesota, 207 Church St. SE, 400 Lind Hall, Minneapolis, MN 55455. Phone: 612-624-6066, e-mail: staff@ima.umn.edu or Web page: http://ima.umn.edu/multimedia/winter/m5.html/.

February 2001

* 12–16 **IMA Workshop 6A: Digital Libraries: Digital Asset Management**, IMA, University of Minnesota, Minneapolis, Minnesota.
Organizers: M. Barnsley (Georgia Tech), G. Cybenko (Dartmouth), D. Du (Univ. of Minnesota).
Information: Institute for Mathematics and its Applications, University of Minnesota, 207 Church St. SE, 400 Lind Hall, Minneapolis, MN 55455. Phone: 612-624-6066, e-mail: staff@ima.umn.edu or Web page: http://ima.umn.edu/multimedia/winter/m6.html/.

* 26–March 2 **IMA Workshop 6B: Digital Libraries: Classification, Retrieval and Visualization**, IMA, University of Minnesota, Minneapolis, Minnesota.
Organizers: D. Forsyth (UC Berkeley), B.-L. Yeo (Intel), other organizers to be announced.
Information: Institute for Mathematics and its Applications, University of Minnesota, 207 Church St. SE, 400 Lind Hall, Minneapolis, MN 55455. Phone: 612-624-6066, e-mail: staff@ima.umn.edu or Web page: http://ima.umn.edu/multimedia/winter/m7.html/.

The following new announcements will not be repeated until the criteria in the next to the last paragraph at the bottom of the first page of this section are met.

April 2001

* 9–13 **Joint IDR-IMA Workshop: Ideal Data Representation**, IMA, University of Minnesota, Minneapolis, Minnesota.
Organizers: R. DeVore (IDR Organizer, Univ. of South Carolina-Columbia), A. Ron (IDR Organizer, Univ. of Wisconsin - Madison), P. Van Fleet (Workshop Organizer, Univ. of Saint Thomas).
Information: Institute for Mathematics and its Applications, University of Minnesota, 207 Church St. SE, 400 Lind Hall, Minneapolis, MN 55455. Phone: 612-624-6066, e-mail: staff@ima.umn.edu or Web page: http://ima.umn.edu/multimedia/spring/idr.html/.

* 19–20 **IMA Tutorial: Geometric Design**, IMA, University of Minnesota, Minneapolis, Minnesota.

Organizers: L. Schumaker (Vanderbilt Univ.), R. Chang.
Information: Institute for Mathematics and its Applications, University of Minnesota, 207 Church St. SE, 400 Lind Hall, Minneapolis, MN 55455. Phone: 612-624-6066, e-mail: `staff@ima.umn.edu` or Web page: `http://ima.umn.edu/multimedia/spring/tut7.html/`.

*23–27 **IMA Workshop 7: Geometric Design**, IMA, University of Minnesota, Minneapolis, Minnesota.
Organizers: L. Schumaker (Vanderbilt) and R. Chang.
Information: Institute for Mathematics and its Applications, University of Minnesota, 207 Church St. SE, 400 Lind Hall, Minneapolis, MN 55455. Phone: 612-624-6066, e-mail: `staff@ima.umn.edu` or Web page: `http://ima.umn.edu/multimedia/spring/m7.html/`.

May 2001

*10–11 **IMA Tutorial: Computer Graphics**, IMA, University of Minnesota, Minneapolis, Minnesota,
Organizers: T. DeRose (Pixar).
Information: Institute for Mathematics and its Applications, University of Minnesota, 207 Church St. SE, 400 Lind Hall, Minneapolis, MN 55455. Phone: 612-624-6066, e-mail: `staff@ima.umn.edu` or Web page: `http://ima.umn.edu/multimedia/spring/tut8.html/`.

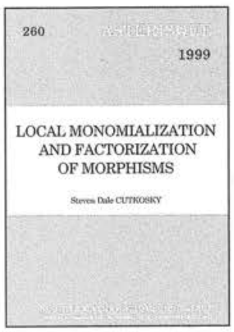
*14–18 **IMA Workshop 8: Computer Graphics**, IMA, University of Minnesota, Minneapolis, Minnesota.
Organizers: T. DeRose (Pixar), P. Schroder (Caltech).
Information: Institute for Mathematics and its Applications, University of Minnesota, 207 Church St. SE, 400 Lind Hall, Minneapolis, MN 55455. Phone: 612-624-6066, e-mail: `staff@ima.umn.edu` or Web page: `http://ima.umn.edu/multimedia/spring/m8.html/`.

June 2001

*11–15 **IMA Workshop: Haptics, Virtual Reality and Human Computer Interaction**, IMA, University of Minnesota, Minneapolis, Minnesota.
Organizers: K. Kolarov (Interval Research Corp.).
Information: Institute for Mathematics and its Applications, University of Minnesota, 207 Church St. SE, 400 Lind Hall, Minneapolis, MN 55455. Phone: 612-624-6066, e-mail: `staff@ima.umn.edu` or Web page: `http://ima.umn.edu/multimedia/spring/m9.html/`.

New Publications Offered by the AMS

Algebra and Algebraic Geometry



Local Monomialization and Factorization of Morphisms

Steven Dale Cutkosky

A publication of Société Mathématique de France.

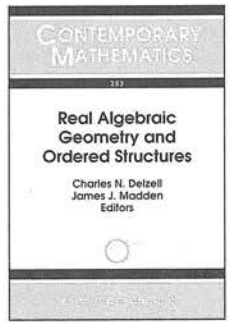
In this volume, the author studies morphisms of algebraic varieties. More specifically, suppose that $R \subset S$ are regular local rings of a common dimension, which are essentially of finite type over a field k of characteristic zero, such that the quotient field K of S is finite over the quotient field of R. If V is a valuation ring of K which dominates S, it is shown that there are sequences of monoidal transforms (blowups of regular primes) $R \to R_1$ and $S \to S_1$ along V such that $R_1 \to S_1$ is a monomial mapping. It follows that a generically finite morphism of nonsingular varieties can be made to be a monomial mapping along a valuation, after blowups of nonsingular subvarieties. Applications are given to factorization of birational morphisms and simultaneous resolution of singularities.

Distributed by the AMS in the United States, Canada, and Mexico. Orders from other countries should be sent to the SMF, Maison de la SMF, B.P. 67, 13274 Marseille cedex 09, France, or to Institut Henri Poincaré, 11 rue Pierre et Marie Curie, 75231 Paris cedex 05, France. Members of the SMF receive a 30% discount from list.

Contents: Introduction; Preliminaries; Uniformizing transforms; Monomialization; Factorization 1; Factorization 2; The Zariski manifold; Bibliography.

Astérisque, Number 260

December 1999, 143 pages, Softcover, 2000 *Mathematics Subject Classification*: 14Exx, 13Bxx, **Individual member $30**, List $33, Order code AST/260N



Real Algebraic Geometry and Ordered Structures

Charles N. Delzell and **James J. Madden**, *Louisiana State University, Baton Rouge*, Editors

This volume contains 16 carefully refereed articles by participants in the Special Semester and the AMS Special Session on Real Algebraic Geometry and Ordered Structures held at Louisiana State University and Southern University (Baton Rouge). The 23 contributors to this volume were among the 75 mathematicians from 15 countries who participated in the special semester.

Topics include the topology of real algebraic curves (Hilbert's 16th problem), moduli of real algebraic curves, effective sums of squares of real forms (Hilbert's 17th problem), efficient real quantifier elimination, subanalytic sets and stratifications, semialgebraic singularity theory, radial vector fields, exponential functions and valuations on nonarchimedean ordered fields, valued field extensions, partially ordered and lattice-ordered rings, rings of continuous functions, spectra of rings, and abstract spaces of (higher-level) orderings and real places.

This volume provides a good overview of the state of the art in this area in the 1990s. It includes both expository and original research papers by top workers in this thriving field. The authors and editors strived to make the volume useful to a wide audience (including students and researchers) interested in real algebraic geometry and ordered structures—two subjects that are obviously related, but seldom brought together.

Contents: **M. E. Alonso** and **M. P. Vélez**, On real involutions and ramification of real valuations; **E. Becker**, **V. Powers**, and **T. Wörmann**, Deciding positivity of real polynomials; **J.-P. Brasselet**, Radial vector fields and the Poincaré-Hopf theorem; **S. Finashin**, A generalization of the Arnold-Viro inequalities for real singular algebraic curves; **P. M. Gilmer**, Floppy curves, with applications to real algebraic curves; **D. Gondard** and **M. Marshall**, Towards an abstract description of the space of real places; **L. Gonzalez-Vega**, A special quantifier elimination algorithm for Pham systems; **M. Henriksen** and **F. A. Smith**, A look at biseparating maps from an algebraic point of view; **J. Huisman**, Real Teichmüller spaces and moduli of real algebraic curves; **J. Huisman**, Correction to "A real algebraic vector bundle is strongly algebraic whenever its total space is affine"; **F.-V. Kuhlmann** and **S. Kuhlmann**, The exponential rank of nonarchimedean exponential fields; **L. Noirel** and **D. Trotman**, Subanalytic and semialgebraic realisations of abstract stratified sets; **J. Ohm**, On the vector space defect for valued field extensions; **G. M. Polotovskii**, On the classification of decomposable 7-th degree curves; **M. J. de la Puente**, The complex spectrum of a ring; **B. Reznick**, Some concrete aspects of Hilbert's 17th problem; **M. Shiota**, Semialgebraic singularity theory.

Contemporary Mathematics, Volume 253

April 2000, 287 pages, Softcover, ISBN 0-8218-0804-4, 2000 *Mathematics Subject Classification*: 00B25, 14Pxx; 01A60, 06Fxx, 11Exx, 12-XX, 13-XX, 32B20, 54C45, 57R25, 58A07, **Individual member $45**, List $75, Institutional member $60, Order code CONM/253N

Analysis

Géométrie Complexe et Systèmes Dynamiques Colloque en L'Honneur D'Adrien Douady

Marguerite Flexor, **Pierrette Sentenac**, and **Jean-Christophe Yoccoz**, Editors

A publication of Société Mathématique de France.

This volume presents written accounts of the lectures given at the University of Paris-Sud (Orsay) during the conference in honor of Adrien Douady's sixtieth birthday. The multi-faceted activity within the field of dynamical systems is reflected in the papers in this volume. Topics covered in the book include iteration of polynomials (specifically quadratic), rational fractions, holomorphic foliations, and non-uniformly hyperbolic dynamics.

This item will also be of interest to those working in geometry and topology.

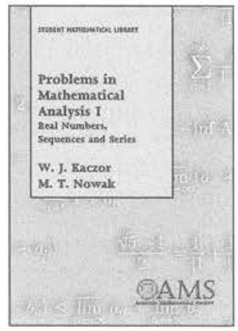
Distributed by the AMS in the United States, Canada, and Mexico. Orders from other countries should be sent to the SMF, Maison de la SMF, B.P. 67, 13274 Marseille cedex 09, France, or to Institut Henri Poincaré, 11 rue Pierre et Marie Curie, 75231 Paris cedex 05, France. Members of the SMF receive a 30% discount from list.

Contents: K. Astala, **Z. Balogh**, and **H. M. Reimann**, Lempert mappings and holomorphic motions in $\mathbf{C}^n$; **M. Benedicks** and **L.-S. Young**, Markov extensions and decay of correlations for certain Hénon maps; **C. Camacho** and **B. A. Scárdua**, Complex foliations with algebraic limit sets; **A. Fathi**, Une caractérisation des stades à virages circulaires; **M. Jakobson** and **S. Newhouse**, Asymptotic measures for hyperbolic piecewise smooth mappings of a rectangle; **G. Levin** and **S. van Strien**, Total disconnectedness of Julia sets and absence of invariant linefields for real polynomials; **M. Lyubich**, Dynamics of quadratic polynomials, III parapuzzle and SBR measures; **S. Luzzatto** and **M. Viana**, Positive Lyapunov exponents for Lorenz-like families with criticalities; **M. Martens** and **T. Nowicki**, Invariant measures for typical quadratic maps; **J.-F. Mattei**, Quasi-homogénéité et équiréductibilité de feuilletages holomorphes en dimension deux; **J. Milnor**, Periodic orbits, externals rays and the Mandelbrot set: An expository account; **J. Palis**, A global view of dynamics and a conjecture on the denseness of finitude of attractors; **K. Pilgrim** and **T. Lei**, Rational maps with disconnected Julia set; **F. Przytycki**, Hölder implies Collet-Eckmann; **D. Schleicher**, Rational parameter rays of the Mandelbrot set.

Astérisque, Number 261

December 1999, 443 pages, Softcover, ISBN 2-85629-081-7, 2000 *Mathematics Subject Classification*: 30Cxx, 30Dxx, 30Fxx, 32Axx, 32Bxx, 32Gxx, 32Sxx, 32Lxx, 37Axx, 37Cxx, 37Dxx, 37Exx, 37-XX, 52Axx, 53Cxx, **Individual member $89**, List $99, Order code AST/261N

Supplementary Reading



Problems in Mathematical Analysis I

Real Numbers, Sequences and Series

W. J. Kaczor and **M. T. Nowak**, *Marie Curie-Skłodowska University, Lublin, Poland*

We learn by doing. We learn mathematics by doing problems. This book is the first volume of a series of books of problems in mathematical analysis. It is mainly intended for students studying the basic principles of analysis. However, given its organization, level, and selection of problems, it would also be an ideal choice for tutorial or problem-solving seminars, particularly those geared toward the Putnam exam. The volume is also suitable for self-study.

Each section of the book begins with relatively simple exercises, yet may also contain quite challenging problems. Very often several consecutive exercises are concerned with different aspects of one mathematical problem or theorem. This presentation of material is designed to help student comprehension and to encourage them to ask their own questions and to start research. The collection of problems in the book is also intended to help teachers who wish to incorporate the problems into lectures. Solutions for all the problems are provided.

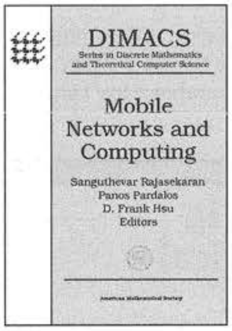
The book covers three topics: real numbers, sequences, and series, and is divided into two parts: exercises and/or problems, and solutions. Specific topics covered in this volume include the following: basic properties of real numbers, continued fractions, monotonic sequences, limits of sequences, Stolz's theorem, summation of series, tests for convergence, double series, arrangement of series, Cauchy product, and infinite products.

Contents: *Problems:* Real numbers; Sequence of real numbers; Series of real numbers; *Solutions:* Real numbers; Sequences of real numbers; Series of real numbers; Bibliography.

Student Mathematical Library

April 2000, approximately 400 pages, Softcover, ISBN 0-8218-2050-8, LC 99-087039, 2000 *Mathematics Subject Classification*: 00A07; 40-01, **All AMS members $31**, List $39, Order code STML-NOWAKN

Applications



Mobile Networks and Computing

Sanguthevar Rajasekaran and **Panos Pardalos**, *University of Florida, Gainesville*, and **D. Frank Hsu**, *Fordham University, Bronx, NY*, Editors

Advances in the technologies of networking, wireless communications, and miniaturization of computers have lead to rapid development in mobile communication infrastructure and have engendered a new paradigm of computing. Users carrying portable devices can now move freely about while remaining connected to the network. This "portability" allows for access to information from anywhere and at any time. The flexibility has resulted in new levels of complexity not encountered previously in software and protocol design for wired networking.

New challenges in designing software systems for mobile networks include location and mobility management, channel allocation, power conservation, and more. In this book, renowned researchers in the field address these aspects of mobile networking.

Contents: **A.-H. A. Abou-Zeid**, **M. Azizoglu**, and **S. Roy**, Stochastic modeling of a single TCP/IP session over a random loss channel; **A. F. Almutairi**, **S. L. Miller**, and **H. A. Latchman**, Tracking of multi-level modulation formats for DS/CDMA systems in a slowly fading channel; **E. Bertino**, **E. Pagani**, and **G. P. Rossi**, An adaptive concurrency control protocol for mobile transactions; **J. Gomez** and **A. T. Campbell**, Supporting adaptive-QOS over multiple time scales in wireless networks; **S. K. S. Gupta** and **P. K. Srimani**, Using self-stabilization to design adaptive multicast protocols for mobile ad hoc networks; **Z. J. Haas** and **A. Warkhedi**, The design and performance of mobile TCP for wireless networks; **A. (Sumi) Helal**, **J. Jing**, and **A. Elmagarmid**, Supporting transaction service handoff in mobile environments; **B. Jaumard**, **C. Meyer**, and **T. Vovor**, How to combine a column and row generation method with a column or row elimination procedure–Application to a channel assignment problem; **A. Joshi**, On mobility and agents; **I. Korpeoglu**, **P. Bhagwat**, **C. Bisdikian**, and **M. Naghshineh**, Multiplexed serial wireless connectivity for palmtop computers; **J.-P. Lin**, **S.-Y. Kuo**, and **Y. Huang**, A cluster-based checkpointing scheme for mobile computing on wide area network; **X. Liu**, **P. M. Pardalos**, **S. Rajasekaran**, and **M. G. C. Resende**, A GRASP for frequency assignment in mobile radio networks; **R. A. Murphey**, **P. M. Pardalos**, and **E. Pasiliao**, Multicriteria optimization for frequency assignment; **T. Hayashi**, **K. Nakano**, and **S. Olariu**, Randomized initialization protocols for packet radio networks; **K. Naik** and **D. S. L. Wei**, Energy-conserving software design for mobile computers; **K. Naik** and **D. S. L. Wei**, Software implementation strategies for power-conscious systems; **R. Prakash** and **M. Singhal**, Impact of unidirectional links in wireless ad-hoc networks; **S. Rajasekaran**, **K. Naik**, and **D. Wei**, On frequency assignment in cellular networks; **X. Yi**, **S. Kitazawa**, **H. Sakazaki**, **E. Okamoto**, and **D. F. Hsu**, An agent-based architecture for securing mobile IP.

DIMACS: Series in Discrete Mathematics and Theoretical Computer Science

April 2000, approximately 313 pages, Hardcover, ISBN 0-8218-1547-4, 2000 *Mathematics Subject Classification*: 68M10, 68M12, 90B18, **Individual member $59**, List $99, Institutional member $79, Order code DIMACS-PARDALOS8N

Differential Equations

Réductibilité des Systèmes Produits-Croisés à Valeurs dans des Groupes Compacts

Raphaël Krikorian, *Centre de Mathématiques de l'École Polytechnique, Palaiseau, France*

A publication of Société Mathématique de France.

This book studies the problem of reducibility (conjugacy to constants) of quasi-periodic skew-product systems with values in compact semisimple groups, as well as the existence of Floquet-type solutions for linear differential quasi-periodic systems with values in compact semisimple algebras.

The main result (Chapter 6) is that for real one-parameter families of quasi-periodic systems with values in the group of rotations of the 3-space, reducibility holds for almost all values of the parameter (provided the family is close enough to some family of constant systems). For the proof of this result (which relies on a resonance removing procedure due to L. H. Eliasson), the author introduces a notion of transversality à la Pyartli, which allows for controlling the dependence of the eigenvalues on the parameter. Also used is a positive measure reducibility theorem, which in case the group is compact semisimple, is proven in Chapter 3. In Chapter 5, again in the compact semisimple group case, the author proves that modulo some finite covering which depends only on the group, the set of reducible systems is dense near the constants. Chapter 4 is devoted to a normal form type theorem which enables recovery of the result of Chapter 3. Finally in Chapter 2, a necessary and sufficient condition (modulo a finite covering) is given for reducibility of skew-product systems and the centralizer of constant systems is studied.

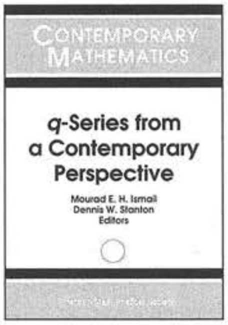
Distributed by the AMS in the United States, Canada, and Mexico. Orders from other countries should be sent to the SMF, Maison de la SMF, B.P. 67, 13274 Marseille cedex 09, France, or to Institut Henri Poincaré, 11 rue Pierre et Marie Curie, 75231 Paris cedex 05, France. Members of the SMF receive a 30% discount from list.

Contents: Introduction; Rappels et notations difféomorphismes produits-croisés, systèmes quasi-périodiques; Réductibilité des systèmes produits-croisés; Méthode K.A.M. classique, résultats en mesure positive; Théorèmes de formes normales et applications; Densité et quasi-densité des systèmes réductibles au voisinage des constantes; Réductibilité presque partout dans le cas $SO(3,\mathbf{R})$; Annexe: Quelques estimées; Bibliographie.

Astérisque, Number 259

November 1999, 216 pages, Softcover, 2000 *Mathematics Subject Classification*: 34-XX, 58-XX, **Individual member $50**, List $55, Order code AST/259N

Discrete Mathematics and Combinatorics



q-Series from a Contemporary Perspective

Mourad E. H. Ismail, *University of South Florida, Tampa*, and **Dennis W. Stanton**, *University of Minnesota, Minneapolis*, Editors

This volume presents the proceedings of the Summer Research Conference on q-series and related topics held at Mount Holyoke College (Hadley, MA). All of the papers were contributed by participants and offer original research. Articles in the book reflect the diversity of areas that overlap with q-series, as well as the usefulness of q-series across the mathematical sciences. The conference was held in honor of Richard Askey on the occasion of his 65th birthday.

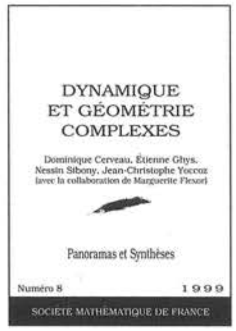
This item will also be of interest to those working in analysis.

Contents: **G. E. Andrews**, Schur's theorem, partitions with odd parts and the Al-Salam-Carlitz polynomials; **K. Aomoto** and **K. Iguchi**, Singularity and monodromy of quasi-hypergeometric functions; **B. C. Berndt**, **H. H. Chan**, and **S.-S. Huang**, Incomplete elliptic integrals in Ramanujan's lost notebook; **W. C. Connett** and **A. L. Schwartz**, Measure algebras associated with orthogonal polynomials; **D. Foata** and **G. Han**, Word straightening and q-Eulerian calculus; **O. Foda**, **K. S. M. Lee**, **Y. Pugai**, and **T. A. Welsh**, Path generating transforms; **G. Gasper**, q-extensions of Erdélyi's fractional integral representations for hypergeometric functions and some summation formulas for double q-Kampé de Fériet series; **R. Wm. Gosper, Jr.** and **S. K. Suslov**, Numerical investigation of basic Fourier series; **M. D. Hirschhorn**, An identity of Ramanujan, and applications; **M. E. H. Ismail** and **D. W. Stanton**, Addition theorems for the q-exponential function; **K. W. J. Kadell**, The Schur functions for partitions with complex parts; **J. Kaneko**, On Forrester's generalization of Morris constant term identity; **A. N. Kirillov**, New combinatorial formula for modified Hall-Littlewood polynomials; **C. Krattenthaler**, Schur function identities and the number of perfect matchings of Holey Aztec rectangles; **S. C. Milne**, A new $U(n)$ generalization of the Jacobi triple product identity; **H. Rosengren**, A new quantum algebraic interpretation of the Askey-Wilson polynomials; **S. Sahi**, Some properties of Koornwinder polynomials; **M. Schlosser**, A new multidimensional matrix inversion in A_r.

Contemporary Mathematics

April 2000, approximately 440 pages, Softcover, ISBN 0-8218-1150-9, 2000 *Mathematics Subject Classification*: 05-XX, 11-XX, 20-XX, 22-XX, 30-XX, 33-XX, 41-XX, 42-XX, 43-XX, 82-XX, **Individual member $56**, List $93, Institutional member $74, Order code CONM-ISMAIL2N

Geometry and Topology



Dynamique et Géométrie Complexes

Dominique Cerveau, *Université de Rennes I, France*, **Étienne Ghys**, *École Normale Supérieure de Lyon, France*, and **Nessim Sibony** and **Jean-Christophe Yoccoz**, *Université de Paris-Sud, Orsay, France*

A publication of Société Mathématique de France.

In the last twenty years, the theory of holomorphic dynamical systems had a resurgence of activity, particularly concerning the fine analysis of Julia sets associated to polynomials and rational maps in one complex variable. At the same time, closely related theories had a similar rapid development, for example the qualitative theory of differential equations in the complex domain.

The meeting, "État de la recherche" held at the ENS Lyon presented the current state of the art in this area, emphasizing the unity linking the various sub-domains. This volume contains four survey articles corresponding to the talks presented at this meeting.

D. Cerveau describes the structure of polynomial differential equations in the complex plane, focusing on the local analysis in neighborhoods of singular points. É. Ghys surveys the theory of laminations by Riemann surfaces which occur in many dynamical or geometrical situations. N. Sibony describes the present state of the generalization of the Fatou-Julia theory for polynomial or rational maps in two or more complex dimensions. Lastly, the talk of J.-C. Yoccoz, written by M. Flexor, considers polynomials of degree 2 in one complex variable and, in particular, with the hyperbolic properties of these polynomials centered around the Jakobson theorem.

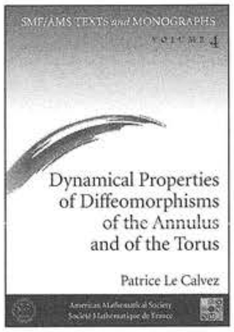
A general introduction gives a basic history of holomorphic dynamical systems, which demonstrates the numerous and fruitful interactions among the topics. In the spirit of the "État de la recherche de la SMF" meetings, articles are written for a broad mathematical audience, especially students or mathematicians working in different fields.

Distributed by the AMS in the United States, Canada, and Mexico. Orders from other countries should be sent to the SMF, Maison de la SMF, B.P. 67, 13274 Marseille cedex 09, France, or to Institut Henri Poincaré, 11 rue Pierre et Marie Curie, 75231 Paris cedex 05, France. Members of the SMF receive a 30% discount from list.

Contents: **E. Ghys**, Les systèmes dynamiques holomorphes; **D. Cerveau**, Feuilletages holomorphes de codimension 1. Réduction des singularités en petite dimensions et applications; **E. Ghys**, Laminations par surfaces de Riemann; **N. Sibony**, Dynamique des applications rationnelles de $\mathbb{P}^k$; **J.-C. Yoccoz**, Dynamique des polynômes quadratiques.

Panoramas et Synthèses, Number 8

December 1999, 222 pages, Softcover, ISBN 2-85629-078-7, 2000 *Mathematics Subject Classification*: 32S65, 37F75, 34Mxx, 37B10, **Individual member $40**, List $44, Order code PASY/8



Dynamical Properties of Diffeomorphisms of the Annulus and of the Torus

Patrice Le Calvez, *University of Paris, Villetaneuse, France*

The first chapter of this monograph presents a survey of the theory of monotone twist maps of the annulus. First, the author covers the conservative case by presenting a short survey of Aubry-Mather theory and Birkhoff theory, followed by some criteria for existence of periodic orbits without the area-preservation property. These are applied in the area-decreasing case, and the properties of Birkhoff attractors are discussed. A diffeomorphism of the closed annulus which is isotopic to the identity can be written as the composition of monotone twist maps.

The second chapter generalizes some aspects of Aubry-Mather theory to such maps and presents a version of the Poincaré-Birkhoff theorem in which the periodic orbits have the same braid type as in the linear case. A diffeomorphism of the torus isotopic to the identity is also a composition of twist maps, and it is possible to obtain a proof of the Conley-Zehnder theorem with the same kind of conclusions about the braid type, in the case of periodic orbits. This result leads to an equivariant version of the Brouwer translation theorem which permits new proofs of some results about the rotation set of diffeomorphisms of the torus.

This is the English translation of a volume previous published as volume 204 in the Astérisque series.

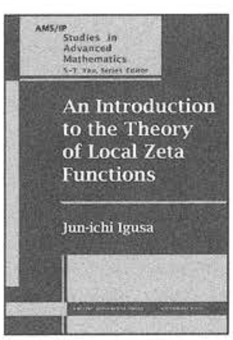
Contents: Presentation and comparison of the different approaches to the theory of monotone twist diffeomorphisms of the annulus; Generating phases of the diffeomorphisms of the torus and the annulus; Bibliography; Index.

SMF/AMS Texts and Monographs, Volume 4

April 2000, 105 pages, Softcover, ISBN 0-8218-1943-7, LC 99-087060, 2000 *Mathematics Subject Classification*: 58-XX, **All AMS members $17**, List $21, Order code SMFAMS/4N

Number Theory

Independent Study



An Introduction to the Theory of Local Zeta Functions

Jun-ichi Igusa, *Johns Hopkins University, Baltimore, MD*

This book is an introductory presentation to the theory of local zeta functions. As distributions, and mostly in the archimedian case, local zeta functions are called complex powers.

The volume contains major results on complex powers by Atiyah, Bernstein, I. M. Gelfand, and S. I. Gelfand. Also included are related results by Sato. The section on p-adic local zeta functions presents Serre's structure theorem, a rationality theorem and many examples by the author. It concludes with theorems by Denef and Meuser.

Prerequisites for understanding the text include basic courses in algebra, calculus, complex analysis, and general topology. The book follows the usual pattern of progress in mathematics: examples are given, conjectures follow, conjectures are developed into theorems.

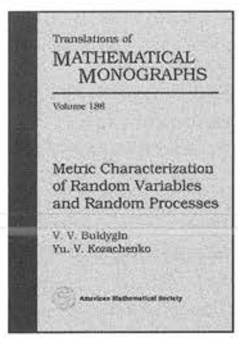
This book is accessible and self-contained. Results illustrate the unity of mathematics by gathering important theorems from algebraic geometry and singularity theory, number theory, algebra, topology, and analysis. The ideas are then employed in essential ways to prove the theorems.

Contents: Preliminaries; Implicit function theorems and K-analytic manifolds; Hironaka's desingularization theorem; Bernstein's theory; Archimedean local zeta functions; Prehomogeneous vector spaces; Totally disconnected spaces and p-adic manifolds; Local zeta functions (p-adic case); Some homogeneous polynomials; Computation of $Z(s)$; Theorems of Denef and Meuser; Bibliography; Index.

AMS/IP Studies in Advanced Mathematics, Volume 14

April 2000, 232 pages, Hardcover, ISBN 0-8218-2015-X, LC 99-087031, 2000 *Mathematics Subject Classification*: 11Sxx, 11S40, 11Mxx, 11Gxx, 14Gxx, **All AMS members $36**, List $45, Order code AMSIP/14N

Probability



Metric Characterization of Random Variables and Random Processes

V. V. Buldygin, *Kyïv Politechnic Institute, Ukraine*, and **Yu. V. Kozachenko**, *Kyïv Taras Shevchenko National University, Ukraine*

The topic covered in this book is the study of metric and other close characteristics of different spaces and classes of random variables and the application of the entropy method to the investigation of properties of stochastic processes whose values, or increments, belong to given spaces. The following processes appear in detail: pre-Gaussian processes, shot noise processes representable as integrals over processes with independent increments, quadratically Gaussian processes, and, in particular, correlogram-type estimates of the correlation function of a stationary Gaussian process, jointly strictly sub-Gaussian processes, etc.

The book consists of eight chapters divided into four parts: The first part deals with classes of random variables and their metric characteristics. The second part presents properties of stochastic processes "imbedded" into a space of random variables discussed in the first part. The third part considers applications of the general theory. The fourth part outlines the necessary auxiliary material.

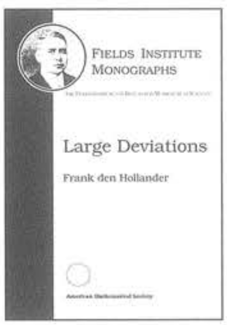
Problems and solutions presented show the intrinsic relation existing between probability methods, analytic methods, and functional methods in the theory of stochastic processes. The concluding sections, "Comments" and "References", gives references to the literature used by the authors in writing the book.

Contents: Sub-Gaussian and pre-Gaussian random variables; Orlicz spaces of random variables; Regularity of sample paths of a stochastic process; Pre-Gaussian processes; Shot noise processes and their properties; Correlograms of stationary Gaussian processes; Jointly sub-Gaussian, super-Gaussian, and pseudo-Gaussian stochastic processes; Appendices; Comments; References; Basic notation; Index.

Translations of Mathematical Monographs, Volume 188

April 2000, approximately 264 pages, Hardcover, ISBN 0-8218-0533-9, LC 99-087766, 2000 *Mathematics Subject Classification*: 60Gxx; 60Exx, **Individual member $57**, List $95, Institutional member $76, Order code MMONO/188N

Recommended Text



Large Deviations

Frank den Hollander,
Nijmegen University, Netherlands

This volume offers an introduction to large deviations. It is divided into two parts: theory and applications. Basic large deviation theorems are presented for i.i.d. sequences, Markov sequences, and sequences with moderate dependence. The rate function is computed explicitly. The theory is explained without too much emphasis on technicalities. Also included is an outline of general definitions and theorems. The goal is to expose the unified theme that gives large deviation theory its overall structure, which can be made to work in many concrete cases. The section on applications focuses on recent work in statistical physics and random media.

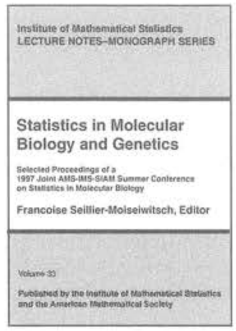
This book contains 60 exercises (with solutions) that should elucidate the content and engage the reader. Prerequisites for the book are a strong background in probability and analysis and some knowledge of statistical physics. It would make an excellent textbook for a special topics course in large deviations.

This item will also be of interest to those working in mathematical physics.

Contents: *Theory:* Large deviations for i.i.d. sequences: Part 1; Large deviations for i.i.d. sequences: Part 2; General theory; Large deviations for Markov sequences; Large deviations for dependent sequences; *Applications:* Statistical hypothesis testing; Random walk in random environment; Heat conduction with random sources and sinks; Polymer chains; Interacting diffusions; Solutions to the exercises; Bibliography; Index; Glossary of symbols.

Fields Institute Monographs, Volume 14

March 2000, 143 pages, Hardcover, ISBN 0-8218-1989-5, LC 99-058913, 2000 *Mathematics Subject Classification*: 60-01, 60F10, 60K35; 82B31, 82B44, **All AMS members $39**, List $49, Order code FIM/14N



Statistics in Molecular Biology and Genetics

Francoise Seillier-Moiseiwitsch,
Editor

This volume contains papers from the Summer Research Conference in the Mathematical Sciences jointly sponsored by the Institute of Mathematical Statistics, the American Mathematical Society, and the Society for Industrial and Applied Mathematics. The theme of the conference was Statistics in Molecular Biology and Genetics.

Articles fall into the following broad categories: population genetics, evolutionary genetics, protein structure, genetic mechanisms, quantitative genetics, human genetics, and sequence motifs. Talks by Professors D. Botstein, M.-C. King, and M. Olson outlined the great need for statistical expertise in cutting-edge biological technology. Their stimulating presentations offered very clear overviews of directions in important areas of genetic research, such as physical mapping, genetic mapping, and functional genetics. Manuscripts went through vigorous review, making this a fine comprehensive volume on the topic.

This item will also be of interest to those working in applications.

Co-published by the American Mathematical Society and the Institute of Mathematical Statistics.

Contents: *Genetic Mechanisms:* **H. Zhao** and **T. Speed**, On a Markov model for chromatid interference; *Population Genetics:* **S. Datta**, Some statistical aspects of cytonuclear disequilibria; **R. Fan** and **K. Lange**, Diffusion process calculations for mutant genes in nonstationary populations; **M. Nordborg**, The coalescent with partial selfing and balancing selection: An applications of structured coalescent processes; *Human Genetics:* **W. Ewens**, Statistical aspects of the transmission/disequilibrium test (TDT); **E. Thompson** and **S. Heath**, Estimation of conditional multilocus gene identity among relatives; *Quantitative Genetics:* **K. Broman** and **T. Speed**, A review of methods for identifying QTL's in experimental crosses; *Evolutionary Genetics:* **M. Newton**, **B. Mau**, and **B. Larget**, Markov chain Monte Carlo for the Bayesian analysis of evolutionary trees from aligned molecular sequences; **J. Felsenstein**, **M. Kuhner**, **J. Yamato**, and **P. Beerli**, Likelihoods on coalescents: A Monte Carlo sampling approach to inferring parameters from population samples of molecular data; **K. Crandall**, Uses of statistical parsimony in HIV analyses; **P. Joyce**, **L. Fox**, **N. Casavant**, and **H. Wichman**, Linear estimators for the evolution of transposable elements; **M. Karnoub**, **F. Seillier-Moiseiwitsch**, and **P. K. Sen**, A conditional approach to the detection of correlated mutations; **A. Lapedes**, **B. Girard**, **L. Liu**, and **G. Stormo**, Correlated mutations in protein sequences: Phylogenetic and structural effects; *Sequence Motifs:* **G. Reinert** and **S. Schbath**, Compound Poisson approximations for occurences of multiple words; *Protein Structure:* **M. Trosset** and **G. Phillips**, Deriving interatomic distance bounds from chemical structure; **L. Edler** and **J. Grassmann**, Protein fold class prediction is a new field for statistical classification and regression.

October 1999, 313 pages, Softcover, ISBN 0-940600-47-1, 2000 *Mathematics Subject Classification*: 60-XX, 62-XX, 92-XX, **Individual member $36**, **All Individuals $36**, List $45, Institutional member $36, Order code SMBGN

Previously Announced Publications

Extension Theory

Hermann Grassmann

The *Ausdehnungslehre* of 1862 is Grassmann's most mature presentation of his "extension theory". The work was unique in capturing the full sweep of his mathematical achievements.

Compared to Grassmann's first book, *Lineale Ausdehnungslehre*, this book contains an enormous amount of new material, including a detailed development of the inner product and its relation to the concept of angle, the "theory of functions" from the point of view of extension theory, and Grassmann's contribution to the Pfaff problem. In many ways, this book is the version of Grassmann's system most accessible to contemporary readers.

This translation is based on the material in Grassmann's "Gesammelte Werke", published by B. G. Teubner (Stuttgart and Leipzig, Germany). It includes nearly all the Editorial Notes from that edition, but the "improved" proofs are relocated, and Grassmann's original proofs are restored to their proper places. The original Editorial Notes are augmented by Supplementary Notes, elucidating Grassmann's achievement in modern terms.

This is the third in an informal sequence of works to be included within the History of Mathematics series, co-published by the AMS and the London Mathematical Society. Volumes in this subset are classical mathematical works that served as cornerstones for modern mathematical thought.

This item will also be of interest to those working in general and interdisciplinary areas.

Co-published with the London Mathematical Society. Members of the LMS may order directly from the AMS at the AMS member price. The LMS is registered with the Charity Commissioners.

History of Mathematics, Volume 19

April 2000, approximately 403 pages, Softcover, ISBN 0-8218-2031-1, 2000 *Mathematics Subject Classification*: 01A55, 15A75, **Individual member $45**, List $75, Institutional member $60, Order code HMATH/19RT003

Introduction to Mathematical Finance

David C. Heath, *Cornell University, Ithaca, NY*, and **Glen Swindle**, *Avista Energy, Houston, TX*, Editors

The foundation for the subject of mathematical finance was laid nearly 100 years ago by Bachelier in his fundamental work, *Théorie de la spéculation*. In this work, he provided the first treatment of Brownian motion. Since then, the research of Markowitz, and then of Black, Merton, Scholes, and Samuelson brought remarkable and important strides in the field. A few years later, Harrison and Kreps demonstrated the fundamental role of martingales and stochastic analysis in constructing and understanding models for financial markets. The connection opened the door for a flood of mathematical developments and growth.

Concurrently with these mathematical advances, markets have grown, and developments in both academia and industry continue to expand. This lively activity inspired an AMS Short Course at the Joint Mathematics Meetings in San Diego (CA).

The present volume includes the written results of that course. Articles are featured by an impressive list of recognized researchers and practitioners. Their contributions present deep results, pose challenging questions, and suggest directions for future research. This collection offers compelling introductory articles on this new, exciting, and rapidly growing field.

This item will also be of interest to those working in probability.

Contributors include: S. E. Shreve, M. Avellaneda, F. Delbaen, W. Schachermayer, D. Heath, Y. Aït-Sahalia, and T. Zariphopoulou.

Proceedings of Symposia in Applied Mathematics, Volume 57

March 2000, 167 pages, Hardcover, ISBN 0-8218-0751-X, 2000 *Mathematics Subject Classification*: 91B28; 60H30, 91B24, 93E20, **All AMS members $24**, List $30, Order code PSAPM/57RT003

Recommended Text

Dynamics in One Complex Variable

John Milnor, *State University of New York at Stony Brook, NY*

A publication of Vieweg Verlag.

The text studies the dynamics of iterated holomorphic mappings from a Riemann surface to itself, concentrating on the classical case of rational maps of the Riemann sphere. It is based on introductory lectures given by the author at SUNY, Stony Brook (NY), over the past 10 years.

The subject is large and rapidly growing. These lecture notes are intended to introduce readers to some key ideas in the field and to form a basis for further study. Readers are assumed to be familiar with the basics of complex variable theory and of two-dimensional differential geometry, as well as some basic topics from topology. The exposition is clear and enriched by many beautiful illustrations.

The AMS is exclusive distributor in North America, and non-exclusive distributor worldwide except in Germany, Switzerland, Austria, and Japan.

Vieweg Monographs

August 1999, 257 pages, Softcover, ISBN 3-528-03130-1, 2000 *Mathematics Subject Classification*: 37Fxx, **All AMS members $26**, List $29, Order code VW/9RT003

p-adic L-Functions and p-adic Representations

Bernadette Perrin-Riou, *Université Paris-Sud, France*

Since the original publication of this book in French (see Astérisque 229, 1995), the field has undergone significant progress. These advances are noted in this English edition. Also, some minor improvements have been made to the text.

SMF members are entitled to AMS member discounts.

SMF/AMS Texts and Monographs, Volume 3

February 2000, 150 pages, Softcover, ISBN 0-8218-1946-1, LC 99-055660, 2000 *Mathematics Subject Classification*: 11E95, 11G40, 11R32, 11R42, **All AMS members $39**, List $49, Order code SMFAMS/3RT003

Mathematical Sciences Professional Directory, 2000

March 2000, approximately 232 pages, Softcover, ISBN 0-8218-2043-5, 2000 *Mathematics Subject Classification*: 00-XX, List $50, Institutional member $40, Order code PRODIR/2000RT003

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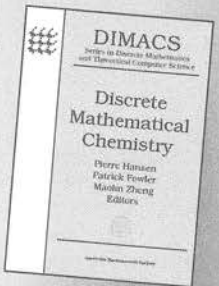

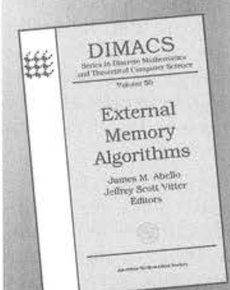

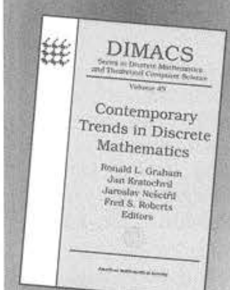

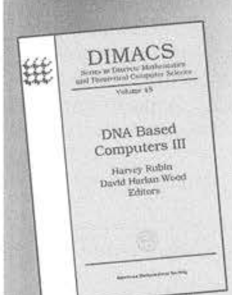

This series contains volumes coming out of programs at the Center for Discrete Mathematics and Theoretical Computer Science (DIMACS), located at Rutgers University. Included are written accounts of the proceedings of conferences and workshops held at the Center. Volumes result from the activities of special years and focus on education in discrete mathematics and theoretical computer science. For more titles in this series, visit the AMS Bookstore at www.ams.org/bookstore/.

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Pierre Hansen, *GERARD, Montreal, PQ, Canada*, **Patrick Fowler**, *University of Exeter, England*, and **Maolin Zheng**, *Lexis-Nexis, Mianmisburg, OH*, Editors

2000; ISBN 0-8218-0987-3; 392 pages; Hardcover; **Individual member $59**, List $99, Institutional member $79, Order Code DIMACS-HANSEN2CT003

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Volume 49; 1999; ISBN 0-8218-0963-6; 389 pages; Hardcover; **Individual member $54**, List $90, Institutional member $72, Order Code DIMACS/49CT003

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Harvey Rubin, *University of Pennsylvania, Philadelphia, PA*, and **David Harlan Wood**, *University of Delaware, Newark, DE*, Editors

Volume 48; 1999; ISBN 0-8218-0842-7; 375 pages; Hardcover; **Individual member $47**, List $79, Institutional member $63, Order Code DIMACS/48CT003

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Volume 47; 1999; ISBN 0-8218-0826-5; 286 pages; Hardcover; **Individual member $39**, List $65, Institutional member $52, Order Code DIMACS/47CT003

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Volume 46; 1998; ISBN 0-8218-1004-9; 249 pages; Hardcover; **Individual member $33**, List $55, Institutional member $44, Order Code DIMACS/46CT003

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Volume 45; 1999; ISBN 0-8218-0992-X; 160 pages; Hardcover; **Individual member $23**, List $39, Institutional member $31, Order Code DIMACS/45CT003

DNA Based Computers II

Laura F. Landweber, *Princeton University, NJ*, and **Eric B. Baum**, *NEC Research Institute, Princeton, NJ*, Editors

Volume 44; 1999; ISBN 0-8218-0756-0; 275 pages; Hardcover; **Individual member $35**, List $59, Institutional member $47, Order Code DIMACS/44CT003

DNA Based Computers

Richard J. Lipton, *Princeton University, NJ*, and **Eric B. Baum**, *NEC Research Institute, Princeton, NJ*, Editors

Volume 27; 1996; ISBN 0-8218-0973-3; 221 pages; Softcover; **Individual member $29**, List $49, Institutional member $39, Order Code DIMACS/27.SCT003

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Qualified applicants should send a detailed curriculum vitae, a brief statement of their specific teaching and research objectives, and three letters of recommendation, at least one of which addresses teaching potential, to: Visitor Search Committee, Mathematical Sciences Department, WPI, 100 Institute Road, Worcester, MA 01609-2280.

Applicants will be considered on a con-

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THE UNIVERSITY OF NORTH CAROLINA AT CHARLOTTE
Chair of Department of Mathematics

Applications and nominations are invited for the position of chair of the department of mathematics, beginning fall 2000. The chair is expected to provide leadership in the department's research endeavors as well as its instructional programs. Qualifications include a strong record of research and a commitment to undergraduate and graduate teaching. Also important is an equal appreciation of scholarly research in applied and pure mathematics, statistics, and mathematics education. Administrative experience is preferable but not mandatory. Appointment is a senior rank on a 12-month basis, and the salary is competitive.

The Department of Mathematics (http://www.math.uncc.edu) has active research programs in applied mathematics (mathematical physics, numerical analysis, inverse problems, probability, statistics, and dynamical systems), pure mathematics (analysis, algebra), and mathematics education. The department currently has forty full-time faculty, ten visiting faculty, thirty graduate students, and 120 undergraduate majors, and offers degree programs leading to B.S. and B.A. degrees in mathematics, M.S. degrees in applied mathematics and applied statistics, M.A. degrees in mathematics and mathematics education, and a Ph.D. degree in applied mathematics. The department strives to continue its quality research and teaching programs during the university's planned transition to a research status institution.

The University of North Carolina at Charlotte, with an enrollment of 17,000 students, is the fourth largest campus in the North Carolina University system and is located in the rapidly growing Charlotte Metropolitan Area with a population in excess of 1.3 million. It enjoys a mild climate and diverse cultural events as well as easy access to the Blue Ridge Mountains to the west and the Atlantic coast to the east.

Candidates should submit a C.V., a statement of professional goals, and the names, addresses, and telephone numbers of four references to: Dr. Wei Cai, Chair of Mathematics Chair Search Committee, Department of Mathematics, UNC Charlotte, Charlotte, NC 28223. Review of the applications will begin in January 2000 and will continue until the position is filled. If you have any questions, e-mail Dr. Cai at wcai@uncc.edu. The University of North Carolina at Charlotte is an AA/EOE Employer.

OHIO

OHIO STATE UNIVERSITY

Director of The Mathematics and Statistics Learning Center (MSLC) The MSLC is an exciting joint venture by the Mathematics and Statistics Departments to enhance the mathematical learning experience at OSU. The MSLC provides the following:

- tutoring services
- small group learning facilities
- computer laboratories
- faculty, teaching assistant (TA), and peer-mentor training programs
- support for pedagogical experimentation
- state-of-the-art teaching materials

The duties and responsibilities of the director of the MSLC will include:

- ongoing development of the MSLC
- working with the associate director to oversee daily operations
- working with faculty and TAs to improve teaching effectiveness
- teaching one lower division mathematics (or statistics) course per quarter
- serving as liaison to other university departments
- community outreach
- writing grant proposals for supplemental resources

We are seeking a dynamic, energetic, individual with innovative ideas and outstanding communication skills. Qualifications include an advanced degree in mathematics, statistics, or math education, demonstrated excellence in teaching mathematical sciences at the college level, and some experience in administration.

The position is a twelve-month, full-time, Administrative and Professional staff appointment, with full Ohio State University benefits. The salary will be commensurate with qualifications and experience. The anticipated starting date is July 1, 2000.

Send letter of interest and résumé to Prof. Sia Wong, Chair MSLC Search Committee, Department of Mathematics, The Ohio State University, 231 West 18th Avenue, Columbus, OH 43210. Evaluation of applications will begin February 2000 and will continue until a suitable candidate is found. THE OHIO STATE UNIVERSITY IS AN EQUAL OPPORTUNITY EMPLOYER. Learn more about the MSLC and the director's duties at http://www.math.ohio-state.edu/mslc.

TEXAS

SOUTHWEST TEXAS STATE UNIVERSITY
Math Education Position

The Department of Mathematics invites applications for a tenure-track associate or full professor position in mathematics education starting September 1, 2000.

Responsibilities will include: teaching a broad range of mathematics courses in both mathematics and mathematics education at undergraduate and graduate levels, including courses for prospective and experienced teachers of mathematics; carrying on research in mathematics education and other scholarly activity that leads to publication. Applicants must have a doctorate in mathematics education with a strong mathematics background or a doctorate in mathematics with a strong mathematics education background. The successful applicant will be expected to spearhead an effort to bolster our Math Education program by attracting more students to the program and being active in the math education community.

To be considered, an application must include a cover letter, curriculum vitae, a research plan, and three or more letters of recommendation. All applications must be received by April 3, 2000. Apply to:

Prof. Xingde Jia
Recruitment Committee Chair
Department of Mathematics
Southwest Texas State University
San Marcos, Texas 78666 USA
Phone: 512-245-2551,
Fax: 512-245-3425
E-mail: math@swt.edu

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INDIA

INDIAN INSTITUTE OF SCIENCE
Department of Mathematics
India

The department has several openings at the assistant professor level in any area of pure or applied mathematics. Candidates should have a Ph.D. with three years of postdoctoral research experience (relaxable in exceptional cases) and should have significant publications. For more details write to: Chairman, Department of Mathematics, Indian Institute of Science, Bangalore 560 012, India; e-mail: prasad@math.iisc.erent.in or visit home page http://www.math.iisc.ernet.in/.

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Pure & appl. adv. & research level, any age, usable cond. Reprints OK. One box to whole libraries sought. Contact: Collier Brown or Kirsten Berg @ **Powell's Technical Bks.**, Portland, OR. Call 800-225-6911, fax 503-228-0505, or e-mail kirsten@technical.powells.com.

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- 12 Field theory and polynomials
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- 14 Algebraic geometry
- 15 Linear and multilinear algebra; matrix theory
- 16 Associative rings and algebras
- 17 Nonassociative rings and algebras
- 18 Category theory, homological algebra
- 19 *K*-theory
- 20 Group theory and generalizations
- 22 Topological groups, Lie groups
- 26 Real functions
- 28 Measure and integration
- 30 Functions of a complex variable
- 31 Potential theory
- 32 Several complex variables and analytic spaces
- 33 Special functions
- 34 Ordinary differential equations
- 35 Partial differential equations
- 37 Dynamical systems and ergodic theory
- 39 Difference and functional equations
- 40 Sequences, series, summability
- 41 Approximations and expansions
- 42 Fourier analysis
- 43 Abstract harmonic analysis
- 44 Integral transforms, operational calculus
- 45 Integral equations
- 46 Functional analysis
- 47 Operator theory
- 49 Calculus of variations and optimal control; optimization
- 51 Geometry
- 52 Convex and discrete geometry
- 53 Differential geometry
- 54 General topology
- 55 Algebraic topology
- 57 Manifolds and cell complexes
- 58 Global analysis, analysis on manifolds
- 60 Probability theory and stochastic processes
- 62 Statistics
- 65 Numerical analysis
- 68 Computer science
- 70 Mechanics of particles and systems
- 74 Mechanics of deformable solids
- 76 Fluid mechanics
- 78 Optics, electromagnetic theory
- 80 Classical thermodynamics, heat transfer
- 81 Quantum theory
- 82 Statistical mechanics, structure of matter
- 83 Relativity and gravitational theory
- 85 Astronomy and astrophysics
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For ***ordinary members*** whose annual professional income is below $65,000, the dues are $99; for those whose annual professional income is $65,000 or more, the dues are $132.

For a ***joint family membership,*** one member pays ordinary dues, based on his or her income; the other pays ordinary dues based on his or her income, less $20. (Only the member paying full dues will receive the *Notices* and the *Bulletin* as a privilege of membership, but both members will be accorded all other privileges of membership.)

Minimum dues for ***contributing members*** are $198. The amount paid which exceeds the higher ordinary dues level and is purely voluntary may be treated as a charitable contribution.

For either ***students*** or ***unemployed individuals,*** dues are $33, and annual verification is required.

The annual dues for ***reciprocity members*** who reside outside the U.S. are $66. To be eligible for this classification, members must belong to one of those foreign societies with which the AMS has established a reciprocity agreement, and annual verification is required. Reciprocity members who reside in the U.S. must pay ordinary member dues ($99 or $132).

The annual dues for ***category-S members,*** those who reside in developing countries, are $16. Members can chose only one privilege journal. Please indicate your choice below.

Members can purchase a ***multi-year membership*** by prepaying their current dues rate for either two, three, four or five years. This option is not available to category-S, unemployed, or student members.

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- ☐ Dansk Matematisk Forening
- ☐ Deutsche Mathematiker-Vereinigung
- ☐ Edinburgh Mathematical Society
- ☐ Egyptian Mathematical Society
- ☐ Gesellschaft für Angewandte Mathematik und Mechanik
- ☐ Glasgow Mathematical Association
- ☐ Hellenic Mathematical Society
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Journal mailing lists must be printed four to six weeks before the issue date. Therefore, in order to avoid disruption of service, members are requested to provide the required notice well in advance.

Besides mailing addresses for members, the Society's records contain information about members' positions and their employers (for publication in the Combined Membership List). In addition, the AMS maintains records of members' honors, awards, and information on Society service.

When changing their addresses, members are urged to cooperate by supplying the requested information. The Society's records are of value only to the extent that they are current and accurate.

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MATH 2000

McMaster University, Hamilton, Ontario, June 10 - 13, 2000

McMaster University, the University of Waterloo, the Canadian Mathematical Society, the Canadian Applied and Industrial Mathematics Society, the Canadian Operational Research Society, the Canadian Society for History and Philosophy of Mathematics, the Canadian Undergraduate Mathematics Conference, and the 14th Canadian Symposium on Fluid Dynamics have joined together to celebrate World Mathematical Year 2000. This joint meeting is to be held at McMaster University, Hamilton, Ontario, June 10-13, 2000, and will bring together researchers, educators, and students from around the world. Please join us at MATH 2000.

All scientific activities will take place from Saturday, June 10, to Tuesday, June 13, at the campus of McMaster University, Hamilton (Ontario), Canada. A diverse program is planned and detailed below.

The most up-to-date information concerning the program, including scheduling, is available at the World Wide Web address `http://camel.math.ca/CMS/Events/math2000/`.

Meeting registration forms, abstract forms, and hotel accommodation forms will be published in the February 2000 issue of the *CMS Notes* and in other Society publications and will also be available on the Web site.

Public Lecture

Sunday, June 11, 7:00 p.m., **James Stewart, McMaster University**, *How to Enliven the Mathematics Classroom.*

Plenary Speakers

Francis Clarke (Lyon), *Control Theory*; **Ioannis Karatzas** (Columbia), *Financial Mathematics*; **P. L. Lions** (Paris), *Nonlinear PDE*; **Dusa McDuff** (SUNY, Stony Brook), *Symplectic Geometry*; **David Mumford** (Brown), *Vision & Imaging*; **G. Myers** (Celera Genomics), *Mathematical Biology*; **R. Pierrehumbert** (Chicago), *Geophysical Fluid Dynamics*; **Carl Pomerance** (Georgia), *Cryptography & Number Theory*; **M. Queyranne** (UBC), *Operations Research*; **L. Shampine** (Southern Methodist Univ.), *Education*; **L. van den Dries** (Urbana), *Logic*; **Shing-Tung Yau** (Harvard), *PDE*–to be confirmed; **Efim I. Zelmanov** (Yale), *Group Theory*.

Prize Lectures

CMS Krieger-Nelson Lecture, CMS Jeffery-Williams Lecture, CAIMS Doctoral Prize.

Symposia

By invitation of the Meeting Committee, there will be symposia in the following areas:

Algebraic Groups (Org: **Vladimir Platonov**, University of Waterloo).

Biofluid Dynamics & Medical Science (Org: **Siv Sivaloganathan**, University of Waterloo).

Control Theory (Org: **Kirsten Morris**, University of Waterloo).

Cryptography & Number Theory (Org: **Hugh Williams**, University of Manitoba, and **Gary Walsh**, CSE, University of Ottawa).

Education (Org: **Eric Muller**, Brock University, and **Robert Corless**, University of Western Ontario).

Financial Mathematics (Org: **Luis Seco**, University of Toronto).

Geophysical Fluid Dynamics (Org: **Kevin Lamb**, University of Waterloo, and **Richard Greatbatch**, Dalhousie University).

Group Theory (Org: **Olga Kharlampovich**, McGill University).

History of Mathematics at the Dawn of a New Millennium (Org: **Tom Archibald**, Acadia University).

Imaging & Vision (Org: **Ed Vrscay** and **Alan Law**, University of Waterloo).

Industrial Statistics (Org: **N. Balakrishnan**, McMaster University).

Logic (Org: **Bradd Hart**, McMaster University, and **Claude Laflamme**, University of Calgary).

Math Biology (Org: **Robert Miura**, University of British Columbia).

Math on the Internet (Org: **June Lester**, Simon Fraser University).

Operations Research (Org: **Rick Caron**, University of Windsor).

Partial Differential Equations (Org: **Pengfei Guan**, McMaster University).

Symplectic Geometry (Org: **Lisa Jeffrey**, University of Toronto).

Topology of Manifolds (Org: **Ronnie Lee**, Yale University, and **Ian Hambleton**, McMaster University).

Contributed Papers Session

Contributed papers of 15 minutes' duration are invited. Abstracts for CMS contributed papers should be prepared as specified below. For an abstract to be eligible, it must be received before **February 28, 2000**. The abstract must be accompanied by its contributor's registration form and payment of the appropriate fees.

Graduate Student Poster Session

There will be a poster session for graduate students, organized by Sue Ann Campbell, University of Waterloo, on

Saturday, June 10, from 6:00 p.m. to 7:30 p.m. A reception will be held during this poster session.

Travel Grants for Graduate Students

Limited funds are available to partially fund the travel and accommodation costs for graduate students. For more information, please contact the Meeting Committee at gradtravel-math2000@cms.math.ca.

Social Events

A welcoming reception will be held during registration on Friday evening, June 9, from 7:00 p.m. to 9:00 p.m. A cash bar will be available.

The Delegates' Luncheon will be held on Saturday, June 10, at McMaster University. A ticket to this luncheon is included in all registration fee categories.

Everyone is also invited to a reception at the Celebration Banquet Hall on Sunday, June 11, from 6:00 p.m. to 7:00 p.m., preceding the public lecture given by James Stewart (McMaster) at 7:00 p.m.

A banquet will be held on Monday, June 12, at 7:30 p.m. at the Royal Botanical Gardens, preceded by a cash bar at 6:30 p.m. Tickets to this event are available at $50 each. Bus service will be provided to and from this event.

Since limited catering facilities are available at McMaster University, luncheon tickets may be purchased for Sunday, Monday, and Tuesday.

Coffee and juice will be available during the scheduled breaks.

Related Activities

MITACS General Meeting: The 1st General Meeting of MITACS will be held on June 6 and 7 at the Toronto Conference Center. For complete information regarding registration, please contact Bradd Hart at bhart@fields.utoronto.ca.

CMS Job Fair: The 2nd CMS Job Fair will be held on June 6 and 7 at the Toronto Conference Center in conjunction with the MITACS General Meeting . For complete information regarding registration and submission of résumés, please contact Bradd Hart at bhart@fields.utoronto.ca.

Symposium on the Legacy of John Charles Fields: This special symposium will take place on June 8 and 9, 2000, at the The Royal Ontario Museum, Toronto, Ontario. For information, please contact The Fields Institute at geninfor@fields.utoronto.ca, or consult the Fields Web site at http://www.fields.utoronto.ca/.

CSHPM: The Canadian Society for History and Philosophy of Mathematics is holding its 2000 Annual Meeting at McMaster University on June 10-12. Participants should complete the MATH 2000 registration form. For more information on the CSHPM program, please contact Thomas Archibald at tom.archibald@acadiau.ca, or consult the CSHPM Web site at http://kingsu.ab.ca/~glen/cshpm/home.htm.

CUMC: The 2000 Canadian Undergraduate Mathematics Conference will take place June 6-10 at McMaster University. For information regarding the program and registration, please contact Gabriella Couto at cumc2000@cms.math.ca, or consult the CUMC Web site at http://cumc.math.ca/cumc2000/.

IMO Alumni Reunion: This special reunion celebrating 20 years of Canadian participation at the International Mathematical Olympiad will be held from 11:30 a.m. to 3:00 p.m. on June 11 in Toronto. For more information on the program, please contact Richard Hoshino at IMO-reunion@cms.math.ca.

Business Meetings

The CMS, CAIMS, and CSHPM will be holding business meetings during the course of MATH 2000. Additional information will be provided in later announcements and may be found on the societies' Web sites.

The CAIMS General Meeting will be held from 12:30 p.m. to 2:00 p.m. on Monday, June 12.

The CMS Executive Committee Meeting will meet on Thursday, June 8, from 9:00 a.m. to 3:00 p.m. in Suite 1111 of the Royal Connaught Howard Johnson Plaza.

The CMS Development Group Luncheon will be held from 11:00 a.m. to 1:00 p.m. on Friday, June 9, in the Dundurn Room of the Royal Connaught Howard Johnson Plaza.

The CMS Board of Directors meeting will be held from 1:30 p.m. to 6:30 p.m. on Friday, June 9, in the Ontario Room of the Royal Connaught Howard Johnson Plaza.

The CMS Annual General Meeting will be held from 12:30 p.m. to 2:00 p.m. on Monday, June 12.

The CSHPM Business Meeting will be held from 12:00 p.m. to 2:00 p.m. on Monday, June 12.

Exhibits

Exhibits will be open during specified hours during the conference.

Submission of Abstracts

Titles for plenary speakers, prize lecturers, invited symposia speakers, and contributed papers will appear in the **April** issue of the *CMS Notes*. An updated list will appear in the **May** issue. All **abstracts** will be published in the meeting program and will also be available at http://camel.math.ca/CMS/Events/math2000/.

All speakers should send the title of their talk to their organizers before January 4, 2000, and submit their abstract as instructed by their organizers.

Plenary Speakers, Prize Lecturers, and Invited Symposia Speakers: Abstracts may be sent electronically, following instructions given below. Abstracts may also be prepared on the standard form available from the session organizer or on the Web site. Abstracts should be sent to the Abstracts Coordinator, MATH 2000, CMS Executive Office, 577 King Edward, Suite 109, Ottawa, Ontario, Canada K1N 6N5, **by February 1, 2000.**

Contributed Papers: Abstracts may be sent electronically, following instructions given below. Abstracts may also be prepared on the standard form available from the **February 2000** issue of the *CMS Notes* or on the Web site. Abstracts should be sent to the Abstracts Coordinator, MATH 2000, CMS Executive Office, 577 King

Edward, Suite 109, Ottawa, Ontario, Canada K1N 6N5, **by February 28, 2000.**

Electronic submission of abstracts: Files including the speaker's name, affiliation, complete address, title of talk, and abstract may be sent to `abstracts@cms.math.ca` (speakers) or `cp-abstracts@cms.math.ca` (contributed papers).

Please note the above deadlines for the submission of your abstract.

Registration

The Canadian Mathematical Society will be handling registrations for MATH 2000. Registration forms will appear in the **February 2000** issue of the *CMS Notes* and will be published in other Society publications as well. Forms are also available from:

MATH 2000 Registration
CMS Executive Office
577 King Edward, Suite 109
P. O. Box 450, Station A
Ottawa, Ontario, Canada K1N 6N5
Tel: 613-562-5702
Fax: 613-565-1539
E-mail: `meetings@cms.math.ca`

Electronic preregistration is available at `http://camel.math.ca/CMS/Events/math2000/`.

Payment for preregistration may be made by check or by VISA or MasterCard. Although registration fees are given in Canadian dollars, delegates may send checks in U.S. dollars by contacting their financial institution for the current exchange rate.

Please note that **payment must be received on or before May 15 in order to qualify for reduced rates.**

	Before May 15	After May 15
Plenary speakers/prize lecturers	\$ 0	\$ 0
Session speakers/organizers	135	135
Delegates with grants	270	350
Delegates without grants	135	175
One-day fee	135	175
Postdocs, retired, students, unemployed	50	50
Banquet (free for plenary/prize speakers)	50	50
Lunch (tickets required for Sunday–Tuesday), each	10.50	10.50

Refund Policy

Delegates wishing to cancel their registration must notify the CMS Executive Office **in writing before June 1** to receive a refund less a \$40 processing fee. Those whose contributed paper has not been accepted will receive a full refund upon request.

Accommodations

It is recommended that those attending the conference book early to avoid disappointment. Blocks of rooms have been reserved at the locations given below and will be held until **May 8, 2000.** Reservations not made by that date will be on a request-only, space-available basis. Rates quoted are in Canadian dollars.

It should be noted that most of the hotels are at some distance from the university but are generally accessible with public transportation. The closest is the Visitors Inn, which is approximately 1 kilometer from the university. The McMaster Residence borders on a wooded area surrounding "Cootes Paradise", the western tip of Lake Ontario.

Reservation Deadline: May 8, 2000.

Ramada Plaza Hotel
150 King Street East, Hamilton, ON L8N 1B2
Check-in: 3:00 p.m.; Check-out: 12:00 noon
Applicable taxes: GST (7%), hotel tax (5%)
Phone: 905-528-3451; Fax: 905-525-8638
Rates: \$99, single/double occupancy; \$10 per additional adult
(continental breakfast included)

Royal Connaught Howard Johnson Plaza
112 King Street East, Hamilton, ON L8N 1A8
Check-in: 3:00 p.m.; Check-out: 12:00 noon
Applicable taxes: GST (7%), hotel tax (5%)
Phone: 905-546-8111; Fax: 905-546-8118
Rates: \$85, single/double/triple occupancy; \$10, rollaway per day

Holiday Inn Burlington
3063 South Service Rd., Burlington, ON L7N 3E9
Check-in: 3:00 p.m.; Check-out: 12:00 noon
Applicable taxes: GST (7%), hotel tax (5%)
Phone: 905-639-4443; Fax: 905-333-4033
Rates: \$109, single/double occupancy

Note: Holiday Inn Burlington is not easily accessible by public transportation. A car may be needed.

Visitors Inn
649 Main Street West, Hamilton, ON L8S 1A2
Check-in: 2:00 p.m.; Check-out: 11:00 a.m.
Applicable taxes: GST (7%), hotel tax (5%)
Phone: 905-529-6979; Fax: 905-529-6979
Rates: \$79, single occupancy; \$84, double occupancy

McMaster Residence
Housing & Conference Services, Commons Building 129B, McMaster University, 1280 Main Street West, Hamilton, ON L8S 4K1
Check-in: 7:00 a.m. to 11:00 p.m.; Check-out: 12:00 noon
Applicable taxes: GST (7%), hotel tax (5%)
Phone: 905-525-9140, ext. 24781; Fax: 905-529-3319
E-mail: `confs@mcmaster.ca`
Rates: \$39.75 per person, single occupancy; \$32.75 per person, double occupancy; rates include breakfast, linens, towels, parking

For McMaster Residence, the Accommodation Reservation Form should be sent with full payment for your entire stay. Only requests accompanied with full payment will be confirmed. When calculating total, please add 12% taxes to the above rates.

In all cases, delegates must make their own reservations. The conference rate is extended up to two days pre- and postconvention. Please mention that you are participating in MATH 2000.

Accommodation cancellations: For the hotels, reservations will be held until 6:00 p.m. on the arrival day only unless you provide a deposit for one night or the reservation is guaranteed by a major credit card. Cancellation may be made up to 6:00 p.m. on the day of arrival.

For McMaster Residence, refunds will be granted if written notice of cancellation is received by Housing & Conference Services 72 hours prior to arrival date. Cancellations are subject to a $10 administrative fee.

Acknowledgements

Support from the following is gratefully acknowledged:

- Centre de recherches mathématiques
- The Fields Institute for Research in Mathematical Sciences
- The Pacific Institute for the Mathematical Sciences
- McMaster University
- University of Waterloo

The participating societies of MATH 2000 wish to acknowledge the contribution of the members of the Meeting Committee for organizing this meeting and presenting these exciting scientific, educational, and social programs. Thanks are also extended to the many session organizers for their participation in this unique event.

Meeting Committee

Program

Meeting Director: Ian Hambleton (McMaster)
Sue Ann Campbell (Waterloo)
Niky Kamran (McGill)
Richard Kane (Western)
William Langford (Guelph)
Anna Lawniczak (Guelph)
Siv Sivaloganathan (Waterloo)
Edward Vrscay (Waterloo)
Graham Wright (CMS ex-officio)

Local Arrangements

Chair: Carl Riehm (McMaster)
N. Balakrishnan (McMaster)
Monique Bouchard (CMS ex-officio)
Bradd Hart (McMaster)
Pamela Penny (McMaster)
Gail Wolkowicz (McMaster)

Come celebrate the achievements of mathematics and contemplate what the future might bring!

MATHEMATICAL CHALLENGES OF THE 21ST CENTURY

UNIVERSITY OF CALIFORNIA LOS ANGELES
AUGUST 7–12, 2000

The purpose of the meeting is to demonstrate, not just to the mathematical community, but to the world at large, the power of mathematical ideas across the landscape of the sciences and practical affairs, while still maintaining a close link to ongoing developments. These leaders in their fields have agreed to give plenary talks. There will be 30 in all who will provide broad perspectives on mathematics in science and practical applications. Ronald L. Graham will give the AMS-MAA President's Lecture.

James G. Arthur
Alexander A. Beilinson
Sir Michael V. Berry
Haim Brezis
Alain Connes
David L. Donoho
Charles L. Fefferman
Michael H. Freedman
Helmut H. W. Hofer
Richard M. Karp
Sergiu Klainerman
Maxim Kontsevich
Peter D. Lax
Simon A. Levin
László Lovász
David Mumford
Peter Sarnak
Saharon Shelah
Peter W. Shor
Yakov G. Sinai
Richard P. Stanley
Dennis P. Sullivan
Clifford Taubes
Jean Taylor
William P. Thurston
Karen Uhlenbeck
S. R. S. Varadhan
Edward Witten
Sing-Tung Yau
Don B. Zagier

For continually updated information on the speakers and events included in this meeting, visit http://www.ams.org/amsmtgs/mathchall.html regularly.

Program Committee: Richard Askey, Spencer Bloch, Felix Browder (chair), Charles Fefferman, Peter Lax, Robert MacPherson, David Mumford, Gian-Carlo Rota (deceased), Peter Sarnak, Audrey Terras, and S. R. S. Varadhan.

Part of World Math Year 2000

AMS
AMERICAN MATHEMATICAL SOCIETY

Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on e-MATH. See `http://www.ams.org/meetings/`. Programs and abstracts will continue to be displayed on e-MATH in the Meetings and Conferences section until about three weeks after the meeting is over. Final programs for Sectional Meetings will be archived on e-MATH in an electronic issue of the *Notices* as noted below for each meeting.

Santa Barbara, California

University of California, Santa Barbara

March 11-12, 2000

Meeting #951

Western Section
Associate secretary: Bernard Russo
Announcement issue of *Notices*: January 2000
Program first available on e-MATH: February 1, 2000
Program issue of electronic *Notices*: May 2000
Issue of *Abstracts*: Volume 21, Issue 2

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

Invited Addresses

Dietmar Bisch, University of California, Santa Barbara, *Analytical and combinatorial aspects of subfactors.*

Svetlana Jitomirskaya, University of California, Irvine, *Title to be announced.*

Yair Minsky, State University of New York, Stony Brook, *Title to be announced.*

Ram Murty, Queen's University, *Zeta functions and Dirichlet series.*

Special Sessions

Automorphic Forms, **Ozlem Imamoglu** and **Jeffrey Stopple**, University of California, Santa Barbara.

Geometric Analysis, **Xian-Zhe Dai**, **Doug Moore**, **Guofang Wei**, and **Rick Ye**, University of California, Santa Barbara.

Geometric Methods in 3-Manifolds, **Daryl Cooper**, **Darren Long**, and **Martin Scharlemann**, University of California, Santa Barbara.

History of Mathematics, **James Tattersall**, Providence College.

Representation Theory of Algebras, **F. W. Anderson**, University of Oregon, **K. R. Fuller**, University of Iowa, and **B. Huisgen-Zimmermann**, University of California, Santa Barbara.

Schrodinger-Type Operators, **Abel Klein** and **Svetlana Jitomirskaya**, University of California, Irvine.

Subfactors and Free Probability Theory, **Dietmar Bisch**, University of California, Santa Barbara, **Sorin Popa**, University of California, Los Angeles, and **Dan Voiculescu**, University of California, Berkeley.

Uniformly and Partially Hyperbolic Dynamical Systems, **Bjorn Birnir**, University of California, Santa Barabara, and **Nicolai T. A. Haydn**, University of Southern California.

Lowell, Massachusetts

University of Massachusetts, Lowell

April 1-2, 2000

Meeting #952

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: February 2000
Program first available on e-MATH: February 24, 2000
Program issue of electronic *Notices*: June 2000
Issue of *Abstracts*: Volume 21, Issue 2

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

Invited Addresses

Walter Craig, Brown University, *Title to be announced.*

Erwin Lutwak, Polytechnic University, *L_p curvature.*

Alexander Nabutovsky, University of Toronto, *Variational problems for Riemannian functionals, arithmetic groups, and noncomputable functions.*

M. Beth Ruskai, University of Massachusetts, Lowell, *Quantum information theory.*

Special Sessions

Combustion Theory, **James Graham-Eagle**, University of Massachusetts, Lowell, and **Daniel A. Schult**, Colgate University.

Discrete Geometry, **Robert Connelly**, Cornell University, **Marjorie Senechal**, Smith College, **Robert M. Erdahl**, Queen's University, and **Walter J. Whiteley**, York University.

Enumerative Geometry in Physics, **Emma Previato**, Boston University.

Ergodic Theory and Dynamical Systems, **Stanley J. Eigen**, Northeastern University, and **Vidhu S. Prasad**, University of Massachusetts, Lowell.

Invariance in Convex Geometry, **Daniel A. Klain**, Georgia Institute of Technology, and **Elisabeth Werner**, Case Western Reserve University.

PDE and Dynamical Systems, **Walter L. Craig**, Brown University, and **C. Eugene Wayne**, Boston University.

Quantum Information Theory, **M. Beth Ruskai**, University of Massachusetts, Lowell, and **Christopher K. King**, Northeastern University.

Syzygies, **Irena Peeva**, Cornell University.

Teaching Mathematics in the New Millennium, **Ronald Brent**, University of Massachusetts Lowell.

Volumes on Minkowski and Finsler Spaces, **Juan Carlos Alvarez**, Université Catholique de Louvain, **Dimitri Burago**, Pennsylvania State University, and **Gaoyong Zhang**, Polytechnic University.

Vorticity in Fluid Flows: Analysis and Methods, **Louis F. Rossi**, University of Massachusetts, Lowell, **Richard B. Pelz**, Rutgers University, and **John Grant**, Naval Undersea Warfare Center.

Notre Dame, Indiana

University of Notre Dame

Alert/Update:
The correct dates of the meeting are **Saturday, April 8**, and **Sunday, April 9**. The registration and AMS Book Exhibit desk will be located on the first floor of McKenna Hall, also known as the Center for Continuing Education (CCE), and will be open from 7:30 a.m. to 4:30 p.m. on Saturday, and 8:00 a.m. to noon on Sunday. All talks will take place in McKenna Hall (CCE).

April 8-9, 2000

Meeting #953

Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: February 2000
Program first available on e-MATH: February 24, 2000
Program issue of electronic *Notices*: June 2000
Issue of *Abstracts*: Volume 21, Issue 2

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: February 15, 2000

Invited Addresses

Peter Bates, Brigham Young University, *Title to be announced.*

Andras Nemethi, Ohio State University, *Title to be announced.*

Charles Radin, University of Texas at Austin, *Title to be announced.*

David Sattinger, Utah State University, *Title to be announced.*

Special Sessions

Algebraic Coding Theory (Code: AMS SS K1), **Judy Walker**, University of Nebraska, and **Jay Wood**, Purdue University, Calumet.

Algebraic Geometry (Code: AMS SS F1), **Karen Chandler** and **Scott Nollet**, University of Notre Dame.

Algebraic Methods in Statistics (Code: AMS SS V1), **Marlos Viana**, University of Illinois, Chicago.

Applications of Invariant Manifold Theory (Code: AMS SS M1), **Peter Bates**, Brigham Young University, and **Clarence Eugene Wayne**, Boston University.

Commutative Algebra (Code: AMS SS A1), **Juan Migliore**, University of Notre Dame, and **Chris Peterson**, Colorado State University.

Cooperative Learning in Undergraduate Mathematics Education (Code: AMS SS T1), **Nahid Erfan**, University of Notre Dame, and **Vic Perera**, Kent State University.

Differential Geometry and Its Applications (Code: AMS SS B1), **Jianguo Cao**, **Brian Smyth**, and **Frederico Xavier**, University of Notre Dame.

Differential Inequalities and Applications (Code: AMS SS N1), **Paul W. Eloe**, University of Dayton.

Geometry and Analysis in Carnot Carathéodory Spaces (Code: AMS SS W1), **Scott Pauls**, Rice University.

Homotopy Theory (Code: AMS SS H1), **William G. Dwyer**, University of Notre Dame, and **Michele Intermont**, Kalamazoo College.

Index Theory and Topology (Code: AMS SS Q1), **John Roe**, Pennsylvania State University, and **Stephan Stolz** and **Bruce Williams**, University of Notre Dame.

Integrable Systems and Nonlinear Waves (Code: AMS SS E1), **Mark S. Alber** and **Gerard Misiolek**, University of Notre Dame.

Microlocal Analysis and Partial Differential Equations (Code: AMS SS D1), **Nicholas Hanges**, CUNY, Lehman College, and **Alex Himonas**, University of Notre Dame.

Nonlinear Partial Differential Equations (Code: AMS SS J1), **Qing Han** and **Bei Hu**, University of Notre Dame, and **Hong-Ming Yin**, Washington State University.

Number Theory, Algorithms, and Cryptography (Code: AMS SS R1), **Eric Bach**, University of Wisconsin, Madison, and **Jonathan Sorenson**, Butler University.

Optimization and Numerical Analysis (Code: AMS SS C1), **Leonid Faybusovich**, University of Notre Dame.

Quasigroups and Loops and Their Applications (Code: AMS SS P1), **Michael K. Kinyon**, Indiana University South Bend, and **J. D. Phillips**, Saint Mary's College of California.

Representations of Groups and Related Objects (Code: AMS SS S1), **Chris Bendel**, University of Wisconsin, Stout, and **George McNinch**, University of Notre Dame.

Several Complex Variables (Code: AMS SS G1), **Jeffrey Diller**, University of Notre Dame, and **Nancy Stanton**, Univeristy of Notre Dame.

Singularities in Algebraic Geometry (Code: AMS SS L1), **Sandor Kovacs**, University of Chicago, and **Andras Nemethi**, Ohio State University.

Symbolic Dynamics (Code: AMS SS U1), **William Geller**, Indiana University-Purdue University Indianapolis, and **Nicholas S. Ormes**, University of Texas, Austin.

Lafayette, Louisiana

University of Louisiana at Lafayette

April 14–16, 2000

Meeting #954

Southeastern Section
Associate secretary: John L. Bryant
Announcement issue of *Notices*: February 2000
Program first available on e-MATH: March 2, 2000
Program issue of electronic *Notices*: June 2000
Issue of *Abstracts*: Volume 21, Issue 2

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: February 22, 2000

Invited Addresses

Paul Aspinwall, Duke University, *Strings, duality, and geometry.*

Michael Renardy, Virginia Polytech Institute & State University, *The spectral problem for linear stability of viscoelastic flows.*

Robin Thomas, Georgia Institute of Technology, *Generalizing the Four-Color Theorem.*

Fernando Rodriguez Villegas, University of Texas at Austin, *Periods, L-functions, and arithmetic.*

Special Sessions

Continuum Theory (Code: AMS SS F1), **Thelma R. West**, University of Louisiana at Lafayette.

Fluid Dynamics (Code: AMS SS H1), **Michael Renardy**, Virginia Polytechnic Institute & State University.

Graph Theory (Code: AMS SS K1), **Robin Thomas** and **Dhruv Mubayi**, Georgia Institute of Technology.

L-functions, Periods, and Arithmetic (Code: AMS SS J1), **Fernando Rodriguez Villegas** and **Jeff Vaaler**, University of Texas at Austin.

Mathematical Models in the Biological and Physical Sciences (Code: AMS SS B1), **Azmy S. Ackleh**, **Lan Ke**, and **Robert D. Sidman**, University of Louisiana at Lafayette.

Nonlinear Differential Equations and Their Applications (Code: AMS SS C1), **C. Y. Chan**, **Keng Deng**, and **A. S. Vatsala**, University of Louisiana at Lafayette.

Quantum Topology (Code: AMS SS E1), **Patrick M. Gilmer**, Louisiana State University.

Recent Advances in Statistics (Code: AMS SS G1), **Calvin Berry**, **Kalimuthu Krishnamoorthy**, and **Nabendu Pal**, University of Louisiana at Lafayette.

Rings and Their Generalizations (Code: AMS SS A1), **Gary F. Birkenmeier** and **Henry E. Heatherly**, University of Louisiana at Lafayette.

Scientific Computing (Code: AMS SS D1), **R. Baker Kearfott**, **Qin Sheng**, and **Christo Christov**, University of Louisiana at Lafayette.

Odense, Denmark

Odense University

June 13-16, 2000

Note: This is a World Math Year 2000 (WMY2000) event.

Meeting #955

First AMS-Scandinaivan International Mathematics Meeting. Sponsored by the AMS, Dansk Matematisk Forening, Suomen matemaattinen yhdistys, Icelandic Mathematical Society, Norsk Matematisk Forening, and Svenska matematiker-samfundet.
Associate secretary: Robert M. Fossum
Announcement issue of *Notices*: March 2000
Program first available on e-MATH: None
Program issue of electronic *Notices*: None
Issue of *Abstracts*: None

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: March 15, 2000

Invited Addresses

Tobias Colding, Courant Institute, New York University, *Embedded minimal surfaces and applications to 3-manifold topology.*

Johan Håstad, Royal Institute of Technology, Stockholm, *Title to be announced.*

Nigel J. Hitchin, University of Oxford, *Title to be announced.*

Elliott Lieb, Princeton University, *Title to be announced.*

Pertti Mattila, University of Jyväskylä, *What has Menger curvature given to complex and harmonic analysis?*

Curtis T. McMullen, Harvard University, *The shape of the moduli space of Riemann surfaces.*

Alexei N. Rudakov, Norwegian University of Science & Technology, *Title to be announced.*

Karen Uhlenbeck, University of Texas at Austin, *Integrable systems in geometry.*

Dan Voiculescu, University of California, Berkeley, *Title to be announced.*

Special Sessions

Algebraic Groups and Representation Theory, **Henning Haahr Andersen** and **Niels Lauritzen**, Aarhus University.

Complex Analysis in Higher Dimensions, **Finnur Larusson**, University of Western Ontario, and **Ragnar Sigurdsson**, University of Iceland.

Differential Geometry, **Claude R. LeBrun**, State University of New York at Stony Brook, and **Peter Petersen**, University of California, Los Angeles.

Discrete Mathematics, **Iiro S. Honkala**, University of Turku, and **Carsten Thomassen**, Technical University of Denmark.

Dynamical Systems, **Michael Benedicks**, Royal Institute of Science, Stockholm, and **Carsten Lunde Petersen**, Roskilde.

Geometric Analysis/PDE, **Gerd Grubb**, University of Copenhagen, and **Bent Ørsted**, Odense University.

Joint EWM and AWM Session, **Lisbeth Fajstrup**, Aalborg University, **Tinne Hoff Kjeldsen**, Roskilde, and **Christina Wiis Tonnesen-Friedman**, Aarhus University.

K-Theory and Operator Algebras, **Soren Eilers**, University of Copenhagen, and **Nigel D. Higson**, Pennsylvania State University.

Linear Spaces of Holomorphic Functions, **Peter L. Duren**, University of Michigan, Ann Arbor, **Michael Stessin**, State University of New York at Albany, and **Harold S. Shapiro**, Royal Institute of Technology, Stockholm.

Mathematical Physics, **Bergfinnur Durhuus**, University of Copenhagen, and **Kurt Johansson**, Royal Institute of Technology, Stockholm.

Mathematics Education, **Claus Michelsen**, Odense, and **Martti E. Pesonen**, University of Joensuu.

Stochastic DE and Financial Mathematics, **Tomas Björk**, University of Stockholm, and **Bernt Øksendal**, University of Oslo.

Contributed Papers

There will be no sessions of contributed papers.

Conference Web Site

The information in this announcement is taken from the Web site maintained by the local organizers. See `http://www.imada.sdu.dk/~hjm/AMS.Scand.2000.html` for additional program details and links to sites for hotels, tours, campus, and much other local information.

The e-mail address for conference information is `ams.scand.2000@imada.sdu.dk`.

Abstracts

All speakers (both plenary and for special sessions) should submit abstracts by e-mail **no later than March 15**, containing the following information:

Lecture title.
Speaker's name, title, affiliation.
TeX version used (LaTex2e is preferred).
Abstract text.
References.

Send your abstract to `ams.scand.2000@imada.sdu.dk`; be sure to include ABSTRACT as the subject of your e-mail submission. Abstracts may also be submitted via the conference Web site.

Abstracts will be made available on the conference Web site ONLY, in PDF format. No printed versions are planned.

Accommodations

Blocks of rooms have been reserved in the following hotels by the conference organizers. In order to be eligible for the conference rate, you must make your reservation and include a deposit when you register for the conference (see registration procedures below) **no later than April 1**. Note that room rates are listed in approximate U.S. dollars and are subject to change. Payment (less deposit) must be made directly to the hotel upon checkout. Approximate distances to the campus meeting site are indicated. Most of the hotels are on a convenient bus route to campus with two to three trips per hour; the approximate bus travel time is indicated. Participants may also find it convenient to share a cab.

The following properties are near the train station in the center of town, about 3.5 miles to campus and a 25-minute bus ride to campus center:

Grand Hotel Odense, \$115/single.
Odense Plaza (Best Western), \$100 single/\$125 double.
City Hotel, \$75 single/\$105 double.
Hotel Ansgar/Windsor, \$75 single/\$95 double.

Other hotels

Knudsens Gaard (Best Western), \$90/single; about 2 miles to campus, nine minutes by bus to the campus perimeter.

Scandic Hotel, \$115 single/\$145 double, about 4.5 miles to campus, ideal if you rent a car.

In addition, dormitory-style rooms are available at Dalum Landbrugsskole, \$40/single (two nice rooms share one bath), located on the perimeter of campus, about 1.5 miles from the Meeting site (a nice walk along rural campus paths, or share a cab). Payment (less deposit) must be made in full at the conference registration desk.

Food Service

Because of busy cafeteria conditions, it is recommended that participants purchase the daily buffet lunch package for \$12 per day (beverages not included). Prepayment for participants is required on the registration form (prepayment is not necessary for persons accompanying participants, but should be noted separately on the form). There is a public university cafeteria also available for lunch only.

Also, there is a special presentation on Thursday evening (see below). Because there is no university cafeteria open for dinner, it is essential that those who wish to stay for the evening program purchase their dinner in advance on the registration form. The cost is \$25 (beverages not included).

Registration and Meeting Information

All sessions will be held at the University of Southern Denmark, main campus, Odense University. Currently, lectures are scheduled to begin on Tuesday afternoon and end Friday at noon.

Advance registration is highly recommended. The registration fee is \$50 (if paid by May 1) or \$75 if paid after May 1. See the conference Web site to register on line. Plenary speakers and invited special session speakers receive complimentary registration. On site the registration office will be in Room U49A, just inside entrance L, P lot 2, and will be open Tuesday, 9:00 a.m. to 6:00 p.m., Wednesday and Thursday, 9:00 a.m. to 3:30 p.m., and Friday, 9:00 a.m. to 1:30 p.m.

Social Events and Tours

Several enjoyable excursions have been planned.

On Tuesday at 7:45 p.m., all participants are invited to a complimentary reception sponsored by the City of Odense at the Town Hall where light refreshments will be served. This is near most of the hotels. Several restaurants are nearby for a late dinner on your own.

On Tuesday afternoon there is a tour of the Hans Christian Andersen Museum. This fascinating museum contains editions of his fairy tales in over 140 languages, and story illustrations in paintings and prints by world renowned artists. The tour leaves campus at 5:45 p.m. From the museum, it is a five-minute walk to the reception at the Town Hall. The cost is \$15 each, including transportation and admission fee.

On Friday afternoon after the conclusion of the scientific program for the meeting, there will be an excursion to Egeskov Castle, the best-preserved Renaissance moat castle throughout all of Europe. Many rooms are open to the public, as are the lovely gardens (including a geometrically interesting sundial and maze, both designed by the Danish poet/mathematician Piet Hein), along with large collections of antique cars, motor bikes, and carriages. The cost is \$25 each, including transportation and admission fee. Buses will leave campus at about 1:30 p.m. and return to the Odense train station about 6:00 p.m.

Special Presentation

On Thursday evening at 8:00 p.m. a new project sponsored by the European Union will be presented, *Large Infrastructure in Mathematics—Enhanced Services* (LIMES). The aim of the project is to enhance the availability of mathematically relevant databases throughout Europe. One of the centers will be at the Danish Technical University in Lyngby. Those who wish to stay on campus for dinner after the special sessions have concluded and before this program begins MUST purchase dinner in advance (\$25, beverages not included).

Travel

Scandinavian Airlines (SAS) is the official airline of the conference (reference **#DK0030**). The SAS agent in the U.S. is Conferences International, Inc., 1101 Worcester Road, Suite 401, Framingham, MA 01701-5249; 800-221-8747 (U.S.and Canada); 508-872-4455 (outside the U.S. and Canada); fax: 508-872-5566. Or see `http://www.conferencesintl.com/` and fill in the form to have the agent propose convenient air schedules and check for the lowest conference airfares.

From Copenhagen (airport) to Odense: The Danish State Railroad has efficient, fast connections directly from the Copenhagen Airport, Kastrup, (and from the Copenhagen main station) to the center of Odense. From some countries you will be able to buy an airline ticket on SAS all the way to Odense, including a coupon for the train, Copenhagen to Odense. Be sure to ask if this is available when you book your airline ticket.

Los Angeles, California

Note: This is a World Math Year 2000 (WMY2000) event.

University of California, Los Angeles

August 7-12, 2000

Meeting #956

Associate secretary: Robert J. Daverman
Announcement issue of *Notices*: May 2000
Program first available on e-MATH: May 24, 2000
Program issue of electronic *Notices*: October 2000
Issue of *Abstracts*: Volume 21, Issue 3

Deadlines

For organizers: Not Applicable
For consideration of contributed papers in Special Sessions: Not Applicable
For abstracts: May 10, 2000

Invited Addresses

James G. Arthur, University of Toronto, *will speak on automorphic forms and the Langlands program.*

Alexander A. Beilinson, University of Chicago, *will speak on the geometric Langlands conjecture.*

Michael V. Berry, University of Bristol, *will speak on waves, geometry, and arithmetic.*

Haim Brezis, University of Paris XI and Rutgers University, *will speak on nonlinear partial differential equations.*

Alain Connes, Collège de France, *will speak on noncommutative geometry.*

David L. Donoho, Stanford University, *will speak on interactions among harmonic analysis, statistical analysis, and information theory.*

Charles L. Fefferman, Princeton University, *will speak on the equations of fluid mechanics.*

Michael H. Freedman, Microsoft Research, *will speak on the physics of computation.*

Ronald L. Graham, University of California at San Diego, *title to be announced* (AMS-MAA President's Lecture).

Helmut H. W. Hofer, Courant Institute, New York University, *will speak on symplectic geometry/dynamical systems.*

Richard M. Karp, University of Washington, *will speak on computational molecular biology.*

Sergiu Klainerman, Princeton University, *will speak on partial differential equations.*

Maxim Kontsevich, Institut des Hautes Études Scientifiques, *will speak on deformations, supermanifolds, and homotopical algebra.*

Peter D. Lax, Courant Institute, New York University, *will speak on mathematics and computing.*

Simon A. Levin, Princeton University, *will speak on complexity of biology.*

László Lovász, Yale University, *will speak on discrete mathematics and algorithms.*

David Mumford, Brown University, *will speak on models of perception and inference.*

Peter Sarnak, Princeton University, *will speak on analysis and number theory.*

Saharon Shelah, The Hebrew University and Rutgers University, *will speak on mathematical logic.*

Peter W. Shor, AT&T Labs, *will speak on quantum computing/quantum information theory.*

Yakov G. Sinai, Princeton University, *will speak on dynamical systems.*

Richard P. Stanley, Massachusetts Institute of Technology, *will speak on algebraic combinatorics.*

Dennis P. Sullivan, The CUNY Graduate School, *will speak on applications of combinatorial topology to geometry.*

Clifford Taubes, Harvard University, *will speak on geometry and topology of the future.*

Jean E. Taylor, Rutgers University, *will speak on applications of geometric analysis.*

William P. Thurston, University of California - Davis, *will speak on three-dimensional topology and geometry.*

Karen Uhlenbeck, University of Texas at Austin, *will speak on a subject to be announced.*

S. R. S. Varadhan, Courant Institute, New York University, *will speak on stochastic analysis and applications.*

Edward Witten, Institute for Advanced Study, *will speak on the mathematical impact of quantum fields and strings.*

Shing-Tung Yau, Harvard University, *will speak on geometry and its relation to physics.*

Don B. Zagier, Max-Planck Institut für Mathematik, *will speak on number theory: modular forms.*

Toronto, Ontario Canada

University of Toronto

September 22-24, 2000

Meeting #957

Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: August 2000
Program first available on e-MATH: August 10, 2000
Program issue of electronic *Notices*: November 2000
Issue of *Abstracts*: Volume 21, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: June 6, 2000
For abstracts: July 14, 2000

Invited Addresses

John H. Conway, Princeton University, *Title to be announced* (Erdos Memorial Lecture).

George Elliott, University of Toronto, *Title to be announced.*

Benson Farb, University of Chicago, *Title to be announced.*

Yongbin Ruan, University of Wisconsin, *Title to be announced.*

Boris Tsyagan, Pennsylvania State University, *Title to be announced.*

Special Sessions

Applied Categorical Structures (Code: AMS SS J1), **Joan Wick Pelletier** and **Walter Tholen**, York University.

Commutative Algebra and Algebraic Geometry (Code: AMS SS A1), **Anthony Geramita**, Queens University, and **William Traves**, United States Naval Academy.

Computational Wavelet Analysis (Code: AMS SS H1), **Sebastian Ferrando** and **Larry Kolasa**, Ryerson Polytechnic University.

Discrete and Applied Geometry (Code: AMS SS L1), **Asia Ivic Weiss** and **Walter Whiteley**, York University.

Ergodic Theory and Dynamical Systems (Code: AMS SS B1), **Andres del Junco**, University of Toronto, and **Blair Madore**, SUNY, Potsdam.

Functional Differential Equations and Applications (Code: AMS SS D1), **Anatoli F. Ivanov**, Pennsylvania State University, and **Jianhong Wu**, York University.

Modern Schubert Calculus (Code: AMS SS K1), **Nantel Bergeron**, York University, and **Frank Sottile**, University of Wisconsin.

Nonabsolute Integration (Code: AMS SS C1), **Patrick Muldowney**, University of Ulster, and **Erik Talvila**, University of Illinois, Urbana.

Pseudo-differential Operators, Wavelet Transforms and Related Topics (Code: AMS SS F1), **M. W. Wong**, York University.

Representation Theory of Infinite Dimensional Lie Algebras (Code: AMS SS E1), **Yun Gao**, York University.

Set Theory and Set-Theoretic Topology (Code: AMS SS G1), **Franklin D. Tall**, University of Toronto.

San Francisco, California

San Francisco State University

October 21-22, 2000

Meeting #958

Western Section
Associate secretary: Bernard Russo
Announcement issue of *Notices*: August 2000
Program first available on e-MATH: September 11, 2000
Program issue of electronic *Notices*: December 2000
Issue of *Abstracts*: Volume 21, Issue 4

Deadlines

For organizers: March 21, 2000
For consideration of contributed papers in Special Sessions: June 21, 2000
For abstracts: August 29, 2000

Special Sessions

Algebraic and Geometric Combinatorics (Code: AMS SS A1), **Jesus De Loera**, University of California, Davis, and **Frank Sottile**, University of Wisconsin.

New York, New York

Columbia University

November 3-5, 2000

Meeting #959

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: September 2000
Program first available on e-MATH: September 28, 2000
Program issue of electronic *Notices*: December 2000
Issue of *Abstracts*: Volume 21, Issue 4

Deadlines

For organizers: April 3, 2000
For consideration of contributed papers in Special Sessions: July 18, 2000
For abstracts: September 12, 2000

Invited Addresses

Paula Cohen, Université des Sciences et Technologies de Lille, France, *will speak on geometry and its relation to physics.*

Brian Greene, Columbia University, *Title to be announced.*

Sergey Novikov, University of Maryland, College Park, and Landau Institute for Theoretical Physics, *Title to be announced.*

Alexander I. Suciu, Northeastern University, *Title to be announced.*

Special Sessions

Arithmetic Geometry and Modular Forms (Code: AMS SS D1), **Dorian Goldfeld**, Columbia University, and **Paula Cohen**, Université des Sciences et Technologies de Lille, France.

Arrangements of Hyperplanes (Code: AMS SS C1), **Michael J. Falk**, Northern Arizona University, and **Alexander I. Suciu**, Northeastern University.

Combinatorial Group Theory (Code: AMS SS A1), **Gilbert Baumslag**, **Alexei Myasnikov**, and **Vladimir Shplirain**, City College of New York (CUNY).

Commutative Algebra (Code: AMS SS F1), **Irena Peeva**, Cornell University.

Differential Algebra and Related Topics (Code: AMS SS E1), **Li Guo** and **William Keigher**, Rutgers University at Newark, and **William Sit**, City College (CUNY).

The Topology of 3-Manifolds (Code: AMS SS B1), **Joan S. Birman** and **Brian S. Magnum**, Columbia University, and **Walter D. Neumann**, University of Melbourne.

Birmingham, Alabama

University of Alabama-Birmingham

November 10-12, 2000

Meeting #960

Southeastern Section
Associate secretary: John L. Bryant
Announcement issue of *Notices*: September 2000
Program first available on e-MATH: October 5, 2000
Program issue of electronic *Notices*: January 2001
Issue of *Abstracts*: Volume 21, Issue 4

Deadlines

For organizers: April 10, 2000
For consideration of contributed papers in Special Sessions: July 25, 2000
For abstracts: September 19, 2000

Special Sessions

Inverse Problems (Code: AMS SS A1), **Ian Walker Knowles** and **Rudi Weikard**, University of Alabama at Birmingham.

Hong Kong, People's Republic of China

December 13-17, 2000

Meeting #961

First Joint International Meeting between the AMS and the Hong Kong Mathematical Society.
Associate secretary: Bernard Russo
Announcement issue of *Notices*: To be announced
Program first available on e-MATH: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Special Sessions

Combinatorial and Computational Methods in Commutative Algebra and Algebraic Geometry, **Vladimir Shpilrain**, City College of New York (CUNY), and **Jie-Tai Yu**, University of Hong Kong.

New Orleans, Louisiana

New Orleans Marriott and Sheraton New Orleans Hotel

January 10-13, 2001

Meeting #962

Joint Mathematics Meetings, including the 107th Annual Meeting of the AMS, 84th Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM).
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: October 2000
Program first available on e-MATH: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: Volume 22, Issue 1

Deadlines

For organizers: April 11, 2000
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
For summaries of papers to MAA organizers: To be announced

Columbia, South Carolina

University of South Carolina

March 16–18, 2001

Southeastern Section
Associate secretary: John L. Bryant
Announcement issue of *Notices*: To be announced
Program first available on e-MATH: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 15, 2000
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Lawrence, Kansas

University of Kansas

March 30–31, 2001

Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: To be announced
Program first available on e-MATH: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: June 28, 2000
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Las Vegas, Nevada

University of Nevada

April 21–22, 2001

Western Section
Associate secretary: Bernard Russo
Announcement issue of *Notices*: To be announced
Program first available on e-MATH: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Hoboken, New Jersey

Stevens Institute of Technology

April 28–29, 2001

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: To be announced
Program first available on e-MATH: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 28, 2000
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Lyon, France

July 17–20, 2001

First Joint International Meeting between the AMS and the Société Mathématique de France.
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: To be announced
Program first available on e-MATH: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Chattanooga, Tennessee

University of Tennessee, Chattanooga

October 5–6, 2001

Southeastern Section
Associate secretary: John L. Bryant
Announcement issue of *Notices*: To be announced
Program first available on e-MATH: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 5, 2001
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Williamstown, Massachusetts

Williams College

October 13–14, 2001

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: To be announced
Program first available on e-MATH: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 11, 2001
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Diego, California

San Diego Convention Center

January 6–9, 2002

Joint Mathematics Meetings, including the 108th Annual Meeting of the AMS and 85th Meeting of the Mathematical Association of America (MAA).
Associate secretary: John L. Bryant
Announcement issue of *Notices*: To be announced
Program first available on e-MATH: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 4, 2001
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
For summaries of papers to MAA organizers: To be announced

Pisa, Italy

June 16–20, 2002

First Joint International Meeting between the AMS and the Unione Matematica Italiana.
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: To be announced
Program first available on e-MATH: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Boston, Massachusetts

Northeastern University

October 5–6, 2002

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: To be announced
Program first available on e-MATH: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 5, 2002
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Meetings and Conferences of the AMS

Associate Secretaries of the AMS

Western Section: Bernard Russo, Department of Mathematics, University of California, Irvine, CA 92697; e-mail: brusso@math.uci.edu; telephone: 949-824-5505.

Central Section: Susan J. Friedlander, Department of Mathematics, University of Illinois at Chicago, 851 S. Morgan (M/C 249), Chicago, IL 60607-7045; e-mail: susan@math.nwu.edu; telephone: 312-996-3041.

Eastern Section: Lesley M. Sibner, Department of Mathematics, Polytechnic University, Brooklyn, NY 11201-2990; e-mail: lsibner@magnus.poly.edu; telephone: 718-260-3505.

Southeastern Section: John L. Bryant, Department of Mathematics, Florida State University, Tallahassee, FL 32306-4510; e-mail: bryant@math.fsu.edu; telephone: 850-644-5805.

The Meetings and Conferences section of the *Notices* gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. **Information in this issue may be dated. Up-to-date meeting and conference information is available on the World Wide Web at** www.ams.org/meetings/.

Meetings:

Important Information regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 106 in the January 2000 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts

Several options are available for speakers submitting abstracts, including an easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX or AMS-LaTeX may submit abstracts with such coding. To see descriptions of the forms available, visit http://www.ams.org/abstracts/instructions.html, or send mail to abs-submit@ams.org, typing help as the subject line; descriptions and instructions on how to get the template of your choice will be e-mailed to you.

Completed abstracts should be sent to abs-submit@ams.org, typing submission as the subject line. Questions about abstracts may be sent to abs-info@ams.org.

Paper abstract forms may be sent to Meetings & Conferences Department, AMS, P.O. Box 6887, Providence, RI 02940. There is a $20 processing fee for each paper abstract. There is no charge for electronic abstracts. Note that all abstract deadlines are strictly enforced. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

Conferences: (See http://www.ams.org/meetings/ for the most up-to-date information on these conferences.)

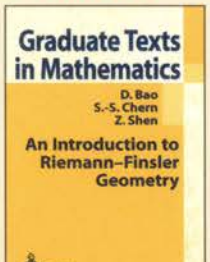
February 17-22, 2000: AAAS Meeting, Marriott Wardman Park Hotel, Washington, DC. (See page 1479, December 1999 issue, for details.)

June 11-July 20, 2000: Joint Summer Research Conferences in the Mathematical Sciences, Mt. Holyoke College, South Hadley, MA. (See pages 1325-30, November 1999 issue, for details.)

SPRINGER FOR MATHEMATICS

DAVID BAO, University of Houston, TX; **SHIING-SHEN CHERN,** University of California at Berkeley; and **ZHONGMIN SHEN,** Indiana University-Purdue University, Indianapolis

AN INTRODUCTION TO RIEMANN-FINSLER GEOMETRY



In Riemannian geometry, measurements are made with both yardsticks and protractors. These tools are represented by a family of inner-products. In Riemann-Finsler geometry (or Finsler geometry for short), one is in principle equipped with only a family of Minkowski norms. So yardsticks are assigned but protractors are not. With such a limited tool kit, it is natural to wonder just how much geometry one can uncover and describe.

It now appears that there is a reasonable answer. Finsler geometry encompasses a solid repertoire of rigidity and comparison theorems, most of them founded upon a fruitful analogue of the sectional curvature. There is also a wide array of explicit examples, illustrating many phenomena which admit only Finslerian interpretations. This book focuses on the elementary but essential items among these results. Much thought has gone into making the account a teachable one.

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GRADUATE TEXTS IN MATHEMATICS, VOLUME 200

PAULO RIBENBOIM, Queen's University, Kingston, Ontario

MY NUMBERS, MY FRIENDS

This collection of expository essays by Paulo Ribenboim covers topics such as Fibonacci numbers, prime numbers, Bernoulli numbers, and historical presentations of the main problems pertaining to elementary number theory, including Kummer's work on Fermat's Last Theorem. The essays are written in a light and humorous style and are thoroughly accessible to everyone with an interest in numbers.

Contents: Fibonacci Numbers and the Arctic Ocean • Representations of Real Numbers by Means of Fibonacci Numbers • Prime Number Records • Selling Primes • Euler's Famous Polynomial and the Class Number of Imaginary Quadratic Fields • Gauss and the Class Number Problem • Consecutive Powers • 1093 • Powerless Facing Powers • The Classical Bernoulli Numbers • Galimatias Arithmetica • The Work of Kummer on Fermat's Last Theorem • An Essay on Irrational Numbers

2000/APP. 450 PP./SOFTCOVER/$37.50 (TENT.)
ISBN 0-387-98911-0

REINHARD DIESTEL, Universität Hamburg, Germany

GRAPH THEORY

Second Edition

This book is a concise—yet most carefully written—introduction to modern graph theory, covering all its major recent developments. It can be used both as a reliable textbook for an introductory course and as a graduate text: on each topic it covers all the basic material in full detail, and adds one or two deeper results (again with detailed proofs) to illustrate the more advanced methods of that field.

This second edition extends the first in two ways. It offers a thoroughly revised and updated chapter on graph minors, which now includes full new proofs of two of the central Robertson-Seymour theorems (as well as a detailed sketch of the entire proof of their celebrated Graph Minor Theorem). Secondly, there is now a section of hints for all the exercises, to enhance their value for both individual study and classroom use.

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HARDCOVER: $59.95 (TENT.)/ISBN 0-387-95014-1
ALSO IN SOFTCOVER: $32.95 (TENT.)
ISBN 0-387-98976-5
GRADUATE TEXTS IN MATHEMATICS, VOLUME 173

LEONHARD EULER (1707-1783), Translated by **JOHN D. BLANTON,** St. John Fisher College, Rochester, NY

FOUNDATIONS OF DIFFERENTIAL CALCLULUS

The positive response to the publication of Blanton's English translation of Euler's Introduction to Analysis of the Infinite confirmed the relevance of this 240 year-old work and encouraged Blanton to translate Euler's Foundations of Differential Calculus as well. The current book constitutes just the first 9 out of 27 chapters. The remaining chapters will be published at a later time. With this new translation, Euler's thoughts will not only be more accessible, but more widely enjoyed by the mathematical community.

Contents: On Finite Differences • On the Use of Differences in the Theory of Series • On the Infinite and the Infinitely Small • On the Nature of Differential of Each Order • On the Differentiation of Algebraic Functions of One Variable • On the Differentiation of Transcendental Functions • On the Differentiation of Functions of Two or More Variables • On the Higher Differentiation of Functions of Differential Formulas • On Differential Equations

2000/APP. 224 PP./HARDCOVER/$59.95
ISBN 0-387-98534-4

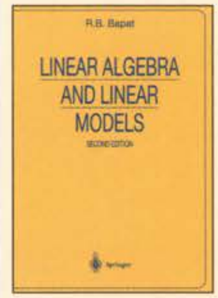
Springer

www.springer-ny.com

R.B. BAPAT, Indian Statistical Institute, New Delhi

LINEAR ALGEBRA AND LINEAR MODELS

Second Edition



The main purpose of *Linear Algebra and Linear Models* is to provide a rigorous introduction to the basic aspects of the theory of linear estimation and hypothesis testing. The necessary prerequisites in matrices, multivariate normal distribution and distributions of quadratic forms are developed along the way.

Contents: Vector Spaces and Matrices • Linear Estimation • Tests of Linear Hypotheses • Singular Values and Their Applications • Block Designs and Optimality • Rank Additivity

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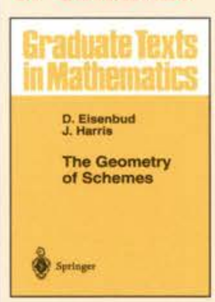
DAVID EISENBUD, Mathematical Sciences Research Institute, Berkeley, CA and **JOE HARRIS,** Harvard University, Cambridge, MA

THE GEOMETRY OF SCHEMES



This book is intended to bridge the chasm between a first course in classical algebraic geometry and a technical treatise on schemes. It focuses on examples, and strives to show "what is going on" behind the definitions. There are many exercises to test and extend the reader's understanding.

Contents: Basic Definitions • Examples • Projective Schemes • Classical Constructions • Local Constructions • Schemes and Functors

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