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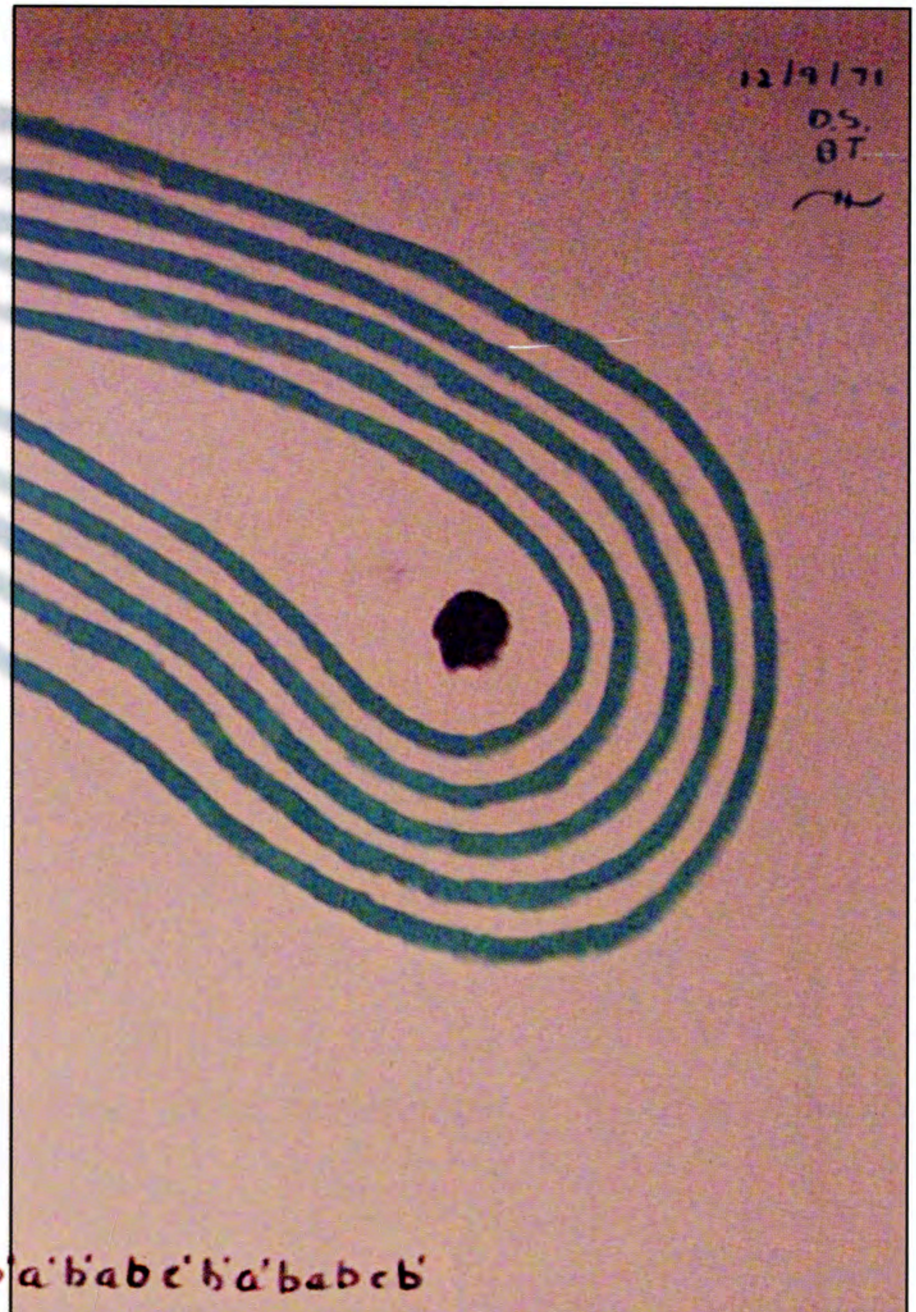

Primitive curve painting (see page 356)

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March 2003

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Departments

From the AMS Secretary

Telling the Truth

I believe there is wide agreement over the following statement: *The fundamental problem with today's college students is that most arrive thinking that college is a simple continuation of high school.* I have been recommending that the difference between high school and college be explained systematically during Freshman Orientation. Further, *there must be a separate orientation for mathematics and the sciences.* It is beneficial to join up with the sciences, but it is important to avoid diluting the message with generalities that must cover the other disciplines as well. A successful presentation must achieve the difficult balance between directness about the change of terrain and reassurance that the students can succeed if they really try.

In 2001 I did my first joint presentation with professors of engineering and physics. It was so well received that we were urged to do it again in 2002. The blurb in the Orientation Week program was: *The biggest difference between high school and college will lie in your math and science courses. Find out what and why, and how the student can adapt and succeed.* The outline of my part is given with the expectation that it is suitable for "peer institutions" and that an adjusted version could be used for science-oriented students at the premier public institution(s) of most states.

I began with a statement from our *Freshman Academic Handbook*: *The key differences between learning at Hopkins and your high school are: 1) learning does not take place primarily in the classroom; and 2) you, and not your professor, are responsible for what you learn.* Referring to #1, I asked, "Where *does* the learning take place?" Answers like "the library," "your room" were offered. Given that, #2 becomes largely obvious; in the past this point was difficult to get across, for freshmen see it as a threat, so alien to their experience. A solid foundation was thereby laid for the rest of the program:

1. *New level of responsibility.* Though guided by your instructors and advisors, *you* are responsible from now on for your own education.

2. *New peer group.* Most of you are no longer well above the majority of your classmates, but in a new environment with people much like yourself. Virtually all of you have the capability to do well (A or B) in your math classes, but talent alone cannot produce success. (Statistics on grade distributions in freshman courses were provided.) The students getting D or F (10+%) were barred from taking the next course. These are the ones who badly fell behind in their coursework, overestimated their effort, or insisted on high-schoolish modes of learning. (It is rarely the fault of the instructor, whatever students say.)

3. *New level of learning.* In *any* subject, the goal in college is to learn *flexibly* so that you can judge what applies in new situations and carry it out. The subject where this is furthest from high school experience is mathematics. Thus, *most students face a new challenge in their math courses.* Flexible learning is especially important, because many other departments require math courses and want their majors to be ready to *use* the material. For that, *the student must start to think conceptually.*

4. *New roles of the instructor and student.* The instructor's is *to guide the students' learning.* It is not to cover the material, for that is the textbook's job. It is not to teach everything to the student: *teaching in college becomes a cooperative effort shared by the instructor and the student.* There is a corresponding change in what is expected from the student. In a typical high school the attentive student is able to pass with modest exertion. In college the vast majority of students can learn well with *reasonable* exertion: two hours per week outside of class for each hour in class is not an unreasonable effort. That includes reading the textbook for both concept and (additional) examples. The course will then be moving a lot faster than in high school, with far less repetition. The exams will cover several weeks of material, even the whole semester on the final. The student should view the learning of math as accumulating a body of knowledge, not just learning isolated facts and problem types.

I will never forget what one student at Rutgers wrote about me in a course survey in the 1970s: *He's an all right teacher, but you have to read the book too in order to understand the material.* "But," eh? One of the most important things an instructor can do for the students is to insist they learn mathematics in part from written sources, so they can get beyond the surface. Most students are excused from that in high school. I have heard too many of our students say that someone gave a good calculus course when—perhaps I should say *because*—they were able to manage easily without reading the textbook, even when important material was cut from the syllabus.

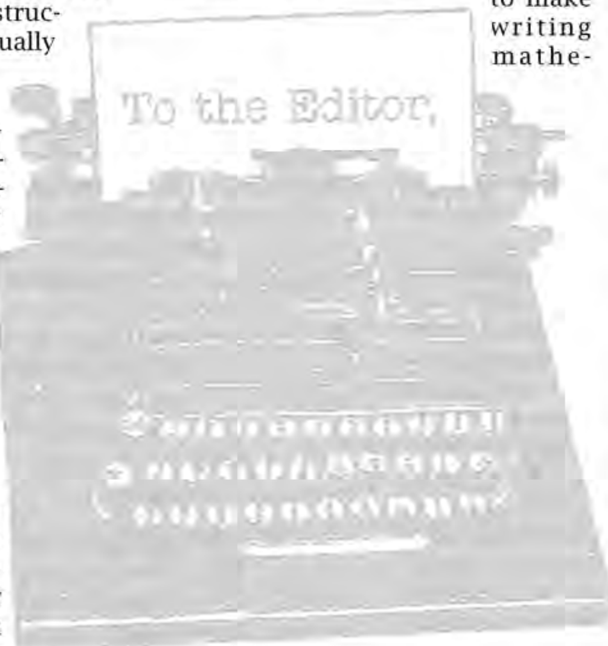
Now, go back to those two fundamental statements from the *Academic Handbook* and ask yourself to what extent you and your colleagues act on them. I know there is pressure to back off from those principles. It is common to give in, to go with the flow. Students become perceived as consumers of our services. But they are also our output, subject to quality control. Wouldn't it be better to preserve the integrity of our colleges? After all, the education of the next generation is at stake.

—Steven Zucker
Johns Hopkins University
See `http://www.math.jhu.edu/~sz`
for address and "orientation material"

Letter to the Editor

Mathematical Word Processing

W. Brian Arthur, a colleague of Donald Knuth at Stanford, has studied dynamical systems of the self-reinforcing or autocatalytic type. These have a multiplicity of possible emergent structures. The initial state, possibly affected by early random events, pushes the dynamics into "selecting" a structure that the system eventually "locks into". Instances abound of this nonoptimal, premature phenomenon capturing industrial and commercial processes to the detriment of both superior processes and users.



The history of TeX-LaTeX2e described in the December 2002 issue by Michael Downes constitutes still another instance. Downes lists six particular reasons for writing material for the AMS in LaTeX, beginning with (1) the inherent logical structure (read software programming); (2) as a source for HTML and PDF, even though these stand independent in their own right with readily available software that is compatible with Microsoft Word, for instance; (3) well established and stable (but compare his section on the history and his closing "Beware of Obsolete Documentation!"); (5) "...a way that seems well suited to mathematical material" (in the view of a programmer); and (6) "easy to feed directly into the AMS production system." Aye, there's the rub! The AMS has been captured by this ponderous, unintuitive, error-ridden software, and we are all hostage to the obsolescence.

Freely writing mathematics has been replaced by programming in a ponderous system of macros. Downes counts this as a virtue: "a non-WYSIWYG approach helps sensitize authors to the kind of discrimination between visual appearances and essential information that they need to make if they do not want what they write to be inadvertently encumbered by limitations of the medium (or software, or printer, or type of computer monitor) in which it is originally produced." Is "essential information" on mathematics to be judged by software programmers? Are we at the mercy of this obsessive bureaucracy? Is it better to "make it as easy as possible for other programs to print or preview DVI files on an arbitrary printing device or computer screen" than to make writing mathematics as easy as possible for human AMS members? Microsoft Word is certainly a program present on more computers worldwide than any version of TeX-LaTeX. From this author's standpoint, its editor capabilities, when supplemented by MathType (advertised on the inside back cover of the December 2002 issue), are certainly more user-friendly. It is WYSIWYG without any ponderous macros that become obsolete from one edition to the next. It has served me as well as any version of TeX, even in mappings between chain complexes and other diagrammatic structures, except of course in submissions to the AMS.

Downes reviews the sorry history in "Some Historical Notes about TeX". Though released to "people in the wild outside Stanford" (what arrogance!) in 1978, Knuth and others, increasingly dissatisfied, were still putting TeX and its variants through ever more changes, in particular the TeX macro language, through 1978, 1980, 1982, 1983, 1990, culminating in the "final" version of TeX indexed by some finite part of the irrational number pi. May we thus expect an infinite number of patchwork changes extending out to eternity? Downes states that after Knuth's death the version number will change from an approximation to pi itself. Perhaps it would be better to bury TeX with its illustrious inventor and have the AMS go to something closer to a typical user-friendly word processor that still does the job. See the content of Downes' closing statement, "Beware of Obsolete Documentation!". He further warns that this is "only one instance of a more general pitfall that LaTeX users should be careful to watch out for." And the final paragraph of this section directly contradicts Downes's point 4 on page 1384, that the format is "readily exchangeable with colleagues."

I realize full well that with this letter I am "plowing the sea", but things could be better.

—*William C. Hoffman*
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(Received November 18, 2002)

The *Notices* invites readers to submit letters and opinion pieces on topics related to mathematics. Electronic submissions are preferred (`notices-letters@ams.org`; see the masthead for postal mail addresses. Opinion pieces are usually one printed page in length (about 800 words). Letters are normally less than one page long, and shorter letters are preferred.

Disks That Are Double Spiral Staircases

*Tobias H. Colding and William P. Minicozzi II**

What are the possible shapes of various things and why?

For instance, when a closed wire or a frame is dipped into a soap solution and is raised up from the solution, the surface spanning the wire is a soap film; see Figure I. What are the possible shapes of soap films and why? Or, for instance, why is DNA like a double spiral staircase? "What?" and "why?" are fundamental questions and, when answered, help us understand the world we live in.

The answer to any question about shape of natural objects is bound to involve mathematics, because as Galileo Galilei[1] observed, the book of Nature is written in the characters of mathematics.

Soap films, soap bubbles, and surface tension were extensively studied by the Belgian physicist and inventor (of the stroboscope) Joseph Plateau in the first half of the nineteenth century. At least since his studies, it has been known that the right mathematical models for soap films are minimal surfaces: the soap film is in a state of minimum energy when it is covering the least possible amount of area.

We will discuss here the answer to the question, What are the possible shapes of embedded minimal disks in $\mathbf{R}^3$ and why?

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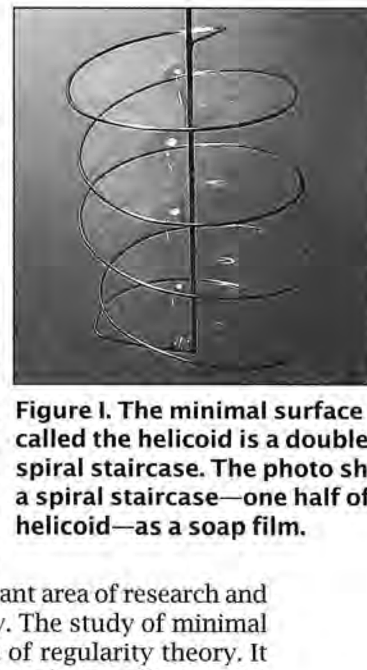
**The article was written by Tobias H. Colding and is about joint work of Colding and Minicozzi. Their research was partially supported by NSF grants DMS 0104453 and DMS 0104187.*

[1]*Italian (from Pisa) Renaissance astronomer, mathematician, and physicist (1564–1642).*



Photograph courtesy of John Oprea.

Figure I. The minimal surface called the helicoid is a double spiral staircase. The photo shows a spiral staircase—one half of a helicoid—as a soap film.

The field of minimal surfaces dates back to the publication in 1762 of Lagrange's famous memoir "Essai d'une nouvelle méthode pour déterminer les maxima et les minima des formules intégrales indéfinies". In a paper published in 1744 Euler had already discussed minimizing properties of the surface now known as the catenoid, but he considered only variations within a certain class of surfaces. In the almost one quarter of a millennium that has passed since Lagrange's memoir, minimal surfaces has remained a vibrant area of research and there are many reasons why. The study of minimal surfaces was the birthplace of regularity theory. It lies on the intersection of nonlinear elliptic PDE, geometry, and low-dimensional topology, and over the years the field has matured through the efforts of many people. However, some very fundamental questions remain. Moreover, many of the potentially spectacular applications of the field have yet to be achieved. For instance, it has long been the hope that several of the outstanding conjectures about the topology of 3-manifolds could be resolved using detailed knowledge of minimal surfaces.

Surfaces with uniform curvature (or area) bounds have been well understood, and the regularity theory is complete, yet essentially nothing was known without such bounds. We discuss here the theory of embedded minimal disks in $\mathbf{R}^3$ without a priori bounds. As we will see, the helicoid, which is a double spiral staircase, is the most important example

Figure II. Leonardo da Vinci's double spiral staircase in Château de Chambord.

Figure III. A model of da Vinci's staircase.

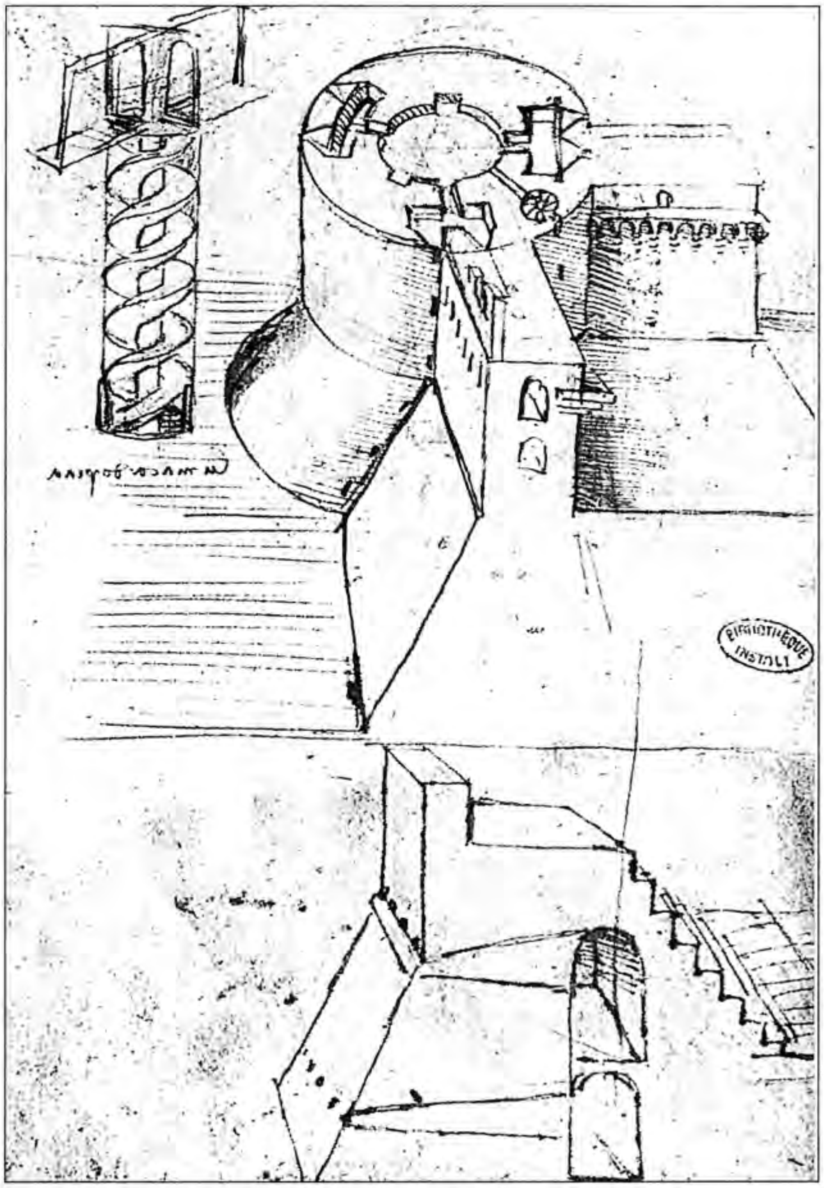


Figure IV. Drawing by Leonardo da Vinci from around 1487–90.

of such a disk. In fact, we will see that every embedded minimal disk is either a graph of a function or is part of a double spiral staircase. The helicoid was discovered to be a minimal surface by Meusnier in 1776.

However, before discussing minimal disks we will give some examples of where one can find double spiral staircases and structures.

Double Spiral Staircases

A *double spiral staircase* consists of two staircases that spiral around one another so that two people can pass each other without meeting. Figure II shows Leonardo da Vinci's double spiral staircase in Château de Chambord in the Loire Valley in France. The construction of the castle began in 1519 (the same year that Leonardo da Vinci died) and was completed in 1539. In Figure III we see a model of the staircase where we can clearly see the two staircases spiraling around one another. The double spiral staircase has, with its simple yet surprising design, fascinated and inspired many people. We quote here from Mademoiselle de Montpensier's[2] memoirs.

"One of the most peculiar and remarkable things about the house [the castle at Chambord] are the stairs, which are made so that one person can ascend and another descend without meeting, yet they can see each other. Monsieur [Gaston of Orléans, the father of Mademoiselle de Montpensier] amused himself by playing with me. He would be at the top of the stairs when I arrived; he would descend when I was ascending, and he would laugh when he saw me run in the hope of catching him. I was happy when he was amused and even more when I caught him."

Leonardo da Vinci had many ideas for designs involving double spiral staircases. Two of his main interests were water flow and military instruments. For Leonardo everything was connected, from the braiding of a girl's hair (see for instance the famous drawing of Leda and the swan) to the flow of water (which also exhibits a double spiral behavior when it passes an obstacle like the branch of a tree). He designed water pumps that were formed as double spiral staircases that supposedly would pump more efficiently than the more standard Archimedean pump then in use. He designed fire escapes formed as double (and greater multiples) spiral staircases where each staircase was inapproachable from another, thus preventing the

[2]*Duchess Montpensier (1627-1693), princess of the royal house of France and author, who played a key role in the revolts known as the Fronde.*

Figure V. Pozzo di San Patrizio in Orvieto. The well was built as a double spiral staircase by the pope between 1527 and 1537.

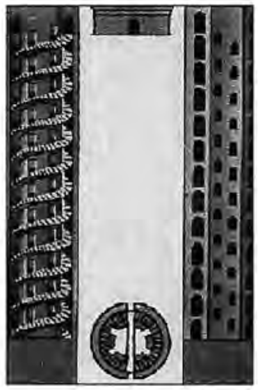


Figure VI. Cutaway view of the Orvieto well.

Figure VII. A double spiral staircase in the Vatican designed by the architect Donato Bramante and built around 1502.

fire from jumping from one staircase to another. That they spiraled together made them very space efficient. This basic design idea for fire escapes has become quite standard; for example, both the fire escapes in Fine Hall at Princeton University and the fire escapes in New York University's Silver Towers use this idea.

Another design of Leonardo's using double spirals staircases was for the protection of a fortress; see Figure IV. Leonardo explains the military purpose of such designs in his notebook:

"He who intends to capture a tower standing on the seashore will contrive to get one of his followers hired by the commandant of the castle, and when it is the turn of that man to mount guard he will throw the rope ladder, given him by the enemy, over the merlons, thus enabling the soldiers to swarm over the walls. And in order to make this impossible I divide the tower into eight spiral staircases and into eight defensive positions and quarters for the garrison, so that, if one is disloyal, the others cannot join him. And each defensive position will be so small that it allows room for no more than four men. The commandant, who lives above, can beat them back, firing through the machicolations and locking them behind the iron grating, and place smoke devices at the head of the spiral staircase. And on no account may foreign soldiers stay with him, but only his immediate entourage."

Another Renaissance double spiral staircase can be found in the hill town Orvieto near Florence in Italy. This is the so-called Pozzo di San Patrizio (St. Patrick's Well), Figures V and VI. Under the entry of Orvieto in Frommer's travel guide, one finds the description:

"Pozzo di San Patrizio (St. Patrick's Well). While Emperor Charles V was sacking Rome in 1527, the Medici pope, Clement VII, took advantage of a dark night and the disguise of a fruit vendor to sneak out of his besieged Roman fortress and scurry up

Figure VIII. Gothic double spiral staircase from 1499 in Graz, Friedrich III's Imperial City.

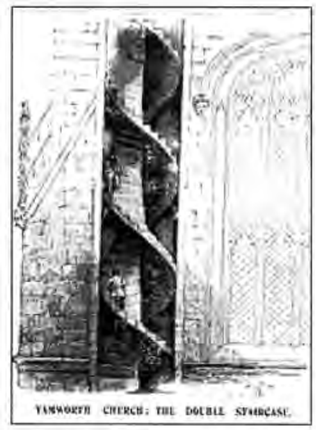


Figure IX. Postcard drawing of the double spiral staircase in St. Editha's Church, Tamworth, England.

to Orvieto. Convinced that the emperor would follow him, Clement set about fortifying his position.

"Orvieto's main military problem throughout history has been a lack of water. Clement hired Antonio Sangallo the Younger to dig a new well that would ensure an abundant supply in case the pope should have to ride out another siege. Sangallo set about sinking a shaft into the tufa at the lowest end of town. His design was unique: He equipped the well with a pair of wide spiral staircases, lit by seventy-two internal windows, forming a double helix so that mule-drawn carts could descend on one ramp and come back up the other without colliding.

"Although Clement and Charles V reconciled in 1530, the digging continued. Eventually, workers did strike water—almost ten years later, at which point Clement was dead and the purpose moot. The shaft was named St. Patrick's Well when some knucklehead suggested it vaguely resembled the cave into which the Irish saint was wont to withdraw and pray. What you get for descending the 248

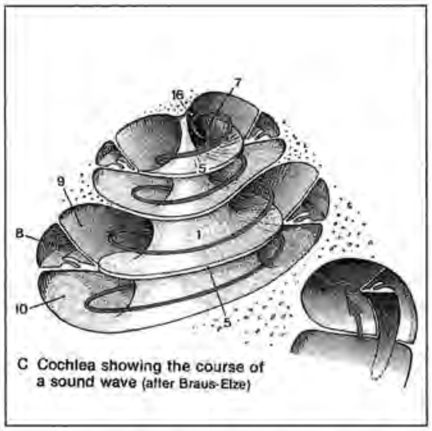


Figure X. The inner ear, the cochlea, is a double spiral staircase. Each of the canals is filled with liquid.

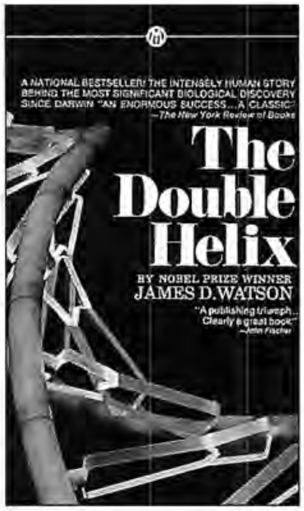


Figure XI. The autobiography of James Watson about his and Francis Crick's discovery of the double spiral structure of DNA.

Figure XII. A Spanish windlass is used on a beach in Africa to pull a boat.

steps is a close-up view of that elusive water, a good echo, and the sheer pleasure of climbing another 248 steps to get out."

Two more Renaissance double spiral staircases are shown in Figures VII and VIII. Figure VII is a staircase designed by the famous architect Donato Bramante. It leads to the Vatican Museum—one enters the museum on one of the two spiral staircases and exits on the other. Figure VIII show a double spiral staircase built by Friedrich III of Habsburg. Friedrich III was elected to be the Emperor of the "Holy Roman Empire of the German nation" in 1452. He made Graz into an imperial city. A castle dates back to this time, of which now only the double spiral staircase from 1499 exists.

An even earlier double spiral staircase can be found in the tower of St. Editha's parish church in the town of Tamworth in Staffordshire, England (see Figure IX). The church was supposedly founded around 963, but burned down in 1345. The tower in its present form was built between 1380 and 1420.

Also the internal ear, the cochlea, is a double spiral staircase; see Figure X or p. 343 of [K]. In the cochlea the two canals wind around a conical bony axis, and after about two and a half rotations they meet at the top and fuse. The canals are filled with fluid, and sound waves travel up one canal, turn around, and come down the other. When the liquid is set into movement, it sets the Basilar membrane and the hair cells into vibration. Different hair cells correspond to different frequencies.

In Figure XI we see "the double helix", which was discovered in 1952 by Francis Crick and James Watson to be the structure of DNA. The double spiral structure represented the culmination of half a century of prior work on genetics and is considered by many to be one of the greatest scientific discoveries of the twentieth century.

Other examples of double spiral structures include parking ramps and Spanish windlasses. A Spanish windlass (see Figure XII) is a device for moving heavy loads such as rocks and logs. It consists of a two-strand rope that one puts a stick or log through and twists around.

What Is a Minimal Surface and What Are the Central Examples?

Let $\Sigma \subset \mathbf{R}^3$ be a smooth orientable surface (possibly with boundary) with unit normal $\mathbf{n}_\Sigma$. Given a function ϕ in the space $C_0^\infty(\Sigma)$ of infinitely differentiable (i.e., smooth), compactly supported functions on Σ, consider the one-parameter variation

$$(1) \qquad \Sigma_{t,\phi} = \{x + t\,\phi(x)\,\mathbf{n}_\Sigma(x) \mid x \in \Sigma\}.$$

The so-called first-variation formula of area is the equation (integration is with respect to d area)

$$(2) \qquad \frac{d}{dt}\Big|_{t=0} \operatorname{Area}(\Sigma_{t,\phi}) = \int_\Sigma \phi\, H,$$

where H is the mean curvature of Σ. (When Σ is noncompact, then $\Sigma_{t,\phi}$ in (2) is replaced by $\Gamma_{t,\phi}$, where Γ is any compact set containing the support of ϕ.) The surface Σ is said to be a *minimal* surface (or just minimal) if

$$(3) \qquad \frac{d}{dt}\Big|_{t=0} \operatorname{Area}(\Sigma_{t,\phi}) = 0 \quad \text{for all } \phi \in C_0^\infty(\Sigma)$$

or, equivalently by (2), if the mean curvature H is identically zero. Thus Σ is minimal if and only if it is a critical point for the area functional. (Since a critical point is not necessarily a minimum, the term "minimal" is misleading, but it is time honored. The equation for a critical point is also sometimes called the Euler-Lagrange equation.) Moreover, a computation shows that if Σ is minimal, then

$$\frac{d^2}{dt^2}\bigg|_{t=0} \operatorname{Area}(\Sigma_{t,\phi}) = -\int_\Sigma \phi\, L_\Sigma \phi, \quad \text{where } L_\Sigma\phi = \Delta_\Sigma\phi + |A|^2\phi \tag{4}$$

is the second-variational (or Jacobi) operator. Here Δ_Σ is the Laplacian on Σ, and A is the second fundamental form. Thus $|A|^2 = \kappa_1^2 + \kappa_2^2$, where κ_1 and κ_2 are the principal curvatures of Σ, and $H = \kappa_1 + \kappa_2$. A minimal surface Σ is said to be *stable* if

$$\frac{d^2}{dt^2}\bigg|_{t=0} \operatorname{Area}(\Sigma_{t,\phi}) \geq 0 \quad \text{for all } \phi \in C_0^\infty(\Sigma). \tag{5}$$

One can show that a minimal graph is stable and, more generally, so is a multivalued minimal graph (see below for the precise definition).

There are two local models for embedded minimal disks (by an *embedded disk* we mean a smooth injective map from the closed unit ball in $\mathbf{R}^2$ into $\mathbf{R}^3$). One model is the plane (or, more generally, a minimal graph), and the other is a piece of a helicoid.

The derivation of the equation for a minimal graph goes back to Lagrange's 1762 memoir.

Example 1. *(Minimal graphs) If Ω is a simply connected domain in $\mathbf{R}^2$ and u is a real-valued function on Ω satisfying the minimal surface equation*

$$\operatorname{div}\left(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}}\right) = 0, \tag{6}$$

then the graph of u, i.e., the set $\{(x_1, x_2, u(x_1, x_2)) \mid (x_1, x_2) \in \Omega\}$, is a minimal disk.

A classical theorem of Bernstein from 1916 says that entire (i.e., where $\Omega = \mathbf{R}^2$) minimal graphs are planes. This remarkable theorem of Bernstein was one of the first illustrations of the fact that the solutions to a nonlinear PDE, such as the minimal surface equation, can behave quite differently from the solutions to a linear equation. In the early 1980s Schoen and Simon extended the theorem of Bernstein to complete simply connected embedded minimal surfaces in $\mathbf{R}^3$ with quadratic area growth. A surface Σ is said to have quadratic area growth if for all $r > 0$, the intersection of the surface with the ball in $\mathbf{R}^3$ of radius r and center at the origin is bounded by Cr^2 for a fixed constant C independent of r.

The second model comes from the helicoid, which was discovered by Meusnier in 1776. Meusnier had been a student of Monge. He also discovered that the surface now known as the catenoid is minimal in the sense of Lagrange, and he was the first to characterize a minimal surface as a surface with vanishing mean curvature. Unlike the helicoid, the catenoid is topologically a cylinder rather than a plane.

The helicoid is a "double spiral staircase".

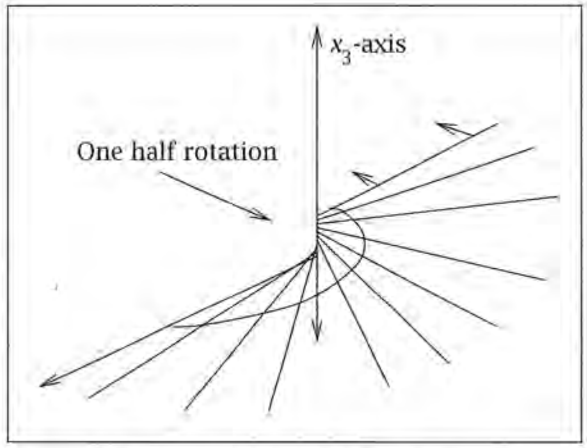


Figure 1. Multivalued graphs. The helicoid is obtained by gluing together two ∞-valued graphs along a line.

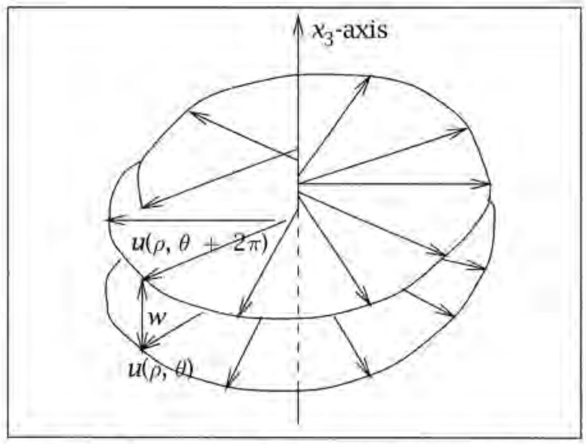


Figure 2. The separation w grows/decays in ρ at most sublinearly for a multivalued minimal graph; see (12).

Example 2. *(Helicoid; see Figure 1) The helicoid is the minimal surface in $\mathbf{R}^3$ given by the parametrization*

$$(s\cos t, s\sin t, t), \qquad \text{where } s, t \in \mathbf{R}. \tag{7}$$

To be able to give a precise meaning to the statement that the helicoid is a double spiral staircase, we will need the notion of a multivalued graph, each staircase will be a multivalued graph. Intuitively, an (embedded) multivalued graph is a surface such that over each point of an annulus the surface consists of N graphs. To make this notion precise, let D_r be the disk of radius r in the plane centered at the origin, and let the universal cover of the punctured plane $\mathbf{C} \setminus \{0\}$ have global polar coordinates (ρ, θ), where $\rho > 0$ and $\theta \in \mathbf{R}$. An *N-valued graph* on the annulus $D_s \setminus D_r$ is a single-valued graph of a function u over $\{(\rho, \theta) \mid r < \rho \leq s, |\theta| \leq N\pi\}$. For working purposes, we generally think of the intuitive picture of a multisheeted surface in $\mathbf{R}^3$, and we identify the single-valued graph over the universal cover with its multivalued image in $\mathbf{R}^3$.

The multivalued graphs that we will consider will all be embedded, which corresponds to a nonvanishing separation between the sheets (or the floors). Here the *separation* is the function (see Figure 2)

$$w(\rho, \theta) = u(\rho, \theta + 2\pi) - u(\rho, \theta). \tag{8}$$

If Σ is the helicoid, then $\Sigma \setminus \{x_3\text{-axis}\} = \Sigma_1 \cup \Sigma_2$, where Σ_1 and Σ_2 are ∞-valued graphs on $\mathbf{C} \setminus \{0\}$. Σ_1 is the graph of the function $u_1(\rho, \theta) = \theta$, and Σ_2 is the graph of the function $u_2(\rho, \theta) = \theta + \pi$. ($\Sigma_1$ is the subset where $s > 0$ in (7), and Σ_2 is the subset where $s < 0$.) In either case the separation $w = 2\pi$. A *multivalued minimal graph* is a multivalued graph of a function u satisfying the minimal surface equation.

Note that for an embedded multivalued graph, the sign of w determines whether the multivalued graph spirals in a left-handed or a right-handed manner, in other words, whether upwards motion corresponds to turning in a clockwise direction or in a counterclockwise direction. For DNA, although both spirals occur, the right-handed spiral is far more common, because of certain details of the chemical structure; see [CaDr].

As we will see, a fundamental theorem about embedded minimal disks is that such a disk is either a minimal graph or can be approximated by a piece of a rescaled helicoid, depending on whether the curvature is small or not; see Theorem 1 below. To avoid tedious details about the dependence of various quantities, we state this, our main result, not for a single embedded minimal disk with sufficiently large curvature at a given point, but instead for a sequence of such disks where the curvatures are blowing up. Theorem 1 says that a sequence of embedded minimal disks mimics the following behavior of a sequence of rescaled helicoids.

> Consider a sequence $\Sigma_i = a_i \Sigma$ of rescaled helicoids, where $a_i \to 0$. (That is, rescale $\mathbf{R}^3$ by a_i, so points that used to be distance d apart will in the rescaled $\mathbf{R}^3$ be distance $a_i d$ apart.) The curvatures of this sequence of rescaled helicoids are blowing up along the vertical axis. The sequence converges (away from the vertical axis) to a foliation by flat parallel planes. The singular set S (the axis) then consists of removable singularities.

Throughout let x_1, x_2, x_3 be the standard coordinates on $\mathbf{R}^3$. For $y \in \Sigma \subset \mathbf{R}^3$ and $s > 0$, the extrinsic and intrinsic balls are $B_s(y)$ and $\mathcal{B}_s(y)$: namely, $B_s(y) = \{x \in \mathbf{R}^3 \mid |x - y| < s\}$, and $\mathcal{B}_s(y) = \{x \in \Sigma \mid \text{dist}_\Sigma(x, y) < s\}$. The Gaussian curvature of $\Sigma \subset \mathbf{R}^3$ is $\mathrm{K}_\Sigma = \kappa_1 \kappa_2$, so if Σ is minimal (i.e., $\kappa_1 = -\kappa_2$), then $|A|^2 = -2\mathrm{K}_\Sigma$.

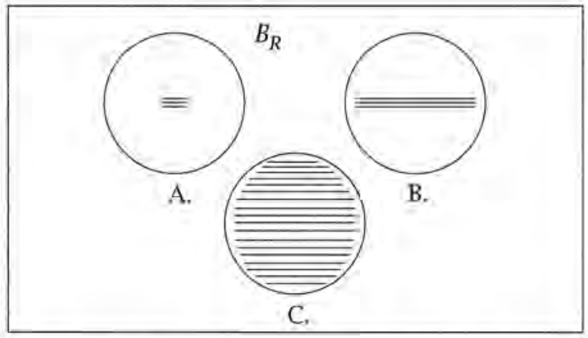


Figure 3. Proving Theorem 1. A) Finding a small N-valued graph in Σ. B) Extending it in Σ to a large N-valued graph. C) Extending the number of sheets.

See [CM1], [O], [S] (and the forthcoming book [CM3]) for background and basic properties of minimal surfaces and [CM2] for a more detailed survey of the results described here and for references. See also [C] for an abbreviated version of this paper intended for a general nonmathematical audience. The article [A] discusses in a simple nontechnical way the shape of various things that are of "minimal" type. These shapes include soap films and soap bubbles, metal alloys, radiolarian skeletons, and embryonic tissues and cells; see also D'Arcy Thompson's influential book, [Th], about form. The reader interested in some of the history of the field of minimal surfaces may consult [DHKW], [N], and [T].

The Limit Foliation and the Singular Curve

In the next few sections we will discuss how to show that every embedded minimal disk is either a graph of a function or part of a double spiral staircase; Theorem 1 below gives precise meaning to this statement. In particular, we will in the next few sections discuss the following (see Figure 3).

A. Fix an integer N (the meaning of "large" curvature in what follows will depend on N). If an embedded minimal disk Σ is not a graph (or equivalently, if the curvature is large at some point), then it contains an N-valued minimal graph, which initially is shown to exist on the scale of $1/\max|A|$. That is, the N-valued graph is initially shown to be defined on an annulus with both inner and outer radius inversely proportional to $\max|A|$.

B. Such a potentially small N-valued graph sitting inside Σ can then be seen to extend as an N-valued graph inside Σ almost all the way to the boundary. That is, the small N-valued graph can be extended to an N-valued graph defined on an annulus whose outer radius is proportional to the radius R of the ball in $\mathbf{R}^3$ whose boundary contains the boundary of Σ.

C. The N-valued graph not only extends horizontally (i.e., tangent to the initial sheets) but also vertically (i.e., transversally to the sheets). That is, once there are N sheets there are many more, and the disk Σ in fact consists of two multivalued graphs glued together along an axis.

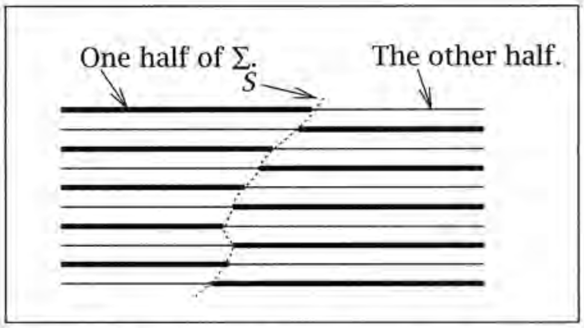


Figure 4. Theorem 1: The singular set and the two multivalued graphs.

These three items, A, B, and C, will be used to demonstrate the following theorem, which is the main result.

Theorem 1. *(See Figure 4). Let $\Sigma_i \subset B_{R_i} = B_{R_i}(0) \subset \mathbf{R}^3$ be a sequence of embedded minimal disks with $\partial\Sigma_i \subset \partial B_{R_i}$, where $R_i \to \infty$. If $\sup_{B_1 \cap \Sigma_i} |A|^2 \to \infty$, then there exists a subsequence Σ_j and a Lipschitz curve $S : \mathbf{R} \to \mathbf{R}^3$ such that after a rotation of $\mathbf{R}^3$:*

1. $x_3(S(t)) = t$ (that is, S is a graph over the x_3-axis).

2. Each Σ_j consists of exactly two multivalued graphs away from S (which spiral together).

3. For each $\alpha > 0$, $\Sigma_j \setminus S$ converges in the C^α-topology to the foliation $\mathcal{F} = \{x_3 = t\}_t$ of $\mathbf{R}^3$ by flat parallel planes.

4. $\sup_{B_r(S(t)) \cap \Sigma_j} |A|^2 \to \infty$ for all $r > 0$ and $t \in \mathbf{R}$. (The curvature blows up along S.)

In 2 and 3 the statement that the $\Sigma_j \setminus S$ are multivalued graphs and converge to $\mathcal{F}$ means that for each compact subset $K \subset \mathbf{R}^3 \setminus S$ and j sufficiently large, $K \cap \Sigma_j$ consists of multivalued graphs over (part of) $\{x_3 = 0\}$, and $K \cap \Sigma_j \to K \cap \mathcal{F}$.

As will be clear in the following sections, A, B, and C alone are not enough to prove Theorem 1. For instance, 1 does not follow from A, B, and C but needs a more precise statement than C of where the new sheets form above and below a given multivalued graph. This requires using the "one-sided curvature estimate".

Here is a summary of the rest of the paper:

First we discuss two key results that are used in the proof of Theorem 1. These are the existence of multivalued graphs, i.e., A and B, and the important one-sided curvature estimate. Following that we discuss some bounds for the separation of multivalued minimal graphs. These bounds are used in both B and C above, and we discuss what they are used for in C. After that we explain how the one-sided curvature estimate is used to show that the singular set S is a Lipschitz curve. The last two sections before our concluding remarks contain further discussion on the existence of multivalued graphs and on the proof of the one-sided curvature estimate.

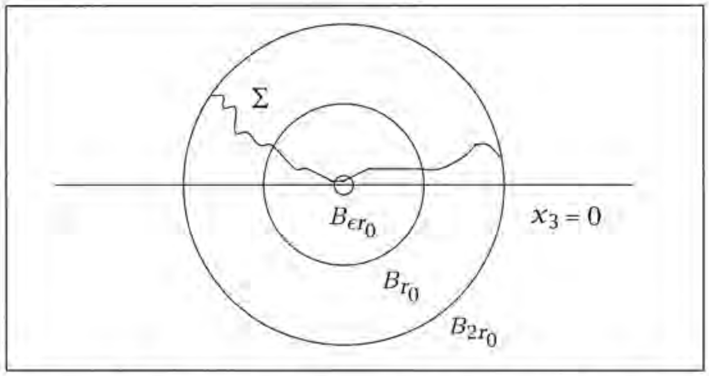


Figure 5. The one-sided curvature estimate for an embedded minimal disk Σ in a half-space with $\partial\Sigma \subset \partial B_{2r_0}$: The components of $B_{r_0} \cap \Sigma$ intersecting $B_{\epsilon r_0}$ are graphs.

Two Key Ingredients in the Proof of Theorem 1: Existence of Multivalued Graphs and the One-Sided Curvature Estimate

We now come to the two key results about embedded minimal disks. The first says that if the curvature of such a disk Σ is large at some point $x \in \Sigma$, then near x a multivalued graph forms (in Σ) and this extends (in Σ) almost all the way to the boundary. Moreover, the inner radius r_x of the annulus where the multivalued graph is defined is inversely proportional to $|A|(x)$, and the initial separation between the sheets is bounded by a constant times the inner radius, i.e., $|w(r_x, \theta)| \leq C\, r_x$.

An important ingredient in the proof of Theorem 1 is that, just like the helicoid, general embedded minimal disks with large curvature at some interior point can be built out of N-valued graphs. In other words, any embedded minimal disk can be divided into pieces, each of which is an N-valued graph. Thus the disk itself should be thought of as being obtained by stacking these pieces (graphs) on top of each other.

The second key result (Theorem 2) is a curvature estimate for embedded minimal disks in a half-space. As a corollary of this theorem, we get that the set of points in an embedded minimal disk where the curvature is large lies within a cone, and thus the multivalued graphs, whose existence was discussed above, will all start off within this cone; see Figure 8 and Figure 9.

The curvature estimate for disks in a half-space is the following:

Theorem 2. *(See Figure 5). There exists $\epsilon > 0$ such that for all $r_0 > 0$, if $\Sigma \subset B_{2r_0} \cap \{x_3 > 0\} \subset \mathbf{R}^3$ is an embedded minimal disk with $\partial\Sigma \subset \partial B_{2r_0}$, then for all components Σ' of $B_{r_0} \cap \Sigma$ which intersect $B_{\epsilon r_0}$*

$$\sup_{x \in \Sigma'} |A_\Sigma(x)|^2 \leq r_0^{-2}\,. \tag{9}$$

Theorem 2 is an interior estimate where the curvature bound (9) is on the ball B_{r_0} of one half of the radius of the ball B_{2r_0} containing Σ. This is just

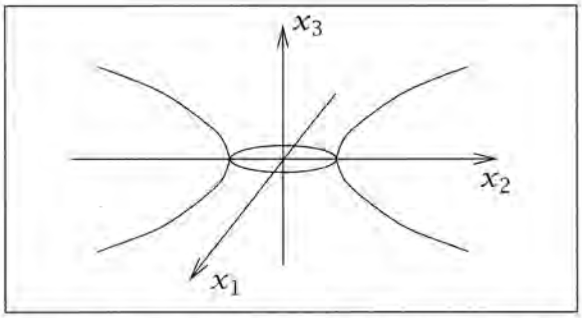


Figure 6. The catenoid given by revolving $x_1 = \cosh x_3$ around the x_3-axis.

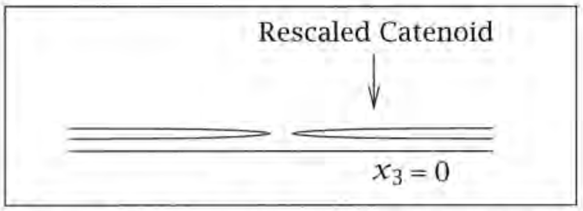


Figure 7. Rescaling the catenoid shows that the property of being simply connected (and embedded) is needed in the one-sided curvature estimate.

like a gradient estimate for a harmonic function where the gradient bound is on one half of the ball where the function is defined.

Using the minimal surface equation and the fact that Σ' has points close to a plane, it is not hard to see that, for $\epsilon > 0$ sufficiently small, (9) is equivalent to the statement that Σ' is a graph over the plane $\{x_3 = 0\}$.

We will often refer to Theorem 2 as *the one-sided curvature estimate* (since Σ is assumed to lie on one side of a plane). Note that the assumption in Theorem 2 that Σ is simply connected (i.e., that Σ is a disk) is crucial, as can be seen from the example of a rescaled catenoid. The catenoid (see Figure 6) is the minimal surface in $\mathbf{R}^3$ given by $(\cosh s \cos t, \cosh s \sin t, s)$, where $s, t \in \mathbf{R}$. Rescaled catenoids converge (with multiplicity two) to the flat plane; see Figure 7. Likewise, by considering the universal cover of the catenoid, one sees that Theorem 2 requires the disk to be embedded and not just immersed.

Definition 1. *(Cones; see Figure 8). If $\delta > 0$ and $x \in \mathbf{R}^3$, then we denote by $\mathbf{C}_\delta(x)$ the (convex) cone with vertex x, cone angle $(\pi/2 - \arctan \delta)$, and axis parallel to the x_3-axis. That is (see Figure 8),*

$$\mathbf{C}_\delta(x) = \{x \in \mathbf{R}^3 \mid x_3^2 \geq \delta^2 (x_1^2 + x_2^2)\} + x\,. \tag{10}$$

In the proof of Theorem 1, the following (direct) consequence of Theorem 2 (with Σ_d playing the role of the plane $\{x_3 = 0\}$; see Figure 9) is needed. (Paraphrased, this corollary says that if an embedded minimal disk contains a 2-valued graph, then the disk consists of multivalued graphs away from a cone with axis orthogonal to the 2-valued graph.

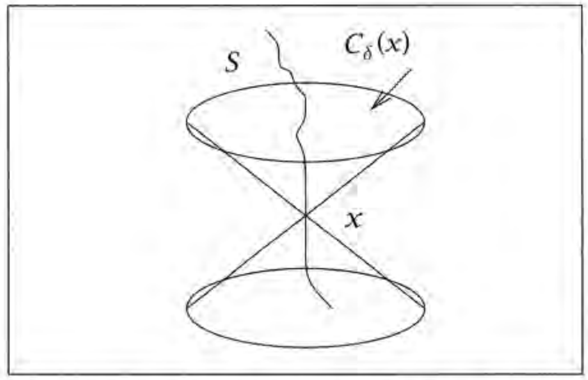


Figure 8. It follows from the one-sided curvature estimate that the singular set has the cone property and hence is a Lipschitz curve.

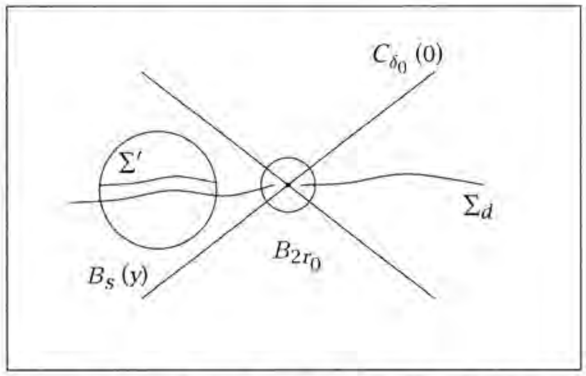


Figure 9. Corollary 1: With Σ_d playing the role of the plane $x_3 = 0$, by the one-sided estimate, Σ consists of multivalued graphs away from a cone.

In Corollary 1 the "d" in Σ_d stands for double-valued.)

Corollary 1. *(See Figure 9.) There exists $\delta_0 > 0$ such that for all $r_0, R > 0$ with $r_0 < R$, if $\Sigma \subset B_{2R}$ with $\partial\Sigma \subset \partial B_{2R}$ is an embedded minimal disk containing a 2-valued graph $\Sigma_d \subset \mathbf{R}^3 \setminus \mathbf{C}_{\delta_0}(0)$ over the annulus $D_R \setminus D_{r_0}$ with gradient $\leq \delta_0$, then each component of $B_{R/2} \cap \Sigma \setminus (\mathbf{C}_{\delta_0}(0) \cup B_{2r_0})$ is a multivalued graph.*

Figure 9 illustrates how this corollary follows from Theorem 2. In this picture, $B_s(y)$ is a ball away from 0, and Σ' is a component of $B_s(y) \cap \Sigma$ disjoint from Σ_d. It follows easily from the maximum principle that Σ' is topologically a disk. Since Σ' is assumed to contain points near Σ_d, we can let a component of $B_s(y) \cap \Sigma_d$ play the role of the plane $\{x_3 = 0\}$ in Theorem 2, and the corollary follows.

Using Theorems 1 and 2, W. Meeks and H. Rosenberg proved in "The uniqueness of the helicoid and the asymptotic geometry of properly embedded minimal surfaces with finite topology" that the plane and the helicoid are the only complete properly embedded simply connected minimal surfaces in $\mathbf{R}^3$. Catalan had proved in 1842 that any complete ruled minimal surface is either a plane or a helicoid. A surface is said to be *ruled* if it has the parametrization

$$X(s,t) = \beta(t) + s\,\delta(t), \qquad \text{where } s,\, t \in \mathbf{R}, \tag{11}$$

and β and δ are curves in $\mathbf{R}^3$. The curve $\beta(t)$ is called the *directrix* of the surface, and a line having $\delta(t)$ as direction vector is called a *ruling*. For the helicoid in (7), the x_3-axis is a directrix, and for each fixed t the line $s \to (s\cos t, s\sin t, t)$ is a ruling.

Towards Removability of Singularities: Analysis of Multivalued Minimal Graphs

Even given the decomposition into multivalued graphs mentioned in the beginning of the previous section, in order to prove Theorem 1 one still needs to analyze how the various N-valued pieces fit together. In particular, we need Theorem 2 and Corollary 1 to show that an embedded minimal disk that is not a graph cannot be contained in a half-space, and thus the subset of points with large curvature lies within a cone. This is still not enough to imply Theorem 1. One also needs to show that part of any embedded minimal disk cannot accumulate in a half-space. This is what we call *properness* below; see Figure 10 and (14) that give an example of an ∞-valued graph whose image lies in a slab in $\mathbf{R}^3$. The property we call properness is the assertion that no limit of embedded minimal disks can contain such a (nonproper) multivalued graph.

In this section, we will discuss bounds for the separation of embedded multivalued graphs and applications to properness and to the proofs of Theorems 1 and 2. Two types of bounds for the growth/decay (as $\rho \to \infty$) of the separation will be needed:

a. The weaker <u>sublinear bounds</u>: There exists $0 < \alpha < 1$ such that for fixed ρ_0 we have the bounds

$$\begin{aligned} (\rho/\rho_0)^{-\alpha}\, |w(\rho_0, \theta)| &\le |w(\rho,\theta)| \\ &\le (\rho/\rho_0)^{\alpha}\, |w(\rho_0,\theta)| \quad \text{as } \rho \to \infty . \end{aligned} \tag{12}$$

These bounds hold for N-valued graphs (where N is some fixed large number). By letting N be large, we can choose α small.

b. The stronger <u>logarithmic bounds</u>: There exist constants c_1 and c_2 such that for fixed ρ_0 we have the bounds

$$\begin{aligned} \frac{c_1}{\log(\rho/\rho_0)}\, |w(\rho_0,\theta)| &\le |w(\rho,\theta)| \\ \le c_2 \log(\rho/\rho_0)\, |w(\rho_0,\theta)| &\quad \text{as } \rho \to \infty . \end{aligned} \tag{13}$$

These bounds will require a growing number of sheets (growing as $\rho \to \infty$) and will be used only to show properness; cf. Figure 10.

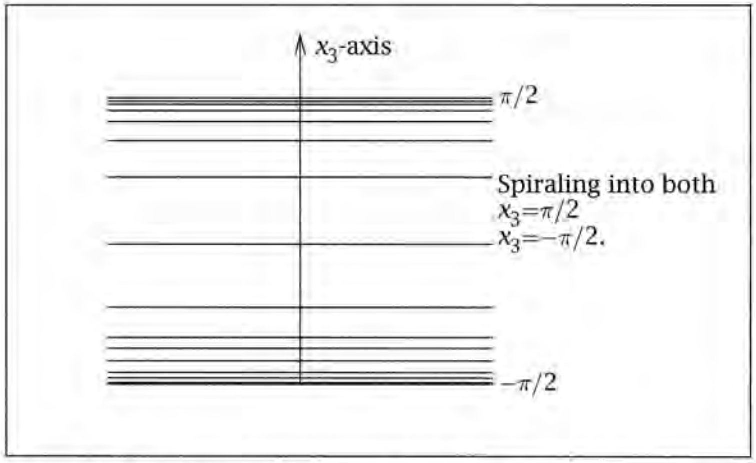


Figure 10. To show properness, one needs to rule out that one of the multivalued graphs can contain a nonproper graph like $\arctan(\theta/\log\rho)$, where (ρ,θ) are polar coordinates. The graph of $\arctan(\theta/\log\rho)$ is illustrated above.

Here are a couple of things that the sublinear bounds are used for. First, as a consequence of the existence of multivalued graphs discussed in the previous section, one easily gets that if $|A|^2$ is blowing up near 0 for a sequence of embedded minimal disks Σ_i, then there is a sequence of 2-valued graphs $\Sigma_{i,d} \subset \Sigma_i$. Here the 2-valued graphs start off defined on a smaller and smaller scale (the inner radius of the annulus where each multivalued graph is defined is inversely proportional to $|A|$). Consequently, by the sublinear separation growth, such 2-valued graphs collapse. Namely, if $\Sigma_{i,d}$ is a 2-valued graph over $D_R \setminus D_{r_i}$, then $|w_i(\rho,\theta)| \le (\rho/r_i)^{\alpha}\, |w_i(r_i,\theta)| \le C\rho^{\alpha} r_i^{1-\alpha}$ for some $\alpha < 1$ and some constant C. (In fact, by making N large, we can choose α small.) Letting $r_i \to 0$ shows that $|w_i(\rho,\theta)| \to 0$ for ρ and θ fixed. Thus as $i \to \infty$, the upper sheet collapses onto the lower, and hence a subsequence converges to a smooth minimal graph through 0. (Here 0 is a removable singularity for the limit.) Moreover, if the sequence of disks is as in Theorem 1, i.e., if $R_i \to \infty$, then the minimal graph in the limit is entire and hence, by Bernstein's theorem, is a plane.

The sublinear bounds are also used in the proof of Theorem 2, which in turn—through its corollary, Corollary 1—is used to show that any multivalued graph contained in an embedded minimal disk can be extended, inside the disk, to a multivalued graph with a rapidly growing number of sheets, and thus we get the better logarithmic bounds for the separation. Namely, by Corollary 1, outside a cone such a multivalued graph extends as a multivalued graph. An application of a Harnack inequality then shows that the number of sheets that it takes to leave the complement of the cone where the disk is a graph must grow in ρ sufficiently fast so that (13) follows. (Recall that for the bounds (13) to hold requires that the number of sheets grows sufficiently fast.)

By (b.), when the number of sheets grows sufficiently fast, the fastest possible decay for $w(\rho,0)/w(1,0)$ is $c_1/\log\rho$. This lower bound for the decay of the separation is sharp. It is achieved for the ∞-valued graph of the harmonic function

$$u(\rho,\theta)=\arctan\frac{\theta}{\log\rho}\,. \tag{14}$$

(One can show that graphs of multivalued harmonic functions are good models for multivalued minimal graphs.) Note that the graph of u is embedded and lies in a slab in $\mathbf{R}^3$ (namely, $|u|\le\pi/2$) and hence in particular is not proper. On the top it spirals into the plane $\{x_3=\pi/2\}$ and on the bottom it spirals into $\{x_3=-\pi/2\}$, yet it never reaches either of these planes; see Figure 10.

The next proposition rules out not only this example as a possible limit of (one half of) embedded minimal disks but, more generally, any ∞-valued minimal graph in a half-space.

Proposition 1. *Multivalued graphs contained in embedded minimal disks are proper: they do not accumulate in finite height.*

Proposition 1 relies in part on the logarithmic bound (13) for the separation.

Regularity of the Singular Set and Theorem 1

In this section we will indicate how to define the singular set S in Theorem 1 and show the regularity of S.

By a very general standard compactness argument, it follows (after possibly passing to a subsequence) that for a sequence of smooth surfaces in $\mathbf{R}^3$ there is a well-defined notion of points where the second fundamental form of the sequence blows up. That is, let $\Sigma_i\subset B_{R_i}$, with $\partial\Sigma_i\subset\partial B_{R_i}$, and $R_i\to\infty$ be a sequence of (smooth) compact surfaces. After passing to a subsequence, Σ_j, we may assume that for each $x\in\mathbf{R}^3$ either (a) or (b) holds:

(a) $\sup_{B_r(x)\cap\Sigma_j}|A|^2\to\infty$ for all $r>0$,

(b) $\sup_j\sup_{B_r(x)\cap\Sigma_j}|A|^2<\infty$ for some $r>0$.

Definition 2. *(Cone property). Fix $\delta>0$. We will say that a subset $S\subset\mathbf{R}^3$ has the* cone property *(or the δ-cone property) if S is closed and nonempty and:*
(i) If $z\in S$, then $S\subset\mathbf{C}_\delta(z)$; see Definition 1 for $\mathbf{C}_\delta(z)$.
(ii) If $t\in x_3(S)$ and $\epsilon>0$, then $S\cap\{t<x_3<t+\epsilon\}\neq\emptyset$ and $S\cap\{t-\epsilon<x_3<t\}\neq\emptyset$.

Note that (ii) just says that each point in S is the limit both of points coming from above and of points coming from below.

When $\Sigma_i\subset B_{R_i}\subset\mathbf{R}^3$ is a sequence of embedded minimal disks with $\partial\Sigma_i\subset\partial B_{R_i}$ $(R_i\to\infty)$, Σ_j is the subsequence as above, and S is the set of points where the curvatures of Σ_j blow up (i.e., where (a) above holds), then, as we will see below, S has the cone property (after a rotation of $\mathbf{R}^3$). Hence (by the next lemma) S is a Lipschitz curve that is a graph over the x_3-axis. Note that if Σ_i is a sequence of rescaled helicoids, then S is the x_3-axis.

Lemma 1. *(See Figure 8). If $S\subset\mathbf{R}^3$ has the δ-cone property, then $S\cap\{x_3=t\}$ consists of exactly one point S_t for each $t\in\mathbf{R}$, and $t\to S_t$ is a Lipschitz parameterization of S.*

With Lemma 1 in hand, we can proceed with the proof of Theorem 1. If Σ_i is as in Theorem 1 and Σ_j and S are as above, then S is closed by definition and nonempty by the assumption of Theorem 1. Centered at any $x\in S$, we can, by the existence of multivalued graphs near points where the curvatures blow up, the sublinear separation growth, and Bernstein's theorem, find a sequence of 2-valued graphs $\Sigma_{d,j}\subset\Sigma_j$ which converges to a plane through x; see the discussion preceding Theorem 2. (This is after possibly passing to a subsequence of the Σ_j's.) Thus (i) above holds by Corollary 1. Therefore, to see that S has the cone property, all we need to see is that (ii) holds. The proof of this relies on Proposition 1. Once the cone property of S is shown, it follows from Lemma 1 that S is a Lipschitz curve and by Corollary 1, away from S, each Σ_j consists of multivalued graphs. It is not hard to see that there are at least two such graphs, and a barrier argument shows that there are not more.

Blow-Up Points and the Existence of Multivalued Graphs

To describe the existence of multivalued graphs in embedded minimal disks, we will need the notion of a blow-up point.

Let Σ be a smooth (minimal or not) embedded (compact) surface in a ball $B_{r_0}(x)$ in $\mathbf{R}^3$ passing through the center x of the ball and with boundary contained in the boundary of the ball. Here $B_{r_0}(x)$ is the extrinsic ball of radius r_0, but it could as well have been an intrinsic ball $\mathcal{B}_{r_0}(x)$, in which case the notion of a blow-up point below would have to be appropriately changed. Suppose that $|A|^2(x)\ge 4\,C^2\,r_0^{-2}$ for some constant $C>0$. We claim that there is $y\in B_{r_0}(x)\cap\Sigma$ and $s>0$ such that $B_s(y)\subset B_{r_0}(x)$ and

$$\sup_{B_s(y)\cap\Sigma}|A|^2\le 4\,C^2\,s^{-2}=4\,|A|^2(y)\,. \tag{15}$$

That is, the curvature at y is large (this just means that C should be thought of as a large constant equal to $s\,|A|(y)$) and is almost (up to the constant 4) the maximum on the ball $B_s(y)$. We will say that the pair (y,s) is a *blow-up pair* and the point y is a *blow-up point.* Later s will be replaced by $8s$ and eventually by a constant times s, and sometimes the extrinsic ball will be replaced by an intrinsic ball, but we will still refer to the pair (y,s) as a blow-up pair.

That there exists such a point y is easy to see; on $B_{r_0}(x) \cap \Sigma$ set $F(z) = (r_0 - r(z))^2 \, |A|^2(z)$ where $r(z) = |z - x|$. Then

$$(16) \quad F(x) \geq 4\,C^2, \quad F \geq 0, \quad \text{and } F|_{\partial B_{r_0}(x) \cap \Sigma} = 0.$$

Let y be a point where the maximum of F is achieved and set $s = C/|A|(y)$. Note that s is at most one-half of the distance from y to the boundary of the ball $B_{r_0}(x)$. One easily checks that y and s have the required properties. Namely, clearly $|A|^2(y) = C^2 s^{-2}$, and since y is where the maximum of F is achieved,

$$(17) \qquad |A|^2(z) \leq \left(\frac{r_0 - r(y)}{r_0 - r(z)}\right)^2 |A|^2(y).$$

Since $F(x) \geq 4\,C^2$ it follows from the choice of s that $|r_0 - r(y)| \leq 2\,|r_0 - r(z)|$ for $z \in B_s(y) \cap \Sigma$. Hence, $|A|^2(z) \leq 4\,|A|^2(y)$. Putting this together gives (15).

The existence of multivalued graphs is shown by combining a blow-up result with an extension result. This blow-up result says that if an embedded minimal disk in a ball has large curvature at a point, then it contains a small (in fact, on the scale of $1/\max|A|$) almost flat N-valued graph nearby; this is A in Figure 3. The extension result allows us to extend the (small) N-valued graphs almost out to the boundary of the "big" ball B_R; this is B in Figure 3. In fact, the blow-up result shows that if (y, s) is a blow-up pair satisfying (15), then the corresponding N-valued function is defined on an annulus whose inner radius is s, and so the initial separation is proportional to s. That is, for positive constants C_1 and C_2

$$(18) \qquad C_1 s \leq |w(s,\theta)| \leq C_2 s.$$

Equation (18) will be used implicitly in the next section.

The extension result is significantly more subtle than the local existence of multivalued graphs. The key for being able to extend is a curvature estimate "between the sheets" for embedded minimal disks; see Figure 11. We think of an axis for such a disk Σ as a point or curve away from which the surface locally (in an extrinsic ball) has more than one component. With this weak notion of an axis, the estimate between the sheets is that if one component of Σ is sandwiched between two others that connect to an axis, then there is a fixed bound for (the norm of) the curvature of the one that is sandwiched. The example to keep in mind is a helicoid, the components being "consecutive sheets" away from the axis. Once the estimate between the sheets is established, it is applied to the "middle" sheet(s) of an N-valued graph to show that even as we go far out to the "outer" boundary of the N-valued graph, the curvature has a fixed bound. Using this a priori bound and additional arguments, one gets better bounds and eventually (with more work) shows that the sheets must remain almost flat and thus the N-valued graph will remain an N-valued graph.

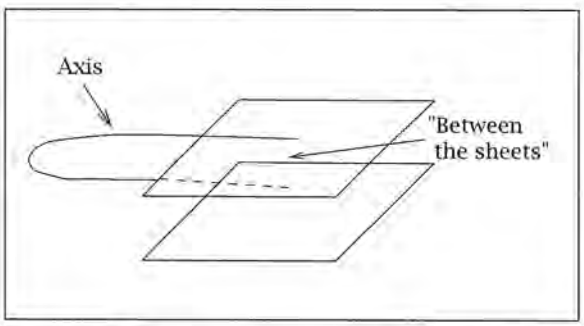


Figure 11. The curvature estimate "between the sheets".

The Proof of the One-Sided Curvature Estimate

Using a blow-up argument and the minimal surface equation, one can prove curvature estimates for minimal surfaces that on all sufficiently small scales lie on one side of, but come close to, a plane. Such an assumption is a scale-invariant version of Theorem 2. However, the assumption of Theorem 2 is not scale invariant, and the theorem cannot be proved this way. The scale-invariant condition is very similar to the classical Reifenberg property. (A subset of $\mathbf{R}^n$ has the Reifenberg property if it is close on all scales to a hyperplane; see the appendix of [ChC]. This property goes back to Reifenberg's fundamental 1960 paper "Solution of the Plateau problem for m-dimensional surfaces of varying topological type".) As explained above (in particular Corollary 1), the significance of Theorem 2 is indeed that it requires closeness only on one scale. On the other hand, this is what makes it difficult to prove (the lack of scale invariance is closely related to the lack of a useful monotone quantity).

Let us give a very rough outline of the proof of the one-sided curvature estimate; i.e., Theorem 2. Suppose that Σ is an embedded minimal disk in the half-space $\{x_3 > 0\}$ intersected with the ball B_{2r_0} and with boundary in the boundary of the ball B_{2r_0}. The curvature estimate is proved by contradiction, so suppose that Σ has low points with large curvature. Starting at such a point, we decompose Σ into disjoint multivalued graphs using the existence of nearby points with large curvature. (The existence of such nearby points is highly nontrivial to establish. We will use that such a nearby point of large curvature can be found below any given multivalued graph, and thus we can choose the "next" blow-up point always to be below the previous one.) The key point is then to show (see Proposition 2 below) that we can in fact find such a decomposition where the "next" multivalued graph starts off a definite amount below where the previous multivalued graph started off. In fact, what we show is that this definite amount is a fixed fraction of the distance between where the two graphs started off. Iterating eventually forces Σ to have points where $x_3 < 0$. This is the desired contradiction.

To show this key proposition (Proposition 2), we use two decompositions and two kinds of blow-up points. The first decomposition uses the more standard blow-up points given as pairs (y, s), where $y \in \Sigma$ and $s > 0$ is such that

$$\sup_{\mathcal{B}_{8s}(y)} |A|^2 \le 4|A|^2(y) = 4\,C_1^2\,s^{-2}\,. \tag{19}$$

(Here $\mathcal{B}_{8s}(y)$ is the intrinsic ball of radius $8s$, so in particular $\mathcal{B}_{8s}(y) \subset \Sigma$.) The point about such a pair (y, s) is that, since Σ is a minimal disk, Σ contains a multivalued graph near y starting off on the scale s. (This is assuming that the curvature at y is sufficiently large, i.e., that C_1 is a sufficiently large constant.) Blow-up points of the second kind are the ones where (except for a technical issue) 8 in the radius of the ball centered at y is replaced by some really large constant C, i.e.,

$$\sup_{\mathcal{B}_{Cs}(y)} |A|^2 \le 4|A|^2(y) = 4\,C_1^2\,s^{-2}\,. \tag{20}$$

Proposition 2. *(See Figure 12). There exists $\delta > 0$ such that if the point 0 and the radius $s > 0$ satisfy (20) and $\Sigma_0 \subset \Sigma$ is the corresponding (to $(0, s)$) 2-valued graph over the annulus $D_{r_0} \setminus D_s$, then there are a point y and a radius t satisfying (20) with $y \in \mathbf{C}_\delta(0) \cap \Sigma \setminus B_{Cs/2}$ and with y below Σ_0.*

The idea for proving Proposition 2 is that we can find blow-up points satisfying (20) such that the distance between them is proportional to the sum of the scales. Moreover, between consecutive blow-up points satisfying (20), we can find many blow-up points satisfying (19); see Figure 13. The advantage is now that if we look between blow-up points satisfying (20), then the height of the multivalued graph given by such a point grows like a small power of the distance, whereas the separation between the

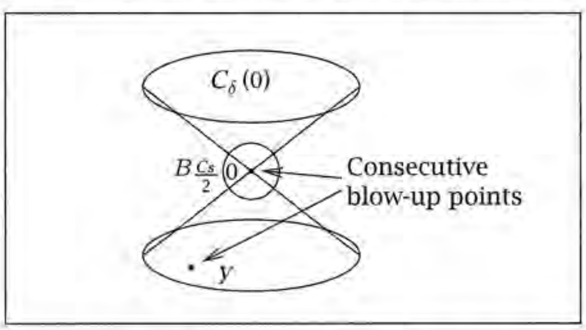


Figure 12. Two consecutive blow-up points satisfying (20).

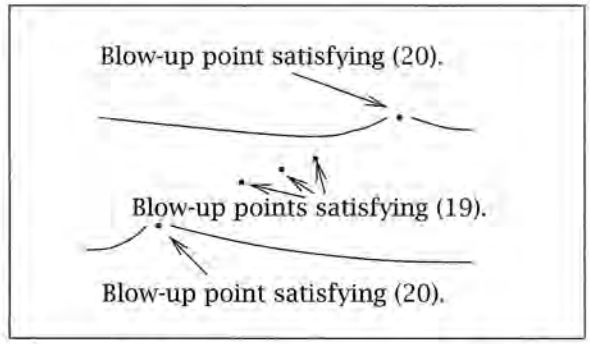


Figure 13. Between two consecutive blow-up points satisfying (20) there are many blow-up points satisfying (19).

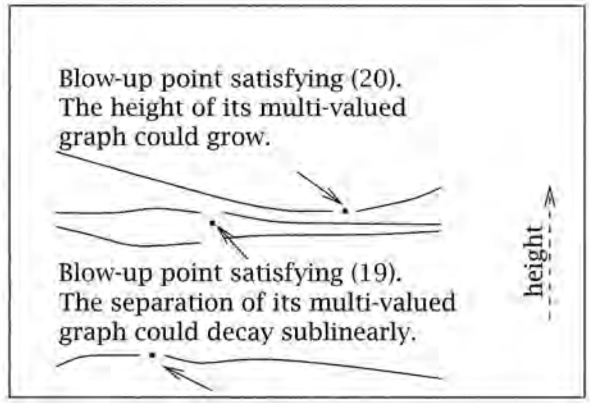


Figure 14. Measuring height. Blow-up points and corresponding multivalued graphs.

sheets in a multivalued graph given by (19) decays (at the worst) like a small power of the distance; see Figure 14. Now, thanks to that the number of blow-up points satisfying (19) (between two consecutive blow-up points satisfying (20)), grows almost linearly, then, even though the height of the graph coming from the blow-up point satisfying (20) could move up (and thus work against us), the sum of the separations of the graphs coming from the points satisfying (19) dominates the other term. Thus the next blow-up point satisfying (20) (which lies below all the other graphs) is forced to be a definite amount lower than the previous blow-up point satisfying (20). This gives the proposition.

Theorem 2 follows from the proposition. Suppose the theorem fails; starting at a point of large curvature and iterating the proposition will eventually give a point in the minimal surface with $x_3 < 0$, which is a contradiction.

Concluding Remarks and Some Possible Future Directions of Research

In this article we have seen that minimal surfaces and surfaces of "minimal" type occur naturally, and we have described why embedded minimal disks are double spiral staircases. We would hope that similar considerations can be used to answer, for things other than minimal disks, the age-old question,

What are the possible shapes of various things and why?

A different possible direction is to describe 3-manifolds. Namely, by a result of B. White for a generic metric on a closed 3-manifold, the area functional—that is, the map that assigns to each closed surface its area—is a Morse function (i.e., a function that has only nondegenerate critical points). As we saw earlier, the critical points of the area functional are precisely the minimal surfaces; thus if one could understand all minimal surfaces in a given 3-manifold M, then one would understand all critical points for the area functional. For a generic metric one could then hope to use Morse-theoretic arguments to understand M. For general embedded minimal surfaces of a given fixed genus in a closed 3-manifold, the key for understanding them is to understand their intersection with a small ball in M. Since locally any fixed 3-manifold is almost Euclidean, this boils down to understanding minimal surfaces in a ball in $\mathbf{R}^3$ with boundary in the boundary of the ball. The key for this is indeed the case where the minimal surfaces are disks; thus the key is the results described here. We will discuss this elsewhere.

The field of minimal surfaces has undergone enormous development since the days of Euler and Lagrange. It has played a key role in the development of many other fields in analysis and geometry. It has had times of intense development followed by times of stagnation before new fundamental results and techniques have been discovered. This has happened over and over again. In closing, we believe that, after nearly a quarter of a millennium, the field of minimal surfaces is at its very peak and of utmost importance in mathematics and its applications. We hope that this expository article has helped convey this. Although as the saying goes, "It is hard to predict—especially about the future",[3] we believe that more magnificent results and techniques are to follow.

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[3]*Quote attributed to Danish humorist Storm P. (Robert Storm Petersen) from the 1920s.*

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The Riemann Hypothesis

J. Brian Conrey

Hilbert, in his 1900 address to the Paris International Congress of Mathematicians, listed the Riemann Hypothesis as one of his 23 problems for mathematicians of the twentieth century to work on. Now we find it is up to twenty-first century mathematicians! The Riemann Hypothesis (RH) has been around for more than 140 years, and yet now is arguably the most exciting time in its history to be working on RH. Recent years have seen an explosion of research stemming from the confluence of several areas of mathematics and physics.

In the past six years the American Institute of Mathematics (AIM) has sponsored three workshops whose focus has been RH. The first (RHI) was in Seattle in August 1996 at the University of Washington. The second (RHII) was in Vienna in October 1998 at the Erwin Schrödinger Institute, and the third (RHIII) was in New York in May 2002 at the Courant Institute of Mathematical Sciences. The intent of these workshops was to stimulate thinking and discussion about one of the most challenging problems of mathematics and to consider many different approaches. Are we any closer to solving the Riemann Hypothesis after these efforts? Possibly. Have we learned anything about the zeta-function as a result of these workshops? Definitely. Several of the participants from the workshops are collaborating on the website (`http://www.aimath.org/WWN/rh/`) which provides an overview of the subject.

Here I hope to outline some of the approaches to RH and to convey some of the excitement of working in this area at the present moment. To begin, let us examine the Riemann Hypothesis itself. In 1859 in the seminal paper "Ueber die Anzahl der Primzahlen unter eine gegebener Grösse", G. B. F. Riemann outlined the basic analytic properties of the zeta-function

$$\zeta(s) := 1 + \frac{1}{2^s} + \frac{1}{3^s} + \cdots = \sum_{n=1}^{\infty} \frac{1}{n^s}.$$

The series converges in the half-plane where the real part of s is larger than 1. Riemann proved that $\zeta(s)$ has an analytic continuation to the whole plane apart from a simple pole at $s = 1$. Moreover, he proved that $\zeta(s)$ satisfies an amazing *functional equation*, which in its symmetric form is given by

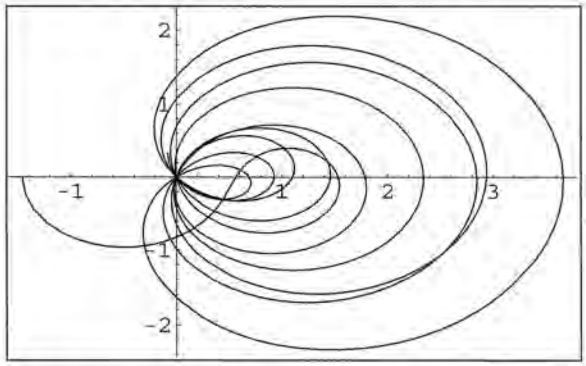

Figure 1. $\zeta(\frac{1}{2} + it)$ **for** $0 < t < 50$.

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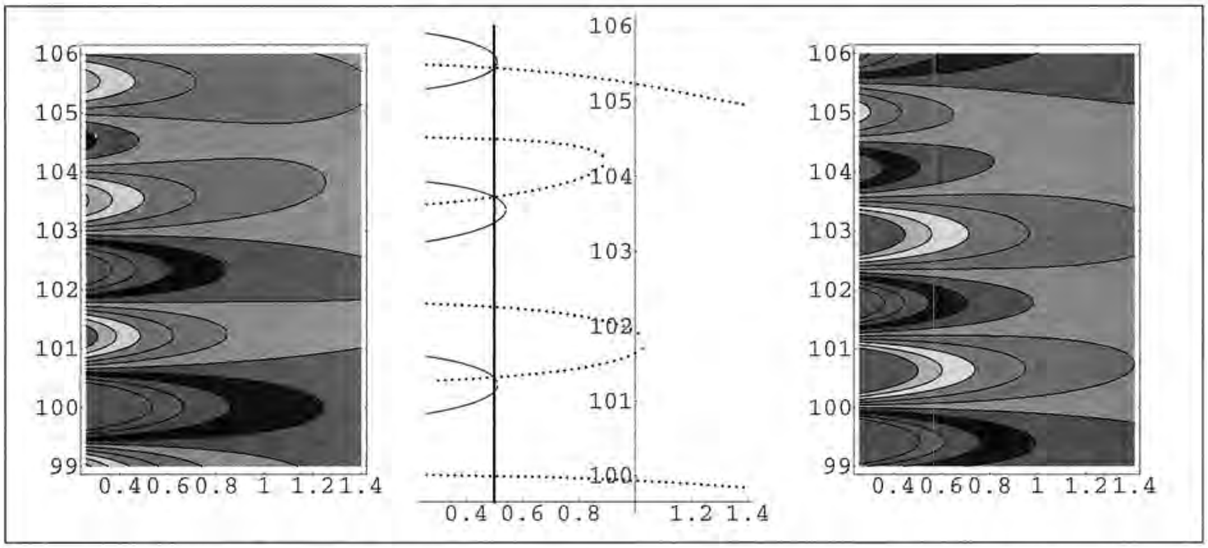

Figure 2. Contour plot of $\Re\zeta(s)$, the curves $\Re\zeta(s) = 0$ (solid) and $\Im\zeta(s) = 0$ (dotted), contour plot of $\Im\zeta(s)$.

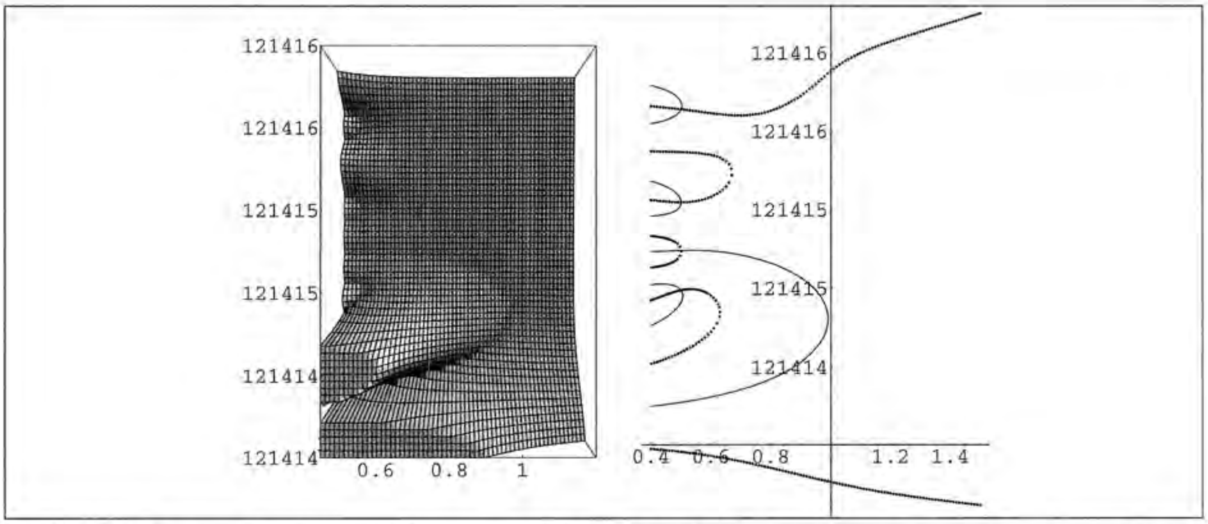

Figure 3. 3-D plot of $|\Re\zeta(s)|$, and the curves $\Re\zeta(s) = 0$ (solid) and $\Im\zeta(s) = 0$ (dotted). This may be the first place in the critical strip where the curves $\Re\zeta(s) = 0$ loop around each other.

$$\xi(s) := \tfrac{1}{2}s(s-1)\pi^{-\frac{s}{2}}\Gamma\left(\frac{s}{2}\right)\zeta(s) = \xi(1-s),$$

where $\Gamma(s)$ is the usual Gamma-function.

The zeta-function had been studied previously by Euler and others, but only as a function of a real variable. In particular, Euler noticed that

$$\begin{aligned}\zeta(s) &= \left(1 + \frac{1}{2^s} + \frac{1}{4^s} + \frac{1}{8^s} + \dots\right) \\ &\quad \times \left(1 + \frac{1}{3^s} + \frac{1}{9^s} + \dots\right)\left(1 + \frac{1}{5^s} + \dots\right)\dots \\ &= \prod_p \left(1 - \frac{1}{p^s}\right)^{-1},\end{aligned}$$

where the infinite product (called the *Euler product*) is over all the prime numbers. The product converges when the real part of s is greater than 1. It is an analytic version of the fundamental theorem of arithmetic, which states that every integer can be factored into primes in a unique way. Euler used this product to prove that the sum of the reciprocals of the primes diverges. The Euler product suggests Riemann's interest in the zeta-function: he was trying to prove a conjecture made by Legendre and, in a more precise form, by Gauss:

$$\pi(x) := \#\{\text{primes less than } x\} \sim \int_2^x \frac{dt}{\log t}.$$

Riemann made great progress toward proving Gauss's conjecture. He realized that the distribution of the prime numbers depends on the distribution of the complex zeros of the zeta-function. The Euler product implies that there are no zeros of $\zeta(s)$ with real part greater than 1; the functional equation implies that there are no zeros with real part less than 0, apart from the *trivial zeros* at

$s = -2, -4, -6, \ldots$. Thus, all of the complex zeros are in the *critical strip* $0 \le \Re s \le 1$. Riemann gave an explicit formula for $\pi(x)$ in terms of the complex zeros $\rho = \beta + i\gamma$ of $\zeta(s)$. A simpler variant of his formula is

$$\begin{aligned}\psi(x) &:= \sum_{n \le x} \Lambda(n) \\ &= x - \sum_\rho \frac{x^\rho}{\rho} - \log 2\pi - \tfrac{1}{2}\log(1 - x^{-2}),\end{aligned}$$

valid for x not a prime power, where the von Mangoldt function $\Lambda(n) = \log p$ if $n = p^k$ for some k and $\Lambda(n) = 0$ otherwise. Note that the sum is not absolutely convergent; if it were, then $\sum_{n \le x} \Lambda(n)$ would have to be a continuous function of x, which it clearly is not. Consequently, there must be infinitely many zeros ρ. The sum over ρ is with multiplicity and is to be interpreted as $\lim_{T \to \infty} \sum_{|\rho| < T}$. Note also that $|x^\rho| = x^\beta$; thus it was necessary to show that $\beta < 1$ in order to conclude that $\sum_{n \le x} \Lambda(n) \sim x$, which is a restatement of Gauss's conjecture.

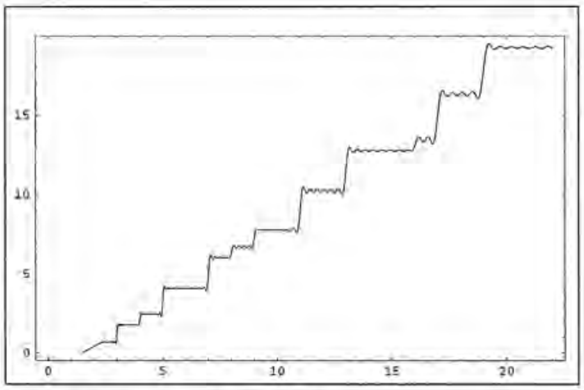

Figure 4. Explicit formula for $\psi(x)$ using the first 100 pairs of zeros.

The functional equation shows that the complex zeros are symmetric with respect to the line $\Re s = \frac{1}{2}$. Riemann calculated the first few complex zeros $\frac{1}{2} + i14.134\ldots, \frac{1}{2} + i21.022\ldots$ and proved that the number $N(T)$ of zeros with imaginary parts between 0 and T is

$$N(T) = \frac{T}{2\pi}\log\frac{T}{2\pi e} + \frac{7}{8} + S(T) + O(1/T),$$

where $S(T) = \frac{1}{\pi}\arg\zeta(1/2 + iT)$ is computed by continuous variation starting from $\arg\zeta(2) = 0$ and proceeding along straight lines, first up to $2 + iT$ and then to $1/2 + iT$. Riemann also proved that $S(T) = O(\log T)$. Note for future reference that at a height T the average gap between zero heights is $\sim 2\pi/\log T$. Riemann suggested that the number $N_0(T)$ of zeros of $\zeta(1/2 + it)$ with $0 < t \le T$ seemed to be about

$$\frac{T}{2\pi}\log\frac{T}{2\pi e}$$

and then made his conjecture that all of the zeros of $\zeta(s)$ in fact lie on the $1/2$-line; this is the Riemann Hypothesis.

Riemann's effort came close to proving Gauss's conjecture. The final step was left to Hadamard and de la Vallée Poussin, who proved independently in 1896 that $\zeta(s)$ does not vanish when the real part of s is equal to 1 and from that fact deduced Gauss's conjecture, now called the Prime Number Theorem.

Initial Ideas

It is not difficult to show that RH is equivalent to the assertion that for every $\epsilon > 0$,

$$\pi(x) = \int_2^x \frac{dt}{\log t} + O(x^{1/2+\epsilon}).$$

However, it is difficult to see another way to approach $\pi(x)$ and so get information about the zeros.

Another easy equivalent to RH is the assertion that $M(x) = O(x^{1/2+\epsilon})$ for every $\epsilon > 0$, where

$$M(x) = \sum_{n \le x} \mu(n)$$

and $\mu(n)$ is the Möbius function whose definition can be inferred from its generating Dirichlet series $1/\zeta$:

$$\frac{1}{\zeta(s)} = \sum_{n=1}^\infty \frac{\mu(n)}{n^s} = \prod_p \left(1 - \frac{1}{p^s}\right).$$

Thus, if $p_1, \ldots, p_k$ are distinct primes, then $\mu(p_1 \ldots p_k) = (-1)^k$; also $\mu(n) = 0$ if $p^2 \mid n$ for some prime p. This series converges absolutely when $\Re s > 1$. If the estimate $M(x) = O(x^{1/2+\epsilon})$ holds for every $\epsilon > 0$, then it follows by partial summation that the series converges for every s with real part greater than $1/2$; in particular, there can be no zeros of $\zeta(s)$ in this open half-plane, because zeros of $\zeta(s)$ are poles of $1/\zeta(s)$. The converse, that RH implies this estimate for $M(x)$, is also not difficult to show.

Instead of analyzing $\pi(x)$ directly, it might seem easier to work with $M(x)$ and prove the above estimate, perhaps by some kind of combinatorial reasoning. In fact, Stieltjes let it be known that he had such a proof. Hadamard, in his famous 1896 proof of the Prime Number Theorem, refers to Stieltjes's claim and somewhat apologetically offers his much weaker theorem that $\zeta(s)$ does not vanish on the 1-line in the hope that the simplicity of his proof will be useful. Stieltjes never published his proof.

Mertens made the stronger conjecture that

$$|M(x)| \le \sqrt{x};$$

clearly this implies RH. However, Mertens's conjecture was disproved by Odlyzko and te Riele in 1985. The estimate $M(x) = O(\sqrt{x})$ is also likely to

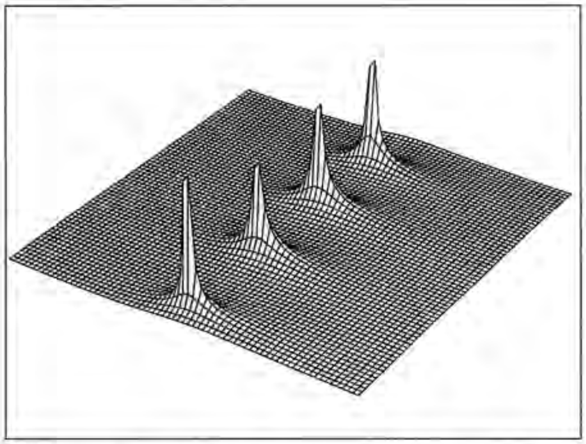

Figure 5. $1/|\zeta(x+iy)|$ **for** $0 < x < 1$ **and** $16502.4 < y < 16505$.

be false, but a proof of its falsity has not yet been found.

Subsequent Efforts

In England in the early 1900s the difficulty of the question was not yet appreciated. Barnes assigned RH to Littlewood as a thesis problem. Littlewood independently discovered some of the developments that had already occurred on the continent. Hardy, Littlewood, Ingham, and other British mathematicians were responsible for many of the results on the zeta-function in the first quarter of the century. Hardy and Littlewood gave the first proof that infinitely many of the zeros are on the 1/2-line. They found what they called the *approximate functional equation* for $\zeta(s)$. Later, Siegel uncovered a very precise version of this formula while studying Riemann's notes in the Göttingen library; the formula is now called the Riemann-Siegel formula and gives the starting point for all large-scale calculations of $\zeta(s)$. Hardy and Littlewood gave an asymptotic evaluation of the second moment of $\zeta(\frac{1}{2}+it)$; Ingham proved the asymptotics for the fourth moment.

Much effort has also been expended on the unproved Lindelöf hypothesis, which is a consequence of RH. The Lindelöf hypothesis asserts that for every $\epsilon > 0$,

$$\zeta(1/2+it) = O(t^{\epsilon}) \qquad \text{as } t \to \infty.$$

Hardy and Littlewood proved that $\zeta(1/2+it) = O(t^{1/4+\epsilon})$. This bound is now called the "convexity bound", since it follows from the functional equation together with general principles of complex analysis (the maximum modulus principle in the form of the Phragmén-Lindelöf theorem). Weyl improved the bound to $t^{1/6+\epsilon}$ with his new ideas for estimating special trigonometrical sums, now called Weyl sums.

Hardy grew to love the problem. He and Littlewood wrote at least ten papers on the zeta-function. Hardy once included proving RH on a list of New Year's goals he set for himself. Hardy even used RH as a defense: he once sent a postcard to his colleague Harald Bohr prior to crossing the English Channel one stormy night, claiming that he had solved RH. Even though Hardy was an atheist, he was relatively certain that God, if he did exist, would not allow the ferry to sink under circumstances so favorable to Hardy!

Hilbert seems to have had somewhat contradictory views about the difficulty of RH. On one occasion he compared three unsolved problems: the transcendence of $2^{\sqrt{2}}$, Fermat's Last Theorem, and the Riemann Hypothesis. In his view, RH would likely be solved in a few years, Fermat's Last Theorem possibly in his lifetime, and the transcendence question possibly never. Amazingly, the transcendence question was resolved a few years later by Gelfond and Schneider, and, of course, Andrew Wiles recently proved Fermat's Last Theorem. Another time Hilbert remarked that if he were to awake after a sleep of five hundred years, the first question he would ask was whether RH was solved.

Near the end of his career, Hans Rademacher, best known for his exact formula for the number of partitions of an integer, thought he had disproved RH. Siegel had checked the work, which was based on the deduction that a certain function would absurdly have an analytic continuation if RH were true. The mathematics community tried to get *Time* magazine interested in the story. It transpired that *Time* became interested and published an article only after it was discovered that Rademacher's proof was incorrect.

Evidence for RH

Here are some reasons to believe RH.

- Billions of zeros cannot be wrong. Recent work by van de Lune has shown that the first 10 billion zeros are on the line. Also, there is a distributed computing project organized by Sebastian Wedeniwski—a screen-saver type of program—that many people subscribe to, which claims to have verified that the first 100 billion zeros are on the line. Andrew Odlyzko has calculated millions of zeros near zeros number $10^{20}, 10^{21}$, and 10^{22} (available on his website).
- Almost all of the zeros are very near the 1/2-line. In fact, it has been proved that more than 99 percent of zeros $\rho = \beta + i\gamma$ satisfy $|\beta - \frac{1}{2}| \leq 8/\log|\gamma|$.
- Many zeros can be proved to be on the line. Selberg got a positive proportion, and N. Levinson showed at least 1/3; that proportion has been improved to 40 percent. Also, RH implies that all zeros of all derivatives of $\xi(s)$ are on the 1/2-line. It has been shown that more than 99 percent of the zeros of the third derivative $\xi'''(s)$ are on the 1/2-line. Near the end of his life, Levinson thought he had a method that allowed for a converse to Rolle's theorem in

this situation, implying that if $\xi'(s)$ has at least a certain proportion of zeros on the line, then so does ξ and similarly for ξ'' to ξ' and so on. However, no one has been able to make this argument work.

- Probabilistic arguments. For almost all random sequences of -1's and $+1$'s, the associated summatory function up to x is bounded by $x^{1/2+\epsilon}$. The Möbius sequence appears to be fairly random.
- Symmetry of the primes. RH tells us that the primes are distributed in as nice a way as possible. If RH were false, there would be some strange irregularities in the distribution of primes; the first zero off the line would be a very important mathematical constant. It seems unlikely that nature is that perverse!

Various Approaches

There is an often-told story that Hilbert and Pólya independently suggested that the way to prove RH was to interpret the zeros spectrally, that is, to find a naturally occurring Hermitian operator whose eigenvalues are the nontrivial zeros of $\zeta(1/2+it)$. Then RH would follow, since Hermitian operators have real eigenvalues. This idea has been one of the main approaches that has been tried repeatedly.

We describe an assortment of other interesting approaches to RH.

Pólya's Analysis

Pólya investigated a chain of ideas that began with Riemann: namely, studying the Fourier transform of $\Xi(t) := \xi(\frac{1}{2}+it)$, which as a consequence of the functional equation is real for real t and an even function of t. RH is the assertion that all zeros of Ξ are real. The Fourier transform can be computed explicitly:

$$\begin{aligned}\Phi(t) &:= \int_{-\infty}^{\infty} \Xi(u)e^{itu}\,du \\ &= \sum_{n=1}^{\infty}(2n^4\pi^2\exp(9t/2) - 3n^2\pi\exp(5t/2)) \\ &\qquad \times \exp\left(-\pi n^2 e^{2t}\right).\end{aligned}$$

It can be shown that Φ and Φ' are positive for positive t. One idea is to systematically study classes of reasonable functions whose Fourier transforms have all real zeros and then try to prove that $\Xi(t)$ is in the class. A sample theorem in this direction is due to de Bruijn:

Let $f(t)$ be an even nonconstant entire function of t such that $f(t) \ge 0$ for real t and $f'(t) = \exp(\gamma t^2)g(t)$, where $\gamma \ge 0$ and $g(t)$ is an entire function of genus ≤ 1 with purely imaginary zeros only. Then $\Psi(z) = \int_{-\infty}^{\infty}\exp\{-f(t)\}e^{izt}dt$ has real zeros only.

In particular, all the zeros of the Fourier transform of a first approximation (see Titchmarsh for details)

$$\begin{aligned}\phi(t) = &(2\pi\cosh\frac{9t}{2} - 3\cosh\frac{5t}{2}) \\ &\times \exp(-2\pi\cosh 2t)\end{aligned}$$

to $\Phi(t)$ are real. These ideas have been further explored by de Bruijn, Newman, D. Hejhal, and others. Hejhal (1990) has shown that almost all of the zeros of the Fourier transform of any partial sum of $\Phi(t)$ are real.

Probabilistic Models

Researchers working in probability are intrigued by the fact that the ξ-function arises as an expectation in a moment of a Brownian bridge:

$$2\xi(s) = E(Y^s)$$

where

$$Y := \sqrt{\frac{2}{\pi}}\left(\max_{t\in[0,1]} b_t - \min_{t\in[0,1]} b_t\right)$$

with $b_t = \beta_t - t\beta_1$ where β_t is standard Brownian motion. See a paper of Biane, Pitman, and Yor (*Bull. Amer. Math. Soc.* (N.S.) **38** (2001), 435–65).

Functional Analysis: The Nyman-Beurling Approach

This approach begins with the following theorem of Nyman, a student of Beurling.

RH holds if and only if

$$\operatorname{span}_{L^2(0,1)}\{\eta_\alpha, 0 < \alpha < 1\} = L^2(0,1)$$

where

$$\eta_\alpha(t) = \{\alpha/t\} - \alpha\{1/t\}$$

and $\{x\} = x - [x]$ is the fractional part of x.

This has been extended by Baez-Duarte, who showed that one may restrict attention to integral values of $1/\alpha$. Balazard and Saias have rephrased this in a nice way:

RH holds if and only if

$$\inf_A \int_{-\infty}^{\infty} \left|1 - A(\tfrac{1}{2}+it)\zeta(\tfrac{1}{2}+it)\right|^2 \frac{dt}{\frac{1}{4}+t^2} = 0,$$

where the infimum is over all Dirichlet polynomials A.

Let d_N be the infimum over all Dirichlet polynomials

$$A(s) = \sum_{n=1}^{N} a_n n^{-s}$$

of length N. They conjecture that $d_N \sim C/\log N$, where $C = \sum_\rho 1/|\rho|^2$. Burnol has proved that

$$d_n \ge \frac{1}{\log N}\sum_{\rho \text{ on the line}} \frac{m_\rho}{|\rho|^2},$$

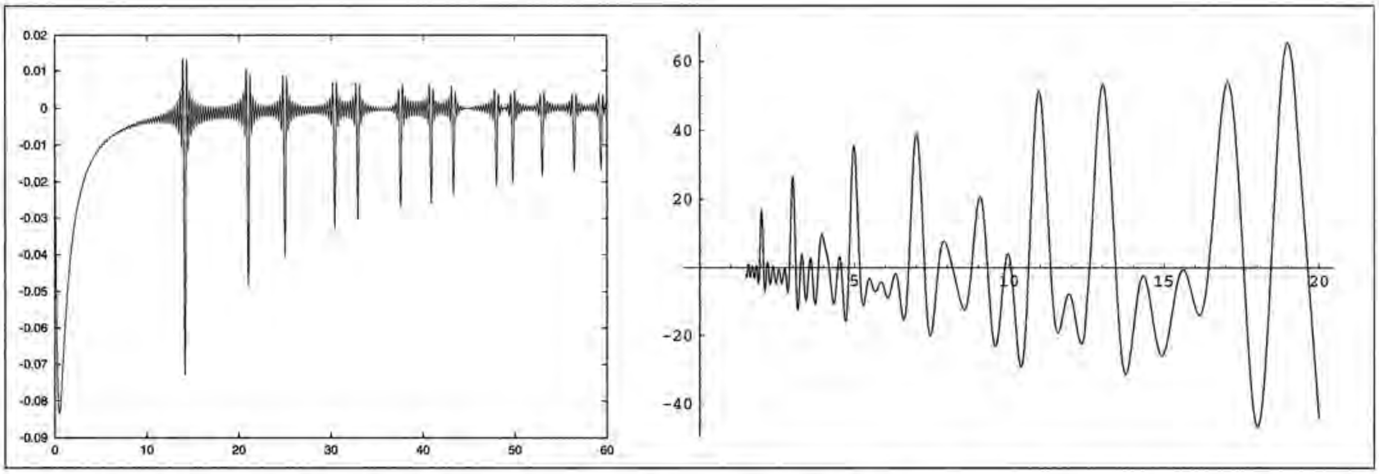

Figure 6. Duality: The Fourier transform of the error term in the Prime Number Theorem (note the spikes at ordinates of zeros) and the sum over zeros $-\sum x^{\rho}$ with $|\rho| < 100$ (note the peaks at primes and prime powers).

where m_ρ is the multiplicity of the zero ρ. If RH holds and all the zeros are simple, then clearly these two bounds are the same.

Weil's Explicit Formula and Positivity Criterion

André Weil proved the following formula, which is a generalization of Riemann's formula mentioned above and which specifically illustrates the dependence between primes and zeros. Suppose h is an even function that is holomorphic in the strip $|\Im t| \le 1/2 + \delta$ and that satisfies $h(t) = O((1 + |t|)^{-2-\delta})$ for some $\delta > 0$, and let

$$g(u) = \frac{1}{2\pi}\int_{-\infty}^{\infty} h(r)e^{-iur}\,dr.$$

Then we have the following duality between primes and zeros of ζ:

$$\begin{aligned}\sum_{\gamma} h(\gamma) =& 2h(\tfrac{i}{2}) - g(0)\log\pi \\ &+ \frac{1}{2\pi}\int_{-\infty}^{\infty} h(r)\frac{\Gamma'}{\Gamma}(\tfrac14 + \tfrac12 ir)\,dr \\ &- 2\sum_{n=1}^{\infty}\frac{\Lambda(n)}{\sqrt{n}}g(\log n).\end{aligned}$$

In this formula, a zero is written as $\rho = 1/2 + i\gamma$ where $\gamma \in \mathbb{C}$; of course RH is the assertion that every γ is real. Using this duality Weil gave a criterion for RH:

RH holds if and only if

$$\sum_{\gamma} h(\gamma) > 0$$

for every (admissible) function h of the form $h(r) = h_0(r)\overline{h_0(\bar{r})}$.

Xian-Jin Li has given a very nice criterion which, in effect, says that one may restrict attention to a specific sequence h_n:

The Riemann Hypothesis is true if and only if $\lambda_n \ge 0$ for each $n = 1, 2, \ldots$ where

$$\lambda_n = \sum_{\rho}(1 - (1 - 1/\rho)^n).$$

As usual, the sum over zeros is $\lim_{T\to\infty}\sum_{|\rho|<T}$. Another expression for λ_n is

$$\lambda_n = \frac{1}{(n-1)!}\,\frac{d^n}{ds^n}(s^{n-1}\log\xi(s))\Big|_{s=1}.$$

It would be interesting to find an interpretation (geometric?) for these λ_n, or perhaps those associated with a different L-function, to make their positivity transparent.

Selberg's Trace Formula

Selberg, perhaps looking for a spectral interpretation of the zeros of $\zeta(s)$, proved a trace formula for the Laplace operator acting on the space of real-analytic functions defined on the upper half-plane $\mathcal{H} = \{x + iy : y > 0\}$ and invariant under the group $SL(2,\mathbb{Z})$ of linear fractional transformations with integer entries and determinant one, which acts discontinuously on $\mathcal{H}$. This invariance is expressed as

$$f\left(\frac{az+b}{cz+d}\right) = f(z);$$

the Laplace operator in this case is

$$\Delta = -y^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial x^2}\right).$$

The spectrum of Δ splits into a continuous part and a discrete part. The eigenvalues λ are all positive and, by convention, are usually expressed as $\lambda = s(1-s)$. The continuous part consists of all $s = 1/2 + it$, $t \ge 0$, and we write the discrete part as $s_j = \frac12 + ir_j$. Then

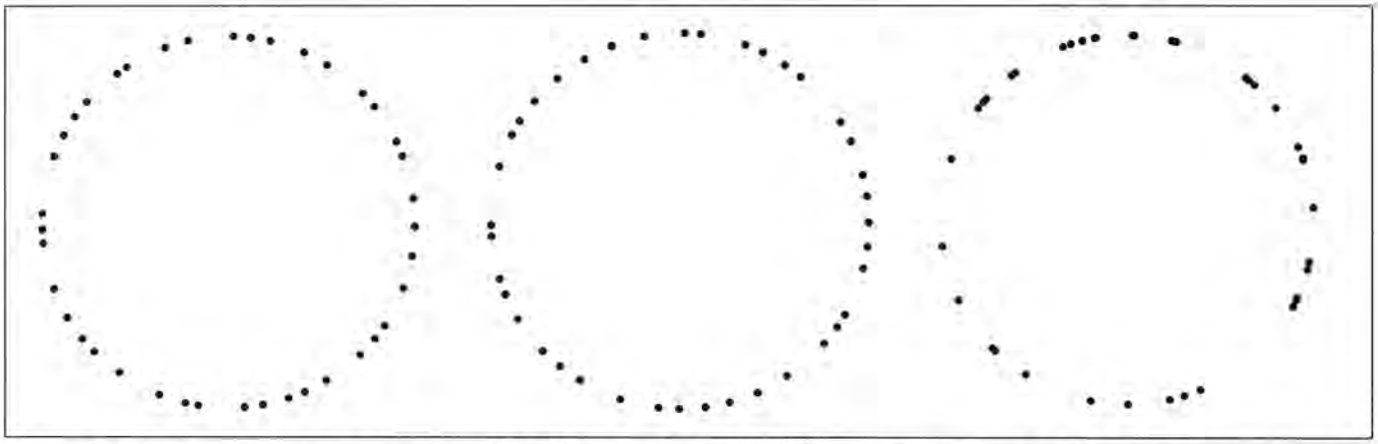

Figure 7. The eigenvalues of a random 40 x 40 unitary matrix, 40 consecutive zeros of $\zeta(s)$ scaled to wrap once around the circle, and 40 randomly chosen points on the unit circle.

$$\sum_{j=1}^{\infty} h(r_j) = -h(0) - g(0)\log\tfrac{\pi}{2} - \frac{1}{2\pi}\int_{-\infty}^{\infty} h(r)G(r)\,dr$$
$$+ 2\sum_{n=1}^{\infty}\frac{\Lambda(n)}{n}g(2\log n)$$
$$+ \sum_{P}\sum_{\ell=1}^{\infty}\frac{g(\ell\log P)\log P}{P^{\ell/2} - P^{-\ell/2}}$$

where g, h, and Λ are as in Weil's formula and

$$G(r) = \frac{\Gamma'}{\Gamma}(\tfrac{1}{2} + ir) + \frac{\Gamma'}{\Gamma}(1 + ir) - \frac{\pi}{6}r\tanh\pi r$$
$$+ \frac{\pi}{\cosh\pi r}(\tfrac{1}{8} + \tfrac{\sqrt{3}}{9}\cosh\tfrac{\pi r}{3}).$$

The final sum is over the norms P of prime geodesics of $SL(2,\mathbb{Z})\backslash\mathcal{H}$. The values taken on by P are of the form $(n + \sqrt{n^2-4})^2/4$, $n \geq 3$, with certain multiplicities (the class number $h(n^2-4)$). H. Haas was one of the first people to compute the eigenvalues $r_1 = 9.533\ldots$, $r_2 = 12.173\ldots$, $r_3 = 13.779\ldots$ of $SL(2,\mathbb{Z})$ in 1977 in his University of Heidelberg Diplomarbeit. Soon after, Hejhal was visiting San Diego, and Audrey Terras pointed out to him that Haas's list contained the numbers $14.134\ldots, 21.022\ldots$: the ordinates of the first few zeros of $\zeta(s)$ were lurking amongst the eigenvalues! Hejhal discovered the ordinates of the zeros of $L(s,\chi_3)$ (see section 7) on the list too. He unraveled this perplexing mystery about six months later. It turned out that the spurious eigenvalues were associated to "pseudo cusp forms" and appeared because of the method of computation used. If the zeros had appeared legitimately, RH would have followed because $\lambda = \rho(1-\rho)$ is positive. (The 1979 IHÉS preprint by P. Cartier and Hejhal contains additional details of the story.)

The trace formula resembles the explicit formula in certain ways. Many researchers have attempted to interpret Weil's explicit formula in terms of Selberg's trace formula.

Some Other Equivalences of Interest

Here are a few other easy-to-state equivalences of RH:

- Hardy and Littlewood (1918): *RH holds if and only if*
$$\sum_{k=1}^{\infty}\frac{(-x)^k}{k!\,\zeta(2k+1)} = O(x^{-1/4}) \quad \text{as } x \to \infty.$$
- Redheffer (1977): *RH holds if and only if for every $\epsilon > 0$ there is a $C(\epsilon) > 0$ such that $|\det(A(n))| < C(\epsilon)n^{1/2+\epsilon}$, where $A(n)$ is the $n \times n$ matrix of 0's and 1's defined by $A(i,j) = 1$ if $j = 1$ or if i divides j, and $A(i,j) = 0$ otherwise.* It is known that $A(n)$ has $n - [n\log 2] - 1$ eigenvalues equal to 1. Also, A has a real eigenvalue (the spectral radius) which is approximately $\sqrt{n}$, a negative eigenvalue which is approximately $-\sqrt{n}$, and the remaining eigenvalues are small.
- Lagarias (2002): Let $\sigma(n)$ denote the sum of the positive divisors of n. *RH holds if and only if*
$$\sigma(n) \leq H_n + \exp(H_n)\log H_n$$
for every n, where $H_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$.

Other Zeta- and L-Functions

Over the years striking analogies have been observed between the Riemann zeta-function and other zeta- or L-functions. While these functions are seemingly independent of each other, there is growing evidence that they are all somehow connected in a way that we do not fully understand. In any event, trying to understand, or at least classify, all of the objects which we believe satisfy RH is a reasonable thing to do. The rest of the article will give a glimpse in this direction and perhaps a clue to the future.

First, some examples of other functions that we believe satisfy RH. The simplest after ζ is the Dirichlet L-function for the nontrivial character of conductor 3:

$$L(s,\chi_3) = 1 - \frac{1}{2^s} + \frac{1}{4^s} - \frac{1}{5^s} + \frac{1}{7^s} - \frac{1}{8^s} + \ldots.$$

This can be written as an Euler product

$$\prod_{p\equiv 1 \bmod 3} (1-p^{-s})^{-1} \prod_{p\equiv 2 \bmod 3} (1+p^{-s})^{-1},$$

it satisfies the functional equation

$$\xi(s,\chi_3) := \left(\tfrac{\pi}{3}\right)^{-\frac{s}{2}} \Gamma(\tfrac{s+1}{2}) L(s,\chi_3) = \xi(1-s,\chi_3),$$

and it is expected to have all of its nontrivial zeros on the $1/2$-line. A similar construction works for any primitive Dirichlet character.

Dedekind, Hecke, Artin, and others developed the theory of zeta-functions associated with number fields and their characters. These have functional equations and Euler products, and are expected to satisfy a Riemann Hypothesis. Ramanujan's tau-function defined implicitly by

$$x\prod_{n=1}^{\infty}(1-x^n)^{24} = \sum_{n=1}^{\infty} \tau(n)x^n$$

also yields an L-function. The associated Fourier series $\Delta(z) := \sum_{n=1}^{\infty} \tau(n)\exp(2\pi i n z)$ satisfies

$$\Delta\left(\frac{az+b}{cz+d}\right) = (cz+d)^{12}\Delta(z)$$

for all integers a, b, c, d with $ad - bc = 1$. A function satisfying these equations is called a *modular form* of weight 12. The associated L-function

$$\begin{aligned} L_\Delta(s) &:= \sum_{n=1}^{\infty} \frac{\tau(n)/n^{11/2}}{n^s} \\ &= \prod_p \left(1 - \frac{\tau(p)/p^{11/2}}{p^s} + \frac{1}{p^{2s}}\right)^{-1} \end{aligned}$$

satisfies the functional equation

$$\xi_\Delta := (2\pi)^{-s}\Gamma(s+11/2)L_\Delta(s) = \xi_\Delta(1-s),$$

and all of its complex zeros are expected to be on the $1/2$-line.

Another example is the L-function associated to an elliptic curve $E : y^2 = x^3 + Ax + B$, where A and B are integers. The associated L-function, called the Hasse-Weil L-function, is

$$\begin{aligned} L_E(s) &= \sum_{n=1}^{\infty} \frac{a(n)/n^{1/2}}{n^s} \\ &= \prod_{p\nmid N} \left(1 - \frac{a(p)/p^{1/2}}{p^s} + \frac{1}{p^{2s}}\right)^{-1} \\ &\quad \times \prod_{p\mid N} \left(1 - \frac{a(p)/p^{1/2}}{p^s}\right)^{-1}, \end{aligned}$$

where N is the conductor of the curve. The coefficients a_n are constructed easily from a_p for prime p; in turn the a_p are given by $a_p = p - N_p$, where N_p is the number of solutions of E when considered modulo p. The work of Wiles and others proved that these L-functions are associated to modular forms of weight 2. This modularity implies the functional equation

$$\xi_E(s) := (2\pi/\sqrt{N})^{-s}\Gamma(s+1/2)L_E(s) = \xi_E(1-s).$$

It is believed that all of the complex zeros of $L_E(s)$ are on the $1/2$-line. A similar construction ought to work for other sets of polynomial equations, but so far this has not been proved.

What is the most general situation in which we expect the Riemann Hypothesis to hold? The Langlands program is an attempt to understand all L-functions and to relate them to automorphic forms. At the very least a Dirichlet series that is a candidate for RH must have an Euler product and a functional equation of the right shape. Selberg has given a set of four precise axioms which are believed to characterize the L-functions for which RH holds. Examples have been given that show the necessity of most of the conditions in his axioms.

L-Functions and Random Matrix Theory

An area of investigation which has stimulated much recent work is the connection between the Riemann zeta-function and Random Matrix Theory (RMT). This work does not seem to be leading in the direction of a proof of RH, but it is convincing evidence that the spectral interpretation of the zeros sought by Hilbert and Pólya is an idea with merit. Moreover, the connection between zeta theory and RMT has resulted in a very detailed model of $\zeta(s)$ and its value distribution.

Montgomery's Pair Correlation Conjecture

In 1972 Hugh Montgomery was investigating the spacings between zeros of the zeta-function in an attempt to solve the class number problem. He formulated his Pair Correlation Conjecture based in part on what he could prove assuming RH and in part on RH plus conjectures for the distribution of twin primes and other prime pairs. This conjecture asserts that

$$\sum_{\frac{2\pi\alpha}{\log T} < \gamma - \gamma' \le \frac{2\pi\beta}{\log T}} 1 \sim N(T)\int_\alpha^\beta \left(1 - \left(\frac{\sin \pi u}{\pi u}\right)^2\right) du.$$

The sum on the left counts the number of pairs $0 < \gamma, \gamma' < T$ of ordinates of zeros with normalized spacing between positive numbers $\alpha < \beta$. Montgomery had stopped in Princeton on his way from St. Louis, where he had presented this result at an AMS symposium, to Cambridge University, where he was a graduate student. Chowla persuaded him to show this result to Freeman Dyson at afternoon tea at the Institute for Advanced Study. Dyson immediately identified the integrand $1 - \left(\frac{\sin \pi u}{\pi u}\right)^2$ as the pair correlation function for eigenvalues of large random Hermitian matrices measured with a Gaussian measure—the Gaussian Unitary Ensemble that physicists had long been studying in connection with the distribution of energy levels in large systems of particles. With this insight, Montgomery went on to conjecture that perhaps all the statistics, not just the pair correlation statistic, would match up for zeta-zeros and eigenvalues of Hermitian matrices. This conjecture is called the GUE conjecture. It has the flavor of a spectral interpretation of the zeros, though it gives no indication of what the particular operator is.

Odlyzko's Calculations

In the 1980s Odlyzko began an intensive numerical study of the statistics of the zeros of $\zeta(s)$. Based on a new algorithm developed by Odlyzko and Schönhage that allowed them to compute a value of $\zeta(1/2+it)$ in an average time of t^ϵ steps, he computed millions of zeros at heights around 10^{20} and spectacularly confirmed the GUE conjecture.

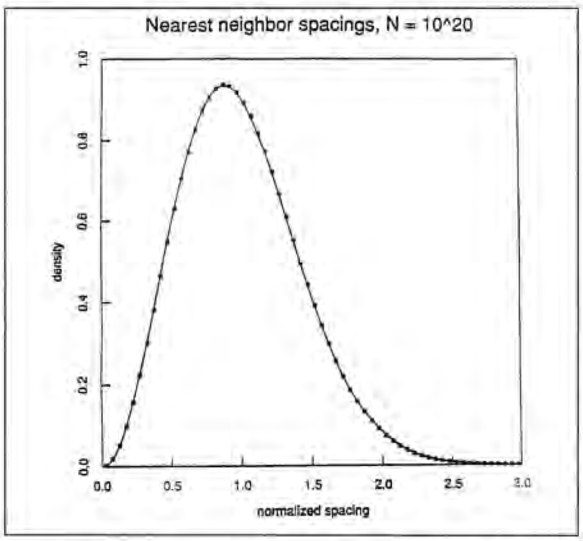

Figure 8a. The nearest neighbor spacing for GUE (solid) and for 7.8×10^7 zeros of $\zeta(s)$ near the 10^{20} zero (scatterplot). Graphic by A. Odlyzko.

Moments of Zeta

More recently, RMT has led to a conjecture for moments of ζ on the critical line. Let

$$I_k(T)=\frac{1}{T}\int_0^T|\zeta(1/2+it)|^{2k}\,dt.$$

Asymptotic formulas for I_1 and I_2 were found by Hardy and Littlewood and Ingham by 1926. In 1995 Ghosh and I formulated a conjecture for I_3 and set up a notation to clarify the part missing from our understanding of I_k. After scaling out the arithmetic parts, we identified a factor g_k which we could not predict. The factor is $g_1=1$ and $g_2=2$ for the second and fourth moments and conjecturally $g_3=42$ for the sixth moment. At RHI in Seattle, Sarnak proposed to Keating that he find a random matrix explanation for these numbers. By 1998 Gonek and I had found a number-theoretic way to conjecture the answer for the eighth moment, namely $g_4=24024$. At RHII in Vienna, Keating announced that he and Snaith had a conjecture for all of the moments which agreed with g_1, g_2, and g_3. Keating, Snaith, and I—moments before Keating's lecture—checked (amid great excitement!) that the Keating and Snaith conjecture also produced $g_4=24024$.

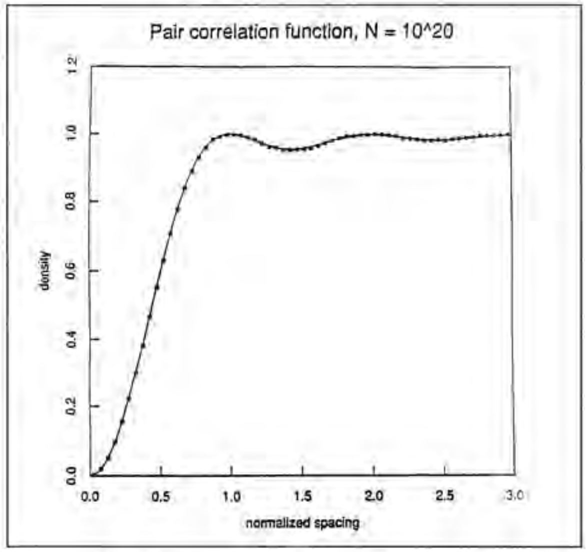

Figure 8b.The pair-correlation function for GUE (solid) and for 8×10^6 zeros of $\zeta(s)$ near the 10^{20} zero (scatterplot). Graphic by A. Odlyzko.

The idea of Keating and Snaith was that if the eigenvalues of unitary matrices model zeta zeros, then perhaps the characteristic polynomials of unitary matrices model zeta values. They were able to compute—exactly—the moments of the characteristic polynomials of unitary matrices averaged with respect to Haar measure by using Selberg's integral, which is a formula found in the 1940s by Selberg that vastly generalizes the integral for the beta-function. Keating and Snaith proposed that

$$g_k=k^2!\prod_{j=0}^{k-1}\frac{j!}{(j+k)!}.$$

Farmer and I (2000) proved that g_k is always an integer and found that it has an interesting prime factorization.

Families

At RHI in Seattle, Sarnak gave a lecture on families of L-functions based on work that he and Katz were doing. They discovered a way to identify a symmetry type (unitary, orthogonal, or symplectic) with various families of L-functions. Their work was based on studying families of zeta-functions over finite fields (for which RH was already proved by Weil for curves and by Deligne for general varieties). For these zeta-functions, Katz and Sarnak proved that the zeros of the family were distributed exactly

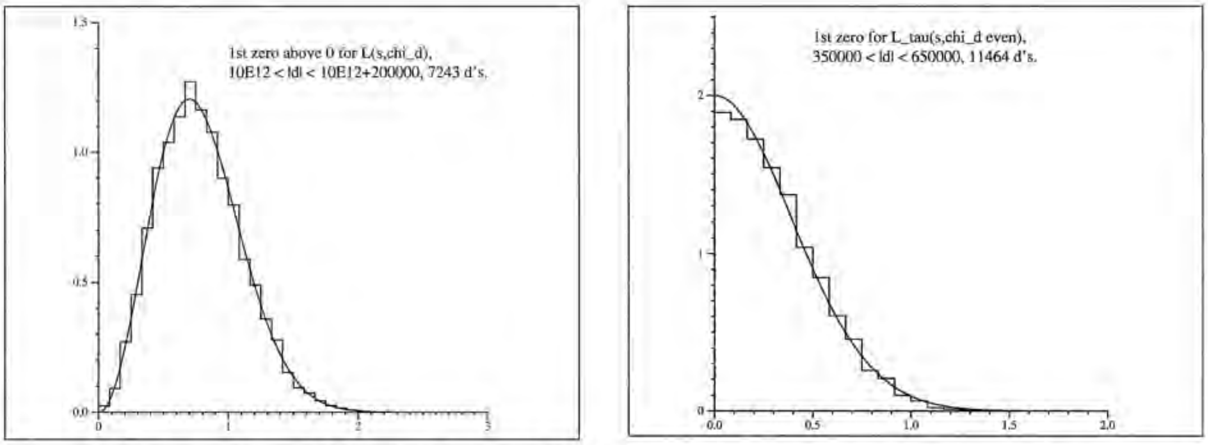

Figure 9. A comparison of the distribution of the lowest lying zero for two families of L-functions. In each case one first needs to suitably normalize the zeros. The first figure compares the distribution of the lowest zero of $L(s,\chi_d)$, Dirichlet L-functions, for several thousand d's of size 10^{12}, against the distribution of the zero closest to 1 for large unitary symplectic matrices. In the second picture we show the same statistic, but for several thousand even quadratic twists d of size 500,000, of the Ramanujan τ cusp form L-function. This is compared to the distribution of the zero closest to 1 for large orthogonal matrices with even characteristic polynomial (in the latter family, one needs to distinguish between even and odd twists). Graphics by M. Rubenstein.

as the RMT distributions of the monodromy group associated with the family.

Katz and Sarnak stress that the proofs of Weil and Deligne use families of zeta-functions over finite fields to prove RH for an individual zeta-function. The modelling of families of L-functions by ensembles of random matrix theory gives evidence for a spectral interpretation of the zeros, which may prove important if families are ultimately used to prove RH. At this point, however, we do not know what plays the role of the monodromy groups in this situation.

RMT and Families

Keating and Snaith extended their conjectures to moments of families of L-functions by computing moments of characteristic polynomials of symplectic and orthogonal matrices, each with their own Haar measure. (It should be mentioned that the orthogonal and symplectic circular ensembles used by the physicists do not use Haar measure and so have different answers. Katz and Sarnak figured out that Haar measure must be used to model L-functions.)

Further works by Farmer, Keating, Rubinstein, Snaith, and this author have led to precise conjectures for all of the main terms in moments for many families of L-functions. These results are so precise that they lead to further conjectures about the distribution of values of the L-functions. We can even predict how frequently we find double zeros at the center of the critical strip of L-functions within certain families.

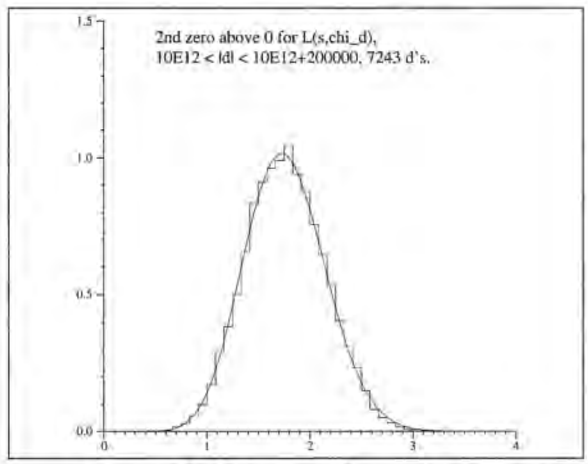

Figure 10. The second zero for $L(s,\chi_d)$ as compared to the RMT prediction. Graphic by M. Rubenstein.

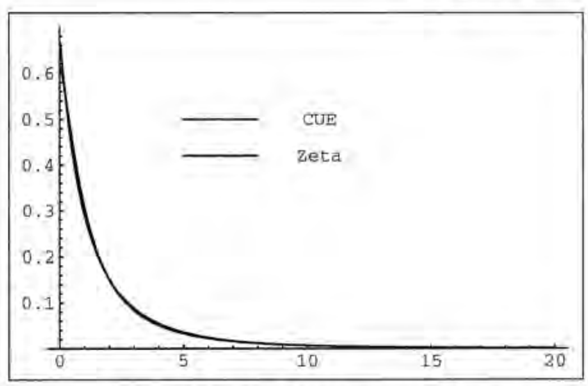

Figure 11. The distribution of values of $|\zeta(1/2+it)|$ near $t=10^6$ compared with the distribution of values of characteristic polynomials of 12×12 unitary matrices. Graphic by N. Snaith.

The Conspiracy of L-Functions

There is a growing body of evidence that there is a conspiracy among L-functions—a conspiracy which is preventing us from solving RH!

The first clue that zeta- and L-functions even know about each other appears perhaps in works of Deuring and Heilbronn in their study of one of the most intriguing problems in all of mathematics: Gauss's class number problem. Gauss asked whether the number of equivalence classes of binary quadratic forms of discriminant $d < 0$ goes to ∞ as d goes to $-\infty$.

The equivalence class of a quadratic form $Q(m,n) = am^2 + bmn + cn^2$ of discriminant $d = b^2 - 4ac$ consists of all of the quadratic forms obtained by a linear substitution $m \to \alpha m + \beta n$, $n \to \gamma m + \delta n$, where $\alpha, \beta, \gamma, \delta$ are integers with $\alpha\delta - \beta\gamma = 1$. The number $h(d)$ of these equivalence classes is called the class number and is known to be finite. Equivalently, $h(d)$ is the number of ideal classes of the imaginary quadratic field $Q(\sqrt{d})$. The history of Gauss's problem is extremely interesting; it has many twists and turns and is not yet finished—we seem to be players in the middle of a mystery novel.

Deuring and Heilbronn were trying to solve Gauss's problem. The main tool they were using was the beautiful class number formula of Dirichlet, $h(d) = \sqrt{|d|}\, L(1, \chi_d)/\pi$ ($|d| > 4$), which gives the class number in terms of the value of the L-function at 1, which is at the edge of the critical strip. So the question boils down to giving a lower bound for $L(1, \chi_d)$; this question, in turn, can be resolved by proving that there is no real zero of $L(s, \chi_d)$ very near to 1.

Hecke had shown that the truth of RH for $L(s, \chi_d)$ implies that $h(d) \to \infty$. Then Deuring proved that the falsity of RH for $\zeta(s)$ implies that $h(d) > 1$ for large $|d|$. Finally, Heilbronn showed that the falsity of RH for $L(s, \chi)$ for any χ implied that $h(d) \to \infty$. These results together proved Gauss's conjecture and gave a first indication of a connection between the zeros of $\zeta(s)$ and those of $L(s, \chi_d)$!

Later Landau showed that a hypothetical zero of $L(s, \chi_{d_1})$ very near to 1 implies that no other $L(s, \chi_d)$, $d \neq d_1$, could have such a zero, further illustrating that zeros of $L(s, \chi_d)$ know about each other. Siegel strengthened this approach to show that for every $\epsilon > 0$ there is a $c(\epsilon) > 0$ such that no zero β of $L(s, \chi_d)$ satisfies $\beta > 1 - c(\epsilon)|d|^{-\epsilon}$. The problem with the arguments of Landau and Siegel is that the constant $c(\epsilon)$ cannot be effectively computed, and so the bound cannot be used to actually calculate the list of discriminants d with a given class number, which presumably is what Gauss wanted. The ineffectivity comes about from the assumption that some L-function actually has a real zero near 1. Such a hypothetical zero of some L-function, which no one believes exists, is called a Landau-Siegel zero.

In fact, one can show that if there is some d_1 such that $L(s, \chi_{d_1})$ has a zero at $\beta < 1$, then it follows that $h(d) > c|d|^{\beta - 1/2} / \log |d|$ for all other d, where $c > 0$ can be effectively computed. Thus, the closer to 1 the hypothetical zero is, the stronger the result. But note also that any zero bigger than 1/2 would give a result. The basic idea behind this approach is that if there is an $L(s, \chi_d)$ with a zero near 1, then $\chi_d(p) = -1$ for many small primes. In other words, χ_d mimics the Möbius function $\mu(n)$ for small n. This is consistent with the fact that

$$\sum_{n=1}^{\infty} \frac{\mu(n)}{n^s}$$

has a zero at $s = 1$ (since $\zeta(s)$ has a pole at $s = 1$).

The Landau-Siegel Zero

Much effort has gone toward trying to eliminate the Landau-Siegel zero described above and so find an effective solution to Gauss's problem. However, the L-function conspiracy blocks every attempt exactly at the point where success appears to be in sight. We begin to suspect that the battle for RH will not be won without getting to the bottom of this conspiracy. Here are some tangible examples which give a glimpse of this tangled web.

The Brun-Titchmarsh theorem. Let $\pi(x; q, a)$ denote the number of primes less than or equal to x that lie in the arithmetic progression $a \bmod q$. Sieve methods can show that for any $1 \le q < x$ the inequality

$$\pi(x; q, a) \le 2 \frac{x}{\phi(q) \log(x/q)}$$

holds, where ϕ is Euler's phi-function. It is believed that the same theorem should be true with 2 replaced by any number larger than 1 and sufficiently large x. Any lowering of the constant 2 would eliminate the Landau-Siegel zero. In particular, Motohashi [1979] proved that if $1 - \delta$ is a real zero of $L(s, \chi_q)$, then if for $x \ge q^c$ the Brun-Titchmarsh theorem is valid in the form $\pi(x; q, a) \le (2 - \alpha)x/(\phi(q)\log(x/q))$, where $\alpha > 0$ is an absolute constant, then $\delta \ge c'\xi/\log q$, where c and c' are certain numerical constants.

The Alternative Hypothesis. This is an alternative to the GUE model for the distribution of zeros. It proposes the existence of a function $f(T)$ that goes to 0 as $T \to \infty$ such that if any two consecutive ordinates γ and γ' of zeros of ζ larger than some T_0 are given, then the normalized gap $2\pi(\gamma \log \gamma - \gamma' \log \gamma')$ between γ and γ' is within $f(T_0)$ of half of an integer. This hypothesis is clearly absurd! However, ruling this out would eliminate the Landau-Siegel zero (Conrey-Iwaniec (2002)), and so for all we know it could be true.

If one could prove, for example, that there is a $\delta > 0$ such that for all sufficiently large T there is a pair of consecutive zeros with ordinates between T and $2T$ whose distance apart is less than $1/2 - \delta$ times the average spacing, then the alternative hypothesis would be violated. Random matrix theory predicts the exact distribution of these neighbor spacings and shows that we should expect that about 11 percent of the time the neighbor gaps are smaller than $1/2$ of the average. These ideas were what led Montgomery to consider the pair-correlation of the zeros of $\zeta(s)$ mentioned above. He showed that there are arbitrarily large pairs of zeros that are as close together as 0.68 of the average spacing. Later works have gotten this bound down to 0.5152. There are indications that using work of Rudnick and Sarnak on higher correlations of the zeros of ζ, one might be able to reach 0.5, but 0.5 is definitely a limit (more like a brick wall!) of all of the known methods.

Vanishing of modular L-functions. The most spectacular example is the work of Iwaniec and Sarnak. They showed that if one could prove that there is a $\delta > 0$ such that more than $1/2 + \delta$ of the modular L-functions of a fixed weight, large level, and even functional equation do not vanish, then the Landau-Siegel zero could be eliminated. It is predicted that all but an infinitesimal proportion of these values are nonzero; they just needed one-half plus δ of them to be nonzero. They can prove that 50 percent do not vanish, but despite their best efforts they cannot get that extra little tiny bit needed to eliminate the Landau-Siegel zero.

A Clue and a Partial Victory

The only approach that has made an impact on the Landau-Siegel zero problem is an idea of Goldfeld. In 1974 Goldfeld, anticipated somewhat by Friedlander, realized that while a zero at $1/2$ would barely fail to produce a lower bound for the class number tending to infinity, a multiple zero at $1/2$ would produce a lower bound which, while not a positive power of $|d|$, still goes to ∞. Moreover, it was believed—by virtue of the Birch and Swinnerton-Dyer conjecture—that zeros of high multiplicity do exist and the place to look for them is among L-functions associated to elliptic curves with large rank. However, it was not until 1985 that Gross and Zagier demonstrated conclusively that there exist L-functions with triple zeros at $1/2$. This led to the lower bound that for any $\epsilon > 0$ there is an effectively computable $c_1(\epsilon) > 0$ such that $h(d) > c_1(\epsilon)(\log|d|)^{1-\epsilon}$. This is a long way from the expected $h(d) > c\sqrt{|d|}/\log|d|$, but it did solve Gauss's problem. The clue that it gave us was to study exotic L-functions, or extremal L-functions, which have zeros of high multiplicity at the center. At present, our best hope for finding these L-functions is to look at elliptic curves with many rational points.

Iwaniec's Approach

Iwaniec, in his lecture at RHIII, proposed a way to take advantage of the above ideas. In a nutshell, his idea is to take a family of L-functions having a multiple zero at $1/2$ and use this family to obtain useful approximations for the Möbius function $\mu(n)$ as a linear combination of the coefficients of the L-functions from the family. In this way, the Möbius function is tamed. One example of a family considered by Iwaniec is the family of L-functions associated to the elliptic curves

$$E_{A,B^2} : y^2 = x^3 + Ax + B^2,$$

which have a rational point $(B, 0)$ and so have rank at least one. Considering A and B in certain arithmetic progressions shows that the associated L-function must have a double zero at the center.

Iwaniec presented three conjectures which together would eliminate the Landau-Siegel zero. The main two theorems needed to complete his program are a bound for the second moment

$$\sum_{A \approx X^{1/3},\, B \approx X^{1/4}} L_{A,B^2}(1/2)^2 = O\left(X^{7/12}(\log X)^C\right)$$

of this family together with a good estimate (square-root cancellation uniform in M, N, and q) for the incomplete exponential sum

$$\sum_{M<m<2M, N<n<2N} \chi_q(mn)\exp\left(2\pi i \frac{m^3 n^{-4}}{q}\right),$$

the kind of estimate that for a completed exponential sum follows from the RH for varieties proved by Deligne. Iwaniec has similar, but more complicated, constructions that would lead to a quasi-Riemann hypothesis, producing a concrete $\beta < 1$ such that there are no zeros to the right of the line through β.

Iwaniec's approach will likely reduce the question of RH, which is ostensibly about zeros or poles, into several subsidiary questions that have a much different flavor, such as finding upper bound estimates for moments and values of L-functions. This approach offers hope of attack by methods from analytic number theory.

Conclusion

A major difficulty in trying to construct a proof of RH through analysis is that the zeros of L-functions behave so much differently from zeros of many of the special functions we are used to seeing in mathematics and mathematical physics. For example, it is known that the zeta-function does not satisfy any differential equation. The functions which do arise as solutions of some of the classical differential equations, such as Bessel functions, hypergeometric functions, etc., have zeros which are fairly regularly spaced. A similar remark holds for the zeros of solutions of classical differential equations regarded as a function of a parameter in the differential equation. For instance, in the Pólya theorem above comparing $\phi(t)$ with $\Phi(t)$, the zeros are actually zeros of a Bessel function of fixed argument regarded as a function of the index. Again the zeros are regularly spaced.

On the other hand, the zeros of L-functions are much more irregularly spaced. For example, the RMT models predict that for any $\epsilon > 0$ there are infinitely many pairs of zeros ρ and ρ' such that $|\rho - \rho'| < |\rho|^{-1/3+\epsilon}$. Generally it is believed that all zeros of all L-functions are linearly independent (in particular, simple), except that certain L-functions can have a zero at $s = 1/2$ of high multiplicity. The conjecture of Birch and Swinnerton-Dyer asserts that the multiplicity of the zero of the L-function associated with a given elliptic curve is equal to the rank of the group of rational points on the elliptic curve. It is known that the latter can be as large as 26, and it is generally believed to get arbitrarily large. None of the methods from analysis seem capable of dealing with such exotic phenomena.

It is my belief that RH is a genuinely arithmetic question that likely will not succumb to methods of analysis. There is a growing body of evidence indicating that one needs to consider families of L-functions in order to make progress on this difficult question. If so, then number theorists are on the right track to an eventual proof of RH, but we are still lacking many of the tools. The ingredients for a proof of RH may well be moment theorems for a new family of L-functions not yet explored; modularity of Hasse-Weil L-functions for many varieties, like that proved by Wiles and others for elliptic curves; and new estimates for exponential sums, which could come out of arithmetic geometry. The study of L-functions is still in its beginning stages. We only recently learned the modularity of the L-functions associated to elliptic curves; it would be very helpful to understand the L-functions for more complicated curves and generally for varieties. It would be useful to systematically compute many new examples of L-functions to get a glimpse of what is out there waiting to be discovered. The exotic behavior of the multiple zeros of L-functions associated to elliptic curves with many rational points could be just the beginning of the story.

Acknowledgements

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—J.B.C.

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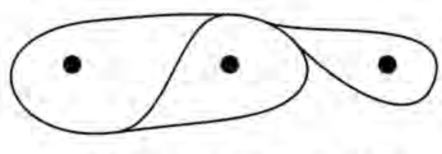
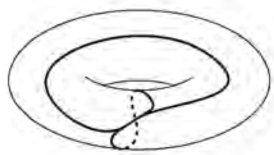
WHAT IS...

A Train Track?

Lee Mosher



(a) On the 4-punctured sphere



(b) On the torus

Figure 1. Examples of train tracks.

On a surface S, train tracks approximate simple closed curves just as partial quotients of continued fraction expansions approximate rational numbers. The simple closed curve on the 4-punctured sphere (see photograph on cover and p. 356) that, in about 1972, was painted on the wall of the UC Berkeley math department by William P. Thurston and Dennis Sullivan is approximated by the train track shown in Figure 1(a). To visualize the approximation, blur your eyes so that parallel strands of the curve merge into branches of the train track and so that diverging strands split apart at switches of the train track. Train tracks were introduced by Thurston in the late 1970s as a means of studying simple closed curves and related structures on surfaces.

In general, the surface S should be of finite type—a compact, connected, oriented surface, possibly with a finite number of punctures. The simple closed curves on S that we study are those which are *essential*, meaning that any disc they bound has at least two punctures. Two essential simple closed curves are considered to be the same if they are isotopic on S, that is, homotopic through simple closed curves.

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A train track on S is a smooth 1-complex τ, whose vertices are called *switches* and whose edges are called *branches*, such that at each switch s there is a unique tangent line and s has an open neighborhood in τ which is a union of smoothly embedded open arcs. The metric completion C of each component of $S - \tau$ is a surface with cusps whose "cusped Euler index" must be negative, that is, $\chi(C) - \frac{1}{2}\#(\text{cusps}) < 0$. The latter condition rules out several possibilities for C: a disc with no cusps and ≤ 1 puncture, a disc with no puncture and one or two cusps, and an annulus with no cusps and no puncture. Sometimes these conditions are slightly relaxed to allow C to be a *bigon*, a disc with no puncture and two cusps, which has cusped Euler index equal to zero. Indeed, on a torus, one must allow bigons or else train tracks do not exist.

A simple closed curve γ is *carried* by a train track τ if γ can be isotoped into an arbitrarily small neighborhood of τ so that each tangent line of γ is arbitrarily close to a tangent line of τ. The requirement that each completed component of $S - \tau$ has negative cusped Euler index (or is a bigon) implies that each simple closed curve carried by τ is essential on S. The statement that a simple closed curve is carried by a train track is an analogue of the statement that a rational number is approximated by a continued fraction partial quotient. On the torus $T^2 = \mathbf{R}^2/\mathbf{Z}^2$ this analogy becomes very precise, as we now describe.

Up to isotopy, simple closed curves on T^2 are in one-to-one correspondence with the extended rationals $\mathbf{Q} \cup \{\infty\}$—a rational number r corresponds to a simple closed curve γ_r which lifts to a line in $\mathbf{R}^2$ of slope r. The basic train track $\tau_{[0,\infty]}$ on T^2, shown in Figure 1(b), is obtained from $\gamma_0 \cup \gamma_\infty$ by flattening the angles at the transverse intersection point $\gamma_0 \cap \gamma_\infty$ until this point has a unique tangent line of positive slope. The train track $\tau_{[0,\infty]}$ has one bigon, and $\tau_{[0,\infty]}$ carries precisely those simple closed curves γ_r with $0 \le r \le \infty$. More generally, consider integers $a, b, c, d > 0$ such that $ad - bc = 1$. The rational numbers $p = \frac{c}{d} < \frac{a}{b} = q$

determine simple closed curves γ_p, γ_q on T^2 which intersect transversely in a single point, and one can flatten the angles at this point to form a train track $\tau_{[p,q]}$ which carries precisely those curves γ_r with $p \le r \le q$.

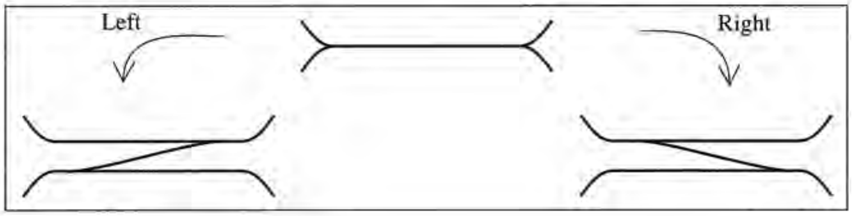

Figure 2. Splittings.

To make sense of any notion of approximation, we must say how the approximation can be refined. With train tracks this is accomplished by the notion of *splitting*. Given a train track τ, first one looks in τ for a splitting locus consisting of two strands of τ that meet tangentially at a point or along a short arc, and then one can define a left splitting $\tau \succ \tau_L$ and a right splitting $\tau \succ \tau_R$, using the model for splitting depicted in Figure 2. Every simple closed curve γ that is carried by τ is also carried by one of τ_L or τ_R, possibly both, and so one can regard τ_L or τ_R as refining the approximation of γ.

On the torus, starting from the train track $\tau_{[0,\infty]} = \tau_{[\frac{0}{1},\frac{1}{0}]}$, a left splitting results in $\tau_{[\frac{1}{1},\frac{1}{0}]} = \tau_{[1,\infty]}$, and a right splitting results in $\tau_{[\frac{0}{1},\frac{1}{1}]} = \tau_{[0,1]}$. More generally, given $p = \frac{c}{d} < \frac{a}{b} = q$ as above, if one takes the Farey sum $r = \frac{c+a}{d+b}$, then $p < r < q$, and there is a right splitting $\tau_{[p,q]} \overset{R}{\succ} \tau_{[p,r]}$ and a left splitting $\tau_{[p,q]} \overset{L}{\succ} \tau_{[r,q]}$. Given a simple closed curve γ_r with $r \in [0,\infty]$, there is a finite sequence of splittings $\tau_{[0,\infty]} = \tau_0 \succ \tau_1 \succ \cdots \succ \tau_{n-1} \succ \tau_n$, where the parity (L or R) of each splitting is chosen inductively so that each τ_i carries the curve γ_r. The sequence halts when it first reaches a train track τ_n that contains an embedded copy of γ_r. This train track sequence is called the *train track expansion* of the closed curve γ_r. For example, the train track expansion of $\gamma_{10/7}$ is given by

$$\begin{aligned}\tau_{[0,\infty]} = \tau_{[\frac{0}{1},\frac{1}{0}]} &\overset{L}{\succ} \tau_{[\frac{1}{1},\frac{1}{0}]} \overset{R}{\succ} \tau_{[\frac{1}{1},\frac{2}{1}]} \overset{R}{\succ} \tau_{[\frac{1}{1},\frac{3}{2}]} \\ &\overset{L}{\succ} \tau_{[\frac{4}{3},\frac{3}{2}]} \overset{L}{\succ} \tau_{[\frac{7}{5},\frac{3}{2}]} \overset{L}{\succ} \tau_{[\frac{10}{7},\frac{3}{2}]}\end{aligned}$$

From the LR sequence of this train track expansion—1 L, 2 Rs, 3 Ls—one can derive the continued fraction expansion $\frac{10}{7} = 1 + \frac{1}{2+\frac{1}{3}}$. Also, the partial quotients $1 = \frac{1}{1}$ and $1 + \frac{1}{2} = \frac{3}{2}$ show up in the train track expansion. More generally, given any rational number $r \in [0,\infty]$, from the train track expansion of γ_r one can derive the partial quotients and the continued fraction expansion of r: from the RL sequence consisting of n_0 Ls, n_1 Rs, n_2 Ls, ..., ending with n_K Ls or Rs depending on whether K is even or odd, one obtains the expansion

$$r = n_0 + \cfrac{1}{n_1 + \cfrac{1}{n_2 + \cfrac{1}{\cdots + \frac{1}{n_K}}}}\,.$$

The dictionary between train track expansions and continued fraction expansions can be extended much further. Thurston discovered that, just as the extended rational numbers $\mathbf{Q} \cup \{\infty\}$ can be completed to the extended real numbers $\mathbf{R} \cup \{\infty\}$ by a compactification which is natural with respect to the fractional linear action of the modular group $SL(2,\mathbf{Z})$, so can the set of isotopy classes of essential simple closed curves on a finite-type surface S be completed to the space $\mathcal{PML}$ of projective measured laminations on S by a compactification which is natural with respect to the action of the mapping class group $\mathcal{MCG}(S)$ [CB88]. Some results about train track expansions of measured laminations on S are described in [Pen92], and a detailed description of the theory is given in [Mos]. For example, just as irrational numbers correspond bijectively with infinite continued fractions, so do projective measured laminations whose leaves are not all closed curves correspond bijectively to infinite train track expansions that satisfy some mild combinatorial condition. One can also use train track expansions to detect finer properties of a measured lamination such as "arationality", which means that the lamination fills the whole surface.

One application of the dictionary is to the study of Thurston's classification of mapping classes on S [CB88]. The set of points in $\mathbf{R} \cup \infty$ fixed by Anosov elements of $SL(2,\mathbf{Z})$are precisely the quadratic irrationalities, which are precisely the numbers with eventually periodic continued fraction expansion; moreover, the periodicity loop can be used to classify Anosov elements of $SL(2,\mathbf{Z})$ up to conjugacy. The set of points in $\mathcal{PML}$ fixed by pseudo-Anosov elements of $\mathcal{MCG}$ can be characterized in terms of periodic behavior of their train track expansions, and the periodicity data can be used to classify pseudo-Anosov mapping classes up to conjugacy [Mos].

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Comments and suggestions may be sent to `notices-whatis@ams.org`.

About the Cover

This month's cover, mentioned in Lee Mosher's article, portrays part of a mural on a wall of Evans Hall at the University of California in Berkeley, painted in the fall of 1971 by Dennis Sullivan and William (Bill) Thurston (signatures at upper right in cover image). The whole mural is shown below. It portrays what I presume to be a more or less randomly chosen path on a two-sphere punctured by three points, and the ideas behind it played a role in the evolution of the concept of train tracks.

Last fall Sullivan wrote to Mosher: "In 1971 I was a guest of the University of California giving lectures in the Math Dept. At the same time there was a confrontation between the trustees and the graduate students et al. The latter planned to continue decorating the walls of the department by painting attractive murals and the trustees forbade it. At tea some students came up and invited me to join their painting the next day. I became enthusiastic when one bearded fellow [W. T.] showed me an incredible drawing of an embedded curve in the triply punctured disk and asked if I thought this would be interesting to paint. I said, 'You bet,' and the next day we spent all afternoon doing it. As we transferred the figure to the wall it was natural and automatic to do it in terms of bunches of strands at a time—as an approximate foliation—and then connect them up at the end as long as the numbers worked out. Thus some years later in '76 when Bill gave an impromptu 3-hour lecture about his theory of surface transformations I absorbed it painlessly at a heuristic level after the experience of several hours of painting in '71."

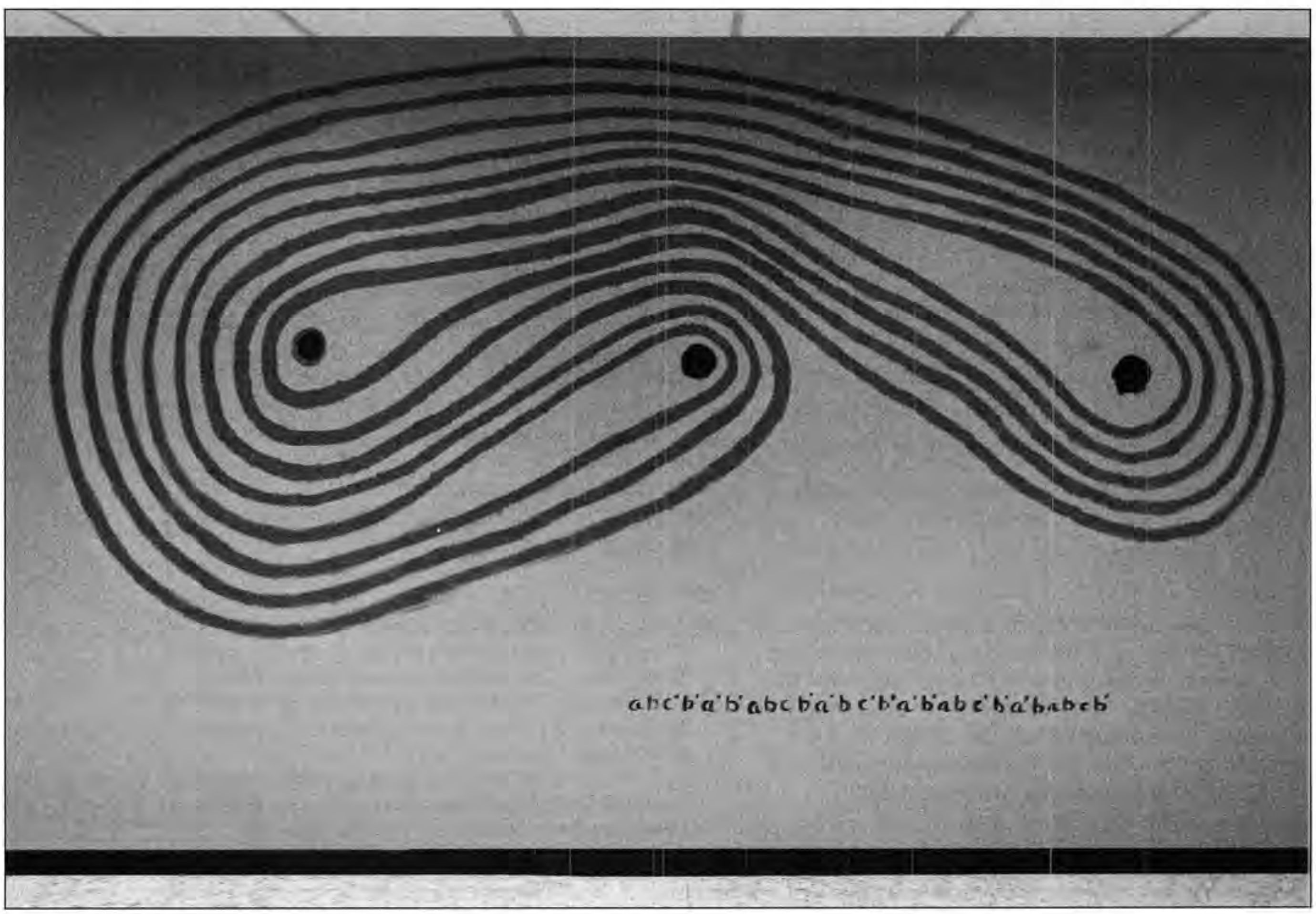
Thurston wrote in a note to Mosher that the project "was in response to a little flurry with administration sanctions of some sort when John Rhodes painted the wall outside his office, I think with a political slogan related to one of the issues of the times (Vietnam war, invasion of Cambodia, People's Park?)."

Later Thurston wrote to add: "The letters refer to a word in the free group on three generators, which is the fundamental group of the plane minus the 3 points. If you imagine 3 'branch cuts' going vertically from the three spots, and label them a, b, and c, then as you trace out the word starting from the left inside (I believe) it will trace out the given word, where a' designates a^{-1}, etc.

"I was excited as a graduate student to rediscover that you could describe simple closed curves such as this by a small number of integer parameters, which I later learned had been earlier investigated by Dehn and Nielsen (i.e., in this case, the three vertical branch cuts intersect 8, 13, and 5 segments of the curve). The fact that these are Fibonacci numbers is related to one method for generating this curve. Start with 3 points in the plane, with a circle enclosing say the right two. Now 'braid' the points, like a standard woman's hair braid, middle over left, then middle over right, etc. If you drag the curve along, these numbers will always be Fibonacci numbers, and you'll get the given curve after a few passes. Generalizing this theory eventually led me to my theory of pseudo-Anosov diffeomorphisms." References for this work of Thurston's are "On the geometry and dynamics of diffeomorphisms of surfaces", *Bull. Amer. Math. Soc.* **19** (1988), 417–31 and "Travaux de Thurston sur les surfaces", *Astérisque* **66–67** (1979).

Both photographs were taken by Kenneth Ribet, to whom we are extremely grateful for the time and effort he spent to obtain them.

—*Bill Casselman* (`notices-covers@ams.org`)



The Zen of Magic Squares, Circles, and Stars: An Exhibition of Surprising Structures across Dimensions

Reviewed by Andrew Bremner

The Zen of Magic Squares, Circles, and Stars
Clifford Pickover
Princeton University Press, 2001
Cloth, $29.95, 400 pp., ISBN 0-691-07041-5

Benjamin Franklin, the youngest son and fifteenth in a family of seventeen children, is renowned for his statesmanship and for his work on electricity. But he has a lesser-known claim to fame, the ability "to fill the cells of any magic square, of reasonable size, with a series of numbers as fast as I can write them, disposed in such a manner, as that the sums of every row, horizontal or perpendicular, or diagonal, should be equal." Such was his facility that one of the outstanding known examples of magic square was constructed by him in a single evening, and he was proud of its intrinsic beauty: "You will readily allow this square of 16 to be *the most magically magical* of any magic square *ever* made by any magician."

Magic squares are ancient and common to several civilizations. They were sometimes used as amulets and viewed as possessing talismanic powers. The first references appear to be Chinese, with the familiar 3×3 square

$$\begin{pmatrix} 4 & 9 & 2 \\ 3 & 5 & 7 \\ 8 & 1 & 6 \end{pmatrix}$$

known as the lo shu. Magic squares can be traced in early Arabic and Indian cultures, appearing in the former towards the end of the first millennium. They began to appear in European writings apparently around the fifteenth century and by the eighteenth century had attracted the study of several well-known mathematicians, including Faulhaber (the "Arithmetician of Ulm" and original discoverer of the Bernoulli numbers) and Euler. There followed a surge of writings. David Singmaster has one of the world's best collections of recreational mathematics and puzzle books (the others are the collections of Will Shortz, editor of the *New York Times* crossword puzzle, and the Strens-Guy Collection at the University of Calgary). Singmaster's source bibliography [5] contains eighteen pages of exhaustive references to magic squares and figures, and the explosion of texts in later centuries is reflected by the statement that "17th–20th century material has generally been omitted." In the current volume Pickover starts out with his own brief history of magic squares, which is perhaps not quite so scholarly in its reference and which aims for popular appeal.

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So we have a delightful description of the eighth-century Islamic alchemist "Geber", who in his attempts to transmute base metal into gold evidently used elixirs based on numbering various substances (including gazelle urine) according to the numbers in his magic square; however, there is no attribution. One passage on page 11, extracted verbatim from what must be a rather dubious source, is startling: "We find it [the lo shu] venerated by civilizations of almost every period and continent. The Mayan Indians of southern Mexico and Central America were fascinated by it, and today it is used by the Hausa people of northwestern Nigeria and southern Niger as a calculating device with mystical associations. The square was respected by the ancient Babylonians and was used as a cosmic symbol in prehistoric cave drawings in northern France." This latter assertion in particular seems quite preposterous. There *are* no Paleolithic cave drawings in the north of France. More fundamentally, the specific existence of a symbol system at this early date is questionable and is the subject of a large and complex area in Paleolithic art research.

It is not clear when the magic square first appeared in English writings. Consulting the *Oxford English Dictionary*, we find the first recorded mention of "magic square" to be an entry in a technical lexicon of 1704. Around this time the adjective "magic" was shifting in meaning towards that of "producing wonderful appearances or results, like those commonly attributed to sorcery" as opposed to the older sense of directly describing the art of the occult, with its intervening spirits or controlling principles of nature. According to Singmaster, one of the first European manuscripts to deal with magic squares is a fifteenth-century Latin manuscript in Cracow (which incidentally gives the famous 4×4 square that occurs in Dürer's print *Melencolia*). The sixteenth century provides a couple of additional references in Latin but also some in German, and Frénicle de Bessy made major contributions in "Des quarrez ou Tables Magiques" of the late seventeenth century. Presumably the English phrase entered the language as a direct translation from either French or German, with "magisches Quadrat" harking back to when magic squares were genuinely associated with the arts of divination and alchemy. Goethe in *Faust* [3] constructs a mysterious magic square that he calls the Hexen-Einmaleins, a direct link to a sorcerous source ("Du mußt versteh'n! / Aus eins mach zehn, / Und zwei laß geh'n, / Und drei mach gleich, / …").[1]

Pickover has written a splendid recreational book, though it contains very little actual mathematics. The author does say in his introduction that "This book is not for mathematicians looking for formal mathematical explanations," and indeed the number of proofs given is minimal. He does, for example, include demonstrations of the propositions that there do not exist perfect magic cubes of orders 3 and 4, but in general the thrust of the book is to provide numerical examples of ever-increasing complexity in a context of entertaining prose for the lay reader. An *N-th order* magic square is an $N \times N$ square containing the integers $1, \ldots, N^2$, with the property that all row, column, and principal diagonal sums are equal. Up to symmetry there are 1, 0, 1, 880, 275305224 magic squares of orders 1 to 5 respectively. It has been estimated (see [4]) that the number of magic squares of order 6 is 1.77×10^{19}. You can find all 880 squares of order 4 in Frénicle de Bessy (posthumous work of 1693) or, more accessibly, categorized in Berlekamp, Conway, and Guy [1]. Pickover's interest, however, lies outside the simple N-th order squares, and we are treated to an amazing menagerie of oftentimes fantastic formations.

It is possible in a brief review to convey only a fraction of the sheer exuberance and variety of squares on display. You will find squares of primes, of consecutive primes, and of consecutive composite integers. There are simultaneous addition and multiplication squares, where multiplication along rows, columns, and diagonals also produces a constant product. There are squares that remain magic when each element is replaced by its square (*bimagic*), and similarly squares that remain magic when elements are replaced by the square or cube (*trimagic*). (In fact, there are recent analogous results (Boyer & Viricel [2]) showing the existence of *quadrimagic* and *quinquemagic* squares, the latter example being of order 1024.) A 10-digit integer is *pandigital* if it contains each of the digits $0, \ldots, 9$; an example is given of a 3×3 square with pandigital entries and pandigital sum. There is a 4×4 square (whose entries are formed from 1's and 8's) with the property that it remains magic when viewed upside down or in a mirror. And these are only the two-dimensional specimens. The range of examples for cubes, tesseracts, circles, spheres, stars, and bizarre geometrical configurations in general (including a magic spider) is extreme. There is even a magic hypercube in five dimensions.

The book in consequence has an appealing visual nature: open it at random, and the eye will be dazzled. The author indeed finds a fruitful source of geometrical patterns in Buddhist mandalas and other meditative images. Using the term *satori* from Zen Buddhism, which the dictionary defines as "a sudden indescribable and uncommunicable inner experience of enlightenment," the author writes that "*Arithmetic satori* is the psychological result and aim of the practice of magic square meditation" and that this practice "induces an awareness, an

[1] *"Now you must ken! From one make ten, And two let free, Make even three …."*

experience of joy emanating from a mind that has transcended its earthly existence." He states in "Some Final Thoughts", page 373, that "while studying and doing research on magic squares, sitting in my home office or in a library or while gazing at the computer screen, I often get a flicker of happiness, or dare I say, 'transcendence' or 'wonder', that seems to bring the magic square to life...for a few seconds, I felt touched and mystically elevated." Perhaps Pickover is just experiencing the primal pleasure in the process of pattern recognition or feeling a thrill similar to that of the professional mathematician as the components of an abstract proof fall into place. Clearly, arithmetic satori will not come to all readers, and the reviewer, for example, on fixedly studying the "super overlapping fifteenth-order magic square" has managed only a state of hypnagogic lassitude. Enlightenment is something that will take a great deal of time and effort! But this is facetiousness on my part, and the author's research and meditation does provide a great treasury of entertaining material.

Some of the figures truly pop the eyes, though many have a certain *arbitrariness* that is vaguely unsettling. The intrinsic underlying mathematical structure of the square, cube, and tesseract seems lacking from the "Circles of Prometheus", the "Cirri of Euripides", and the magic spider. Nonetheless, it is the examples that render the book most valuable and at the same time provide a salutary lesson in what extraordinary insight and talent the mathematical "amateur" can possess. Many of the stunning squares, cubes, and higher-dimensional forms were discovered by John Hendricks, a former employee of the Canadian Meteorological Service who retired in 1984. Several other examples were constructed by prison inmates (notably, a 7×7 square of primes, remaining magic when the rightmost digit of each entry is deleted). It is intriguing to ponder the reaction of our nation's sheriffs to the author's penological rumination: "One wonders what effect there would be on magic square research, mathematics, and society if prisoners were rewarded for any novel magic squares they created."

Occasionally one glimpses intriguing aspects of underlying mathematics, for instance in the following remark. Identify a magic square with its corresponding matrix, so that a multiplication is defined between magic squares of the same dimension. Then the cube of a 3×3 magic square is also a magic square! (This highly nonobvious observation is due to Frank E. Hruska, professor of chemistry at the University of Manitoba.) For instance, if A is equal to the lo shu square of sum 15, then

$$A^3 = \begin{pmatrix} 1149 & 1029 & 1197 \\ 1173 & 1125 & 1077 \\ 1053 & 1221 & 1101 \end{pmatrix},$$

which is a magic square of constant sum $3375 = 15^3$. The author remarks that "raising any third-order magic square to any odd power seems to yield a magic square. (We used a computer to test this up to the fifteenth power.)" A proof is furnished for the third power, using a parametrization of all 3×3 magic squares (curiously, the author states that such a square may be parametrized in one of two forms; he displays the forms apparently without realising that they are directly equivalent). However, this fascinating thread is not developed further.

A dutiful student of linear algebra recognises that if M is a matrix representing a magic square, then necessarily both M and M^t have the vector $(1, 1, \dots, 1)$ as eigenvector, with corresponding eigenvalue the constant magic sum (which equals the trace of M). So it follows immediately that if M_1 and M_2 are magic squares with constant sums m_1 and m_2, then $(1, 1, \dots, 1)$ is an eigenvector of both $M_1 M_2$ and its transpose, with eigenvalue $m_1 m_2$. Equivalently, $M_1 M_2$ is semimagic (has row and column sums all equal, namely, equal to $m_1 m_2$). What happens to the two diagonal sums is less obvious, but for 3×3 magic squares we can resort to the general parametrization and observe by direct computation that the product of three such magic squares is always magic, with constant sum equal to the product of the three original sums. An induction argument now proves that the k-th power (k odd) of a 3×3 magic square of sum a is magic with constant sum a^k.

Similar analysis for 4×4 squares will fail, for it is no longer true that the cube of such a magic square is still magic. However, imposing further symmetry conditions does result in some findings. There are four principal types of magic squares: Simple, Associated, Nasik, and Semi-Nasik. A Simple square has the minimal property of equal row, column, and diagonal sums. In an Associated N-th order square, any two cells symmetric about the centre of the square have sum $N^2 + 1$. In a Nasik square all the broken diagonals also have the constant sum, and a Semi-Nasik square has a less strong condition on the broken diagonals. By playing with a generic parametrization for the 4×4 case, it can be shown that the product of three Associated squares is Associated and that the product of three Nasik squares is Nasik (and hence the cube of an Associated or Nasik square is respectively Associated or Nasik); however, no such property seems to hold for products of Semi-Nasik squares. There is much to investigate here, and more in higher orders. A bright undergraduate student could fashion an enjoyable research experience from this aspect of the subject in itself.

Several errors in the book can be attributed to poor proofreading, with others perhaps ascribable to hurried writing. For example, the "Yin" 4×4 square on page 278 has an entry 227 misprinted as 277, and 2^3 is printed on page 22 as 89 in the "cube" of the Dürer square (and Dürer is referred to as the *fourteenth*-century painter and print-maker, despite the stress given to the Dürer square containing 1514, the year of its composition). The famous Jaina square is found in the temples at *Khajuraho* rather than Klajuraho, and there is a reference to Cantor's Theorem on the *unaccountability* of the real numbers. In the section "Further Reading" there is recommendation of an article by "H. E. Du" in earlier editions of the *Encyclopaedia Britannica*, but this was the abbreviation used by the famous puzzle master Henry Ernest Dudeney. And the classic reference work *Magic Squares and Cubes* by W. S. Andrews is listed as published in 1917, whereas the first edition appeared in 1908. More alarmingly, a fifth-order Nasik square is given on page 70 with the assertion that it has "many marvellous properties," but the properties given, involving sums of squares of row elements, simply fail to hold. One hopes that transcription of the vast number of numerical diagrams throughout the book has been done with more care. Still, for the most part these are the quibbles of a pedantic curmudgeon, and it would be churlish to deny the enormous appeal of this book. It is an extremely alluring page-turner. For anyone who loves numbers and puzzles, *The Zen of Magic Squares, Circles, and Stars* is, to quote Ian Stewart, "compulsive (and compulsory) reading."

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[1] E. R. Berlekamp, J. H. Conway, and R. K. Guy, *Winning Ways for Your Mathematical Plays*, Academic Press, 1982.

[2] C. Boyer, Les premiers carrés tétra et pentamagiques, *Pour la Science*, no. 286 (August 2001), 98–102; see also `http://www.multimagie.com/`.

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It Must Be Beautiful: Great Equations of Modern Science

Reviewed by William G. Faris

It Must Be Beautiful: Great Equations of Modern Science
Graham Farmelo, editor
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It Must Be Beautiful describes eleven great equations of modern science. (It is not claimed that these are the only great equations.) To qualify as great, the equation must summarize a key principle of a domain of knowledge and have significant implications for scientific and general culture. The book is thus organized around eleven essays, each devoted to one equation. The authors of the individual essays typically state the equation (but just once, and often in an appendix), explore what it says in nontechnical terms, describe its discovery and discoverer, and explain its importance. In some cases they describe their own personal involvement. They write for the general reader; there are no technical derivations either of the equations or of their consequences.

The authors are divided among science journalists, academic writers on science, and practicing scientists. The latter category includes Roger Penrose, Frank Wilczek, John Maynard Smith, Robert May, and Steven Weinberg (who contributes a brief afterword). The writing is good; this is a book you can recommend to friends who may wonder what theoretical science is about.

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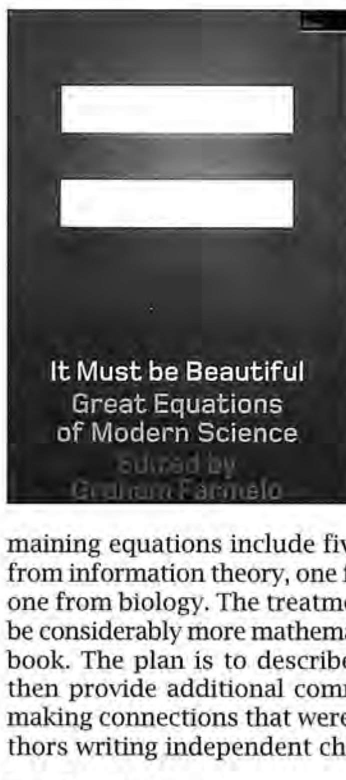


This review will not attempt to comment on all the chapters. In particular, it will skip the Schrödinger equation, evolutionary game maps, and the balance equations for ozone concentration in the atmosphere. The remaining equations include five from physics, one from information theory, one from astronomy, and one from biology. The treatment in the review will be considerably more mathematical than that of the book. The plan is to describe each equation and then provide additional commentary, sometimes making connections that were not possible for authors writing independent chapters of a book.

Einstein's $E = mc^2$

The first equation is Einstein's $E = mc^2$ from his special theory of relativity. This equation relates the energy E of a particle at rest to its mass m through a conversion factor involving the speed of light c. The release of energy from atoms has obvious social consequences; part of the essay in this chapter of the book is devoted to this aspect of Einstein's discovery.

This way of writing the equation obscures the underlying four-dimensional geometry. The energy E is the first component of a vector $(E, c\mathbf{p}) = (E, cp_1, cp_2, cp_3)$. Here $\mathbf{p}$ is the vector that describes the momentum of the particle. The relation for a particle in motion is the equation of a hyperbola:

$$E^2 - (c\mathbf{p})^2 = (mc^2)^2. \tag{1}$$

This more general equation exhibits the mechanism of the conversion of mass into relative motion. Here is an example. A particle of mass m at rest with energy E is converted into two moving particles of masses m' with energies E' and opposite nonzero momenta $\pm\mathbf{p}'$. Conservation of energy $2E' = E$, together with relation (1) for each of the particles, leads to the inequality $2m' < m$. The particles can fly apart only if mass is lost.

The Planck-Einstein Equation $E = \hbar\omega$

The second equation is due to Planck, but its deeper significance was realized by Einstein. If anything, it is even more important for modern science. This equation is $E = \hbar\omega$, where ω is the angular frequency (frequency measured in radians per second), and $\hbar$ is a conversion constant (rationalized Planck's constant). This equation has the same flaw; it deals with only one component of a vector. The accompanying equation for the other three components was found by de Broglie; it is $\mathbf{p} = \hbar\mathbf{k}$, where $\mathbf{k}$ is the wave number (measured in radians per meter). What these equations say is that a moving particle is described by a wave. The wave associated with a particle of energy E and momentum $\mathbf{p}$ varies in time at the corresponding angular frequency ω and in space at the corresponding wave number $\mathbf{k}$. These are fundamental ideas of quantum mechanics.

Quantum mechanics is an even greater conceptual revolution than special relativity. There is an atmosphere of mystery to the subject. It is sometimes said that quantum mechanics is about measurement, but this is strange for what should be a theory of nature. However, there is no doubt that quantum mechanics gives the explanation of much of what we experience in the world, from the solidity of a table to the brilliant color of a flame.

In quantum mechanical wave equations energy and momentum are expressed by differential operators:

$$E = i\hbar\frac{\partial}{\partial t} \tag{2}$$

and

$$\mathbf{p} = -i\hbar\nabla. \tag{3}$$

Here ∇ is the gradient operator with components $\partial/\partial x_j$ for $j = 1, 2, 3$. To see how this works, apply these differential operators to a plane wave, that is, to a function of time t and space $\mathbf{x}$ of the form $\exp(-i\omega t + i\mathbf{k}\cdot\mathbf{x})$. The result is the relations $E = \hbar\omega$ and $\mathbf{p} = \hbar\mathbf{k}$. However, the differential operator formulation is more general, since the operators may be applied to functions ψ of t and $\mathbf{x}$ that are not plane waves.

Substitute equations (2) and (3) in the nonrelativistic formula $E = 1/(2m)\,\mathbf{p}^2$, and apply the result to a complex-valued function ψ. The result is the Schrödinger equation that describes the waves associated with freely moving nonrelativistic particles. Instead, substitute equations (2) and (3) in the relativistic Einstein formula (1), and again apply the result to a function ψ. The result is the Klein-Gordon equation, which describes the waves associated with freely moving relativistic particles.

Both these equations describe spin zero particles, that is, particles without intrinsic angular momentum. In nature many particles do have an intrinsic angular momentum. This angular momentum has the seemingly paradoxical property that along each axis (as determined by the imposition of a magnetic field) it has values that are integer multiples of $\hbar/2$. The simplest such case (other than spin zero) is that of spin $1/2$ particles, which can have only the two angular momentum values $\pm\hbar/2$ along each axis. The equation for describing nonrelativistic spin $1/2$ particles is the Pauli equation, a modification of the Schrödinger equation in which the function ψ has two components. The equation for describing relativistic spin $1/2$ particles is the Dirac equation, but its discovery required a new idea.

The Dirac Equation

Dirac invented his equation in 1928 to describe the motion of electrons. The equation has had a remarkable success; it seems also to describe most of the other elementary particles that make up matter, not only protons and neutrons, but even the constituent quarks.

The idea of Dirac was to rewrite the quadratic Einstein relation (1) as a linear relation. This would seem impossible. But here is the solution:

$$\gamma^0 E + c\sum_{j=1}^{3}\gamma^j p_j = mc^2 I. \tag{4}$$

What makes it work is that this is a matrix equation. There are four matrices, γ^0, γ^1, γ^2, and γ^3. They anticommute: $\gamma^j\gamma^k = -\gamma^k\gamma^j$ for $j \neq k$. Furthermore, they satisfy $(\gamma^0)^2 = I$ and $(\gamma^j)^2 = -I$ for $j \neq 0$. In these formulas I denotes the identity matrix. All this is designed so that squaring both sides of the equation gives the quadratic Einstein relation. The anticommuting property makes the cross terms cancel.

The idea of an algebra associated with a quadratic form was known before Dirac. Such algebras are known as Clifford algebras, and a standard fact about the Clifford algebra over four-dimensional space-time is that it may be represented by 4 by 4 matrices. Four components might seem to be a nuisance, but they turned out to be of great interest for Dirac. He could interpret them as describing two kinds of particles, each with two components of spin. That is, electron spin emerged naturally from this way of writing a relativistic wave equation.

Insert (2) and (3) into (4) and apply to a function ψ. The result is the Dirac equation for the wave associated with a freely moving spin 1/2 particle:

$$\gamma^0 i\hbar\frac{\partial}{\partial t}\psi - c\sum_{j=1}^{3}\gamma^j i\hbar\frac{\partial}{\partial x_j}\psi = mc^2\psi. \tag{5}$$

This makes sense if the solution ψ is a function of t and $\mathbf{x}$ with values that are four-component column vectors.

This is not the ultimate version of the equation, since it describes only the motion of a freely moving particle. The real problem is to describe motion in the context of an electric potential ϕ (a function of t and $\mathbf{x}$) and a magnetic vector potential $\mathbf{A}$ (a vector function of t and $\mathbf{x}$). There is a surprisingly simple recipe for doing this, which has become known as the gauge principle. The principle is to replace the energy E by $E - e\phi$ and the momentum $\mathbf{p}$ by $\mathbf{p} - e\mathbf{A}$. Here e is the charge of the particle. In the language of quantum mechanics, the gauge principle says that the quantum mechanical equations should be expressed in terms of the differential operators

$$i\hbar D_t^{\phi} = i\hbar\frac{\partial}{\partial t} - e\phi \tag{6}$$

and

$$-i\hbar\nabla^{\mathbf{A}} = -i\hbar\nabla - e\mathbf{A}. \tag{7}$$

This prescription works equally well for the Schrödinger equation, the Klein-Gordon equation, and the Dirac equation. Furthermore, it suggests the structure of the Maxwell equation that governs the electromagnetic potentials and fields. The Yang-Mills equation (discussed below) is a generalization of the Maxwell equation that is based on an extension of this idea.

The Dirac equation is the fundamental equation for matter, and similarly the Yang-Mills equation is the fundamental equation for force. Both equations have a geometric flavor. However, this complacent summary omits one essential feature: these are equations, not for functions, but for more complicated objects called quantum fields. There are warnings about this complication in the book. For instance, Weinberg writes:

> The more important an equation is, the more we have to be alert to changes in its significance. Nowhere have these changes been more dramatic than for the Dirac equation. Here we have seen not just a change in our view of why an equation is valid and of the conditions under which it is valid, but there has also been a radical change in our understanding of what the equation is about.

Wilczek writes the Dirac equation in the form with potentials. He employs relativistic notation, where the time and space coordinates are treated on an equal basis. However, since he uses units with $c = 1$ and $\hbar = 1$, it is possible that readers may not make the connection with the $E = mc^2$ of special relativity and the $E = \hbar\omega$ of quantum mechanics encountered in earlier chapters.

Contents of *It Must Be Beautiful*

Listed below are the book's chapters and chapter authors.

The Yang-Mills Equation

The Yang-Mills equation was invented in 1953 to describe the forces between elementary particles. It developed from an idea that Hermann Weyl published in 1929 and that was familiar to many theoretical physicists. Yang was aware of this idea, the fact that gauge invariance determines the electromagnetic interactions. However, Yang did not know its origin. As Sutton explains in her chapter: "Yang was initially unaware that these ideas were due to Weyl, and still did not realize it when both men were at the Institute for Advanced Study in Princeton and even met occasionally. Weyl had left Germany in 1933 and taken up a position at Princeton, becoming a U.S. citizen in 1939, whereas Yang joined the Institute in 1949. It seems that Weyl, who died in 1955, probably never knew of the remarkable paper that Yang wrote with Mills—the paper that demonstrates for the first time how the symmetry of gauge invariance could indeed specify the behavior of a fundamental force."

The Yang-Mills principle can be summarized in a slogan: force is curvature. Curvature is associated with surfaces. It represents the turning or rotating that takes place when some object is transported around the closed curve bounding a surface. An example of curvature is a nonzero magnetic field. Suppose $|B|$ is the total flux of magnetic field through a particular surface. The quantum mechanical phase rotates by $(e/\hbar)|B|$ as it is transported around the boundary of the surface. Here e is the electric charge, and $\hbar$ is the rationalized Planck's constant. In macroscopic contexts this is such a huge number of rotations that it is difficult to exhibit the effect directly, but the consequences on the atomic scale are quite apparent.

The technical formulation of these ideas is in a framework of differential operators inspired by the gauge principle of formulas (6) and (7). The fundamental idea is that of a connection, which is a rule for differentiating a vector field. The vector field is defined on four-dimensional space-time, with coordinates $x^0 = ct, x^1, x^2, x^3$. However, the values of the vector field belong to some N-dimensional space having to do with the internal structure of a class of elementary particles.

The gauge invariance idea may be expressed by the principle that the basis for the N-dimensional vector space may vary from point to point in space-time. Express the vector field in such a "moving basis" as $u = \sum_k u^k \mathbf{e}_k$. The connection applied to the vector field is $d^A u = \sum_k du^k \mathbf{e}_k + \sum_k u^k\, d\mathbf{e}_k$. It is determined by functions $A^j_{k\mu}$. Their purpose is to give a way $d\mathbf{e}_k = \sum_{j\mu} A^j_{k\mu} \mathbf{e}_j\, dx^\mu$ of differentiating the basis vectors. The covariant differential of the vector field is a vector-valued 1-form:

$$d^A u = \sum_{j\mu} \left(\frac{\partial u^j}{\partial x^\mu} + \sum_k A^j_{k\mu} u^k \right) \mathbf{e}_j\, dx^\mu. \tag{8}$$

In this formula the sums over j and k range over N values corresponding to basis vectors in the vector space. The sum over μ is over the four dimensions of space-time. Roughly speaking, this first covariant differential is a generalization of the idea of gradient, except that it acts on vectors rather than on scalars, and it depends on the choice of connection. In the context of Yang-Mills the connection is called the gauge potential.

The covariant derivative may also be applied to vector-valued 1-forms. The result is a vector-valued 2-form. There is an antisymmetrization in the definition of 2-forms, and so this derivative may be thought of as a generalization of the idea of curl. However, the analog of the fact that the curl of the gradient is zero no longer holds. The measure of this failure is the curvature: the linear-transformation-valued 2-form F determined by

$$d^A d^A u = Fu \tag{9}$$

for arbitrary u. Due to the antisymmetry property $dx^\nu\, dx^\mu = -dx^\mu\, dx^\nu$ of differential forms, the terms involving derivatives of u cancel. The result is

$$Fu = \sum_{kj\nu\mu} \left(\frac{\partial A^j_{k\mu}}{\partial x^\nu} + \sum_m A^j_{m\nu} A^m_{k\mu} \right) u^k \mathbf{e}_j\, dx^\nu\, dx^\mu. \tag{10}$$

The second term is a matrix product, and this is responsible for the fact that F is nonlinear in A. The curvature F describes, for each point and for each small two-dimensional surface near the point, how a vector transported around the surface must necessarily twist. In the context of Yang-Mills the curvature is called the field.

The Yang-Mills equation constrains the curvature: it says that the divergence of the field F is the current J. There is an operator δ^A that plays the role of a divergence, and the equation is

$$\delta^A F = J. \tag{11}$$

This scheme is a generalization of electromagnetism. In electromagnetism the current consists of the ordinary current and charge. The curvature is a combination of electric and magnetic field. The connection is made of electric and magnetic potentials. The electromagnetic example has one special feature: the vector space has complex dimension $N = 1$, and so the matrices commute. This means that the nonlinear term is not present in the electromagnetic case. The striking feature of the general Yang-Mills equation is that the matrices do not commute; the nonlinear term is a necessary feature, imposed by the geometry. A solution is then called a nonabelian gauge field.

The equations of physics considered thus far form a closed system. Solve the Dirac equation with connection A for a function ψ. From ψ construct the current J. Integrate the Yang-Mills divergence equation with source J to get the curvature field F. Integrate the equation for F to get back to the connection A. The results should be consistent.

Einstein's General Relativity

Einstein's general theory of relativity describes gravitational force. Again there are notions of connection and curvature, but the theory is much more geometrical. The reason is that the connection differentiates vectors, but now these vectors are tangent to space-time itself.

The fundamental quantity is the metric tensor g that attaches to each space-time point a quadratic form in these space-time tangent vectors. There is a unique symmetric connection Γ with the property that

$$d^{\Gamma} g = 0. \tag{12}$$

This is used to express the connection Γ in terms of the metric g. As before, the curvature R is determined by the connection Γ by

$$d^{\Gamma} d^{\Gamma} u = Ru \tag{13}$$

for arbitrary u.

The curvature tensor R is a two-form whose values are linear transformations. Einstein wanted to find an equation that relates R to the energy-momentum tensor T. However, this tensor is a one-form with values that are vectors. It satisfies the conservation law $\delta^{\Gamma} T = 0$; that is, it has zero divergence. He struggled with this issue for some time and made a false start, proposing an incorrect equation involving the Ricci tensor, a tensor constructed from algebraic manipulations on R. Penrose explains what happened: "This equation is indeed what Einstein first suggested, but he subsequently came to realize that it is not really consistent with a certain equation, necessarily satisfied by T_{ab}, which expresses a fundamental *energy conservation* law for the matter sources. This forced him, after several years of vacillation and uncertainty, to replace the quantity R_{ab} [the Ricci tensor] on the left by the slightly different quantity $R_{ab} - \frac{1}{2} R g_{ab}$ [a form of the Einstein tensor] which, for purely mathematical reasons, rather miraculously *also* satisfies the same equation as T_{ab}!" [The italics are in the original text.]

The Einstein tensor G represents a three-dimensional curvature obtained by averaging the two-dimensional curvature $g^{-1}R$ over planes. The way G is constructed from the curvature tensor ensures that it satisfies the identity $\delta^{\Gamma} G = 0$ necessary for consistency with the conservation law. The Einstein equation relating G and T is thus simply

$$G = -\kappa T, \tag{14}$$

where κ is a physical constant. It says that the source of the three-dimensional curvature is the density of energy-momentum.

Technical note: The averaging in the definition of the Einstein tensor is seen in the explicit formula $G^a_b = -\frac{1}{4} \delta^{ast}_{bij} R^{ij}_{st}$, where $R^{ij}_{st} = g^{ik} R^j_{kst}$, and repeated indices are summed. This works out to be $G^a_b = R^{aj}_{bj} - \frac{1}{2} R^{ij}_{ij} \delta^a_b$. The corresponding quadratic form is $G_{ab} = R^j_{abj} - \frac{1}{2} R^{ij}_{ij} g_{ab}$.

Even knowing the right equation, there is still a leap from four-dimensional geometry to the fall of an apple. In his essay Penrose helps bridge this gap by giving a picture of how the geometry of the Einstein tensor is related to the "tidal effect" of gravity.

The most obvious analogy between electric field and gravitational field turns out to be misleading. The reason is that the Yang-Mills equation has a different structure from Einstein's equation. In Yang-Mills the potential A is the connection, which is related to curves. The field F is the derivative of A, so F is the curvature, which is related to surfaces. The divergence of the field F is then related to the source J. Contrast this with Einstein's equation. The potential is the metric g. The field Γ is the derivative of g, so Γ is the connection, which is related to curves. The derivative of the field Γ is a curvature R associated with surfaces. The Einstein tensor G is constructed from an average of curvatures associated with surfaces bounding a volume, and it is this tensor that is proportional to the source T. The mathematical ingredients in the Yang-Mills and Einstein theories are similar, but they are used in different ways. The relation between these two theories is a puzzle. Penrose expresses his own attitude as follows. "We know, in any case, that Einstein's theory cannot be the last word concerning the nature of space-time and gravity. For at some stage an appropriate marriage between Einstein's theory and quantum mechanics needs to come about."

Shannon's Channel Capacity Equation

Two equations are featured in this chapter. The first equation is the definition of information. If X is a random variable with probability density $p(x)$, then the corresponding information is

$$I(X) = -\int p(x) \log p(x)\, dx. \tag{15}$$

(There is also a discrete analog of this equation, where the integral is replaced by a sum.)

The second equation is Shannon's formula for channel capacity. This gives the rate at which information can be transmitted in the presence of noise. If S is the signal strength, N is the noise strength, and W is the bandwidth of the channel, then the capacity is

$$C = W \log(1 + \frac{S}{N}). \tag{16}$$

The essay stresses the role of this theory in technology. Aleksander writes: "Shannon's equations are not about nature, they are about systems that engineers have designed and developed. Shannon's contribution lies in making engineering sense of a medium through which we communicate. He shares the same niche as other great innovators such as his boyhood hero Thomas Edison (who turned out to be a distant relative, much to Shannon's delight) and Johann Gutenberg." It is only fair to say that he has also played a role as a mathematician.

The channel capacity equation is the expression of a nontrivial theorem, but a simple example can give an idea of why it takes this form. Information is to be transmitted in a time interval of length T. The input X and the output Y are vectors of real numbers of length $2WT$. (Take this as the definition of the bandwidth W, at least for the purposes of this rough computation.) The noise Z is also such a vector. It is natural to take Z to be a vector with random components. The output Y is the input X corrupted by the noise Z in the additive form

$$Y = X + Z. \tag{17}$$

It is not so obvious that it is also reasonable to model the input X as a random vector. This is justified, however, since well-encoded information looks random to a disinterested observer. Thus each of the three vectors is to consist of mean-zero Gaussian random variables. The signal vector X has total variance ST, while the noise vector Z has total variance NT. If the signal and noise are independent, then the output vector Y has total variance $ST + NT$. The information transmitted in time T is CT. This is

$$\begin{aligned} CT &= I(Y) - I(Z) \\ &= WT\log\left(\frac{\pi e(S+N)}{W}\right) - WT\log\left(\frac{\pi e N}{W}\right). \end{aligned} \tag{18}$$

The first term $I(Y)$ is the total information in Y, while the second term $I(Z)$ is the amount of this information in Y that is useless in determining X (since it is only information about the noise Z). The difference is the information in Y that is transmitted successfully from X. The result is equivalent to that given in formula (16), and thus it illustrates the Shannon formula in a special example.

The Drake Equation for Artificial Radio Sources in the Galaxy

The Drake equation is an equation for the expected number N of radio sources in the galaxy produced by intelligent civilizations. Let r be the average rate at which such sources are produced. Let L be the average length of time that a civilization persists. Then

$$N = rL. \tag{19}$$

The importance of this equation is the research program it suggests. The rate of production r is equal to the rate of production of planets in the galaxy times the conditional probability (given that the planet exists) that there will be life on a planet times the conditional probability (given that life exists) that the life will evolve to a suitable civilization. This chain of conditional probabilities can be broken down even further. The task is identified: find numerical estimates for each of these quantities. It was a valuable insight. It has also produced something of a puzzle, since many reasonable estimates give a rather large value of N. Yet we, as yet, see no such radio emissions.

The reasoning behind this equation illustrates a more general mathematical point, the importance of balance equations and of detailed balance equations. A balance equation for a stationary probability says that the rate of probability flow from all other states into a given state is equal to the rate of probability flow from that state into all other states. A detailed balance equation is a stronger and more special condition: between every pair of states the probability flows balance.

The derivation follows a standard pattern in queueing theory. The state of the galaxy is the number of civilizations n. The average rate at which civilizations are created by random fluctuations is r. The average rate of loss for a single civilization would be $1/L$, where L is the average lifetime. If there are n civilizations, then the total rate at which civilizations dissipate is n/L. If p_n is the steady-state probability of having n such civilizations, then the detailed balance equation is

$$p_{n-1}\, r = p_n \frac{n}{L}, \tag{20}$$

and since $N = rL$, this says that

$$p_n = \frac{N}{n} p_{n-1}. \tag{21}$$

The meaning of equation (20) is that the expected rate of transitions from a state of the galaxy with $n-1$ civilizations to a state with n civilizations (by having $n-1$ civilizations and then spontaneously generating a new one at expected rate r) is equal to the expected rate of transitions from a state with n civilizations to a state with $n-1$ civilizations (by having n civilizations, choosing one of the n, and having it die out at expected rate $1/L$). Then equation (21) immediately implies that the steady-state probabilities p_n have a Poisson distribution with expectation N.

It was Einstein who found a method of exploiting detailed balance in statistical mechanics in the form of fluctuation-dissipation relations. He used these relations in a particularly ingenious way to calculate the parameters describing Brownian motion and thus find the size of atoms. Special relativity, general relativity, quantum theory, Brownian motion—enough great equations for one man.

The Quadratic Map in Ecology: Chaos

It is easy to find a function f that maps an interval into itself and has one interior maximum. It can even be a second-degree polynomial in one variable. The iteration of such a function can lead to fixed points. It can also lead to periodic cycles. Of course a cycle of period n is a fixed point of the n-fold composition f^n. But there is another possibility: irregular long range behavior, that is, chaos. May describes his excitement in introducing these ideas in ecology in a phrase of Tom Stoppard: "It's the best possible time to be alive, when almost everything you know is wrong."

What did we know that was wrong? Here is how May puts it: "Situations that are effectively unpredictable—a roulette ball whose fate, the winning number, is governed by a complex concatenation of the croupier's hand, the spinning wheel and so on—were thought to arise only because the rules were many and complicated." What the quadratic map shows is that even the simplest nonlinear dynamics can produce long-range unpredictability. There is no need for complexity.

There is order in chaos. May mentions the numerical invariants of the period-doubling approach to chaos and the renormalization-group explanation of this given by Feigenbaum. The renormalization group (actually a one-parameter semigroup T^k of transformations) consists of looking at the dynamics on a longer time-scale and then rescaling to compare with the previous scale. It is a dynamical system that acts on a class of dynamical systems. In this case the dynamical systems belong to a particular space of functions f with one interior maximum. The renormalization transformation T is defined by

$$(Tf)(x) = \frac{1}{s} f(f(sx)), \tag{22}$$

where s is a scaling parameter. The Feigenbaum fixed-point equation $Tf = f$ should qualify as a great equation. Its solution f^* is a function that encodes universal properties of the approach to chaos for other functions f. There is an irony: fixed points of T describe chaotic behavior of functions f.

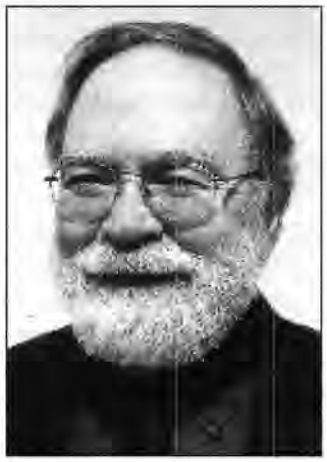
Conclusion

There are threads of unity linking these great equations. The equations of physics are brought together by the idea of gauge potential or connection. The formula for information is related to the formulas for entropy in statistical mechanics and to similar quantities in quantum field theory. The renormalization-group idea is now central not only to current viewpoints on statistical mechanics and dynamical systems but to quantum field theory itself.

The authors of the chapters in this volume do a remarkable job of showing how each of the great equations is situated in a broad cultural context. The equation itself is at the center. But the geometry is something like that of a black hole; the actual equation remains nearly invisible to the general reader. One of the privileges of being a mathematician is that one is allowed a glimpse inside.

Mandelbrot and Yorke Receive 2003 Japan Prize

Benoit B. Mandelbrot



James A. Yorke

Benoit B. Mandelbrot and James A. Yorke were named Laureates for the 2003 Japan Prize for Science and Technology of Complexity. Awarded by the Japan Science and Technology Foundation, the Japan Prize carries a cash award of 50 million Japanese yen (about US$400,000).

Citation

What follows is the full text of the citation for the prize.

The world we live in is so complex that it is an enormous challenge to understand the fundamental nature of its complexities. Modern science has so far been successful in explaining the world by breaking it down into its constituent elements and then analyzing their properties. However, there are phenomena which emerge only when elements are connected into systems, and which the elements do not have in themselves. Modern science has taken up the challenge to examine those properties, going beyond the reductionistic approach. This is called the science and technology of complexity.

Nature is filled with complex geometrical shapes such as seashore lines, branching patterns of rivers, biological shapes, and even the curves of currency exchange rates. There is a common feature in such complex shapes: their self-similarity. This is the property that, when a part of a shape is enlarged, the same type of structure appears again. Dr. Mandelbrot discovered that self-similarity is the universal property that underlies such complex shapes, and he coined the expression "fractal". Furthermore, he has illustrated its properties mathematically and founded a new methodology for analyzing complex systems.

Numerous time-varying, complex patterns of behavior are found in dynamic phenomena such as the motion of the planets, turbulence in water and air, variations of the populations of species in ecological systems, and many other instances. These patterns of behavior are described by nonlinear evolution equations. Dr. Yorke has found the universal mechanism underlying such nonlinear phenomena. He named it "chaos", and he has elucidated its properties mathematically. He has played a leading role in further development of research into chaos, including its controls and applications.

It is still a challenge to understand complex phenomena. The two concepts—chaos and fractal—have been established as universal concepts underlying such phenomena, irrespective of specific fields. Their applicability has been extended even to modern technology, the arts, economics, and the social sciences.

Dr. Mandelbrot and Dr. Yorke found, respectively, that fractals and chaos are the universal structures existing in complex systems, and they elucidated their fundamental properties. They have furnished us with new frameworks for understanding complex phenomena, and they have contributed both by establishing fundamentals and by providing us with applications. Therefore, Dr. Mandelbrot and Dr. Yorke deserve the 2003 Japan Prize.

Biographical Sketch: Benoit Mandelbrot

Benoit Mandelbrot was born November 20, 1924, in Warsaw, Poland. He studied at the École Polytechnique and the California Institute of Technology before receiving his doctorate from the Université de Paris in 1952. He worked in France before moving to the IBM T. J. Watson Research Center in 1958. Since 1987 he has also been on the faculty of Yale University. He is currently an IBM Fellow Emeritus and the Sterling Professor of Mathematical Sciences at Yale.

Mandelbrot has received numerous prizes and awards, including the Harvey Prize for Science and Technology (1989), the Wolf Foundation Prize for Physics (1993), and the Sigma Xi William Proctor Prize for Scientific Achievement (2002). He is a member of the U.S. National Academy of Sciences, a fellow of the American Academy of Arts and Sciences, and a foreign member of the Norwegian Academy of Science and Letters.

Biographical Sketch: James Yorke

James Yorke was born on August 3, 1941. He received his bachelor's degree at Columbia University (1963) and his Ph.D. from the University of Maryland (1966). He has spent his career at the University of Maryland, where he is currently Distinguished University Professor of Mathematics and Physics, Institute for Physical Sciences and Technology. He was the director of that institute from 1988 to 2001. Yorke is a fellow of the American Association for the Advancement of Science.

About the Prize

The Japan Prize is awarded to people from all parts of the world whose original and outstanding achievements in science and technology are recognized as having advanced the frontiers of knowledge and served the cause of peace and prosperity for mankind. The prize was established by the government of Japan in 1983. Previous recipients include Marvin Minsky (1990) and Jacques-Louis Lions (1991).

—Allyn Jackson

Presidential Views: Interview with David Eisenbud

Every other year, when a new AMS president takes office, the *Notices* publishes interviews with the incoming and outgoing presidents. What follows is an edited version of an interview with AMS president David Eisenbud, whose term began on February 1, 2003. The interview was conducted in October 2002 by *Notices* senior writer and deputy editor Allyn Jackson. Eisenbud is director of the Mathematical Sciences Research Institute in Berkeley and professor of mathematics at the University of California, Berkeley.

An interview with AMS past president Hyman Bass appeared in the February 2003 issue of the *Notices*, pages 232–4.

Notices: You are director of a major math institute, you travel a lot, you talk to a lot of mathematicians. What kinds of problems or challenges do mathematicians tell you they are facing?

Eisenbud: One concern is about the flow of young people into the profession. Both here and in Europe people say the flow of youngsters into mathematics has dried up over the last years. There are bright spots and dim spots, and it's very hard to get a good integrated picture. But it seems pretty clear there is a serious problem.

In this country there has been recently a lot of concern about VIGRE [Vertical Integration of Research and Education in the Mathematical Sciences, a program of the Division of Mathematical Sciences at the National Science Foundation (NSF)]. There is also a lot of concern about how people get supported over the long term. We've added a great deal of money for support of young people in recent times, and there is concern that there is no mechanism to follow through for mid-career people.

But the amount of mathematics to do doesn't seem to be drying up, and there are plenty of new opportunities for interactions with other sciences, particularly in biology.

Notices: *Do people worry about funding being drained off to applied areas?*

Eisenbud: There has been such a shortage of funding for all of mathematics that people worry whenever something new comes under the umbrella. Despite all the talk about interactions with applied mathematics, many mathematicians don't really know how mathematics is involved with other sciences. I think that the connections with other sciences and applications will continue to develop rapidly and that this is very healthy. If there were a better level of funding overall, it would be easier for people to accept this with confidence.

Of course, NSF math funding has seen wonderful increases lately. But there is a sustainability problem: Much of the increase is gotten on the basis of new activities, and if the increases don't keep coming, the new activities can spell trouble for the existing programs in the short run. That doesn't mean one shouldn't be bold. I think we have to take this risk. Overall, it's very healthy for the profession to have done so. The only way to get out of the hole is to be open to these new possibilities. In the long run I believe that all mathematical activities will profit from this.

I think that the AMS's activities in advocacy for mathematics funding are much more effective than they were. Sam [Rankin, director of the AMS Washington Office] has done a wonderful job, and Monica Foulkes [a member of the Washington Office staff] is terrific. That's an effort that I want to support and encourage. [AMS Committee on Science Policy chair] Jane Hawkins and I have talked about this, and that committee is moving toward a more active stance. I think the committee's contacts in Washington have been useful, and I think it could increase its usefulness by encouraging more "at home" contacts between mathematicians and people in the Congress.

The flow of talent into mathematics is a hard problem to address. The AMS has been actively trying to help with programs like the "Epsilon Program", to which a very large number of AMS members contribute. I'd like to see the AMS do more at the graduate level, too. For example, there are a couple of programs for department chairs (the Chairs' Colloquia [of the National Research Council's

Board on Mathematical Sciences and Applications] and the AMS workshops for new chairs at the annual meetings), and perhaps such things could be done for graduate advisors. The AMS might also help to advertise good practices: both novel ideas for graduate programs and simply things that work well. A lot of what is done in graduate education in mathematics in this country is very good.

The Carnegie Foundation is initiating a major study of the doctorate. One of the essays they have commissioned will be written by Hy Bass, and there have been institutional contacts with the AMS. Perhaps the AMS can play a useful role in the Carnegie study.

Notices: *Looking back over the past couple of decades, what do you think accounts for the slacking off of the flow of talent into the profession?*

Eisenbud: There are surely many causes. It became very popular to make money. Business schools absorbed very talented young people when it seemed they could make a lot of money easily and quickly. We may see some flow back, now that the bubble has burst.

In the seventies computer science became an exciting thing to do—and it's still an exciting thing to do. Some of the people who used to go into mathematics (and the other theoretical sciences) now go into computer science.

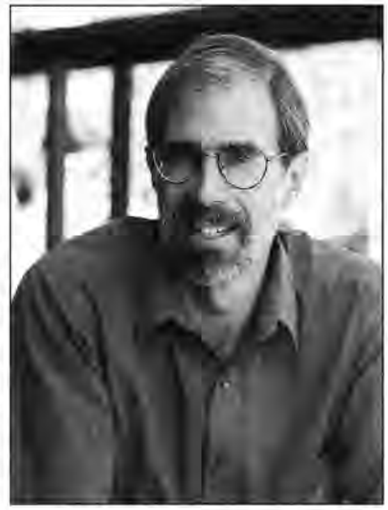
Another cause is the unpredictability and big fluctuations of the job market. People don't rush back in as soon as the job market gets better. When youngsters struggle with several postdoc positions in a row, the example is before the students: Those who are struggling are their teachers! It's no mystery that mathematics students today worry about the job market for mathematicians.

Perhaps another part of the problem is the rejection of a culture that favored hard work and deep thought and effort put into scholarly things. Many people prefer to do things that are easy and safe.

Notices: *You got your Ph.D. in 1970. In terms of the sense of idealism or the willingness to work hard, do you think the climate was different then?*

Eisenbud: I do. In my circle of friends we worried about finding a grand dream to pursue, not about making money. There was a sense that the universe was opening up and that science was the great frontier. You couldn't do anything more exciting or more important for society's future. Government support was also very important in this. In the ten years after Sputnik, from 1957 to 1967, money poured into science and science education. It was clear that you would be adequately compensated, and that was enough. That now seems an extraordinary period. It happened that I came of age in that period, so I thought for a long time that it was the norm. But it was not.

Notices: *Many math departments depend on foreign students to keep their graduate programs going, and often the foreign graduate students are the best students.*



David Eisenbud

Eisenbud: That's a little unfair. They usually come with at least one extra year's preparation. It is true that they are quite often the best prepared.

Notices: *That's true. But are people in the U.S. concerned about this? It's a delicate issue.*

Eisenbud: Many foreign graduate students become Americans; they are as important a part of the next generation of Americans as they are of this generation. My own feeling is that we should be incredibly grateful that we are sent the best students from other countries. Of course we should do the necessary things to encourage a stronger flow of Americans into mathematics. But we should be very open to foreigners.

Notices: *One difficulty with VIGRE is that the students supported have to be American.*

Eisenbud: At the beginning of VIGRE I was very worried about this, but I've come to think I need not have been. There are statistics showing that the number of foreign graduate students studying here has increased under VIGRE, along with the number of American students. Even if a new source of support is entirely directed to one group, it may help other groups by relieving older sources.

I would like to comment on the importance of the AMS data collection efforts. Take the question of whether the number of foreign graduate students in mathematics has gone down because of VIGRE. That's something that only the AMS is studying. Mathematicians are particularly aware of, interested in, and sensitive to data of that kind. The articles in the *Notices* that give data on the state of the profession are excellent, but I think that the data could be made more accessible. I could imagine having the data available online in a cumulative form that would allow one to search and make one's own table of numbers of, say, Ph.D.'s in algebra each year since 1975.

Notices: *Some years ago the issue of women and minorities in mathematics was very big. This issue seems to be much less at the forefront now. Why has this changed?*

Eisenbud: I also see a diminution in official expressions of interest in this issue. I think this comes partly from the political climate. For example, the University of California is forbidden to use affirmative action in deciding which students to admit. A number of states have such laws. Some people don't want to take strong public stands, because they think it might be counterproductive in the present climate.

The AMS is sensitive to the issue of underrepresentation, and the number of women and minorities speaking at the meetings, for example, has increased quite a bit over the last fifteen years. The AMS has done a pretty good job in making sure that there are women on committees and involved in many other ways with the work of the Society. Still, the improvements are not uniform, and I think even more can be done.

The problem of minority involvement is a bit different from that for women, because the absolute number of minority mathematicians is still tiny. I'm proud that the first CAARMS [Conference of African-American Researchers in the Mathematical Sciences] conference was held ten years ago at MSRI; there have been CAARMS conferences annually ever since. A big focus for CAARMS and for organizations like SACNAS [Society for the Advancement of Chicanos and Native Americans in Science] is on getting undergraduates in science to go on to graduate school.

I think the AMS should not be discouraged by the fact that the government is in some sense moving out of this arena. Our involvement is important. One of the ways the AMS president can influence the organization is through appointments to committees. I plan to pay close attention to such appointments.

Notices: *What about electronic publishing and things like the Digital Math Library. Are you interested in these matters?*

Eisenbud: Yes, very interested. I think John [Ewing, AMS executive director] is a great resource for this activity, and he will play a significant role. But the issue is not at all confined to the AMS! For example, the CEIC [Committee on Electronic Information and Communication of the International Mathematical Union] has a lot of energy right now.

It seems clear that something must change. There is practically *no* really complete mathematics library anymore. Perhaps there is no library on the face of the earth that has all the mathematics journals reviewed in *Math Reviews*. I'm lucky to live near pretty big libraries, and I find most papers I need, but in the last five years half a dozen times I have tried to find articles that are not in any University of California library. And if it's hard to get journals here, it's much harder in many other places. This means that mathematics is fragmenting. The published literature is supposed to be an accessible repository for what we know. If I publish in a journal you can't get, I might as well have not published, as far as you are concerned.

Notices: Mathematical Reviews *gets everything. If library collections continue to worsen, tools like MathSciNet will become even more important.*

Eisenbud: That's true. And the number of institutions that can now access MathSciNet has increased a lot because of the AMS's consortium pricing policy. MathSciNet has figured out a way to make its information available to many, many more mathematicians. I think people are not aware how important this step has been elsewhere in the world.

If I had to name the tools that changed my research life the most in the last ten to fifteen years, they would be: some of the computer algebra systems (Macaulay and Macaulay2 in particular), the arXiv, and MathSciNet.

Notices: *Do you think the arXiv is a threat to the AMS?*

Eisenbud: No. In what way?

Notices: *It could cut into the journal revenues.*

Eisenbud: I think no one knows how this will play out. First of all, the arXiv's coverage is far from universal. At the moment it is just wild fantasy to think that it could take over. But I could imagine a future in which that would become true. Things journals do—the sorting of good papers from bad, the refereeing process, the stamp of approval, the role in career advancement, their longevity—are not trivial at all. We should all be concerned about keeping those functions healthy as the system changes. On the other hand, some publishers have raised prices too much. I think the AMS has been very good about its journal pricing, but that's not universal.

Libraries are in real trouble. The status quo, with prices increasing exponentially and library budgets being slashed, just cannot go on. I feel sure that twenty years from now things will be different in some major ways.

Speaking of publishing, I am enthusiastic about the AMS book publishing [program]. Some time ago it was seen as a rather stodgy program, with poor distribution, but that has changed. The AMS is now one of the top math publishers. It seems to me that the AMS can be a very good steward for the publishing needs of mathematicians.

We are living at a time when there is a lot of public awareness of mathematics, as witnessed by the many plays, movies, books, and popular press articles about mathematicians and mathematical subjects. It's a time when individual mathematicians or mathematics departments can do more in public outreach. This can be extremely useful in continuing the momentum for increasing mathematics funding that we have at the moment.

Let me say that I am extremely impressed by the multitude of AMS activities and the professional way that they are handled. I think John Ewing is terrific and runs a very tight and functional ship. The office in Washington has been a big success. Bob Daverman and the associate secretaries, backed up by a great staff, do a remarkable job with the meetings. The network of committees for journals, prizes, professional issues...functions amazingly well. There is a lot going on.

MSRI Celebrates Its Twentieth Birthday

The past twenty years have seen a great proliferation in mathematics institutes worldwide. An inspiration for many of them has been the Mathematical Sciences Research Institute (MSRI), founded in Berkeley, California, in 1982. An established center for mathematical activity that draws researchers from all over the world, MSRI has distinguished itself for its programs in both pure and applied areas and for its wide range of outreach activities. MSRI's success has allowed it to attract many donations toward financing the construction of a new extension to its building. In October 2002 MSRI celebrated its twentieth year with a series of special events that exemplified what MSRI has become—a focal point for mathematical culture in all its forms, with the discovery and delight of new mathematical knowledge the top priority.

The Founding of MSRI

From the 1930s through the 1970s, the only large-scale mathematics institute in North America was the School of Mathematics at the Institute for Advanced Study (IAS) in Princeton. In the 1970s there were discussions between leaders in the mathematical sciences community and the National Science Foundation (NSF) about the need to come up with alternative modes of funding support, including mathematics institutes. The NSF's 1978 call for proposals for alternative modes of support resulted in the funding of two national mathematics institutes: MSRI and the Institute for Mathematics and its Applications at the University of Minnesota. In a recompetition for institute funding held by the NSF in 1997, both IMA and MSRI won renewed support. Since then, the NSF has launched four more institutes: the Institute for Pure and Applied Mathematics at the University of California, Los Angeles; the AIM Research Conference Center at the American Institute of Mathematics (AIM) in Palo Alto, California; the Mathematical Biosciences Institute at the Ohio State University; and the Statistical and Applied Mathematical Sciences Institute, which is a partnership of Duke University, North Carolina State University, the University of North Carolina at Chapel Hill, and the National Institute of Statistical Sciences.

Shiing-Shen Chern, Calvin C. Moore, and I. M. Singer, all on the mathematics faculty at the University of California, Berkeley, initiated the original proposal for MSRI; Chern served as the founding director, and Moore was the deputy director. In its first year MSRI held two programs, one in nonlinear partial differential equations and one in mathematical statistics. Since then, two to four programs, running over semesters or over the whole academic year, have been held each year. Irving Kaplansky succeeded Chern as director and served until 1992, when William Thurston took the position. The current director of MSRI is David Eisenbud, who started a two-year term as president of the AMS in February 2003 (see the interview with Eisenbud elsewhere in this issue of the *Notices*). While Eisenbud is on sabbatical for the 2002–2003 academic year, Michael Singer is serving as acting director.

MSRI is a large operation, with about 1,300 visitors coming through each year and about 85 in residence at any one time. It is also large in terms of its coverage of mathematics. Over the years it

David Hilbert: Boy, Did He Have Problems!

As part of its twentieth-anniversary celebration, MSRI held a panel discussion entitled "The Honors Class: Hilbert's Problems in Perspective". The panelists were Benjamin Yandell, author of *The Honors Class*, a book about the people who solved Hilbert's problems; Constance Reid, author of the acclaimed biography of Hilbert; Paul Cohen, a Fields Medalist at Stanford University who solved one of Hilbert's problems, pertaining to the independence of the axiom of choice and of the continuum hypothesis; and Sir Michael Atiyah, Fields Medalist at Edinburgh University. Organized by MSRI's associate director for external collaborations, David Hoffman, the panel provided an occasion for some fascinating discussion about the culture of mathematics.

Growing up in New York, Cohen said, he felt very close to European traditions, so "when I thought of mathematics, I thought of Göttingen and Cambridge." Hilbert was a towering figure in this tradition. Cohen noted that Hilbert's way of thinking about mathematics was characterized by a freshness and an impatience with any sort of politicking. Hilbert had his finger on all of mathematics, except topology, Cohen said, a feat that would be impossible today, given the enormous growth in mathematics since 1900.

Atiyah spoke of one of Hilbert's great contemporaries, Henri Poincaré, thereby reopening a perennial debate over the relative importance and influence of the two. Without denying Hilbert's great achievements, Atiyah seemed to believe Poincaré was the more creative and visionary. He summed up the heritage of the two: "The most famous disciple of Hilbert, I think, was Bourbaki, and the most famous disciple of Poincaré is Arnold....The two are very controversial!" Cohen by contrast said he feels a "closer kinship" to Hilbert and expressed awe at, for example, Hilbert's rigorous proof that the Dirichlet principle is correct. This proof took care of what was one of the dominant problems of nineteenth-century mathematics. "When I look at [Hilbert's] achievements, the raw brain power is overwhelming," Cohen said.

Reid noted that Hilbert always referred to Poincaré as "the greatest mathematician of his [Poincaré's] generation"; this statement neatly sidesteps a comparison of the two, for Poincaré was eight years older than Hilbert. Reid said she once asked Richard Courant who was the greater mathematician, Hilbert or Poincaré (Courant was a student of Hilbert's). Courant replied that there was no question it was Poincaré. Reid's book on Hilbert quotes Courant as saying: "But you cannot compare him with Hilbert. He did not have the intensity that radiated from Hilbert and which was so wonderful. If he had had that..."

The good-humored sparring over Hilbert and Poincaré was mixed with discussion of problem solving versus theory building in mathematics. Cohen said he feels particularly close to Hilbert, because "he loved problems. He thought about mathematics in quite a similar way to how I do." Atiyah pointed out that there is no real dichotomy between problems and theory. "No theory is any good unless it solves problems, and no problem is really good unless it leads to a theory," he said. Wiles's solution to Fermat's Last Theorem was important because it emerged as part of a much larger and richer theory, Atiyah noted, whereas the proof of the four-color theorem did not. Said Atiyah, "It's like a mountain. Until you get to the top, you don't know whether it was important to climb that mountain."

—*A. J.*

has hosted programs in mathematical economics, mathematical biology, string theory, and statistics, as well as in a wide variety of areas in pure mathematics. Indeed, Eisenbud notes that a distinctive feature of MSRI in the world of mathematics institutes is its combination of pure and applied areas. As he puts it, "We have continued to have a fundamental emphasis, and we mix it with applied areas." Perhaps the closest cousin in this regard is the Isaac Newton Institute in Cambridge, England, but there the emphasis is more strongly in applied areas.

Another hallmark of MSRI is the way it promotes collaboration. Even the building, with its atrium design in which three levels of offices open onto the central lobby, is intended to get people to interact. By contrast, places like the IAS or the Institut des Hautes Études Scientifiques in Paris have a more monastic feel. "MSRI tends to be much more collaborative" than those institutes, Eisenbud says. "You can get away by yourself, but those who are happiest here are interested in collaboration." Some visitors find the rather bustling atmosphere distracting, while others thrive on it.

MSRI is a nonprofit corporation separate from the University of California, but the ties between the two are close. There is plenty of interaction between the university's mathematics department and MSRI, as the two are only about ten minutes apart by shuttle bus. One of the great attractions of MSRI is its location, perched on a hillside with commanding views of the San Francisco Bay Area. But the location also has its drawbacks: cost of living is high in Berkeley. MSRI does not have its own housing, though it does provide extensive help for visitors navigating the area's tight housing market. Pay for visitors is adequate but not lavish, and MSRI has sometimes had difficulty attracting well-paid senior mathematicians. This difficulty has an impact on MSRI's postdoctoral program, which attracts twenty to thirty young people each year and which has sometimes suffered from a lack of mentors for the postdocs. "This is something we struggle with each semester," Eisenbud remarks. "[Senior] people have to make sacrifices to come to MSRI—it's sad, but true." This problem has been eased to some extent in recent years through visiting professorships paid for by Hewlett-Packard and by the mathematician and philanthropist James Simons. Hewlett-Packard and Microsoft also sponsor postdoctoral appointments, and UC Berkeley sponsors a one-semester visiting professorship.

One of the biggest changes in MSRI in recent years is the growth of its fundraising activity. Two years ago MSRI hired its own full-time development officer; currently 25–30 percent of MSRI's budget comes from non-NSF sources. Right now MSRI is in the midst of a capital campaign to raise the remaining \$1 million needed to finance a \$7.3 million extension to its building. The initial \$6.3 million was donated by foundations and private individuals. Construction will begin in fall 2003 and is scheduled to be completed a year later. The extension will double the capacity of the library and add new seminar rooms plus a common room and a large lecture hall. The addition of a professional kitchen should provide much needed relief for the food service quandaries that arise from MSRI's somewhat isolated location, far from the many restaurants for which the Bay Area is so famous. Because the amount of office space will remain the same with the extension, MSRI's policy of having visitors share offices will not change.

Photograph by David Zetland.

Cutting the birthday cake, left to right, David Eisenbud, MIchael Atiyah, and Robert Bryant.

Celebrating Twenty Years

"Brainiacs heat up screen at Cinemath", reads a headline in the October 2, 2002, *San Francisco Chronicle*. The article described a mathematics film festival called Cinemath held in celebration of MSRI's twentieth anniversary. It was organized by the Pacific Film Archive in Berkeley and by Robert Osserman, MSRI's special projects director and professor emeritus at Stanford University. Among the films were π (Darren Aronofsky, 1998), *Death of a Neapolitan Mathematician* (Mario Martone, 1992), *Drowning by Numbers* (Peter Greenaway, 1988), and a biographical film about Paul Erdős called *N Is a Number* (George Paul Csicsery, 1993). Most of the films were accompanied by talks by mathematicians, including Osserman, Dave Bayer of Barnard College, Keith Devlin of Stanford University, and Ronald Graham of the University of California, San Diego. Osserman reported that Cinemath attracted mathematicians and nonmathematicians alike.

The film festival is very much in keeping with the tradition of outreach activities begun by Thurston, who served as MSRI director from 1992 to 1997 and is now at the University of California, Davis. This part of the MSRI agenda got an enormous boost from the success of the 1993 "Fermat Fest", a celebration of Andrew Wiles's proof of Fermat's Last Theorem, which combined brief talks by mathematicians with lighter fare such as songs by mathematician-songwriter Tom Lehrer. Held in a science museum in San Francisco, the Fermat Fest sold out the 1,000-seat auditorium. Among the other MSRI outreach programs are the Journalist-in-Residence program, occasional lectures for the general public, and sponsorship of the Bay Area Mathematics Olympiad. One of the most unusual activities is a series of "conversations" in which Osserman discusses mathematics with artists, writers, and performers whose work has touched on the field. The most recent conversation, with the comedian Steve Martin, was held in December 2002. As these activities have become more prominent, there has been concern that they could overshadow MSRI's core mission of mathematics research. "They could," Eisenbud said. "One has to be vigilant and keep the balance right. The truth is that we have such good mathematics and such a good structure that we are not in danger. The quality of the mathematics programs remains our first priority."

In addition to the film festival, the MSRI birthday celebration included a colloquium lecture by Sir Michael Atiyah and a panel discussion about Hilbert's problems (see sidebar). There was also a public lecture by Atiyah, presented on a sunny Saturday afternoon to an audience of about 300 people assembled in a lecture hall on the Berkeley campus. The lecture, entitled "Geometry and Physics from Plato to Hawking", made the point, as Atiyah put it, that "mathematics is the creation of people, not of a machine." He discussed the symmetry of the Platonic solids, tracing a thread of ideas from Kepler's incorrect model of the solar system as nested polyhedra, to the connections between the Platonic solids and the exceptional Lie groups. One of these groups, which is connected to the icosahedron, turns out to play a role in string theory. Although Kepler's model of the solar system was wrong, his intuition about a deep connection between the Platonic solids and physics has proved to be essentially correct. This shows, Atiyah said, that "a good idea lives on to fight another day." And MSRI, as good an idea today as it was twenty years ago, lives on to provide a place where mathematics can flourish.

—*Allyn Jackson*

Mathematics People

Iglehart and Derman Awarded von Neumann Prize

The 2002 John von Neumann Theory Prize, the highest prize given in the field of operations research and management science, has been awarded to DONALD L. IGLEHART, professor emeritus at Stanford University, and CYRUS DERMAN, professor emeritus at Columbia University, for their fundamental contributions to performance analysis and optimization of stochastic systems. The award, presented by the Institute for Operations Research and the Management Sciences (INFORMS), carries a cash award of $5,000.

The citation for Iglehart reads, in part, "Iglehart pioneered and, in subsequent papers with his student Ward Whitt, led the development of diffusion limits and approximations for heavily congested stochastic systems. The importance of these ideas is that they provide tractable limiting processes and readily computable approximations for complex queueing and other stochastic systems for which closed-form or even numerical solutions have proved intractable. His work in this area transformed the field, with literally hundreds of papers subsequently continuing the development of his ideas."

The prize citation for Derman states that he "fundamentally advanced finite-state-and-action Markovian decision processes. He took the lead role in showing that starting from a state, the set of state-action frequencies over all policies is the convex hull of the finite set of state-action frequencies over all stationary deterministic Markov policies. This work plays a fundamental role in solving such problems in the presence of linear constraints on the state-action frequencies, e.g., reflecting desired limits on the frequency of unfavorable events like failures, rejects, shortages and accidents, and has been widely used in practice."

—From an INFORMS announcement

Mathematics Project Wins Siemens Westinghouse Competition

The top prize in the 2002–03 Siemens Westinghouse Competition in Math, Science, and Technology has been awarded to STEVEN J. BYRNES, a senior at Roxbury Latin High School in West Roxbury, Massachusetts, for a mathematics project titled "Poset-Game Periodicity". The prize carries a $100,000 scholarship.

The winning project analyzes a class of two-player games known as poset games, which play an important role in the field of discrete mathematics for their potential applications in artificial intelligence, error correcting, and computer networks. Byrnes was the only student in the United States in 2002 to be a prizewinner in both the U.S. Mathematics Olympiad and the U.S. Physics Olympiad. He plans to study mathematics in college.

The annual competition, administered by the College Board and funded by the Siemens Foundation, recognizes outstanding talent among high school students in science, mathematics, and technology.

—From a Siemens Foundation announcement

Rhodes Scholarships Awarded

Three mathematics students are among the thirty-two American men and women chosen as Rhodes Scholars by the Rhodes Scholarship Trust. The Rhodes Scholars were chosen from 981 applicants who were endorsed by 341 colleges and universities in a nationwide competition. The names and brief biographical descriptions of the mathematics scholars follow.

MATT LANDREMAN of St. Paul, Minnesota, is a senior at Swarthmore College, where he majors in physics. He is a Goldwater Scholar and has published research in plasma physics in major academic journals. He is a tutor in the Philadelphia Upward Bound program and established a commercial bakery at Swarthmore College, the proceeds of which are donated to charity. He is a cross-country and marathon runner. He plans to read mathematics at Oxford University.

Jacob G. Foster of Winchester, Virginia, is a physics major at Duke University. He is also an actor and musician who plays piano and organ, is president of Duke's musical theater company, and practices kung fu. He plans to study for the M.Sc. in mathematics.

Heidi L. Williams of Williston, North Dakota, majors in pure mathematics at Dartmouth College. She has been a Truman Scholar and a U.S. Presidential Scholar, won a national cryptology competition in high school, and in 2002 participated in the Director's Summer Program at the National Security Agency. She founded a program to address educational barriers for middle school girls, and she participates in ballet and modern dance. She plans to study for the M.Sc. in mathematical foundations of computer science.

Rhodes Scholarships provide two or three years of study at the University of Oxford in England. The value of the Rhodes Scholarship varies depending on the academic field, the degree (B.A., master's, doctoral), and the Oxford college chosen. The Rhodes Trust pays all college and university fees and provides a stipend to cover students' necessary expenses while in residence in Oxford, as well as during vacations, and transportation to and from England. The total value averages approximately $28,000 per year.

—Allyn Jackson

Mathematics Opportunities

Project NExT: New Experiences in Teaching

Project NExT (New Experiences in Teaching) is a professional development program for new and recent Ph.D.'s in the mathematical sciences (including pure and applied mathematics, statistics, operations research, and mathematics education). It addresses all aspects of an academic career: improving the teaching and learning of mathematics, engaging in research and scholarship, and participating in professional activities. It also provides the participants with a network of peers and mentors as they assume these responsibilities. Each year about sixty faculty members from colleges and universities throughout the country are selected to participate in a workshop preceding the Mathematical Association of America (MAA) summer meeting, in activities during the summer MAA meetings and the Joint Mathematics Meetings in January, and in an electronic discussion network. Faculty for whom the 2003–2004 academic year will be the first or second year of full-time employment with significant teaching responsibilities at the college or university level are invited to apply to become Project NExT Fellows.

The application deadline is **April 11, 2003**. For more information, see the Project NExT website, `http://archives.math.utk.edu/projnext/`. Project NExT receives major funding from the ExxonMobil Foundation, with additional funding from the Dolciani-Halloran Foundation, the American Mathematical Society, the Educational Advancement Foundation, the American Statistical Association, the Association of Mathematics Teacher Educators, and the Greater MAA Fund.

—Christine Stevens, Project NExT

Women's International Science Collaboration Program

The American Association for the Advancement of Science (AAAS) announces the Women's International Science Collaboration (WISC) Program. Supported by the National Science Foundation (NSF), this program aims to increase the participation of women in international scientific research through travel awards to locations around the world. The awards are to foster new research partnerships between U.S. scientists and colleagues overseas. *Both male and female scientists are eligible to apply.*

Men and women who have their Ph.D.'s or equivalent research experience are eligible. Graduate students (Ph.D. candidates) are also eligible as long as they will be conducting research in an established Ph.D. program in the United States and will be traveling with their Ph.D. advisors and serving as co-PIs on future proposals. The deadline for applications is **July 15, 2003**.

For further information on fields eligible for funding, please visit the NSF website at `http://www.nsf.gov/` or contact one of the AAAS administrators listed below. The administrators are: for Central and Eastern Europe, and Newly Independent States (NIS) of the former Soviet Union: Curtis Cook, email: `ccook@aaas.org`, telephone 202-326-7027; for East Asia and Pacific: Suteera Nagavajara, email: `snagavaj@aaas.org`, telephone 202-326-6496; for Africa, Middle East, Near East, and South Asia: Alan Bornbusch, email: `abornbus@aaas.org`, telephone 202-326-6651; for the Americas and the Caribbean: Marina Ratchford, email: `mratchfo@aaas.org`, telephone 202-326-6490.

For further application information and region-specific guidelines, see `http://www.aaas.org/international/wiscnew.shtml`, or contact one of the above administrators or WISC Travel Grant, American Association for the Advancement of Science, Directorate for International Programs, 1200 New York Avenue, NW, Washington, DC 20005.

—From an AAAS announcement

Partners Program for Research in the FSU

For several years the U.S. Department of State has helped support two science centers in Russia and Ukraine. Serving as research project "clearinghouses", the International

Science and Technology Center (ISTC) in Moscow and the Science and Technology Center in Ukraine (STCU) in Kiev provide former Soviet Union (FSU) weapons scientists the opportunity to redirect their talents to peaceful research. In 1997 the Science Centers introduced the Partners Program, enabling U.S. companies, universities, scientific institutions, and other entities to "partner" with the centers and fund projects at FSU research institutes.

Partners enjoy the benefits and protection of the centers' infrastructure, including tax and customs exemptions, protection of intellectual property, a legally binding project agreement, and more. Developing projects with FSU colleagues through the Partners Program not only has commercial advantages but also supports the Department of State's effort to stem the global proliferation of weapons of mass destruction and their delivery vehicles.

Scientists and institutes working in the program are engaged in a broad range of research and development projects, including projects in pure and applied mathematics. Examples of project areas include high-performance computing, turbulent mixing, equation of state modeling, multidimensional grids for adaptive computation, discrete dynamical systems analysis, and studies of plasma instablities.

To request Partner status, interested organizations need only submit to the Department of State a brief letter providing two or three sentences of basic background information on the Partner applicant and a website address (if available), a short explanation of the prospective Partner's project-funding plans (if known), and a statement that the Partner applicant accepts the nonproliferation objectives of the center and the terms of the center's Statute and Agreement. Becoming a Partner costs nothing—Partners incur costs only when they decide to fund a project—and does not obligate the Partner to fund a project at all.

For additional information on science center activities and the Partners Program, visit the websites for ISTC (`http://www.istc.ru/`) or STCU (`www.stcu.kiev.ua`), call the Department of State at 202-736-7694, or write to: Andy Hood, U.S. Department of State, NP/PTR, Room 2428, 2201 C Street, NW, Washington, DC 20520.

—From a U.S. Department of State announcement

National Academies Internship Program

The Christine Mirzayan Science and Technology Policy Internship Program of the National Academies is designed to engage graduate science, engineering, medical, veterinary, business, and law students in the analysis and creation of science and technology policy and to familiarize them with the interactions of science, technology, and government. As a result, students develop essential skills different from those attained in academia and make the transition from being a graduate student to a professional. There are two programs in 2003: the summer program, June 2 through August 8; and fall, September 8 through November 26.

Applications for the internships are invited from graduate students through postdoctoral scholars in any physical, biological, or social science field or any field of engineering, medicine and health, or veterinary medicine, as well as business, law, education, and other graduate and professional programs. Postdoctoral scholars should have received their Ph.D.'s within the past five years.

The stipend for the 12-week September program is $5,700 and for the 10-week June program, $4,800. The internship stipend is to cover all living expenses for the period. In addition, a travel stipend of up to $500 will be provided.

Deadlines for receipt of materials for the summer program is **March 1, 2003**, and for the fall program, **June 1, 2003**. More information and application forms and instructions can be found on the website `http://www7.nationalacademies.org/internship/index.html` or by contacting The National Academies Christine Mirzayan Science and Technology Policy Internship Program, 500 5th Street, NW, Room 508, Washington, DC 20001; telephone: 202-334-2455; fax: 202-334-1667.

—From a National Academies announcement

Maria Mitchell Women in Science Award

The Maria Mitchell Association offers an annual award to recognize an individual, program, or organization that encourages the advancement of girls and women in studies and careers in science and technology. Maria Mitchell (1818–1889) was the first woman astronomer and first woman astronomy professor in the United States.

The award may be given in the natural and physical sciences, mathematics, engineering, computer science, or technology. The winner will be chosen by a national jury of distinguished educators and scientists and will receive a cash award of $10,000. Funding for the award has been provided by an anonymous donor.

Guidelines and nomination forms are available from the Association's website at `http://www.mmo.org/` or by contacting the Maria Mitchell Women in Science Award Committee at the Maria Mitchell Association, 2 Vestal Street, Nantucket, MA 02554; telephone 508-228-9198. Deadline for nominations is **April 30, 2003**.

—From a Maria Mitchell Association announcement

Inside the AMS

AMS Supports ICM Travel Grants

Between 1998 and 2001, contributions from individuals, totaling about US\$134,000, were collected by the AMS for the Special Development Fund of the International Mathematical Union (IMU). This fund is used primarily to support travel grants for young mathematicians from developing countries to attend the International Congress of Mathematicians (ICM). For ICM 2002, held in Beijing in August 2002, ninety-five young mathematicians received grants. In addition, a small portion of the funds were used for senior mathematicians from developing countries and Eastern Europe.

The IMU Special Development Fund is made up of contributions from mathematics organizations around the world. Contributions collected by the AMS accounted for about 70 percent of the total contributions to the Special Development Fund between 1998 and 2001.

The IMU hopes to be able to support the travel of about 120 mathematicians to attend ICM 2006, which will be held in Madrid, Spain, in August 2006.

—Allyn Jackson

AMS-MER Workshop on Undergraduate Mathematics

Beginning in 2001, the American Mathematical Society (AMS) and the Mathematicians and Education Reform (MER) Forum began a three-year project, funded by the National Science Foundation, to help mathematics/mathematical science departments strengthen their undergraduate programs. Entitled "Excellence in Undergraduate Mathematics: Confronting Diverse Student Interests", the project focuses on the different groups of students in mathematics and their particular mathematical needs. The goal is to develop curricula and instruction that is valuable to these different student populations. The project seeks to build a network of departments that are committed to (1) reviewing and assessing how well their undergraduate program is working for their students, and (2) revising existing courses and developing or adapting new courses to afford all their students a meaningful experience in learning mathematics.

The heart of the project is six integrated 4-day workshops, two in each academic year, each of which will bring together faculty teams of two to four members from some twenty or more colleges and universities to discuss curricular, instructional, and implementation issues relating to the mathematics offerings for different groups of students.

Three workshops have been held so far: at Arizona State University, Washington University in St. Louis, and Louisiana State University. The next workshop will be held March 13-16, 2003, at Ithaca College. For further information and application forms for participation, visit the MER website, `http://www.math.uic.edu/MER/`.

—Naomi Fisher, MER Forum

Bass Attends Signing of NSF Reauthorization

On December 19, 2002, Hyman Bass, then AMS president, was present in the White House Roosevelt Room to witness President Bush sign into law H.R. 4664, a five-year reauthorization bill for the National Science Foundation (NSF).

Although the bill does not provide funds for the NSF, it does provide the authority to obligate funds for specified activities. It authorizes increasing funding for the NSF from the current (fiscal year 2002) level of \$4.79 billion to \$9.84 billion in FY 2007. The funding levels of the last two years of the authorization are contingent on the NSF making satisfactory progress in meeting the goals of the president's management agenda.

"We are quite pleased that President Bush has endorsed the efforts in Congress to support this much-needed growth of the NSF budget," Bass said. "This is a proven high-leverage investment in the national welfare—the economy, education, national security, and public health. This event is a tribute to the sustained efforts of many people and organizations, including the Coalition for National Science Funding chaired by our own Sam Rankin [director of the AMS Washington office]. We hope that the same wisdom will guide similar federal investment in research sponsored by other government agencies."

In 1998 the NSF was authorized by the 105th Congress for FY 1998, 1999, and 2000. H.R. 4664 is significant because of the level of funding authorized and because of its unanimous passage by the House and Senate.

The full bill text can be found at `http://www.nsf.gov/od/lpa/congress/107/final_authorization_language.pdf`.

—Allyn Jackson

Deaths of AMS Members

Henry L. Alder, professor emeritus, University of California, Davis, died on November 6, 2002. Born on March 26, 1922, he was a member of the Society for 59 years.

Sheila Brenner, senior fellow, University of Liverpool, died on October 10, 2002. Born on January 17, 1930, she was a member of the Society for 12 years.

Paul F. Cherenack, of the University of Cape Town, South Africa, died on January 17, 2002. Born on June 19, 1942, he was a member of the Society for 31 years.

George F. D. Duff, professor emeritus, University of Toronto, died on March 2, 2001. Born on July 28, 1926, he was a member of the Society for 49 years.

Gunnar Fogelberg, of Linköping University, Sweden, died on December 23, 2001. Born on March 24, 1944, he was a member of the Society for 9 years.

Barry Johnson, retired, of Newcastle upon Tyne, England, died on May 5, 2002. Born on August 1, 1937, he was a member of the Society for 13 years.

Carl Kossack, professor emeritus, of Vero Beach, FL, died on October 1, 2002. Born in May 1915, he was a member of the Society for 64 years.

Howard G. McCutcheon, of Mayo, MD, died on October 10, 2002. He was a member of the Society for 4 years.

Bernhard H. Neumann, professor emeritus, Australian National University, died on October 21, 2002. Born on October 15, 1909, he was a member of the Society for 55 years.

Lee E. Pray, retired, of Naples, FL, died on September 21, 2002. He was a member of the Society for 4 years.

Arnold Ross, of The Ohio State University, Columbus, died on September 25, 2002. Born on August 24, 1906, he was a member of the Society for 75 years.

Jean E. Rubin, of Purdue University, died on October 25, 2002. Born on October 29, 1926, she was a member of the Society for 52 years.

Frank Smithies, of Cambridge, England, died on November 16, 2002. He was a member of the Society for 64 years.

Oxana Ziza, of the Moscow Institute of Electronics and Mathematics, died on November 8, 2002. Born on October 16, 1930, she was a member of the Society for 22 years.

Reference and Book List

*The **Reference** section of the* Notices *is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.*

Contacting the *Notices*

The preferred method for contacting the *Notices* is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The managing editor is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar". Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are `notices@math.tamu.edu` in the case of the editor and `notices@ams.org` in the case of the managing editor. The fax numbers are 979-845-6028 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Upcoming Deadlines

March 1, 2003: Applications for Christine Mirzayan Science and Technology Policy Internship Program. See "Mathematics Opportunities" in this issue.

March 1, 2003: Applications for 2003 George Washington University Summer Program for Women in Mathematics. See `http://www.gwu.edu/~math/spwm.html`, or contact the director, Murli M. Gupta (`mmg@gwu.edu`), Department of Mathematics, George Washington University, Washington, DC 20052; telephone 202-994-4857; fax 202-994-6760.

March 1, 2003: Nominations for Third World Academy of Science Prizes. See `http://www.ictp.trieste.it/~twas/twas_prizes.html`.

March 3, 2003: Applications for EDGE Summer Program. See `http://www.brynmawr.edu/Acads/Math/edge/edge.html`.

March 31, 2003: Nominations for the 2003 Prize for Achievement in Information-Based Complexity. For more information, contact Joseph Traub, `traub@cs.columbia.edu`.

Where to Find It

A brief index to information that appears in this and previous issues.

April 8, 2003: Proposals for 2004 NSF-CBMS Regional Conferences. Contact Conference Board of the Mathematical Sciences, 1529-18th Street, NW, Washington, DC 20036-1385; telephone: 202-293-1170; fax: 202-293-3412; World Wide Web: http://www.cbmsweb.org/NSF/2004_call.htm; email: kolbe@math.georgetown.edu or rosier@math.georgetown.edu.

April 11, 2003: Applications for Project NExT. See "Mathematics Opportunities" in this issue.

April 15, 2003: Applications for National Research Council Research Associateship Program. See http://www4.nationalacademies.org/pga/rap.nsf/, or contact the National Research Council, Associateship Programs (TJ 2114), 2101 Constitution Avenue, NW, Washington, DC 20418; telephone 202-334-2760; fax 202-334-2759; email: rap@nas.edu.

April 18, 2003: Full proposals for NSF IGERT program. See http://www.nsf.gov/pubsys/ods/getpub.cfm?nsf02145/.

April 30, 2003: Nominations for Maria Mitchell Women in Science Award. See "Mathematics Opportunities" in this issue.

May 1, 2003: Applications for NSF/AWM Travel Grants for Women. See http://www.awm-math.org/travelgrants.html; telephone 301-405-7892; email: awm@math.umd.edu.

May 15, 2003: Applications for fall semester of Math in Moscow and for AMS scholarships. See http://www.mccme.ru/mathinmoscow/, or contact Math in Moscow, P.O. Box 524, Wynnewood, PA 19096; fax +7095-291-65-01; email: mim@mccme.ru. For information about and application forms for the AMS scholarships, see http://www.ams.org/careers-edu/mimoscow.html, or contact Math in Moscow Program, Professional Services Department, American Mathematical Society, 201 Charles Street, Providence, RI 02904; email: prof-serv@ams.org.

June 1, 2003: Applications for Christine Mirzayan Science and Technology Policy Internship Program. See "Mathematics Opportunities" in this issue.

June 30, 2003: Nominations for the Fermat Prize for Mathematics Research. See http://www.ups-tlse.fr/ACTUALITES/Sciences/Prix_Fermat_2003/Areglement.html.

July 15, 2003: Applications for Women's International Science Collaboration (WISC) Program. See "Mathematics Opportunities" in this issue.

August 15, 2003: Applications for National Research Council Research Associateship Program. See http://www4.nationalacademies.org/pga/rap.nsf/, or contact the National Research Council, Associateship Programs (TJ 2114), 2101 Constitution Avenue, NW, Washington, DC 20418; telephone 202-334-2760; fax 202-334-2759; email: rap@nas.edu.

December 31, 2003: Entries for *Cryptologia* paper competitions. See http://www.dean.usma.edu/math/pubs/cryptologia/, or contact *Cryptologia*, Department of Mathematical Sciences, United States Military Academy, West Point, NY 10996; email: Cryptologia@usma.edu.

Board on Mathematical Sciences and Their Applications, National Research Council

Dimitris Bertsimas, MIT Sloan School of Management
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Douglas Ravenel, University of Rochester
Stephen M. Robinson, University of Wisconsin, Madison
S. R. Srinivasa Varadhan, New York University
Scott Weidman (Director, BMSA), National Academy of Sciences

The postal address for BMSA is: Board on Mathematical Sciences and Their Applications, National Academy of Sciences, Room TNA 924, 500-5th Street, NW, Washington, DC 20001; telephone 202-334-2421; fax 202-334-2422, 2101; World Wide Web http://www7.nationalacademies.org/bms/BMSA_Members.html.

Book List

The Book List highlights books that have mathematical themes and are aimed at a broad audience potentially including mathematicians, students, and the general public. When a book has been reviewed in the Notices, *a reference is given to the review. Generally the list will contain only books published within the last two years, though exceptions may be made in cases where current events (e.g., the death of a prominent mathematician, coverage of a certain piece of mathematics in the news) warrant drawing readers' attention to older books. Suggestions for books to include on the list may be sent to* notices-booklist@ams.org.

*Added to "Book List" since the list's last appearance.

The Algorithmic Beauty of Seaweeds, Sponges and Corals, by Jap Kaandorp and Janet Kübler. Springer-Verlag, January 2001. ISBN 3-540-67700-3.

The Annotated Flatland: A Romance of Many Dimensions, Edwin A. Abbott; introduction and notes by Ian Stewart. Perseus Publishing, November 2001. ISBN 0-7382-0541-9. (Reviewed November 2002.)

* *The Art of the Infinite: The Pleasures of Mathematics*, by Robert Kaplan and Ellen Kaplan. Oxford University Press, March 2003. ISBN 0-195-14743-X.

Behind Deep Blue: Building the Computer That Defeated the World Chess Champion, by Feng-hsiung Hsu. Princeton University Press, November 2002. ISBN 0-691-09065-3.

The Bit and the Pendulum: How the New Physics of Information Is Revolutionizing Science, by Tom Siegfried. John Wiley & Sons, February 2000. ISBN 0-47132-174-5. (Reviewed August 2002.)

The Book of Nothing: Vacuums, Voids, and the Latest Ideas about the Origins of the Universe, by John D.

Barrow. Pantheon Books, April 2001. ISBN 0-375-42099-1. (Reviewed June/July 2002.)

Codes and Ciphers: Julius Caesar, the Enigma, and the Internet, by Robert Churchhouse. Cambridge University Press, January 2002. ISBN 0-521-81054-X.

The Colossal Book of Mathematics: Classic Puzzles, Paradoxes, and Problems, by Martin Gardner. W. W. Norton & Company, August 2001. ISBN 0-393-02023-1. (Reviewed October 2002.)

Conned Again, Watson! Cautionary Tales of Logic, Math, and Probability, by Colin Bruce. Perseus Publishing, January 2001. ISBN 0-7382-0345-9. (Reviewed November 2002.)

The Constants of Nature: From Alpha to Omega—The Numbers That Encode the Deepest Secrets of the Universe, by John D. Barrow. Jonathan Cape, September 2002. Pantheon Books, January 2003. ISBN 0-375-42221-8.

Correspondance Grothendieck-Serre, Pierre Colmez and Jean-Pierre Serre, editors. Société Mathematique de France, 2001. ISBN 2-85629-104-X.

Curve Ball: Baseball, Statistics, and the Rules of Chance in the Game, by Jim Albert and Jay Bennett. Copernicus-Springer Verlag, July 2001. ISBN 0-387-98816-5.

Damned Lies and Statistics: Untangling Numbers from the Media, Politicians, and Activists, by Joel Best. University of California Press, May 2001. ISBN 0-520-21978-3. (Reviewed February 2003.)

Does God Play Dice? The New Mathematics of Chaos, by Ian Stewart. Blackwell, revised second edition, January 2002. ISBN 0-631-23251-6. (Reviewed December 2002.)

* *Dr. Riemann's Zeros: The Search for the $1 Million Solution to the Greatest Problem in Mathematics*, by Karl Sabbagh. Atlantic Books, November 2002. ISBN 1-843-54100-9.

Entanglement: The Greatest Mystery in Physics, by Amir D. Aczel. Four Walls Eight Windows, October 2002. ISBN 1-56858-232-3.

Euclid's Window: The Story of Geometry from Parallel Lines to Hyperspace, by Leonard Mlodinow. Free Press, April 2001. ISBN 0-684-86523-8. (Reviewed May 2002.)

Flatterland: Like Flatland, Only More So, by Ian Stewart. Perseus Publishing, May 2001. ISBN 0-7382-0442-0. (Reviewed April 2002.)

The Fractal Murders, by Mark Cohen. Muddy Gap Press, May 2002. 0-9718986-0-X.

Fragments of Infinity: A Kaleidoscope of Math and Art, by Ivars Peterson. John Wiley & Sons, October 2001. ISBN 0-471-16558-1. (Reviewed October 2002.)

A Gardner's Workout: Training the Mind and Entertaining the Spirit, by Martin Gardner. A K Peters, June 2001. ISBN 1-56881-120-9.

Geometry: Our Cultural History, by Audun Holme. Springer, April 2002. ISBN 3-540-41949-7.

The Glass Wall: Why Mathematics Can Seem Difficult, by Frank Smith. Teachers College Press, July 2002. ISBN 0-807-74241-4 (paperback), 0-807-74242-2 (cloth).

Go To: The Story of the Math Majors, Bridge Players, Engineers, Chess Wizards, Scientists and Iconoclasts Who Were the Hero Programmers of the Software Revolution, by Steve Lohr. Basic Books, October 2001. ISBN 0-465-04225-2.

God in the Equation: How Einstein Became the Prophet of the New Religious Era, by Corey S. Powell. Free Press, August 2002. ISBN 0-684-86348-0.

Gödel's Proof, by Ernest Nagel and James R. Newman. New York University Press, revised edition, February 2002. ISBN 0-8147-5816-9.

* *The Golden Ratio: The Story of Phi, the World's Most Astonishing Number*, by Mario Livio. Broadway Books, October 2002. ISBN 0-767-90815-5.

The Hilbert Challenge, by Jeremy J. Gray. Oxford University Press, December 2000. ISBN 0-198-50651-1. (Reviewed September 2002.)

Hinged Dissections: Swinging and Twisting, by Greg N. Frederickson. Cambridge University Press, September 2002. ISBN 0-521-81192-9.

The Honors Class, by Benjamin Yandell. A K Peters, December 2001. ISBN 1-568-81141-1. (Reviewed September 2002.)

How the Other Half Thinks: Adventures in Mathematical Reasoning, by Sherman Stein. McGraw-Hill, July 2001. ISBN 0-071-37339-X. (Reviewed September 2002.)

How the Universe Got Its Spots, by Janna Levin. Princeton University Press, April 2002. ISBN 0-691-09657-0.

Indra's Pearls: The Vision of Felix Klein, by David Mumford, Caroline Series, and David J. Wright. Cambridge University Press, January 2002. ISBN 0-521-35253-3. (Reviewed January 2003.)

It Must Be Beautiful: Great Equations of Modern Science, Graham Farmelo, editor. Granta Books, February 2002. ISBN 1-862-07479-8. (Reviewed in this issue.)

The Lady Tasting Tea: How Statistics Revolutionized Science in the Twentieth Century, by David Salsburg. W. H. Freeman & Co., April 2001. ISBN 0-716-74106-7.

Lebesgue's Theory of Integration: Its Origins and Development, by Thomas Hawkins. AMS, September 2001. ISBN 0-8218-2963-7.

Linked: The New Science of Networks, by Albert-Lásló Barabási. Perseus Publishing, May 2002. ISBN 0-738-20667-9.

The Mathematical Explorer, by Stan Wagon. Electronic book, Wolfram Research, Inc., 2001. (Reviewed June/July 2002.)

Mathematical Reflections, by Peter Hilton, Derek Holton, and Jean Pedersen. Springer, December 1996. ISBN 0-387-94770-1. (Reviewed February 2003.)

Mathematical Vistas, by Peter Hilton, Derek Holton, and Jean Pedersen. Springer-Verlag, January 2002. ISBN 0-387-95064-8. (Reviewed February 2003.)

A Mathematician Grappling with His Century: The Autobiography of Laurent Schwartz. Translated from the French by L. Schneps. Birkhäuser, 2001. ISBN 3-7643-6052-6.

The Mathematician Sophus Lie: It Was the Audacity of My Thinking, by Arild Stubhaug. Springer, 2002. ISBN 3-540-42137-8.

Mathematics and the Roots of Postmodern Thought, by Vladimir Tasic. Oxford University Press, 2001. ISBN 0-195-13967-4.

Mathematics Elsewhere: An Exploration of Ideas across Cultures, by Marcia Ascher. Princeton University Press, September 2002. ISBN 0-691-07020-2.

Mathematics Galore: Masterclasses, Workshops, and Team Projects in Mathematics and Its Applications, by C. J. Budd and C. J. Sangwin. Oxford University Press, June 2001. ISBN 0-198-50769-0 (hardcover), 0-198-50770-4 (paperback). (Reviewed September 2002.)

Mathematics in a Postmodern Age: A Christian Perspective, Russell W. Howell and W. James Bradley, editors. Wm. B. Eerdmans Publishing Company, May 2001. ISBN 0-802-84910-5.

The Mathematics of Oz: Mental Gymnastics from beyond the Edge, by Clifford Pickover. Cambridge University Press, October 2002. ISBN 0-521-01678-9.

The Millennium Problems: The Seven Greatest Unsolved Mathematical Puzzles of Our Time, by Keith J. Devlin. Basic Books, October 2002. ISBN 0-465-01729-0.

More Mathematical Astronomy Morsels, by Jean Meeus. Willmann-Bell Inc., 2002. ISBN 0-943396-743.

A New Kind of Science, by Stephen Wolfram. Wolfram Media, Inc., May 2002. ISBN 1-579-55008-8. (Reviewed February 2003.)

Nexus: Small Worlds and the Groundbreaking Science of Networks, by Mark Buchanan. W. W. Norton & Company, May 2002. ISBN 0-393-04153-0.

Niels Hendrik Abel and His Times: Called Too Soon by Flames Afar, by Arild Stubhaug; translated by R. Daly. Springer, May 2000. ISBN 3-540-66834-9. (Reviewed August 2002.)

Political Numeracy: Mathematical Perspectives on Our Chaotic Constitution, by Michael Meyerson. W. W. Norton & Company, March 2002. ISBN 0-393-04172-7.

Puzzlers' Tribute: A Feast for the Mind, Tom Rodgers and David Wolfe, editors. A K Peters, December 2001. ISBN 1-56881-121-7.

The Rainbow Bridge: Rainbows in Art, Myth, and Science, by Raymond L. Lee Jr. and Alistair B. Fraser. Pennsylvania State University Press and SPIE Press, 2001. ISBN 0-271-01977-8. (Reviewed December 2002.)

The Riddle of the Compass, by Amir Aczel. Harcourt Brace, August 2001. ISBN 0-151-00506-0.

Science and an African Logic, by Helen Verran. University of Chicago Press, January 2002. ISBN 0-226-85389-6 (cloth), 0-226-85391-8 (paperback).

The Science of Conjecture: Evidence and Probability before Pascal, by James Franklin. Johns Hopkins University Press, June 2001. ISBN 0-8018-6569-7.

Signs of Life: How Complexity Pervades Biology, by Richard Solé and Brian Goodwin. Basic Books, January 2001. ISBN 0-465-01927-7.

Spaceland, by Rudy Rucker. Tor Books, June 2002. ISBN 0-765-30366-3.

Statisticians of the Centuries, C. C. Heyde and E. Seneta, editors. Springer, September 2001. ISBN 0-387-953283-7.

The Story of Mathematics, by Richard Mankiewicz. Princeton University Press, February 2001. ISBN 0-691-08808-X. (Reviewed April 2002.)

Such Silver Currents: The Story of William and Lucy Clifford, 1845–1929, by M. Chisholm. Lutterworth Press, March 2002. ISBN 0-7188-3017-2.

The Unfinished Revolution: Human-Centered Computers and What They Can Do for Us, by Michael L. Dertouzos. Harperbusiness, January 2001. ISBN 0-066-62067-8.

The Universe in a Nutshell, by Stephen Hawking. Bantam Doubleday Dell, November 2001. ISBN 0-553-80202-X . (Reviewed May 2002.)

Wavelets through a Looking Glass: The World of the Spectrum, by Ola Bratteli and Palle Jorgensen. Birkhäuser/ Springer, 2002. ISBN 0-8176-4280-3.

What Are the Odds? The Chances of Extraordinary Events in Everyday Life, by Jefferson Hane Weaver. Prometheus Books, February 2002. ISBN 1-573-92933-6.

What Shape Is a Snowflake?, by Ian Stewart. W. H. Freeman & Co., November 2001. ISBN 0-716-74794-4. (Reviewed December 2002.)

The Zen of Magic Squares, Circles, and Stars: An Exhibition of Surprising Structures across Dimensions, by Clifford A. Pickover. Princeton University Press, January 2001. ISBN 0-691-07041-5. (Reviewed in this issue.)

From the AMS Secretary

Each of the five AMS policy committees, which report to the AMS Council, has one face-to-face meeting annually. Two of them meet in the spring; three in the fall. This is a report on the fall 2002 meetings of the Committee on Publications, held in Chicago on September 13–14, 2002; the Committee on the Profession, held in Chicago on September 21, 2002; and the Committee on Education, held in Washington, DC, on October 25–26, 2002.

Committee on Publications Highlights

As part of the regular review cycle of the Society's publishing program, a subcommittee, chaired by Gail Ratcliff, carried out a review of the AMS electronic-only journals. A final report was given at the September meeting of the Committee on Publications (Cpub). The subcommittee received many positive remarks as a result of its surveys and concluded that the journals were doing well. The report mentions that authors are happy with the speed of publication, the two specialized journals are attracting authors whose work fits into their specific areas, and all the editorial boards are working well.

As a result of discussions held by Cpub in 2001, the AMS now features a page specially designed for journal authors, and access to AMS electronic journal back files will now be free five years after publication. The AMS journals posted on JSTOR do not fall under this policy.

The executive director mentioned that momentum is building for a coordinated effort to put "all past mathematical literature" online as a Digital Mathematics Library (DML). Participants have agreed that the project must be international in scope, loosely under the International Mathematical Union (IMU). Some of the key goals of this initiative are to coordinate all ongoing efforts and to propose a good initial model which will concentrate on journal collections.

One major issue connected with this project is the archiving of material in a suitable format under the control of the academic community. There is currently little specific action needed from the AMS for the DML other than to continue its basic support of the concept. The Society has adopted a policy on copyright, however, that ultimately may impede future work on the DML. In 2003 this policy will be reviewed by the executive director and the Committee on Publications in light of the changing environment.

Cpub discussed the ongoing focused planning process by AMS staff and was asked to make suggestions about the planning process in the areas of transition to electronic journals, the production process within the publication environment, the issues for *Math Reviews*, and the transition from paper to electronic publishing. Data is being gathered for a cost benefit discussion and the practices of other publishers. The committee will also look at the proposed author resource center and Web-based upload tool that would allow journal authors to submit manuscripts more efficiently to the AMS.

The publisher provided the traditional report on the Society's book program, stating that the AMS staff had carried out a detailed analysis of the book program finances as well as many of the associated business practices. As a result, a lot of changes have been made in marketing and distribution and to the production environment. The analysis shows that even though the book program is bigger and better than ten years ago, it is still paying only for its direct expenses. As a result, further changes will need to be made in distribution, pricing, acquisitions, and the overall program mission and goals. Other recommendations include the renegotiation of many of the copublication and distribution agreements, and a better process for ushering book projects through the system, from initial contract to final publication. One clear conclusion is that the Society must expand the scope of the book program if it wants to expand its financial base. This will mean publishing more books in certain areas that traditionally have not been the strength of the AMS book publishing program.

The publisher reported on journal backlogs, concluding that the problem mostly results from the refereeing side. The proposed central processing of papers at the AMS, already successful with the *Bulletin*, is seen as a possible solution.

Also presented were a report on *Math Reviews* by the executive editor and a report on the *Bulletin* and *Notices* by the publisher.

Committee on the Profession Highlights

The Committee on the Profession (CoProf) discussed a proposal for a new AMS Book Prize; a recommendation that the AMS Council establish the prize was approved by email following the meeting. The Book Prize would recognize a single, relatively recent, outstanding research book that makes a seminal contribution to the research literature, reflects the highest standards of research exposition, and promises to have a deep and long-term impact in its area. Books published within the six calendar years prior to the year in which nominations are due would be eligible. The prize would be awarded every three years, and the amount would be $5,000. CoProf also discussed a proposal to establish an AMS award to recognize outstanding achievement by a mathematics department, but ultimately decided to hold this proposal over for further consideration at its meeting next year.

CoProf endorsed a recommendation that the AMS implement the option of electronic voting for the 2003 elections by hiring an electronic balloting firm to handle the process via the Internet. Other societies, such as the American Physical Society, have recently implemented electronic balloting, with a significant increase in member voting rates and a reduction in costs.

CoProf approved recommending to the Council changes in the bylaws governing life membership. The recommendation offers two options for rewording the relevant section of the bylaws. One gives the Council the authority to determine eligibility and dues levels for this category of membership, subject to the usual approval of the dues by the Board of Trustees. The other proposal includes language that describes factors the Council must consider in establishing the dues amounts. Both of the options will be forwarded for Council consideration at its January 2003 meeting.

The Committee reviewed the draft document outlining the focused planning process for membership, now scheduled to take place during 2003. Members of the Committee viewed the document quite favorably, and they suggested several new issues for exploration during the planning effort. The chair will appoint a subcommittee to react to specific topics, questions, and issues that arise as the project progresses over the coming year. In addition, CoProf selected membership as the area for annual review during the coming year.

The committee reviewed two areas of Society activity that fall within its charge: human rights and professional ethics. Separate subcommittees formed to review these areas each presented reports. No significant problems were reported for either area of activity.

Committee on Education Highlights

The Committee on Education (CoE) discussed the impact that President Bush's "No Child Left Behind" law may have on K–12 mathematics education, the professional training and development of teachers of mathematics, and the funding of research in mathematics education through the National Science Foundation (NSF) and the Department of Education. Visitors included representatives from the NSF, the Department of Education, and other mathematical and educational groups. Panels of mathematicians involved in education offered inside views on teaching mathematics courses designed for future K–12 teachers, and the impact of NSF's VIGRE (Vertical Integration of Research and Education in the Mathematical Sciences) program on U.S. mathematics departments. Chairs of doctorate-granting departments of mathematics were again invited to participate in the meeting. A total of thirty-nine participants were involved in the meeting.

National Science Foundation

Judith Ramaley, assistant director of the NSF's Directorate of Education and Human Resources, outlined the new Math and Science Partnerships (MSP) program, created to implement an important part of the president's "No Child Left Behind" initiative. The NSF will be the lead funding agency, with some funding coming through the Department of Education. Janice Earle, of NSF's Elementary, Secondary and Informal Education Division, also provided details on the MSP program's goals: (1) to ensure that all students have access, are prepared, and are encouraged to participate and succeed; (2) to enhance the quality, quantity, and diversity of teachers; and (3) to develop evidence-based outcomes. Ramaley discussed the projects funded in the first round of awards: seven comprehensive projects, seventeen targeted, and twelve smaller awards for development of an MSP Learning Network. Successful proposals were partnership-driven, in addition to addressing teacher quality and quantity, offering challenging courses, promising evidence-based design and outcomes, and providing evidence of likely institutional change and sustainability. The year 2003 will see the creation of the Learning Network, the second round of awards, and development of teacher institutes. Solicitations had not yet been written, and Ramaley and Earle invited CoE input, especially on the proposed teacher institutes and the Learning Network. Earle also described the Centers for Learning and Teaching program and mentioned the opportunities and the frustrations of NSF's current focus on large partnership-oriented programs. Calvin Williams, Division of Undergraduate Education, and Eric Sheppard, Division of Graduate Education, discussed details of programs within their divisions. There was discussion of the impact of increased stipends in the graduate research fellowship program.

U.S. Department of Education

Patricia O'Connell Ross described the department's participation in the Math and Science Partnerships Program, which will provide more money to state and local authorities, but also require accountability. There will be a major rethinking of Title I programs, with schools held to much more rigorous standards in mathematics and reading. Teacher development will receive more funding. President Bush will announce a major initiative in mathematics and science in early December. A concern is that, unlike reading, there is not a strong research base in mathematics education, and one of the department's goals is to develop a major research program on successful learning in mathematics. There was discussion about a design proposal approved in September to examine the mathematical preparation of teachers, the people to be involved in that project, and its connection to the Conference Board of Mathematical Sciences Mathematical Education of Teachers project. Ross announced that OERI (Office of Education Research and Improvement) will be replaced by an Institute for Educational Sciences, created as a more autonomous organization to conduct program evaluation.

Although both the NSF and the Department of Education fiscal year 2003 appropriations are still held up in Congress, Ross noted that the 50 percent increase for research in the department's budget has held up so far in the federal budget process.

Panel Discussion on Teacher Development

Scott Baldridge (Indiana University), Sybilla Beckmann (University of Georgia), and Gary Jensen (Washington University) talked about courses and methods used in their departments for education majors and stimulated discussion on the thought necessary for course design, development of materials, the mathematical content, the importance of knowing the students' backgrounds, teaching future teachers what they will need in their classrooms, and the issues of grading.

Panel Discussion on NSF's VIGRE (Vertical Integration of Graduate Research and Education) Program

Richard Hain (Duke University) and Robert Lazarsfeld (University of Michigan) discussed the impact of VIGRE programs on their departments. CoE members also received the report from a recent workshop on the program. Both visitors felt the VIGRE program had been a positive experience for their institutions, but they discussed with CoE members some concerns and possible consequences: for instance, the long-term stability of the program and the impact on a department that suddenly loses such a large award, the sustainability of support for postdoctorates, the emphasis on change rather than on quality of instruction, and changes in the VIGRE management team and the possibility of changes in the criteria for evaluation.

Participation of Research Mathematicians in Mathematics Education

Roger Howe distributed a draft proposal to develop a cadre of mathematicians experienced and willing to act as professional consultants on mathematics education, to provide support for them so they can successfully interact with educators, and to introduce educational administrators to the support that mathematicians can provide. The idea arose as a result of the many ad hoc requests he and other mathematicians receive for help on educational projects and would provide a way to professionalize and recognize this activity. Howe invited CoE members to comment on the draft proposal.

Brookings Institution

Tom Loveless discussed the recent Brown Center Report "How Well Are American Students Learning?" His analysis of trends in NAEP (National Assessment of Education Progress) test data (for ages 9, 13, and 17) led him to conclude that computation skills (especially fractions) have either stagnated or lost ground since 1990. Discussion included possible reasons, including the impact of NCTM (National Council of Teachers of Mathematics) standards and the use of calculators.

Achieve, Inc.

Achieve, Inc., was created in 1996 after a governors' summit meeting on education as an independent body working to achieve agreement across states on strategies to improve K–12 education. Fifteen states are currently involved. Matt Gandal described the problems of misalignment of standards, tests, and assessment. State standards demonstrated coverage of content but not focus, tests are not measuring what is being taught, and therefore cross-state comparison of results is not possible. Maria Santos discussed the further challenges posed by the new "No Child Left Behind" law, which requires every state to test in mathematics in grades 3–8 and high school. Achieve's Mathematics Achievement Partnership (MAP) focuses on middle school grades, offering support to states on standards, professional development for teachers, testing and assessment, and cross-state comparisons. CoE members received a consultation draft of "Foundations for Success: Mathematics Expectations for the Middle Grades" and sample items from the MAP proto-test. Many mathematicians have been involved in the advisory panel for the development of the "Foundations for Success" report. CoE members were interested in plans for the long-term continuation of this very ambitious undertaking.

MAA Guidelines for Programs and Departments in Undergraduate Mathematical Sciences (rev 2000)

MAA has requested the committee's endorsement of the report "MAA Guidelines for Programs and Departments in Undergraduate Mathematical Sciences". After discussion, CoE approved the endorsement of the principles on which the report was based and recommended that the AMS Council make a similar endorsement. This recommendation has been submitted to the Council.

Mathematical Sciences Education Board (MSEB)

Carole Lacampagne reported on MSEB's planned activities for the next ten years. Priority areas include learning, instruction, assessment, equity, attracting and retaining students, evidence of effectiveness, teacher development, and public perceptions. Current projects include a review of evaluation data on effectiveness of curriculum materials. She will send the committee information on the texts to be reviewed and on forthcoming workshops and conferences.

MAA Committee on Undergraduate Programs in Mathematics (CUPM)

Michael Pearson reported that CUPM is working on an update of curriculum guidelines, to appear in 2003, and outlined some general recommendations in the current draft (a publicly releasable draft was not available at the time of the meeting). Several CoE members had provided reactions to the draft guidelines at the summer 2002 MathFest in Burlington, and Pearson said that the current draft reflects reviews received at that time, although further rewriting was still to be done.

Review of AMS Educational Activities

CoE established a five-year cycle of reviews, beginning with a review of the Young Scholars Program in 2003, to be conducted by a subcommittee appointed by the chair. In 2004 CoE will look at graduate education, and in 2005 it will review education committees in which the AMS is involved. Areas for review in 2006 and 2007 will be decided at a future time.

AMERICAN MATHEMATICAL SOCIETY

Associate Secretary

(Southeastern Section)

Position

The American Mathematical Society is seeking applications and nominations of candidates for the post of Associate Secretary of the Southeastern Section. That section consists of the states southeast of the boundary determined by Virginia, West Virginia, Kentucky, Arkansas, and Louisiana, each of which is in the section. John L. Bryant, the current Associate Secretary there, wishes to step down when his present term ends on 31 January 2005. Whenever possible, the AMS identifies a new Associate Secretary well before the appointment begins, in order for him/her to observe a full year's cycle of the duties.

An Associate Secretary is an officer of the Society and is appointed by the Council to a two-year term, ordinarily beginning on 01 February. In this case the term would begin 01 February 2005 and end 31 January 2007. Reappointments are possible and desirable. All necessary expenses incurred by an Associate Secretary in performance of duties for the Society are reimbursed, including travel and communications.

Duties

The primary responsibility of an Associate Secretary is to oversee scientific meetings of the Society in the section. Once every four years an Associate Secretary has primary responsibility for the Society's program at the January Joint Mathematics Meetings. An Associate Secretary is a member of the Secretariat, a committee consisting of the four Associate Secretaries and the Secretary, which approves all applications for membership in the Society and approves all sites and dates of meetings of the Society. Occasionally an Associate Secretary is in charge of an international joint meeting. Associate Secretaries are the principal contact between the Society and its members in the various sections. They are invited to all Council meetings and have a vote on the Council on a rotating basis.

Applications

An Associate Secretary is appointed by the Council upon recommendation by the Executive Committee and Board of Trustees. Applications—including a brief CV and names of three references—should be sent to: Robert J. Daverman, Secretary, American Mathematical Society, 312D Ayres Hall, University of Tennessee, Knoxville TN 379961330; email: daverman@math.utk.edu.

Applications received by 30 June 2003 will be assured full consideration.

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AMS
AMERICAN MATHEMATICAL SOCIETY

Add this Cover Sheet to all of your Academic Job Applications

How to use this form

1. Using the facing page or a photocopy, (or visit the AMS web site for a choice of electronic versions at `www.ams.org/coversheet/`), fill in the answers which apply to *all* of your academic applications. Make photocopies.

2. As you mail each application, fill in the remaining questions neatly on one cover sheet and include it *on top of* your application materials.

The purpose of the cover form is to aid department staff in tracking and responding to each application for employment. Mathematics departments in Bachelor's-, Master's-, and Doctorate-granting institutions are expecting to receive the form from each applicant, along with the other application materials they require.

The AMS suggests that applicants and employers visit the Job Application Database for Mathematicians (`www.mathjobs.org`), a new electronic resource being offered by the AMS (in partnership with Duke University) for the second year in 2002-03. The system provides a way for applicants to produce printed coversheet forms, apply for jobs, or publicize themselves in the "Job Wanted" list. Employers can post a job listing, and once applications are made, search and sort among their applicants. Note-taking, rating, e-mail, data downloading and customizable EOE funnc-tions are available to employers. Also, reference writers can submit their letters online. A paperless application process is possible with this system, however; employers can choose to use any portion of the service. There will be annual employer fees beginning this year. This system was developed at the Duke University Department of Mathematics.

Please direct all questions and comments to: `emp-info@ams.org`.

Academic Employment in Mathematics

AMS Standard Cover Sheet

Last Name ______________________

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Middle Names ______________________

Address *through next June* ______________________ **Home Phone** ______________________

______________________ **e-mail Address** ______________________

Current Institutional Affiliation ______________________ **Work Phone** ______________________

Highest Degree Held or Expected ______________________

Granting Institution ______________________ **Date (optional)** ______________________

Ph.D. Advisor ______________________

Ph.D. Thesis Title (optional) ______________________

This form is provided courtesy of the American Mathematical Society.

This cover sheet is provided as an aid to departments in processing job applications. It should be included with your application material.

Please print or type. Do not send this form to the AMS.

***Indicate** the mathematical subject area(s) in which you have done research using the Mathematics Subject Classification printed on the back of this form or on the AMS website. Use the two-digit classification which best fits your interests in the Primary Interest line and additional two-digit numbers in the Secondary Interest line.*

Primary Interest ☐☐ ______________________

Secondary Interests *optional* ☐☐ ☐☐ ______________________

***Give a brief synopsis** of your current research interests (e.g. finite group actions on four-manifolds). Avoid special mathematical symbols and please do not write outside of the boxed area.*

____

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Most recent, if any, position held *post Ph.D.*

University or Company ______________________

Position Title ______________________

Indicate the position for which you are applying and position posting code, if applicable

____

If unsuccessful for this position, would you like to be considered for a temporary position?

☐ Yes ☐ No If yes, please check the appropriate boxes.

☐ Postdoctoral Position ☐ 2+ Year Position ☐ 1 Year Position

***List the names, affiliations, and e-mail addresses** of up to four individuals who will provide letters of recommendation if asked. Mark the box provided for each individual whom you have already asked to send a letter.*

☐ ______________________

☐ ______________________

☐ ______________________

☐ ______________________

2000 Mathematics Subject Classification

00 General
01 History and biography
03 Mathematical logic and foundations
05 Combinatorics
06 Order, lattices, ordered algebraic structures
08 General algebraic systems
11 Number theory
12 Field theory and polynomials
13 Commutative rings and algebras
14 Algebraic geometry
15 Linear and multilinear algebra, matrix theory
16 Associative rings and algebras
17 Nonassociative rings and algebras
18 Category theory, homological algebra
19 *K*-theory
20 Group theory and generalizations
22 Topological groups, Lie groups
26 Real functions
28 Measure and integration
30 Functions of a complex variable
31 Potential theory
32 Several complex variables and analytic spaces
33 Special functions
34 Ordinary differential equations
35 Partial differential equations
37 Dynamical systems and ergodic theory
39 Difference and functional equations
40 Sequences, series, summability
41 Approximations and expansions
42 Fourier analysis
43 Abstract harmonic analysis
44 Integral transforms, operational calculus
45 Integral equations
46 Functional analysis
47 Operator theory
49 Calculus of variations and optimal control, optimization
51 Geometry
52 Convex and discrete geometry
53 Differential geometry
54 General topology
55 Algebraic topology
57 Manifolds and cell complexes
58 Global analysis, analysis on manifolds
60 Probability theory and stochastic processes
62 Statistics
65 Numerical analysis
68 Computer science
70 Mechanics of particles and systems
74 Mechanics of deformable solids
76 Fluid mechanics
78 Optics, electromagnetic theory
80 Classical thermodynamics, heat transfer
81 Quantum theory
82 Statistical mechanics, structure of matter
83 Relativity and gravitational theory
85 Astronomy and astrophysics
86 Geophysics
90 Operations research, mathematical programming
91 Game theory, economics, social and behavioral sciences
92 Biology and other natural sciences
93 Systems theory, control
94 Information and communication, circuits
97 Mathematics education

Mathematics Calendar

The most comprehensive and up-to-date Mathematics Calendar information is available on e-MATH at `http://www.ams.org/mathcal/`.

March 2003

3-7 **Computational Aspects of Algebraic Curves, and Cryptography**, Gainesville, Florida. (Feb. 2003, p. 290)

3-7 **Mini-courses in Cryptography**, Gainesville, Florida. (Feb. 2003, p. 290)

6-8 **Sixth New Mexico Analysis Seminar**, University of New Mexico, Albuquerque, New Mexico. (Jan. 2003, p. 79)

10-12 **International Conference on Mathematics and Its Applications**, Kuwait University, Kuwait. (Sept. 2002, p. 998)

10-14 **International Conference on High Performance Scientific Computing**, Hanoi, Vietnam. (Feb. 2003, p. 290)

11 **IMA Tutorial: Semidefinite Programming and Robust Optimization**, Minneapolis, Minnesota. (Sept. 2002, p. 998)

12-14 **DIMACS Working Group Meeting on Mathematical and Computational Aspects Related to the Study of the Tree of Life**, DIMACS Center, Rutgers University, Piscataway, New Jersey. (Jan. 2003, p. 79)

12-19 **IMA Workshop 5: Semidefinite Programming and Robust Optimization**, Minneapolis, Minnesota. (Sept. 2002, p. 998)

14-16 **AMS Southeastern Section Meeting**, Louisiana State University, Baton Rouge, Louisiana. (Sept. 2002, p. 998)

15-19 **Arizona Winter School on "Logic and Number Theory"**, University of Arizona, Tucson, Arizona. (Jan. 2003, p. 79)

15-20 **Recent Developments in Superstring Theory**, Banff International Research Station, Banff, Alberta, Canada. (Dec. 2002, p. 1418)

*16-17 **13th Iranian Seminar of Analysis and Its Application**, University of Isfahan, Isfahan, Iran.
Organizer: M. R. Pouryayevali.
Information: email: `analysis@ui.ac.ir`.

17-19 **DIMACS Workshop on Network Information Theory**, DIMACS Center, Rutgers University, Piscataway, New Jersey. (Feb. 2003, p. 290)

17-20 (REVISED) **SIAM Conference on Mathematical and Computational Issues in the Geosciences**, Radisson Hotel and Suites Austin, Austin, Texas. (Sept. 2002, p. 998)

17-June 13 **Symplectic Geometry and Physics**, Institute for Pure and Applied Mathematics, Los Angeles, California. (Oct. 2002, p. 1134)

20-22 **Spring Topology and Dynamical Systems Conference 2003**, Texas Tech University, Lubbock, Texas. (Oct. 2002, p. 1134)

21-23 **Fourth International Conference on Intelligent Data Engineering and Automated Learning (IDEAL'03)**, Hong Kong Convention and Exhibition Centre, Hong Kong, P.R. China. (Feb. 2003, p. 291)

*21-23 **Number Theory and Combinatorics in Physics**, University of Florida, Gainesville, Florida.
Conference Topics: q-Hypergeometric functions, Rogers-Ramanujan identities, exact integrable models and conformal field theory; Quantum computing; Riemann zeta-function and random matrices; Enumerative combinatorics and lattice models; Alternating sign matrices and determinants.
Plenary Speakers: G. Andrews, Penn. State Univ.; R. Calderbank, Amer. AT&T Lab. Res.; P. Di Francesco, CEA, Saclay; D. Gottesman,

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This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.

An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.

In general, announcements of meetings and conferences held in North America carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. Meetings held outside the North American area may carry more detailed information. In any case, if there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences in the mathematical sciences should be sent to the Editor of the *Notices* in care of the American Mathematical Society in Providence or electronically to `notices@ams.org` or `mathcal@ams.org`.

In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the *Notices* prior to the meeting in question. To achieve this, listings should be received in Providence **six months** prior to the scheduled date of the meeting.

The complete listing of the Mathematics Calendar will be published only in the September issue of the *Notices*. The March, June, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.

The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through the AMS website on the World Wide Web. To access the AMS website, use the URL: `http://www.ams.org/`.

Berkeley and Perimeter Inst. in Waterloo; B. McCoy, Stony Brook; T. Miwa, RIMS, Kyoto; J. Propp, Univ. of Wisconsin; M. Rubinstein, AIM, Palo Alto; Y. Stroganov, IHEP, Protvino; F. Y. Wu, Northeastern Univ.
Organizers: F. Garvan and K. A. Muttalib.
Information: http://www.math.ufl.edu/~frank/qmiftconf.html.

22-27 **Scattering and Inverse Scattering**, Banff International Research Station, Banff, Alberta, Canada. (Dec. 2002, p. 1418)

23-29 **INGO2003: Invariant Theory and Its Interactions with Related Fields**, University of Göttingen, Germany. (Jan. 2003, p. 79)

24-26 **The 2nd International Conference "Functional Spaces. Differential Operators. Problems of Mathematical Education"**, Russian People Friendship University (RPFU), Moscow, Russia. (Feb. 2003, p. 291)

24-28 **Mathematical Theory of Hyperbolic Systems of Conservation Laws**, Newton Institute of Mathematical Sciences, Cambridge, UK. (Jan. 2003, p. 79)

24-28 **Symplectic Geometry and Physics: Workshop I: Symplectic Geometry**, Institute for Pure and Applied Mathematics, UCLA, Los Angeles, California. (Oct. 2002, p. 1134)

* 24-28 **Workshop on Proof Theory and Algorithms**, International Centre for Mathmatical Sciences, Edinburgh, UK.
Program: The aim of the workshop is to promote interaction between traditional proof theory and a more structural mathematical proof theory. The organisers hope to encourage the application-oriented to consider their tools more abstractedly and those with a foundational leaning to focus on possible applications.
Participation: The meeting is by invitation only (numbers are strictly limited).
Information: http://www.ma.hw.ac.uk/icms/meetings/2003/proof/.

27-30 **Quadrature Domains and Related Topics**, University of California, Santa Barbara. (Sept. 2002, p. 998)

27-30 **Sixth IMACS International Symposium on Iterative Methods in Scientific Computing**, University of Colorado, Denver, Colorado. (Oct. 2002, p. 1134)

28-30 **Southeast Geometry Conference**, College of Charleston, Charleston, South Carolina. (Nov. 2002, p. 1287)

29-April 3 **Commutative Algebra and Geometry**, Banff International Research Station, Banff, Alberta, Canada. (Dec. 2002, p. 1418)

* 30-April 5 **Workshop on SDEs and SPDEs: Numerical Methods and Applications**, International Centre for Mathematical Science, Edinburgh, UK.
Program: The workshop will focus on recent advances in stochastic (partial) differential equations, especially numerical methods and applications.
Deadline: The application period closes on January 10, 2003.
Information: http://www.ma.hw.ac.uk/icms/meetings/2003/sde/.

31-April 2 **Tutorial on Olfaction, Audition, and the Sensory Motor System**, Mathematical Biosciences Institute (MBI), The Ohio State University, Columbus, Ohio. (Feb. 2003, p. 291)

31-April 4 **Multiphase Fluid Flows and Multi-dimensional Hyperbolic Problems**, Newton Institute of Mathematical Sciences, Cambridge, UK. (Feb. 2003, p. 291)

April 2003

2-4 **DIMACS Workshop on Medical Applications in Computational Geometry**, DIMACS Center, Rutgers University, Piscataway, New Jersey. (Feb. 2003, p. 291)

3-5 **Workshop on Olfaction**, Mathematical Biosciences Institute (MBI), The Ohio State University, Columbus, Ohio. (Feb. 2003, p. 291)

4-6 **AMS Central Section Meeting**, Indiana University, Bloomington, Indiana. (Sept. 2002, p. 999)

* 5 **Announcement and Call for Participation: East Coast Computer Algebra Day 2003**, Clemson University, South Carolina.
Themes: Algebraic algorithms, hybrid symbolic-numeric computation, computer algebra systems and generic programming, mathematical communication, complexity of algebraic problems.
Organizers: S. Gao, sgao@math.clemson.edu; D. Jacobs, dpj@cs.clemson.edu.
Invited Speakers: R. Brent, Oxford Univ., England, Primitive and almost primitive trinomials over GF(2); J. Johnson, Drexel Univ., USA, TBA; W. Schreiner, Johannes Kepler Univ., Austria, Distributed Maple—Lessons Learned on Parallel Computer Algebra in Distributed Environments.
Deadlines: Room reservations: February 19, 2003. Submitting poster abstracts: March 22, 2003. Travel support: February 19, 2003. Registration begins: February 1, 2003.
Information: http://www.math.clemson.edu/~sgao/ECCAD03/.

5-6 **CombinaTexas: Combinatorics Conference in the South-Central U.S.**, Southwest Texas State University, San Marcos, Texas. (Jan. 2003, p. 80)

5-10 **BIRS Workshop on Noncommutative Geometry**, Banff International Research Station, Banff, Alberta, Canada. (Dec. 2002, p. 1419)

6 **IMA Tutorial: Network Management and Design**, Minneapolis, Minnesota. (Sept. 2002, p. 999)

7-11 **EuroConference—Multiscale Modelling, Multiresolution and Adaptivity**, Isaac Newton Institute for Mathematical Sciences, Cambridge, England. (Feb. 2003, p. 291)

7-11 **IMA Workshop 6: Network Management and Design**, Minneapolis, Minnesota. (Sept. 2002, p. 999)

10-13 **Louisiana Conference on Mathematical Control Theory**, Louisiana State University, Baton Rouge, Louisiana. (Oct. 2002, p. 1134)

12-13 **AMS Eastern Section Meeting**, Courant Institute, New York, New York. (Sept. 2002, p. 999)

12-17 **Quantum Mechanics on the Large Scale**, Banff International Research Station, Banff, Alberta, Canada. (Dec. 2002, p. 1419)

14-17 **Numerical Methods for Free Boundary Problems**, Isaac Newton Institute for Mathematical Sciences, Cambridge, England. (Feb. 2003, p. 291)

17-19 **The Sixth Asian Symposium on Computer Mathematics (ASCM 2003)**, Beijing, China. (Sept. 2002, p. 999)

19-24 **Computational Fuel Cell Dynamics-II**, Banff International Research Station, Banff, Alberta, Canada. (Dec. 2002, p. 1419)

26-May 1 **The Many Aspects of Mahler's Measure**, Banff International Research Station, Banff, Alberta, Canada. (Dec. 2002, p. 1419)

* 27-May 3 **Spring School on Analysis: Variational Analysis**, Paseky nad Jizerou, Czech Republic.
Topics: Eigenvalue optimization, nonsmooth analysis, analysis of value functions, variational analysis in metric spaces, sequential variational analysis in infinite dimensions.
Main Speakers: A. Ioffe (Technion Univ.), A. Lewis (Simon Fraser Univ.), B. S. Mordukhovich (Wayne State Univ.), J.-P. Penot (Univ. of Pau, France).
Organizers: J. Lukes, M. Fabian, J. Outrata (Charles Univ. and Acad. of Sci., Prague, Czech Republic).
Deadlines: For a reduced fee and support: January 15, 2003. Grants: Probably support for a limited number of students.

Information: email: `paseky@karlin.mff.cuni.cz`; `http://www.karlin.mff.cuni.cz/katedry/kma/ss/apr03/ss.htm`.

28–June 20 **Bimestre Intensivo: Microlocal Analysis and Related Subjects**, University of Torino, and Politecnico of Torino, Italy. (Feb. 2003, p. 291)

May 2003

1–3 (REVISED) **SIAM International Conference on Data Mining**, Cathedral Hill Hotel, San Francisco, California. (Sept. 2002, p. 999)

* 1–4 **Great Lakes Geometry Conference**, University of Wisconsin-Madison.

Focus: On geometric analysis and will be held in honor of E. Calabi's 80th birthday.

Invited speakers: J. P. Bourguignon (IHES); H. Bray (MIT); J. Y. Chen (UBC); T. Colding (Courant Institute); F. Hirzebruch (MPI); J. Jost (MPI); B. Lawson (SUNY at Stony Brook); Z. Lu (UC at Irvine), J. Milgram (Stanford); X. Rong (Rutgers); Y. T. Siu (Harvard); C. L. Terng (Northeastern Univ.); G. Tian (MIT); J. P. Wang (Univ. of Minnesota).

Organizers: A. Adem (Univ. Wisconsin-Madison); X. X. Chen (Univ. Wisconsin-Madison); J. Cheeger (Courant Institute); H. Glucker (Univ. of Pennsylvania).

Information: Blocks of rooms have been reserved at Friedrich Center, 1950 Willow Drive, phone: 608-231-1341 and Lowell Hall, 610 Langdon St., phone: 608-256-2621. Participants are responsible for making their own lodging arrangements.

Deadline: The deadline for making reservations is March 30, 2003.

* 2–13 **International Conference and Spring School in Conjunction with the 18th Shanks Lectures on "Noncommutative Geometry and Applications"**, Vanderbilt University, Nashville, Tennessee.

Description: The meeting will feature a lecture series by A. Connes and several minicourses by leading experts in noncommutative geometry and its applications to physics and geometry. In addition there will be a number of invited research talks and short contributions. Graduate students and postdocs are especially encouraged to participate. We expect that (partial) funding will be available.

Principal Speaker: A. Connes, IHES and College de France.

Invited Speakers: J. Bellissard, Georgia Inst. of Tech.; J. Cuntz, Univ. of Muenster; N. Higson, Penn State Univ.; G. Kasparov, Vanderbilt Univ.; A. Konechny, Hebrew Univ.; D. Kreimer, Boston Univ.; H. Moscovici, Ohio State Univ.; M. Pimsner, Univ. of Pennsylvania; M. Rieffel, UC Berkeley; J. Roe, Penn State Univ.; J. Varilly, Univ. de Costa Rica.

Organizers: D. Bisch, B. Hughes, G. Kasparov, and G. Yu.

Information: `http://atlas.math.vanderbilt.edu/~shanks/`.

3–4 **AMS Western Section Meeting**, San Francisco State University, California. (Sept. 2002, p. 999)

3–5 **First International Conference on Smarandache Geometries**, Griffith Univ., Gold Coast Campus, Australia. (Dec. 2001, p. 1368)

3–8 **Recent Advances in Algebraic and Enumerative Combinatorics**, Banff International Research Station, Banff, Alberta, Canada. (Dec. 2002, p. 1419)

5 **IMA Tutorial: Data Analysis and Optimization**, Minneapolis, Minnesota. (Sept. 2002, p. 999)

5–9 **Théorie des Nombres et Applications**, Université Hassan II of Casablanca and Université de Marrakech, Morocco. (Feb. 2003, p. 291)

5–9 **Workshop on the Auditory System**, Mathematical Biosciences Institute (MBI), The Ohio State University, Columbus, Ohio. (Feb. 2003, p. 292)

6–9 **IMA Workshop 7: Data Analysis and Optimization**, Minneapolis, Minnesota. (Sept. 2002, p. 999)

10–15 **Statistical Mechanics of Polymer Models**, Banff International Research Station, Banff, Alberta, Canada. (Dec. 2002, p. 1419)

11–16 **International Conference on General Control Problems and Applications (GCP 2003): Dedicated to the 100th Anniversary of A. N. Kolmogorov**, Tambov State University, Tambov, Russia. (Sept. 2002, p. 999)

11–18 **Conference on Topological Algebras, Their Applications, and Related Topics**, Poznan, Poland. (Sept. 2002, p. 999)

12–14 **CMEN 2003: Eleventh International Conference on Computational Methods and Experimental Measurements**, Halkidiki, Greece. (Dec. 2002, p. 1419)

15–16 **Partial Differential Equations and Computational Material Science and Solid Mechanics**, Isaac Newton Institute for Mathematical Sciences, Cambridge, England. (Feb. 2003, p. 292)

18–21 **Second International Workshop on Computer Graphics and Geometric Modeling, CGGM'2003**, Delta Centreville Hotel, Montreal, Canada. (Jan. 2003, p. 80)

18–23 **Applied Inverse Problems: Theoretical and Computational Aspects**, Institute for Pure and Applied Mathematics, UCLA, Los Angeles, California. (Oct. 2002, p. 1134)

19–21 **OPTI 2003: Eighth International Conference on Computer Aided Optimum Design of Structures**, Detroit, Illinois. (Dec. 2002, p. 1419)

19–21 **BETECH 2003: 15th International Conference on Boundary Element Technology**, Detroit, Illinois. (Dec. 2002, p. 1419)

19–22 **Workshop on Dynamics and Bifurcations of Patterns in Dissipative Systems**, Colorado State University, Fort Collins, Colorado. (Jan. 2003, p. 80)

* 19–23 **International Turbulence Workshop**, University of Central Florida, Orlando, Florida.

Announcement: Turbulence arises in practically all flow situations that occur naturally or in modern technological systems. On the other hand, the turbulence problem poses many formidable intellectual challenges and occupies a central place in modern nonlinear mathematics and statistical physics. The American Mathematical Society has listed the turbulence problem as one of the four top unsolved problems of mathematics. But, deep problems still remain to challenge conventional methodologies and concepts. There is also an ongoing need to bridge the gap between the grand theories of idealized turbulence and the harsh realities of practical applications. The turbulence problem continues to command the attention of physicists, applied mathematicians and engineers.

The workshop program envisages introductory overviews, specialist talks, contributed talks, as well as group discussions (one every day) to insure a positive experience for graduate students. We propose to bring out a document containing these summaries as well as overview talks for the benefit of the turbulence research community at large. We propose also to incorporate several intersession breaks in the Workshop program to provide excellent opportunities for graduate students and junior researchers to interact with leading researchers in the subject.

Invited Speakers: E. Bodenschatz (Cornell), G. Carnevale (Scripps Institution), P. Constantin (Univ. Chicago), G. Eyink (Univ. Arizona), G. Falkovich (Weizmann Institute), T. Gatski (NASA Langley), J. Gibbon (Imperial College), S. Girimaji (Texas A&M), J. Gollub (Haverford), S. Grossmann (Univ. Marburg), J. Herring (NCAR), D. Holm (Los Alamos), F. Hussain (Univ. Houston), S. Kida (Nagoya Univ.), J. Krommes (Princeton), M. Lesieur (Grenoble), D. Montgomery (Dartmouth), K. Ohkitani (Kyoto Univ.), J. Riley (Univ. Washington), S. Sarkar (Univ. Calif. San Diego), E. Sharon (Univ. Texas Austin), S. Shkoller (Univ. Calif. Davis), E. Titi (Univ. Calif. Irvine), Z. Warhaft (Cornell), V. Yakhot (Boston Univ.).

Information: Contributed oral presentations as well as participation in the workshop are invited. One-page abstracts (preferably by email) will be accepted until March 14, 2003. Acceptance of abstracts will be notified by April 1, 2003. Limited financial assistance may be available for graduate students and junior researchers. Please send all inquiries regarding the Workshop to: S. Shivamoggi, Dept. of Math., Univ. of Central Florida, Orlando, FL 32816-1384; tel: 407-823-2754; email: ijmms@pegasus.cc.ucf.edu.

19-23 **Symplectic Geometry and Physics: Workshop II: Chaotic Dynamics and Transport**, Institute for Pure and Applied Mathematics, UCLA, Los Angeles, California. (Oct. 2002, p. 1134)

19-24 **The V International Conference "Algebra and Number Theory: Modern Problems and Applications"**, Tula State Pedagogical University, Tula, Russia. (Feb. 2003, p. 292)

20-23 **31st Annual Canadian Operator Algebra Symposium**, UNB, Fredericton, New Brunswick, Canada. (Dec. 2002, p. 1419)

20-24 **31st Annual Canadian Operator Theory and Operator Algebras Symposium**, UNB, Fredericton, New Brunswick, Canada. (Sept. 2001, p. 910)

21-24 **Dynamic Systems and Applications**, Atlanta, Georgia. (Dec. 2002, p. 1419)

21-25 **Logic and Mathematics: Connections and Interactions**, University of Illinois at Urbana-Champaign. (Oct. 2002, p. 1134)

23-26 **International Conference on Numerical Analysis & Computational Mathematics (NACoM-2003)**, Anglia Polytechnic University (APU), Cambridge, United Kingdom. (Feb. 2003, p. 292)

24-29 **Constraint Programming, Belief Revision and Combinatorial Optimization**, Banff International Research Station, Banff, Alberta, Canada. (Dec. 2002, p. 1420)

24-30 **Conference in Number Theory in Honour of Professor H. C. Williams**, The Banff Center, Banff, Alberta, Canada. (Feb. 2003, p. 292)

25-29 **Dynamical Systems, Denton 2003**, University of North Texas, Denton, Texas. (Nov. 2002, p. 1287)

*25-31 **Workshop on Singularity Theory**, International Centre for Mathematical Science, Edinburgh, UK.
Program: The major aims of the workshop are to disseminate new results and ideas, to foster new developments within singularity theory, and to seek new areas of application and interaction.
Topics: Complex and real singularities, study of discriminant spaces and Vassiliev type variations, singularities in symplectic and contact spaces, singularities in differential geometry, computer packages on singularities.
Participation: The meeting is by invitation only (numbers are strictly limited).
Information: http://www.ma.hw.ac.uk/icms/meetings/2003/singth/.

26-30 **SampTA03 (Sampling Theory and Applications 2003)**, Strobl, Salzburg, Austria. (Sept. 2002, p. 1000)

27-31 (REVISED) **SIAM Conference on Applications of Dynamical Systems**, Snowbird Ski and Summer Resort, Snowbird, Utah. (Sept. 2002, p. 1000)

28-30 **Lattices, Universal Algebra and Applications**, Centro de Algebra da Universidade de Lisboa, Lisboa, Portugal. (Oct. 2002, p. 1134)

28-31 **ACMS 14th Biennial Conference**, Point Loma Nazarene University, San Diego, California. (Jan. 2003, p. 81)

28-31 **Quantum and Reversible Computation**, State University of New York at Stony Brook, Stony Brook, New York. (Jan. 2003, p. 81)

*29-June 1 **6th Panhellenic Conference in Geometry**, Limassol, Cyprus.
Description: The 6th Panhellenic Conference in Geometry will take place in Limassol, Cyprus, from May 29 to June 1, 2003. The main topics include Riemannian, symplectic and Poisson geometry, integrable systems and Lie groups. Deadline for registration is February 29 and for abstracts, March 31.
Information: More information and online registration on the website of the conference, http://www.ucy.ac.cy/~geom/, or by email to geom@ucy.ac.cy.

30-June 1 **Annual Meeting of the Canadian Society for History and Philosophy of Mathematics (CSHPM)**, Dalhousie University, Halifax, Nova Scotia, Canada. (Feb. 2003, p. 292)

*30-June 1 **Midwest Geometry Conference 2003**, Washington University, St. Louis, Missouri.
Topics: Three-manifolds, hyperbolic geometry and geometric group theory; Geometry, analysis and probability on discrete groups; Minimal submanifolds; Surface immersions in space.
Program: Each half-day session will be devoted to a topic. A list of invited speakers is on the conference website given below.
Supporters: The NSF and Washington University. Support is available for transportation and lodging for all registered participants. A registration form is available on the conference website: http://www.math.wustl.edu/MGC2003/.
Organizers: R. Roberts, email: roberts@math.wustl.edu; R. Feres, email: feres@math.wustl.edu; Q.-S. Chi, email: chi@math.wustl.edu; G. Jensen, email: gary@math.wustl.edu.
Information: The conference website provides more information and more details about the conference. If you have additional questions, please contact the conference secretary, T. Schneider, email: terri@math.wustl.edu or one of the organizers.

31-June 5 **Symmetry and Bifurcation in Biology**, Banff International Research Station, Banff, Alberta, Canada. (Dec. 2002, p. 1420)

June 2003

1-4 **2003 ASL Annual Meeting**, University of Illinois at Chicago, Chicago, Illinois. (Sept. 2002, p. 1000)

1-6 **International Conference on Group Theory: Combinatorial, Geometric, and Dynamical Aspects of Infinite Groups**, Gaeta, Italy. (Sept. 2002, p. 1000)

*1-7 **Spring School in Analysis: Function Spaces and Applications**, Paseky nad Jizerou, Czech Republic.
Topics: Function spaces, integral inequalities, Schrödinger operator, rearrangement estimates, Sobolev inequalities.
Main Speakers: I. Verbitsky (Univ. of Missouri), L.-I. Hedberg (Univ. of Linkoping, Sweden).
Organizers: J. Lukes, L. Pick (Charles Univ., Prague, Czech Republic).
Deadlines: For a reduced fee and support: February 15, 2003. Grants: Probably support for a limited number of students.
Information: email: pasejune@karlin.mff.cuni.cz; http://www.karlin.mff.cuni.cz/katedry/kma/ss/jun03/ss.htm.

*1-30 **Statistical Methods in Microarray Analysis**, Institute for Mathematical Sciences, National University of Singapore, Singapore.
Organizing Committee: Chair: T. Speed (Univ. of California at Berkeley, and Walter & Eliza Hall Inst. of Medical Research, Australia); Co-chairs: M.-Y. Leung (Univ. of Texas at San Antonio) and L. Zhang (National Univ. of Singapore).
Members: A. Kuk (National Univ. of Singapore), A. Owen (Stanford Univ.), S. Richardson (Imperial College), and W. H. Wong (Harvard Univ.).
Program: The development and applications of the microarray technology have given rise to many problems that need to be addressed by the collective knowledge and skills of the mathematical and biological scientists. This program aims to study the many new statistical methods tailored to microarrays that is being developed and adapted in the last few years. Its main objective is to bring together a group of leading researchers in microarrays to discuss among themselves as well as with local researchers their

current work and exchange ideas for addressing the main statistical challenges in the future.
Format: The program will consist of a tutorial on background material and a workshop at research level, in addition to seminars and informal discussions. The tutorial will be held from June 16 to 19, 2003, and the workshop from June 23 to 27, 2003.
Registration: Registration forms for the tutorial/workshop are available at `http://www.ims.nus.edu.sg/Programs/microarray/index.htm` and should be received at least one month before commencement of each activity. Registration is free. Membership is not required for participation.
Membership: Membership application for visiting the institute under the program is also available from the above website. Members do not need to register for specific activities.
Contacts: For general enquiries, please send email to `ims@nus.edu.sg`, while for enquiries on scientific aspects of the program, please email Ming-Ying Leung at `MLeung@utsa.edu` or L. Zhang at `matzlx@nus.edu.sg`. More information is available at `http://www.ims.nus.edu.sg/Programs/microarray/index.htm`.

2-4 **International Workshop on Computer Algebra Systems and Their Applications, CASA'2003**, Saint Petersburg, Russian Federation. (Jan. 2003, p. 81)

2-4 **Technical Session on Computer Graphics, TSCG'2003**, Saint Petersburg, Russian Federation. (Jan. 2003, p. 81)

* 2-6 **Conference on Representation Theory for Algebraic Groups**, University of Aarhus, Aarhus, Denmark.
Sponsor: Danish Natural Science Research Council.
Information: P. Abrahamsen, Dept. of Math., Univ. of Aarhus, Ny Munkegade, DK8000 Aarhus C, Denmark.

2-6 **Symplectic Geometry and Physics: Workshop III: Symplectic Geometry and String Theory**, Institute for Pure and Applied Mathematics, UCLA, Los Angeles, California. (Oct. 2002, p. 1135)

5-8 **Call for Papers: Hawaii International Conference on Statistics and Related Fields**, Sheraton Waikiki Hotel, Honolulu, Hawaii. (Jan. 2003, p. 81)

* 7-12 **Applicable Harmonic Analysis**, Banff International Research Station, Banff, Alberta, Canada.
Organizers: R. Jia (Univ. Alberta), S. Riemenschneider (West Virginia Univ.), M. Wickerhauser (Washington Univ.).
Information: `http://www.pims.math.ca/birs/`.

* 8-15 **41st International Symposium on Functional Equations**, de la Motte Castle, Noszvaj, Hungary.
Topics: Functional equations and inequalities; mean values; functional equations on algebraic structures; Hyers-Ulam stability; regularity properties of solutions; conditional functional equations; iteration theory; functional-differential equations; applications of the above, in particular to the natural, social, and behavioral sciences.
Organizer: Z. Páles, Inst. of Math. and Inform., Univ. of Debrecen, Pf.12, H-4010 Debrecen, Hungary (`pales@math.klte.hu`).
Scientific Committee: J. Aczél (Honorary Chair; Waterloo, ON, Canada), Z. Daróczy (Debrecen, Hungary), R. Ger (Katowice, Poland), J. Rätz (Chair; Bern, Switzerland), L. Reich (Graz, Austria), and A. Sklar (Chicago, IL, USA).
Information: Participation at these annual symposia is by invitation only. Those wishing to be invited should send details of their interest and, preferably, publications (paper copies) and/or manuscripts, and their postal and email addresses to: Professor Jürg Rätz, Math. Inst., Univ. Bern, Sidlerstr. 5, CH-3012 Bern, Switzerland (`math@math-stat.unibe.ch`) before April 7, 2003. A short history of the symposia can be found on the websites `http://riesz.math.klte.hu/~isfe/` and `http://riesz.math.klte.hu/~isfe41/`.

9-13 **Workshop on the Sensory Motor System**, Mathematical Biosciences Institute (MBI), The Ohio State University, Columbus, Ohio. (Feb. 2003, p. 292)

* 9-14 **29th International Summer School "Application of Mathematics in Engineering and Economics"**, Sozopol, Bulgaria.
Organizer: Faculty of Applied Mathematics and Informatics, Technical University of Sofia, Sofia, Bulgaria.
Chairman: M. Marinov (chair); L. Karandzhulov (vice chair).
Organizing Committee: M. Hardalov, V. Pasheva, K. Peeva, I. Petrov, K. Prodanova, M. Slavkova, E. Varbanova, G. Venkov.
Description: The program of the 29th Summer School will focus on the present purpose and results, and future perspectives of application of mathematics in different fields of engineering and economics.
Deadlines: March 31, 2003: abstract submission; April 30, 2003: declaration of participation; May 15, 2003: reservation form; October 1, 2003: full paper submission.
Topics: Mathematical analysis and application, Differential equations and application, Geometry and topology, Probability and statistics, Numerical methods and operations research, Mathematical modelling and simulation, Information technologies, Education in applied mathematics. There will be two types of lecturers: the first type could be considered as an introduction in large research domains, and the second type will be authors' contributions.
Submission: Potential participants are invited to submit a full paper describing their work to be presented at the Summer School. The submitted papers will be reviewed by members of the scientific committee for presentation and will be published in the proceedings volume of the Summer School.
Information: Details available from: `http://www.tu-sofia.acad.bg/FPMI/school29/amee03.html`.

10-14 **Fifth Dublin Differential Equations Conference**, Dublin City University, Dublin, Ireland. (Feb. 2003, p. 292)

10-14 **Groups and Group Rings X**, Wisla, Poland. (Feb. 2003, p. 292)

10-14 **XIX Rolf Nevanlinna Colloquium**, University of Jyäskylä, Jyväskylä, Finland. (Dec. 2002, p. 1420)

* 11-14 **Curvature in Geometry, in Honour of Prof. L. Vanhecke**, Grand Hotel Tiziano e dei Congressi, Lecce, Italy.
Information: `http://www.diffgeo.unile.it`.

* 11-16 **Conference on Partial Differential Equations and Applications in Celebration of Aizik I. Volpert's 80th Birthday**, Technion-I.I.T., Haifa, Israel.
Information: See `http://www.math.technion.ac.il/~shafrir/volpert.html`.

13-22 **Poisson Geometry, Deformation Quantisation and Group Representations (PQR2003)**, Universersité Libre de Bruxelles, Belgium. (Jan. 2003, p. 82)

* 14-19 **Integration on Arc Spaces, Elliptic Genus and Chiral de Rham Complex**, Banff International Research Station, Banff, Alberta, Canada.
Organizers: M. Kapranov (Univ. Toronto), A. Libgober (Univ. Illinois at Chicago), F. Loeser (École Normale Supérieure).
Information: `http://www.pims.math.ca/birs/`.

* 15-22 **Functional Analysis VIII**, IUC, Dubrovnik, Croatia.
Topics: Operator theory, representation theory, probability theory, harmonic analysis.
Organizer: Department of Mathematics, Univ. of Zagreb, Croatia.
Deadline: For submitting an abstract: March 1, 2003.
Information: `http://www.math.hr/~congress/Dubrovnik03/`.

16-20 **2003 SIAM Annual Meeting**, Queen Elizabeth Hotel, Montreal, Quebec, Canada. (Sept. 2002, p. 1000)

16-21 **International Conference: Kolmogorov and Contemporary Mathematics**, Moscow, Russia. (Sept. 2002, p. 1000)

17-21 **Fourth Geoffrey J. Butler Memorial Conference**, University of Alberta, Edmonton, Alberta, Canada. (Sept. 2002, p. 1000)

17-24 **Furstenfest—Conference on Probability in Mathematics in Honor of H. Furstenberg**, Jerusalem and Beer Sheva, Israel. (Feb. 2003, p. 293)

18-21 **First Joint International Meeting between the American Mathematical Society and the Real Sociedad Matematica Española**, Seville, Spain. (Sept. 2002, p. 1000)

* 18-28 **Instructional Conference in the Mathematical Analysis of Hydrodynamics**, International Centre for Mathematical Science, Edinburgh, UK.

Program: The aim of this 10-day course is to instruct young mathematicians in topics involving mathematical analysis of hydrodynamics. The conference takes the form of a series of courses (each course being made up of between two and four 1-hour lectures), the courses being structured so that the more basic material is presented during the first week, leading to more advanced lectures in the second week. The courses are supplemented by single research-level talks on recent advances.

Topics: Euler equations, quasi-geostrophic equations, statistical hydrodynamics, related models, mathematical aspects of physical turbulence, water waves.

Deadline: Registration closes on February 3, 2003.

Information: Please regiser using the website `http://www.ma.hw.ac.uk/icms/meetings/2003/hydro/`.

19-26 **Some Intermediate Questions of Model Theory and Universal Algebra: ERLOGOL-2003**, Novosibirsk State Technical University, Novosibirsk, Siberia/Russia. (Dec. 2002, p. 1420)

21-24 **Mathematics of Finite Elements and Applications: Satellite Conference at Brunel University**, Isaac Newton Institute for Mathematical Sciences, Cambridge, England. (Feb. 2003, p. 293)

* 21-26 **Point Processes—Theory and Applications**, Banff International Research Station, Banff, Alberta, Canada.

Organizers: P. Guttoro (Univ. Washington), B. Smith (Dalhousie Univ.).

Information: `http://www.pims.math.ca/birs/`.

22-25 **IEEE Symposium on Logic in Computer Science 2003: Call for Workshop Proposals**, Ottawa, Ontario, Canada. (Dec. 2002, p. 1420)

23-25 **SIAM Conference on Mathematics for Industry: Challenges and Frontiers**, The Metropolitan Hotel, Toronto, Ontario, Canada. (Nov. 2002, p. 1287)

23-27 **15th International Conference on Formal Power Series and Algebraic Combinatorics**, Linköping University, Vadstena, Sweden. (Jan. 2003, p. 82)

23-27 **Hyperbolic Models in Astrophysics and Cosmology**, Newton Institute of Mathematical Sciences, Cambridge, UK. (Jan. 2003, p. 82)

23-27 **Workshop on Extremal Graph Theory (Miklos Simonovits Is 60)**, Lake Balaton, Hungary. (Oct. 2002, p. 1135)

23-27 **Workshop on Harmonic Analysis and Partial Differential Equations**, Puerto Vallarta, Mexico. (Oct. 2002, p. 1135)

23-28 **Tools for Mathematical Modelling**, Saint-Petersburg State Technical University, Saint-Petersburg, Russia. (Nov. 2002, p. 1287)

23-29 **Symmetry in Nonlinear Mathematical Physics**, Institute of Mathematics, Kiev, Ukraine. (Feb. 2003, p. 293)

24-26 **Fluid Structure Interaction 2003**, Cadiz, Spain. (Dec. 2002, p. 1420)

24-27 **Day on Diffraction-2003 (DD'03)**, St. Petersburg Univ., St. Petersburg, Russia. (Oct. 2002, p. 1135)

* 24-28 **Nonlinear Modeling and Control, an International Seminar**, Nayanova University, Samara, Russia.

Purpose: The seminar's aim is the exchange of information about recent trends in mathematical modeling and control theory and their applications to various problems in physics, chemistry, biology, medicine, economics, and industrial concerns.

Call for Papers: Original papers related to the aim of the seminar are solicited. Potential speakers should submit an abstract before April 30. The cover page should contain title, affiliation, and email address of each author. Electronic submissions in LaTeX are encouraged.

Sponsors: Samara Municipal Nayanova Univ., Samara State Univ., Russian Academy of Natural Sciences, International Federation of Nonlinear Analysts, Univ. College Cork (Ireland).

Organizers: A. Pokrovskii (Cork, Ireland), V. Sobolev (Samara, Russia).

Information and Submission: V. Sobolev (`sable@ssu.samara.ru`) or H. Gorelova (seminar coordinator, e-mail: `gorhel@ssu.samara.ru`); Nayanova Univ., Molodogvardeiskaya 196, Samara 443001, Russia.

* 25-28 **Current Geometry 2003**, Palazzo Serra di Cassano, Naples, Italy.

Topics: Current Geometry 2003, the IV edition of the international conference on problems and trends of contemporary geometry.

Invited Speakers (preliminary list): E. Arbarello (Rome), M. Berger (Paris), Ph. Griffiths (Princeton), A. Sossinski (Moscow), A. Vinogradov (Salerno).

Scientific Committee: E. Arbarello (Rome), F. Baldassarri (Padova), U. Bruzzo (Trieste), C. Ciliberto (Rome), A. Collino (Torino), M. Cornalba (Pavia), C. De Concini (Rome), B. Dubrovin (Trieste), L. van Geemen (Pavia), Ph. Griffiths (Princeton), V. Kac (Boston), K. O' Grady (Rome), C. Procesi (Rome), J. Stasheff (Chapel Hill), A. Vinogradov (Salerno).

Sponsors: Istituto Italiano per gli Studi Filosofici, Naples; Università degli studi di Salerno; Gruppo Nazionale per le Strutture Algebriche, Geometriche, e le loro Applicazioni; Dipartimento di Matematica ed Applicazioni dell'Università degli studi di Napoli "Federico II".

Organizing Committee: A. De Paris (Naples), G. Rotondaro (Naples), G. Sparano (Salerno), A. Vinogradov (Salerno), R. Vitolo (Lecce).

Deadline: May 15, 2003.

Information: `http://www.diffiety.ac.ru/conf/curgeo03/` or `http://www.diffiety.org/conf/curgeo03/`; email: `curgeo@diffiety.org` or `curgeo@diffiety.ac.ru`; surface mail: Prof. A. M. Vinogradov, Dipartimento di Matematica e Informatica, Università di Salerno, Via Salvador Allende, 84081 Baronissi (SA), Italy; fax: +39 089 965226; phone: +39 089 965395.

25-28 (REVISED) **Mathematics in XXI Century**, Novosibirsk University, Novosibirsk, Russia. (Sept. 2002, p. 1000)

* 25-30 **2003 International Workshop on Small Sets in Analysis**, Center for Mathematical Sciences, Technion-Israel Institute of Technology, Haifa, Israel.

Brief Description: In recent years there has developed a growing interest in the role of small sets in (infinite-dimensional) analysis. Such sets can be small in several senses, e.g., measure-theoretic (Gauss null sets, Haar null sets), topological (sets of the first Baire category), a combination of both (Gamma-null sets), and metric (sigma-porous sets). These concepts have found application in many areas of analysis, e. g., approximation theory, the calculus of variations, convexity, differentiability theory, Fourier analysis, the geometry of Banach spaces, nonlinear analysis, and optimization. The purpose of this workshop is to bring together researchers to present and discuss recent developments and to assess future directions.

Organizing Committee: E. Matouskova, S. Reich, and A. Zaslavski.

Participants: Y. Benyamini, J. Borwein, A. Cellina, S. Cobzas, M. Csornyei, B. Dacorogna, F. De Blasi, R. Deville, T. Dominguez Benavides, V. Fonf, A. Ioffe, S. Konyagin, J. Lindenstrauss, G. Lopez

Acedo, R. Lucchetti, P. Mankiewicz, J. Myjak, G. Pianigiani, D. Preiss, J. Revalski, A. Rubinov, S. Solecki, T. Szarek, J. Tiser, L. Zajicek, and M. Zeleny.

Information: S. Schur (workshop coordinator), Dept. of Math., Technion-Israel Inst. of Tech., 32000 Haifa, Israel; email: `cms@math.technion.ac.il`.

* 28-July 3 **Joint Dynamics**, Banff International Research Station, Banff, Alberta, Canada.

Organizers: D. Lind (Univ. Washington), D. Rudolph (Univ. Maryland), K. Schmidt (Univ. Vienna), B. Solomyak (Univ. Washington).

Information: `http://www.pims.math.ca/birs/`.

* 29-July 4 **BCC19: 19th British Combinatorial Conference**, University of Wales, Bangor, North Wales, UK.

Description: The British Combinatorial Conference is held every two years at one of the UK universities. All areas of combinatorics are covered. The conference has been invited to Bangor in 2003 to honour the 100th anniversary of the birth of D. E. Littlewood, professor of mathematics at Bangor from 1948 until 1970.

Organizers: The British Combinatorial Committee; local organizer: C. Wensley.

Invited Speakers: L. Andersen (Aalborg), S. Blackburn (Royal Holloway), A. Borovik (UMIST), P. Hell (Simon Fraser), D. Jungnickel (Augsburg), I. Leader (Cambridge), A. Ram (Wisconsin-Madison), A. Street (Queensland), G. Ziegler (TU, Berlin). See the website for titles and abstracts. The invited speakers will contribute survey articles to the conference book *Surveys in Combinatorics, 2003*, London Math. Soc. Lecture Note Series, published by Cambridge Univ. Press, which will be distributed to all participants at the conference.

Call for Papers: Contributed talks of 15 minutes in length are invited. See the website for submission details.

Financial Support: The London Mathematical Society.

Information: email: `bcc2003@informatics.bangor.ac.uk`, or visit `http://www.informatics.bangor.ac.uk/public/mathematics/bcc2003/`.

* 29-July 19 **IAS/Park City Mathematics Institute 13th Annual Summer Session**, Park City, Utah.

Description: Annual 3-week Summer Session for researchers, graduate students, undergraduate students, high school teachers, college teaching faculty, and math education researchers.

Topic: Mathematics research topic for 2003: Harmonic analysis and partial differential equations.

Organizers: M. Christ, C. Kenig, W. Schalm.

Deadline and Support: Application deadline is February 15, 2003. Financial support is available.

Information: Program information and applications are available at `http://www.ias.edu/parkcity/`.

July 2003

1-December 31 **Mathematics and Computation in Imaging Science and Information Processing**, Institute for Mathematical Sciences (IMS), National University of Singapore, Singapore. (Dec. 2002, p. 1420)

* 1-4 **International T. I. Amanov Memorial Conference "Theory of Functions, Functional Analysis and Applications"**, Shakarim Semipalatinsk State University, Semipalatinsk, Kazakhstan.

Description: The conference honors T. I. Amanov (25.8.1923-21.6.1978), corresponding member of Academy of Science (AS) of the Kazakh Republic, a well-known mathematician all over the world for his significant contribution to the theory of functional spaces.

Workshop Topics: Function theory; Functional analysis; Differential equations and the equations of mathematical physics; Algebra, geometry and mathematical logics; Fundamental issues in the study of physical processes; Computational mathematics and its applications; Theory and the contemporary technology of teaching the basics of physical and mathematical sciences.

Speakers: S. M. Nikolski (Russia), O. V. Besov (Russia), Yu. G. Reshetnyak (Russia), V. N. Burenkov (England), U. M. Sultangazin (Kazakhstan), M. O. Otelbaev (Kazakhstan), A. A. Zhensykbaev (Kazakhstan), N. K. Bliev (Kazakhstan), S. K. Vodopyanov (Russia), G. A. Kalyabin (Russia).

Deadlines: Registration forms deadline: February 1, 2003. Electronic version of abstracts (up to 5 pages) deadline: March 1, 2003.

Registration Fee: $10 for participants from Kazakhstan and CIS (in KZT according to exchange rate of NBRK) and $50 for participants from other countries; should be paid upon registration.

Information: M. A. Prmaganbetovich (executive secretary), Semipalatinsk State Univ., 1 Tanirbergenov Street, Semipalatinsk, 490035 Kazakhstan; email: `amanov@semgu.kz`; phone/fax: (3222) 458466, (3222) 665764; `http://www.semgu.kz/amanov.html`.

1-5 **EuroConference VBAC 2003: Dedicated to Andrei Tyurin**, Porto, Portugal. (Feb. 2003, p. 293)

1-10 **PI-Rings: Structure and Combinatorial Aspects (Summer Course)**, Bellaterra, Barcelona, Spain. (Jan. 2003, p. 82)

* 3-8 **Wavelets and Splines**, Euler International Mathematical Institute, St. Petersburg, Russia.

Topics: Wavelets and wavelet methods, signal analysis and processing, wavelet bases and frames, spline theory and its applications, related topics in approximation theory and functional analysis.

Organizers: St. Petersburg Dept. of Steklov Inst. of Math., St. Petersburg State Univ.

Program Committee: A. Averbuch (Israel), O. Davydov (Germany), Yu. Demjanovich (Russia), R. DeVore (USA), L. Gori (Italy), D. Leviatan (Israel), V. Malozemov (Russia), I. Novikov (Russia), A. Petukhov (USA), V. Zheludev (Israel), V. Zhuk (Russia).

Deadline: Deadline for registration is March 1, 2003.

Information: `ws@imi.ras.ru`; `http://www.pdmi.ras.ru/EIMI/2003/ws/`.

* 5-10 **Mathematical Biology**, Banff International Research Station, Banff, Alberta, Canada.

Organizers: L. Keshet (UBC), S. Levin (Princeton), M. Lewis (Univ. Alberta).

Information: `http://www.pims.math.ca/birs/`.

6-12 **Journées Arithmétiques XXIII**, Universität Graz, Graz, Austria. (Jan. 2003, p. 82)

* 7-10 **Announcement and Call for Papers: Fourth Panhellenic Logic Symposium**, Thessaloniki, Greece.

Description: The Panhellenic Logic Symposium is a biannual scientific event established in 1997. It is open to researchers from all countries who work on logic broadly conceived. The language of the symposium is English. The Fourth Panhellenic Logic Symposium will be hosted at the Conference Center of the International Fair of Thessaloniki, located downtown in close proximity to the University campus. The scientific program of the symposium will consist of hour-long invited talks, tutorials, and presentations of accepted papers.

Original papers on all aspects of logic are solicited. Authors are invited to submit an extended abstract of at most five pages. In addition, the contact author of each paper should send a cover page with his/her addresses (postal and email) and a statement classifying the paper in one of the following areas: mathematical logic, set theory, logic in computer science, history of logic, philosophy of logic, other logic related area (specify).

Information: L. Kirousis, Dept. of Computer Engineering and Informatics, University of Patras, University Campus, GR-265 04 Patras, Greece; phone: +30 2610-99 7702; fax: +30 2610-99 1909; email: `kirousis@ceid.upatras.gr`.

7-11 **ICIAM 2003, 5th International Congress on Industrial and Applied Mathematics,** Sydney, Australia. (Sept. 2002, p. 1000)

* 7-11 **International Conference on Algebraic and Topological Methods in Non-Classical Logics,** Tbilisi State University, Tbilisi, Georgia.
Information: See the conference website http://piscopia.nmsu.edu/morandi/TbilisiConference/.

7-18 **Discrete Dynamical Systems and Their Applications to Population Dynamics,** University of Wyoming, Laramie, Wyoming. (Feb. 2003, p. 293)

* 7-18 **SMS-NATO ASI: Structural Theory of Automata, Semigroups, and Universal Algebra,** Université de Montréal, Canada.
Program: Completeness of automata; quantum finite automata; algebraic classifications of regular tree languages; profinite semigroups in computer science; definability, decidability, complexity in semigroup varieties and varieties of universal algebra.
Invited Speakers: J. Almeida, J. Dassow, R. Freivalds, J. Jezek, A. Krokhin, V. B. Kudryavtsev, A. Letichevsky, R. McKenzie, I. Rosenberg, L. N. Shevrin, M. Steinby, M. V. Volkov.
Organizers: J. Almeida, A. Korkhin, V.B. Kudryavtsev, I. Rosenberg, G. Sabidussi, Y. Saint-Aubin.
Application Deadline: February 21, 2003.
Information: http://www.dms.umontreal.ca/sms/; email: sms@dms.umontreal.ca.

8-11 **Applications of Plausible, Paradoxical, and Neutrosophical Reasoning for Information Fusion (FUSION 2003),** Radisson Hotel, Cairns, Queensland, Australia. (Jan. 2003, p. 82)

9-12 **2003 Summer Conference on Topology and Its Applications,** Howard University, Washington, DC. (Jan. 2003, p. 82)

* 12-17 **Perspectives in Differential Geometry,** Banff International Research Station, Banff, Alberta, Canada.
Organizers: R. Schoen (Stanford), G. Tian (MIT), J. Chen (UBC).
Information: http://www.pims.math.ca/birs/.

12-17 **What Comes beyond the Standard Model? Symmetries beyond the Standard Model,** Hotel Histrion, Portoroz, Slovenia. (Jan. 2003, p. 82)

14-15 **Mathematics of Computation and Approximation: A Conference in Honour of the 65th Birthday of Professor Ian Sloan,** University of New South Wales, Sydney, Australia. (Nov. 2002, p. 1287)

14-18 (NEW DATE) **International Conference on Algebras, Modules and Rings,** University of Lisbon, Lisbon, Portugal. (Oct. 2002, p. 1135)

14-18 **Wavelet Theory and Applications: New Directions and Challenges,** Institute for Mathematical Sciences, National University of Singapore, Singapore. (Dec. 2002, p. 1421)

* 14-26 **Summer School on Applications of Advanced Mathematical and Computational Methods to Atmospheric and Oceanic Problems,** National Center for Atmospheric Research (NCAR), Boulder, Colorado.
Brief Description: The purpose of this interdisciplinary summer school is to bring together graduate students and young researchers on one hand and, on the other hand, specialists of meteorology and oceanography and applied mathematicians interested in geophysical fluid dynamics (GFD). It is hoped in this way, firstly, to initiate or further develop the communication and interactions between specialists of different fields working on diverse frontiers of GFD and to discuss new ideas and methods that will advance the field in the next decade. Secondly, to equip the students in these fields with the necessary tools and to bring them to the frontiers of this challenging and important field. The lectures will take place at the National Center for Atmospheric Research (NCAR), located in Boulder, Colorado, in a scenic place at the foot of the majestic Rocky Mountains.
Topics: The summer school consists of background pedagogical lectures in the mornings, invited lectures and informal discussions in the afternoons. The background pedagogical lectures cover topics in mathematical methods (nonlinear systems, functional analysis, asymptotic techniques, and stochastic methods), computational methods (multilevel modeling and adaptive methods, implicit methods, and Lagrangian methods), and science problems (atmospheric basics, Hamiltonian fluid dynamics, reduced systems, oceanography, and turbulence). The invited talks as well as the informal discussions will cover several important scientific issues in the atmospheric and oceanic sciences; both state-of-the-art knowledge and future directions will be addressed.
Coordinators: R. Temam (Indiana Univ.), J. Tribbia (NCAR), S. Wang (Indiana Univ.)
Information: Both students and young scientists in the relevant fields are encouraged to participate, and partial financial support is available. For information, contact one of the coordinators or B. Hansford at NCAR (barbm@ucar.edu), or visit http://php.indiana.edu/~iuisc/mcao.htm.

* 15-19 **Complex Hyperbolic Geometry,** CIRM, Luminy, France.
Description: Following the conferences (including AMS Special Sessions) on complex hyperbolic geometry and related topics in Oklahoma in 1994, Okayama in 1998, and Washington in 2000, we will hold a similar conference in Luminy from July 15 to July 19, 2003.
Workshop Topics: Geometry of the Heisenberg group, discrete groups of complex hyperbolic isometries and their deformations, real hypersurfaces and polyhedra, generalization to other rank-1 spaces.
Organizers: E. Falbel, Univ. Pierre et Marie Curie, Paris, France; J. R. Parker, Univ. of Durham, England.

15-19 **SIAM Conference on Applied Linear Algebra,** William and Mary College, Williamsburg, Virginia. (Sept. 2002, p. 1001)

* 19-24 **Differential Invariants and Invariant Differential Equations,** Banff International Research Station, Banff, Alberta, Canada.
Organizers: N. Kamran (McGill), P. Olver (Univ. Minnesota).
Information: http://www.pims.math.ca/birs/.

* 20-25 **XV Latinamerican Colloquy of Algebra,** Instituto de Matematicas, Universidad Nacional Autonoma de Mexico, Mexico City, Mexico.
Information: Contact: C. Geiss, christof@math.unam.mx, or visit http://cla.matem.unam.mx/.

* 20-26 **Hodge Theory in a New Century: A Euro Conference Celebrating the Centenary of Sir William Hodge (1903-1975),** International Centre for Mathematical Science, Edinburgh, UK.
Program: Hodge theory was described by Hermann Weyl as "one of the landmarks in the history of mathematics in the 20th century." It is fitting that a meeting to celebrate Sir William Hodge's achievements should take place in the city of his birth and in the university where he received his undergraduate education.

The general theme will be the influence and development of Hodge theory in various branches of mathematics and physics. The meeting will also incorporate an afternoon (hosted by the London and Edinburgh Mathematical Societies) with talks tied into the general theme but suitable for a general mathematical audience.
Scientific Organizing Committee: M. Atiyah, chair (Edinburgh), S. Bloch (Chicago), J. P. Bourguignon (IHES), S. Donaldson (Imperial College), P. Griffiths (IAS), E. Witten (IAS).
Deadline: Registration closes on April 18, 2003. (Applications for financial support should be received by March 1, 2003.)
Information: You may register at: http://www.ma.hw.ac.uk/icms/meetings/2003/HODGE/.

21–25 **Infinite Dimensional Algebras and Quantum Integrable Systems**, Faro, Portugal. (Feb. 2003, p. 293)

21–25 **The Władysław Orlicz Centenary Conference and Function Spaces VII**, Faculty of Mathematics and Computer Science, Adam Mickiewicz University, Poznań, Poland. (Jan. 2003, p. 83)

21–August 1 **IMA 2003 Summer Program: Probability and Partial Differential Equations in Modern Applied Mathematics, Data Analysis and Optimization**, University of Minnesota, Minneapolis, Minnesota. (Dec. 2002, p. 1421)

*21–August 15 **Spaces of Kleinian Groups and Hyperbolic 3-Manifolds**, Isaac Newton Institute for Mathematical Sciences, Cambridge, England.
Description: Kleinian groups stand at the meeting point of several different parts of mathematics. Classically, they arose as the monodromy groups of Schwarzian equations, in modern terminology projective structures, on Riemann surfaces. The action by Möbius maps on the Riemann sphere provides further intimate links not only with Riemann surfaces but also complex dynamics and fractals, while Thurston's revolutionary insights have made hyperbolic 3-manifolds, quotients of the isometric action on hyperbolic 3-space, central to 3-dimensional topology.
Topics: Specific topics to be addressed include relationships between the analytic, combinatorial, and geometric structure of hyperbolic 3-manifolds; topology of deformation spaces and the arrangement of their components; classification of hyperbolic 3-manifolds by asymptotic invariants; complex projective structures; convex hull boundaries; cone manifolds, orbifolds, and knot groups; the combinatorial structure of Teichmüller spaces, mapping class groups, and spaces of curves on surfaces; and the challenge of extending recent advances from one punctured tori to higher genus.
Organizers: C. Series (Warwick), Y. Minsky (Stonybrook), M. Sakuma (Osaka).
Information: Isaac Newton Institute for Mathematical Sciences, 20 Clarkson Road, Cambridge, CB3 OEH UK; tel: +44 (0) 1223 335999; fax: +44 (0) 1223 330508; email: `info@newton.com.ac.uk`; `http://www.newton.cam.ac.uk/`.

*21–December 19 **Interaction and Growth in Complex Stochastic Systems**, Isaac Newton Institute for Mathematical Sciences, Cambridge, England.
Description: Random processes are often understood by means of an asymptotic in which the number of basic random components becomes large. Classical examples of behaviour which can be manifest in such a limit are the law of large numbers and the central limit theorem. These two examples apply in contexts where the interaction between typical random components becomes negligible in the limit. However, there are many extremely important physical models, where such an asymptotic decorrelation does not hold. Such models correspond to a critical behaviour where interaction is strong and statistical scaling properties are highly nontrivial. Understanding of asymptotic properties for such systems is one of the most important problems in the modern probability theory. Renormalization group ideology, which was developed by physicists in the last 30 years, predicts appearance of the universal scaling and conformal invariant random fields characterizing statistical behaviour in critical situations. However, rigorous results in this direction were out of reach until very recently.

The programme will focus on problems from a number of contexts where strong random interactions give rise to non-classical limiting behaviour. These will include random walk in a random environment and interactive random walk, interfaces in growth models and equilibrium statistical mechanics, coagulation and fragmentation, conformally invariant scaling limits, random matrices.
Organizers: E. Bolthausen (Zurich), K. Khanin (BRIMS/Heriot-Watt), J. Norris (Cambridge), Y. Suhov (Cambridge), G. Lawler (Duke).
Information: Isaac Newton Institute for Mathematical Sciences, 20 Clarkson Road, Cambridge, CB3 OEH UK; tel: +44 (0) 1223 335999; fax: +44 (0) 1223 330508; email: `info@newton.com.ac.uk`; `http://www.newton.cam.ac.uk/`.

*22–26 **Equadiff 2003, International Conference on Differential Equations**, Hasselt, Belgium.
Organizing committee: F. Dumortier (Chair), H. W. Broer, J. P. Gossez, J. Mawhin, A. Vanderbauwhede, S. Verduyn Lunel.
Scientific committee: A. Ambrosetti, A. Doelman, E. Feireisl, B. Fiedler, M. Fila, J. Hale, Y. Ilyashenko, J. Palis, B. Peletier, C. Simo, F. Takens.
Invited speakers and organizers of mini-symposia include: A. Albouy, V. Araujo, P. Bonckaert, R. Farwig, M. Golubitsky, H. Hanbmann, A.J. Homburg, R. Johnson, H. Th. Jongen, V. Kaloshin, T. Kaper, A. Katok, H. Kokubu, B. Krauskopf, P. Krejci, K. Lust, R. MacKay, J. Mallet-Paret, H. Matano, A. Mielke, R. Moeckel, R. Obaya, R. Ortega, R. Peeters, P. Polácik, D. Rand, G. Raugel, M. Roberts, R. Roussarie, B. Sandstede, K. Schmitt, G. Sell, D. Serre, J. Sotomayor, C. Stuart, P. Szmolyan, S. Terracini, J.F. Toland, F. Verhulst, M. Viana, O.J. Vrieze, M. Wiegner, Y. Yi, K. Zumbrun.
Information and registration: `http://www.equadiff.be/equadiff@luc.ac.be/`; `http://www.equadiff.be/` or by email: `equadiff@luc.ac.be`.

23–24 **Workshop on Categorification and Higher-Order Geometry**, Instituto Superior Tecnico, Lisbon, Portugal. (Feb. 2003, p. 293)

23–26 **The Joint Meeting of ISAMA 2003 and the 6th Annual BRIDGES Conference**, University of Granada, Spain. (Feb. 2003, p. 293)

24–26 **First Joint ISBA-IMS Meeting**, Intercontinental Hotel, Isla Verde, San Juan, Puerto Rico. (Feb. 2003, p. 294)

*26–31 **Analysis and Geometric Measure Theory**, Banff International Research Station, Banff, Alberta, Canada.
Organizers: A. Granados (Univ. Washington), H. Pajot (Univ. Cergy-Pontoise), T. Toro (Univ. Washington).
Information: `http://www.pims.math.ca/birs/`.

26–31 **Algebra and Discrete Mathematics—Symmetries and Ordered Structures under the Influence of Model Theory and Combinatorics**, Hattingen, Germany. (Dec. 2002, p. 1421)

27–31 **First Announcement of the 11th International Conference on Finite or Infinite Dimensional Complex Analysis and Applications**, Chiangmai University, Chiangmai, Thailand. (Dec. 2002, p. 1421)

*27–August 2 **V International Conference "Nonassociative Algebra and Its Applications"**, Morelos State University, Cuernavaca, Morelos, Mexico.
Description: The V International Conference on Nonassociative Algebra and Its Applications will be a continuation of a 15-year-old sequence of international conferences devoted to nonassociative algebra and its applications. As in previous years it will focus on:

1. Lie algebras.
2. Nonassociative rings and algebras.
3. Quasigroups and loops (algebraic and smooth) and related systems.
4. Applications of nonassociative algebra in geometry.
5. Applications of nonassociative algebra in physics and natural sciences.

It will have 30-minute talks and a number of one-hour talks. It is our intention to publish the proceedings of the conference. This conference is a satelite conference of the XV Latino-American Algebraic Colloquium (July 0 20–25, 2003, Cocoyoc, Mexico); `http://www.cla.matem.unam.mx/`.
Information: `http://loops.intranets.com/`; email: `committee@loops.intranets.com`; fax: 52-777-3134466.

27–August 9 (NEW DATE) **Banach Algebras and Their Applications: Banach Algebras 2003**, Edmonton, Alberta, Canada. (Sept. 2002, p. 1001)

* 28-31 **9th International Conference on Applications of Computer Algebra**, North Carolina State University, Raleigh, North Carolina.

Organizing Committee: S. Steinberg, M. Wester.

General Chairs: H. Hong, E. Kaltofen, A. Szanto.

Program Chair: Proposals for organizing a session should be directed to: M. Giesbrecht (mwg@uwaterloo.ca).

Scientific Committee: ACA Working Group: A. G. Akritas (Greece), J. Calmet (Germany), V. Edneral (Russia), V. Ganzha (Germany), V. Gerdt (Russia), M. Giesbrecht (Canada), H. Hong (USA), E. Kaltofen (USA), I. S. Kotsireas (Canada), B. Kutzler (Austria), R. Liska (Czech Republic), B. Pletsch (USA), E. Roanes-Lozano (Spain), S. Steinberg (USA), Q.-N. Tran (USA), N. Vassiliev (Russia), M. Wester (USA).

Call for Sessions: The Scientific Committee is soliciting proposals to organize sessions at the conference. Session chairs are expected to organize four or more speakers on a theme consistent with that of the conference.

Deadlines: While there is a target date of March 1 for session proposals, organizing earlier will help guarantee your choice of session date and time in the ACA schedule. March 1, 2003: Target to submit a session proposal (see above); May 1, 2003: Target date for submission of speaker list and abstracts; May 15, 2003: Deadline to submit an application for financial support; June 15, 2003: Notification of decisions for financial support; June 15, 2003: Deadline for early registration; July 15, 2003: Deadline for regular registration.

Information: ACA'2003 conference email: aca2003@math.unm.edu. ACA'2003 conference website: http://math.unm.edu/ACA/2003. ACA conferences main website: http://math.unm.edu/aca.html. Please see http://math.unm.edu/ACA/2003/Sessions.html for details.

* 28-August 1 **The 8th International Conference on Difference Equations and Applications**, Brno, Czech Republic.

Sponsor: The 8th ICDEA will be held under the auspices of the International Society of Difference Equations.

Topics: All the topics included in the *Journal of Difference Equations and Applications*, including: general theory of difference equations, asymptotic behavior, oscillation theory, stability theory, discrete dynamical systems, chaotic and complex dynamics, control theory, combinatorics, bifurcation theory, iterated function systems, numerical analysis, functional equations, stochastic processes, dynamics equations on time scale, special functions and orthogonal polynomials, and discrete analogues of continuous mathematics. Applications in economic theory, biology, physics, chemistry, engineering, computer science, and other disciplines that use difference equations in a significant way.

Contacts: To get information about the conference, you may contact one of the following: B. Aulbach (aulbach@math.uni-augsburg.de), Inst. of Math., Univ. of Augsburg, D-8900 Augsburg, Germany; O. Došlý (dosly@math.muni.cz), Dept. of Math., Masaryk Univ., Janáčkovo nám. 2a, CZ-662 95 Brno, Czech Republic; G. Ladas (gladas@math.uri.edu), Dept. of Math., Univ. of Rhode Island, Kingston, RI 02881-0816; S. Elaydi (selaydi@trinity.edu), Dept. of Math., Trinity Univ., San Antonio, TX 78212-7200.

Organizers: If you are interested in organizing a special session on any of the above topics or other related topics, please send your proposal to one of the contacts listed above.

Information: For updated information on registration, housing, and deadline for abstract submission, visit http://www.math.muni.cz/~icdea2003/.

* 28-August 31 **Stein's Method and Applications: A Program in Honor of Charles Stein**, Institute for Mathematical Sciences, National University of Singapore, Singapore.

Program: Due to its broad range of application, Stein's method has become particularly important, not only in the future development of probability theory, but also in a wide range of other fields, some theoretical, some extremely practical. This program aims to refocus interest on understanding the essence of the method and on the various open problems associated with it. It also seeks to foster collaboration in the many fields of application now being studied.

Format: The program will consist of a tutorial on background material and a workshop at research level, in addition to seminars and informal discussions. The tutorial will be held from August 4 to 8, 2003, and the workshop from August 11 to 15, 2003.

Confirmed Participants (as of December 17, 2002): C. Stein (Stanford Univ.), A. Baddeley (Univ. of Western Australia), Z. Bai (National Univ. of Singapore), A. D. Barbour (Univ. of Zürich), T. Brown (Australian National Univ.), L. Chen (National Univ. of Singapore), K.-P. Choi (National Univ. of Singapore), L. Devroye (McGill Univ.), P. Diaconis (Stanford Univ.), T. Erhardsson (Royal Institute of Technology, Stockholm), L. Goldstein (Univ. of Southern California), P. Hall (Australian National Univ.), S. Holmes (Stanford Univ.), L. Holst (Royal Institute of Technology, Stockholm), H. Huang (Harvard Univ.), W.-L. Loh (National Univ. of Singapore), M. Penrose (Univ. of Durham), G. Reinert (Univ. of Oxford), Y. Rinott (Hebrew Univ. of Jerusalem), V. Rotar (San Diego State Univ.), Q.-M. Shao (National Univ. of Singapore and Univ. of Oregon), J. M. Steele (Univ. of Pennsylvania), S. Utev (Univ. of Nottingham), A. Xia (Univ. of Melbourne).

Registration: Registration forms for the tutorial/workshop are available at http://www.ims.nus.edu.sg/Programs/stein/index.htm and should be received at least one month before commencement of each activity. Registration is free. Membership is not required for participation.

Membership: Membership application for visiting the institute under the program is also available from the above website. Members do not need to register for specific activities.

Information: For general enquiries, please send email to: ims@nus.edu.sg, while for enquiries on scientific aspects of the program, please email A. Barbour at adb@amath.unizh.ch or L. Chen at lhychen@ims.nus.edu.sg. More information is available at http://www.ims.nus.edu.sg/Programs/stein/index.htm.

29-August 1 **Tenth Workshop on Logic, Language, Information and Computation (WoLLIC'2003)**, Ouro Preto, Minas Gerais, Brazil. (Feb. 2003, p. 294)

29-August 2 **IMS New Researchers Conference**, University of California, Davis, California. (Feb. 2003, p. 294)

August 2003

* 2-7 **Monge-Ampere Type Equations and Applications**, Banff International Research Station, Banff, Alberta, Canada.

Organizers: A. Chang (Princeton), P. Guan (McMaster), P. Yang (Princeton).

Information: http://www.pims.math.ca/birs/.

3-9 **29th Stochastic Processes and Their Applications**, Hotel do Frade, Angra dos Reis, State of Rio de Janeiro, Brazil. (Dec. 2002, p. 1421)

4-9 **4th International Algebraic Conference**, Lviv, Ukraine. (Nov. 2002, p. 1287)

* 4-9 **Transformation Groups and Related Topics: An International Conference in Honor of Sören Illman to Celebrate His 60th Birthday**, Helsinki, Finland.

Invited Speakers: A. Adem, Univ. of Wisconsin-Madison; W. Browder, Princeton Univ., NJ; G. Carlsson, Stanford Univ., CA; T. tom Dieck, Georg-August-Univ., Göttingen, Germany; B. Oliver, Univ. Paris 13, France; E. K. Pedersen, State Univ. of New York at Binghamton; F. Quinn, Virginia Polytechnic Inst. and State Univ.; S. Weinberger, Univ. of Chicago.

Scientific and Organizing Committee: T. Matumoto, Chairman, Univ. of Hiroshima, Japan, email: matumoto@math.sci.hiroshima-u.ac.jp; E. Elfving, Univ. of Helsinki, Finland, email: Erik.Elfving@helsinki.fi; I. Hambleton, McMaster Univ., Ontario, Canada, email: ian@math.mcmaster.ca; B. Jahren, Univ. of Oslo, Norway, email:

bjoernj@math.uio.no; M. Kreck, Ruprecht-Karls Univ., Heidelberg, Germany, email: kreck@mathi.uni-heidelberg.de; M. Morimoto, Okayama Univ., Japan, email: morimoto@ems.okayama-u.ac.jp; K. Pawalowski, Adam Mickiewicz Univ., Poznan, Poland, email: kpa@main.amu.edu.pl; R. Lee (honorary member), Yale Univ., New Haven, CT.

Information: Further information will be available on the conference home page: http://www.astagor.net/illman/.

*5-9 **13th Capricornio Mathematical Congress**, Universidad Católica del Norte, Antofagasta, Chile.

Description: The congress will provide an opportunity for the presentation and exchange of the latest research by those mathematicians interested in topics such as dynamical systems, matrix theory, control theory, algebra, complex geometry, stochastic processes, applied mathematics, and others. The congress is planned to include short courses, plenary and subplenary talks, special invited sessions, and paper presentations.

Organizer: Dept. of Math. at Univ. Católica del Norte (R. L. Soto-Montero, head).

Preliminary List of Speakers (confirmed): R. Brualdi (Univ. of Wisconsin), W. Kliemann (Iowa State Univ.), L. S. Martin (Univ. Estadual de Campinas, Brazil), I. P. Shestakov (Univ. de Sao Paulo, Brazil), J. C. Yoccoz (Collège de France), V. Ayala (Univ. Católica de Norte, Chile), R. Correa (Univ. de Chile), G. Gatica (Univ. de Concepción, Chile), P. Iglesias (Pontificia Univ. Católica de Chile), S. Martinez (Univ. de Chile), R. Rebolledo (Pontificia Univ. Católica de Chile), J. Rivera (Univ. Católica del Norte, Chile), R. Rodríguez (Pontificia Univ. Católica de Chile).

Call for Papers: Authors are kindly invited to submit abstracts before April 15, 2003, to J. R. Letelier (comca2003.ucn.cl). The abstract should fit on one page and should include the title of the talk; the name, affiliation, full postal address, and email address of the speaker; and a summary of the talk which provides sufficient information to assess the relevance and novelty of the results. The preferred mode is electronic submission in LaTeX (Scientific Workplace) or PostScript format.

Information: Dept. of Math., Univ. Católica del Norte, Casilla 1280, Antofagasta, Chile; phone: 56-55-355597, 56-55-355573; fax: 56-55-355599; comca2003@ucn.cl.

*5-10 **Workshops Loops '03**, Prague, Czech Republic.

Description: Lectures on quasigroups, loops and Latin squares.

Program and Organizing Committee: D. Bedford (Keele Univ.), O. Chein (Temple Univ.), A. Drapal [secretary] (Charles Univ.), M. Kinyon (Western Michigan Univ.), G. Nagy (Bolyai Institute, Szeged), M. Niemenmaa (Univ. of Oulu), H. Pflugfelder [honorary chair] (Santa Rosa), J. D. Phillips (Wabash College), V. Shcherbacov (Moldovian Academy of Sciences), P. Vojtechovsky (Univ. of Denver).

Local Organizing Committee: A. Drapal (chair), P. Jedlicka (visas), T. Kepka (finances), P. Nemec (accommodation), D. Stanovsky (website), P. Vojtechovsky (website, registration).

Deadline: June 20, 2003.

Information: http://www.karlin.mff.cuni.cz/~loops03/workshops.html; email: loops03@karlin.mff.cuni.cz.

*9-14 **Combinatorics, Linear Algebra & Graph Coloring**, Tehran, Iran.

Organizers: S. Hedayat (Univ. of Illinois at Chicago); H. Kharaghani (Univ. of Lethbridge, Canada); G. B. Khosrovshahi (IPM and Univ. of Tehran, Iran); and S. Shahriari (Pomona College).

Invited Speakers: R. Brualdi, P. Cameron, Z. Furedi, C. Johnson, C. Thomassen, and R. M. Wilson.

Deadline: For contributed papers: June 15, 2003.

Information: Contact person: G. B. Khosrovshahi, rezagbk@ipm.ir; http://www.ipm.ac.ir/combinatorics/.

*9-16 **Defects and Their Dynamics**, Banff International Research Station, Banff, Alberta, Canada.

Organizers: P. Bates (Brigham Young), L. Bronsard (McMaster), C. Gui (Univ. Connecticut).

Information: http://www.pims.math.ca/birs/.

*9-16 **Localization Behavior in Reaction-Diffusion Systems and Applications to the Natural Sciences**, Banff International Research Station, Banff, Alberta, Canada.

Organizers: A. Bernoff (Harvey Mudd College) P. Fife (Univ. Utah), T. Hille (Univ. Alberta), M. Ward (UBC), J. Wei (Chinese Univ.).

Information: http://www.pims.math.ca/birs/.

*10-17 **Loops '03**, Prague, Czech Republic.

Information: International conference on quasigroups, loops and Latin squares.

Organizers: See Workshops Loops '03 (August 5-10).

Main Speakers: Galina B. Belyavskaya (Moldovian Academy of Sciences), Michihiko Kikkawa (Shimane University), Alexander Kreuzer (University of Hamburg), Kenneth Kunen (University of Wisconsin), Curt C. Lindner (Auburn University), Gary L. Mullen (Pennsylvania State University), Daniel A. Robinson (Georgia Institute of Technology), Jonathan D. H. Smith (Iowa State University).

Deadline for registration and abstracts: June 20, 2003.

Information: email: loops03@karlin.mff.cuni.cz, http://www.karlin.mff.cuni.cz/~loops03.

14-20 **2003 ASL European Summer Meeting (Logic Colloquium '03)**, Helsinki, Finland. (Sept. 2002, p. 1001)

*15-20 **12th Summer St. Petersburg Meeting in Mathematical Analysis: Dedicated to the 70th Anniversary of Victor Petrovich Havin**, Euler International Mathematical Institute, St. Petersburg, Russia.

Organizer: St. Petersburg Dept. of Steklov Inst. of Math., Euler Internat. Math. Inst.

Organizers: N. Nikolski, S. Kisliakov, V. Vasyunin.

Information: mathanal@pdmi.ras.ru; http://www.pdmi.ras.ru/EIMI/2003/analysis12/.

*16-21 **Current Trends in Arithmetic Geometry and Number Theory**, Banff International Research Station, Banff, Alberta, Canada.

Organizers: I. Chen (SFU), B. Conrad (Univ. Michigan), E. Goren (McGill), A. Iovita (Univ. Washington), C. Skinner (Univ. Michigan), N. Vatsal (UBC).

Information: http://www.pims.math.ca/birs/.

18-22 **7th International Symposium on Orthogonal Polynomials, Special Functions and Applications**, Copenhagen, Denmark. (Dec. 2002, p. 1421)

18-22 **ENUMATH 2003: The European Conference on Numerical Mathematics and Advanced Applications**, Prague, Czech Republic. (Feb. 2003, p. 294)

*23-28 **Computational Techniques for Moving Interfaces**, Banff International Research Station, Banff, Alberta, Canada.

Organizers: R. LeVeque (Univ. Washington), R. Russell (SFU), S. Ruuth (SFU).

Information: http://www.pims.math.ca/birs/.

*24-26 **Second St. Petersburg Days of Logic and Computability: Devoted to the Centennial of Andrei Andreevich Markov Jr.**, Euler International Mathematical Institute, St. Petersburg, Russia.

Organizer: St. Petersburg Dept. of Steklov Inst. of Math., Euler Internat. Math. Inst.

Program Committee: S. Adian (Russia); S. Artemov (Russia/USA); N. Kosovskii (Russia); M. Margenstern (France); G. Mints (USA); Y. Matiyasevich (Russia), chairman; N. Nagorny (Russia); V. Orevkov (Russia); A. Slissenko (France).

Information: LogicDays@logic.pdmi.ras.ru; http://logic.pdmi.ras.ru/2ndDays/.

28-30 **International Conference on Computability and Complexity in Analysis**, University of Cincinnati, Cincinnati, Ohio. (Feb. 2003, p. 294)

* 30-September 4 **A Workshop in Creative Scientific Writing**, Banff International Research Station, Banff, Alberta, Canada.
Organizers: M. Senechal (Smith College), C. Davis (Univ. Toronto).
Information: `http://www.pims.math.ca/birs/`.

* 30-September 4 **Locally Finite Lie Algebras**, Banff International Research Station, Banff, Alberta, Canada.
Organizers: Y. Bahturin (Memorial Univ.), G. Benkart (Univ. Wisconsin-Madison), I. Penkov (Univ. Calif., Riverside), H. Strade (Hamburg Univ.), A. Zalesskii (Univ. North Anglia).
Information: `http://www.pims.math.ca/birs/`.

September 2003

1-6 **The Sixth International Workshop on Differential Geometry and Its Applications and The Third German-Romanian Seminar on Geometry**, Cluj-Napoca, Romania. (Oct. 2002, p. 1135)

* 1-December 19 **Granular and Particle-Laden Flows**, Isaac Newton Institute for Mathematical Sciences, Cambridge, England.
Description: The programme will be concerned with theoretical issues in the burgeoning areas of dry granular and particle laden flows. The flow of such materials is of considerable practical interest, with applications ranging from the geosciences to industry. In recent years it has also become a very active area in the international physics community, as it provides rich structure in the form of novel pattern-forming processes. Much of the progress that is made in these studies relies upon relatively simple mathematical modelling which is often ad hoc in nature. In all cases, a fundamental difficulty which makes essential theoretical input difficult is the availability of suitable continuum models. Indeed, appeal is often made to microscopic descriptions, although these are often limited in scope. Therefore, the time is now ripe for a workshop where mathematicians and physicists can be brought together to discuss these fundamental issues at a detailed level. Three specific topics will be addressed:

1. Localisation and type change: The mathematical issues raised at the interface between appropriate models for different flow regimes.
2. Pattern formation and chaos: The mathematical modelling of patterns and their dynamics in controlled experiments on segregation and vibrated layers.
3. Geophysical and particle-laden flows: Theoretical modelling of problems ranging from the formation of sedimentary rock to avalanching.

Organizers: N. Gray (Manchester), K. Hutter (Darmstadt), J. T. Jenkins (Cornell), T. Mullin (Manchester).
Information: `http://www.newton.cam.ac.uk/programmes/CPD/`, Isaac Newton Institute for Mathematical Sciences, 20 Clarkson Road, Cambridge, CB3 OEH or CB3 0EH; tel: +44 (0) 1223 335999; fax: +44 (0) 1223 330508; email: `info@newton.cam.ac.uk`.

2-5 **Symposium for the Developments of the Cantorian Set Theory**, Paris, France. (Jan. 2003, p. 83)

2-6 **The Barcelona Conference on Asymptotic Statistics**, Bellaterra, Barcelona, Spain. (Jan. 2003, p. 83)

4-9 **Analytic Methods of Analysis and Differential Equations (AMADE-2003)**, Belarusian State University, Minsk, Belarus. (Jan. 2003, p. 83)

8-10 **25th World Conference on Boundary Element Methods: BEM 25: Incorporating Electromagnetic Effects on Human Beings and Equipment Seminar**, Split, Croatia. (Oct. 2002, p. 1135)

8-December 13 **Inverse Problems: Computational Methods and Emerging Applications**, Institute for Pure and Applied Mathematics, UCLA, Los Angeles, California. (Oct. 2002, p. 1135)

10-12 **Sixth International Conference on Computational Methods for the Solution of Electrical and Electromagnetic Engineering Problems: ELECTROCOMP 2003**, Split, Croatia. (Oct. 2002, p. 1135)

12-16 **International Conference of Computational Methods in Sciences and Engineering (ICCMSE 2003)**, Kastoria, Greece. (Feb. 2003, p. 294)

15-19 **Colloquium on the Occasion of the 200th Anniversary of Charles-François Sturm and Workshop on Sturm-Liouville Theory**, University of Geneva, Geneva, Switzerland. (Oct. 2002, p. 1135)

* 15-19 **IMA Tutorial: Open Week Tutorial**, University of Minnesota, Minneapolis, Minnesota.
Organizer: S. Keller-McNulty (Los Alamos National Labs), M. Newton (Wisconsin-Madison), S. Tavaré (USC).
Information: Institute for Mathematics and its Applications, Univ. of Minnesota, 207 Church St. SE, 400 Lind Hall, Minneapolis, MN 55455; phone: 612-624-6066; email: `visit@ima.umn.edu`; `http://www.ima.umn.edu/complex/fall/t1.html`.

16-20 **The Barcelona Conference on Set Theory**, Bellaterra, Barcelona, Spain. (Feb. 2003, p. 294)

* 22-26 **Analytic Dynamical Systems, Summability of Divergent Series and Galois Theories**, Université Paul Sabatier, Toulouse, France.
Description: International Conference on the occasion of the 60th Anniversary of Jean-Pierre Ramis.
Topics: Complex analysis, analytic dynamical systems (Painvelé equations, d-modules, differences and q-differences equations, foliations, p-adic differential equations), Summability of divergent series and applications, Galois theories.
Speakers: Y. André (Paris); J.-B. Bost (Orsay); B. Braaksma (Groningen); A. Connes (Collège de France); O. Costin (Rutgers); B. Dubrovin (Sissa); Y. Il'Yashenko (Moscou and Cornell); V. Kaloshin (Princeton); T. Kawai (RIMS-Kyoto); Y. Laurent (Grenoble); A. Lins Neto (IMPA -Rio); J. Morales (Barcelone); R. Moussu (Dijon); K. Okamoto (Tokyo); J. Sauloy (Toulouse); D. Sauzin (Paris); M. Singer (MSRI - Berkeley); H. Umemura (Nagoya); M. Van der Put (Groningen); A. Verjovski (Cuernavaca).
Organizers: A. Duval (Lille), F. Fauvet (Strasbourg), M. Loday-Richaud (Angers), M. Klughertz (Toulouse), J.-F. Mattei (Toulouse), L. Stolovitch (Toulouse).
Information: email: `ramisconf@picard.ups-tlse.fr`; `http://picard.ups-tlse.fr/`.

24-26 **International Conference on Differential Equations Devoted to the 100th Anniversary of K. P. Persidskii**, Institute of Mathematics of the ME&S of the RK, Almaty, Kazakhstan (CIS). (Sept. 2002, p. 1001)

* 29-October 3 **IMA Workshop I: Statistical Methods for Gene Expression: Microarrays and Proteomics**, University of Minnesota, Minneapolis, Minnesota.
Organizers: M. A. Newton (Wisconsin-Madison), G. Parmigiani (Johns Hopkins).
Information: Institute for Mathematics and its Applications, University of Minnesota, 207 Church St. SE, 400 Lind Hall, Minneapolis, MN 55455; phone: 612-624-6066; email: `visit@ima.umn.edu`; `http://www.ima.umn.edu/complex/fall/c1.html`.

30-October 7 **Mathematics in Armenia—Advances and Perspectives**, Institute of Mathematics of National Academy of Sciences of Armenia, Yerevan, Armenia. (Feb. 2003, p. 295)

October 2003

11-12 **AMS Eastern Section Meeting**, SUNY-Binghamton, Binghamton, New York. (Sept. 2002, p. 1001)

13-17 **2003 IEEE/WIC International Conference on Web Intelligence (WI 2003)**, Tianlun Dynasty Hotel, Beijing, China. (Feb. 2003, p. 295)

15-20 **Von Neumann Centennial Conference: Linear Operators and Foundations of Quantum Mechanics**, Budapest, Hungary. (Sept. 2002, p. 1001)

*20-24 **IMA Workshop 2: Comparative Genomics**, University of Minnesota, Minneapolis, Minnesota.
Organizers: J. Lagergren (Royal Inst. of Tech., Stockholm), B. Moret (UNM), D. Sankoff (Ottawa).
Information: Institute for Mathematics and its Applications, University of Minnesota, 207 Church St. SE, 400 Lind Hall, Minneapolis, MN 55455; phone: 612-624-6066; email: visit@ima.umn.edu; http://www.ima.umn.edu/complex/fall/c2.html.

24-25 **AMS Southeastern Section Meeting**, University of North Carolina, Chapel Hill, North Carolina. (Sept. 2002, p. 1001)

28-31 **Fourteenth International Symposium on Methodologies for Intelligent Systems**, Maebashi TERRSA, Maebashi City, Japan. (Feb. 2003, p. 295)

November 2003

3-5 **Second International Conference on Computational Methods in Multiphase Flow**, Santa Fe, New Mexico. (Oct. 2002, p. 1135)

4-6 **Seventh International Conference on Computational Modelling of Free and Moving Boundary Problems**, Santa Fe, New Mexico. (Oct. 2002, p. 1135)

10-13 **SIAM Conference on Geometric Design & Computing**, Elliott Grand Hyatt, Seattle, Washington. (Nov. 2002, p. 1287)

*17-21 **IMA Workshop 3: Networks and the Population Dynamics of Disease Transmission**, University of Minnesota, Minneapolis, Minnesota.
Organizers: M. Morris (Washington), C. Neuhauser (UMN).
Information: Institute for Mathematics and its Applications, University of Minnesota, 207 Church St. SE, 400 Lind Hall, Minneapolis, MN 55455; phone: 612-624-6066; email: visit@ima.umn.edu; http://www.ima.umn.edu/complex/fall/c3.html.

December 2003

*14-16 **International Conference on Matrix Analysis and Applications**, Nova Southeastern University, Fort Lauderdale, Florida.
Description: The aim of this meeting is to stimulate research and interaction of researchers interested in all aspects of linear algebra and matrix analysis and applications, and to provide an opportunity for researchers to exchange ideas and recent developments on the subjects.
Keynote Speaker: R. Horn, Univ. of Utah.
Organizing Committee: T. Ando (Hokkaido Univ., Japan), C.-K. Li (College of William and Mary, USA), G. P. H. Styan (McGill Univ., Canada), H. Woerdeman (College of William and Mary, USA, and Catholic Univ., Belgium), F. Zhang (Nova Southeastern Univ., USA).
Contact: ckli@math.wm.edu or zhang@nova.edu.
Information: http://www.resnet.wm.edu/~cklixx/nova03.html.

15-19 **Numerical Methods in Imaging Science and Information Processing**, Institute for Mathematical Sciences, National University of Singapore, Singapore. (Dec. 2002, p. 1422)

17-20 **First Joint International Meeting between the American Mathematical Society and Various Indian Mathematical Societies**, Bangalore, India. (Sept. 2002, p. 1001)

January 2004

*7-9 **IMA Short Course: The Internet for Mathematicians**, University of Minnesota, Minneapolis, Minnesota.
Organizer: W. Willinger (AT&T Research), B. Hajek (Illinois).
Information: Institute for Mathematics and its Applications, University of Minnesota, 207 Church St. SE, 400 Lind Hall, Minneapolis, MN 55455; phone: 612-624-6066; email: visit@ima.umn.edu; http://www.ima.umn.edu/complex/winter/sc1.html.

7-10 **Joint Mathematics Meetings**, Phoenix Civic Plaza, Phoenix, Arizona.

*11 **IMA Tutorial: Measurement, Modeling and Analysis of the Internet**, University of Minnesota, Minneapolis, Minnesota.
Organizer: B. Hajek (Illinois), D. Towsley (Massachusetts).
Information: Institute for Mathematics and its Applications, University of Minnesota, 207 Church St. SE, 400 Lind Hall, Minneapolis, MN 55455; phone: 612-624-6066; email: visit@ima.umn.edu; http://www.ima.umn.edu/complex/winter/t2.html.

*12-16 **IMA Workshop 4: Measurement, Modeling and Analysis of the Internet**, University of Minnesota, Minneapolis, Minnesota.
Organizers: B. Hajek (Illinois), D. Towsley (Massachusetts).
Information: Institute for Mathematics and its Applications, University of Minnesota, 207 Church St. SE, 400 Lind Hall, Minneapolis, MN 55455; phone: 612-624-6066; email: visit@ima.umn.edu; http://www.ima.umn.edu/complex/winter/c4.html.

*19-July 9 **Statistical Mechanics of Molecular and Cellular Biological Systems**, Isaac Newton Institute for Mathematical Sciences, Cambridge, England.
Description: The programme will be structured along four linked themes:

1. Single molecule biophysics (including protein dynamics, mechanical force spectroscopy).
2. Membrane/cortical dynamics and self-assembly (including lipid phase separation, motility and interaction with the extracellular matrix).
3. Molecular motors (including modelling of single-molecule motors in the presence of noise, cooperative behaviour, etc.).
4. Molecular and cellular aspects of gene expression (including DNA binding proteins and complexes, cell division, transmembrane signalling, networks or polymerisation and depolymerisation).

While all four will be worked on throughout the 6-month period, there will be periods of more focus on each when the theoretically based scientists and mathematicians of the long-term programme will be visited on a shorter-term basis by key experimentalists working in these areas and their boundaries.
Organizers: T. McLeish (Leeds), J. Trinick (Leeds), P. Stockley (Leeds), J. Molloy (York), W. Poon (Edinburgh), T. Duke (Cambridge).
Information: Isaac Newton Institute for Mathematical Sciences, 20 Clarkson Road, Cambridge, CB3 OEH UK; tel: +44 (0) 1223 335999; fax: +44 (0) 1223 330508; email: info@newton.com.ac.uk; http://www.newton.cam.ac.uk/.

*26-July 16 **Random Matrix Approaches in Number Theory**, Isaac Newton Institute for Mathematical Sciences, Cambridge, England.
Description: The programme will mainly focus on how random matrix theory can further contribute to unanswered questions in number theory and on how to put the connection between random matrices and number theory on a rigorous footing. However, both random matrix theory and number theory individually play significant roles in theoretical physics, and probability random matrix statistics appear in the spectra of quantum systems whose classical limit is chaotic; the problem of quantum unique ergodicity has connections with the theory of modular surfaces and algebraic number theory; many of the main results on the statistics of ensembles of random matrices have been the work of probabilists; the Riemann zeta function even shows up in the theory of Brownian motion—and this is just to name a few. These themes will also be developed through focused workshops.
Organizers: B. Conrey (Palo Alto), P. Diaconis (Stanford), F. Mezzadri (Bristol), P. Sarnak (Princeton).

Information: Isaac Newton Institute for Mathematical Sciences, 20 Clarkson Road, Cambridge, CB3 0EH UK; tel: +44 (0) 1223 335999; fax: +44 (0) 1223 330508; email: info@newton.com.ac.uk; http://www.newton.cam.ac.uk/.

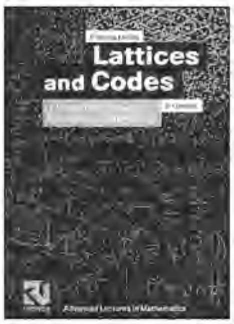
The following new announcements will not be repeated until the criteria in the next to the last paragraph at the bottom of the first page of this section are met.

July 2004

* 5-9 **Graphes et Combinatoire, un Colloque a la Memoire de Claude Berge**, Université Paris 6, Paris, France.
Program: Graph theory and combinatorics.
Organizers: A. Bondy, J.-C. Fournier, and M. Las Vergnas.

New Publications Offered by the AMS

Algebra and Algebraic Geometry



Lattices and Codes

A Course Partially Based on Lectures by F. Hirzebruch, Second Edition

Wolfgang Ebeling, *Universität Hannover, Germany*

The purpose of coding theory is the design of efficient systems for the transmission of information. The mathematical treatment leads to certain finite structures: the error-correcting codes. Surprisingly, problems that are interesting for the design of codes turn out to be closely related to problems studied earlier and independently in pure mathematics. In this book, examples of such connections are presented. The relation between lattices studied in number theory and geometry and error-correcting codes is discussed. At the same time, the book provides an introduction to the theory of integral lattices and modular forms and to coding theory.

In this second edition, additional basic material has been included to make the text even more self-contained. There is a new section on autmorphism groups related to the Leech lattice. Some hints to new results have been incorporated. Finally, several new exercises have been added.

It is suitable for graduate students and researchers in mathematics and computer science.

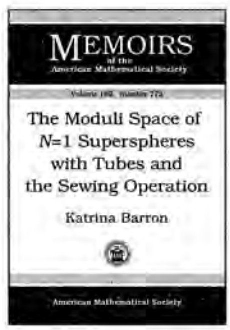
This item will also be of interest to those working in discrete mathematics and combinatorics.

A publicaton of Vieweg Verlag. The AMS is exclusive distributor in North America. Vieweg Verlag Publications are available worldwide from the AMS outside of Germany, Switzerland, Austria, and Japan.

Contents: Lattices and codes; Theta functions and weight enumerators; Even unimodular lattices; The Leech lattice; Lattices over integers of number fields and self-dual codes; Bibliography; Index.

Vieweg Advanced Lectures in Mathematics

July 2002, 188 pages, Softcover, ISBN 3-528-16497-2, 2000 *Mathematics Subject Classification*: 11H06, 11H31, 11H55, 11F11, 11F41, 11R04, 11R18, 94B05, 94B15, 94B75, 51F15, 51E10, **All AMS members $32**, List $35, Order code VWALM/9N



The Moduli Space of $N = 1$ Superspheres with Tubes and the Sewing Operation

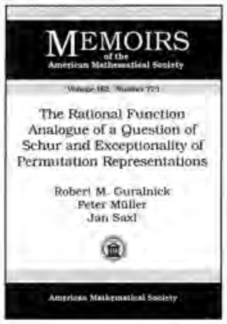
Katrina Barron, *University of Notre Dame, IN*

This item will also be of interest to those working in mathematical physics.

Contents: Introduction; An introduction to the moduli space of $N = 1$ superspheres with tubes and the sewing operation; A formal algebraic study of the sewing operation; An analytic study of the sewing operation; Bibliography.

Memoirs of the American Mathematical Society, Volume 162, Number 772

March 2003, 135 pages, Softcover, ISBN 0-8218-3260-3, LC 2002038386, 2000 *Mathematics Subject Classification*: 17B65, 17B68, 17B81, 32A05, 32C11, 58A50, 81R10, 81T40, 81T60; 17B69, 30F10, 32G15, **All AMS members $44**, List $55, Order code MEMO/162/772N



The Rational Function Analogue of a Question of Schur and Exceptionality of Permutation Representations

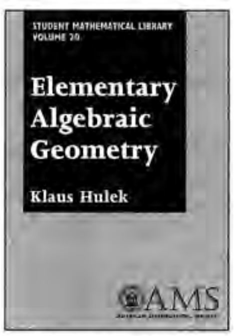
Robert M. Guralnick, *University of Southern California, Los Angeles*, **Peter Müller**, *University of Heidelberg, Germany*, and **Jan Saxl**

Contents: Introduction; Arithmetic-Geometric preparation; Group theoretic exceptionality; Genus 0 condition; Dickson polynomials and Rédei functions; Rational functions with Euclidean signature; Sporadic cases of arithmetic exceptionality; Bibliography.

Memoirs of the American Mathematical Society, Volume 162, Number 773

March 2003, 79 pages, Softcover, ISBN 0-8218-3288-3, LC 2002038385, 2000 *Mathematics Subject Classification*: 12E30, 20B15; 11G05, 14H25, 14H30, **All AMS members $39**, List $49, Order code MEMO/162/773N

Recommended Text



Elementary Algebraic Geometry

Klaus Hulek, *Universität Hannover, Germany*

This is a true introduction to algebraic geometry. The author makes no assumption that readers know more than can be expected of a good undergraduate. He introduces fundamental concepts in a way that enables students to move on to a more advanced book or course that relies more heavily on commutative algebra.

The language is purposefully kept on an elementary level, avoiding sheaf theory and cohomology theory. The introduction of new algebraic concepts is always motivated by a discussion of the corresponding geometric ideas. The main point of the book is to illustrate the interplay between abstract theory and specific examples. The book contains numerous problems that illustrate the general theory.

The text is suitable for advanced undergraduates and beginning graduate students. It contains sufficient material for a one-semester course. The reader should be familiar with the basic concepts of modern algebra. A course in one complex variable would be helpful, but is not necessary. It is also an excellent text for those working in neighboring fields (algebraic topology, algebra, Lie groups, etc.) who need to know the basics of algebraic geometry.

Contents: Introduction; Affine varieties; Projective varieties; Smooth points and dimension; Plane cubic curves; Cubic surfaces; Introduction to the theory of curves; Bibliography; Index.

Student Mathematical Library, Volume 20

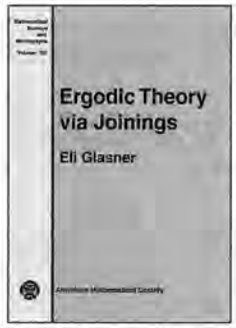
March 2003, 213 pages, Softcover, ISBN 0-8218-2952-1, LC 2002038457, 2000 *Mathematics Subject Classification*: 14-01, **All AMS members $28**, List $35, Order code STML/20N

Analysis

Supplementary Reading

Independent Study

Recommended Text



Ergodic Theory via Joinings

Eli Glasner, *Tel Aviv University, Israel*

This book introduces modern ergodic theory. It emphasizes a new approach that relies on the technique of joining two (or more) dynamical systems. This approach has proved to be fruitful in many recent works, and this is the first time that the entire theory is presented from a joining perspective.

Another new feature of the book is the presentation of basic definitions of ergodic theory in terms of the Koopman unitary representation associated with a dynamical system and the invariant mean on matrix coefficients, which exists for any acting groups, amenable or not. Accordingly, the first part of the book treats the ergodic theory for an action of an arbitrary countable group.

The second part, which deals with entropy theory, is confined (for the sake of simplicity) to the classical case of a single measure-preserving transformation on a Lebesgue probability space.

Topics treated in the book include:

- The interface between topological dynamics and ergodic theory;
- The theory of distal systems due to H. Furstenberg and R. Zimmer—presented for the first time in monograph form;
- B. Host's solution of Rohlin's question on the mixing of all orders for systems with singular spectral type;
- The theory of simple systems;
- A dynamical characterization of Kazhdan groups;
- Weiss's relative version of the Jewett-Krieger theorem;
- Ornstein's isomorphism theorem;
- A local variational principle and its applications to the theory of entropy pairs.

The book is intended for graduate students who have a good command of basic measure theory and functional analysis and who would like to master the subject. It contains many detailed examples and many exercises, usually with indications of solutions. It can serve equally well as a textbook for graduate courses, for independent study, supplementary reading, or as a streamlined introduction for non-specialists who wish to learn about modern aspects of ergodic theory.

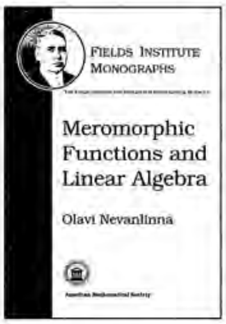
Contents: Introduction; *General group actions:* Topological dynamics; Dynamical systems on Lebesgue spaces; Ergodicity and mixing properties; Invariant measures on topological systems; Spectral theory; Joinings; Some applications of joinings; Quasifactors; Isometric and weakly mixing extensions; The Furstenberg-Zimmer structure theorem; Host's theorem;

Simple systems and their self-joinings; Kazhdan's property and the geometry of $M_\Gamma(\mathbf{X})$; *Entropy theory for* $\mathbb{Z}$*-systems:* Entropy; Symbolic representations; Constructions; The relation between measure and topological entropy; The Pinsker algebra, CPE and zero entropy systems; Entropy pairs; Krieger's and Ornstein's theorems; Prerequisite background and theorems; Bibliography; Index of symbols; Index of terms.

Mathematical Surveys and Monographs, Volume 101

March 2003, 384 pages, Hardcover, ISBN 0-8218-3372-3, LC 2002043617, 2000 *Mathematics Subject Classification*: 37Axx, 28Dxx, 37Bxx, 54H20, 20Cxx, **All AMS members $71**, List $89, Order code SURV/101N

Independent Study



Meromorphic Functions and Linear Algebra

Olavi Nevanlinna, *Helsinki University of Technology, Finland*

This volume describes for the first time in monograph form important applications in numerical methods of linear algebra. The author presents new material and extended results from recent papers in a very readable style.

The main goal of the book is to study the behavior of the resolvent of a matrix under the perturbation by low rank matrices. Whereas the eigenvalues (the poles of the resolvent) and the pseudospectra (the sets where the resolvent takes large values) can move dramatically under such perturbations, the growth of the resolvent as a matrix-valued meromorphic function remains essentially unchanged. This has practical implications to the analysis of iterative solvers for large systems of linear algebraic equations.

First, the book introduces the basics of value distribution theory of meromorphic scalar functions. It then introduces a new nonlinear tool for linear algebra, the total logarithmic size of a matrix, which allows for a nontrivial generalization of Rolf Nevanlinna's characteristic function from the scalar theory to matrix- and operator-valued functions. In particular, the theory of perturbations by low rank matrices becomes possible. As an example, if the spectrum of a normal matrix collapses under a low rank perturbation, there is always a compensation in terms of the loss of orthogonality of the eigenvectors. This qualitative phenomenon is made quantitative by using the new tool. Applications are given to rational approximation, to the Kreiss matrix theorem, and to convergence of Krylov solvers.

The book is intended for researchers in mathematics in general and especially for those working in numerical linear algebra. Much of the book is understandable if the reader has a good background in linear algebra and a first course in complex analysis.

This item will also be of interest to those working in applications.

Fields Institute Monographs, Volume 18

April 2003, 136 pages, Hardcover, ISBN 0-8218-3247-6, LC 2002041519, 2000 *Mathematics Subject Classification*: 30G30, 47A10, 47B10, 65F10, **All AMS members $39**, List $49, Order code FIM/18N

Applications

Stochastic Petri Nets

An Introduction to the Theory, Second Edition

Falko Bause and **Pieter S. Kritzinger**, *University of Cape Town, South Africa*

Stochastic Petri nets are a modeling paradigm for the functional and performance analysis of systems. This book provides all the information necessary for understanding stochastic Petri nets, including a short refresher on probability theory, Markov processes, and single queues. The authors explain generalized stochastic Petri nets in detail and also show how to incorporate queueing aspects into the theory. The book analyzes stochastic Petri nets stressing a combined functional and quantitative examination of systems.

It is suitable for advanced undergraduates, graduate students, researchers, software developers, and systems engineers interested in mathematics, computer science, and the functional and performance evaluation of systems.

This item will also be of interest to those working in probability.

A publicaton of Vieweg Verlag. The AMS is exclusive distributor in North America. Vieweg Verlag Publications are available worldwide from the AMS outside of Germany, Switzerland, Austria, and Japan.

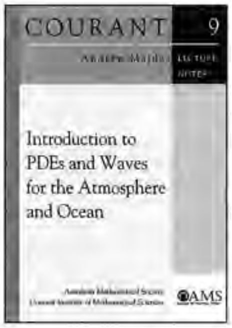
Contents: Preface; *Part I. Stochastic Theory:* Random variables; Markov processes; General queueing systems; Further reading; *Part II. Petri Nets:* Place-transition nets; Coloured Petri nets; Further reading; *Part III. Time-augmented Petri nets:* Stochastic Petri nets; Generalized stochastic Petri nets; Queueing Petri nets; Further reading; Application examples; Solutions to selected exercises; Bibliography; Index.

Vieweg Monographs

August 2002, 218 pages, Softcover, ISBN 3-528-15535-3, 2000 *Mathematics Subject Classification*: 60-XX, 68-XX, **All AMS members $32**, List $35, Order code VW/12N

Supplementary Reading

Independent Study



Introduction to PDEs and Waves for the Atmosphere and Ocean

Andrew Majda, *New York University-Courant Institute of Mathematical Sciences, New York*

Written by a leading specialist in the area of atmosphere/ocean science (AOS), the book presents an excellent introduction to this important topic. The goals of these lecture notes, based on courses presented by the author at the Courant Institute of Mathematical Sciences, are to introduce mathematicians to the fascinating and important area of atmosphere/ocean science (AOS) and, conversely, to develop a mathematical viewpoint on basic topics in AOS of interest to the disciplinary AOS community, ranging from graduate students to researchers. The lecture notes emphasize the serendipitous connections between applied mathematics and geophysical flows in the style of modern applied mathematics, where rigorous mathematical analysis as well as asymptotic, qualitative, and numerical modeling all interact to ease the understanding of physical phenomena. Reading these lecture notes does not require a previous course in fluid dynamics, although a serious reader should supplement these notes with material such as additional information on geophysical flows, as suggested in the preface.

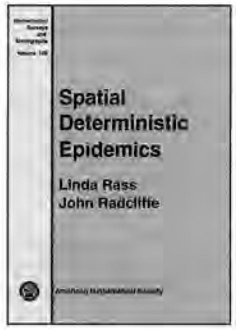
The book is intended for graduate students and researchers working in interdisciplinary areas between mathematics and AOS. It is excellent for supplementary course reading or independent study.

Titles in this series are copublished with the Courant Institute of Mathematical Sciences at New York University.

Contents: Introduction; Some remarkable features of stratified flow; Linear and nonlinear instability of stratified flows with strong stratification; Rotating shallow water theory; Linear and weakly nonlinear theory of dispersive waves with geophysical examples; Simplified equations for the dynamics of strongly stratified flow; The stratified quasi-geostrophic equations as a singular limit of the rotating Boussinesq equations; Introduction to averaging over fast waves for geophysical flows; Waves and PDEs for the equatorial atmosphere and ocean; Bibliography.

Courant Lecture Notes, Volume 9

February 2003, 234 pages, Softcover, ISBN 0-8218-2954-8, LC 2002042674, 2000 *Mathematics Subject Classification*: 34-XX, 35-XX, 65-XX, 76-XX, 86-XX, **All AMS members $26**, List $32, Order code CLN/9N



Spatial Deterministic Epidemics

Linda Rass and **John Radcliffe**, *Queen Mary, University of London*

The study of epidemic models is one of the central topics of mathematical biology. This volume is the first to present in monograph form the rigorous mathematical theory developed to analyze the asymptotic behavior of certain types of epidemic models.

The main model discussed is the so-called spatial deterministic epidemic in which infected individuals are not allowed to again become susceptible, and infection is spread by means of contact distributions. Results concern the existence of traveling wave solutions, the asymptotic speed of propagation, and the spatial final size. A central result for radially symmetric contact distributions is that the speed of propagation is the minimum wave speed. Further results are obtained using a saddle point method, suggesting that this result also holds for more general situations.

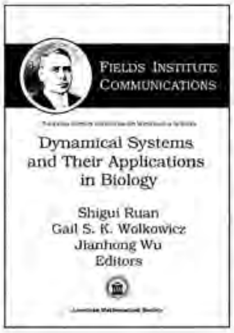
Methodology, used to extend the analysis from one-type to multi-type models, is likely to prove useful when analyzing other multi-type systems in mathematical biology. This methodology is applied to two other areas in the monograph, namely epidemics with return to the susceptible state and contact branching processes.

This book presents an elegant theory that has been developed over the past quarter century. It will be useful to researchers and graduate students working in mathematical biology.

Contents: Introduction; The non-spatial epidemic; Bounds on the spatial final size; Wave solutions; The asymptotic speed of propagation; An epidemic on sites; The saddle point method; Epidemics with return to the susceptible state; Contact branching processes; Appendices; Bibliography; Index.

Mathematical Surveys and Monographs, Volume 102

March 2003, 261 pages, Hardcover, ISBN 0-8218-0499-5, LC 2002038456, 2000 *Mathematics Subject Classification*: 92D30; 92D25, **All AMS members $55**, List $69, Order code SURV/102N



Dynamical Systems and Their Applications in Biology

Shigui Ruan, *Dalhousie University, Halifax, NS, Canada*, **Gail S. K. Wolkowicz**, *McMaster University, Hamilton, ON, Canada*, and **Jianhong Wu**, *York University, North York, ON, Canada*, Editors

This volume is based on the proceedings of the International Workshop on Dynamical Systems and their Applications in Biology held at the Canadian Coast Guard College on Cape Breton Island (Nova Scotia, Canada). It presents a broad picture of the current research surrounding applications of dynamical systems in biology, particularly in population biology.

The book contains 19 papers and includes articles on the qualitative and/or numerical analysis of models involving ordinary, partial, functional, and stochastic differential equations. Applications include epidemiology, population dynamics, and physiology.

The material is suitable for graduate students and research mathematicians interested in ordinary differential equations and their applications in biology. Also available by Ruan, Wolkowicz, and Wu is *Differential Equations with Applications to Biology*, Volume 21 in the AMS series Fields Institute Communications.

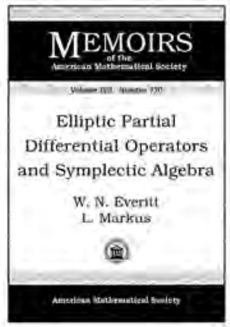
This item will also be of interest to those working in differential equations.

Contents: **J. Atamanyk** and **W. F. Langford**, A compartmental model of Cheyne-Stokes respiration; **M. Bachar** and **O. Arino**, Integrated semigroup and linear ordinary differential equation with impulses; **C. Bauch** and **D. J. D. Earn**, Interepidemic intervals in forced and unforced SEIR models; **E. Beretta**, **H. Sakakibara**, and **Y. Takeuchi**, Stability analysis of time delayed chemostat models for bacteria and virulent phage; **J. Best**, **C. Castillo-Chavez**, and **A.-A. Yakubu**, Hierarchical competition in discrete time models with dispersal; **F. Brauer**, Stability and instability theorems for a characteristic equation arising in epidemic modeling; **F. Brauer** and **P. van den Driessche**, Some directions for mathematical epidemiology; **Y. Chen**, Global attractivity of a population model with state-dependent delay; **Z. Feng**, **Y. Yi**, and **H. Zhu**, Metapopulation dynamics with migration and local competition; **S. A. Gourley**, Oscillations and convergence in a harvesting model with sawtooth delay; **W. Li** and **M. Zhang**, Rigidity for differentiable classification of one-dimensional dynamical systems; **X. Liu**, Management of biological populations via impulsive control; **C. C. McCluskey**, Stability for a class of three-dimensional homogeneous systems; **I. Ncube**, **S. A. Campbell**, and **J. Wu**, Change in criticality of synchronous Hopf bifurcation in a multiple-delayed neural system; **Y. Saito** and **Y. Takeuchi**, Sharp conditions for global stability of Lotka-Volterra systems with delayed intraspecific competitions; **H. L. Smith** and **B. Li**, Competition for essential resources: A brief review; **X. H. Tang**, **L. Wang**, and **X. Zou**, 3/2 type criteria for global attractivity of Lotka-Volterra discrete system with delays; **P. van den Driessche** and **J. Watmough**, Epidemic solutions and endemic catastrophies; **X.-Q. Zhao**, Persistence in almost periodic predator-prey reaction-diffusion systems.

Fields Institute Communications, Volume 36

March 2003, 268 pages, Hardcover, ISBN 0-8218-3163-1, LC 2002038530, 2000 *Mathematics Subject Classification*: 34-XX; 92-XX, 35-XX, 58-XX, **All AMS members $63**, List $79, Order code FIC/36N

Differential Equations



Elliptic Partial Differential Operators and Symplectic Algebra

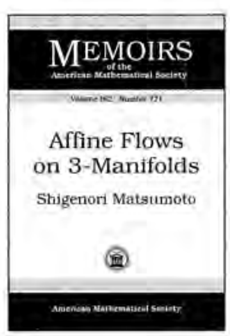
W. N. Everitt, *University of Birmingham, England*, and **L. Markus**, *University of Minnesota, Minneapolis*

Contents: Introduction: Organization of results; Review of Hilbert and symplectic space theory; GKN-theory for elliptic differential operators; Examples of the general theory; Global boundary conditions: Modified Laplace operators; Appendix A. List of symbols and notations; Bibliography; Index.

Memoirs of the American Mathematical Society, Volume 162, Number 770

March 2003, 111 pages, Softcover, ISBN 0-8218-3235-2, LC 2002038388, 2000 *Mathematics Subject Classification*: 35J40, 35P05, 51A50; 47B25, 35J67, **All AMS members $42**, List $52, Order code MEMO/162/770N

Geometry and Topology



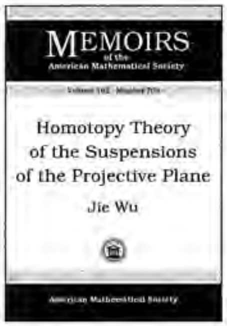
Affine Flows on 3-Manifolds

Shigenori Matsumoto, *Nihon University, Tokyo*

Contents: Introduction; Complete affine flows; Luxuriant foliations; SL-flows; SA-flows; Bibliography.

Memoirs of the American Mathematical Society, Volume 162, Number 771

March 2003, 94 pages, Softcover, ISBN 0-8218-3257-3, LC 2002038387, 2000 *Mathematics Subject Classification*: 57R25, 53C12, 37C10, **All AMS members $41**, List $51, Order code MEMO/162/771N



Homotopy Theory of the Suspensions of the Projective Plane

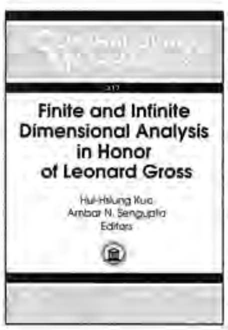
Jie Wu, *National University of Singapore*

Contents: Preliminary and the classical homotopy theory; Decompositions of self smash products; Decompositions of the loop spaces; The homotopy groups $\pi_{n+r}(\Sigma^n \mathbb{R}P^2)$ for $n \geq 2$ and $r \leq 8$; The homotopy theory of $\Sigma\mathbb{R}P^2$; Bibliography.

Memoirs of the American Mathematical Society, Volume 162, Number 769

March 2003, 130 pages, Softcover, ISBN 0-8218-3239-5, LC 2002038389, 2000 *Mathematics Subject Classification*: 55Q52; 55P35, 55P40, 55Q20, 55R05, 20F38, 20C20, 57T05, 57T20, **All AMS members $42,** List $53, Order code MEMO/162/769N

Probability



Finite and Infinite Dimensional Analysis in Honor of Leonard Gross

Hui-Hsiung Kuo and **Ambar N. Sengupta,** *Louisiana State University, Baton Rouge,* Editors

This book contains the proceedings of the special session in honor of Leonard Gross held at the annual Joint Mathematics Meetings in New Orleans (LA). The speakers were specialists in a variety of fields, and many were Professor Gross's former Ph.D. students and their descendants.

Papers in this volume present results from several areas of mathematics. They illustrate applications of powerful ideas that originated in Gross's work and permeate diverse fields. Topics include stochastic partial differential equations, white noise analysis, Brownian motion, Segal-Bargmann analysis, heat kernels, and some applications.

The volume should be useful to graduate students and researchers. It provides perspective on current activity and on central ideas and techniques in the topics covered.

This item will also be of interest to those working in analysis.

Contents: L. Accardi, Meixner classes and the square of white noise; **S. Albeverio, Y. Kondratiev**, and **M. Röckner**, Strong Feller properties for distorted Brownian motion and applications to finite particle systems with singular interactions; **H. Allouba** and **V. Goodman**, Market price of risk and random field driven models of term structure: A space-time change of measure look; **N. Asai, I. Kubo**, and **H.-H. Kuo**, Gaussian and Poisson white noises with related characterization theorems; **B. K. Driver**, Analysis of Wiener measure on path and loop groups; **M. Gordina**, Stochastic differential equations on noncommutative L^2; **B. C. Hall**, The Segal-Bargmann transform and the Gross ergodicity theorem; **B. C. Hall** and **M. B. Stenzel**, Sharp bounds for the heat kernel on certain symmetric spaces of non-compact type; **T. Hida**, Laplacians in white noise analysis; **M. Hino**, On Dirichlet spaces over convex sets in infinite dimensions; **C. King**, Information capacity of quantum channels; **Y.-J. Lee** and **C.-Y. Shih**, The Riesz representation theorem on infinite dimensional spaces; **J. J. Mitchell**, Asymptotic behavior in heat kernel analysis on manifolds; **M. Redfern**, Complex stochastic calculus; **S. B. Sontz**, Recent results and open problems in Segal-Bargmann analysis; **A. Stan**, A new Heisenberg inequality for white noise analysis.

Contemporary Mathematics, Volume 317

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Classified Advertisements

Positions available, items for sale, services available, and more

FLORIDA

UNIVERSITY OF FLORIDA
Mathematics Department
Gainesville, FL 32611

Subject to budget approval, we expect to have five Special Fellowships for U.S. citizens, U.S. nationals, or permanent residents beginning graduate study in mathematics in our department in fall 2003. These fellowships are renewable for up to five years pending budget approval and satisfactory progress. They will carry a 11-month stipend of $18,000 and are expected to include tuition and partial fee remission. Florida residents under similar fellowships paid about $300 per semester and nonresidents about $500 per semester. There is no teaching during the first year. Duties include a one-third time teaching assistant assignment during the second and third academic years. Subject to budget approval, there will be no or reduced during the last two years of the fellowship. Applications completed by March 1, 2003, will receive full consideration.

To apply online, to obtain paper application forms, or for more information about our department, visit `http://www.math.ufl.edu/` or contact Professor Scott McCullough at `sam@math.ufl.edu`.

IOWA

ST. AMBROSE UNIVERSITY
Department of Mathematics

Tenure-track position in mathematics, starting August 2003; rank and salary commensurate with experience. Responsibilities: teach full spectrum of traditional undergraduate classes. An ability to teach operations research helpful but not required. Qualifications: Ph.D. in mathematics or applied mathematics preferred, but other candidates will be considered. St. Ambrose University is a Catholic, liberal arts, diocesan university that emphasizes excellence in teaching and ongoing professional development. Total institutional enrollment is 3,500 students. St. Ambrose is located in Davenport, Iowa, one of the Quad Cities, a vibrant and diverse metropolitan area with a population of more than 350,000. The Mississippi River joins the two-state community, creating a very affordable, manageable, and culturally rich urban setting distinguished by friendly people and unique river vistas. Review of applications will begin March 1, 2003, and will continue until the position is filled. Please send cover letter, transcripts, vita, three letters of recommendation, and teaching evaluations for three recent courses to: Director of Human Resources, St. Ambrose University, 518 West Locust Street, Davenport, IA 52803. AA/EOE.

MASSACHUSETTS

BOSTON UNIVERSITY
Department of Mathematics and Statistics

The Department of Mathematics and Statistics at Boston University invites exceptionally strong candidates to apply for a position in the general area of stochastic processes at the level of tenure-track assistant professor or associate professor with tenure, to begin in September 2003. Preference will be given to candidates with some experience in the area of mathematical finance. Candidates must demonstrate strong research potential and commitment to excellence in teaching. Responsibilities include teaching, research, and involvement in the M.A. in Mathematical Finance Program. Applicants should submit a vita, the AMS application cover sheet, and at least three letters of recommendation to: Prof. Eric Kolaczyk (Hiring Committee Chair), Department of Mathematics and Statistics, Boston University, 111 Cummington Street, Boston, MA 02215. Review of applications will begin January 16, 2003, and will continue until the position is filled. Boston University is an Affirmative Action/Equal Opportunity Employer.

Suggested uses for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services.

The 2003 rate is $100 per inch or fraction thereof on a single column (one-inch minimum), calculated from top of headline. Any fractional text of $1/2$ inch or more will be charged at the next inch rate. No discounts for multiple ads or the same ad in consecutive issues. For an additional $10 charge, announcements can be placed anonymously. Correspondence will be forwarded.

Advertisements in the "Positions Available" classified section will be set with a minimum one-line headline, consisting of the institution name above body copy, unless additional headline copy is specified by the advertiser. Headlines will be centered in boldface at no extra charge. Ads will appear in the language in which they are submitted.

There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified advertising.

Upcoming deadlines for classified advertising are as follows: April 2003 issue–January 28, 2003; May 2003 issue–February 27, 2003; June/July issue–April 29, 2003; August 2003 issue–May 28, 2003; September 2003 issue–June 26, 2003; October 2003 issue–July 30, 2003.

U.S. laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the U.S. cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to U.S. laws. Details and specific wording may be found on page 1373 (vol. 44).

Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the U.S. and Canada or 401-455-4084 worldwide for further information.

Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02940; or via fax: 401-331-3842; or send email to `classads@ams.org`. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 02904. Advertisers will be billed upon publication.

MICHIGAN

MICHIGAN STATE UNIVERSITY
proMSc Program in Industrial Mathematics
East Lansing, MI 48824

Direct your students toward one of the professional M.Sc. programs. Industry needs business-savvy mathematicians. See http://www.sciencemasters.com/.

PENNSYLVANIA

SWARTHMORE COLLEGE
Mathematics and Statistics Department

Faculty Position: The Department of Mathematics and Statistics at Swarthmore College announces an opening for a one-year Visiting Assistant Professor of Statistics, beginning in September 2003. The teaching load is five semester courses per year. A Ph.D. in statistics or a closely related field is required. Candidates must demonstrate excellence in undergraduate teaching and show evidence of an active research program. A complete application will contain a cover letter, curriculum vitae, statement of teaching philosophy, a summary of research interests, and at least three letters of recommendation. Review of applications will begin no later than January 15, 2003. Send applications to: Statistics Search Committee, Swarthmore College, 3 Whittier Place, Swarthmore, PA 19086. For more information, contact Phil Everson (everson@swarthmore.edu).

TENNESSEE

VANDERBILT UNIVERSITY
Department of Mathematics
1326 Stevenson Center
Nashville, TN 37240

We invite applications for several non-tenure-track assistant professor positions beginning fall 2003. Each position is at the non-tenure-track assistant professor level with a 2-2 teaching load. These are two-year appointments, normally renewable for a third year. These positions are intended for recent Ph.D.'s with demonstrated research potential and a strong commitment to excellence in teaching who would like to spend time in a department with a vigorous research atmosphere. Submit your application and supporting material in a single mailing, inclusive of an email address, a fax number, an AMS standardized curriculum vitae, and a research summary. Do not send additional information (including letters of recommendation) unless requested to do so after our initial screening. Evaluation of the applications will commence on January 1, 2003, and will continue until the positions are filled. Prospective applicants are invited to visit the Vanderbilt University Department of Mathematics website (http://www.math.vanderbilt.edu/) for the research interests of the faculty. Vanderbilt University is an Affirmative Action/Equal Opportunity Employer.

WISCONSIN

UNIVERSITY OF WISCONSIN-MADISON
Great Lakes Geometry Conference in Honor of E. Calabi's 80th Birthday

This year's Great Lakes Geometry Conference will be held at the University of Wisconsin-Madison from May 1 to May 4, 2003. It will focus on geometric analysis and will be held in honor of E. Calabi's 80th birthday. Invited speakers include: J. P. Bourguignon (IHES); H. Bray (MIT); J. Y. Chen (UBC); T. Colding (Courant Institute); F. Hirzebruch (MPI); J. Jost (MPI); B. Lawson (SUNY at Stony Brook); Zhiqin Lu (UC at Irvine); J. Milgram (Stanford); X. Rong (Rutgers); Y. T. Siu (Harvard); C. L. Terng (Northeastern Univ.); G. Tian (MIT); J. P. Wang (Univ. of Minnesota).

Conference organizers: A. Adem (UW-Madison); X. X. Chen (UW-Madison); J. Cheeger (Courant Institute); H. Gluck (University of Pennsylvania).

Blocks of rooms have been reserved at Friedrich Center, 1950 Willow Drive, phone: 608-231-1341; and Lowell Hall, 610 Langdon St., phone: 608-256-2621. Participants are responsible for making their own lodging arrangements. The deadline for making reservations is March 30, 2003.

Graduate students, new Ph.D.'s, women and minorities are encouraged to attend. Limited financial support is available.

TURKEY

KOÇ UNIVERSITY
Department of Mathematics

Applications and nominations are invited for faculty positions in all areas of mathematics; preference will be given to the applicants who have been working in numerical analysis or number theory. Appointment will be effective September 1, 2003. Koç University, founded in 1993, is a private Turkish institution of higher education, committed to the pursuit of excellence in both research and teaching. Its aim is to provide a world-class education to a highly select group of students in a liberal arts setting. The medium of instruction is English. Koç University is also considering M.A./M.S. degree programs in the coming year in selected areas, including mathematics, where there is particular strength.

Successful candidates are expected to have a strong research and publication record. More information about the faculty and their research activities is available on the university's Webpage at http://www.ku.edu.tr/.

The compensation package is competitive. All information on candidates will be kept confidential. For further information you may contact any one of the mathematics faculty. Letter and vita should be sent to:

Professor Ersin Yurtsever
Dean of the College
of Arts and Sciences
Koç University
Rumelifeneri Yolu, 34450
Sariyer-Istanbul, Turkey
eyurtsev@ku.edu.tr
phone: (90-212) 338-1401
fax: (90-212) 338-1559

International Conference (Call for Papers):

Applications of Plausible, Paradoxical, and Neutrosophic Reasoning for Information Fusion

8-11 July 2003, Radisson Hotel, Cairns, Queensland, Australia

Topics:

1) Applications of Neutrosophic Logic in Information Fusion
2) Generalization of Dempster-Shafer Theory of Evidence to Dezert-Smarandache Theory of Plausible and Paradoxical Reasoning

Description:

The processing of uncertain information has always been a hot topic of research since the 18th century and deep theoretical advances have been obtained for the theory of probability theory and statistics. During the second half of the 20th century, several new and interesting mathematical theories have emerged in parallel with the development of computer science and technology in order to combine many types of information (fuzzy, uncertain, imprecise, etc.) provided by different sources (human expertise, sensor measurements, AI expert systems, neural network, quantum theory, economics predictions). The problem of combination of such diverse information is very difficult and is great challenge for all researchers working in this field. The information fusion is very important in many fields of applications and particularly in all modern defense systems. Up to now, the principal theories available for data fusion are the axiomatic probability theory (Kolmogorov 1933), the fuzzy set theory (Zadeh 1965), the possibility theory (Dubois and Prade 1985) and the theory of evidence developed by G. Shafer in 1976.

Only recently, in 1995, Dr. Smarandache has introduced in philosophy the notion of 'neutrosophy', as a generalization of Hegel's dialectic, which is the basement of his researches in mathematics and economics, such as 'neutrosophic logic', 'neutrosophic set', 'neutrosophic probability', and 'neutrosophic statistics' (1995-2002). Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. Neutrosophic Logic is a logic in which each proposition is estimated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F, where T, I, F are standard or non-standard intervals included in]-0, 1+[. There is no boundary restriction on sup(T)+sup(I)+sup(F), neither on inf(T)+inf(I)+inf(F), which leave room for the fusion of incomplete and respectively paraconsistent information too. Dr. Smarandache also defined the neutrosophic logic connectors. Neutrosophic Logic is a generalization of the fuzzy logic (especially of IFL), intuitionistic logic (which supports incomplete theories), paraconsistent logic (which deals with paraconsistent information), dialetheism (which says that some contradictions are true), faillibilism (which asserts that uncertainty/indeterminacy belongs to every proposition), etc. and tries to unify all existing logics in a common mathematical framework. In neutrosophic logic it is possible to characterize contradictions, antitheses, antinomies, paradoxes (while in the fuzzy logic it was not), and to distinguish between relative, and espectively, absolute truth. Similarly, Dr. Smarandache proposed an extension of the classical probability and the imprecise probability to the 'neutrosophic probability', that he defined as a tridimensional vector whose components are subsets of the non-standard interval]-0, 1+[. Also, he generalized the fuzzy set to the 'neutrosophic set' (and its derivatives: 'intuitionistic set', 'paraconsistent set', 'dialetheist set', 'paradoxist set', 'tautological set') and defined the neutrosophic set operators.

In parallel, Dr. Jean Dezert has developed a new theory for plausible and paradoxical reasoning that can be interpreted as a generalization of the Dempster-Shafer Theory. The neutrosophical information processing can be regarded as a prelude to the plausible and paradoxical inference developed in the DSmT, acronym for Dezert-Smarandache Theory - as called by researchers. It has been recently proved that the DSmT is able to correctly solve many problems where the classical Dempster-Shafer theory fails. The main idea of the DSmT is basically not to accept the third exclude principle and to deal directly in the formalism with the possible paradoxical, inconsistent (and even incomplete or redundant) nature of the information. Doing this, the DSmT allows us to get easily results without approximations or requirement of heuristics for combining any sources of information (even for those appearing as in full contradiction). Invited Speakers: M. Khoshnevisan, S. Bhattacharya, F. Liu, J. Brenner, etc.

Details about neutrosophic logic and DSmT can be found in the following web site with free e-books and articles: http://www.gallup.unm.edu/~smarandache/DSmT.htm. Potential authors can also ask organizers for additional references.

The goal of this session is to present and discuss theoretical advances in neutrosophic logic and DSmT, together with applications in information fusion. The session will focus on fundamental aspects of processing of uncertain and paradoxical information, architecture of intelligent hybrid systems, and applications of DSmT to solution of military as well as non-military problems. Authors are encouraged to submit their questions and contributions for this session (LaTeX, ps, pdf, or MS Word files) directly to organizers through email at Jean.Dezert@onera.fr and smarand@unm.edu. The contributed papers have to be ready for print by May 15, 2003, in order to meet the printing schedule (see http://fusion2003.ee.mu.oz.au/call_for_papers.html). All submitted papers must follow the paper guidelines given at http://fusion2003.ee.mu.oz.au/paper_submission.html.

Abstracts of papers can be submitted to http://atlas-conferences.com/cgi-bin/abstract/submit/cajx-01 which is a web site at York University, Canada, and to view submitted abstracts at http://atlas-conferences.com/cgi-bin/abstract/cajx-01.

Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See `http://www.ams.org/meetings/`. Programs and abstracts will continue to be displayed on the AMS website in the Meetings and Conferences section until about three weeks after the meeting is over. Final programs for Sectional Meetings will be archived on the AMS website in an electronic issue of the *Notices* as noted below for each meeting.

Baton Rouge, Louisiana

Louisiana State University

March 14–16, 2003

Meeting #984

Southeastern Section
Associate secretary: John L. Bryant
Announcement issue of *Notices*: January 2003
Program first available on AMS website: January 30, 2003
Program issue of electronic *Notices*: March 2003
Issue of *Abstracts*: Volume 24, Issue 2

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

Invited Addresses

Bruce K. Driver, University of California San Diego, *Heat equations on loop groups.*

Gunter Lumer, University of Mons-Hainaut, *Stability analysis, Paley-Wiener criteria, and related problems in material science/ecology.*

Barry M. McCoy, SUNY at Stony Brook, *Evaluation representations, Drinfeld polynomials, and Bethe's ansatz.*

Stephen C. Milne, The Ohio State University, *A new look at sums of squares, Jacobi elliptic functions, and Ramanujan's tau function.*

Special Sessions

Algebraic Number Theory and K-Theory, **Jurgen Hurrelbrink** and **Jorge F. Morales**, Louisiana State University, and **Robert Osburn**, McMaster University.

Applied Mathematics and Materials Science, **Robert Lipton**, **Stephen Shipman**, **Blaise Bourdin**, and **Yuri Antipov**, Louisiana State University.

Arrangements in Topology and Algebraic Geometry, **Daniel C. Cohen**, Louisiana State University, and **Alexander I. Suciu**, Northeastern University.

Asymptotic Analysis, Stability, and Generalized Functions, **Ricardo Estrada** and **Frank Neubrander**, Louisiana State University, and **Gunter Lumer**, University of Mons-Hainaut.

Commutative Ring Theory, **James B. Coykendall**, North Dakota State University, and **Bernadette Mullins**, Birmingham State College.

Frames, Wavelets, and Tomography, **Gestur Olafsson**, Louisiana State University.

Graphs and Matroids, **Bogdan S. Oporowski** and **James G. Oxley**, Louisiana State University.

Induced Representations: Connections to Graphs, Number Theory, Geometry, **J. William Hoffman**, **Robert V. Perlis**, and **Neal W. Stoltzfus**, Louisiana State University.

Low Dimensional Topology, **Oliver T. Dasbach**, **Patrick M. Gilmer**, and **Richard A. Litherland**, Louisiana State University.

Mathematical Techniques in Musical Analysis, **Judith L. Baxter**, University of Illinois at Chicago, and **Robert Peck**, Louisiana State University.

Q-Series in Number Theory and Combinatorics, **Mourad E. H. Ismail**, University of South Florida, and **Stephen C. Milne**, The Ohio State University.

The Role of Mathematics Departments in Secondary Education, **James J. Madden** and **Frank Neubrander**, Louisiana State University.

Stochastic Analysis and Applications, **H.-H. Kuo** and **P. Sundar**, Louisiana State University.

Stochastics, Quantization, and Segal-Bargmann Analysis, **Bruce K. Driver**, University of California San Diego, **Brian C. Hall**, University of Notre Dame, and **Jeffrey J. Mitchell**, Baylor University.

Bloomington, Indiana

Indiana University

April 4–6, 2003

Meeting #985

Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: February 2003
Program first available on AMS website: February 20, 2003
Program issue of electronic *Notices*: April 2003
Issue of *Abstracts*: Volume 24, Issue 2

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

Invited Addresses

Daniel J. Allcock, University of Texas, *Title to be announced.*

Brian D. Conrad, University of Michigan, *Title to be announced.*

Robin A. Pemantle, Ohio State University, *Title to be announced.*

Sijue Wu, University of Maryland, *Title to be announced.*

Special Sessions

Algebraic Topology, **Randy McCarthy**, University of Illinois, Urbana-Champaign, and **Ayelet Lindenstrauss**, Indiana University.

Applications of Teichmüller Theory to Dynamics and Geometry, **Christopher M. Judge** and **Matthias Weber**, Indiana University.

Codimension One Splittings of Manifolds, **James F. Davis**, Indiana University, and **Andrew A. Ranicki**, University of Edinburgh.

Cryptography and Computational and Algorithmic Number Theory, **Joshua Holden** and **John Rickert**, Rose-Hulman Institute of Technology, **Jonathan Sorenson**, Butler University, and **Andreas Stein**, University of Illinois at Urbana-Champaign.

Differential Geometry, **Jiri Dadok** and **Bruce Solomon**, Indiana University, and **Ji-Ping Sha**, Indiana university.

Ergodic Theory and Dynamical Systems, **Roger L. Jones** and **Ayse A. Sahin**, DePaul University.

Extremal Combinatorics, **Dhruv Mubayi**, University of Illinois at Chicago, and **Jozef Skokan**, University of Illinois at Urbana-Champaign.

Geometric Topology, **Paul A. Kirk** and **Charles Livingston**, Indiana University.

Graph and Design Theory, **Atif A. Abueida**, University of Dayton, and **Mike Daven**, Mount Saint Mary College.

Graph Theory, **Tao Jiang**, **Zevi Miller**, and **Dan Pritikin**, Miami University.

Harmonic Analysis in the 21st Century, **Winston C. Ou** and **Alberto Torchinsky**, Indiana University.

Holomorphic Dynamics, **Eric D. Bedford** and **Kevin M. Pilgrim**, Indiana University.

Mathematical and Computational Problems in Fluid Dynamics and Geophysical Fluid Dynamics, **Roger Temam** and **Shouhong Wang**, Indiana University.

Operator Algebras and Free Probability, **Hari Bercovici**, Indiana University, and **Marius Dadarlat**, Purdue University.

Operator Algebras and Their Applications, **Jerry Kaminker** and **Ronghui Ji**, Indiana University-Purdue University Indianapolis.

Particle Models and Their Fluid Limits, **Robert T. Glassey** and **David C. Hoff**, Indiana University.

Probability, **Russell D. Lyons**, Indiana University, and **Robin A. Pemantle**, Ohio State University.

Recent Trends in the Analysis and Computations of Functional Differential Equations, **Paul W. Eloe** and **Qin Sheng**, University of Dayton.

Representations of Infinite Dimensional Lie Algebras and Mathematical Physics, **Katrina Deane Barron**, University of Notre Dame, and **Rinat Kedem**, University of Illinois, Urbana-Champaign.

Stochastic Analysis with Applications, **Jin Ma** and **Frederi Viens**, Purdue University.

Weak Dependence in Probability and Statistics, **Richard C. Bradley** and **Lanh T. Tran**, Indiana University.

New York, New York

Courant Institute

April 12–13, 2003

Meeting #986

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: February 2003
Program first available on AMS website: February 27, 2003
Program issue of electronic *Notices*: April 2003
Issue of *Abstracts*: Volume 24, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: February 18, 2003

Invited Addresses

Matthias Aschenbrenner, University of California Berkeley, *Title to be announced.*

John Etnyre, University of Pennsylvania, *Legendrian knots.*

Hans Foellmer, Humboldt University Berlin, *Title to be announced.*

Wilfrid Gangbo, Georgia Institute of Technology, *Title to be announced.*

Special Sessions

Algebraic Geometry, Integrable Systems, and Gauge Theory (Code: AMS SS C1), **Marcos Jardim** and **Eyal Markman**, University of Massachusetts, Amherst.

Algebraic and Topological Combinatorics (Code: AMS SS E1), **Eva-Maria Feichtner**, ETH, Zurich, Switzerland; and **Dmitry N. Kozlov**, University of Bern, Switzerland, and KTH, Stockholm, Sweden.

Analytical and Computational Methods in Electromagnetics (Code: AMS SS G1), **Alexander P. Stone**, University of New Mexico, and **Peter A. McCoy**, U. S. Naval Academy.

Combinatorial and Statistical Group Theory (Code: AMS SS B1), **Alexei Myasnikov** and **Vladimir Shpilrain**, City College, New York.

Contact and Symplectic Geometry (Code: AMS SS K1), **John B. Etnyre** and **Joshua M. Sabloff**, University of Pennsylvania.

Galois Module Theory and Hopf Algebras (Code: AMS SS F1), **Daniel R. Replogle**, College of Saint Elizabeth, and **Robert G. Underwood**, Auburn University.

The History of Mathematics (Code: AMS SS D1), **Patricia R. Allaire**, Queensborough Community College, CUNY, and **Robert E. Bradley**, Adelphi University.

Hopf Algebras and Quantum Groups (Code: AMS SS A1), **M. Susan Montgomery**, University of Southern California, **Earl J. Taft**, Rutgers University, and **Sarah J. Witherspoon**, Amherst College.

Low-Dimensional Topology (Code: AMS SS M1), **James Conant**, Cornell University, **Slava Krushkal**, University of Virginia, and **Rob Schneiderman**, Courant Institute, NYU.

Nonlinear Partial Differential Equations in Differential Geometry (Code: AMS SS L1), **John C. Loftin** and **Mu-Tao Wang**, Columbia University.

Rigidity in Dynamics, Geometry, and Group Theory (Code: AMS SS J1), **David Fisher**, Herbert H. Lehman College, CUNY, and **Steven E. Hurder** and **Kevin M. Whyte**, University of Illinois at Chicago.

Topological Aspects of Complex Singularities (Code: AMS SS H1), **Sylvain E. Cappell**, Courant Institute, NYU, and **Walter D. Neumann** and **Agnes Szilard**, Barnard College, Columbia University.

San Francisco, California

San Francisco State University

May 3–4, 2003

Meeting #987

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: March 2003
Program first available on AMS website: March 20, 2003
Program issue of electronic *Notices*: May 2003
Issue of *Abstracts*: Volume 24, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: March 11, 2003

Invited Addresses

Joe P. Buhler, Reed College, *A problem in symmetric functions arising from phase determination in crystallography.*

Raymond C. Heitmann, University of Texas at Austin, *The direct summand conjecture in dimension three.*

Alexei Y. Kitaev, California Institute of Technology, *Quantum media and topological quantum computation.*

Arkady Vaintrob, University of Oregon, *Higher spin curves and Gromov-Witten theory.*

Special Sessions

Beyond Classical Boundaries of Computability (Code: AMS SS E1), **Mark Burgin**, University of California Los Angeles, and **Peter Wegner**, Brown University.

Combinatorial Commutative Algebra and Algebraic Geometry (Code: AMS SS C1), **Serkan Hosten**, San Francisco State University, and **Ezra Miller**, Mathematical Sciences Research Institute.

Commutative Algebra (Code: AMS SS L1), **Raymond C. Heitmann**, University of Texas at Austin, and **Irena Swanson**, New Mexico State University.

Efficient Arrangements of Convex Bodies (Code: AMS SS H1), **Dan P. Ismailescu**, Hofstra University, and **Wlodzimierz Kuperberg**, Auburn University.

Geometry and Arithmetic over Finite Fields (Code: AMS SS G1), **Bjorn Poonen**, University of California Berkeley, and **Joe P. Buhler**, Reed College.

Gromov-Witten Theory of Spin Curves and Orbifolds (Code: AMS SS M1), **Tyler Jarvis**, Brigham Young University, **Takashi Kimura**, Boston University, and **Arkady Vaintrob**, University of Oregon.

The History of Nineteenth and Twentieth Century Mathematics (Code: AMS SS A1), **Shawnee McMurran**, California State University, San Bernardino, and **James A. Tattersall**, Providence College.

Numerical Methods, Calculations and Simulations in Knot Theory and Its Applications (Code: AMS SS J1), **Jorge Alberto Calvo**, North Dakota State University, **Kenneth C. Millett**, University of California Santa Barbara, and **Eric J. Rawdon**, Duquesne University.

PDEs and Applications in Geometry (Code: AMS SS K1), **Qi S. Zhang**, University of California Riverside.

Q-Series and Partitions (Code: AMS SS B1), **Neville Robbins**, San Francisco State University.

Qualitative Properties and Applications of Functional Equations (Code: AMS SS F1), **Theodore A. Burton**, Southern Illinois University at Carbondale.

Topological Quantum Computation (Code: AMS SS D1), **Alexei Kitaev**, California Institute of Technology, and **Samuel J. Lomonaco**, University of Maryland, Baltimore County.

Accommodations

Participants should make their own arrangements directly with a hotel of their choice. Special rates have been negotiated at the hotels listed below. Rates quoted do not include sales tax of 10% plus a $2 tourism assessment per room per night. The AMS is not responsible for rate changes or for the quality of the accommodations. When making a reservation, participants should state that they are with the American Mathematical Society group (AMS Meeting).

Days Inn, 2600 Sloat Blvd., San Francisco, CA; 415-665-9000. Rates start at $89.25/single and $106.25/double. Rates include a special discount of 15%. Reservations are on a space-available basis only, and participants must make reservations directly with this hotel, using the phone number listed in this announcement.

The Great Highway Inn, 1234 Great Highway, San Francisco, CA 94112; 800-624-6644, 415-731-6644; fax: 415-731-5309. Rates are $98/single and $125/double. Cancellation policy requires a 48-hour notice.

Sheraton Gateway Hotel-San Francisco Airport, 600 Airport Blvd., Burlingame, CA 94010; 650-340-8500; `http://www.sheratonsfo.com/`. Rate is $100 single/double. There is an additional $15 fee per person for triple or quad occupancy.

Food Service

A list of restaurants will be available at the registration desk.

Local Information

Please visit the website maintained by San Francisco State University at `http://www.sfsu.edu/` and the site maintained by the San Francisco Convention and Visitors Bureau at `http://www.sfvisitor.org/`.

Other Activities

AMS Book Sale: Examine the newest titles from AMS! Most books will be available at a special 50% discount offered only at meetings. Complimentary coffee will be served, courtesy of AMS Membership Services.

AMS Editorial Activity: An acquisitions editor from the AMS Book Program will be present to speak with prospective authors. If you have a book project that you would like to discuss with the AMS, please stop by the book exhibit.

TA Development Using Case Studies: A Workshop for Faculty. Organizer: Diane Herrmann, University of Chicago. Saturday, May 3.

Parking

Parking is available in the university parking structure located on South State Street. For more information regarding parking, please visit `http://www.sfsu.edu/~parking/text/tocampus.html`.

Registration and Meeting Information

The registration desk will be located on the third (main) floor of Thornton Hall and will be open from 7:30 a.m. to 4:30 p.m. on Saturday and from 8:00 a.m. to noon on Sunday. Talks will take place in the Science Building and Thornton Hall.

Registration Fees: (payable on-site only) $40/AMS and CMS members; $60/nonmembers; $5/emeritus members, students, or unemployed mathematicians. Fees are payable by cash, check, VISA, MasterCard, Discover, or American Express.

Travel

By Air: The San Francisco International Airport (SFO) is served by all major airlines. Shuttles from the airport to the campus are available and cost $12–$15. Taxi service is approximately $40 one way. If driving from the airport, go north on 101 to 380 North to 280 North. From 280 North follow the signs for 19th Ave. and SFSU. As the freeway ends, remain in the right lanes of traffic and exit onto Junipero Serra Blvd. From Junipero Serra turn left on Holloway Ave., then turn right onto Font Blvd., then right onto Lake Merced Blvd., and then right onto South State Street Drive. The university parking structure is on South State Street Drive.

Driving: From the North: Take Highway 101 South; cross the Golden Gate Bridge. Take 19th Ave./Highway 1 exit. Follow 19th Ave. to campus at Holloway Ave.

From the South: Take I-280 North, exit at 19th Ave. Take Junipero Serra Blvd. to Holloway Ave.; turn left on Holloway Ave. to campus at 19th Ave.

To get directly to the parking garage from 19th and Holloway, continue down Holloway to Font Blvd. Take a right on Font Blvd. until you come to Lake Merced Blvd. Take a right onto Lake Merced and then take another immediate right onto State Drive. The public parking garage is straight ahead.

From the East: Take I-80 West across the Bay Bridge to Highway 101 South. Take 101 South to I-280 toward Daly City. Take the Mission St./Daly City exit, bearing right onto Sagamore St. to Brotherhood Way to Junipero Serra Blvd. North. Take Junipero Serra Blvd. to Holloway Ave., turn left on Holloway Ave. to campus at 19th Ave. To get directly to the parking garage, stay on Brotherhood Way and turn right onto Lake Merced Blvd. Two stoplights up is the entrance to the public parking garage (turn right).

Car Rental: Special rates have been negotiated with Avis Rent A Car for the period April 26–May 11, 2003, beginning at $23.99/day for a subcompact car at the weekend rate (the weekend rate is available from noon Thursday until midnight Monday). All rates include unlimited free mileage. Rates do not include state or local surcharges, tax, optional coverages, or gas refueling charges. Renter must meet Avis's age, driver, and credit requirements, and return to the same renting location. Make reservations by calling 800-331-1600 or online at `http://www.avis.com/`. Please quote **Avis Discount Number B159266** when making reservations.

Weather

The weather in May is variable, with temperatures from 70° F. to 85° F. The weather can turn cold, overcast, and windy due to the close proximity of the SFSU campus to the ocean.

Seville, Spain

June 18–21, 2003

Meeting #988

First Joint International Meeting between the AMS and the Real Sociedad Matematica Española (RSME).
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: February 2003
Program first available on AMS website: Not applicable
Program issue of electronic *Notices*: Not applicable
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: February 10, 2003
For abstracts: February 10, 2003

Invited Addresses

Xavier Cabre, Universidad Politècnica de Cataluña, Barcelona, *Title to be announced.*

Charles Fefferman, Princeton University, *Title to be announced.*

Michael Hopkins, Massachusetts Institute of Technology, *Title to be announced.*

Ignacio Sols, Universidad Complutense, Madrid, *Title to be announced.*

Luis Vega, Universidad del Pais Vasco, Bilbao, *Title to be announced.*

Efim I. Zelmanov, Yale University, *Title to be announced.*

Special Sessions

Affine Algebraic Geometry, **Jaime Gutierrez**, University of Cantabria, **Vladimir Shpilrain**, City College of New York, and **Jie-Tai Yu**, University of Hong Kong.

Algebraic Geometry, **Felix Delgado**, Universidad de Valladolid, and **Andrey N. Todorov**, University of California Santa Cruz.

Algebraic Toplogy, **Alejandro Adem**, University of Wisconsin, **J. Aguade**, Universitat Autònoma de Barcelona, and **Eric M. Friedlander**, Northwestern University.

Banach Spaces of Analytic Functions, **Daniel Girela**, University of Malaga, and **Michael Stessin**, SUNY at Albany.

Biomolecular Mathematics, **Thomas J. Head** and **Fernando Guzman**, SUNY at Binghamton, **Mario Perez**, Universidad de Sevilla, and **Carlos Martin-Vide**, Rovira i Virgili University.

Classical and Harmonic Analysis, **Nets Katz**, Washington University, **Carlos Perez**, Universidad de Sevilla, and **Ana Vargas**, Universidad Autònoma de Madrid.

Combinatorics, **Joseph E. Bonin**, George Washington University, and **Marc Noy**, Universitat Politècnica de Cataluña.

Commutative Algebra: Geometric, Homological, Combinatorial and Computational Aspects, **Alberto Corso**, University of Kentucky, **Philippe Gimenez**, Universidad de Valladolid, and **Santiago Zarzuela**, Universitat de Barcelona.

Computational Methods in Algebra and Analysis, **Eduardo Cattani**, University of Massachusetts, Amherst, and **Francisco Jesus Castro-Jimenez**, Universidad de Sevilla.

Constructive Approximation Theory, **Antonio Duran**, Universidad de Sevilla, and **Edward B. Saff**, Vanderbilt University.

Control and Geometric Mechanics, **Manuel de Leon**, Instituto de Matemáticas y Fisica Fundamental, **Alberto Ibort**, Universidad Carlos III, and **Francesco Bullo**, University of Illinois at Urbana-Champaign.

Differential Galois Theory, **Teresa Crespo** and **Zbigniew Hajto**, Universitat de Barcelona, and **Andy R. Magid**, University of Oklahoma.

Differential Structures and Homological Methods in Commutative Algebra and Algebraic Geometry, **Gennady Lyubeznik**, University of Minnesota, and **Luis Narvaez-Macarro**, Universidad de Sevilla.

Discrete and Computational Geometry, **Ferran Hertado**, Universitat Politècnica de Cataluña, and **William Steiger**, Rutgers University.

Dynamical Systems, **George Haller**, Massachusetts Institute of Technology, **Zbigniew H. Nitecki**, Tufts University, **Enrique Ponce**, Universidad de Sevilla, **Tere M. Seara**, Universitat Politècnica de Cataluña, and **Xavier Jarque**, Universitat Autònoma de Barcelona.

Effective Analytic Geometry over Complete Fields, **Luis-Miguel Pardos**, Universidad de Cantabria, and **J. Maurice Rojas**, Texas A&M University.

Geometric Methods in Group Theory, **José Burillo**, Universitat Politècnica de Cataluña, **Jennifer Tayback**, University of Albany, and **Enric Ventura**, Universitat Politècnica de Cataluña.

History of Modern Mathematics—Gauss to Wiles, **Jose Ferreiros**, Universidad de Sevilla, and **David Rowe**, Universitat Mainz.

Homological Methods in Banach Space Theory, **Jesus M. F. Castillo**, Universidad de Extremadura, and **N. J. Kalton**, University of Missouri.

Homotopy Algebras, **Pedro Real**, Universidad de Sevilla, **Thomas J. Lada**, North Carolina State University, and **James Stasheff**, University of North Carolina.

Interpolation Theory, Function Spaces and Applications, **Fernando Cobos**, University Complutense de Madrid, and **Pencho Petrushev**, University of South Carolina.

Lorentzian Geometry and Mathematical Relativity, **Luis J. Alias**, Universidad de Murcia, and **Gregory James Galloway**, University of Miami.

Mathematical Aspects of Semiconductor Modeling and Nano-technology, **Irene Martinez Gamba**, University of Texas, Austin, and **Jose Antonio Carrillo**, Universidad de Granada.

Mathematical Fluid Dynamics, **Diego Cordoba**, CSIC, Madrid, and Princeton University, **Susan Friedlander**, University of Illinois at Chicago, and **Marcos Antonio Fontelos**, Universidad Rey Juan Carlos.

Mathematical Methods in Finance and Risk Management, **Santiago Carrillo Menendez**, Universidad Autònoma de Madrid, **Antonio Falcos Montesinos**, Universidad Cardenal Herrera CEU, **Antonio Sanchez-Calle**, Universidad Autònoma de Madrid, and **Luis A. Seco**, University of Toronto at Mississauga.

The Mathematics of Electronmicroscopic Imaging, **Jose-Maria Carazo**, Centro Nacional de Biotecnologia-CSIC, and **Gabor T. Herman**, City University of New York.

Moduli Spaces in Geometry and Physics, **Steven B. Bradlow**, University of Illinois at Urbana-Champaign, and **Oscar Garcia-Prada**, Universidad Autònoma de Madrid.

Nonassociative Algebras and Their Applications, **Efim I. Zelmanov**, Yale University, **Santos Gonzalez**, Universidad de Oviedo, and **Alberto Elduque**, Universidad de Zaragoza.

Nonlinear Dispersive Equations, **Gustavo Ponce**, University of California Santa Barbara, and **Luis Vega**, Universidad del Pais Vascos.

Numerical Linear Algebra, **Lothar Reichel**, Kent State University, and **Francisco Marcellan**, University Carlos III de Madrid.

Operator Theory and Spaces of Analytic Functions, **Jose Bonet**, Universidad Politècnica de Valencia, **Pedro Paul**, Universidad de Sevilla, and **Cora S. Sadosky**, Howard University.

PDE Methods in Continuum Mechanics, **Juan L. Vazquez**, Universidad Autònoma de Madrid, and **J. W. Neuberger**, University of North Texas.

Polynomials and Multilinear Analysis in Infinite Dimensions, **Richard M. Aron**, Kent State University, **J. A. Jaramillo** and **Jose G. Llavona**, Universidad Complutense de Madrid, and **Andrew M. Tonge**, Kent State University.

Quantitative Results in Real Algebra and Geometry, **Carlos Andradas** and **Antonio Diaz-Cano**, Universidad Complutense, **Victoria Powers**, Emory University, and **Frank Sottile**, University of Massachusetts, Amherst.

Recent Developments in the Mathematical Theory of Inverse Problems, **Russell Brown**, University of Kentucky, **Alberto Ruiz**, Universidad Autònoma de Madrid, and **Gunther Uhlmann**, University of Washington.

Riemannian Foliations, **Jesus Antonio Alvarez Lopez**, Universidade de Santiago de Compostela, and **Efton L. Park**, Texas Christian University.

Ring Theory and Related Topics, **Jose Gomez-Torrecillas**, University of Granada, **Pedro Antonio Guil Asensio**, University of Murcia, **Sergio R. Lopez-Permouth**, Ohio University, and **Blas Torrecillas**, University of Almeria.

Variational Problems for Submanifolds, **Frank Morgan**, Williams College, and **Antonio Ros**, Universidad de Granada.

Boulder, Colorado

University of Colorado

October 2–4, 2003

Meeting #989

Joint Central/Western Sections
Associate secretaries: Susan J. Friedlander and Michel L. Lapidus
Announcement issue of *Notices*: August 2003
Program first available on AMS website: August 21, 2003
Program issue of electronic *Notices*: October 2003
Issue of *Abstracts*: Volume 24, Issue 4

Deadlines

For organizers: March 3, 2003
For consideration of contributed papers in Special Sessions: June 6, 2003
For abstracts: August 12, 2003

Invited Addresses

J. Brian Conrey, American Institute of Mathematics, *Title to be announced.*

Giovanni Forni, Northwestern University, *Title to be announced.*

Juha M. Heinonen, University of Michigan, *Title to be announced.*

Joseph D. Lakey, New Mexico State University, *Title to be announced.*

Albert Schwarz, University of California Davis, *Title to be announced.*

Avi Wigderson, Institute for Advanced Study, *Title to be announced* (Erdős Memorial Lecture).

Special Sessions

Algebras, Lattices and Varieties (Code: AMS SS A1), **Keith A. Kearnes**, University of Colorado, Boulder, **Agnes Szendrei**, Bolyai Institute, and **Walter Taylor**, University of Colorado, Boulder.

Applications of Number Theory and Algebraic Geometry to Coding (Code: AMS SS B1), **David R. Grant**, University of Colorado, Boulder, **Jose Felipe Voloch**, University of Texas at Austin, and **Judy Leavitt Walker**, University of Nebraska, Lincoln.

Geometric Methods in Partial Differential Equations (Code: AMS SS C1), **Jeanne N. Clelland**, University of Colorado, Boulder, and **George R. Wilkins**, University of Hawaii.

Groupoids in Analysis and Geometry (Code: AMS SS D1), **Lawrence Baggett**, University of Colorado, Boulder, **Jerry Kaminker**, Indiana University-Purdue University Indianapolis, and **Judith Packer**, University of Colorado, Boulder.

Homotopy Theory (Code: AMS SS F1), **Daniel Dugger**, University of Oregon, and **Brooke E. Shipley**, Purdue University.

Noncommutative Geometry and Geometric Analysis (Code: AMS SS E1), **Carla Farsi**, **Alexander Gorokhovsky**, and **Siye Wu**, University of Colorado.

Structured Population and Epidemic Models: Periodicity, Chaos, and Extinction (Code: AMS SS G1), **Linda J. S. Allen**, Texas Technical University, and **Sophia R.-J. Jang**, University of Louisiana at Lafayette.

Binghamton, New York

Binghamton University

October 11–12, 2003

Meeting #990

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: August 2003
Program first available on AMS website: August 28, 2003
Program issue of electronic *Notices*: October 2003
Issue of *Abstracts*: Volume 24, Issue 4

Deadlines

For organizers: March 10, 2003
For consideration of contributed papers in Special Sessions: June 24, 2003
For abstracts: August 19, 2003

Invited Addresses

Peter Kuchment, Texas A&M University, *Title to be announced.*

Zlil Sela, Einstein Institute of Mathematics, *Title to be announced.*

Zoltan Szabo, Princeton University, *Title to be announced.*

Jeb F. Willenbring, Yale University, *Title to be announced.*

Special Sessions

Biomolecular Mathematics (Code: AMS SS A1), **Thomas J. Head** and **Dennis G. Pixton**, Binghamton University, **Mitsunori Ogihara**, University of Rochester, and **Carlos Martin-Vide**, Universitat Rovira i Virgili.

Boundary Value Problems on Singular Domains (Code: AMS SS C1), **Juan B. Gil**, Temple University, and **Paul A. Loya**, Binghamton University.

Geometric Group Theory (Code: AMS SS B1), **Zlil Sela**, Einstein Institute of Mathematics, and **Ross Geoghegan**, Binghamton University.

Infinite Groups and Group Rings (Code: AMS SS D1), **Luise-Charlotte Kappe**, Binghamton University, and **Derek J. S. Robinson**, University of Illinois at Urbana-Champaign.

Lie Algebras, Conformal Field Theory, and Related Topics (Code: AMS SS E1), **Chongying Dong**, University of California Santa Cruz, and **Alex J. Feingold** and **Gaywalee Yamskulna**, Binghamton University.

Probability Theory (Code: AMS SS F1), **Miguel A. Arcones**, Binghamton University, and **Evarist Gine**, University of Connecticut.

Chapel Hill, North Carolina

University of North Carolina at Chapel Hill

October 24–25, 2003

Meeting #991

Southeastern Section
Associate secretary: John L. Bryant
Announcement issue of *Notices*: August 2003
Program first available on AMS website: September 11, 2003
Program issue of electronic *Notices*: October 2003
Issue of *Abstracts*: Volume 24, Issue 4

Deadlines

For organizers: March 24, 2003

For consideration of contributed papers in Special Sessions: July 19, 2003
For abstracts: September 3, 2003

Bangalore, India

India Institute of Science

December 17-20, 2003

Meeting #992

First Joint AMS-India Mathematics Meeting
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: To be announced
Program first available on AMS website: Not applicable
Program issue of electronic *Notices*: Not applicable
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Invited Addresses

R. Balasubramanian, Institute for Mathematical Sciences, *Title to be announced.*

George C. Papanicolaou, Stanford University, *Title to be announced.*

M. S. Raghunathan, Tata Institute of Fundamental Research, *Title to be announced.*

Peter Sarnak, Princeton University and Courant Institute, New York University, *Title to be announced.*

K. B. Sinha, India Statistical Institute, *Title to be announced.*

Vladimir Voevodsky, Institute for Advanced Study, *Title to be announced.*

Special Sessions

Algebraic and Geometric Methods in Multivariable Operator Theory, **Ronald G. Douglas**, Texas A&M University, and **Gadadhar Misra**, Indian Statistical Institute.

Algebraic and Geometric Topology, **Parameswaren Sankaran**, Institute of Mathematical Sciences, and **P. B. Shalen**, University of Illinois.

Buildings and Group Theory, **N. S. Narasimha Sastry**, Indian Statistical Institute, and **Richard M. Weiss**, Tufts University.

Commutative Algebra and Algebraic Geometry, **Sudhir Ghorpade**, Indian Institute of Technology, Bombay, **Hema Srinivasan**, University of Missouri, and **Jugal K. Verma**, Indian Institute of Technology, Bombay.

Cycles, K-Theory, and Motives, **Eric M. Friedlander**, Northwestern University, **Steven Lichtenbaum**, Brown University, **Kapil Paranjape**, Institute of Mathematical Sciences, and **Vasudevan Srinivas**, Tata Institute of Fundamental Research.

Differential Equations and Applications to Population Dynamics, Epidemiology, Genetics and Microbiology, **Bindhyachal Rai**, University of Allahabad, **Sanjay Rai**, Jacksonville University, **Terrance Quinn**, Ohio University Southern, and **Sunil Tiwari**, Sonoma State University.

The Many Facets of Linear Algebra and Matrix Theory, **Richard Brualdi**, University of Wisconsin, and **Rajendra Bhatia**, Indian Statistical Institute.

PDE and Applications, **Susan B. Friedlander**, University of Illinois, and **P. N. Srikanth**, Tata Institute of Fundamental Research.

Spectral and Inverse Spectral Theories of Schrödinger Operators, **Peter David Hislop**, University of Kentucky, and **Krishna Maddaly**, Institute of Mathematical Sciences.

Phoenix, Arizona

Phoenix Civic Plaza

January 7-10, 2004

Meeting #993

Joint Mathematics Meetings, including the 110th Annual Meeting of the AMS, 87th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2003
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2004
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 2, 2003
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
For summaries of papers to MAA organizers: To be announced

Request for Proposals from AMS Special Session Organizers

Michel L. Lapidus, the associate secretary responsible for the AMS program in Phoenix, is soliciting proposals for Special Sessions for this meeting. Each proposal must include the full names, email addresses, and institutions of all members of the organizing committee and identify the one organizer who will serve as contact for all communications about the session; the title and a brief (two or three paragraphs) description of the proposed session; and a *sample* list of speakers whom the proposed organizers plan to invite (please note that it is not at all necessary to have confirmed commitments from these speakers).

It is expected that each Special Session will be allotted 10 hours over two days of the meeting in which to schedule speakers. In order to allow the maximum movement between sessions for all participants, Special Session speakers will be scheduled for either a 20-minute talk, 5-minute discussion, and 5-minute break; or a 40-minute talk, 10-minute discussion, and 10-minute break. All talks must begin on the hour or half-hour. Any combination of 20-minute and 40-minute talks is allowed, provided the schedule conforms to beginning on the hour and half-hour (except on the first afternoon, for technical reasons).

Proposals for AMS Special Sessions must be received by the deadline for organizers, **April 2, 2003**, and submitted (preferably by email) to the AMS associate secretary, Michel L. Lapidus (`lapidus@math.ucr.edu`). Late proposals will not be considered.

There is limited space available for Special Sessions on the AMS program, so it is likely that not all proposals will be accepted. Please be sure to submit as detailed a proposal as possible for review by the program committee. All proposed organizers will be notified of acceptance or rejection no later than May 1, 2003.

Tallahassee, Florida

Florida State University

March 12-13, 2004

Meeting #994

Southeastern Section
Associate secretary: John L. Bryant
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 13, 2003
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Athens, Ohio

Ohio University

March 26-27, 2004

Meeting #995

Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 26, 2003
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Los Angeles, California

University of Southern California

April 3-4, 2004

Meeting #996

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Special Sessions

Contact and Symplectic Geometry (Code: AMS SS A1), **Dragomir Dragnev**, **Ko Honda**, and **Sang Seon Kim**, University of Southern California.

Lawrenceville, New Jersey

Rider University

April 17-18, 2004

Meeting #997

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 17, 2003
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Houston, Texas

University of Houston

May 13–15, 2004

Sixth International Joint Meeting of the AMS and the Sociedad Matemática Mexicana (SMM).

Associate secretary: John L. Bryant
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
For summaries of papers to MAA organizers: To be announced

Nashville, Tennessee

Vanderbilt University

October 16–17, 2004

Southeastern Section
Associate secretary: John L. Bryant
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 16, 2004
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Pittsburgh, Pennsylvania

University of Pittsburgh

November 6–7, 2004

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 7, 2004
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Atlanta, Georgia

Atlanta Marriott Marquis and Hyatt Regency Atlanta

January 5–8, 2005

Joint Mathematics Meetings, including the 111th Annual Meeting of the AMS, 88th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association of Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL).

Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: October 2004
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2005
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 5, 2004
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
For summaries of papers to MAA organizers: To be announced

Mainz, Germany

Deutsche Mathematiker-Vereinigung (DMV) and the Osterreichische Mathematische Gesellschaft (OMG)

June 16–19, 2005

Second Joint AMS-Deutsche Mathematiker-Vereinigung (DMV) Meeting

Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
For summaries of papers to MAA organizers: To be announced

San Antonio, Texas

Henry B. Gonzalez Convention Center

January 12–15, 2006

Joint Mathematics Meetings, including the 112th Annual Meeting of the AMS, 89th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL).

Associate secretary: John L. Bryant
Announcement issue of *Notices*: October 2005
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2006
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 12, 2005
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
For summaries of papers to MAA organizers: To be announced

New Orleans, Louisiana

New Orleans Marriott and Sheraton New Orleans Hotel

January 4–7, 2007

Joint Mathematics Meetings, including the 113th Annual Meeting of the AMS, 90th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL).

Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: October 2006
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2007
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 4, 2006
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
For summaries of papers to MAA organizers: To be announced

San Diego, California

San Diego Convention Center

January 6–9, 2008

Joint Mathematics Meetings, including the 114th Annual Meeting of the AMS, 91st Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL).

Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2007
Program first available on AMS website: November 1, 2007
Program issue of electronic *Notices*: January 2008
Issue of *Abstracts*: Volume 29, Issue 1

Deadlines

For organizers: April 6, 2007
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
For summaries of papers to MAA organizers: To be announced

Washington, District of Columbia

Marriott Wardman Park Hotel and Omni Shoreham Hotel

January 7–10, 2009

Joint Mathematics Meetings, including the 115th Annual Meeting of the AMS, 92nd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL).

Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: October 2008
Program first available on AMS website: November 1, 2008
Program issue of electronic *Notices*: January 2009
Issue of *Abstracts*: Volume 30, Issue 1

Deadlines

For organizers: April 7, 2008
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
For summaries of papers to MAA organizers: To be announced

Meetings and Conferences of the AMS

Associate Secretaries of the AMS

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Sproul Hall, Riverside, CA 92521-0135; e-mail: `lapidus@math.ucr.edu`; telephone: 909-787-3113.

Central Section: Susan J. Friedlander, Department of Mathematics, University of Illinois at Chicago, 851 S. Morgan (M/C 249), Chicago, IL 60607-7045; e-mail: `susan@math.nwu.edu`; telephone: 312-996-3041.

Eastern Section: Lesley M. Sibner, Department of Mathematics, Polytechnic University, Brooklyn, NY 11201-2990; e-mail: `lsibner@duke.poly.edu`; telephone: 718-260-3505.

Southeastern Section: John L. Bryant, Department of Mathematics, Florida State University, Tallahassee, FL 32306-4510; e-mail: `bryant@math.fsu.edu`; telephone: 850-644-5805.

The Meetings and Conferences section of the *Notices* gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. **Information in this issue may be dated. Up-to-date meeting and conference information** at `www.ams.org/meetings/`.

Meetings:

Important Information regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 108 in the January 2003 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts

Several options are available for speakers submitting abstracts, including an easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX may submit abstracts with such coding, and all math displays and similarily coded material (such as accent marks in text) must be typeset in LaTeX. To see descriptions of the forms available, visit `http://www.ams.org/abstracts/instructions.html`, or send mail to `abs-submit@ams.org`, typing `help` as the subject line; descriptions and instructions on how to get the template of your choice will be e-mailed to you.

Completed abstracts should be sent to `abs-submit@ams.org`, typing `submission` as the subject line. Questions about abstracts may be sent to `abs-info@ams.org`.

Paper abstract forms may be sent to Meetings & Conferences Department, AMS, P.O. Box 6887, Providence, RI 02940. There is a $20 processing fee for each paper abstract. There is no charge for electronic abstracts. Note that all abstract deadlines are strictly enforced. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

Conferences: (See `http://www.ams.org/meetings/` for the most up-to-date information on these conferences.)

June 8 - July 24, 2003: Joint Summer Research Conferences in the Mathematical Sciences, Snowbird, Utah.

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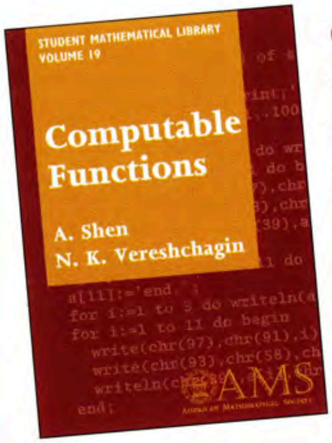

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This book presents a lively and concise introduction to the central facts and basic notions of the general theory of computation. The authors complement the main text with over 150 problems. They also cover specific computational models, such as Turing machines and recursive functions.

The intended audience includes undergraduate students majoring in mathematics or computer science, and all mathematicians and computer scientists who would like to learn the basics of the general theory of computation.
The book also would be an ideal reference source for designing a course.

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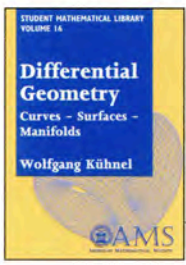

Differential Geometry
Curves - Surfaces - Manifolds

Wolfgang Kühnel, *University of Stuttgart, Germany*

Student Mathematical Library, Volume 16; 2002; 358 pages; Softcover;
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Basic Set Theory

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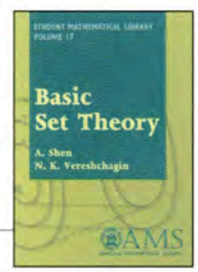

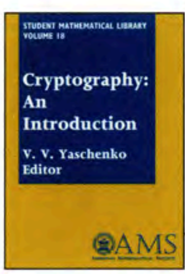

Cryptography: An Introduction

V. V. Yaschenko, *Moscow Center for Continuous Mathematics Education,* Editor

Student Mathematical Library, Volume 18; 2002; 229 pages; Softcover;
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